FINAL REPORT

VOLUME IIb
GEOTECHNICAL ASPECTS

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CHAPTER 1: INTRODUCTION

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This volume is part of the final report of the MAST III project PROVERBS, PRObabilistic design tools for VERtical BreakwaterS (February 1996 – January 1999) under contract no. MAS3-CT95-0041. The various parts of the final report are as follows (this volume in bold letters):

- **Volume I**
  

- **Volume IIa**


- **Volume IIb**


- **Volume IIc**


- **Volume IIId**

Apart from the final report, numerous reports have been produced in the framework of this project which will be referred to in the final report.

This volume deals with geotechnical aspects. It is produced by the PROVERBS Task 2 group, in which 6 institutes from 5 European countries co-operated. It should be read in combination with Volume I which deals with all aspects of probabilistic design tools for vertical breakwaters, among which are the geotechnical aspects. Thus, a survey of the geotechnical failure modes and phenomena, as well as the framework of geotechnical analysis are discussed in Volume I, Chapter 3. The chapters of this volume have the same titles as the sections of Chapter 3 of Volume I, in order to facilitate the study of the different aspects. The key issues addressed in this volume are as follows:

- guidelines on soil investigations and detailed data base on various sands and clays beneath vertical breakwaters;
- dynamic response of vertical breakwaters subjected to breaking waves;
- guidelines on instantaneous pore pressures and uplift forces;
- quantification of degradation and residual pore pressures;
- development of sophisticated limit state equations for vertical breakwaters;
- discussion of geotechnical stability of breakwaters;
- discussion of uncertainties associated with soil parameters;
- alternative foundation methods

The various chapters of this volume cover the aforementioned aspects and are briefly summarised in the following.

Chapter 2 describes the soil investigations that can be performed in several design stages. The corresponding sub-sections 3.3.1 to 3.3.4 in Volume I are mainly limited to the investigations needed for a feasibility study. The rest of section 3.3 of Volume I presents a survey of relevant soil parameters. There was no need to discuss this issue in this volume any more.

Chapter 3 deals with the dynamics. The corresponding section 3.4 in Volume I only presents some general guidelines and equations to be used for a simplified analysis. Here, the subject is discussed more extensively, including a brief survey of experience from the past, the results of large-scale tests and the results of full-scale tests on several Italian breakwaters. Special attention is payed to the role of the 3rd dimension: as wave impacts often attack no more than one caisson, the influence of the adjacent non-attacked caissons should be taken into account. Guidelines to quantify this influence are included.

Chapter 4 discusses the instantaneous pore pressures and uplift forces. The background of the guidelines formulated in section 3.5 of Volume I, are explained. The results of hindcasts of large-scale tests and full-scale measurements are extensively discussed. Also more detailed
equations and graphs are presented for the quantification of the influence of apron slabs, non-stationary flow in the rubble foundation, characteristic drainage period of the subsoil and pore pressures in the undrained part of the subsoil.

**Chapter 5** deals with the quantification of degradation and residual pore pressures. The corresponding section 3.6 of Volume I has a more qualitative character. It describes the phenomena and gives brief guidelines for the designer to find out whether degradation and residual pore pressures are relevant or not. If they are relevant, the designer can find further guidelines in Chapter 5 of this volume, including several analytical methods and a description of the background of some numerical methods.

**Chapter 6** concentrates on the presentation of 10 sophisticated limit state equations for rupture surfaces through the rubble foundation and the drained or undrained subsoil. Just 5 simplified limit state equations are presented in the corresponding section 3.7 in Volume I. That section, however, also discusses several aspects of geotechnical stability and deformation which are not discussed in this chapter, like the schematisation of the loads, 3-dimensional rupture surfaces and numerical methods. Chapter 6 in this volume, on the other hand, yields guidelines for quantification of stepwise failure due to repetitive loading (section 6.4), of the undrained shear strength of sand (section 6.5) and of the permanent deformation which may occur during an extreme wave impact exceeding the critical load during a very brief period and which remains limited due to the inertia of the caisson (section 6.6).

**Chapter 7** discusses the uncertainties associated with the soil parameters and with the use of the different models. It is an extension of the content of section 3.8 of Volume I, as it discusses more of the backgrounds of the uncertainties and deals with uncertainties of more parameters.

**Chapter 8** reports the results of a study in which the design and load parameters of a vertical breakwater on a thin bedding layer on coarse grained subsoil have been systematically varied to find the relevance of the different failure modes. Sub-section 3.9.2 of Volume I summarises the results.

**Chapter 9** discusses the possibilities for an alternative foundation method: a skirted caisson directly founded in the clayey subsoil, including formulation of the most important limit state equations. The results are summarised in sub-section 3.10.3 of Volume I.

Each chapter contains a list of references which points the reader to further and more detailed information on the respective subject. Finally the authors apologise for discrepancies between some formulations or definitions found in different parts of the final report. The final edition was made simultaneously with the final editions of the other volumes at the end of the project. The limited duration of the project did not allow for the incorporation of the last results of other parts of the project in each of the chapters of this volume.
CHAPTER 2: SOIL INVESTIGATIONS AND SOIL PARAMETERS

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ABSTRACT

Design of breakwater structures requires identification of the soil conditions and assessment of the soil parameters needed for design calculations. The soil investigation strategy is discussed, and methods to acquire soil data through field and laboratory testing and from existing databases and empirical relationships are summarised.

Key words: Soil investigation, soil parameters, cone penetration test, strength, stiffness

2.1. GENERAL

The geotechnical design requirements relevant for breakwater structures will normally be achieved by use of calculations, possibly supplemented by experimental model tests and monitoring of behaviour during construction, installation and operation.

The specification of the design situation and the geotechnical calculation models require knowledge of the water depth and seabed topography over the construction site. The soil stratification and hydrogeological conditions must be identified with classification of layers and zones involved in the calculation models. Finally the stress conditions and the relevant soil parameters for these zones and layers will have to be assessed.

The design process evolves from feasibility studies through preliminary design and finally detailed design. The requirements to geotechnical design calculations and cost restrictions will change during the design process and govern the phasing and the amount and detail of the site and soil investigations.
2.2. SOIL INVESTIGATION STRATEGY

2.2.1. Feasibility phase

In the feasibility phase the collection of site and soil data will have to be adapted to the decision process which may involve selection of breakwater location, type and size of the structure and requirements for the construction schedule. The geotechnical calculations are rough, mainly addressing sizing of foundation width to ensure stability and evaluation of settlements in cases with soft soils.

The acquisition of available information from regional and local authorities, local geological information and reports from previous soil investigations and construction projects should be used as a starting point. Additional information may be required.

A geophysical survey with bathymetric and seismic profiling will provide information about water depth and seabed topography, the stratification and variability in layer thickness and depth to bedrock over the area in question. The surveys may also detect special features like boulders, rock outcrops, infilled channels, wrecks etc. which may influence the choice of location and/or type of breakwater structure.

A soil investigation with cone penetration tests (CPT), preferably with pore pressure measurement (CPTU) is recommended. In areas with soft clays field vane tests will give more reliable data regarding undrained shear strength. Soil borings with sampling and classification tests may be an alternative or a supplement to the CPTU tests.

When combining the information from the above described methods for data acquisition with empirical relationships and information contained in geotechnical databases, it is possible to establish a picture of soil layering and depth to bedrock over the area and to assess soil parameters for the feasibility study.

2.2.2. Preliminary design phase

Prior to the preliminary design phase the location of the breakwater and the type of structure has normally been decided or limited to a few alternatives. The design calculations regarding stability and deformations/displacements will be more extensive and wave-structure-soil interaction with dynamic and cyclic loading effects will be considered. The requirements to accuracy and detailing of site and soil data will increase. The information from bathymetric and seismic profiling, the CPTU tests and/or the soil sampling in the feasibility phase should be used to plan the most optimal program for additional soil investigations.
Laboratory testing is the only way to provide soil parameters with high accuracy for the site specific soil, and the preliminary design requirements necessitates soil sampling to provide samples for laboratory testing. The soil samples will enable a more exact classification of the soil types and a verification of the layering determined from CPTU testing and seismic profiling. Testing of the stress-strain-strength behaviour of the soil samples under relevant static and cyclic loading conditions provides the soil data required for design calculations and may allow a site-specific adjustment of the empirical relationships to cone resistance and vane strength.

Comparison with and use of empirical relationships between classification data and strength and deformation properties may reduce uncertainties connected to the inevitable interpolation required to transform the discrete information from in situ test points and soil samples to a continuous spatial description of the soil conditions.

The complex behaviour of soils subjected to cyclic loading has been studied in detail for a few selected soil types. The compilation of available information in the database developed for this project (NGI, 1998) will be of considerable help to establish reliable predictions of permanent pore pressure and strength and stiffness degradation of clays and sands without a comprehensive test program for each individual site. (See Section 2.4.6). Supplementary seismic profiling and in-situ testing (CPTU, field vane and other special tests) might be necessary to supplement the information from the feasibility phase.

2.2.3. Detailed design phase

The detailed design phase may require additional and or confirmatory testing to be performed. The need may arise from adjustments in position of the structure, changes in dimensions and weight during the preliminary design phase or in order to reduce uncertainties connected to interpolation and the inevitable spread of test data. Sufficient soil sampling during the soil investigation for the preliminary design phase with careful preservation and adequate storage (temperature, humidity etc.) may be a cost effective solution and save the cost involved in an additional field investigation.

2.3. FIELD INVESTIGATIONS

2.3.1. Geophysical surveys

The seismic profiling should cover the area(s) that will be influenced by the structural alternatives in question. Continuous seismic profiling with boomer equipment will in most cases give relevant information on soil stratification. The interpretation of layer thickness is
dependent on the wave propagation velocity, which can vary considerably dependent on the stiffness of the different soil layers. The interpretation should be tied to layer thickness interpreted from CPTs or borings in order to minimise errors. The bathymetric survey can be performed with echo-sounder equipment, and the side scan sonar is a valuable tool to identify objects protruding from the seafloor.

2.3.2. Cone penetration tests

In shallow water depths down to about 40 m standard onshore CPT(U) equipment can be used working from a moored barge or a jack up work platform. See Figure 1. A comprehensive description of the cone penetration test and the interpretation of the test results is given by Lunne & al. (1997). In this report only a few relevant relationships will be presented.

![Fig. 1: Shallow water CPTU testing, a) from moored barge, b) from jack up work platform](image)

2.3.2.1. Stratification and characterisation

The continuous or near continuous measurement of cone penetration resistance, $q_c$, sleeve friction, $f_s$, and pore pressure, $u$, vs. depth allows identification of changes in strength and permeability and can give a precise identification of stratigraphic changes as shown in Figure 2. When combined with measurement of sleeve friction this allows in most cases a clear identification of the soil type (i.e. gravel, sand, silt, clay or peat). The diagram shown in Figure 3 shows an example of a classification methodology based on CPTU tests. Diagrams for estimates of soil unit weight established by Larsson & Mulabdic (1991) are shown in Figure 4.
Fig. 2: Example CPTU results showing excellent profiling capacity (after Zuidberg & al. 1982)
Fig. 3: Classification chart based on normalised CPT/CPTU data (after Robertson, 1990)

Fig. 4: Soil unit weight from CPTU results (after Larsson and Mulabdic, 1991)
2.3.2.2. Relative density and friction angles of sand

The cone resistance, $q_c$, can be converted to information about relative density of sand as shown in Figure 5. The relative density, $D_r$, is defined as:

$$D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}} \quad (1)$$

where $e_{\text{max}}$ = maximum void ratio
$e_{\text{min}}$ = minimum void ratio
$e$ = void ratio

The cone resistance can be used to estimate the friction angle, $\varphi'$, (corresponding to drained triaxial tests) as shown in Figure 6 (Robertson and Campanella 1983). The relationship is sensitive to the compressibility of the sand as reported by Jamiolkowski & al. (1985).

Fig. 5: Relationship $D_r$-$\sigma_m$-$q_c$ for CPT tests in sand (Baldi & al. 1986)
2.3.2.3. Undrained shear strength of clay

The estimate of undrained shear strength of clays is based on the corrected total cone resistance, $q_t$, from CPTUs. The total cone resistance is corrected for the effect of pore pressure acting behind the cone in the following way:

$$ q_t = q_c + (1-a)u $$

where
- $q_t$ = corrected total cone resistance
- $q_c$ = measured cone resistance
- $a$ = effective area ratio = area of load cell piston/area of cone
- $u$ = measured pore pressure

Empirical correlations have been established based on compilation and comparison of a large number of tests. For feasibility studies it is recommended to use the following relationship:

$$ c_u^{\text{Average}} = \frac{(q_t - \sigma_{vo})}{N_{kt}} $$

where
- $c_u^{\text{Average}}$ = average undrained shear strength from triaxial compression, triaxial extension and direct simple shear tests $= (c_u^C + c_u^D + c_u^E)/3$
- $\sigma_{vo}$ = in situ vertical total stress
- $N_{kt}$ = cone factor which can be estimated from Figure 7
Fig. 7: Cone factor, $N_{kt}$, to estimate $c_u^\text{Average}$ as a function of plasticity index (after Aas & al. 1986)

For normally to lightly overconsolidated soft clays the excess pore pressure $\Delta u$ can be used to estimate the undrained shear strength. The relationship has the form:

$$c_u = \frac{\Delta u}{N_{\Delta u}}$$

where

- $c_u$ = undrained shear strength corresponding to CAUC test
- $\Delta u$ = excess pore pressure = $u_2 - u_o$
- $u_2$ = pore pressure measured behind the cone
- $u_o$ = equilibrium pore pressure
- $N_{\Delta u}$ = cone factor which can be estimated from Figure 8
2.3.2.4. Deformation and consolidation properties of sand

Through combination of experience and empirical relationships between relative density and strength and from tests in calibration chambers estimates of deformation and consolidation properties have been established. General reference is made to Lunne & al. (1997). The following relationship is recommended for normally consolidated unaged and uncemented predominantly silica sands:

\[
M_o = \begin{cases} 
4q_c & \text{for } q_c < 10\text{MPa} \\
2q_c + 20 \text{ (MPa)} & \text{for } 10\text{MPa} < q_c < 50\text{MPa} \\
4q_c + 120 \text{ MPa} & \text{for } q_c > 50\text{MPa}
\end{cases}
\]

and for overconsolidated sands:

\[
M_o = \begin{cases} 
5q_c & \text{for } q_c < 50\text{MPa} \\
250 \text{ MPa} & \text{for } q_c > 50\text{MPa}
\end{cases}
\]

where \(M_o\) = constrained modulus at the in situ effective vertical stress, \(\sigma_{vo}^'\).
For an additional stress, $\Delta \sigma_{vo}'$, the following is recommended for the stress range $\sigma_{vo}'$ to $\sigma_{vo}' + \Delta \sigma_{vo}'$:

$$M = M_0 \sqrt{\frac{\sigma_{vo}' + \Delta \sigma_{vo}'}{\sigma_{vo}'}}$$  \hspace{1cm} (7)

2.3.2.5. Constrained modulus of soft clay

For feasibility calculations of expected settlements, rough estimates of the constrained modulus, M, of soft clays might be required. The following relationship has been proposed by several authors:

$$M = \alpha \cdot (q_t - \sigma_{vo})$$  \hspace{1cm} (8)

For normally consolidated stress range in the Senneset & al. (1989) reports $\alpha$ to be in the range 4 to 8, while Kulhawy and Maine (1990) proposes a more general relationship:

$$M = 8.25 (q_t - \sigma_{vo})$$  \hspace{1cm} (9)

For feasibility calculations, $\alpha$ values in the range 4 to 6 is expected to give conservative (i.e. upper bound) settlement estimates. The uncertainty connected to the constrained modulus evaluated with the above expressions is considerable.

2.3.3. Field vane shear test

In very soft to stiff clays ($c_u$ less than 150 -200 kPa) the field vane shear test (Figure 9) may be better suited for determination of the undrained shear strength. The vane is normally pushed into the soil and tests are run at 0.5m intervals. The vane test will thus not give a continuous profile like the CPT(U) test, but the size of the blades and the failure mode of the soil is considered to give a more direct and more accurate measurement of the shear strength than the CPT.

The tests are run with a constant rate of rotation (Norwegian practise is $12^\circ$/min. at the top of the rods). The associated strain rate in the failure zone is high and the shear strength is mainly associated with the vertical cylindrical failure surface.
To arrive at representative strength values for stability analysis of embankments and foundations the vane strength, $c_u(F_v)$, must be corrected for strain rate and anisotropy effects:

$$c_{u,corrected} = \mu \cdot c_u(F_v)$$  \hspace{1cm} (10)

Aas et al. (1986) gave a thorough review of vane shear test and correction factors. Figure 10 shows the recommended correction factor as a function of $c_u(F_v)/\sigma'_{vo}$ from this reference.

**Fig. 9:** Field vanes tests, equipment and typical test result (after Aas & al, 1986)

**Fig. 10:** Correction factor $\mu$ vs. normalised field vane shear strength $c_u(F_v)/\sigma'_{vo}$ (after Aas et al. 1986).
2.3.4. Supplementary in situ testing

Other in situ test methods exist and may provide useful additional information if they are available. The most important supplementary in situ tests are listed in Table 1.

**Tab. 1: Supplementary in situ test methods**

<table>
<thead>
<tr>
<th>Test type</th>
<th>Specially suited to determine:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seismic cone</td>
<td>Small strain shear modulus, ( G_o )</td>
</tr>
<tr>
<td>Dilatometer</td>
<td>In situ horizontal stress</td>
</tr>
<tr>
<td>Pressuremeter</td>
<td>In situ horizontal stress</td>
</tr>
<tr>
<td></td>
<td>Stress-strain properties</td>
</tr>
<tr>
<td>Nuclear density probe</td>
<td>In situ density</td>
</tr>
<tr>
<td>Electric density probe</td>
<td>In situ density</td>
</tr>
</tbody>
</table>

Of these tests, the use of the seismic cone test to determine the small strain modulus, \( G_o \) is particularly relevant when dynamic behaviour is important. The seismic cone tests can be done in connection with the CPTU testing.

2.3.5. Sampling from the seabed

Simple sampling techniques like the gravity coring, vibrocoring and grab sampling can be performed from smaller and cheaper vessels or be combined with the geophysical surveys to get information about the uppermost section of the soil profile. The penetration depth of the gravity and vibrocorers is normally limited to less than 3 to 6 meters in soft clays and may be restricted to a few centimetres in dense sands and hard clays. The sample disturbance can be considerable.

2.3.6. Soil borings and soil sampling

In water depths less than about 40 meters standard onshore equipment can be used working from an anchored barge or a jack up work platform. At this depth it is possible to mount the drilling rig at the top of a reasonably sized casing. Reference is made to Andresen and Lunne (1986). In deeper waters offshore soil investigation techniques will be required. Work is carried out from special soil drilling vessels equipped with drillstring tensioning systems, heave compensators and seabed reaction frames. Details regarding equipment and sampling techniques can be found in Lunne & Powell (1992).

Standard onshore sampling equipment can be extended by rods or rods and pipes. The samples are taken by pushing or percussion depending on the soil conditions. Several types of
samplers are available and one should generally use the type that gives the least sample disturbance. This is especially important in soft to stiff clays where piston samplers are preferred. When piston sampler cannot be used, thin walled push samplers should be tried. In harder clays and dense sands hammer samplers have to be used, and in some case like boulder clays with stones, rock-coring techniques may be required.

2.4. LABORATORY TESTING

2.4.1. Classification tests

Classification tests should be performed to identify the soil. The classification tests should include grain size distribution, water content, w, unit weight of the soil, \( \gamma \), and specific density, \( \rho_s \), of the mineral. This allows the void ratio, e, to be determined

For cohesive soils the Atterberg limits (plastic limit, \( w_p \) and liquid limit \( w_L \)), index undrained strength, \( c_u \), (from torvane, fall cone, miniature vane and pocket penetrometer tests) and sensitivity, \( S_t \), should be determined.

For non-cohesive soils maximum and minimum void ratio (\( e_{\text{max}} \) and \( e_{\text{min}} \)), grain angularity and mineralogy should be determined.

The number of classification tests should be sufficient to describe the spatial variability of the different soil layers when combined with the results from in situ tests like the CPTU and the field vane.

2.4.2. Consolidation tests

Consolidation tests should be performed to determine the preconsolidation pressure (cohesive soils), \( p_c \), the constrained modulus \( M \) (or the compression index, \( C_c \)), the coefficient of consolidation, \( c_v \), and the permeability, \( k \), of the materials. The coefficient of consolidation is normally calculated from the equation:

\[
c_v = \frac{M \cdot k}{\gamma_w}
\]

(11)

The compression index and the constrained modulus are related through:
The consolidation tests can be run as incremental loading or constant rate of strain tests. Unloading/reloading loops should be included to determine the unloading/reloading modulus, $M_r$ (or the swell/reload index $C_r$). For soils with significant drained creep, rest periods for measurements of creep characteristics should also be included to determine the secondary compression index, $C_\alpha$.

2.4.3. Permeability tests

Permeability tests should be performed to determine the permeability, $k$, of the materials. The tests may be run as a part of consolidation tests or triaxial tests.

2.4.4. Monotonic (static) strength tests

Undrained monotonic direct simple shear tests (DSS), triaxial compression and triaxial extension tests should be performed. For sand, drained monotonic triaxial tests should also be included. The test results will give the information required for assessment of the undrained and drained strength parameters and the strain dependent deformation parameters of the material as listed below:

- Undrained shear strength $c_u$ (often designated $s_u$)
- Angle of internal friction $\phi$
- Cohesion $c'$
- Dilation angle $\Psi$
- Shear modulus $G$
- Elasticity modulus $E$
- Bulk modulus $K$
- Poisson ratio $\Gamma$ (normally designated by $\nu$ in geotechnical literature)

The relationships between the different moduli and the Poisson ratio are:

$$G = \frac{E}{2 \cdot (1 + \Gamma)}$$

$$K = \frac{E}{3 \cdot (1 - 2\Gamma)}$$
\[ M = \frac{E \cdot (1 - \Gamma)}{(1 + \Gamma) \cdot (1 - 2\Gamma)} = K + \frac{4}{3} G \]  

(15)

The specimens should be anisotropically consolidated to the in situ effective stresses under the weight of the soil. In cases with staged construction, i.e. placement of a berm on soft clay one or two years prior to the installation of the breakwater structure, tests should also be consolidated to the expected in situ stresses under the berm. Sand specimens should be prepared to the in situ relative density, \( D_r \), and then consolidated to the effective stresses under the weight of soil, berm and structure. Some undrained tests should also be performed on specimens consolidated to effective stresses below the in situ stresses, in order to determine the effect a reduction in effective stresses due to cyclic loading will have on the negative pore pressure due to dilatancy and on the undrained shear strength.

The DSS specimens should be preloaded close to the preconsolidation stress, and then unloaded to the desired consolidation stress. This should be done to achieve representative horizontal stresses in the specimen.

The triaxial tests should preferably be consolidated to the in situ effective stresses, but in case of non-structured clays with sample disturbance, one may consider preloading to the preconsolidation stresses with subsequent unloading to the in situ stresses.

The tests on sand should be precycled with a small cyclic shear stress under drained conditions prior to shearing.

### 2.4.5. Cyclic tests

Cyclic tests are required to describe the stress-strain behaviour of soils under combined static and cyclic loading. During the tests pore pressure and strain development are monitored. The execution and interpretation of cyclic tests is time consuming and expensive.

At the Preliminary Design level the cyclic testing of site-specific samples may be limited to DSS testing with undrained symmetrical ("two-way") cyclic loading. A comparison with the cyclic test data on comparable materials in the database may reduce the uncertainties connected to the (often limited) test programs carried out for one specific project.

In a detailed design cyclic triaxial tests and DSS tests with cycling around various average shear stress levels may also be needed and again a comparison with the database materials may lead to a reduced need for extensive test series and a improved confidence to the test results.
The specimens should be consolidated as described for monotonic testing and the tests should be run stress-controlled with constant cyclic stress amplitude in each test and with a 10s load period.

If wave impact forces are of special importance the test program should preferably be complemented with a few tests with 1 s load period or less to evaluate the effect of high strain rates on the cyclic strength of the soil.

2.4.6. The PROVERBS database

For sand and clay materials the database developed within this project (NGI, 1998) provides a comprehensive set of data from direct simple shear and triaxial static and cyclic tests performed on 2 clay types and on a number of different sands. The data have been presented in a normalised form and provides the user with interpreted charts for determination of pore pressure and cyclic and permanent strain development under various combinations of cyclic and static shear stress.

The database contains the most relevant data needed for foundation analyses of breakwater structures on sandy, silty and clayey soils:

- Bearing capacity analysis under cyclic and dynamic loading
- Assessment of shear modulus and stiffness
- Cyclic displacements
- Settlements caused by cyclic loading

The database contains data from laboratory tests on soils from 23 locations:

- 17 sand materials (offshore, inshore, beach, tailing and man made)
- 3 silty materials (offshore, tailing)
- 1 gravel (dam material)
- 2 marine clays (offshore and onshore)

The database is organised a main report with summary tables followed by a series of appendices A through W where data for each material are compiled:

- Summary table (origin, mineralogy, grain shape, specific gravity, \(e_{\text{min}}, e_{\text{max}}, D_{50}\), etc.)
- Grain size distribution curves
- Triaxial test results (static and cyclic)
- Direct simple shear test results (static and cyclic)
- The clay data have been presented in NGI papers, which have been copied and included in the relevant appendices.
The amount and quality of the data varies considerable from one material to another. Within the PROVERBS project a series of tests on Oosterschelde sand were performed at Aalborg University (triaxial static and cyclic tests) and at NGI (static and cyclic direct simple shear tests). The information contained in the database regarding Oosterschelde sand is comprehensive.

Figure 11 show the dependency between friction and dilation angle and the confining stress determined in static triaxial tests for Oosterschelde sand, and Figures 12 a-d shows examples from the database of interpreted cyclic triaxial tests data. Further data are shown in Chapter 5 and 7.

![Friction and Dilation Angle vs. Confining Stress](image)

**Fig. 11:** Friction and dilation angle of Oosterschelde sand vs. confining stress (Data from AU triaxial tests)
Fig. 12: Interpretation of cyclic triaxial tests on Oosterschelde sand
a) Permanent pore pressure vs. stress level and number of cycles
b) Average shear strain vs. stress level and number of cycles
c) Average and cyclic shear strains at N=10
d) Average and cyclic shear strains at N=100
REFERENCES


1. BACKGROUND KNOWLEDGE

The impact of breaking waves on caissons causes impulsive forces that do not last normally as long as necessary to make inertia forces irrelevant. Actually the impulse is often so short that it is balanced essentially by caisson inertia and only a small fraction of the force applied by wave to the caisson is transmitted to the foundation. For particular impulse duration however it can not be excluded that the maximum force applied by the caisson to the foundation might be greater than the maximum force applied by the wave to the caisson (dynamic amplification). In any case, in order to represent properly the force transmitted by caissons to the foundation (and therefore their stability) under the effect of breaking waves, a dynamic approach to their equilibrium (including inertia forces) must be followed.

In this chapter the existing knowledge on the subject is reviewed, pointing out the relevant and positive points on the basis of the indications obtained from prototype tests carried out at Genoa Voltri and Brindisi Punta Riso during 1997.

1.1. Dynamic qualitative response

As further introduction to the following paragraphs, a brief and simple overview of the qualitative dynamic response of a system to forces of different duration is presented.

It is sufficient, for a qualitative overview, to study the response of a simple Mass-Spring-Dash-pot (MSD) system to an external force:

\[ M \ddot{x} + D \dot{x} + Kx = F(t) \]  (1)
Indeed even complicated linear systems with several degrees of freedom, provided that damping is not a dominant factor, can be described as a collection of independent natural oscillation modes. Every system eigen-mode is in fact a solution of the simple equation for some natural frequency \( \omega (\omega^2 = K/M) \) or natural oscillation period \( T_n = 2\pi/\omega \) and relative damping \( \zeta \) 

\[
\ddot{\xi} + 2\zeta\omega \dot{\xi} + \omega^2 \xi = 0
\]  

(2)

The unit impulse response is easily obtained, imposing null initial condition on displacement \( x(0^+)=0 \) and a condition on velocity derived from the momentum balance integrated over the impulse duration:

\[
M \dot{x}, (0^+) = 1
\]  

(3)

The unit impulse response is:

\[
x = \frac{1}{M\omega\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \sin (\omega\sqrt{1-\zeta^2} t),
\]  

(4)

A reduction of amplitude per oscillation cycle slightly lower than 50\% was observed to be typical of breakwater caissons; the corresponding relative damping \( \zeta=0.1 \) is therefore adopted as a default.

In general the solution (satisfying initial conditions) can be obtained as the convolution of the unit impulse response with the applied force history.

The exciting force of interest is the force caused by a wave breaking against a vertical wall, which as a first approximation can be schematised as a triangular impulse characterised by some ratio between the raise time and the total duration \( T_d \). Two extreme cases are actually considered, a null rise time and a symmetrical impulse shape (rise time equal half duration).

Fig. 1 shows the resulting dynamic response factor \( \nu_d \) (called dynamic load factor in de Groot et al., 1995) defined as the ratio between the maximum actual response and maximum static response. The maximum displacement is lower than the static response whenever the duration \( T_d \) of the exciting force is a small fraction of the natural period of oscillation of the system \( T_n \); it is moderately greater depending on system damping and loading pattern whenever loading duration is significantly longer than the natural oscillation period.
Fig. 1 A Mass Spring system subjected to a unit impulse loading responds independently from the impulse shape when the mean force duration $T_d$ is shorter than half the natural period of oscillation $T_n$.

For $T_d/T_n < 0.5$ the response to every impulsive force shape is similar and only moderately reduced by damping. For $T_d/T_n > 2$ the response to a symmetric (or moderately asymmetric) loading is well represented by the static model, while abrupt loading cases of long duration, similarly to the step loading, do cause a relevant overshooting.

The asymptotic cases of impulse loading and abrupt load increase are analysed in more detail aiming to show the effect of damping and of rise time.

Fig. 2 shows how the maximum damped response to a step force increase is influenced by the rise time. If the raise time is shorter than the natural oscillation period, a relevant overshoot takes place, whereas if the rise time is longer, the response is almost static.
Fig. 2 Max. damped response to 'step' loading of raise time $T_r$.

Fig. 3 Displacement response factor as function of damping for step loading.

Fig. 4 Displacement response factor for impulse loading.

Fig. 3 shows the effect of damping for a step loading. For the assumed standard damping ratio, the maximum displacement does not differ much from the undamped case: dynamic response factor is 1.72 (overshoot) rather than 2.00 (undamped case).

Fig. 4 shows the effect of damping for an impulsive loading. The relative reduction of the dynamic response factor compared to an undamped system is similar $\approx 14\%$, and remains moderate for any damping ratio reasonable for caissons.

### 1.2. Existing models for isolated caissons


Goda (1994)$^1$ based his model on small scale tests in which a concrete block resting upon a crushed stone mound was hit at different elevations by a pendulum with a known momentum. The structure cross section is given in Fig. 5: note that the rubble mound is rather thick.

The tests showed that the top and bottom of the upright section moved in the same direction regardless of the elevation where the force was applied; this implies that every observable rotation centre was placed below the foundation.

---

$^1$ The paper is the English version of Goda (1973)
In order to obtain a system with two low rotation centres, a relevant added mass, representing the virtual mass of the rubble mound and foundation (moving almost rigidly with the caisson), was included in the model. For the sake of simplicity, the magnitude of the added mass was assumed to be equal to the caisson mass, see Fig. 6.

The model was calibrated aiming to represent the observed caisson sliding. The optimised restitution coefficient of the pendulum impact resulted equal to 0.2 (a rather low value compensating for other model approximations).
The model described in Oumeraci & Kortenhaus (1994) is conceptually represented by the two degrees of freedom system shown in Fig. 7. The mass of the system includes the caisson, geodynamic and hydrodynamic masses.

This model was validated against large-scale model tests conducted in the Large Wave Flume in Hannover (Oumeraci et al., 1992), with breakwater cross section given in Fig. 8. The model structure is characterised by a stiff and homogeneous foundation, and a good calibration could be obtained.

1.3. Empirical knowledge about caisson dynamics

Muraki (1966) performed some prototype measurements on Haboro Harbor breakwater (see Fig. 9) loaded by quasi standing waves.
The published data present some very interesting aspects. The Power Spectrum Density (example presented in Fig. 10) shows three peaks: one is slightly more (1.5 sec) and one slightly less (0.8 sec) than 1 sec and the third is at high frequency (about 8 Hz). The higher frequency seems to appear also in absence of the exciting force and could be just a non-linear response, due to compression of the very first layer of the rubble mound.
Fig. 10 Example of row signal published by Muraki: $\tau$ is 1 sec

Fig. 11 Example analysis on data from Muraki. The 7 Hz component may be due to non-linear effects

1.4. State of the art for designers

All the existing guidelines for vertical wall breakwater design regard the structural behaviour as static: i.e. the force applied by waves to the caisson, increased by caisson weight, are transmitted to foundation. Wave impacts and the dynamic behaviour of the structure are actually included in the Japanese coded approach, Goda (1994), but in the form of equivalent
static forces referring to the usual design in Japan, i.e. to caissons based on a consistent rubble mound.

MCS project, de Groot & al. (1995), has introduced and supported in western Europe at least the necessity to represent the dynamic behaviour of the caisson-foundation interaction introducing the dynamic load response factor. They provide tools for the evaluation of the natural oscillation period of caissons and a graphs that, based on the Kortenhaus & Oumeraci model of caisson dynamics, for triangular loading of varying asymmetry gives the load response factor as function of the ratio between the impulse duration and the natural oscillation period. Formulae for the evaluation of mass/damping/stiffness coefficients are based on 2-D flow in the water and on the concept of isolated caissons as far as the foundation is concerned. Equations are quoted or revised later on.

2. CONCEPTUAL DYNAMIC ANALYSIS

Caissons are essentially monolithic and rigid elements, in the sense that their deformation is much smaller than the deformation of the surrounding media (water and foundation) and their resistance is normally sufficient to stand even the strongest impact loads applied by waves. We do not affirm that there is no risk for caisson to break down, but simply that this is rare and its analysis is not the subject of this chapter.

As a rigid body every caisson has 6 degrees of freedom (3 translations and 3 rotations); caissons actually form long arrays and since the action of waves (horizontal and vertical forces) is intrinsically normal to the array axis, forces do not excite the longitudinal displacement and can, at least as a first approximation, be thought as applied at the mid length of each caisson, so that rotation along transversal axes (vertical and harbour directed) can be neglected. In conclusions the caisson degrees of freedom of interest are three, relative to the movements in the plane perpendicular to the longitudinal direction.

A breakwater is an array of caissons each of which has its own degrees of freedom and is normally more or less linked the adjacent ones. The breakwater is therefore characterised in principle by a huge number of degrees of freedom.

Isolated caissons dynamics can be represented by a Mass, Spring and Dash-pot (MSD) model where contributions to mass, stiffness and damping are partially due to rubble mound, foundation and sea-water.

Indeed some particles in the foundation or in water do move rigidly with the caisson and form the added mass, that can build up to a relevant fraction of the apparent caisson total mass and need for a proportionate attention.
The caisson dimensions are normally of the order of 20 m. Due to the static weight they apply to foundation, the shear modulus of the foundation is around 200 MPa and the shear waves speed is of the order of magnitude of 300 m/s. Caisson oscillations show frequencies in the range 1-5 Hz in prototype conditions. The volume in the foundation subject to relevant stress and strain due to wave action is therefore normally small compared to elastic wave length (<20%) and particles subject to relevant movements in the foundation move proportionally to and in phase with the caisson, i.e. displacements of the caisson are representative also of the displacement of part of the foundation. A similar conclusion can be drawn also for water masses adjacent to the caisson.

2.1. Co-ordinate systems

An isolated caisson has three relevant natural oscillation modes (see Fig. 12):
1. the sway mode (m1): an almost horizontal translation (or rotation around a low centre);
2. the roll mode (m2): a rotation around a higher centre;
3. the heave mode (m3): an almost vertical translation.

Two different co-ordinate system or poles are adopted for the analysis of prototype tests and for dynamic modelling.

The choice of the pole ‘O’ (the point of which displacement \{\xi_o, \eta_o\} is given together with rotation \(\theta\) in order to describe the rigid body movement) is arbitrary and the relation between any other pole position ‘C’ of co-ordinates \{x^c, y^c\} relative to ‘O’ and its displacement is trivial (see Fig. 13):

\[
\begin{align*}
\xi_c &= \xi_o + \theta_o \ y^c_o \\
\eta_c &= \eta_o - \theta_o \ x^c_o \\
\theta_c &= \theta_o \ (\text{index is then usually omitted for rotations})
\end{align*}
\]

When prototype caisson movements are described, the assumed reference system has the origin ‘C’ in proximity of the instrumentation (i.e. at the quay level) in order to reduce the noise level; components are:
1. sway, or harbour directed translation (\(\xi_c\)),
2. *heave*, or vertical translation ($\eta_c$),
3. *roll*, or rotation around a longitudinal axis ($\theta$).

For the dynamic modelling the pole ‘O’ is assumed to be at the caisson base centre in order to have the simplest representation of the mass matrix (hydrodynamic and geodynamic terms are naturally given for such pole) and of the stiffness matrix (which results diagonal); the assumed Lagrangian co-ordinates $q_i$ are:

1. the horizontal displacement of the centre of the caisson base $q_1$;
2. the longitudinal rotation of the caisson $q_2$ (equal to roll);
3. the vertical displacement of the centre of the caisson base $q_3$ (almost equal to heave).

Heave is placed last since it is often disregarded.

When we shall desire to represent the possibility that caissons may move not solidarily with the foundation essentially by sliding over it, two horizontal displacement components are defined: one for the caisson $\{q_{1c}\}$ and the other for the foundation $\{q_{1f}\}$.

Making use of the tools of analytical mechanics, we have to represent:
- the relation between kinetic energy, position and velocity of the system;
- the work performed by the active force system during any admissible displacement.

Kinetic energy is spread over caisson (index ‘c’), foundation (index ‘g’) and water (index ‘h’) causing a contribution of the three subsystem to the inertia of the system (mass matrix).

Generalised forces proportional to velocity components (pseudo-viscous resistance matrix), representing the energy loss due to wave radiation at the water surface and in the foundation...
and dissipation in the foundation granular material (due to hysteresis loops in loading/unloading cycles), contain terms only for the water and foundation subsystems.

Finally generalised forces proportional to system displacement (corresponding to the stiffness matrix) include only terms related to the foundation.

\[ M = M_c + M_h + M_g; \quad D = D_h + D_g; \quad K = K_g \]  

(6)

### 2.2. Caisson mass coefficients

Caisson behaves as a rigid body. For the dynamic analysis the caisson mass and moment of inertia (included the mixed term) must be supplied. The symmetric mass matrix coefficients can be easily computed from elementary mass geometry. It may not be completely useless to remind that, as in any dynamic equations, the real mass must be considered, without any reduction for buoyancy.

For a schematic parallelepiped caisson of width \( B \) (average body width) they are:

\[
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}_{\text{caisson}} = \rho_c h_c B L_c \begin{bmatrix}
1 & h_c / 2 \\
h_c^2 + B^2 / 4 & 3
\end{bmatrix}
\]

(7)

It must be also noticed that not all the terms of the dynamic analysis are exactly proportional to the length of the caisson, \( L_c \), and consequently it is not possible to make an analysis per unit length.

### 2.3. Hydrodynamic added mass terms

Linear wave theory is used to describe water motion around the caisson

The (symmetrical) added hydrodynamic mass matrix relative to a roto-translating paddle in contact with water on both sides is:

\[
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}_{\text{hydrodynamic}} = 2\rho_w d^2 L_c \begin{bmatrix}
0.543 & 0.333d \\
0.333d & 0.210d^2
\end{bmatrix}
\]

(8)

The mass relative to the vertical movement \( M_{33\text{hydrodynamic}} \) certainly exists and represents the inertia effect of water seeping in the compressed rubble mound but no information is available on it. The indexes refer to the coordinate system described in paragraph 3.2.1 \( \{q_1, q_2\} \).
Note that this added mass matrix has a non null mixed term. See de Groot et al (1995) for a discussion on diagonal terms and Lamberti and Martinelli (1996) for the off-diagonal term.

### 2.4. Geodynamic added mass terms

The homogeneous elastic half-space approximation is used to describe, at least as a first approximation, the foundation reaction to caisson movements.

The added geodynamic masses relative to a rigid circular foundation on a homogeneous elastic half space are given by the following relation (Richart et al. 1970):

\[
\begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{bmatrix}_{\text{geodynamic}} = \rho_s \begin{bmatrix}
\frac{0.76R^3}{2-\Gamma} & \approx 0 & 0 \\
0.64R^5 & 1-\Gamma & 0 \\
1.08R^3 & 0 & 1-\Gamma
\end{bmatrix}
\]

(9)

\( \Gamma \) is the foundation Poisson coefficient and \( \rho_s \) its density.

In case of a rectangular foundation, equivalent radii are considered:

- \( R \), radius of circle with equivalent area of caisson base: \( \pi R^2 = B_c L_c \)
- \( R_i \), radius of circle with equivalent inertia of caisson base: \( \pi R_i^4 = \frac{B_c^3 L_c}{12} \)

The caisson base width \( B_c \) must be supplied characterising the caisson-foundation contact area.

### 2.5. Foundation stiffness terms

The stiffness coefficients (of a slice of length \( L_c \)) of a strip foundation of width \( B_c \) are (Wolf, 1988):

\[
\begin{align*}
K_{11} &= (1 + 5 \Gamma^2) G L_c; \\
K_{22} &= (0.45 + 1.30 \Gamma^2) G L_c B_c^2; \\
K_{33} &= (1 + 4 \Gamma^2) G L_c;
\end{align*}
\]

(10)

The stiffness terms relative to an isolated rigid rectangular foundation on a homogeneous elastic half space are given in the following relations (Barkan, 1962):
\begin{align*}
K_{ii} &= 2(1 + \Gamma) G \beta_x \sqrt{B_c L_c} ; \\
K_{ij} &= \frac{G \beta_y L_c B_c^2}{1 - \Gamma} ; \\
K_{ij} &= \frac{G \beta_z \sqrt{B_c L_c}}{1 - \Gamma} ;
\end{align*}

\(L_c\) caisson length
\(B_c\) caisson base width
\(\beta_x, \beta_y, \beta_z\) coefficients depending on the shape of the base

The beta coefficients for a rectangular foundation are given in Fig. 15.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{beta_coefficients.png}
\caption{Beta coefficients}
\end{figure}

Wolf (1988) provides equations for a rectangular foundation including the effect of embedding \(e\), i.e. the effect of being the caisson base somewhat below the rubble mound surface. They are for the usual case \(L_c \geq B_c\)

\begin{align*}
K_{11} &= \frac{G B_c}{2 - \Gamma} [6.8(L_c/B_c)^{0.65} + 0.8L_c/B_c + 1.6] \frac{1 + (0.33 + 1.34 + L_c/B_c)(e/B_c)^0.8}{1 + L_c/B_c} ; \\
K_{22} &= \frac{G B_c^3}{1 - \Gamma} [3.2L_c/B_c + 0.8] \frac{1 + e/B_c + \frac{1.6}{0.35 + L_c/B_c}(e/B_c)^2}{1 + e/B_c} ; \\
K_{33} &= \frac{G B_c}{1 - \Gamma} [3.1(L_c/B_c)^{0.75} + 1.6] \frac{1 + 0.25(1 + B_c/L_c)(e/B_c)^0.8}{1 + 0.25(1 + B_c/L_c)} ; \\
K_{12} &= K_{11} e/3;
\end{align*}

In each formula the last term in brackets represents the effect of embedding and is equal to 1 if there is no embedding. The last equation represents the eccentricity of the horizontal spring in the case of embedding.
The rigidity of an isolated caisson should be evaluated through the equations for rectangular foundation, while the whole breakwater stiffness is better approximated by the strip foundation case when all the caissons move in phase. Consequentially when we shall describe the rigidity of a caisson moving in phase with all the others, the relative rigidity should be given by a slice of strip foundation as long as the caisson.

Elastic coefficients can be obtained knowing the nature of the rubble mound and accounting for vertical pressure $\sigma_v$ and degree of confinement $K_o$.

$$ E \quad \text{Young modulus of elasticity} \quad \frac{E}{\sqrt{\frac{\sigma_v}{3} + 2K_o}} $$

$$ G \quad \text{Shear Modulus} \quad G = \frac{E}{2(1+\nu)} $$

Typical values of the Young modulus are presented in the table below.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Relative density</th>
<th>Young Modulus (Confining pressure=100 kPa) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screened crushed quartz, fine angular</td>
<td>Loose-Dense</td>
<td>117-207</td>
</tr>
<tr>
<td>Screened Ottawa sand, fine rounded</td>
<td>Loose-Dense</td>
<td>179-310</td>
</tr>
<tr>
<td>Ottawa standard sand, medium, rounded</td>
<td>Loose-Dense</td>
<td>207-669</td>
</tr>
<tr>
<td>Screened sand, medium, subangular</td>
<td>Loose-Dense</td>
<td>138-241</td>
</tr>
<tr>
<td>Screened crushed quartz, medium angular</td>
<td>Loose-Dense</td>
<td>124-186</td>
</tr>
<tr>
<td>Well graded sand, coarse, subangular</td>
<td>Loose-Dense</td>
<td>103-193</td>
</tr>
<tr>
<td>Ticino sand</td>
<td>40 %-90 %</td>
<td>120-180</td>
</tr>
<tr>
<td>Hokksund sand</td>
<td>40 %-90 %</td>
<td>210-300</td>
</tr>
</tbody>
</table>

2.6. Damping

The damping is due mainly to:
- the propagation of shear waves along the foundation, the most evident effects being the propagation of the caisson oscillations along the breakwater;
- hysteretic cycles in the foundation material.

Damping due to propagation of surface water waves can be considered negligible (Pedersen, 1997).
Material damping is of the same order of magnitude as radiation damping, it becomes dominant at very low frequencies, but can not be easily represented nor is empirically well known.

On the other hand in prototype conditions damping of the response was observed to be of the order of 40% per cycle and nearly constant from case to case.

For systems where the damping is not dominant the dynamic solution is generally found identifying a set of undamped independent equations (the natural modes) and assuming that damping does not provide any relevant coupling among modes. Damping was therefore applied directly to the natural oscillation modes applying to each mode the observed reduction per cycle (3 empirical parameters) rather than through a calibrated pseudo-viscous matrix (potentially 9 calibration parameters).

3. HINDCAST OF LARGE SCALE TESTS

3.1. Survey of large scale tests

Hindcasts of several large scale tests have been performed aiming to check the effects of geotechnical simplifications of the mass-spring model. Hindcasts have been done with the analytical equations mentioned above and with the finite element computer code TITAN.

Hindcasts have been performed in three cases:

1. Model tests on a caisson in the Delta flume in the Netherlands [Meijers, 1994]. The model caisson had a width of 8.3 m. The typical width of a breakwater caisson is 20 m. The subsoil consisted of 0.3 m thick layer of course sand overlying a 2.5 m thick layer of fine sand. This means that the concrete floor of the flume could be considered as bedrock at a depth of 1/3 of the caisson width. The caisson was loaded with regular, non-breaking waves up to 2.6 m. No impacts occurred. The measuring program was very extensive. All relevant soil parameters (stiffness, density, saturation friction angle) were measured. This allows for a reliable hindcast of the spring coefficients and the natural periods.

2. Model tests on a caisson in the Large Wave Flume in Hannover [Hölscher et al., 1998]. The model caisson had a smaller width than in the Deltaflume (3 m), but the subsoil was relatively much thicker: 0.6 m fine rubble ($D_{50} \approx 30$ mm) overlying a sand bed with a thickness of 2.5 m. This means that here the “bedrock” was at a depth of about the caisson width. The caisson was loaded by breaking and non-breaking waves up to 1.3 m. Several large impacts were observed. The test program allows for a reliable hindcast of the spring coefficients and the natural periods. The hindcast included calculations by the finite element computer code TITAN (example shown in Figure 3-X). TITAN models inertia, stress and strain in the two phases, grain skeleton and pore water.
3. Outdoor test in Kats on a 1:10 model of one of the Oosterschelde barrier piers [Meijers, 1994]. The caisson width was 5 m; its length, however, (direction perpendicular to the direction of the loads) was no more than 2.5 m. It was placed directly on top of the natural subsoil of relatively dense sand, the parameters of which were derived from an extensive program of ept’s, borings and laboratory tests. A ca 1 m thick rubble layer was placed around the bottom of the caisson. The water level was just above the rubble layer. The caisson was cyclically loaded in horizontal direction by a plunger at 2.2 m above the caisson bottom. Many different frequencies and amplitudes were applied.

The movements of the caissons were carefully monitored in all above tests.

![Graph](image)

*Fig. 16 Hindcast of Hannover breakwater with TITAN: horizontal motion of caisson*

### 3.2. Spring coefficients

The hindcasts first concentrated on the spring coefficients. The measured spring coefficients in both flume tests were those found from the quotient of the force or moment and the movements of the caisson in case of quasi-stationary wave loading. The loading could be considered quasi-stationary, because the wave period was 20 to 50 times the largest natural period. In the Deltaflume also a phase difference between load and movement was observed, from which the damping was derived. The damping was considerable, especially for the rotational movement, and was taken into account for the determination of the spring coefficients.

The accuracy of the measurements could be estimated from the variations found. From 10 different regular waves in the Deltaflume 10 different values of both spring coefficients could be derived. The largest measured horizontal spring coefficient was 2 times the smallest. The largest measured rotational spring coefficient was 5 times the smallest. Similar variations were found in the Hannover tests.
With the Kats tests two sets of spring coefficients were derived: one by applying a stationary load and one by extrapolating the amplitude of the response curve to 0 Hz. The larger horizontal spring coefficient was 1.2 times the smaller one. The larger rotational spring coefficient was 1.6 times the smaller one.

Thus, the inaccuracy of the measured horizontal spring coefficients could be expressed by a factor $\approx 1.3$, that of the rotational spring coefficients by a factor $\approx 2$.

The average measured spring coefficients were compared with those found by the equations and those found by the numerical model (table). The coefficients found for the Delta flume by the numerical model are better than those found by the equations. Unlike the equations, the numerical model takes into account the small thickness of the soil layer present between caisson and flume bottom. The large differences between measured and calculated coefficients for Hannover are probably due to the inaccuracies of the measured values as discussed above, rather than unreliability of the calculations.

The spring coefficients measured in Kats for 50% of the failure load were half the coefficients found for 20%. The shear modulus used as input for the calculations was valid for ca 20% of the load. This explains the low values of the quotient in the last column.

<table>
<thead>
<tr>
<th></th>
<th>Delta flume</th>
<th>Hannover</th>
<th>Kats at 20% of failure load</th>
<th>Kats at 50% of failure load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal spring coefficient, $K_{11}$</td>
<td>1.3 (0.8*)</td>
<td>1.7 (1.6*)</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>Rotational spring coefficient, $K_{22}$</td>
<td>2.3 (1.3*)</td>
<td>0.5 (0.5*)</td>
<td>1.2</td>
<td>0.6</td>
</tr>
</tbody>
</table>

* Calculated with the numerical model

3.3. Natural periods

The measured natural periods were compared for the Hannover flume tests and the Kats tests with those predicted by the equations and those found by TITAN. The order of magnitude of the differences corresponded to the differences between measured and calculated spring coefficients.

