## DELFT UNIVERSITY OF TECHNOLOGY

**Department of Electrical Engineering**

**Telecommunication and Traffic**

**Control Systems Group**


Title: **Monte Carlo Simulation of Co-channel Interference in a Shadowed Rician Indoor Environment.**


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**Abstract.**

The co-channel interference probability has been determined by using system simulation. For this purpose the Monte Carlo method has been used, which consists of generation of random numbers from probability distribution functions. The simulation results have been verified by comparing them with previous obtained analytical results. The simulation program has been used to investigate the influence of the Rice factor, the local or (area) mean power and the logarithmic variance on the outage probability in an indoor shadowed Rician environment. The outage probability has been also determined when the interfering signals have different local or (area) mean powers. In order to accelerate the simulation process, the importance sampling technique is investigated. The cluster structure of the $19^{th}$ floor of the Electrical Engineering building of Delft University is determined by system simulation and using the measurements that has been taken there before.



Indexing terms:

Outage probability, random, fast fading, slow fading, simulation, carrier-to-interference ratio.

## Preface

The graduation research for my study at the Technical University in Delft, is fulfilled at the Department of Electrical Engineering, Laboratory of Telecommunication and Traffic Control Systems Group.

The task assigned to me is to Simulate the Co-channel Interference probability in a Shadowed Rician Indoor wireless Communication Environment.

The results of this research are achieved thanks to the input of many people. I am especially indebted to my mentors ir. A. Kegel and Prof. Dr. R. Prasad for their counsel and guidance.

Further I would like to thank Hami Çamkerten for the many discussions we had.

Finally, I would like to thank my mother and family for the grateful support.

## Summary

Radio signals in an indoor environment are faded signals because of a hostile environment. Propagation measurements in this environment have shown that signals are fast and slow faded and because the communication cells are taken very small, the signals are Rician rather than Rayleigh distributed.

In this report the outage probability in a shadowed Rician environment has been determined by system simulation. Therefore two propagation models were proposed which are corresponding with the case when the signals are only fast faded and the case when the signals are slow and fast faded. For each model the influence of the model parameters on the system performance is investigated. This parameters are the Rice factor and the local mean power for model 1 and the logarithmic variance and the area mean power for model 2.
As we have mentioned, the signals in an indoor environment are random rather than deterministic, so in order to simulate the outage probability, random numbers which are Rician distributed are first generated using the Monte Carlo method. Before using the simulation method, the simulation results have been verified by comparing them with the previous obtained analytical results [7,10]. For both models 1 and 2, we found that the simulation results agreed with the analytical results.

The simulation results have shown that the Rice factor of the desired signal is an important parameter for the calculation of the outage probability. However the Rice factor of the interfering signals is less significant for the outage probability. Further the results have also shown that when the variance of the local mean power of the desired signals increases, the outage probability increases also. The influence of the logarithmic variance of the local mean power of the interferers have various consequences for the outage probability depending on the value of the total carrier-to-interference ratio.
In order to accelerate the simulation process, the importance sampling techniques is used. This technique consists of increasing the number of errors in an artificial way so that less number of samples are needed to determine low values of the outage probability.

When the simulation program has been used to determine the cluster structure of the 19$^{th}$ floor of the Electrical Engineering building of Delft University, we found out that if an outage probability lower than 10% is required, the smallest possible cluster size is 3. This result is based on the measurements that were done at this floor in [1].

## List of abbreviations and symbols.

| | |
|---|---|
| CDF | cumulative distribution function |
| CIR | carrier-to-interference ratio |
| PDF | probability density functions |
| Pout | outage probability |
| $P_d$ | the instantaneous power of the desired signal |
| $P_i$ | the instantaneous power of the $i^{th}$ interferer |
| S | The peak value of the dominant component. |
| $\sigma^2$ | The variance of the random component |
| $\sigma_d{}^2$ | The variance of the desired random component |
| $\sigma_i{}^2$ | The variance of the undesired random component |
| $\sigma_{ld}{}^2$ | The logarithmic variance of the local mean power of the desired signal |
| $\sigma_{li}{}^2$ | The logarithmic variance of the local mean power of the undesired signal |
| $K_d$ | The Rice factor of the desired signal |
| $K_i$ | The Rice factor of the undesired signal |
| $\alpha$ | The protection ratio |
| ß | The Rice factor reduction ratio |
| $m_{ld}$ | The area mean power of the desired signal |
| $m_{li}$ | The area mean power of the undesired signal |
| n | The number of interferers |
| N | The number of samples |
| N(0,1) | The standard normal distribution |
| $\mu$ | The mean value of the probability density function |
| $W(P_d)$ | The weight factor |

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# 1. Introduction.

The demand for indoor cellular radio communication possibility is growing, because of the many advantages that those systems are offering. The most important advantages are :

- mobility of the users,
- elimination of cabling,
- flexibility of changing or creating new communication services.
- frequency reuse possibility.

An examples of such cordless systems are: Radio local-area-networks (radio-LAN) and the Digital European Cordless systems (DECT). When using such systems, the available frequency channels to cover a certain area are very limited, so investigation of methods that allow to increase the system capacity is necessary. One solution for this problem is the frequency reuse strategy which allows that many users can use at the same time the same frequency channel, providing that the outage probability ( probability that the communication because of the interferers is not more possible) is below a certain acceptable value. Therefore when designing a system using the frequency reuse strategy, the outage probability must be calculated. However calculation of the outage probability demands a good knowledge about the behaviour of radio signals. In an indoor environment, radio signals are random rather than deterministic and the local mean power of the interferers are not always equal. This all makes the calculation of the outage probability very complicated. Furthermore radio signals in an indoor environment suffers from shadowing and in each building the radio signals are differently attenuated. In order to solve these problems without any restrictions on the channel characteristics, simulation of the outage probability in an indoor radio environment is proposed.

In this report, the different steps that are used to simulate the outage probability are discussed. After a short description of the cellular concept, the frequency reuse strategy and the signals characteristics in an indoor environment in the second chapter, follows in chapter 3 how random numbers from some kind of probability distribution function can be generated with the purpose to be used later to simulate the outage probability. In chapter 4 and 5, the simulation results of the outage probability in an indoor Rician and shadowed Rician environment respectively are presented. Chapter 6 gives a description of the importance sampling techniques that is used to accelerate the simulation process. As described in chapter 7, the simulation program has been used to determine the cluster structure of the 19$^{th}$ floor of the Electrical Engineering building of Delft University, based on measurements that were taken there earlier. Finally in chapter 8 the most important conclusions are discussed.

# 2. Cellular mobile radio communication and channel characteristics.

## 2.1 Cellular concept.

The cellular concept is introduced in mobile communication systems in order to assign a number of frequency channels to a particular cell within a cluster of cells. So the same frequency channels can be reused in other cluster as much as needed to cover all area where the mobile communication system will be used [15]. The frequency reuse strategy is used in such a way providing that the co-channel interference is kept below an acceptable level.
An example of the cellular system is given in fig.1 below

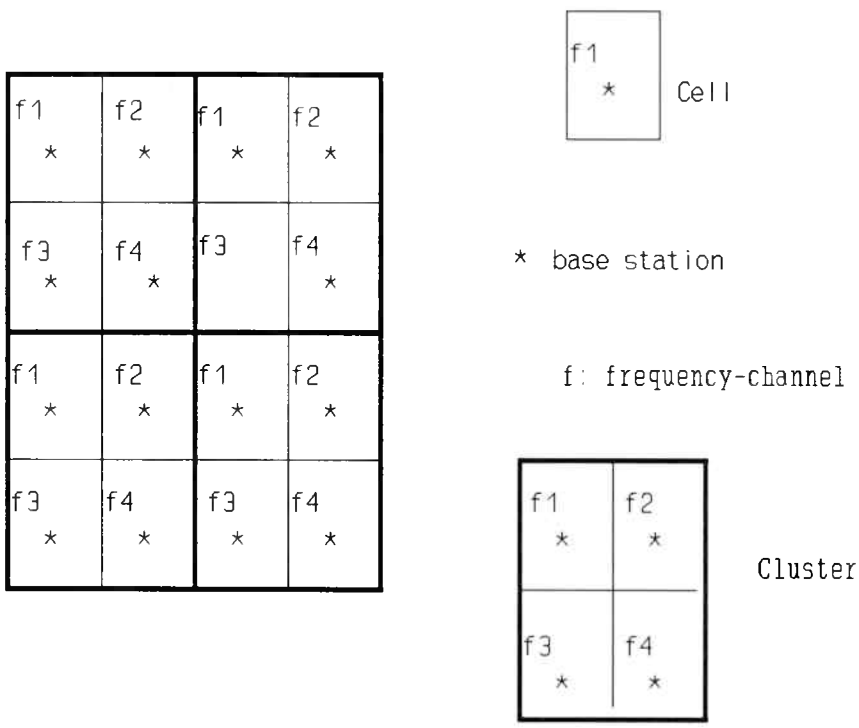


**Figure.1:** The cellular concept.

The mobile user inside cell 1 will be served by the base station located in that cell. The same frequency channel can be used in cell 1 of other clusters. The distance between two co-channel cells must be chosen in such a way that the interference level caused by the interferers is lower than a certain acceptable level.

### 2.2 The outage probability.

A measure of the effect of the co-channel interference on the system quality is given by the outage probability. The outage probability is defined as the probability that the ratio of the desired signal power and the interfering signal power is below a certain protection value. When the signal distribution is known, the outage probability conditional upon n can be written as:

$$Pr(\frac{P_d}{P_n}<\alpha)=\int_0^\infty\int_0^{\alpha P_n} f(P_d)f(P_n)dP_d dP_n \tag{1}$$

Where $P_d$= the desired instantaneous signal power,

$P_n$= the interfering instantaneous signal power,

$\alpha$ = the protection ratio, required for the radio system,

$f(P_d)$= the probability density function (PDF) of $P_d$,

$f(P_n)$= the probability density function of $P_n$.

n = the number of the interferers that are at the same time active.

In the case when many interferers are active, $f(P_n)$ will be taken as the PDF of the joint interference signal powers.

### 2.3 Spectrum efficiency.

The spectrum efficiency is a measure of the performance of a mobile communication system. The system with the highest spectrum efficiency is the one with the highest performance. The Spectrum efficiency is defined as [12].

$$SE=\frac{traffic\ per\ cell}{system\ freq.bandwith\ .\ cell\ area} \quad erlang/MHz/Km^2. \tag{2}$$

We see from equation (2), that the spectrum efficiency can be increased by decreasing the cluster area. When the cell area is fixed, the cluster area can be decreased by decreasing the number of cells inside each cluster.

### 2.4 The channel characteristics.

Measurements in mobile indoor radio environment have shown that the mobile radio channels are faded channels. The channel fading is characterized by the multipath and shadow fading [15].

#### 2.4.1 Multipath fading.

The multipath fading is caused by rapid changes in the amplitude of the signal between the base station and the mobile unit. The rapid changes follows from the signal reflections in an indoor environment. Figure.2 shows the measured signal variation in time.

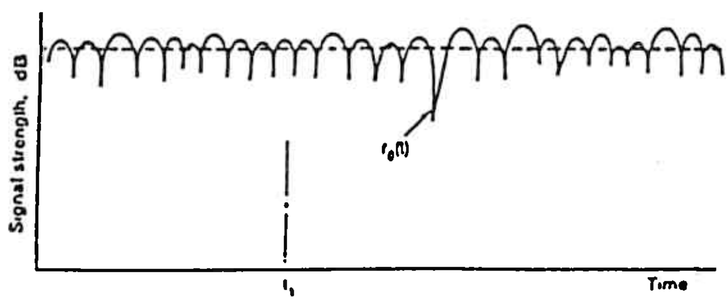


**Figure.2:** Multipath fading.

### 2.4.2 Shadow fading.

Shadow fading is caused by the slow changes in the local mean power because of the presence of obstacles between the base station and the mobile unit in an indoor environment. Figure.3 shows the measured variation of the local mean signal power in time.

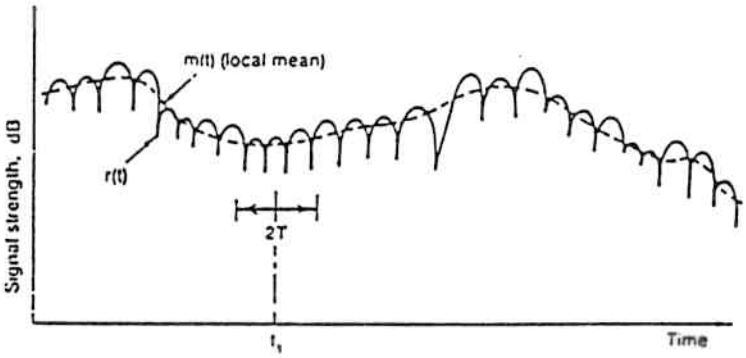


**Figure.3:** The shadow fading.

### 2.5 The log-normal probability density function.

The slow variation of the local mean power in an indoor environment, can be described by a log-normal PDF given by

$$f(P_0)=\frac{1}{\sigma_l P_0\sqrt{2\pi}}\exp\left[-\frac{[\ln(P_0)-m_l]^2}{2\sigma_l^2}\right]. \quad (3)$$

where $P_0$= local mean power,

$\sigma_l^2$= the logarithmic variance of $\ln(P_0)$,

$m_l$= the logarithmic mean of $\ln(P_0)$.

