Abstract

Wireless communications have numerous applications in terrestrial and space environments, but for one environment the number of civil wireless applications is small. This is the underwater environment where for wireless communications acoustic waves are used instead of electromagnetic waves. The underwater acoustic channel is a very difficult medium for wireless communications and is subject to severe multipath and Doppler effects. It is possible that each multipath component may have a unique delay and Doppler shift, so called multi-scale/multi-lag channels. This is at present a limiting factor for wireless communications underwater. At the moment always a single Doppler rate is assumed which converts to a narrowband system when re-sampled. In this report a receiver is introduced which can remove the effects of multipath and Doppler where the novelty lies in the fact that it is especially designed for a multi-scale/multi-lag channel. This is done by introducing equalizers followed by channel estimation which are both designed for a multi-scale/multi-lag channel. The designed receiver shows promising results and is able to recover the original transmitted symbols according to the simulations.
A multi-lag/multi-scale receiver for underwater acoustic communications

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Abstract

Wireless communications have numerous applications in terrestrial and space environments, but for one environment the number of civil wireless applications is small. This is the underwater environment where for wireless communications acoustic waves are used instead of electromagnetic waves. The underwater acoustic channel is a very difficult medium for wireless communications and is subject to severe multipath and Doppler effects. It is possible that each multipath component may have a unique delay and Doppler shift, so called multi-scale/multi-lag channels. This is at present a limiting factor for wireless communications underwater. At the moment always a single Doppler rate is assumed which converts to a narrowband system when re-sampled. In this report a receiver is introduced which can remove the effects of multipath and Doppler where the novelty lies in the fact that it is especially designed for a multi-scale/multi-lag channel. This is done by introducing equalizers followed by channel estimation which are both designed for a multi-scale/multi-lag channel. The designed receiver shows promising results and is able to recover the original transmitted symbols according to the simulations.
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Introduction

Wireless communication has numerous applications in terrestrial environments, examples are mobile phones, bluetooth, wireless Internet, wireless sensor networks, the global positioning system (GPS) etc. The terrestrial wireless environment has been charted extensively and there is a wide collection of available models [1, 2]. Such channel models exist for the country side, several urban environments, indoor environments and space. There is one environment though for which there is still limited knowledge about the channel, because since recently there was little interest for civil applications in this environment. This is the underwater acoustic channel (UAC). At the moment, only a limited number of civil applications exist and a good understanding of the UAC is not available, which is shown in the lack of channel models for the UAC. Recently, this environment has gained more attention as the need arose for underwater communications, such as in underwater sensor networks for environmental observation and communications between underwater autonomous vehicles. Unfortunately the UAC is a very challenging environment for wireless propagation. The main cause is the nature of the UAC, which is highly time varying and a lot of parameters influence the underwater channel. Other effects that can normally be neglected in a terrestrial environment should be taken into account in the UAC since they may severely distort the signal.

1.1 Underwater communications

The reason for using acoustic signals in underwater communications is that the use of standard radio frequency techniques is very difficult or may even be impossible. The electromagnetic waves experience a considerable amount of fading underwater, especially in the oceans due to the presence of salt water. The losses occur due to the high permittivity of the water which has a relative value of 81, other losses are due to the high conductance of seawater which is around 4 Siemens/meter, whereas the one for freshwater is in the range of several milliSiemens/meter [3]. These two properties of the underwater environment have the consequence that the propagation losses are large and the usage of low frequencies is necessary in order to propagate over long distances. For example for a signal with a carrier frequency of 10 kHz, in seawater, the distance related to 87 dB attenuation is only 25 meters. Even for a signal with a carrier frequency as low as 100 Hz the distance at which the attenuation is equal to 87 dB is only 250 meters [4]. In another reference [5], the distances are theoretically calculated to find the maximum attainable communication ranges. They assume that the conductivity of the water is 3.2 Siemens/meter and find that for a signal with a carrier frequency of 100 Hz the calculated distance for 100 dB attenuation is 323 meters.

Although electromagnetic waves have the advantages of a faster propagation speed, over large distances underwater they are not suitable. The electromagnetic waves that
do propagate relatively well underwater are of low frequency. These low frequency electromagnetic waves have the disadvantage that they require large antennas, in the order of tens of thousands kilometers for a frequency of 100 Hz, a high transmit power for the same signal to noise ratio (SNR) due to a higher presence of noise in this region and have a low available bandwidth [4]. Worth mentioning is that there exists one successful application for underwater submarine communications in oceans with electromagnetic waves. It transmits several characters per minute and the operating frequency is around 80 Hz. But in general electromagnetic waves are not used for underwater communications [3, 4].

Another possible option for underwater communications is optical communications, the use of light to transfer information. Optical communications is less affected by attenuation but more by scattering and absorption. Optical communications has also shown good results for short to medium distances. However there are other reasons for not using optical communications. The water needs to be clear, in very cloudy waters, visibility is almost zero and light may be totally blocked, so for some ocean environments optical communication is not an option. Plus in the upper layers of the water level there is a high amount of ambient light. In general the performance of optical communications for underwater communications is sometimes limited and only applicable under certain conditions, such as clear deep waters. [22].

