Note on the Determination of the In-plane Shear Stress-Strain Relation of a Unidirectional Composite

by

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ABSTRACT

A simple relation between the shear response of a unidirectional composite and the tensile stress-strain behavior of a $+45^\circ$ angleply laminate makes direct recordings of the shear stress-strain relation of the U.D. material possible by means of a simple tensile test on a $+45^\circ$ angleply specimen.

For practically all stress and strain calculations on composite laminates the value of the basic material Engineering Constants $E_{\alpha\alpha}$, $E_{\beta\beta}$, $\nu_{\alpha\beta}$ and $G_{\alpha\beta}$ must be known. These quantities may be easily determined by simple experiments, only the method of measuring $G_{\alpha\beta}$ is rather controversial. Several methods have been published in the literature [1, 2, 3]. It is not the intention of this short note to discuss the pro's and contra's of these tests, but to describe a procedure to establish the (non-linear) relation between inplane shear stress $\tau_{\alpha\beta}$ and strain $\gamma_{\alpha\beta}$ that has the advantage of extreme simplicity of the test specimen as well as the testing equipment. In fact, it is a further simplification of the test described by Petit in [4].

In this test a $+45^\circ$ angleply is built up from the U.D. material of which $G_{\alpha\beta}$ is to be established; from this angleply simple tension test coupons are cut. The following relations are used by Petit:

\[ G_{\alpha\beta} = \frac{2U_1 E_{xx}}{8U_1 - E_{xx}} \]  

(1)

\[ \gamma_{\alpha\beta} = (1 + \nu_{xy}) \varepsilon_x \]  

(2)

in which the X-axis is in the direction of the tensile load and the Y-axis is perpendicular to it and lies in the plane of the laminate, and in which

\[ U_1 = \frac{1}{8} [\Omega_{11} + \Omega_{22} + 2\Omega_{12}] \]  

(3)

where $\Omega_{ij}$ are elements of the stiffness matrix of the basic U.D. material. Their values are assumed to be known from previous tests on the U.D.-material.
Then, from the (varying) ratio between the load $P_x$ and the strain $e_x$, the tangent modulus $E_{xx}$ can be found, from which $G_{x\beta}$ follows with (1). From strain measurements and (2) the value of $\gamma_{\alpha\beta}$ can be derived with (2).

It is remarkable that Petit seems to have been unaware of a much simpler relation yielding the same results with less effort and more accuracy, without using $Q_{11}$, $Q_{12}$ and $Q_{22}$. The following formula is easily derived from the well-known transformation relations for stiffness in case of rotation over $45^\circ$:

$$G_{x\beta} = \frac{E_{xx}}{2(1 + \nu_{xy})}$$  \hspace{1cm} (4)

This relation with its surprising similarity to the relation for isotropic materials shows that $G_{x\beta}$ can be calculated from the results of the tensile test of the angleply specimen alone. More specifically:

$$G_{x\beta} = \frac{P/A}{2e_x(1 + \nu_{xy})} = \frac{P/2A}{e_x - e_y}$$  \hspace{1cm} (5)

According to (2) the shear strain $\gamma_{x\beta} = (1 + \nu_{xy}) e_x = e_x - e_y$, so the shear stress $\tau_{x\beta}$ must be $P/2A$ or half the tensile stress $\sigma_x$.

This implies that a direct recording of $P$ versus $(e_x - e_y)$ represents the relation between $\tau_{x\beta}$ and $\gamma_{x\beta}$. Since for an angleply the value of $\nu_{xy}$ is in the range of .75 to .85 the value of $(e_x - e_y)$ is 1.75 $e_x$ to 1.85 $e_x$ which implies good conditions for a precise registration of the $\tau$-$\gamma$ relation viz. $G_{x\beta}$ determination.

The max. strain of a $+45^\circ$ angleply of ca/ep material is of the order of 1% at a tensile stress of over 12 kgf/mm$^2$ (17,103 psi) so measurements with the usual strain gauges may cover a large range of stress values with great accuracy.

Doubt is expressed by Petit about the validity of the results of this test, since none of the two sets of layers is in a pure shear condition; beside the shear stress $\tau_{x\beta}$ there are also tensile stresses of comparable magnitude $\sigma_{\alpha}$ and $\sigma_{\beta}$. Taking the $x$- and $y$-axes as a reference, there is a normal stress $\sigma_x$ and a shear stress $\tau_{xy} \approx .45 \sigma_x$ instead of a normal stress $\sigma_x$ and a normal compressive stress $\sigma_y = -\sigma_x$. Or, in terms of strains, $e_y/e_x$.\]
is not -1 as in pure shear but -.75 to -.85. The effect becomes noticeable at higher stress levels.

The warning of Petit is repeated here, that the stress field in the $± 45°$ angleply is non-homogeneous at the free edges, hence strain measurements must be made at sufficient distance from the sides of the specimen.

REFERENCES