The mean value and the variance of $P_0$ are calculated by Fenton [16], and are given by

$$\mu=\exp(m_l+\frac{\sigma_l^2}{2}) \tag{4}$$

and

$$\sigma^2=[\exp(\sigma_l^2)-1]\ \exp(2m_l+\sigma_l^2) \tag{5}$$

The median of $P_0$ is

$$\hat{P}_0=\exp(m_l) \tag{6}$$

**2.6 The Rician probability density function.**

Due to the multipath fading of the signal amplitude in an indoor environment, the signal amplitude will have a Rician probability density function given by

$$f(r)=\frac{r}{\sigma^2}\exp[-\frac{r^2+S^2}{2\sigma^2}]I_0[\frac{Sr}{\sigma^2}] \qquad r\geq 0 \tag{7}$$

The corresponding PDF of the instantaneous power will be given by

$$f(P)=\frac{1}{\sigma^2}\exp[-\frac{2P+S^2}{2\sigma^2}]I_0[\frac{\sqrt{2P}S}{\sigma^2}] \tag{8}$$

Where

$I_0(x)$ = The modified bessel function of the first kind and zero order,

$S^2/2$ = average power of the dominant component,

$\sigma^2$ = average power of the random component.

The local mean power is given by

$$P_0=\frac{S^2}{2}+\sigma^2 \tag{9}$$

Further we define the Rice factor given by

$$K=\frac{S^2}{2\sigma^2}. \tag{10}$$

When the Rice factor **K=0**, the instantaneous power will have the negative-exponential PDF given by

$$f(P)=\frac{1}{\sigma^2}\exp[-\frac{P}{\sigma^2}].$$

**2.7 Propagation characteristics.**

Measurements in an indoor radio environment show that the received signal in an indoor radio environment is slow and fast faded because of the small cell size and the presence of many obstacles between the base station and the mobile or movable unit.

**2.7.1 Propagation model of the received signal without shadow fading**

If we assume that the received signal is only fast faded which means that the local mean power $\mathbf{P_0}$ is constant, the PDF of the received desired signal power $\mathbf{P_d}$ will be given by

$$f(P_d)=\frac{1}{\sigma_d^2}\exp\left[-\frac{2P_d+S_d^2}{2\sigma_d^2}\right]I_0\left[\frac{\sqrt{2P_d}S_d}{\sigma_d^2}\right] \tag{12}$$

Where

$\sigma_d^2 = P_{0d}/(K_d+1)$.

The PDF of the received interfering power has the form (12) with the substitutions:

$P_d \rightarrow P_i$

$P_{0d} \rightarrow P_{0i}$

$K_d \rightarrow K_i$

$S_d \rightarrow S_i$

$\sigma_d \rightarrow \sigma_i$

Equation (12) shows that the stochastic behaviour of the received signal, when the signal is only fast faded, can be modeled by the following parameters:

- Rice factor (K),
- The average power of the random component ($\sigma^2$).

**2.7.2 Propagation model of the received signal including shadow fading.**

When the received signal is not only fast faded but also slow faded which means that the local mean power $\mathbf{P_0}$ is log-normal distributed, the PDF of the desired instantaneous power will be given by [10]

$$f(P_d/P'_{0_d})=\frac{1}{P'_{0_d}}\exp\left[-\frac{2P_d+S_d}{2P'_{0_d}}\right]I_0\left[\frac{\sqrt{2P_dS_d}}{P'_{0_d}}\right] \tag{13}$$

Here $\mathbf{P'_{0d}= P_{0d}/(K_d+1)}$,

The slow varying local mean power $\mathbf{P_{0d}}$ has the log-normal PDF given by

$$f(P_{0_d})=\frac{1}{\sigma_{l_d}P_{0_d}\sqrt{2\pi}}\exp\left[-\frac{(lnP_{0_d}-m_{l_d})^2}{2\sigma_{l_d}^2}\right]. \tag{14}$$

Here, $\boldsymbol{\sigma_{ld}^2}$ and $\mathbf{m_{ld}}$ are the logarithmic variance and the area mean of $\mathbf{ln(P_{0d})}$ respectively. On the same way, we can find the PDF of the received interfering signal.

Equations (13) and (14), show that the stochastic behaviour of the received signal, when the signal is fast and slow faded, can be modeled by the following parameters:

- The Rice factor (K),
- The logarithmic area mean of $\ln(P_0)$ ($\mathbf{m_l}$),
- The logarithmic variance of $\ln(P_0)$ ($\boldsymbol{\sigma_l^2}$).

When the variance $\sigma_l^2=0$, the received signal will suffer only from fast fading. Thus in this case the propagation model goes over into the model presented in paragraph (2.7.1).

# 3. Generation of random numbers

In this chapter, random numbers from some kind of probability density function will be generated with the purpose to be later used for the simulation of the outage probability.

### 3.1 Simulation.

The use of simulation has been increased enormously in the recent years. Simulation has been found to be an extremely important tool for investigators. Recent advances in simulation methodologies, availability of software, and technical developments have made simulation one of the most widely used and accepted tools in system analysis. Simulation can be used to studies the behaviour of complex systems. It usually requires a large amount of time and great expense for analysis and programming.

There are many kinds of simulation. One of them is called Monte Carlo simulation [1], it includes generation of stochastic numbers from a particular probability distribution.

### 3.2 Monte Carlo method.

Monte Carlo method is known as a technique of using random numbers from the computer to simulate a model [1]. Monte Carlo method uses uniform distributed random numbers to obtain various desired approximated distributions.

The most important characteristics of Monte carlo method are:

1. The time does not play an essential role.
2. The observations in the Monte Carlo method, are independent. So the outcomes are not correlated.

### 3.3 Generation of random numbers with a particular distribution

There are many procedures for generating random numbers with a particular distribution. All of them make use of uniform distributed random numbers.

### 3.3.1 Uniform distributed random numbers

Many techniques for generating uniform random numbers make use of digital computers. The most important techniques are the **congruential techniques** [1]. A variety of those techniques are implemented on computers and are called random number generators. The most of those techniques generate uniform distributed numbers from the interval [0,1].

As we have mentioned, uniform random numbers from the interval [0,1] exist already in the computer and they can be used any time. But if we need uniform random numbers from some particular interval, some kind of transformation is required. An example of transformation to get uniform random numbers from the interval [a,b] is given by

$$x=(b-a)u+a. \qquad (15)$$

where u is uniform distributed on [0,1].

When using the standard function ( Random ) from the computer program language Pascal, uniform distributed numbers have been generated. The corresponding PDF and CDF are plotted in fig.4 and 5, generating $10^4$ numbers.

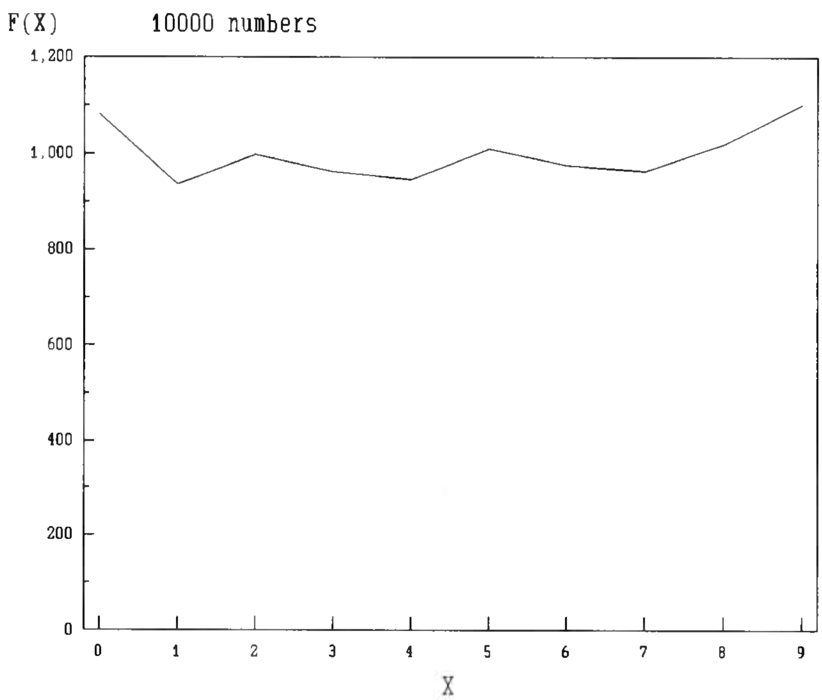


**Figure.4:** The PDF of the simulated Uniform distribution.

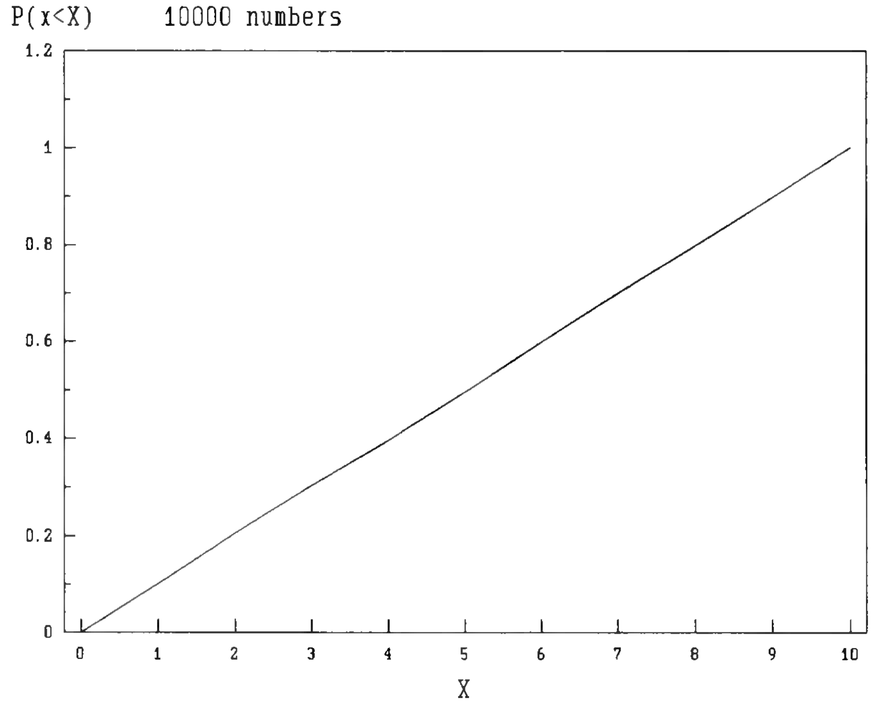


**Figure.5:** The CDF of the simulated Uniform distribution.

From fig.4 it seems that the PDF is not a straight line. In fact the values increase and decrease between certain small margins. This error will be accepted if when using these numbers a good approximation for the other distributions can been obtained.

### 3.3.2 Normal-distributed random numbers.

A random variate X has a normal distribution if the PDF is

$$f_X(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]. \qquad (16)$$

and is denoted $N(\mu,\sigma^2)$. Here $\mu$ is the mean and $\sigma^2$ is the variance.

X may be written as:

$$X=\mu + \sigma Z,$$

where Z is the standard normal distributed variable denoted by N(0,1). Thus X may be easily generated from N(0,1).

There are many procedures for the generation of normal distributed numbers [1]. One of them is given below:

**Procedure**

In this procedure lets $U_1$ and $U_2$ been independent random variates from u(0,1), then it can be shown ( see appendix A) that the variates

$$Z_1=(-2\ln U_1)^{1/2}\cos 2\pi U_2$$
$$Z_2=(-2\ln U_1)^{1/2}\sin 2\pi U_2 \tag{17}$$

are independent standard normal distributed variables [1].

**Algorithm:**

1. Generate two independent uniform random numbers $U_1$ and $U_2$ from u(0,1).
2. Compute $Z_1$ and $Z_2$ simultaneously by substituting $U_1$ and $U_2$ in equation (17).
3. Substitute $Z_1$ and $Z_2$ in

$$X_1= \mu_1+\sigma Z_1,$$
$$X_2= \mu_2+\sigma Z_2$$

for a given $(\mu_1,\mu_2,\sigma)$.

4. Deliver $X_1$, $X_2$.

When using this algorithm, normal distributed numbers have been generated. The CDF of the generated numbers has been simulated and is plotted in fig.6.