The final logical possibility is to use sound waves in order to transmit information underwater. This is also known as underwater acoustic communications and at the moment is the preferred technique for underwater communications. Underwater acoustic communications has shown the best results of all the earlier mentioned methods especially with respect to the attainable range. The achievable communication range is in the order of tens of kilometers [6] and clearly outperforms the earlier mentioned methods. Underwater acoustic communication has the disadvantages of a slow propagation speed compared to electromagnetic waves. But for long underwater distances it is the moment the only practical choice for wireless communication [22].

One problem that still remains is that the acoustic underwater channel is a very challenging environment for wireless communications. Effects that normally would have a limited effect using electromagnetic waves have to be taken into account in the acoustic underwater channel. The main characteristics of the UAC are the same as in any wireless environment and these characteristics include fading, shadowing, noise, multipath and Doppler spread, but all these effects have to be adjusted to the UAC.

1.2 Previous work

Previous research has already been done for wireless communications over the UAC, where the goal was to obtain reliable communications using underwater acoustic communications. The previous research extends from single carrier techniques, Direct Sequence Spread Spectrum (DSSS) techniques [13]-[15] and to multicarrier techniques mainly orthogonal frequency division multiplexing (OFDM) [7]-[12].

Several of the first successful attempts at underwater communications are made in [16]-[18]. In [18] an algorithm is proposed which performs optimal phase synchronization and channel equalization jointly. A suboptimal but low complexity decision
feedback equalizer is derived in combination with channel tracking through the use of a recursive least squares (RLS) and a second order phase locked loop (PLL). The parameters of the DFE are adaptively adjusted using the RLS algorithm in combination with the second order PLL. The same approach was utilized in [16, 17] which extended [18] and tried to enhance and reduce the complexity of the earlier mentioned method with still using a DFE. In [19, 20] single carrier techniques are also used in acoustic underwater communications and this is combined with frequency domain equalization (FDE). All these methods were tested and have shown relatively good results. For more detailed information concerning the history of acoustic underwater communications the reader is referred to [16]-[18] and references therein.

In [13]-[15] DSSS techniques are used, this technique is less susceptible to narrow-band interference due to the use of spreading codes. Furthermore due to the large time-bandwidth product of DSSS signals it is possible to recover discrete multipath signal components that can be combined to allow diversity reception with a RAKE receiver. Also DSSS offers the possibility of covert underwater communications but this is not the main issue for this report. The DSSS methods have also shown reasonable results and DSSS is a possible contender for underwater acoustic communications.

Recently there is raising interest in multicarrier techniques for acoustic underwater communications. These techniques also show promising results as a possible modulation technique in acoustic underwater communications. Especially OFDM has received a lot attention lately [7]-[11] where several receivers based on OFDM are introduced for underwater acoustic communications. The usage of OFDM is interesting since it has the ability to cope with severe channel conditions but still offers the possibility for low complexity channel equalization. However multicarrier techniques suffer from inter carrier interference (ICI) caused by Doppler shifting of the respective carrier frequencies of the subcarriers. Furthermore the Doppler shifts are not uniform and every subcarrier is shifted with a different amount. Some of these receiver from [7]-[11] are of low complexity and are only able to cope with mild Doppler effects while others are also able to deal with more severe Doppler effects. At the moment almost all references [8]-[11] compensate the Doppler shift by resampling the received signal so that Doppler shifts translates to an equal frequency shift for all the subcarriers. This is only true when the rate of the Doppler shift is equal on all paths. If this is not the case and there is a different Doppler rate on each path making the resampling factor not equal to the Doppler rates there will still be residual ICI after the resampling operation. This remaining interference is often treated as colored noise.

### 1.3 Motivation

The main topic of this report is to develop a receiver, using single carrier techniques, that is capable of handling multipath components which each having their own delay and Doppler rate. Such a channel is called a multiscale/multilag channels. Such environments can occur in the UAC and the receiver that is described in this report may be interesting for wireless acoustic underwater communication.
Within this topic, two main objectives can be distinguished and these are:

1. Performing equalization to remove the channel effects.

2. Channel estimation in order to obtain the channel information necessary to construct the equalizers.

The equalizers shall be based on the already existing equalizers mentioned in [21] where the performance of these equalizers is tested theoretically. Here in this report these equalizers shall be simulated using Matlab. The second novelty lies in the channel estimation needed for construction of the equalizers. The channel estimation is performed on a multiscale/multilag channel model for which at the moment there is no standard approach on obtaining all the channel information.

1.4 Outline of the report

These two objectives are treated in the report as follows:

• In chapter 2, background is given about the underwater environment and the main physical phenomena.