4. PROTOTYPE TESTS
Prototype tests were performed in Genoa Voltri and Brindisi during 1997, stirring artificially some prototype caissons and measuring their acceleration. The purpose of the tests was to verify existing models of caisson dynamics and to check errors in the estimated values of soil parameters.

4.1. Description of tests

The tests were carried out hitting a vertical breakwater with a heavy sac and/or with a tug-boat in order to produce a significant dynamic response.

The sac, half filled with sand and weighting 2 tons, see Fig. 16, could fall completely free from a height of 5 m (or partially slowed down from a greater height) and it hit the caisson in proximity of its harbour edge, i.e. with a strong eccentricity. One accelerometer was installed on a “buoy” right into the sac on the sand surface, in order to give information on the sac deceleration during impact and on the applied force.

Fig. 17 shows the excitation due to the 100 tons tug boat hitting a caisson in Genoa Voltri (a 500 tons tug boat was used in Brindisi). The speed before the impact was close to 0.3 m/sec, varying on the occasions, and the impact force lasting approximately 0.5 sec for the smaller tug (1.0 sec for the bigger one) was measured by the accelerometer fixed on the tug-boat in the longitudinal direction.

The accelerations of the excited caisson and of the two adjacent ones were measured by 15 accelerometers placed as shown in Fig. 18. 9 accelerometers are placed on the central caissons, 1 describing the movements in the longitudinal direction and 8 describing the move-
ments in the perpendicular plane. The adjacent caissons were monitored with 3 accelerometers each. In total, seven groups of three adjacent caissons were monitored, belonging to three vertical breakwaters (the main breakwater in Voltri, the western lee breakwater in Voltri and the main breakwater in Brindisi, see Fig. 3.19) having similar shape (common to most breakwaters of this type in Italy) but different caisson size: the caisson mass is $3.0 \times 10^7$ kg, $1.0 \times 10^7$ kg and $2.0 \times 10^7$ kg respectively). The joints between caissons are wide ($5$-$10$ cm) only for the case of the main breakwater in Genoa, but in all cases the superstructure has joints at the ends of each caissons. The rubble mound height vary from $2$ m to $15$ m for the different cases, and the under-lying bed has a first layer of clayey-silty sand over a more fine material and rock below. A detailed description of the hydrodynamic conditions, caisson geometry, structural and foundation aspects are given in Lamberti et al (1998) and Lamberti & Archetti (1998).

![Fig. 18 Position of accelerometers.](image)

In order to reduce the high frequency vibrations produced by sharp impacts, and thus avoid amplifier saturation, the accelerometers of the central caisson were fixed on mechanical filters. Since vibrations are greatly reduced passing from the central excited caisson to the adjacent ones, instruments placed on the lateral caissons were not mechanically filtered.
Test sites and dates
V1, V2, V3  3rd - 5th Jun 1997
B1, B2  30th Sep - 1 Oct 1997
W1, W2  19th - 20th Nov 1997

Fig. 3.19 Position of tested caissons in tested breakwaters. Symbols starting with letters S, P or F are relative to several geotechnical drillings, analysed in Lamberti & Archetti (1998) and Lamberti et al. (1998)

The filter (see Fig. 20) is formed by a concrete cube (side of 20 cm) glued to the superstructure through 4 round rubber disks. The response of this filter to an impulse is shown in Fig. 21. Note that the natural frequency of oscillation (≈20 Hz) is out of the range of interest ([1÷8] Hz) and that the damping is sufficiently strong.

Fig. 20 The accelerometers of the central caissons were fixed on a concrete cube

Fig. 21 The concrete block was excited by a hammer, and this is the response of one of the accelerometers fixed on the block. The eigen-frequency of the mechanical filter is about 20 Hz and the damping is moderate: amplitude is halved in one cycle.
4.2. Analysis of tests

The analysis started with the quantitative definition of the original applied force signal; this is done with the aid of an accelerometer placed on the hitting body. For the sand sac excitation case the accelerometer direction was controlled fixing the accelerometer to a round and flat disk inserted into the sac, that could ‘float’ over the sand. The sac is however not rigid and the average acceleration could not be measured; the signal was used for the evaluation of the length of the impact and for the force-response synchronisation. The exact force history was then defined assuming an anelastic sac impact behaviour, knowing its weight and velocity before the impact and its duration. For the tug-boat excitation, the acceleration during the contact is actually proportional to the real applied force (a 10% hydrodynamic added mass was considered).

All the 16 channel registrations of the same type (same caisson, same kind of excitation) were cut 2 seconds before the impact and 8 seconds after it, pasted into a unique sequence, analysed in the frequency domain, or band-pass filtered (in Tab. 2 the range for the different tests is given) and analysed in the time domain.

Furthermore, phase averaged signals were created averaging each channel for the same kind of excitation over the different strokes. The records were synchronised maximising the cross correlation.

The rigid body movements of the excited caisson were evaluated combining its eight horizontal and vertical accelerations with a best fit procedure: the sway signal is approximately obtained by averaging the four original horizontal signals; the heave is approximately the average of the vertical signals and the roll is similar to the difference between the upper and lower horizontal and vertical accelerations divided by the respective distance of the instruments.

A similar procedure was applied to the adjacent caissons, whose rigid movements were averaged over the two caissons.

The following analysis consisted in assessing the Power Spectral Density of the responses and the Transfer Functions between force and response, both for the phase averaged signal and the sequence of tests. The power peaks coherent with the force were then evaluated for each signal, see Fig. 22 for instance. Since the force showed a rather flat spectrum around acceleration peaks, frequencies of the response peaks were interpreted as the natural frequencies of oscillation.

Signals were then band-pass filtered around the natural frequency. Vibrations are only partially filtered out from the rigid body signal; they are finally recognised comparing the single and fitted rigid body signals in the appropriate frequency range.
The damping of the natural oscillations was assessed measuring, for each mode, the rate of amplitude decrease of the *phase averaged signals* after the impact. Similarly, comparing for each mode the acceleration amplitude and phase of the central and adjacent caissons, the amount of energy that travels along the breakwater was evaluated.

The upper graph of Fig. 23 shows the acceleration at quay level (precisely the phase averaged sway signal) induced in Brindisi by the tug-boat. It is possible to note immediately the presence of two different harmonics. The harmonics are presented in the lower graph of the same figure, and they are obtained bandpass filtering the signal around [0.5-1.8 Hz] and [1.8-6 Hz] (e.g. around the frequency peaks observed in the roll and sway signals, see Tab. 2).

The sum of this two harmonics reproduces almost exactly the original signal (almost undistinguish dotted line in upper graph). The same two harmonics are present in the ‘roll signal’ describing two rigid body rotations, i.e. two modes.

The position of the rotation centre can be evaluated for each harmonic from the ratio between sway and roll, and/or, for the central caissons, evaluating the acceleration directions at the four corners. A further method can be followed: the power of the horizontal acceleration at any point can be easily evaluated combining sway and roll signals, the power of the horizontal acceleration is minimum if the pole is placed at the height of the rotation centre (Eq. 5 governs the effect of changing the pole).
Fig. 23  The sway acceleration induced in Brindisi by the tug-boat (upper graph) is the sum of two harmonics, presented in the lower graph, and they are obtained bandpass filtering the signal around [0.5-1.8 Hz], dotted line, and [1.8-6 Hz], full line. The sum of these two harmonics reproduces almost exactly the original sway signal (dotted line in upper graph).

Fig. 24 shows an example of the last method for one case in Brindisi. The relative power of the horizontal acceleration at different heights is represented for the two different frequency bands: it is clear that the minimum of the curve, i.e. the position of the rotation centre, is placed below the caisson base for both the harmonics. Similar results are found for the cases in Voltri.
Fig. 24  The horizontal acceleration power (divided by its minimum value) in the two frequency bands (see Fig. 23) is presented as function of the distance from quay level. The minimum value of the acceleration power is considered as a good estimation of the height of the rotation centre. The rotation centre is placed well below the caisson base (caisson height = 21.7 m).

Fig. 25 represents, maybe more clearly, the same effect: within the frequency range of the 2nd observed natural frequency, the corners of the caissons move in the directions shown by the two graphs, obtained plotting the horizontal vs vertical acceleration. The rotation centre is located at the intersection of the two radii orthogonal to displacements or accelerations: evidently a rotation around a very low centre is taking place in this case.

Fig. 25  The accelerations of the corners of the caissons describe a rotation around a very low centre. The graphs are obtained plotting the horizontal vs vertical acceleration in the higher frequency band [1.8-6 Hz], which according to the single caisson model is instead associated to a mode with a high rotation centre.

Regarding the natural oscillation modes Table 2 summarises the results obtained in all tests. The conclusion can be drawn that two most frequently observed modes correspond to low rotation centres, in total disagreement with the assumed mathematical model described in Fig. 12, according to which the higher frequency mode (present in the sway and roll signal) should have a high rotation centre.
The natural periods of oscillation were assessed as well as the associated rigid movements: for the almost horizontal oscillation, which are rotations around low centres, the actual depth below the base of rotation centre (R.C.) could be identified comparing the sway and the roll signal. Superstructure vibrations (v) are present only in case of sharp impacts and if the band of frequency, given in the rightmost column, is large enough. Non accurate data are given in square brackets.

<table>
<thead>
<tr>
<th>Test Type</th>
<th>1st Frequency Sway/Roll signal</th>
<th>R.C. below base</th>
<th>2nd Frequency Sway/Roll signal</th>
<th>R.C. below base</th>
<th>Frequency present in Heave signal</th>
<th>Vibrations</th>
<th>Band of frequencies considered</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[ Hz ]</td>
<td>[m]</td>
<td>[ Hz ]</td>
<td>[m]</td>
<td>[ Hz ]</td>
<td>[Hz]</td>
<td>[ Hz ]</td>
</tr>
<tr>
<td>V1A</td>
<td>not evident</td>
<td></td>
<td>not evident</td>
<td></td>
<td>not evident</td>
<td></td>
<td>0.5-9</td>
</tr>
<tr>
<td>V1B</td>
<td>2.3</td>
<td></td>
<td>3.2</td>
<td></td>
<td>0.5-9</td>
<td></td>
<td>0.5-9</td>
</tr>
<tr>
<td>V1C</td>
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<td></td>
<td>2.3</td>
<td></td>
<td>0.5-9</td>
<td></td>
<td>0.5-9</td>
</tr>
<tr>
<td>V2A</td>
<td>[1.2]</td>
<td></td>
<td>2.5</td>
<td></td>
<td>3.0</td>
<td></td>
<td>0.5-9</td>
</tr>
<tr>
<td>V2B</td>
<td>1.4</td>
<td></td>
<td>2.7</td>
<td></td>
<td>0.5-9</td>
<td></td>
<td>0.5-9</td>
</tr>
<tr>
<td>V2C</td>
<td>1.4</td>
<td></td>
<td>2.5</td>
<td></td>
<td>0.5-9</td>
<td></td>
<td>0.5-9</td>
</tr>
<tr>
<td>V3A</td>
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<td></td>
<td>3.6</td>
<td></td>
<td>4.3</td>
<td></td>
<td>0.5-9</td>
</tr>
<tr>
<td>V3C</td>
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<td></td>
<td>3.6</td>
<td></td>
<td>4.3</td>
<td></td>
<td>0.5-9</td>
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<td>2.4</td>
<td>7</td>
<td>10</td>
<td>10</td>
<td>0.2-19</td>
</tr>
<tr>
<td>B1C</td>
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<td>10</td>
<td>2.4</td>
<td>[0]</td>
<td>10</td>
<td>10</td>
<td>0.2-19</td>
</tr>
<tr>
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<td>17</td>
<td>2.5</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>0.2-19</td>
</tr>
<tr>
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<td>17</td>
<td>2.4</td>
<td>[0]</td>
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<td></td>
<td>0.2-19</td>
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<td>10</td>
<td>10</td>
<td>0.2-19</td>
</tr>
<tr>
<td>W2A</td>
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<td>10</td>
<td>2.7</td>
<td>[0]</td>
<td>2.7</td>
<td></td>
<td>0.2-19</td>
</tr>
<tr>
<td>W2C</td>
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<td>10</td>
<td>2.7</td>
<td>6</td>
<td>2.7</td>
<td></td>
<td>0.2-19</td>
</tr>
</tbody>
</table>

4.3. Effect of adjacent caissons and interpretation of the identified modes of oscillation

The oscillations of the caissons adjacent to the excited one were one third as intense as the central one and significantly delayed; a large amount of energy is apparently subtracted from the central caisson by waves propagating along the breakwater. The effect of this wave can not be completely described by the considered single caisson model.

Comparing the horizontal oscillation of the central and adjacent caissons in a single frequency band (see Fig. 26), it was possible to observe that the oscillations were near to be in phase in the frequency band [0.5-1.8 Hz] (around 1st observed frequency) and near to phase opposition in the frequency band [1.8-6 Hz] (around 2nd observed frequency).

A possible interpretation of the two identified sway modes involves the longitudinal dimension: the 1st observed mode represents the movement of all the caisson in phase, the 2nd observed mode represents their alternate movement.
4.4. Conclusions derived from tests

Tab 2 summarises the results of the analysis: the identified frequencies and modes. One natural frequency of oscillation was identified in the heave signal and two in both the sway and roll signals (see Fig. 22), recognised as rotations around low centres.

Even disregarding the position of the rotation centres, a 3 DOF model for isolated caisson could not be calibrated interpreting the observed oscillations as modes $m_1$ and $m_2$; the ratio between the measured eigen-frequencies was lower than foreseen by the model for any value of shear stress. Also using the model of the foundation suggested by Goda (1994), it was impossible to obtain a calibration without assuming a not realistic anisotropy of the foundation.

In conclusion the test analysis pointed out three main modes, two of which are rotational oscillations around low centres (two sway modes, in disagreement with the system described in Fig. 12) and one is a vertical oscillation ($m_3$). The two sway modes are different combinations of the central and adjacent caisson oscillations: for the mode with lower frequency such oscillations appear almost in phase, while for the higher frequency mode they appear almost in opposition of phase.
5. MODEL OF A CAISSON ARRAY

A very good simulation of the prototype measures was obtained considering the rigid body movements of 7 caissons, i.e. with 12 DOF (see Fig. 28). Since the system is symmetrical and is excited symmetrically, only the symmetric movements are considered in the model.

Fig. 28

Natural frequencies and oscillation modes of the system are the eigen values and eigen vectors of the inverse mass matrix multiplied by the stiffness matrix, i.e. of $M^{-1}K$. The eigenvectors describe the modes, and consequently also the linear transformation from the original co-ordinates to a new co-ordinate system, representing the amplitude of each natural oscillation mode. Eigen vector can be collected as columns of the matrix $P$, representing the transformation.

In the eigen-vectors reference system, eq. 1 becomes diagonal representing all uncoupled equations. The uncoupled and undamped equation is: $M^* \ddot{\xi} + K^* = P^{-1} F$. In this base, the transformed mass matrix $M^* (=P^{-1}MP)$ and stiffness matrix $K^* (=P^{-1}KP)$ have the property that $M^{-1}K^*$ is the diagonal eigen-values matrix.

In order to easily solve the damped system, a diagonal matrix (formed by damping coefficients) can approximate the non diagonal term $M^{*,1}D^*$.

In order to find the impulse responses, the system is excited by all the possible independent impulse forces, with the following initial conditions given at time zero:
- null displacements,
- initial velocities given by
\[ \begin{bmatrix} \ddot{\xi}_1 \\ \ddot{\xi}_2 \\ \vdots \\ \ddot{\xi}_n \end{bmatrix}^{0+} = \begin{bmatrix} 1 \\ 0 \\ \ldots \\ 0 \end{bmatrix}, \quad M^* \begin{bmatrix} \ddot{\xi}_1 \\ \ddot{\xi}_2 \\ \vdots \\ \ddot{\xi}_n \end{bmatrix}^{0+} = 0, \quad \ldots \quad M^* \begin{bmatrix} \ddot{\xi}_1 \\ \ddot{\xi}_2 \\ \vdots \\ \ddot{\xi}_n \end{bmatrix}^{0+} = 0 \quad (13) \]

The above set of initial conditions determinate the set of impulse responses that can be convoluted with any force system. Note that since the mass \( M^* \) is not diagonal, one impulse excites normally more modes.

The final displacement at time \( t \), in term of the initial lagrangian co-ordinates, is obviously obtained applying the matrix \( P \) to the vector containing the displacement at time \( t \) in the base of the eigenvectors.

### 5.1. New features

The modules are defined according to the typical MSD model described in chapter 3.2, except that the stiffness terms should be evaluated for a slice of strip foundation as long as a caisson. Each module is connected to the adjacent ones (see Fig. 29) by an extra spring and dash-pot element, describing the stiffness and damping relative to the movements of the adjacent caissons. Such added stiffness was assessed as a fraction of the stiffness between caisson and foundation, and the proportionality coefficient was better calibrated so that the adjacent caisson accelerations fit well with the actually measured ones. The damping coefficient was not calibrated separately, since the computed approximate solution describes the damping effect directly on the uncoupled modes.

A lateral condition must be inserted if the array of caisson is shorter than the actual number of caissons. If it is assumed that the last caisson is still, the stiffness between the end caissons and the adjacent ones \( (K_L) \) is equal to the stiffness between any two other caissons \( (K_A) \); an opposite case is found if the last caissons are free to move, where \( K_L=0 \). The assumed lateral condition was than a compromise, consisting of a reduction of the last stiffness coefficient to 50% of the others \( (K_L/K_A=0.5) \)

The effect of this assumption is reduced if an higher number of caissons is considered, and for an array of 7 caissons these 2 limit cases do not affect significantly the results in terms of acceleration of the central caisson.
This model was set up in order to describe the movements of the central caisson, but, on the basis of the prototype measurements, it proved to be sufficiently effective also in describing the effect of the first caisson adjacent to the central one.

The 12 equations are rather trivial, although long to write down if not condensed in a matrix form. They result from an extension of eq. (22) relative to simplified model.

The simulated system has 12 DOF and consequently 12 eigen-modes. These can be divided into 3 categories:

- 4 modes $m_1$, which are rotations around low centres, e.g. below the caisson base (see Fig. 30);
- 4 modes $m_2$, which are rotations around high centres, e.g. slightly above the gravity centre;
- 4 modes $m_3$, which are vertical movements.

Within the same category, each mode is a combination of amplitudes (and phases).

When the adjacent caissons move in phase, the geodynamic added mass is reasonably bigger than in case of a movement in opposition of phase. This effect can explain the possible

\[ \begin{align*}
\text{1.4 Hz} & \quad m_1^{\text{low}} \\
\text{1.9 Hz} & \quad m_1^{\text{high}} \\
\text{2.4 Hz} & \quad m_1^{\text{vert}} \\
\text{2.6 Hz} & \quad m_1^{\text{vert}}
\end{align*} \]
different heights of the rotation centres of the modes \( m_1 \): such difference was 9 m in Brindisi and only 3 m for the case of Voltri, where the longitudinal dimension of the caisson is bigger (the caisson length is 30.1 m in Voltri, 21.0 m, in Brindisi).

### Tab. 3 Computed eigenmodes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigen-frequency [Hz]</th>
<th>Rotation centre [m above the base]</th>
<th>Mode</th>
<th>Eigen-frequency [Hz]</th>
<th>Rotation centre [m above the base]</th>
<th>Mode</th>
<th>Eigen-frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1 ++</td>
<td>1.4</td>
<td>-7.5</td>
<td>m2 ++</td>
<td>3.9</td>
<td>17.6</td>
<td>m3 ++</td>
<td>2.5</td>
</tr>
<tr>
<td>m1 +−</td>
<td>1.9</td>
<td></td>
<td>m2 +−</td>
<td>5.2</td>
<td></td>
<td>m3 +−</td>
<td>3.8</td>
</tr>
<tr>
<td>m1 −+</td>
<td>2.4</td>
<td></td>
<td>m2 −+</td>
<td>6.8</td>
<td></td>
<td>m3 −+</td>
<td>5.0</td>
</tr>
<tr>
<td>m1 −−</td>
<td>2.6</td>
<td></td>
<td>m2 −−</td>
<td>7.9</td>
<td></td>
<td>m3 −−</td>
<td>5.7</td>
</tr>
</tbody>
</table>

### 5.2. Calibrations

The calibration for the Genoa Voltri main breakwater case was obtained with:
- \( E = 350 \text{ MPa} \) for average stress of 100 kPa, the actual vertical stress is \( \sigma_v = 350 \text{ kPa} \)
- \( \gamma = 0.30 \) Poisson coefficient
- \( K_0 = 1 \) coefficient of lateral confinement

The parameters used for the Brindisi case, presented in the above paragraph, are exactly the same of the Voltri case with the exception of \( E = 220 \text{ MPa} \).

The spring coefficient for horizontal, rotational and vertical oscillations (indicated with \( K \), for brevity) are obtained according to the procedure described in chapter 3.2 for the strip foundation case.

For the adjacent caisson, the calibration was obtained with:
- \( K_A = 0.4 K \) for horizontal and rotational stiffness
- \( K_A = 0.6 K \) for vertical stiffness
- \( K_L = 0.5 K_A \)

Only two damping coefficients were calibrated and applied to each eigenmode separately: a reduction per cycle of 47% for vertical modes and 40% for the others. This agrees with the literature guidelines, suggesting the vertical as the more damped oscillations.

### 5.3. Comparison with prototype tests
The simulation of the prototype tests through the 12DOF model confirmed the validity of the test analysis interpretation. The model outputs, that even visually showed good agreement with the prototype results, were reanalysed in order to check possible discrepancies with the interpretation of the system behaviour. The Power Spectral Density and the Transfer Functions of the simulations pointed out the same frequency peaks found in the tests, and confirmed that the not identified modes were little excited, less than the measured environmental noise.

The simulation, presented in Fig.31, is compared with the recorded horizontal acceleration averaged over the tests of the same type. Not only the simulation of the horizontal acceleration fits well with the recorded data; also the recorded rotational oscillations (not presented here) were found to be congruent with the simulations, being substantially proportional to the sway. This follows from the correct evaluation of the low rotation centres of all the modes type m1.

The transfer function (TF) from the applied force to acceleration, presented in Fig. 32 is extremely relevant. The first peak of the TF is placed at 1.4 Hz and it is relative to the eigenmode that describes rotations around low centres and all the caissons in phase (m1+++). The last peak is placed at 2.7 Hz and the relative eigenmode describes all rotations around low centres and alternate movements of the caissons (m1+−). Since the model has 4 independent modules (the central one plus 3 couples of elements) there are 4 modes associated to rotations around low centres, which are modes m1+++ , m1−+, and the two modes (m1−− and m1+++) with intermediate eigenfrequency (1.9 Hz and 2.4 Hz, in the simulation). The four modes relative to the high centres are not excited by the tug, as well as the four vertical modes.

**Fig.31** Simulation of horizontal acceleration at quay level for the case of Voltri, induced by the tug-boat excitation.
CHAPTER 3

TRANSFER FUNCTION Force → Sway

Fig. 32 Transfer Function between exciting force and outputs shown in Fig. 31

In prototype, the breakwater is formed by many caissons and thus it is feasible that many modes are placed between mode \( m_{1\text{all}}^{++} \) and \( m_{1\text{alternate}}^{+-} \), and this explains the small differences, also due to errors in the measurements.

The tug-boat impact excites only the rotations around the low centre, mainly for two reasons: the point of application of the force is very close to the high centre and the length of the impact is not very short compared to mode \( m_2 \) eigen-frequencies. Tab. 4 shows the relative power of the excited modes (in terms of rotations).

The time history of the force due to the sand sac is also reconstructed, see Fig. 33. The simulated response matches the recorded acceleration with adequate approximation for the first cycle.

Tab. 4 The modes are excited differently by the tug boat excitation. The damping coefficient was 40\% (expressed as reduction of the oscillation per cycle), globally calibrated.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigen-frequency [Hz]</th>
<th>Relative power [%]</th>
<th>Mode</th>
<th>Eigen-frequency [Hz]</th>
<th>Relative power [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1^{+++} )</td>
<td>1.4</td>
<td>12.7</td>
<td>( m_2^{+++} )</td>
<td>3.9</td>
<td>21.1</td>
</tr>
<tr>
<td>( m_1^{++} )</td>
<td>1.9</td>
<td>9.3</td>
<td>( m_2^{++} )</td>
<td>5.2</td>
<td>13.0</td>
</tr>
<tr>
<td>( m_1^{+} )</td>
<td>2.4</td>
<td>4.6</td>
<td>( m_2^{+} )</td>
<td>6.8</td>
<td>5.8</td>
</tr>
<tr>
<td>( m_1^{-} )</td>
<td>2.6</td>
<td>9.0</td>
<td>( m_2^{-} )</td>
<td>7.9</td>
<td>7.5</td>
</tr>
</tbody>
</table>

The impact strongly excites the superstructure vibrations, identified at 10-15 Hz. The rigid body oscillations were assessed in Fig. 33 low-pass filtering below 10 Hz the recorded signal.
Fig. 33 Comparison of the vertical accelerations induced by the sand-sac excitations simulated (array of 7 caissons) and measured. The vibrations induced by the sharp impact were filtered out (lowpass 10 Hz) for a better comparison.

Tab. 5 The sand sac eccentric fall excites the vertical modes and the rotational modes. The relative power of each mode is given below (for a meaningful comparison, the rotational oscillations were multiplied by the lever arm of the force, i.e. the eccentricity of the free falling sac). The applied reduction per cycle was globally calibrated as 47% for the vertical modes, 40% for the rotational modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigen-frequency [Hz]</th>
<th>Relative power [%]</th>
<th>Mode</th>
<th>Eigen-frequency [Hz]</th>
<th>Relative power [%]</th>
<th>Mode</th>
<th>Eigen-frequency [Hz]</th>
<th>Relative power [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>m₁ ̂ ̂ ̂</td>
<td>1.4</td>
<td>12.7</td>
<td>m₂ ̂ ̂ ̂</td>
<td>3.9</td>
<td>21.1</td>
<td>m₃ ̂ ̂ ̂</td>
<td>2.5</td>
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<tr>
<td>m₁ ̂ ̂</td>
<td>1.9</td>
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<td>m₁ ̂ ̂</td>
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<td>m₂ ̂ ̂</td>
<td>7.9</td>
<td>7.5</td>
<td>m₃ ̂ ̂</td>
<td>5.7</td>
<td>2.1</td>
</tr>
</tbody>
</table>
Fig. 34  PSD of Computed roll, solid line, and PSD of roll derived from tests W1a: the simulated modes 'm2 type' (frequencies in [3.9-7.9 Hz], solid line) might be present but hidden by the environmental noise. Accelerations above 10 Hz represent structure vibrations.

Tab. 5 shows how the sand sac impact excite the various simulated modes in terms of displacements. In general the ‘all in phase’ and the ‘alternate’ motions are the most excited modes in terms of accelerations, the former more than the latter.

Note that all the modes are excited, even the ‘m2’ modes that were not identified.

In order to better understand why the ‘m2’ modes could not be identified, the PSD of the simulated roll induced by the sand sac excitation was compared to the recorded case (see Fig. 34): in the records a 2.5 Hz peak is much higher than in the simulations, which might be effect of the vertical oscillation motion (with same frequency) that was not perfectly identified by the combination of signals that described the rigid body ‘roll’, or it could be induced by resonance between the two modes with same frequency. Above 10 Hz a lot of energy is present, relative to superstructure vibration. It is evident that in between, some kind of noise could hide the peaks of the ‘m2’ modes.

The Brindisi tests confirmed all the considerations given above.

The sway averaged over all the tests of the same kind was computed and presented in Fig. 35 (the mean initial speed before the impact was calibrated as 0.25 m/sec)
Fig. 35  Simulation of Brindisi tests

Fig. 36  Simulated and measured sway. The peaks relative to modes \( m_1 \) are in the range 1.3-2.5 Hz. At higher frequencies, some noise is present; in the simulations there is some numerical noise too, induced by the fact that the force spectrum tends to zero in correspondence of the harmonics with period multiple of the impact duration.

6. SIMPLIFIED MODEL

The described caisson array model was implemented with the purpose to best simulate the observed prototype measurements, confirming the given physical explanation of the observed natural modes. The model can be simplified if the purpose is restricted to the evaluation of
the maximum load applied to the foundation: all what is below the maximum value becomes irrelevant as well as the exact timing.

6.1. Description of the 2DoF model

A simplified version can be obtained considering only the sway modes, which were found the most excited ones by a horizontal force applied near sea level. The longitudinal effects can be accounted for considering just the ‘in phase’ and ‘in opposition of phase’ movements between three adjacent caissons (see Fig. 37).

Only 1 Degree of Freedom (DoF) per caisson is considered and the combination of the central and the two adjacent caissons can eventually be examined setting up a 2 DoF model that describes the essential features of the caisson array structure. This extreme approximation is based on the conclusion that the roll modes are far less important than the modes originated by the forces exchanged by adjacent caissons.

Fig. 37  The simplified model consider only 1DoF per caisson: the rotation around the low centres relative to the sway modes. The damping effect is applied to the uncoupled modes.

Returning a step back in the approximation process, we can first solve the problem for an isolated caisson. The natural frequencies are:

\[
\begin{align*}
\omega_S^2 &= \frac{M_{11}K_{22}+M_{22}K_{11}}{2(M_{11}M_{22}-M_{12}^2)} - \frac{\sqrt{(M_{11}K_{22}-M_{22}K_{11})^2+4M_{12}^2K_{11}K_{22}}}{2(M_{11}M_{22}-M_{12}^2)} \\
\omega_R^2 &= \frac{M_{11}K_{22}+M_{22}K_{11}}{2(M_{11}M_{22}-M_{12}^2)}
\end{align*}
\]

where \(\omega_R^2\) is relative to the roll mode and \(\omega_S^2\) (lower eigenvalue) to the sway mode. The associated eigen-vector defines the heights of the rotation centre \(h_S\) of the sway mode (placed below the base) and \(h_R\) of the roll mode:

\[
h_S = \frac{-M_{12}K_{22}}{M_{22}K_{11}+\omega_S^2(M_{11}M_{22}-M_{12}^2)}
\]

\[
h_R = \frac{-M_{12}K_{22}}{M_{22}K_{11}+\omega_R^2(M_{11}M_{22}-M_{12}^2)}
\]
If the system movements are described using the natural modes as co-ordinate system, the dynamic equations are uncoupled; they are generally called normal equations (Flügge, 1962); assuming that the impact wave force is applied at mean water level (at elevation \( d_w \) over the base) the equation for the undamped rotation (\( \theta_3 \)) around the low centre (\( h_3 \)) is:

\[
\ddot{\theta}_3 + \omega_3^2 \theta_3 = \frac{-M_{22}+M_{12}(d_w+h_R)-d_ah_R}{M_{11}(M_{11}M_{22}-M_{12}^2)} (h_R+h_3) F(t) \tag{16}
\]

If a damping factor \( \exp(\omega_3 \zeta_3 t) \) is applied to the impulse response of eq. (16), we represent the simplified dynamic of an isolated caisson with only 1DOF; for instance if the sway mode is the only excited one, displacement at the base is given by \( q_1(t)=h_3 \theta_3(t) \) (sway rotation multiplied by distance of base from sway rotation centre). A normal equation similar to eq. (16) holds for the rolling mode but, if the point of application of the force is close to the height of the roll centre \( h_R \), the roll mode oscillation is irrelevant and the result derived from the sway normal equation is similar to the 2 DOF mass spring dash-pot model by Oumeraci and Kortenhaus (1994)

When also the effect of the elastic force exchanged by the two adjacent caissons is considered, the solution becomes slightly more complicated.

We shall now introduce an auxiliary problem that can give useful results. This problem solves the intuitive case of just one lagrangian co-ordinate \( \theta_j \) describing for instance the sway mode of each isolated caisson, with rotational stiffness \( K_\phi \) and moment of inertia \( M_\phi \) (\( \omega_5^2 \) is their ratio)

If a pair of adjacent caisson is considered, the model has 2 DOF, e.g. the rotation of the central caissons \( \theta_1 \) and of the two adjacent caissons \( \theta_2 \) (moving symmetrically).

Let \( M \) be the mass matrix and \( K \) the stiffness matrix, being function of \( a \) and \( l \), e.g the ratio \( K_\phi/K \) and \( K_1/K \), respectively, that describe the elastic force exchanged by the caisson (see Fig. 37).

In order to follow better the procedure, some results will be given between braces relative to the case \( l=0 \) (an isolated triplet of caissons).

The matrixes of the Mass Spring model are:
\[
M = \begin{bmatrix} M_\phi & 0 \\ 0 & 2M_\phi \end{bmatrix} \\
K = \begin{bmatrix} k_\phi (1+2a) & -2k_\phi a \\ -2k_\phi a & 2k_\phi (1+a+l) \end{bmatrix}
\]

(17)

(18)

The equations are coupled and in order to separate them we shall seek for a combination of the sway modes of central and adjacent caissons solving again the eigenvalue problem:

\[
det(M^{-1} K - \lambda I) = 0, \Rightarrow \lambda_1 = \frac{1}{2} \frac{k_\phi}{M_\phi} (2+3a+l+\sqrt{9a^2-2al+l^2}) \quad \{l=0 \Rightarrow \omega^2 \} \\
\lambda_2 = \frac{1}{2} \frac{k_\phi}{M_\phi} (2+3a+l-\sqrt{9a^2-2al+l^2}) \quad l=0 \Rightarrow \omega^2 (1+3a) \}
\]

(19a)

(19b)

Where the ratio \( \frac{k_\phi}{M_\phi} \) is the eigenvalue relative to the isolated caisson. It is equal to \( \omega^2 \) if the sway oscillation is considered.

The first (lower) eigen-value of eq. (19) describes the ‘in phase mode’ and is similar to the eigen-value of the isolated caisson (\( \lambda_1 \) is actually identical to \( \omega^2 \) for the example given). The second (higher) eigen-value originates by the presence of the elastic forces exchanged between adjacent caissons.

Once the 2 eigen-values \( \lambda_j \) of the sway modes are determined, the eigen-vectors \([\xi_{j1}, \xi_{j2}]\) describe the mode in the co-ordinate system \( \{\theta_1, \theta_2\} \) are obtained solving the:

\[
[T^{-1} K - \lambda_j I] \begin{bmatrix} \xi_{j1} \\ \xi_{j2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \xi_{j1} = \xi_{j2} \frac{2a}{l+2a-\lambda_j M_\phi} \frac{k_\phi}{k_\phi}
\]

(20)

the resulting eigenvectors are given as rows:

\[
\begin{bmatrix}
\xi_{11} & \xi_{12} \\
\xi_{21} & \xi_{22}
\end{bmatrix} = \begin{bmatrix}
1 & \frac{l}{4a}(a-l+\sqrt{9a^2-2al+l^2}) \\
1 & \frac{l}{4a}(a-l-\sqrt{9a^2-2al+l^2})
\end{bmatrix} \quad \{l=0 \Rightarrow \begin{bmatrix} 1 & 11 \\ 0.5 & \end{bmatrix} \}
\]

(21)

and looking at the example case, \( l=0 \) (given in braces), it is clear that the first eigenvector describes an equal oscillation of all the caissons (with frequency \( \omega^2 \), see eq. (19)); while according to the second eigen-vector (with frequency \( \omega^2 (1+3a) \)) the adjacent caissons are in opposition of phase and have half the amplitude of the central one.
We are now ready to face the whole problem. If we forget the slight difference among the heights of the rotation centres of the sway modes (which can be taken equal to that of the ‘in phase’ mode), the mass matrix is simplified and the system equations can be stated as:

\[
\begin{bmatrix}
M_{11} & M_{12} & 0 & 0 \\
M_{12} & M_{22} & 0 & 0 \\
0 & 0 & 2M_{11} & 2M_{12} \\
0 & 0 & 2M_{12} & 2M_{22}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_{1c} \\
\ddot{q}_{2c} \\
\ddot{q}_{1a} \\
\ddot{q}_{2a}
\end{bmatrix}
+ \begin{bmatrix}
K_{11}(1+2a) \\
0,2K_{22}(1+2a) \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{q}_{1c} \\
\dot{q}_{2c} \\
\dot{q}_{1a} \\
\dot{q}_{2a}
\end{bmatrix}
\begin{bmatrix}
F(t) \\
d\omega F(t) \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
q_{1c} \\
q_{2c} \\
q_{1a} \\
q_{2a}
\end{bmatrix}
\]

where,

\[
\begin{bmatrix}
q_{1c} \\
q_{2c} \\
q_{1a} \\
q_{2a}
\end{bmatrix}
= \begin{bmatrix}
\text{rotation around low centre of central caisson} \\
\text{rotation around high centre of central caisson,} \\
\text{rotation around low centre of adjacent caisson,} \\
\text{rotation around high centre of adjacent caisson,}
\end{bmatrix}
\]

The four eigen-values \( \omega_{1S}^2, \omega_{2S}^2, \omega_{1R}^2, \omega_{2R}^2 \) are given by eq. (19a,b) in analogy with the previous case. They are computed with \( \frac{k_b}{M_b} = \omega_r^2 \) for the two roll modes and with \( \frac{k_b}{M_b} = \omega_s^2 \) for the two sway modes:

\[
\begin{align*}
\omega_{1S}^2 &= \frac{1}{2} \omega_s^2 (2+3a+l-\sqrt{9a^2-2al+l^2}) \\
\omega_{2S}^2 &= \frac{1}{2} \omega_s^2 (2+3a+l+\sqrt{9a^2-2al+l^2}) \\
\omega_{1R}^2 &= \frac{1}{2} \omega_r^2 (2+3a+l-\sqrt{9a^2-2al+l^2}) \\
\omega_{2R}^2 &= \frac{1}{2} \omega_r^2 (2+3a+l+\sqrt{9a^2-2al+l^2})
\end{align*}
\]
The eigen-vector matrix describes the natural modes in the initial reference basis \(\{q_i\}\) (eigen-vectors are rows):

\[
\begin{bmatrix}
  h_S & 1 & h_S\xi_{12} & \xi_{12} \\
  h_S & 1 & h_S\xi_{22} & \xi_{22} \\
  -h_R & 1 & -h_R\xi_{12} & -h_R & 1 & -h_R & \xi_{22} & \xi_{22}
\end{bmatrix}
\]

- \(\xi_{12} \equiv +1.0\): Sway mode in phase
  - eigenvalue: \(\omega_{1S}\)
- \(\xi_{12} \equiv -0.5\): Sway mode in opposition of phase
  - eigenvalue: \(\omega_{2S}\)
- \(\xi_{12} \equiv +1.0\): Roll mode in phase
  - eigenvalue: \(\omega_{1R}\)
- \(\xi_{22} \equiv -0.5\): Roll mode in opposition of phase
  - eigenvalue: \(\omega_{2R}\)

The eigenvector constant of proportionality is arbitrary: they are here presented in a simple form, so that they all describe a unit rotation around the central caisson.

For instance the eigen-vector in the first row describes a horizontal movement of the base of the central caisson equal to \(h_S\) times the rotation, with the adjacent caissons describing movements and rotations of amplitude \(\xi_{12}\) with respect to the central one.

In order to find the impulse response, we shall the four modes that satisfy the initial conditions. We shall seek them among the four generic modes that satisfy the condition on the initial positions, i.e. with null initial phase: \(A_1 \sin(\omega_{1S} t), A_2 \sin(\omega_{2S} t), A_3 \sin(\omega_{1R} t), A_4 \sin(\omega_{2R} t)\). The second condition on velocities defines the amplitudes \(A_1,..A_4\). We can apply this condition to the equivalent generic vector \(Q\) in the \(\{q_i\}\) co-ordinate system, describing the same movements of the four generic modes:

\[
Q \equiv \begin{bmatrix} q_{1c} \\ q_{2c} \\ q_{1a} \\ q_{2a} \end{bmatrix} = \begin{bmatrix} +h_S & +h_S & -h_R & -h_R \\ 1 & 1 & 1 & 1 \\ +h_S\xi_{12} & +h_S\xi_{22} & -h_R\xi_{12} & -h_R\xi_{22} \\ \xi_{12} & \xi_{22} & \xi_{12} & \xi_{22} \end{bmatrix} \begin{bmatrix} A_{1S}\sin(\omega_{1S} t) \\ A_{2S}\sin(\omega_{2S} t) \\ A_{1R}\sin(\omega_{1R} t) \\ A_{2R}\sin(\omega_{2R} t) \end{bmatrix}
\]

(23)

Note that if we consider just the two principal modes (the sway modes), the component \(q_{2c}\) describing the rotation of the central caisson is:

\[
q_{2c} = [A_{1S} \sin(\omega_{1S} t) + A_{2S} \sin(\omega_{2S} t)]
\]

(24)

The impulse response to a horizontal load applied to the central caisson is given by
\[
\begin{bmatrix}
1 \\
d_w \\
0 \\
0
\end{bmatrix}
\]

(condition on velocity) which implies that:

\[
A_{1S} = -\frac{\xi_{22}}{\omega_{1S} \mu} \tag{25a}
\]
\[
A_{2S} = -\frac{\xi_{12}}{\omega_{2S} \mu} \tag{25b}
\]

where:

\[
\mu = \frac{(M_{11} M_{22} - M_{12}^2) (h_R + h_S) (\xi_{22} - \xi_{12})}{-M_{22} + M_{12} (d_w + h_R) - d_w h_R M_{11}} \tag{26}
\]

and the impulse damped response \( (R_{2c}) \) in terms of rotation around the lower centre is therefore:

\[
R_{2c}(t) \equiv A_{1S} \sin(\omega_{1S} t) \exp(-\zeta_{1S} \omega_{1S} t) + A_{2S} \sin(\omega_{2S} t) \exp(-\zeta_{2S} \omega_{2S} t) \tag{27}
\]

For a generic horizontal loading \( f_h(t) \) applied at height \( d_w \) (as already imposed in eq. (26)) the rotation is:

\[
q_{2c}(t) \equiv \sum_{k=0}^{t} f_h(t) \ R_{2c}(t-k) \ dt \tag{28}
\]

the horizontal displacement at caisson base being \( q_{1c}(t) \equiv h_R q_{2c}(t) \).

In conclusion it is necessary to solve eq. (14) for \( \omega_R \) and \( \omega_S \), eq. (15) for \( h_R \) and \( h_S \), eq. (19c,d) for \( \omega_{1S}^2 \) and \( \omega_{2S}^2 \), eq. (21) for \( \xi_{ij} \), eq. (26) for \( \mu \), eq. (25) for \( A_1 \) and \( A_2 \) and finally eq. (27) for the impulse response \( R_{2c}(t) \) and eq. (28) for the actual rotation \( q_{2c}(t) \) around the low centre (at height \( h_S \) below the base).

6.2. Comparison with prototype tests

The simulations carried out according to the simplified 2DOF model are presented in Fig. 38. The main eigen-modes were correctly assessed and the amplitudes of the first cycle is accurately represented.
When the 1DOF model representing sway of isolated caissons is used for the simulations, the stiffness of the central caisson is slightly lower and the displacement induced by forces of long duration is higher than for the 2DOF model. The opposite happens for forces of short duration, since the force excites also the higher frequency component which is neglected by the 1DOF model. Fig. 39 shows the comparison with prototype tests.

In Voltri the tug impact lasts 0.5 sec, in Brindisi it lasts 1.0 sec

In terms of accelerations the higher frequency component (due to the longitudinal array effect) is slightly more important (see Fig. 40 for the Voltri case and Fig. 23 for the Brindisi case). In terms of elastic force transmitted to the foundation, e.g. of displacement, the ratio between the maximum values reached by higher and lower frequency component is almost halved.

Fig. 40 shows the different simulated modes for the Voltri breakwater hit by the tug.
7. DESIGN DIAGRAMS BASED ON THE CAISSON ARRAY MODEL

7.1. Scaling

Due to traditions in the design procedures, caisson dimensions within a tradition area are very well correlated one to the other and each of them with depth, with the exception of the caisson length which is mainly determined by construction capacity. For similar caissons on the same rubble mound material (having the same elastic parameters at the standard average pressure), natural oscillation frequencies scale according to very simple laws.

Two caisson shapes are considered, one representing the average European design as codified by Larras (1961) and the other the average Japanese design (Takahashi, 1996). The cross sections are given below. The ratio between $L_c$ and $B_c$ is usually within the range 1:2, but recently new projects have considered increasing such ratio aiming to resist to local wave impacts, therefore a high value ($L_c/B_c=5$) is considered in the following computations beside the typical one ($L_c/B_c=1$).
7.2. Design diagrams for natural oscillation periods and displacement response factors

Fig. 41 Scheme of the adopted average European design: $H_d = H_{1/10} = 1.27 \, H_s$

Fig. 42 Design diagram giving the eigenperiods of the sway modes $T_1^+$ and $T_1^-$ function $h_c$ (caisson height, superstructure included) computed on the basis of the typical European design, with a significant rubble mound and sufficiently stiff underlying foundation.

Fig. 41 shows the considered cross section for the ‘average European design’ and Fig. 42 represents the natural oscillation periods relative to the two sway modes described by the simplified model. The ‘in phase’ mode does not depend on the ratio $L/B$, but if only a limited number of caissons are considered, the lateral condition could affect this result. Furthermore the geodynamic added mass terms suggested in literature (obtained by a circular foundation hypothetical case) are not suitable for the description of a strip foundation case. In order to maintain the independence of $T_1^+$ with the ratio $L_c/B_c$ the lateral condition is taken as null and an expression of the geodynamic added mass proportional to $L_c$ was considered. The ratio between the stiffness terms $K_A$ and $K$ was considered proportional to $B_c/L_c$. The relevance of the longitudinal array effect can be evaluated comparing cases with different ratio $L_c/B_c$, since when this ratio tend to infinity the scheme collapses into the plain strain case.

The curves given in Fig. 42 representing the main periods of the European average design are given by (caisson height $h_c$ in metres, natural periods $T$ in sec):

$$ T_1^+ = 0.0565 \, h_c^{0.75} $$
$$ T_1^- = \alpha_E \, h_c^{0.75} $$

(29a)
(29b)

<table>
<thead>
<tr>
<th>$L_c/B_c$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_E$</td>
<td>0.0381</td>
<td>0.0447</td>
<td>0.0477</td>
<td>0.0495</td>
<td>0.0507</td>
</tr>
<tr>
<td>$T_1^-/T_1^+$</td>
<td>0.67</td>
<td>0.79</td>
<td>0.84</td>
<td>0.88</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Once the longer sway eigen-period $T_1^+$ is evaluated, it is possible to assess the displacement response factor $\nu_c$ from the graphs given below, defined as the ratio between the maximum actual displacement and the static displacement relative to the maximum force.
The static displacement is proportional to the static force transmitted to the foundation, although, since also the adjacent caissons are displaced (15-20% of the loaded one) an extra stiffness is activated and the system becomes more rigid. The static displacement of reference must be evaluated through eq. (22) with null accelerations, resulting as:

\[q_{1c}(\text{static}) = \frac{F}{K_{11}} \frac{1+a+l}{(1+3a+l+2al)} ,\]

with \(K_{11}\) calculated for a strip foundation, with the adjacent ones moving of

\[q_{1d}(\text{static}) = \frac{F}{K_{11}} \frac{a}{(1+3a+l+2al)} ,\]

and total force transmitted from the central caisson to the foundation being the unchanged static force \(F\), as can be easily checked applying the stiffness matrix eq. (22) to such displacements.

Note that this displacement resulted just slightly lower than that computed for the isolated caisson, where the stiffness of a rectangular foundation can be evaluated with Barkan formula (see paragraph 3.2.4) with same shear modulus: \(q_{1c}(\text{static}) = \frac{F}{K_{11}(\text{Barkan})}\)

The above graphs show only small differences, due to the fact that the first (smaller period) modes are equal and dominant in the two cases.
Similar results for the Japanese average design are given below. Comparing this design with the European one we can note that for a given caisson height \( h_c \) and ratio \( L_c/B_c \), the considered average Japanese case has a (almost) quadruplicated caisson mass, (almost) quadruplicated geodynamic mass, (almost) same hydrodynamic mass; the stiffness is more than doubled, being proportional to the caisson width and dependent on the vertical pressure of the caisson on the foundation (this effect leads to an increase of the shear modulus, but not so much to become proportional to the caisson base area, see de Groot et. al, 1995).

The design diagrams for the Japanese case, referring to the proper main oscillation period, are almost equal to those derived for the European case.

The curves given in Fig. 46 representing the main periods of the Japanese average design are given by (caisson height \( h_c \) in metres, natural periods \( T \) in sec):

\[
T_1^+ = 0.0825 h_c^{0.75} \quad (30a)
\]

\[
T_1^- = \alpha_J h_c^{0.75} \quad (30b)
\]

The conclusion that can be drawn is that the Japanese design is associated to a definitely longer eigen-periods for the same caisson height and the difference is quantifiable comparing Fig. 42 and Fig. 47. The ratio between significant wave height and caisson height however is quite different in the two cases and a similar comparison performed for equal wave height produces the opposite result: typical periods are 7% shorter in the average Japanese design.
7.3. Design diagrams for sliding response factors

Sliding is one of the most critical failure modes in dynamic conditions. The failure mechanism is rather simple: wave impact produces a strong horizontal force that accelerates horizontally the caisson and a vertical uplift force that has the effect of reducing the limit friction force. Sliding occurs if the limit friction is lower than the horizontal force transferred to the foundation, i.e. the wave horizontal force reduced of the caisson inertia (included the inertia of the hydrodynamic added mass).

The force transferred from caisson to its foundation is defined as sliding force $F_s(t)$, and its maximum value divided by the maximum horizontal wave force is defined as sliding response factor $\nu_s$. The sliding force is balanced by the elastic reaction of the foundation proportional to the induced displacement and by foundation inertia, proportional to the induced acceleration of the foundation.

As the ratio $t_d/T_n$ approaches to zero, impact providing the same impulse correspond to progressively greater forces tending to infinity, being proportional to $1/t_d$. For these impacts in absence of sliding the displacement oscillations of the caissons are unchanged and so the elastic forces transferred to the foundation. Generally the foundation inertia force is small compared to the elastic one and much smaller than caisson inertia. Anyway below a certain limit of $t_d/T_n$ the impulsive load spent to accelerate foundation grows so much that it make up the maximum sliding forces when the elastic force is still negligible. When these conditions are reached sliding force and wave force are respectively proportional to foundation and caisson plus hydrodynamic masses: $\nu_s$ reaches its lower limit.

For the European average design this limit results negligible, since the foundation added mass for translation is very low compared to the caisson mass. In case the force is applied exactly in correspondence of the roll centre, the sliding force is:
\[ F_{s(lim)} = \frac{F_{h(max)}}{1 + \frac{M_{11c} + M_{11h} + M_{12c} + M_{12h}}{hS} + \frac{M_{11g} + M_{12g}}{hS}} = \begin{cases} 0.03 F_{h(max)} & \text{(European Design)} \\ 0.12 F_{h(max)} & \text{(Japanese Design)} \end{cases} \quad (31) \]

If \( F_s(t) \) tends at some instant to grow above the friction limit, sliding occurs, the force applied to the foundation equals the friction limit, while the caisson inertia will withstand the unbalanced horizontal wave force. A mathematical model will be given in next paragraph.

\[ q_{1c} \quad q_{1f} \quad q_{2c} \quad q_{1a} \quad q_{2a} \]

**Fig. 49**

**Fig. 50**

**7.4. The evaluation of sliding**

When the force transferred to the foundation exceeds the friction, the caisson slides. A numerical integration model computes the sliding distance.

\[ + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & K_{11}(1+2a) & 0 & -2aK_{11} & 0 \\ 0 & 0 & 2K_{22}(1+2a) & 0 & -2aK_{22}, 0, -2aK_{11}, 0, 0, 0, -2aK_{22}, 0, 2K_{22}(1+a+l) \end{bmatrix} \]
\[
\begin{bmatrix}
F(t) - F_s(t) \\
F_s(t) \\
d_n F(t) \\
0 \\
0
\end{bmatrix}
\]

with zero initial conditions and the following relation boundary conditions:

- \( F_s \leq \) friction limit (function of time since wave uplift forces are applied) anyway;
- adherence of caisson to foundation (equal velocities) until \( F_s < \) friction limit (in this case the dynamic description is equivalent to eq. (22));
- \( F_s = \) friction limit during sliding (unequal velocities);
- sliding starts when \( F_s \) tends to cross over the friction limit;
- sliding stops when velocities cross each other and bodies can stick together (after adherence sliding force tends to move within friction limits).