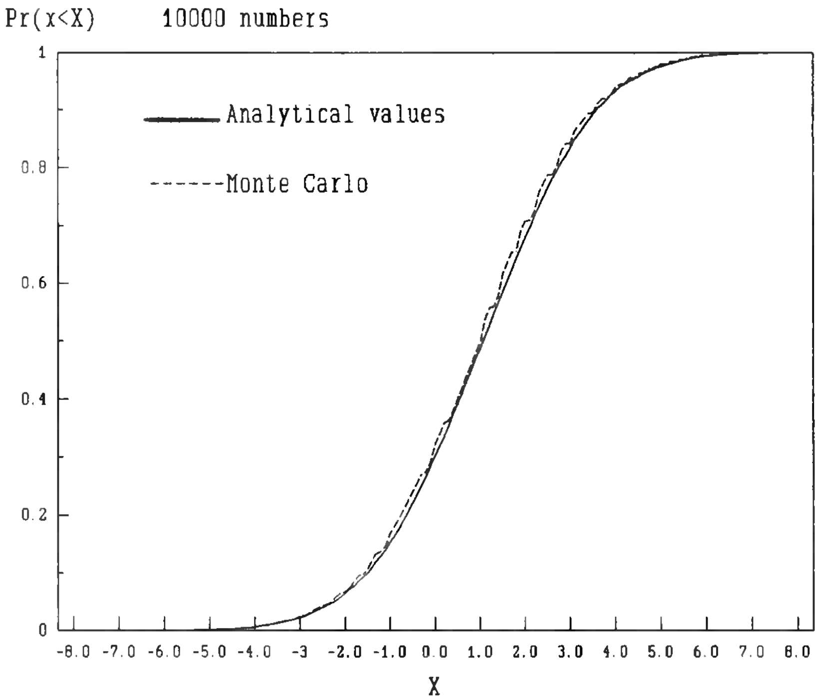


**Figure.6:** The CDF of the normal distributed random numbers for $\sigma=2$ and $\mu=1$.

From fig.6, it appears that the simulated CDF of the normal distributed numbers is nearly the same as the analytical one. So the generated numbers can been accepted as having the normal distribution.

### 3.3.3 Log-normal distributed numbers

When x is normal distributed with mean **m** and variance $\sigma^2$, then $p_0=\exp(x)$ is log-normal distributed with mean $\exp(m+\sigma^2/2)$ and variance $\{\exp(\sigma^2)-1\}\exp(2m+\sigma^2)$ (see Appendix B). Let x have a PDF given by:

$$f_x(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp[-\frac{(x-m)^2}{2\sigma^2}]. \tag{18}$$

The log-normal PDF can be obtained using the following transformation [2].

$$f_{p0}(p_0)=f_x(x)\left|\frac{dx}{dp_0}\right| \tag{19}$$

with $x=\ln(p_0)$.

we obtain the following PDF for $p_0$

$$f_{p0}(p_0)=\frac{1}{\sqrt{2\pi}\sigma p_0}\exp\left[-\frac{[\ln(p_0)-m]^2}{2\sigma^2}\right]. \tag{20}$$

**Algorithm**

1. For X normal distributed variate, deliver $p_0=\exp(X)$.

In order to check whether the generated numbers have really a log-normal distribution, the CDF of the generated numbers has been simulated and compared with the analytical CDF. The results are presented in fig.7.

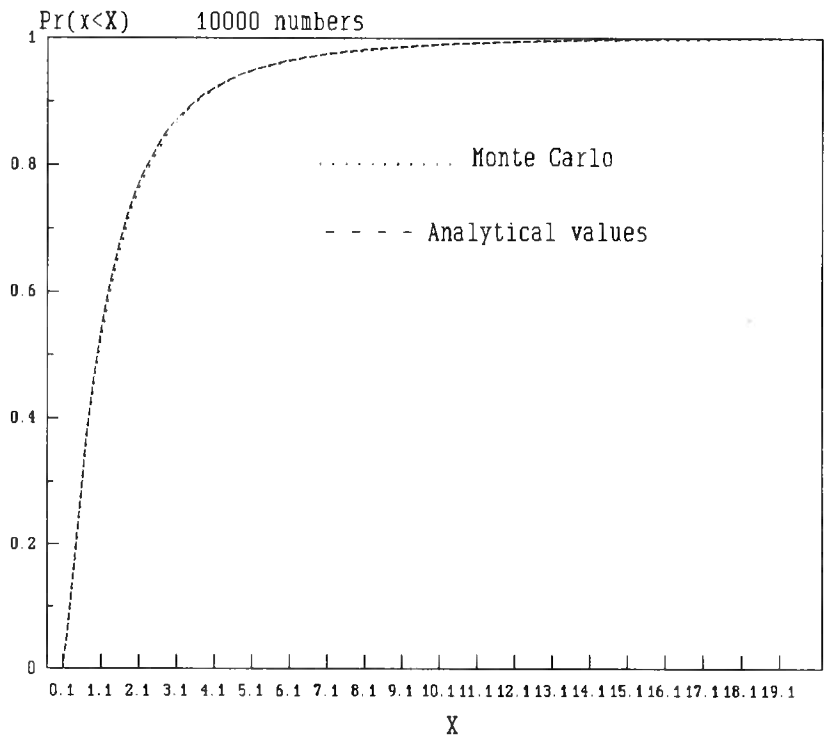


**Figure. 7:** The CDF of a log-normal distributed numbers

### 3.3.4 Rician distributed numbers

For $X_1$ and $X_2$ normal distributed numbers from $N(0,\sigma)$ and $N(\mu,\sigma)$ respectively, it can be shown that

$$R=\sqrt{X_1^2+X_2^2} \tag{21}$$

is Rician distributed [9] with Rice factor given by

$$K=\frac{\mu^2}{2\sigma^2} \tag{22}$$

For the case $\mu=0$, the Rice factor becomes $K=0$, thus the generated numbers will have a Rayleigh distribution.

**Algorithm**

1. For generating $X_1$ and $X_2$ from the normal distributions $N(0,\sigma)$ and $N(\mu,\sigma)$ respectively, deliver $R=(X_1^2+X_2^2)^{1/2}$.

Generation of Rician distributed random numbers can be achieved using the following model.

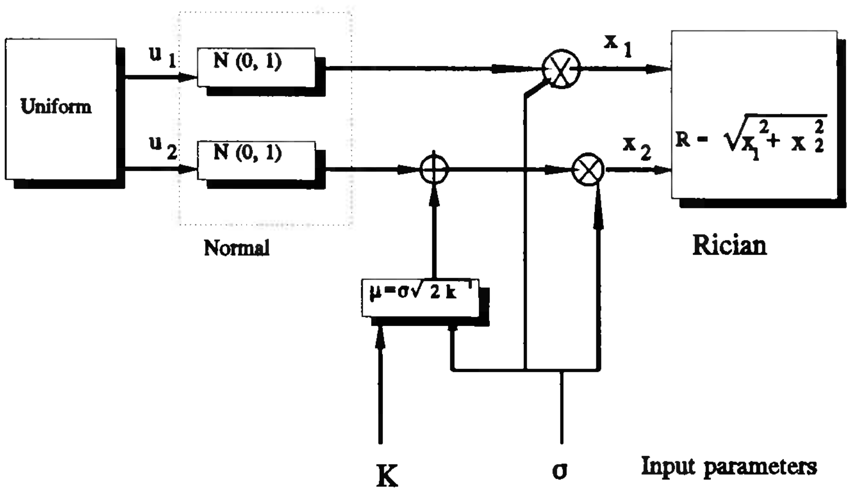


**Figure.8:** Rician number generation.

In fig.9 and fig.10, the simulated CDF of the generated numbers are plotted. Fig.9 represents the CDF of the generated random numbers when the Rice factor K=0 and fig.10 for K=8.

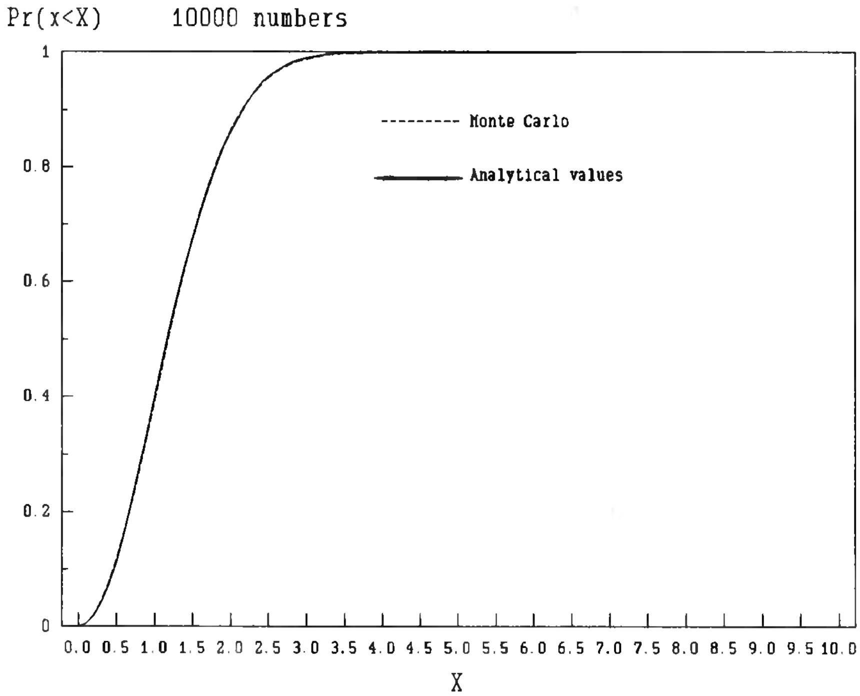


**Figure.9:** The CDF of a Rayleigh distribution for $\sigma=1$.

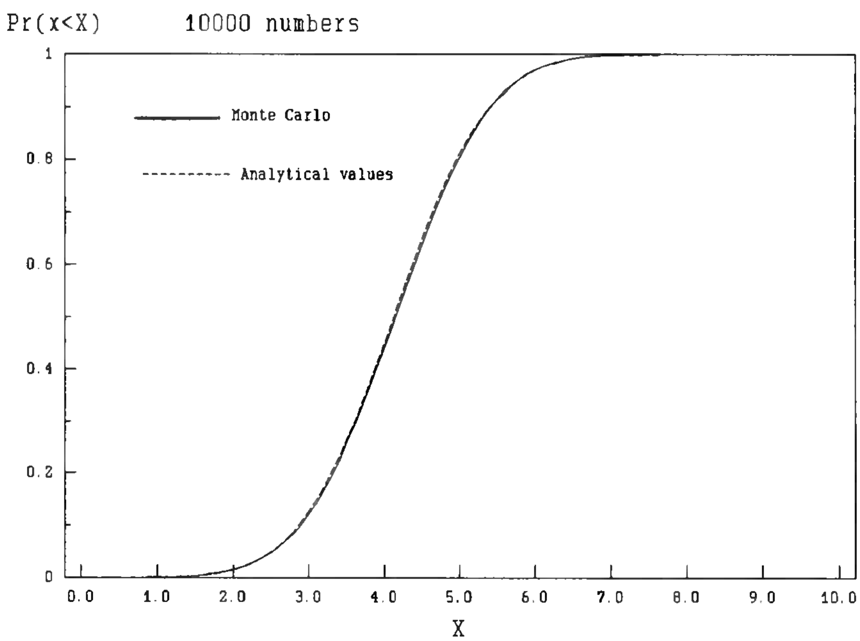


**Figure.10:** The CDF of a Rice distribution for K=8.

From fig.9 and fig.10, we see that the simulated CDF is very close to the analytical calculated CDF.

### 3.4 Conclusions

Due to the generation of the uniform random numbers from the interval [0,1] and using some transformation techniques, random numbers from some particular distributions have been obtained. Further the CDF of the obtained numbers has been simulated and compared with the analytically calculated CDF. In all cases we have seen that the simulated CDF approaches very well the analytical calculated CDF. This results show the reliability of using those numbers to simulate the outage probability in an shadowed Rician environment.

# 4. Simulation of the outage probability in an indoor Rician environment.

In this chapter, the outage probability in an indoor Rician environment will be simulated. The radio signals in an indoor environment have a stochastic behaviour. So in order to simulate the outage probability, sampling variates from probability distributions are required. Therefore the Monte Carlo method which consist of generating random numbers from some particular distribution will be used.

### 4.1 Simulation method

The simulation method depends on the used communication system which consists of many mobile users and a base station. In the base station all signals arrive with some instantaneous power. The signal coming from the appropriate cell will be considered as the desired signal and the other signals coming from the neighbouring cells will be considered as interferers. The outage probability can be simulated by every time taking a signal sample from the desired signal and compare it with the sum of the samples from the interfering signals. Every time that some desired sample is lower than an interfering sample times certain protection ratio, an error will be detected and an error counter will be increased by one. This situation is depicted in the following model given in fig.11.