• Chapter 3 contains the system model, the used equalizers and the performance of the equalizers.

• After the equalizers of chapter 3, channel estimation is discussed in chapter 4, which includes the method used and the performance evaluation using the equalizers in combination with the information obtained from the channel estimation.

• Finally in chapter 5 the work is concluded and some future recommendations are made.
In this chapter, the main characteristics of the UAC are introduced in order to provide a good understanding of the phenomena that are present in the UAC and to explain why certain choices later in the report are made. Furthermore the channel conditions can be varying per location. For example the Atlantic Ocean may have a totally different sound channel profile then the Mediterranean. The sound channel profile is dependent on many factors which can be divided in 2 classes. The first one are the chemical/physical properties of the water such as temperature, salinity, underwater currents, density and the cloudiness. The second class is more dependent on the environment and include geographical properties, the depth of the water layer, structure of the bottom and the weather conditions. All the conditions combined can make the channel conditions fast varying in time. It is reported that within seconds the sound channel profile can change substantially and the possible number of channel taps that are present can be in the hundreds [6, 23].

This fast time-varying property of the UAC makes it difficult to obtain proper channel estimates and to maintain a good tracking of the sound channel. At the moment there is no universal model that can represent all the effects of the UAC [25]. Furthermore it is likely that a model that may work well for the Atlantic Ocean may not work the Mediterranean due to the location dependency. In general underwater acoustic communications is characterized by transmission losses due to spreading and absorption, a low propagation speed of sound underwater, in some cases severe multipath and the presence of noise [24]. Next some of these characteristics are discussed in more detail.

2.1 The transmission losses

The three primary mechanisms which attenuate underwater acoustic signals are spreading loss, absorption loss and scattering loss. The spreading losses and absorption losses are caused by the spreading of the signal power over a larger area in the medium and the transfer from acoustic energy in to heat respectively. The spreading losses are dependent on the depth of the water column and can be spherical for deep waters with a decay rate of $R^{-2}$. For shallow waters a cylindrical spreading is chosen to be the best model, which decays at a rate of $R^{-1}$ where $R$ is the distance from the source. In shallow waters the signal energy is bounded between the surface of the water and the bottom. Therefor, the spreading of the signal energy is not spherically since it is trapped between these two layers and mainly spreads out in the horizontal directions. For deep waters a cylindrical spreading model can also be used if the distance $R$ is very large compared to the depth of the water column.
The absorption losses are losses due to the water’s ability to convert acoustic energy into heat. The main physical phenomena behind the absorption losses are the chemical properties of the water, for example in [26, 27] a study is done on how boric acid and magnesium sulphate concentrations influence the absorption losses. The details behind the causes of absorption losses are too extensive to all explain in this report and are also not important for our planned objectives. The only aspect of interest is that the absorption losses in ocean waters are frequency dependent and are increasing with frequency. The frequencies suitable for underwater acoustic communications are therefore limited since for high frequencies the absorption losses are too high to allow long distance communications. In Figure 2.1 [28], the dependency of frequency on the absorption losses is shown, as can be seen only for the lower frequencies the absorption losses are small enough to allow long distance communications.

The final mechanism which induces transmission losses for acoustic signals are the scattering losses. One main cause of scattering losses are air bubbles originating from breaking waves at the water surface, this is mainly present in heavy weather conditions with high wind speeds. Under calm conditions the scattering losses that occur due these air bubbles are not present. The scattering losses are small compared to the absorption and spreading losses and the latter two are the main mechanisms that are accountable for the transmission losses especially the absorption losses [6]. In most underwater acoustic systems the operating frequency range is 100 Hz to 100 kHz since for this frequency range the losses are still bearable, see also Figure 2.1.
Another phenomenon of the UAC is that of shadow zones where there is no signal energy at all. Communication with receivers located in these shadow zones is very difficult. These zones can occur in deep waters as well as in shallow waters. In Figure 2.2 [6], traces of propagation paths for a typical deep water environment with a typical deep water sound profile are shown, where there is a high variance in the spread of the signal energy over the depth of the water column. At some ranges the signal energy is spread over the entire water column and at other ranges is concentrated around a certain depth. This shows the appearance of shadow zones, although the distances in this figure are not realistic it shows the concept of such zones. For example if a receiver would be placed at a depth of around 4000 meters and a distance of 60 kilometers from the receiver it would receive nothing. However placed at a depth of 1000 or 2000 meter communication would be possible at the same distance of 60 kilometers. This means these zones, if they can be found, should be taken into account in placing the receiver. If the receiver is placed in such a shadow zone it would be unable to communicate with the transmitter. Of course, this is more important in designing network topologies than for the physical layer design.