In the vertical direction an independent set of two analogue equations should be considered in order to evaluate the vertical acceleration of the caisson and thus the real weight that determines the limit friction force. Fig. 51 shows an example output of the simulation of sliding.

7.5. Remarks about non-linearity and plasticity of soil

The model is calibrated on the basis of the prototype tests where the applied load was much lower than the design wave load. The calibrated shear modulus is consequently a tangent value to stress conditions at rest and is not completely representative of the behaviour under extreme stress conditions. Longer eigen-periods are expected near failure conditions; according to experiments quoted in point 3.3.4 shear modulus could be about half the calibrated one. Cyclic loading during caisson lifetime, which took place at a significant intensity in the case of prototype tests causing settlement and not at the same level in the quoted experiment, are likely to increase stiffness almost up to failure conditions.

Plasticity conditions in the rubble mound and foundation are reached if the sliding force, though lower than the friction force, is sufficiently high to produce failure. The limit boundary must be evaluated in dynamic conditions remarking that small sliding distance (small compared to sliding surface roughness) could not lead to actual failure but could be recovered, see Lamberti (1998) and related discussion.
7.6. Application to Genoa Voltri

In order to make clear with an example the use of the described results a deterministic check of stability is carried out for the case of Genoa Voltri.

Genoa Voltri main breakwater caissons are 27.4 m high; the caisson aspect ratio is $L_c/B_c=1.5$; the design off-shore significant wave height was 8.4 m and period 12.3 s; due to refraction and shoaling wave significant design wave height at the breawater was 7.8 m.

Lifetime of the breakwater is supposed to be 50 years. The following wave conditions are assumed: $H_{os}=6.2$ m, wave steepness $s_{op}=3.5\%$ (wave period 10.7 s), storm duration 9 hrs, frontal attack.

The model from Allsop & Calabrese is used for the determination of the breaking wave height $H_b= L_p \left(0.1025+0.0217 \frac{(1-C_r)}{(1+C_r)} \tanh(2 \pi \frac{\xi}{h_s/L_p})\right)$ assuming full reflection, the depth in front of the structure being $h_s=30$m.

The incident breaking wave height is 12 m equivalent to 13 m off-shore wave height, the probability of impact is 0.03%, i.e. probability is low but breaking can not be excluded, actually one breaking wave in the lifetime extreme storm is expected.

The force time history due to impact is determined on the basis of Kortenhaus method: $\alpha_A=2.2632; \beta_A=.7360; \gamma_A=1.1860$; describe the LogWeibull distribution of maximum wave forces.

Provided that a 10% hazard is accepted, $P_f_A=.9$, the extreme impact characteristics can be estimated:

$$F_{max}^* = \exp\left(\frac{1}{\gamma_A} (-\log(1-P_f_A))^\alpha (1/\alpha_A)+\beta_A\right); \quad \text{and} \quad F_{max} = (F_{max}^* \rho_w g H_b^2) L_c;$$

The resulting maximum force is 300 MN.

Assuming a deterministic value of the non dimensional impulse: $k_P=2.24$; typical durations of impact are estimated:

$$t_{Fh} = k_P/F_{max}^* \sqrt{d_{eff}/g}; \quad \tau_r=0.8 \ t_{Fh}; \quad \tau_b=1.0 \ (t_{Fh}+0.35 \ (1-exp(-20 \ t_{Fh})))$$

Rise time is 0.42 sec, total duration 0.87 sec.

From the size of caissons, corresponding to the European design, and from Fig. 3.42 we can estimate the fundamental oscillation period: $T_1^+=0.7$ s.
From Fig. 43 the displacement response factor for the European case with $L_c/B_c=1$ and $t_d/T_1^+=1.24$ results 1.2.

The force applied to the foundation reaches 360MN while the friction force in static condition (neglecting uplift wave forces) is 110 MN. Sliding does certainly occur; the associated sliding distance is assessed in Fig. 51, where the results of simulation with different wave period are also shown.

![Graph](image)

*Fig. 51 On the left forces and velocities of the caissons are shown during a sliding event under an extreme wave of the 50 years storm in Voltri. Since due to breaking condition only the period influences the breaker height, on the right the resulting sliding distance is presented as a function of the highest wave period.*

Sliding exceeds 0.8 m and is relevant. We may conclude that under the lifetime storm a single breaking wave may occur; in this case it would cause failure (relevant sliding of caissons) of the Voltri breakwater.

### 8. DESIGN GUIDELINES AND CONCLUSIONS

#### 8.1. Which model should be used at different design levels

For feasibility study, the design diagrams giving the qualitative dynamic response factor should be used.

For preliminary design the dynamic simplified model (2 sway DoF) presented above is suggested.
For the final design a full model including several caissons and vertical oscillations should be implemented.

8.2. Conclusions

Prototype experiments showed that:

- the sway oscillation mode is clearly the most important mode excited by horizontal loading applied near to the mean water surface;
- more than one oscillation mode is present due to the caisson array structure of a breakwater, but the relevant modes are all of the sway type;
- relative damping remains apparently almost constant from case to case.

The different sway oscillation periods result almost proportional to the fundamental period representing all the caissons moving equally and in phase, the proportionality constant resulting only weekly dependent on caisson length or mutual caisson link.

As a consequence the dynamic response factor is fundamentally dependent only on the ratio between impact duration and fundamental oscillation period, with minor influence of caisson shape, i.e. of caisson height to width and width to length ratios. Diagrams providing response factors are given for the different situations.

The wider base of typical Japanese design compared with a typical European design for the same wave conditions causes a slightly stiffer behaviour and greater inertia of the foundation making sliding under short impact comparatively more easy.

REFERENCES


CHAPTER 4: INSTANTANEOUS PORE PRESSURES AND UPLIFT FORCES

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4.1. QUASI-STATIONARY FLOW IN RUBBLE FOUNDATION

4.1.1. Quasi-stationary flow models

The assumption of quasi-stationary flow is based on the assumption that pressure gradients are completely balanced by flow resistance in each phase of the wave cycle.

The most simple prediction method for quasi-stationary flow in the rubble foundation is also based on the assumption of 1-dimensional flow and evenly distributed flow resistance. This yields the triangular pore pressure distribution along the bottom of the wall, as illustrated in Figure 3-12 of section 3.5.2 of Volume I.

\[
\frac{p(x)}{p_{u}} = 1 - \frac{\int_{0}^{x} \frac{1}{k(x)} dx}{\int_{0}^{\infty} \frac{1}{k(x)} dx} \tag{1}
\]

With one-dimensional flow, but non-homogeneous, linear flow resistance, \(k(x)\), a simple analytical expression is found by integration of the Darcy equation:

Where \(p(x)\) is the pressure(head) and \(p_u\) is the pressure(head) at the seaward edge of the wall bottom, assuming zero pressure (head) at the harbour side edge.

For 2-dimensional flow with non-homogeneous, but linear flow resistance (Darcy flow) of incompressible fluid, the Laplace equations must be solved. This can be done by analogue or numerical models, in particular Finite Element models, such as MSEEP, discussed in subsection 4.1.2.

A serious limitation of such models for the flow in the rubble foundation is the assumed linearity of the flow resistance or permeability. In reality the flow is largely turbulent,
especially with coarse rubble during wave crest or wave trough. The effect of turbulence can, however, be taken into account by correcting the permeability of the different elements, depending on the local head gradients. An example of the influence is presented in subsection 4.1.3 (Figure 5).

The permeability can be predicted as a function of the grain size and porosity with the Forchheimer equation (*Volume I, subsection 3.3.6, equations 3-2 and 3-4*). It can be linearised by calculating a characteristic flow velocity \( v_0 \), e.g. for the characteristic average head gradient during wave crest, \( i_0 = \frac{H}{B_c} \) and adopting the linearised Darcy coefficient, \( k_{\text{lin}} = \frac{1}{(A + B \cdot v_0)} \).

The assumption of incompressibility of the pore water is no serious limitation here.

### 4.1.2. Hindcast Hannover tests with 2-D Darcy flow model

At the end of 1993, large scale tests on a vertical breakwater have been performed in the Large Wave Flume in Hannover [Kortenhaus e.a. 1994]. Hindcasts of these tests have been done with several models. The hindcast with the stationary flow model MSEEP [Hölscher et al 1998] is discussed in this subsection. The hindcast with the non-stationary, two-phase model TITAN is discussed in section 4.4.

A test caisson had been constructed in the Large Wave Flume. The caisson height was about 2.76 m and the caisson width 3.0 m. The caisson was placed on a rubble foundation with a thickness underneath the caisson of 0.60 m, a height at the front of 1.02 m and 0.77 m at the harbourside. Below the rubble foundation and separated by a geotextile a sand layer with a thickness of about 2 m was present. This sand layer was placed on the concrete floor of the flume.

The water pressures have been measured at several locations in the model. Directly underneath the caisson 6 rows of pore pressure transducers were located. The first row on top of the rubble, in the caisson bottom. The next row in the middle of the rubble layer. The following rows were located in the sand layer.

Four water pressure transducers have been placed in the front slope of the rubble. The readings of these transducers have been used to determine the boundary conditions for the numerical model calculations. The pressures measured underneath the caisson have been used to validate the calculation results.

A large number of waves have been applied on the test model. Two regular waves have been selected for this hindcast, first a smooth non-breaking wave, \( H = 0.7 \) m and \( T_p = 6.5 \) s, secondly a breaking wave including wave impacts, \( H = 0.9 \) m and \( T_p = 3.5 \) s. Despite
differences in wave characteristics, the maximum wave induced pressures in the rubble at the front of the caisson were nearly equal for both waves.

For the validation of the quasi-stationary calculation technique four moments in time have been selected from the continuously measured time-series. The moments are indicated by the water level at the front of the caisson:
- wave crest
- falling water level, passing still water level
- wave trough
- rising water level, passing still water level

The exact moments in time are selected from the reading of the pore pressure transducer, closest to the caisson front.

In Figure 1 the validation results for the wave with wave-impact are presented for each of the four conditions. From these graphs the following can be notified:
- Differences between the measured and calculated pore pressures in the rubble may be neglected for wave trough conditions.
- Differences up to 20% are found for the wave crest with its wave impact.
- Significant differences are found for the situation at which the water level at the front passes still water level. For the rising water level more difference is found than for the falling water level.

With the non-breaking wave similar tendencies are found as with the breaking wave, with one exception: no significant differences are found for the wave crest.

Non-stationary effects can explain the differences during rising and falling water level. When the water level passes still water level, the acceleration of the water flow reaches its peak value and the time-depending effects of inertia and elastic storage become important.

The differences in pore pressures found for the wave crest with its temporarily high peak forces during the wave impact can also be explained by non-stationary effects, as will be discussed in section 4.4.
4.1.3. Application of 2-D Darcy flow model to Porto Torres breakwater

In the Hannover tests a homogeneous rubble layer with constant thickness had been applied. Consequently, an almost linear pressure distribution was found for the wave crest and the wave trough. In practice, the phenomena, summarised in section 3.5.2 of Volume I, may cause another pressure distribution. Pore pressure measurements performed in the bottom of one of the caissons of the Porto Torres breakwater [Franco and Neroni 1997] showed such another pressure distribution: nearly equal pressures along a large part of the caisson bottom.

All the phenomena mentioned in section 3.5.2 of Volume I could have contributed to this pressure distribution. The information about the grainsize distribution was limited (probably quarry run up to 50 kg), as was the information about the original seabed shape underneath the rubble foundation and the possibility of local settlement of the rubble foundation due to scour of any sand pockets between the bedrock (no filter layer had been constructed). Apron slabs (“armour units”) with some holes in it had been placed close to the footing of the
caisson at the sea side. The permeability of the holes and the permeability of the slits between footing and slabs and between the different slabs were not known. If the dimensions of the holes and the slits would have been known, the permeability could have been calculated with the formulas presented in [Bezuijen et al 1990]. See also subsection 4.1.4

Calculations with the MSEEP model have been done to quantify these qualitative explanations [Van Hoven 1998]. Some results are presented in the Figures 2 to 5 for locally varying grainsizes, thickness variation of the layer, a non-flat top of the rubble foundation and flow concentrations around the corners. The caisson is 17 m wide. The seaside is at x = 17 m. If non of the effects is present, the pressure distribution is nearly triangular, as can be seen from the uninterrupted lines in Figures 3 and 4.

Fig. 2 Influence of unequal grainsize distribution and consequent unequal flow resistance
Fig. 3 Influence of thickness variation of rubble foundation layer
Fig. 4 Influence of non-flat top of rubble foundation leaving space underneath caisson bottom: permeability of indicated layer larger than core ($k_{\text{core}} = 1.2 \text{ m/s}$)

Fig. 5 Influence of flow concentration around caisson edges with turbulence
4.1.4. Modelling influence of apron slabs

The extreme influence of apron slabs would be an extension of the triangular pressure distribution to the outer edges of the slabs. If the slab has a width of 1/3 of $B_c$, the uplift force, $F_u$, would reduce to $\frac{3}{4}$ of the value without a slab. This is favourable at wave crest. At wave trough, the absolute value of $F_u$ also reduces to $\frac{3}{4}$, which is unfavourable. The reductions at wave crest and wave trough are even larger due to the lower (absolute) values of $p_u$ at the edge of the slab (See Volume 1, Chapter 2).

In reality water can flow through the slits between the apron slabs and the edge of the caisson and the slits between the slabs and the reduction of the uplift force is less. To study the possible real influence, a more refined analysis is presented below for the case of Genoa Voltri V1. See also Volume IIId, Chapter 5.

The base of the caisson has a width of 22.5 m. The rubble foundation consists of a 1.5 m thick layer of 50 – 500 kg on top of a ca 12 m thick layer of 0 – 500 kg. At the seaside two apron slabs with a width of each 5 m are placed. Thus, the total slab length is $0.44 \cdot B_c$. The slab thickness is 1 m. Thus, the (vertical) length of the slits is 1 m. The width of the slits is estimated to be $B(slit) = 0.02$ m.

Slits parallel to the breakwater axis are present between the caisson and the first slab, between the first and the second slab. Slits perpendicular to the axis are present probably each 5 m (square slabs). Thus, the total length of the slits, including those perpendicular to the breakwater axis, is 4 m per unit meter length of the breakwater. The flow resistance may be represented by one imaginary slit of $L(slit) = 1m/4 = 0.25$ m length at some distance from the seaward edge of the caisson. As the slit closest to the edge influences the flow pattern most, the distance is assumed to be zero here.

Now the flow pattern during a high wave, causing 10 m pressure head difference across the breakwater, may be schematised as presented in Figure 6. There are 5 “flow channels”, with the discharges $q_s$ (through the slit), $q_{50,s}$ (through the sea side part of the coarse layer), $q_{50,h}$ (through the harbour side part of the coarse layer), $q_{0,s}$ (through the sea side part of the 0 – 500 kg) and $q_{0,h}$ (through the harbour side part of the 0 – 500 kg). The influence of the lower part of the 0-500 kg on the flow pattern close to the caisson bottom is relatively small. Therefore only circa half the layer is modelled: 6 m. The head differences over the slit and the two seaward flow channels are the same: $\Delta H_{sea}$. Those over the two harbour side channels are also the same: $\Delta H_h$.
Turbulent flow will be present in all channels. First the flow resistance for each flow channel will be calculated: $\Delta H_{\text{sea}}/q_s^2$, $\Delta H_{\text{sea}}/q_{50,s}^2$ etc.

**Slit**
Chezy formula: $i = v^2/(0.5B \cdot C^2)$ or $\Delta H_{\text{sea}}/L = q_s^2/(0.5B^3 \cdot C^2)$, where $L = 0.25$ m, $B = 0.02$ m, $C = 18 \text{ m}^{0.5}/\text{s} \cdot 10^{\log(3B/k_s)} \approx 32 \text{ m}^{0.5}/\text{s}$ (assuming $k_s \approx 0.001$ m). This yields:
$\Delta H_{\text{sea}}/q_s^2 = 61 \text{ s}^2/\text{m}^3$.

Seaward part of coarse layer
Forchheimer equation, second term: $i = b v^2$ or $\Delta H_{\text{sea}}/L = b \cdot q_{50,s}^2/B^2$,
where $b = 2.9 \text{ s}^2/\text{m}^2$, $L = 10$ m, $B = 1.5$ m. This yields:
$\Delta H_{\text{sea}}/q_{50,s}^2 = 13 \text{ s}^2/\text{m}^3$.

Seaward part of 0 – 500 kg
Forchheimer equation, second term: $i = b v^2$ or $\Delta H_{\text{sea}}/L = b \cdot q_{0,s}^2/B^2$,
where \( b = 12 \, \text{s}^2/\text{m}^2 \), \( L = 10 \, \text{m} \), \( B = 6 \, \text{m} \). This yields:
\[
\Delta H_{\text{sea}}/q_{0,s}^2 = 3.3 \, \text{s}^2/\text{m}^3.
\]

Harbourside part of coarse layer
Forchheimer equation, second term: \( i = b \, v^2 \) or
\[
\Delta H_{h}/L = b \cdot q_{50,h}^2/B^2 ,
\]
where \( b = 2.9 \, \text{s}^2/\text{m}^2 \), \( L = 22.5 \, \text{m} \), \( B = 1.5 \, \text{m} \). This yields:
\[
\Delta H_{h}/q_{50,h}^2 = 29 \, \text{s}^2/\text{m}^3.
\]

Harbourside part of 0 – 500 kg
Forchheimer equation, second term: \( i = b \, v^2 \) or
\[
\Delta H_{h}/L = b \cdot q_{0,h}^2/B^2 ,
\]
where \( b = 12 \, \text{s}^2/\text{m}^2 \), \( L = 22.5 \, \text{m} \), \( B = 6 \, \text{m} \). This yields:
\[
\Delta H_{h}/q_{0,h}^2 = 7.5 \, \text{s}^2/\text{m}^3.
\]

With \( \Delta H_{\text{sea}} + \Delta H_{h} = 10 \, \text{m} \) and \( q_{s} + q_{50,s} + q_{0,s} = q_{50,h} + q_{0,h} \), the following is found:
\[
\Delta H_{\text{sea}} = 2.5 \, \text{m} \quad \text{and} \quad \Delta H_{h} = 7.5 \, \text{m}.
\]
This means that the pressure distribution underneath the caisson can be approximated by a triangular pressure distribution with \( p_u \) at an imaginary distance of \((2.5/7.5) \cdot 22.5 = 7.5 \, \text{m}\) seaward of the edge of the caisson (lower part of Figure 6), yielding a reduction of the uplift force \( F_u \) to \( 22.5\text{m}/(22.5\text{m} + 7.5\text{m}) = 75\% \).

If the slit width is not 0.02m, but \( B(\text{slit}) = 0.04\text{m} \), the flow resistance in the slits reduces to 5.6 \( \text{s}^2/\text{m}^3 \). Then, \( \Delta H_{\text{sea}} = 1.6 \, \text{m} \) and \( \Delta H_{h} = 8.4 \, \text{m} \) is found. The corresponding imaginary distance seaward of the edge of the caisson at which \( p_u \) could be thought to be present, is now \((1.6/8.4) \cdot 22.5 = 4.3 \, \text{m}\) and \( F_u \) is reduced to \( 22.5/(22.5 + 4.3) = 84\% \) of the value without a slab.

If the slit width \( B(\text{slit}) = 0.06 \, \text{m} \), the imaginary distance is 2.4 \( \text{m} \) and \( F_u \) reduces to 90\% of the value without a slab.

### 4.1.5. Conclusions about quasi-stationary flow through rubble foundation

Quasi-stationary flow may be assumed during wave crest and wave trough if no wave impacts occur. Non-stationary effects may be relevant during wave impacts and half way the wave cycle.

Quasi-stationary flow may be modelled with 2-dimensional models for linear flow resistance, applying linearised permeability of the rubble. The following potential causes of a significant deviation from the triangular pressure distribution and the uplift force, can be quantified with these models:
- Locally varying grainsizes
- Large variation of layer thickness
Non-flat top of rubble foundation
- Flow concentrations around caisson edges

The influence of apron slabs on the pore pressure distribution and uplift force can be modelled in the way worked out in subsection 4.1.4. The influence is only significant if the slits are very narrow.

4.2. ANALYTICAL EXPRESSIONS FOR NON-STATIONARY FLOW IN RUBBLE FOUNDATION

4.2.1. Summary of non-stationary flow effects

In many cases the prediction of the pore pressures in the rubble foundation can be done by considering the flow quasi-stationary. In some (wave impact) cases, however, the effects of non-stationary flow need to be taken into account. See section 3.5.3 of Volume I. This can be done by calculating corrections to the quasi-stationary pressure distribution. A first estimate of the required corrections can be derived from the analytical expressions presented in this section. If a more refined analysis is needed, sophisticated (numerical) models must be used or carefully scaled physical models. The requirements are discussed in section 4.3. In section 4.4 a hindcast of the Hannover tests with both analytical expressions and the sophisticated F.E. model TITAN is discussed. The hindcast is more extensively reported in [Hölscher et al 1998].

A qualitative description of the main effects of non-stationary flow are presented in section 3.5.3 of Volume I (Fig.3.15). Three effects are distinguished, two direct effects and one indirect effect:
- the direct effect of a pressure wave or sound wave (inertia, one-phase)
- the direct effect of the elastic storage wave (no inertia, two phases)
- the indirect effect of “suction”.

The quantitative description of these effects can be found subsequently in the following subsections.

4.2.2. First direct effect: sound pressure wave

The propagation of the wave impact pressure variation through the rubble foundation can be described as a sound wave, which means that inertia and compression of the rubble-water mixture are modelled, if the mixture can be considered as a one-phase material. This condition is met either if both phases move together (“no drainage”), as occurs with fine-grained material, or if the water phase moves alone, hardly hindered by the rubble (“complete drainage”), as occurs with very coarse rubble. In the “no drainage” case, the sound wave
propagation velocity, $c_p$, can be expressed as $c_{p1}$ according to equation (2); if “complete drainage occurs” it can be expressed as $c_{p2}$ according to equation (3).

\[
c_p = c_{p1} = \sqrt{\frac{K_w}{n + K + \frac{G}{2}}} \sqrt{\frac{n\rho_w + (1 - n)\rho_s}{\rho_w}}
\]

where $K_w$ is the compression modulus of the pore water, $K$ the compression modulus of the skeleton and $G$ the shear modulus of the skeleton, $\rho_w$ the density of the pore water and $\rho_s$ the density of the grains.

The propagation of the sound wave from the seaside to the harbourside and back, is illustrated in Figure 7 for the case of a triangular wave impact that is relatively short: $t_d << B_c$. The assumption is made that no deformation of the sound wave occurs during the propagation. In reality, deformation occurs, especially with small values of $t_r$ and $t_d$. As soon as the sound wave reaches the harbourside of the mound, where the pressure head has the constant value of mean sea level, it reflects partly and the reflected part starts to run back as a negative wave. During the reflection, the negative pressures of the front of the reflected wave partly compensate the positive pressures of the tail of the same wave, still running in the positive $x$-direction.

Before the start of the reflection the total uplift force, $F_{u,\text{max}}$, equals:

\[
F_{u,\text{max}} = \frac{1}{2} p_{u,\text{max}} t_d c_p
\]  

After the start of the reflection it is smaller and reaches its minimum value:

\[
F_u = - \frac{1}{2} p_{u,\text{max}} t_d c_p
\]  

Remark: the largest value of $F_{u,\text{max}}$ for these relatively short wave impacts, is found when $t_d c_p = B_c$. Then $F_{u,\text{max}}$ reaches the same value as in the (quasi)-stationary case with triangular pressure distribution. Only the working line may differ, being closer to the middle of the caisson instead of 1/3 from the sea side edge.

Reasoning in the same way for the case with $t_d c_p > B_c$, peak values of $F_u$ larger than the quasi stationary value of $\frac{1}{2} p_{u,\text{max}} B_c$ can be reached. The largest peak values are found for $t_d c_p \approx 3 B_c$. Then, the peak values are ca 1.5 times the quasi-stationary value if $t_r > t_d/4$, which is usually the case. See Figure 8. In this way $F_{u,\text{max}}$ can be presented as a function of $t_d c_p / B_c$. See Figure 9.
4.2.3. Second direct effect: elastic storage wave

According to the assumption used for the sound wave, no energy loss, thus no damping occurs with the pressure wave transmitted through the rubble. For the completely undrained case it is assumed that an important part of the water pressure variation at the seaside is transferred from the water to the skeleton without relative movement of the water through the skeleton. For the completely drained case, it is assumed that the fluid moves freely through the skeleton without any stress transfer (“friction”) from the water to the skeleton. Both cases are rather unlikely with the grainsizes usually applied.
There will be some relative movement of the pore water with respect to the skeleton together with friction and, consequently, energy loss. This effect, in combination with the compression of the water, but without inertia, can be modelled with the one-dimensional storage equation (6) for elastic compression of the pore water:

\[
\frac{\partial^2 p}{\partial x^2} = \frac{1}{c_{vp}} \frac{\partial p}{\partial t}
\]

where \( p \) is the excess pore pressure, \( c_{vp} = k \cdot K_w / (n \cdot \gamma_w) \) is the consolidation coefficient due to the compression of the pore water, \( k \) is the (Darcy) permeability, \( n \) the porosity and \( K_w \) the compression modulus of the pore water. If the pressure at the seaside would vary harmonically, i.e. \( p(0,t) = p_{u,max} \cos(2\pi t/T) \), then the pressure variation would be as indicated in equation (7) for \( x < B_c \):
\[ p(x,t) = p_{u,max} \exp\left(-\frac{x}{L_{ESP}}\right) \cos\left(\frac{2\pi}{T} - \frac{x}{L_{ESP}}\right) \] (7a)

where

\[ L_{ESP} = \sqrt{\frac{4\pi}{T}} \] (7b)

with

\[ c_{p3} = \frac{2\pi L_{ESP}}{T} = \sqrt{\frac{4\pi v_p}{T}} \] (7c)

The propagation velocity of this pressure wave is defined by \( c_{p3} \). The progress of such wave due to a wave impact may take place as illustrated in Figure 10. The progress is now largely determined by the parameter \( L_{ESP} \). If \( L_{ESP} \gg B_c \) the elastic storage is not relevant and the pore pressure distribution may be quasi-stationary. With \( L_{ESP} < B_c \), the uplift force, \( F_{u,max} \), will be reduced, as illustrated in Figure 11.

Fig. 10  Progress of pressure wave due to elastic storage
4.2.4. **Indirect effect: suction and expel of pore water**

a) Movements of the wall

The indirect effect consists of the pore pressure variation, $p_{u,\text{indir}}$, due to the movements of the wall, which are, during a wave impact, rotation and uplift, both resulting in an uplift of the front edge.

The prediction first requires a prediction of these movements. And these movements result from the horizontal wave load, $F_h$, the uplift force, $F_u$ and the inertia. The uplift force is the resultant of the instantaneous pore pressures in the rubble foundation, which cannot completely be determined before determination of the indirect component $p_{u,\text{indir}}$. Consequently, a loop back to $F_u$ is needed after a preliminary determination of the indirect effect. It is likely, however, that the correction to $F_u$ will be small, and so will the consequent correction to $p_{u,\text{indir}}$.

4.2.5. **Indirect effect: suction and expel of pore water**

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effect. It is likely, however, that the correction to \( F_u \) will be small, and so will the consequent correction to \( p_{u,\text{indir}} \).

The rotation and uplift of the caisson considered here, are not the total values of rotation and uplift, but the values relative to the top of the subsoil. Indeed, for the indirect component only the increase and decrease of the distance between the top of the subsoil and the floor of the caisson are relevant. Consequently the determination of the indirect effect can be done by considering the skeleton of the mound as an elastic medium (or a row of springs) and by considering the caisson as a (stiff) mass loaded by the external forces \( F_h(t) \) and \( F_u(t) \). The inertia components of the caisson force can best be incorporated by the dynamic load factor. Remark: this approach is not completely correct, because the dynamic load factor is also determined by the elasticity of the subsoil.

The elasticity of the skeleton of the rubble foundation is likely to be far from linear. The following Equation (8), may give a good approximation. In that equation \( K_z \) is the coefficient of subgrade reaction, \( K_{z,0} \) is the initial value, i.e. the value without any uplift, \( \sigma' \) is the effective stress and \( \sigma'_{0} \) its initial value. This approximation is good until the effective stress, \( \sigma' \), would become negative and a gap starts to be present. This will be discussed in subsection 4.5.

\[
K_z = K_{z,0} \left( \frac{\sigma'}{\sigma'_{0}} \right)^{0.5} \quad \text{thus} \quad \Delta z = \frac{2\sigma'_{0}}{K_{z,0}} \left[ 1 - \left( \frac{\sigma'}{\sigma'_{0}} \right)^{0.5} \right]
\]  

(8)

b) Influence of wall movements on pore pressures
The effect of the indirect component can be found by considering the case where the “external” forces, together with caisson inertia, cause uplift and rotation of the caisson, whereas the pressure heads at both outer edges of the caisson remain equal to mean sea level, defined to be zero: right hand part of Figure 3-15 of section 3.5.3 of Volume I.

Uplift causes an increase in pore volume; rotation causes an increase and a decrease in pore volume. Uplift and rotation together cause a change in pore volume which is linearly distributed in x-direction, as illustrated at the right hand part of Figure 3-15 of Volume I. The mass balance requires a flow velocity \( q \) which will vary more or less parabolically in x-direction. This flow causes friction resistance, which is compensated by a pressure head gradient to meet the impulse balance. With the given zero pressure head at both ends of the caisson, the pressure distribution may look like the one sketched in the figure: a porepressure decrease where the caisson rises and an increase where it descends.

A first approximation of the amplitude of pore pressure decrease can be found by assuming a harmonically fluctuating movement with period \( T \) and by assuming that neither inertia of the pore water, nor compression of the pore water are relevant. This is so if \( T >> T_{\text{sound}} \) and \( T >> \)
T_{ESP}. It means that only continuity and Darcy flow are relevant. These first approximations are indicated with the suffix "0".

For the case of pure uplift (no rotation) with fluctuation amplitude \( z \) (Figure 12), the following is found for the minimum excess pore pressure:

\[
p_{u,\text{indir, min}, 0} = -\frac{\pi}{4} \frac{\Delta z}{h_r} \frac{\rho_u g B_c^2}{k T} \tag{9}
\]

If the uplift of the caisson is completely caused by the uplift force and not by the caisson inertia, \( z/h_r \) can be related to \( F_{u,\text{max}} \) by considering the bedding layer as a row of elastic springs with spring coefficient \( (K + 4G/3)B_c/h_r \). Then, the following is found:

\[
p_{u,\text{indir, min}, 0} = -\frac{\pi}{4} \frac{F_{u,\text{max}} B_c}{c_{ys} T} \quad \text{with} \quad c_{ys} = \frac{k(K + \frac{4}{3} G)}{\rho_u g} \tag{10}
\]
With the increase/reduction of the uplift force, \( F_{u,\text{indir}} = 2/3 \cdot p_{u,\text{indir}} \cdot B_c \), and defining \( T_{\text{ESS}} = (0.5 B_c)^2/c_{\text{vs}} \) the following is found:

\[
\frac{\Delta F_{u,\text{indir},\text{min},0}}{F_{u,\text{max}}} = -\frac{\pi}{6} \frac{B_c^2}{c_{\text{vs}} T} \approx -2 \frac{T_{\text{ESS}}}{T} \tag{11}
\]

For the case \( F_u \) acts eccentrically, also rotation occurs and the part of the caisson that is lifted up is smaller. With \( l_{F_u} = 2/3 B_c \), i.e. an eccentricity of 1/6, the largest reduction of the uplift force and its lever arm (distance to harbour side corner), \( l_{F_u} \), can be estimated to be:

\[
\frac{\Delta F_{u,\text{indir},\text{min},0}}{F_{u,\text{max}}} \approx -0.8 \frac{T_{\text{ESS}}}{T} \quad \& \quad l_{\Delta F_u} \approx 0.7 B_c \tag{12}
\]

For the case of only a horizontal force \( F_h \) at \( l_h \) above the level of the caisson bottom only pure rotation occurs, which causes uplift over half the caisson. The largest reduction of the uplift force and its lever arm can be estimated to be:

\[
\frac{\Delta F_{u,\text{indir},\text{min},0}}{F_{u,\text{max}}} \approx -1.2 \frac{l_h}{B_c} \frac{T_{\text{ESS}}}{T} \quad \& \quad l_{\Delta F_u} \approx 0.8 B_c \tag{13a}
\]

\[
\frac{\Delta F_{u,\text{indir},\text{min},0}}{F_{h,\text{max}}} = -\frac{\pi}{6} \frac{B_c^2}{c_{\text{vs}} T} \approx -0.5 \frac{T_{\text{ESS}}}{T} \quad \text{where} \quad c_{\text{vd}} = \frac{kG}{\rho_w g \tan \psi} \tag{13b}
\]

The horizontal force may also cause a volume increase due to dilation of the rubble. The dilation can be characterised by the dilation angle \( \psi \). Equation (13b) is derived for this case in the same way as Equation (11). This volume increase occurs simultaneously with the volume increase due to caisson rotation. Thus, the corresponding reductions of the uplift forces, according to (13a) and (13b) should be added.

The period \( T \) may be taken equal to \( T \approx 2t_d \) if the natural period \( T_N \) of the caisson-foundation system, \( T_N << t_d \). If \( T_N >> t_d, T \approx T_N \). Then, however, uplift and rotation are also determined by the inertia of the caisson, and the absolute values of \( p_{u,\text{indir},\text{min}} \) and \( F_{u,\text{indir},\text{min}} \) will probably be smaller than the values given in equations (10) to (13).

If inertia of the pore water is relevant, the values found in formulas (9) to (13) need a correction similar to the one presented in Figure 9. If pore water compression is relevant, the values that found in these formulas need a correction similar to the one presented in Figure 11. The Figures 9 and 11 should be changed such that \( t_d \) at the horizontal axis is replaced by \( T \) and the parameter at the vertical axis is replaced by \( p_{u,\min}/p_{u,\min,0} \) or \( F_{u,\min}/F_{u,\min,0} \).
4.3. SOPHISTICATED ANALYSIS OF NON-STATIONARY FLOW IN RUBBLE

4.3.1. Physical scale models

Scale models can be used to quantify the non-stationary effects. The scale must be relatively large, for several reasons:

Froude scaling is needed for the pressure, gravity and inertia. This must be combined, however, with scaling of the flow resistance in the porous medium, which is largely turbulent. A completely correct scaling (same Reynolds number) is not possible. With large scale models, the Reynolds number remains large enough to keep completely turbulent flow during the most important part of the wave cycle with the grain size scaled with the length scale. With smaller scales, the relatively larger grain size must be selected.

The indirect effect can only be modelled correctly if also the stiffnesses are correctly scaled: the shear modulus must be proportional to the scale of stresses, which equals the length scale according to Froude. This requirement will usually not be met as the stiffness of natural rubble is proportional to the square root of the effective stress, rather than proportional to it. Thus the skeleton in the model will be relatively too stiff. Remark: other stress-strain properties of soil materials have other scaling rules. Therefore correct scaling in geotechnical modelling is usually done with a centrifuge to have the same stresses in the model as in the prototype.

Wave impacts are essential for the non-stationary effects and they cannot be modelled correctly. See Chapter 2 of Volume I.

Pore pressure measurements require the use of accurate pore pressure transducers with the necessary electronic equipment.

4.3.2. Numerical models

Essential for the numerical modelling are the modelling of the two phases: porewater and skeleton. The compressibility of both must be taken into account, as it is vital for all three effects. Finally inertia must be modelled, apart from gravity, pressures and flow resistance. Preferably non-linear flow resistance (turbulence) should be modelled. Otherwise a careful selected linearised flow resistance must be chosen and, depending on the results be adapted.
There is no need to model the phreatic surface in the porous medium, as no such surface is present in the rubble foundation of a vertical breakwater. In this respect the situation is less complicated than the situation of a crownwall on top of a rubble foundation breakwater.

The simultaneous modelling of the wave movement, including wave impacts, is not needed if the pressures as a function of time against the front wall and along the seaward slope of the rubble foundation can be derived from other sources. See Section 2 of Volume I and Volume II.

An example of a model that meets the above requirements, is TITAN, which is used for the hindcast of the large scale tests in Hannover. (section 4.4).

### 4.4. HINDCAST OF LARGE SCALE TESTS PERFORMED IN HANNOVER

#### 4.4.1. Summary of observations in Hannover

The following phenomena are discussed in [Kortenhaus e.a. 1994]:

- The propagation of the pressure wave in the rubble under the caisson. See also their Figure 11/12 (copied here as Figure 13) and 12/13.

Fig. 13 Hindcast Hannover tests: analysis of compression wave underneath the structure
Non-linear pressure distributions (their figures 8 - 10/10 - 11)

"Suction" underneath the structure (their Figure 13/14 and the Figure 14/15 copied here as Figure 14); the authors suggest "suction" to occur together with a gap.

"Gap" underneath the structure (their Figure 13/14).

A reduction, for larger values of the upward movement, of the apparent foundation stiffness, i.e. the ratio between uplift force and upward movement of the caisson front ("\(d_{vf}\)"), found from the results of many waves and many tests (their Figure 15 or 16, copied here as Figure 15); the authors believe this reduction to be caused by "suction".

**Fig. 14** Hindcast of Hannover tests: time histories of uplift force and vertical displacement at caisson front
Fig. 15  Hindcast of Hannover tests: reduction of apparent foundation stiffness with increasing uplift

First, a short discussion about the last 3 phenomena. They are likely to be phenomena of the same type as the "indirect effects" discussed here (subsection 4.2.4). During each wave crest, some uplift of the caisson front occurs and water is sucked into the additional room. "Suction" in this sense occurs during any wave crest.

The additional room usually consists of an increase of the pore volume of the rubble, associated with the uplift induced reduction of the effective stress. In case of very high wave loads, however, the effective stress can be reduced to zero. Then, not only additional room in the pore volume appears, but also a real "gap" between the crests of the upper rubble stones and the caisson bottom. The possible influence on the flow is discussed below. The reduction of the effective stresses may cause a reduction of the soil skeleton stiffness, even before a gap occurs. The question whether this reduction may explain the reduction of the ratio between uplift force and upward movement of the caisson front will be discussed in subsection 4.4.5.

4.4.2. Comparison measured and calculated wave propagation velocity

a) Observed propagation velocities
The propagation of the pressure waves in the rubble under the caisson is studied by [Kortenhaus e.a. 1994]. See also their Figure 11/12, here copied in Figure 13. With Figure 12/13 of the same publication a wave propagation celerity of 100 - 150 m/s is found. The mean value, 125 m/s, is indicated in Figure 16, together with some calculated values, by the straight line referred to with “Kortenhaus”.
b) Analytical approximations

According to the analytical approach, wave propagation only occurs with the direct component. The equations (2) and (3) are applied for the acoustical wave and (7) for the elastic storage.

With shear modulus $G_{\text{rubble}} = 45.6 \text{ MPa}$, Poisson ratio $\Gamma = 0.33$, the compression modulus for porewater with 5% of air of $K_w = 4 \text{ MPa}$, permeability $k = 0.05 \text{ m/s}$, $n = 0.4$ and $T \approx 2 t_d = 0.84 \text{ s}$, the following values are found:

$$c_{p1} = 319 \text{ m/s}, \ c_{p2} = 63 \text{ m/s} \text{ and } c_{p3} = 27 \text{ m/s}.$$ 

The first two values ($c_{p1} = 319 \text{ m/s}$ and $c_{p2} = 63 \text{ m/s}$) have been indicated in Figure 16 by straight lines.

The air content, however, was measured in the sand and not in the rubble. It probably was much smaller in the rubble, e.g. 1% yielding $K_w = 20 \text{ MPa}$. With the grain size of the rubble of $D_{50} \approx 31 \text{ mm}$ and the observed head gradients, the permeability is likely to be higher, e.g. $k \approx 0.3 \text{ m/s}$. With these adjustments, the following is found:

$$c_{p1} = 350 \text{ m/s}, \ c_{p2} = 140 \text{ m/s} \text{ and } c_{p3} = 150 \text{ m/s}.$$ 

Fig. 16 Hindcast of Hannover tests: measured and calculated wave propagation velocity
CHAPTER 4  INSTANTANEOUS PORE PRESSURES AND UPLIFT FORCES

It should be noticed that a large part of $c_{p1}$ is generated by the skeleton stiffness and small part by the pore water stiffness: in the first case (air content = 5%) 95% of is generated by the skeleton and 5% by the pore water; in the second case (air content = 1%) these percentages are 75% and 25%, respectively. The measured values, however, are lower than the $c_{p1}$ values, showing that partial drainage and/or elastic storage are relevant.

c) Calculations with TITAN

Figure 16 shows the arrival times of the peak pressure in five locations in the rubble for wave number 27129303 (with impact), calculated with TITAN and based on skeleton stiffness $G_{rubble} = 45.6$ MPa and Poisson ratio $\gamma = 0.33$, compression modulus for porewater with 5% of air of $K_w = 4$ MPa. and $k = 0.05$ m/s. With the non-linear calculations $G$ reduces to 20 MPa after a shear deformation of 2%.

The average propagation velocity found from the TITAN simulation, is about 100 m/s. In the middle the value is much lower (50 m/s). Under the front and the rear it is much higher.

An additional calculation made with TITAN for $K_w = 40$ kPa showed a much higher wave propagation velocity. According to the equations (2), (3) and (7) such increase in stiffness of the water yields an increase with a factor 1.5 ($c_{p1}$), respectively a factor 3 ($c_{p2}$ and $c_{p3}$).

4.4.3. Uplift force influenced by direct effects?

a) Relationship between direct effect and uplift force

The limitation of the propagation velocity of the pressure wave causes a varying pressure distribution in the rubble foundation during the wave impact. Thus, the pressure distribution is non-triangular most of the time, as illustrated in the Figures 8 to 10 (10 and 11) of [Kortenhaus e.a.1994]. The resulting value of $F_u$ and the lever arm vary. The question discussed in this subsection is, how much this value is influenced by the direct effects.

b) Analytical approximations

The influence of the sound wave parameter, $t_d/T_{sound} = t_d c_p/B_c$, on the uplift force can be estimated from Figure 9. With $t_d \approx 0.42$ s, $c_p \approx 125$ m/s and $B_c = 3.3$ m, $t_d/T_{sound}$ is 16 and no significant influence is to be expected. With the low value of $c_p = c_{p2} = 63$ m/s, $t_d/T_{sound}$ is 8 and the influence would be small: an increase or a decrease of no more than 10%.

The influence of the elastic storage parameter, $\sqrt{t_d/T_{ESP}} = \sqrt{(t_d c_{vp})/B_c}$, on the uplift force can be estimated from Figure 11. With $t_d \approx 0.42$ s, $c_{vp} = (0.05$ m/s 4 MPa)/0.4 10 kN/m$^3$) = 50 m$^2$/s and $B_c = 3.3$ m, $\sqrt{t_d/T_{ESP}}$ is 1.4 and a reduction of a few percent may be expected. With $k \approx 0.3$ m/s and/or $K_w = 40$ MPa, $t_d/T_{ESP}$ would be much larger and no influence is to be expected.
c) TITAN simulations
The increase in pore water stiffness from 4 MPa to 40 MPa yields an increase of circa 5% in the value of $F_{u,max}$ according to TITAN. This agrees with the conclusions from the analytical approximations. With such a high pore water stiffness, no influence is to be expected: a practically quasi-stationary situation. With the low pore water stiffness, a reduction of a few percent occurs by the sound wave effect and/or by the elastic storage effect.

All observations make clear that the resulting effect in the Hannover tests was small.

4.4.4. Reduction of uplift force due to indirect effect?

a) Observations
In [Kortenhaus e.a. 1994] a drop of the uplift force in the order of 5 to 10 kN/m is observed simultaneously with an increasing upward movement of the caisson front during the peak of the extreme wave 27129303 (their Figure 14 or 15, copied here as Figure 14). Could this be explained by the indirect effect of "suction"?

b) Analytical approximations
Equation (9) can be applied to get an indication of the reduction of the pore pressure that may be expected by the observed uplift, $z$. Assume just the front half of the caisson is lifted over a distance of $z \approx 1$ mm in 100 ms (Figure 14). Then, $T \approx 0.2$ s, $h_c = 0.59$ m, $B_c = 3.3$ m/2 and, with $k = 0.05$ m/s, a pore pressure reduction of circa 3.5 kPa is found; with $k = 0.3$ m/s, a reduction of circa 0.6 kPa. The first value is a significant value; the second is hardly noticeable.

Equation (12) can be applied to get an impression of the reduction of the total uplift force. With $B_c = 3.3$ m, $T \approx 0.2$ s and $c_{vs} = k(K + 4G/3)/(\gamma_w \approx k \cdot 4G/(\gamma_w \approx 0.05)$ m/s . $200$ MPa / $10$ kN/m$^3 = 1000$ m$^2$/s, the reduction appears to be just 1% of the total uplift force, or circa 3 kN/m. With $k \approx 0.3$ m/s, it is even much smaller: 0.2% or 0.5 kN/m. Also equation (13) can be applied. With $l_0 \approx 1/3$ $B_c$, also the percentages become 1/3, and values of circa 2.5 kN/m and 0.4 kN/m respectively are found. Thus, these equations yield force reductions smaller than the observed drop.

c) TITAN simulations
The vertical displacement of the front wall has been calculated (with the first in subsection 4.4.2 presented soil parameters) and compared with the measured one, indicated with “$d_i$” in Figure 14. The differences are small. Consequently the TITAN calculations simulate the measurements reasonably well and a detailed analysis of the calculated pressures and velocities must show “suction” if it really occurred during the observed drop of the uplift force.
Suction due to uplift movement would mean a reduction of pore pressures underneath the caisson bottom in the region near the seaward edge and an increase in discharges underneath the front. The calculations, however, just show the opposite. These calculated phenomena can well be explained by the direct effects, because the pore pressures at the toe (measured and taken as boundary condition for the calculation) strongly decrease in this period. Also the observed small drop of the uplift force can be explained by this sudden decrease. Probably no significant “suction” effect occurred.

This was confirmed by an additional TITAN calculation with a vertically fixed caisson. The indirect effects could now be quantified by considering the difference in values found between these calculation and the original one. The difference in pore pressures and the difference in total uplift force appeared to be extremely small.

4.4.5. Formation of a gap

a) Origin and effects of a gap
In case of very high wave loads the effective stress underneath the caisson front can be reduced to zero. Then, not only additional room in the pore volume appears, but also a real "gap" between the crests of the upper rubble stones and the caisson bottom. Such gap may have two sub-effects:
- a reduction of the flow resistance and
- no further reduction of the effective stresses, which cannot become negative.

The first sub-effect starts to be significant as soon as the gap is much larger than the pores. In case of the test considered here, this is not the case: the total uplift, which consists of an increase of pore volume of sand, an increase in pore volume of the rubble and the gap, is just a bit more than 1 mm in test 27129303) and is in no test larger than 1.8 mm, whereas the pores in the rubble (D50 = 31 mm) have diameters in the order of 10 mm. A similar conclusion is probably found with real breakwaters: failure in some way occurs long before a gap of such dimensions occurs.

The second sub-effect of a gap, could be modelled by reducing the stiffness to zero after a certain increase of the thickness of the bedding layer. Even more realistic would be the gradual reduction of the stiffness to zero. With a gradual stiffness reduction model (e.g. as equation 8 in section 4.2.4) the formation of a gap would be postponed. Usually, however, the skeleton of the rubble is modelled by a linear elastic medium or a non-linear elastic medium, yielding negative effective stresses as soon as a gap would occur. Therefore, usually too large effective stress reductions and too small vertical movements of the caisson are found.
b) TITAN simulation

TITAN doesn’t model the decrease to zero stiffness, but the non-linear soil stiffness model may model the stiffness reduction rather well. Therefore, the TITAN simulation for test 27129303 offers a good possibility to see whether such a gap may have occurred. The vertical effective stress just underneath the caisson is presented in Figure 17 (non-linear shear modulus) as a function of the location at 0.20, 0.25 and 0.30 s which correspond to 171.42, 171.47 and 171.52 s in the test (see Figure 15). At 0.25 s, the vertical effective stress is just zero under the front wall of the caisson. At this moment the vertical displacement of the front wall of the caisson is maximum. So, the model predicts a small gap during a period in the order of 10 ms.

Fig. 17 Hindcast of Hannover tests: effective stress underneath caisson bottom calculated with TITAN

4.4.6. Reduction of apparent foundation stiffness with increasing uplift

a) Observations

The relationship between the uplift force and the upward movement of the front of the caisson, $d_{v_f}$, as observed by Kortenhaus et al, is illustrated in Figure 15. The ratio is about $28/0.0004 = 70,000$ kN/m$^2$ for $0 < d_{v_f} < 0.4$ mm (say $d_{v_f} \approx 0.2$ mm) and it is circa $15,000$ kN/m$^2$ for $0.4$ mm $< d_{v_f} < 1.6$ mm (say $d_{v_f} \approx 1.0$ mm), that is circa 1/5 of the first ratio.

It should be noted that $d_{v_f}$ at the horizontal axis concerns the uplift of the front of the caisson; the average uplift of the caisson is certainly less and may increase quicker or slower than $d_{v_f}$. The uplift force, $F_v$ ($F_{u_{max}}$ in present symbols), is not the only cause of the upward movement of the caisson front. The horizontal force $F_{h_{max}}$ may even contribute more. Apart from these
two forces with their moments, also the elastic response of the skeleton and the inertia (against uplift and against rotation) must be taken into account when trying to predict the relationship between upward movement and uplift force.

b) Possible explanations:
A With large values of the upward movement a gap occurs yielding a zero stiffness of the foundation bed over a certain width and/or the stiffness of the skeleton just reduces by the reducing effective stress.
B The considered upward movement is mainly caused by the horizontal force. The uplift force does increase less than proportional to the horizontal force, e.g. by suction.
C The role of inertia increases with higher horizontal and uplift forces.

Analytical approximation of explanation A
During test 27129303 the maximum value of $d_{vf} \approx 1.2$ mm and just a small gap occurred (see section 4.5). Probably no gap occurred with smaller values of $d_{vf}$. According to Figure 15, however, the reduction of the uplift force/upward movement ratio started at $d_{vf} \approx 0.4$ mm. Therefore, it does not seem likely that a gap alone caused the phenomenon concerned.

Reduction of the skeleton stiffness, however, occurs with any uplift. The question is, whether it is sufficient to cause the above found large reduction of the ratio. A reasonable estimate of the stiffness reduction can be found by assuming the coefficient of subgrade reaction, $K_z$, proportional to the effective stress, $\sigma'$, to the power 0.5, according to equation (8).

