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The SHIP STRUCTURE COMMITTEE is constituted to prosecute a research program to improve the hull structures of ships and other marine structures by an extension of knowledge pertaining to design, materials, and methods of construction.

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GUIDE TO DAMAGE TOLERANCE ANALYSIS OF MARINE STRUCTURES

Ship structures are subjected to various sources of wave-induced cyclic loading which may cause fatigue cracks to initiate at welded details during the service life of a ship. The propagation of these cracks may eventually compromise the structural integrity and watertightness of a ship. Therefore, the current practice is to repair severe fabrication flaws and any cracks detected in service as soon as possible. However, such a strategy could lead to prohibitive through-life maintenance costs. A useful tool for optimizing the maintenance and inspection of ship structures without compromising the structural integrity and water-tightness of a ship is damage tolerance analysis. The latter makes use of fracture mechanics to quantitatively assess the residual strength and residual life of a cracked structural member.

This guide is intended to provide naval architects and structural engineers with detailed guidance on the application of damage tolerance analysis to ship structures. The guide begins with a review of the essential elements of damage tolerance assessment. This is followed by guidance on: (i) the use of Failure Assessment Diagrams to assess the local residual strength of a cracked structural detail, (ii) the use of linear elastic fracture mechanics models for fatigue crack growth to predict the residual life of a cracked structural member, (iii) the estimation of peak and cyclic loads over the assessment interval of interest, and (iv) the calculation of stresses and crack driving forces. Towards the end of this guide, the aforementioned guidance is demonstrated by means of two numerical examples.

ROBERT C. NORTH
Rear Admiral, U.S. Coast Guard
Chairman, Ship Structure Committee
16. Abstract

Ship structures are subjected to various sources of wave-induced cyclic loading which may cause fatigue cracks to initiate at welded details during the service life of a ship. The propagation of these cracks may eventually compromise the structural integrity and watertightness of a ship. Therefore, the current practice is to repair severe fabrication flaws and any cracks detected in service as soon as possible. However, such a strategy could lead to prohibitive through-life maintenance costs. A useful tool for optimizing the maintenance and inspection of ship structures without compromising the structural integrity and watertightness of a ship is damage tolerance analysis. The latter makes use of fracture mechanics to quantitatively assess the residual strength and residual life of a cracked structural member. Unfortunately, most naval architects and structural engineers have little or no experience with damage tolerance analysis and fracture mechanics.

This guide is intended to provide naval architects and structural engineers with detailed guidance on the application of damage tolerance analysis to ship structures. The guide begins with a review of the essential elements of damage tolerance assessment. This is followed by guidance on: (i) the use of Failure Assessment Diagrams to assess the local residual strength of a cracked structural detail, (ii) the use of linear elastic fracture mechanics models for fatigue crack growth to predict the residual life of a cracked structural member, (iii) the estimation of peak and cyclic loads over the assessment interval of interest, and (iv) the calculation of stresses and crack driving forces. Towards the end of this guide, the aforementioned guidance is demonstrated by means of two numerical examples.
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(Approximate conversions to metric measures)

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1.0 INTRODUCTION

1.1 Background

Metal fatigue is the progressive failure of metal under cyclic loading. This type of failure can be divided into three basic stages:

1. the initiation of microscopic cracks at microscopic or macroscopic stress concentrations;

2. the growth of microscopic cracks into macroscopic cracks; and

3. the growth of macroscopic cracks to a critical size for failure (e.g., plastic collapse, fracture, or oil leakage).

The absolute and relative magnitudes of these stages depend on material, notch severity, structural redundancy, and environment [1.1].

Fatigue cracks in steel ships generally initiate at welded structural details. The initiation and subsequent propagation of these cracks can be driven by several sources of cyclic loading including: (1) longitudinal bending, transverse bending, and torsion of the hull girder as a result of wave loading; (2) fluctuating hydrostatic pressure on side shell plating, cargo hold boundaries, and tank walls; and (3) machinery and hull vibration [1.2]. Exposure to corrosive media, such as sour crude oil or sea water, can accelerate the initiation and propagation of fatigue cracks, either directly through corrosion fatigue mechanisms or indirectly through the higher cyclic stresses that result from localized and general corrosion. Fatigue-prone areas in bulk carriers include hatch corners, coamings, bracketed connections between hold frames and wing ballast tanks, the intersections of transverse corrugated bulkheads with top-side structure, and the intersections of inner bottom plating with hopper plating, whereas fatigue-prone areas in tankers include the intersections of side shell longitudinals and transverse structure and the end connections of deck and bottom longitudinals [1.3-1.5].

Although most fatigue cracks in steel ships are not detected by conventional inspection techniques until they are at least several inches long and through the thickness of plating, catastrophic brittle fractures rarely initiate from undetected fatigue cracks because of the relatively good fracture toughness of modern ship steels, the inherent redundancy of ship structures, the use of crack arrestors, and the relatively low rate of normal service loads. Nevertheless, any detected cracks are usually repaired at the earliest opportunity to prevent other problems from arising.

Historically, ship structures have been designed to meet minimum scantling requirements which have included allowances for general corrosion and uncertainties in design methods. Until recently, fatigue cracking was not explicitly considered by designers because fatigue cracking was rarely detected in ships less than 10 years old and because the...
frequency and costs of repairing fatigue cracks in older ships was acceptable to owners. Since the late 1970's, however, fatigue cracking has occurred more frequently in relatively new ships. This change has been attributed to the design and construction of more structurally optimized ships with thinner scantlings. This optimization, which has been motivated by commercial demands to reduce the fabrication costs and weight of hull structures, has been achieved through the greater use of high strength steels, the use of more sophisticated design tools, and the greater exploitation of classification society rules which have permitted design stresses to increase with tensile strength up to a fraction of the tensile strength defined by the so-called material factor. Unfortunately, stress concentrations of structural details have not been adequately reduced to compensate for the higher design stresses and higher local bending stresses associated with thinner scantlings. Furthermore, the fatigue strength of as-welded steel joints is essentially independent of tensile strength. Therefore, local cyclic stresses at structural details have been permitted to increase without a matching increase in fatigue strength of these details. In addition, corrosive environments have exacerbated this mis-match since the flexibility of thin structure promotes the flaking of rust which accelerates the wastage process and further increases the flexibility of thin structure [1.6-1.8].

In response to safety concerns and escalating maintenance costs for owners, classification society rules have recently introduced explicit fatigue design criteria for welded structural details in steel ships [1.5, 1.9, 1.10]. These criteria, which are largely based on well-established fatigue design procedures for welded joints in bridges and offshore structures [1.11-1.13], are intended to ensure that there is a low probability of fatigue failures occurring during the design life of a ship, where failure is generally considered to be the initiation of a through-thickness crack several inches long. However, premature fatigue cracking as a result of fabrication or design errors can still occur. Furthermore, some fatigue cracking can still be expected in properly designed ships. Therefore, quantitative techniques for predicting the residual life and residual strength of cracked structural welded details are needed to develop safe but cost-effective inspection schedules at the design stage. These techniques can also be used to optimize the scheduling of repairs for cracks found in service and to assess whether the operation of existing ships can be extended beyond their original design lives.

1.2 Objective of Project

In principle, existing techniques based on fracture mechanics for assessing the residual strength and residual life of cracked structure in aircraft, pipelines, bridges, and offshore structures can be adapted to ship structures, and these practices have been reviewed in previous SSC projects [1.14, 1.15] and other documents [1.1, 1.16-1.20]. However, it should be noted that many years have been spent in the development of standardized load histories, material data bases, failure criteria, and crack growth models to enable damage tolerance assessment of aircraft, bridges, offshore platforms, pressure vessels and pipelines. Still, the application of these techniques to the design of welded structures has been limited by the complexity of stress fields around welded details, the presence of welding residual stresses, and the complexity of crack growth. The
adaptation of these practices to ship structures is further complicated by the added complexity of ship details, uncertainty of operational loads, uncertainty of welding residual stress assumptions, and the redundancy of ship structures. As a result, this adaptation is still under development. Nevertheless, there is an immediate need for structural engineers and naval architects to have the capability to assess the damage tolerance of ship structures at the design stage, in service, and when extending the service lives of older ships. Therefore, the Ship Structure Committee has requested the preparation of an engineering guide based on the current state-of-the-art.

The objective of the present project is to prepare an engineering guide that will:

1. lead structural engineers and naval architects through the application of damage tolerance analysis to ship structures at the design and fabrication stages, in service, and when extending the service life of older ships;

2. provide a framework and guidance/commentary for performing detailed calculations of residual strength and residual life; and

3. present examples that follow the guide in a step by step manner to illustrate the applications of the damage tolerance methodology.

1.3 Scope of the Guide

The scope of this guide is restricted to the damage tolerance assessment of cracked ship structures since the fracture mechanics background for analyzing cracks is not usually part of a structural engineer’s or naval architect’s formal training. The guide is further restricted to the assessment of fatigue crack propagation and the local residual strength of structural details in ships. It is recognized that in-service damage to ship structures could also be sustained from corrosion wastage, corrosion pitting, and accidental collision and that there could be cases where structural integrity is governed by the ultimate strength of the entire ship hull rather than the local residual strength of a structural detail. However, structural engineers and naval architects generally have the technical background to assess such damage, and assessment procedures for such damage have been covered in previous SSC projects [1.22-1.26].

Topics that are covered by this guide are the subject of on-going research. Although the procedures presented in this guide are based on the current state of technology, a number of simplifications have been made to ensure that the guide is practical and accessible to a broad range of users. Where uncertainties exist, conservative assumptions have been made to ensure that a working guide is available now.

It is assumed that users of this guide will properly document their damage tolerance assessments and provide sufficient analysis or correlation against test data to assure that their assessments are conservative.
This guide is not intended to replace normal design procedures when such procedures are applicable. For example, it is not intended to circumvent the normal requirements for good workmanship.

1.4 Organization of the Guide

The guide is organized into seven sections starting with Section 2 herein. Section 2 reviews the essential elements of damage tolerance assessment and identifies potential applications of damage tolerance assessment for ship structures. Section 3 specifies a three level procedure based on the Failure Assessment Diagram for assessing the local residual strength of cracked structural details in ships, while Section 4 provides a two level procedure based on linear elastic fracture mechanics for predicting the growth of a fatigue crack under variable amplitude loading from an initial crack or flaw. Section 5 gives three options for estimating peak loads and cyclic loads due to wave loading on a ship hull, while Section 6 provides guidance on the calculation of stresses and crack driving forces for input into assessments of fatigue crack growth and local residual strength. The emphasis of Sections 3 to 6 is to outline basic assessment procedures and to explain the underlying basis for the procedures. Wherever possible, details are relegated to Appendices at the end of the guide.

1.5 References


2.0 APPLICATIONS OF DAMAGE TOLERANCE ASSESSMENT

2.1 Design

Damage tolerance is the ability of a damaged structure to withstand anticipated operational loads without failure or loss of functionality. There are three basic ways that damage tolerance can be designed into structures [2.1, 2.2]:

1. **The safe-life approach** designs a structure for a finite life and requires the imposition of large factors of safety on design loads and material properties to ensure that there is a low probability of failure during the design life. Machine components, bridges, offshore platforms, aircraft landing gear, and aircraft engine mounts are typically designed with this approach.

2. **The fail-safe approach** allows a structural component to be designed with lower factors of safety and, therefore, a higher probability of failure during its service life. However, multiple load paths (i.e., structural redundancy), crack arrestors, and accessibility for inspection must be built into the structure so that damage is detected before the failure of one or more individual components leads to overall failure. This approach was initially developed by the aircraft industry for airframes because the additional weight of a safe-life design was unacceptable.

3. **The damage tolerant design approach** is a refinement of the fail-safe approach. Damage is assumed to be initially present in critical structural elements, and explicit analyses are conducted to predict the spread of this damage and to assess residual strength. The results of the analyses are used to develop an inspection program for critical structural elements that will ensure that damage will never propagate to failure prior to detection. If necessary, the structure is re-designed to obtain practical inspection intervals and to improve the durability of the structure (i.e., damage over the service life is limited and can be economically repaired). This approach was developed by the aircraft industry in recognition that the spread of initial and subsequent damage can degrade the integrity of redundant members in a fail-safe structure and relies heavily on fracture mechanics to predict the residual life and residual strength of cracked structure. Since the late 1970’s, regulations for civil and military aircraft have required that these damage tolerance assessment techniques be used to design most components and to re-qualify aging aircraft [2.3-2.7].

---

1 As stated at the beginning of this section, safe-life, fail-safe, and damage tolerant design approaches are all intended to produce damage tolerant structures. In this regard, the term “damage tolerant design approach” is a potential source of confusion because it may imply to some readers that safe-life and fail-safe approaches do not produce damage tolerance structures. Nevertheless, the term has been retained in this guide for historical consistency.
The *damage tolerant design approach* is shown schematically in Figure 2.1, and the basic steps of this approach are summarized below:

1. Critical structural elements and potential crack initiation sites in these elements are identified.

2. A crack-like flaw is assumed to be initially present at each of the aforementioned sites. The size of these flaws is assumed to be the smallest size that can be reliably detected by conventional non-destructive inspection techniques.

3. The criticality of each initial flaw is evaluated using stress analysis based on maximum expected service loads or design loads, fracture mechanics, and failure criteria for fracture and other possible failure modes.

4. If the *residual strength* of a critical structural element is smaller than the design strength (i.e., initial flaws exceed a critical size), then the structural member is redesigned with lower stresses and/or more damage-resistant materials. Otherwise, the design of the structural element is accepted, and fracture mechanics analysis of fatigue crack propagation is carried out to determine the *residual life* of the structural element (i.e., the time period/voyages after which the initial flaw will grow to a critical size).

5. Each critical structural element is inspected before the end of the calculated residual life. The inspection interval includes an adequate margin of safety, and it is repeated if no flaws are detected. Detected flaws are repaired immediately.

As discussed earlier, classification societies have recently introduced explicit fatigue design criteria for welded joints in ships structures. These procedures are largely based on well-established fatigue design procedures (S-N design curves and joint classifications) for welded joints in bridges and offshore structures, and they are intended to ensure that there is a low probability of fatigue failures occurring during the design life of a ship (i.e., ensure a safe-life design).

In principle, a damage tolerance design approach would permit designers to exploit the redundancy of ship structures, and some design codes for bridges and offshore structures now permit designers to use damage tolerance assessment techniques in lieu of conventional fatigue design procedures. However, the widespread use of these techniques has been hindered by the difficulty of analyzing stress fields around complex structural details, particularly in redundant structures, and by the complexity of the fatigue cracking process in welded joints. Therefore, the use of damage tolerance assessment techniques in the design of these structures has been restricted to situations where normal fatigue assessment procedures are inappropriate (e.g., the geometry, size, loading, or operational environment of the structural detail under consideration is unusual, and the detail cannot be reliably assessed with available joint classifications, S-N design curves, and stress concentration factors) and to the development of inspection intervals for designs produced by the conventional approach. In both types of
applications, it is necessary to assume that there is a pre-existing initial crack, the size and shape of which is determined by the detection capability of conventional non-destructive evaluation techniques. Until designers have greater access to powerful analysis tools and until there is a greater understanding of the fatigue and fracture process in welded joints, it is expected that ship designers will use fracture mechanics in the same way.

2.2 Fitness-for-Service, Fitness-for Purpose, or Engineering Critical Assessment

Modern ship structures are mainly fabricated by fusion arc welding processes. These processes enable continuous water-tight connections to be produced in an efficient and economical manner. Unfortunately, welding processes can introduce planar and volumetric flaws from which fatigue cracks or brittle fracture could initiate. Such flaws can occur despite careful training of welders and careful design of structures for easy access by welders. Therefore, ship fabricators must rely heavily on inspectors to ensure the quality of fabricated welds. The current practice is to repair defects that do not pass workmanship-based acceptance criteria. These criteria, however, tend to be very conservative, and damage tolerance assessment could be used to screen out unnecessary repairs.

The fatigue design procedures recently introduced by classification societies are consistent with a safe life design philosophy. Such a philosophy will ensure a low probability of fatigue cracking in new ships but it will not completely eliminate it. For example, fatigue cracks could initiate from flaws that have escaped detection during fabrication or from delayed hydrogen-assisted cracking in the heat affected zone of welds. It should also be noted that the majority of existing ships were designed without explicit consideration of fatigue cracking. Fatigue cracking has occurred in these ships, particularly in high strength steel ships, and they will continue to occur as these ships age. As mentioned earlier, the current practice is to repair detected cracks at the earliest opportunity. Damage tolerance assessment could be used to screen out unnecessary repairs, to determine whether needed repairs can be delayed (e.g., to the next scheduled maintenance or port-of-call), to minimize repair costs and down-time, and to establish safe but efficient inspection schedules for unrepaired flaws.

The aforementioned applications of damage tolerance assessment fall within a broader group of applications commonly referred to as fitness-for-service, fitness-for-purpose, or engineering critical assessments. Such assessments have been permitted by design codes for piping and pressure vessels in oil and gas transmission systems, petrochemical installations, and power generation systems for many years now [2.10,2.11]. The basic procedure is similar to the damage tolerant design procedure shown schematically in Figure 2.1, and it is summarized below:

1. The criticality of the detected flaw is evaluated using stress analysis based on maximum expected service loads or design loads, fracture mechanics, and failure criteria for fracture and other possible failure modes.
Figure 2.1: Flow chart of procedure for damage tolerance assessment.
2. If a detected flaw exceeds a critical size (i.e., the residual strength of a critical structural element is lower than its design strength), the flaw is repaired before the structure is returned to service. Otherwise, a fracture mechanics analysis of fatigue crack propagation is carried out to determine the residual life of the structural element (i.e., the time period/voyages after which the initial flaw will grow to a critical size), and the structure is returned to service.

3. Critical structural elements with unrepaired flaws are re-inspected before the end of their calculated residual life. The inspection interval must include a suitable margin of safety, and the criticality of the flaws must be re-evaluated at the end of the interval. The flaw can also be repaired at any convenient time before the end of the inspection interval.

The aforementioned process differs from damage tolerance assessment at the design stage in a few respects. The most obvious difference is that real flaws rather than assumed flaws are considered in a fitness-for-service assessment. In addition, fitness-for-service assessments are usually based on more specific, more up-to-date, and less conservative inputs (e.g., load, material properties, and scantlings) than damage tolerance assessments at the design stage. For example, whenever possible, actual material properties and scantlings rather than design values are used in fitness-for-service assessments to account for any degradation during service.

2.3 Life Extension and Changes in Operational Profile

There is a strong economic incentive for ship owners to extend the service lives of existing ships beyond their original design lives and to maximize the utilization of their fleets by using ships in roles for which they were not originally designed. If a hull condition survey is conducted when the original design life of a ship expires or before the operational profile of a ship is altered, then residual strength assessments could be carried out to determine the criticality of detected flaws and to determine which flaws need to be repaired before the ship is returned to service. Residual life assessments could also be used to assess the residual life of critical structural elements with non-critical flaws and to establish inspection and repair schedules for these flaws. By assuming that initial flaws equal in size to the smallest detectable flaw exist in critical structural members with no detected flaws and by extending the residual strength and residual life assessments to these elements, inspection and repair schedules could be established for the entire ship hull. Therefore, the aforementioned process combines the elements of fitness-for-service assessment with some elements of damage tolerance assessment at the design stage.
2.4 References


[2.5] Airworthiness Information Leaflet AD/IL/0067/1-5, "Continuing Structural Integrity of Transport Aeroplanes", Civil Aviation Authority, United Kingdom, 18 August, 1978.


3.0 ASSESSMENT OF RESIDUAL STRENGTH

3.1 Preamble

As discussed in Section 2, a key element of damage tolerance assessment is the estimation of the residual strength of a damaged structure at a particular point in time (i.e., load bearing capacity of the structure in the presence of a crack of known size).

The main purpose of residual strength assessment of a structural member containing a crack is to ensure that it does not lead to unstable brittle fracture or local plastic collapse. The importance of such an assessment is obvious for flaws in primary structure. Residual strength assessment of flaws in secondary structural members may not seem as important since there is a greater possibility of stress redistribution in secondary structural members. However, a brittle fracture that initiates in secondary structure may run into adjoining primary structure before being arrested by stress relaxation or tougher material. For example, brittle fracture of a poorly fabricated splice weld in a longitudinal [3.1] or brittle fracture initiation from a fatigue crack at the toe of a bracket welded to a longitudinal frame [3.2] can penetrate the shell and affect the overall structural integrity and water-tightness of a ship.

The residual strength of a structural member containing a crack depends on the potential failure mode (e.g., brittle cleavage fracture, cleavage fracture preceded by ductile tearing, and plastic collapse). Brittle fractures are of greatest concern since low material toughness and/or local stress concentrations can precipitate the initiation of fast catastrophic fracture at nominal stresses that are far below the uni-axial yield strength of the material. Local plastic collapse, on the other hand, occurs when the stresses in the remaining ligament adjacent to the crack exceeds the flow stress of the material. Local collapse should be differentiated from global structural collapse because local collapse may be preceded by structural collapse at some other smaller flaw located in a region of higher stresses (e.g., smaller flaw subject to hoop stress in a pressure vessel compared to another flaw subject to longitudinal stresses). Local collapse may indeed lead to structural collapse if the affected member is a non-redundant one. In between the possibilities of brittle fracture and local plastic collapse, there can be situations where some ductile tearing at crack tips may precede unstable fracture.

3.2 Residual Strength Assessment using the FAD Concept

The Failure Assessment Diagram (FAD) is a graphical model of the potential for failure by brittle fracture or local plastic collapse for different combinations of crack driving force and net-section stress (i.e., stress across remaining uncracked ligament). This diagram consists of two elements (Figure 3.1): the Failure Assessment Point (FAP) and the Failure Assessment Curve (FAC). The FAP defines the state of a member containing a flaw under specific service conditions. The vertical co-ordinate of this point is defined by the ratio of the applied crack driving force to the fracture toughness of the material, while the horizontal co-ordinate of the point is defined by the ratio of the applied net section stress to the yield strength or flow strength of the
material. The FAC, on the other hand, represents critical combinations of the non-dimensionalized crack driving force and non-dimensionalized net section stress. The structure being analyzed is deemed to be safe if the FAP lies within the region bounded by the FAC and axes of the FAD. Failure is predicted if the FAP lies outside of the region bounded by the FAC and axes of the FAD. The failure mode is expected to be brittle fracture initiation in the upper left corner of the FAD, by plastic collapse in the lower right corner of the FAD, and by a mixed mode in between.

The FAD shown in Figure 3.1 is a commonly used FAD based on the strip yield model for a crack in an infinitely wide plate. In this diagram, the vertical co-ordinate \( K_r \) is the ratio of the crack tip stress intensity factor \( K_{app} \) to the material's fracture toughness \( K_{max} \). The horizontal co-ordinate \( S_r \) is the ratio of the applied net section stress \( \sigma_n \) to the material flow strength \( \sigma_f \). The crack tip stress intensity factor quantifies the severity of the asymptotic stress-strain field at a crack tip in linear elastic material (i.e., K-field). The derivation of the FAC in this FAD is briefly discussed below.

\[
\begin{align*}
K_r &= \frac{K_{app}}{K_{max}} \\
S_r &= \frac{\sigma_n}{\sigma_f}
\end{align*}
\]

If the material were to behave in a perfectly linear elastic manner, then the shape of the FAD would be a square bounded by lines at \( K_r = 1.0 \) and \( S_r = 1.0 \). The actual driving force for brittle fracture in this case \( K_{app} \) would be given by:

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where $K_{app}$ (also denoted as $K_1$) is the driving force for crack initiation from a through-thickness crack of half length $a$ that is present in a structural member subject to an applied stress $\sigma$. $Y$ in the above equation is a constant that is dependent on the geometry of the structural member and the crack. In practice, most structural steels display at least some degree of elastic-plastic behaviour so that a certain amount of plasticity develops at the crack tip. In the presence of this plastic zone, the effective driving force for brittle fracture ($K_{eff}$) is in fact greater than $K_{app}$ calculated on the assumption of linear elastic behaviour. In Irwin's approach [3.4], this difference is accounted for in the following manner:

$$K_{eff} = Y'\sigma\sqrt{\pi a}$$  \hspace{1cm} (3.2)

where, $r_y$ is the radius of the plastic zone size at the crack tip, and the geometry dependent constant $Y'$ now depends on the effective flaw size ($a + r_y$). The above correction for plastic zone size becomes significant when the applied stress magnitude exceeds about half the material's yield strength ($\sigma_y$) and becomes inaccurate when it exceeds about 75 to 80% of the yield strength [3.5].

A more accurate model of the effect of crack tip plasticity on the effective crack driving force is given by the strip yield model [3.6] for a through-thickness crack of length $2a$ in an infinite plate of an elastic-perfectly plastic material:

$$K_{eff} = \sigma_y \sqrt{\pi a} \left[ \frac{8}{\pi^2} \ln \sec \left( \frac{\pi \sigma_y}{2\sigma_f} \right) \right]^{-0.5}$$  \hspace{1cm} (3.3)

By replacing the yield strength ($\sigma_y$) in Equation 3.3 by the flow strength ($\sigma_f$), one can show from Equations 3.1 and 3.3 that:

$$\frac{K_{app}}{K_{eff}} = \frac{\sigma_n}{\sigma_f} \left[ \frac{8}{\pi^2} \ln \sec \left( \frac{\pi \sigma_n}{2\sigma_f} \right) \right]^{-0.5}$$

The ratio $\frac{K_{app}}{K_{eff}}$ is less than unity in the presence of crack tip plasticity. Also, at the critical point for brittle fracture initiation, $K_{eff} = K_{mat}$, $K_{app}/K_{mat} = K_r$, and $\sigma_n/\sigma_f = S_r$. The above equation may be rewritten as:

$$K_r = S_r \left[ \frac{8}{\pi^2} \ln \sec \left( \frac{\pi}{2} S_r \right) \right]^{-0.5}$$  \hspace{1cm} (3.4)
This equation defines the FAC in the FAD in Figure 3.1 (i.e., the ultimate state of a given cracked detail of a given material). This curve or the failure locus for any given value of $S_r$ lies below the line $K_r = 1$ by an amount by which $K_{eff}$ exceeds $K_{app}$. There are at least two advantages of this approach. First, the non-dimensional crack driving force ($K_r$) can still be calculated based on linear elastic calculation for $K_{app}$ whereas the material fracture toughness ($K_{mat} = K_{eff}$) can be obtained from full thickness specimens even though these may display crack tip plasticity. Secondly, the approach takes into account failure by brittle fracture as well as plastic collapse. If the structural material has high fracture toughness (high $K_{mat}$), $K_r$ tends to be small and failure usually occurs by local plastic collapse ($S_r \approx 1$). In the case of a brittle material (low value of $K_{mat}$), $K_r$ will approach unity very quickly and failure will occur in a brittle mode. In the intermediate region, fracture and collapse interact and fracture occurs in an elastic-plastic manner.

Finally, it should be noted that other FAD's besides that based on the strip yield model have been developed. Some of these FAD's can be used for application to ship structures, depending on the quality of the input parameters and the accuracy desired, and they are discussed in the commentary in Sections 3.3.1 and 3.3.3.

3.2.1 Limitations in Application to Ship Structures

The FAD approach for residual strength assessment of a structural member or detail assumes that the far field stresses (away from the flaw but local to the structural member) are well-defined and that these do not change as the crack grows. Thus, the FAD, itself, cannot take into account the effect of any redistribution of loads/stresses that might occur as a result of flaw growth or structural redundancy.

In comparison, ship structures are recognised as having a significant degree of structural redundancy, however, quantification of its effect on the stresses in such structures is in its early research stage. Therefore, it is customary to ignore any stress distribution effects, and thus conduct a local residual strength assessment as if the flawed member is isolated from the rest of the structure. If such local residual strength analysis indicates the crack present in a structural member to be larger than the critical size, then, as mentioned in a previous SSC study [3.7], a normal practice in assessing global structural strength is to completely disregard that member from further consideration.

Another limitation of the FAD approach is that it does not consider buckling which is a common failure mode in ship structures. At this stage, there is little definitive knowledge on the effect of crack like flaws on the buckling residual strength and further research is needed in this area.

Finally, a common limitation in applying the FAD approach to structures is the limited availability of fracture toughness data. This limitation is a particular concern for ship structures because most fracture toughness tests, to the extent that these are indeed performed on materials relevant to ships, are conducted at a quasi-static loading rate whereas the loading rates of extreme
wave loads in ship structures are in the intermediate range [3.8]. Clearly, it is necessary to
determine fracture toughness values of ship structural steels and weldments at an appropriate
loading rate if the application of damage tolerance methodology for residual strength assessment is
envisaged. In the meantime, there is no choice but to use the available fracture toughness values
with the hope that any degree of unconservatism introduced due to neglect of the loading rate effect
will be compensated by conservatism introduced in the selection of the other input parameters.

3.2.2 General Procedure for Determining Residual Strength

There are two principal ways in which FAD's and residual strength assessment procedures
can be used. In the first, commonly referred to as fitness for purpose analysis or engineering critical
assessment, all the input parameters (applied/service stresses, flaw size, material toughness
properties) are known and the main objective is to establish if this particular combination of input
parameters is sub-critical (safe) or not. From the known input parameters, \( K_f \) and \( S_f \) values can be
computed and an actual failure assessment point (FAP) can be plotted on the FAD. If this point is
within the FAD (e.g., point A in Figure 3.1), then the structure is safe and its location with respect
to failure assessment curve is indicative of the safety margin. In a deterministic analysis and in the
absence of residual stresses, the safety factor on load is \( OB/OA \). An assessment point outside the
FAD, point C in Figure 3.1, would indicate unstable fracture initiation before the peak service stress
magnitude is reached.

The second application requires determination of the critical combination of parameters that
will lead to failure (i.e., combination of parameters that will lie on the failure assessment curve.).
Generally, two of the three inputs would be known and the objective then is to determine the
critical value for the third. Thus, for a given known flaw, structural geometry and material
properties (strength and fracture toughness), the failure assessment curve can be used to compute
the residual strength (maximum allowable applied stress). Conversely, if the maximum magnitude
of the in-service applied stresses were known, then the FAC can be used to compute the critical
flaw size. Since both the ordinate and the abscissa in the FAD depend on the flaw size and the
applied stress, these computations will require an iterative procedure to obtain the final solution. It
is, therefore, useful to have a simple computer program to perform these calculations.

The procedure for performing residual strength assessment is detailed in the commentary in
the next section and can be summarised for the time being as follows:

1. **Determine the flaw size.** This may be already known from inspection reports for evaluating a
current flaw. However, if the assessment is for some point in time in the future, then its
determination will take into account a flaw's growth under cyclic loading using the guidance in
Section 4.

2. **Establish material properties.** The material properties of interest are the yield strength,
tensile strength, and fracture toughness of the material where the flaw resides. Guidance on this
subject is offered in the commentary in Section 3.5.
3. Determine the magnitude of applied/service stresses and residual/assembly stresses: See Section 5 and 6 for Guidance.

4. Select FAD for residual strength assessment. See Section 3.3.3 for Guidance.

5. Calculate driving force parameters ($K_{app}$ and $\sigma_d$) and resistance parameters ($K_{mat}$ and $\sigma_y$ or $\sigma_f$). See Section 6 for calculation of driving force parameters and Section 3.5 for resistance parameters.

6. Perform residual strength assessment. Calculate $K_r$ and $S_r$.

   As mentioned earlier, the above procedure is for fitness for purpose analysis (all parameters known). If a critical condition corresponding to the failure assessment curve is to be determined, then an iterative procedure will need to be used since both the ordinate and abscissa ($K_r$ and $S_r$) will be a function of the one unknown parameter to be determined, the other two being known.

3.3 Commentary on the Use of FADs

3.3.1 Other Commonly Used FADs

The previous section focused on one commonly used FAD based on the strip yield model to explain the FAD concept. However, there are several other FAD's and analysis procedures available for residual strength or critical flaw size analysis. This guide covers FAD's included in Level 1 and Level 2 analysis procedures in the draft PD6493-1996 document since the analysis procedures in 1981 and 1991 editions of this document have been used extensively for flaw assessment and inspection scheduling for offshore structures, bridges, pipelines, storage tanks and pressure vessels. The latest edition of this Published Document was issued in 1991 [3.3] and several revisions and additions are being currently discussed for inclusion in the next edition (planned for 1996). To the extent that this information is in the public domain [3.9], it has been taken into consideration in preparing this Guide.

There are other more sophisticated FAD's and analysis procedures (e.g., Level 3 analysis in the draft version of PD6493-1996 [3.9] and analysis based on Deformation Plasticity FAD [3.10]) but these are quite complex requiring non-linear, 3-dimensional finite element analysis and specific material properties. These have been developed for tough, ductile materials and enable one to consider ductile tearing, and constraint and weld mismatch effects. These are used mostly in the nuclear industry and are not appropriate for application to ship structures where the accuracy of such procedures is likely to be negated by the uncertainties in the magnitude of the input parameters.

The FAD for Level 1 analysis is shown in Figure 3.2, and the flow diagram for assessing the significance of a known flaw (knowing the service stresses, material fracture toughness, flaw and structural geometry) is shown in Figure 3.3. The fracture assessment is based on a semi-
empirical crack driving force relationship referred to as the CTOD design curve [3.11] which in turn has been shown to represent an upper bound for the experimental data from a large number of wide plate tests on structural steels and weld metals.

The FAC in this case is defined by two straight lines: $K_r$ or $\sqrt{\delta_r} = 1/\sqrt{2}$ and $S_r = 0.8$ where, $K_r$ is the ratio of the applied crack driving force in terms of the crack tip stress intensity factor ($K_1$) to the material fracture toughness ($K_{mat}$), and $\delta_r$ is the ratio of the applied crack driving force in terms of CTOD ($\delta_1$) to the corresponding material fracture toughness ($\delta_{mat}$). The value of $1/\sqrt{2}$ for $K_r$ or $\sqrt{\delta_r}$ arises simply from an inclusion of a safety factor of 2 on flaw size in fracture assessment when the applied stress is $\leq 0.5\sigma_r$. At higher applied stresses, the safety factor on flaw size can be slightly different from 2. The CTOD design curve considers fracture only and not failure by plastic collapse and, therefore, to adapt it to the FAD format, an arbitrary cut off for $S_r$ has been established at 0.8. Since there already are safety factors built into this FAD, both on flaw size and on stress ratio, it is advised against the application of additional safety factors in assessing critical stress (residual strength) or critical flaw size.

![Assessment Curve](image)

Figure 3.2: Level 1 failure assessment diagram (from Reference [3.3])
Figure 3.3: Flow Chart for Level 1 Assessment (from Reference [3.9])
An assessment based on Level 1 FAD employs upper bound estimates for loading and flaw size, and lower bound estimate for material toughness. In addition, the through-thickness stress distribution at the assessment site is assumed to be uniform for calculating stress intensity factors and net section stresses. These features and the safety factors built into the Level 1 FAD imply that the results of a Level 1 assessment are quite conservative. Since Level 1 assessment is also relatively easy to perform, it is usually referred to as a preliminary assessment. If it finds a flaw to be safe, then no further analysis is deemed necessary. Conversely, if the flaw is found to be unsafe, then one can perform additional assessment based on more complex but more accurate FAD’s described in the paragraphs below. Determination of the various driving force input parameters ($K_{\text{app}}$ and $\sigma_n$) is detailed in Section 6, while the determination of the various material resistance parameters ($K_{\text{mat}}$ and $\sigma_y$ or $\sigma_f$) in Section 3.5.

Under Level 2, there are three FAD’s that one can potentially use for analysis. These are shown in Figures 3.1, 3.4 and 3.5, respectively and the analysis approach for all these three FAD’s is summarised in the flow diagram shown in Figure 3.6. Overall, these three approaches are more accurate than Level 1 assessment, and PD6493 refers to Level 2 analyses as normal assessment to assess the susceptibility of a flawed member to unstable fracture. Unlike the Level 1 FAD, there are no built-in safety factors in these FAD’s so that any parameters calculated from the FAC (residual strength, flaw size) will be critical values. Therefore, the conservatism of these values in a deterministic assessment will be largely determined by the selected input variables (material fracture toughness, service loads). Guidance on the values for these inputs is provided later in Sections 3.5 and 5.0.
Figure 3.5a: Stress-strain curve and material dependent failure assessment diagram for a quenched and tempered steel (from Reference [3.3]).
Figure 3.5b: Stress-strain curve and material dependent failure assessment diagram for a carbon steel (from Reference [3.3])

\[ \sigma_y = 255 \text{ N/mm}^2 \]
\[ \sigma_u = 429 \text{ N/mm}^2 \]
I, Structure Has Been Demonstrated to be Safe at Level 2

Define Stresses

Determine $K_{\text{max}}$

Determine Material Tensile Properties

Characterize Flaw

Select FAD

Calculate $S$, or $L$

Calculate $K_I$

Plot Assessment Point on FAD

Assess Significance of Results

Defect Tolerable?

Yes

No

Can a Material Specific FAD be Used?

Yes

No

Can Flaw be Recharacterized?

Yes

No

Can Stress Analysis be Refined?

Yes

No

Structure Cannot be Demonstrated to be Safe at Level 2

Figure 3.6: Flow Chart for Level 2 Assessment (from Reference [3.9])
As mentioned earlier, the FAD in Figure 3.1 is based on the strip yield model and the FAC is given by Equation 3.4. Because of the elastic-perfectly plastic material assumption, it is suitable for low work hardening materials and therefore recommended for welded steel structures. However, one situation where this Level 2 FAD can become unsafe, is when the material displays a yield plateau (Luder band extension) and the applied stresses exceed the yield level so that considerable local strains are involved. To address such situations, one can either impose a cut off for $S_r$ value ≤ 0.83 (1/1.2) or use a material specific FAD (Figure 3.5).

The other two Level 2 FAD’s in the 1996 edition of PD6493 (Figures 3.4 and 3.5) are included in the current (1991) edition of PD6493 as Level 3 FAD’s and are more suitable for high work hardening materials (e.g., stainless steels, some low strength ferritic pressure vessel steels). The difference between the FAD’s in Figures 3.4 and 3.5 is that the former is a lower bound, material non-specific FAD to be used when the stress-strain curve for the material is not available or cannot be easily established (e.g., for heat affected zone), whereas the one in Figure 3.5 has to be constructed from the actual stress-strain behaviour of the material. The FAC’s for these two FAD’s are given by the following equations:

**Material Non-specific**

\[
K_r = \begin{cases} 
(1 - 0.14L_r^2)(0.3 + 0.7 \exp(-0.65L_r^4)) & \text{for } L_r < L_r \text{ max} \\
0 & \text{for } L_r \geq L_r \text{ max.}
\end{cases}
\]  

**Material Specific**

\[
K_r = \left( \frac{E \ln(1 + \varepsilon)}{\varepsilon(1 + \varepsilon)} + \frac{\sigma^2(1 + \varepsilon)}{2\sigma_y E \ln(1 + \varepsilon)} \right)^{-0.5}
\]

\[
L_r = \frac{\sigma(1 + \varepsilon)}{\sigma_y}
\]

where; $\sigma$ is any value of stress along the materials engineering stress strain curve at a strain of $\varepsilon$, and $\sigma_y$ is the material's lower yield strength or 0.2% offset yield strength. It should be noted that the abscissa in these two FAD's is $L_r$ rather than $S_r$ as in the FAD's in Figures 3.1 and 3.2. In the term $L_r$, now called the load ratio, the net section stress is normalised with respect to the materials yield strength rather than flow stress as for the applied stress ratio, $S_r$. Thus,

\[
L_r = \frac{\sigma_n}{\sigma_y}
\]

where $\sigma_n$ is the net section stress as defined and calculated for Level 2 strip yield model FAD (Figure 3.1). The maximum value of for $L_r$ is, however, no longer limited to 1.2 and is given by
However, if the material displays discontinuous yielding, then $L_r$ is limited to a maximum value of unity. The ordinate of the FAD, $K_r$, is calculated in exactly the same manner as for the FAD in Figure 3.1 and the assessment procedure for residual strength or critical flaw size also follow the same approach.

### 3.3.2 Use of FADs in Other Industries

In the late 70's and early 80's, the CTOD design curve was the main basis for conducting engineering critical assessments. This approach to evaluate potential for unstable fracture has also been incorporated in non-mandatory appendices in pipeline standards (e.g., CSA Z662, Appendix K, API 1104, Appendix A) to establish flaw acceptance criteria that are usually less restrictive than workmanship criteria. A separate check, however, is needed for considering plastic collapse and the criteria are based on large scale pipe bend tests. By the mid 80's, a methodology based on the strip yield model FAD had been formalized and used more often for assessments of flaws in offshore structures and pressure vessels.

More recently, Anderson [3.12] has formalized an engineering critical assessment approach for pressure vessel steels that is based on the material non-specific FAD shown in Figure 3.4. Since pressure vessel design is based on the ultimate strength of the steel, pressure vessel steels tend to have a relatively higher ultimate strength to yield strength ratio ($\sigma_u/\sigma_y$, greater work hardening) compared to structural steels, especially for higher strength structural steels (yield strength of 350 MPa or more). As mentioned earlier, for such steels it is more appropriate to use one of the FAD's shown in Figures 3.4 or 3.5. The unconservatism of the failure locus resulting from the use of the strip yield model FAD (Figure 3.1) for materials with $\sigma_u/\sigma_y > 1$ is shown in Figure 3.7. Here material specific FAC's were calculated by Reemsnyder [3.13] for steels with different $\sigma_u/\sigma_y$ ratios and then after adjusting the load ratio to stress ratio, plotted on to the strip yield model FAD. It is evident that as the $\sigma_u/\sigma_y$ ratio increases beyond unity, the strip yield model FAD increasingly becomes more unconservative.