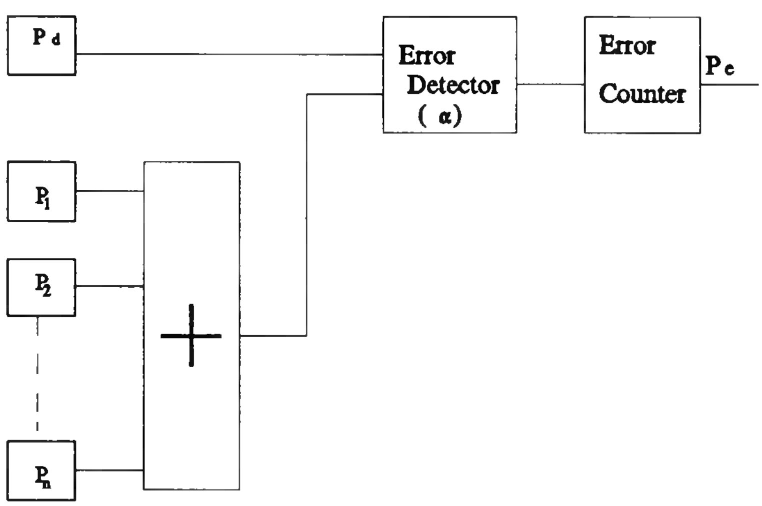


**Figure.11:** Simulation model

Let us say that an error will occur if $\mathbf{P_d \leq \alpha P_n}$. The probability of errors [6] is then :

$$P_e = \int_0^{\infty} f(P_n)\, dP_n \int_0^{\alpha P_n} f(P_d)\, dP_d \tag{23}$$

Suppose that $h_e(\mathbf{P_d})$ is given by

$$\begin{aligned} h_e(P_d) &= 1, \quad if \quad P_d \leq \alpha P_n \\ &= 0, \quad if \quad P_d \geq \alpha P_n \end{aligned} \tag{24}$$

Equation (23) is equivalent to

$$P_e = E\left[h_e(P_d)\right] \tag{25}$$

$h_e(\mathbf{P_d})$ is an **error detector**. If the number of observations is given by N, then $\mathbf{\Sigma^N_{i=1}\ h_0(P_{d^i})}$ is an **error counter**.

A simple and natural unbiased **estimator**, can be given by

$$\hat{p}_e = \frac{n}{N}. \tag{26}$$

where n is the number of observations where an error has been occurred.

In the limit $N \rightarrow \infty$ the estimate $\mathbf{\hat{P}_e}$ converge to the true value $\mathbf{P_e}$. For finite N we quantify the reliability of the estimator in terms of **confidence intervals.** The confidence interval is given by two numbers $h_1$ and $h_2$ which for given probability satisfies $h_2 \leq \mathbf{P_e} \leq h_1$. The **confidence level, 1-ß** is defined as

$$Prob[h_2 \leq p \leq h_1] = 1 - \beta. \tag{27}$$

In [6] it has been shown that if we set $P_e=10^{-k}$ and $N=\mu 10^k$, we find for $(h_2,h_1)$

$$h_1^2=10^{-k}\left\{1+\frac{d_\beta^2}{2\mu}\left[1\pm\sqrt{\frac{4\mu}{d_\beta^2}+1}\right]\right\} \tag{28}$$

where $d_\beta$ is chosen so that

$$\int_{-d_\beta}^{d_\beta}\frac{1}{\sqrt{2\pi}}\exp(-\frac{t^2}{2})dt=1-\beta. \tag{29}$$

From the plots of the confidence interval in [6], it seems that N for the confidence level of 90%, 95% and 99% can be approximated between **[10/$P_e$, 100/$P_e$]**. thus for **$P_e$= $10^{-k}$** and a confidence level of 90%, a number of samples **N= $10^{k+1}$** is required.

**4.2 Co-channel interference model**

In this case we assume that all signals are Rician distributed with a specified value of the Rice factor K and the spread $\sigma$ of the random component. The co-channel model that will be simulated is given below

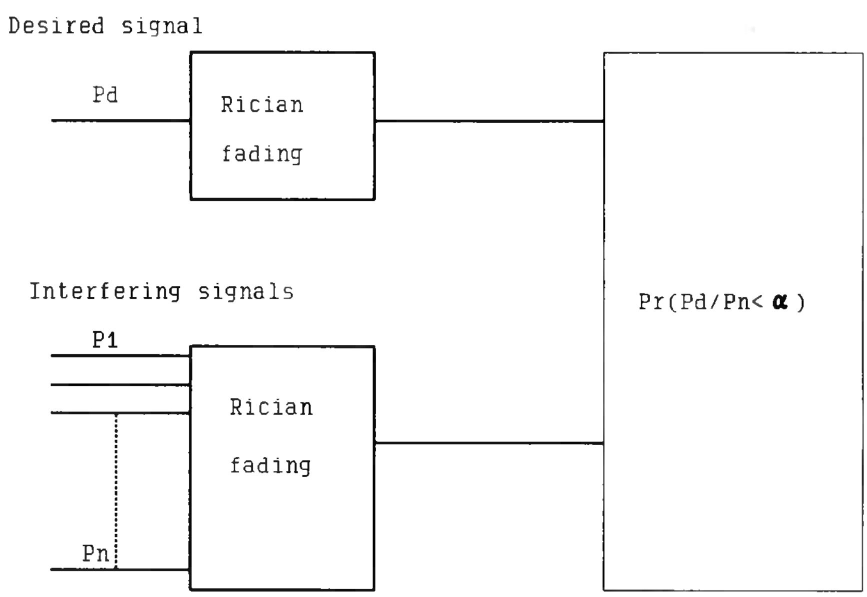


**figure.12:** The co-channel model

When using this model, the behaviour of the outage probability as function of the carrier-to-interference ratio (CIR) will be investigated.

Lets consider the case where all signals are Rician distributed, then the CIR will be given by

$$CRI = \frac{P_{0_d}}{\sum_{i=1}^{n}(1+K_i)\sigma_i^2} \qquad (30)$$

here

$$P_{0_d} = (1+K_d)\sigma_d^2 \qquad (31)$$

**n** is a number of interferers, **K** is the Rice factor and $\boldsymbol{\sigma^2}$ is the average power of the random component.

Using equation (30) and (31), we find out

$$\sigma_d^2 = CIR\,\frac{\sum_{i=1}^{n}(1+K_i)\sigma_i^2}{(1+K_d)} \qquad (32)$$

If we suppose that $\mathbf{K_d}$ and all $\mathbf{K_i}$ and $\boldsymbol{\sigma_i^2}$ are specified, then the outage probability can been simulated as a function of the total CIR.

**4.3 Simulation procedure.**

The outage probability can been simulated as a function of the total CIR, by using the following procedure:

- Generate for each interferer, a sample $P_i$ using the values ($K_i$, $\sigma_i$).
- For each value of CIR, determine $\sigma_d$ using equation (32).
- Generate a desired sample $P_d$ using the values ($K_d$, $\sigma_d$).
- Compare $P_d$ and $\alpha$ times $\Sigma P_i$.
- If error then increase counter.

### 4.4 Simulation results.

In this paragraph the influence of the Rice factors $K_d$ and $K_i$ of the desired signal and the interfering signal respectively on the outage probability will be investigated using the simulation method. Comparison with the previous analytical method [7] will be made in order to check the validity of the simulation method.

#### 4.4.1 The interferers have the same Rice factor and the same local mean power.

If we assume that all interfering signals have the same Rice factor $K_i$ and the same mean power $P_{0_i}$ then equation (32) becomes

$$\sigma_d^2 = CRI \, \frac{n(1+K_i)}{(1+K_d)} \sigma_i^2$$

**a) The influence of $K_i$ and $K_d$.**

In this case we have based our choice for the values of $K_i$ and $K_d$ on the measurements taken in the electrical engineering department of the Delft university of technology [7]. In [7] it was found that the Rice factor varies between 0 and 7. The simulation results are presented in figure.13.

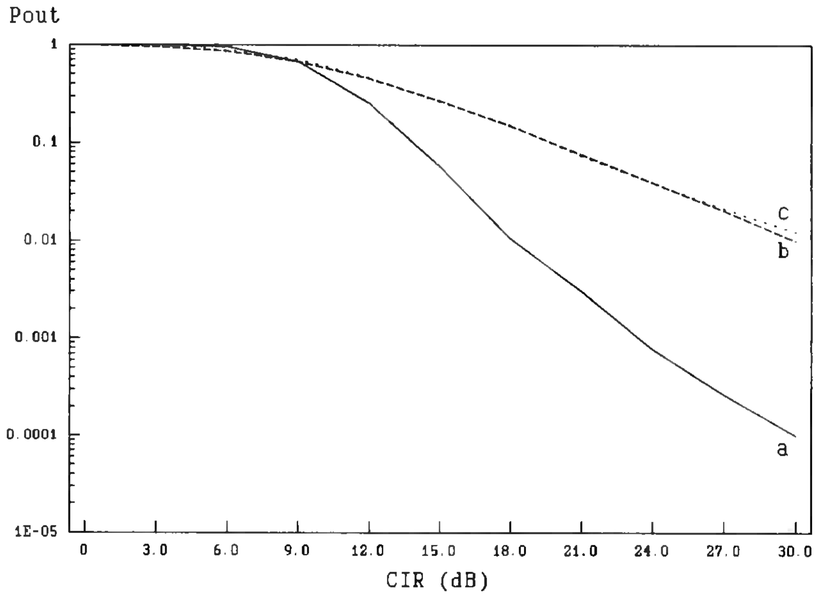


**Figure.13:** The simulated outage probability versus CIR for $\alpha$=10 dB, n=4 and N=$10^4$

a) $K_d$=7 and $K_i$=3.

b) $K_d$=0 and $K_i$=0.

c) $K_d$=0 and $K_i$=3.

The analytical results obtained in [7] are also presented below,

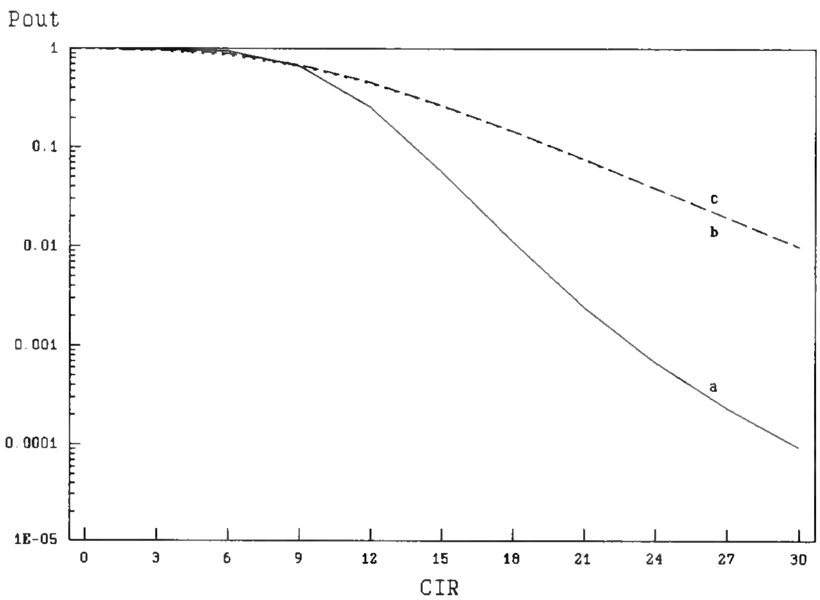


**Figure.14:** The analytically computed outage probability versus CIR for $\alpha$=10 dB, n=4.

From the plots it appears that the simulation results agree almost with the analytical results. So we can conclude that the simulation method gives suitable results. Thus it can be used to investigate the outage probability for the cases where the analytical method no solution can offer.

Further from the plots above, the following points are noticed:

1. IF $P_{out} \leq 10\%$ is required, we see that when $K_d=0$ the minimum allowed CIR is 20 dB and when $K_d=7$ the minimum allowed CIR is 14 dB.
2. For fixed value of $K_d$, the difference between $K_i=0$ and $K_i=3$ is not noticeable.
3. For CIR values between 0 and 3 dB, the outage probability still maximum and equal 1.

### 4.4.2 The interfering signals have different local mean powers.

When the transmitters of the interfering signals are located at different distances from the receiver, the local mean powers of the interfering signals will be different. If this can not be neglected, calculation of the outage probability analytically will be very complicated. Therefore simulation of this case may be the only way to solve this problem.

In table.1 below we have tried to see what will happen for the outage probability if the local mean power of interferer 3 increases.

| | Desired signal | interferer 1 | Interferer 2 | Interferer 3 | $P_{out}$ |
|---|---|---|---|---|---|
| $P_0$<br>K | 20 dB<br>7 | 0 dB<br>3 | 0 dB<br>3 | 0 dB<br>3 | $5.48\ 10^{-2}$ |
| $P_0$<br>K | 20 dB<br>7 | 0 dB<br>3 | 0 dB<br>3 | 6 dB<br>3 | $2.34\ 10^{-1}$ |
| $P_0$<br>K | 20 dB<br>7 | 0 dB<br>3 | 0 dB<br>3 | 12 dB<br>3 | $7.35\ 10^{-1}$ |

Table 1: The outage probability when the local mean power of one interferer increases.