2.2 Multipath environment

Multipath occurs if the transmitted signal encounters objects which then produce reflected, diffracted and scattered copies of the transmitted signal. Each one of these copies is attenuated in power, shifted in phase and has its own delay respectively to
the direct line of sight (LOS). At the receiver these copies of the transmitted signal are all summed up which then produces the total received signal. The presence of multipath can lead to severe performance degradation due to distortion of the received signal which is inter symbol interference (ISI). The ISI is a form of distortion in which data symbols interfere with subsequent data symbols due to delayed paths arriving at the same time at the receiver. The ISI makes communication less reliable due to the distortion of the original transmitted data symbols and removal is necessary in order to obtain the original data symbols [1].

In the UAC the multipath originates from two main sources namely the bottom of the water column and the surface. Especially in shallow waters there are numerous multipath components which results in ISI. In deep waters multipath can occur due to the reflections caused by different water layers with a different temperature and/or salinity. But in general deep waters experience less multipath.

The multipath that is present in underwater acoustic communications is often more severe than that in the case of the terrestrial environment. This is caused by the larger delay spread of the UAC which is dependent on the location. The delay spread is the range of the possible delays that can occur and it is equal to the difference between the minimum delay and maximum delay. The propagation speed of acoustic waves underwater is much lower 1500 meters/second than that of electromagnetic waves and hence the delays shall be larger than in the case of electromagnetic waves and can be considerable. In [6] it is mentioned that delay spreads of 80 ms and 100 ms can occur. In case of a symbol rate of 5000 symbols/second this can induce a ISI hundreds of symbols. This shall limit the performance of the communication system since 1 symbol will have interference from hundreds of other symbols. It is also important for equalizer design since it means that a lot of information needs to be taken into account to remove all the present ISI [24]. In [17] it is mentioned however that the typical delay spread of a UAC is 10 ms which is lower than the previous example.

2.3 The available bandwidth

The practical usable frequency range for acoustic communications is limited between 100 Hz and 100 kHz approximately. The lower bound is chosen because below 100 Hz the the region is more susceptible for noise. The upper boundary is chosen with respect to the earlier mentioned absorption losses which increase significantly above a frequency of 100 kHz, see Figure 2.1 [28]. The UAC channel has the peculiar bandwidth distance relationship derived from dependency on frequency due to the absorption losses and the noise sources. In Figure 2.3 [29], this can be seen as it shows the the bandwidth that would be available if a certain distance needs to be attained while maintaining the same signal to noise ratio(SNR). The bandwidth dependency on distance is important since for larger distance the bandwidth is smaller and the optimal frequency range of the bandwidth shifts down [29].
Figure 2.3: The effect of distance on the available bandwidth.

Due to this the available frequency range is small and further more is in the lower part of the frequency spectrum. The small bandwidth results in that the achievable data rates for UAC are low and are likely in the order of kbits/s. The second consequence is that for communications the signals used are inherently wideband. Although the used bandwidth may be small but it is in the order of the center frequency. Therefor the signals used in acoustic underwater communications are inherently wideband. This is important later when discussing the Doppler effects since for wideband signals certain approximations cannot be made. [28][6].

2.4 Doppler effects

Doppler effects occur when the transmitter and/or the receiver are moving during the time a transmission takes place. In the UAC Doppler originates from the drifting of the transmitter and receiver on the surface due to waves or wind and below the water surface due to currents. It is also possible that Doppler effects originate from non stationary objects present in the medium such as floating debris and moving underwater currents with different temperature and or salinity. If the transmitter and or receiver are underwater autonomous vehicles then the main Doppler originates from the transmitter and receiver itself. But even without intentional motion, underwater instruments are subject to drifting with waves, currents and tides, which may occur at comparable velocities. In the case of the UAC a stationary situation is highly unlikely because of the motion of the water due to currents and drifting at the surface. The Doppler mentioned here can be better described as an equivalent Doppler shift in which all the individual shifts are incorporated into one final equivalent Doppler shift.
In the UAC the Doppler spread is also much larger, due to the low propagation speed of sound underwater. For example, the transmitted has a speed of 5 m/s in the terrestrial environment where the propagation speed of the used electromagnetic waves is \(3 \times 10^8\) meters/second, it will hardly have any effect. But in the case of acoustic communications underwater, with a propagation speed of 1500 meters/second, the effect is several orders larger. Doppler effects cause degradation of the performance in digital communications since it distorts the received signal by frequency shifting and spreading. For the UAC an example of the Doppler shift is given in [6], where it is mentioned that for the speed of 2 meters per second and a center frequency of 25 kHz the Doppler shift was 33 Hz. It is necessary that a communication system is that takes the Doppler effects into account [28].

In multicarrier communications Doppler shifts may cause carrier frequencies to start interfering with each other. This will result in interference between carrier, since their respective carrier frequencies may start overlapping [11][10].

From the chapter about the bandwidth availability in the UAC it was mentioned that the used signals will certainly be wideband. This results in non uniform Doppler shifts throughout the bandwidth. For narrowband signals the Doppler effect can be approximated as a frequency shift which makes it easier to model it and also easier to remove the Doppler effects on the signals. In wideband signals this cannot be done and the distorted Doppler signal for wideband signals needs to be used.