### 3.3.3 Selection of FAD for Residual Strength Assessment of Ship Structural Members

In light of the comments made in the previous sections, it is recommended that wherever possible, the material specific FAD defined by Equation 3.6 be used for residual strength assessment. This FAD is henceforth referred to as the Level 2c FAD. This FAD, however, requires the stress-strain curve for the material of interest which is often unavailable for steel base materials and never for the heat affected zone. Under such circumstances, either the material non-specific FAD defined by Equation 3.5 (henceforth referred to as the Level 2b FAD) or the strip yield model FAD defined by Equation 3.4 (henceforth referred to as the Level 2a FAD) can be used. These two FAD's are comparable in ease of application, but the Level 2a FAD is less suitable for high work hardening materials since: (i) it is less conservative at relatively high $S_r$ values ($< 1$), and (ii) it does not permit net section stresses to exceed $1.2\sigma_y$. 

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The Material Specific, FAC Curves represent: (1) elastic-perfectly plastic material, i.e., $\sigma_f/\sigma_v = 1$, (2) A36, i.e., $\sigma_f/\sigma_v = 1.403$, (3) HSLA 50, i.e., $\sigma_f/\sigma_v = 1.20$, (4) HSLA 60, i.e., $\sigma_f/\sigma_v = 1.125$, and (5) HSLA 80 i.e., $\sigma_f/\sigma_v = 1.063$.

Figure 3.7: Comparison of failure assessment diagrams for steels with different yield/ultimate strength ratios (from Reference [3.13])
Use of any one of the three Level 2 FADs presupposes that the material fracture toughness data is available. If that is not the case, then an estimate of the material fracture toughness is obtained indirectly via empirical and conservative correlations between CVN toughness and fracture toughness. In such cases, it is recommended that Level 2 FAD's not be used. Instead, assessments should be based on a Level 1 FAD.

In using any of the FAD's described above, it should be kept in mind that these cover failure due to mode I loading (principal stress perpendicular to the crack surface) only.

3.4 Crack Driving Force Calculations

The driving force for brittle fracture ($K_{\text{app}}$) and local plastic collapse ($\sigma_a$) are required inputs for each of the failure assessment diagrams described above. $K_{\text{app}}$ depends on the local stress state around a crack tip due to applied loads, welding residual stresses, and fabrication residual stresses, whereas $\sigma_a$ depends on the local stress state around a crack due to applied loads and only those residual stresses that do not relax as a result of local net section yielding. Guidance on the calculation of these driving forces and the relevant local stress is given in Section 6.0. Because of the stochastic nature of these calculations, it is necessary to base these calculations on the maximum expected applied loads over the assessment period of interest (usually the inspection period). Guidance on the calculation of these loads is given in Section 5.0.

3.5 Resistance to Crack Initiation (Material Property) Inputs

The material property data required for the analysis are the yield strength (lower yield or 0.2% offset), ultimate strength and fracture toughness of the material (weld metal, heat affected zone or base metal) where the flaw tips reside. Guidance for obtaining appropriate values for these inputs is as follows:

3.5.1 Tensile Properties

The yield strength and ultimate strength of weld metal and base metal are easily determined following standard test procedures. (When multiple tensile tests are conducted, the scatter in results is usually minimal so that damage tolerance assessment results are not significantly affected when the tensile properties used are obtained from a single specimen or as a lower bound from multiple specimens.) Once the yield and ultimate strength values are established, the flow stress ($\sigma_f$) is computed as the lower of $1.2 \times \sigma_y$ or $(\sigma_y + \sigma_u)/2$ for FAD's in Figures 3.1 and 3.2, and as $(\sigma_y + \sigma_u)/2$ for FAD's in Figures 3.4 and 3.5.

For heat affected zones, tensile properties are not easily determined. To obtain conservative assessments, it is recommended that the HAZ tensile properties be assumed to be the lower of adjacent base metal or weld metal in calculating $S_r$ or $L_r$ values, and higher of adjacent base and weld metals when required for calculating HAZ fracture toughness in experimental procedures (CTOD, K or J tests).

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3.5.2 Fracture Toughness

Since the damage tolerance assessment results are sensitive to the input material toughness and since there usually is a significant scatter in the fracture toughness measurements of a given material (especially weld metal and HAZ), selection of fracture toughness input value has warranted considerable thought. In selecting the input material fracture toughness value for deterministic analysis, the following guidelines are provided:

- For most marine structural steels and weldments, the fracture toughness is most commonly measured as a critical CTOD value ($\delta_{\text{mat}}$) which corresponds to either unstable fracture initiation in the specimen without any crack extension ($\delta_{\text{s}}$), unstable fracture after some ductile crack extension ($\delta_{\text{u}}$), or maximum load behaviour in a test ($\delta_{\text{m}}$) \[3.14\]. For an analysis based on the Level 1 FAD, this is an ideal choice since the experimentally established CTOD design curve, the basis of the Level 1 FAD, also uses such values.

- The fracture toughness value, $K_{\text{mat}}$, should be computed from critical values of CTOD ($\delta_{\text{mat}}$) using Equation 3.9 below which is less conservative than the equation implicit in PD 6493 (see Section 3.6):

$$K_{\text{mat}} = \frac{1.6 \sigma_{f} \delta_{\text{mat}} E}{\sqrt{(1 - \nu^2)}}$$

(3.9)

Occasionally, the fracture toughness may be available as $J_{\text{mat}}$, a critical value of $J$ determined in accordance with standards like ASTM E 1737 \[3.15\]. In such cases, $K_{\text{mat}}$ can be inferred from Equation 3.10 below:

$$K_{\text{mat}} = \frac{J_{\text{mat}} E}{\sqrt{(1 - \nu^2)}}$$

(3.10)

- The CTOD tests can be conducted according to ASTM Standard E 1290 \[3.14\] and $J$ tests according to ASTM Standard E 1737 \[3.15\]. In near future, when a current draft standard \[3.16\] is finalized, both fracture parameters will be calculable from the same test procedure. The fracture toughness tests for determining the fracture toughness must be conducted at the design temperature on full thickness specimens machined from the same material as the welded structure and at the same stress intensity factor rate as that anticipated in service. For welds and heat affected zone, it means that the welding procedure (welding consumables, heat input, restraint during welding, post-weld heat treatment, etc.) for the test weld for preparing the specimens should be the same as for the production welds. The crack plane and location in the specimen should be the same as that anticipated for the flaw in service. For HAZ specimens, post-test metallography ought to be performed to ensure that the crack tip indeed resided in the microstructural region of interest.

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It is a common practice to determine fracture toughness using rectangular $B \times 2B$ (preferred geometry), three point loaded, single edge notched beam specimens where $B$ is the specimen and plate thickness.

The fracture toughness is most frequently measured at a quasi-static loading rate whereas the loading rates that ship structural members are subjected to are in the intermediate loading rate (strain rate of about $5 \times 10^{-5}$) as mentioned earlier [3.8]. In the absence of such data, it is common practice to input fracture toughness values based on quasi-static loading rate tests though it does introduce a degree of non-conservatism in the assessment.

- An important issue is the number of fracture toughness tests that ought to be done and then which value should be used as a representative fracture toughness. For Level 1 assessment, PD 6493 recommends conducting at least three tests and then using the lowest one in analysis. The minimum value from a set of three corresponds to a 33rd percentile value (mean minus one standard deviation) with 70% confidence. Further tests are recommended if there is too much scatter within the three results. Excessive scatter is indicated when the minimum CTOD value is less than half the average of three values or if the maximum value is more than twice the average of the three values. Further testing would normally comprise an additional set of three specimens and then selecting the second lowest value for material fracture toughness from six values available from the two sets of tests.

- For the remaining FADs described earlier, there are no safety factors built into them, and once again the lowest fracture toughness value obtained from a set of three is normally used subject to the qualifications stated above and provided worst case estimates for stress and flaw size are used. However, it is generally desired that greater volume of fracture toughness data be available. When more than three test results are available, then the statistically equivalent value to the minimum of three which should be used in the damage tolerance assessment is given in Table 3.1.

<table>
<thead>
<tr>
<th>Number of Fracture Toughness Tests</th>
<th>Equivalent Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 to 5</td>
<td>Lowest</td>
</tr>
<tr>
<td>6 to 10</td>
<td>Second Lowest</td>
</tr>
<tr>
<td>11 to 15</td>
<td>Third Lowest</td>
</tr>
<tr>
<td>16 to 20</td>
<td>Fourth Lowest</td>
</tr>
</tbody>
</table>

(In reliability based analyses, log normal or Weibull distribution could be fitted to the available data, assuming that all data points represent the same failure mode ($\delta_c$, $\delta_w$, or $\delta_m$) and then a
characteristic value equal to mean minus one standard deviation established. Further, it is recommended that a partial safety factor be applied to this value depending on the consequences of the member’s failure. Thus, for moderate consequences of failure, the partial safety factor suggested is 1 and it is 1.4 when the fracture toughness is expressed as CTOD (1.2 for $K_{\text{mat}}$) and the failure consequences are severe.)

- An alternate approach to handle the scatter in the fracture toughness value is being considered in a draft ASTM Standard on fracture toughness measurement in the ductile-brittle transition region [3.16]. The basis of this approach lies in two observations. First, it has been shown that at any test temperature, the cleavage fracture toughness distribution can be described by a three parameter Weibull distribution of slope 4 so that:

$$F = 1 - \exp\left[-\left(\frac{K_{Jc} - 20}{\Theta_k - 20}\right)^4\right]$$

(3.11)

where, $F$ is the cumulative probability, $K_{Jc}$ is the fracture toughness obtained from J integral and $\Theta_k$ is the 63rd percentile toughness. Generally, six tests at any one test temperature are expected to be sufficient to establish the $\Theta_k$ value first, and then the median (or any other percentile) $K_{Jc}$ value by setting $F$ equal to 0.5 (or appropriate fraction).

Secondly, according to Wallin and co-workers [3.17], the temperature dependence of fracture toughness can be expressed by the equation:

$$K_{Jc(\text{median})} = 30 + 70 \exp\left[0.019(T - T_0)\right]$$

(3.12)

According to this equation, when $T = T_0$, $K_{Jc} = 100 \text{ MPa}\cdot\text{m}$. Once this is established, then the $K_{Jc}$ value (any percentile) can be plotted as a function of temperature. This approach should be used only for the ductile-brittle region as it is not suitable for the upper shelf region and it may not fit the data well in the lower shelf region.

- Frequently, it is the case that no fracture toughness data is available at all and none can be generated due to material unavailability. On the other hand, CVN toughness for the desired region may be available or could be generated with the limited material available. In such cases, lower bound CVN-$K_{\text{mat}}$ correlation may be used but at the risk of obtaining very conservative assessments (small critical flaw size or low residual strength).

PD6493 provides two graphs to estimate $K_{\text{mat}}$ from the CVN test results. When the CVN absorbed energy (20, 27 or 40 J) transition temperature of the region of interest is known and it is different from the service or design temperature then Figure 3.8(a) enables one to estimate $K_{\text{mat}}$ as a function of (design temperature - transition temperature).
The curve in Figure 3.8(a) is based on a lower bound to the data generated for ASTM A533 grade B, nuclear pressure vessel steel and includes data from crack arrest and dynamic fracture toughness test, and is therefore quite conservative for relatively thinner ship steels subject to intermediate loading rates rather than dynamic. The transition temperature used in the ASME lower bound curve is the drop weight nil ductility transition temperature (NDTT) as determined by ASTM E208 procedure. However, for ship and structural steels in general, CVN data is more frequently available than the NDTT and therefore PD 6493 recommends the use of CVN transition temperature though some recent work suggests that for modern, clean, low carbon steels, the NDTT can be higher than the CVN transition temperature. (Anderson [3.12] uses a similar lower bound curve for pressure vessel steels but based on quasi-static fracture toughness data only. Obviously, this approach gives higher $K_{mat}$ value for the same CVN toughness, however, since the applicable loading rates for ships are in the intermediate range, it is prudent to use the lower bound curve in PD 6493 rather than the one used by Anderson).

If the CVN absorbed energy at the design or operating temperature is known, then Figure 3.8(b) can be used directly to estimate $K_{mat}$. If both these pieces of information are available then the lower of the two resulting $K_{mat}$ values is recommended to be used. Secondly, these correlations should be used only for steels with less than 480 MPa yield strength.

Care should be taken to ensure that the CVN data is from specimens that represent the same fracture path and microstructural region as the region of the structure containing the flaw.

3.5.3 Practical Examples of Estimating $K_{mat}$ Values in Various Scenarios

(a) Steel of unknown specification, and not available for any toughness tests

In such a situation, ideally no damage tolerance analysis will be performed. If one must be performed for whatever reason, then there is no choice but to estimate a lower bound material toughness value. Anderson in the MPC procedure recommends that for steel of unknown origin, one should presuppose a hot rolled steel and assume the 20 J transition temperature to be 38°C. For a design temperature of 0°C, the $K_{mat}$ value using the correlation provided in PD 6493 (Figure 3.8(a)) would be about 1025 Nmm$^{-1.5}$ (32.5 MPa√m).

In practice, a similar situation can arise when it is known that the steel used conforms to Grade A which does not have any CVN toughness requirements at all. According to Yajima and Tada [3.18] in developing the guidelines for steel grade application to different regions of the ships, it was assumed that Grade A steels would meet a transition temperature of 10°C. Once again, Figure 3.8(a) would suggest that the $K_{mat}$ value for a design temperature of 0°C for such a steel would be about 1250 Nmm$^{-1.5}$ (39.5 Mpa√m).

$K_{mat}$ values of 32 to 40 MPa√m are quite small and are likely to indicate unsafe conditions except in most benign conditions (very low stresses or very small flaws).
Figure 3.8a: K\text{mat} estimate at design temperature based on design temperature difference with respect to the CVN transition temperature (from Reference [3.9])

Figure 3.8b: Lower bound K\text{mat} estimate from energy absorbed in Charpy V notch test at the operating temperature (from Reference [3.3])
Steel or Weld Metal of Known CVN Toughness or Specification

Two cases can be envisaged in this scenario. The steel grade or fabrication specification detailing the minimum requirements is known but the actual values from the mill test report or procedure qualification record may or may not be known. Or, the actual CVN toughness values at a particular test temperature might be available or be determinable, with or without the knowledge of the governing material specifications. Again it is assumed that material is not available for testing and generating the fracture toughness data.

For example, it may be known only that the steel used in fabrication was specified to be EH 36 which is required to meet a requirement of 34 J at -40°C (i.e., the design temperature of 0°C is 40°C above the CVN test temperature) in the longitudinal direction. Then, assuming the flaw orientation to be consistent with flaw propagation in a direction perpendicular to the rolling direction, Figure 3.8(a) indicates that lower bound $K_{mat}$ for such a steel is about 2000 Nmm$^{-1.5}$ (63 MPa√m) at 0°C.

The actual data may indicate that the steel in fact had a CVN toughness range of say, 100 to 110 J at -40°C. Clearly, the CVN toughness would be higher at the design temperature of 0°C. Still, using the 100 J number (lowest value) and Figure 3.8(b), one would estimate the $K_{mat}$ value to be 3500 Nmm$^{-1.5}$ (110 MPa√m). Unfortunately, PD 6493 implies that one is limited to using the lower of the two $K_{mat}$ values obtained from Figures 3.8(a) and (b), respectively.

However, if only the actual CVN toughness were available, without knowledge of the steel grade or other specification requirements, then it is allowable to use $K_{mat}$ value obtained from Figure 3.8(b). Thus, in the example in the preceding paragraph, it would be acceptable to use a value of 3500 Nmm$^{-1.5}$ (110 MPa√m) based on the actual CVN toughness value at -40°C. Similarly, if the CVN toughness were available at 0°C, say 150 J, then it would be acceptable, based on Figure 3.8(b) to use a $K_{mat}$ value of 4300 Nmm$^{-1.5}$ (135 MPa√m).

PD6493 goes on to caution that before using the above correlations in Figures 3.8(a) and (b) for Level 2 analyses, one should validate them for the particular material of interest based on data that might be available in the literature.
Fracture Toughness Data at Design Temperature Available

This of course is the ideal situation. In the context of marine materials, CTOD tests as per BS 5762 or ASTM E1290 are usually carried out to give critical CTOD values which might correspond to c, u or m type behaviour. Invariably, three valid tests would have been conducted and based on the guidance provided earlier, the lowest of the three would be used to estimate $K_{\text{mat}}$ from equation 3.9, assuming that the scatter between the three values is acceptable.

For example, at the time of fabrication, weld procedure qualification may have required weld metal CTOD tests and the three valid values might have been 0.23 (u), 0.31 (u) and 0.12 (c) mm. The CTOD value used for calculating $K_{\text{mat}}$ in Level 2 analysis would then be 0.12 mm since the scatter in the results is acceptable. (The average value of 0.22 mm is within a factor of 2 of the minimum and maximum values.) If on the other hand the three values were 0.75 (m), 0.64 (m) and 0.18 (u) mm, then one ought to do additional tests for a useable value since the average of the three values (0.52 mm) is more than twice the minimum value.

Similarly, if a large heat affected zone fracture toughness investigation had been undertaken at the time of steel procurement and initial fabrication, then it is conceivable that several microstructurally verified, valid HAZ CTOD fracture toughness values at the design temperature are available for the HAZ using a particular welding procedure of interest. For the purposes of illustration, let these values (in mm) at the design temperature be: 0.27 (u), 0.23 (u), 0.16 (u), 0.52 (u), 0.47 (u), 0.06 (c), 0.09 (c), 0.29 (u), 0.21 (u), 0.31 (u) and 0.18 (u). Since eleven values are available, according to Table 3.1 presented earlier, the third lowest value should be used in the analysis which in this case would be 0.16 mm. Alternatively, one could establish the Weibull distribution for these CTOD values and then use a value corresponding to a certain percentile value.

The only limitation in the above data is likely the loading rate at which the data had been obtained. Unfortunately, at this time there are no reliable means to correct these values for the potentially higher loading rate that may be encountered in ship service.

3.6 Commentary on Fracture Toughness Input

Traditionally, the notch toughness of steels and weldments has been assessed on the basis of absorbed energy in the blunt notched Charpy Vee notch specimen, and the minimum requirements for material specification are based primarily on experience. Unfortunately, the CVN notch toughness values can not be used directly in fracture mechanics analysis described in the previous section. The required input has to be in terms of fracture toughness which is a measure of the material’s resistance to fracture initiation from sharp flaws under specific loading conditions. It is conveniently measured by subjecting a single edge (fatigue sharpened) notched specimen to a three point bending load at the test temperature of interest and monitoring load and crack (notch) mouth opening displacement and/or the load line displacement until a fracture occurs in the test or a maximum load condition is reached.
The fracture toughness of a material can be presented in the form of one of three parameters, viz., critical stress intensity factor (K<sub>k</sub>), crack tip opening displacement (CTOD), or the J integral. These parameters and detailed test procedures to determine them were initially developed to measure fracture toughness in the three different regimes of fracture toughness-temperature transition curve (Figure 3.9). At low temperatures, the material behaves in a brittle, linear elastic fashion and the extent of plasticity at the crack tip is small compared to specimen thickness. Fracture toughness under these conditions can be expressed in terms of K<sub>k</sub> and measured as per ASTM Standard E399. However, one generally endeavours to avoid using steels that satisfy the requirements of ASTM E399 for valid K<sub>k</sub> values at the design temperature since it would otherwise imply the use of a relatively brittle steel for the intended application.

The CTOD procedure was developed to measure fracture toughness in the ductile-brittle transition region where there is plasticity and stable ductile tearing at the crack tip before the initiation of the brittle cleavage factor. The CTOD toughness can be measured using ASTM Standard E1290 and BSI Standard 7448. Since the extent of plasticity at the crack tip and therefore the measured CTOD fracture toughness can depend on the specimen thickness (crack tip constraint), it is recommended that CTOD fracture toughness should be determined using full thickness specimens.

The J integral on the other hand was devised for materials that display fully ductile behaviour at the design temperature such as the nuclear pressure vessel steels. The J values are also material thickness dependent and therefore full thickness specimen should be employed for assessing J value. The ASTM Standards covering measurement of J values are E813, E1152 and E1737, and currently ASTM is preparing a draft standard so that CTOD and J can be obtained from the same test [3.16].

In calculating J or CTOD toughness for elastic-plastic materials, another consideration is the stage of the load versus crack mouth opening displacement at which the fracture toughness value should be computed. Referring to the CTOD test load vs CMOD trace shown in Figure 3.10, four CTOD values can be defined. For brittle materials, cleavage fracture is initiated in the elastic load range and an unambiguous CTOD toughness, δ<sub>p</sub>, can be calculated. However, in the presence of extensive crack tip plasticity, there are three potential values of CTOD toughness that can be defined. Thus, δ<sub>p</sub> denotes CTOD toughness corresponding to the peak load at fracture in specimens that display some ductile tearing at the crack tip before the fracture. Similarly, δ<sub>m</sub> refers to the CTOD value corresponding to the maximum load reached in the test for specimens that display ductile tearing only and wherein no cleavage fracture intervenes. In between δ<sub>p</sub> and δ<sub>m</sub>, there is a potential CTOD value, δ<sub>t</sub>, that just corresponds to onset of ductile tearing. Its determination requires a different test procedure so that a CTOD (or J) vs crack growth (Δa) curve (also called the CTOD-R or J-R curve) is generated and then CTOD or J value determined for Δa = 0.2 mm.
Figure 3.9: Usually applicable measures of fracture toughness in the different regimes of the fracture toughness versus temperature curve (from Reference [3.12])

Figure 3.10: Types of load versus crack mouth opening displacement records (from ASTM Standard E1290-93)
In the linear elastic (plane strain) regime, one can obtain a theoretical relationship between the above three measures of fracture toughness:

\[ K_{te} = \frac{\sqrt{J_{te}E}}{(1-\nu^2)} = \frac{2\sigma_y \delta_c E}{\sqrt{(1-\nu^2)}} \]  

(3.13)

where; \( K_{te}, J_{te} \) and \( \delta_c \) are critical values for fracture toughness expressed in terms of stress intensity factor, \( J \) integral and CTOD, \( E \) is the Young’s modulus, \( \nu \) is the Poisson’s ratio and \( \sigma_y \) is the yield strength.

In the presence of crack tip plasticity, however the relationship between \( K, J \) and CTOD breaks down and then one can use the following:

\[ K_{mat} = \frac{\sqrt{J_{mat}E}}{(1-\nu^2)} \]  

(3.14)

where \( J_{mat} \) is the \( J \) toughness corresponding to 0.2 mm crack extension, i.e., \( J_{te} \), though arguments are being developed [3.12] to accept the use of critical \( J \) value corresponding to the load at brittle fracture initiation after some stable crack extension.

Based on tests wherein both CTOD and \( J \) were measured, Anderson et al propose [3.12] the following for \( K_{mat} \) - critical CTOD relationship:

\[ K_{mat} = \frac{1.6\sigma_f \delta_{mat} E}{\sqrt{(1-\nu^2)}} \]  

(3.15)

where; \( \sigma_f \) is the flow stress, and \( \delta_{mat} \) is the critical CTOD for \( c, u, \) or \( m \) type fracture behaviour. It is presumed that any error caused by the use of these critical values instead of \( \delta_i \) is small and acceptable.

In the present Guide, following Anderson, it is recommended that \( K_{mat} \) be estimated from critical CTOD value using Equation 3.15. In comparison, PD 6493 recommends that if the fracture toughness is available as critical CTOD, it should not be converted to \( K_{mat} \). Instead, the driving force is to be computed in CTOD terms (\( \delta_{app} \)) using the equation:

\[ \delta_{app} = \frac{K_{app}^2}{\sigma_f E} \]  

(3.16)

Implicit in the above expression is a following relationship between \( K_{mat} \) and \( \delta_{mat} \):
Equation 3.17 provides a more conservative estimate of the fracture toughness to be used in the analysis. Anderson however takes issue with this approach because when material toughness data is available both as CTOD and J, their respective use will lead to different answers. The Anderson approach (Equation 3.15) recommended in the Guide provides similar answers when the fracture toughness data used is as CTOD or J from the same test. While Anderson’s approach recommended in this Guide is less conservative than the PD 6493 approach, his validation studies based on analysis of numerous wide plate tests has indicated that when a lower bound toughness is used (based on a relationship similar to that in Figure 3.8(a) but a quasi-static loading rate), the predictions with respect to the non-specific material FAD are still safe.

Finally, as mentioned earlier, the use of fracture toughness value, $K_{mat}$, derived from CVN toughness leads to overly conservative assessment because the $K_{mat}$ - CVN toughness correlation is based on data from thick steels representing plane strain conditions and includes fracture toughness data at dynamic loading rates as well as from crack arrest toughness tests. Therefore, while this method of estimating the $K_{mat}$ value may be fast and convenient, it is the least preferred as well.
3.7 REFERENCES


[3.13] Personal Communication between Dr. H. Reemsnyder (Bethlehem Steel) and Mr. Peter Noble (ABS).


4.0 ASSESSMENT OF FATIGUE CRACK GROWTH

This section describes a two level procedure based on linear elastic fracture mechanics for predicting fatigue crack growth in ship structures from an assumed initial crack or from flaws detected in service or during fabrication by non-destructive evaluation. Guidance is given on the preparation of inputs for the procedure, the execution of the procedure and the use of the results to establish safe and efficient schedules for inspection and repairs.

4.1 Background

4.1.1 Characterization of Fatigue Crack Growth by Linear Elastic Fracture Mechanics

The resistance of a metal to fatigue crack propagation is normally characterized by a log-log plot of crack growth rate ($da/dN$) under tensile loading (Mode I) versus the range of the crack tip stress intensity factor ($\Delta K$). Crack growth rates for such plots are extracted from discrete measurements of crack length during fatigue tests of standard specimens with through-thickness edge cracks or center cracks subjected to Mode I constant amplitude loading, and the corresponding stress intensity factor ranges are calculated by linear elastic fracture mechanics. Although fatigue cracks can also propagate by an in-plane shearing mechanism (Mode II) or an out-of-plane tearing mechanism (Mode III), Mode I cracking usually predominates in engineering structures.

The correlation of $da/dN$ against $\Delta K$ assumes that identical stress-strain fields exist at the tips of different cracks regardless of crack size, crack shape, applied loads, and structural geometry if the crack tip stress intensity factor, material, R-ratio, and environment remain the same. The crack tip stress intensity factor quantifies the severity of the asymptotic stress-strain field at a crack tip in linear elastic material (i.e., $K$-field), and $\Delta K$ is generally defined in the following manner:

$$\Delta K = Y\Delta\sigma\sqrt{\pi a}$$  (4.1)

where; $a$ is a measure of crack size, $\Delta\sigma$ is the tensile portion of the range of the applied stress (e.g., hot spot stress, nominal stress, or local nominal stress) plus total residual stress (due to welding and fabrication) over a load cycle, and $Y$ is a dimensionless factor that depends on the geometry of a crack, the location along a crack front, and the geometry and loading of a structure. The corresponding R-ratio is the ratio of the minimum stress to maximum stress (applied stress plus residual stress) around a crack over a load cycle. Although a plastic zone inevitably develops at a crack tip in ductile materials subjected to cyclic loading, similitude is maintained if the plastic zone is small compared to crack size and surrounded by the elastic $K$-field. These conditions are usually satisfied in high cycle fatigue problems.
A typical log-log plot of has a sigmoidal shape which can be divided into three regions (Figures 4.1 and 4.2):

**Region I -** Crack growth in Region I (<$10^{-5}$ mm/cycle) can be strongly influenced by microstructure and R-ratio. These rates diminish rapidly with decreasing $\Delta K$, and fatigue cracks are assumed to be non-propagating below a threshold value of the stress intensity factor range ($\Delta K_{th}$) which is usually defined at a growth rate of $10^{-8}$ mm/cycle to $10^{-7}$ mm/cycle.

**Region II -** Crack growth in Region II is characterized by a nearly linear relationship between log $da/dN$ and log $\Delta K$. This relationship is usually approximated by the following power relationship, which is often referred to as the Paris equation [4.1],

$$\frac{da}{dN} = C\Delta K^m$$  \hspace{1cm} (4.2)

where $C$ and $m$ are empirical constants. Crack growth rates in Region II ($10^{-5}$ mm/cycle to $10^{-3}$ mm/cycle) are less sensitive to microstructure and R-ratio than crack growth rates in Region I.

**Region III -** Crack growth rates in Region III increase asymptotically with increasing $\Delta K$. This acceleration of crack growth is related to the emergence of static failure modes such as fracture, ductile tearing, and plastic collapse, and it is accompanied by an increased sensitivity of crack growth rates to microstructure and R-ratio.

### 4.1.2 Prediction of Crack Propagation Under Constant Amplitude Loading

The following equation generalizes the relationship between $da/dN$ and $\Delta K$ under constant amplitude loading for a given material, R-ratio, and environment:

$$\frac{da}{dN} = f(\Delta K)$$  \hspace{1cm} (4.3)

If the variation of $\Delta K$ with crack size for an idealized two-dimensional edge crack or center crack is known, then the number of load cycles to propagate the crack from an initial length $a_i$ to a final crack length $a_f$ can be determined by integrating equation in the following manner:

$$\Delta N = \int_{a_i}^{a_f} \frac{da}{f(\Delta K)}$$  \hspace{1cm} (4.4)
Figure 4.1: Log-log plot of da/dN vs ΔK data.

Figure 4.2: Basic regions of da/dN vs ΔK curve.
Conversely, the incremental crack growth from an initial number of constant amplitude stress cycles $N_1$ to a final number of constant amplitude load cycles $N_f$ can be determined by integrating the equation in the following manner:

$$\Delta a = \int_{N_1}^{N_f} (\Delta K) dN$$  \hspace{1cm} (4.5)$$

A number of empirical equations are available to describe the entire sigmoidal relationship between $da/dN$ and $\Delta K$. This relationship can also be described piecewise by a series of linear segments. However, for many practical applications, it is sufficiently accurate to fit the Paris equation to all values of $\Delta K$ from $\Delta K_{th}$ up to failure:

$$\frac{da}{dN} = C(\Delta K)^m \quad \text{for } \Delta K > \Delta K_{th}$$  \hspace{1cm} (4.6)$$

$$\frac{da}{dN} = 0 \quad \text{for } \Delta K \leq \Delta K_{th}$$

In actual engineering structures, edge cracks and through-thickness cracks usually have an irregular or curved crack front. Furthermore, surface cracks and embedded cracks with smooth and irregular curved crack fronts are frequently encountered in such structures. As discussed in Section 4.1.1, $\Delta K$ depends on crack size as well as crack shape, and crack shape development can have a significant influence on crack growth rates and accumulated crack growth. In principle, changes in crack shape as well as crack size could be tracked by predicting the incremental crack growth at various locations along the crack front. However, such an approach is time-consuming and impractical. Usually, an embedded flaw is idealized as an elliptic crack, and crack growth is only predicted along the major and minor axes of the idealized flaw. Similarly, a surface flaw is idealized as a semi-elliptic crack, and crack growth is only predicted at the deepest point and surface. It is also customary to idealize a through-thickness edge crack or center crack as a straight-fronted crack and to only predict the average growth along the actual crack front.
4.1.3 Prediction of Crack Propagation Under Variable Amplitude Loading

Most engineering structures, including ships, are subjected to variable amplitude loading rather than constant amplitude loading. Variable amplitude loading can complicate the prediction of fatigue crack growth in several ways:

1. Interaction effects between load cycles of different amplitude can produce temporary departures from \( \frac{da}{dN} \) versus \( AK \) data for constant amplitude loading. In particular, an over-shooting load spike can retard subsequent crack growth, and to a lesser extent, an under-shooting load spike can temporarily accelerate subsequent crack growth. These effects tend to cancel out under narrow-banded stationary random loading and under certain types of stationary and non-stationary broad-banded random loading, but they can have a significant influence on accumulated crack growth if there are long sequences of load cycles between one-sided load spikes (Figure 4.3).

2. The value of \( AK \) for a given load cycle varies with crack size and shape. In addition, a load cycle that is too small to propagate a small crack (i.e., \( AK < \Delta K_{th} \)) may be large enough to significantly propagate a larger crack (i.e., \( AK > \Delta K_{th} \)). As a result, the crack growth produced by each load cycle in a load-time history and the total crack growth over a given number of load cycles can depend on the sequence of the load cycles even if interaction effects are negligible.

3. Individual load cycles in certain load-time histories (e.g., broad-banded random histories) are difficult to define and counting methods such as rainflow and reservoir techniques are needed to decompose such histories into individual load cycles (Figure 4.4).

4. There is no unique load-time history for forecasting fatigue crack growth under random and pseudo-random loading.

In principle then, a realistic sequence of properly counted load cycles and a crack growth model that accounts for interaction effects between load cycles are needed to predict fatigue crack growth under variable amplitude loading. Furthermore, probabilistic simulation methods and/or calibrated standard load-time histories are required for forecasting fatigue crack growth under random and pseudo-random loading. In practice, however, a rigorous approach is not always needed. A few examples are listed below:

1. If the numbers of load cycles between one-sided spikes in a load history are short, then interaction effects following the spikes have a negligible effect on the accumulated crack growth because the spikes are directly responsible for most of the accumulated crack growth.
Figure 4.3: Effects of different overload patterns on fatigue crack growth in 7075-T6 aluminum [4.17].
Figure 4.4: Random load versus time histories (a) narrow banded (b) broad banded.
2. Retardation and acceleration effects tend to cancel out under narrow-banded stationary random loading and under certain types of stationary and non-stationary broad-banded random loading.

3. The total crack growth over a given number of load cycles is independent of load sequence if interaction effects are negligible and if the $\Delta K$ value for each load cycle exceeds $\Delta K_{th}$. Under these conditions, the total crack growth over a given number of variable amplitude load cycles can be predicted by a cycle by cycle integration of the Paris equation over an arbitrary sequence of the load cycles and their corresponding $\Delta K$ values. Alternatively, a weighted average of the stress ranges ($\Delta \sigma_{eq}$) associated with the load cycles in the variable amplitude load history can be used to calculate an equivalent stress intensity factor range ($\Delta K_{eq}$) for cycle by cycle integration with the Paris equation [4.2]:

$$\Delta K_{eq} = Y \Delta \sigma_{eq} \sqrt{\pi a}$$  \hspace{1cm} (4.9)$$

$$\Delta \sigma_{eq} = \left[ \sum_{j=1}^{N} \frac{\Delta \sigma_{ij} n_j}{N_T} \right]^{1/m}$$  \hspace{1cm} (4.10)$$

where; $n_j$ is the number of cycles of magnitude $\Delta \sigma_{ij}$ in the random history, $m$ is the material exponent in the Paris equation, and $N_T$ is the total number of cycles.

4.2 Application to Ship Structures

As evident in Figure 4.5, the variation of stresses at a given point in a ship can be described as a broad-banded non-stationary pseudo-random process. Broad-banded means that the frequency content is wide, and non-stationary means that the statistics of the process do not remain constant. Pseudo-random means that there are deterministic as well as random cyclic stress components. Random components arise from wave-induced bending and torsion of the ship's hull, fluctuations of the external pressure on shell plating, fluctuations of the internal pressure on tank and cargo boundaries as a result of wave-induced motions, and wave-induced dynamic effects such as springing, slamming, and whipping. Deterministic components, on the other hand, arise from thermal effects, changes in still water bending moment as a result of changes in cargo and ballast conditions, seasonal variations in sea states, changes in heading to avoid rough seas, and reductions in speed to minimize slamming.

Guide to Damage Tolerance Analysis of Marine Structures
Figure 4.5: Variation of midship stresses versus time in SS R.G. Follis [4.18].
The prediction of fatigue crack growth in engineering structures is the subject of on-going research, and further work is needed in the following areas before rigorous methods are available for the prediction of fatigue crack growth in ships:

1. Wave-induced loads are responsible for the majority of stress cycles experienced by the hull of a ship over its operational life, and considerable attention has been given to quantifying the statistical distributions of wave-induced cyclic stresses over the short term and long term. In contrast, much less attention has been given to understanding and quantifying the sequence of cyclic stresses in ship structures over time. Realistic sequences of these stresses cannot be re-constructed from the short-term and long-term statistical distributions of wave-induced cyclic stresses by probabilistic simulation methods without an understanding of the deterministic nature of these stresses (e.g., a large peak is generally followed by a large trough, the build up and decay of sea states is gradual rather than random). In addition, little is known about the significance and nature of other cyclic stresses in ship structures. For example, changes in still water bending moment can cause relatively large changes in stresses at certain locations in a ship that could retard or accelerate subsequent crack growth. These stress cycles are infrequent and make little direct contribution to the total fatigue damage and accumulated crack growth but the associated retardation effects could have a significant effect on crack growth.

2. Interaction effects under variable amplitude loading are generally attributed to cycle by cycle variations of residual stresses and crack closure at the crack tip. The complexities of these effects have so far precluded a complete theoretical treatment of the problem. Several variants of the Paris equation (e.g., Wheeler and Willenborg models) have been successfully used to model interaction effects, but these empirical models have been calibrated with fatigue crack growth data for specific types of variable amplitude loading and material. In principle, these models could be adapted to ship structures but they would have to be re-calibrated against data for fatigue crack growth in ships and representative variable amplitude loading for such structures.

3. As fatigue cracks propagate away from their initiation sites, load is continuously shed from the damaged areas to surrounding material. The inherent redundancy of ship structures enables them to tolerate a considerable amount of load shedding. Unfortunately, users of this guide cannot fully exploit this redundancy at this time. Stress intensity factor solutions for cracks in basic welded details (handbook solutions or direct calculation by analytical or numerical methods) only account for load shedding around small cracks in ship structures, and little is known about load shedding around large cracks in ship structures. There are many possible paths for the propagation of large fatigue cracks in ships, and a number of large finite element models would be required to quantify load shedding along any given path.
Until further advancements are made in the above areas, procedures for predicting fatigue crack growth in ships structures should be consistent with the sophistication and assumptions of fatigue design procedures recently introduced by classification societies. These procedures only consider wave-induced cyclic stresses and ignore other cyclic stresses in ships. It is assumed that the short-term variation of wave height is a narrow-banded stationary process, and that the structural response is linear. These assumptions enable the short-term distributions of stress range for all possible sea states over a specific voyage route or over the operational life of a ship to be generated by spectral methods. A long-term distribution of stress ranges over a specific voyage route or over the life of a ship is then built-up from the weighted sum of the short-term distributions. Alternatively, the long-term distribution is assumed to be a Weibull distribution characterized by an assumed shape factor and a reference stress range corresponding to a specific probability of exceedance. The long-term distribution of stress ranges is then used in conjunction with fatigue design curves to predict the initiation of relatively large fatigue cracks. These calculations ignore load shedding and interaction effects. It is assumed that the interaction effects are mitigated by the narrow-banded nature of short-term sea states and by the gradual build up and decay of sea states. The calculations also assume that fatigue cracks continue to propagate during the compressive portions of applied stress cycles because of the presence of tensile residual stresses.

The following procedure is, therefore, recommended for predicting fatigue crack growth in ship structures for the purpose of establishing inspection and maintenance schedules or assessing the fitness-for-service of detected flaws.

1. Define the size and shape of an initial crack. See Section 4.3 for guidance on assuming an idealized initial crack at the design stage, idealizing the size and shape of a crack detected in service by non-destructive evaluation, and selecting the points along the idealized crack front where crack growth will be simulated.

2. Define the statistical distribution of the appropriate stress range (e.g., hot spot stress range, nominal stress range, or local nominal stress range) for stress intensity factor calculations over the interval of interest (e.g., inspection period, voyage route). See Section 6 for guidance on identifying the appropriate stress range, calculating this stress range from applied loads, and estimating the statistical distribution of this stress range.

3. Approximate the statistical distribution of the applied stress range with a histogram consisting of 10 to 20 levels. Assume that the stress range in each level is constant and equal to the maximum value of the range.

4. Arrange the blocks of stress ranges in the histogram into at least three different sequences including: high-to-low, low-to high, low-high-low. For each stress history, carry out Steps 5 to 9.
5. Calculate the stress intensity factor range $\Delta K$ for the first applied stress range in a particular stress history at the simulation points along the idealized crack front. For Level 1 analysis, assume that the through-thickness stress distribution is uniform and equal to the magnitude of the maximum peak stress (See Section 6). For Level 2 analysis, use the actual through-thickness stress distribution. The Level 1 crack growth analysis is consistent the Level 1 residual strength assessment described in Section 3, whereas the Level 2 crack growth analysis is consistent with the Level 2a, 2b and 2c residual strength assessments described in Section 3.

6. Calculate the corresponding increments of crack growth $\Delta a$ by integrating the Paris equation over the stress range assuming that the crack growth rate is constant over the stress cycle and equal to the crack growth rate at the beginning of the stress cycle.

$$\Delta a = C(\Delta K)^m \quad \text{for} \quad \Delta K > \Delta K_{th}$$

$$\Delta a = 0 \quad \text{for} \quad \Delta K \leq \Delta K_{th}$$

See Section 4.4 for guidance on generating $C$, $m$, and $\Delta K_{th}$ values for a specific steel. Upper bound values for steels are also given in Section 4.4.

7. Update crack size and crack shape.

8. Check to see whether crack has reached a critical size. See Section 3 for guidance on residual strength assessment.

9. Repeat Steps 6 to 9 for subsequent stress ranges in a given stress history.

The results for each stress history and tracking location along a crack front can be used to construct a curve or table of the accumulated crack growth versus the number of applied stress ranges. In general, the crack growth that accumulates up to any given point of the stress history will depend on the sequence of the applied stress ranges. Therefore, the worst predicted case should be used to establish inspection and maintenance schedules or to assess the fitness-for-service of detected cracks.

Note: If the value of $\Delta K$ for every applied stress range in a sequence exceeds $\Delta K_{th}$, the total crack growth would be independent of the sequence of applied stress ranges. Furthermore, the same total crack growth would be predicted by the weighted average stress range approach described in Section 4.1.