From table.1, we see that when the local mean power of one interferer is increasing, the outage probability becomes larger.

Table.2, represents the simulated outage probability when the individual interfering local mean powers are different, but the total amount of interfering power still the same.

| | Desired signal | interferer 1 | interferer 2 | interferer 3 | total CIR | $P_{out}$ |
|---|---|---|---|---|---|---|
| $P_0$<br>K | 40 dB<br>7 | 10 dB<br>3 | 10 dB<br>3 | 10 dB<br>3 | 25.2 dB | $2.0\ 10^{-4}$ |
| $P_0$<br>K | 40 dB<br>7 | 0 dB<br>3 | 13 dB<br>3 | 9.5 dB<br>3 | 25.2 dB | $2.0\ 10^{-4}$ |

Table.2: The outage probability when the individual interfering local mean powers are different and the total interfering power is the same.

From table.2, it can be seen that when the individual local mean powers becomes different, but the total amount interfering power is still the same, the outage probability doesn't change.

**4.5 conclusions.**

We have simulated the outage probability in an Rician indoor environment. When we compare the simulation results with the previous obtained analytical results, we see that the difference is very small. Therefore we decided to use the simulation method to investigate the influence of the Rice factor and the local mean power of the signals on the outage probability. From the results it can be concluded that:

- The influence of the Rice factor of the interferers on the outage probability in an indoor Rician environment is minimal.
- The Rice factor of the desired signal has a large influence on the outage probability. When $P_{out} \leq 10\%$ is required, we see that for $K_d=0$ the minimum allowed CIR equals 20 dB and for $K_d=7$ the minimum CIR is 14 dB.

_ When the local mean power of one interferer increases, the outage probability becomes worse.

- When the individual interfering local mean powers become different, the outage probability doesn't change if the total amount interfering power is still the same.

# 5. Simulation of the outage probability in an shadowed Rician environment.

In the previous chapter, the outage probability in an indoor Rician environment has been simulated when the interfering signals have different Rice factors and different local mean power. But recent measurements in an indoor environment show that signal fading consists of fast and slow varying components. The slow variation of the signal component can be described by a log-normal distribution of the local mean power.

In this report log-normal distributed numbers have been generated and further they are used to obtain the local mean power of the signals. This last is used to simulate the outage probability in an indoor shadowed Rician environment. The influence of the model parameters will be investigated. Finally the most important conclusions are presented.

### 5.1 Co-channel interference model.

In an shadowed Rician environment, the radio signals have a Rician distributed instantaneous power with certain specified Rice factor **K** superimposed by a log-normal distributed local mean power with also certain specified logarithmic variance $\sigma_l^2$ and logarithmic area mean $\mathbf{m_l}$. The co-channel interference model is given below.

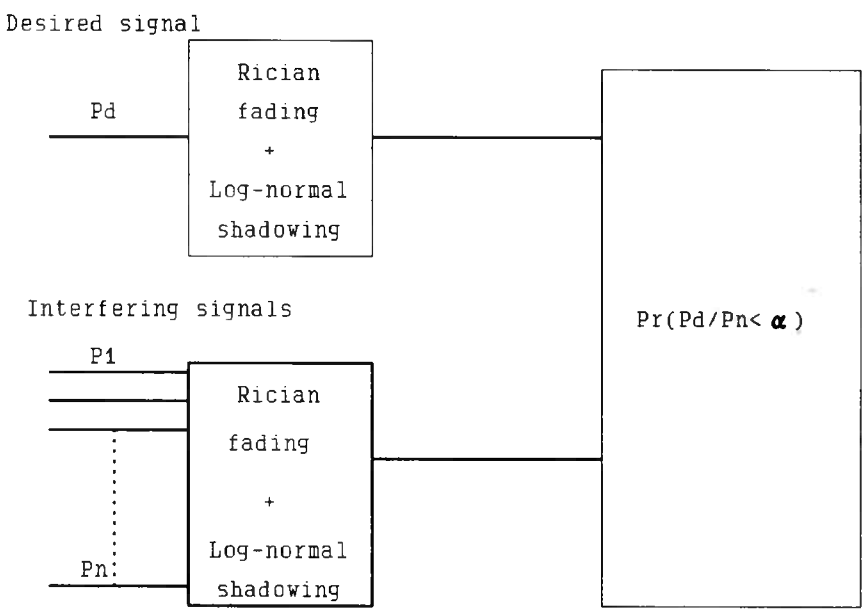


**Figure.15:** Co-channel interference model

### 5.2 Generation of the shadowed Rician distributed samples.

As we have mentioned, the signal amplitude is Rician distributed with a specified Rice factor K and variance $\sigma^2$ of the random component given by

$$\sigma^2=\frac{P_0}{(1+K)} \tag{34}$$

where $P_0$ is the local mean power which is log-normal distributed with certain specified logarithmic variance $\sigma_l^2$ and mean $m_l$ [10]. Thus generation of the shadowed Rician distributed samples can be achieved using the following algorithm:

**Algorithm**

- Generate a number $P_0$ from a log-normal distribution with logarithmic variance $\sigma_l^2$ and mean $m_l$.
- Generate a sample from a Rician distribution with

$$\sigma^2=\frac{P_0}{(1+K)} \tag{35}$$

and

$$S^2=2K\sigma^2 \tag{36}$$

When $\sigma_l=0$, the local mean power $P_0$ becomes constant and the signal will be only fast faded.

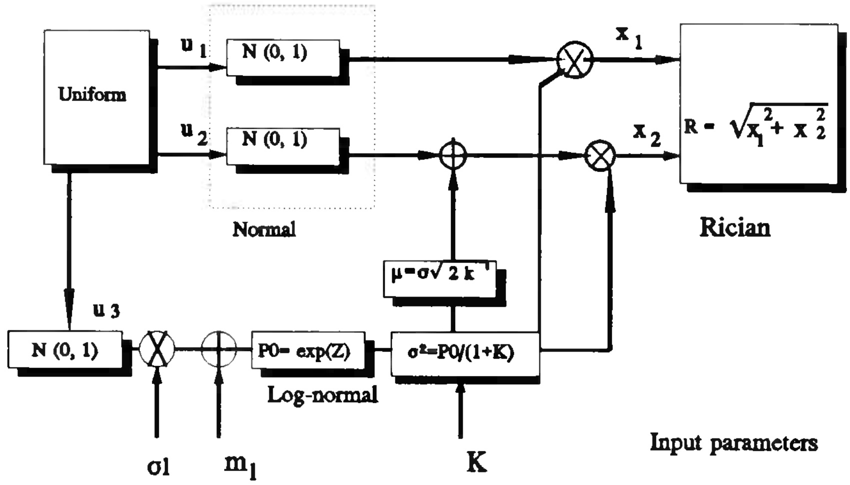


**Figure.16:** Generation of a shadowed Rician distributed sample.

The outage probability will be simulated as a function of the carrier-to-interference ratio (CIR). The carrier-to-interference ratio is obtained by

$$CIR = \frac{\sum_{j=1}^{N} P_{0_{dj}}}{\sum_{j=1}^{N} P_{0_{nj}}} \tag{37}$$

where $P_{0dj}$ = the generated sample from the desired local mean power,

$P_{0nj}$ = the sum of the generated samples from the interfering local mean powers,

N= The number of the used samples.

### 5.3 Simulation procedure.

The outage probability can been simulated as a function of the total CIR, by using the following procedure:

- Generate for each interferer a value $P_{0i}$ by using ($\sigma_{li}$, $m_{li}$).
- Generate for each interferer a sample $P_i$ by using ($P_{0i}$, $K_i$).
- For each value of $m_{ld}$, generate a desired $P_{0d}$ by using $\sigma_{ld}$.
- Determine the total CIR by using equation (37).
- Generate a desired sample $P_d$ by using ($P_{0d}$, $K_d$).
- Compare $P_d$ and $\alpha$ times $\Sigma P_i$.
- If error then increase counter.

### 5.4 Simulation results.

In this paragraph, the effect of the slow fading of de desired signal and of the interfering signals respectively on the outage probability will be investigated. But first we are going to check whether the simulation method is correct. Therefore the outage probability has been also analytically calculated for the case that the desired signal is only fast faded and only one slow and fast faded interferer is active.

Supposing that $K_i$=0, the formula for the outage probability given in [10] by equation (9) becomes:

$$F(C/I|1)=\int_0^{\infty}\frac{1}{\sqrt{2\pi}\;\sigma_{I_i}\,P_{0_i}}\exp\left[-\frac{(\ln P_{0_i}-m_{I_i})^2}{2\sigma_{I_i}^2}\right]\cdot\frac{\alpha}{\frac{\sigma_d^2}{P_{0_i}}+\alpha}\exp\left(-K_d\,\frac{\sigma_d^2/P_{0_i}}{\alpha+\sigma_d^2/P_{0_i}}\right)dP_{0_i} \qquad (38)$$

Equation (38) has been computed and is plotted in fig.17 together with the simulation results.

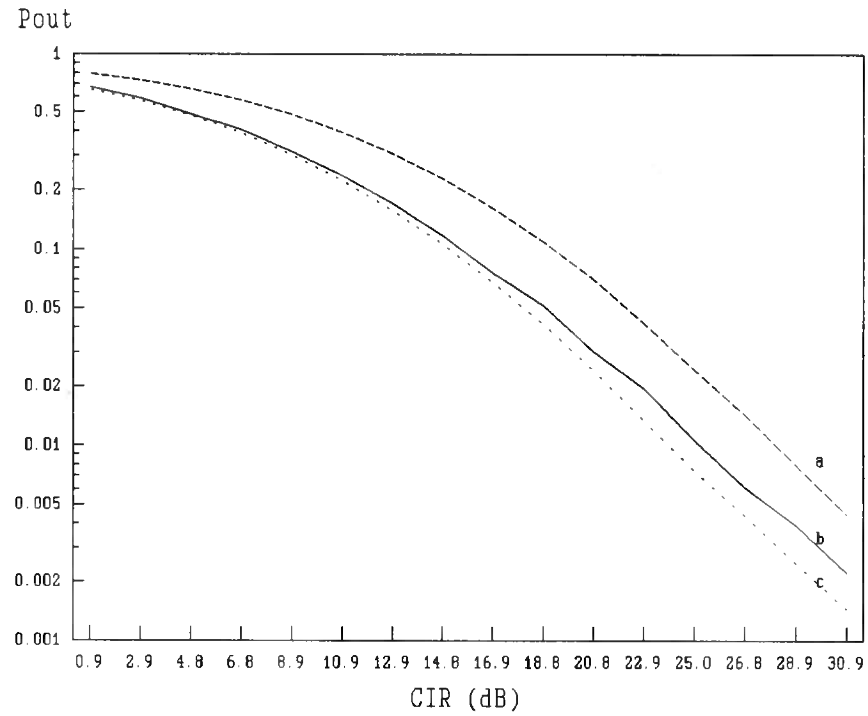


**Figure.17:** Outage probability versus CIR for n=1, $K_i$=0, $\sigma_{l_d}$=0, $\sigma_{l_i}$=6 dB,$m_{l_i}$=0 dB, $K_d$=1 and $\alpha$=10 dB

a) Analytical 1,

b) simulation

c) Analytical 2.

Fig.17 depicts the analytical and the simulation results. The first analytical result is obtained by taking the CIR given by

$$CIR=\frac{(1+K_d)\,\sigma_d^2}{\exp(m_l\,)}. \qquad \textbf{(39)}$$

where **exp(m$_l$)** is the median of the log-normal distribution.

The second analytical result is obtained by taking the CIR given by

$$CIR=\frac{(K_d+1)\sigma_d^2}{\exp(m_l+\sigma_l^2/2)}. \qquad \textbf{(40)}$$

where **exp(m$_l$+$\sigma_l^2$/2)** is the mean value of the log-normal distribution given by Fenton [16].

The difference between the first analytical result and the simulation result is very large. However the second analytical result fit the simulation result very well for low CIR and there is a little difference for high CIR. This difference can be reduced by taking a larger number of samples.

From this results we can conclude that the simulation method approaches the analytical method when the carrier-to-interference ratio is given by equation (40).

### 5.4.1 The interfering signals have the same logarithmic variance and the same area mean power.

In the following simulation results, we assume that when the interfering signals are slowly faded, they have all the same logarithmic variance ($\sigma_{li}^2$) and the same area mean power ($m_{li}$).