The initial effective stress $= W/B_c \approx 40$ kPa and the effective stress is just zero with $\sigma' \approx 1$ mm (section 4.3). This yields $K_{z,0} \approx 80$ MN/m$^3$.

Assume that the value of the average uplift $\sigma'$ is half the uplift of the front, i.e. $\sigma' \approx 0.5$ $d_{vf}$. Then, the ratio $\sigma'/\sigma'_{0}$ reduces from $\approx 0.9$ with $d_{vf} = 0.2$ mm to $\sigma'/\sigma'_{0} \approx 0.5$ with $d_{vf} = 1$ mm and the ratio $\partial F_u/\partial d_{vf}$ reduces to circa 75% if going from $d_{vf} = 0.2$ mm to $d_{vf} = 1$ mm. This is not sufficient to explain the observed variation of the reduction of the uplift force/upward movement ratio completely.

c) Explanation B
The reduction of the uplift force due to suction during test 27129303 was probably smaller than a few percent, as explained in section 4.4. The ratio $F_{u,max}/F_{h,max}$ during test 27129303 was nearly equal to the ratios during tests 22129304 and 29129308, tests with much smaller values of $F_{h,max}$ and $F_{u,max}$. Consequently, it is not very likely that explanation B is correct.

d) Explanation C
The tests with the higher values of $d_{vf}$ are likely to be the tests with wave impacts. According to the hindcast the dynamic load factor is 1.0 to 1.3. This means an enlargement of the
displacements (horizontal, rotation and uplift) with a factor 1.0 to 1.3 compared to the quasi-stationary value. This also holds for $d_{r}$ and explains part of the reduction of the uplift force/upward movement ratio.

e) Conclusion
Explanation A and explanation C are probably both partly correct. The reduction of the uplift force/upward movement ratio is caused by both effects together.

4.5. CONCLUSIONS ABOUT NON-STATIONARY EFFECTS OF FLOW IN RUBBLE

- Non-stationary effects are only relevant under specific circumstances: the direct effect may only be relevant if $t_{d}/T_{\text{sound}} < 10$ or $t_{d}/T_{\text{ESP}} < 2$; the indirect effect only if $t_{d}/T_{\text{ESS}} < 10$ or $t_{d}/T_{\text{ESD}} < 10$, where $T_{\text{sound}} = B_{c}/c_{p}$, $T_{\text{ESP}} = (0.5B_{c})^{2}/c_{vp}$, $T_{\text{ESS}} = (0.5B_{c})^{2}/c_{vs}$, $T_{\text{ESD}} = (B_{c})^{2}/c_{vd}$. Definitions of $c_{p}$ etc are presented in the equations (2), (3), (6), (7c), (10), (13b).
- A preliminary estimate of the direct effects may be found with Figures 9 and 11; that of the indirect effect with the equations (12) and (13).
- Modelling of the non–stationary effects with a scale model is difficult due to several scale effects: the scaling of permeability, skeleton stiffness and wave impacts cannot be done (completely) correct.
- Modelling the non–stationary effects numerically requires a sophisticated computer code that models the stresses and deformations of the two phases, pore water and skeleton, the compressibility of both phases, the inertia and the permeability for turbulent flow (at least by a carefully linearised permeability).
- The TITAN model meets these requirements. The hindcast of the Hannover tests with this model yields satisfactory results, although not all observations are clearly explained.

4.6. INSTANTANEOUS PORE PRESSURES IN SANDY OR SILTY SUBSOIL

4.6.1. Characteristic drainage period as a function of depth

Whether the soil at a certain depth, $A$, should be considered to be drained or undrained, depends on the ratio of the characteristic load period to the characteristic drainage period. The soil may be considered drained if it is much larger than one and undrained if it is smaller than
The characteristic drainage period is the largest of the following periods:

- \( T_{ESP} = A^2/c_{vp} \) where \( c_{vp} = k \cdot K_w/(\gamma_w) \)
- \( T_{ESS} = A^2/c_{vs} \) where \( c_{vs} = k \cdot (K + 4/3G)/\gamma_w \)
- \( T_{ESD} = A^2/c_{vd} \) where \( c_{vd} = k \cdot G/\tan\psi \cdot \gamma_w \)

where \( c_{vp}, c_{vs} \) and \( c_{vd} \) [m²/s] are the consolidation coefficients for elastic deformation, functions of the stiffness parameters, \( K_w \) (compression modulus of the pore water), \( K \) (compression modulus of the skeleton) and \( G \) (shear modulus of the skeleton), of the permeability, \( k \), and of the dilation angle, \( \psi \).

With high density and consequent high value of \( \psi \), \( c_{vd} \approx c_{vs} \). Otherwise \( c_{vd} > c_{vs} \). Thus, \( T_{ESD} \leq T_{ESS} \) and no attention need to be paid here to \( T_{ESD} \).

An impression of the values of \( T_{ESP} \) and \( T_{ESS} \) can be gained from the ranges of the relevant parameters. The permeability of (medium) fine sand usually varies between \( 10^{-5} < k < 10^{-4} \) m/s; with a high silt content it may drop to \( 10^{-6} \) m/s; with silt \( 10^{-8} < k < 10^{-6} \) m/s, with the lower value for clayey silt and the higher for sandy silt. The value of the pore water stiffness of the soil skeleton usually varies between \( 30 < (K + 4/3G) < 300 \) MPa, with the lowest value for clayey silt and the highest for dense sand. The value may vary between \( 10 < K_w < 10^1 \) MPa. It strongly depends on the degree of saturation. Not much is known about the degree of saturation. However, in the seabed at a depth larger than 10 m, it is likely to be rather high and \( 10^2 < K_w < 10^3 \) MPa. Thus, \( T_{ESP} \leq T_{ESS} \) and the characteristic drainage period equals \( T_{ESS} \). Table 1 can be used for a first impression, based on the assumptions: \( T/\pi \approx 5 \) s and \( t_d \approx 1 \) s.
Table 1: Order of magnitude of characteristic drainage period. Advise on drained or undrained conditions.

<table>
<thead>
<tr>
<th></th>
<th>A = 0.1 m</th>
<th>A = 1 m</th>
<th>A = 10 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense, coarse sand</td>
<td>3.10^{-4}</td>
<td>0.03</td>
<td>3 s drained for wave impacts and pulsating loads</td>
</tr>
<tr>
<td>k ≈ 10^{-3} m/s</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>K+4/3G ≈ 300 MPa</td>
<td></td>
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<tr>
<td>cv_s ≈ 30 m^2/s</td>
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<tr>
<td></td>
<td>0.01</td>
<td>1</td>
<td>100 s undrained for wave impacts and pulsating loads</td>
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<tr>
<td>Medium dense,</td>
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<tr>
<td>medium fine sand</td>
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<tr>
<td>k ≈ 10^{-4} m/s</td>
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<td>K+4/3G ≈ 100 MPa</td>
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<td>cv_s ≈ 1 m^2/s</td>
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<td></td>
<td>0.1</td>
<td>3</td>
<td>1000 s undrained for wave impacts and pulsating loads</td>
</tr>
<tr>
<td>Medium dense,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>silty sand</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k ≈ 10^{-5} m/s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K+4/3G ≈ 100 MPa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cv_s ≈ 0.1 m^2/s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>300</td>
<td>30 000 s undrained for wave impacts and pulsating loads</td>
</tr>
<tr>
<td>Loose, sandy silt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k ≈ 10^{-6} m/s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K+4/3G ≈ 30 MPa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cv_s ≈ 0.003 m^2/s</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.6.2. Pore pressures due to mean total stress variations

Below the layer with direct influence of the WATER pressure variations at the boundary, only the TOTAL stress variations determine the pore pressure variations in the subsoil. Two components of the total stress variations may be distinguished: those of the mean stress and those of the shear stress. The effects of the mean stress variations are discussed in this subsection.

a) General guidelines for modelling

The mean total stress variations can be calculated as a function of the vertical total stress variations at the upper boundary of the subsoil. A first approximation can be found assuming the vertical total stress at the boundary varying like a sinus with wavelength L, then the vertical total stress at a certain depth does likewise, however with an amplitude reduced according to a negative exponential function with characteristic length L/2π. A similar vertical total stress distribution is found if any variation of stress at the boundary is supposed to be distributed into the depth according to 1 : 1 or 2 : 1.
A better approximation, especially with layered soil, can be found with a rather simple numerical model in which each soil layer is modelled as a one-phase elastic medium.

The mean stress variations cause compression or decompression of the soil, which yield an increase or decrease of the mean effective stress and the pore pressure proportional to the respective stiffnesses: $K$ and $K_w/n$.

b) Application of first approximation to moment from wave loaded caisson on thin bedding layer

The wave load to a caisson causes a moment $M$, which the caisson transfers via the bedding layer to the subsoil. This moment causes an increase in the vertical stress underneath one edge of the caisson and a decrease under the other edge. The vertical total stress variations directly underneath the caisson bottom, $\Delta \sigma_v (x,0)$, are assumed to vary linearly over the caisson width, $B_c$:

$$\Delta \sigma_v (edges,0) = \pm \frac{M}{B_c^2 / 6} \quad (14)$$

The vertical stress variations reduce with depth, just like the moment transferred from higher layers to lower soil layers. The soil in between remains stable due to the compensating normal stress and shear stresses along vertical planes. Here, the “wave length” may be assumed to be $L \approx 2 B_c$ and the reduction with depth will be according to a negative exponential function:

$$\Delta \sigma_v (edges, z) = \pm \frac{M}{B_c^2 / 6} \frac{1}{\exp(z \pi / B_c)} \quad (15)$$

The horizontal total stress variations will probably be smaller than the vertical ones. Consequently the mean total stress variations will be smaller, e.g.:

$$\Delta \sigma (total, z) = \frac{\Delta \sigma_v + \Delta \sigma_{hx} + \Delta \sigma_{hy}}{3} \approx 0.75 \Delta \sigma_v (z) \quad (16)$$

These mean total stress variations are partly met by pore pressure variations and partly by the skeleton, depending on the ratio of the stiffnesses of pore water ($K_w/n$) and skeleton ($K$). This yields Equation (17):

$$u(edges, z) = 0.75 \frac{K_w}{K_w + nK} \Delta \sigma_v (z) = \pm 0.75 \frac{K_w}{K_w + nK} \frac{M}{B_c^2 / 6} \frac{1}{\exp(z \pi / B_c)} \quad (17)$$

A triangular region with its top at the edge of the caisson bottom (slope 2:1) is assumed in between the region right underneath the caisson and the regions adjacent to the caisson. Seaward of this region in front of the breakwater high total stresses are present at wave crest
due to the wave load. The pore pressures in this region are supposed to be according to linear interpolation between the pore pressures found in the other regions.

An example for the case of highly incompressible pore water, \( \frac{K_w}{(K_w + nK)} \approx 1 \), is presented for wave crest in Figure 18.

### 4.6.3. Negative pore pressures due to dilation

The shear stress variations may cause both increasing or decreasing instantaneous pore pressures in the subsoil, if this consist of sand or silt. During continuous shearing of drained sand and silt, usually first some contraction (volume decrease) occurs, then dilation (volume increase). When approaching the failure condition the dilation is larger than the contraction, unless the sand or silt is very loosely packed. It can be quantified by the dilation angle \( \psi \). With undrained soil, the dilation causes negative pore pressures, i.e. reduction of the pore pressure.

Assuming a shear stress due to the wave load of \( \tau(z) \approx \frac{F_h}{B_c \exp\left(\frac{z \pi}{B_c}\right)} \), the following excess pore pressure \( u \) (negative, i.e. pore pressure reduction) is found:

\[
 u = - \left[ \frac{F_h}{B_c \exp\left(\frac{z \pi}{B_c}\right)} \tan \psi \right] \left[ \frac{1}{K + \frac{4}{3}G + \frac{n}{K_w}} \right]
\]

(18)
The pore pressure reduction continues as long as shearing continues until the end of the dilation (corresponding to maximum volume increase) or until the absolute pore pressure has
become zero. The first limit is not relevant for vertical breakwaters if the sand or silt is very dense (density index 80% or more) and if the pore water is highly saturated (saturation 99.5% or more). In other cases it may be. The second limit yields a negative pore pressure, $p_{\text{min}}$, equal to the original absolute pore pressure, as expressed in Equation (19):

$$p_{\text{min}} = -p_{\text{atm}} - \rho_w g h_{\text{slidingplane}}$$  \hspace{1cm} (19)

where $p_{\text{atm}}$ is the atmospheric pressure at sea level ($p_{\text{atm}} \approx 100$ kPa) and $h_{\text{slidingplane}}$ is the depth of the sliding plane below sea level. This limit is clearly observed in the centrifuge tests (subsection 6.5), where the negative pore pressures were limited to $p_{\text{min}} = -230$ kPa. The density index was $I_D \approx 80\%$ (at the moment of extreme loadings) and the saturation $S > 99.9\%$.

The value of $p_{\text{min}}$ according to Equation (19) must be considered as an upper limit. With dense sand or silt and high saturation, it can be used to estimate the upper limit of the corresponding increase in shear strength, as will be discussed in section 4.6.5.

4.6.4 Pore pressures found in Hannover tests

The air content of the pore water in the sand bed of the Hannover tests was very high, thus the saturation, $S$, was very low $S \approx 95\%$. This yields a low stiffness of the pore water ($K_w \approx 4000$ kPa) and a consequent relatively high value of the characteristic drainage period for elastic storage in the pore water. At a depth of 0.1 m, $T_{\text{ESP}} \approx 1$ s; at a depth of 0.2 m, $T_{\text{ESP}} \approx 4$ s. This should be compared to the characteristic load period of $T/\pi = 1.5$ s. A second effect of this low pore water stiffness is the low value of $K_w/(K_w + nK)$ and the consequent low value of the pore pressure variation at the depth where the sand can be considered to behave undrained (Equation 17).

This corresponds to the results of the measurements. The pore pressure variations at the level of 0.05 m below the rubble/sand interface were nearly the same as those in the rubble just above this interface: nearly completely drained behaviour. At the depth of 0.25 m and deeper no significant pore pressure variations were measured. The TITAN calculations, performed in the framework of the hindcast of these tests, did also show no significant pore pressure variations at these depths.

4.6.4. Pore pressures found in centrifuge tests

The influence of the wave action on the pore pressure development is among other things studied by centrifuge tests [van der Poel and de Groot, 1998] and [van der Poel and de Groot, 1997]. These tests were carried out in the centrifuge of Delft Geotechnics in Delft. For these
Tests a 13 m (prototype) wide caisson was directly placed on the sand layer. The model was enclosed by a gravel layer, which covered the sandy subsoil. A layout of the prototype caisson is presented in Figure 19. Below the caisson pore pressure transducers were placed. First a row is placed in the caisson floor. Secondly a row of pore pressure transducers was placed at 2.70 m (prototype) below the bottom of the caisson.

A typical reading of the transducer indicated in Figure 19 by a circle is presented in Figure 20. From this figure both the residual pore pressures and the instantaneous pore pressures can be observed. The gradual increase of average pore pressure can be considered as the residual pore pressures. These will be discussed in chapter 4.5. The instantaneous pore pressures follow the wave action at sea directly and can be distinguished from the reading by the sharp high rise peaks. These fluctuations occur rapidly in time. For this reason the instantaneous pore pressures are hardly influenced by drainage effects. Only in a small top layer some drainage influences the pore pressures.

The positive peak in the instantaneous pore pressure of Figure 20 is probably completely due to the compression (subsection 4.6.2). A large part of the negative peak found in this transducer, is certainly due to decompression during wave trough. Whether a part is also due to dilation (subsection 4.6.3) is not clear.
Simultaneous with the positive peak measured in this transducer, negative peaks are found not only underneath the caisson front, but also underneath the middle of the caisson. Dilation may certainly have contributed to some of these negative peaks, because no significant simultaneous uplift of the middle of the caisson was observed during several wave crests.

During the largests waveloads a minimum pore pressure of $p_{\text{min}} = -230$ kPa was found underneath the caisson front. This was very close to zero absolute pressure (vacuum). In the centrifuge tests, the sand was prepared in such a way as to arrive at the highest possible saturation. The result was $S > 99.9\%$.

**Fig. 20** Example of pore pressure measurements underneath caisson back wall. Pore pressures in kPa found along vertical axis.

**REFERENCES**


CHAPTER 5: DEGRADATION AND RESIDUAL PORE PRESSURES

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ABSTRACT:

The foundations of vertical breakwaters will be subjected to large cyclic and impact loads from waves. The wave forces are counteracted by the resistance of the rubble mound and the subsoil. Cyclic stress reversals tend to compact the grain structure of the soil and a gradual accumulation of pore water pressure may develop. The resulting reduction in effective stresses will lead to reduced strength and stiffness. The degradation effect depends on the magnitude and number of stress reversals, i.e. the wave load history, as well as the drainage conditions. This chapter discusses methods to derive a realistic wave load history and methods to evaluate the effect of cyclic loading and pore pressure generation on strength and stiffness of soils.

Keywords: Vertical breakwaters, cyclic loads, degradation, pore pressure, strength, stiffness

5.1. INTRODUCTION

Soil subjected to cyclic loading will generally have a tendency to undergo volumetric compression. Volumetric compression will generate excess pore pressure in the soil. The pore pressure increase depends on the intensity and duration of the cyclic load history and the drainage conditions, i.e. soil permeability, compressibility and drainage distance. If the drainage conditions are limited a gradual accumulation of excess pore pressure, i.e. a residual pore pressure, may develop. This will cause a gradual reduction of the effective stress in the soil and lead to reduced strength and stiffness.

Clayey soils will be undrained during a storm and possibly also over periods of several storms while sand may experience partial drainage during a storm. The amount of drainage depends on the permeability and compressibility of the sand and the drainage boundary conditions. Thin interbedded layers and bands of silty/clayey fine sand may reduce the drainage in vertical direction considerably and change the main drainage to the horizontal direction.
When the pore pressure dissipates, the subsoil will undergo compression. For sands and normally to lightly overconsolidated clays this will normally lead to an increased strength and a reduced susceptibility to pore pressure generation and strength degradation under further cyclic loading. For highly overconsolidated clays, cyclic loading may lead to a permanent reduction of strength and stiffness (see Bjerrum, 1973, Andersen & al., 1976, Smits & al, 1978).

Prior to the "design storm" the breakwater will normally be subjected to a series of smaller storms. In sandy soils the pore pressure generated under the cyclic loading from these storms and even from the first part of the design storm will dissipate. This is normally referred to as "precycling". The beneficial effect of this "precycling" should be accounted for in the design, and should therefore be included in the test program for sand by applying some precycling prior to the main cyclic loading.

The low permeability of clays will prevent dissipation of pore pressure prior to the design storm. The beneficial effect of precycling is thus not accounted for in normally and lightly overconsolidated clays. In highly overconsolidated clays the negative precycling effect will be most unfavourable if the design storm occurs after several years with precycling from smaller storms. During this time, however, the clay will also have consolidated under the weight of the breakwater, and the effect of consolidation and increased effective stresses will in most cases compensate for the negative effect of precycling.

To resist the combined load effect of the breakwater weight and extreme waves the soil must be able to mobilise sufficient shear strength. The shear strength depends on the effective stresses in the soil, and thus on the excess pore pressure generated during the storm. The shear strength is further dependent on whether the soil is contractive or dilative during the shearing under the extreme loads and whether this shearing takes place under drained, partly drained or undrained conditions. If the soil is dilative and saturated, a negative pore pressure will develop when the soil is sheared under undrained conditions. This will give a higher shear strength than for drained conditions.

Clays will be undrained during the period of one cycle, and negative pore pressures due to cyclic loading can be relied upon.

Sands will be undrained during the short peak loads from wave impacts, but may be partly drained during the maximum load, especially close to the rubble mound. It is thus necessary to consider whether the undrained, the partly drained or the drained shear strength should be used in the stability calculations.
The gravels and rockfills of the rubble mound will be drained during pulsating wave loading, and drained or close to drained even during the short term peak impact loading, somewhat dependent on the grain size distribution and void ratio of the material.

For a dilating sand or gravel one should be careful about relying on higher shear strength due to negative pore pressures, especially if the soil is not saturated. For saturated soils the amount of drainage during a cycle may be estimated from the diagrams given in this chapter.

The design diagrams presented in this report are based on simplified load histories and selected soil data from the soil database developed within the frame of the PROVERBS project, NGI (1998a). To cover a wide range of structural geometries and soil conditions in the design diagrams simplified analytical methods and normalisation of parameters have been applied. In Sections 5.2 through 5.5 the assumptions and boundary conditions are described.

The effect of pore pressure generation on the soil stiffness has been evaluated and presented as stiffness reduction diagrams for different soils and breakwater geometries in Section 5.6.

Design diagrams for preliminary estimates of residual pore pressure in sand and strength degradation in clays have been worked out for some of the typical soils and a range of breakwater geometries in Section 5.7.

5.2. COMPOSITION OF DESIGN STORM LOADING FOR BREAKWATERS

5.2.1. General

The wave load history will have a significant impact on the rate of pore pressure generation and strength reduction in fine sands and silty/clayey soils. Dependent on the geometric conditions (i.e. water depth, berm height and berm width) and wave heights, breakwaters will be subjected to a combination of cyclic loads and impact loads. The cyclic loads are quasi sinusoidal with a period corresponding to the wave period and can be treated as static loads, while the impact from breaking waves shows a peak load of short duration and requires dynamic analyses.

The distribution of wave loads during a stationary sea state has for offshore structures been assumed to follow a Rayleigh distribution. This assumption may be questionable for breakwater structures. For cyclic/pulsating loads Goda’s method predicts a slightly non-linear relationship between wave height and horizontal force. With increasing of $H_{si} / d$ or $H_{si} / h_s$ wave breaking will occur and may result in impact loading. The prediction methods described
by Allsop et al. (1997) show relationship between the horizontal impact load and the wave height to the power of about 3.1.

Analysis of the probability distribution of the amplitudes of pulsating and impact horizontal forces from hydrodynamic test series shows that pulsating loads can be well described by a Weibull distribution while the impact forces are better represented by a Log-normal distribution (Vicinanza, 1997). The probability distributions are still under evaluation at the time of writing. Kortenhaus (1998) has given preliminary recommendations for using a Weibull-3 distribution for non-impact wave loading and a log-Weibull distribution for impact conditions. See also further considerations and discussions on this matter by McConnell and Allsop (1998).

The use of two different distributions with validity ranges which depend strongly on the geometry of the breakwater (Hsi /d or Hsi /hs, berm length etc.) can be considered in detail engineering phases. In the early design phases this would complicate the situation. In order to develop design diagrams, simplifications has to be made with due consideration of the errors involved. In the following a simplified approach based on Rayleigh distribution of wave heights and load calculation with Goda’s method is evaluated, and the results compared with the method of Allsop & al (1997) and with test data.

5.2.2. Storm history and wave height distributions

The development of wave heights during a storm is dependent on the local weather conditions. For this study the NPD (1987) practise has been adopted. There is limited statistical material regarding duration of extreme storms periods and the build-up and decay of the significant wave height, Hs, but the NPD recommendation shown in Fig. 1 is considered to give a conservative description of a design storm.
CHAPTER 5  CYCLIC DEGRADATION AND RESIDUAL PORE PRESSURE

Storm history, (NPD)

Fig. 1: Recommended storm history for evaluation of cyclic load effects on strength and stiffness of soil consisting of 18 hours build-up from $0,5 \, H_{s,\text{max}}$ to $H_{s,\text{max}}$, 6 hours design storm with $H_{s,\text{max}}$ and 18 decay back to $0,5 \, H_{s,\text{max}}$.

In order to arrange the wave history in order of descending height and sort the waves in groups or parcels with similar wave height the build-up and decay periods are each split in 5 periods of 3.6 hours duration with constant $H_{s}$, i.e. stationary conditions. For each stationary condition a Rayleigh distribution of wave heights is assumed.

The average wave period of a seastate may vary considerably but will generally increase with increasing significant wave height. For this study the relationship shown in Tab. 1 has been assumed:

Tab. 1: Assumed connection between mean wave period and significant wave height

<table>
<thead>
<tr>
<th>$H_s/H_{s,\text{max}}$</th>
<th>0.55</th>
<th>0.65</th>
<th>0.75</th>
<th>0.85</th>
<th>0.95</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_m$</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

In addition the distributions where checked with $T_m = \text{const.} = 10$ seconds and 12 seconds.

For evaluation of strength and stiffness degradation the situation at the end of the peak storm period is normally considered to be most critical. Applying the maximum wave or maximum wave load at the end of this period, i.e. after 18hrs build up and 6hrs design storm loading, is considered to be conservative. The waves are sorted in 12 parcels and the calculations are summarised in the Table 2.

Tab 2: Number of waves vs. relative wave height $H/H_{\text{max}}$ for 3 different mean wave period assumptions; $T_m = 10$ s, $T_m = 12$ s and $T_m$ according to Table 1
The difference between the three wave period assumptions is increasing with decreasing wave height or wave load level. As the cyclic load effect strongly decreases with decreasing load level the differences between the three assumptions will not lead to significant differences in cyclic strength and stiffness.

5.2.3. Transfer function wave height - wave load for pulsating loads and moderate impact

Goda’s (1985) method is considered to give reasonable predictions of pulsating wave loads and to some extent include impact loading. For the purpose of developing cyclic load histories for foundation design Goda’s method was applied for the individual wave parcels using the average $H_{max}$ in each parcel as design wave height. This procedure may not be statistically correct, however, it assumed to give a reasonable physical connection between wave heights and wave loads.

The wave pressures and forces calculated with Goda’s method will be dependent on the breakwater geometry and the wave height. In order to cover a broad range of possible situations the geometries of the Gela and Genoa Voltri breakwaters have been evaluated with wave heights varied in the range 3 to 12 m and 4 to 16 m respectively. The wave period used for load calculation was 12 seconds. The calculated normalised load ($F/F_{max}$) distributions are summarised in Table 3.

**Tab. 3:** Relative wave height vs. relative load level, Gela and Genoa V1.
The table above shows that there is not a linear relationship between relative wave height and relative load when using Goda’s method. The number of waves related to the relative wave height from Tab. 2 could be applied here as shown in the last column to give recommended connection between load level and number of load cycles. The corresponding distributions are shown in Fig. 2.

![Distribution of normalised wave height (H/Hmax) and normalised horizontal forces based on Goda’s method (Fh/Fh,max) for Gela (Hmax = 6m and 12 m) and for Genoa Voltri V1 (Hmax = 8 and 16 m).](image)

**Fig. 2:** Distribution of normalised wave height ($H/H_{max}$) and normalised horizontal forces based on Goda’s method ($F_h/F_{h,max}$) for Gela ($H_{max} = 6$ m and $12$ m) and for Genoa Voltri V1 ($H_{max} = 8$ and $16$ m).
Alternatively the load levels are kept the same as for the relative wave heights and the number of cycles is reduced correspondingly. This is shown in Tab. 4, where a comparison with the MCS load distribution (MCS report Table 5.5.1), (De Groot & al., 1996) is shown as well.

**Tab. 4: Number of cycles vs. load (rel. wave height) level for a NPD storm**

<table>
<thead>
<tr>
<th>Relative load level</th>
<th>No. of waves or cycles</th>
<th>(NPD storm 18 hr build-up + 6 hrs $H_{s,max}$)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H/$H_{max}$ or F/F$_{max}$</td>
<td>H/$H_{max}$ distr. NPD 24 hrs</td>
<td>$F_h/F_{h,max}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NPD 24 hrs Genoa V1 (H$_{max}$=8 m)</td>
<td>Genoa V1 (H$_{max}$=8 m)</td>
</tr>
<tr>
<td>1,00</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0,975</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0,925</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>0,85</td>
<td>16</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>0,75</td>
<td>51</td>
<td>36</td>
<td>24</td>
</tr>
<tr>
<td>0,65</td>
<td>140</td>
<td>82</td>
<td>55</td>
</tr>
<tr>
<td>0,55</td>
<td>334</td>
<td>222</td>
<td>140</td>
</tr>
<tr>
<td>0,45</td>
<td>700</td>
<td>440</td>
<td>333</td>
</tr>
<tr>
<td>0,35</td>
<td>1268</td>
<td>790</td>
<td>628</td>
</tr>
<tr>
<td>0,25</td>
<td>1874</td>
<td>1727</td>
<td>1524</td>
</tr>
<tr>
<td>0,15</td>
<td>1943</td>
<td>2312</td>
<td>2494</td>
</tr>
<tr>
<td>0,05</td>
<td>870</td>
<td>1544</td>
<td>1952</td>
</tr>
<tr>
<td>Sum cycles</td>
<td>7200</td>
<td>7200</td>
<td>7200</td>
</tr>
</tbody>
</table>

A comparison of the three last columns of Table 4 shows that there are only marginal differences between the load compositions used in the MCS project and the ones derived for Genoa V1 and Gela with the approach described above.

**5.2.4. Transfer functions wave height to wave load for impact loading**

Allsop et al. (1997) presented the following relationship for prediction of impact loading from breaking waves based on hydraulic model tests on simple vertical walls and vertical walls with low mounds:

$$\frac{F_{h_{1/250}}}{(\rho_{wg}g d^2)} = 15 \left( \frac{H_{s1}}{d} \right)^{3.134}$$  \hspace{1cm} (1)

The number of waves in the test series was 500. $F_{h_{1/250}}$ will then be the average of the two highest loads i.e. approximately the maximum measured load. Assuming a Rayleigh distribution of $H$ the relationship between significant and maximum wave height is
CHAPTER 5  CYCLIC DEGRADATION AND RESIDUAL PORE PRESSURE

\[ H_{si} = \frac{H_{\text{max}}}{1.76} \]  \hspace{1cm} (2)

Inserting Eq. 5-1 into Eq. 5-2, replacing \( F_{h1/250} \) with \( F_{\text{max}} \) and rearranging:

\[ F_{\text{max}} = 2.55 \rho_w g d^2 \left( \frac{H_{\text{max}}}{d} \right)^{3.134} \]  \hspace{1cm} (3)

If this transfer function is applied for single waves and is applied whenever the force exceeds the corresponding force calculated with Goda's method the result will be distributions as shown in Figure 3. The curves were calculated for Gela geometry with \( h_s = 12.8 \) and \( d = 8 \) m for 4 different stationary storms each with 500 load cycles. The lower bound of the distributions is determined by Eq. 5-3 (Allsop et al., 1997), while the upper bound is determined by Goda's method. As can be seen the distribution transforms from a "Goda distribution" at low \( H_{si}/d \) ratios (no impacts) towards "Allsop distributions" with increasing \( H_{si}/d \) and increasing number of impacts.

![Distribution Fh/Fhmax vs log N](image)

**Fig. 3:** Distribution of normalised wave loads \((F_h/F_{h,max})\) vs. \( H_{si}/d \) for combined Goda and Allsop prediction methods. Storms with 500 waves.

In order to compare the distributions with test results, data from test series 04 and 05 conducted at GWK (Kortenhaus and Oumeraci, 1997) has been reviewed and presented in the same way. Test series 04 and 05 were selected, as they are the GWK test series closest to the geometries of the Gela and the Genoa Voltri I breakwaters. Figure 4 shows the normalised distributions of the measured horizontal forces \( F_h \) vs. \( \log_{10} N \). As can be seen the distributions approach the "Goda distribution" at low \( H_{si}/d \) values and gradually transforms to "Allsop distributions" with increasing \( H_{si}/d \).
5.2.5. **Recommended load histories for evaluation of cyclic loading effects**

The storm histories to be applied for evaluation of cyclic load effects on the build-up of pore pressure and degradation of strength of the soils underlying a breakwater should preferably be based on data describing the local wave climate regarding storm history. If this information is not available a distribution according to NPD (Fig. 1) can be applied.

For cases with predominantly pulsating loads the load history described in the MCS project (see last column in Tab. 4) is considered to give a good description of the load history to be expected during the build-up period (18 hrs) and the design storm period (6 hrs). This approach is adopted for the development of design diagrams for use in the early engineering phases.

For detailed design in cases with a high number of impact wave loads the two sets of transfer functions should considered as described in Ch. 5.2.4 or distributions described by Vicinanza (1997) for pulsating and impact waves respectively should be considered. General reference is made to the work of Task 1 reported in Volume I and IIa. The approach would then be to predict wave load distributions for each of the stationary storm periods based on the two sets of transfer functions or distributions and then to sort the wave loads in parcels as described in Section 5.2.2.

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**Fig. 4:** Distribution of measured horizontal forces from GWK Test Series 04 and 05.

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**5.3. GEOMETRY**
The design diagrams have been developed for a breakwater structure installed on a relatively thin permeable and draining bedding layer or directly on the subsoil assuming an impervious base.

The subsoil is modelled as a layer with thickness D over a very stiff and impermeable layer.

The breakwater is modelled as a 2-D rectangular block, width $B_c$ and height, $h_c$.

### 5.4. SOIL CONDITIONS

The soil conditions under a breakwater may vary from soft clay (Gela) and loose sand to very stiff clay and very dense sand or layers of different types of these materials (Genoa Voltri). In some cases the breakwaters are founded directly on rock (Hanstholm). Degradation of strength and stiffness is mainly connected to sands and clays.

Development of design diagrams requires simplification and selection of a few typical cases. For this study the following cases have been selected.

- a) Stiff overconsolidated (Drammen) clay: OCR 1 and 4
- c) Medium dense sand: $Dr = 0.55$
- d) Very dense silty sand: $Dr = 85 - 95\%$

### 5.5. STRESS CONDITIONS IN THE SUBSOIL

The in situ effective stresses are generated by the submerged weight of the soil. Within the considered layer a constant unit weight is assumed.

The vertical stress due the weight of the structure is assumed to be distributed 1:2 to 1:3 with depth.

The cyclic wave forces and moments at mudline level (i.e. at the base of the structure) are assumed to be symmetric and quasi sinusoidal.

### 5.6. DESIGN DIAGRAMS FOR REDUCTION OF FOUNDATION STIFFNESS USED IN DYNAMIC ANALYSIS
5.6.1. Elastic solutions

The stiffness of the soil is dependent on the soil type, the consolidation stresses, the shear stress level under static and dynamic loads and the load history. Lumped stiffness, mass and damping parameters have been developed and are used in simplified dynamic analysis of structures to assess dynamic amplification. These solutions have primarily been developed for circular, rectangular or strip foundations on an isotropic, elastic half-space or on a finite layer and the soil parameters required are an average or representative G-value and a Poisson’s ratio which for undrained conditions will be very close to 0.5 (incompressible material).

Gazetas (1983) presented a comprehensive state-of-the-art paper on elastic solutions for footings on a finite layer of thickness D. The recommended expressions are summarised in Tab. 5. For undrained loading Poisson's ratio, ν, will be very close to 0.5, i.e. constant volume.

**Tab. 5: Spring stiffness solutions; strip and circle on finite layer (after Gazetas,1983)**

<table>
<thead>
<tr>
<th></th>
<th>Strip on finite layer</th>
<th>Circle on finite layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$K_v = \frac{1.23G}{1-\nu} \left(1 + 1.75 \frac{B}{D}\right)$</td>
<td>$K_v = \frac{1.23GR}{1-\nu} \left(1 + 1.28 \frac{R}{D}\right)$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$K_h = \frac{2.1G}{2-\nu} \left(1 + \frac{B}{D}\right)$</td>
<td>$K_h = \frac{8GR}{2-\nu} \left(1 + \frac{1}{2} \frac{R}{D}\right)$</td>
</tr>
<tr>
<td>Rocking</td>
<td>$K_\theta = \frac{\pi GB^2}{2(1-\nu)} \left(1 + 0.1 \frac{B}{D}\right)$</td>
<td>$K_\theta = \frac{8GBR^3}{3(1-\nu)} \left(1 + \frac{1}{6} \frac{R}{D}\right)$</td>
</tr>
</tbody>
</table>

Rectangles of width B in direction of the load and length L can be approximated by the expressions for a circular foundation by inserting an "equivalent" radius $R_0$. For translational modes the "equivalent radius is obtained by equating the areas of the contact surfaces:

$$R_0 = \sqrt{\frac{BL}{\pi}}$$  \hspace{1cm} (4)

For rocking the "equivalent radius" gives corresponding area moments of inertia:

$$R_0 = \sqrt{\frac{LB^3}{3\pi}}$$  \hspace{1cm} (5)

The consolidation stresses and shear stress level vary with load level and depth and distance from the base. The pore pressure build-up and the shear modulus of the soil will thus vary within the soil volume.
Detailed design will normally require FE-analyses. In order to account for pore pressure increase and stiffness degradation specialised material models have been developed as in NGI’s programs FEAST, INFIDEL and BIFURC.

The design diagrams described in the following were developed to overbridge the gap between the elastic half space spring models and the FE-approach and to assist the designer in the feasibility and the preliminary design phase.

5.6.2. Simplified non-linear spring stiffness approach

In order to approximately account for the variation in cyclic shear stress level and thus G modulus in the soil mass, «trough»- models of the soil as shown in Figures 5, 7 and 14 have been used. The model is described in more detail in NGI (1998b). The soil mass is divided in a finite number of troughs over the depth D. The foundation load is assumed to generate a constant shear strain within each individual trough and the displacement can be calculated by summing up the corresponding shear displacements from depth D to the surface.

The trough shapes are selected to give a stress distribution that on one hand approximates the elastic half space or finite layer solutions and on the other hand includes shapes that are close to the failure surfaces representative for the three load cases; vertical load, horizontal load and moment load.

The constant strain requirement within each trough can be fulfilled in different ways. The simplest solution is to assume constant shear stress and G modulus within each trough. In order to evaluate the effect of stress level this implies also the assumption of constant shear strength.

With the above simplifications and restrictions the "trough" model allows calculation of stiffness and shear displacement within each trough as a function of the stress level. The corresponding surface displacements, and thus the spring stiffness, can be expressed as a function of the load level.

For clay the effect of cyclic loading on the stiffness can be found by evaluation of the degradation within each trough using the strain accumulation method.

For sand the effect of cyclic loading on the stiffness requires calculation of pore pressure generation and dissipation during a storm. The "trough model" requires calculation of an average pore pressure within each trough. Pore pressure generation can be calculated with the pore pressure accumulation method. The simultaneous pore pressure dissipation requires analysis of transient seepage.
5.6.3. **Vertical spring stiffness**

The vertical spring stiffness is not of main interest for breakwaters under predominantly horizontal wave loading. However, the "trough"-model is evaluated here mainly for comparisons with more rigorous elastic solutions for a strip foundation on a finite layer based on analytical solutions and/or FE analyses.

The model for vertical spring stiffness is shown in Fig. 5.

![Fig. 5: Trough model for vertical load](image)

The shear stress, $\tau_i$, the shear strain, $\gamma_i$, and the displacement, $\delta_i$, is expressed as

\[
\tau_i = \frac{F_{vi}}{(N_{\tau} \cdot b_i)} \quad (6)
\]
\[
\gamma_i = \frac{\tau_i}{G_i} \quad (7)
\]
\[
\delta_i = \gamma_i \cdot h_i \quad (8)
\]

where $\tau_i$ = shear stress within trough no. i  
$\gamma_i$ = shear strain within trough no. i  
$\delta_i$ = displacement ($\delta_h = \delta_v$) caused by shear deformation of trough no. i  
$F_{vi}$ = vertical load on trough no. i  
$N_{\tau}$ = shear stress factor (equivalent to bearing capacity factor $N_c$)  
$b_i$ = half width of trough  
$G_i$ = shear modulus  
$h_i$ = thickness of horizontal section of trough
A $N_e$ value of 5.5 has been used for a strip foundation. This is somewhat below the upper bound solution of 6 for this failure surface and above the lower bound solution $5.14 = 2 + \pi$.

![Graph showing the normalised spring stiffness $K_v/G$ for vertically loaded strip on finite layer. Trough solution compared with linear elastic solution.](image)

**Fig. 6:** Normalised spring stiffness $K_v/G$ for vertically loaded strip on finite layer. Trough solution compared with linear elastic solution.

The shear stress, strain and displacements are calculated by means of a spreadsheet. The normalised vertical spring stiffness, $K_v/G$, has been calculated for varying values of $D/B$ as shown in Figure 6 which also shows the corresponding values based on equations from Table 5. The correspondence is acceptable over the total range of $D/B$ values.

### 5.6.4. Horizontal spring stiffness

The trough model for horizontal spring stiffness is shown in Fig. 7.
The shear stress is expressed as

$$\tau_i = \frac{F_h}{(B + N_{\tau} \cdot z_i)}$$  \hspace{1cm} (9)

where
- $\tau_i$ = shear stress within trough no. i
- $F_h$ = horizontal load
- $B$ = width of foundation
- $N_{\tau}$ = shear stress factor (equivalent to bearing capacity factor $N_c$)
- $z_i$ = depth of trough

The same approach as described for the vertical spring stiffness is applied using the same value of $N_{\tau} = 5.5$ is applied to account for the active and passive wedges. In this case the lower bound is 4 neglecting the vertical earth pressure component and 5.66 including vertical earth pressure components. The upper bound is 6.

5.6.4.1. Effect of static stress level

The effect of the static stress level, $\tau/\tau_{max}$, is evaluated applying a hyperbolic relation between stress and strain.

$$G_{sec} = G_{max} \cdot (1 - \tau/\tau_{max})$$ \hspace{1cm} (10)

where
- $G_{sec}$ = secant shear modulus for two way loading
- $G_{max}$ = maximum (small strain shear modulus)
\( \tau \) = shear stress
\( \tau_{\text{max}} \) = shear strength

Assuming undrained conditions during one cycle for clay:

\[
\tau_{\text{max}} = c_u \\
G_{\text{max}} = K c_u
\]

(11) (12)

For sand \( \tau_{\text{max}} \) and \( G_{\text{max}} \) are dependent on the consolidation stress as shown below.

\[
\tau_{\text{max}} = c' + \sigma'_{n,c} \cdot \tan \varphi' \\
G_{\text{max}} = K' \cdot (\sigma'_m)^{0.5}
\]

(13) (14)

To evaluate the effect of the load level reasonable estimates of \( \sigma'_{n,c} \) and \( \sigma'_m \) will have to be established. In the trough model this has been approximated by:

\[
\sigma'_{n,c} = \gamma' \cdot z + F_v / (B + 2 \cdot z) \\
\sigma'_m = 0.6 \cdot \sigma'_{n,c}
\]

(15) (16)

For a horizontally loaded foundation on the surface of a clay layer the load level is defined as the ratio:

\[
F_h/F_{h,\text{ult}} = F_h/B \cdot c_u
\]

(17)

This corresponds to the inverse value of the safety factor against sliding. On sand the load level is defined as:

\[
F_h/F_{h,\text{ult}} = F_h/ (F_v \cdot \tan \varphi')
\]

(18)

The effect of the load level has been evaluated using the above assumptions. The result is shown in Fig. 8 as normalised stiffness \( K/G \) for a strip on a finite clay layer with constant undrained shear strength for a range of \( D/B \) values.

The effect is also shown in Fig. 9 as \( K_h/K_{h,\text{max}} \) vs. load level \( F_h/F_{h,\text{ult}} \) for a range of \( D/B \) values.
Fig. 8: Normalised horizontal spring stiffness \( K_h/G \) vs. \( D/B \) and stress level.

Fig. 9: Reduction in horizontal spring stiffness \( K_h/K_{h,max} \) vs. load level, \( F_h/F_{h,ult} \) for different values of \( D/B \). Clay layer with constant undrained shear strength.

A similar evaluation for a foundation on a sand layer is shown in Fig. 10.
5.6.4.2. Effect of cyclic loading

The degradation of the soil stiffness is primarily a function of the cyclic stress level within the soil mass. The cyclic stress level caused by a cyclic horizontal load drops rather quickly with depth.

For clays the strain accumulation method allows calculation of the effect of cyclic loading on the stiffness. The MCS storm composition has been applied for different levels of maximum shear stress using strain contour data for Drammen clay with OCR = 1 and 4. The resulting reduction in shear modulus for the maximum load at the end of the storm is shown in Figure 11.
Fig. 11: Reduction in shear modulus for maximum wave at end of MCS storm vs. load level. Drammen clay with OCR 1 and 4.

The reduction is presented in Fig. 11 as \( \frac{G_{sec}}{G_{max}} \) vs. stress cyclic stress level \( \frac{\tau_{c,max}}{\tau_{f,cyclic}} \) (\( \tau_{\text{max}} \) = max cyclic shear stress during storm, \( \tau_{f,cyclic} \) = cyclic shear strength). As can be seen the curves for OCR = 1 and OCR = 4 coincides. This curve which is fitted well by

\[
\frac{G_{sec}}{G_{max}} = (1-\frac{\tau_{\text{max}}}{\tau_{f,cyclic}})^{0.75}
\]

is inserted in the trough model for evaluation of cyclic degradation effects on the horizontal stiffness. The result is shown in Figure 12.

Fig. 12: Reduction in horizontal spring stiffness \( \frac{K_h}{K_{h,max}} \) vs. load level, \( \frac{F_h}{F_{h,ult}} \) for different values of D/B. Clay layer with constant shear strength. Cyclic loading with MCS storm.
For sands the pore pressure accumulation method allows calculation of pore pressure generation under cyclic loading. The pore pressure accumulation is then combined with a simultaneous dissipation analysis. The pore pressure distribution vs. depth below the base will vary dependent on the stress level and the consolidation properties of the soil. The storm simulation method may have a considerable influence on the results. Reference is made to Ch. 5.7. A general normalisation is not possible. There are two many parameters involved.

5.6.5. Rotational stiffness

The trough shapes for evaluation of rotational stiffness of a strip foundation on a finite layer are shown in Fig. 13. As can be seen a series of cylindrical segment troughs with centre B/2 above the centre of the foundation simulates one set of shear stress trajectories, while a series of "bearing capacity" troughs simulates the shear stress trajectories connected with the vertical pressure under the edge of the footing. As can be seen the trough-shaped trajectories intersect each other close to 90° over the major part of the soil volume. However, in the active and passive zones to the right and the left of the footing the model shows some inconsistency.

![Trough model for rotational moment](image)

Fig. 13: Trough model for rotational moment

The distribution of the contact stress is non-uniform and has been approximated by a quadratic distribution from the centre to the edge:
\[ q(x) = 32M/B^4 \cdot x^2 \quad -B/2 < x < B/2 \tag{20} \]

where
- \( q(x) \) = contact pressure
- \( M \) = moment
- \( B \) = width of strip
- \( x \) = distance from centre line

This gives a correct moment load and an edge pressure of \( 8 \cdot M/B^2 \)

The shear stress is at the surface of the cylindrical segment troughs can be expressed as

\[ \tau_i = M/(2\alpha_i \cdot R_i^2) \tag{21} \]

where
- \( \tau_i \) = shear stress within trough no. \( i \)
- \( M \) = moment
- \( R_i \) = radius of cylinder segment no. \( i \)
- \( 2\alpha_i \) = opening angle of cylinder segment

The maximum shear stress develops at the cylinder surface having \( R = B\sqrt{2} \) and \( 2\alpha = \pi/2 \), i.e. the cylinder segment touching the edge of the footing.

\[ \tau_{\text{edge,cyl}} = (4/\pi) \cdot M/B^2 \approx 1.27 \cdot M/B^2 \tag{22} \]

The shear stress on the complementary "bearing capacity" troughs are equivalent to the expressions derived for vertical load. The shear stress corresponding to the edge pressure equals

\[ \tau_{\text{edge,bc}} = 8 \cdot M / (B^2 \cdot N_i) \tag{23} \]

with \( N_i = 5.5 \) this gives

\[ \tau_{\text{edge,bc}} = 1.45 \cdot M / B^2 \tag{24} \]

which is very close to the complementary edge shear stress.

The shear stress, strain and displacements are calculated by means of a spread-sheet. The normalised rotational spring stiffness, \( K_r/(G \cdot B^2) \), has been calculated for a strip footing on a clay layer with constant undrained shear strength for varying values of \( D/B \) as shown in Figure 14, which also shows the corresponding values based on the relevant expression from
Table 3. The same approach as for the horizontal spring stiffness has been applied in order to account for the effect of load and stress level.

**Fig. 14:** Normalised rotational spring stiffness vs. D/B and load level for strip footing on clay layer with constant undrained shear strength
Fig. 15: Reduction in rotational spring stiffness $K_r/K_{r,max}$ vs. load level, $F_r/F_{r,ult}$ for different values of $D/B$. Clay layer with constant undrained shear strength.

The effect is also shown in Figure 15 showing $K_r/K_{r,max}$ vs. load level $F_r/F_{r,ult}$ for a range of $D/B$ values.

The trough model gives a realistic model of the failure surface for pure sliding under horizontal loads at the base and pure rotational loading under moment loading. The model is thus capable of modelling elastic stiffness behaviour at low load levels and transforms into a plastic failure model under failure loads. Interaction between moment and horizontal force is not accounted for.

5.7. DESIGN DIAGRAMS FOR RESIDUAL PORE PRESSURE AND DEGRADATION OF STRENGTH

5.7.1. Shear stress distribution

The shear stress distribution may have a considerable impact on the prediction of residual pore pressure. For the "feasibility design phase" a uniform cyclic stress level may be assumed (i.e. $\tau_c/\sigma_{vc}' = \text{const.}$). The diagrams in Figure 17 have been developed with this simplified assumption.

For the "preliminary design phase" a simple shear stress distribution will normally be sufficient. The "trough"-model described in Ch. 5.6 or similar methods may be used.
5.7.2. Residual pore pressure generation in sand

Both the tendency for pore pressure generation and the rate of pore pressure dissipation of the generated excess pore pressure will depend on the sand characteristics (e.g. the grain size distribution, grain shape, relative density, permeability and compressibility of the sand) and the drainage boundary conditions. The generation also depends on the magnitude of the design wave and the design storm composition and the previous loading history.

Sands may be partly drained during a single load cycle. The amount of drainage during a cycle may be estimated from the diagrams in Figures 16.c and .d for saturated soils. If dilative behaviour and negative pore pressures are considered in evaluation of the undrained strength of a dilative sand, the drainage effect will have to be considered carefully. This is especially so if the soil is not saturated.

The pore pressure accumulation method (Andersen, 1976) can be used to determine the generation of residual pore pressure caused by cyclic loading. The method is based on the use of pore pressure contour diagrams. The diagrams should be site specific, but relevant test data from the Database (NGI 1998) may be used.

For the "feasibility phase", diagrams have been developed can be used for estimate of the residual pore pressure at the end of a typical design storm (see Ch. 5.2). These diagrams were developed for a MCS storm (see Ch. 5.2) presented in the MCS report (De Groot & al. 1996) and are reproduced here in Fig. 17. The diagrams were based on calculations with the finite element program BIFURC (NGI, 1995) with soil parameters from symmetrical cyclic loading and uniform $\tau_{cy}/\sigma_{vc}'$ within the soil layer (i.e. the effect of the reduction in $\tau_{cy}/\sigma_{vc}'$ with increasing depth below the base is not considered). The pore pressure in the diagrams is the pore pressure just prior to the maximum wave. The waves have been sorted parcels in order of increasing amplitude. The diagrams cover several of the parameters mentioned above and were based on the pore pressure contour data shown in Fig. 18.
Fig. 16: Diagrams to estimate pore pressure dissipation in sand.
Fig. 17a: Diagrams for preliminary estimate of residual pore pressure at end of storm in medium dense fine sand.
Fig. 17b: Diagrams for preliminary estimate of residual pore pressure at end of storm in very dense, silty sand.
For the "preliminary design phase" the following simplified calculations are suggested:

1. Construct the pore pressure contour diagram based on site specific undrained DSS tests with symmetrical two-way cyclic loading. Eventually complete the diagram with information from similar soils in the Database (NGI, 1998) which contains a considerable amount of pore pressure contour diagrams from tests on different soils with varying densities.