The cycle-by-cycle integration procedure can be easily implemented on a personal computer either on a spreadsheet or as a stand-alone program if the variation of the crack tip stress intensity factor with crack size and shape is parametrically defined. If stress intensity factor solutions are only available in tabular or graphical form or if users of this
guide do not have access to computing resources, then a manual assessment can be performed with the following block integration procedure:

1. Divide the stress history into blocks of stress ranges of the same magnitude, and carry out Steps 2 and 3 for each block in turn.

2. Calculate $\Delta K$ using the crack size, crack shape, and stress range at the beginning of the block.

3. Calculate the crack growth increment ($\Delta a_B$) at each simulation point along a crack front over the number of stress ranges in the block ($\Delta N_B$) assuming that the crack growth rate is constant and equal to the crack growth rate at the beginning the block.

$$
\Delta a_B = C (\Delta K)^m \Delta N_B \quad \text{for } \Delta K > \Delta K_{th}
$$

$$
\Delta a_B = 0 \quad \text{for } \Delta K \leq \Delta K_{th}
$$

(4.9)

Although the approach is inherently non-conservative, results will be close to that obtained by cycle-by-cycle integration if the block size is relatively short (up to 0.1% of the total fatigue life obtained, or the increment of crack growth does not exceed 0.5% of the crack depth at the start of a block).

4.3 Flaw Characterization

4.3.1 Idealization of Detected Flaws

Flaws that are detected in service by non-destructive evaluation may be planar or volumetric. The fracture mechanics procedure described in Section 4.2 is inherently conservative for volumetric flaws and planar flaws that are not cracks because it does not account for the cyclic loading required to initiate fatigue cracks from such flaws.

The fracture mechanics procedure described in Section 4.2 only considers fatigue crack growth under Mode I loading. However, detected flaws are often inclined with respect to the principal stress direction, and fatigue cracks that originate at such flaws may initially propagate under a mixture of Mode I, Mode II, and Mode III loading. Although it is possible to incorporate mixed-mode loading into fatigue crack growth calculations, fatigue cracks tend to curve towards a trajectory that is perpendicular to the principal stress direction. It is simpler albeit conservative to project the detected flaws onto a plane normal to the principal stress direction and to treat the projected flaws as cracks subjected to Mode I loading.
The shapes of detected flaws are often irregular, and a number of closely spaced flaws may be detected. As discussed in Section 4.1.1, crack shape development can have a significant influence on crack growth rates and accumulated crack growth. In principle, the growth of multiple cracks could be simulated simultaneously with stress intensity factors that account for interaction effects between closely spaced cracks, and the shape of individual cracks could be tracked by predicting the incremental crack growth at various locations along the crack front. However, such an approach is time-consuming and impractical. In order to minimize the number of simulations and to simplify stress intensity factor calculations, it is necessary to idealize detected flaws in the following manner:

1. Idealize the projected profiles of surface, embedded, and through-thickness flaws as semi-elliptic, elliptic, and straight-fronted cracks, respectively.

2. Re-characterize closely spaced cracks as a single crack.

3. Assume that the shape of an idealized crack develops in a self-similar manner so that crack growth only needs to be tracked at the major and minor axes of an elliptic crack front, the deepest point and one of the two surface points of a semi-elliptic crack front, or a single point along a straight through-thickness crack front.

4. When an elliptic embedded crack breaks through the top or bottom surface of a plate, re-characterize the elliptic crack as a surface crack for subsequent crack growth calculations. Similarly, when a semi-elliptic surface crack breaks through the top or bottom surface of a plate, re-characterize the surface crack as a straight-fronted through-thickness crack for subsequent crack growth calculations.

Conservative circumscription methods for idealizing the shape of projected flaws and the re-characterization of idealized cracks during crack growth predictions are given in Appendix D.

4.3.2 Assumed Initial Crack

Fatigue cracks in ship structures with properly designed and fabricated welds generally initiate along the toe of a transverse fillet weld or transverse butt weld, usually along the hot spot region of the weld toe where structural stresses (i.e., total stresses minus the stress concentration effect of local weld geometry) are highest. Within this region, multiple surface cracks initiate at microscopic stress raisers such as slag intrusions and macroscopic stress raisers such as weld ripples and undercuts. These cracks coalesce as they propagate through the thickness of a plate, and a dominant crack usually emerges before the fatigue cracking is detected. The spacing and number of crack initiation sites along a weld toe depend on local stresses and welding process. These factors, in turn, influence the size and shape of fatigue cracks during the crack coalescence stage.
Since the fracture mechanics procedure described in Section 4.2 does not explicitly consider fatigue crack initiation, the size, shape, and location of one or more initial cracks must be assumed for fatigue crack growth predictions at the design stage. If statistical information about the size, shape, number, and spacing of initial fatigue cracks along a weld toe are available, then Monte Carlo simulation methods can be used to define an initial array of fatigue cracks along the weld toe. The subsequent growth of these cracks can be simulated simultaneously, and the re-characterization criteria given in Appendix D can be used to conservatively model the coalescence of adjacent cracks. Alternatively, a mean relationship between the aspect ratio and depth of surface cracks along a weld toe can be constructed from experimental observations of crack shape development. This empirical relationship can be used as a forcing function to prescribe the shape of a single surface crack as the growth at the deepest point of the crack is simulated.

In practice, relevant forcing functions and statistical information about crack initiation will rarely be available to users of this guide. In this event, users should assume that a semi-elliptic crack of depth \(a_i\) extends across the length of the hot spot region. If the hot spot region extends across the full width of plate, then the surface crack should be re-characterized as an edge crack of constant depth \(a_i\). For comparative assessment of welded joints failing from the weld toe, it is often assumed that \(a_i\) lies within the range 0.1 mm to 0.25 mm unless a larger size is known to be relevant. Researchers have found that predicted fatigue lives based on these initial sizes are comparable to the experimental fatigue lives of laboratory specimens. For establishing inspection intervals at the design stage, however, \(a_i\) should be taken as the minimum defect size that can be reliably detected using the relevant NDT technique. The reliability of the inspection technique should be taken into account in the determination of this minimum defect size. This entails the use of probability of detection (POD) curves for a given confidence. A 90% POD with a 95% confidence limits has been found to be appropriate in most cases [4.3].

### 4.4 Fatigue Crack Growth Data

#### 4.4.1 Specific Data

The fracture mechanics procedure described in Section 4.2 assumes that the Paris equation uniquely characterizes the relationship between \(\frac{da}{dN}\) and \(\Delta K\) for all values of \(\Delta K\) above a threshold value \(\Delta K_{th}\), and that fatigue cracks do not propagate at \(\Delta K\) values less than \(\Delta K_{th}\):

\[
\frac{da}{dN} = C(\Delta K)^m \quad \text{for} \quad \Delta K > \Delta K_{th}
\]

\[
\frac{da}{dN} = 0 \quad \text{for} \quad \Delta K \leq \Delta K_{th}
\]

(4.10)
Whenever possible, specific values of $C$, $m$, and $\Delta K_{th}$ for the relevant combination of material, direction of crack growth, environment, $R$-ratio ($\sigma_{min}/\sigma_{max}$), and frequency of cyclic loading should be used in fatigue crack growth predictions, and the chosen values should include a sufficient factor of safety to account for the variability of fatigue crack growth data. If there is any doubt about the applicability of available values in the open literature, then specific $da/dN$ versus $\Delta K$ data should be produced in accordance with a relevant test standard such as ASTM E647 [4.4] or BS 6835 [4.5].

As discussed in Section 4.1.1, $da/dN$ versus $\Delta K$ data is generated from discrete measurements of crack length during fatigue tests of standard specimens with through-thickness edge cracks or center cracks subjected to Mode I constant amplitude loading. Moving average techniques are used to extract crack growth rates from these measurements, and the corresponding $\Delta K$ values are calculated by linear elastic fracture mechanics. It is customary to fit a least squares regression line through $\log da/dN$ versus $\log \Delta K$ data for Region II crack growth and to report the corresponding $C$ and $m$ values. It is also customary to define an operational value of $\Delta K_{th}$ by fitting a least squares regression line through $\log da/dN$ versus $\log \Delta K$ data for Region I crack and by extrapolating the fitted line to the smallest detectable crack growth rate (typically $10^{-10}$ m/cycle). These values characterize the mean fatigue crack growth behaviour of a test sample, and are usually the values reported in the open literature. Although they can be used as inputs for relative fatigue crack growth assessment, more conservative values are generally required for absolute fatigue crack growth assessments to account for measurement errors, local variations of material properties within a batch of material, and general variations of material properties between different batches of material. If the test specimens and structure being analyzed are fabricated from the same batch of material, then absolute fatigue crack growth assessments should be based on $C$ and $\Delta K_{th}$ values that correspond to the mean values of $\log da/dN$ in the test sample plus two standard deviations. If it is not possible to fabricate the test specimens from the same batch of material as the structure being analysed, then the test specimens should be fabricated from several other batches of material to account for the variability of material properties between different batches of material.

4.4.2 $C$ and $m$ Values for Region II Crack Growth in Steels in Air

Although Region II crack growth rates for steels in air tend to increase with increasing $R$-ratio, this dependency is small compared to the dependency of Region I crack growth on $R$-ratio and it is usually ignored in comparisons of Region II crack growth rates for different steels. For example, Rolfe and Barsom [4.6] compiled $da/dN$ versus $\Delta K$ data for Region II crack growth in a wide range of steels tested at various $R$-ratios, and divided this data into three groups according to microstructural differences (viz., martensitic, ferritic-pearlitic, or austenitic). They found that most of the measured crack growth rates within each group varied by less than a factor of two at any given $\Delta K$ value. Considering the wide range of mechanical properties and chemical compositions represented within each group, Rolfe and Barsom suggested that engineering estimates of crack growth rates in
martensitic, austenitic, and ferritic-pearlitic steels could be obtained from the following upper bound relationships:

martensitic steels

\[
\frac{da}{dN} = 1.2 \times 10^{-0} \Delta K^{1.25}
\]  (4.11)

ferritic-pearlitic steels

\[
\frac{da}{dN} = 4.92 \times 10^{-13} \Delta K^{3}
\]  (4.12)

austenitic steels

\[
\frac{da}{dN} = 1.73 \times 10^{-13} \Delta K^{3.25}
\]  (4.13)

Most investigations of fatigue crack growth in steels have not been accompanied by fractographic examinations of fatigue crack growth mechanisms. The few studies [6.18] that have involved such examinations have shown that Region II fatigue crack growth in a wide range of microstructures occurs by a transgranular striation mechanism, and that crack growth rates associated with this mechanism fall within a common scatter band regardless of R-ratio and tensile strength. Departures from the striation mechanism (e.g., microcleavage in coarse pearlitic steels and steels with brittle second phase particles such as spheroidized carbides, intergranular cracking in tempered martensite tested at low \(\Delta K\), void coalescence in tempered martensitic steels tested at high \(\Delta K\)) are invariably associated with higher crack growth rates that tend to increase with increasing tensile strength and R-ratio.

The BS 7608 [4.7] and PD6493 [4.8] recommend, in the absence of specific data, the following relationship for engineering assessments of fatigue crack growth in ferritic structural steels (including plain plate, weld metal, and heat affected zone metal with yield strengths below 600 MPa) operating in air at temperatures up to 100°C:

\[
\frac{da}{dN} = 3.0 \times 10^{-13} \Delta K^{3}
\]  striation  (4.14)

This relationship represents an upper bound on published \(\frac{da}{dN}\) versus \(\Delta K\) data for crack growth by a striation mechanism. If there is a potential for crack growth by non-striation mechanisms (e.g., certain weld metals and heat affected zones as \(K_{\text{max}}\) approaches its critical value), then both references recommend the following equation:

\[\text{\footnotesize Units for } \frac{da}{dN} \text{ and } \Delta K \text{ are mm/cycle and MPa\text{\text-s}/mm respectively.}\]
The former equation may be overly conservative for certain steels since the rate of crack growth rate by a striation mechanism in different steels can vary by as much as factor of five for a given $\Delta K$ value while the latter equation should be used with caution since it is less conservative than Rolfe and Barsom's upper bound relationship for martensitic steels.

Note: Equations 4.14 and 4.15 correspond to the mean of log $\frac{da}{dN}$ plus two standard deviations for pooled data.

4.4.3 $\Delta K_{th}$ values for Steels In Air

$\Delta K_{th}$ values for steels are essentially independent of R-ratio for R-ratios less than 0.1, but tend to decrease with increasing R-ratio for R-ratios above 0.1. Several investigators [4.9, 4.10] have compiled $\Delta K_{th}$ values for a wide range of steels in air, and Rolfe and Barsom [4.6] have found that the following equations

\[
\Delta K_{th} = 190 \text{ MPa} \cdot \text{mm} \quad \text{for } R < 0.1 \tag{4.16}
\]

\[
\Delta K_{th} = 221(1 - 0.85R) \text{ MPa} \cdot \text{mm} \quad \text{for } R \geq 0.1 \tag{4.17}
\]

define a reasonable lower bound on this data (Figure 6.2). The range of compiled $\Delta K_{th}$ values for a given R-ratio is nearly 300 MPa\cdot mm at R-ratios less than 0.1, but narrows with increasing R-ratio to about 60 MPa\cdot mm at an R-ratio of 0.9. The greater range of $\Delta K_{th}$ values at low R-ratios appears to be related to the strong influence of microstructure on $\Delta K_{th}$ for some steels loaded at low R-ratios. In particular, Taylor [4.11] and Ritchie [4.12] have noted that $\Delta K_{th}$ values for martensitic steels, bainitic steels, and ferritic-pearlitic steels with high ferrite content decrease significantly with increasing yield strength at low R-ratios whereas $\Delta K_{th}$ values for ferritic-pearlitic steels with high pearlite content are relatively insensitive to yield strength. In addition, several investigators [4.12] reported a marked increase in $\Delta K_{th}$ for various low strength ferritic-pearlitic steels loaded at low R-ratios when ferrite grain size was increased, while other investigators [4.11] found little effect of prior austenite grain size on $\Delta K_{th}$ values for martensitic and bainitic high strength steels loaded at low R-ratios.
4.4.4 C, m, and $\Delta K_{th}$ Values for Fatigue Crack Growth in Steels in a Marine Environment

Unprotected areas of steel marine structures are prone to general corrosion as a result of exposure to sea water. Wastage can lead to higher stresses as a result of reductions in net section and load re-distribution away from severely corroded structure, and gross corrosion pitting can introduce significant stress concentrations in plating. In addition to these factors, which effectively increase the driving force for fatigue crack propagation, the resistance of steels to fatigue crack propagation can be reduced by various corrosion fatigue mechanisms.

Various experimental studies have shown that the fatigue crack growth resistance of freely corroding steels in sea water differs from that in air [4.13, 4.14]. Fatigue crack growth rates under free corrosion conditions approach those in air at high $\Delta K$ values (> 1500 MPa$^\text{mm}$) and as $\Delta K$ approaches $\Delta K_{th}$ (< 300 MPa$^\text{mm}$). At intermediate $\Delta K$ values, however, crack growth rates under free corrosion conditions are higher than those in air and can be characterized by a bi-linear relationship between log $da/dN$ and log $\Delta K$. The difference between crack growth rates in air and under free conditions is highest at the knee of the bi-linear relationship and increases with decreasing loading frequency, increasing temperature, and increasing oxygen content. For example, it has been observed that growth rates under free corrosion conditions in 0°C sea water are only marginally higher than crack growth rates in air at room temperature, whereas crack growth rates under free corrosion conditions in sea water at room temperature are 3 to 4 times faster than those in air at room temperature. This acceleration of crack growth has been attributed to anodic dissolution at the crack tip which is enhanced by higher temperatures, lower loading frequency, and higher oxygen content. It is also believed that the diffusion of hydrogen to the crack tip contributes to the acceleration of crack growth, but it is not clear whether this is through an embrittlement mechanism or through some other form of hydrogen-assisted cracking. It is also worth noting that the knee of the bi-linear relationship occurs at a higher $\Delta K$ value with decreasing frequency. Furthermore, crack growth rates above this knee increase with decreasing frequency. In contrast, crack growth rates seem to be independent of frequency although there is relatively little data on frequency effects in this regime (Region I).

Cathodic protection is used to control the general corrosion process in steel marine structures. Although it is believed that cathodic protection helps to nullify the anodic dissolution process at a crack tip as well, experimental studies indicate that cathodic protection does not completely restore fatigue crack growth rates in steels to their in-air values [4.14 - 4.16]. In Region I, cathodic protection reduces crack growth rates in sea water below crack growth rates in air and increases $\Delta K_{th}$ values in sea water above $\Delta K_{th}$ values in air. Increasing the negativity of impressed potentials increases $\Delta K_{th}$ and decreases crack growth rates. These beneficial effects of cathodic protection have been attributed to the precipitation of calcareous deposits which wedge the crack closed at $\Delta K$ values near $\Delta K_{th}$. In Region II, crack growth approaches a plateau of constant rate. Above this plateau,
growth rates approach in-air values. Crack growth rates along this plateau increase with increasing impressed potential, decreasing loading frequency, and increasing R-ratio. Impressed potentials of -0.7V to -0.8V (Ag/AgCl) have been found to reduce fatigue crack growth rates in sea water close to air values, whereas highly negative impressed potentials (-1.1 V) have been found to elevate crack growth rates in sea water above growth rates under free corrosion conditions. It is believed that the more negative potentials increase the amount of hydrogen available for adsorption and diffusion to the crack tip and, therefore, promotes hydrogen-assisted cracking.

Recommendations of $\frac{da}{dN}$ versus $\Delta K$ relationships for engineering predictions of fatigue crack propagation in steels in a marine environment have been complicated by the sensitivity of crack growth rates to impressed potential, loading frequency, R-ratio, and the complex relationship between $\frac{da}{dN}$ versus $\Delta K$. In the absence of specific corrosion fatigue data, PD6493 [4.8] recommends the following relationships$^2$ for engineering assessments of fatigue crack growth in ferritic structural steels in a marine environment:

$$\frac{da}{dN} = 2.3 \times 10^{-12} \Delta K^3 \quad \text{for } \Delta K > \Delta K_{th} \quad (4.18)$$

$$\frac{da}{dN} = 0 \quad \text{for } \Delta K \leq \Delta K_{th} \quad (4.19)$$

$$\Delta K_{th} = 63 \quad \text{for } R > 0.5 \quad (4.20)$$

$$\Delta K_{th} = 170 - 214R \quad \text{for } 0 < R < 0.5 \quad (4.21)$$

$$\Delta K_{th} = 170 \quad \text{for } R < 0 \quad (4.22)$$

These equations, which are similar to relationships recommended by reference [4.2], define an upper bound on crack growth rates over a wide range of $\Delta K$ values in structural ferritic steels that are loaded at high R-ratios and cathodically protected at highly negative impressed potentials. Although Equation 4.18 does not clear all of the experimental data for cathodically protected steels in the plateau region, the value of $C$ in this equation has been chosen to ensure conservative fatigue life predictions for steels loaded at high R-ratios and cathodically protected at highly negative impressed potentials.

$^2$ Units for $da/dN$ and $\Delta K$ are mm/cycle and MPa√mm respectively.
4.4.5 Effect of Residual Stresses

A fatigue crack will not propagate in a compressive stress field, and the stress intensity factor for such a crack has no physical meaning. Nevertheless, it is generally accepted that fatigue cracks in welded steel structures such as bridges, pipelines, and pressure vessels continue to propagate during the compressive portions of applied stress cycles, and it is common practice to use the complete applied stress range to calculate stress intensity factors for cracks in such structures. It is also common practice to use $C$, $m$, and $\Delta K_{th}$ values for high R-ratios in engineering assessments of fatigue crack growth in such structures, regardless of the applied R-ratio. It is assumed that large tensile residual stresses are initially present along the toes of welded joints in large steel structures such as bridges, pipelines, and pressure vessels. It is also assumed that these residual stresses are only partially re-distributed by local yielding at peak loads (i.e., shake-down) under normal service and that post-shake-down residual stresses are sufficiently high for the weld toe to remain in tension during the compressive portions of applied loading cycles.

Experience has shown that the aforementioned practices are not overly conservative for bridges, pressure vessels, and pipelines, and these practices are recommended for ships structures if the magnitude of residual stresses around a crack is unknown. However, peak loads over the operational life of a ship may be large enough to cause significant changes in the magnitude and sign of initial residual stresses. Furthermore, fabrication residual stresses, which may be tensile or compressive, will be more relevant as fatigue cracks propagate away from weld details in ship structures. If the re-distribution of welding residual stresses or the propagation of a fatigue crack beyond the zone of influence of welding residual stresses occurs early in the operational life of a ship, it may be overly conservative to use the complete applied stress range and crack growth constants for high R-ratios in fatigue crack growth calculations. Unfortunately, there is little quantitative data on the magnitude of residual stresses (due to welding and fabrication) in ship structures over the short term and long term.

4.5 References


5.0 LOAD ESTIMATION

The purpose of this section is to provide guidelines for determining the loads that a ship will experience during the period of interest. In the context of a damage tolerance assessment, this period may vary from a few days up to the life of the ship. Furthermore, within this period, estimates of both the extreme load and the long-term statistical distribution (or spectrum) of load range are required for an assessment.

The loading imposed on ships depends on several parameters, many of which are highly variable. The dominant load on ships arises from waves, the computation of which is problematic. Apart from the randomness associated with the engineering parameters that determine loading, particularly parameters that derive from climatic phenomena, additional parameters associated with the way operators of ships respond to extreme weather have to be quantified either explicitly or implicitly. Quantifying any process with a significant human element is always difficult.

While wave loads are the predominant load effect experienced by ships, any loading that can result in significant stress levels are potentially relevant for damage tolerance assessment. Nevertheless, certain cyclic load types are ignored in this study. Vibration loads are not considered since they tend to be local and generally do not impact the structural integrity of the hull. Also ignored are thermal loads which are difficult to quantify and model. Thermal loads also cycle very slowly and hence their contribution to calculated fatigue damage by linear damage summation is generally small. This observation also applies to the stillwater bending moment. However, when these loads are superimposed on wave loads significant interaction effects on crack growth are possible as discussed earlier in Section 4.2.

Wave loads may impose loads in several different ways. The primary mechanism is through hull girder bending. The loading is cyclic with periods of the order of several seconds. In severe sea states, phenomena such as slamming may occur which result in transient impact loads; the response to this type of load is characterized by frequencies that are considerably higher than those associated with normal wave loading. Waves may also impose significant loads on a more local scale. The primary examples include dynamic pressure loads near the waterline, and ship motion which causes acceleration forces on liquid or solid cargoes and other masses.

The direct calculation of loads on ships is difficult and onerous. It requires consideration of a large number of relevant parameters, many of which are highly variable. The primary source of this variability is the environment. Virtually all environmental phenomena are random in character. In addition, the response of ships to waves is a complex fluid-structure interaction problem which is very difficult to model. Nevertheless, methodologies have been developed for this purpose and have met with a measure of success in terms of predicting response with sufficient accuracy for engineering purposes. However, such methodologies rely on a considerable degree of skill and require sophisticated software tools.
Direct calculation methods may not always be appropriate. In circumstances where resources are unavailable and/or data is limited, for example, in the early stages of design, simpler, less rigorous, methods are more appropriate.

Guidelines for three methods of load computation with varying degrees of complexity are presented below. The methods presented are believed to represent the best currently available in terms of accuracy, practicality and cost effectiveness. The methods are termed Level 1, 2 and 3 in order of increasing sophistication. Section 5.1 opens with a brief discussion of the various load types which may require consideration in determining both the extreme load and also the stress range spectrum. This is followed in Section 5.2 which provides an overview of load estimation methods. Sections 5.3, 5.4 and 5.5 present the Level 3, Level 2 and Level 1 methods respectively. Section 5.6 presents a commentary on wave load estimation methods, and discusses the main assumptions and limitations of the methods presented herein. Additional information in support of the wave loading methodologies presented in this section is provided in Appendix F.

5.1 Load Components

Ships are required to resist a variety of loads. For damage tolerance assessment, the relevant loads can be categorized as global or local. The primary global loads are:

- still water loads
- wave loads
  - low frequency steady-state, response largely rigid-body
  - high frequency steady-state (springing), response largely elastic
  - high frequency transient (wave impact or slamming), response largely elastic

The primary local loads are:

- hydrostatic pressure loads
- pressure loads due to waves
  - low frequency steady-state
  - high frequency transient (wave impact or slamming)
- inertia loads from cargo induced by ship motion
- inertia loads from fluids induced by ship motion (sloshing)

The significance of each type of load depends, among other things, upon the ship type, the payload, structural configuration and location of structure. Tables 5.1 to 5.5 provide guidance in identifying the important loads for a selection of ship types. Additional discussion on the subject is provided by Chen and Shin (1995).
Table 5.1: Highly Loaded Structural Elements - Tankers (Cramer et al., 1995)

<table>
<thead>
<tr>
<th>STRUCTURE MEMBER</th>
<th>STRUCTURAL DETAIL</th>
<th>LOAD TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side-, bottom- and deck plating and longitudinalss</td>
<td>Butt joints, deck openings and attachment to transverse webs, transverse bulkheads and intermediate longitudinal girders</td>
<td>Hull girder bending, stiffener lateral pressure load and support deformation</td>
</tr>
<tr>
<td>Transverse girder and stringer structures</td>
<td>Bracket toes, girder flange butt joints, curved girder flanges, panel knuckles including intersecting transverse girder webs, etc. Single lug slots for panel stiffeners, access and lightening holes</td>
<td>Sea pressure load combined with cargo or ballast pressure load</td>
</tr>
<tr>
<td>Longitudinal girders of deck and bottom structure</td>
<td>Bracket terminations of abutting transverse members (girders, stiffeners)</td>
<td>Hull girder bending, and bending/deformation of longitudinal girder and considered abutting member</td>
</tr>
</tbody>
</table>

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Table 5.2: Highly Loaded Structural Elements - Bulk Carriers (Cramer et al., 1995)

<table>
<thead>
<tr>
<th>STRUCTURE MEMBER</th>
<th>STRUCTURAL DETAIL</th>
<th>LOAD TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hatch corners</td>
<td>Hatch corner</td>
<td>Hull girder bending, hull girder torsional deformation</td>
</tr>
<tr>
<td>Hatch side coaming</td>
<td>Termination of end bracket</td>
<td>Hull girder bending</td>
</tr>
<tr>
<td>Main frames</td>
<td>End bracket terminations, weld main frame web to shell for unsymmetrical main frame profiles</td>
<td>External pressure load, ballast pressure load as applicable</td>
</tr>
<tr>
<td>Longitudinals of hopper tank and top wing tank</td>
<td>Connection to transverse webs and bulkheads</td>
<td>Hull girder bending, sea- and ballast pressure load</td>
</tr>
<tr>
<td>Double bottom longitudinals (1)</td>
<td>Connection to transverse webs and bulkheads</td>
<td>Hull girder bending stress, double bottom bending stress and sea-, cargo- and ballast pressure load</td>
</tr>
<tr>
<td>Transverse webs of double bottom, hopper and top wing tank</td>
<td>Slots for panel stiffener including stiffener connection members, knuckle of inner bottom and sloped hopper side including intersection with girder webs (floors). Single lug slots for panel stiffeners, access and lightening holes</td>
<td>Girder shear force, and bending moment, support force from panel stiffener due to sea-, cargo- and ballast pressure load</td>
</tr>
</tbody>
</table>

(1) The fatigue life of bottom and inner bottom longitudinals of bulk carriers is related to the combined effect of axial stress due to hull girder- and double bottom bending, and due to lateral pressure load from sea or cargo.
Table 5.3: Highly Loaded Structural Elements - Ore Carriers (Cramer et al., 1995)

<table>
<thead>
<tr>
<th>HULL MEMBER</th>
<th>STRUCTURAL DETAIL</th>
<th>LOAD TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper deck plating</td>
<td>Hatch corners and side coaming terminations</td>
<td>Hull girder bending</td>
</tr>
<tr>
<td>Side-, bottom- and deck longitudinals</td>
<td>Butt joints and attachment to transverse webs, transverse bulkheads, hatch openings corners and intermediate longitudinal girders</td>
<td>Hull girder being, stiffener lateral pressure load and support deformation</td>
</tr>
<tr>
<td>Transverse girder and stringer structures</td>
<td>Bracket toes, girder flange butt joints, curved girder flanges, panel knuckles at intersection with transverse girder webs, etc. Single lug slots for panel stiffeners, access and lightening holes</td>
<td>Sea pressure load combined with cargo or ballast pressure</td>
</tr>
<tr>
<td>Transverse girders of wing tank (1)</td>
<td>Single lug slots for panel stiffeners</td>
<td>Sea pressure load (in particular in ore loading condition)</td>
</tr>
</tbody>
</table>

(1) The transverse deck-, side- and bottom girders of the wing tanks in the ore loading condition are generally subjected to considerable dynamic shear force- and bending moment loads due to large dynamic sea pressure (in rolling) and an increased vertical racking deflection of the transverse bulkheads of the wing tank. The rolling induced sea pressure loads in the ore loading condition will normally exceed the level in the ballast (and a possible oil cargo) condition due to the combined effect of a large GM-value and a small rolling period. The fatigue life evaluation must be considered with respect to the category of the wing tank considered (cargo oil tank, ballast tank or void). For ore-oil carriers, the cargo oil loading condition should be considered as for tankers.
Table 5.4: Highly Loaded Structural Elements - Container Carriers
(Cramer et al., 1995)

<table>
<thead>
<tr>
<th>HULL MEMBER</th>
<th>STRUCTURAL DETAIL</th>
<th>LOAD TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side- and bottom</td>
<td>Butt joints and attachment to transverse webs, transverse bulkheads and intermediate longitudinal girders</td>
<td>Hull girder bending, torsion (1), stiffener lateral pressure load and support deformation</td>
</tr>
<tr>
<td>longitudinal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper deck</td>
<td>Plate and stiffener butt joints, hatch corner curvatures and support details welding on upper deck for container pedestals, etc.</td>
<td>Hull girder bending- and torsional warping stress (2)</td>
</tr>
</tbody>
</table>

(1) Torsion induced warping stresses in the bilge region may be of significance from the forward machinery bulkhead to the forward quarter length.
(2) The fatigue assessment of upper deck structures must include the combined effect of vertical and horizontal hull girder bending and the torsional warping response. For hatch covers, additional stresses introduced by the bending of transverse (and longitudinal) deck structures induced by the torsional hull girder deformation must be included in the fatigue assessment.

Table 5.5: Highly Loaded Structural Elements - Roll on/Roll off- and Car Carriers
(Cramer et al., 1995)

<table>
<thead>
<tr>
<th>HULL MEMBER</th>
<th>STRUCTURAL DETAIL</th>
<th>LOAD TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side- and bottom</td>
<td>Butt joints and attachment to transverse webs, transverse bulkheads and intermediate longitudinal girders</td>
<td>Hull girder bending, stiffener lateral pressure load and support deformation</td>
</tr>
<tr>
<td>longitudinal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Racking constraining girders, bulkheads, etc.</td>
<td>Stress concentration points at girder supports and at bulkhead openings</td>
<td>Transverse acceleration load (1)</td>
</tr>
</tbody>
</table>

(1) It should be noted that the racking constraining girders and bulkheads are in many cases largely unstressed when the ship is in the upright condition. Thus the racking induced stresses may be entirely dynamic, which implies that fatigue is likely to be the primary design criterion. For designs which incorporate "racking bulkheads", the racking deformations are normally reduced such that the fatigue assessment may be limited to stress concentration areas at openings of the racking bulkheads only. If sufficient racking bulkheads are not fitted, racking deformations will be greatly increased, and the fatigue assessment of racking induced stresses should be carried out for primary racking constraining members and vertical girder structures over the ship length as applicable.
5.2 Load Estimation Methods

The key elements of the process for determining the loading that ships are required to endure was discussed in the opening paragraphs of this section. Loads can be determined by calculation, or can be estimated from full-scale or model-scale tests. In the context of damage tolerance assessment, the most practicable method is through calculation. Data gathered in full-scale or model-scale tests are, of course, very useful. However, in the present context, their value is greatest when used for calibrating and validating methods for establishing loads based on calculation. Methods for estimating loads on ships range from simple algebraic expressions to sophisticated analytical approaches that require computer programs to yield results.

In engineering, it is generally assumed that the accuracy to which a parameter is estimated is related to the degree of sophistication of the model used to make the prediction. It is reasonable to suppose that the same applies to ship loading models. In engineering, it is quite common to select a model, and there may be several potential choices to suit the application. The engineer implicitly acknowledges that the uncertainty in the result calculated using a simple rule-of-thumb is probably greater than that computed using a more complex methodology based on first principles. These approaches are equally valuable depending on the purpose of the analysis, and depending on the stage of the life cycle of the ship. Early in design, for example, when the design parameters are not well established, detailed methods are not justified.

Methods for damage tolerance assessment at different levels of complexity are presented in this document. It is intended that the appropriate level be selected on the basis of the quality and detail of the information available, and the accuracy of the assessment required. It is of course possible, and perhaps desirable, in some circumstances to use methods of different levels of complexity at different stages of the analysis. Generally, however, it is most cost-effective if the levels of accuracy for each stage are broadly consistent throughout the analysis.

Two sets of load data are required for damage tolerance assessment:

- extreme load
- load (stress) range spectrum

The importance of assessing both can be illustrated by considering the behaviour of a structure with a crack. The structure starts life with a certain design strength which degrades with time as the crack grows under the action of cyclic loads; this crack growth can only be estimated with knowledge of the stress range spectrum the structure is subjected to. The structure may degrade to the point where the residual strength is so low that any large (extreme) load will cause the structure to fail; the process is illustrated in Figure 5.1. In order to be able to assess the risk of the structure failing during a given period of time an estimate of both extreme load and the stress range are required.
Direct methods of wave load estimation are usually categorized as follows:

- short-term estimation
- long-term estimation

Both methods can, in principle, yield estimates of extreme load for arbitrary periods of time, a key requirement for damage tolerance assessment. However, of the two, only the long-term method can generate load (stress) range spectra required for crack growth calculation. The short-term method seeks to establish the extreme wave height that will be encountered during the period of interest. This wave height is then used to compute the extreme load. The implicit assumption is that the highest wave height yields the highest load effect. This is not always the case.

A brief description of the long-term method follows as the method most suitable for damage tolerance assessment. A brief overview is given in Section 5.2.1, and a fuller description is presented in Section 5.3. Supporting data and discussion is provided in Appendix F. Approximate indirect methods are introduced in Section 5.2.2. Any specified wave load has an associated probability of occurring. Section 5.2.3 discusses this issue in the context of damage tolerance assessment.

![Figure 5.1: Degradation of Structure With Time](taken from Broek, D., Elementary Engineering Fracture Mechanics, 4th edition, Kluwer Academic Publishers, 1986)
5.2.1 Direct Methods of Wave Load Estimation

As outlined above, the long-term method is appropriate for damage tolerance assessment. The basic steps in the process are described in outline below.

The long-term method takes advantage of the results of random vibration theory. The main advantage is that the characteristics of the output (response) can be computed very simply when certain requirements in regard to the dynamical system (ship) and the statistical nature of the input (load) are met. The key requirements are that the system is linear and that the input process is statistically ergodic and stationary. Once this assumption is made the responses from different sea conditions, which individually satisfy the requirement of stationarity and ergodicity, can be superimposed to yield an estimate of overall response. Combining this process with certain statistical techniques will yield both an estimate of extreme load and stress range spectrum.

Certain simplifying assumptions are necessary in applying the methodology outlined above. The limitations associated with these assumptions are discussed in Section 5.6.

5.2.2 Approximate Methods of Wave Load Estimation

The application of the long-term method requires the gathering of a large amount of input data and the use of advanced software tools. This level of effort may not be possible, or even appropriate. An “engineering” alternative is to use parametric equations that yield the load quantities required for damage tolerance assessment. Parametric equations are often based on the fitting of results from calculations, model tests, and full scale measurements. The results can only be representative of the ship configuration from which the results are obtained and, hence, cannot be generally applicable. It is important to be aware of the limitations; discussion in this regard is contained in Section 5.5 where three methods based on parametric equations are presented.

5.2.3 Risk Level for Extreme Loads

The wave loads ships experience are highly variable. This is because wave loads result from climatic phenomena which can only be expressed quantitatively in statistical terms. Furthermore, the load levels ship experience also depends on the behaviour of the operator which is also a variable quantity. Hence, wave loads can only be expressed as probabilistic values. While the damage tolerance assessment approach presented in this document in not formally cast in probabilistic terms, it is necessary to be aware of the probability levels inherent in the guidance.
The statistics of loads from environmental sources are often expressed in terms of the “return period”. The return period is the average time between two successive statistically independent events. Hence a particular significant wave height, $H_s$, with a return period of 50 years means that $H_s$ will be exceeded, on average, once every 50 years. It can be shown that the probability of a particular parameter with a return period of $T$ years occurring in a period of $T$ years is almost two-thirds. Hence, in the design of structures such as offshore platforms and civil engineering structures, the practice is to design for environmental loads which have a return period considerably in excess of the design life of the structure.

Ochi (1978) developed the concept of a “risk parameter” as a means for specifying a lower probability of occurrence of response consistent with design practice. The value of the risk parameter is the probability that a certain extreme load will be exceeded during the period of interest. Ochi recommends a value of 0.01 which he notes is generally consistent with current design practice for marine vehicles. This approximately amounts to increasing the time period to which the structure is exposed, by a factor of $1/0.01$, or 100 for design purposes. It appears (Brown and Chalmers, 1990) that the value of 0.01 is broadly consistent with the design of large steel structures in general.

This value is adopted for present purposes. Where possible, a risk parameter of 0.01 is applied in specifying the extreme load. In other words, the extreme load in question has a 0.01 probability of occurring in the duration of assessment. In one case, a Level 1 method, it was not possible to systematically apply this risk level; this is discussed at the appropriate point.

5.2.4 Overview of Presentation Methodology

The steps to be followed in each of the Levels are described below. The methodologies are presented in decreasing order of complexity. Level 1 relies on parametric equations; three methods are presented under this level. Levels 2 and 3 use the long-term load estimation method. The only difference between these two levels is the level of detail required to define the routes, and hence the wave climate. Level 3 assumes detailed knowledge of the route(s) the ship will ply.

Supporting information is provided in two places. First, a commentary on the procedures is presented in Section 5.6, and second, sources of data required as input for the particular element under discussion are described in Appendix F.

The primary purpose of the commentary is to make the reader aware of general limitations of the procedures, and also to make the reader aware of the current trends and issues in the calculation of wave loads on ships. The procedures presented for estimating extreme loads and stress range spectra can only be made practicable by making several assumptions, the implications of some of which are poorly understood. Hence, a clear understanding of the assumptions and limitations are prerequisites for successfully applying the methods presented herein.
5.3 Load Estimation - Level 3

The overall procedure for Level 3 is illustrated in Figure 5.2. This is an example of a direct method of wave load estimation sometimes known as the “spectral method”. The procedure involves a number of steps each comprising several tasks and requiring several sets of data. These are summarized in the steps below. The procedure assumes the hull form and the structure are defined.

The procedure summarized in Figure 5.2 assumes that only the sea states and the headings are variable. If it is necessary to assume that other parameters, for example ship speed, are variable then they should be included in the summation process.

For each task, the guidance presented is supported by discussion in Appendix F. In certain cases quantitative data is provided in Appendix F; it is only in these cases that explicit reference is made in the text below.

5.3.1 Definition of Problem

As discussed above, a large number of sources are required to perform an analysis to determine loading using direct methods. The primary data required are:

- Route
- Duration for which assessment is to be performed
- Percentage of time at sea
- Parts of structure to be subject to damage tolerance assessment
- Significant load types

The wave climate experienced by ships varies considerably depending on the areas of the oceans it operates in. Wave data is available for most parts of the world. This also applies to large bodies of water such as the Great Lakes. Perhaps the most comprehensive compilation of wave data is published by British Maritime Technology (Hogben et al., 1986). Other compilations includes ones by Bales et al., (1981) and Gilhousen et al. (1983). Wave climate atlases typically divide the world's oceans into blocks sometimes referred to as “Marsden zones”.
The degree to which the wave climate can be defined depends on how well the route is defined. In cases where the wave climate is unknown, it is advisable to employ "standard" wave climate data for areas known to experience severe weather. A good choice in this circumstance is to use data gathered in the North Atlantic, not only because it experiences severe weather compared with many other areas, but also because the quality of data for this area of the world's oceans is very high as it is particularly well instrumented.

Wave climate data is usually expressed in terms of "scatter diagrams" which express the probability of certain combinations of wave height and period occurring. Hence, using statistical terminology, the diagram is the joint probability density function for wave height and period. A typical scatter diagram is shown in Figure 5.3.

The duration of the damage tolerance assessment may vary from a few days to the lifetime of the ship. In general, particularly for longer periods, the ship will not be at sea for the total duration. This should be taken into account in the analysis. This data may be gathered from records. Limited guidance is provided in Section F2.2 of Appendix F.

For vessels with significantly varying loading conditions, it is necessary to determine the percentage of time in loaded and ballasted conditions. This applies particularly to tankers, container vessels, and bulk carriers. If specific data is not available, data presented in Section F2.2 of Appendix F can be used.
Figure 5.2: Overview of Level 3 Procedure for Computing Wave Loads
It is important to establish early in the assessment process which parts of the structure are to be subject to analysis and what load types are relevant. These two parameters are, of course, related. This determines the scope of the analysis and will vary from ship type to ship type. Tables 5.1 to 5.5 provides guidance in this regard for a wide range of commercial vessels.

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Figure 5.3: Typical Scatter Diagram (Hogben et al., 1986)

5.3.2 Definition of Operational Profile

In order to use the wave climate data referred to in the previous section it is necessary to select a spectral model for wave height, $S_h(u|H_s, T_z)$, where $u$, $H_s$ and $T_z$ are the wave height, the significant wave height and the corresponding zero crossing period respectively. Several such models have been developed and sources of data are identified in Appendix F.
The loading that ships experience depend several factors beyond those listed in the previous section including:

- load condition
- ship speed
- heading angle

In the assessment, it is necessary to establish which of these parameters vary significantly for the duration under consideration and develop an operational profile. It should be noted that the computational demands increase dramatically as the number of variable parameters increases.