**a) The influence of the logarithmic variance of the interfering signals.**

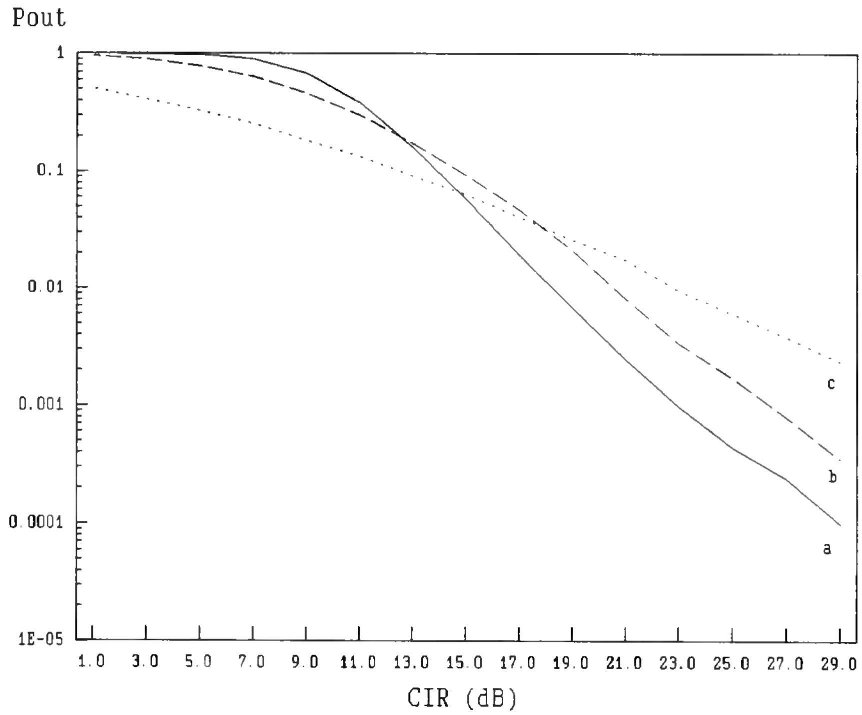


**figure.18:** The outage probability versus CIR for $\alpha$= 10 dB, $K_d$=7, $\sigma_{ld}$=0 dB, n=4, $K_i$=3 and $m_{li}$=0 dB.

a) $\sigma_{li}$= 0 dB,

b) $\sigma_{li}$= 6 dB,

c) $\sigma_{li}$= 12 dB.

From fig.18, we observe that the outage probability doesn't decrease each time that the logarithmic variance of the local mean power of the interferers increases. The consequences of an increase of $\sigma_{li}$ on the outage probability are variable and depend on the corresponding CIR. For the example in fig.18, we see that $\sigma_{li}$=0 dB gives the worst outage probability when CIR

≤ 12 dB and the best outage probability when CIR ≥ 15 dB.

This phenomena is recommended as a subject for further investigation.

**b) The influence of the logarithmic variance of the desired signal.**

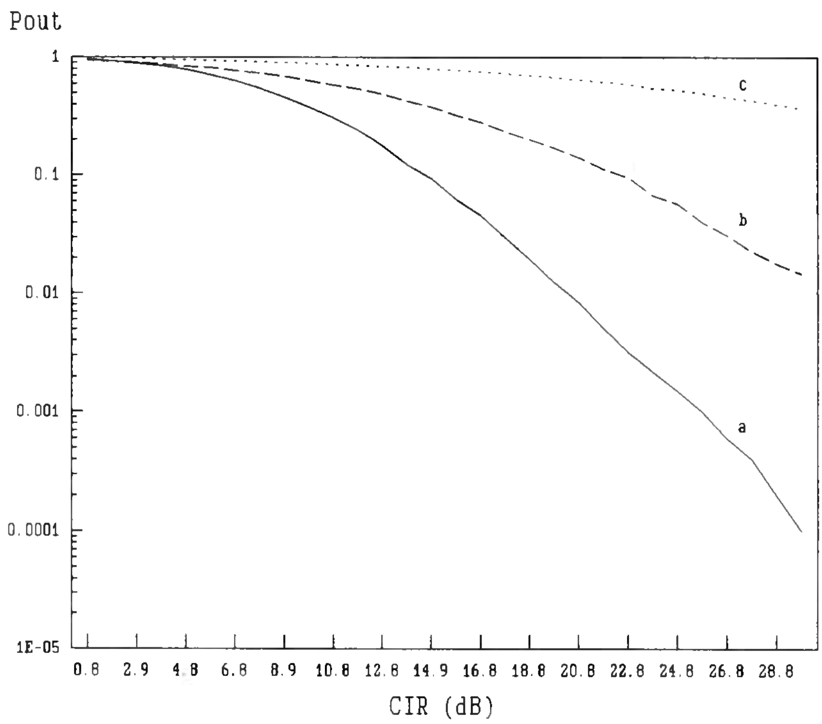


**figure.19:** The outage probability versus CIR for $\alpha$=10 dB, $K_d$=7, n=4, $K_i$=3, $m_{l_i}$=0 dB and $\sigma_{l_i}$=6 dB.

a) $\sigma_{ld}$= 0 dB,

b) $\sigma_{ld}$= 6 dB,

c) $\sigma_{ld}$= 12 dB.

We see from fig.19 that the best outage probability is obtained when the desired signal is not slow faded (plot a) and it becomes worse when the logarithmic variance becomes larger.

### 5.4.2 The interferers have different area mean power.

When the signals are also slow faded, the area mean powers will be different for each interferer if the transmitters of the interfering signals are located at different distances.
On the same way as before, this effect on the outage probability can be simulated. When the area mean power of one interferer increases, the resulted outage probability is presented in table.3 below.

| | Desired signal | Interferer 1 | Interferer 2 | Interferer 3 | Pout |
|---|---|---|---|---|---|
| $m_I$<br>K<br>$\sigma_I$ | 20 dB<br>7<br>6 dB | 0 dB<br>3<br>6 dB | 0 dB<br>3<br>6 dB | 0 dB<br>3<br>6 dB | $3.51\ 10^{-1}$ |
| $m_I$<br>K<br>$\sigma_I$ | 20 dB<br>7<br>6 dB | 0 dB<br>3<br>6 dB | 0 dB<br>3<br>6 dB | 6 dB<br>3<br>6 dB | $4.66\ 10^{-1}$ |
| $m_I$<br>K<br>$\sigma_I$ | 20 dB<br>7<br>6 dB | 0 dB<br>3<br>6 dB | 0 dB<br>3<br>6 dB | 12 dB<br>3<br>6 dB | $6.49\ 10^{-1}$ |

**Table.3**: The outage probability when the area mean power of one interferer increases.

Table.3 shows that if the area mean power of one interferer increases, the outage probability becomes worse. But if we compare this results with the results obtained when the signals were only fast faded ( table 1), we see from table.3 that the outage probability increases slower than when the signals were only fast faded.

The simulated $P_{out}$ for the case that the individual interfering area mean powers are different, but the total amount of the interfering mean power is the same, is given in table.4.

| | desired signal | interferer 1 | interferer 2 | interferer 3 | total CIR | $P_{out}$ |
|---|---|---|---|---|---|---|
| $m_l$ | 40 dB | 10 dB | 10 dB | 10 dB | 25.2 dB | 5.1 $10^{-2}$ |
| K | 7 | 3 | 3 | 3 | | |
| $\sigma_l$ | 6 dB | 6 dB | 6 dB | 6 dB | | |
| $m_l$ | 40 dB | 0 dB | 13 dB | 9.5 dB | 25.2 dB | 5.1 $10^{-2}$ |
| K | 7 | 3 | 3 | 3 | | |
| $\sigma_l$ | 6 dB | 6 dB | 6 dB | 6 dB | | |

**Table.4:** The outage probability when the individual interfering area mean powers are different but the total mean power is still the same.

From table.4 we see that when the individual area mean powers become different, the outage probability doesn't changes if the total interfering power is still the same.

### 5.5 Comparison of the outage probability for different propagation models.

The outage probability for different propagation models has been simulated using the simulation program. For each model, the corresponded model parameters which characterize the signals behaviour are used as input parameters for the simulation program.
The following models have been investigated:

| Propagation model | Distribution desired signal | Distribution interfering signals |
|---|---|---|
| a | Rician | Rician |
| b | Rician | Shadowed-Rician |
| c | Rayleigh | Rician |
| d | Shadowed-Rician | Shadowed-Rician |
| e | Shadowed-Rician | Rician |
| f | shadowed-Rayleigh | Rician |

**Table.5:** Propagation models.

For all propagation models we have assumed that the interfering signals have the same local mean power (or area mean power) and the same logarithmic variance if they are log-normal slow faded.
The simulation results are plotted in fig.20.

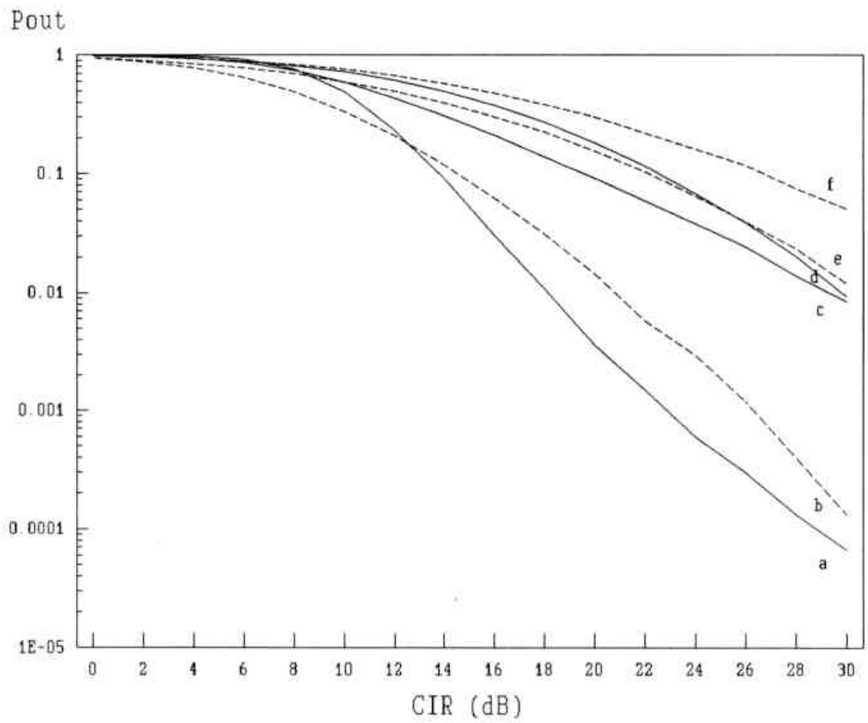


**Figure 20:** The outage probability versus CIR for $\alpha$= 10 dB, n=3 and $m_{li}$=0 dB.

a) $K_d$=7, $\sigma_{ld}$=0 dB, $K_i$=3, $\sigma_{li}$=0 dB.

b) $K_d$=7, $\sigma_{ld}$=0 dB, $K_i$=3, $\sigma_{li}$=6 dB.

c) $K_d$=0, $\sigma_{ld}$=0 dB, $K_i$=3, $\sigma_{li}$=0 dB.

d) $K_d$=7, $\sigma_{ld}$=6 dB, $K_i$=3, $\sigma_{li}$=6 dB.

e) $K_d$=7, $\sigma_{ld}$=6 dB, $K_i$=3, $\sigma_{li}$=0 dB.

f) $K_d$=0, $\sigma_{ld}$=6 dB, $K_i$=3, $\sigma_{li}$=0 dB.

From fig.20, the following points are noticed:

1. Model b gives the best outage probability when the CIR is lower than 12 dB and model a becomes the best when the CIR is larger than 12 dB.
2. The worst outage probability is obtained by model f ( the desired signal is shadowed Rayleigh distributed and the interferers are Rician distributed).
3. When the desired signal suffer from slow fading, the outage probability increases very fast (model d,e,f).

### 5.6 Conclusions.

Monte Carlo method has been used to generate shadowed Rician distributed samples. Those samples are used to simulate the outage probability in an shadowed Rician indoor environment. From the simulation results, we can conclude that:

- The effect of the slow fading of the desired signal leads to increasing of the outage probability. This increase depends on the value of the logarithmic variance of local mean power of the desired signal. For large value of the logarithmic variance $\sigma_{ld}^2$, the outage probability becomes dramatically higher.
- The effect of slow fading of the interfering has different consequences on the outage probability depending on the value the CIR. For very low value of CIR, this effect leads to better outage probability. But for large value of CIR, this effect leads to an increase of the outage probability.

Further we have used the simulation program to calculate the outage probability when the area mean power of the interferers are different. The simulation results have showed that:

- When the area mean power of one interferer increases, the outage probability increases also. This increase in outage probability is less fast than when the signals were only fast faded.
- The outage probability doesn't change if the individual area mean powers become different but the total amount of the interfering power is still the same.

After comparing the outage probability for different propagation models, we found that:

- The best outage probability can be obtained when the desired signal is only fast faded.
- The worst outage probability is obtained when the desired signal is slow and fast faded and the interferers are only fast faded.

# 6. Simulation of the outage probability using Importance sampling technique.

### 6.1 Importance sampling.

The Monte Carlo simulation can be very time consuming. From the previous results it seemed that a sample size of $10/P_{out}$ is required to estimate a outage probability of $P_{out}$. This means that for $P_{out} = 10^{-5}$ a number of $10^6$ samples are needed. Importance sampling can be achieved by increasing the number of errors in the simulation process. This can done by modifying the PDF of the sampled signals [3,6]. In our case this can be achieved using one of the following methods:

1) Modifying the PDF of the desired signal in such way that samples with low value occur often than samples with high values.
2) Modifying the PDF of the interferers in such way that samples with high values occur often than samples with low values.