2. Determine the residual pore pressure due to cyclic loading just prior to the maximum wave by performing pore pressure accumulation for the given cyclic load history and the site specific drainage conditions. The maximum wave is not included in this accumulation. The diagrams in Figure 16a and b can be used to estimate pore pressure dissipation.

3. Calculate the effective stresses in the soil mass just prior to the maximum wave load by subtracting the residual pore pressures from the initial in situ effective stresses due to the weight of the soil and the caisson.

4. Use the calculated effective stresses as input to the failure load calculations.

5.7.3. Cyclic degradation of the strength of clay

The undrained shear strength of clay normally refers to the static (monotonic) shear strength from undrained strain controlled tests with about 1 to 2 hours to failure. In design of monolithic coastal structures one should therefore correct this static shear strength for effects of undrained cyclic loading in the design storm and the high rate of loading from the maximum wave.

In the "feasibility design phase" the cyclic effect can be approximately accounted for by the diagram in Figure 19. The input to the diagram is the ratio between the average wave load and the failure load based on monotonic shear strength. The monotonic failure load can be calculated as described in Ch. 5.6.

The correction for cyclic load effects is a function of the clay type and the composition of the design storm. The diagram in Figure 19 is based on Drammen Clay, which is a plastic clay with a plasticity index, $I_p$, of 27%. Further information on this clay type and other clays is compiled in the database. As the MCS storm is considered to be representative for evaluation of cyclic loading effects also in the PROVERBS study there has been no changes and this diagram is the same as presented in the diagram as presented in the MCS report (De Groot & al., 1996).
Fig. 18: Strain contour diagrams for a) medium dense sand and b) Very dense silty sand
In the "preliminary design phase" it is suggested to express the strength of clays by the cyclic shear strength, determined by the strain accumulation procedure. The cyclic strain accumulation procedure and the cyclic shear strength concept is described by Andersen and Lauritzen (1988).

The cyclic strength has the advantage over a reduced static strength that it can include the effect of average load on the cyclic degradation in an approximate manner. A discussion of cyclic strength versus reduced static strength can be found in Andersen & al. (1982).

The cyclic strength can be determined either by pore pressure accumulation or by strain accumulation. In clays there will be no significant amount of pore pressure dissipation during a storm, and the strain accumulation procedure can be used. The strain accumulation procedure is preferable for clays because the pore pressure contours for overconsolidated clays often have shapes which makes pore pressure accumulation difficult.
A simplified way to determine the cyclic shear strength for clays is explained in the following:

1. Construct the cyclic strain contour diagram from site specific undrained DSS laboratory tests with symmetrical (two-way) cyclic loading. Examples of shear strain contour diagrams are shown in Figure 20 and further data can be found in the Database.
2. Perform the strain accumulation for a given cyclic load history. The cyclic load history should be expressed in the form of $\tau_{cy}/c_u$, assuming proportionality between $\tau_{cy}/c_u$ and the wave load. Determine the accumulated cyclic shear strain, $\gamma_c$.
3. Vary the intensity of the storm load history by scaling of $\tau_{cy}/c_u$ and define the locus end points as shown in Figure 21. The intersection of this locus and the 15% strain contour is defined as the normalised cyclic strength of the clay, $\tau_{f, cy}/c_u$, which in the example in Figure 22 is $\tau_{f, cy}/c_u^{DSS} = 0.75$. This normalised shear strength is valid for symmetrical, cyclic DSS loading and the cyclic load history (the MCS storm) described in Table 4.
4. The normalised cyclic shear strength can be used to determine the failure load under cyclic loading in two different ways:

- **Alternative 1** is to first calculate the failure load with monotonic shear strengths as described in Ch. 5.6, and then to correct this calculated failure load for cyclic effects. The correction factor can be established as shown in Figure 22. The normalised cyclic shear strength is plotted in the failure load diagram, and a curve for the site specific soil is extrapolated by means of the Drammen clay curves. This curve defines the factor that can be used to correct the calculated monotonic failure load for cyclic effects. The input to the diagram is the ratio between the average wave load and the failure load calculated based on monotonic strength. For clays with $I_p < 20\%$ one should limit the correction factor from the diagram to 1.0.

- **Alternative 2** is to apply the normalised cyclic shear strength as a correction factor to the monotonic shear strengths prior to performing the failure load analysis. With the information available at this design level the same factor has to be used on the active, DSS and passive monotonic shear strengths. The beneficial effect of the average load (or average shear stress) in Figure 22 is not included in this alternative. For the example shown in Figures 21 and 22 the correction factor is 0.75.

The rate effect is partly accounted for in the cyclic correction above, as the diagram in Figure 22 assumes a cyclic load period of 10 seconds. If the load period deviates from 10 s, the additional rate effect can be corrected for by increasing the calculated cyclic failure load by 10% for each tenfold decrease in the load period. This correction is valid for plastic clays with an $I_p$ of more than 20%. For less plastic clays one should be careful about relying on this additional rate effect. It is therefore recommended to limit the correction factor from the diagram in Figure 25 to 1.0 for clays with $I_p$ less then 20%.
The duration of peak loads caused by wave impacts is short. The resistance of a clay to this type of load can be estimated with the above recommendation. This means a further increase of the cyclic strength in the order of 10 to 20%.

Fig. 20: Strain contour diagrams for Drammen clay, OCR = 1 and 4.
Fig. 21: Example of cyclic strain accumulation and determination of cyclic shear strength for symmetrical DSS loading.

Fig. 22: Diagram to correct monotonic (static) failure load for effect of cyclic loading on clay.
5.8. REFERENCES


6 LIMIT STATE EQUATIONS FOR STABILITY AND DEFORMATION

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6.1 Introduction

The soil beneath vertical breakwaters is subjected to a combination of forces induced by the waves. These forces can be characterised as (1) static load due to the submerged weight of the structure, (2) quasi-static forces induced by cyclic wave loading, and (3) wave impact from breaking waves on the vertical wall. A detailed explanation of the wave induced force is given in volume II a “Hydraulic Aspects”. This chapter describes numerous geotechnical failure modes and corresponding limit state equations for monotonic, and dynamic loading of caisson breakwaters. The limit state equations presented herein are all estimated using the upper bound theory of classical plasticity theory. This approach, to formulate the limit state equations, is suitable for implementation in computational reliability programs.

The behaviour of granular materials can normally be accurately characterised by results from drained tests, since the materials can be considered fully drained in many practical problems. However, if the rate of loading is very high, such as that resulting from impact forces from breaking waves on vertical breakwaters, then essentially undrained conditions can exist. In addition, if the dimension of the breakwater is large and the permeability of the cohesionless soil is relatively low, significant pore-pressure changes may develop as a result of the quasi-static forces induced by cyclic wave loading, as described in Chapters 4 and 5. It will be explained how the limit state equations can be used for (1) quasi-static forces induced by cyclic wave loading, (2) wave impact from breaking waves on the vertical wall.

6.2 General

In the following different failure modes for caisson breakwaters and the corresponding limit state equations will be presented. The failure modes have to be both statically and kinematically admissible. It is, however, difficult to find solutions that fulfil both conditions.
In practice this problem is solved by introducing different calculation tools in the design of large foundation structures, (Sørensen et al. 1993).

Normally the following calculation tools are used:

- Upper Bound Theory
- Limit Equilibrium Analysis
- Finite Element Analysis

Bearing capacities calculated by the Upper Bound Theory will be on the unsafe side of the correct solution compared to the Limit Equilibrium and the Finite Element Analysis. Normally the range between the three methods is very small.

6.2.1 Assumptions and Simplifications

The elaborated limit state equations can be used for pure frictional or cohesive materials. The contribution to the bearing capacity from cohesive layers is therefore expressed by the undrained shear strength \( c_u \) whereas the contribution from layers consisting of friction materials is due to its dilative behaviour, \( \psi \), and the gravity \( \gamma' \) of the displaced soil mass. In traditional bearing capacity analysis this fundamental description of soils corresponds to an expectation of drained and undrained failure in the soil under static loading. When dealing with time varying loads it is, however, necessary to consider the possibility of instantaneous pore pressure build up during repetitive loading (see Section 4.1). For soils with low permeability the undrained condition and generation of pore pressure are evident, whereas for more permeable soils the possibility of undrained or partly undrained behaviour must be evaluated. This possibility of pore pressure build-up can be expressed in terms of the characteristic drainage period \( T_{ES} \) and the period of the wave load (see Section 4.2). The terms drained and undrained subsoil are adopted throughout the presentation of the limit state equations and it is up to the user to evaluate the actual drainage conditions.

A basic principle of the upper bound theory is the assumption of associated flow in the soil during shear. This prerequisite is fulfilled during undrained failure but is indubitably violated during drained failure, as the friction and dilation angle for frictional materials are quite different. This discrepancy is taken into account by use of a reduced effective friction angle \( \phi_d \), see Hansen (1979):

\[
\tan \phi_d = \frac{\sin \phi' \cos \psi}{1 - \sin \phi' \sin \psi}
\]
6.2.2 Foundation Loads

The foundation loads are due to the net weight of the caisson, wave generated horizontal force on the caisson and seepage forces in the rubble mound. The wave generated horizontal force, $F_h$, is estimated using the formulas given in Volume II a “Hydraulic Aspects”. The wave generated seepage forces, is as mentioned in Chapter 4, rather complicated to describe. It is for simplicity assumed that the seepage forces in the rubble mound can be described by one or two vertical components acting on impermeable horizontal boundaries and a horizontal component acting on the rupture boundary. As illustrated in Figure 1 has a simple linear approach been adopted for determination of the forces. As illustrated in the figure will the horizontal force on the rupture boundary and the downward force on the interface between rubble mound and subsoil depend on the height of the rubble mound and failure mode.

The bottom of the caisson forms an impermeable horizontal boundary and is therefore subjected to a seepage generated vertical uplift. The determination of the vertical uplift, $F_U$, is discussed in Chapter 4 and given by equation 2.

$$ F_U = \frac{B}{2} p_u $$

where

- $B$ is the width of the caisson.
- $p_u$ is the uplift pressure at the edge of the caisson.
The horizontal seepage force acting on the rupture boundary is rather complicated to determine as the variation of the pressure along the rupture boundary depends on several mechanical and geometrical quantities. The horizontal seepage force, $F_{HU}$, is for simplicity derived from equation 4, which is based on the approximated model illustrated in Figure 1.

$$F_{HU} = \begin{cases} \frac{B_z^2 - d^2}{2B}p_u \tan \theta & d > 0 \\ \frac{B_z^2}{2B}p_u \tan \theta & d \leq 0 \end{cases}$$

where
- $B_z$ is the effective width of the caisson.
- $d$ is the horizontal distance from the rear edge of the caisson to the point where the failure progresses into the subsoil.
- $\theta$ is the angle between the bottom of the caisson and the rupture boundary.

In cases where failure progresses into an undrained or impermeable subsoil can the seepage pressure generate a driving force on the interface between the rubble mound and subsoil. The force is for simplicity assumed to have the same distribution and magnitude as the uplift pressure. The downward pressure only affects the part of the subsoil that is included in the failure. Based on the principles outlined in Figure 1 it is possible to express the downward force, $F_D$, as in equation 3.

$$F_D = \begin{cases} \frac{d^2}{2B}p_u & d > 0 \\ 0 & d \leq 0 \end{cases}$$

From the figure and formulas above it is noticed that the principle of effective foundation width is adopted for derivation of the limit state functions and determination of seepage forces. This principle is known to be conservative, but it leads to limit state equations that are more applicable to engineering practice.

### 6.3 Limit State Equations

The limit state equations are obtained from three different groups of failure modes expresses the relation between external forces, geometric quantities and soil properties.

For the majority of the equations an optimisation on the geometric quantities is required for determination of the critical loads or bearing capacity. Optimisation parameters and constraints are given in these cases. Furthermore, a description of the mode of operation for each limit state equation is given. As the derived limit state equations are very voluminous only the principles of the limit state equations are stated. A thorough derivation of the limit state equations is found in Annex A.
6.3.1 Failure Mode: Sliding Over Foundation

Limit state equation 1

\[ g = \frac{(F_G - F_U)\omega_{IV} - F_H\omega_{IH}}{G7A} \]

Fig. 2. Sliding between caisson and rubble mound.

The failure progresses as a horizontal displacement of the caisson due to lack of frictional resistance between the caisson and the bedding layer/rubble mound. The limit state equation, which is identical to Coulomb's frictional hypothesis, is given in equation 3.

\[ g = (F_G - F_U)\omega_{IV} - F_H\omega_{IH} = 0 \]  

6.3.1 Failure Mode: Bearing Capacity Failure in Rubble Mound

Limit state equation 2

Fig. 3. Sliding through rubble mound.

Failure occurs as a simple rupture line from the bottom of the caisson and through the rubble mound. The failure is described by the optimisation parameter \( \theta \). As the resistance against failure solely depends on the work from the displaced rubble mound material, the angle \( \theta \) must be within the limits given in 6. It is hereby ensured that a positive work contribution is
produced and that the failure lies within the rubble mound. The derived limit state equation is given in 7, where $\omega_1V$, $\omega_{1H}$ are the vertical and horizontal displacement of the soil mass in region 1. $W_1$ is the work due to the gravity of displaced soil mass in region 1, see Annex A.

$$0 \leq \theta \leq \min \left( \phi_d; \tan^{-1} \left( \frac{h_H}{B_z + a + b} \right) \right)$$

$$(6)$$

$$g = W_1 + (F_G - F_U)\omega_{1V} - (F_H + F_{HU})\omega_{1H} = 0$$

$$(7)$$

**Limit state equation 3**

![Diagram](image)

**Fig. 4:** Sliding in the transition zone between rubble mound and undrained subsoil.

Failure occurs as sliding in the transition zone between the rubble mound and undrained subsoil. As the caisson and region 1 are subjected to a horizontal displacement the resistance against failure is solely due to the shear resistance of the undrained subsoil along the rupture line $l_{BC}$. The angle between the bottom of the caisson and failure line through the rubble mound is given by the friction angle of the rubble mound material. The limit state equation is given in 8.

$$g = W_1 - (F_H + F_{HU})\omega_{1H} = 0$$

$$(8)$$

**Limit state equation 4**

The failure occurs as a combination of line and zone failure in the rubble mound. Region 1 and 3 are displaced as rigid bodies, whereas region 2 consists of a zone failure rotating about point D. The limit state function is given in 9.

$$g = \sum_{i=1}^{3} W_i + (F_G - F_U)\omega_{1V} - (F_H + F_{HU})\omega_{1H} = 0$$

$$(9)$$
Fig. 5. Line and zone failure in rubble mound.

The geometry of the failure is controlled by the 2 free parameter's $\theta$ and $\alpha$ used for optimisation of the mechanism. To ensure that the failure will progress within the rubble mound the constraints given in 10-13 must be imposed.

\[
\theta \geq 0 \\
0 \leq \alpha \leq \theta \\
\beta + \alpha - \theta > 0 \\
A_3 \geq 0
\] (10)  (11)  (12)  (13)

Limit state equation 5 (Constant volume)

Fig. 6. Failure in rubble mound.

Limit state equation 5 is based on an assumption of constant volume conditions in the rubble mound. The limit state equation is therefore only valid if the failure load is reached within a short period of time. Under these circumstances the failure can be described by a single
circular slip line from the bottom of the caisson to a point located on the surface of the rubble mound. The resistance against failure is obtained from the shear resistance along the circular arc from A to B. In cases where point B is located on the inclined part of the rubble mound the displaced soil mass will affect the bearing capacity due to asymmetry and actually precipitate the failure. The limit state function is given in 14

\[ g = \sum_{i=1}^{3} W_i - (F_G - F_U) \left( x_D - \frac{B_z}{2} \right) \beta - (F_H + F_{HU}) y_D \beta = 0 \]  

(14)

The limit state equation must be optimised with respect to the centre of rotation \((x_D, y_D)\). The constraints given in 15-17 must be imposed.

\[ y_D \geq 0 \]  

(15)

\[ \frac{B_z}{2} \leq x_D \leq B_z + a + b \]  

(16)

\[ c \leq h_H \]  

(17)

6.3.1 Failure Mode: Bearing Capacity Failure in Subsoil

Limit state equation 6

**Fig. 7.** Failure in rubble mound and drained subsoil.

Assuming that the rubble mound material has a higher strength than the drained subsoil and that the angle between the failure lines \(l_{AB}\) and \(l_{BC}\) correspond to the difference between the friction angles of the two materials, regions 1 and 2 will be displaced as one rigid body. Region 3 consists of a rupture zone rotating about point F. As the zone failure takes place in both the rubble mound and drained subsoil, the development of the zone is described by the weaker subsoil. Region 4 brings the rupture to the free surface of the subsoil by a rigid body.
displacement. For regions 2 to 4 a possible difference in gravity between rubble mound and subsoil material is taken into account. The limit state function is given in 18.

$$g = \sum_{i=1}^{4} W_i + (F_G - F_U)\omega_{1V} - (F_H + F_{HU})\omega_{1H} = 0$$ \hspace{1cm} (18)$$

The geometry of the failure is controlled by the free parameter $\theta$ used for optimisation of the mechanism. To ensure that the failure will progress within the rubble mound the constraints given in 19-20 must be imposed.

$$\theta \geq \tan^{-1}\left(\frac{h_I}{B_z + a + b}\right)$$ \hspace{1cm} (19)$$

$$\theta_1 \geq \frac{\pi}{2} - \varphi_{d_1}$$ \hspace{1cm} (20)$$

Limit state equation 7

![Diagram](image)

**Fig. 8.** Failure in rubble mound and drained subsoil.

Assuming that the rubble mound material has a higher strength than the drained subsoil and that the angle between the failure lines $l_{AB}$ and $l_{BC}$ corresponds to the difference between the friction angles of the two materials, regions 1 and 2 will be displaced as one rigid body. Region 3 consists of a rupture zone rotating about point F. As the zone failure takes place in both the rubble mound and drained subsoil, the development of the zone is described by the weaker subsoil. Region 5 brings the rupture to the free surface of the subsoil by a rupture zone rotating about the heel of the rubble mound (point G). Region 4, placed between the two rupture zones, moves as a rigid body. For regions 2 to 4 a possible difference in gravity between rubble mound and subsoil material is taken into account. The limit state equation is given in 21.
The geometry of the failure is controlled by the free parameter $\theta$ used for optimisation of the mechanism. To ensure that the failure will progress within the rubble mound the constraints given in 22-23 must be imposed.

\[
\theta \geq \tan^{-1}\left(\frac{h_B}{B_z + a + b}\right)
\]  

(22)

\[
\theta_1 \geq \frac{\pi}{2} - \varphi_d
\]

(23)

Limit state equation 8

Fig. 9. Sliding through rubble mound and failure in undrained subsoil.

The failure in the clayey subsoil is identical to the traditional translation mechanism. To preserve the compatibility between region 1 and 4, the angle between the failure lines $l_{AB}$ and $l_{BC}$ must be identical to the friction angle of the rubble mound material. At failure regions 1, 3 and 4 move as rigid bodies. Region 2, which consists of a rupture zone rotating about the heel of the rubble mound (point F), brings the failure to the free surface of the undrained subsoil. The resistance against failure is in part due to the gravity of the displaced soil in region 4, but also the internal work due to the shear resistance of the undrained subsoil along the line rupture from point B to E and in the rupture zone. The derived limit state equation is given in 23. The limit state equation is optimised with respect to $\theta$. To ensure that the rupture enters the clayey subsoil the constraint given in 24 is imposed.

\[
g = \sum_{i=1}^{5} W_i + (F_G - F_U)\omega_{1V} - (F_H + F_{HU})\omega_{1H} = 0
\]

(21)
\[
\theta \geq \max \left( 0; \tan^{-1}\left( \frac{h}{B_z + a + b} \right) - \phi_{d1} \right) \tag{24}
\]

Limit state equation 9

The failure occurs as overturning of the breakwater. The failure consists of a logarithmic slip line in the rubble mound material and a circular slip line in the undrained subsoil. The mechanism is controlled by the centre of rotation and the corresponding distance to the heel of the rubble mound (point C). The resistance against failure is obtained from the gravity of the displaced soil in region 1 and the shear resistance along the circular arc from B to C.

Fig. 10. Failure in rubble mound and undrained subsoil.

The derived limit state equation is given in 25, whereas the necessary constraints are given in 26-30.

\[
g = \sum_{i=1}^{2} W_i - (F_G - F_U + F_D) \left( x_D - \frac{B_z}{2} \right) \beta - (F_H + F_{HU}) y_D \beta = 0 \tag{25}
\]

\[
y_D \geq 0 \tag{26}
\]

\[
\frac{B_z}{2} \leq x_D \leq B_z + a + b \tag{27}
\]

\[
r_{BD} \cos a = y_D + h_H \tag{28}
\]

\[
a \geq 0 \tag{29}
\]

\[
\theta \geq 0 \tag{30}
\]
Limit state equation 10 (Constant volume)

Limit state equation 10 differs from limit state equation 9 in the description of the failure through the rubble mound. In the present case the failure is presumed to take place under constant volume conditions. For frictional materials this applies that failure loads must be reached within a short period of time, for example impact loading due to breaking waves. Under these circumstances the failure can be described by a single circular slip line from the bottom of the caisson to the heel of the rubble mound (point C).

![Diagram](image)

Fig. 11. Failure in rubble mound and undrained subsoil.

The resistance against failure is obtained from the gravity of the displaced soil in region 1 and the shear resistance along the circular arc from A to C. The derived limit state equation is given in 31, whereas the necessary constraints are identical to those given for limit state equation 9.

\[
g = \sum_{i=1}^{2} W_i - (F_G - F_U + F_D)(x_D - \frac{B_z}{2})\beta - (F_H + F_{HU})y_D\beta = 0
\]

(31)

6.4 Influence of cyclic wave loading

As discussed in Chapter 5, the pore pressure increase depends on the intensity and duration of the cyclic load history and the drainage conditions, i.e. soil permeability, compressibility and drainage distance. If the drainage condition is limited a gradual accumulation of excess pore pressure may develop. This will cause a gradual reduction or increase of the effective stress in the soil. The influence of cyclic wave loading, i.e. the effect of “precycling” and accumulation of excess pore pressure, is taken into account by following the guidance given in Chapter 5. The shear strength depends on the effective stresses in the soil, and thus on the excess pore pressure generated during the storm. The shear strength is further dependent on whether the soil is contractive or dilative during the shearing under the extreme loads and whether this takes place under drained, partly drained or undrained conditions. If the breakwater is not exposed to impact wave loading the increased or reduced strength parameters, due to the influence of cyclic wave loading should be used.
The gravel and the rockfills of the mound will be drained and clay will be undrained during the period of one wave cycle. The stability of the breakwater is evaluated by using limit state equations 1, 2, 3, 4, 8 and 9. Sand may be partly drained during the period of one wave cycle and it is necessary to consider whether the partial drained or the drained shear strength should be used in the stability calculations. The stability of the breakwater is evaluated by using limit state equations 1, 2, 4, 6 and 7. After soil is loaded close to failure and subsequently unloaded, a small permanent deformation remains. Significant permanent deformations remain if several cycles of extreme loading and subsequent unloading occur briefly after each other, so briefly that hardly any drainage takes place during the considered cycles. This phenomenon is usually referred to as “cyclic mobility”, and it results in cyclic residual deformations.

Cyclic mobility may result in unacceptable deformation of the foundation of a vertical breakwater on a subsoil of clay, silt or fine sand (characteristic drainage period $T_{ES}$ larger than approximately 10 times the wave period, see Section 4.2). In design of breakwaters the cyclic residual deformations need to be restricted to an allowable measure. This means that the cyclic stress amplitude has to be limited to such a value that within the expected number of wave loads a certain residual shear strain is not exceeded. The Database (NGI, 1999) contains diagrams that give the cyclic shear strain relative to the average shear stress and shear stress amplitude. Some diagrams are presented in chapter 2 of this volume.

The analysis to restrict cyclic residual strain requires the following steps:

- Select the maximum acceptable residual shear strain
- Perform a sliding plane analysis with the load induced by the weight load and the average between extreme wave load (at wave crest) and the minimum wave load (at wave trough) to calculate the average shear load along potential sliding planes.
- Derive the acceptable cyclic shear amplitude for all parts of each potential sliding plane from the Database graphs, with respect to the selected maximum acceptable residual shear strain.
- Perform sliding plane analysis with the extreme wave load, find the resulting shear stress and compare it with the allowable.

A complete description of the required steps can be found in Andersen and Lauritzen (1988).

6.5 Influence of impact wave loading

In practice undrained failure in frictional materials is of rare occurrence. This failure state must, however, be considered in a design situation where there is a possibility for impact wave loading. Whether drained or undrained failure occurs depends on the grain size distribution and void ratio of the material, but for fine frictional materials as sand undrained failure may actually occur during the short peak loads from wave impacts. The undrained shear strength of the frictional material should under these circumstances be evaluated as described below. For coarse materials as the gravel and rock fill of the mound the occurrence
of an undrained failure is questionable. It could instead be argued that the failure in these materials progresses under constant volume as the duration of the impact is so short that dilation hardly occurs. Under these conditions the shear strength of the gravel and rock fill should be used. Limit state equations 5 and 10 are especially derived for evaluation of constant volume and undrained failure.

6.5.1 Undrained shear strength of frictional materials

![Graph showing undrained shear strength](image)

**Fig. 12.** Results of undrained triaxial test performed on Aalborg University Sand No. 1. The test was performed with a constant deformation rate of 100 % per hour. The specimen was prepared with e=0.55 corresponding to D_R=100%.

It is known from constant volume tests that the tendency to contraction or dilation of the soil skeleton is forcefully resisted by the pore fluid creating a positive or negative increase in the
pore pressure. In the majority of the cases the pore fluids resistance against volume changes will lead to a strengthening of the soil. The phenomenon static liquefaction only occurs for very loose deposits. An example of undrained soil response is given in Figure 12. The figure shows that the pore pressure initially increases to prevent the sand from contraction, $\delta u > 0$. When the effective stress state approaches the characteristic stress state (CS), point A, $\delta u \to 0$. Between point A and B the pore water manages to prevent the sand from dilating and the pore pressure generation is negative, $\delta u < 0$. Up to point B the effective stress path follows the common stress path (CSP) defined by the drained stress states where $\Sigma \delta \varepsilon = 0$. When the pore pressure becomes negative the response changes character. The pore water can no longer fully prevent the dilation and the response starts to deviate from the common stress path. At point C the pore water provides the maximum resistance and cavitation occurs at approximately -90 to -100 kPa, corresponding to the atmospheric pressure. From this state the effective confining pressure remains constant, and further strengthening of the material depends entirely on the dilatation. At point D failure occurs, while the negative pore pressure remains constant and does not vanish between point C and D.

Based on the characteristics given above the undrained shear resistance for frictional materials can easily be determined at a given point beneath the structure. The determination requires knowledge of the effective friction angle, initial effective state of stress corrected for the influence of the cyclic loading ($\sigma_{3i}$, $\tau_{i}$) and the maximum pore pressure that can be generated during shear. The maximum negative pore pressure that can be generated during shear equals the initial hydrostatic pressure $u_{h}$ plus the atmospheric pressure $p_{a}$. The principle of the method is illustrated in Figure 13.

\[\text{Fig. 12. Principle of the determination of the undrained shear strength.}\]
The undrained shear strength of the frictional material is given by the following relation:

$$c_u = 	au_f = \sigma'_{zy} \frac{\sin \phi'}{1 - \sin \phi'} = (\sigma'_{zy} + u_k + p_a) \frac{\sin \phi'}{1 - \sin \phi'}$$

(32)

If the dilating sand and gravel are not fully saturated one should be careful about relying on the higher shear strength due to negative pore pressures.

**6.6 Deformations due to impact wave loading**

During extreme high wave impacts, the loading may exceed the static bearing capacity of the foundation for a short period. This exceeding will cause permanent deformations of the foundation, but due to inertia or dynamic strengthening of the soil the deformations may be small enough to be acceptable. The model presented in this section is developed for evaluation of such permanent deformations. The model is based on the upper bound theory of classical plasticity theory and settles with the classical elastic methods. This change of approach is necessary, as the elastic method is inadequate in cases where the impact load is so significant that it doubtless will cause permanent plastic deformations of the foundation.

Experience shows that the soil under impact loading normally will produce a reaction, which exceeds the static capacity. This additional capacity emanates from inertia forces, geometrical changes and an additional strengthening of the soil due to the dynamic loading. The model takes these contributions into account and allows for impact loading that exceeds the static bearing capacity.

The principle of the model is given in the succeeding sections and an example of the application is given in Annex B.

**6.6.1 Principle of the Dynamic Model**

As described above the dynamic model is based on the upper bound theory of classical plasticity theory. Example on the use of this theory was given in Section 6.3 for various failure modes and corresponding limit state equations for caisson breakwaters subjected to static or quasi-static loading.

The first step in the dynamic analysis includes a determination of the critical limit state equation under static loading. The knowledge gained from this analysis, e.g. bearing capacity, displacement field and geometrical quantities describing the failure, forms basis of the further investigation.

As the foundation is subjected to a dynamic impulse, I(t), that exceeds the static bearing capacity, F, a failure state is reached and the structure and the subsoil will be accelerated and displaced. The deformation of the system will progress until the dynamic contributions at a given time counteract the dynamic impulse. As the model is based on the assumption of rigid
plasticity the deformation stops when the deformation velocity reaches zero and the permanent deformation is directly obtained.

6.6.2 Dynamic Impulse

The description of the dynamic impulse or dynamic load history is in general attended with great difficulties due to the highly stochastic processes involved. For impact wave loading the deficiency is mainly due to the large number of influencing parameters related to the geometry of the structure and foreshore, as well as the effect of entrapped air in breaking waves. Results, however, show that for waves breaking on the structure the load history contains one or two peaks followed by rapid decay to the quasi-static load level, Oumeraci and Kortenhaus (1995). These characteristics have lead to two simple descriptions of the impulse given in Figure 14.

Fig. 13. a) Triangular load history, Oumeraci and Kortenhaus (1995). b) Exponential load history.

It may be argued that the descriptions are oversimplified, but with the difficulties attached to the characterisation of the impulse and the fact that the present dynamic model only accounts for loads that exceed the static bearing capacity of the foundation the procedures are found reasonable. The mathematical formulation for the two load histories is given in equation 33. The triangular load history is given by the peak force $Q$, the rise time $t_r$ and the decay time $t_d$.

$$I(t) = \begin{cases} 
Q_d \frac{t}{t_r}, & 0 \leq t < t_d \\
Q_d \left(1 - \frac{t - t_r}{t_d - t_r}\right), & t_r \leq t < t_d \\
0, & t_d \leq t
\end{cases}$$

(33)

In cases where the rise time is very short it is found convenient to describe the load history by a simple exponential function given by the peak force and a constant of decay $k$.

$$I(t) = Q_d e^{-kt}$$

(34)
For use with the dynamic model it is convenient to express the magnitude of the impact relative to the static bearing capacity by an overloading factor $S$:

$$
S = \frac{Q_d}{F}
$$

(35)

A more thorough description is found in Annex B.

6.6.3 Dynamic Contributions

The necessary information about displacements and accelerated masses is fully deduced from the static analysis in form of predefined displacement diagrams and geometrical quantities.

Inertia forces due to acceleration of the structure and the soil mass will, in accordance with Newton’s 2 law, contribute to the system’s resistance against deflection. The inertia of the system is found by summing up the inertia from structure and the different failure regions:

$$
\Delta F^m(\dot{\omega}) = \sum_{i=1}^{i_{in}} \dot{\omega}_i A_i \rho_i
$$

(36)

Whenever the bearing capacity is exceeded soil will be pushed up at the side of the foundation and increase the stabilising earth pressure. The displacement of the soil will, however, at the same time lead to a reduction of the shear resistance due to a shortening of the failure lines. This effect of geometrical changes can be expressed as:

$$
\Delta F^k(\omega) = \gamma \Delta A(\omega) - \int_0^{\omega} c_u(s) ds
$$

(37)

The effect of the geometrical changes will either lead to a strengthening of the foundation or precipitate failure. Thus the effect must be evaluated in each case.

Besides these contributions due to inertia and geometrical changes the displacement velocity might affect the strength of the soil. Several tests have been performed in order to investigate the effect of deformation rates on the strength of both cohesive and frictional soils. For frictional materials the strength seems to be independent of the displacement velocity (Ibsen, 1995), whereas the strength of cohesive soils is found to increase with increasing displacement velocity. This partially viscous behaviour is described by Bjerrum (1971) and by Kulhawy and Mayne (1990) and may be expressed as:

$$
c_u^d(\dot{\omega}) = c_u + \kappa c_u \log \dot{\omega} \approx c_u + \Delta c_u^d \dot{\omega}
$$

(38)

The effect on the failure mechanisms shear resistance can be formulated as in 39 by summing up contributions from the different failure lines and regions:
6.6.4 Equation of Motion

The dynamic terms given in 36, 37 and 39 together with the static bearing capacity and the dynamic impulse yield the equation of motion with one degree of freedom:

\[ I(t) = F + \Delta F^m(\dot{\omega}) + \Delta F^c(\omega) + \Delta F^k(\omega) \]  

(40)

The permanent deformation of the system can be determined by solving the equation of motion and following the principles outlined in Section 6.6.1. The solution to the equation of motion depends on the actual failure mode and the dynamic contributions. An example of the application of the model is given in Annex B.

6.7 References


6.8 Symbols

The list only includes the most common symbols. Additional quantities are explained in the text or will appear from the figures.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[m²]</td>
<td>Area</td>
</tr>
<tr>
<td>B</td>
<td>[m]</td>
<td>Foundation width</td>
</tr>
<tr>
<td>Bz</td>
<td>[m]</td>
<td>Effective foundation width</td>
</tr>
<tr>
<td>cu</td>
<td>[kPa]</td>
<td>Undrained shear strength</td>
</tr>
</tbody>
</table>
F [kN/m]: Force
F_G [kN/m]: Force due to gravity
F_U [kN/m]: Force due to upward seepage pressure
F_D [kN/m]: Force due to downward seepage pressure
F_H [kN/m]: Horizontal force
F_HU [kN/m]: Horizontal component of wave induced pressure
H [m]: Height of structure
J [kgm]: Mass moment of inertia
M [kNm/m]: Moment
p_u [kPa]: Upward pressure
p_d [kPa]: Downward pressure
p_a [kPa]: Atmospheric pressure
Q_d [kN/m]: Magnitude of dynamic pulse
u [kPa]: Pore pressure
u_h [kPa]: Hydrostatic pore pressure
S [-]: Dynamic overloading factor
W [kNm/m]: External and internal work
x [m]: Horizontal distance
y [m]: Vertical distance
δ [m]: Unit displacement
β [°]: Angle of rotation
φ [°]: Friction angle
φ_d [°]: Reduced friction angle
ψ [°]: Angle of dilatation
γ [kN/m³]: Specific weight
ω [m]: Displacement
ρ [kg/m³]: Density
σ_3 [kPa]: Confining pressure, horizontal stress
τ [kPa]: Shear stress, shear strength

Subscripts
i : Summational parts, layer, initial state or region
G : Parameter relative to centre of rotation or centre of gravity
f : Failure
H : Horizontal
v : Vertical
AB : Paired subscripts in capital letters are used for the length between adjacent points (e.g. point A and B)

Superscripts
' : Denotes effective parameter
k : Dynamic contribution due to geometric changes
d : Dynamic property (strengthening due to dynamic loading)
m : Dynamic contribution due to inertia
ANNEX A

FAILURE MODES - LIMIT STATE EQUATIONS FOR STABILITY

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A.1 Introduction

The present appendix contains the derivation of ten different limit state equations divided on three different failure modes. Five of the limit state equations can be used independently of the characteristics of the subsoil, whereas the remaining five can be used for either drained or undrained subsoils. The presumed geotechnical conditions appear from the illustrations. The used theory and its limitations is described in Sections 6.2 and 6.3 and the notation appears from the list of symbols in Section 6.8.

A.2 Failure Mode: Sliding Over Foundation

A.2.1 Limit state equation 1

\[
\frac{B_x}{2} \quad \frac{F_n - F_v}{\text{Bedding Layer}} \quad \text{Rubble Mound}
\]

Fig. 1: Sliding between caisson and rubble mound.
The displacement field for the slip failure plane is given in Figure 2. A unit displacement, $\delta=1$, is imposed along the interface between caisson and rubble mound.

Fig. 2: Displacement field.

The directional displacements become:

$$\omega_1 = \frac{1}{\cos \phi_{d_1}}$$  \hspace{1cm} (1)

$$\omega_{1V} = \omega_1 \sin \phi_{d_1} = \frac{\sin \phi_{d_1}}{\cos \phi_{d_1}}$$  \hspace{1cm} (2)

$$\omega_{1H} = 1$$  \hspace{1cm} (3)

Limit state equation 1:

$$g = (F_G - F_U)\omega_{1V} - F_H\omega_{1H} = 0$$  \hspace{1cm} (4)

A.3 Failure Mode: Bearing Capacity Failure in Rubble Mound

A.3.1 Limit state equation 2

Fig. 3: Sliding through rubble mound.

The displacement field for region 1 is given in Figure 4. A unit displacement, $\delta=1$, is imposed along failure line $l_{AB}$. 

- 2 -
Directional displacements for region 1:

\[ \omega_1 = \frac{1}{\cos \varphi_{d_1}} \]  \hspace{1cm} (5)

\[ \omega_{1H} = \omega_1 \cos(\theta - \varphi_{d_1}) = \frac{\cos(\varphi_{d_1} - \theta)}{\cos \varphi_{d_1}} \]  \hspace{1cm} (6)

\[ \omega_{1V} = \omega_1 \sin(\theta - \varphi_{d_1}) = \frac{\sin(\varphi_{d_1} - \theta)}{\cos \varphi_{d_1}} \]  \hspace{1cm} (7)

Area of region 1:

\[ A_1 = \frac{1}{2}(B_z + a)^2 \left( \cos \theta \sin \theta + \sin^2 \theta \tan \left( \frac{\pi}{2} + \theta - \tan^{-1} \left( \frac{h_H}{b} \right) \right) \right) \]  \hspace{1cm} (8)

Work due to the gravity of displaced soil mass in region 1:

\[ W_1 = \gamma_1 A_1 \omega_{1V} \]  \hspace{1cm} (9)

Limit state equation 2:

\[ g = W_1 + (F_G - F_u) \omega_{1V} - (F_H + F_{HU}) \omega_{1H} = 0 \]  \hspace{1cm} (10)

The limit state equation must be optimised with respect to \( \theta \). To ensure that the rupture line lies within the rubble mound, the following constraint must be fulfilled:

\[ 0 \leq \theta \leq \min \left( \varphi_{d_1}; \tan^{-1} \left( \frac{h_H}{B_z + a + b} \right) \right) \]  \hspace{1cm} (11)
A.3.2 Limit state equation 3

Fig. 5: Sliding in the transition zone between rubble mound and undrained subsoil.

A unit displacement, \( \delta = 1 \), is imposed along failure line \( l_{BC} \) and the directional displacements become:

\[
\omega_1 = \omega_{1H} = 1 \quad (12)
\]

\[
\omega_{1V} = 0 \quad (13)
\]

Length of failure line \( l_{BC} \):

\[
l_{BC} = B_z + a + b - \frac{h_I}{\tan \phi_{d_1}} \quad (14)
\]

Internal work from rupture along \( l_{BC} \):

\[
W_1 = \omega_{1H}l_{BC}c_{u_2} \quad (15)
\]

Limit state equation 3:

\[
g = W_1 - (F_H + F_{HU})\omega_{1H} = 0 \quad (16)
\]
A.3.3 Limit state equation 4

Fig. 6: Line and zone failure in rubble mound.

The failure is described by the geometrical quantities in equations 17-25. A detailed geometry of region 3 is given in Figure 7.

\[ r_{BD} = \frac{B_z \sin \theta}{\cos \varphi_{d_i}} \]  \hspace{1cm} (17)

\[ r_{CD} = r_{BD} e^{\alpha \tan \varphi_{d_i}} \]  \hspace{1cm} (18)

\[ l_{AB} = \frac{B_z \cos(\theta - \varphi_{d_i})}{\cos \varphi_{d_i}} \]  \hspace{1cm} (19)

\[ \zeta = \frac{\pi}{2} - \varphi_{d_i} - a + \theta \]  \hspace{1cm} (20)

\[ \beta = \tan^{-1}(\frac{h_I}{b}) \]  \hspace{1cm} (21)

\[ \Delta a = \frac{r_{CD} \sin(\zeta - \beta)}{\sin \beta} \]  \hspace{1cm} (22)

\[ l_{CE} = \frac{(a + \Delta a) \sin \beta}{\sin(\beta + a - \theta)} \]  \hspace{1cm} (23)

\[ \Delta r_{CD} = \frac{a \sin(\theta - a)}{\cos \varphi_{d_i}} \]  \hspace{1cm} (24)
Fig. 7: Detailed geometry of region 3.

The displacement field for regions 1 to 3 is given in Figure 8. A unit displacement, $\delta=1$, is imposed along failure line $l_{AB}$.

The directional displacements become:

$$\omega_1 = \frac{1}{\cos \phi_{d_1}}$$  \hspace{1cm} (26)

$$\omega_{1V} = \frac{\sin(\phi_{d_1} - \theta)}{\cos \phi_{d_1}}$$  \hspace{1cm} (27)

$$\omega_{1H} = \frac{\cos(\phi_{d_1} - \theta)}{\cos \phi_{d_1}}$$  \hspace{1cm} (28)

$$\omega_{2V}(r, \tau) = \omega_1 e^{\alpha \tan \phi_{d_1}} \sin(\phi_{d_1} - \theta + \tau) \quad \tau \in [0, a]; \quad r \in [0, r_{CD}]$$  \hspace{1cm} (29)

$$\omega_3 = \omega_1 e^{\alpha \tan \phi_{d_1}}$$  \hspace{1cm} (30)
\[ \omega_{3V} = \omega_3 \sin(\phi_{d_1} + a - \theta) = \frac{\sin(\phi_{d_1} + a - \theta)}{\cos \phi_{d_1}} e^{a \tan \phi_{d_1}} \]  

(31)

Area of region 1:

\[ A_1 = \frac{1}{2} r_{BC} l_{AB} \sin(\frac{\pi}{2} - \phi_{d_1}) \]  

(32)

Work due to the gravity of displaced soil mass in region 1:

\[ W_1 = A_1 \gamma'_1 \omega_{1V} \]  

(33)

Work due to gravity of displaced soil mass in region 2:

\[ W_2 = \gamma'_1 \int_0^{r_{CD}} \int_0^a \omega_{2V}(r, \tau) r dr d\tau \]  

(34)

\[ = \gamma'_1 \omega_1 \frac{r_{CD}^2 e^{a \tan \phi_{d_1}} (\tan \phi_{d_1} \sin(\phi_{d_1} - \theta + a) - \cos(\phi_{d_1} - \theta + a))}{2 \tan^2 \phi_{d_1} + 2} \]  

\[ - \gamma'_1 \omega_1 \frac{r_{CD}^2 (\tan \phi_{d_1} \sin(\phi_{d_1} - \theta) - \cos(\phi_{d_1} - \theta))}{2 \tan^2 \phi_{d_1} + 2} \]

Area of region 3:

\[ A_3 = \frac{1}{2} a r_{CD} \sin \zeta + \frac{1}{2} l_{CE} l_{EF} \sin(\beta + a - \theta) \]  

(35)

Work due to gravity of displaced soil mass in region 3:

\[ W_3 = A_3 \gamma'_1 \omega_{3V} \]

Limit state equation 4:

\[ g = \sum_{i=1}^{3} W_i + (F_G - F_U) \omega_{1V} - (F_H + F_{HU}) \omega_{1H} = 0 \]  

(36)

The limit state equation must be optimised with respect to \( \theta \) and \( \alpha \). Introductions of the following constraints ensure that the correct optimum is obtained:

\[ \theta \geq 0 \]  

(37)

\[ 0 \leq a \leq \theta \]  

(38)

\[ \beta + a - \theta > 0 \]  

(39)
A.3.4 Limit state equation 5 (Constant volume)

Fig. 9: Failure in rubble mound.

The limit state equation is obtained by a unit rotation, \( \beta = 1 \), about point D. The radius of the circular failure line is given by the location of the centre of rotation:

\[
r_{AD} = \sqrt{x_D^2 + y_D^2}
\]  

The location of point B is given by the solution to the system of equations given in 42. The system describes the inclination of the rubble mound and the circular failure line relative to the centre of rotation:

\[
\begin{cases}
  y = -y_0 + sx, & y_0 = \frac{y_D}{s} - (B_z + a - x_D) \\
  x^2 + y^2 = r_{AD}^2
\end{cases}
\]  

The solution yields:

\[
x_1 = \frac{2y_0s + \sqrt{(2y_0s)^2 - 4(1 + s^2)(y_0^2 - r_{AD}^2)}}{2(1 + s^2)}
\]  

\[
c = x_1 - y_D
\]  

\[
y_1 = -y_0 + sx_1
\]
\[ a = \tan^{-1}\left(\frac{y_1}{x_1}\right) + \tan^{-1}\left(\frac{x_D}{y_D}\right) \] (46)

If point B is located on the inclined part of the rubble mound, that is \( c > 0 \), the displaced soil mass will affect the bearing capacity due to asymmetry. Thus the centre of gravity of the displaced soil mass is located to the left of the centre of rotation, \( D \), and the soil mass will precipitate the failure. This effect is numerically equivalent to the work contributions from the “missing” soil mass in regions 1-2 as illustrated in Figure 10.

**Fig. 10:** Detailed geometry of regions 1 and 2.

Area of region 1:

\[ A_1 = \frac{1}{2} cb_1 \] (47)

Position of centre of gravity for region 1 relative to point D:

\[ R_1 = \sqrt{\left(y_1 - \frac{b_1}{3}\right)^2 + \left(\frac{y_D}{3} + c\right)^2} \] (48)

\[ \theta_1 = \tan^{-1}\left(\frac{3y_1 - b_1}{3y_D + c}\right) \] (49)

Work due to the gravity of displaced soil mass in region 1:

\[ W_1 = -\beta A_1 \gamma' R_1 \sin \theta_1 \] (50)

Area of region 2 (approximately):

\[ A_2 = \frac{1}{2} cb_2 \] (51)
Position of centre of gravity for region 2 relative to point D (approximately):

\[ R_2 = \sqrt{\left( y_1 + \frac{b_2}{3} \right)^2 + \left( y_D + \frac{c}{3} \right)^2} \]  

(52)

\[ \theta_2 = \tan^{-1}\left( \frac{3y_1 + b_2}{3y_D + c} \right) \]  

(53)

Work due to the gravity of displaced soil mass in region 2:

\[ W_2 = -\beta A_2 \gamma' R_2 \sin \theta_2 \]  

(54)

Internal work from rupture along \( l_{AB} \) due to shear strength of rubble mound material:

\[ W_3 = \beta r_{AD} \int_0^{ar_{AD}} \tau_i(s) ds \]  

(55)

Limit state equation 5:

\[ g = \sum_{i=1}^{3} W_i - (F_G - F_U) \left( x_D - \frac{B_z}{2} \right) \beta - (F_H + F_{HU}) y_D \beta = 0 \]  

(56)

The limit state equation must be optimised with respect to the centre of rotation \( (x_D, y_D) \). The following constraints must be imposed:

\[ y_D \geq 0 \]  

(57)

\[ \frac{B_z}{2} \leq x_D \leq B_z + a + b \]  

(58)

\[ c \leq h_H \]  

(59)
A.4 Failure Mode: Bearing Capacity Failure in Subsoil

A.4.1 Limit state equation 6

Fig. 11: Failure in rubble mound and drained subsoil.

The failure is given by the geometrical quantities in equations 60-75. A detailed geometry of regions 4 to 5 is given in Figure 12.

\[ l_{AB} = \frac{h_H}{\sin \theta} \] (60)

\[ \theta_0 = \tan^{-1} \left( \frac{h_H}{B_L - \sqrt{l_{AB}^2 - h_H^2}} \right) \] (61)

\[ \theta_1 = \pi - \theta - \theta_0 \] (62)

\[ \theta_2 = \theta + \theta_0 - \varphi_{d_1} + \varphi_{d_2} \] (63)

\[ \theta_3 = \frac{\pi}{4} - \frac{\varphi_{d_1}}{2} \] (64)

\[ \theta_4 = \theta_1 + \varphi_{d_1} - \frac{\pi}{2} \] (65)

\[ \theta_5 = \frac{\pi}{4} - \frac{\varphi_{d_1}}{2} + \theta + \theta_0 \] (66)
\[
\theta_6 = \tan^{-1}\left(\frac{h_{II}}{a + b}\right)
\]

(67)

\[
l_{BF} = \frac{\sin \theta - B_z}{\sin \theta_1}
\]

(68)

\[
l_{BC'} = B_z - h_{II} \cot \theta + \tan\left(\theta_0 + \theta_4 - \frac{\pi}{2}\right) h_{II}
\]

(69)

\[
r_{CF} = \frac{l_{BF} \sin \theta_2}{\cos \varphi_{d_2}}
\]

(70)

\[
r_{DF} = r_{CF} \tan \theta_{d_1}
\]

(71)

\[
l_{DF} = \frac{h_{II}}{\cos\left(\frac{\pi}{4} + \frac{\varphi_{d_2}}{2}\right)}
\]

(72)

\[
l_{DD} = r_{DF} - l_{DF}
\]

(73)

\[
l_{DF} = l_{DD'} \frac{\cos \varphi_{d_2}}{\sin\left(\frac{\pi}{4} - \frac{\varphi_{d_2}}{2}\right)}
\]

(74)

\[
l_{FH} = \sqrt{(a + b)^2 + h_{II}^2}
\]

(75)

Fig: 12: Detailed geometry of regions 2 and 4.

The displacement field for region 1 and regions 2 to 4 is given in Figure 13. A unit displacement, \(\delta = 1\), is imposed along failure line \(l_{AB}\).
Fig. 13: Displacement diagram for regions 1 to 4.