2.5 The noise

The noise present in underwater channels has two major sources: man-made from machinery and shipping activity, and ambient noise which is caused by natural sources like wind, rain, tides, currents and biological life [24]. Further there is of course the noise contribution of the hardware and possible interference from other users. At the moment however interference from other users is probably not a major problem due to the still limited number of other applications.

In [28][29] a more detailed description of noise sources present in the UAC is given. The noise contribution consists of 4 sources which are the turbulence noise which influences only the very low frequency region below 10 Hz, the shipping noise which present is dominant in 10 to 100 Hz region, surface motion caused by wind driven waves which is dominant in the 100 Hz to 100 kHz region and the thermal noise which has the main contribution above the 100 kHz. The summation of these 4 noise is the total noise contribution. For certain frequency ranges the noise can be dominated by a particular noise source such as surface waves while for another frequency range the main contribution can come from shipping noise. The total noise contribution is the summation of these 4 sources and is decaying with frequency to about 100 kHz and increases after that [29]. This in combination with the frequency dependency of the absorption losses makes the SNR also frequency dependent, see Figure 2.3. For every distance there is a certain frequency range at which the SNR is maximized. Most noise sources can be described by Gaussian statistics with a continuous power spectral density, for more information see [29].
2.6 Conclusion

The UAC is generally recognized as one of the most difficult communication media in use today [6]. In this chapter the main phenomena of the UAC were discussed and the difficulties are presented if reliable communication needs to be achieved. This is done in order to provide insight in the choices made later in for example modeling the channel. In the remaining part of the report the focus shall be on how to design a communication system that can cope with Doppler effects and removes the ISI so that reliable communication can be achieved.
In this chapter the total system model that is used for this particular environment is introduced. First the communication system and the used transmission scheme are presented. After the introduction and motivation behind the choice for the structure of the transmitted signal, the channel effects on the transmitted signal are discussed. A mathematical description of the channel is derived and applied to the transmitted signal. Followed by the introduction of the receiver which performs matched filtering followed by equalization to remove the ISI. For more detailed information about the physical background the reader is referred to Chapter 2. The main characteristics are the Doppler effects that dilate or extend the original transmitted signal, fading, and multipath components that will cause ISI.

### 3.1 The transmitter

The transmitter is the part of the system that creates the transmitted signal and sends this signal to the receiver. A schematic representation of the entire communication system is given in Figure 3.1 below.

![Figure 3.1: A schematic overview with the most important steps of the communication system](image)
In Figure 3.1 the transmitter is shown as the upper part of the system. First the information bits are rescheduled according to an error correcting scheme, such as interleaving in combination with parity bits. Adding an error correcting code is optional but increases the bit error rate performance. The disadvantage is that it increases complexity and can decrease the bandwidth efficiency since extra non data bits are added.

This is followed by modulation in which each codeword of bits is represented by a symbol from the modulation alphabet of the used modulation scheme. Modulation can improve the bandwidth efficiency since multiple bits will be represented by one symbol from the modulation alphabet. The higher the modulation level, meaning an increase in the size of the modulation alphabet, the more bandwidth efficient the transmission becomes \cite{1}. There exists a variety of modulation schemes that are used such as M-ary phase shift keying (M-PSK) techniques, where $M$ stands for the modulation level. PSK modulation encodes the information in the phase difference between the symbols. Another example is M-ary pulse-amplitude modulation (M-PAM) where the information is encoded in the amplitude of the symbols or for example frequency shift keying (FSK) which encodes by shifting the frequency. Combinations of modulation schemes are also possible such as quadrature amplitude modulation (QAM) which uses 2 PAM signals which are 90 degrees out of phase and encodes the information as well in phase and amplitude. QAM can also be extended to larger size alphabets such as 16-QAM, 32-QAM, 64 QAM etc. For more information about modulation techniques see \cite{1}.

After the information bits have been coded and modulated they shall be converted from the digital to analog domain. This is done in the pre-coding block by modulating the symbols onto an analog signal which later shall be discussed in more detail. Finally the signal is up-converted to the carrier frequency, which is done in the mixer where the signal is multiplied with the carrier frequency from the local oscillator.

### 3.1.1 Structure of the transmitted signal

The transmission takes place on a block per block basis where the information bits will be divided in blocks of length $N$ data symbols. The reason for doing this is due to the time-varying channel conditions of the UAC. The time in which the channel parameters remain constant, the channel coefficients and Doppler rates, shall influence the choice of the the block length $N$, since the longer the length of 1 block the more chance the channel changes within that block.

![Figure 3.2: The used transmission scheme](image)

Between each block of data symbols a guard time is added in order to prevent inter block interference (IBI). The guard time is dependent on the delay spread of the channel and the guard time shall be equal to the maximum delay present in the
channel. Although this extra guard time decreases the symbol rate and efficiency, it makes sure that two blocks won’t overlap. Schematically the transmission scheme is shown in Figure 3.2. The assumption shall be made that the channel parameters shall remain constant within the duration of one block.