These parameters are not necessarily independent of each other. In severe sea states, the practice is to reduce speed and to orient the ship in preferred directions. The nature of this response depends very much on the operating practice of the captain. Nevertheless, general trends have been established through questionnaires and other means. Lloyd (1989) has summarized research on this subject and the key results are presented in Appendix F.

Once the operational profile has been established, then a composite wave scatter may be developed:

\[
\left( H_S, T_z \right)_{\text{composite}} = \sum_{i=1}^{N} \mu_i \left( H_S, T_z \right),
\]

where \( H_S \) and \( T_z \) are the significant wave height and mean crossing period respectively, \( \mu_i \) is the proportion of time spent in the \( i \)th Marsden zone, and \( N \) is the total number of Marsden zones in the route.

### 5.3.3 Calculation Response Amplitude Operators & Stress Coefficients

The next stage requires the calculation of the response characteristics in terms of the transfer functions and stress coefficients which together will yield values of field stress, in the vicinity of the details of interest, for unit wave amplitudes for a range of wave periods.

The number of transfer functions (Response Amplitude Operators) and stress coefficients that need to be calculated depend on which load types are relevant and which parts of the ship structure are subject to the analysis.
The procedure is to compute transfer functions (Response Amplitude Operators) for selected load types. In general, the following transfer functions will be required:

- $H_v(\omega|\theta)$: transfer function for vertical bending moment
- $H_h(\omega|\theta)$: transfer function for horizontal bending moment
- $H_t(\omega|\theta)$: transfer function for torsional bending moment
- $H_p(\omega|\theta)$: transfer function for external pressure
- $H_c(\omega|\theta)$: transfer function for liquid loads

Each transfer function represents the load quantity experienced by the ship at a particular section or part, in response to waves of unit amplitude. Transfer functions are complex with real and imaginary components which express the phase relationship between the wave load and response. In general, transfer functions vary with wave frequency and heading. Transfer functions can be determined by model tests, full-scale measurements, or by computer programs.

The next stage requires the stress coefficients to be computed. The stress coefficients corresponding to the transfer functions listed above are:

- $A_v$: stress per unit vertical bending moment
- $A_h$: stress per unit horizontal bending moment
- $A_t$: stress per unit torsional bending moment
- $A_p$: stress per unit external pressure
- $A_c$: stress per unit liquid loads

Stress coefficients are normally determined from a global finite element model of the ship, or a large part of the ship, and loading the model with waves of unit height with a range of wave periods. The stress coefficient expresses the value of the stress component at the point of interest normalized by the unit load (e.g. horizontal bending moment).

The RAOs and stress coefficients are then combined to yield transfer functions for parts of structure that are being subject to damage tolerance assessment:

$$H_s(\omega|\theta) = A_v H_v(\omega|\theta) + A_h H_h(\omega|\theta) + A_t H_t(\omega|\theta) + A_p H_p(\omega|\theta) + A_c H_c(\omega|\theta)$$

where $H_s(\omega|\theta)$ is the stress transfer function for the part of the structure which is to be subjected to damage tolerance assessment.

In general, a damage tolerance assessment will investigate several parts of the ship. There will be at least one stress transfer function for each part. Stress transfer functions will be required for each stress component of interest. For example, if axial stress and shear stress are required for a damage tolerance assessment of a part, then two stress transfer functions will be required for each relevant load type.
5.3.4 Computation of Response

At this point, all the data required for computing response for a given sea state and ship heading are available. The input data comprise:

- Wave spectrum, \( S_H(\omega | H, T) \)
- Ship speed and heading
- Wave scatter diagram \((H, T)_{\text{composite}}\)
- Stress transfer function(s)

Wave height spectra typically refer to the wave climate at a stationary point in the ocean. The spectrum being used in the analysis needs to be modified to account for this fact. The frequency of waves that the ship experiences differs from the frequency a stationary observer would experience. The former is usually referred to as the “encounter frequency”. In general, the ship direction will not be aligned with the direction of the waves. The encounter frequency that accounts for these two effects is given by:

\[
\omega_e = \omega \left( 1 - \frac{\omega V}{g} \cos \theta \right) \tag{5.3}
\]

where

\( V = \) ship speed  \\
\( g = \) acceleration due to gravity

The expression for the wave height spectrum also needs to be modified to account for the transformation of the axes system from a fixed point to one that is translating with the ship. The modified wave spectrum is given by:

\[
S_H(\omega_e) = S_H(\omega) \frac{1}{1 - (2\omega V / g) \cos \theta} \tag{5.4}
\]
In addition, it is often the practice to account for the "short-crestedness" of the seas. A more complete description of the sea is given by two-dimensional spectra which express wave energy in terms of frequency and direction. Where measured two-dimensional spectra are unavailable, the usual practice is to apply a cosine-squared spreading function as follows:

\[ S_q(\omega, \theta) = S_q(\omega_s) \times \frac{2}{\pi} \cos^2(\theta) \]  \hspace{1cm} (5.5)

The response spectrum is given by:

\[ S_\sigma(\omega|H_s, T_s, \theta) = |H_\sigma(\omega|\theta)|^2 S_q(\omega_s, \theta|H_s, T_s) \]  \hspace{1cm} (5.6)

where \( S_\sigma(\omega|H_s, T_s, \theta) \) is the spectrum of the stress for a given combination of \( H_s, T_s \) and \( \theta \).

The moments of the spectrum of stress for the \( i \)th sea state and \( j \)th heading are given by:

\[ m_{kj} = \int_0^\infty \omega^k S_\sigma(\omega|H_s, T_s, \theta) d\omega \]  \hspace{1cm} (5.7)

\( m_{kj} \) is the k'th moment. \( m_0 \) is the zeroth moment and is equal to mean square stress response.

### 5.3.5 Computation of the Spectrum of Stress Range

A key assumption of the spectral method is that for each sea state the input wave forces are ergodic and statistically stationary. In this circumstance, the response is narrow-banded and is Rayleigh-distributed. Hence, the stress range distribution for the \( i \)th sea state and \( j \)th heading is given by:

\[ F_{\sigma_{ij}}(\sigma) = 1 - \exp \left[ -\frac{\sigma^2}{8m_{0j}} \right] \]  \hspace{1cm} (5.8)

The response zero crossing frequency in the \( i \)th sea state and \( j \)th heading is given by:

\[ \nu_j = \frac{1}{2\pi} \sqrt{\frac{m_{2g}}{m_{0g}}} \]  \hspace{1cm} (5.9)
5.3.6 Computation of Extreme Response

There are several methods available for computing the long term response of ships to wave loading. These range from the application of simple parametric equations to rigorous first-principle approaches. Simple approaches are discussed in later parts of this section.

The method presented below was developed by Ochi (1978) and is based on combining the “short term responses” that occur during the duration of interest. In the present context, each short term response has special characteristics. The main assumptions are that the input process (wave loading) is statistically stationary and ergodic, and the system (dynamical model of the ship) is linear. These assumptions are discussed in Section 5.2.1.

Ochi showed that the probability density function of the long term response can be expressed as follows:

\[
 f(\sigma_a) = \frac{\sum_i \sum_j n_i p_i p_j f_*(\sigma_a)}{\sum_i \sum_j n_i p_i p_j} \tag{5.10}
\]

where

- \( \sigma_a \) = stress amplitude
- \( f_* \) = probability density function for short-term response
- \( n_* \) = average number of responses per unit time of short-term response

\[
 n_* = \frac{1}{2\pi} \sqrt{\frac{m_{2ij}}{m_{0ij}}} , \text{ where } m_{0ij} \text{ and } m_{2ij} \text{ are as defined in Section 5.3.4}
\]

- \( p_i \) = weighting factor for ith sea state
- \( p_j \) = weighting factor for jth heading

Implicit in the above expression is that sea state and heading are the only variable parameters. Other parameters, such as ship speed and wave spectrum could vary significantly over the duration of interest. If this is the case, the expression would have to be modified to include an additional summation for each further variable.

Integration of the probability density function of the long term response yields the cumulative distribution function:

\[
 F(\sigma_a) = \int_0^{\sigma_a} f(\sigma_a) d\sigma_a \tag{5.11}
\]
The total number of cycles can be calculated from the following expression:

\[ n = \left( \sum_i \sum_j n_{i,j} \right) \times T \times (60)^2 \]  

(5.12)

where \( T \) is total exposure time to the sea over the duration of interest.

Applying the asymptotic distribution of extreme values a “design” extreme value can be computed from the following expression:

\[ \frac{1}{1 - F\left( \frac{\hat{\sigma}_a}{\alpha} \right)} = \frac{n}{\alpha} \]  

(5.13)

where

- \( \hat{\sigma}_a \) = design extreme stress
- \( \alpha \) = risk parameter discussed in Section 5.2.3; recommended value 0.01

Caution needs to be applied in calculating an estimate of the extreme response for periods of duration much less than the typical life of a ship. The wave climate model as expressed in terms of scatter diagrams represents an average gathered over many years. Clearly, such a model is not reliable for very short periods, and hence, in these cases, it is incorrect to merely substitute the actual duration for “\( T \)” in the expression given above. This is a subject that has not been extensively researched and, therefore, definitive guidance cannot be provided. A limited discussion on one aspect of this subject is presented in DNV (1992).

For very short periods of duration, in the order of days or weeks, a very conservative approach is recommended. In the case of “on-the-spot” analyses, it may be feasible to use weather forecasts upon which to base the wave climate model. Several national and international agencies provide weather forecasts which include forecasts of wave heights over much of the globe. For the approach to be generally applicable, there is no alternative but to assume an extreme wave climate model.

In the case of intermediate periods of duration there is little alternative but to use the wave scatter diagrams together with conservative assumptions in regard to the duration. Strictly, an analysis should be performed in which the statistical variability of the data upon which the wave scatter diagrams are based is accounted for. As very tentative guidance for periods of less than five years, a period of duration of no less than five years should be used. Where the data allows it, the seasonal variations in wave climate should be included in the analysis.

The treatment of short and intermediate periods of duration outlined above is not based on rigorous analysis, and it is recommended that further analysis be performed before applying any of the above.
5.3.7 Compilation of Total Stress Range Spectrum

The purpose of this part of the work is to compile the total stress range spectrum for the part(s) of the ship structure that are to be subjected to a damage tolerance assessment.

The stress range spectrum for each combination of sea state and heading were calculated as described above. In order to express the total stress range spectrum the individual spectra need to combined as follows:

\[
F_{\Delta \sigma} = \sum_{i=1, j=1}^{\text{all headings, sea states}} r_{ij} F_{\Delta \sigma ij}(\sigma) p_{ij}
\]  

(5.14)

where

- \( p_{ij} \) = probability of occurrence of a given sea state and heading
- \( r_{ij} = v_{ij} / v_0 \) = the ratio between the crossing rates in a given sea state and the average crossing rate
- \( v_0 = \sum \frac{p_{ij}}{v_{ij}} \) = the average crossing rate

The method as described above does not retain information on sequencing of stress ranges. The sequencing of stress range data is problematic since it is so variable. However, the variations of wave climate from one Marsden zone to another can be captured in an average sense. The stress range data for each combination of \( H_s, T_s \) is known and the probability of occurrence of the combination with each Marsden zone is also known. What is still not known is the sequence within each Marsden zone. It may be acceptable to gather the stress range data for each Marsden zone and order the stress range data from high to low. While this is conservative (for certain types of damage assessment) it is not as conservative as sequencing the total stress range data for the duration of the damage tolerance assessment. The degree of conservatism can only be determined by trial and error through simulation exercises.

5.3.8 Slamming Loads

The spectral method, which relies on linearity, is not a convenient framework within which to account for slamming loads. Alternative methods are more conveniently applied. These range from specialist computer programs that compute slamming loads and ship response, to simple empirical rules. As an example of the latter, Mansour et al. (1996a) suggests that to account for slamming the wave bending moment for commercial ships should be increased by 20%, and by 30% for warships. When more sophisticated approaches are employed in predicting the response to slamming it is necessary that a dynamic analysis be performed in which the elasticity of the ship structure is accounted for.
5.4 Load Estimation - Level 2

The Level 2 approach is identical to the Level 3 approach except for the wave climate model used. While the ideal is to compile a composite wave scatter diagram that reflects the intended route of the ship, this information is not necessarily available. In this situation, it is necessary to use average pre-compiled data. Two such scatter diagrams are presented below. Table 5.6 is intended for use for routes in the North Atlantic. This is significantly more severe than the "world average" wave climate presented in Table 5.7.

Table 5.6: Scatter Diagram for North-Atlantic for Use in Fatigue Computations
(Cramer et al., 1995)

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5.5 Load Estimation - Level 1

The methodology for determining global stresses described for Levels 2 and 3 is a first principles approach which requires considerable resources to apply. This applies particularly to the data required and the software tools needed. There are several reasons why such a detailed approach may not be justified. These reasons may include lack of data and lack of appropriate tools. In addition, despite the rigour of the methodology, there are limitations in the method which are not easily overcome.

Several alternative approaches have been developed that are much less demanding in terms of effort required but which are nearly as effective as first principles approaches as long the limitations are recognized and catered for. These methods are largely empirical in origin and hence are applicable to the ship types from which the data was derived in the development of the methods.

Table 5.7: Scatter Diagram Describing World Wide Trade for Use in Fatigue Computations (Cramer et al., 1995)

<table>
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<th>2.5</th>
<th>3.5</th>
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Methods from three sources are presented below. A short description of each follows:

Method A A method for applicable to a wide range of commercial vessels has been developed by Det Norske Veritas (Cramer, 1995) and is adapted here for damage tolerance assessment purposes. The method is based on an estimate of extreme load derived from the DNV Rules, and the stress range spectrum is based on the Weibull model. This is similar in approach to the method devised by the American Bureau of Shipping (ABS, 1992), which forms the basis of the approach used in the ABS SafeHull system. The two approaches are interchangeable in this guide.

Method B A method based on data gathered on frigate/destroyer size warships was developed by Clarke (1985, 1986) and described by Chalmers (1993).

Method C A method based on data from both warships and commercial ships was developed by Sikora et al. (1983) and further extended by Sikora and Beach (1986).

The methods are presented in order of the effort required and comprehensiveness. Method A provides parametric expressions for load calculation for load types considered relevant for several types of ship; these are listed in Section 5.5.1. below. Methods B and C are limited to predicting extreme vertical bending moments. Hence these methods cannot predict stresses due to for example, horizontal or torsional bending. Furthermore, these methods are unable to predict stresses due to local wave action or due to acceleration forces induced by ship motion. These limitations can be a serious drawback in applying the methods unless they are supplemented by other calculation methods where required.

While, in general, wave-induced vertical bending is the major source of high stresses in ship structures there are potentially other loading modes that may be also be significant sources. For example, torsional moments can cause high stresses at hatch corners in container ships. While this mode can be included in the analysis using Level 2 and 3 methods parametric equations for predicting moments other that due to vertical bending do not appear to be available except in Class Society rules. Therefore, in situations where considering vertical bending alone is insufficient, and where Level 2 and 3 methods cannot be applied, it is recommended that Class Society rules be employed. The reference from which Method A is adapted, provides guidance in the use of DNV rules for this purpose.

The same observations apply to the calculation of local loads which can be significant for several classes of large commercial vessels which tend to have limited internal structure.

Methods B and C do not provide explicit models for the stress range spectrum. This is a less serious drawback since the well-established Weibull model can be used. The primary decision to be made in this regard is the choice of a value for the Weibull shape parameter. Again the Class societies are a good source of information for commercial ship structure (see for
The same observations apply to the calculation of local loads which can be significant for several classes of large commercial vessels which tend to have limited internal structure.

Methods B and C do not provide explicit models for the stress range spectrum. This is a less serious drawback since the well-established Weibull model can be used. The primary decision to be made in this regard is the choice of a value for the Weibull shape parameter. Again the Class societies are a good source of information for commercial ship structure (see for example DNV (1995) and ABS (1992) where values for shape parameter are recommended. In the case of warships, a limited amount of full-scale data for vertical hull girder bending suggests a value of unity (Clarke, 1986). Mansour et al. (1996) using a computer program based on second-order strip theory predicted shape parameters somewhat less than unity for two cruisers.

5.5.1 Method A

Almost all major classification societies have developed or are in the process of developing structural design methodologies that are a significant departure from past practice. Traditional design methods as expressed in the rules are empirically based and have evolved over many decades of use. While such methods yield adequate designs they suffer from being inflexible and poor at accommodating new materials, innovative structural configurations, and novel hull forms.

In response to this weakness, and for other reasons, the major classification societies have invested considerable resources in developing first-principles methods of analysis and design. These methods have several characteristics in common; principal among them are:

- computer-based systems
- explicit calculation of loads
- explicit calculation of structural resistance
- capability to assess fatigue performance

While these systems are based on first-principles approaches, they are not generic in terms of the ship types that the approaches can be applied to. The classification societies have developed systems primarily for larger vessels, the main ship types being tankers, bulk carriers, and container ships. Implicit in many of the systems is an assumption of a specific structural configuration.

The degree to which the classifications societies have published the background on the methods in the open literature is variable. It would not be appropriate to describe all the approaches developed by the classification societies. The method described here is taken from the methodology for fatigue assessment developed by Det Norske Veritas (Cramer et al., 1995). While the published approach does not exclude any ship type, it appears by implication to be intended for:
tankers
bulk carriers
ore carriers
container ships
RO/RO and car carriers

The DNV document presents two methods which are termed "simplified analysis" and "direct analysis". The latter is essentially the long-term method presented earlier in Section 5.3. The methodology reproduced below is essentially the simplified analysis presented in the DNV document. It should be noted that in approach the method is similar to that presented by ABS (1992).

The overall approach is to use the DNV Rules to define the following load components based on the Rules:

- vertical bending moment
- horizontal bending moment
- torsional bending moment
- dynamic external pressure loads
- internal pressure loads due to ship motions

The stress range spectrum used for fatigue assessment purposes is modelled using the Weibull distribution. As discussed elsewhere in this document, the Weibull distribution has become the preferred model for representing the stress range spectrum and is illustrated in Figure 5.4.

The stress range distribution is expressed as follows:

$$\Delta \sigma = \Delta \sigma_0 \left[1 - \frac{\log n}{\log n_0}\right]^h$$  \hspace{1cm} (5.15)

where

- $\Delta \sigma$ = stress range
- $\Delta \sigma_0$ = reference stress range
- $n$ = number of cycles
- $n_0$ = number of cycles associated with reference stress range
- $h$ = Weibull distribution shape parameter
Cramer et al. (1995) provide guidance on values for shape parameters (for tankers and bulk carriers)

\[
\begin{align*}
  h &= h_0 \
  h &= h_0 + h_a (D - z) / (D - T_{act}) \
  h &= h_0 + h_a \
  h &= h_0 + h_a z / T_{act} - 0.005 (T_{act} - z) \
  h &= h_0 - 0.005 T_{act} \
  h &= h_0 + h_a
\end{align*}
\]

where

- \(D\) = moulded depth of ship (See Figure 5.5)
- \(z\) = height above baseline (See Figure 5.5)
- \(T_{act}\) = actual draught
- \(h_0\) = basic shape parameter \(= 2.21 - 0.54 \log_{10}(L)\)

(In the absence of alternatives, \(h_0\) can be taken as 1.05 for open type vessels.)

\(h_a\) = additional factor depending on motion response period
- \(= 0.05\) in general
- \(= 0.00\) for plating subjected to forces related to roll motions for vessels with roll (period greater than 14 secs)

In order to avoid sensitivities associated with the Weibull shape parameter, Cramer et al. (1995) recommend that the reference number of cycles be taken at \(10^6\). The methodology described is formulated accordingly.

This section outlines a simplified approach for calculation of dynamic loads. Formulae are given for calculation of global wave bending moments, external sea pressure acting on the hull and internal pressure acting on the tank boundaries.

The simple formula for calculation of loads in this section are based on the linear dynamic part of the loads as defined in the 1993 edition of the Rules [DNV, 1993]. The design loads as defined in the Rules also include non-linear effects such as bow-flare and roll damping and are not necessarily identical with the dynamic loads presented herein.

Fatigue damage should in general be calculated for all representative load conditions accounting for the expected operation time in each of the considered conditions. For tankers, bulk carriers and container vessels, it is normally sufficient to consider the only ballast- and fully loaded conditions. The loads are calculated using actual draughts, \(T_{act}\), metacentic heights \(GM_{act}\) and roll radius of gyration, \(k_{T,act}\) for each considered loading condition.
Figure 5.4: Weibull distribution representing stress range
Wave induced hull girder bending moments

The vertical wave induced bending moments may be calculated using the bending moment amplitudes specified in the Rules Pt. 3, Ch. 1 (DNV, 1993). The moments, at $10^{-2}$ probability level of exceedance, may be taken as:

$$M_{ds} = -0.11 f_r k_{wm} C_w L^2 B (C_B + 0.7) \text{ (kNm)}$$

$$M_{dh} = 0.19 f_r k_{wm} C_w L^2 B C_B \text{ (kNm)}$$

where

$M_{ds}$ = wave sagging moment amplitude

$M_{dh}$ = wave hogging moment amplitude

$C_w$ = wave coefficient

- $= 0.0792 L; \quad L < 100 \text{ m}$
- $= 10.75 - \left(\frac{(300 - L)}{100}\right)^{3/2}; \quad 100 \text{ m} < L < 300 \text{ m}$
- $= 10.75; \quad 300 \text{ m} < L < 350 \text{ m}$
- $= 10.75 - \left(\frac{(L - 350)}{150}\right)^{3/2}; \quad 350 \text{ m} < L$

$k_{wm}$ = moment distribution factor

- $= 1.0 \text{ between } 0.40 L \text{ and } 0.65 L \text{ from A.P., for ships with low/moderate speed}$
- $= 0.0 \text{ at A.P. and F.P. (Linear interpolation between these values.)}$

$f_r$ = factor to transform the load from $10^{-6}$ to $10^{-4}$ probability level.

- $= \frac{1}{5^h}$

$h_0$ = long-term Weibull shape parameter

- $= 2.21 - 0.54 \log(L)$

$L$ = rule length of ship (m)

$B$ = greatest moulded breath of ship in meters measured at the summer waterline

$C_B$ = block coefficient

For the purpose of calculating vertical hull girder bending moment by direct global finite element analyses, simplified loads may be obtained from Appendix C in Cramer (1995).
The horizontal wave bending moment amplitude \( M_{H} \) at the \( 10^{-4} \) probability level may be taken as follows (ref. Rules Part 3, Chapter 1 (DNV, 1993)):

\[
M_{H} = 0.22 f_{L}^{9/4} (T_{\text{sec}} + 0.30 B) C_{B} (1-\cos(2\pi x/L)) \quad \text{(kNm)} 
\]  

(5.18)

where \( x \) is the distance in metres from A.P. to section considered, and \( L, B, C_{B}, f_{L} \) are as defined above.

Wave torsional load and moment which may be required for analyses of open type vessels (e.g., container vessels) are defined in Appendix C and D of Cramer (1995).

**Dynamic external pressure loads**

Due to intermittent wet and dry surfaces, the range of the pressure is reduced above \( T_{\text{sec}} - C_{t} \) (see Figure 5.5). The dynamic external pressure amplitude (half pressure range), \( p_{e} \), related to the draught of the load condition considered may be taken as:

\[
p_{e} = r_{p} p_{d} \quad \text{(kN/m}^{2}) 
\]

(5.19)

where

\[ p_{d} = \text{dynamic pressure amplitude below the waterline} \]

The dynamic pressure amplitude may be taken as the largest of the combined pressure dominated by pitch motion in head/quartering seas, \( p_{dp} \), or the combined pressure dominated by roll motion in beam/quartering seas, \( p_{dr} \), as:

\[
p_{d} = \max (p_{dp}, p_{dr}) \quad \text{(kN/m}^{2}) 
\]

(5.20)

where

\[
p_{dp} = p_{1} + 135y/(B + 75) - 1.2(T - z) 
\]

\[
p_{dr} = 10[y\phi/2 + C_{B}(y + k_{d})(1 + 2z/T)/30] 
\]

\[
p_{1} = k_{s} C_{w} + k_{r} = (k_{s} C_{w} + k_{d}) (0.8 + 0.15 V/\sqrt{L}) \quad \text{if } V/\sqrt{L} > 1.5 
\]

\[
k_{s} = 3C_{B} + 2.5/\sqrt{C_{B}} \quad \text{at AP and aft} 
\]

\[
= 3C_{B} \quad \text{between 0.2 L and 0.7 L from AP} 
\]

\[
= 3C_{B} + 4.0/C_{B} \quad \text{at FP and forward} 
\]

(between specified areas \( k_{s} \) is to be varied linearly)
\( z \) = vertical distance from the baseline to the loadpoint
\( = \) maximum \( T \) (m)

\( y \) = horizontal distance from the centre line to the loadpoint
\( = \) minimum \( B/4 \) (m)

\( C_w \) = wave coefficient as defined earlier

\( k_f \) = the smallest of \( C_w \cdot T \) and \( f \)

\( f \) = vertical distance from in from the waterline to the top of the ship’s side at transverse section considered
\( = \) maximum \( C_w \) (m)

\( L \) = ship length

\( \phi \) = rolling angle, single amplitude (rad) as defined later in this section

\( V \) = vessels design speed in knots

\( r_p \) = reduction of pressure amplitude in the surface zone
\( = 1.0 \) for \( z < T_{act} - c \)
\( = \frac{T_{act} + c - z}{2c} \) for \( T_{act} - c < z < T_{act} + c \)
\( = 0.0 \) for \( T_{act} + c < z \)

\( c \) = distance in m measured from actual waterline. In the area of side shell above \( z = T_{act} + c \) it is assumed that the external sea pressure will not contribute to fatigue damage.
\( = p_d/\rho g \)

\( T_{act} \) = the draught in m of the considered load condition

\( \rho \) = density of sea water \( = 1.025 \) (t/m³)
Partly dry surfaces

$T_{\text{act}} - c$

$2p_e$

Figure 5.5: Reduced Pressure Ranges in the Surface Region
Internal pressure loads due to ship motion

The dynamic pressure from liquid cargo or ballast water should be calculated based on the combined accelerations related to a fixed coordinate system. The gravity components due to the motions of the vessel should be included. The dynamic internal pressure amplitude, \( p_i \) in kN/m\(^2\), may be taken as the maximum pressure due to acceleration of the internal mass:

\[
p_i = f_a \max \left\{ p_1 = \rho a_h h, p_2 = \rho a_l |y|, p_3 = \rho a_l |x| \right\} \quad (\text{kN/m}^2) \tag{5.21}
\]

where

- \( p_1 \) = pressure due to vertical acceleration (largest pressure in lower tank region)
- \( p_2 \) = pressure due to transverse acceleration
- \( p_3 \) = pressure due to longitudinal acceleration
- \( \rho \) = density of sea water, 1.025 \((\text{t/m}^3)\)
- \( x \) = longitudinal distance from centre of free surface of liquid in tank to pressure point considered (m)
- \( y \) = transverse distance from centre of free surface of liquid in tank to the pressure point considered (m), see Figure 5.9.
- \( h \) = vertical distance from point considered to surface inside the tank (m), Figure 5.9.
- \( a_h, a_l \) = accelerations in vertical-, transverse-, or longitudinal direction \((\text{m/s}^2)\)
- \( f_a \) = factor to transform the load effect to probability level \(10^{-4}\), when the accelerations are specified at the \(10^{-8}\) probability level

\[
f_a = 0.5^{1/h}
\]

\[
h = h_0 + 0.05
\]
Note: The factor $f_a$ is estimated for ships with a roll period $TR < 14\,\text{sec.}$, and may otherwise be less for roll induced pressures and forces. See also above for guidance on the Weibull shape parameter $h$. The accelerations $a_r$, $a_t$ and $a_l$ are given in the following sub-section.

The effect of ullage (void space in top of tank) will add to the pressure in one half cycle and subtract in the other and is, therefore, omitted in the above description of the half pressure range. Similarly, the effect of the tank top geometry may be omitted. For partly subdivided tanks where the fluid is prevented to flow through swash bulkheads during one half motion cycle, the pressures may be reduced accordingly.

Note: The above scaling of pressures by use of the factor $f_a$ is only valid for fatigue assessment and may be justified as the dominating fatigue damage is caused mainly by moderate wave heights.

For bulk and ore cargoes, only $p_1$ need to be considered. The appropriate density and pressure height should be specially considered.

For similar tank filling conditions on both sides of a bulkhead (e.g., for a bulkhead between two cargo tanks), the following apply:

(a) the effect of vertical acceleration is cancelled and may be set to zero; and
(b) the pressures due to motion are added for bulkheads normal to the direction (plane) of the motions.

The net pressure range may be taken as:

\[ p = 2p_2 \] for longitudinal bulkheads between cargo tanks

\[ p = 2p_3 \] for transverse bulkheads between cargo tanks

(Note that $\Delta p = 2p$, when liquid on both sides)

As a simplification, sloshing pressures may normally be neglected in fatigue computations. However, if sloshing is to be considered, the sloshing pressures in partly filled tanks may be taken as given in the Rules (DNV, 1993), Part 3, Chapter 1, Section 4, C306. The pressure amplitude is defined at the probability level of $10^{-4}$. In case of partly filled tanks on both sides of a bulkhead, the pressure range may be taken as the sum of the pressure amplitudes in the two tanks. Otherwise the range may be taken equal to the amplitude.

Unless otherwise specified, it may be assumed that tanks (in tankers) are partly filled 10% of the vessels design life.
Figure 5.6: Distribution of Pressure Amplitudes for Tankers in the Fully Loaded Condition

Figure 5.7: Distribution of Pressure Amplitudes for Tankers in Ballast Condition
Figure 5.8: Distribution of Pressure Amplitudes for a Bulk Carrier in the Ore Loading Condition
Ship accelerations and motions

The formula for ship accelerations and motions given below are derived from the Rules, Ch. 1, Pt. 3, Sec. 4, (DNV, 1993). The acceleration and motions are extreme values corresponding to a probability of occurrence of $10^{-8}$.

Combined accelerations:

\[ a_t = \text{combined transverse acceleration (m/s}^2) \]
\[ a_t = \sqrt{a_z^2 + \left(g_0 \sin \phi + a_{yw}\right)^2} \]

\[ a_l = \text{combined longitudinal acceleration (m/s}^2) \]
\[ a_l = \sqrt{a_x^2 + \left(g_0 \sin \phi + a_{px}\right)^2} \]

\[ a_v = \text{combined vertical acceleration (m/s}^2) \]
\[ a_v = \max\left(\frac{\sqrt{a_{yw}^2 + a_z^2}}{\sqrt{a_{px}^2 + a_x^2}}\right) \]

Acceleration components:

\[ a_x = \text{surge acceleration (m/s}^2) \]
\[ = 0.2 \ g \ a_0 \sqrt{C_b} \]

\[ a_y = \text{acceleration due to sway and yaw (m/s}^2) \]
\[ = 0.3 \ g \ a_0 \]

\[ a_z = \text{heave acceleration (m/s}^2) \]
\[ = 0.7 \ g \ a_0 / \sqrt{C_b} \]

\[ a_0 = \text{acceleration constant} \]
\[ = 3C_w/L + C_V V / \sqrt{L} \]

\[ C_V = \sqrt{L} / 50, \text{ max 0.2} \]

\[ V = \text{ship design speed (knots)} \]
Roll motions:

\[ a_{ry} = \text{horizontal component of roll acceleration (m/s}^2) = \varphi \left( \frac{2 \pi}{T_R} \right)^2 R_{RZ} \]

\[ a_{rz} = \text{vertical component of roll acceleration (m/s}^2) = \varphi \left( \frac{2 \pi}{T_R} \right)^2 R_{RY} \]

\[ R_R = \text{distance from the axis of rotation to centre of mass (m)} \]

The distance is related to the roll axis of rotation that may be taken at \( z_r \) (m) above the baseline, where \( z_r \) is the smaller of \( [D/4 + T/2] \) and \( [D/2] \).

For double hull ballast tanks, the \( R_R \) may be approximated by the horizontal distance from the centreline to the tank surface centre.

\[ R_{RZ} = \text{vertical distance from axis of rotation to centre of tank/mass (m)} \]

\[ R_{RY} = \text{transverse distance from axis of rotation to centre of tank/mass (m)} \]

\[ T_R = \text{period of roll} = 2 k_r / \sqrt{GM} \text{, maximum 30 (s)} \]

In case the values of roll radius, \( k_r \), and metacentric height, \( GM \), have not been calculated for the relevant loading conditions, the following approximate values may be used:

\[ k_r = \text{roll radius of gyration (m)} = 0.39 \text{ B for ships with even distribution of mass and double hull tankers in ballast} = 0.35 \text{ B for single skin tankers in ballast} = 0.25 \text{ B for ships loaded with ore between longitudinal bulkheads} \]

\[ GM = \text{metacentric height (m)} = 0.07 \text{ B in general} = 0.12 \text{ B for single skin tankers, bulk carriers and fully loaded double hull tankers} = 0.17 \text{ B for bulk and ore carriers in the ore loading condition} = 0.33 \text{ B for double hull tankers in the ballast loading condition} = 0.04 \text{ B for container carriers} \]
\[ \varphi \] = maximum roll angle, single amplitude (rad)
\[ = 50 \, c / (B + 75) \]
\[ c \] = \((1.25 - 0.25 \, T_R) \) k
\[ k \] = 1.2 for ships without bilge keel
\[ = 1.0 \] for ships with bilge keel
\[ = 0.8 \] for ships with active roll damping capabilities

Pitch motions:
\[ a_p \] = tangential pitch acceleration (m/s^2)
\[ = \theta \left( \frac{2\pi}{T_p} \right)^2 R_p \]
\[ a_{px} \] = longitudinal component of pitch acceleration (m/s^2)
\[ = \theta \left( \frac{2\pi}{T_p} \right)^2 R_{px} \]
\[ a_{pz} \] = vertical component of pitch acceleration (m/s^2)
\[ = \theta \left( \frac{2\pi}{T_p} \right)^2 R_{pz} \]

\[ R_p \] = distance from the axis of rotation to the tank centre (m)
\[ R_{pz} \] = vertical distance from axis of rotation to centre of tank/mass (m)
\[ R_{px} \] = longitudinal distance from axis of rotation to centre of tank/mass (m)

\[ T_p \] = period of pitch (s)
\[ = 1.80 \sqrt{L / g} \]

\[ \theta \] = maximum pitch angle (rad)
\[ = 0.25 \, a_v / C_B \]

Axis of rotation may be taken as 0.45L from A.P., at centreline, z, above baseline.

The material presented above summarizes the calculation of loads. For guidance on the computation of stresses and the combining stresses from the different loads summarized above the reader is referred to Cramer et al. (1995).
Figure 5.9: Illustration of Acceleration Components

Figure 5.10: Illustration of Acceleration Components and Centre of Mass for Double Hull Tankers or Bulk Carriers With Connected Top Wing and Hopper/Bottom Ballast Tanks
Estimates of extreme load

The methodology summarized above is intended for fatigue analysis and not specifically for estimating extreme loads. However, the fatigue analysis is based on estimates for extreme load for each load component. The DNV methodology uses a reference probability level of $10^{-4}$ for this purpose and hence a conversion factor is applied to expressions for extreme load which are generally based on a probability level of $10^{-8}$ which is representative of lifetime of ship. The exception is external pressure loads which are specified directly at probability level of $10^{-4}$.

The extreme load levels for a duration other than the lifetime of the ship can be computed by factoring the load as follows:

$$\text{Factor} = \left[ \frac{\log p_2}{\log p_1} \right]^{\frac{1}{h}}$$

(5.22)

where

- $p_2 =$ probability level that quantity is to be changed to
- $p_1 =$ probability level that quantity specified at
- $h =$ Weibull parameter

Caution should be exercised in using the above expression for high probability levels (i.e., periods of short duration). The reasons for this are discussed in Section 5.3.6.

5.5.2 Method B

This method is based on strain measurements made on several warships in the range of 100-200m length. The method was developed by Clarke (1985, 1986) and described by Chalmers (1993). The strain data, which was gathered over a limited time period, was extrapolated to predict a lifetime maximum. Based on a lifetime of 25 years and a 30% exposure the bending moment expected to be exceeded once in $3 \times 10^7$ wave encounters was computed. This was found to have a close correlation with the bending moment calculated by balancing the ship on a 8m wave.
Lifetime Design Load

On this basis the following expressions were derived:

\[ M_{ds} = M_{sw} + 1.54(M_s - M_{sw}) \]  \hspace{1cm} (5.23)

\[ M_{dh} = M_{sw} + 1.54(M_h - M_{sw}) \]  \hspace{1cm} (5.24)

where \( M_{ds} \) and \( M_{dh} \) are the design sagging and hogging bending moments, \( M_s \) and \( M_h \) are the sagging and hogging bending moments from static balance on an 8m wave, and \( M_{sw} \) is the still water bending moment making proper allowance for its sign.

The multiplier, 1.54, accounts for two effects. The first is a factor of 1.12 which accounts for the systematic biases arising from the mean bias against the 8m wave balance, the underestimate in the measured strain inherent in the type of gauges used, and inaccuracies in the calculation of hull section modulus. The remaining factor is a statistical multiplier which is applied to convert the value of the expected bending moment, which has a probability of being exceeded once in its lifetime, to a probability of exceedance of 1% in its lifetime.

Inherent in the 1.54 multiplier is \( 3 \times 10^7 \) wave encounters. The value of the multiplier for other values of wave encounter are given in Table 5.8 below.

Table 5.8: Bending Moment for Various Numbers of Wave Encounters
(Chalmers, 1993)

<table>
<thead>
<tr>
<th>Number of Wave Encounters</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3 \times 10^7 )</td>
<td>1.54</td>
</tr>
<tr>
<td>( 4 \times 10^7 )</td>
<td>1.57</td>
</tr>
<tr>
<td>( 5 \times 10^7 )</td>
<td>1.59</td>
</tr>
<tr>
<td>( 7 \times 10^7 )</td>
<td>1.63</td>
</tr>
<tr>
<td>( 10 \times 10^7 )</td>
<td>1.67</td>
</tr>
</tbody>
</table>
Limited Duration Design Bending Moment

The expressions presented above are intended for design purposes, and hence the numbers of wave encounters are typical of the lifetime of a ship. Damage tolerance assessments may be required for shorter periods. Using an approximation of the methodology upon which the table above is based, multipliers for smaller numbers of wave encounters are derived and presented in the Table 5.9 below. For reasons discussed in Section 5.3.6, caution must be applied in reducing the multiplier to account for short periods of duration.

Table 5.9: Bending Moment for Various Numbers of Wave Encounters
(adapted from Chalmers, 1993)

<table>
<thead>
<tr>
<th>Number of Wave Encounters</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^6$</td>
<td>1.25</td>
</tr>
<tr>
<td>$3 \times 10^6$</td>
<td>1.34</td>
</tr>
<tr>
<td>$7 \times 10^6$</td>
<td>1.41</td>
</tr>
<tr>
<td>$10 \times 10^6$</td>
<td>1.45</td>
</tr>
</tbody>
</table>

The expressions presented above can be used to predict bending moments for any point along the length of the ship. This, however, does not account for slamming effects. To allow for slamming, Clarke (1986), recommends extending the length over which the maximum bending moment applies forward by a distance of 15% of ship length and then reducing the bending moment linearly to zero at the forward perpendicular. This is illustrated in Figure 5.11 below.

The expressions for the hog and sag bending moment can be added, converted to yield a stress range that has a 1% probability of being exceeded in the duration of interest can be calculated. This together with the appropriate selection of the appropriate Weibull shape parameter are sufficient to define the stress range spectrum.

The methodology described above is based almost entirely on full-scale data gathered on warships ranging in length from about 100m to 200m. Hence the approach should, strictly, not be applied to other types of vessel. The methodology presented in the next section, Method C, is broadly similar in approach and includes a variety of ship types although the number of ships is small. However, the success of Method C in representing the response of a range of ship types suggests that the present method may be applicable to ship types other than those from which the data was gathered.
Stress Range Spectrum

The Weibull distribution, described in Section 5.5.1, can be used to express the stress range spectrum. For shape parameters guidance can be taken from Section 5.5.1.

5.5.3 Method C

This method is based on work originally reported by Sikora et al. (1983) in which a method for predicting lifetime extreme loads and stress range spectra was presented. The method was further developed and reported by Sikora and Beach (1986).

The method relies on a generalized response amplitude operator for vertical bending moment at midships. Sikora et al. examined response data from model-scale results and full-scale trials for a variety of ships and found that the RAO for vertical bending at midships could, after appropriate normalization, be represented by a single curve. This generalized RAO is expressed as a function of the following parameters of the ship:
The maximum wave induced bending moment was computed for several ships using the generalized RAO together with assumptions in regard to the wave climate, wave height spectrum, ship speed and direction. The key assumptions are summarized below:

### Wave climate

Wave climates from three areas of the world's oceans were used in the investigation. The frequency of occurrence of sea states for the three areas are presented in Table 5.10. The primary purpose of this part of the work was to establish the sensitivity of structural response, in terms of extreme response and fatigue behaviour, to different wave climates.

### Wave height spectrum

The six-parameter wave height spectrum developed by Ochi (1977, 1978) was used in the analysis.

### Ship speed and heading

The operational profile a ship sees depends on a number of parameters. A key element is the reduction of speed which usually accompanies high sea states. The degree to which speed is reduced by the captain depends on factors such as slamming, deck wetness, propeller emergence, and accelerations levels experienced by the ship. Depending on the mission the captain may tolerate some of these phenomena. Similarly, depending on the hull forms, size of ship and mission, the captain may alter heading in high sea states. This subject is discussed in detail in Lloyd (1989).