Because when using the second method all PDF's of the interferers must be modified which is very time consuming, it will be better to use the first method where only one PDF must be modified.

The desired signal has a Rice PDF. The importance sampling can be achieved by modifying a Rician PDF. This can be done by decreasing the constant S (line of sight component) [5]. The constant S is given by:

$$S=\sigma_d \sqrt{2K_d} \tag{41}$$

where $K_d$ is a Rice factor and $\sigma_d^2$ is the average power of the random component.

In order to find out which parameter $\sigma_d$ or $K_d$ must be decreased to achieve this purpose, the PDF of a Rician distribution has been computed and plotted in fig.21 for different values of $\sigma_d$ and $K_d$.

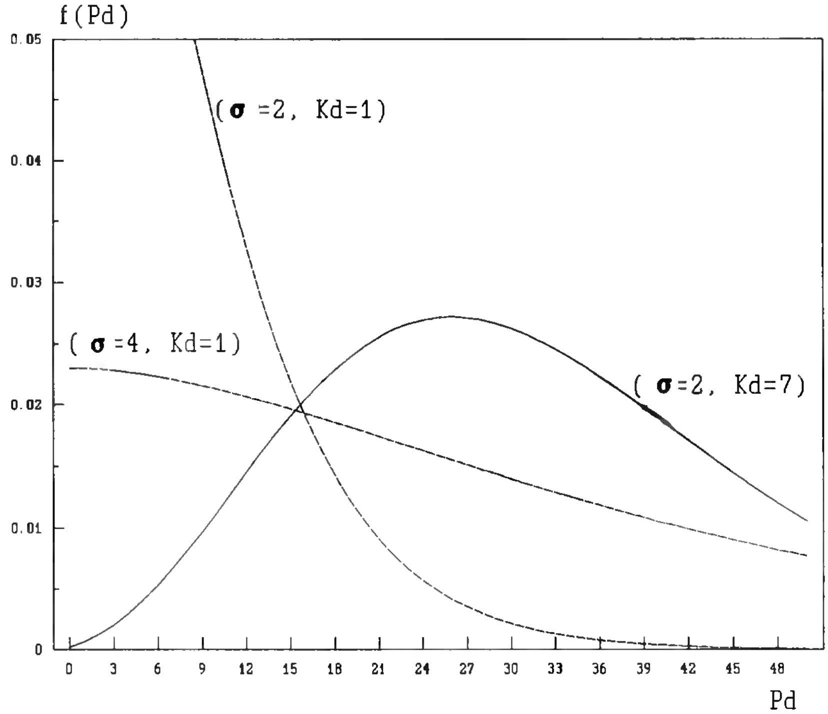


**Figure.21:** The PDF of Pd for:

a) $\sigma_d=2$, $K_d=1$.

b) $\sigma_d=4$, $K_d=1$.

c) $\sigma_d=2$, $K_d=7$.

When analyzing fig.21, it seems that the best way to modify the PDF of the Rice distribution in order that low values more occurs than high values, can be done by keeping $\sigma_d$ very small and decreasing the value of $K_d$.

Let us say that an error will occur if $P_d \leq \alpha P_n$. The probability of errors is given by

$$P_e=\int_0^{\infty} f(P_n)dP_n \int_0^{\alpha P_n} f(P_d)dP_d. \quad \textbf{(42)}$$

Equation (42) is equivalent to

$$P_e=E\,[h(P_d)]. \quad \textbf{(43)}$$

$h(P_d)$ is an error detector. A simple and natural **unbiased** estimator, can be given by

$$P_e=\frac{n}{N}. \tag{44}$$

where n is the number of observations where an error has been occurred, and N is the total number of observations.

When using the importance sampling method, the PDF of the desired signal has been modified and the new probability of errors becomes [6]

$$P_e^*=\int_0^\infty f(P_n)dP_n\int_0^{\alpha P_n} f^*(P_d)dP_d \tag{45}$$

where $f^*(P_d)$ is the new **biased** PDF of the desired signal.

Because the probability of errors with the use of importance sampling must be equal to the case when no importance sampling is used, $P_e^*$ must be divided by some weight function.

Rewriting $P_e$ as

$$P_e=\int_0^\infty f(P_n)dP_n\int_0^{\alpha P_n}\frac{f(P_d)}{f^*(P_d)}\quad f^*(P_d)dP_d \tag{46}$$

The ratio $\mathbf{f(P_d)/f^*(P_d)}$= **W(Pd)** is called the weight function and $\mathbf{f(P_d)}$ is given by

$$f(P_d)=\frac{1}{\sigma_d^2}\exp\left(-\frac{P_d}{\sigma_d^2}-K_d\right)I_0\left(\sqrt{\frac{4P_dK_d}{\sigma_d^2}}\right) \tag{47}$$

When using the importance sampling, the Rice factor of $P_d$ has been decreased with some factor **ß**, so the new Rice factor will be given by

$$K_d^*=\beta\ K_d \qquad 0<\beta<1. \tag{48}$$

and

$$f^{*}(P_d)=\frac{1}{\sigma_d^2}\exp\left(-\frac{P_d}{\sigma_d^2}-\beta K_d\right) I_0\left(\sqrt{\frac{4P_d\beta K_d}{\sigma_d^2}}\right) \tag{49}$$

using equation (47), (48) and (49), $W(P_d)$ is found to be:

$$W(P_d)=\exp[k_d(\beta-1)]\frac{I_0\left[\sqrt{\frac{4P_dK_d}{\sigma_d^2}}\right]}{I_0\left[\sqrt{\frac{4P_d\beta K_d}{\sigma_d^2}}\right]}. \tag{50}$$

and the estimator of the outage probability can be obtained as

$$P_e=\frac{1}{N}\sum_{i=1}^{K^*} W(P_{d_i}). \tag{51}$$

Where

$K^*$ = Number of observation with the use of importance sampling where an error has occurred.

N= The total used samples.

The new obtained simulation model is given below:

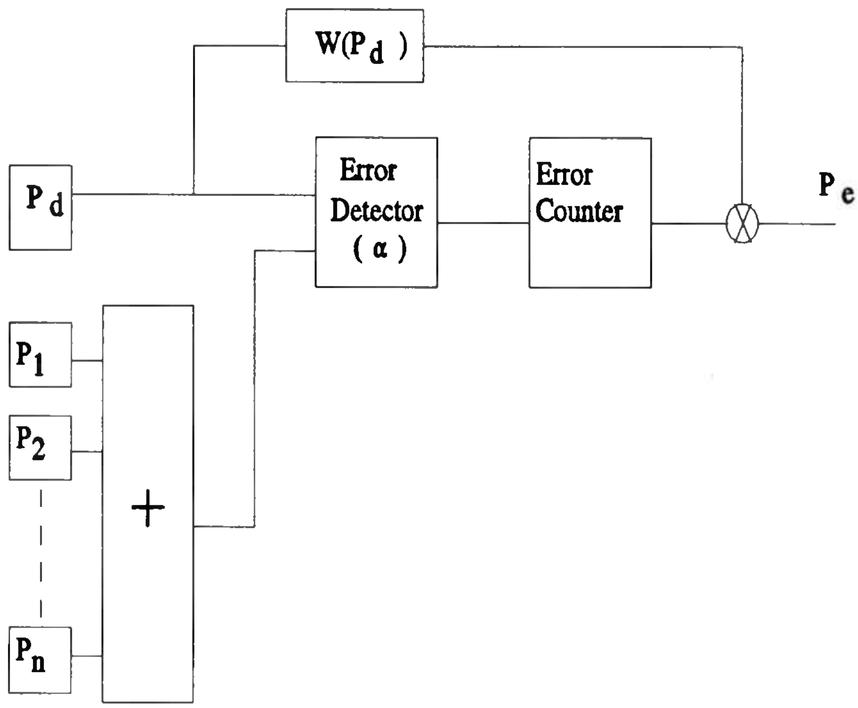


**Figure.22:** Simulation model

**6.2 Simulation results.**

When using the importance sampling, the outage probability has been simulated taking a low number of samples N=1000. In order to determine the efficiency of this method, the results have been compared with the case when no importance sampling is used. The simulation results are given in fig.23.

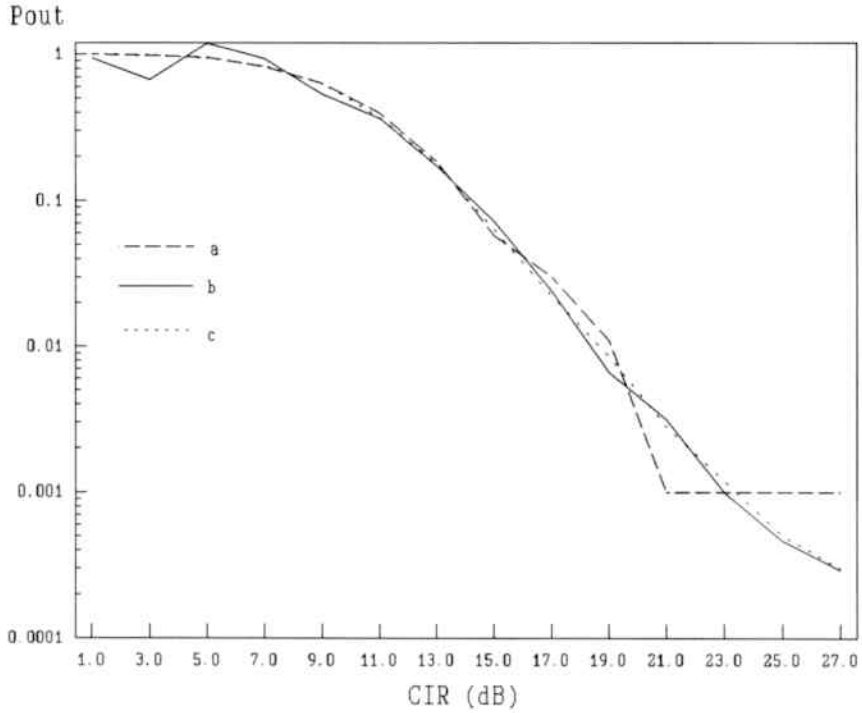


**figure.23:** The outage probability versus CIR for $\alpha=10$ dB, $K_d=7$, $n=4$, $K_i=0$.

a) $N=10^3$ without use of importance sampling,

b) $N=10^3$ using importance sampling,

c) $N=10^4$ without use of importance sampling.

From fig.23, we can see that when using importance sampling, values of $P_{out}$ lower than $10^{-3}$ can be obtained. For the same value, we see also that we need ten times more samples when using the classical method. For low value of the CIR, we see that the results of importance sampling show a large variance. For this interval the outage probability is maximum and approaches one. But as we have found before, the outage probability is given by the sum of the values of the weight function. This weight function can not be exactly calculated, it can only be approximated which introduce those inaccuracies in the outage probability.

Further we have observed that the simulation time using the classical method is about 8 minutes and for the importance sampling is 5 minutes. The time reduction factor is lower than we have expected. This is because when using the importance sampling, for each observation where an error occurred, the weight function must be calculated which is very time consuming. Considering this disadvantage of the importance sampling, we have decided to make only use of the classical method which is more reliable.

# 7. Determination of the cellular structure for the 19$^{th}$ floor of the Electrical Engineering building of Delft University using the simulation method.

In this chapter a cellular structure for the 19$^{th}$ floor of Electrical Engineering building will be determined using the measurement results obtained in [7]. These measurement results are also presented in Appendix C.

### 7.1 The features of the 19$^{th}$ floor.

The 19$^{th}$ floor of the building is divided in two sections that are separated by a long corridor. Each section is separated by walls in a certain number of rooms. The objects that are located in a certain room, are computers, tables and cupboards. A layout of this floor is given below

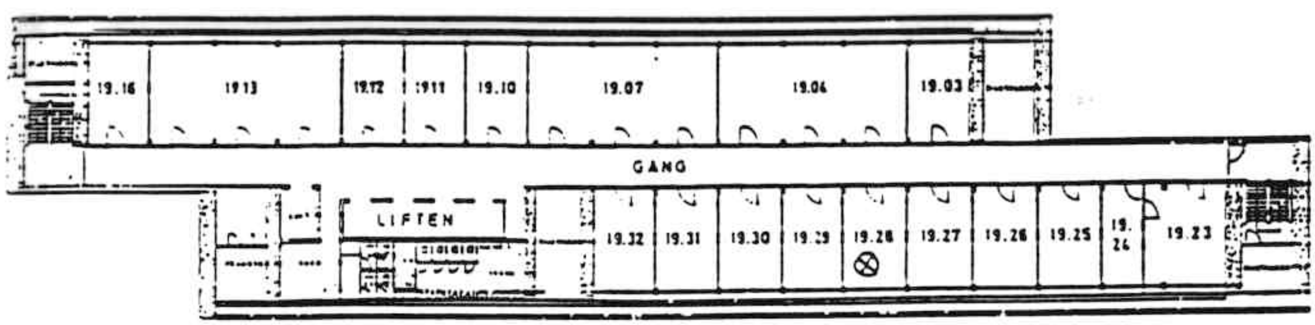


**Figure.24:** The floor layout of the 19$^{th}$ floor.

The transmitter has be positioned in room 19.28 and in each room including room 19.28, the Rice factor, the area mean power and the logarithmic variance of the local mean power were measured. The measurement results are presented in Appendix C. For further details concerning the measurement method see [7].