### 3.1.2 The pulse shaping

Since it is very difficult to transmit the modulated symbols directly it is necessary to modulate the data symbols on an analog signal, shown in Figure 3.1 as the pre-coding block. The symbols will be modulated on a pulse with the characteristic that it is limited within the available bandwidth so that there is no interference to other users outside the assigned bandwidth. A single carrier system is used here and thus the bandwidth is not subdivided into different subcarriers but there is one carrier that comprises the entire available bandwidth.

\[
s(t) = \sum_{n=0}^{N-1} b_n g(t - nT)
\]

In (3.1) a description of the entire transmitted signal for one block is shown. The pulse shape for transmitting is noted as \( g(t) \), the vector \( b \) denotes the vector with the to be transmitted symbols. Again \( N \) is the number of symbols present in 1 block and \( T \) is the duration of one symbol. The pulse \( g(t) \) should preferably have the properties denoted in (3.2) and (3.3).

\[
G(f) = \begin{cases} 
\neq 0 & -1/2T < f < 1/2T \\
0 & f < -1/2T \text{ or } f > 1/2T
\end{cases}
\]

\[
g(t) = \begin{cases} 
\neq 0 & 0 < t < T \\
0 & \text{otherwise}
\end{cases}
\]

The Fourier transform of \( g(t) \) should be satisfying the condition in (3.2), so it causes no interference to other users outside the bandwidth. The second preferable property is that there is no ISI meaning that \( g(t) \) should also satisfy the condition given in (3.3). The first requirement is best achieved with a sinc pulse given in (3.5), since it’s Fourier transform has a rectangular shape. For the second requirement the perfect pulse would be a rectangular wave described as in (3.6). This is also known as a zero-order hold filter, it repeats the current symbol till the next symbol arrives where the symbols are modulated on rectangular pulses.

\[
g(t) = \text{sinc} \left( \frac{t}{T} \right) = \frac{\sin \left( \frac{\pi t}{T} \right)}{\frac{\pi t}{T}}
\]

\[
g(t) = \begin{cases} 
1 & 0 < t < T \\
0 & \text{otherwise}
\end{cases}
\]
However the rectangular pulse has severe side lobes in the frequency domain. It is not limited within the available bandwidth, so it does not fulfill the first requirement. Although the sinc function is bandwidth limited, it is susceptible to synchronization errors and noise \([30]\). Therefore this pulse is not suitable for practical uses and often another type of pulse is chosen. In general a root raised cosine pulse is chosen which is defined in (3.6) with the Fourier transform shown in (3.7).

\[
g(t) = \left( \frac{\sin \left( \frac{\pi t}{T} \right)}{\frac{\pi t}{T}} \right) \left( \frac{\cos \left( \frac{\beta \pi t}{T} \right)}{1 - \left( \frac{2\beta}{T} \right)^2} \right) \tag{3.6}
\]

\[
G(f) = \begin{cases} 
T & |f| \leq \frac{1 - \beta}{2T} \\
T \cos^2 \left( \frac{\pi T}{2\beta} \left( |f| - \frac{1 - \beta}{2T} \right) \right) & \frac{1 - \beta}{2T} < |f| \leq \frac{1 + \alpha}{2T} \\
0 & |f| > \frac{1 + \alpha}{2T}
\end{cases} \tag{3.7}
\]

This pulse fulfills the two requirements reasonably, not perfectly and is less susceptible to synchronization errors. It is a trade off between the ideal pulse and what is achievable in practice. A root raised cosine is easily adjustable for different symbol times and bandwidth. The factor \(\beta\) in (3.6) and (3.7) controls the amount of how much the pulse 'leaks' outside the bandwidth.

![Figure 3.3: Example of some root raised cosine pulses for different roll off factors, for the prevention of ISI a high roll off factor is preferable](image)

In Figure 3.3 several raised cosine pulses are shown for different roll off factors and in Figure 3.4 their respective Fourier transforms are shown. As can be seen from these two
Figure 3.4: Example of some Fourier transforms of root raised cosine pulses for different roll of factors, for the prevention of ISI a high roll of factor is preferable.

The figures are that the two requirements are partially full filled. As can be seen from the roll off factor $\beta$ that for large roll off factors the pulse becomes more robust against ISI but more susceptible to synchronization errors while also requiring a larger bandwidth. For small roll of factors the pulse becomes more bandwidth limited but more susceptible to ISI.

The final part of the transmitter is the up converting of the signal from baseband to passband with respect to the carrier frequency denoted as $f_c$. The transmitted signal is the real part of the complex baseband signal and the final transmitted signal is shown in (3.8).

$$s(t) = \text{Re} \left[ \sum_{n=0}^{N-1} b_n g(t - nT) e^{-2\pi j f_c t} \right]$$  \hspace{1cm} (3.8)

The next chapter shall introduce the channel and model the effect it has on the original transmitted signal.