For the purposes of developing parametric equations predicting response it is necessary to make assumptions in regard to the frequency of occurrence of combinations of sea state, ship speed, heading and ship type. Based on an analysis (Sikora et al., 1983) Sikora and Beach (1986) developed estimates of frequency of occurrence for combinations of these parameters. These are presented below in Table 5.11.

### Slamming

The role of slamming was also investigated. Using measurements from several ships a simple algorithm was developed for predicting the whipping bending moment. Slamming was assumed to occur on the basis of the criteria presented in Table 5.12.
Table 5.10: Frequency of Occurrence of Sea States  
(Sikora and Beach, 1986)

<table>
<thead>
<tr>
<th>Significant Wave Height (meter)</th>
<th>Frequency of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Area A</td>
</tr>
<tr>
<td>&lt;1</td>
<td>0.0503</td>
</tr>
<tr>
<td>1-2</td>
<td>0.2665</td>
</tr>
<tr>
<td>2-3</td>
<td>0.2603</td>
</tr>
<tr>
<td>3-4</td>
<td>0.1757</td>
</tr>
<tr>
<td>4-5</td>
<td>0.1014</td>
</tr>
<tr>
<td>5-6</td>
<td>0.0589</td>
</tr>
<tr>
<td>6-7</td>
<td>0.0346</td>
</tr>
<tr>
<td>7-8</td>
<td>0.0209</td>
</tr>
<tr>
<td>8-9</td>
<td>0.0120</td>
</tr>
<tr>
<td>9-10</td>
<td>0.0079</td>
</tr>
<tr>
<td>10-11</td>
<td>0.0054</td>
</tr>
<tr>
<td>11-12</td>
<td>0.0029</td>
</tr>
<tr>
<td>12-13</td>
<td>0.0016</td>
</tr>
<tr>
<td>13-14</td>
<td>0.00074</td>
</tr>
<tr>
<td>14-15</td>
<td>0.00045</td>
</tr>
<tr>
<td>&gt;15</td>
<td>0.00041</td>
</tr>
</tbody>
</table>

Area A - North Atlantic  
Area B - Combined Atlantic, Mediterranean and Caribbean  
Area C - Combined Pacific
Table 5.11: Frequency of Occurrence of Heading Speed Combinations

<table>
<thead>
<tr>
<th>Frigates and Small Ships (Displacement &lt;10,000 LT)</th>
<th>Speed (Kts)</th>
<th>Heading</th>
<th>Significant Wave Height (m)</th>
<th>0-5</th>
<th>6-10</th>
<th>&gt;10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>Head</td>
<td>0.013</td>
<td>0.025</td>
<td>0.025</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bow</td>
<td>0.025</td>
<td>0.375</td>
<td>0.808</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quartering</td>
<td>0.025</td>
<td>0.050</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Following</td>
<td>0.013</td>
<td>0.025</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>Head</td>
<td>0.088</td>
<td>0.023</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bow</td>
<td>0.175</td>
<td>0.338</td>
<td>0.142</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quartering</td>
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<td>0.045</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Following</td>
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<td>0.023</td>
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<td>0.0</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>Head</td>
<td>0.025</td>
<td>0.0025</td>
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<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bow</td>
<td>0.050</td>
<td>0.038</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quartering</td>
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<td>0.005</td>
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<td>0.0</td>
</tr>
<tr>
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<td></td>
<td>Following</td>
<td>0.025</td>
<td>0.0025</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>High Speed Cargo Ships</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Head</td>
<td>0.010</td>
<td>0.125</td>
<td>0.175</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bow</td>
<td>0.020</td>
<td>0.125</td>
<td>0.175</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quartering</td>
<td>0.020</td>
<td>0.125</td>
<td>0.175</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Following</td>
<td>0.010</td>
<td>0.063</td>
<td>0.088</td>
<td>0.088</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>Head</td>
<td>0.096</td>
<td>0.115</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bow</td>
<td>0.193</td>
<td>0.115</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quartering</td>
<td>0.193</td>
<td>0.115</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Following</td>
<td>0.096</td>
<td>0.058</td>
<td>0.038</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>Head</td>
<td>0.019</td>
<td>0.010</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bow</td>
<td>0.038</td>
<td>0.010</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quartering</td>
<td>0.038</td>
<td>0.010</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Following</td>
<td>0.019</td>
<td>0.005</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Commercial Cargo Ships</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Head</td>
<td>0.010</td>
<td>0.125</td>
<td>0.175</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bow</td>
<td>0.020</td>
<td>0.125</td>
<td>0.175</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quartering</td>
<td>0.020</td>
<td>0.125</td>
<td>0.175</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
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<td>Following</td>
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<td>0.063</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>Head</td>
<td>0.115</td>
<td>0.125</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bow</td>
<td>0.231</td>
<td>0.125</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quartering</td>
<td>0.231</td>
<td>0.125</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Following</td>
<td>0.115</td>
<td>0.063</td>
<td>0.038</td>
<td>0.038</td>
</tr>
</tbody>
</table>
Table 5.12 Operational Conditions in Which Whipping May Occur

<table>
<thead>
<tr>
<th>Head and Bow Seas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement (tn)</td>
</tr>
<tr>
<td>&lt;10,000</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>&gt;10,000</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

**Lifetime Design Load**

The methodology described above was applied to a range of ships and simple expressions for predicting the extreme lifetime bending moment including the effects of whipping. These expressions are:

\[ M_{dh} = M_{sw} + 0.0006L^{2.5}B \]  \hspace{1cm} (5.27)

\[ M_{ds} = M_{sw} + 0.0009L^{1.5}B \]  \hspace{1cm} (5.28)

where

- \( M_{dh} \) = design hog moment (ton-ft)
- \( M_{ds} \) = design sag moment (ton-ft)
- \( M_{sw} \) = stillwater bending moment (ton-ft)
- \( L \) = length of ship (ft)
- \( B \) = breadth of ship (ft)

The equivalent expressions in SI units are as follows:

\[ M_{dh} = M_{sw} + 0.000115L^{2.5}B \]  \hspace{1cm} (5.27)

\[ M_{ds} = M_{sw} + 0.000172L^{1.5}B \]  \hspace{1cm} (5.28)

where \( L \) and \( B \) are in metre units and the moments are given in MNm units.

Implicit in the expressions presented above is a duration of 3600 days. The risk parameter was found to range from 0.03 to 0.08 for the ships considered. Sikora and Beach argue that in contrast to the method developed by Ochi (1987) where the risk parameter is introduced, their method explicitly includes a method for including the loads arising from whipping. The implication is that the actual risk parameter, based on normal wave loads alone, is considerably less.
Limited Duration Design Bending Moment

The expressions given above are for an operating lifetime of 3600 days. Assuming an average of eight second zero-crossing period, this translates to approximately $3.888 \times 10^7$ encounters. Assuming an exponential distribution (i.e., Weibull parameter of unity) the limited duration design bending moments can be shown to be:

$$M_{dh} = M_{rw} + 0.00015L^{2.5}B \left( \frac{\log \frac{1}{n}}{7.59} \right)$$ (5.29)

$$M_{ds} = M_{rw} + 0.000172L^{1.5}B \left( \frac{\log \frac{1}{n}}{7.59} \right)$$ (5.30)

where $n$ is the number of wave encounters.

As stressed elsewhere, and discussed in Section 5.3.6, caution must be applied in reducing the multiplier to account for short periods of duration.

Stress Range Spectrum

The Weibull distribution, described in Section 5.5.1, can be used to express the stress range spectrum. For shape parameters guidance can be taken from Section 5.5.1.

5.6 Commentary

Several methods for computing the expected load and stress range spectrum for arbitrary periods of duration have been presented earlier in this section. The methods presented are believed to represent the best currently available in terms of accuracy, practicability and cost effectiveness. However, the subject of wave loading cannot be regarded as entirely mature; it is an area of active research and it is only relatively recently that direct methods of wave load calculation have been applied in the design environment.
Historically, the primary interest was in the development of methods for predicting the extreme loads a ship would likely experience in its lifetime. More recently, interest in estimating the stress range spectrum has also developed. The latter was not a significant issue in the past because fatigue failures were less of a concern. But with the trend towards lighter, optimized structure and the use of high strength steels, the need to explicitly consider fatigue grew.

A parallel development has been the growth in interest in the application of reliability theory principles to the structural design of ships. In concept at least, reliability theory provides an appealing framework within which to develop design methodologies that rely on direct methods. This is because of the strongly probabilistic nature for the main variables that characterize wave loads on ships. However, the application of these methodologies requires the explicit characterization of all relevant variables. This requires much more data than is typically required using traditional design methods.

The purpose of this commentary is to make the reader aware of the limitations of the methods presented, the implications of the key assumptions and outline some of the progress being made in improving load prediction techniques. The primary interest is in the Level 2 and 3 methods which are based on long-term estimation using spectral techniques. The Level 1 methods are semi-empirical in character and their limitations are evident from the limited data sets upon which their load estimation expressions are based.

The key issues in regard to wave loading in the context of damage tolerance assessment can be categorized as follows:

- Arbitrary assessment periods
- Non-linearities in the wave/load relationship

Each topic is discussed in turn.

5.6.1. Arbitrary Assessment Periods

Virtually all the research in the application of direct methods of wave load calculation has been focused on the lifetime of the ship. In the present context, damage tolerance assessments will be required for shorter periods of duration, perhaps measured in days or weeks. This presents a particular problem because all the methodologies, and the supporting data, have been developed in a framework in which the duration is measured in tens of years.
As discussed by Chen and Shin (1995), the spectral approach is considered the most appropriate method for fatigue calculations, and, by extension it is reasonable to suggest that is also so for damage tolerance assessment. A key advantage of the spectral approach is its flexibility, in principle, in handling durations of arbitrary length.

Having said that, however, it must be noted that the wave climate data, as exemplified by the typical scatter diagram, is a variable quantity itself. This is much less an issue when 20 or 30 years is the time scale of interest. The problem of using typical wave scatter data for limited duration assessments can only be resolved by further investigation. A related issue in applying the spectral approach is the discretization of wave climate into histograms of wave height and period. Changes between sea states of course occur gradually. The impact of discretization on the estimate of extreme load does not appear to have been investigated. Time domain programs could presumably be employed to investigate this aspect.

A Ship Structure Committee project is underway to develop sea operational profiles. A major element of the project is the development of operational profiles for arbitrary periods of duration. The statistical variability of the profiles are also to be established; the intention of this is to provide data for future reliability analyses. The results of this project should alleviate, to some extent, the current uncertainty in regard to wave load estimates for short- and medium-term time scales.

5.6.2 Non-linearities in the Wave-Load Relationship

There are other fundamental assumptions made in applying the spectral method. The principle one is linearity in the wave-load relationship as expressed by response amplitude operators (RAO). This may be acceptable for fatigue damage estimation because, generally, moderate seas cause the most fatigue damage. However, the assumption of linearity in computing extreme wave loads is questionable particularly for fast fine-formed ships.

Response amplitude operators (RAOs) are normally developed through model-scale experiments or numerically using computer programs based on strip theory. Several such programs have been developed over the years. The most general of these programs yield RAOs for ship motion parameters and also bending moments and shear forces. A key assumption in virtually all such programs is that of linearity. In particular, the oscillatory motion is assumed to be linear and harmonic. For ships with a vertical plane of symmetry though the ship centreline the vertical response is assumed to be uncoupled to lateral responses. Despite these severe assumptions linear strip theory generally yields good agreement between numerical prediction and corresponding full- and model-scale measurement for moderate seas. In severe seas, the wave excitation and the ship response are both non-linear. Because of the wall-sided assumption in linear strip theory the predicted hog and sag bending moments are identical; measurements indicate the sag moments are generally numerically greater than the hog moments. The
tendency is for linear strip theory to under-predict the sag moments and over-predict the hog moments.

The prediction of wave loads on ships is an active area of research. As such providing guidance that is generally applicable is problematic. However, it is reasonable to suggest that for slow full-bodied ships the predictions of linear strip theory are adequate. Predicting wave loads on faster, finer ships will, in general, require more sophisticated approaches where phenomena such as slamming may need to be accounted for. These observations apply primarily to estimation of the extreme load. In the case of fatigue the limitations of linear strip theory are much less serious. This topic was investigated by Chen and Shin (1995) and they conclude that there is a "strong argument" that linearity can be assumed for the purpose of fatigue assessment.

Various corrections have been applied to the linear theory to improve prediction in extreme seas. One of the more successful has been the use of second order strip theory. Second order terms arise from the wave excitation and the angle with the vertical of the side of the ship (zero in linear strip theory). Such a theory was developed by Jensen and Pederson (1970). Mansour has developed a simple method for correcting the results of linear strip theory. The method is described in a series of papers and in summary form in a recent report (Mansour, 1996b). Three dimensional panel programs and time domain simulation programs are other approaches into which research is being conducted.

The possibility of slamming exists particularly for faster, finer-formed ships. The Level 1 methods presented account for this phenomenon either explicitly or implicitly. The spectral method is not a convenient framework within which to account for such effects. Specialist programs are available to compute slamming loads. Alternatively, simple multipliers can be applied as described by Mansour et al. (1996a).

The spectral approach is a demanding methodology in terms of data and tools required. It is not always to appropriate, or possible, to employ an analysis of this detail. Therefore, alternative much simpler methods of lead estimation have been presented. The primary limitation that these methods suffer from is the question of their applicability beyond the ship types upon which their expressions are based. While such methods developed from the Class Societies are quite comprehensive, other methods generally only address vertical hull girder bending.
5.7 References


DET NORSKE VERITAS, Structural Reliability Analysis of Marine Structures, Classification Notes No. 30.6, DNV, July, 1992.


6.0 STRESS ANALYSIS

6.1 Introduction

Section 5 has outlined several approaches for estimating the statistical distributions or spectra of ship loads over a given assessment interval. Section 5 has also outlined how maximum local stresses and the spectrum of the local stress range history may be obtained from the combination of the loads distributions. This requires the development of local stress coefficients, $A_i$, which relate the local stresses required for the damage tolerance assessment to the global hull girder bending moments, external sea pressures acting on the hull, and internal pressures acting on the tank boundaries.

The stress coefficients are evaluated by calculating the local field stresses at the point of interest for a unit value of each load component (e.g., vertical, horizontal and torsional bending moment loads, internal and external pressure loads). In general, this will involve conducting stress analyses for unit loads considering each type of loading individually. Strictly, the stress coefficients are a function of wave frequency. However, it appears (Cramer et al., 1995) that it is acceptable practice to compute stress coefficients for one particular wave frequency, and heading for that matter, and apply it to all wave frequencies and/or headings. The total stress spectrum at the location of interest can then be estimated by combining the stress coefficients and loads spectra using the methods outlined in Section 5.

The damage tolerance analysis procedures for residual strength and residual life assessment also require the determination of certain crack driving forces which are used to describe the crack behaviour in fracture and fatigue. Residual strength assessment (Section 3) requires the maximum value of the stress intensity factor ($K_i$) and the local effective net section stress ($\sigma_{en}$) expected within the assessment interval. For a given crack size, the stress intensity factor is proportional to the local stress state at the crack location (see Section 6.4). Hence the maximum stress intensity factor can be related to the maximum stress obtained for the extreme loading condition in the assessment interval, bearing in mind that local residual stresses may have to be taken into account. Residual life assessment (Section 4) requires the stress intensity factor range ($\Delta K$) spectrum which is in turn related to the local stress range spectrum ($\Delta \sigma$) for the assessment interval.

This section outlines methodologies to determine the local stresses from unit loads (hence determining the stress coefficients), as well as methods of determining the effective net section stresses and stress intensity factors required for damage tolerance assessments.
6.2 Definition of Stress Categories

Damage tolerance assessment procedures for fatigue and fracture require knowledge of the stress field local to the crack. The stresses to be considered may be treated directly, or after resolution into four components as shown in Figure 6.1 and described below.

![Figure 6.1: Stress Components in a Welded Joint](image)

a) Local Nominal Membrane and Bending Stress \((\sigma_m \text{ and } \sigma_b)\): The local nominal membrane stress is the uniformly distributed stress which is equal to the average value of stress across the section thickness. The local bending stress is the component of nominal stress due to applied loading which varies linearly across the section thickness. The nominal stresses satisfy the simple laws of equilibrium of forces and moments from applied loads. They may be derived from simple
formulae, beam element models, or coarse mesh finite element analysis (FEA) as described in Section 6.3.2. The term local nominal stress is used because stress concentrations resulting from the gross shape of the structure surrounding the local detail under consideration will affect the magnitude of the local field stresses (e.g. shear lag effects) and must be included in the local nominal stresses.

b) Peak Stress ($\sigma_p$) : is the component of stress due to applied loads due to stress concentrations at local discontinuities in the vicinity of the crack. The peak stress represents the highest value, usually at the surface at a notch (e.g., weld toe). Peak stresses arise from stress concentrations due to the following effects:

1) Geometric Stress Concentrations ($K_g$) : due to the gross geometry of the detail considered. The effect of the geometric stress concentration typically decays over distances of the order of the section thickness.

2) Notch Stress Concentrations ($K_n$) : due to the local geometry of the notch (e.g. weld geometry). The effect of the notch stress concentration typically decays over distances of the order 10% to 20% of the section thickness.

3) Misalignment Stress Concentrations ($K_{te}, K_{ta}$) : due to bending stresses caused by misalignments including eccentricity tolerance ($K_{te}$), and angular mismatch ($K_{ta}$). These are normally used for plate connections only. The effect of the misalignment stress concentrations typically decay over distances of the order of the section thickness.

c) Residual Stresses ($\sigma_r$) : are local self-equilibrating stresses that arise from fabrication and welding. In general, residual stresses are strain/displacement limited phenomena and, as such, do not contribute to plastic collapse if they relax. However, they do add to the tensile stress field in the vicinity of the crack and have to be included in the calculation of the stress intensity factor for residual strength assessment. Residual stresses may also be resolved into membrane and bending components. However, since there is only limited quantitative data on the distribution of welding residual stresses in ship structural details, it is normal practice to assume a uniform (membrane) residual stress field near tensile yield strength (i.e., $\sigma_t = \sigma_y$). This is discussed further in Section 6.3.4.

d) Total Stress : is the total sum of the various stress components. The maximum value of total stress at the crack location is referred to as the peak total stress ($\sigma_t$). The peak total stress can be evaluated by:

$$\sigma_t = \sigma_m + \sigma_g + \sigma_p + \sigma_r$$

$$= K_g \cdot K_w \cdot (K_{te} \cdot K_{ta} \cdot \sigma_m + \sigma_p) + \sigma_T$$

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The Level 1 residual strength assessment procedure is based on this value of total peak stress which, for the purposes of the assessment, is conservatively assumed to be uniformly distributed through the plate thickness. Other levels of assessment for fracture and fatigue require taking into account the variation of stress through the load bearing section containing the crack as discussed in Section 6.4.

The nominal membrane, bending and peak stress components due applied loadings (excluding residual stresses) may be derived, for a given stress distribution $\sigma(x)$ for $x=0$ at the surface to $x=B$ at through thickness, by the following analytical expressions (Hobbacher, 1993):

\[
\sigma_m = \frac{1}{B} \int_0^B \sigma(x) \cdot dx
\]

\[
\sigma_b = \frac{6}{B} \int_0^B \sigma(x) \cdot \left( \frac{B}{2} - x \right) \cdot dx
\]

\[
\sigma_p(x) = \sigma(x) - \sigma_m - \sigma_b(x)
\]

6.3 Determination of Stresses and Stress Coefficients

6.3.1 General

As with other elements of the calculations there are a number of approaches with varying degrees of complexity and accuracy that can be used to calculate the stresses (or stress coefficients) required for damage tolerance assessments. The approach employed should, in general, be consistent with the complexity and accuracy applied to other elements of the assessment process.

As previously mentioned, the Level 1 fracture assessment is simply based on the maximum total stress ($\sigma_1$) at the crack location as defined in Equation 6.1 and Figure 6.1. Since Level 1 is suitable for a basic screening assessment, it is preferable that the stress analysis be kept as simple as possible. The simple approach is to calculate nominal global stresses at the stations of interest using the computed hull girder bending moments and shear forces and the relevant sectional properties. Estimates of stress can be improved somewhat to account for gross effects such as shear lag, openings in decks and the effect of the superstructure using various rules of thumb. Hughes (1983) discusses methods of accounting for some of these effects. The total stress at the crack site can then be estimated from available stress concentration factors (i.e., $K_w$, $K_z$, $K_e$ and
K_{rc}) for ship details. Alternatively, a combination of coarse mesh finite element methods and available stress concentration factor (SCF) solutions may be used to calculate the maximum total stress.

The higher levels of assessments for fatigue and fracture require a more accurate description of the actual stress distribution. When published solutions are used for evaluating stress intensity factors (SIF) (see Section 6.4.2.2), it is usually sufficient to determine the local nominal membrane and bending stress components (i.e., \( \sigma_m \) and \( \sigma_b \)), and residual stresses (i.e., \( \sigma_{rm} \) and possibly \( \sigma_{rs} \)). The effects of the local structural geometry (i.e., \( K_g \) and \( K_w \), but not stress concentrations due to misalignments) are normally taken into account by the stress intensity magnification factor (\( M_k \)) used to determine the SIF. The \( M_k \) factor is defined as

\[
M_k = \frac{K_{ic} \text{ for crack in welded detail}}{K_{ic} \text{ for same crack in a flat plate}}
\]  

(6.5)

In the limit, for crack depths approaching zero, it can be shown that the \( M_k \) factor is equal to the product of the notch and geometric SCF.

\[
M_k = K_w \cdot K_g \quad \text{as crack depth } a \to 0
\]

(6.6)

The local nominal stresses may be calculated based on global nominal stresses and available factors for global stress concentrations and misalignment effects. Alternatively, coarse mesh FEA which accounts for gross stress concentrations and secondary bending stresses may be used to derive local nominal stresses at the crack location. Where \( M_k \) factors are available, it is not necessary to model the local geometry of the detail in the coarse mesh FEA.

Where appropriate SIF and \( M_k \) solutions are not available, finite element or weight function methods can be used. When weight function methods are used, the actual stress distribution due to applied loads may be derived from local fine mesh FEA of the uncracked detail, upon which an assumed residual stress distribution can be superimposed to calculate the SIF. When finite element methods are used to compute the SIF, the local detail including the crack is modelled. The actual total stress distribution at the crack location is therefore accounted for directly in the calculation of the SIF, although it is usually difficult to include residual stresses in the FEA. The application of weight function and finite element methods for determining stress intensity factors is described in Section 6.4.2.

6.3.2 Determination of Local Nominal Stresses

As discussed in the previous section, the local nominal stresses are defined as the stresses that would be calculated in the section containing the crack, in the absence of the crack and the stress concentration due to the local structural detail and weld; that is, the local nominal stresses
include the stress concentration effects of the overall geometry of the structure surrounding the detail, but not the detail itself.

The local nominal stresses may be calculated, for unit loads, using a combination of parametric formulae for simple structural assemblies and global stress concentration factors to account for the gross geometry of the structure and effects of misalignment. Alternatively, frame models or coarse mesh global FEA may be used to obtain a more precise estimation of the local nominal stresses. The following subsections outline methodologies for evaluating the local nominal stresses using these approaches.

### 6.3.2.1 Simplified Analysis

Calculation of hull girder stresses is the simplest way of getting reasonable approximations to the stress levels in longitudinal hull girder elements and connections and can be used for quick evaluation of stress levels in important details. This approach is most suitable for a Level 1 screening assessment. Global hull girder stresses may be calculated based on gross scantlings. Local stress components should be calculated based on net scantlings, i.e. gross scantlings minus corrosion allowances.

Formulae for calculating hull girder stresses are included in Classification Society Rules. Alternatively, the following formulae derived from formulae presented by Cramer et al. (1995) may be used.

**Vertical Hull Girder Bending**

\[
\sigma_{m,v} = K_G \cdot M_v \cdot z / I_v
\]  

(6.7)

where

- \(\sigma_{m,v}\) = nominal hull girder membrane stress due to vertical bending
- \(M_v\) = vertical (sagging or hogging) bending moment amplitude at the location under consideration (taken as unity for evaluation of stress coefficients)
- \(z\) = vertical distance from the neutral axis of the hull cross section to the location under consideration.
- \(I_v\) = moment of inertia of the hull cross section about the transverse neutral axis.
- \(K_G\) = global stress concentration factor to account for gross structural geometry (e.g., hatch openings, shear lag) that affects the local nominal stress field.
Horizontal Hull Girder Bending

\[ \sigma_{m,h} = K_G \cdot M_h \cdot y / I_h \]  

(6.8)

where

\[ \sigma_{m,h} = \text{nominal hull girder membrane stress due to horizontal bending.} \]
\[ M_h = \text{horizontal bending moment amplitude at the location under consideration.} \]
\[ y = \text{transverse distance from the neutral axis of the hull cross section to the location under consideration.} \]
\[ I_h = \text{moment of inertia of the hull cross section about the vertical neutral axis.} \]

Stresses due to Internal and External Pressure Loads

Local secondary bending stresses are the results of bending, due to lateral pressure, of stiffened single skin or double hull cross-stiffened panels between transverse bulkheads (see Figure 6.2). This may be bottom or deck structures, sides or longitudinal bulkheads.

The preferred way of determining secondary stresses is by means of FEA or alternatively frame analysis models. Alternatively, secondary bending stresses may be estimated from parametric equations such as the following equations recommended by Cramer et al. (1995). Similar equations are given in the American Bureau of Shipping’s “Guide for the Fatigue Assessment of Tankers (ABS, 1992).

a) Longitudinal Secondary Bending Stress in Double Bottom Panels

Longitudinal secondary bending stresses in double bottom panels at the intersection of transverse bulkheads may be estimated by the following formulae.

**Double Bottom Wider than Long (b > a): Case 1 and 2, Table 6.1**

\[ \sigma_m = (k_6 \cdot \rho \cdot b^2 \cdot r_t) / (i_a \cdot i) \]
\[ \rho = (a/b) \cdot (i_h / i_o)^{1/4} \]  

(6.9)

**Double Bottom Longer than Wide (a > b): Case 3 and 4, Table 6.1**

\[ \sigma_m = (k_6 \cdot \rho \cdot a^2 \cdot r_t) / i_a \]
\[ \rho = (b/a) \cdot (i_h / i_o)^{1/4} \]  

(6.10)

where

\[ k_6 = \text{coefficient dependant on the aspect ratio } \rho, \eta=1, \text{ and actual boundary condition in Table 6.1.} \]
\( p \) = effective lateral pressure - taken as unity for evaluation of stress coefficients.

\( r_a \) = distance from point considered to neutral axis of the panel

\( a \) = longitudinal length of double bottom panel (Figure 6.3)

\( b \) = transverse width of double bottom panel.

\( i_a \) = smeared out stiffness per girder about transverse neutral axis of double bottom

\( I_a \) given by \( i_a = I_a/s_a \) (see Table 6.3)

\( i_b \) = smeared out stiffness per girder about longitudinal neutral axis of double bottom

\( I_b \) given by \( i_b = I_b/s_b \) (see Table 6.3)

\( I_a, I_b \) = section moduli of inertia about the double bottom neutral axis in transverse and
longitudinal axes respectively, including effective width of plating.

\( s_a, s_b \) = spacing between girders in longitudinal and transverse directions respectively.

b) Transverse Secondary Bending Stress in Double Bottom Panels

Transverse secondary bending stresses in double bottom panels at the intersection of
transverse bulkheads may be estimated by the following formulae.

**Double Bottom Longer than Wide (\( a > b \)) : Case 3 and 4, Table 6.1**

\[
\sigma_m = \frac{(k_b \cdot p \cdot b^2 \cdot r_b)}{i_b} \\
\rho = \frac{(a/b)}{(i_b/i_a)^{1/4}} \tag{6.11}
\]

**Double Bottom Wider than Long (\( b > a \)) : Case 1 and 2, Table 6.1**

\[
\sigma_m = \frac{(k_b \cdot p \cdot b^2 \cdot r_b)}{i_a} \\
\rho = \frac{(b/a)}{(i_b/i_a)^{1/4}} \tag{6.12}
\]

c) Secondary Bending Stress in Single Skin Panels

The stresses at transverse and longitudinal bulkheads may be estimated from the same
formulae as for double bottom configurations. However, the parameters \( \rho \) and torsion coefficient
\( \eta \) should be taken as given in Table 6.2 (also see Table 6.3).

d) Bending Stress of Stiffeners Between Transverse Supports (e.g. Frames, Bulkheads)

The local bending stress of stiffeners with effective flange between transverse supports may
be estimated by:

\[
\sigma_b = \frac{(M)}{(Z_s)} + \frac{(6 \cdot E \cdot l / 12 \cdot Z_s)}{\delta} \tag{6.13}
\]
where

\[ M = \text{moment at stiffener support} = \left( p \cdot s \cdot 1^2 \right) / (12 - r_p) \]

\[ p = \text{lateral pressure (external or internal pressure load)} - \text{taken as unity for evaluation of stress coefficients).} \]

\[ s = \text{stiffener spacing} \]

\[ l = \text{effective span of stiffener or longitudinal} \]

\[ Z_1 = \text{section modulus of stiffener or longitudinal} \]

\[ I = \text{moment of inertia of stiffener or longitudinal with associated effective plate width} \]

\[ \delta = \text{deformation of nearest frame relative to transverse bulkhead} \]

\[ r_s, r_p = \text{moment interpolation factors for interpolation to crack location along stiffener length} \]

\[ r_s = 1 - 2(x/l) \]

\[ r_p = 6(x/l)^2 - 6(x/l) + 1 \]

\[ x = \text{distance from end of stiffener to crack location}. \]

It is of great importance for reliable assessments that bending stresses in longitudinals caused by relative deformation between supports are not underestimated. The appropriate value of relative deformation, \( \delta \), has to be determined for each particular case (Figure 6.4). This usually will require 2-D or 3-D frame analysis or coarse mesh FEA.

e) Tertiary Bending Stress of Plates Bounded by Stiffeners

The local longitudinal tertiary plate bending stress in the weld at the plate/transverse frame/bulkhead intersection midway between longitudinals is given by:

\[ \sigma_b = 0.343 \cdot p \cdot (s / t_n)^2 \]  \hspace{1cm} (6.14)

where

\[ p = \text{lateral pressure (external or internal pressure load)} - \text{taken as unity for evaluation of stress coefficients}. \]

\[ s = \text{stiffener spacing} \]

\[ t_n = \text{net plate thickness} \]

Similarly the transverse stress at stiffener mid length is:

\[ \sigma_b = 0.5 \cdot p \cdot (s / t_n)^2 \]  \hspace{1cm} (6.15)
Figure 6.2: Simplified Stress Analysis of Hull Girder (Cramer et al, 1995)
Figure 6.3: Definition of Geometric Parameters for Hull Configurations (Cramer et al. 1995)
Figure 6.4: Stresses in Stiffener (Cramer et al, 1995)
Table 6.1: Support Bending Stress Coefficients $k_c$ - Double Bottom Panels
(Cramer et al, 1995)

(For intermediate values, use linear interpolation)

<table>
<thead>
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<th>Case no. &amp; Stress location</th>
<th>Boundary conditions</th>
<th>$\rho$</th>
<th>$\eta = 0.0$</th>
<th>$\eta = 0.5$</th>
<th>$\eta = 1.0$</th>
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<tr>
<td>direction at middle of short end</td>
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Table 6.2: Support Bending Stress Coefficients $k_b$ - Single Skin Panels
(Cramer et al, 1995)

(For intermediate values, use linear interpolation)

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<tr>
<th>Case no. &amp; Stress location</th>
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<th>$\rho$</th>
<th>$\eta=0.0$</th>
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Table 6.3: Definition of Stiffness and Geometry Parameters
(Cramer et al, 1995)

<table>
<thead>
<tr>
<th>Type</th>
<th>Sketch</th>
<th>Formulas for $\rho$ and $\eta$</th>
</tr>
</thead>
</table>
| A: Cross stiffening | ![Sketch](image) | $i_s = \frac{I_{ax}}{s_a} + 2 \left( \frac{I_a - I_{ax}}{b} \right)$  
$\rho = \frac{a_s}{b} \sqrt{\frac{i_s}{i_a}}$  
$\eta = \frac{I_{p0} I_{p0}}{I_{ax} I_{ax}}$ |
| Middle girder /  
stiffener in  
both directions are  
stiffer than the  
others | | |
| B: Modified cross  
stiffening | ![Sketch](image) | $i_s = \frac{I_{ax}}{s_a} + 2 \left( \frac{I_a - I_{ax}}{a} \right)$  
$\rho = \frac{a_s}{b} \sqrt{\frac{i_s}{i_a}}$  
$\eta = 0.124 \sqrt{\frac{I_{p0}^2}{I_{ax} I_{ax} s_b}}$ |
| One girder /  
stiffener in  
a-direction only | | |
| C: Single stiffening | ![Sketch](image) | $i_s = 0$  
$i_s = \frac{I_{ax}}{s_b}$  
$\rho = \infty$  
$\eta = \text{indeterminate}$ |
| Girders / stiffeners  
in b-direction only | | |
| D: Unstiffened plate | ![Sketch](image) | $i_s = \frac{a}{b}$  
$\eta = 1.0$ |

Guide to Damage Tolerance Analysis of Marine Structures
6.3.2.2 Finite Element Analysis

In more critical assessments, or where the global structure is too complicated for simple parametric formulae, finite element analysis (FEA) may be used to obtain a reliable description of the overall stiffness and global stress distribution in the hull.

The global FEA is generally carried out with a relatively coarse mesh, the main objective being to obtain a good representation of the overall membrane panel stiffness in the longitudinal and transverse directions and for shear, sufficient for determination of nominal stresses. Stiffened panels may be modelled by means of anisotropic elements or, alternatively, using a combination of plate and beam elements.

The extent of the model is dependent on the type of response to be considered and the structural arrangement of the hull. In damage tolerance assessments, the local region to be considered is dependent on the stiffness variation of the hull over a certain length and this has to be captured in the global FEA model.

For horizontal and torsional bending response of the hull of an open hatch ship, it is generally required that the extent of the global model covers the complete hull length, depth and breadth (a half breadth model may be used if antisymmetric boundary conditions can be assumed at the centerline). A complete finite element model may also be required for the evaluation of vertical hull girder bending of ships with complex arrangements of superstructures (e.g., warships, passenger ships), and for ships of complex cross-section (e.g., catamarans).

Alternatively, a part of the hull (for example, the midship area) may be modelled. Hull girder loads should be applied individually at each end of the model to result in a value of unit load (e.g., bending moment) at the location of interest. Unit pressure loads will normally be distributed over the appropriate section of the hull. The loads should be balanced in order to give a minimum of reaction forces at the supports (boundary conditions). The loads and boundary conditions in the hull cross section should be evaluated carefully when modelling only a part of the hull to avoid unrealistic stiffness from the forebody/afterbody.

Figure 6.5 shows an example of a global finite element model of a section of a bulk carrier. This model may be used to calculate nominal global stresses and deformations away from areas with stress concentrations. In areas where local stresses in web frames, girders or other areas (for example hatch corners) are to be considered, the global model should have a mesh producing deformations applicable as boundary conditions for local stress analysis. In such cases the global and local models should be compatible. The local model may be directly applied as a substructure or super-element in the global model (if such techniques are available with the FEA software). The substructure technique ensures that forces and deformations in the global and local models are compatible and, if the substructure is detailed enough, local stress results may be obtained directly.
The substructure technique is very effective where local structural assemblies (i.e., the substructure) are repeated several times in the overall assembly, but it does present added complexity into the analysis.

More commonly, the global and local analyses are conducted separately. Nodal forces and/or displacements obtained from the global model are applied as boundary conditions for the local model. In general the stiffness of the local model should be comparable to that of the global model representation so that forces and displacements between the two models are compatible. However, due to the greater level of geometric detail and mesh refinement of the local model, this is rarely achievable. As such it is preferable that nodal forces be transferred from the coarse model to the local model rather than forced displacements. It is important that the extent of the local model is sufficiently large that boundary effects due to prescribed forces or displacements are away from the areas where accurate stresses need to be determined.

The loads to be applied in the global analysis can be produced using any of the methodologies presented in Section 5. The global analysis should be conducted for each load case (i.e., vertical bending, horizontal bending, torsional bending, external pressure, internal pressure) individually. Each load case should be analyzed for a unit value of the applied load at the location being considered. In this manner, the stresses derived from subsequent local analysis will correspond to unit loading and therefore be equal to the stress coefficients, $A$, which are required to generate the local stress spectrum from the combined loading spectra.
6.3.3 Determination of Peak Stresses

Peak stresses may be estimated based on parametric approximations of stress concentration factors for ship details, when these are available. Alternatively, they may be determined based on local fine mesh FEA stress analysis of the joint.

6.3.3.1 Stress Concentration Factors for Ship Details

Stress concentration factors (SCF) for a range of details typical of ship structures are given by Stambaugh et al (1994), ABS (1992), Cramer et al (1995), and Yoneya et al (1992) for example. Appendix A presents some solutions for notch stress concentrations \( K_n \). Stress concentration factors for typical ship structural details \( K_p \) and for misalignment effects \( K_{\text{mis}}, K_{\text{w}} \) are presented in Appendix B.

The analyst must exercise extreme care when applying stress concentration factors from different sources to ensure that the correct definitions for nominal stress used. For example, in some cases the nominal stress is defined at the intersection point of a connection, in other cases the global nominal stress may be defined at the weld toe or some distance from the weld toe.

Furthermore, the analyst should be aware that sometimes the published stress concentration factor solutions are designed to calculate the "hot spot" stress or the "notch" stress as opposed to the local nominal stress. The analyst should make certain which form of peak stress will result from the application of the SCF.

6.3.3.2 Local Finite Element Analysis

If appropriate stress concentration factors are not available, the total stress distribution including local peak stresses may be calculated by local FEA. The crack itself is usually not modelled unless the local FEA is going to be used to calculate stress intensity factors directly (see Section 6.4.2.4). As discussed previously in the section on global FEA, the extent of the local model should be large enough that the calculated results are not significantly affected by assumptions made for boundary conditions and application of loads.

Figure 6.5 shows a local finite element model of a ship detail. The local model should have a relatively fine mesh, especially in areas of stress concentration. It is important to have a continuous and not too steep change in the density of the element mesh in the areas where the local stresses are to be analyzed. The geometry of the elements (aspect ratio, corner angles, skewness and warp) at the point of interest should be as near optimal as possible (for example: length/breadth aspect ratio less than 2, corner angles between 60° and 120°, avoid use of triangular elements with reduced order shape functions).
Local FEA of a joint is usually conducted to determine the local nominal and hot spot stress at the location of interest, and seldom for direct evaluation of peak notch stress since the weld geometry itself is usually not modelled. If the peak notch stress has to be determined (i.e. for a Level 1 assessment), then the most common approach is to use local FEA to evaluate the hot spot stress. The hot spot stress value is then factored by a weld notch factor, $K_w$, derived from parametric equations or tables (see Appendix A) to provide an estimate of the peak notch stress in the joint.

Requirements for element size at the hot spot region is dependent on the type of element. The mesh size may be determined based on experience or by benchmark testing a similar mesh for a case where results have been presented in the literature. Figure 6.7 provides some guidance on element sizes for 20-node solid, 8-node shell and 4-node shell element types suitable for determining the hot spot stress.
Figure 6.7: Examples of Local Detail FEA With Recommended Element Sizes
(Cramer et al, 1995)
Normally the element stresses are derived at the gaussian integration points. Depending on the element type, it may be necessary to perform several extrapolations in order to determine the stress at the weld toe. Reference is made to Figure 6.8, principal stresses are used for the extrapolation. First an extrapolation of the stresses at the gauss points to the surface is performed. Then a further extrapolation of these stresses to the line A-B is conducted. It is assumed that the final extrapolation of stresses is performed by linear extrapolation of surface stresses at line A-B at a distance t/2 and 3t/2 from the weld toe.

If FEA is to be used to determine the notch stress, then it should be realised that an extremely fine mesh will be required in order to obtain accurate stresses (much more so than that required for the determination of hot spot stresses). The notch is a relatively severe form of stress concentration and stresses rise very rapidly as the notch root is approached. For example, the calculated stress in a linear elastic analysis of a right angle corner will approach infinity as the element size is decreased to zero. Therefore the local (micro) geometry of the notch (i.e., weld toe radius, angle, etc.) has to be included in the model to obtain reasonable stresses that account for this geometry. Since the notch radius is typically of the order of 1 mm (0.04") and at least one node per 15° of the notch arc radius is required for accurate stresses, a considerable degree of mesh refinement is required which results in a relatively large computer model. Some advantage can be taken by the fact that the effect of the notch on the stresses is very localized, typically only affecting stresses within 10% of the plate thickness (B) at a weld toe. The mesh need not be as refined outside this region, however care must be taken to ensure that the transition from the less refined region to the fine mesh region at the notch is smooth and does not affect the results of interest. Elements within 10% B of the weld toe should be as close to optimal shape as possible.

The stresses obtained from a 2D or 3D local FEA of a joint containing a notch are not evenly distributed through the plate thickness direction. The total notch stress can be separated into different components $\sigma_m$, $\sigma_b$, and $\sigma_p$ using Equations 6.2 - 6.4.
Figure 6.8: Stress Distribution at an Attachment and Extrapolation of Stresses at Hot Spot (Cramer et al, 1995)
6.3.4 Residual Stresses

Residual stresses caused by welding and fabrication are self equilibrating stresses necessary to satisfy compatibility in the structure. These stresses in themselves do not cause plastic collapse since they arise from strain/displacement limited phenomena, and therefore do not influence the abscissa in the FAD (S, or Lc). However, residual stresses do add to the crack driving force and therefore have to be included in the calculation of $K_{\text{app}}$ for residual strength assessments. Residual stresses need not be considered for fatigue assessments since they are accounted for in the constants for fatigue crack growth law.