The directional displacements become:

\[
\omega_1 = \frac{1}{\cos \varphi_{d_1}} \tag{76}
\]

\[
\omega_{1V} = \frac{\sin(\varphi_{d_1} - \theta)}{\cos \varphi_{d_1}} \tag{77}
\]

\[
\omega_{1H} = \frac{\cos(\varphi_{d_1} - \theta)}{\cos \varphi_{d_1}} \tag{78}
\]

\[
\omega_{2V} = \omega_{1V} \tag{79}
\]

\[
\omega_{3V}(r, \tau) = \omega_1 e^{\tau \tan \varphi_{d_2}} \sin(\varphi_{d_1} - \theta + \tau) \quad \tau \in [0, \theta_5]; \quad r \in [0, r_{DF}] \tag{80}
\]

\[
\omega_4 = \omega_1 e^{\theta_5 \tan \varphi_{d_2}} \tag{81}
\]

\[
\omega_{4V} = \omega_4 \sin(\varphi_{d_1} + \theta_5 - \theta) = \frac{\sin(\varphi_{d_1} + \theta_5 - \theta)}{\cos \varphi_{d_1}} e^{\theta_5 \tan \varphi_{d_2}} \tag{82}
\]

Area of region 1:

\[
A_1 = \frac{1}{2} B \cdot h_{II} \tag{83}
\]

Work due to the gravity of displaced soil mass in region 1:

\[
W_1 = A_1 \gamma' \omega_{1V} \tag{84}
\]
Area of region 2:

\[ A_2^I = \frac{1}{2} l_{BC} h_{II} \]  

\[ A_2^H = \frac{1}{2} (l_{BF} r_{CF} \sin \theta_4 - l_{BC} h_{II}) \]  

Work due to the gravity of displaced soil mass in region 2:

\[ W_2 = (A_2^I \gamma_4 + A_2^H \gamma_2') \omega_2 V \]  

Work due to the gravity of displaced soil mass in region 3:

\[ W_3 = \gamma_1' \int_0^\theta_1 \int_0^{\theta_5} \omega_3 V(r, \tau)rdrd\tau + \gamma_2' \int_{l_{DF}}^{r_D} \int_0^{\theta_5} \omega_3 V(r, \tau)rdrd\tau \]

\[ = \gamma_1' \omega_1 \frac{l_{DF}^2 e^{\theta_5 \tan \varphi_{d_2}} (\tan \varphi_{d_2} \sin (\varphi_{d_2} - \theta + \theta_5) - \cos (\varphi_{d_2} - \theta + \theta_5))}{2 \tan^2 \varphi_{d_2} + 2} \]

\[ - \gamma_1' \omega_1 \frac{l_{DF}^2 (\tan \varphi_{d_2} \sin (\varphi_{d_2} - \theta) - \cos (\varphi_{d_2} - \theta))}{2 \tan^2 \varphi_{d_2} + 2} \]

\[ + \gamma_2' \omega_1 \frac{(r_D^2 - l_{DF}^2) e^{\theta_5 \tan \varphi_{d_2}} (\tan \varphi_{d_2} \sin (\varphi_{d_2} - \theta + \theta_5) - \cos (\varphi_{d_2} - \theta + \theta_5))}{2 \tan^2 \varphi_{d_2} + 2} \]

\[ - \gamma_2' \omega_1 \frac{(r_D^2 - l_{DF}^2) (\tan \varphi_{d_2} \sin (\varphi_{d_2} - \theta) - \cos (\varphi_{d_2} - \theta))}{2 \tan^2 \varphi_{d_2} + 2} \]

Area of region 4:

\[ A_4^I = \frac{1}{2} l_{FH} a \sin \theta_6 + \frac{1}{2} l_{FH} l_{DF} \sin (\theta_3 - \theta_6) \]  

\[ A_4^H = \frac{1}{2} l_{DF} l_{DD'} \sin \left( \frac{\pi}{4} - \frac{\varphi_{d_2}}{2} \right) \]
Work due to the gravity of displaced soil mass in region 4:

\[ W_4 = (A_{41}' + A_{42}'' + A_{43}''') \omega_{4V} \]  

(91)

Limit state equation 6:

\[ g = \sum_{i=1}^{4} W_i + (F_G - F_U) \omega_{1V} - (F_H + F_{HU}) \omega_{1H} = 0 \]  

(92)

The limit state equation must be optimised with respect to \( \theta \). To ensure that the rupture enters the subsoil the following constraints must be imposed:

\[ \theta \geq \tan^{-1} \left( \frac{h_H}{B_z + a + b} \right) \]  

(93)

\[ \theta_1 \geq \frac{\pi}{2} - \varphi_{d_1} \]  

(94)

A.4.2 Limit state equation 7

Fig. 14: Failure in rubble mound and drained subsoil.

The failure is given by the geometrical quantities in equations 95-116. A detailed geometry of regions 4 and 5 is given in Figure 15.
Fig. 15: Detailed geometry of regions 2, 4 and 5.

\[ l_{AB} = \frac{h_{II}}{\sin \theta} \]  
(95)

\[ \theta_0 = \tan^{-1} \left( \frac{h_{II}}{B_z - \sqrt{l_{AB}^2 - h_{II}^2}} \right) \]  
(96)

\[ \theta_1 = \pi - \theta - \theta_0 \]  
(97)

\[ \theta_2 = \theta + \theta_0 - \varphi_{d_1} + \varphi_{d_2} \]  
(98)

\[ \theta_3 = \frac{\pi}{4} - \frac{\varphi_{d_1}}{2} \]  
(99)

\[ \theta_4 = \theta_1 + \varphi_{d_1} - \frac{\pi}{2} \]  
(100)

\[ \theta_5 = \frac{\pi}{4} - \frac{\varphi_{d_1}}{2} + \theta + \theta_0 \]  
(101)

\[ \theta_6 = \tan^{-1} \left( \frac{h_{II}}{a + b} \right) \]  
(102)

\[ \theta_7 = \theta_3 \]  
(103)
\[ \theta_8 = \frac{\pi}{4} + \frac{\varphi_d}{2} - \varphi_d \]  
\[ l_{BI} = \frac{\sin \theta}{\sin \theta_1} B_z \]  
\[ l_{BC'} = \frac{\sin \theta_4}{\sin \left(\frac{\pi}{2} - (\varphi_d + \varphi_d)\right)} l_{BI} \]  
\[ r_{CI} = l_{BI} \frac{\sin \theta_2}{\sin \left(\frac{\pi}{2} - \varphi_d\right)} \]  
\[ r_{DI} = r_{CI} \varphi_3 \tan \varphi_d \]  
\[ l_{D'I} = \frac{h_{II}}{\sin \theta_3} \]  
\[ l_{DD'} = r_{DI} - l_{D'I} \]  
\[ l_{D'F'} = l_{DD'} \frac{\sin \left(\frac{\pi}{2} + \varphi_d\right)}{\sin \theta_8} \]  
\[ l_{GI} = \sqrt{h_{II}^2 + (a + b)^2} \]  
\[ l_{D'R} = l_{GI} \frac{\sin(\theta_3 - \theta_3)}{\sin \theta_3} \]  
\[ l_{GP'} = l_{D'F'} - l_{D'R} \]
\[ l_{GE} = l_{GF} \frac{\sin \theta_8}{\sin \left( \frac{\pi}{2} + \varphi_{d2} \right)} \] (115)

\[ r_{GF} = l_{GE} e^{\varphi_1 \tan \varphi_{d2}} \] (116)

The displacement fields for regions 1 to 4 is given in Figure 16. A unit displacement, \( \delta = 1 \), is imposed along failure line \( l_{AB} \).

**Fig. 16:** Displacement diagrams for regions 1 to 4.

The directional displacements become:

\[ \omega_1 = \frac{1}{\cos \varphi_{d1}} \] (117)

\[ \omega_{1V} = \frac{\sin(\varphi_{d1} - \theta)}{\cos \varphi_{d1}} \] (118)

\[ \omega_{1H} = \frac{\cos(\varphi_{d1} - \theta)}{\cos \varphi_{d1}} \] (119)

\[ \omega_{2V} = \omega_{1V} \] (120)

\[ \omega_{3V}(r, \tau) = \omega_1 e^{\tau \tan \varphi_{d2}} \sin(\varphi_{d1} - \theta + \tau) \quad \tau \in [0, \theta_4]; \quad r \in [0, r_{DI}] \] (121)

\[ \omega_4 = \omega_1 e^{\varphi_1 \tan \varphi_{d2}} \] (122)

\[ \omega_{4V} = \omega_4 \sin(\varphi_{d1} + \theta_5 - \theta) = \frac{\sin(\varphi_{d1} + \theta_5 - \theta)}{\cos \varphi_{d1}} e^{\theta_5 \tan \varphi_{d2}} \] (123)
\[ \omega_{SV}(r, \tau) = \omega_4 e^{\tau \tan \varphi_d} \sin(\varphi_{d1} - \theta + \varphi_5 + \tau) \quad \tau \in [0, \theta_1]; \quad r \in [0, r_{GF}] \]  

(124)

Area of region 1:

\[ A_1 = \frac{1}{2} B_z h_{II} \]  

(125)

Work due to the gravity of displaced soil mass in region 1:

\[ W_1 = A_1 \gamma_1 \omega_{1V} \]  

(126)

Area of region 2:

\[ A_2^I = \frac{1}{2} l_{BC} h_{II} \]  

(127)

\[ A_2^II = \frac{1}{2} (l_{BR} - l_{BC} h_{II}) \]  

(128)

Work due to the gravity of displaced soil mass in region 2:

\[ W_2 = (A_2^I \gamma_1 + A_2^II \gamma_2) \omega_{2V} \]  

(129)

Work due to the gravity of displaced soil mass in region 3:

\[ W_3 = \gamma_1 \int_0^{\theta_1} \int_0^{\theta_2} \omega_{3V}(r, \tau) r dr d\tau + \gamma_2 \int_0^{\theta_1} \int_0^{\theta_2} \omega_{3V}(r, \tau) r dr d\tau \]  

(130)

\[
= \gamma_1 \omega_1 \frac{r_{II}^2 e^{\theta_2 \tan \varphi_{d2}} (\tan \varphi_{d2} \sin(\varphi_{d1} - \theta + \varphi_5) - \cos(\varphi_{d1} - \theta + \varphi_5))}{2 \tan^2 \varphi_{d2} + 2}
\]

\[-\gamma_1 \omega_1 \frac{r_{II}^2 (\tan \varphi_{d2} \sin(\varphi_{d1} - \theta) - \cos(\varphi_{d1} - \theta))}{2 \tan^2 \varphi_{d2} + 2} \]

\[+\gamma_2 \omega_1 \frac{(r_{II}^2 - r_{II}^2) e^{\theta_2 \tan \varphi_{d2}} (\tan \varphi_{d2} \sin(\varphi_{d1} - \theta + \varphi_5) - \cos(\varphi_{d1} - \theta + \varphi_5))}{2 \tan^2 \varphi_{d2} + 2}
\]

\[-\gamma_2 \omega_1 \frac{(r_{II}^2 - r_{II}^2) (\tan \varphi_{d2} \sin(\varphi_{d1} - \theta) - \cos(\varphi_{d1} - \theta))}{2 \tan^2 \varphi_{d2} + 2} \]
Area of region 4:

\[ A_4' = \frac{1}{2} l_G I_a \sin \theta_6 + \frac{1}{2} l_G l_{D'} \sin(\theta_3 - \theta_6) \]  
(131)

\[ A_4'' = \frac{1}{2} l_{D'} l_{D''} \sin\left(\frac{\pi}{4} - \frac{\varphi_{d_2}}{2}\right) - \frac{1}{2} l_{GF'} l_{GE} \sin \theta_7 \]  
(132)

Work due to the gravity of displaced soil mass in region 4:

\[ W_4 = (A_4' \gamma_1' + A_4'' \gamma_2') \omega_{AV} \]  
(133)

Work due to the gravity of displaced soil mass in region 5:

\[ W_5 = \gamma_2' \int_0^{r_{GF}} \int_0^{\theta_1} \omega_{SV}(r, \tau) r d\tau d\tau \]  
(134)

\[ = \gamma_2' \omega_{AV} \frac{r_{GF}^2 \epsilon^{\theta_2} \tan \phi_{d_2} \sin(\phi_{d_2} - \theta + \theta_5 + \theta_7) - \cos(\phi_{d_1} - \theta + \theta_5 + \theta_7)}{2 \tan^2 \phi_{d_2} + 2} \]

\[ - \gamma_2' \omega_{AV} \frac{r_{GF}^2 \tan \phi_{d_2} \sin(\phi_{d_1} - \theta + \theta_5) - \cos(\phi_{d_1} - \theta + \theta_5)}{2 \tan^2 \phi_{d_2} + 2} \]

Limit state equation 7:

\[ g = \sum_{i=1}^{5} W_i + (F_G - F_U) \omega_{1V} - (F_H + F_{HU}) \omega_{1H} = 0 \]  
(135)

The limit state equation must be optimised with respect to \( \theta \). To ensure that the rupture enters the subsoil the following constraints must be imposed:

\[ \theta \geq \tan^{-1}\left(\frac{h_H}{B_z + a + b}\right) \]  
(136)

\[ \theta_1 \geq \frac{\pi}{2} - \phi_{d_1} \]  
(137)
A.4.3 Limit state equation 8

\[ l_{BF} = B_z + a + c - h_H \cot(\theta + \varphi_{d_i}) \] (138)

\[ l_{BC} = l_{BF} \cos \theta \] (139)

\[ r_{CF} = l_{BF} \sin \theta \] (140)

\[ l_{DE} = r_{CF} \] (141)

\[ d = B_z - h_H \cot(\varphi_{d_i} + \theta) \] (142)

A unit displacement, \( \delta = 1 \), is imposed along failure line \( l_{BC} \). The directional displacements for region 1 to 4 become:

\[ \omega_1 = \omega_2 = \omega_3 = \omega_4 = 1 \] (143)

\[ \omega_{4V} = \sin \theta \] (144)

\[ \omega_{4H} = \cos \theta \] (145)

**Fig. 17:** Sliding through rubble mound and failure in undrained subsoil.

The failure is given by the geometrical quantities in equations 138-142.
Internal work from rupture along \( I_{BC} \):

\[
W_1 = \int_0^{l_{BC}} \omega_1 c_{u_2}(s) ds
\]  

(146)

Internal work from rupture along \( I_{CD} \) and from the rupture zone in region 2:

\[
W_2 = \int_0^{(\frac{z + \delta}{r_{CF}})} \omega_2 c_{u_2}(s) ds + \int_0^{r_{CF}} \omega_2 c_{u_2}(s) dsd\tau
\]  

(147)

Internal work from rupture along \( I_{DE} \):

\[
W_3 = \int_0^{l_{DE}} \omega_3 c_{u_2}(s) ds
\]  

(148)

Area of region 4:

\[
A_4 = \frac{1}{2} (B_z + a) h_{II} + \frac{1}{2} l_{BF} h_{II}
\]  

(149)

Work due to the gravity of displaced soil mass in region 4:

\[
W_4 = -A_4 g_1 \omega_{4v}
\]  

(150)

Limit state equation 8:

\[
g = \sum_{i=1}^{4} W_i + (F_G - F_U + F_D) \omega_{4v} - (F_H + F_{HU}) \omega_{4H} = 0
\]  

(152)

The limit state equation must be optimised with respect to \( \theta \). The following constraints must be imposed:

\[
\theta \geq \max \left( 0; \tan^{-1} \left( \frac{h_{II}}{B_z + a + b} \right) - \varphi_{\delta_i} \right)
\]  

(153)
A.4.4 Limit state equation 9

Fig. 18: Failure in rubble mound and undrained subsoil.

The limit state equation is obtained by a unit rotation, $\beta=1$, about point D. The corresponding geometrical quantities are given in equations 154-161.

$$r_{AD} = \sqrt{x_D^2 + y_D^2}$$ (154)

$$a = \tan^{-1}\left(\frac{B_z + a + b - x_D}{h_H + y_D}\right)$$ (155)

$$\zeta = \tan^{-1}\left(\frac{y_D}{x_D}\right)$$ (156)

$$\theta = \frac{\pi}{2} - a - \zeta$$ (157)

$$r_{BD} = r_{AD}e^{\theta\tan\phi_{11}}$$ (158)

$$l_{BC} = 2(y_D + h_H)\tan a$$ (159)

$$l_{AE} = B_z + a + b - l_{BC}$$ (160)

$$d = (2(y_D + h_H)\tan a - (a + b))$$ (161)
Area of region 1 (approximately):

\[ A_1 = \frac{1}{2} h_I (2B_z + 2a - l_{AE} + b) \]  

(162)

Position of centre of gravity for region 1 relative to point D (approximately):

\[
x_{CG} = \frac{b(2b + 6a + 6B_z - 6x_D)h_I - 2(B_z + a - l_{AE})(6x_D - 3(l_{AE} - B_z - a))h_I}{12A_1}
\]  

(163)

\[
-\frac{l_{AE}(6x_D - 2l_{AE})h_I}{12A_1}
\]

Work due to the gravity of displaced soil mass in region 1:

\[ W_1 = \beta A_1 \gamma'_{i} x_{CG} \]  

(164)

Internal work from rupture along l_{BC}:

\[ W_2 = \beta r_{BD} \int_{0}^{2 \alpha r_{BD}} c_{\nu_{i}}(s) ds \]  

(165)

Limit state equation 9:

\[
g = \sum_{l=1}^{2} W_l - (F_G - F_U + F_D)(x_D - \frac{B_z}{2})\beta - (F_H + F_{HU})y_D \beta = 0
\]  

(167)

The limit state equation must be optimised with respect to the centre of rotation \((x_D, y_D)\). The following constraints must be imposed:

\[ y_D \geq 0 \]  

(168)

\[ \frac{B_z}{2} \leq x_D \leq B_z + a + b \]  

(169)

\[ r_{BD} \cos a = y_D + h_I \]  

(170)

\[ a \geq 0 \]  

(171)

\[ \theta \geq 0 \]  

(172)
A.4.5 Limit state equation 10 (Constant volume)

Fig. 19: Failure in rubble mound and undrained subsoil.

The limit state equation is obtained by a unit rotation, $\beta=1$, about point D. The corresponding geometrical quantities are given in equations 173-179.

\[
\begin{align*}
r_{AD} &= \sqrt{x_D^2 + y_D^2} \quad (173) \\
\alpha &= \tan^{-1}\left(\frac{B_z + a + b - x_D}{h_{II} + y_D}\right) \quad (174) \\
\zeta &= \tan^{-1}\left(\frac{y_D}{x_D}\right) \quad (175) \\
\theta &= \frac{\pi}{2} - a - \zeta \quad (176) \\
r_{BD} &= r_{AD} \quad (177) \\
l_{BC} &= 2(y_D + h_{II}) \tan \alpha \quad (178) \\
l_{AE} &= B_z + a + b - l_{BC} \quad (179) \\
d &= 2(y_D + h_{II}) \tan \alpha - (a + b) \quad (179)
\end{align*}
\]
Area of region 1 (approximately):

\[ A_1 = \frac{1}{2} h_H (2B_z + 2a - l_{AE} + b) \]  
(180)

Position of centre of gravity for region 1 relative to point D (approximately):

\[ x_{CG} = \frac{b(6a + 6B_z - 6x_D)h_H - 2(B_z + a - l_{AE})(6x_D - 3(l_{AE} - B_z - a))h_H}{12A_1} \]  
(181)

\[ -\frac{l_{AE}(6x_D - 2l_{AE})h_H}{12A_1} \]

Work due to the gravity of displaced soil mass in region 1:

\[ W_1 = \beta A_1 \gamma_1 x_{CG} \]  
(182)

Internal work from rupture along \( l_{AC} \) due to shear strength of rubble mound material and undrained shear strength of subsoil:

\[ W_2 = \beta r_{BD} \left[ \int_0^{\theta_{BD}} \tau_1(s)ds + \int_0^{2\pi_{BD}} c_u(s)ds \right] \]  
(183)

Limit state equation 10:

\[ g = \sum_{i=1}^{2} W_i - (F_G - F_U + F_D)\left(x_D - \frac{B_z}{2}\right)\beta - (F_H + F_{HU})y_D \beta = 0 \]  
(185)

The limit state equation must be optimised with respect to the centre of rotation \((x_D, y_D)\). The following constraints must be imposed:

\[ y_D \geq 0 \]  
(186)

\[ \frac{B_z}{2} \leq x_D \leq B_z + a + b \]  
(187)

\[ r_{BD} \cos \alpha = y_D + h_H \]  
(188)

\[ a \geq 0 \]  
(189)

\[ \theta \geq 0 \]  
(190)
ANNEX B

PERMANENT DEFORMATIONS DUE TO IMPACT LOADING

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B.1 Introduction

In this annex the application of the dynamic model for determination of permanent deformations due to impact loading is described. The application is based on the model description found in Section 6.6. The mode of operation of the dynamic model can briefly be summarised as:

1) Determination of the critical limit state equation, i.e. static bearing capacity and geometrical quantities.
2) Determination of dynamic contributions for establishment of the equation of motion.
3) Description of impact loading.
4) Determination of permanent deformations by solving the equation of motion.

In order to illustrate the application of the dynamic model a single limit state equation is considered and the steps described above are followed.

B.2 Limit State Equation Used for Illustration of the Dynamic Model

The considered limit state equation is based on a somewhat simpler geometrical configuration than the limit state equations given in Section 6.3 and Annex A. Since caissons in general are subjected to eccentric loads a rotational mechanism is considered. It is for simplicity assumed that the caisson is resting directly on a undrained subsoil. The failure is illustrated in Figure B.1.
The centre of rotation is presumed to be situated below the free soil surface. AC is a line rupture, for which reason region 1 constitutes a stiff body. Region 2 is a simple Prandtl zone and region 3 is a Rankine zone. With the present failure mode it is possible to account for strongly eccentric loads, thus only a part of the structure, $B_z$, is in contact with the soil and the remaining part is lifted from the soil surface during failure. The failure mode is geometrically described by two parameters, namely by the angle $\lambda$ and $\alpha$, which is the inclination of the chord $k_{AC}$ and half the angle at the centre of circle arc with centre $O$, respectively. The displacement field in the rupture figure is defined by the rigid body movement of the structure and of the body ACP of soil. The failure mode is given by the geometrical quantities in equation 1-6:

$$k_{AC} = B_z \frac{\cos(\lambda + a)}{\cos a}$$

(1)  

$$r_{PC} = B_z \frac{\sin \lambda}{\cos a}$$

(2)  

$$l_{DE} = r_{PD} = r_{PC}$$

(3)  

$$2\theta = \frac{\pi}{4} + \lambda + a$$

(4)
The rotation about point O implies that, if a unit sliding displacement, $\delta=1$, is imposed along the boundary ACDE, the displacement at point P equals $1+r_{PC}/r_{OC}$, whereas the clockwise rotation of the structure, $\beta$, equals $1/r_{OA}$. The directional displacements and rotation for regions 1 to 3 become:

$$\omega_1 = 1$$ (7)

$$\beta = \frac{\omega_1}{r_{OA}}$$ (8)

$$\omega_2(r) = \omega_1 + \frac{r_{PC} - r}{r_{OC}} \quad r \in [0, r_{PC}]$$ (9)

$$\omega_3(r) = \omega_1 + \frac{r_{PD} - r}{r_{OC}} \quad r \in [0, r_{PD}]$$ (10)

Internal work from rupture along the boundary line $l_{ACDE}$:

$$W_1 = \int_0^{2\pi r_{OA}} \omega_1 c_u(s) ds + \int_0^{2\pi r_{PC}} \omega_2(r_{PC}) c_u(s) ds + \int_0^{l_{DE}} \omega_3(r_{PD}) c_u(s) ds$$ (11)

Internal work from the rupture zone corresponding to region 2:

$$W_2 = \int_{r_{OA}}^{r_{OA} + r_{PC}} \int_0^{2\pi r_{OA} + r_{PC}} \frac{r_{OA} + r_{PC}}{r_{OA}} c_u(s) ds d\tau$$ (12)

Internal work from the rupture zone corresponding to region 3:

$$W_3 = \int_0^{r_{PD} l_{DE}} \int_0^{l_{DE}} \frac{1}{r_{OA}} c_u(s) ds ds$$ (13)

Limit state equation:

$$g = \sum_{i=1}^{3} W_i - [(F_G - F_U)(B - B_z) - F_H(r_{OA} + \eta)]\beta = 0$$ (14)
The unknown quantities $\lambda$, $\alpha$, $B_z$ are used to establish the extremum condition. Due to the moveable boundary at point A, (i.e. determination of the effective width, $B_z$) uncontrolled optimisation will give erroneous results. The problem is avoided by keeping $B_z$ at a fixed value and taking $\lambda$ and $\alpha$ as unknowns. The value of $B_z$ is varied until $\lambda$ is equal to $\alpha$.

**B.3 Determination of Static Bearing Capacity and Geometrical Quantities**

The static bearing capacity and the geometrical quantities are determined for the structure given in Figure 2. It is presumed that failure will occur as described in Section B.1.

A vertical breakwater with a height and width ($H \times B$), of $9.00 \times 8.70$ m and an average density ($\rho_c$) of $2400$ kg/m$^3$ is considered. The breakwater is founded on clay with a constant undrained shear strength ($c_u$) of $55$ kPa and a density ($\rho_s$) of $2000$ kg/m$^3$. The breakwater is subjected to a horizontal wave load, acting $7.00$ m ($\eta$) above the seabed.

![Fig. 2. Dimension of structure and soil properties.](image)

The static bearing capacity, expressed by the maximum permissible horizontal load ($F_H$), can be determined from the limit state function given in 14. An optimisation yields the bearing capacity and geometrical quantities listed below.

- $\lambda = \alpha = 24.2^\circ$
- $\rho_s = 2400$ kg/m$^3$
- $c_u = c_u^d = 55$ kPa
- $\rho_s = 2000$ kg/m$^3$
- $r_{OA} = 7.00$ m
- $r_{PC} = 3.54$ m
- $B_z = 7.88$ m
- $F_H = 216.7$ kN/m

**B.4 Dynamic Contributions**

Provided that the transgression of the static bearing capacity does not effect the shape and size of the soil bodies, the dynamic contributions can be calculated from the geometrical quantities.
given in Section B.1. At failure a rotation $\beta$ about point O will cause the system to move in accordance with the displacement field as illustrated in Figure 1.

As the undrained shear strength is assumed to be constant and unaffected by the deformation rate, the dynamic properties of the failure mode will in the present case consist of three contributions:

1) Decrease of shear resistance due to shortening of the boundary line.
2) Upheaval of the subsoil on the back of the structure (region 3).
3) Acceleration of the system’s mass.

The contributions are calculated in accordance with the guidelines in Section 6.6.

B.4.1 Decrease of shear resistance due to shortening of the boundary line.

The failure line $l_{DE}$ is shortened due to the upheaval and rotation of region 3. The shortening of the failure line corresponds to the directional displacement $\omega_3$ equal to $\beta_{OA}$. The changes in moment capacity due to the shortening of the boundary line can be expressed as:

$$\Delta M^I_{f}(\beta) = -\beta r_{OA}^2 c_u$$  (15)

B.4.2 Upheaval of the subsoil on the back of the structure.

The upheaval of region 3 is composed by a displacement of $\beta R$ along the boundary line $DE$ and the body is afterwards rotated $\beta$ about point D (see Figure 3).

Fig. 3. Schematic drawing of the upheaval of region 3.

The gravity of the soil above the original soil surface acts as a surface load and increases the bearing capacity. Small deformations are assumed, wherefore second order terms are neglected. The area of the body that is lifted above the soil surface becomes:

$$\Delta A^I = \beta r_{OA}^2 - \frac{1}{2} \beta^2 r_{OA}^2 \approx \beta r_{OA}^2$$  (16)
\[ \Delta A' = \frac{1}{2} \beta r_{OA} r_{PD} - \frac{1}{2} \beta^2 r_{PD} r_{OA} = \frac{1}{2} \beta r_{OA} r_{PD} \]  

(17)

\[ A = \frac{r_{OA}(r_{PD} + 2r_{OA})}{2} \beta \]  

(18)

Neglecting the displacement of the centre of gravity of the body due to the rotation, the distance from the rear edge of the structure to the centre of gravity equals \( r_{PD}/\sqrt{2} \). The effect of the upheaval hereby becomes:

\[ \Delta M^i_{ik}(\beta) = \frac{r_{OA}(r_{PD} + 2r_{OA})}{2} \left( B_z + \frac{r_{PD}}{\sqrt{2}} \right) \beta g \rho_s \]  

(19)

B.4.3 Acceleration of the system’s mass

The mass that must be accelerated is equivalent to the system’s mass moment of inertia about the centre of rotation.

\[ J_G = J_G^{ACP} + J_G^{CDP} + J_G^{DEP} + J_G^{Struc.} \]  

(20)

\[ J_G^{ACP} = J_G^{OAP} - J_G^{OAC} = \frac{r_{OA}(r_{OA}^2 B_z + B_z^3 - 6r_{OA}^3)}{12} \rho_s \]  

(21)

\[ J_G^{CDP} = \frac{r_{PC}^2 \theta (r^2 + 2(r_{OA} + r_{PC})^2)}{2} \rho_s \]  

(22)

\[ J_G^{DEP} = \left( \frac{r_{PD}^2}{12}(4r_{PD}^2 + 6B_z(B_z + \sqrt{2} r_{PD})) + \frac{r_{PD}^2 r_{OA}}{12}(6r_{OA} - \sqrt{2} r_{PD}) \right) \rho_s \]  

(23)

\[ J_G^{Struc.} = \rho_s BH \left( \frac{H^2 + B^2}{4} + r_{OA}(r_{OA} + H) + B_z(B_z - B) \right) \]  

(24)

The dynamic contribution can be expressed as:

\[ \Delta M^m(\beta) = J_G \beta \]  

(25)

B.5 Equation of Motion
The equation of motion is established by demanding a balance between the moments of the internal and external forces about the centre of rotation. The angular displacement of the system is determined by solving the equation of motion for a system with one degree of freedom. Similar to equation 35 in the Section 6.6 the equation of motion for the system yields:

\[
I(t) - M = \Delta M^m(\dot{\beta}) + \sum_{i=1}^{i=2} \Delta M_i^t(\beta)
\]

\[
= J_G \ddot{\beta} + \left[ \frac{r_{OA}(r_{PD} + 2r_{OA})}{2} \left( B_z + \frac{r_{PD}}{\sqrt{2}} \right) g \rho_s - r_{OA}^2 c_u \right] \beta
\]

The equation of motion is rearranged by introducing the dynamic impulse as an exponential function related to the static bearing capacity by the overloading factor \( S \), the amplitude of the system and the circular eigenfrequency:

\[
A = \frac{M}{J_G}
\]

\[
\omega^2 = \frac{r_{OA}(r_{PD} + 2r_{OA})}{2} \left( B_z + \frac{r_{PD}}{\sqrt{2}} \right) g \rho_s - r_{OA}^2 c_u
\]

\[
I(t) = Q(\eta + r_{OA}) e^{-kt} = S Me^{-kt}
\]

\[
\ddot{\beta} + \omega^2 \beta = A(Se^{-kt} - 1)
\]

**B.6 Example 1: Permanent Rotation of Breakwater due to a Dynamic Impulse**

The breakwater given in Figure 2 is subjected to an impact with a magnitude \( Q_d \) of 282 kN/m, acting 7.00 m above the seabed. The dynamic impulse is described by an exponential load history with a constant of decay that equals 0.29 s\(^{-1}\). The objective is to determine the irreversible rotation of the structure due to the impact. The geometrical quantities that describe the mode of operation are found in Section B.2, where the static bearing capacity is determined. These parameters constitute a base for the equation of motion on the form given in 30.

\[
S = \frac{Q_d}{F_H} = 1.30
\]

\[
M = F_H(\eta + r_{OA}) + F_G \left( sB_z - \frac{B}{2} \right) = 9541 \text{ kNm/m}
\]
\[ \Delta M^k = 3558 \text{ kNm/m} \]  
\[ J_G = 3.39 \cdot 10^4 \text{ kgm}^2/\text{m} \]  
\[ A = \frac{M}{J_G} = 2.81 \cdot 10^{-1} \text{ s}^{-2} \]  
\[ \omega^2 = \frac{\Delta M^k}{J_G} = 1.05 \cdot 10^{-1} \text{ s}^{-2} \]

By inserting the constants given in 30-35 in the equation of motion results in the solution illustrated in Figure 4.

![Fig. 4. Angular acceleration, velocity and displacement of breakwater subjected to impact loading.](image)

The irreversible rotation of the breakwater is found whenever the velocity equals zero. The impact, described by an exponential load history, produces a permanent rotation of 2.5°.

**B.7 Example 2: Design Diagram**

In cases where the breakwater is subjected to several impacts with different load parameters (i.e. overloading factors or constant of decay) the final irreversible deformation can with advantage be determined by use of design diagrams. The design diagram is based on the static and dynamic properties of the structure, determined in Example A and given in Figure 5.
Fig. 5. Design diagram for caisson subjected to a dynamic impulse.
ANNEX C

COMBINED EFFECT OF DILATANCY IN RUBBLE MOUND AND CAISSON INERTIA

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C.1. INTRODUCTION

Prototype experience (Oumeraci, 1994) shows that approximately in 2 failure cases out of 3 sliding is the failure mechanism, whereas computations performed within PROVERBS, see vol. 2-d, show quite often that a deep foundation failure is the critical failure mode. This can be explained in several ways, for instance:

- a stabilising three dimensional effect (the friction on sliding body exerted by adjacent rubble mound) which increases with the depth of the sliding surface;
- a behaviour of rubble mound material not conforming with the model assumed.

The second cause is examined here.

The elastic behaviour of the foundation is represented by the MSD dynamic model, reducing in the case of short and strong impacts the horizontal force applied by caissons to the rubble mound due to the combined effect of foundation elasticity and caisson inertia. The dynamic response factor represents the ratio between the maximum force applied by caissons to foundation and the maximum force applied by breakers to caissons. During elastic oscillations following the impact, sliding force may come over the stability threshold for a short time interval (short compared to oscillation period: in prototype conditions of the order of magnitude of 0.1 s).

In this annex the problem is analysed whether some mechanisms other than elasticity may exist that increase the structure stability due to the limited duration of instability conditions;
the problem was posed in Lamberti (1998) and is presented here after the discussion within the project. The attention is focused on dilation necessary to allow irreversible shear in the rubble mound. In this sense the approach in this annex is different from the approach in Appendix B where any plastic deformation however small is supposed to be permanent. In the following point it is explained how, particularly under wave impacts and resulting caisson oscillation, sliding along the smooth caisson base might be easier than sliding within the rough rubble mound.

In the geotechnical praxis non-stationarity of the load conditions are mainly analysed looking at the effects on and of the pore pressure, i.e. the effect of soil drainage conditions. Vertical forces are essentially the static weight of the structure and no account is normally provided for accelerations which could be strictly inherent to the failure mechanism.

At the same time the size of the grains in a geotechnical layer is normally several order of magnitude smaller than the layer thickness and an order of magnitude smaller than the elastic or plastic deformation of the layer. Geotechnical layers are essentially treated as continua composed of particle of infinitesimal size: this does not apply to the rubble mound of a caisson breakwater, where stones as big as 0.5 m are normally used in a layer approximately 10 m thick. The Young modulus of the foundation is around 300 MPa and pressure at the base of the caisson do vary as a consequence of waves of something about 300 kPa; the elastic deformation of the rubble mound layer and short term displacements of caissons are rather small compared to the rubble mound stone size, so that it is hard to believe that stones contact pattern in the rubble mound can substantially change in a short time at the expense of the subsoil deformation and without any displacement of the caisson. The necessity of the rubble mound to dilate and raise the caisson in a short time is the cause of some extra vertical force which increase the sliding resistance.

C.2. EFFECT OF STONE SIZE AND DILATANCY ON RUBBLE MOUND DYNAMIC RESPONSE

The main point of the analysis is that, in order to make sliding possible, at least a one grain layer around the sliding surface should dilate so that the roughness of the two sliding surfaces do not interlock each other or a certain volume around the sliding surface must expand in order to make grain rotation and shear strain possible. Dilation is normally obtained partly through an upward displacement of the breakwater structure and partly by a compression of the subsoil, both caused by a temporary increase of the normal pressure through the sliding surface.
Let figure 1 represent the schematic mechanism. A horizontal force, the load $S$, is applied to a heavy mass $M$ resting on a rough surface. The contact surface is characterised by plane elements, forming an angle $\delta$ -the dilation angle- with the macroscopic sliding surface; interlocking of roughness elements takes place only when dilation is smaller than the small but finite roughness height $h$. Among the sliding body imperfect rolling elements are interposed causing a certain obliquity $\varphi$ -the true friction angle- between the force transmitted through the contact and the contact plane. Relative to the macroscopic sliding surface the transmitted force has obliquity $\vartheta = \delta + \varphi$ -the apparent friction angle-. The obliquity cause a relation between the normal force through the sliding surface and the tangential force $R$ balancing the load.

According to a static approach to stability, the normal force balances the weight of the structure and the tangential force balances the load. In the schematised case

\begin{align}
V - M \cdot g &= 0 \\
S - R &= 0
\end{align}

with $R \leq V \tan \vartheta$

The fact that no solution exists of the equilibrium conditions, can be transformed into a failure equation, and one or the other can be reworked into a virtual work equation by evaluating the work $\delta W$ produced by an infinitesimal and kinematically admissible displacement: if the work is positive the system accelerates and diverge from the equilibrium position.

Failure $\iff \delta W > 0$

In all what is said above time plays no role: forces are stationary; if they are not in equilibrium, however small the unbalance is, the system will diverge indefinitely from the equilibrium configuration.

If the non-equilibrium conditions last only for a short time interval, the question must be posed of what will happen during and after that time interval. The dilation mechanism is often reversible (mechanically not thermodynamically) -i.e. $\delta > \psi$ holds- and, if the system has not passed the roughness threshold it will return back to the initial position. In the case that this does not occur just after the unbalance period, it is likely that due to the reversal of wave
force this will occur after a short time. If the system was displaced more than one roughness element, dilation recover is not complete since it never exceeds one roughness element.

If the work produced by the load during the non-equilibrium period is great enough, so that the system is accelerated during this period and then, due to inertia, continues the displacement and reaches the top of the roughness, some permanent displacement will take place as a consequence of the temporary unbalance. I.e. in order that a failure does occur the work of all the forces -the load, the dissipative friction forces and the conservative gravity forces- should be positive at the end of the schematised dilation phase (when the mass is on top of the roughness). During the dilation phase friction co-operates with gravity causing resistance to load.

According to the energetic approach, the failure equation should be therefore:

$$W_S - W_R > 0 \quad \text{at the end of the finite dilation phase},$$

and not just at the beginning of movement as accounted for by virtual work.

The same problem can be analysed according to a momentum approach, and actually the energetic approach is not simpler in this case since friction resistance depends on the acceleration. It is useful to separate the case of a rigid foundation from the case of an elastic foundation. In the first case dilation is possible only by raising the caisson; in the second dilation can be obtained by raising the caisson and compressing the foundation.

C.3. THE CASE OF A RIGID FOUNDATION

Let us analyse the limit condition when the system reaches just (with zero velocity) the end of dilation, assuming for the sake of simplicity that $S$ is constant and greater than $R$ during the acceleration phase lasting the time $t_a$ and zero after. Let $z_a$ be the vertical displacement at the end of this phase, and let $\dot{z}_a$ be the vertical velocity in this moment. Let $z_*$ be the height reached at the end of the following deceleration phase; failure occurs if $z_* > h$.

The momentum equation are in this case:
ANNEX C  COMBINED EFFECT OF DILATANCY AND CAISSON INERTIA

\[ V - M \cdot g = M \cdot \ddot{z} \]  \hspace{1cm} (3)
\[ S - R = M \cdot \ddot{x} \]  \hspace{1cm} (4)

with \( R \leq V \tan \delta \) and \( z = x \tan \delta \)

With the aid of simple passages one obtains:
\[ R = V \tan \delta = M \cdot (g + \ddot{z}) \cdot \tan \delta \]
\[ S - M \cdot g \cdot \tan \delta = M \cdot (\ddot{x} + \ddot{z} \cdot \tan \delta) = M \cdot \ddot{x} \cdot (1 + \tan \delta \cdot \tan \delta) \]

Denoting by \( R_s \) the static resistance \( R_s = M \cdot g \cdot \tan \delta \), the horizontal momentum equation becomes:
\[ \ddot{x} = \frac{S - R_s}{M \cdot (1 + \tan \delta \cdot \tan \delta)} \]

Denoting by \( \varepsilon \) the relative load excess \( \varepsilon = (S - R_s)/R_s \) the vertical momentum equation during the acceleration phase becomes:
\[ \ddot{z} = \varepsilon \cdot g \cdot \frac{\tan \delta \cdot \tan \delta}{1 + \tan \delta \cdot \tan \delta} = \varepsilon \cdot g^* \]  \hspace{1cm} (5)

During the following deceleration phase, when \( S = 0 \)
\[ \ddot{z} = -g^* \]  \hspace{1cm} (6)

The duration of the deceleration phase is \( t_d = \varepsilon \cdot t_a \).

The vertical velocity at the end of the acceleration phase is \( \dot{z}_a = \varepsilon \cdot g^* \cdot t_a \)

The vertical displacement at the end of the acceleration phase is
\[ z_a = \frac{1}{2} \cdot \dot{z}_a \cdot (t_a + t_d) = \frac{1}{2} \cdot g^* \cdot t_a^2 \cdot \varepsilon \cdot (1 + \varepsilon) \]  \hspace{1cm} (7)

Failure occurs only if \( z_a > h \), i.e. if
\[ t_a^2 \cdot \varepsilon \cdot (1 + \varepsilon) > 2h/g^* \]  \hspace{1cm} (8.1)

or equivalently
\[ \varepsilon \cdot (1 + \varepsilon) \left( \frac{t_a}{t_a^*} \right)^2 \Leftrightarrow \varepsilon = \frac{1}{2} \left( \sqrt{1 + 4 \left( \frac{t_a^*}{t_a} \right)^2} - 1 \right) \]  \hspace{1cm} (8.2)

providing an approximate reciprocity relation between the duration of the unbalance conditions and the relative intensity necessary to cause failure; the time scale appearing in the right hand side depends essentially on the surface roughness.
In the limit for impulsive load the previous approach provides a minimum impulse necessary to cause sliding:

\[ I > R_s \cdot \sqrt{2h/g} \]  

(9)

C.4. THE CASE OF AN ELASTIC FOUNDATION

If account is taken of foundation stiffness by means of an elastic reaction characterised by the elastic constant \( k_z \), the terms of eqn. (7) are equal to the increment of the vertical reaction of the foundation. Assuming that foundation reacts instantaneously the application of the load cause the instantaneous foundation settlement \( \frac{Mg_s}{k_z} \); if at the end of the acceleration phase this settlement plus the vertical displacement of the structure is greater than the surface roughness failure will occur, i.e.

\[ \varepsilon g_s \cdot \left( \frac{M}{k_z} + \frac{1}{2} \frac{t_a^2}{t^*} \right) > h \Rightarrow Failure \]  

(10)

\( k_z/M \) is the squared angular frequency of heave oscillations, that in prototype conditions was observed to be approximately 20 rad/s, the effect of foundation elasticity may be therefore relevant if load duration is less than 0.1 s..

During the following deceleration phase the foundation shows a negative settlement (elastic rebound) compared to static conditions and a failure occurs during this phase if:

\[ g_s \cdot \left( -\frac{M}{k_z} + \frac{1}{2} \frac{t_a^2}{t^*} \cdot \varepsilon \cdot (1 + \varepsilon) \right) > h \Rightarrow Failure \]  

(11)

Under the assumption of an instantaneous reaction, foundation elasticity may increase the risk of sliding during the acceleration phase but has always a positive effect during the deceleration phase.

<table>
<thead>
<tr>
<th>( t_a/t^* )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>5.0</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1+\varepsilon )</td>
<td>10.5</td>
<td>5.52</td>
<td>2.56</td>
<td>1.62</td>
<td>1.21</td>
<td>1.04</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Tab. 1 Relation between load relative duration and critical relative load
C.5. ACTUAL THRESHOLD VALUES

The essential features of the mechanism described is the finite recover of dilation. No permanent displacement remains after the impact if impact force and duration do not cause the recover limit to be exceeded.

For typical values of the apparent friction and dilation angles ($\theta \approx 45^\circ, \delta \approx 30^\circ$) and assuming that the roughness height in the 2-D mechanism of fig. 1 is a small fraction of the grain size, let us assume 1/10, the time scale $t* = \sqrt{2h/g*}$ is 7 ms for a 1 mm sand, but is as great as 166 ms for 0.5 m (330 kg) stones, providing virtually an extra safety factor of 2 when load exceeds static resistance for 0.1 s in absence of foundation elasticity.

This is certainly an upper bound to the effect. Actually threshold values seem to be significantly lower because the 3rd dimension make always some lower saddle point available, and because some irreversible permanent deformation show up at stress level around half the yield strength.

As pointed out by De Groot (1998) many experiments on sand (Baskarp sand, Oster Scheldt sand), Andersen & al. (1997, 1998), and gravel (Portland gravel), Jacobsen (1996), show however that, when dilation has taken place, a hysteretic cycle is followed in the stress-strain plane having width in the strain direction approximately equal to 0.5%. This hysteretic cycle looks remarkably similar to the cycle followed by the conceptual system in figure 1 and dilating conditions in the rubble mound are likely to be present in prototype after the initial settlement (contracting behaviour) that takes place during the first few storms after placement.

![Fig. 3 Loading unloading cycles in granular media.](image)

Eastern Scheldt sand on the left, Baskarp sand n. 15 on the right

Let us assume that the shearing zone in prototype is 10 grain (5 m) thick as it certainly is in the experiments, the displacement of the opposite faces of the shearing zone during reversible shearing cycles is .05 grain diameters, corresponding to a dilation of .025 grain diameters.
Below this level of dilation no permanent effects are expected; this reduce the time scale appearing in eqns 8 or 9 by a factor of 2.

Geotechnical experiments apply normally the load gradually (strain rate less than 100%/hour) and if a movement among grains is possible, as far kinematics and statics are concerned, it takes place, inertia having no significant role in hindering movements. Experiments show also that material resistance increases (it is clearly demonstrated for clay) if load are applied rapidly. Even if the time scale is not clear there is no doubt that prototype dynamic loading under impacts (full loading in approximately 0.2 s) is substantially more rapid than during normal geotechnical tests.

The presented analysis is qualitative, assumes that stones do not break down under impact forces and is not experimentally tested; it shows however how the rubble mound could resist more than foreseen by a continuum theory under impact of short duration.

C.6. CONCLUSIONS

The boulder size of the rubble mound might have a significant effect on the dynamic behaviour of the foundation, imposing some threshold proportional to their size on a combination of load intensity and duration similar to impulse. Below this threshold deformation should be substantially recovered, either spontaneously at the end of the impact, or during the following seawards pushing trough conditions; above this threshold permanent sliding does occur.

The duration of dilating condition and the load relative intensity above static resistance are approximately inversely proportional, every contact surface being characterised by a time constant proportional to the square root of its roughness. The time constant of typical geotechnical materials seems to be so short that the phenomenon has no practical relevance, but for rubble mound and impulsive loads the time constant is greater and the duration of unbalanced conditions during caisson oscillations induced by impact may be short, leading to an extra-resistance factor compared to stationary conditions that may reach values around 2. The contact between caisson and rubble mound is normally rather smooth and the sketched mechanism does not increase substantially its resistance, making sliding under impact easier along this plane than along a deep surface in the rubble mound.

Increasing the roughness of the contact would increase the static and moreover the dynamic resistance.
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REFERENCES


CHAPTER 7: UNCERTAINTIES

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ABSTRACT

The foundation design of a breakwater will require assessment of soil parameters to be applied in the analysis models of foundation capacity and deformations. A series of failure modes will normally be investigated to ensure that the breakwater foundation can resist the effect of gravity and environmental forces with an adequate safety margin against failure (sliding, bearing capacity failure) and without unacceptable deformations (tilt, settlement, horizontal displacements). Probabilistic design of a breakwater will require definition of limit state functions describing the failure modes. These functions are idealised and simplified models of the physical problems, and the parameters entering the functions are connected with uncertainty. In this chapter the uncertainties connected with the foundation are treated.

Keywords: Vertical breakwaters, foundation design, model uncertainty, soil parameters

7.1. GENERAL

Methods for soil investigation and assessment of soil parameters were described in Chapter 2. Geophysical, field test and laboratory data are collected, analysed and translated to a representative soil model, which covers the soil volume affected by the breakwater and contains the soil parameters required for design.

In Chapters 3 through 6 calculation models for evaluation of dynamic effects, instantaneous and residual pore pressure, soil degradation and limit state equations for stability and deformation have been described. The calculation models have limitations and include idealisations and simplifications of the physical problem. This applies to the calculation of stress distributions as well as the constitutive model describing the material behaviour and pore water flow under the gravity and environmental loading.
In order to carry out a probabilistic analysis of the reliability of breakwater structures, the uncertainties connected to the soil parameters involved and the uncertainties connected to the calculation models have to be described. Uncertainties associated with the foundation design can be divided in two groups: inherent (or natural) uncertainty and uncertainty due to lack of knowledge. The inherent uncertainty represents the natural randomness of the property and cannot be reduced or eliminated. The uncertainty caused by lack of knowledge can, however, be reduced and perhaps eliminated by increasing the number of observations (more borings, more sampling will reduce the statistical uncertainty), improving the measurement accuracy and reducing the model uncertainty by improved physical formulation of the problem.

### 7.2. SOURCES OF UNCERTAINTIES IN SOIL PARAMETERS

The collection of data will introduce inaccuracies. Electrical and mechanical signals from geophysical surveys, field tests and laboratory tests are sampled and translated to physical soil parameters (step I in Figure 1). The quality of the test and soil sampling equipment and the experience and qualification of the personnel involved may affect the results. Sampling disturbance of cohesive soils and preparation of cohesionless soil samples will inevitably introduce deviations from the in situ material behaviour, but also laboratory procedures may vary such that significant deviations develop between different laboratories.

![Fig. 1: Transformation process from measured to relevant soil parameters](flowso.dwg)

A large part of the gathered data describes parameters that are not directly involved in the calculation models. The translation to relevant parameters for the calculations will be based on correlations between one or several parameters based on calibration tests and experience, and is associated with uncertainties (step II in Figure 1). Typical examples are the correlations between cone resistance and relative density of sand (which again is correlated to the friction angle) and the undrained shear strength of clay. Numerous empirical relationships have been developed which correlate classification properties like grain size distribution, water content and plasticity to shear strength and deformation properties.
A further complication in assessment of these material properties and their uncertainties, is the fact that they are dependent on the nature, magnitude and time history of the effective stress. The uncertainty in the stress-strain-strength properties are affected by the uncertainties connected to the loads and the load history and the drainage conditions. Normalisation with respect to effective stress (i.e. in situ vertical effective stress, \( \sigma'_{vo} \), or mean normal stress, \( \sigma'_m \)) will often give a clearer picture of the variability under drained conditions. In undrained or partly drained situations, the uncertainty connected to development of pore pressure and associated strength reduction will have to be considered.

In a soil volume there will always be a natural spatial fluctuation of the material properties. The site investigation and number of observations will be limited, and the inevitable extrapolation required to describe the soil parameters in the soil volume affected by the breakwater will introduce uncertainties (step III in Figure 1). The uncertainty will depend on the local soil conditions and the degree of detail and quality of the soil investigations performed.

7.3. **STATISTICAL TREATMENT OF GEOTECHNICAL DATA**

Uncertain soil properties and model uncertainty can be defined as random variables described by a mean value and a standard deviation (or coefficient of variation) and a probability distribution function. The model uncertainty is defined as the ratio of the actual quantity to the quantity described by the model. A mean value different from 1.0 expresses a bias in the model, while the standard deviation expresses the variability in predictions by the model.

The statistical estimates give a mean value and an estimate of the uncertainty in the data. When selecting design parameters the statistical estimates should be combined with engineering judgement. Statistics apply within “homogenous” layers and identification of the main layers should be done prior to statistical treatment of data.

Statistical methods belong to either traditional statistics (mean value, variance, histograms, probability density etc., see Ang and Tang, 1975) or geostatistical approaches see (Lacasse & Nadim, 1996). If a soil parameter is obtained from a complex calculation involving many random parameters (e.g. the cyclic strength of clay under cyclic loading), simulation methods like Monte-Carlo simulation should be considered.