### 3.2 The channel model

The causes behind the channel effects present in the medium were mentioned earlier in the previous chapter. It is assumed that multipath is present and ISI shall occur and that all paths experience fading, Doppler effects and are delayed. Many references about the UAC mention that these effects occur in acoustic underwater communications [12][6][24][19].
The Doppler effect is presented by the term $\alpha$ as shown in (3.9) which is called the Doppler rate. The Doppler rate is defined as $\alpha = 1 + \frac{2v}{c}$ where $c$ is propagation speed of sound underwater and $v$ is the total speed of the transmitter and/or receiver \[21\][31]. The Doppler rate is an indication of the rate at which the signal is compressed or dilated compared to the original signal. The received signal $r(t)$ is a compressed or dilated version of the original transmitted signal $s(t)$. The factor $\sqrt{\alpha}$ is such that the total power remains equal since a Doppler shift does not change the power of the signal and hence it serves as a sort of normalization.

If $\alpha$ is 1 there are no Doppler effects present in the channel. Per definition this always means that $\alpha > 0$ because a signal cannot be dialted to a duration of zero seconds. If $\alpha$ is between 0 and 1 it means that the signal is dilated if $\alpha$ is larger then 1 it means that the signal is compressed \[10\][21].

In the previous chapter it is also noted that the signals used in UAC are inherently wideband. For narrowband signals the Doppler effects can be approximated by a frequency shift as follows $f_d \approx (\alpha - 1)f_c$ where $f_c$ denotes the carrier frequency \[31\]. This approximation can only be made in the case of narrowband signals. One of the conditions for a signal being narrowband is that $\frac{2v}{c} \ll \frac{1}{TW}$ holds. With $TW$ is denoted as the time bandwidth product \[31\]. Another condition is that $B \ll f_c$ which means that the bandwidth, denoted as $B$, is much smaller compared to the carrier frequency. In the UAC these 2 conditions do not hold and a narrowband approximation cannot be made. The ratio $\frac{2v}{c}$ is usually in the region of $10^{-1} - 10^{-3}$ but the main reason why in underwater acoustic communications the used signals are wideband is that the carrier frequency is in general of the same order as the bandwidth. For proper modeling of the Doppler effects the wideband notation for Doppler effect on a signal is used \[31\]. Using the wideband model for Doppler effects the received signal at the receiver can be written as in (3.10).

$$r(t) = \sqrt{\alpha} s(\alpha t)$$  \hspace{0.5cm} (3.9)

The channel coefficients $h_l$ are presumed to be Rayleigh fading \[21\] \[17\]. The delays are called $\tau_l$ and represent the time at which delayed versions of the original signal arrive at the receiver. A finite number of $L$ paths are presumed present in the channel, with each path having its own delay $\tau_l$, channel coefficient $h_l$ and Doppler rate $\alpha_l$.

The received signal in (3.10) is still without the up converting of the signal from base-band to pass-band with the carrier frequency $f_c$. The Doppler effect also affects the carrier frequency which result in a shift in frequency of the carrier frequency depending on the parameter $\alpha_l$. After conversion to base-band at the receiver there is still a residual term present in the base-band signal since the carrier frequency has
shifted causing a mismatch compared to the local oscillator. The noise is added which is denoted as \( n(t) \) and it is assumed that the noise is white Gaussian random noise \( \mathcal{CN}(0, \sigma^2) \) and is uncorrelated with the signal. Substituting (3.8) in (3.10), adding the earlier mentioned Doppler effects on the carrier frequency and adding the noise, (3.10) becomes (3.11) \[21\]:

\[
r(t) = \sum_{n=0}^{N-1} b_n \sum_{l=0}^{L-1} h_l \sqrt{|\alpha_l|} [g(\alpha_l(t - \tau_l) - \alpha_l(nT))] e^{2\pi j f_c ((\alpha_l-1)t - \alpha_l \tau_l)} + n(t) \quad (3.11)
\]

The conversion from pass-band to base-band at the receiver is done by removing the carrier frequency, the removing of the carrier frequency is performed by multiplying the received signal with \( e^{-2j\pi f_c t} \). If this is done on the transmitted signal as shown in (3.8) it can be easily seen that the carrier frequency is eliminated. After the conversion from pass-band to base-band as seen in (3.12) there is still a residual term present.

\[
e^{-2j\pi f_c \alpha_l(t-\tau)} \times e^{2j\pi f_c t} = e^{2j\pi f_c ((\alpha_l-1)t - \alpha_l \tau)} \quad (3.12)
\]