Ideally, one would establish the residual stress magnitude based on actual measurements and resolve them into their membrane and bending components (i.e. $\sigma_{\text{rm}}$ and $\sigma_{\text{rb}}$). However that is impractical and therefore conservative estimates of residual stresses based on findings in the technical literature and on the location of the flaw (weld zone or base metal) and orientation with respect to the weld, are incorporated in the analysis.

The following guidelines can be used to estimate the magnitude of residual stresses to be incorporated into the residual strength assessment.

Level 1 FAD

- In the as welded condition, and with the flaw plane transverse to the weld axis, tensile (weld longitudinal) residual stress is assumed to be the room temperature yield strength of the material in which the flaw tips are located. However, once the flaw tips grow out of the weld metal and the heat affected zone, and are about one plate thickness from the weld fusion line, the weld longitudinal stresses become compressive and may be neglected. For flaw planes parallel to the welding direction, the tensile (weld transverse) residual stress is assumed the lesser of the yield strengths of the base metal and the weld metal.

- If the welded assembly has been uniformly heated and cooled for a post-weld heat treatment (PWHT) to affect stress relief, then the residual stresses parallel to the weld (for flaws that are transverse to the weld axis) are assumed to be 0.3 times the weld metal yield strength. The residual tensile stresses after PWHT in a direction perpendicular to the weld are suggested to be 0.2 times the weld metal yield strength.

Level 2 FAD

- If the actual distribution of residual stresses is known, then these can be incorporated by linearizing the distribution such that the assumed residual stresses are greater than the actual (measured) stresses over the flaw depth. The linearized residual stress distribution can then be separated into its membrane and bending components.
• A reasonable estimate of residual stresses can be based on some typical residual stress distributions given in PD6493 (1991) for butt, fillet and pipe welds (see Figure 6.9). Parametric equations have been developed corresponding to these distributions and their use can reduce the conservatism in the assumption of "yield strength residual stresses in as-welded joints". Still, the use of these parametric equations pre-supposes some knowledge of the weld joint restraint during fabrication.

• The most conservative approach remains the assumption of uniform, yield strength level residual stresses as in the Level 1 analysis.

If the net section stress is deemed high enough to cause plasticity at the crack tips, a certain amount of residual stress relief occurs and the residual stress can be appropriately reduced to the minimum of:

a) $\sigma_y$
b) $\sigma_r$ based on approximate distributions
c) $(1.4 - \sigma_y/\sigma_r) \sigma_y$ for Level 2a FAD
d) $(1.2 - \sigma_y/1.2\sigma_y) \sigma_y$ for Level 2b or 3 FAD’s

The evaluation of net section stress, $\sigma_n$, is presented in Section 6.6. Clearly, the net section stress must be of the order of 50% of the yield strength in order to get any residual stress relief due to plasticity.

When the flaw tips are in the base metal and away from the weld (2 to 3 plate thicknesses), then the weld residual stresses are negligible. However, there are some longer range assembly and construction stresses that still may be present. These may be relieved to some extent with service (shake down effect) or as the crack grows. However, this effect is difficult to predict and therefore, as a conservative measure, longer range residual stresses equal to 20% of the yield strength are recommended to be included in the damage tolerance analysis.
(a) Variation through-thickness of base plate of transverse residual stresses at toe of fillet or T-butt weld

(b) Idealized distribution of transverse residual stresses at toe of fillet or T-butt weld

For carbon manganese steel

\[ y = \left( \frac{122 \phi}{\sigma_f} \right)^{0.5} \]

where \( \phi \) is the heat input of the weld run adjacent to flaw (in J/mm), \( y \) (in mm), \( \sigma_f \) (in N/mm²) and \( y < B \). If \( y > B \), assume residual stress \( \sigma_y \) throughout thickness.

Then for \( a < y \)

\[ Q_m = \sigma_y \left[ 1 - \left( \frac{B}{2y} \right)^{3} \right] \]

\[ Q_b = \sigma_y \left( \frac{B}{2y} \right)^{3} \]

and for \( a > y \)

\[ Q_m = 0 \]

(c) Variation through-thickness of transverse residual stresses at unrestrained butt weld

(d) Variation through-thickness of transverse residual stresses at butt weld with bonding restraint

Second side welded

First side welded

(e) Variation through-thickness of transverse residual stresses at butt weld with membrane restraint

(f) Variation across panel of longitudinal residual stresses at butt weld

Figure 6.9: Typical Distributions of Residual Stresses at Welds (PD6493, 1991)
6.4 Determination of Stress Intensity Factors

A key requirement of local damage tolerance assessment for fatigue and fracture is the ability to evaluate stress intensity factors (SIF) for ship structural details containing cracks. The following sections review the basic concept of the SIF, and present methods which can be used to calculate SIF's for damage tolerance assessments.

6.4.1 General Concepts

The rigorous derivation of the SIF can be found in most advanced texts on fracture mechanics and so only a brief overview will be presented here. A crack represents a very sharp notch (ie. notch radius - 0) and in an ideal elastic body the stresses approach infinity at the crack tip. By studying the conditions near the tip of a crack in an elastic body, it can be shown that the stress and displacement fields can be expressed in terms of three elastic SIF's corresponding to the three modes of fracture (Figure 6.10) : $K_1$ for Mode I (Opening Mode), $K_II$ for Mode II (Sliding Mode), and $K_III$ for Mode III (Tearing Mode). Any crack problem can be considered to be a combination of these three basic modes of fracture. However, since there is always a tendency for a brittle fracture to propagate in the direction which minimizes the shear loading, the first mode is generally regarded as the most important.

The SIF may be described as the amplitude or strength of the stress singularity at the crack tip (Figure 6.11) and includes the influence of loading, crack size, and structural geometry. Since the SIF governs the magnitude of the forces acting in the crack tip region, it plays an essential role in the prediction of brittle strength of bodies containing cracks. The mode I SIF, $K_1$, can be correlated to the onset of fracture in brittle materials when it reaches some critical value, denoted $K_{IC}$, referred to as the plane strain fracture toughness of the material. The cyclic SIF range, $\Delta K$, has also been determined to correlate fatigue crack growth.

Figure 6.10: Three Modes of Cracking (Almar-Naess, 1985)
The use of the SIF to define the fatigue and fracture behaviour of cracks is the basis of Linear Elastic Fracture Mechanics (LEFM). LEFM follows a similitude approach where identical crack growth and fracture behaviour are assumed to occur for cracks having the same value of SIF. The theoretical basis for LEFM can be justified for brittle materials from thermodynamic arguments. Extension of these arguments to more ductile materials, such as steels used in ship construction, requires simplifying assumptions.

In ductile materials, some non-linear plastic deformation occurs in the highly stressed crack tip region. Provided this plastic zone is "small" in relation to the crack size and well contained within an elastic stress field, the stresses outside this zone will still resemble the K-field stress singularity, and the LEFM approach will suffice to describe the crack behaviour. This condition is generally satisfied in fatigue problems, where cyclic stresses remain well within the elastic range (generally the case for ship structures). In such cases, the LEFM approach based on ΔK can be used for predicting fatigue crack growth behaviour as outlined in Section 4 for residual life assessment.

However, elasto-plastic conditions usually dominate the fracture behaviour of ship steels at service temperatures, and direct application of LEFM is generally inappropriate. Increasingly attention has focussed on the limitations of LEFM to characterize ductile tearing. Other fracture mechanics theories have also attempted to describe the crack behaviour in terms of a single parameter which accounts for the nonlinear plastic deformation occurring at the crack tip (eg. J-integral, crack-tip opening displacement, energy release rate). These approaches, generally referred to as Elastic-Plastic Fracture Mechanics (EPFM), continue to be the subject of intense investigations and correlations but, due to added complexity, are not as widely used as LEFM approaches.

The FAD approach (presented in Section 3 for residual strength assessment) provides an alternative, and convenient, method of assessing fracture behaviour using linear stress and LEFM
analysis techniques. The vertical axis of the FAD measures the propensity for brittle fracture using the ratio of the applied crack driving force to material fracture toughness, whereas the horizontal axis of the FAD measures the propensity for stable tearing and plastic collapse using the ratio of the net section stress to the yield strength or flow strength of the material. The failure assessment curve (FAC) represents critical combinations of these ratios.

6.4.2 Methods of Calculating Stress Intensity Factors

Various techniques are available to calculate stress intensity factors. However, the method used should be consistent with the level of assessment being applied. The PD6493 Level 1 approach for calculating SIF, outlined in Section 6.4.2.1, should be used when applying the Level 1 FAD for residual strength assessment. For other levels of assessment, techniques including numerical methods (e.g., finite element analysis, boundary element analysis) and weight function approaches, can be used to directly calculate $K_I$ at a particular point along a crack front for a given applied stress range, crack size and shape, and structural configuration. When time and resources do not permit the direct calculation of $K_I$, estimates can be obtained using handbook solutions for simplified geometries and loadings which most closely resemble the actual conditions at the crack location.

A discussion of the various techniques for calculating $K_I$ is presented in the following sections. Key parameters in the following discussion are graphically defined in Figure 6.12. A selection of SIF solutions for basic plate and weld joint configurations are presented in Appendix C.

![Figure 6.12: Definition of Parameters for Evaluating Stress Intensity Factors](image)

6.4.2.1 PD6493 Level 1 - Peak Stress Method

The PD6493 (1991) Level 1 FAD for residual strength assessment (see Section 3) is suitable
for a basic screening assessment. It uses an approximate approach for estimating SIF's assuming the maximum total stress ($\sigma_1 = \sigma_m + \sigma_b + \sigma_r + \sigma_p$) is applied as a uniform stress at the crack location and applying the SIF solution for a crack in a finite plate. No account is taken of the local stress profile through the section and, since the maximum or peak value of tensile stress is used, the estimated SIF is generally quite conservative. The basic $K_{app}$ solutions for through thickness and partial thickness cracks are given as follows.

a) Through Thickness Cracks

\[
K_1 = \sigma_1 \sqrt{\frac{\pi a}{\ell}} \cdot f_w
\]  

\[
\sigma_1 = \sigma_m + \sigma_b + \sigma_r + \sigma_p
\]

\[
= K_w \cdot \sigma_{HS} + \sigma_r
\]

\[
= K_w \cdot K_g \cdot (K_{m,1} \cdot \sigma_m + \sigma_b) + \sigma_r
\]

\[
= K_G \cdot K_w \cdot K_g \cdot (K_{m,1} \cdot \sigma_m + \sigma_b) + \sigma_r
\]

where

$2a =$ flaw length.

$f_w =$ finite width correction for flaws greater than 10% of the plate width

$= \{\sec(\pi a/W)\}^{0.5}$

$W =$ width of the load bearing section containing the crack.

b) Partial Thickness Flaws (Elliptical Embedded or Semi-Elliptical Surface)

\[
K_1 = Y_m \sigma_1 \sqrt{\frac{\pi a}{\ell}} \cdot f_w
\]  

where

$Y_m =$ flaw shape factor given in Appendix C for flaws under membrane loading

$f_w =$ finite width correction when flaw area is greater than 10% of $A_i$

$= \{\sec(2\pi a/A_i)\}^{0.5}$ for embedded flaws

$= \{\sec(\pi a/A_i)\}^{0.5}$ for surface flaws

$A_i =$ cross-sectional area of the load bearing section containing the crack.

6.4.2.2 Published Solutions

Stress intensity factor solutions for general crack geometries and stress fields are included in compendia and handbooks by Murikami (1987), Tada et. al (1984), Rooke and Cartwright (1976), Sih (1973), and PD6493(1991). The SIF solutions are obtained either from a simple
graphical representation or by evaluating a simple polynomial or analytic expression with given coefficients. The analyst should be careful when using such solutions to ensure that the selected model adequately represents the geometry and boundary conditions of the actual problem.

Stress intensity factor solutions are commonly presented in the following form:

\[ K_I = \sigma \cdot Y \cdot \sqrt{\pi a} \]  

where
- \( \sigma \) = a reference local nominal or "field" stress at the crack location
- \( Y \) = stress intensity magnification factor
- \( a \) = relevant crack length

The stress intensity magnification factor, \( Y \), is a function of crack geometry, structural geometry and mode of loading. The reference nominal stress at the crack location is determined from a local stress analysis of the uncracked body. For residual strength assessments, the reference nominal stress corresponds to the stresses under the extreme load condition (including residual stresses). For residual life assessment, the reference stress range is required to calculate \( \Delta K \) from

\[ \Delta K = K_{I,max} - K_{I,min} = \Delta \sigma \cdot Y \cdot \sqrt{\pi a} \]  

where \( \Delta \sigma \) is the reference nominal stress range due to applied cyclic loadings. Welding residual stresses are not included in the calculation of \( \Delta \sigma \) since they are usually taken into account in the constants of the crack growth relationship. It should be noted that the reference nominal stress (or stress range), \( Y \) factor, and the crack length in Equations 6.18 and 6.19 must be consistently defined for a particular problem.

The membrane and bending components of stress usually require separate correction functions. In addition, self-limiting residual stresses should not be factored by stress concentration factors and therefore need to be separated from the stresses due to applied loading. As a result, Equation 6.18 becomes:

\[ K_I = (Y\sigma) \sqrt{\pi a} \]  

\[ (Y\sigma) = \{Y_m \cdot (M_{km} \cdot \sigma_m + \sigma_{m0}) + Y_b \cdot (M_{kb} \cdot \sigma_b + \sigma_{b0})\} \]  

where
- \( Y_m, Y_b \) = magnification factors accounting for the flaw geometry
- \( M_{km}, M_{kb} \) = magnification factors accounting for stress concentrations of the detail
\( \sigma_m, \sigma_b = \) local nominal stresses due to the applied loading
\( \sigma_{rm}, \sigma_{rb} = \) local residual stresses

The subscripts \( m \) and \( b \) refer to membrane and bending stress components respectively. The stress intensity magnification factors, \( Y_m \) and \( Y_b \), account for the crack size and shape and are equivalent to the reference SIF solutions for flaws in a flat plate. \( M_{km} \) and \( M_{kb} \) are the stress intensity magnification factors due to the stress concentration of the detail, and are functions of the local geometry (i.e., joint configuration, weld toe radius, angle) as well as crack geometry (shape and depth).

Reference solutions for \( Y_m \) and \( Y_b \) for through-thickness cracks, elliptical embedded, and semi-elliptical surface cracks in membrane and bending loading have been published in PD6493 (1990) and are included in Appendix C. \( M_{km} \) and \( M_{kb} \) solutions for several basic weld joint configurations are also presented in Appendix C. In practice, \( M_{kb} \) solutions are not available for many configurations. In such cases, it is usually conservative to assume \( M_{kb} = M_{km} = M_k \).

Most \( M_k \) solutions for cracks at welds have been calculated by 2-D finite element or weight function methods. The 2-D \( M_k \) solutions are generally presented as follows:

\[
M_k = \alpha (a/B)^\beta
\]  \hspace{1cm} (6.22)

where

- \( a = \) crack length
- \( B = \) plate thickness
- \( \alpha, \beta = \) functions of crack size and weld geometry.

The 2-D \( M_k \) solutions are, strictly speaking, applicable to the case of a straight fronted crack (i.e., \( a/2c = 0 \)). Due to the complexity and costs of the analyses, only a few 3-D solutions exist for semi-elliptical cracks at welds, see for example Bell (1987), Nykanen (1987) and Straalen et al (1988). However experience indicates that 2-D solutions can be applied for semi-elliptical cracks (provided \( 0 \leq a/2c \leq 0.5 \)) as described below.

For most practical cases, the analysis of semi-elliptical cracks requires only the solutions at the point of deepest penetration (i.e., \( K_{La} \) at \( \phi = \pi/2 \)) and at the surface (i.e., \( K_{Le} \) at \( \phi = 0 \)):

\[
K_{La} = \{ Y_{m,a} \cdot (M_{km,a} \cdot \sigma_m + \sigma_{rm}) + Y_{b,a} \cdot (M_{kb,a} \cdot \sigma_b + \sigma_{rb}) \} \cdot \sqrt{\pi a}
\]  \hspace{1cm} (6.23)

\[
K_{Le} = \{ Y_{m,c} \cdot (M_{km,c} \cdot \sigma_m + \sigma_{rm}) + Y_{b,c} \cdot (M_{kb,c} \cdot \sigma_b + \sigma_{rb}) \} \cdot \sqrt{\pi a}
\]  \hspace{1cm} (6.24)

where
If 3-D solutions are available, the values of $M_{k,a}$ and $M_{k,c}$ can usually be obtained directly. Alternatively, 2-D solutions for $M_k$ may be used to estimate the semi-elliptical crack solutions using the formulae given by Pang (1990):

$$M_{k,a} = M_k(2-D) = \alpha(a/B)^\beta$$  \hspace{1cm} (6.25)

$$M_{k,c} = M_{k,a} + 1.15 \exp(0.74 a/B)$$  \hspace{1cm} (6.26)

The available SIF solutions for welded joints are generally limited to simple basic weld joint configurations (e.g., butt joint, T-joint, cruciform joint, etc.). Ship details may be considered to be built up of various simple joints, however the stress distributions in actual ship details are somewhat more complicated than that of the simple joints due to the load flow in the structure surrounding the detail as well as the local stress concentration effect of the basic detail. The reference stresses to be used in the SIF solutions should correspond to the local nominal stresses at the crack location. When applying published SIF solutions to actual ship structural details, it is not always clear which stress(es) are to be used as the reference stress.

As noted previously, the SIF solutions for basic welded joints account for the stress concentration for weld ($K_w^{\text{basic}}$) and basic joint configuration ($K_g^{\text{basic}}$) through the $M_k$ factor for the joint. In the limit, as the crack depth approaches zero, it can be shown that the $M_k$ factor approaches the value of the stress concentration factor for the basic detail, including the notch effect of the weld toe; that is:

$$M_k = K_w^{\text{basic}} \cdot K_g^{\text{basic}} \text{ as } a \to 0$$  \hspace{1cm} (6.27)

When stress intensity factor solutions for basic welded joints are applied to complex ship details, then global nominal membrane stresses ($\sigma_{G,m}$) and global nominal bending stresses ($\sigma_{G,b}$) must be corrected with: i) a stress concentration factor ($K_G$) that accounts for the gross structural configuration that surrounds the detail, and ii) a stress concentration factor ($K_g^*$) that accounts for the difference between the stress concentration of the detail’s configuration ($K_g^{\text{detail}}$) and the contribution to the latter from $K_g^{\text{basic}}$.

$$\sigma_m = K_g^* \cdot K_G \cdot \sigma_{G,m}$$  \hspace{1cm} (6.28)
\[ \sigma_b = K_g^{*} \cdot K_G \cdot \sigma_{G,b} \]  \hfill (6.29)

As evident in Table B2 of Appendix B, \( K_g^{\text{basic}} \) is approximately equal to unity for most basic joint configurations, and it is more convenient but not unduly conservative to use \( K_g^{\text{detail}} \) instead of \( K_g^{*} \) in Equations 6.28 and 6.29.

6.4.2.3 PD6493 Level 2 - Linearized Stress Method

The PD6493 (1991) Level 2 assessment procedure provides an approximate method for evaluating SIF’s based on taking the linearized stress distribution across the flaw (as determined from local stress analysis of the uncracked body) and applying the basic SIF solutions for flaws in a plate. In this manner the stress profile at the crack is more accurately accounted for, resulting in a more accurate evaluation of the SIF than the Level 1 procedure. The equations used to calculate the SIF using this method are as follows:

\[ K_i = \{ Y_m \cdot (M_{km} \cdot \sigma_m + \sigma_m) + Y_b \cdot (M_{kb} \cdot \sigma_b + \sigma_b)\} \cdot \sqrt{\pi a} \]  \hfill (6.30)

where

- \( Y_m, Y_b \) = flaw shape factors for cracks in plates in tension and bending respectively
- \( M_{km}, M_{kb} \) = magnification factors accounting for the local joint geometry
- \( \sigma_m, \sigma_b \) = membrane and bending components of the local stress from applied load
- \( \sigma_m, \sigma_b \) = membrane and bending components of the local residual stresses

Parametric formulae for \( Y_m, Y_b, M_{km}, \) and \( M_{kb} \) are presented in Appendix C.

The application of this Level 2 approach requires linearization of the stress distribution over the crack length as opposed to linearization of the stresses over the plate thickness or width. The method used for linearization of stresses is different for fracture assessments and fatigue assessments and is summarized in Figure 6.13.
Examples of linearization of stress distributions for surface flaws

\[ \sigma_m = \frac{\sigma_1 - \sigma_2}{2} \quad \sigma_o = \frac{\sigma_1 + \sigma_2}{2} \]

NOTE. Any linearized distribution of stress is acceptable provided that it is greater than or equal to the magnitude of the real distribution over the flaw surface.

(a) Linearization of stress distributions in fracture assessments

Examples of linearization of stress range distributions for surface flaws

\[ \Delta \sigma_m = \frac{\Delta \sigma_1 - \Delta \sigma_2}{2} \quad \Delta \sigma_o = \frac{\Delta \sigma_1 + \Delta \sigma_2}{2} \]

(b) Linearization of stress range distributions in fatigue assessments

Figure 6.13: Linearization of Stress Distributions (PD6493, 1991)
6.4.3 Finite Element Methods

In cases where published solutions are not readily available for the detail under consideration, finite element methods may be used to calculate SIF solutions. General review of the finite element method relating to fracture mechanics are provided by Bell and Kirkhope (1981), and Cartright and Rooke (1975).

The application of FEM to LEFM requires modelling the stress singularity that occurs at the crack tip. The first attempts to model cracks were simply the use of very large numbers of conventional elements. No attempt was made to take into account the stress singularity in the element formulation. It was clearly demonstrated by Chan et al (1970) that many hundreds of elements are required to achieve perhaps 5% accuracy. As a result, this approach has been abandoned in favour of elements which take explicit account of the crack tip stress singularity. The most important of these formulations include classical solution based singularity elements, polynomial singularity function elements, and modified isoparametric elements.

Isoparametric elements are perhaps the most important of these due to their wide availability in commercial FEM programs. Their application to LEFM is based on the ability to represent the $1/\sqrt{r}$ stress singularity by a very simple modification to the standard isoparametric element. Henshell and Shaw (1975) were the first to recognize that by shifting the "mid-side" nodes to the quarter point in a quadratic isoparametric triangular or quadrilateral element, the required singularity resulted at the nearest node. Barsoum (1976) in a most important paper, investigated two and three-dimensional quadratic isoparametric elements. He introduced the idea of "collapsing" nodes along one edge of the element, and placing the adjacent nodes at the quarter point (Figure 6.14). These collapsed or degenerate elements were later shown by Barsoum (1977) to contain the required stress singularity along any ray from the crack tip, whereas the simple modified elements exhibit the singularity only along the boundaries of the element. The demonstrated accuracy of the collapsed form of isoparametric element, together with their wide availability and ease of application, makes them the preferred choice for elastic crack analysis.

Figure 6.14: Collapsed Node Isoparametric Crack Tip Element
The application of FEM for determining SIF is similar to that described for local stress analysis in Section 6.3.2.2 in terms of extent of model and application of loads and boundary conditions. In general, a local model of the detail containing the crack is required with special crack-tip elements applied at the crack tip. Shell elements models may be used to derive SIF for through-thickness and 2-D straight-fronted (i.e., $a/2c = 0$) cracks. The analysis of partial thickness elliptical cracks is somewhat more complicated and requires the use of 3-D solid elements. Figures 6.15 and 6.16 show typical FEM meshes for 2-D and 3-D cracks.

The 2-D crack mesh shown in Figure 6.15 was used to model an edge cracked plate. Four triangular crack tip elements are located at the crack tip in the arrangement shown. The rest of the model uses conventional isoparametric plate or shell elements. In this particular example the crack face lies on a plane of symmetry, therefore only half of the crack is modelled. The nodes between the crack tip and the far edge of the plate are prescribed symmetry displacement conditions, nodes along the crack surface are free to move. Note that the crack tip elements are relatively small (typically about 2% of the crack length) and that elements gradually get larger as the distance from the crack tip increases. This is to ensure that the rapid stress gradient at the crack tip is adequately represented.

![Figure 6.15: Example of 2-D Crack Model of an Edge Cracked Plate](image-url)
Figure 6.16 shows a 3D FEM model of a semi-elliptical surface crack in a plate. The design of the 3D crack mesh requires analogous considerations for element placing and sizing to those discussed for the 2D crack mesh. As a guide, the size of the crack tip elements normal to the crack front should be less than 5% of the crack length, a, for acceptable accuracy (2-5%). The length to width aspect ratio of solid crack-tip elements should not exceed 4, where the length dimension of the element is measured along the crack front. The 3D crack model is considerably more complex than the 2D problem. In general modelling of 3D semi-elliptical cracks requires the use of computerized "mesh generation" programs or FEM preprocessors with advanced solids modelling features to facilitate their preparation.
6.4.4 Weight Function Methods

The weight function \( m(x,a) \) for a crack in mode I is a unique property of geometry and it enables an alternative economical method of calculating SIF solutions for complex geometries and stress profiles. It is particularly well suited for allowing the effects of residual stresses to be incorporated into the SIF solution. The weight function for a 2-D cracked body can be written in the form:

\[
K_1 = \int_0^a \sigma(x) \cdot m(x) \cdot dx
\]  

(6.31)

\[
m(x,a) = \frac{H \cdot du_r}{Kr} \quad \text{(6.32)}
\]

\[H = E \quad \text{for plane stress}; \quad H = \frac{E}{1 - v^2} \quad \text{for plane strain.} \quad \text{(6.33)}
\]

In order to derive the weight function, a reference stress intensity factor \( K_r \) for a given geometry and stress system needs to be known together with the corresponding crack opening displacement field \( u_r(x,a) \). An appropriate solution for \( K_r \) can usually be found from the literature (e.g., \( K_r \) solution for a partial thickness crack in a flat plate). However, reference solutions for \( u_r(x,a) \) are usually more difficult to find. To overcome this difficulty, Petroski and Achenbach (1978) proposed an approximation for the crack opening displacement function as follows:

\[
u_r(x,a) = \frac{\sigma_o}{H} \left[ 4Y(a/t) \sqrt{a} \left( a - \frac{t}{2} \right) + \frac{G(a/t) (a - x)^{2/3}}{\sqrt{a}} \right]
\]  

(6.34)

Generalized weight function expressions were derived assuming this displacement function and are summarized on the following page. Knowing the weight function \( m(x,a) \), a stress intensity factor \( K_{\text{new}} \) can be calculated for any new local stress system \( \sigma_{\text{new}}(x) \). The local stress distribution \( \sigma_{\text{new}}(x) \) has to be obtained for the prospective crack plane in the actual structural configuration for which \( K_{\text{new}} \) is to be derived. This may be achieved by conventional local detail FEA of the uncracked geometry to derive the local stress field, upon which the residual stress field may be superimposed, to achieve the total stress field at the crack location.
Further details of this approach are provided by Niu and Glinka (1987), and Albrecht and Yamada (1977). Hobbacher (1993) illustrates the use of weight function technique to calculate SIF solutions for weld details as presented in Appendix C.

\[ m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[ 1 + m_1 \left( \frac{a-x}{a} \right) + m_2 \left( \frac{a-x}{a} \right)^2 \right] \]  
(6.35)

\[ m_1 = \frac{4Y'(a/t)a + 2Y(a/t) + 3/2G(a/t)}{2Y(a/t)} \]  
(6.36)

\[ m_2 = \frac{G'(a/t)a + 1/2G(a/t)}{2Y(a/t)} \]  
(6.37)

\[ Y(a/t) = \frac{K_r}{\sigma_o} \sqrt{\pi a} \]  
(6.38)

\[ Y'(a/t) = \frac{d}{d(a/t)} \{ Y(a/t) \} \]  
(6.39)

\[ G(a/t) = \frac{[I_1(a) - 4Y(a/t)\sqrt{a} \cdot I_2(a)]\sqrt{a}}{I_3(a)} \]  
(6.40)

\[ I_1(a) = \sigma_o \pi \sqrt{2} \int_0^a [Y(a/t)]^2 \cdot (a - da) \]  
(6.41)

\[ I_2(a) = \int_0^a \sigma_o(x) \cdot (a - x)^{1/2} \, dx \]  
(6.42)

\[ I_3(a) = \int_0^a \sigma_o(x) \cdot (a - x)^{3/2} \, dx \]  
(6.43)

\[ G'(a/t) = \frac{d}{d(a/t)} \{ G(a/t) \} \]  
(6.44)
6.5 Net Section Stresses

The horizontal co-ordinate of a failure assessment diagram (FAD), the ratio of the net section stress ($\sigma_n$) to the material yield strength or flow strength (see Section 3), measures the plastic collapse strength of the cracked section or structure. The net section stress for a uniform applied tensile load and symmetric crack configuration is simply the applied load divided by the net cross-sectional area. In contrast, the net section stress for unsymmetric crack configurations and/or applied bending loads is not obvious and usually defined by an imaginary stress that is obtained in the following manner. The distribution of net section stresses prior to plastic collapse is assumed to be identical in form to the distribution of net section stresses at plastic collapse according to limit load analysis with an elastic perfectly plastic material model. The net section stress prior to plastic collapse is then obtained by simultaneously solving the equilibrium equations for moment and forces across the remaining uncracked ligament. Closed form solutions for simple cases are given in PD 6493 (1991) and repeated below. Additional guidance and formulae for other geometries are given by Willoughby and Davey (1989) and Miller (1984), and calculations for a side shell longitudinal are presented in Section 7.0 of this guide.

a) Through-Thickness Flaw

$$\sigma_n = \left\{ \sigma_b + \left( \sigma_b^2 + 9\sigma_m^2 \right)^{0.5} \right\} / \left\{ 3 \left[ 1 - (2a/W) \right] \right\}$$

(6.46)

where

- $\sigma_n$ = effective net section stress.
- $\sigma_b$ = local nominal bending stress at the flaw.
- $\sigma_m$ = local nominal membrane stress at the flaw.
- $2a$ = the crack length.
- $W$ = the width of the load bearing section containing the crack.

b) Surface Flaw - Normal Bending Restraint

$$\sigma_n = \left\{ \sigma_b + \left[ \sigma_b^2 + 9\sigma_m^2(1 - \alpha^2) \right]^{0.5} \right\} / \left\{ 3(1 - \alpha^2) \right\}$$

(6.47)

where

- $\alpha = (a/B) / \left\{ 1 + (B/c) \right\}$ for $W \geq 2(c+B)$
- $\alpha = (2a/B) (c/W)$ for $W < 2(c+B)$
- $a$ = the crack depth in the plate thickness direction
- $2c$ = the crack length at the surface
- $W$ = the width of the load bearing section containing the crack
- $B$ = the plate thickness

c) Surface Flaw - Negligible Bending Restraint (eg. pin-jointed)
\[ \sigma_a = \left\{ \sigma_b + 3\sigma_m\alpha + \left[ (\sigma_b + 3\sigma_m\alpha)^2 + 9\sigma_m^2(1 - \alpha^2) \right]^{0.5} \right\} / \left\{ 3(1 - \alpha^2) \right\} \]  

(6.48)

d) Embedded Flaw

\[ \sigma_a = \left\{ \sigma_b + \sigma_m\alpha + \left[ (\sigma_b + \sigma_m\alpha)^2 + 9\sigma_m^2(1 - \alpha^2) + 4\pi\alpha/B \right]^{0.5} \right\} / \left\{ 3(1 - \alpha^2) + 4\alpha/B \right\} \]  

(6.49)

where:
\[ \alpha = \frac{2a/B}{1 + (B/c)} \] for \( W \geq 2(c+B) \)
\[ \alpha = \frac{4a/B}{c/W} \] for \( W < 2(c+B) \)

\[ d = \text{dimension of the nearest distance of the flaw to the plate surface.} \]

6.6 References


Bell, R., “Stress Intensity Factors for Weld Toe Cracks in T Plate Joints”, DSS Contract Serial No. OST84-00125, Faculty of Engineering, Carleton University, Canada, May 1987.


7.0 EXAMPLES

This section demonstrates, by way of two hypothetical examples, the damage tolerance assessment procedures presented in this guide. The first example demonstrates the application of these procedures in service, while the second example demonstrates the application of these procedures at the fabrication stage. The fracture assessment procedure presented in Section 3 of this guide and the fatigue assessment procedure presented in Section 4 of this guide are fully implemented in both examples. In addition, indirect and direct approaches for stress analysis from Section 6 of this guide are used in these examples. However, only a Level 1 approach for the determination of loads from Section 5 of this guide and a simplified approach for the determination of stress intensity factors from Section 6 of the guide are used in these examples as they will be the most efficient, if not the only possible, approaches in most practical situations. Demonstration of spectral load analysis is limited to a qualitative “walk-through” of the various steps of such an analysis in Appendix E.

7.1 Description of the Problem

A common platform has been selected for the two examples; namely, an 85,000 tons single-skin tanker that was previously analyzed in SSC Report 381 “Residual Strength of Damaged Marine Structures” [7.1]. The layout and particulars of this tanker are shown in Figure 7.1, while the mid-ship structural configuration of this tanker is shown in Figure 7.2.

Example No.1 involves a through-thickness fatigue crack in side shell longitudinal No. 8, which is located in the starboard No. 5 wing ballast tank, about 6 m below the upper deck, near the summer load line (refer to Figures 7.2 and 7.3). The crack is located midway between Frames 66 and 67 in the weld metal of a transverse splice weld with ground-off reinforcements, and the plane of this crack is more or less perpendicular to the longitudinal axis of the tanker. The crack had initiated at the outside corner of the longitudinal and had propagated 10 mm into the flange and 10 mm into the web before being detected during a scheduled survey. The ship operator wants to determine if the repairs can be delayed until the next scheduled inspection which is 4 years away.

Example No. 2 considers the fillet weld that joins the flat bracket at Frame 67 to the flange of the aforementioned side shell longitudinal (refer to Figures 7.2 and 7.3). A 1.0 mm long undercut has been found along the weld toe at the end of the bracket during a post-fabrication inspection. The undercut is about 0.5 mm deep at its deepest point. Similar undercuts have been found at other brackets throughout the tanker. Although undercuts of this size are normally found to be acceptable under typical defect acceptance criteria, fatigue cracks initiating at such defects have been found in a number of sister ships after a few years of service. The owner and fabricator of the tanker agree that the welds should be repaired before delivery of the vessel because the undercuts are located in regions of high stress concentrations. However, the project is already several months
Figure 7.1: Platform for examples - profile, plan view, and particulars of 85,000 ton tanker.

<table>
<thead>
<tr>
<th>PARTICULARS</th>
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<tbody>
<tr>
<td><strong>Length (O.A.)</strong></td>
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<tr>
<td><strong>Length (B.P.)</strong></td>
</tr>
<tr>
<td><strong>Breadth (MLD)</strong></td>
</tr>
<tr>
<td><strong>Depth (MLD)</strong></td>
</tr>
<tr>
<td><strong>Draft (Summer)</strong></td>
</tr>
<tr>
<td><strong>C_b</strong></td>
</tr>
<tr>
<td><strong>Velocity</strong></td>
</tr>
</tbody>
</table>
Figure 7.2: Midship structural configuration of tanker in Figure 7.1.
Case 2
0.5 mm deep x 1.0 mm long undercut

Figure 7.3: Damage sites along side shell longitudinal No. 8 between Frames 66 and 67.
behind schedule and considerably over budget. The fabricator and owner have, therefore, opted for a damage tolerance assessment to determine whether such repairs are necessary.

The side-shell longitudinal, flat bracket, and side-shell plating are fabricated from Grade A mild steel. The nominal yield strength (σ_{YS}) and nominal ultimate tensile strength (UTS) of this material are 235 and 440 MPa respectively. The following CTOD values were obtained from fracture toughness tests on the steel at 0°C (the minimum design temperature): 0.32, 0.37, and 0.25 mm. The failure mode in all three tests was initial ductile tearing followed by unstable cleavage (i.e., Type u).

### 7.2 Load Analysis

Extreme stresses at the damage site for the interval of interest and corresponding to a specific probability of exceedance are required for fracture assessment, while the statistical distribution of stress range at the damage site over the interval of interest are required for fatigue assessment. Method A of the three Level 1 approaches described in Section 5.5 of this guide will be used to estimate the statistical stress distribution. This method assumes the basic form of the distribution, and a reference stress range corresponding to a probability of exceedance per wave encounter of 10^{-4} is used as a reference point for the distribution. The extreme stress will correspond to a probability of exceedance of 0.01 over the interval of interest (with the corresponding probability of exceedance per wave encounter equal to 1/n, where n is the number of wave encounters over the interval of interest). This is consistent with the generally accepted value of 0.01 for the ship design life which is typically twenty years. The starting point for the load calculations will, therefore, be the determination of loads for an arbitrary probability of exceedance per wave encounter. The corresponding stresses can then be transformed to the required probability of exceedance for the fatigue and fracture calculations.

The following wave-induced loads can be estimated from parametric equations in Appendix A of the American Bureau of Shipping’s (ABS), “Guide for Fatigue Strength Assessment of Tankers” [7.2]:

1. hull girder bending moments (vertical and horizontal);
2. external hydrodynamic pressure range;
3. internal pressure range of tank loads (inertial fluid loads and added static head due to vessel motion).

The ABS guide does not specify the corresponding probability of exceedance for these loads. However, the equations are intended to give extreme loads so they will be taken to be 10^{-8} per wave encounter. The guide does not take into account wave impact loads, whipping, springing, tank fluid sloshing, or vibrating forces due to machinery or
propellers. In this example, however, the latter loads are secondary or insignificant compared to the former three types of loads.

The ABS guide divides the ship cross-section into Zones A and B as shown schematically in Figure 7.4. The appropriate stress range for each zone is calculated based on the fluctuating load due to two (acting together) of the eight combinations of internal tank loading and draft shown in Figure 7.5. For Zone A, the greater value of LC1 and LC2 or LC3 and LC4 is used and for Zone B, the greater of LC5 and LC6 or LC7 and LC8. The area of interest in both examples is Zone B so the greater values for LC5 and LC6 or LC7 and LC8 should be used. The values were computed in Reference 7.1 and are repeated here in Table 7.1.