**7.2 Determination of the cluster size.**

When designing some cellular system, one tries to maximize the spectrum efficiency. This can be achieved by minimizing the cluster area. In paragraph (2.1) we saw that a cluster consists of certain number of cells. In this case because the measurements were taken in each room, the cell area is fixed and will be considered to be equal to one room.

In order to determine the minimal possible cluster size, we start first with a cluster size of 2 cells and then a cluster size of three cells and so on. The corresponding cluster structures are presented below:

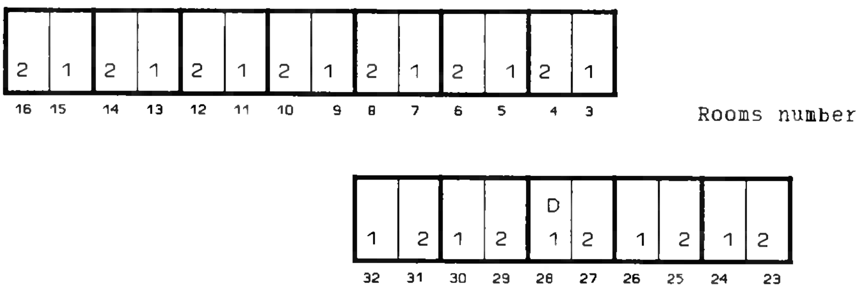


**figure.25 :** Cluster structure of 2 cells.

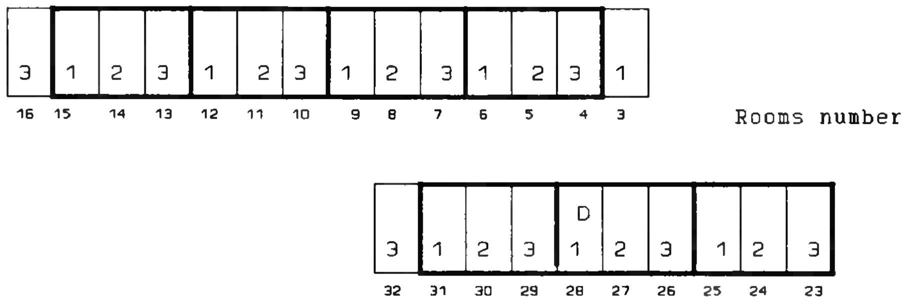


**Figure.26:** Cluster structure of 3 cells.

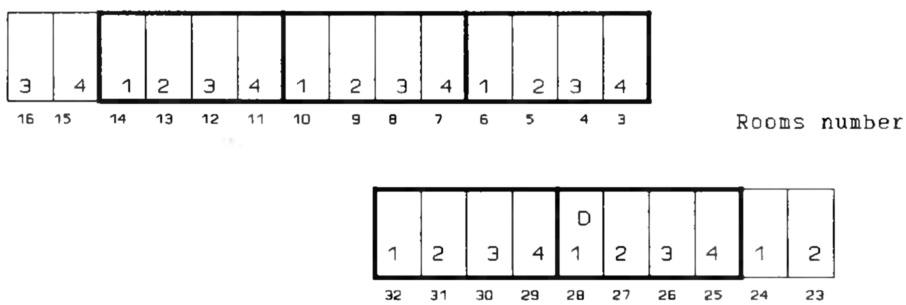


**Figure.27:** Cluster structure of 4 cells.

The user inside room 28, will be considered as the desired user, which is using the frequency-channel number 1. When we choose a cluster structure of 2 cells as in fig.25, we see that the desired user will have to do with 11 interferers, from room numbers {24, 26, 30, 32, 3, 5, 7, 9, 11, 13, 15}. For a cluster structure of 3 cells as in fig.26, the number of the interferers is 7 from rooms {25, 31, 3, 6, 9, 12, 15}. This number of interferers can be reduced by taking a cluster size of 4 cells as in fig.27. So the number of interferers becomes 5 from rooms {24, 32, 6, 10, 14}.

When using the simulation program, for each cluster size the outage probability and the CIR were determined. The results are presented in the table below:

| Cluster size | Outage probability | CIR (dB) |
|---|---|---|
| 2 | $2.69\ 10^{-1}$ | 13.5 |
| 3 | $6.37\ 10^{-2}$ | 18.4 |
| 4 | $8.6\ \ 10^{-3}$ | 23.2 |

**Table 6:** Cluster size versus the outage probability for the 19$^{th}$ floor.

If a outage probability lower than 10% is required, we see that a cluster size of 2 gives an outage probability of 27% which is larger than the system requirement. A cluster size of 3 and 4 gives an outage probability of 6.37% and 0.86% respectively which is lower than the required value. But a cluster size of 4 gives a lower spectrum efficiency. Therefore a cluster size of 3 is the smallest cluster with the highest possible capacity at the 19$^{th}$ floor.

# 8. Conclusions and recommendations.

## 8.1 Conclusions.

In this report the outage probability in an shadowed Rician indoor radio environment has been simulated. Because radio signals in this environment are characterized as faded signals with an appropriate distribution function, sampling from this distribution function was required. To achieve this, Monte Carlo method has been used.

The outage probability has been simulated for two propagation models which characterize the case when the signals are only fast faded and the case when the signals are slow and fast faded. For each model, the influence of the model parameters on the outage probability has been investigated.

From the simulation results of the first model we can conclude that:

- The Rice factor of the interfering signals has no significant influence on the outage probability. The difference in outage probability between $k_i$=0 and $K_i$=3 is not noticeable.
- However the Rice factor of the desired signals has an important influence on the outage probability. For $P_{out}$<10%, we found that when $K_d$=0 a minimal value of CIR= 20 dB is required, however for $K_d$=7 a CIR=14 dB is sufficient.

When simulating the outage probability for the second model, which corresponds with the case when the signals are not only fast faded but also slow faded, the following conclusions can been made:

- When the desired signal is slow faded, the outage probability becomes very poor. This increase in outage probability is related to the value of the logarithmic variance of the desired local mean power. When this logarithmic variance is large, the outage probability becomes dramatically large.
- The effect of the slow fading of the interferers improves not always the outage probability. This effect has a different consequences on the outage probability depending on the value of the total CIR. For a very low value of CIR, we saw that an increase of the logarithmic variance of the local mean power of the interferers induces better outage probability. But

when the CIR becomes very large, each increase of this logarithmic variance makes the outage probability worse.

The simulation program is used also to investigate the case when the local mean power and the area mean power of the interferers for fast faded signals and slow and fast faded signals respectively are different to each other. From the simulation results, we saw that if the local mean power of area mean power of one interferer increases, this results in an increase of the outage probability. Further we saw also that this increase in outage probability is more faster when the signals are only fast faded than when they are at the same time slow and fast faded. However the outage probability doesn't changes if the individual local or (area) mean powers are different but the total amount interfering power is still the same. Hence it appears that when we have to do with interfering signals, we must treat them all together at the same time and not each individually.

### 8.2 Recommendations.

When we have simulated the outage probability in an shadowed Rician environment, and in order to verify the validity of the used simulation method, we have compared the simulation results with the analytical results for the case that the desired signal is only fast faded and only one slow and fast faded interferer is active. Thus for a general validity of the simulation method, we recommend to verify the simulation results analytically for the case that all signals are slow and fast faded and more than one interferers are at the same time active.

Further we found that an increase of the logarithmic variance of the local mean power of the interferers, does notalways result in an improvement of the outage probability. But this depends on the total CIR. This phenomena was not expected, so we recommend to verify this analytically.

The simulation program is developed to simulate the outage probability in an indoor environment. This program can be further developed to be used also in an outdoor environment by introducing the signal attenuation model.

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## Appendix A: Generation of the normal distribution.

When $U_1$ and $U_2$ are independent uniform distributed random variates from U(0,1), then the variates

$$Z_1=(-2lnU_1)^{1/2}cos2\pi U_2.$$
$$Z_2=(-2lnU_1)^{1/2}sin2\pi U_2. \qquad (A.1)$$

are independent standard normal deviates. To show this let us rewrite the system (A.1) as

$$Z_1=(2V)^{1/2}\cos2\pi U$$
$$Z_2=(2V)^{1/2}\sin2\pi U, \qquad (A.2)$$

where $V=\exp(-U_1)$ and $U_2=U$. It follows from (A.1) that

$$Z_1^2+Z_2^2=2V \quad \text{and} \quad Z_2/Z_1= \tan 2\pi U.$$

The Jacobian of the transformation is

$$J = \begin{vmatrix} \frac{\partial u}{\partial z_1} & \frac{\partial u}{\partial z_2} \\ \frac{\partial v}{\partial z_1} & \frac{\partial v}{\partial z_2} \end{vmatrix} = \begin{vmatrix} \frac{-z_2\cos^2 2\pi u}{2\pi z_1^2} & \frac{\cos^2 2\pi u}{2\pi z_1} \\ z_1 & z_2 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{-z_2}{4\pi v} & \frac{z_1}{4\pi v} \\ z_1 & z_2 \end{vmatrix} = -\frac{1}{4\pi v}(z_2^2+z_1^2)= -\frac{1}{2\pi}$$

and

$$f_{z_1,z_2}(z_1,z_2) = f_{u,v}(u,v)|J|=\frac{1}{2\pi}\exp(-\frac{z_1^2+z_2^2}{2}) \qquad (A.3)$$

The last formula represents the joint PDF of two independent standard normal deviates.

## Appendix B: Calculation of the mean value and the variance of the log-normal distribution.

The PDF of the log-normal distribution is given by

$$f(P_0)=\frac{1}{\sigma_l P_0\sqrt{2\pi}}\exp\left(\frac{-[\ln(P_0)-m_l]^2}{2\sigma_l^2}\right) \qquad \textbf{(B.1)}$$

the mean value is defined as

$$\begin{aligned}\mu&=\int_0^\infty P_0\, f(P_0)\, dP_0\\&=\frac{1}{\sigma_l\sqrt{2\pi}}\int_0^\infty \exp\left(-\frac{[\ln(p_0)-m_l]^2}{2\sigma_l^2}\right) dP_0\end{aligned} \qquad \textbf{(B.2)}$$

solution of the integral gives:

$$\mu=\exp(m_l+\frac{\sigma_l^2}{2}) \qquad \textbf{(B.3)}$$

The variance is defined as:

$$\sigma^2=\int_0^\infty (P_0-\mu)^2\, f(P_0)\, dP_0. \qquad \textbf{(B.3)}$$

After lengthily calculation we find out

$$\sigma^2=\left[\exp(\sigma_l^2)-1\right]\exp(2m_l+\sigma_l^2). \qquad \textbf{(B.5)}$$

# Appendix C: The measurement results of the 19th floor.

| Room number | Rice factor K | $-P_0$ (dBm) | Standard deviation of $P_0$ (dB) |
|---|---|---|---|
| 03 | 1 | 67.2 | 3.0 |
| 04 | 1.4 | 57.5 | 5.7 |
| 05 | 0.8 | 66.0 | 2.3 |
| 06 | 0.6 | 68.1 | 1.4 |
| 07 | 1.4 | 72.3 | 2.1 |
| 08 | 0 | 74.2 | 1.1 |
| 09 | 0 | 76.8 | 1.2 |
| 10 | 0.8 | 81.3 | 1.6 |
| 11 | 1 | 87.4 | 1.8 |
| 12 | 0.6 | 90.8 | 1.6 |
| 13 | 0 | 95.2 | 1.0 |
| 14 | 0 | 97.0 | 1.6 |
| 15 | 0 | 98.0 | 1.1 |
| 16 | 0.4 | 99.5 | 1.9 |
| 23 | 1.2 | 69.4 | 2.1 |
| 24 | 0.8 | 67.7 | 1.5 |
| 25 | 0.8 | 65.0 | 1.5 |
| 26 | 0.8 | 61.3 | 1.5 |
| 27 | 0 | 50.0 | 1.5 |
| **28 •** | **5.8** | **37.3** | **2.9** |
| 29 | 6.6 | 43.2 | 2.6 |
| 30 | 1.8 | 51.6 | 2.6 |
| 31 | 1.4 | 58.0 | 4.3 |
| 32 | 2.4 | 63.5 | 3.4 |

**Table C.1**: The room measurements of the 19th floor ( • transmitter).

**Appendix D: Principle of the simulation method.**