Often the correlation between two parameters is of relevance for a calculation. Typical examples are the variation of undrained shear strength with depth and cone penetration resistance vs. undrained shear strength. Statistical evaluation can be done using linear regression analysis. The goodness-of-fit is given by how close the correlation coefficient is to unity.
Properly accounting for the uncertainty related to the spatial variability may substantially reduce the computed risk of failure associated with a particular design. Stochastic interpolation and/or spatial averaging can reduce the uncertainty associated with a particular variable. These techniques require knowledge of the spatial structure of the soil property, which can be described by the autocorrelation function of the variable. Keaveny et al. (1989) analysed data from several offshore sites with respect to the vertical and horizontal autocorrelation of a number of soil properties. In general the correlation lengths will be different in horizontal and vertical direction due to soil stratification. Identification of the stratification from geophysical surveys and knowledge of the local geology will be valuable in reducing the uncertainty connected to continuity of layers.

The uncertainties due to measurements of soil properties can be influenced by errors and misinterpretations. Consistent data populations should be used. Inconsistency may be introduced using data from different soils, different stress conditions, different test methods and different codes of practise, testing errors and non-reported imprecision, sampling disturbance etc. Lacasse & Nadim (1996) presented the results of a review on test results in NGI’s files and data available from the literature. The uncertainties obtained, in terms of coefficient of variation (COV), and the probability distribution functions arrived at are shown in Table 1 was obtained.

**Tab. 1**: Coefficient of variation of different soil properties (from Lacasse and Nadim 1996).

<table>
<thead>
<tr>
<th>Soil property</th>
<th>Soil type</th>
<th>Prob. distr. function</th>
<th>Mean</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone resistance</td>
<td>Sand, Clay</td>
<td>LN, N/LN</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Undrained shear strength, $c_u$**</td>
<td>Clay (triax) Clay (index $c_u$) Clayey silt</td>
<td>LN, LN, N</td>
<td>*</td>
<td>5 – 20% 10-35% 10-30%</td>
</tr>
<tr>
<td>Ratio $c_u/\sigma_{cp}$</td>
<td>Clay</td>
<td>N/LN</td>
<td>*</td>
<td>5-15%</td>
</tr>
<tr>
<td>Plastic limit</td>
<td>Clay</td>
<td>N</td>
<td>0.13-0.23</td>
<td>3-20%</td>
</tr>
<tr>
<td>Liquid limit</td>
<td>Clay</td>
<td>N</td>
<td>0.30-0.80</td>
<td>3-20%</td>
</tr>
<tr>
<td>Submerged unit weight</td>
<td>All soils</td>
<td>N</td>
<td>5 – 11 kN/m$^3$</td>
<td>0-10%</td>
</tr>
<tr>
<td>Friction angle</td>
<td>Sand</td>
<td>N</td>
<td>*</td>
<td>2-5%</td>
</tr>
<tr>
<td>Soil property</td>
<td>Soil type</td>
<td>Prob. distr. function</td>
<td>Mean</td>
<td>COV</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------</td>
<td>-----------------------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>Void ratio, porosity, initial void ratio</td>
<td>All soils</td>
<td>N</td>
<td>*</td>
<td>7-30%</td>
</tr>
<tr>
<td>Overconsolidation ratio</td>
<td>Clay</td>
<td>N/LN</td>
<td>*</td>
<td>10-35%</td>
</tr>
</tbody>
</table>

N/LN  Normal and lognormal distribution  
*   Values are site- and soil type-dependent  
** Undrained shear strength is anisotropic and depends on the stress imposed. The coefficient of variation for good quality tests (consolidated triaxial compression/extension, direct simple shear, true triaxial, plain strain) is expected to be 5-20%. For extension conditions, because of fewer data available and at times more difficult testing conditions, the coefficient of variation may be higher.

7.4. UNCERTAINTIES OF SOIL PARAMETERS IN BEARING CAPACITY CALCULATIONS

7.4.1. Soil types evaluated

The bearing capacity calculations for a breakwater shall demonstrate that the foundation can resist the large static, cyclic and impact loads with an adequate safety margin and without excessive deformations. Failure during cyclic loading may be associated with large cyclic displacements or rotations and/or large cyclically induced average displacements (horizontal sliding, vertical settlements) or rotations or a combination of such displacements.

The pore pressure accumulation and the strain accumulation method described in Chapter 5 allows prediction of strain and pore pressure development and thus the cyclic strength and associated shear strains of a soil element under a defined (static and cyclic) stress history. The methods can be combined with bearing capacity analysis as described by Andersen and Lauritzen (1988) to predict the cyclic capacity of foundation subjected to a defined storm load history.

The uncertainty connected to these type of calculations has been evaluated for Drammen clay, for a soft offshore clay and for Oosterschelde sand, NGI (1998-1). These materials are included in the database, NGI (1998-2). Some comments and references regarding uncertainties connected to the material properties of gravel and rockfill are included.
7.4.2. Clay

To derive guidelines for the uncertainty in the cyclic capacity of a clay, the clay materials included in the database were evaluated. The following steps were considered:

- Determination of cyclic and average strain contours
- Determination of equivalent number of cycles by strain accumulation
- Determination of cyclic shear strength
- Uncertainty in static undrained shear strength
- Calculation of failure envelope

The coefficient of variation, COV, was estimated for typical shear strain contours. For the offshore clay the COV can be taken as 10 to 15% and for Drammen clay a COV of 5-10% is more typical.

The cyclic shear strength is defined as the maximum shear stress $\tau = (\tau_a + \tau_{cy})$ of a cyclic load history where an unacceptable shear strain $\gamma = (\gamma_a \pm \gamma_{cy})$ is reached. A series of Monte Carlo simulations were carried out for the Offshore Clay. The load histories contained the same number of waves of a given amplitude, their sequence was simply arranged differently for each simulation, ascending, descending and random. The strain contours were varied; average contours, contours systematically at the 90-percentile boundary of the average contours and strain contours that were a "low asymptote" at a number of cycles greater than 10000.

The evaluation showed that the large variation in load sequence resulted in essentially the same average cyclic shear strength and a coefficient of variation of maximum 5%, or in other words a bias of 1 with a COV of 5%. Using strain contours at the 90-percentile boundary of the average contours resulted in a reduction in cyclic shear strength of 5% and a COV of 3% and finally the "low asymptote" strain contours gave a maximum reduction of cyclic strength of 12% and a COV of 2%.

The uncertainty of static undrained shear strength was evaluated for the two sites (the Offshore site and Drammen) by simple statistical analyses for laboratory strength tests and for in-situ field tests (CPT and field vane). This indicates a COV in the order of 5 to 10% for undrained shear strength from triaxial compression and direct simple shear and 10 to 15% for the field tests and for triaxial extension tests.

The uncertainties connected to cyclic strength envelopes (see Figures 2 and 3) and strength correction curves (see Figure 4) based on triaxial and direct simple shear tests, were evaluated using the above described uncertainties. For both Offshore clay and Drammen clay a bias of 1.0 and a COV of 15% are recommended as the most representative values to be used in bearing capacity analysis.
Fig. 2: Cyclic strength envelopes and recommended uncertainty for Offshore clay (Data from By and Skomedal, 1992)
Fig. 3: Bias and COV for curves showing number of cycles to failure as a function of average and cyclic shear stress for Drammen clay, DSS tests. (Data from Andersen and Heien 1986)
7.4.3. Sand

The uncertainty study performed within the PROVERBS project, NGI (1998-1), considered only the diagrams of permanent pore-pressure build-up under cyclic loading of Oosterschelde sand. Other components entering the stability analysis of breakwaters were not included. The
uncertainty connected to the assessment of a representative relative density, $D_r$, and the friction angle $\phi'$ is thus not treated here. Figure 5 shows relations between $\phi'$, porosity and mean normal consolidation stress and gives an impression of the range of variation.

**Fig. 5:** Relationship and variation range of friction angle vs. relative density and normal stress for sands

Even though a rather comprehensive cyclic test program has been conducted on Oosterschelde sand it was not possible to evaluate a statistical uncertainty based on the data alone. The uncertainty assessment had to be based on a combination of experience from extensive cyclic testing of sand at NGI during the last 15 years. It turned out that it was not possible to differentiate neither between relative densities nor between test types (triaxial vs. simple shear).

It is believed that the uncertainties in the cyclic behaviour of sand arise primarily from factors such as sample preparation method, effect of precycling and representativeness of the laboratory test specimen for the in situ conditions. The uncertainty in the pore pressure contours for sand is probably not larger than for the strain contours for clay.
The best estimate and recommended uncertainty was set to a bias of 1.0 and a coefficient of variation of 15%. Figures 6 and 7 illustrate this variability for two of the pore pressure diagrams for Oosterschelde sand and the numbers should be used with caution.

Fig. 6: Uncertainty in permanent pore pressure contours for a COV of 30%, Oosterschelde sand, Triaxial tests, Dr = 60%
7.4.4. Gravel and rockfill

For gravel and rockfill the main input parameters in the stability analysis for static and pulsating loads are the unit weight and the friction angle. The two parameters are correlated. A dense gravel will have a higher friction angle than a loose gravel. Figure 8 gives a summary of available rockfill data gathered by Leps (1970). The lower bound and upper bound ranges give a rough indication of the range of variation and may be used to estimate a COV.
7.5. UNCERTAINTIES CONNECTED TO SOIL PARAMETERS IN FOUNDATION STIFFNESS CALCULATION

7.5.1. General

The soil parameters entering a stiffness calculation for submerged soils are primarily the shear modulus. Poisson's ratio can be assumed to be 0.5 for undrained loading during a wave cycle or during impact loading caused by breaking waves. The shear modulus depends primarily on the void ratio and the consolidation stresses, the load history and the stress or strain level. In FE analysis the stress distribution can be modelled with good accuracy. For lumped spring stiffness solutions an additional uncertainty is introduced in the attempt to model a highly non-uniform stress and strain regime with one single modulus value.

Fig. 8: Relationship and variation range of friction angle vs. relative density and normal stress for gravel and rockfill (Leps, 1970).
7.5.2. Clay

The uncertainties in the shear modulus of an Offshore clay and Drammen clay were evaluated based on available laboratory data. Since not enough data were available to do traditional statistical estimates, the uncertainty evaluation was also based on a range of data available for other clays and correlations developed at NGI for a number of offshore clays. For the initial shear modulus, $G_{\text{max}}$, a bias of 1.0 and a COV of 15% are recommended. The same bias and COV are also recommended for the shear modulus during cycling. Figures 9 and 10 illustrates the uncertainty connected to $G_{\text{max}}$ for the two clays sites considered while Figures 11 and 12 show the bias and COV for shear modulus vs. cyclic stress level and number of waves for Drammen clay and offshore clay respectively.

**Fig. 9:** Bias and COV in initial shear modulus ($G_{\text{max}}$) for Offshore clay. Direct simple shear tests (Data from NGI files)
CHAPTER 7  UNCERTAINTIES

Fig. 10: Bias and COV in initial shear modulus ($G_{\text{max}}$) for Drammen clay. Direct simple shear tests (Data from Andersen and Heien, 1986)

Fig. 11: Bias and COV in secant shear modulus for 1 to 1000 cycles, Drammen clay, direct simple shear tests, OCR = 1, 4 and 10 (Andersen and Heien, 1986).
Fig. 12: Bias and COV in secant shear modulus for 1 to 1000 cycles, Offshore clay, triaxial tests, OCR = 1.5 (Data from NGI files)
7.5.3. Sand

The shear modulus of sand is strongly related to the void ratio and relative density, the effective stress and thus to the pore pressure development and to the stress level. The pore pressure accumulation method allows prediction of pore pressure generation and an equivalent number of cycles $N_{eq}$ for a given stress history. Diagrams showing the cyclic shear strain as a function of average and cyclic shear stress and number of cycles can then be used to calculate the shear modulus.

The uncertainties connected to prediction of shear moduli using this procedure has not been explicitly evaluated. The uncertainty connected to the modulus is expected to be of the same order as the prediction of pore pressure. The main uncertainties are probably connected to factors such as sample preparation method, effect of pre-cycling and representativeness of the laboratory test specimen for the in situ conditions regarding relative density, stress situation and fabric.

7.5.4. Gravel and rockfill

The amount of test data on shear modulus and damping relationships based on cyclic loading on gravel and rockfill materials is increasing, see Rollins et al. (1998). The PROVERBS database contains only results from triaxial tests on Oroville rounded to subrounded river gravel with $D_r = 84\%$.

In many cases the rubble mound will consist of blasted and crushed rock of angular shape and rough surface. The installed relative density of the rubble mound will normally be connected with uncertainty and will be difficult to control. Cyclic and impact wave loading on the breakwater structure will gradually densify the material underneath the base. A statistical treatment is not possible and the effect of a relative wide range of shear moduli should be investigated when evaluating the spring stiffness contribution of the rubble mound.

7.6. UNCERTAINTIES IN TO SOIL PARAMETERS IN SETTLEMENT/DISPLACEMENT CALCULATIONS

7.6.1. Types of settlements and displacements

The settlement (i.e. the vertical displacement) and the lateral and rotational (tilt) displacements of a breakwater structure can be significant design factors for breakwaters on
normally and slightly overconsolidated clays and loose sandy and silty subsoils. For medium
dense sands and hard clays the compressibility is low and the elastic moduli so high that the
installation tolerances probably will dominate provided the design against foundation
instability is adequately performed.

The displacements can be described as a sum of displacement contributions as shown in
Figure 13 and explained below:

a) Under static load
- Initial undrained settlement/displacement when installing and ballasting a breakwater
  structure
- Primary consolidation settlement under added weight of rubble mound and breakwater
  structure; pore pressure dissipation
- Secondary (long term creep) settlement

b) Under cyclic load
- Accumulated displacements due to plastic shear strains caused by cyclic loading, partly
  yielding and redistribution of stresses and partly pore pressure induced reduction in
  strength and stiffness,
- Settlements/displacements caused by wave load induced compaction (volumetric strain)
  of rubble mound and subsoil as generated pore pressure dissipates
Fig 13: Principal sketch of settlement contributions.

The assessment of the uncertainty connected to the calculation of the different settlement/displacement contributions will depend on the local soil conditions, the quality of the soil investigation and the analysis methods applied. The uncertainty connected to the settlement contribution caused by wave loading effects, is inevitably influenced by the uncertainty connected to the load level and load history.

For each of the listed settlement contributions various calculation methods exist spanning from simple elastic solutions for strip or rectangle on elastic halfspace to nonlinear 1-2D finite element consolidation analysis methods and empirical methods for assessment of accumulated strains and pore pressure. In the following the soil parameters involved in calculation of the different are commented on and in Table 2 at the end of this chapter a summary of recommended values for bias and COV are presented for the different soil parameters involved in calculation of settlements and consolidation time.
### 7.6.2. Initial settlements

Initial settlements are calculated based on assessment of the undrained Young's modulus, $E_u$, of the soil. Under undrained conditions no volumetric strain takes place and thus Poisson ratio, $\nu = 0.5$ and $E_u = 3\,G$

For clays $G$ and $E_u$ is related to the undrained shear strength, $c_u$, and the overconsolidation ratio, OCR. The undrained shear strength is again related to the effective overburden pressure, the preconsolidation stress and the plasticity index. The value is further dependent on the stress level, $\tau/\tau_u$. A high weight compared to the bearing capacity, i.e. a low safety factor, will give lower values of $G$ and $E_u$.

In sands $G$ and $E_u$ is related to the relative density, $D_r$, the effective normal stress, $\sigma'_v$, and the stress level.

### 7.6.3. Primary consolidation settlements

The primary consolidation settlements are normally calculated based on assessment of the confined compression modulus from oedometer tests. In the stress range from the present effective overburden pressure, $p'_o$, to the apparent preconsolidation stress, $p'_c$, the reload modulus could be used, and above $p'_c$ the virgin confined compression modulus should be used.

\[
s = \int (p'_c - p'_o)/M_r(z, \sigma'_v) \cdot dz + \int (\sigma'_v - p'_c)/M(z) \cdot dz
\]

where 
- $s$ = consolidation settlement
- $p'_c$ = (apparent) preconsolidation stress = OCR($z$) $\cdot p'_o$
- $p'_o$ = effective overburden pressure = $\gamma(z) \cdot dz$
- $\sigma'_v(z)$ = effective stress under weight of soil, gravel mound and structure
- $M_r$ = reload modulus of soil in the range $p'_o$ to $p'_c$
- $M(z)$ = virgin confined compression modulus above $p'_c$

### 7.6.4. Consolidation time and degree of consolidation

The consolidation process will be of great importance for projects where staged construction is required to achieve the required stability of the rubble mound and structure. Monitoring of pore pressure dissipation and settlement development is recommended for this type of projects.

\[
t = T \cdot H^2/c_v
\]
where \( t \) = time since application of load
\( T \) = time factor dependent on the degree of consolidation
\( H \) = length of drainage path (i.e. half the thickness of a layer with drainage on both sides)
\( c_v \) = coefficient of consolidation = \( k \cdot M / \gamma_w \)
\( k \) = permeability
\( M \) = modulus of confined compression
\( \gamma_w \) = unit weight of water

7.6.5. Secondary long term creep settlements

\[
s = H \cdot C_\alpha \cdot \Delta \log_{10}(t)
\]

where \( s \) = secondary settlement
\( H \) = thickness of layer
\( C_\alpha \) = rate of secondary compression
\( t \) = time

7.6.6. Accumulated displacements due to plastic shear strains caused by cyclic loading

There exist several material models that have been developed with the aim to predict generation of displacements caused by cyclic loading. The strain accumulation method and the pore pressure accumulation method (Andersen & al., 1992 and Andersen 1995) allows prediction of the pore pressure generation and shear strain development in clays and sands under cyclic loading. The method can also account for pore pressure dissipation caused by drainage and redistribution of pore pressure that occurs simultaneously with the pore pressure generation. The method is implemented in the FE program BIFURC (NGI, 1995) and is described in more detail in Chapter 5.

The uncertainty connected to prediction of cyclic shear strength and stiffness of two slightly overconsolidated offshore clays, Drammen clay and pore pressure generation of Oosterschelde sand was evaluated and presented as a part of PROVERBS in NGI (1998-1). The evaluation indicated that there is more uncertainty connected to the strain contour diagrams with respect to accumulated (average) shear strain than to cyclic shear strain. The accumulated shear strain can be interpreted as a reduced shear modulus under average loading. Comparing with the recommended Bias =1.0 and COV = 0.15 for the secant shear modulus for cyclic loading (see Chapter 7.5.2) the prediction of accumulated strain should be carried out with a somewhat higher COV = 0.20.
7.6.7. **Settlements caused by wave load induced compaction of rubble mound and subsoil**

For contractive materials, which have potential for considerable volume reduction (i.e. loose sands and silts and soft clays), cyclic loading may generate a gradual accumulation of settlement.

The method described by Yasuhara and Andersen (1991) can be used to estimate recompression of normally consolidated Drammen clay after cyclic loading. The recommended expression for calculation of the increase in vertical strain caused by dissipation of pore pressure generated by cyclic loading is as follows:

\[
\varepsilon_{vt} = 0.67 \frac{C_r}{(1+e_c)} \cdot \log \left( \frac{1}{1-\frac{\Delta u}{\sigma'_{vc}}} \right) 
\]

where
- \( \varepsilon_{vt} \) = recompression volumetric strain
- 0.67 = reduction factor for \( C_r \) based on observations in laboratory test
- \( C_r \) = recompression index from oedometer tests
- \( e_c \) = void ratio of clay at start of pore pressure generation
- \( \Delta u \) = excess pore pressure that dissipates
- \( \sigma'_{vc} \) = current vertical effective stress

The settlement is calculated by integration of the vertical strain over the thickness of the clay layer(s).

The material uncertainty of this model is thus composed of the uncertainties connected to \( C_r \), \( e_c \) and \( \Delta u \) and the reduction factor. The contribution of \( e_c \) is considered to be small and neglectable compared with the other contributions. In the tests on Drammen clay the reduction factor was estimated by regression analysis and varied in the range 1.2 to 2. The uncertainty connected to \( \Delta u \) is evaluated in Ch. 7.4.3.

For non-cohesive soils a similar model can be used as shown by Allard & al. (1994) and Andersen & al (1994) based on interpretation of centrifuge model tests on a gravity platform on very dense sand. For loose sands the uncertainty connected to \( C_r \) and the reduction factor to be applied is expected to increase considerably. However, no statistical evaluation of these parameters is presently possible.

The work by Youd (1972) showed that cyclic shear strain reversals could compact a uniform sand well beyond a relative density of 100%. Numerous studies of the cyclic compaction have been reported in connection with evaluation of the liquefaction potential of sandy soils and large number of references could be added on this matter. The long term cyclic compaction of a loose cohesionless subsoil can thus be substantial over the lifetime of a breakwater.
Tab. 2: Summary of uncertainties involved in settlement analysis with indicated range of uncertainty in soil parameters

<table>
<thead>
<tr>
<th>Parameter/Calculation</th>
<th>Sand</th>
<th>Clay</th>
<th>Gravel/ Rockfill</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial conditions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Void ratio</td>
<td>$e_o$</td>
<td>1.00/10-15</td>
<td>1.00/10-30</td>
</tr>
<tr>
<td>Maximum void ratio</td>
<td>$e_{\max}$</td>
<td>1.00/5-10</td>
<td>-</td>
</tr>
<tr>
<td>Minimum void ratio</td>
<td>$e_{\min}$</td>
<td>1.00/5-10</td>
<td>-</td>
</tr>
<tr>
<td>Relative density</td>
<td>$D_r$</td>
<td>1.00/5-15</td>
<td>-</td>
</tr>
<tr>
<td>Submerged unit weight</td>
<td>$\gamma'$</td>
<td>1.00/5-10</td>
<td>1.00/3-10</td>
</tr>
<tr>
<td>Plasticity index</td>
<td>$I_p$</td>
<td>-</td>
<td>1.00/5-10</td>
</tr>
<tr>
<td>In situ vertical eff. stress</td>
<td>$p_o'$</td>
<td>1.00/5-10</td>
<td>1.00/5</td>
</tr>
<tr>
<td>Preconsolidation stress</td>
<td>$p_c'$</td>
<td>-</td>
<td>1.00/10-20</td>
</tr>
<tr>
<td><strong>Initial settlements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Undrained Young's modulus</td>
<td>$E_u$</td>
<td>1.00/10-20</td>
<td>1.00/10-20</td>
</tr>
<tr>
<td>Poisson's ratio (undrained)</td>
<td>$\nu$</td>
<td>1.00/5-10</td>
<td>1.00/5-10</td>
</tr>
<tr>
<td><strong>Primary consolidation settlement</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conf. Compr. modulus $p_o'$ to $p_c'$</td>
<td>$M_o$</td>
<td>1.00/15-25</td>
<td>1.00/20</td>
</tr>
<tr>
<td>Avg. conf. compr. modulus</td>
<td>$M$</td>
<td>1.00/15-20</td>
<td>-</td>
</tr>
<tr>
<td>Virgin compression index</td>
<td>$C_c$</td>
<td>1.00/10-20</td>
<td>1.00/10-15</td>
</tr>
<tr>
<td><strong>Consolidation time</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permeability</td>
<td>$k$</td>
<td>1.00/20-40</td>
<td>1.00/20-40</td>
</tr>
<tr>
<td>Coefficient of consolidation</td>
<td>$c_v$</td>
<td>1.00/10-30</td>
<td>1.00/10-20</td>
</tr>
<tr>
<td><strong>Secondary settlement</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Rate of secondary compression</td>
<td>$C_\alpha$</td>
<td>-</td>
<td>1.00/15-25</td>
</tr>
<tr>
<td><strong>Accumulated displacements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Accumulated strains</td>
<td>$\gamma_p$</td>
<td>-</td>
<td>1.00/10-20</td>
</tr>
<tr>
<td>Accumulated pore pressure</td>
<td>$U_p$</td>
<td>1.00/10-20</td>
<td>1.00/25</td>
</tr>
<tr>
<td><strong>Cyclic compaction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Void ratio at start of loading</td>
<td>$E_c$</td>
<td>1.00/10-20</td>
<td>1.00/10-20</td>
</tr>
<tr>
<td>Recompression index</td>
<td>$C_r$</td>
<td>1.00/20-30</td>
<td>1.00/20-30</td>
</tr>
</tbody>
</table>
7.7. **MODEL UNCERTAINTIES**

7.7.1. **Survey of model uncertainties**

Model uncertainties are presented in Section 3.8 of Volume I (Tab.3-2). These uncertainties are, strictly speaking, only valid for the models presented in that volume. It concerns a limited number of relatively simple equations. Apart from these equations also more sophisticated models, like numerical models, are discussed Chapters 3, 4, 5 and 6 of this volume. These uncertainties will be discussed in this section with the help of Figure 14, which presents principally the same scheme as used in Volume I (Figure 21).

The meaning of the different boxes is as follows:
- The parameters with two circles around are the input parameters with stochastic character
- The models are indicated in the rectangular boxes
- The parameters with one circle around are stochastic parameters, which are found with the help of these models. The final output concerns such parameters: the values of the limit state function \( g_a, g_b, g_c \) and \( g_d \).

7.7.2. **“Dyn I” models for determination of spring coefficients**

These models are discussed in subsection 3.4.3 of Volume I and in Section 3 of this Volume. The inaccuracies (Volume I, table 3-2) have been estimated based on several hindcasts of large-scale model tests (section 3.3.2 in this volume) and also on full scale measurements reported for offshore gravity platforms.

The simplest models are analytical equations as presented in subsection 3.4.3 of volume I. They are based on the assumption that the foundation of the wall can be simplified to behave as an elastic homogenous half-space. This seems reasonable if the rubble mound is relatively thick \( (h_r > 0.5B_c) \) or if it is thin \( (h_r < 0.2B_c) \) and overlying a homogeneous subsoil to a depth of about 0.5 \( B_c \). The lower values of \( m_{k_x} \) and \( m_{k_y} \), presented in table 3-2 of Volume I are valid for these models. In other cases the larger values need to be assumed. Finite element (FE) models are especially useful for cases with a more complicated geometry of rubble mound and layered subsoil. The results of those models can reduce the uncertainty to the lower values of table 3-2.
Fig. 14: “Operational fault trees” (for explanation: see text)
7.7.3. “Dyn II” models for determination of natural period

These models are discussed in subsection 3.4.3 of Volume I and Chapter 3 of this report. The inaccuracies have been estimated by considering the results of several hindcasts of large-scale tests (section 3.3.3 in this volume). The absolute values of the ratios between calculated and measured natural periods were not larger and sometimes smaller than those corresponding to differences in spring coefficients.

The prototype tests discussed in section 3.4 of this Volume confirm this. All measured deviations, for as far not due to inaccuracies in the measurement system, can be explained by the uncertainties in the soil parameters and the uncertainties in the function "Dyn I". Therefore no additional uncertainty need to be taken into account for this submodel.

7.7.4. “Dyn III” models for determination of dynamic load factor

The inaccuracy of the function expressed by Figure 3-10 of Volume I is mainly due to the simplification of the load as a triangular force-time function. According to table 3-2 in Volume I, this can be expressed by a COV of 5% to 10%. The smaller value is justified for the low values of \( \nu_D \), because the \( \nu_D \)-value is not very sensible for the force-time function in that region. The 5% is also justified for the larger values of \( \nu_D \) because \( \nu_D \) cannot be much larger than 1.5. A more accurate prediction can be found with a 2DOF mass-spring model and force-time functions as found from flume tests. See Chapter 3. Then, the lower value of the COV may also be justified for \( 0.5 < \nu_D < 1.5 \).

7.7.5. “Instant" models for determination of instantaneous pore pressure

The expected values of uplift force \( F_u \) and its lever arm, \( l_{Fu} \), in a feasibility study are derived from the triangular pressure distribution (Volume I, equations 3-22). The uncertainty expressed in the coefficients \( m_{Fu} \) and \( m_{IFu} \) presented in section table 3-2 of Volume I, is derived from calculations with other pressure distributions, as discussed in section 3.5 of Volume I and chapter 4 of this Volume.

The estimate of inaccuracy can be reduced considerably by performing more sophisticated calculations as discussed in chapter 4 of this volume with variation of the geometry and the soil parameters. Then a COV of 5% for both parameters seems to be possible.
7.7.6. **“Degra” models for determination of degradation or residual pore pressure.**

The models are described in Chapter 5 of this volume. The uncertainties associated with the “pore pressure accumulation method” for prediction of residual pore pressure and the “strain accumulation method” for prediction of strength degradation of clay under cyclic loading are discussed in detail in NGI (1998-1). The uncertainties connected to the prediction of pore pressure generation and strength degradation due to cyclic loading is believed to be best represented by a bias of 1.0 and a coefficient of variation of 15% to be applied over the predicted shear strength, $c_{uy}$. These estimates apply to structures on medium dense to dense sand and soft clays with a low overconsolidation ratio, and when the result are combined with the approach for bearing capacity analysis specifically developed for consideration of degradation effects.

7.7.7. **“Stab I” models for determination of $F_h$, $F_u$, $B_z$ and $F_{hu}$**

No uncertainties need to be considered for the equations to find $F_h$, $F_u$ and $B_z$ from previously found parameters. The equation for the horizontal seepage force in the rubble, $F_{hu}$, proposed in Volume I (eq. 3-26) and also applied here in chapter 6 (eq. 7-4), is, however, a rough approximation. Its uncertainty is partly due to the uncertainty about the location of the sliding plane and partly due to the uncertainty about the pore pressure distribution in the rubble foundation. This distribution is based on the assumption of a triangular pressure distribution in horizontal direction and hydrostatic pressure distribution in vertical direction. With the more sophisticated calculations, mentioned in subsection 7.7.5, a correction factor could be derived for $F_{hu}$, such that the COV can be reduced to ca 10%.

7.7.8. **“Stab II” models for determination of limit state functions**

The most simple of these models are discussed in subsections 3.7.2 to 3.7.4 of Volume I; the more sophisticated models are briefly mentioned in subsection 3.7.5 of Volume I and are discussed extensively in chapter 6 of this Volume.

No explicit uncertainty needs to be taken into account for failure mode “a” – “sliding of structure along the base”. The uncertainty is included in the uncertainty of the friction angle.

The model inaccuracies for the “bearing capacity failure” modes have to do with a large number of assumptions explicitly or implicitly made in these models. The main assumptions deal with:

- the shape of the rupture surface in two dimensions
- idem in the 3rd dimension
- the soil behaviour in the rupture surface: direction of principal stresses, role of dilatancy, role of pore pressure, size of grains, thickness of shear zone, etc.
- the interaction between different soil layers, especially if the stiffness differs considerably

An idea of the effect of the assumptions can be gained by comparing different models with different assumptions, e.g. pure plastic models for straight rupture surfaces (e.g. some of the equations presented in Volume I), pure plastic models for circular rupture surfaces including the effect of dilatancy, elastic-plastic FE models.

The best verification of the reliability would be the comparison with well-monitored tests at large scale or in a centrifuge. Unfortunately, such tests are very rare. The large test in Hannover did not clearly show failure, although near failure probably occurred in one of the analysed tests. Failure also occurred in the centrifuge tests. This certainly gives a good impression of the reliability of the whole prediction method for a foundation on sand. However, the complicated role of (negative) pore pressures (“Instant” model) does not allow for a firm conclusion about the “Stab II” model alone.

The estimated inaccuracies presented in table 3-2 of Volume I (COV = 20%) are based on comparisons with the simple limit state equations with the more sophisticated models, as reported by Hölscher et al. (1998). The more sophisticated methods include FE models using PLAXIS. Large differences in safety factors are found in cases with large differences in soil properties between a stiff and strong rubble mound over a flexible and weak soil. In more homogenous cases the differences are smaller, but still significant.

The equations discussed in chapter 6 of this volume are significantly more sophisticated than those discussed in Volume I. As a first approximation a COV = 10% may be assumed. However, in preliminary and in detailed design, it is recommended to apply several different models and to vary loads and soil properties to get a more reliable estimate of the uncertainty in the models.

Even for detailed design, usually only 2-D models will be used. An idea of the possibly favourable 3-D effects discussed in subsection 3.7.4 of Volume I, can be found with the equations presented there. The accuracy however is very limited. In cases where the effects may be very significant (e.g. for wave impacts and deep sliding planes) and thus essential for the design, it is advised to perform (semi) 3-D calculations.

7.7.9. Model uncertainty associated with settlement calculations

The model uncertainty connected with settlement calculation depends on the sophistication of the analysis model. Prediction of stress distribution can vary from simple spread of load
assumption, elastic half space (Boussinesq equations) to full 3-dimensional FE analysis. In
general the prediction of stress distributions under vertical loads are not very sensitive to the
material stiffness and the uncertainty in settlement predictions is primarily connected to the
uncertainty in the soil parameters.

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CHAPTER 8: INFLUENCE OF DESIGN PARAMETERS
- STABILITY ANALYSIS ON FEASIBILITY LEVEL -

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ABSTRACT

Within this chapter the influence of design parameters on the relevance of particular load cases and failure modes for the stability of a common breakwater type has been investigated. This type is a vertical breakwater on a thin rubble layer on top of a subsoil of coarse sand or gravel. Many different designs of this breakwater type under many different wave conditions - however, no wave impacts - have been considered.

The parameter studies have shown that especially the eccentricity of the caisson dead load, wave height and shear strength of the subsoil determine the relevance of different load cases and failure modes. Bearing capacity failure in the subsoil and sliding along the base are the most important failure modes. Beside this, the limitation of eccentricity is in many cases the decisive design criterion. The most relevant load case is wave crest, but wave through should not be neglected at all.

Key words: Design process, foundation stability, relevant failure modes, parameter study

8.1. INTRODUCTION

Within this report the stability of a vertical breakwater is analysed in order to specify the influence of different combinations of input parameters on the relevance of the different modes of failure. This study gives a survey of critical design conditions and helps the designer to get a better understanding of the difficult and manifold relations between input
parameters and breakwater stability to make design more convenient. The concept of the failure analysis is introduced in detail. A general idea about the design of vertical breakwaters from viewpoint of geotechnical engineers is given as well as some hints on relevant failure modes. Detailed explanations can be found in (GOLÜCKE; PERAU; RICHWIEN, 1998).

8.1.1. General considerations

In this study the stability of breakwaters is described by a sum of different failure mechanisms concerning the subsoil beneath the breakwater, which have to be investigated separately. Different from common problems the bearing capacity is additionally influenced by seepage forces in the subsoil. A set of equations and inequalities therefore has to cover all possible failure modes and will give the breakwater dimensions with respect to the most relevant type of failure.

Since breakwaters are special constructions with typical subsoils and very special relevant loading, it is possible to reduce the number of parameters, because many of them are correlated. Therefore the first step is to explicite the correlations of the parameters in order to reduce their number. In a second step the remaining system will be examined to reduce the range within each parameter can vary to find relevant parameter combinations for the later design process.

8.1.2. Load cases and failure modes

The loading sequence of a breakwater due to waves may be studied in three specific cases: still water level, wave crest and wave trough. Within load case wave crest loading in case of non-breaking waves (quasi-static loads) is calculated according to Vol. IIa, Section 4.1. Since only thin rubble layers are considered, the probability of impact loads is assumed to be very low. So, impact loads are not investigated within this study. Load case wave trough is calculated according to Vol. IIa, Section 4.2. The different load cases are presented in Fig. 1.
In this study only the limit state of subsoil is considered whereas serviceability is not investigated. The analysis is confined to feasibility level. Herein the failure modes and restrictions introduced in Vol. 1, Section 3.7 are considered. Within failure mode bearing capacity failure in rubble mound only a sliding plane through the rubble mound and along top of the subsoil is investigated because the rubble mound is assumed to be thin. Additionally, uplift and overturning of the caisson are checked (see Fig. 2).
The limit state equations used in this study are given below. They correspond with the limit state equations described in Vol. I, Section 3.7.2. The variables are named according to the notations of this document. No safety concept is involved.

- Sliding along the base:
  \[ F_{v,\text{base}} \cdot \mu_{\text{base}} - |F_{h,\text{base}}| > 0 \]  
  with \( \mu_{\text{base}} \) = friction coefficient between caisson and rubble mound: 
  \[ \mu_{\text{base}} = \tan\left(\frac{2}{3} \cdot \phi'_R\right) \]  

- Bearing capacity failure in rubble mound
  \[ F_{v,\text{sub}} \cdot \mu_{\text{sub}} - |F_{h,\text{sub}}| > 0 \]  
  with \( \mu_{\text{sub}} \) = friction coefficient between rubble mound and subsoil:
  \[ \mu_{\text{sub}} = \tan\left(\min\{\phi'_R, \phi'_S\}\right) \]  

- Bearing capacity failure in subsoil
  \[ F_{v}^R \left( F_{h,\text{sub}}, F_{v,\text{sub}}, M_{\text{sub}}, \phi'_S, c'_S \right) - F_{v,\text{sub}} > 0 \]  
  with:
  \[ F_{v}^R = \frac{1}{2} \cdot \gamma'_S \cdot \left( B_{c,\text{eff}} \right)^2 \cdot N_\gamma \cdot i_\gamma \]  
  \[ N_\gamma = 2 \cdot \left[ \cdot \frac{1 + \sin \phi'_S}{1 - \sin \phi'_S} \right] \cdot \tan \phi'_S \]  
  \[ i_\gamma = \left( 1 - \frac{F_{h,\text{sub}}}{F_{v,\text{sub}}} \right)^3 \]  

- Limitation of eccentricity
  \[ \left| \frac{M_{\text{base}}}{B_c \cdot F_{v,\text{base}}} \right| \leq 0.3 \]  

- Overturning
  \[ \left| \frac{M_{\text{base}}}{B_c \cdot F_{v,\text{base}}} \right| \leq 0.5 \]  

- Uplift
8.1.3. Assumptions and simplifications

Some general assumptions should be made in order to reduce the complexity of the problem. The breakwater on its subsoil is suggested as a two-dimensional problem. So, it is treated as a classical strip foundation. Furthermore the base of the breakwater is assumed as plane and horizontal. Subsoil and rubble layer are assumed to be constantly below water level.

Overall instabilities due to breakwater dead load, local instabilities like erosion and scour are not considered in this study. They have to be excluded constructively in advance. Otherwise they may affect the failure modes. Since the study concentrates on breakwaters with relatively thin rubble layers (vertical breakwaters), the contribution of the rubble layer's shear strength to the bearing capacity of the system can be neglected.

All soils are assumed to underlie the Mohr-Coulomb failure criterion with homogeneous and isotropic properties in the vicinity of the breakwater. Also only drained conditions of subsoil and rubble mound are taken into account. Beside this, the parameter study is restricted to non-cohesive subsoils.

The pressure head in the thin rubble layer is assumed to vary linearly from the value at the seaward edge of the caisson bottom to mean sea level at the harbourside edge. The same horizontal pressure head distribution is assumed in the subsoil. The vertical variation of the pore pressures is assumed to be hydrostatic. However, the horizontal resultant of the pore pressure gradient in the subsoil is not taken into account, i.e. seepage forces in the subsoil are neglected.

8.2. PARAMETER STUDY

Before performing the failure analysis of caisson breakwaters all input parameters have to be specified. Additionally, some restrictions have to be made concerning the relevant range of the parameters in order to avoid unrealistic parameter combinations.

8.2.1. Dimensional parameters

All parameters which are needed to calculate the stability of a vertical breakwater are listed in Tab. 1. Two new parameters have been introduced: Firstly, the eccentricity of the caisson dead load \( e_c \), which is regarded positive in seaward direction (see Fig. 1), and secondly, the
geometry factor $\alpha_c$, which describes the relation of the real cross-section of the caisson to a rectangular cross-section (see Fig. 3):

$$\alpha_c = \frac{\text{area of real caisson cross-section}}{B_c \cdot h_c} \leq 1.0 \quad (9)$$

The unit weight $\gamma_c$ of the caisson is the average value over its concrete frame and sand fill.

![Fig. 3: Geometry factor $\alpha_c$ (GOLÜCKE; PERAU; RICHWIEN, 1998)](image)

<table>
<thead>
<tr>
<th>Tab. 1: Dimensional parameters</th>
</tr>
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<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td><strong>water and wave parameters</strong></td>
</tr>
<tr>
<td>height of still water level (SWL)</td>
</tr>
<tr>
<td>Direction of wave propagation</td>
</tr>
<tr>
<td>height of SWL in deep water</td>
</tr>
<tr>
<td>Significant inshore wave height</td>
</tr>
<tr>
<td>wave period of spectral peak</td>
</tr>
<tr>
<td><strong>Geometric parameters</strong></td>
</tr>
<tr>
<td>width of caisson in x-direction</td>
</tr>
<tr>
<td>Geometry factor of caissons cross-section</td>
</tr>
<tr>
<td>Eccentricity of caissons dead load</td>
</tr>
<tr>
<td>height of caisson</td>
</tr>
<tr>
<td>height of rubble mound</td>
</tr>
<tr>
<td>Inclination of berm slope</td>
</tr>
<tr>
<td>width of rubble berm, averaged over berm height</td>
</tr>
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<td><strong>unit weights</strong></td>
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<td>caisson</td>
</tr>
<tr>
<td>rubble mound</td>
</tr>
<tr>
<td>subsoil</td>
</tr>
<tr>
<td><strong>shear strength parameter</strong></td>
</tr>
<tr>
<td>angle of internal friction of rubble mound</td>
</tr>
<tr>
<td>angle of internal friction of subsoil</td>
</tr>
<tr>
<td>cohesion of subsoil</td>
</tr>
</tbody>
</table>
For the parameter study the variation range of each parameter has to be fixed. Here existing pragmatical values are considered, e.g. from the representative structures (LÖFFLER; KORTENHAUS, 1997). So, the variation ranges are set as listed in Tab. 2.

Tab. 2: Variation ranges of dimensional parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>water and wave parameters</td>
<td></td>
</tr>
<tr>
<td>$h_s$</td>
<td>$5.0 , m \leq h_s \leq 40.0 , m$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0^\circ \leq \beta \leq 60^\circ$</td>
</tr>
<tr>
<td>$h_{s0}$</td>
<td>$h_s \leq h_{s0} \leq 1.2 \cdot h_s$; min $h_{s0} = 15.0 , m$</td>
</tr>
<tr>
<td>$H_{si}$</td>
<td>$0.2 , m &lt; H_{si} \leq 13.0 , m$</td>
</tr>
<tr>
<td>$T_p$</td>
<td>$7.0s \leq T_p \leq 14.0s$</td>
</tr>
<tr>
<td>geometric parameters</td>
<td></td>
</tr>
<tr>
<td>$B_c$</td>
<td>design parameter</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>$0.7 \leq \alpha_c \leq 1.0$</td>
</tr>
<tr>
<td>$e_c/B_c$</td>
<td>$-0.2 \leq e_c / B_c \leq 0.2$</td>
</tr>
<tr>
<td>$h_c$</td>
<td>$0.8 \cdot (h_s + h_{c\text{rest}} - h_t) \leq h_c \leq 1.2 \cdot (h_s + h_{c\text{rest}} - h_t)$</td>
</tr>
<tr>
<td>$h_r$</td>
<td>$0.1 , m &lt; h_r \leq 2.0 , m$</td>
</tr>
<tr>
<td>$1:m$</td>
<td>$1:3 \leq 1:m \leq 1:15$</td>
</tr>
<tr>
<td>$B_{eq}$</td>
<td>$5.0m &lt; B_{eq} &lt; 30.0m$</td>
</tr>
<tr>
<td>unit weights</td>
<td></td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>$10 , kN/m^3$</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>$20 , kN/m^3 \leq \gamma_c \leq 23 , kN/m^3$</td>
</tr>
<tr>
<td>$\gamma'_R$</td>
<td>$8 , kN/m^3 \leq \gamma'_R \leq 12 , kN/m^3$</td>
</tr>
<tr>
<td>$\gamma'_S$</td>
<td>$6 , kN/m^3 \leq \gamma'_S \leq 11 , kN/m^3$</td>
</tr>
<tr>
<td>shear strength parameter</td>
<td></td>
</tr>
<tr>
<td>$\varphi'_R$</td>
<td>$30^\circ \leq \varphi'_R \leq 45^\circ$</td>
</tr>
<tr>
<td>$\varphi'_S$</td>
<td>$25^\circ \leq \varphi'_S \leq 45^\circ$ for $c_u = 0$</td>
</tr>
<tr>
<td></td>
<td>$15^\circ \leq \varphi'_S \leq 32^\circ$ for $c_u \neq 0$</td>
</tr>
<tr>
<td></td>
<td>$\varphi'_S = 0^\circ$ for $c_u$</td>
</tr>
</tbody>
</table>

- only thin rubble mounds considered

- not considered
Here quantity $h_{\text{crest}}$ describes the height of wave crest related to SWL (Fig. 1) and can be calculated as follows:

$$h_{\text{crest}} = H_{\text{max}} + h_0$$

with $h_0$ according to:

$$h_0 = \frac{\pi \cdot H_{\text{max}}^2}{L} \cdot \coth \left( \frac{2 \cdot \pi \cdot h'}{L} \right)$$

Within the calculation of different load cases maximum wave height $H_{\text{max}}$ and wave length $L$ have been derived according to:

$$H_{\text{max}} = 1.8 \cdot H_{\text{si}}$$

$$L = T_p \cdot \sqrt{g \cdot h^*} \quad \text{for } h^*/L < 0.05$$

$$L = \frac{g \cdot T_p^2}{2\pi} \cdot \tanh \left( \frac{2\pi \cdot h^*}{L} \right) \quad \text{for } 0.05 \leq h^*/L \leq 0.5$$

$$L = 1.56 \cdot T_p^2 \quad \text{for } h^*/L > 0.5$$

with:

$$h^* = \begin{cases} h_s \text{ for calculation of } h_{\text{crest}}/h_{\text{rough}} \\ h' \text{ for calculation of non\textendash breaking wave load} \end{cases}$$

Additionally, some restrictions are necessary to get appropriate parameter combinations. Tab. 2 shows the calculation of the caisson height $h_c$. Variations with $h_c < 1.0 \cdot (h_s + h_{\text{crest}} - h_r)$ lead to structures which are exposed to wave overtopping. Simultaneously in some cases - especially at small waves - the height of the caisson calculated in that way may be lower than the water depth in front of the caisson $h'$. To exclude such absurd variations it has to be guaranteed that the caisson height always exceeds $h'$:

$$h_c \geq h'$$

The wave steepness $s$ should be limited by the following restriction in order to get realistic wave loads:

$$0.018 \leq s = \frac{H_{\text{max}}}{L} \leq 0.1$$
The water level draw down in case of wave trough $h_{\text{trough}} = H_{\max} - h_0$ (see Fig. 1) can definitely not exceed the water depth at the caisson base:

$$h_{\text{trough}} \leq h'$$  \hspace{1cm} (16)

### 8.2.2. Dimensionless parameters

Usually it is more feasible to transfer dimensional parameters into dimensionless parameters which will be the input parameters of the calculations. Since the unit weight of water $\gamma_w$ is constant and the height of SWL $h_s$ is always given, these two parameters can be used as denominators in the dimensional analysis. The other parameters, which may be described as design parameters, have to be set in relation with them. The resulting ranges of the dimensionless parameters are listed in Tab. 3. Obviously, wave period $T_p$ could not be transformed into dimensionless form.

#### Tab. 3: Variation ranges of dimensionless parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
<th>in this report:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>water and wave parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0^\circ \leq \beta \leq 60^\circ$</td>
<td></td>
</tr>
<tr>
<td>$h_{s0}/h_s$</td>
<td>$\min h_{s0}/h_s = 15.0/h_s$</td>
<td></td>
</tr>
<tr>
<td>$H_{si}/h_s$</td>
<td>$0.0 &lt; H_{si}/h_s \leq 0.6$</td>
<td></td>
</tr>
<tr>
<td>$T_p$</td>
<td>$7.0s \leq T_p \leq 14.0s$</td>
<td></td>
</tr>
<tr>
<td><strong>Geometric parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_c/h_s$</td>
<td>design parameter</td>
<td></td>
</tr>
<tr>
<td>$e_c/B_c$</td>
<td>$-0.2 \leq e_c/B_c \leq 0.2$</td>
<td></td>
</tr>
<tr>
<td>$h_c/h_s$</td>
<td>$0.8 \cdot (h_s + h_{\text{crest}} - h_r)/h_s \leq h_c/h_s \leq 1.2 \cdot (h_s + h_{\text{crest}} - h_r)/h_s$</td>
<td></td>
</tr>
<tr>
<td>$h_v/h_s$</td>
<td>$0.1 &lt; h_v/h_s \leq 0.2$</td>
<td>only thin rubble mounds consid.</td>
</tr>
<tr>
<td>$1:m$</td>
<td>$1:3 &lt; 1:m \leq 1:1.5$</td>
<td></td>
</tr>
<tr>
<td>$B_{eq}/h_s$</td>
<td>$0.125 \leq B_{eq}/h_s \leq 6.0$</td>
<td></td>
</tr>
<tr>
<td><strong>unit weights</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_c \gamma_c/\gamma_w$</td>
<td>$1.4 \leq \alpha_c \cdot \gamma_c/\gamma_w \leq 2.3$</td>
<td></td>
</tr>
<tr>
<td>$\gamma'_{r}/\gamma_w$</td>
<td>$0.8 \leq \gamma'_{r}/\gamma_w \leq 1.2$</td>
<td></td>
</tr>
<tr>
<td>$\gamma'_{s}/\gamma_w$</td>
<td>$0.6 \leq \gamma'_{s}/\gamma_w \leq 1.1$</td>
<td></td>
</tr>
<tr>
<td><strong>shear strength parameter</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi'_r$</td>
<td>$30^\circ \leq \phi'_r \leq 45^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$25^\circ \leq \phi'_s \leq 45^\circ$</td>
<td>for $c'_s = 0$</td>
</tr>
</tbody>
</table>
### 8.3. DESIGN PROCESS

The main objective of a design process is to restrict the resulting loads due to breakwater and waves to such an intensity that the subsoil does not fail. In a given case some quantities especially wave and soil parameters are fix and can not be changed, whereas other parameters can be influenced, e. g. the unit weight and the geometry of the caisson or the thickness of the rubble layer. So, design of the caisson means to choose these parameters so that the limit state of the subsoil does not occur.

#### 8.3.1. Failure function

A relation of all dimensionless parameters (design variables and fixed quantities) has to be stated which describes the limit state of the structure at a certain parameter combination. This relation is given by a function \( F \) depending on the dimensionless parameters of Tab. 3:

\[
F(..., ..., ..., ..., ...) \geq 0
\]  

(17)

If this inequality is fulfilled, the structure will not fail. That means the present parameter combination is allowed and the design values can be chosen in that way.