Table 7.1: Wave-Induced Bending Moments and Pressure Acting on Side Shell Longitudinal No. 8 Between Frames 36 and 37

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>still-water bending moment ($M_{sw}$)</td>
<td>$+2.52 \times 10^7$ N-m</td>
</tr>
<tr>
<td>vertical sagging moment ($M_{vs}$)</td>
<td>$-1.14 \times 10^7$ N-m</td>
</tr>
<tr>
<td>vertical hogging moment ($M_{vh}$)</td>
<td>$+1.07 \times 10^7$ N-m</td>
</tr>
<tr>
<td>horizontal bending moment ($M_h$)</td>
<td>$+/ - 1.38 \times 10^7$ N-m</td>
</tr>
<tr>
<td>side shell pressure range (p)</td>
<td>0.045 MPa</td>
</tr>
</tbody>
</table>

7.3 Stress Analysis

7.3.1 Global Nominal Stresses

Estimates of the hull girder bending stresses produced by the bending moments in Table 7.1 are tabulated in Table 7.2. These estimates are based on the flexure formula:

$$\sigma_{sw} = \frac{M_{sw}y}{I_{yy}}$$

(7.1)

$$\sigma_{vs} = \frac{M_{vs}y}{I_{yy}}$$

(7.2)

$$\sigma_{hs} = \frac{M_{hs}y}{I_{yy}}$$

(7.3)

$$\sigma_{h} = \frac{M_{h}z}{I_{zz}}$$

(7.4)

where

$I_{yy}$ = moment of inertia about the horizontal neutral axis = $2.82 \times 10^6$ m$^2$-cm$^2$

$I_{zz}$ = moment of inertia about the vertical neutral axis = $5.51 \times 10^6$ m$^2$-cm$^2$

$y$ = vertical distance of side longitudinal No. 8 from horizontal neutral axis = 2.44 m

$z$ = horizontal distance of side longitudinal No. 8 from vertical neutral axis = 16.0 m
Figure 7.4: Break down of a ship's cross-section for load analysis in accordance with Reference [7.1].
a) Load Cases # 1, 3, and 7, 2/3 Design Draft

b) Load Cases # 2, 4, and 8, Design Draft

c) Load Cases # 5, 2/3 Design Draft

d) Load Cases # 6, 2/3 Design Draft

Figure 7.5: Loading cases for load analysis in accordance with Reference [7.1].
Table 7.2: Stresses Produced By Wave-Induced Bending Moments and Pressure Acting on Side Shell Longitudinal No. 8 Between Frames 36 and 37

<table>
<thead>
<tr>
<th>Stress Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>still-water bending stress ($\sigma_{sw}$)</td>
<td>+22.0 MPa</td>
</tr>
<tr>
<td>vertical sagging stress ($\sigma_{vs}$)</td>
<td>-10.0 MPa</td>
</tr>
<tr>
<td>vertical hogging stress ($\sigma_{vh}$)</td>
<td>+9.0 MPa</td>
</tr>
<tr>
<td>horizontal bending stress ($\sigma_{h}$)</td>
<td>+/- 40.0 MPa</td>
</tr>
</tbody>
</table>

The estimated global nominal stresses are treated as membrane stresses ($\sigma_{G,m}$) across the cross-section of the longitudinal because the distances between side shell longitudinal No.8 and the horizontal and vertical neutral axes of the ship's cross-section are large with respect to the width and depth of the longitudinal. The worst case from the fatigue and fracture point of view is when the horizontal bending stress is in phase with the vertical bending stress:

$$\sigma_{G,m}^{\text{max}} = |\sigma_{sw}| + |\sigma_{hs}| + |\sigma_{h}| = 22 + 9 + 40 = 71 \text{ MPa}$$  \hspace{1cm} (7.5)

$$\sigma_{G,m}^{\text{min}} = |\sigma_{sw}| - |\sigma_{vs}| - |\sigma_{h}| = 22 - 10 - 40 = -28 \text{ MPa}$$  \hspace{1cm} (7.6)

The side shell pressure produces a combination of bending and torsion in side shell longitudinal No. 8. The resulting maximum principal stresses peak at the corner between the flange and web of the longitudinal as evident in the finite element results shown in Figure 7.6. The range of this peak stress ($f_{r2}^*$) is estimated by the following equations from Section 3.3.3 of Reference [7.2] to be 55.4 MPa at the mid-span and 56.6 MPa at the frame ends

$$f_{r2}^* = C_t M/SM$$  \hspace{1cm} (7.7)

where:

$$M = k \cdot p \cdot s \cdot l^2$$  \hspace{1cm} (7.8)

- $C_t$ = correction factor for combined bending and torsional stress = 1.5
- $SM$ = sectional modulus about the vertical axis of the longitudinal and its associated effective side shell plating
  - = 473400 mm$^3$ at mid-span and 463700 mm$^3$ at frame ends
- $b_e$ = effective breadth of side shell plating as defined by Figures 3 and 4 of Reference [7.2]
  - = 351.8 mm at mid-span and 493.7 mm at frame ends
- $M$ = bending moment at the supported ends of the longitudinal
- $k$ = factor accounting for the fixity of the stiffener = 1.15/12
- $s$ = stiffener spacing = 800 mm
- $l$ = unsupported span of the stiffener = 2250 mm
Figure 7.6: Longitudinal stresses (MPa) in side shell longitudinal No.8 subjected to unit inward pressure (MPa) as predicted by finite element analysis (ANSYS) with plate element model.
A horizontal bending stress ($\sigma_{G,b}$) of range $f_{r2}$ is conservatively assumed to act across the cross-section of the longitudinal. The side shell pressure is also assumed to fully reverse so:

$$\sigma_{G,b}^{\text{max}} \text{ @ mid-span} = +27.7 \text{ MPa} \quad (7.9)$$

$$\sigma_{G,b}^{\text{min}} \text{ @ mid-span} = -27.7 \text{ MPa} \quad (7.10)$$

$$\sigma_{G,b}^{\text{max}} \text{ @ frame ends} = +28.3 \text{ MPa} \quad (7.11)$$

$$\sigma_{G,b}^{\text{min}} \text{ @ frame ends} = -28.3 \text{ MPa} \quad (7.12)$$

### 7.3.2 Stress Concentrations

The total stresses ($\sigma_t$) and local nominal stresses ($\sigma_L$) at the damage sites are related to the global nominal stresses ($\sigma_G$) as follows:

$$\sigma_L = K_G \cdot \sigma_G + \sigma_r \quad (7.13)$$

$$\sigma_t = K_G \cdot K_w \cdot K_g \cdot K_{te} \cdot K_\alpha \cdot \sigma_G + \sigma_r \quad (7.14)$$

where:

- $\sigma_r$ = welding residual stress
- $K_G$ = stress concentration to account for shear lag and gross section changes
- $K_w$ = local stress concentration of weld
- $K_g$ = stress concentration of local structural detail
- $K_{te}$ = stress concentration due to eccentricity
- $K_\alpha$ = stress concentration due to angular mismatch

For both sites, welding residual stresses at the damage site are assumed to be uniform through the thickness of the web and flange of the longitudinal and equal in magnitude to the yield strength of the base metal (235 MPa). The determination of $K_G$ requires a global hull finite element analysis, whereas $K_{te}$ and $K_\alpha$ can be estimated from Appendix A of this guide. For the purposes of this demonstration, however, $K_{te}$, $K_\alpha$ and $K_G$ are assumed to be unity at both damage sites. The product of $K_w$ and $K_g$ at the splice weld in Example 1 is also assumed to be unity. However, the product of $K_w$ and $K_g$ at the toe of the flat bracket in Example 2 is estimated to be 2.2 from Table B2 of Appendix B in this guide.
The values of $\sigma_L$ and $\sigma_t$ corresponding to $\sigma_{G,m}^{\max}$, $\sigma_{G,b}^{\max}$, $\sigma_{G,m}^{\min}$, and $\sigma_{G,b}^{\min}$ are tabulated in Table 7.3:

### Table 7.3: Local Nominal Stresses and Total Stresses at Damage Sites

<table>
<thead>
<tr>
<th></th>
<th>@ mid-span</th>
<th>@ toe of flat bracket</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{L,m}^{\max}$</td>
<td>$71 + 235 = +306 \text{ MPa}$</td>
<td>$71 + 235 = +306 \text{ MPa}$</td>
</tr>
<tr>
<td>$\sigma_{L,m}^{\min}$</td>
<td>$-28 + 235 = +207 \text{ MPa}$</td>
<td>$-28 + 235 = +207 \text{ MPa}$</td>
</tr>
<tr>
<td>$\Delta\sigma_{L,m} = \sigma_{L,m}^{\max} - \sigma_{L,m}^{\min}$</td>
<td>99 MPa</td>
<td>99 MPa</td>
</tr>
<tr>
<td>$\sigma_{L,b}^{\max}$</td>
<td>$+27.7 \text{ MPa}$</td>
<td>$+28.3 \text{ MPa}$</td>
</tr>
<tr>
<td>$\sigma_{L,b}^{\min}$</td>
<td>$-27.7 \text{ MPa}$</td>
<td>$-28.3 \text{ MPa}$</td>
</tr>
<tr>
<td>$\Delta\sigma_{L,b} = \sigma_{L,b}^{\max} - \sigma_{L,b}^{\min}$</td>
<td>55.4 MPa</td>
<td>56.6 MPa</td>
</tr>
<tr>
<td>$\sigma_{l,m}^{\max}$</td>
<td>$71 + 235 = +306 \text{ MPa}$</td>
<td>$2(71) + 235 = +376 \text{ MPa}$</td>
</tr>
<tr>
<td>$\sigma_{l,m}^{\min}$</td>
<td>$-28 + 235 = +207 \text{ MPa}$</td>
<td>$2(-28) + 235 = +179 \text{ MPa}$</td>
</tr>
<tr>
<td>$\Delta\sigma_{l,m} = \sigma_{l,m}^{\max} - \sigma_{l,m}^{\min}$</td>
<td>99 MPa</td>
<td>198 MPa</td>
</tr>
<tr>
<td>$\sigma_{l,b}^{\max}$</td>
<td>$+27.7 \text{ MPa}$</td>
<td>$2(28.3) = +56.6 \text{ MPa}$</td>
</tr>
<tr>
<td>$\sigma_{l,b}^{\min}$</td>
<td>$-27.7 \text{ MPa}$</td>
<td>$2(-28.3) = -56.6 \text{ MPa}$</td>
</tr>
<tr>
<td>$\Delta\sigma_{l,b} = \sigma_{l,b}^{\max} - \sigma_{l,b}^{\min}$</td>
<td>55.4 MPa</td>
<td>113.2 MPa</td>
</tr>
</tbody>
</table>

#### 7.3.3 Stress Intensity Factors

In the first example, a fatigue crack has initiated at the corner of side shell longitudinal No. 8 in a splice weld located midway between Frames 36 and 37, and propagated through the thickness of the flange and web before being detected. In the second example, a small undercut has been found along the weld toe at the base of the flat bracket at the Frame 37 end of side shell longitudinal No. 8. It is assumed that a fatigue crack will rapidly initiate from the undercut, so the latter is treated as a pre-existing quarter-elliptic corner crack of the same depth and length as the undercut. It is also assumed that the corner crack will propagate through the thickness of the flange and web in a self-similar manner and that the corner crack will evolve into a through-thickness crack once the flange and web are penetrated.

Matoba and Inoue [7.3] have developed a relatively simple model for estimating stress intensity factors for the aforementioned types of cracks. Their model considers a semi-elliptic surface crack in an imaginary flat plate subjected to the hot spot stress distribution $(K_L \cdot \sigma_L + \sigma_t)$ over the cross-section of an actual longitudinal down to a depth $B$, where $B$ is the width of the longitudinal’s flange (Figure 7.7). The width and thickness of the imaginary plate are $2B$ and $B$, respectively, while the imaginary surface crack has a surface length and depth of $2c$ and $a$, respectively, where $a$ and $c$ are the surface lengths of a corner crack or through-thickness crack in the longitudinal (see Figure 7.7).
Figure 7.7: Matoba and Inoue's model [7.3] for calculation of stress intensity factors of crack in side shell longitudinal No. 8.
The stress intensity factor at the deepest point of the semi-elliptic surface crack ($K_{i,a}$) and the stress intensity factor at the ends of the crack ($K_{i,c}$) are defined by the following equations:

**for Level 1 FAD (assume through-thickness stress distribution is uniform)**

\[
K_{i,a} = [Y_{m,a}M_{km,a}yK_g(\sigma_{L,m} + \sigma_{L,b})]\sqrt{\pi a} \tag{7.15}
\]

\[
K_{i,c} = [Y_{m,c}M_{km,c}yK_g(\sigma_{L,m} + \sigma_{L,b})]\sqrt{\pi a} \tag{7.16}
\]

**for Level 2a and Level 2b FAD**

\[
K_{i,a} = (Y_{m,a}M_{km,a}yK_g\sigma_{L,m} + Y_{b,a}M_{kb,a}yK_g\sigma_{L,b})\sqrt{\pi a} \tag{7.17}
\]

\[
K_{i,c} = (Y_{m,c}M_{km,c}yK_g\sigma_{L,m} + Y_{b,c}M_{kb,c}yK_g\sigma_{L,b})\sqrt{\pi a} \tag{7.18}
\]

**for Level 1 fatigue analysis (assume through-thickness stress distribution is uniform)**

\[
\Delta K_{i,a} = [Y_{m,a}M_{km,a}yK_g(\Delta \sigma_{L,m} + \Delta \sigma_{L,b})]\sqrt{\pi a} \tag{7.19}
\]

\[
\Delta K_{i,c} = [Y_{m,c}M_{km,c}yK_g(\Delta \sigma_{L,m} + \Delta \sigma_{L,b})]\sqrt{\pi a} \tag{7.20}
\]

**for Level 2 fatigue analysis**

\[
\Delta K_{i,a} = (Y_{m,a}M_{km,a}yK_g\Delta \sigma_{L,m} + Y_{b,a}M_{kb,a}yK_g\Delta \sigma_{L,b})\sqrt{\pi a} \tag{7.21}
\]

\[
\Delta K_{i,c} = (Y_{m,c}M_{km,c}yK_g\Delta \sigma_{L,m} + Y_{b,c}M_{kb,c}yK_g\Delta \sigma_{L,b})\sqrt{\pi a} \tag{7.22}
\]

where: $Y_{m,a}$, $Y_{m,c}$, $Y_{b,a}$, and $Y_{b,c}$ are geometry factors for a semi-elliptic crack in a flat plate; $M_{km,a}$, $M_{km,c}$, $M_{kb,a}$, and $M_{kb,c}$ are magnification factors accounting for $K_g$ and $K_w$ in a simple fillet welded joint; and $\gamma$ is an empirical correction for the local nature of $K_g$ ($\gamma = 1$ for $K_g < 1.17$ and $.85$ for $K_g > 1.17$).

$M_{km,a}$ and $M_{kb,a}$ can be estimated from Equation 6.22 of Section 6, where $\alpha$ and $\beta$ are summarized in Table 7.4, and $M_{km,c}$ and $M_{kb,c}$ are approximated by the values of $M_{km,a}$ and $M_{kb,a}$ for a $0.15$ mm deep crack of aspect ratio $a/c$. $Y_{m,a}$, $Y_{b,a}$, $Y_{m,c}$ and $Y_{b,c}$ can be estimated from parametric equations given in Appendix C.
The value of $K_g$ in Example 1 is set to unity since the crack is located in the weld metal of a ground butt joint. In Example 2, the damage site is located along the weld toe at the base of a flat bracket. The product of $K_w$ and $K_g$ for such a detail is 2.2 according to Table B2 of Appendix B. Although equations for estimating the stress concentration of the weld ($K_w$) itself are given in Appendix A, $K_g$ is conservatively set to 2 for the purposes of this demonstration. As evident in the finite element results previously presented in Figure 7.6, the stress concentration at the base of the flat bracket decays rapidly across the width of the flange of the adjoining longitudinal. This decay is taken into account by the $\gamma$ correction factor in equations.

Table 7.4: $\alpha$ and $\beta$ values for membrane and bending loads where $M_{k,a} = \alpha(a/T)^{\beta}$

<table>
<thead>
<tr>
<th>Loading</th>
<th>$a/T$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>membrane</td>
<td>$\leq .073$</td>
<td>.615</td>
<td>-.31</td>
</tr>
<tr>
<td></td>
<td>$&gt;.073$</td>
<td>.83</td>
<td>-.20</td>
</tr>
<tr>
<td>bending</td>
<td>$\leq .03$</td>
<td>.45</td>
<td>-.31</td>
</tr>
<tr>
<td></td>
<td>$&gt;.03$</td>
<td>.68</td>
<td>-.19</td>
</tr>
</tbody>
</table>

7.3.4 Statistical Distribution of Local Nominal Stress Range

The design life for the tanker in Examples 1 and 2 is 20 years. It is assumed that the tanker will encounter $10^8$ waves over this period and 500,000 waves per year on average. It is also assumed the ranges of the membrane and bending components of wave-induced local nominal stresses ($\Delta \sigma_{L,m}$ and $\Delta \sigma_{L,b}$) at the damage sites in Examples 1 and 2 follow a Weibull distribution

$$n/n_0 = \exp[-(\Delta \sigma/\Delta \sigma_o)^h \ln(n_o)]$$

or

$$\Delta \sigma/\Delta \sigma_o = [1 - \log(n)/\log(n_o)]^{1/h}$$

where: $\Delta \sigma$ is either $\Delta \sigma_{L,m}$ or $\Delta \sigma_{L,b}$; $\Delta \sigma_o$ is the stress range exceeded once in $n_o$ cycles (i.e., probability of exceedance = $1/n_o$); $n$ is the number of times $\Delta \sigma$ is exceeded in $n_o$ encounters; and $h$ is the shape factor.

As discussed in Section 5.0, the shape factor depends on the location of interest in a ship and the particulars of that ship. The side shell longitudinal in Examples 1 and 2 is located just below the water-line, and the corresponding shape factor is approximately unity according to the following equation from Section 5

$$h = h_0 + h_k z/T_{act} - 0.005(T_{act} - z)$$

where:
\[ h_0 = 2.21 - 0.51 \log_{10} L \]
\[ h_a = 0.05 \]
\[ z = \text{vertical distance from baseline to load point} = 11.01 \text{ m} \]
\[ T_{act} = \text{draught in m of load condition} = 13.26 \text{ m} \]
\[ L = \text{ship length} = 239.6 \text{ m} \]

A Weibull distribution with a shape factor of unity reduces to an exponential distribution (Figure 7.8). Tables 7.5 and 7.6 discretize an exponential distribution of stress ranges over 20 years into 21 stress levels. These tables also discretize an exponential distribution of stress ranged over one year into 17 stress levels - the highest four stress levels in the twenty year distribution being clipped off. Histograms of these two distributions are superimposed in Figure 7.9.

The wave-induced bending moments and pressure acting on the side shell longitudinal and corresponding to a probability of exceedance per wave encounter of \(10^{-8}\) are tabulated in Table 7.1, and the associated ranges of total stress and local nominal stress are tabulated in Table 7.2. Because the majority of fatigue damage is usually inflicted by stress ranges corresponding to probability of exceedances between \(10^{-3}\) and \(10^{-6}\), it is recommended that \(n_0\) be set to \(10^4\) in order to minimize errors associated with differences between the assumed and actual shape factor. The following equation can be used to convert the stress ranges in Table 7.3 from one probability level \((p_2)\) to another probability level \((p_1)\)

\[ \Delta\sigma_2 = \Delta\sigma_1 \left[\frac{\log(p_2)}{\log(p_1)}\right]^{1/h} \]  
(7.26)

where:

\[ \Delta\sigma_2 = 0.5\Delta\sigma_1 \]  
(7.27)

for \(p_2 = 10^{-8}\) and \(p_1 = 10^{-4}\).
Figure 7.8: Probability of exceedance diagram for an exponential distribution of stress ranges with a reference stress range corresponding to a probability of exceedance per wave encounter of $10^{-4}$. 
Figure 7.9: Histogram of stress ranges over one year and twenty years according to an exponential distribution with a reference stress range corresponding to a probability of exceedance per wave encounter of $10^{-4}$. 
Table 7.5: Probability of Exceedance

<table>
<thead>
<tr>
<th>$\Delta \sigma_{\text{rms}} / \Delta \sigma_{10^{-4}}$</th>
<th>probability of exceedance</th>
<th>no. of exceedances in $5 \times 10^6$ cycles (1 year)</th>
<th>no. of exceedances in $1 \times 10^8$ cycles (20 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>5,000,000</td>
<td>100,000,000</td>
</tr>
<tr>
<td>0.1</td>
<td>$3.981 \times 10^{-1}$</td>
<td>1,990,054</td>
<td>39,810,717</td>
</tr>
<tr>
<td>0.2</td>
<td>$1.585 \times 10^{-1}$</td>
<td>792,447</td>
<td>15,848,932</td>
</tr>
<tr>
<td>0.3</td>
<td>$6.310 \times 10^{-2}$</td>
<td>315,479</td>
<td>6,309,573</td>
</tr>
<tr>
<td>0.4</td>
<td>$2.512 \times 10^{-2}$</td>
<td>125,594</td>
<td>2,511,886</td>
</tr>
<tr>
<td>0.5</td>
<td>$1.000 \times 10^{-2}$</td>
<td>50,000</td>
<td>1,000,000</td>
</tr>
<tr>
<td>0.6</td>
<td>$3.981 \times 10^{-3}$</td>
<td>19,905</td>
<td>398,107</td>
</tr>
<tr>
<td>0.7</td>
<td>$1.585 \times 10^{-3}$</td>
<td>7,925</td>
<td>158,489</td>
</tr>
<tr>
<td>0.8</td>
<td>$6.310 \times 10^{-4}$</td>
<td>3,155</td>
<td>63,096</td>
</tr>
<tr>
<td>0.9</td>
<td>$2.512 \times 10^{-4}$</td>
<td>1,256</td>
<td>25,119</td>
</tr>
<tr>
<td>1.0</td>
<td>$1.000 \times 10^{-4}$</td>
<td>500</td>
<td>10,000</td>
</tr>
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<td>1.1</td>
<td>$3.981 \times 10^{-5}$</td>
<td>199</td>
<td>3,981</td>
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<tr>
<td>1.2</td>
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<td>79</td>
<td>1,585</td>
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<td>1.3</td>
<td>$6.310 \times 10^{-6}$</td>
<td>32</td>
<td>631</td>
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<td>251</td>
</tr>
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<td>100</td>
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<tr>
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<td>$3.981 \times 10^{-7}$</td>
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<td>40</td>
</tr>
<tr>
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<td>$6.310 \times 10^{-8}$</td>
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<td></td>
</tr>
<tr>
<td>1.9</td>
<td>$2.512 \times 10^{-8}$</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>$1.000 \times 10^{-8}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.3.5 Extreme Stress

Extreme values of $\sigma_{L,m}$ and $\sigma_{L,b}$ are required for the calculation of stress intensity factors for residual strength assessment. Design loads for a ship typically correspond to a probability of exceedance of .01 over the design life of the ship, and the associated probability of exceedance per wave encounter is $0.01/n$ where $n$ is the number of wave encounters over the design of the ship. As mentioned earlier, the tanker in Examples 1 and 2 has a twenty year design life and it is assumed that the ship encounters $10^8$ waves over this life. Therefore, the values of $\sigma_{L,m}^{\text{max}}$ and $\sigma_{L,b}^{\text{max}}$ in Table 7.3, which correspond to a probability of exceedance of $10^{-8}$ per wave encounter, are suitable for residual strength assessment over the design life of the vessel but too conservative for much shorter assessment intervals. A more sensible approach is to use extreme values that correspond to a constant probability of exceedance of .01 over the assessment interval. To this end, Equation 7.26 can be used to convert the values of $\sigma_{L,m}^{\text{max}}$ and $\sigma_{L,b}^{\text{max}}$ in Table 7.3 to values corresponding to a probability of exceedance per wave encounter of $1/n^*$, provided the

<table>
<thead>
<tr>
<th>$\Delta \sigma/\Delta \sigma_{10^4}$</th>
<th>no. of occurrences in $5 \times 10^6$ cycles (1 year)</th>
<th>$\Delta \sigma/\Delta \sigma_{10^4}$</th>
<th>no. of occurrences in $1 \times 10^8$ cycles (20 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 0.1</td>
<td>3,009,464</td>
<td>0.0 - 0.1</td>
<td>60,189,283</td>
</tr>
<tr>
<td>0.1 - 0.2</td>
<td>1,198,089</td>
<td>0.1 - 0.2</td>
<td>23,961,785</td>
</tr>
<tr>
<td>0.2 - 0.3</td>
<td>476,968</td>
<td>0.2 - 0.3</td>
<td>9,539,358</td>
</tr>
<tr>
<td>0.3 - 0.4</td>
<td>189,884</td>
<td>0.3 - 0.4</td>
<td>3,797,687</td>
</tr>
<tr>
<td>0.4 - 0.5</td>
<td>75,594</td>
<td>0.4 - 0.5</td>
<td>1,511,886</td>
</tr>
<tr>
<td>0.5 - 0.6</td>
<td>30,095</td>
<td>0.5 - 0.6</td>
<td>601,893</td>
</tr>
<tr>
<td>0.6 - 0.7</td>
<td>11,981</td>
<td>0.6 - 0.7</td>
<td>239,618</td>
</tr>
<tr>
<td>0.7 - 0.8</td>
<td>4,770</td>
<td>0.7 - 0.8</td>
<td>95,393</td>
</tr>
<tr>
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<td>0.8 - 0.9</td>
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<td>0.9 - 1.0</td>
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<td>0.9 - 1.0</td>
<td>15,119</td>
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<tr>
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<td>1.0 - 1.1</td>
<td>6,019</td>
</tr>
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<td>1.1 - 1.2</td>
<td>120</td>
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<td>2,396</td>
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<tr>
<td>1.2 - 1.3</td>
<td>48</td>
<td>1.2 - 1.3</td>
<td>954</td>
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<tr>
<td>1.3 - 1.4</td>
<td>19</td>
<td>1.3 - 1.4</td>
<td>380</td>
</tr>
<tr>
<td>1.4 - 1.5</td>
<td>8</td>
<td>1.4 - 1.5</td>
<td>151</td>
</tr>
<tr>
<td>1.5 - 1.6</td>
<td>3</td>
<td>1.5 - 1.6</td>
<td>60</td>
</tr>
<tr>
<td>&gt;1.6</td>
<td>1</td>
<td>1.6 - 1.7</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.7 - 1.8</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.8 - 1.9</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.9 - 2.0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; 2.0</td>
<td>1</td>
</tr>
</tbody>
</table>
statistical distribution of stresses remains more or less constant, where n is the number of
wave encounters over the assessment interval. In Examples 1 and 2, the assessment
interval is determined by the number of cycles to propagate a fatigue crack from an initial
 crack size to a critical crack size, so it is necessary to calculate the extreme stresses and
carry out the residual strength assessments simultaneously with the fatigue crack growth
calculations.

7.3.6 Net Section Stress

As discussed in Section, the horizontal co-ordinate for a failure assessment
diagram (S_r or L_v) is the ratio of the net section stress (σ_n) to either the yield stress (σ_Y)
or flow stress ((σ_Y + σ_UTS)/2). The net section stress for a uniform applied tensile load
and symmetric crack configuration is simply the applied load divided by the net cross-
sectional area. In contrast, the net section stress for unsymmetric crack configurations
and/or applied bending loads, is not obvious and usually defined by an imaginary stress
that is obtained in the following manner. The distribution of net section stresses prior to
plastic collapse is assumed to be identical in form to the distribution of net section
stresses at plastic collapse according to a limit load analysis with an elastic-perfectly
plastic material model. The net section stress prior to plastic collapse is then obtained by
solving the equilibrium equations for moment and forces across the net section
simultaneously. Closed form solutions for simple cases are given in Section 6.5.
However, an iterative solution is generally required for more complex cases, such as the
side shell longitudinal in Examples 1 and 2. See Figure 7.10 for the development of
equilibrium equations for the longitudinal.

7.4 Residual Life Assessment Procedure

1. Select the level of analysis. Use Level 1 fatigue analysis with Level 1 FAD and Level
2 fatigue analysis with either Level 2a FAD or Level 2b FAD.

2. Arrange the individual blocks of stress ranges in the histograms constructed in
Section 7.3.4 into three different sequences: high-to-low, low-to high, low-high-low
(Figure 7.11).

3. Further divide the blocks of stress ranges in each sequence into sub-blocks of length
ΔN_B, where ΔN_B is the lesser of the block length or 10% of the block length. For
each sequence of stress ranges, carry out Steps 4 to 7.

4. Use the approximate model described in Section 7.3.3, the relevant values of Δσ_L,m
and Δσ_L,b in Table 7.3 for the damage site of interest, and the appropriate through-
thickness stress distribution for the selected level of fatigue crack growth analysis to
calculate the stress intensity factor range for the crack tip in the flange of the
longitudinal (ΔK_a) and the stress intensity factor range for the crack tip in the web of
the longitudinal (ΔK_c).
Figure 7.10: Calculation of net section stress by simultaneous solution of equilibrium equations for longitudinal forces and bending moments.

Equilibrium of Longitudinal Forces

\[
\sigma_m = \left( b - t_m \right) + t_w + t_{w_1} + t_{w_2} = \sigma_n \left( t_1 \cdot \left( b - a \right) \right) + \frac{t_{s_1} + t_{s_2}}{2}.
\]

Equilibrium of Bending Moments

\[
Y_m \left( t_f \left( b - t \right) + t_{w_1} \cdot \left( d - a \right) + \frac{t_{s_1} + t_{s_2}}{2} \right) = \left( b - c \right) \left( d + t_{s_2} \right) + \frac{t_{s_1} + t_{s_2}}{2} + \frac{b \cdot \left( t_{s_1} + t_{s_2} \right)}{2}.
\]
Figure 7.11: Sequence of stress ranges for fatigue crack growth analysis.
5. Calculate the incremental crack growth in the web of the longitudinal ($\Delta a$) over each sub-block and the incremental crack growth in the flange of the longitudinal ($\Delta c$) over each sub-block by integrating the Paris equation assuming that the crack growth rate is constant over the sub-block and equal to the crack growth rate at the beginning of the sub-block.

\[
\text{for } \Delta K > \Delta K_{th}
\]

\[\Delta a = C(\Delta K_a)^m \Delta N_B \quad (7.28)\]

\[\Delta c = C(\Delta K_c)^m \Delta N_B \quad (7.29)\]

\[
\text{for } \Delta K \leq \Delta K_{th}
\]

\[\Delta a = 0 \quad (7.30)\]

\[\Delta c = 0 \quad (7.31)\]

In the absence of specific values for $C$ and $m$, use the recommended upper bound values for structural steels in sea water from Section 4 ($C = 2.3 \times 10^{12}$ and $m = 3.0$ where: the units for $\Delta a$ and $\Delta c$ are mm and the units for $\Delta K$ are MPa\$\text{mm}$.). Yield-level tensile residual stresses are assumed to exist at the damage site in both examples so the local $R$-ratios are greater than 0.5. Use the recommended upper bound value of $\Delta K_{th}$ (63 MPa\$\text{mm}$) for such $R$-ratios from Section 4 (Equation 4.20).

6. Update crack size and crack shape.

7. Determine whether the crack has reached a critical size. Use either the Level 1 FAD with Level 1 fatigue analysis, or the Level 2a or Level 2b FAD with Level 2 fatigue analysis. Calculate $S_r$ and $L_r$ using the tensile properties defined Table 7.1 and the net section stresses determined in accordance with Section 7.3.3. Calculate $K_r$ using (i) stress intensity factors for extreme stresses calculated in accordance with Section 7.3.5 and (ii) a material fracture toughness ($K_{mat}$) of 5065 MPa\$\text{mm}$. The latter value comes from converting a CTOD value of 0.25 mm, which is the minimum value of the three test results obtained for the steel at the minimum service temperature of the tanker (0°C), to an equivalent $K_{mat}$ value using Equation 3.9.

8. Repeat Steps 3 to 7 for subsequent sub-blocks in each stress history.

9. Repeat Steps 2 to 8 with smaller sub-blocks until crack growth converges.
7.5 Results

At both damage sites and for each stress history, the crack tip in the flange becomes critical before the crack tip in the web. Level 1 predictions of the daily crack advance in the flange are plotted in Figures 7.12 and 7.13, while Level 2 predictions of this advance are plotted in Figures 7.14 and 7.15. Superimposed on these plots are the predicted critical crack sizes. Failure assessment curves (FAC) and failure assessment points (FAP) for the cases involving lo-hi stress histories are presented in Figures 7.16 and 7.17. The predicted critical crack sizes are also tabulated in Table 7.7, while the predicted residual lives are tabulated in Table 7.8.

The following observations can be made about the results in Figures 7.12 and 7.17 and Tables 7.7-7.8:

- For each damage site and level of fatigue analysis, the crack growth curve associated with a hi-lo stress history forms a lower bound on predicted fatigue lives, while the crack growth curve associated with the lo-hi stress history forms an upper bound on predicted fatigue lives. The crack growth curve associated with the lo-hi-lo stress history falls within the aforementioned envelope. The three curves diverge at the beginning of each passing year (i.e., the return period of the stress history) but nearly converge by year’s end. Within each year, the maximum difference between the fatigue lives predicted with the lo-hi stress history and the fatigue lives predicted with the hi-lo stress history is about 350 days.

- The predicted residual lives for the damage site at the mid-span of side shell longitudinal No. 8 range from 2238 days to 2784 days, while the predicted residual lives for the damage site at the toe of the flat bracket range from 376 to 949 days. Because of the large geometric stress concentration associated with the flat bracket, the former lives are substantially longer than the latter lives despite the larger initial flaw at the mid-span site. It is also worth noting that the predicted residual lives are greater than the one year return period of the load sequences so the analyses do not have to be repeated with load sequences having shorter return period periods.

- The predicted critical crack sizes (a/c) for the damage site at the mid-span of side shell longitudinal No.8 range from 37.6/43.7 mm to 47.4/58.2 mm, while the predicted critical crack sizes (a/c) for the damage site at the toe of the flat bracket range from 23.1/21.7 mm to 35/38.2 mm. The critical crack sizes at the mid-span are larger than the critical crack sizes at the flat bracket because of the large geometric stress concentration at the toe of the bracket.

- As expected, the shortest residual lives and smallest critical crack sizes are predicted by the Level 1 FAD and Level 1 fatigue analysis, while the longest residual lives and largest critical crack sizes are predicted by the Level 2a FAD and Level 2 fatigue analysis. In most cases, however, the critical crack sizes predicted by the Level 2a
FAD and Level 2 fatigue analysis are only 25% longer than the critical crack sizes predicted by the Level 1 FAD and Level 1 fatigue analysis, and the percentage difference in terms of residual life is even smaller, viz., 10%.

- The variation of $K_r$ with $S_r$ or $L_r$ is non-linear. This can be attributed to two factors: (i) net section stresses have been calculated for a probability of exceedance per wave encounter that has been constantly updated to maintain a constant probability of exceedance over the assessment interval of 0.01; (ii) the non-linear correction for crack plastic tip plasticity has been applied in both examples because of the assumed presence of yield level welding residual stresses. Therefore, in both examples, the distance of any given failure assessment point (FAP) from the failure assessment curve (FAC) should not be interpreted as a measure of the margin of safety.

### Table 7.7: Critical Crack Lengths

<table>
<thead>
<tr>
<th>Load History</th>
<th>Damage Site</th>
<th>Level 1</th>
<th>Level 2a</th>
<th>Level 2b</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a(mm)</td>
<td>c(mm)</td>
<td>a(mm)</td>
<td>c(mm)</td>
</tr>
<tr>
<td>lo-hi</td>
<td>mid-span</td>
<td>37.6</td>
<td>43.7</td>
<td>47.4</td>
</tr>
<tr>
<td></td>
<td>flat bracket</td>
<td>23.1</td>
<td>21.7</td>
<td>33.3</td>
</tr>
<tr>
<td>hi-lo</td>
<td>mid-span</td>
<td>37.8</td>
<td>43.7</td>
<td>47.4</td>
</tr>
<tr>
<td></td>
<td>flat bracket</td>
<td>32.9</td>
<td>33.8</td>
<td>35</td>
</tr>
<tr>
<td>lo-hi-lo</td>
<td>mid-span</td>
<td>37.7</td>
<td>43.6</td>
<td>46.2</td>
</tr>
<tr>
<td></td>
<td>flat bracket</td>
<td>32.6</td>
<td>33.3</td>
<td>34.3</td>
</tr>
</tbody>
</table>

### Table 7.8: Residual fatigue life in days ($5 \times 10^6$ cycles per year)

<table>
<thead>
<tr>
<th>Load History</th>
<th>Damage Site</th>
<th>Level 1</th>
<th>Level 2a</th>
<th>Level 2b</th>
</tr>
</thead>
<tbody>
<tr>
<td>lo-hi</td>
<td>mid-span</td>
<td>2525</td>
<td>2784</td>
<td>2545</td>
</tr>
<tr>
<td></td>
<td>flat bracket</td>
<td>723</td>
<td>949</td>
<td>725</td>
</tr>
<tr>
<td>hi-lo</td>
<td>mid-span</td>
<td>2191</td>
<td>2238</td>
<td>2331</td>
</tr>
<tr>
<td></td>
<td>flat bracket</td>
<td>376</td>
<td>423</td>
<td>405</td>
</tr>
<tr>
<td>lo-hi-lo</td>
<td>mid-span</td>
<td>2356</td>
<td>2379</td>
<td>2370</td>
</tr>
<tr>
<td></td>
<td>flat bracket</td>
<td>546</td>
<td>565</td>
<td>547</td>
</tr>
</tbody>
</table>

#### 7.6 Interpretation of Results

The probability that the stress ranges in Tables 7.5 and 7.6 naturally occur in order of increasing magnitude or decreasing magnitude is extremely rare (orders of magnitude less than than $10^{-5}$). In view of this rarity and the conservatism of the input material properties for fatigue crack growth analysis and residual strength assessment, it is highly unlikely that a critical crack will develop before the residual life predicted by the
combination of a Level 1 FAD, Level 1 fatigue analysis, and a hi-lo stress sequence (about 6 years for the mid-span site, just over 1 year for the flat bracket). It is very likely that a critical crack will develop before the residual life predicted by the combination of a Level 2a FAD, Level 2 fatigue analysis, and lo-hi stress sequence (7.5 years for the mid-span site, 1, 2.5 years for the bracket end). The next scheduled survey is in four years. Therefore, the crack detected at the mid-span of side shell longitudinal No. 8 need not be repaired until that time provided the area is subjected to special inspections in the interim. On the other hand, the undercut at the toe of the flat bracket should be repaired before the new ship is delivered by the fabricator.

7.7 References


Figure 7.12: Level 1 prediction of flange crack length at mid-span of side shell longitudinal No. 8 (Example 1) versus number of days of operation.
Figure 7.13: Level 1 prediction of flange crack length at flat bracket at Frame 37 end of side shell longitudinal No. 8 (Example 2) versus number of days of operation.
Figure 7.14: Level 2 prediction of flange crack length at mid-span of side shell longitudinal No. 8 (Example 1) versus number of days of operation.
Figure 7.15: Level 2 prediction of flange crack length at flat bracket at Frame 37 end of side shell longitudinal No. 8 (Example 2) versus number of days of operation.
Figure 7.16: Failure assessment diagrams for damage site at mid-span of side shell longitudinal No. 8 (Example 1) where fatigue crack growth has been predicted with a lo-hi stress history.
Figure 7.17: Failure assessment diagrams for damage site at the toe of the flat bracket at the Frame 37 end of side shell longitudinal No. 8 (Example 2), where fatigue crack growth has been predicted with a lo-hi stress history.
APPENDIX A

NOTCH STRESS CONCENTRATION FACTORS

This appendix provides guidance on the estimation of notch stress concentration factors ($K_w$) for welded details. The notch stress concentration factor is that part of the stress concentration due to the geometry of the weld itself. Notch stress concentration factors are required to determine the peak stress at the toe of the weld where cracks tend to initiate. With reference to Figure A, the notch SCF is defined as:

$$K_w = \frac{\sigma_{\text{notch}}}{\sigma_z}$$

The notch SCF may be determined from local fine mesh FEA, as outlined in Section 6.3, or using parametric formulae.

Figure A1: Definition of Notch Stress Concentration Factor

The main parameters affecting $K_w$ are the weld toe radius ($\rho$), weld toe angle ($\theta$), plate thickness ($B$), weld height ($h$). Niu and Glinka proposed the following relationship:

$$K_w = 1 + 0.5121 \cdot \theta^{0.572} \cdot (B/\rho)^{0.469}$$

where the angle $\theta$ is measured in radians.

An alternative simpler relationship, independent of the weld toe angle, was proposed by Lawrence et al:

$$K_w = 1 + (1 + 0.019 \cdot (B/\rho))^{0.5}$$

their SCF solutions for welded joints applicable to ship structural details (see Appendix B). However, this value is very low in comparison to the notch SCF values determined from the above equations.

References


This appendix provides guidance on the estimation of stress concentration factors \((K_g, K_{te}, K_{ta})\) for ship structural details where: \(K_g\) is a stress concentration factor due to the gross geometry of the detail, \(K_{te}\) is an additional stress concentration factor due to eccentricity tolerance (normally used for plate connections only), and \(K_{ta}\) is an additional stress concentration factor due to angular mismatch (normally used for plate connections only).

These stress concentration factors account for the local geometry of the detail, excluding the weld \((K_w - \text{see Appendix A})\). They do not account for the global stress concentration effects of the structure surrounding the detail to be analyzed \((K_G)\). The latter should be determined by global FEA or additional published solutions. The total stress concentration factor for the location, used to determine the peak stress in the load carrying section containing the flaw, is thus defined as follows:

\[
K_o = K_G \cdot K_w \cdot K_g \cdot K_{te} \cdot K_{ta}
\]

The following SCF solutions have been adapted from Cramer et al. (1995). Alternate solutions may be found in Classification Society documents for fatigue analysis, and previous ship structure committee reports.
### Table B1 SCF For Flange Connections

<table>
<thead>
<tr>
<th>Geometry</th>
<th>K-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1.a Flange connection with</td>
<td>$K_z \cdot K_w = 2.2$</td>
</tr>
<tr>
<td>softening toe</td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image1.png" alt="Diagram" /></td>
</tr>
<tr>
<td>B.1.b Crossing of flanges</td>
<td>$K_z \cdot K_w = 2.2$</td>
</tr>
<tr>
<td></td>
<td>$R \geq 0.1t$</td>
</tr>
<tr>
<td></td>
<td>$t = \text{thickness of flange}$</td>
</tr>
<tr>
<td></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td>B.1.c $R/b &gt; 0.15$</td>
<td>$K_z \cdot K_w = 1.9$</td>
</tr>
<tr>
<td></td>
<td><img src="image3.png" alt="Diagram" /></td>
</tr>
<tr>
<td>B.1.d Overlap connection</td>
<td>$K_z = \left( \frac{t_f^2}{t_p} + 3 \left( \frac{t_s}{t_f} \right)^2 + 6 \frac{\Delta t}{t_f^2} \right)$</td>
</tr>
<tr>
<td></td>
<td>$\Delta = \text{gap = tolerance}$</td>
</tr>
<tr>
<td></td>
<td>Default: $\Delta = 10 \text{ mm}$</td>
</tr>
<tr>
<td></td>
<td>$K_{wp} = 1.5$</td>
</tr>
</tbody>
</table>

*Guide to Damage Tolerance Analysis of Marine Structures*
### Table B2 SCF For Stiffener Supports

<table>
<thead>
<tr>
<th>Geometry</th>
<th>K-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.2.a</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="" /></td>
<td>For supporting members welded to stiffener flange:</td>
</tr>
</tbody>
</table>
| &nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbsp;&nbs
Table B3 SCF For Termination of Stiffeners on Plates

<table>
<thead>
<tr>
<th>Geometry</th>
<th>K-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.3.a Local elements and stiffeners welded to plates</td>
<td>$K_t \cdot K_w = 2 \left(1 + \frac{t_w \theta}{t_p \times 160}\right)$</td>
</tr>
<tr>
<td></td>
<td>$\theta = \text{angle in degrees of sloping termination}$</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram of B.3.a" /></td>
</tr>
<tr>
<td>B.3.b Snipping of top flanges:</td>
<td>$K_t \cdot K_w = \frac{3A_f}{lt_s}$</td>
</tr>
<tr>
<td></td>
<td>and $K_t \cdot K_w = \text{min}3.0$</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram of B.3.b" /></td>
</tr>
</tbody>
</table>
Table B4 SCF For Butt Welds

<table>
<thead>
<tr>
<th>Geometry</th>
<th>K-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.4.a</td>
<td>Angular mismatch in joints between flat plates results in additional stresses at the butt weld and the stiffener</td>
</tr>
<tr>
<td></td>
<td>$K_{ta} = 1 + \frac{\lambda}{4} \frac{s}{t}$</td>
</tr>
<tr>
<td></td>
<td>where:</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 6$ for pinned ends</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 3$ for fixed ends</td>
</tr>
<tr>
<td></td>
<td>$\alpha =$ angular mismatch in radians</td>
</tr>
<tr>
<td></td>
<td>$s =$ plate width</td>
</tr>
<tr>
<td></td>
<td>$t =$ plate thickness</td>
</tr>
<tr>
<td>Default: $e = 6$ mm</td>
<td></td>
</tr>
</tbody>
</table>

B.4.b Welding from both sides

<table>
<thead>
<tr>
<th>K-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_e = 1.0$</td>
</tr>
</tbody>
</table>

| $K_\omega = 1.0 + 0.5(\tan \theta)^{1/4}$ |
| Default value $K_\omega = 1.5$ for $\theta = 45$ deg. |

$K_{ta}$ from 7.4.a

| $K_{ta} = 1 + \frac{3e}{t}$ |

Diagram showing geometry and variables for B.4.a and B.4.b.
### Table B4 SCF For Butt Welds (Continued)

**B.4.c**  
Plate not restricted in out-of-plane movement

\[ e = 0.15 t_1 \]

\[
K_x = 1 + \frac{3 \frac{\Delta t}{t_1}}{1 + \left(1 + \frac{\Delta t}{t_1}\right)^3}
\]

\[
K_w = 1.4 \left(1 + \frac{\Delta t + e}{2a}\right)
\]

**B.4.d**  
Plate restricted in out-of-plane movement (e.g. flanges)

\[ \Delta t = t_2 - t_1 \]

\[ K_x = 1.4 \]

\[ K_w = 1.4 \left(1 + \frac{\Delta t + e}{2a}\right) \]

**B.4.e**  
Welding from one side

Welding from one side is not recommended in areas prone to fatigue due to sensitivity of workmanship and fabrication

\[ K_x = 10 \]

Default value: \( K_w = 2.2 \)

\[ K_u = 1 + \frac{2e}{t} \]

\[ K_{ta} \text{ from B.4.a} \]
Table B5 SCF For Doubling Plates

<table>
<thead>
<tr>
<th>Geometry</th>
<th>K-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B.5.a</strong></td>
<td><strong>Welded at its end with throat thickness ( a )</strong></td>
</tr>
<tr>
<td>Cover plates on beams</td>
<td>For ( a \geq \frac{t_p \cdot t_D}{t_r + t_D} )</td>
</tr>
<tr>
<td></td>
<td>( K_s \cdot K_w = 1.8 ) ( d \leq 50 )</td>
</tr>
<tr>
<td></td>
<td>( K_s \cdot K_w = 1.9 ) ( 50 &lt; d \leq 100 )</td>
</tr>
<tr>
<td></td>
<td>( K_s \cdot K_w = 2.0 ) ( 100 &lt; d \leq 150 )</td>
</tr>
<tr>
<td></td>
<td>( K_s \cdot K_w = 2.2 ) ( d &gt; 150 )</td>
</tr>
</tbody>
</table>

| **B.5.b**                       | **Doubling plates welded to plates**                                     |
|                                 | For \( a \geq \frac{t_p \cdot t_D}{t_r + t_D} \)                         |
|                                 | \( K_s \cdot K_w = 1.8 \) \( d \leq 50 \)                                |
|                                 | \( K_s \cdot K_w = 1.9 \) \( 50 < d \leq 100 \)                         |
|                                 | \( K_s \cdot K_w = 2.0 \) \( 100 < d \leq 150 \)                        |
|                                 | For \( l > 150 \): \( K_s \cdot K_w = 2.5 \left(1 + \frac{t_p}{2t_p}\right) \) |
|                                 | if a more detailed analysis is not performed                           |

Note: If the welds of the doubling plates are placed closer to the member (flange, plate) edges than 10 mm, the K-factors in Table B.5 should be increased by a factor 1.3
### Table B6 SCF For Cruciform Joints

<table>
<thead>
<tr>
<th>Geometry</th>
<th>K-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.6.a</td>
<td>$K_a = 1 + \frac{6t^2 \cdot e}{t_1 \left( \frac{t_1}{l_1} + \frac{t_2}{l_2} + \frac{t_3}{l_3} + \frac{t_4}{l_4} \right)}$</td>
</tr>
</tbody>
</table>
| B.6.b    | $K_z = 1.0$  
$K_a = 0.90 + 0.90(tan \theta)^{1/4}$  
Default value: $K_a = 1.8$  
$K_{ta}$ from B.6.a  
$K_{ta} = 1.0$ |
| B.6.c    | $K_z = 1.0$  
$K_a = 0.90 + 0.90(tan \theta)^{1/4}$  
Default value: $K_a = 1.8$  
$K_{ta} = 1.0$  
$K_{ta} = 1.0$  
Applicable also for fillet welds |
Table B6 SCF For Cruciform Joints (Continued)

| B.6.d | $K_x \cdot K_w = 1.2 + 1.3 (\tan \theta)^{1/4}$
|       | Default value: $K_x \cdot K_w = 2.5$
|       | $K_w$ from B.6.a with $e$ as given in B.6.b
|       | $K_{ta} = 1.0$

| B.6.e | Based on nominal stress in member with thickness $t_1$
|       | $K_x \cdot K_w = 1.2 \frac{t_1}{a}$
|       | $K_w$ from B.6.a with $e$ as given in B.6.b
|       | $K_{ta} = 1.0$

| B.6.f | $K_x \cdot K_w = 1.2 \frac{t_1}{a}$
|       | $K_w$ from B.6.a with $e$ as given in B.6.b
|       | $K_{ta} = 1.0$
Table B7  SCF For Cut Outs

Guide to Damage Tolerance Analysis of Marine Structures
APPENDIX C

STRESS INTENSITY FACTORS FOR SHIP STRUCTURAL DETAILS

Stress intensity factor solutions for general crack geometries and stress fields are included in compendia and handbooks by Murikami (1987), Tada et. al (1984), Rooke and Cartwright (1976), Sih (1973), and PD6493:1991. When using such compendia, the SIF solution is obtained either from a simple graphical representation or by evaluating a simple polynomial or analytic expression with given coefficients. The analyst should be careful when using such solutions to ensure that the selected mode adequately represents the geometry and boundary conditions of the actual problem.

The following subsections summarize a few published SIF solutions that are relevant to ship structure fracture mechanics problems.

C.1 Semi-Elliptical Surface Cracks in A Plate

Newman and Raju (1984) developed empirical solutions for $Y_m$ and $Y_b$ derived from a systematic curve-fitting procedure based on their three-dimensional finite element work. The functions $Y_m$ and $Y_b$ correspond to the basic SIF solutions for a semi-elliptical surface crack in a plane plate. Their formulae are generally accepted to be the most comprehensive and are also used in PD6493 (1991). They are as follows:

$$Y_m = 1/\Phi \cdot \left\{ M_1 + M_2 \cdot (a/B)^2 + M_3 \cdot (a/B)^4 \right\} \cdot g \cdot f_{\phi} \cdot f_w$$

$$Y_b = H \cdot Y_m$$

At the deepest point on the crack front, $\phi = \pi/2$

$$g = f_{\phi} = 1$$
$$H = H_1$$

At the ends of the crack, $\phi = 0$

$$g = 1.1 + 0.35 \cdot (a/B)^2$$
$$f_{\phi} = (a/c)^{0.5}$$
$$H = H_2$$
$$H_2 = 1 + G_1 \cdot (a/B) + G_2 \cdot (a/B)^2$$

If $W \geq 20c$, it can be assumed that $c/W = 0$, so that $f_w = 1$. Otherwise use

$$f_w = \left\{ \sec[(\pi \cdot c/W) \cdot (a/B)^{0.5}] \right\}^{0.5}$$

A Guide to Damage Tolerance Analysis of Marine Structures
For Cracks With Aspect Ratios $a/2c \leq 0.5$

$\Phi = \{1.0 + 1.464(a/c)^{1.65}\}^{0.5}$
$M_1 = 1.13 - 0.09(a/c)$
$M_2 = 0.89 / \{0.2 + (a/c)\} - 0.54$
$M_3 = 0.5 - 1 / \{0.65 + (a/c)\} + 14 \cdot \{1 - (a/c)\}^{24}$
$H_1 = 1 - 0.34(a/B) - 0.11 \cdot (a/c) \cdot (a/B)$
$G_1 = 1.22 - 0.12(a/c)$
$G_2 = 0.55 - 1.05(a/c)^{0.75} + 0.14 \cdot (a/c)^{1.5}$

For Cracks With Aspect Ratios $0.5 < a/2c \leq 1.0$

$\Phi = \{1.0 + 1.464(c/a)^{1.65}\}^{0.5}$
$M_1 = \{1 + 0.04(c/a)\} \cdot (c/a)^{1.65}$
$M_2 = 0.20(c/a)^4$
$M_3 = -0.11(c/a)^4$
$H_1 = 1 - \{0.04 + 0.41(c/a)\} \cdot (a/B) + \{0.55 - 1.93(c/a)^{0.75} + 1.38(c/a)^{1.5}\} \cdot (a/B)^2$
$G_1 = -2.11 + 0.77(c/a)$
$G_2 = 0.55 - 0.72(c/a)^{0.75} + 0.14(c/a)^{1.5}$

Figure C1: Semi-Elliptical Surface Crack
C.2 Weld Toe Crack at Transverse Non-Load Carrying Attachment

Parametric formulae for stress intensity factors for 2-D cracks at the toe of a transverse, non-load carrying attachment were developed by Hobbacher (1993) using weight function techniques. The geometry of the problem is shown in Figure C2. Hobbacher states that the effect of the thickness ratio of the two plates is less than 5% for \( b/B < 2 \). The same applies for the weld throat \( A \). The only significant variable apart from the crack depth is the transition angle, \( \theta \), at the weld toe which is related to the weld dimensions \( h \) and \( w \). The solutions were derived for membrane loading, but it is conservative to apply it for bending.

\[
M_k = \alpha (a/B)^\beta \quad \text{and} \quad M_k \geq 1
\]

\[
\alpha = 0.8068 - 0.1554(h/B) + 0.0429(h/B)^2 + 0.0794(w/B)
\]

\[
\beta = -0.1993 - 0.1839(h/B) + 0.0495(h/B)^2 + 0.0815(w/B)
\]

The above equations are valid for:

- \( a/B \geq 0.0025 \)
- \( 0.2 \leq h/B \leq 1 \)
- \( 0.2 \leq w/B \leq 1 \)
- \( 15^\circ \leq \theta \leq 60^\circ \)
- \( 0.175 \leq A/B \leq 0.72 \)
- \( 0.125 \leq b/B \leq 2 \)

![Figure C2: Weld Toe Crack at Transverse Non-Load Carrying Attachment](image)
C.3  **Weld Toe Crack at Cruciform Joint with Full Penetration K-Butt Weld**

This problem is shown in Figure C3 and was also solved by Hobbacher using weight function techniques. The main variables are the crack depth ratio, \( a/B \), and the transition angle, \( \theta \), at the weld toe which is related to the weld dimensions \( h \) and \( w \). The solutions were derived for membrane loading, but it is conservative to apply it for bending.

\[
M_k = \alpha (a/B)^\beta \quad \text{and} \quad M_k \geq 1
\]

\[
\begin{align*}
\alpha &= 0.7061 - 0.4091(h/B) + 0.1596(h/B)^2 + 0.3739(w/B) - 0.1329(w/B)^2 \\
\beta &= -0.2434 - 0.3939(h/B) + 0.1536(h/B)^2 + 0.3004(w/B) - 0.0995(w/B)^2
\end{align*}
\]

The above equations are valid for:

\[
\begin{array}{ccc}
a/B \geq 0.0025 & 0.2 \leq h/B \leq 1 & 0.2 \leq w/B \leq 1 \\
15^\circ \leq \theta \leq 60^\circ & 0.175 \leq A/B \leq 1.3 & 0.5 \leq b/B \leq 20
\end{array}
\]

---

**Figure C3: Weld Toe Crack at Cruciform Joint with Full Penetration K-Butt Weld**
C.4 Weld Toe Crack at Cruciform Joint with Partial Penetration Fillet Welds

This problem is shown in Figure C4 and was also solved by Hobbacher using weight function techniques. The main variables are the crack depth ratio, $a/B$, the weld throat dimension $A$, and the transition angle, $\theta$, at the weld toe. Both $A$ and $\theta$ can be related to the weld dimensions $h$ and $w$. The solutions were derived for membrane loading, but it is conservative to apply it for bending.

\[ M_k = \alpha (a/B)^{\beta} \quad \text{and} \quad M_k \geq 1 \]

For $0.2 < h/B < 0.5$ and $0.2 < w/B < 0.5$ and $0.0025 < a/B < 0.07$

\[ \alpha = 2.0175 - 0.8056(h/B) - 1.2856(w/B) \]
\[ \beta = -0.3586 - 0.4062(h/B) + 0.4654(w/B) \]

For $0.2 < h/B < 0.5$ and $0.2 < w/B < 0.5$ and $a/B > 0.07$

\[ \alpha = 0.2916 - 0.0620(h/B) + 0.6942(w/B) \]
\[ \beta = -1.1146 - 0.2312(h/B) + 1.4319(w/B) \]

For $0.5 < h/B < 1.5$ or $0.2 < w/B < 0.5$

\[ \alpha = 0.9055 - 0.4369(h/B) + 0.1753(h/B)^2 - 0.0665(w/B)^2 \]
\[ \beta = -0.2307 - 0.5470(h/B) + 0.2167(h/B)^2 - 0.2223(w/B) \]

![Figure C4: Weld Toe Crack at Cruciform Joint with Partial Penetration Fillet Welds](image)

A Guide to Damage Tolerance Analysis of Marine Structures
C.5 Root Crack at Cruciform Joint with Partial Penetration Fillet Welds

The solution for this problem, shown in Figure C5, is given in PD6493 (1991). The solution is derived for membrane loading, but it is conservative to apply it for bending.

\[ K_1 = \sigma_m \cdot M_k \cdot \left\{ \pi a \cdot \sec(\pi a/W) \right\}^{0.5} \]

where

\[ M_k = \frac{\{A_1 + A_2(2a/W)\}}{\{1 + (2h/B)\}} \quad \text{and} \quad M_k < 1.0 \]

For \(0.2 < h/B < 1.2\) and \(0 < 2a/W < 0.7\)

\[ A_1 = 0.528 + 3.287(h/B) - 4.361(h/B)^2 + 3.696(h/B)^3 - 1.875(h/B)^4 + 0.415(h/B)^5 \]

\[ A_2 = 0.218 + 2.717(h/B) - 10.171(h/B)^2 + 13.122(h/B)^3 - 7.755(h/B)^4 + 1.783(h/B)^5 \]

Figure C5: Root Crack at Cruciform Joint with Partial Penetration Fillet Welds
C.6 Lap Joint with Fillet Welds

This problem is shown in Figure C6 and was also solved by Hobbacher using weight function techniques. The solutions were derived for membrane loading, but it is conservative to apply it for bending.

\[ M_k = \alpha (a/B)^\beta \quad \text{and} \quad M_k \geq 1 \]

\[ \alpha = 1.0210 - 0.3772(h/B) + 0.1844(h/B)^2 - 0.0187(w/B)^2 - 0.1856(u/B) + 0.1362(u/B)^2 \]
\[ \beta = -0.4535 - 0.1121(h/B) + 0.3409(h/B)^2 - 0.0824(w/B)^2 - 0.0877(u/B) - 0.0417(u/B)^2 \]

Figure C6: Lap Joint with Fillet Welds
C.7 Longitudinal Non-Load Carrying Attachments

This problem is shown in Figure C7 and was also solved by Hobbacher using weight function techniques. The solutions were derived for membrane loading, but it is conservative to apply it for bending.

\[ M_k = \alpha(a/B)^\beta \quad \text{and} \quad M_k \geq 1 \]

\[ \alpha = 0.9089 - 0.2357(t/B) + 0.0249(L/B) + 0.0004(L/B)^2 + 0.0186(W/B) - 1.1414(\theta/B) \]
\[ \beta = -0.0229 - 0.0167(t/B) + 0.3863(\theta/45^\circ) + 0.1230(\theta/45^\circ)^2 \]

Figure C7: Longitudinal Non-Load Carrying Attachment
APPENDIX D

FLAW CHARACTERIZATION CRITERIA

(ASME Boiler and Pressure Vessel Code, Section XI)
MULTIPLE PLANAR FLAWS ORIENTED IN PLANE NORMAL TO PRESSURE RETAINING SURFACE

Guide to Damage Tolerance Analysis of Marine Structures
SUBSURFACE PLANAR FLAWS ORIENTED IN PLANE NORMAL TO PRESSURE RETAINING SURFACE
SURFACE PLANAR FLAWS ORIENTED IN PLANE NORMAL TO PRESSURE RETAINING SURFACE

\[ a = \frac{a}{2} \]

If \( S < 0.4d \), \( a = 2d + S \)

Liquid metal or cover gas retaining surface
NONPLANAR ELLIPTICAL SUBSURFACE FLAWS

GENERAL NOTE:
Flaw area shall be projected in planes normal to principal stresses $\sigma_1$ and $\sigma_2$ to determine critical orientation for comparison with allowable indication standards.
NONALIGNED COPLANAR FLAW IN PLANE NORMAL TO PRESSURE RETAINING SURFACE

$d_1, 2d_1, 2d_2, $ and $2d_3$ = depths of individual flaws
MULTIPLE ALIGNED PLANAR INDICATIONS
APPENDIX E

PROCEDURE FOR LOAD ANALYSIS BY SPECTRAL APPROACH

E.1 Introduction

The elements in the calculation of ship loading due to waves were presented in Section 5. Three procedures, representing three levels of sophistication, were presented. Level 3 is based on a full spectral method and yields detailed load information for arbitrary ship configurations, wave climates and operational profiles. Level 2 is also based on the spectral method but requires less detailed information on the wave climate. Level 1 methods, of which three are presented, are based on parametric equations. The three methods vary in terms of their comprehensiveness and the effort required to exercise them. The most comprehensive of these methods is based on the DNV approach recommended for fatigue analysis. The DNV methodology is broadly similar to approaches developed by other prominent Classification Societies. The second most refined of the Level 1 methods requires a static wave balance calculation to be performed, the result from which is substituted in parametric equations. The third and final Level 1 method relies entirely on parametric equations where values for a very limited number of basic ship parameters are input. The latter two Level 1 methods are only capable of predicting vertical bending moments.

Clearly, the latter two Level 1 methods are inappropriate for parts of ship structure where the stresses are not dominated by those deriving from overall vertical hull girder bending. In these cases, there is little option but to apply one of the other methods. The example presented below is based on the Level 3 method.

The steps in the calculation of loading is as follows:

1. Problem definition
2. Operational profile definition
3. Wave climate definition
4. Calculation of RAUs and stress coefficients
5. Computation of response for each combination of $H_s$ and $T_z$
6. Compilation of stress range spectrum
7. Computation of extreme stress

The purpose of this section is to describe, step-by-step, the calculations to be undertaken in order to compute an estimate of extreme stress and stress range spectrum. The steps are presented in the context of the two examples of damage tolerance assessment presented in Section 7.0. In both examples, the platform is a 85,000 ton displacement single skin tanker, and the structural member of interest is a side shell longitudinal. As discussed in Section 5, the computations involved are complicated because a large number of variables need consideration. The summary provided below identifies the main steps. For detailed discussions of the process the reader is referred to...
the references given. Particularly relevant are documents published by DNV (Cramer, 1994) and ABS (ABS, 1993, etc.).

The consensus is that linear spectral methods are adequate for fatigue damage calculations, and presumably for crack propagation calculations as well. However, for the calculation of extreme load non-linear effects may become important. Where they are significant it may be necessary to apply corrections to account for phenomena such as slamming. This consideration also applies to loads induced by rolling which is known to behave non-linearly. The procedure outlined below does not address non-linear effects.

E.2 Problem Definition

The primary tasks under this heading are to define the problem to be analyzed, and gather the required physical ship data.

The following data shall be gathered:

1. Lines plans and/or offset table - to define geometry of hull
2. General arrangement
3. Weights - to define mass distribution of ship
4. Loading arrangements to be considered
5. Scantlings
6. Steel material properties
7. Structural elements selected for study
8. Relevant load types
9. Duration for which assessment is to be performed

Items 1, 2 and 3 are required for the calculation of RAOs. Items 1 through 8 are required for conducting both global and local finite element analyses the purpose of which is to determine stress coefficients. Items 6 through 9 are required for the damage tolerance assessment phase of the work.

The relevant types of loads for the side shell longitudinal analyzed in Section 7 are:

- stillwater bending moment
- vertical hull girder bending moment
- horizontal hull girder bending moment
- external hydrodynamic pressure
- internal tank loads (inertial fluid loads and added static head due to vessel motion)
E.3 Operational Profile Definition

The primary tasks under this heading are to compile operational ship data. This data can be expressed in terms of an operational profile matrix the dimension of which depends on the number of parameters that vary significantly. In general the following data are required:

1. trading patterns
2. loading patterns
3. speed patterns
4. heading angles
5. time at port

The steps in defining the operational profile are as follows:

1. Establish trading patterns by reviewing routes the ship will ply for the duration under consideration. The percentage of time spent in each part of the route (i.e. in each “Marsden” zone as discussed in Section 5.3.1) shall be compiled. In the absence of detailed information the North Atlantic could be assumed.

2. Select loading patterns by considering cargo weight information, including weight, location and centre of gravity, for each weight item and determining likely loading arrangements.

3. Determine speed variations for the vessels. Speed variations are generally not significant for damage tolerance assessment purposes in the case of tankers, bulk carriers, and other large commercial ships.

4. Determine heading angles for route for each “Marsden” zone that falls on the route.

5. Estimate proportion of time that the ship will spend alongside. In the absence of detailed information 15% may be assumed for commercial ships such as tankers, bulk carriers and container ships.

6. Compile operational profile matrix, the dimension of which will depend on the number of parameters that are considered to be variable.

E.4 Wave Climate Definition

The primary tasks under this heading are to compile operational ship data. The following data are required:

1. Select wave spectrum. In the absence of measured spectra specific to the route of interest standard spectra presented in Section F.5 of Appendix F.
2. Select “wave spreading” model to account for short-crestedness.

3. Compile wave scatter diagrams for route. Using wave scatter diagrams from sources such as those identified in Section 5.3.1 and route and heading data a composite wave scatter diagram may be compiled.

E.5 Calculation Of RAOs And Stress Coefficients

The ultimate objective of this part of the process is to develop stress transfer functions for the structural elements of interest, the general form of which is for a particular point in the structure:

\[ H_o(\omega, \phi) = \sum_{i=1}^{n} A_i H_i(\omega, \phi) \]

where:

- \( H_o \) = stress transfer function
- \( H_i \) = response amplitude operator, or load transfer function for load “I”
- \( A_i \) = stress coefficient for load “I”
- \( n \) = number of load types relevant for structural element

There are two distinct but related tasks under this heading:

1. Calculation of RAOs
2. Calculation of stress coefficients

The steps for each calculation are summarized below:

Calculation of RAOs

1. Select a ship motion and ship load program that is capable of computing ship motion and ship load frequency response functions for all relevant components. Programs that employ linear strip theory in the frequency domain are adequate although more advanced programs, which in principle should give better results, are also available. See Section 5 for discussion.

2. Select the load types relevant to the analysis. These will depend upon the location of structure as discussed under “Problem Definition” above.
3. Select wave headings for which RAOs to be calculated. This will be based on decisions made in defining the Operational Profile. Generally 15° increments for headings ranging from 0° to 180° is sufficient.

4. Select range of frequencies for which RAUs to be calculated. Generally wave frequencies in the range 0.2 to 2 rad/sec in increments of 0.05 rad/sec is sufficient.

5. Select speed/s for which RAOs to be calculated. This will be based on decisions made in defining the Operational Profile.

6. Select loading arrangements for which RAUs to be calculated. This will be based on decisions made in defining the Operational Profile.

7. Use shipmotion and seaload program to calculate RAOs for load types identified under “Problem Definition” above. In the cases of local load types (e.g. external hydrodynamic pressure) it is only necessary to calculate the load for the set of wave period and heading angle that produces the maximum value of the corresponding global load type. In general this will vary from load arrangement to load arrangement.

Calculation of stress coefficients

Stress coefficients should in general be computed from finite element analyses and this approach is summarized below. Alternative simpler approaches based on hand calculation methods are published by several classification societies.

The steps involved are:

1. Develop global finite element model of the whole ship or part thereof. Exercise the model to determine the stress due to unit sectional load for each load type identified as significant. In general the finite element model shall be designed to allow the following loading effects to be modelled:
   - vertical hull girder bending including shear lag effects
   - vertical shear forces distribution between ship sides and bulkheads
   - horizontal bending moment including shear lag effects
   - torsion of hull girder (particularly for ships with large openings)

2. As an approximation it is often possible to restrict calculation, for each load type, to a single wave frequency and a single heading, and apply the same stress to other frequencies and headings.
3. Develop local finite element models for each part of the ship structure of interest. Boundary conditions determined from the global finite element model should be applied.

The reader is referred to Section 6 for guidance on global and local finite element analysis.

5. Computation Of Response For Each Combination of Seastate and Heading

The steps to be followed are:

1. For each combination of seastate and heading compute the following:

\[ S_\omega(H,T,\theta) = |H_\omega(\theta)|^2 \cdot S_\omega(H,T) \]

where the parameters are defined in Section 5.3.4.

2. Establish stress range spectrum for each combination of seastate and heading according to:

\[ F_{\Delta\sigma_y}(\sigma) = 1 - \exp\left( -\frac{\sigma^2}{8m_{\sigma_y}} \right) \]

where the parameters are defined in Section 5.3.4.

6. Compilation of Stress Range Spectrum

1. The step performed in this section is to combine individual stress range spectra computed in the previous step:

\[ F_{\Delta\sigma}(\sigma) = \sum_{y=1}^{t_y} r_y \cdot F_{\Delta\sigma_y}(\sigma) \cdot p_y \]

where the parameters are defined in Section 5.3.4.

7. Computation of extreme stress

The following steps shall be followed:

1. Compute probability density function for long-term response:
Parameters are defined in Section 5.3.6.

2. Compute the number of cycles in the period of interest:
\[ n = \left( \sum_i \sum_j n_ip_ip_j \right) \times T \times (60)^2 \]

3. Compute cumulative distribution function incorporating risk parameter:
\[ \frac{1}{1 - F(\sigma_o)} = \frac{n}{\alpha} \]
Parameters are defined in Section 5.3.6.

4. Compute extreme stress due to wave load.

5. Add stress due to stillwater bending moment.

References

AMERICAN BUREAU OF SHIPPING, "Analysis Procedure Manual for Dynamic Loading Approach (DLA) for Bulk Carriers", ABS, April 1993. (Similar manuals exist for tankers and container ships)

AMERICAN BUREAU OF SHIPPING, "Guide on Fatigue Strength Assessment of Tankers", ABS, 01995.


APPENDIX F

F.1 Introduction

The purpose of this Appendix is to provide information on the input data required for the wave load estimation methodologies presented in Section 5. This refers primarily to the Level 2 and 3 methodologies. The Level 1 methodologies, which are based on parametric equations, are self-sufficient.

F.2 Trading Patterns

In cases where trading patterns are not known, generic trading patterns presented in Table F.1 may be used.

Table F.1: Fatigue Wave Environment Trading Patterns
(Lloyd's Register of Shipping, 1996)

<table>
<thead>
<tr>
<th>SHIP TYPE/TRADE</th>
<th>TRADING PATTERN</th>
<th>AS % OF SERVICE LIFE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EXPORTING AREA</td>
<td>IMPORTING AREA</td>
</tr>
<tr>
<td>Large crude oil tanker</td>
<td>Persian Gulf</td>
<td>Europe</td>
</tr>
<tr>
<td></td>
<td>(Ballast via Suez, fully loaded via Cape)</td>
<td>Persian Gulf</td>
</tr>
<tr>
<td></td>
<td>Persian Gulf</td>
<td>US Gulf</td>
</tr>
<tr>
<td>Bulk carrier (Coal trade)</td>
<td>Australia</td>
<td>Japan</td>
</tr>
<tr>
<td></td>
<td>Australia</td>
<td>Europe</td>
</tr>
<tr>
<td></td>
<td>USA</td>
<td>Europe</td>
</tr>
<tr>
<td></td>
<td>South Africa</td>
<td>Europe</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>Japan</td>
</tr>
<tr>
<td>Bulk Carrier (Ore trade)</td>
<td>Australia</td>
<td>Far East</td>
</tr>
<tr>
<td></td>
<td>Australia</td>
<td>Europe</td>
</tr>
<tr>
<td></td>
<td>South America</td>
<td>Europe</td>
</tr>
<tr>
<td></td>
<td>South America</td>
<td>Japan</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>Europe</td>
</tr>
<tr>
<td>Large bulk carrier</td>
<td>Ore trade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coal trade</td>
<td></td>
</tr>
<tr>
<td>Panamax bulk carrier</td>
<td>Ore trade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coal trade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Grain trade</td>
<td></td>
</tr>
</tbody>
</table>
F.3 Load Conditions

In the absence of specific data, the data in Table F.2 may be used:

Table F.2: Fraction of Time at Sea in Loaded and Ballasted Condition
Assuming 85% of Time at Sea (Cramer et al., 1995)

<table>
<thead>
<tr>
<th>Vessel Type</th>
<th>Tankers</th>
<th>Container Vessels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loaded Condition</td>
<td>0.45</td>
<td>0.65</td>
</tr>
<tr>
<td>Ballast Conditions</td>
<td>0.40</td>
<td>0.20</td>
</tr>
</tbody>
</table>

OBO (oil/bulk/ore) carriers are likely to have a considerably smaller proportion of ballasted voyages.

F.4 Operational Parameters

Lloyd (1989) has summarized seakeeping criteria from a variety of sources that indicate the parameters used by operators to limit speed in severe weather. The table is reproduced below in Table F.3.

F.5 Wave Spectra

The Level 2 and 3 approaches require a wave height spectral model. Many wave spectra have been developed since the first ones were proposed in the early fifties. In a recent review, Ochi (1993) notes that it is highly desirable to use wave spectra that require the minimum number of input parameters. If this restriction is applied the number of possible wave spectra is reduced to less than half a dozen or so. A limited selection is presented below. For further information on wave spectra, the reader is referred to the aforementioned reference and to texts by Sarpkaya and Isaacson (1981) and Faltinsen (1990) as examples of discussions on the subject.

One of the most well known wave spectra was developed by Pierson-Moskowitz (1964)

\[ S_n(f) = \frac{8.10 g^2}{10^3 (2\pi)^4 f^3} \exp \left[ -0.74 \left( \frac{g}{U_{19.5} \cdot 2\pi f} \right)^4 \right] \]

where
\[ g = \text{acceleration due to gravity (m/s}^2) \]
\[ U_{19.5} = \text{mean wind speed at a height of 19.5 m (m/s)} \]
An alternative formulation was developed by Bretschneider (1959):

\[
S_n(f) = \frac{5H_s^2}{16f_m} \left( \frac{f}{f_m} \right)^5 \exp \left[ -\frac{5\left( \frac{f_m}{4f} \right)^4}{f} \right]
\]

where

- \( H_s \) = significant wave height
- \( f_m \) = wave modal frequency, the frequency at which the spectrum is maximum

<table>
<thead>
<tr>
<th>AUTHOR</th>
<th>SHIP TYPE</th>
<th>SLAMMING</th>
<th>WETNESS</th>
<th>PROPELLER EMERGENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kehoe (1973)</td>
<td>Warship</td>
<td>60/hour at 0.15L(^1)</td>
<td>60/hour at FP(^2)</td>
<td>Probability</td>
</tr>
<tr>
<td>Ochi and Motter (1974)</td>
<td>Merchant</td>
<td>Probability 0.03</td>
<td>Probability 0.07</td>
<td></td>
</tr>
<tr>
<td>Shipbuilding Research Association of Japan (1975)</td>
<td>Merchant</td>
<td>Probability 0.01</td>
<td>Probability 0.02</td>
<td></td>
</tr>
<tr>
<td>Lloyd and Andrew (1977)</td>
<td>Warship</td>
<td>36/hour</td>
<td></td>
<td>Probability 0.1</td>
</tr>
<tr>
<td>Lloyd and Andrew (1977)</td>
<td>Merchant</td>
<td></td>
<td></td>
<td>120/hour</td>
</tr>
<tr>
<td>Aertssen (1963, 1966, 1968, 1972)</td>
<td>Merchant</td>
<td>Probability 0.03 or 0.04</td>
<td></td>
<td>Probability 0.25</td>
</tr>
<tr>
<td>Andrew and Lloyd (1981)</td>
<td>Warship</td>
<td>90/hour</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comstock et al. (1982)</td>
<td>Warship</td>
<td>20/hour</td>
<td>30/hour</td>
<td></td>
</tr>
<tr>
<td>Yamamoto (1984)</td>
<td>Merchant</td>
<td>Probability 0.02</td>
<td>Probability 0.02 at FP</td>
<td></td>
</tr>
<tr>
<td>Walden and Grundmann (1985)</td>
<td>Warship</td>
<td>Probability 0.03</td>
<td>Probability 0.07</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\)L = length of ship; \(^2\)FP = forward perpendicular
This form has two variables, \( H \) and \( m \), and is suitable for partially developed wind-generated seas as well as fully developed seas. In the present context, this formulation is most useful for determining loads using the spectral approach because it can be conveniently used with scatter diagrams which are essentially an expression of the joint probability density function of wave height and wave period. For that purpose, the wave modal frequency can be replaced by the average zero-crossing period using the following relationship:

\[
f_m = 0.71 / T
\]

Other two-parameter spectra, very similar in form to the Bretschneider spectrum, have been proposed by the International Ships and Offshore Structures Congress (1964) and the International Towing Tank Conference (1978); both are described in Ochi (1993).

Ochi and Hubble (1976) developed a six-parameter spectrum. All parameters are functions of \( H \). This spectrum is able to model the double peaks evident in spectra in which both wind generated waves and swell are significant.

The spectra discussed to this point are intended for the open ocean. However, measurements suggest that the general form of spectra based on measurements taken in more confined locations are significantly different. Perhaps the most prominent of this class of spectral model is the so-called JONSWAP spectrum which is based on measurements of waves in the North Sea (Hasselman et al., 1973)

\[
S_n(f) = \frac{a g^2}{(2\pi)^4 f^5} \exp \left[ - \frac{5}{4} \left( \frac{f_m}{f} \right)^4 \right] \gamma^a
\]

where \( \gamma \) is the ratio of the maximum spectral density to that of the corresponding Pierson-Moskowitz spectrum and is typically about 3.3. Hence, the JONSWAP spectrum is much "peakier" than the Pierson-Moskowitz spectrum which is representative of the open ocean. The other parameters in the JONSWAP spectrum are defined below:

\[
a = \exp \left[ \frac{-(f - f_m)^2}{2\sigma^2 f_m^2} \right]
\]

\[
\sigma = \begin{cases} 
0.07 & \text{for } f \leq f_m \\
0.09 & \text{for } f > f_m 
\end{cases}
\]
F is the fetch, and the wave modal frequency is given by:

\[ f_m = 2.84 \left( \frac{gF}{U^2} \right)^{-0.33} \]

The parameter \( \alpha \) is given by:

\[ \alpha = 0.066 \left( \frac{gF}{U^2} \right)^{-0.22} \]

F.6 Response Amplitude Operators

The "response amplitude operator" is a term used in the naval architectural word to represent what is more generally known as a "transfer function" or "frequency response function". Transfer functions as developed for electronic and communication theory were first applied to the ship motion problem by St. Denis and Pierson (1953). The essential idea is that an input signal, which may contain energy at a range of frequencies, applied to a transfer function will be yield an output signal comprising several frequency components each of which will either be amplified or attenuated. An important feature of transfer functions is that they are complex and contain both a real and an imaginary term; this captures the phase relationship between input and response. The importance of this manifests itself when the response is due to multiple inputs.

In terms of ship motion the input signal is usually represented by a spectrum of wave height. The transfer function (or response amplitude operator) will typically express a range of values of a response parameter (e.g. roll amplitude) as a function of frequency for unit wave amplitude. The output signal will be the spectrum of the response parameter. The reader is referred to Lloyd (1989) and Bhattacharyya (1978) for discussion on response amplitude operators. A typical example for relative motion of the forefoot of a frigate is shown in Figure F.1. The effect on response of applying wave spectra with different dominant frequencies is shown in the figure.

In addition to programs based on strip theory several programs have been developed using three-dimensional diffraction theory. Several time domain programs have also been developed that account for non-linearities in wave loading. Specialist programs that model slamming and hydro-elastic response have been developed. However, such programs are still the subject of research and are not widely used in industry. A summary review of the current state-of-the-art is contained in a paper by Guedes Soares et al. (1996).
Figure F.1: Effect on Relative Response of Wave Spectra With Different Dominant Frequencies (Lloyd, 1989)
A summary of selected programs for predicting wave loads is presented below in Table F.4.

<table>
<thead>
<tr>
<th>ORGANIZATION</th>
<th>PROGRAM/SYSTEMS</th>
<th>SHORT DESCRIPTION</th>
<th>REFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Det Norske Veritas (DNV)</td>
<td>WAVESHIP</td>
<td>Linear strip theory, frequency domain</td>
<td>Valsgård, et al., 1995</td>
</tr>
<tr>
<td>DNV</td>
<td>NV1418</td>
<td>Non-linear strip theory, time domain</td>
<td>Valsgård, et al., 1995</td>
</tr>
<tr>
<td>DNV</td>
<td>WADAM</td>
<td>3D linear diffraction theory with zero forward speed, frequency domain</td>
<td>Valsgård, et al., 1995</td>
</tr>
<tr>
<td>DNV</td>
<td>FASTSEA</td>
<td>2½D high speed theory, valid for Fn above 0.4, frequency domain</td>
<td>Valsgård, et al., 1995</td>
</tr>
<tr>
<td>DNV</td>
<td>SWAN</td>
<td>Linear and non-linear frequency domain with forward speed, time domain with zero or forward speed</td>
<td>Valsgård, et al., 1995</td>
</tr>
<tr>
<td>Germanischer Lloyd (GL)</td>
<td>GL2D</td>
<td>Strip theory</td>
<td>Guedes Soares, et al., 1996</td>
</tr>
<tr>
<td>Instituto Superior Tecnico</td>
<td></td>
<td></td>
<td>Guedes Soares, et al., 1996</td>
</tr>
<tr>
<td>Registro Italiano Navale</td>
<td></td>
<td></td>
<td>Guedes Soares, et al., 1996</td>
</tr>
<tr>
<td>Technical University of Denmark</td>
<td>SOST</td>
<td>Second order strip theory</td>
<td>Jensen and Pederson, 1979</td>
</tr>
<tr>
<td>Defence Research Establishment Atlantic</td>
<td>SHIPMO 7</td>
<td>Linear strip theory</td>
<td></td>
</tr>
<tr>
<td>Naval Surface Warfare Centre, Carderock</td>
<td>SMP</td>
<td>Linear strip theory</td>
<td></td>
</tr>
<tr>
<td>Hydromechanics, Inc.</td>
<td>SCORES</td>
<td>Linear strip theory</td>
<td></td>
</tr>
<tr>
<td>Technical University of Nova Scotia</td>
<td>SMCA4</td>
<td>3D, frequency domain</td>
<td></td>
</tr>
<tr>
<td>Technical University of Nova Scotia</td>
<td>NOSMA</td>
<td>3D, time domain</td>
<td></td>
</tr>
<tr>
<td>American Bureau of Shipping</td>
<td>ABS/SEAKEEPING AND LOAD suite</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
F.7 Stress Coefficients

The stresses caused by the global and local hull girder loads need to be calculated at some point in the damage tolerance assessment process. As with other elements of the calculations there a number of approaches with varying degrees of complexity and accuracy. The approach employed should, in general, be consistent with complexity and accuracy applied to other elements of the assessment process. The general requirement is to develop stress coefficients which express the field stress at the point of interest for a unit value of the load component (e.g., vertical bending moment). Strictly, the stress coefficients are a function of wave frequency. However, Cramer et al. (1995) indicate that it is acceptable practice to compute stress coefficients for one particular wave frequency, and heading for that matter, and apply it to all wave frequencies and/or headings.

The simple approach is to calculate stresses at the station/s of interest using the computed hull girder bending moments and shear forces and the relevant sectional properties. Estimates of stress can be improved somewhat to account for gross effects such as shear lag, openings in decks and the effect of the superstructure using various rules-of-thumb. Hughes (1983) discusses methods for accounting for some of these effects. This simple approach for computing global stresses is more appropriate for Level 1 methods for damage tolerance assessment.

Stresses at the structural assembly level may be calculated either using hand calculation methods if the structure is fairly regular (e.g., rectangular plates) or can be modelled as a frame. In regard to the latter, Cramer et al. (1995) provide guidance for modelling frame and girder structures. In cases where the structure is irregular and cannot be represented in simple form the finite element method is the only practical method for determining stresses.

For Levels 2 and 3 the use of finite element methods are appropriate for determining stress coefficients.

Finite element methods offer the most convenient approach for calculating stress coefficients especially for ship structures with significant discontinuities. The application of finite element methods to ship structures is a large subject which cannot be addressed satisfactorily in this appendix. Readers are referred to Hughes (1983) and Cramer et al. (1995) for general guidance of ship structure finite element analysis, and Basu et al. (1995) for guidelines on the application of the method.
F.8 References


PROJECT TECHNICAL COMMITTEE MEMBERS

The following persons were members of the committee that represented the Ship Structure Committee to the Contractor as resident subject matter experts. As such they performed technical review of the initial proposals to select the contractor, advised the contractor in cognizant matters pertaining to the contract of which the agencies were aware, and performed technical review of the work in progress and edited the final report.

Chairman:

Mr. Peter Noble | American Bureau of Shipping
Mr. John Porter  | Defence Research Establishment Atlantic
LCDR Patrick Little | United States Coast Guard
Mr. Robert Sedat | United States Coast Guard
Ms. Lisa Hecker | United States Coast Guard
Mr. Trevor Butler | Memorial University of Newfoundland

Technical Advisor:

Mr. Harold Reemsnyder | Bethlehem Steel

Contracting Officer’s Technical Representative:

Mr. William Siekierka | Naval Sea Systems Command

Marine Board Liaison:

Dr. Robert Sielski | National Academy of Science

Executive Director Ship Structure Committee:

LT Thomas Miller | United States Coast Guard
CDR Steve Sharpe | United States Coast Guard