Actually, it is not possible to describe a consistent failure criterion for the limit state of the subsoil, so the failure function \( F \) includes implicitly several sub-functions. These sub-functions represent every possible combination of load cases and failure modes on the basis of the input parameters. All sub-functions together form the limit state of the subsoil at a certain parameter combination. So, all possible load case-failure mode combinations, which have to be investigated for each parameter combination, can be combined in the following matrix:

<table>
<thead>
<tr>
<th>( \varphi_s' )</th>
<th>( 15^\circ \leq \varphi_s' \leq 32^\circ ) for ( c_s' \neq 0 )</th>
<th>not considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_s' = 0^\circ ) for ( c_u )</td>
<td>not considered</td>
<td></td>
</tr>
<tr>
<td>( c_s' / (\gamma_s' \cdot h_s) )</td>
<td>( 0 \leq c_s' / (\gamma_s' \cdot h_s) \leq 16 )</td>
<td>( c_s' = 0 )</td>
</tr>
<tr>
<td>( c_u / (\gamma_s' \cdot h_s) )</td>
<td>( 0.03 \leq c_u / (\gamma_s' \cdot h_s) \leq 50 )</td>
<td>not considered</td>
</tr>
</tbody>
</table>

Filter mechanism for realistic combinations:

\[
B_c / h_c \quad 0.5 \leq B_c / h_c \leq 2.0
\]
Tab. 4: Matrix of possible load case-failure mode combinations

<table>
<thead>
<tr>
<th>load cases</th>
<th>failure modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bearing capacity failure in subsoil</td>
</tr>
<tr>
<td></td>
<td>limitation of eccentricity</td>
</tr>
<tr>
<td></td>
<td>overturning</td>
</tr>
<tr>
<td></td>
<td>sliding along the base</td>
</tr>
<tr>
<td></td>
<td>bearing capacity failure in rubble mound</td>
</tr>
<tr>
<td></td>
<td>uplift</td>
</tr>
<tr>
<td>still-water-level</td>
<td>1-1 1-2 1-3 1-4 1-5 1-6</td>
</tr>
<tr>
<td>wave crest</td>
<td>2-1 2-2 2-3 2-4 2-5 2-6</td>
</tr>
<tr>
<td>wave trough</td>
<td>3-1 3-2 3-3 3-4 3-5 3-6</td>
</tr>
</tbody>
</table>

For a certain load case function F is governed by the different failure modes of Tab. 4. Under special conditions some of them are more restrictive than others. Understanding the function F means to investigate which of the failure modes are the more restrictive ones and which could be neglected or should only be considered in special cases.

8.3.2. Design parameter Bc

The next step within the parameter study is to find a criterion of relevancy of the single failure mode. From practical engineering the caisson width Bc is usually a relevant design parameter, so it is chosen in function F as calculation quantity whereas the other ones have to be varied. Bc usually works favourably for example within the calculation of bearing capacity failure in subsoil or the limitation of eccentricity, because it limits the eccentricity caused by the wave loading. Furthermore, in the case of sliding along the base it raises the vertical load. So this parameter gives information about the relevancy of a load case-failure mode combination. Including these considerations the failure function F can be expressed in the following way:

\[
F = \frac{B_c}{h_s} - \frac{B_{c,\text{crit}}}{h_s} \geq 0
\]

Bc/hs is the caisson width chosen in the design process and Bc,\text{crit}/hs is the caisson width which describes the limit state of the subsoil. Since an explicitly analytical function of Bc,\text{crit}/hs does not exist, it has to be determined by numerical iteration.

However, in some cases the caisson width Bc,\text{crit}/hs has no influence on the calculation of the failure mechanism, e.g. in the load case-failure mode combination 1-6 (SWL-uplift). In this case only pressure from caisson weight and uplift water pressure are of interest. So, at the beginning of the design process it has to be checked if the following restriction is fulfilled:

\[
\frac{h_s - h_i}{h_s} \leq \frac{h_c}{h_s} \cdot \alpha \cdot \gamma_c \cdot \gamma_w
\]
The construction of a breakwater which does not pass this demand would make no sense. For the same reason it is additionally necessary to limit the related eccentricity of the caissons’ dead load \( e/B_c \) in case of SWL. So, the following inequality must be fulfilled:

\[
\left| \frac{e}{B_c} \right| \leq 0.3 \cdot \left( 1 - \frac{(h_s - h_c) \cdot \gamma_w}{h_c \cdot \alpha_c \cdot \gamma_c} \right)
\] (20)

This inequality has been derived from the limitation of eccentricity. The overturning mechanism is included in this demand (see GOLÜCKE; PERAU; RICHWIEN, 1998).

Beside this, within the load case-failure mode combination 2-2 (wave crest-limitation of eccentricity) increasing width results in decreasing stability. The reason is that the horizontal wave loading is independent of the width. So, within the proof of the allowable eccentricity according to Eq. (20), where the moments of wave loading and dead load act in opposite direction for positive eccentricities (eccentricities to seaside), a greater width may result in a loss of stability, because the moment of the caissons’ dead load becomes too large whereas the moment of the horizontal wave load is unaffected.

8.3.3. Relevant failure mode

The failure function \( F \) for each parameter combination provides a matrix containing the results of \( B_{c,\text{crit}}/h_s \) for each load case-failure mode combination. The combination which leads to \( \max(B_{c,\text{crit}}/h_s) \) is defined as the relevant load case-failure mode combination which defines the limit state of subsoil.

As an example a stability analysis for a breakwater construction type as example A (DE GROOT et al., 1996) was carried out for a maximum wave height \( H_{\text{max}} = 10.0 \text{ m} \) (\( \cong H_u = 5.60\text{m} \)) with \( T_p = 12 \text{ s} \). The input parameters were taken from Tables 4.1, 4.2 and from Section 5.7.4 in (DE GROOT et al., 1996).

The direction of wave propagation has been set to \( \beta = 0^\circ \), the offshore water level to \( h_o = h_c = 15.5 \text{m} \), because these quantities were not given. The shear resistance of the rubble mound has been neglected. Tab. 5 shows the results of the calculation. Here, the required width of the caisson \( B_{c,\text{crit}} \) [m] for each load case-failure mode combination is stated. Obviously, our calculation has led to a relevant width of 16.60 m (for bearing capacity failure in subsoil at wave crest), which is smaller than the design width given in example A (\( B_c = 17.50 \text{ m} \)). This result is in good congruence with those of the stability analysis for example A in (DE GROOT et al., 1996).
CHAPTER 8 INFLUENCE OF DESIGN PARAMETERS

Tab. 5: Results of the stability analysis for example A (DE GROOT et al., 1996): relevant width $B_{c,\text{crit}}$ [m] for each load case-failure mode combination

<table>
<thead>
<tr>
<th>load cases</th>
<th>failure modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bearing capacity failure in subsoil 1</td>
</tr>
<tr>
<td>still-water level 1</td>
<td>0.90</td>
</tr>
<tr>
<td>wave crest 2</td>
<td>16.60</td>
</tr>
<tr>
<td>wave trough 3</td>
<td>11.40</td>
</tr>
</tbody>
</table>

8.4. RESULTS

At first a general survey concerning relevant load cases and failure mechanisms within the design process is presented. Secondly, the parameters are graded according to their influence on the caisson stability. Finally some interesting results of the parameter study are presented which show the effects of the most important parameters on the design of caisson breakwaters.

8.4.1. General survey

First of all the parameters in Tab. 3 have been varied in very small variation steps. Thus over 20 million combinations have been calculated within this variation. Tab. 6 shows the statistical results. Here, the numbers indicate how often a certain load case-failure mode combination has become relevant within the failure analysis. One can see that the load case still-water-level is not relevant in any parameter combination as expected due to the restrictions made before (see chapter 8.3.2). The failure modes overturning and uplift are not relevant, too. So, they are not considered in further calculations.
Tab. 6: Statistic file of a fine parameter variation

<table>
<thead>
<tr>
<th>load cases</th>
<th>bearing capacity failure in subsoil</th>
<th>limitation of eccentricity</th>
<th>overturning</th>
<th>sliding along the base</th>
<th>bearing capacity failure in rubble mound</th>
<th>uplift</th>
</tr>
</thead>
<tbody>
<tr>
<td>still-water-level</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>wave crest</td>
<td>2</td>
<td>7,074,7921</td>
<td>6,900,192</td>
<td>3,482,838</td>
<td>41,310</td>
<td>0</td>
</tr>
<tr>
<td>wave trough</td>
<td>3</td>
<td>1,984,086</td>
<td>626,742</td>
<td>720</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

number of all variations: 20,110,680

It is obvious that bearing capacity failure in the subsoil and limitation of eccentricity at wave crest are the most critical failure mechanisms (relevant in 35.2% and 34.3% of all variations). The next critical one is sliding along the base at wave crest. Although wave crest is the most unfavourable load case in general, there are a lot of cases in which load case wave trough is relevant - especially with failure mode 1 (bearing capacity failure in subsoil). The combinations 2-5 (wave crest-bearing capacity failure in rubble mound) and 3-4 (wave trough-sliding along the base) are only relevant in a very few special cases which will be specified later. The results also show that bearing capacity failure in the rubble mound at wave trough does not occur at any parameter combination and can be neglected therefore.

Tab. 6 includes results with caisson widths $B_{c, crit}$ which are not of practical importance. So, in a second step an additional filter mechanism was applied which excludes all results with a relation of $B_{c, crit}/h_c<0.5$ and $B_{c, crit}/h_c>2.0$, since these breakwaters are assumed to be not realistic. Tab. 7 contains the statistical results of this second variation. It can be seen that again wave crest is the most critical load case. Nevertheless, it is not advisable to neglect the load case wave trough. Now, bearing capacity failure in the subsoil and sliding along the base are the most critical failure mechanisms rather than the limitation of eccentricity which is not as relevant as before (only in 15.0 % of all variations).
In the following the relevance of different failure modes in relation to the input parameters has to be discussed.

### 8.4.2. Importance of input parameters

As a first result of the parameter study it can be noticed that the influence of the input parameters is quite different. Parameters which determine the caissons’ dead load and particularly its moment are very important, because they strongly influence the caisson stability. These parameters are the unit weight of the caisson $\gamma_c$, the geometry factor $\alpha_c$, the height of the caisson $h_c$ and especially the eccentricity of the caissons’ dead load $e_c$. Besides, the influence of wave and soil parameters on the relevant failure mode is quite significant as well. Herein, especially the shear strength parameters of subsoil and rubble mound $\phi'_S$ and $\phi'_R$ and the wave height $H_w$ have to be investigated in detail.

Since the variation range of the unit weight of the rubble layer $\gamma'_R$ is very small, changes concerning the relevant failure modes are negligible. The same applies to the offshore still-water-level $h_{s0}$ as well as to the height of the rubble layer $h_r$. The unit weight of the subsoil $\gamma'_S$ is important mainly for the relevance of bearing capacity failure in the subsoil. With increasing $\gamma'_S$ this failure mode becomes less critical. Since the influence of this parameter on the relevant failure mode is not very strong, it is not particularly investigated. For that reasons the parameters $\gamma'_R / \gamma_w$, $\gamma'_S / \gamma_w$, $h_r / h_c$ and $h_{s0} / h_s$ are set to fixed values with $\gamma'_R / \gamma_w = 1.0$, $\gamma'_S / \gamma_w = 0.8$, $h_r / h_c = 0.15$ and $h_{s0} / h_s = 1.1$ and not varied any more.

### 8.4.3. Relevance of different failure modes for the caisson stability
In the following the relevance of different failure modes as a function of the variation ranges of the most important input parameters is described. Parameters which are not mentioned explicitly are also varied according to Sections 8.2.2 and 8.4.2, but do not significantly influence the relevance of a special failure mode.

The results have been divided into failure modes which may become relevant only in very special cases and those which could be relevant in general. The denotation of the load case-failure mode combinations again refers to Tab. 4.

8.4.3.1. Less important failure modes

Bearing capacity failure in rubble mound at wave crest (combination 2-5) may become relevant only under special design conditions. It has been found that the relevance of this failure mode depends mainly on the relation of $\varphi'_s / \varphi'_k$. According to the ranges of parameters given within this study (see Tab. 3) this failure mechanism may be relevant in the range of $25^\circ \leq \varphi'_s < 30^\circ$ and simultaneously $37.5^\circ < \varphi'_k \leq 45^\circ$ only, where the following relation must always be valid:

$$\varphi'_s < \frac{2}{3} \varphi'_k$$  \hspace{1cm} (21)

or exactly for the calculated parameter combinations:

$$\varphi'_s \leq 0.663 \cdot \varphi'_k$$  \hspace{1cm} (22)

If Eq. (22) is not valid, bearing capacity failure in the rubble mound will not occur. If Eq. (22) is valid, combination 2-5 may be relevant for positive eccentricities within the range of $0.5 \leq e_c / B_c \leq 0.15$, but only in cases where other load case-failure mode combinations are not critical (low weight of caisson: $\alpha_c \cdot \gamma_c / \gamma_w \leq 1.4$ and $h_c / h_s < 1.0 \cdot (h_s + h_{crest} - h_r) / h_s$).

Here, especially bearing capacity failure in the subsoil is not critical although the friction angle of subsoil is relatively low. Otherwise, as one can see, combination 2-5 may occur only under design conditions which are not realistic ($\alpha_c \cdot \gamma_c / \gamma_w \leq 1.4$). So, under normal conditions bearing capacity failure in the rubble mound at wave crest need not to be checked.

The second load case-failure mode combination which may become relevant only under special design conditions is sliding along the base at wave trough (combination 3-4). This combination might be relevant if:

$0 \leq e_c / B_c \leq 0.05$ (limitation of eccentricity not very critical)

and $\varphi'_s \geq 35^\circ$ (bearing capacity failure in subsoil not critical) and $\varphi'_k \leq 35^\circ$

and $\alpha_c \cdot \gamma_c / \gamma_w \leq 1.7$ and $h_c / h_s < 0.9 \cdot (h_s + h_{crest} - h_r) / h_s$ (low weight of caisson)

and $H_w / h_s \leq 0.2$ , $T_p \leq 10$ s (higher wave load at wave trough in opposite to wave crest)
It has to be mentioned that within the parameter limits given above other load case-failure mode combinations may become relevant as well, but the relevance of combinations 2-5, 3-4 is restricted to these parameter ranges only. Within these limits they can be relevant, but they must not.

8.4.3.2. Important failure modes

The relevance of bearing capacity failure in subsoil, limitation of eccentricity at wave crest and wave trough (combinations 2-1, 2-2, 3-1, 3-2) and sliding along the base at wave crest (combination 2-4) mainly depends on the design values of \( e_c/B_c, \phi'_S \) and \( H_{si}/h_a \).

The eccentricity of caissons' dead load \( e_c/B_c \) is most decisive firstly for the relevance of different load cases. Calculations have shown that for \( e_c/B_c \leq 0 \) (eccentricity to harbourside) only load case wave crest is relevant, because here negative eccentricities are unfavourable (moment of caissons' dead load and moment of wave load act in same direction). The influence of \( e_c/B_c \) is so strong that even for low wave heights, where negative forces (wave trough) may be greater than positive forces (wave crest) (see Vol. IIa, Section 4.2), this load case is still relevant.

For \( e_c/B_c > 0 \) (eccentricity to seaside) both, wave crest and wave trough, may become relevant. Positive eccentricities are favourable for load case wave crest but unfavourable for load case wave trough. Here the relevance of a certain load case depends especially on the wave height \( H_{si} \). It has to be mentioned that sometimes very high positive eccentricities may be unfavourable for load case wave crest, too. In these cases the moment of the caisson dead load is too large in relation to the moment of the wave load, especially at low waves. For high waves this effect is not so obvious.

Secondly, the eccentricity of caissons' dead load determines the relevance of some failure modes. If \( e_c/B_c < -0.05 \) only bearing capacity failure in subsoil and limitation of eccentricity at wave crest (combinations 2-1 and 2-2) are relevant because of the large resultant moment, which reduces the effective width of the caisson and enlarges the resultant eccentricity. If \( e_c/B_c \geq -0.05 \) every failure mode can be relevant (Fig. 4). Here their relevance is determined by other parameters. For \( e_c/B_c \geq 0.05 \) combination 2-2 (limitation of eccentricity at wave crest) is no longer relevant, because positive eccentricities work favourably within this failure mode.

Beyond the influence of \( e_c/B_c \) especially the angle of internal friction of subsoil \( \phi'_S \) and the dimensionless wave height \( H_{si}/h_a \) are determinant. Parameter \( \phi'_S \) is the decisive criterion for the relevance of bearing capacity failure in subsoil. Generally, its importance decreases with increasing \( \phi'_S \), whereas the limitation of eccentricity and sliding along the base become more and more relevant.
The present value of parameter $H_{si}/h_s$ decides if wave crest or wave trough is the relevant load case. In the following the influence of these parameters is investigated in detail. Frequency distributions of the load case-failure mode combinations for several values of $e_c/B_c$ are presented in relation to $\varphi'_s$ to show the influence on the relevant failure modes.

**Fig. 4:** Frequency distribution related to $e_c/B_c$

For $e_c/B_c < -0.05$ combination 2-2 may become relevant over the whole range of $\varphi'_s$ whereas combination 2-1 may become relevant only if $\varphi'_s \leq 42.5^\circ$. Figs. 5 to 7 show that with increasing $\varphi'_s$ the frequency of relevance of combination 2-1 decreases (higher shear resistance of subsoil) whereas combination 2-2 becomes increasingly relevant. The influence of other input parameters is quite vague and can not be clarified. Therefore it is not possible to state that under special conditions only combination 2-1 or only combination 2-2 is relevant. So, both failure modes should be investigated.
Fig. 5: Frequency distribution related to $\phi_S$ for $e_c/B_c = -0.2$

Fig. 6: Frequency distribution related to $\phi'_S$ for $e_c/B_c = -0.15$
For \( e_c/B_c = -0.05 \) (see. Fig. 8) the relevance of different combinations is determined as follows:

Combination 2-1 may become relevant only for \( \varphi_S \leq 42.5^\circ \), the influence of other parameters is not significant. Combination 2-2 may become relevant over the whole range of \( \varphi_S \), but only for \( h_c/h_s \leq 1.1 \cdot \left( h_s + h_{c\text{rest}} - h_r \right) / h_s \). Its significance increases with increasing \( \varphi_S \) (when bearing capacity failure in subsoil is less critical). Combination 2-4 is relevant only if:

\[
\varphi_h \leq 37.5^\circ \\\text{and} \\
h_c/h_s < 1.1 \cdot \left( h_s + h_{c\text{rest}} - h_r \right) / h_s
\]

Here with increasing unit weight of the caisson the load case-failure mode combination 2-4 becomes less critical. When the unit weight of caisson is high enough, the relevance of this combination is restricted to low friction angles of rubble mound and low caisson heights.
Fig. 8: Frequency distribution related to $\varphi_s$ for $e_c/B_c = -0.05$

For $e_c/B_c = 0.0$ (Fig. 9) combination 2-1 can be relevant over the whole range of $\varphi_s$, no significant influence of other parameters has been found. Combination 2-2 may become relevant only if $\varphi_s \geq 37.5^\circ$ and $H_{ui}/h_s \leq 0.5$. Combination 2-4 is restricted to $\alpha_c \cdot \gamma_c / \gamma_w \leq 1.5$ and $h_c / h_s < 0.8 \cdot (h_s + h_{crest} - h_t) / h_s$. So, the sliding mechanism is critical especially for a low weight of the caisson and in case of wave overtopping. Additionally, combination 3-1 may become relevant, but only for low wave heights ($H_{ui}/h_s \leq 0.3$), because here negative forces (wave trough) may be greater than positive forces (wave crest) (see Vol. IIa, Section 4.2). Beside this, for $\varphi_s \geq 35^\circ$ combination 3-1 may be relevant only if $T_p \leq 11$ s and $\beta \geq 40^\circ$. 
Fig. 9: Frequency distribution related to $\varphi'_S$ for $e_c/B_c = 0.0$

For $e_c/B_c = 0.05$ (Fig. 10) combination 2-1 can be relevant over the whole range of $\varphi'_S$, but only if $H_{si}/h_s \geq 0.2$. Other parameters do not significantly influence the relevance of this combination. Combination 2-2 may become relevant only for $\varphi'_S \geq 45^\circ$ and here only if all other failure modes are not critical ($\varphi'_R \geq 37.5^\circ$, $0.3 \leq H_{si}/h_s \leq 0.4$, $\alpha_c \cdot \gamma_c/\gamma_w \leq 1.7$ and $h_c/ h_s < 1.0 \cdot (h_s + h_{crest} - h_t)/h_s$). Sliding along the base (combination 2-4) is relevant over the whole range of $\varphi'_S$ as well, but only if $H_{si}/h_s \geq 0.2$. For $\varphi'_S \leq 32.5^\circ$ it is relevant if $\alpha_c \cdot \gamma_c/\gamma_w \leq 2.0$ and $h_c/ h_s < 1.1 \cdot (h_s + h_{crest} - h_t)/h_s$). In Figs. 10 and 11 it is apparent that the significance of combination 2-4 increases with increasing $\varphi'_S$, because bearing capacity failure in subsoil is less critical in these cases.

Combination 3-1 may become relevant over the whole range of $\varphi'_S$, but its significance decreases with increasing $\varphi'_S$, where its relevance is simultaneously restricted to decreasing wave heights. Combination 3-2 can be relevant for $\varphi'_S \geq 40^\circ$ and $H_{si}/h_s \leq 0.2$, but only if $\alpha_c \cdot \gamma_c/\gamma_w \leq 2.1$ and $h_c/ h_s < 1.1 \cdot (h_s + h_{crest} - h_t)/h_s$).
CHAPTER 8  INFLUENCE OF DESIGN PARAMETERS

For \( e_c/B_c \geq 0.1 \) it is obvious that the importance of combination 2-1 decreases (Fig. 11 to 13), because due to the high positive eccentricity of the caisson dead load its moment significantly reduces the moment caused by the wave load. With increasing eccentricity \( e_c/B_c \) this combination is restricted to higher values of \( \varphi'_s \), what is not clear at first (compare Fig. 12 and 13). The reason is that for lower values of \( \varphi'_s \) combination 3-1 is relevant more often, because a positive eccentricity acts unfavourably for this load case. On the other hand combination 2-1 becomes relevant especially for higher wave heights (\( H_{si}/h_s \geq 0.3 \) for \( 0.1 \leq e_c/B_c \leq 0.15 \) and \( H_{si}/h_s \geq 0.5 \) for \( 0.15 < e_c/B_c \leq 0.2 \)), where positive wave forces are greater than negative wave forces.

Fig. 13 shows that combination 2-1 is only relevant if \( \varphi'_s \geq 35^\circ \). On the other hand it can be seen here that for \( \varphi'_s \leq 30^\circ \) only combination 3-1 is relevant. So, for such an extreme eccentricity \( \varphi'_s \) seems to be the decisive criterion for the relevance of a certain failure mode.

Further on, Fig. 11 to 13 show that with increasing \( \varphi'_s \) the significance of 3-1 decreases whereas combination 3-2 becomes more and more relevant. This behaviour corresponds to the one observed for negative eccentricities at load case wave crest (see Fig. 6 to 8), but here it is not well developed, because especially for higher waves the resultant loading at wave trough is smaller in opposite to the loading at wave crest. So, other failure modes may be relevant as

---

**Fig. 10:** Frequency distribution related to \( \varphi'_s \) for \( e_c/B_c = 0.05 \)

-23-
well. Otherwise combination 3-1 has to be checked at all, since it is not restricted to certain parameter ranges for $e_c/B_c \geq 0.1$.

![Graph showing frequency distribution related to $\phi'$ for $e_c/B_c = 0.1$](image)

**Fig. 11:** Frequency distribution related to $\phi'$ for $e_c/B_c = 0.1$
For $e_c/B_c \geq 0.1$ combination 2-4 can be relevant over the whole range of $\phi'_S$. It is clear that the frequency of 2-4 increases with increasing $\phi'_S$, because bearing capacity failure in subsoil becomes less critical. Otherwise for $0.1 \leq e_c/B_c \leq 0.15$ and $\phi'_S \leq 30^\circ$ combination 2-4 may become relevant only if $\alpha_c \cdot \gamma_c / \gamma_w \leq 2.0$ and $h_c / h_s < 1.1 \cdot (h_s + h_{rest} - h_i) / h_i$. Here, it could be seen again, that with increasing unit weight of the caisson combination 2-4 is less critical. Additionally, for $e_c/B_c \geq 0.1$ combination 2-4 is only relevant if $H_{si} / h_s \geq 0.3$. Significant influences of other parameters can not be specified.
Fig. 13: Frequency distribution related to $\phi'_s$ for $e_c/B_c = 0.2$

For $0.1 \leq e_c/B_c \leq 0.15$ combination 3-2 may be relevant if $H_{si}/h_s \leq 0.4$. For $e_c/B_c = 0.2$ its relevance is restricted to $\phi'_s > 32.5^\circ$ and $H_{si}/h_s \leq 0.5$. Again the restriction of load case wave trough to lower wave heights and of the limitation of eccentricity to higher shear strength of subsoil, where other combinations are no longer critical, is advisable.

8.4.4. Summary of results

At first it has to be pointed out that all results should be used very carefully. It can not be guaranteed, that for other parameter combinations than those varied here the statements given above, especially the limits within the different load case-failure mode combinations can become relevant, are still valid. It is possible that results differ from those presented here. If it is doubtful, which failure mode is the relevant one in a certain design case, each of them should be proved.

Nevertheless some significant dependencies have been found which can be summarised as follows: Load case still-water-level is negligible at all. The most critical load case is wave crest, but load case wave trough can not be neglected, particularly if the eccentricity of the caisson dead load is directed to seaside which is common practice. With large seaward eccentricities, wave trough is in particular critical with relatively small values of $H_{si}/h_s$. Failure modes overturning and uplift are not relevant. Bearing capacity failure in the rubble
mound at wave crest and sliding along the base at wave trough may become relevant only under very special design conditions. Otherwise they need not to be investigated. The eccentricity of the caisson dead load has been shown as the most important parameter. It determines the relevance of a certain load case-failure mode combination very strongly. Beside this, angle of internal friction of subsoil and wave height are very important. Within certain ranges of these parameters unit weight and height of the caisson or the angle of internal friction of rubble mound could determine the relevance of certain failure modes. Significant influences of other input parameters have not become apparently.

In Tab. 8 the results are summarised in a parameter map to identify relevant failure modes. This map gives a survey of those parameter ranges where certain failure modes are of special importance and others are negligible. The ranges describe those areas to which a certain load case-failure mode combination is restricted. So it can only become relevant within this range but it must not. This also means that other load case-failure mode combinations can become relevant as well, if they are not explicitly excluded from these ranges. It has to be mentioned that the map can not be complete, because it is not possible to describe all influences and dependencies on the caisson stability exhaustively.

Finally, following hints for the design of vertical breakwaters can be given. Because wave and soil conditions are fix in a certain design case, the variable parameters \((h_c, a_c, \gamma_c, e_c, B_c)\) should be chosen to achieve economic caisson dimensions. First of all wave overtopping should be avoided \((h_c / h_s > 1.0 \cdot (h_s + h_{\text{crest}} - h_t) / h_s)\), since the studies have shown that low crested breakwaters are in principle unfavourable for the stability of the foundation. Secondly, low unit weights of the caisson and low geometry factors should be excluded. RICHTER (1998) recommends values between \(17 \leq \alpha_c \cdot \gamma_c / \gamma_w \leq 20\) for \(H_x / h_s < 0.5\), especially in case of low shear strength of subsoil, and \(20 \leq \alpha_c \cdot \gamma_c / \gamma_w \leq 23\) for \(H_x / h_s \geq 0.5\).

The eccentricity of the caisson dead load should be limited to \(0 \leq e_c / B_c \leq 0.06\), because within these limits the eccentricity has positive influence on the breakwater stability (RICHTER; 1998). Positive eccentricities up to \(e_c / B_c \leq 0.1\) are acceptable as well, but not as favourable as those due to the restriction above.
Tab. 8: Parameter map of relevant failure modes (see Tab. 8 continued for legend)
Tab. 8 continued: Parameter map of relevant failure modes

Definitions:
- $h^* = (h_t + h_{max})/h_t$
- $\phi < 32.5'$
- $H_{PA} < 0.3$
- $H_{PA} < 0.3$
- $H_{PA} < 0.3$
- $H_{PA} < 0.3$

Remarks:
- For relevance of bearing capacity failure in stubble mound at wave crest and sliding along the base at wave trough, see chapter 8.4.3.1

[Diagram of parameter map showing failure modes and criteria]
8.5. CONCLUSIONS

Despite of reduced number and range of parameters varied within this study their influence on the overall stability of the caisson is very contradictory. Therefore the stability analysis should be treated as an integrated problem, since the correlations between the parameters involved are very manifold and sometimes quite complicated. After all it is not possible to define one failure mechanism under a special load case as the most relevant one. Only tendencies can be pointed out for those parameter ranges, in which some special load case-failure mode combinations are negligible whereas others are very critical. In case of doubt all failure modes at least the most critical ones should be checked.

A consistent failure equation, however, is not available up to now. So within the conventional process of design the comprehensive problem of subsoil failure is split up into different and clearly defined modes, completely independent of each other.

In this situation it may be helpful to analyse the influence of the design parameters to get an information, which of the separately defined modes may be the most critical one. Within the design process engineers should distinguish the fixed parameters describing the present loading and soil situation and the variable parameters ($h_c$, $a_c$, $\gamma_c$, $e_c$, $B_c$) which should be determined to get most economic caisson dimensions. After that stability should be also proved for the other failure modes with the design parameters chosen before.

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CHAPTER 9: ALTERNATIVE FOUNDATIONS

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ABSTRACT

Traditional breakwaters are placed on a rubble mound. On soft clay sites the bearing capacity of the soil can be so low that staged construction or soil replacement or soil reinforcement measures have to be taken. Skirted foundations have been used with success on soft soils in the offshore industry. In this chapter the possibility of using skirted foundations on soft clay sites is discussed and evaluated for certain conditions.

Key words: Vertical breakwaters, soft clay, skirts, bearing capacity

9.1. GENERAL

Skirted foundations have been used with success for offshore gravity platforms and more recently also for the individual leg foundations of steel jackets and suction anchors. At locations where the natural soil consist of soft clays of clayey silty sands, the bearing capacity will in many cases not be sufficient to support a gravel/rockfill berm and the breakwater structure. To overcome the bearing capacity problem, soil improvement or replacement might be required. One solution is a staged construction method where the rock fill is placed first in one or more stages. The soil will then consolidate and increase its strength under the weight of the berm. The time required for consolidation can be several months and even years before the superstructure can be installed. The construction method will normally lead to considerable settlements during the operational period of the breakwater

Skirted concrete caissons have a number of advantages from a geotechnical point of view:

- The sliding and bearing capacity can be improved immediately
- Underbase erosion is prevented
- The underbase water pressure and corresponding overturning moment acting on traditional breakwater structures is avoided
- A caisson cantilever base can reduce the overturning moment
• A considerable weight reduction can be achieved with increased safety against bearing capacity failure and with strongly reduced settlements
• Quick and proven installation method

There are certainly also limitations and disadvantages:
• The additional concrete work and cost compared with rock/gravel fills
• Skirt depth restrictions during tow out
• The water depth variations along the breakwater will require design and engineering of several sets of concrete structures although a certain water depth variation can be covered by adjustment of penetration and underbase grouting

9.2. PROPOSED STRUCTURAL CONFIGURATIONS

The structural configurations may vary from a box type structure similar to many of the representative structures with skirts under the periphery to a sea wall with a skirted base plate as outlined in Figure 1.

The installation will require structures with sufficient buoyancy for self-floating with adequate clearance between skirts and seabed during installation. Alternatively this may however be achieved with a barge support during float out.

Contrary to breakwater structures on sandy soils, which normally have a considerable bearing capacity, the presence of clay may represent a weight restriction caused by limited bearing capacity as well as long term settlements (Figure 1a)

Skirts prevent the generation of underbase pressure typical for breakwater structures placed on berms. This leads to a reduction in overturning moment, but the water pressure is acting directly on the seabed outside the skirt has to be considered.

A cantilevered base plate on the seaward side of the breakwater may give further reduction in overturning moment caused by dynamic vertical water pressure on the cantilever plate acting against the overturning moment on the wall.

9.3. FAILURE MODES TO BE CONSIDERED

The presence of skirts in a clayey subsoil will prevent intrusion of seawater under the base under moment loading even for low vertical loads. The direct uplift force under the base is
thus eliminated, however, the wave pressure at the seabed will tend to push the soil down and in under the base as shown in Figure 2.b.

Sliding at base level and possible bearing capacity failure under the most loaded part (Figure 2a) are typical failure modes for a gravity type breakwater structure. The skirts allow utilisation of the suction capacity under the heel of the base, and mobilisation of passive earth pressure in front of the skirt. The weight can thus be reduced compared with a traditional structure. This change in proportion between horizontal and vertical force and will influence the shape of the critical failure surfaces. With increasing water depth and increasing overturning moment the critical failure surface will change from bearing capacity type failure towards Figure 2d toward combined suction/bearing capacity failure Figure 2b towards a pure rotational failure with a circular failure surface.

The suction capacity under the heel is in addition limited by cavitation, which will develop when the underpressure approaches -1 atm.
Fig. 1: Possible configurations for skirted breakwater structures on clayey soils.
Fig. 2: Possible structural configurations and failure modes
9.4. DESCRIPTION OF EVALUATED FAILURE MODES

9.4.1. General

The effect of skirts on the foundation capacity has been evaluated for tree different failure modes

a) Bearing capacity failure, wedge analysis (includes sliding along the base of the skirts)
b) Deep seated bearing capacity failure, rotation about a point under the heel of the base
c) Rotational failure with a circular failure surface with centre co-ordinates \( x_c \) and \( z_c \) relative to the centre line of the base at seabed level.

Plane strain 2-D geometry is considered

9.4.2. Failure mechanism 1: wedge type failure (bearing capacity under combined vertical and horizontal loading)

The failure mode is shown in Figure 3 showing external forces, \( F_h \) and \( F_v \) and the external wave pressure, \( p_w \), on the seabed in front of the breakwater. The failure surface consists of three parts; an active wedge in front of the breakwater, a main wedge underneath the breakwater and a passive wedge behind the breakwater. The undrained shear strength of the clay is mobilised at the interface between the wedges and the underlying soil and at the interfaces between the wedges and the skirt wall.

Fig. 3: Wedge model for evaluation of bearing capacity
Assuming a virtual horizontal displacement, $\delta_h$, of the breakwater leads to generation of external and internal work as summarised in Table 1a and b respectively.

**Tab. 1a:** Failure mechanism 1; external work

<table>
<thead>
<tr>
<th>Force</th>
<th>External force</th>
<th>Displacement</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal force $F_h$</td>
<td>$\delta_h$</td>
<td>$F_h \cdot \delta_h$</td>
<td></td>
</tr>
<tr>
<td>Vertical force $F_v$</td>
<td>$\delta_h \cdot \tan \beta$</td>
<td>$F_v \cdot \delta_h \cdot \tan \beta$</td>
<td></td>
</tr>
<tr>
<td>Wave pressure force $p_w \cdot D$</td>
<td>$\delta_h$</td>
<td>$p_w \cdot D \cdot \delta_h$</td>
<td></td>
</tr>
</tbody>
</table>

**Tab. 1b:** Failure mechanism 1; internal work ($A_i = c_{u,i} \cdot l_i \cdot \delta_i$)

<table>
<thead>
<tr>
<th>Interface</th>
<th>Strength, $c_{u,i}$</th>
<th>Displacement, $\delta_i$</th>
<th>Length, $l_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active wedge-soil</td>
<td>$c_{u0} + k \cdot D/2$</td>
<td>$\delta_a = \delta_h \cdot \sqrt{2}$</td>
<td>$D \cdot \sqrt{2}$</td>
</tr>
<tr>
<td>Interface:active wedge-skirt</td>
<td>$\alpha \cdot (c_{u0} + k \cdot D/2)$</td>
<td>$\delta_{aw} = \delta_h$</td>
<td>$D$</td>
</tr>
<tr>
<td>Main wedge-soil</td>
<td>$c_{u0} + k \cdot (D + 0.5B \cdot \tan \beta)$</td>
<td>$\delta_p = \delta_h / \cos \beta$</td>
<td>$B / \cos \beta$</td>
</tr>
<tr>
<td>Main wedge-passive wedge</td>
<td>$r_f [c_{u0} + k \cdot (D + 0.5B \cdot \tan \beta)]$</td>
<td>$\delta_{pw} + \delta_v = \delta_h \cdot (1 + \tan \beta)$</td>
<td>$B \cdot \tan \beta$</td>
</tr>
<tr>
<td>Passive wedge-soil</td>
<td>$c_{u0} + k \cdot (D + B \cdot \tan \beta)/2$</td>
<td>$\delta_p = \delta_h \cdot \sqrt{2}$</td>
<td>$(B \cdot \tan \beta + D) \cdot \sqrt{2}$</td>
</tr>
<tr>
<td>Passive wedge-skirt</td>
<td>$\alpha \cdot (c_{u0} + k \cdot D/2)$</td>
<td>$\delta_{pw} + \delta_v = \delta_h \cdot (1 + \tan \beta)$</td>
<td>$D$</td>
</tr>
</tbody>
</table>

![Graph](image)

**Fig. 4:** Vertical bearing capacity vs. horizontal load. Comparison between failure mechanism 1 and Brinch-Hansen’s (1971) method for bearing capacity analysis. ($B=20m$, $c_{u0} = 20$ kPa, $k=0$)

The expressions in Table 1a and b were inserted in a spreadsheet. The solution was adjusted against Brinch-Hansen’s (1971) bearing capacity solution for surface foundations using a
correction factor $r_f = 0.605$ on the main wedge-passive wedge interface strength. This gave a very close fit for an example calculation as shown in Figure 4. The skin friction factor $\alpha$ was set to 0.5.

9.4.3. Failure mechanism 2: Bearing capacity failure with rotation about point below edge

The model for this failure mode is shown in Figure 5. This failure mode accounts for the effect of the overturning moment from the wave load. When $R_1$ approaches infinity, the model converges towards failure mechanism 1.

![Figure 5: Failure mechanism 2; Bearing capacity failure with centre of rotation under left edge.](image)

The failure mechanism is composed of an active failure wedge in front of the breakwater, a main wedge with a cylindrical failure surface beneath the breakwater and a passive wedge with a cylindrical failure surface. A unit virtual rotation, $d\beta_1$, around the rotation centre $O_1$ is applied. The breakwater and the main wedge will rotate and force the passive wedge to rotate, $d\beta_2 = d\beta_1$ around its rotation centre $O_2$. The active wedge in front of the breakwater is approximated by a triangular wedge. A circular failure surface should have been adopted here as well, however, the error involved is small.

Geometric relationships:
A rotation about \( d\beta_1 \) about \( O_1 \) should cause an equally large rotation \( d\beta_2 \) about \( O_2 \).

\[
d\beta_1 = d\beta_2 = d\beta \quad (1)
\]

The angle of the secant of the passive wedge arc is chosen to be 45°. The horizontal distance from the right edge of the structure to the centre of rotation must then be equal to the vertical distance from seabed to the centre of rotation below the left edge, which is \( R_1+D \). This gives the geometry shown in Figure 5, and the following relations can be derived:

\[
\beta_1 = 2\alpha_1 \quad (2)
\]

\[
R_1 = B \sin 2\alpha_1 \quad (3)
\]

\[
\tan \alpha_2 = R \cos \beta_1 / (R_1+D) \quad (4)
\]

\[
\beta_2 = \pi/2 - 2\alpha_2 \quad (5)
\]

\[
R_2 = (R+D)/\cos \alpha_2 \quad (6)
\]

**External and internal work**

Applying a virtual rotation, \( d\beta \), about the two rotation centres causes displacements and corresponding internal and external work. The active wedge sinks in along the 45°-wedge angle, the structure and the underlying wedge are displaced and rotated and the passive wedge is pushed up and rotated. This leads to the external and internal work contributions summarised in Table 2a and b.

**Table 2a:** Failure mechanism 2; external work

<table>
<thead>
<tr>
<th>Force</th>
<th>External force</th>
<th>Displacement.</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal force</td>
<td>( F_h )</td>
<td>( \delta_h = (R_1 + D + h) \cdot d\beta )</td>
<td>( F_h \cdot \delta_h )</td>
</tr>
<tr>
<td>Vertical force</td>
<td>( F_v )</td>
<td>( \delta_v = \delta_h \cdot (R_1 + D + hw) \tan \alpha_c \cdot d\beta )</td>
<td>( F_v \cdot \delta_v )</td>
</tr>
<tr>
<td>Wave pressure force</td>
<td>( p_w \cdot D )</td>
<td>( \delta_{pw} \approx (R_1 + D/2) \cdot d\beta )</td>
<td>( p_w \cdot D \cdot \delta_{pw} )</td>
</tr>
</tbody>
</table>

**Table 2b:** Failure mechanism 2; internal work

<table>
<thead>
<tr>
<th>Interface</th>
<th>Strength, ( s_{u,i} )</th>
<th>Displacement, ( \delta_i )</th>
<th>Length, ( l_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active wedge-soil</td>
<td>( c_{u0} + k \cdot D/2 )</td>
<td>( \delta_s = (R_1 + D/2) \cdot d\beta \cdot \sqrt{2} )</td>
<td>( D \cdot \sqrt{2} )</td>
</tr>
<tr>
<td>Interface: active wedge-skirt</td>
<td>( \alpha \cdot (c_{u0} + k \cdot D/2) )</td>
<td>( \delta_{uw} = (R_1 + D/2) \cdot d\beta )</td>
<td>( D )</td>
</tr>
<tr>
<td>Main wedge-soil</td>
<td>( c_{u0} + k \cdot D + kR_1(1-\sin \beta_1/\beta_1) )</td>
<td>( \delta_p = R_1d\beta )</td>
<td>( R_1 \cdot \beta_1 )</td>
</tr>
<tr>
<td>Main wedge-passive wedge</td>
<td>( r_f \cdot [c_{u0} + k \cdot D + 0.5B \cdot \tan \alpha_c] )</td>
<td>( R_1 \cdot \sin \beta_1 \cdot d\beta + R_2 \cdot \cos \beta_2 \cdot d\beta )</td>
<td>( B \cdot \tan \alpha_c )</td>
</tr>
</tbody>
</table>
These expressions have been inserted in a spreadsheet using the same reduction factor as for failure mechanism 1 on the interface between main wedge and passive wedge.

### 9.4.4. Failure mechanism 3; rotational overturning failure

With increasing lever arm the overturning moment may generate an overturning failure where the breakwater and a cylindrical segment of the soil rotates around point with co-ordinates \((x_c, z_c)\) as shown in Figure 6. The wave pressure on the seabed in front of the breakwater will counteract this failure mode. By establishing moment equilibrium about the centre of rotation the failure load can be established.

By establishing moment equilibrium about the centre of rotation the failure load can be established.

Geometrical relationships:

\[
R^2 = \left( \frac{B}{2} + |x_c| \right)^2 + (D + z_c)^2, \text{ if } z_c > -D/2
\]

\[
R^2 = (B/2 + \left| x_c \right|)^2 + (z_c)^2, \text{ if } z_c > -D/2
\]

\[
\alpha = \arcsin(z_c/R)
\]

\[
b_{pw} = R \cos \alpha - B/2 - x_c
\]

\[
x_{pw} = B/2 + x_c + b_{pw}/2
\]
Fig. 6: Failure mechanism for rotational overturning failure

**Moment equilibrium:**

\[
M_B = \int_{\alpha}^{\pi - \alpha} s_{\alpha}(\alpha) \cdot R^2 \cdot d\alpha = (s_{\alpha 0} - k \cdot z_c) \cdot R^2 (\pi - 2\alpha) + 2 \cdot k \cdot R^3 \cdot \cos \alpha 
\]  

(12)

\[
M_D = F_h \cdot (l_h - z_c) - F_v \cdot x_c - p_{pw} \cdot b_{pw} \cdot x_{pw} 
\]  

(13)

The expressions are inserted in a spreadsheet and allow evaluation of combinations of \( F_h \) and \( F_v \) giving rotational overturning failure. The rotational centre co-ordinates are varied to determine the minimum capacity.

### 9.5. EXAMPLE CALCULATIONS

#### 9.5.1. Example structures

Two example structures were selected for parametric evaluation of the effect of skirts. Structure A would be suitable at Gela breakwater site regarding water depth and freeboard while Structure B has dimensions that would be relevant for the Genoa Voltri V1 breakwater site. The main parameters are summarised in Table 3 and illustrated in Figure 7.
Tab. 3: Main dimensions of example structures A and B

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Struct. A</th>
<th>Gela</th>
<th>Struct. B</th>
<th>Genoa V1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth outside breakwater</td>
<td>12.0</td>
<td>12.8</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Freeboard</td>
<td>7.0</td>
<td>7.5</td>
<td>10.0</td>
<td>8.8</td>
</tr>
<tr>
<td>Base width of structure</td>
<td>Variable</td>
<td>17.0</td>
<td>variable</td>
<td>22.5</td>
</tr>
</tbody>
</table>

9.5.2. Wave loading

The wave load was calculated using Goda’s method. The wave height was varied in the range 8 to 14 m for example structure A, and in the range 10 to 14 m for example structure B. The resulting wave force, $F_h$, water pressure at seabed level, $p_w$, and lever arm of the wave force, $l_h$, vs. wave height, $H$, is shown in Tab 4 and 5 for structure A and B respectively.

Tab. 4: Wave force, water pressure at seabed and lever arm of wave force vs. wave height for example structure A (Goda’s method).

<table>
<thead>
<tr>
<th>$H$ (m)</th>
<th>$F_h$ (kN/m)</th>
<th>$p_w$ (kPa)</th>
<th>$l_h$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1120</td>
<td>57</td>
<td>9.8</td>
</tr>
<tr>
<td>10</td>
<td>1435</td>
<td>71</td>
<td>10.1</td>
</tr>
<tr>
<td>12</td>
<td>1750</td>
<td>85</td>
<td>10.4</td>
</tr>
<tr>
<td>14</td>
<td>2067</td>
<td>100</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Tab. 5: Wave force, water pressure at seabed and lever arm of wave force vs. wave height for example structure B (Goda’s method).

<table>
<thead>
<tr>
<th>$H$ (m)</th>
<th>$F_h$ (kN/m)</th>
<th>$p_w$ (kPa)</th>
<th>$l_h$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2274</td>
<td>45</td>
<td>20.9</td>
</tr>
<tr>
<td>12</td>
<td>2778</td>
<td>54</td>
<td>21.7</td>
</tr>
<tr>
<td>14</td>
<td>3282</td>
<td>63</td>
<td>22.2</td>
</tr>
</tbody>
</table>
9.5.3. Selected soil parameters

The effect of skirts on the stability was evaluated for a soil profiles with the following undrained shear strength vs. depth relationship:

\[ s_u(z) = 20 \text{ kPa} + 2\text{kPa/m} \cdot z \]  

(14)

where \( z \) = depth below seabed.
9.6. RESULTS OF PARAMETRIC STUDY

The two example structures A and B have been analysed for various combinations of foundation width, B, and skirt depth, D when subjected to wave loads and wave pressure as described in Chapter 9.5.2. The soil profile described in Chapter 9.5.3 was applied, and the three failure modes described in Chapter 9.4 were evaluated to determine the minimum capacity.

Example structure A

![Graph of Example structure A](image)

Fig. 8: Example structure A: Required foundation width vs. skirt depth and wave height.

The results of the analyses for Example structure A are presented in Figure 8 showing required foundation width B vs. skirt depth D for wave heights in the range 8 to 14 m.

As can be seen from the Figure 8 there is a nearly linear relationship between skirt depth and reduction in required foundation width. For the shear strength profile investigated each meter skirt gives approximately 6m reduction in foundation width. A breakwater structure placed on the seabed, i.e. D=0, directly would require a foundation width in the order of 60 to 100 m which will not be feasible. The application of skirts has the potential of bringing the foundation width towards acceptable values in the order of 20 to 40m. The submerged weight of the structure was kept at 1000 kN/m in the analyses.

Example structure B

The evaluation described for example structure A was also carried out for example structure B. The results are shown in Figure 9.
This structure in 30m water depth is showing a slightly different behaviour. The reduction in required foundation width is in the order of 8 to 10 m per m skirt length. However, the overturning moment is considerably higher for this structure. With increasing skirt length the rotational overturning failure mode (Failure type 3) will be critical. With skirt lengths in the range 10 to 14 m (dependent on the wave height) the curves flattens and there is not much to be gained from further increases in skirt length. This tendency can also be seen in Figure 8 for the lowest wave height at a skirt length of about 6 to 7 m and a foundation width of about 20m.

![Graph showing required foundation width vs. skirt depth and wave height.](image)

**Fig. 9:** Example structure B: Required foundation width vs. skirt depth and wave height.

### 9.7. INSTALLATION METHOD AND PENETRATION RESISTANCE

The typical installation method of a skirted breakwater foundation would be float out from a dry dock, towing to location, exact positioning and ballasting. The skirts will then penetrate until the penetration resistance of the skirts reaches the ballast capacity, i.e. the maximum submerged weight, \( W' \), of the structure. Further penetration can be achieved by pumping out the water in the skirt compartments. The differential pressure (suction), \( \Delta p \), between the inside of the skirt compartment and the outside will give an additional penetration force.

The penetration resistance is composed of the side friction, \( Q_s \), along the skirt walls (inside and outside) and the resistance against the skirt tip, \( Q_t \). The resistance can be estimated from the following expressions:

\[
Q_s = A_s \cdot \alpha \cdot s_{u,av}(D) \quad (15)
\]

\[
Q_t = A_t \cdot N_c \cdot [s_o(D) + \gamma' \cdot D] \quad (16)
\]

where \( A_s = \) side area (inside and outside) of skirt
\[ \alpha = \text{skin friction factor (normally in the order of 0.3)} \]
\[ c_{u,av}(D) = \text{average undrained shear strength over penetrated skirt depth} \]
\[ D = \text{skirt penetration depth} \]
\[ A_t = \text{tip area of skirt} \]
\[ N_c = \text{bearing capacity factor } \approx 9 \]
\[ c_u(D) = \text{undrained shear strength at depth of skirt tip} \]
\[ \gamma' = \text{submerged unit weight of soil} \]

The required underpressure can be calculated as:
\[ \Delta p = \frac{(Q_s + Q_t - W')}{BL} \]  (17)

The structural skirt geometry will depend on the superstructure design and the load conditions. For an example evaluation the skirt geometry shown in Figure 10 has been assumed.

Fig 10: Plan view and cross section of example skirt configuration
Using the same soil profile as for the capacity analysis and the dimensions given in Figure 10, (B=40m, L=30m, t=0.5m) and assuming a submerged weight of 1000 kN/m² the relationship between penetration depth and required underpressure shown in Figure 11 is found:

![Graph of required underpressure vs. skirt penetration depth]

Fig. 11: Required underpressure (suction) vs. penetration depth for example skirt configuration

It can be seen from Figure 11 that the required underpressure to achieve 10 to 15 m skirt penetration depth is in the order of 70 to 120 kPa, which is well within the available underpressure (water pressure + atmospheric pressure) for the water depths in question. The base slab and the skirt walls have to be designed with sufficient capacity to resist the differential pressure.

9.8. CONCLUSIONS AND RECOMMENDATIONS

The evaluation of the effect of skirts on vertical breakwater structures on clayey soils has demonstrated that the foundation capacity can be increased considerably by use of skirts. Skirted breakwater structures can be installed quickly and replace composite breakwaters with a caisson structure on top of a rubble mound, which often will require staged construction in order to allow the subsoil to gain sufficient strength.

The installation evaluation has demonstrated that penetration resistance should normally not be a restriction and that moderate underpressure will be sufficient to achieve full penetration.

The evaluations were carried out for two example structures in 12 and 30m water depth and for one set of soil conditions. The conclusions will therefore not be generally valid, but it has been shown that skirted structures may be a viable alternative to composite breakwaters from a geotechnical point of view. Construction cost has not been considered in this study.
REFERENCES