Equation (3.11) represents the entire received signal after conversion to base-band with all the channel effects incorporated into the signal. For notational convenience later on this shall be rewritten as follows:

\[
r(t) = \sum_{n=0}^{N-1} b_n q_n(t) + n(t) \quad (3.13)
\]

\[
q_n(t) = \sum_{l=0}^{L-1} v^l_n(t) \quad (3.14)
\]

\[
v^l_n(t) = h_l \sqrt{|\alpha_l|} [g(\alpha_l(t - \tau_l) - \frac{nT}{\alpha_l}) e^{2\pi j f_c ((\alpha_l-1)t - \alpha_l \tau_l)}] \quad (3.15)
\]

The received signal \( r(t) \) is the total received signal and is a summation of \( q_n(t) \) functions which are called the channel functions for the \( N \) symbols. Each symbol has its own respective channel function \( q_n(t) \) which reflects how that symbol is received at the receiver. \( q_n(t) \) is equal to the summation of the \( L \) multipath components present which are given by \( v^l_n(t) \), which are called the path functions. For every symbol and every path present there is a \( v^l_n(t) \) which represent the received signal for a symbol on path \( l \).

With the presence of Doppler, the ISI that a symbol experiences is dependent on the symbol index. The entire block is dilated or compressed as shown in Figure 3.5. Here the situation is shown for a channel with three paths for which the upper path is compressed, the middle path is the normal block duration without Doppler effects and the lower path is the dilated version. The ISI span for symbols 2 and 6 is marked by a
Figure 3.5: Showing schematically the ISI dependency of the index of a symbol

rectangle. As can be seen, the ISI for symbol 2 is different than the ISI for symbol 7. For symbol 2 three symbols are causing ISI while for symbol 7 four symbols contribute to the ISI. From this example it can be seen that the ISI is dependent on the symbol index. Especially for very long blocks and/or if there is a large difference in Doppler rates, the ISI can change significantly over the block length.

The channel model introduced here represents a channel that has a different Doppler rate on each path and also has a unique arrival time. This is called a multi-scale/multi-lag channel where the scale stands for the Doppler effects and the lag stands for the delays. Such channels are not extensively studied and at the moment there is no standard approach of handling such channels. In the case of 1 common Doppler rate for each path, resampling with a factor \( \frac{1}{\alpha} \) can compensate for that specific Doppler rate. However, at the moment for multiple Doppler rates there is no standard solution on how to compensate for all Doppler effects. With resampling only the part of the signal is removed having the Doppler rate equal to the one that is compensated by the resampling factor. However it is shown in [32] that resampling the signal with the Doppler rate equal to that of the path which contains the highest amount of power is then the best solution. This remains a non-optimal approach though since it is unable to compensate for the effects of multiple unique Doppler rates on each path.

3.3 The receiver

In this part the receiver is introduced, it is shown in Figure 3.1 as the lower part of the communication system. First the conversion to base-band is made which removes the carrier frequency. The next step the receiver performs is to do equalization which removes any ISI and channel estimation. Next the symbols are demodulated and decoded.

After the conversion to baseband is made the receiver performs matched filtering, which is subsequently the same as ‘sampling’ the original received pulse. By taking every symbol period \( T \) a sample of the received signal, where the result of matched filtering is the sample. The results of the matched filtering can be seen as conversion from the analog domain to the digital domain. The received signal is matched filtered with the channel function associated with the current symbol:
\[ y_n = \int_{-\infty}^{\infty} r(t) q_n^*(t) \, dt \quad (3.16) \]

Where the vector \( y = [y_0 \cdots y_{N-1}]^T \) is the vector which contains the result of the matched filtering per symbol [21]. The character * denotes conjugation throughout the report. The results in the vector \( y \) do not contain the original transmitted symbols yet due to distortion by ISI and Doppler. These still need to be equalized as mentioned before to remove these effects. For this purpose it is desirable that (3.16) is written in matrix form.

\[ y = Rb + n \quad (3.17) \]

\[ R(i,j) = \int q_i(t) q_j^*(t) \, dt \quad (3.18) \]

In order to be able to write (3.17), a matrix needs to be found such that the vector \( y \) obtained from (3.16) is equal to the one in (3.17). The original symbols are in the vector \( b \) which is given as \( b = [b_0 \cdots b_{N-1}]^T \). The ‘channel’ matrix \( R \) should be such that the condition in (3.17) holds and is given in (3.18). The noise is represented by adding the noise term \( n \) in (3.17), which is equal to \( n = [n_0 \cdots n_{N-1}]^T \), which is the white Gaussian noise vector after matched filtering which has a variance of \( \sigma^2 R \) but still zero mean. A more detailed version of (3.17) is shown below where the content of the matrices is shown:

\[
\begin{bmatrix}
y_0 \\
\vdots \\
y_{N-1}
\end{bmatrix}
= 
\begin{bmatrix}
R \\
\vdots \\
R
\end{bmatrix}
\begin{bmatrix}
b_0 \\
\vdots \\
b_{N-1}
\end{bmatrix}
+ 
\begin{bmatrix}
n_0 \\
\vdots \\
n_{N-1}
\end{bmatrix}
\quad (3.19)
\]

\[
R = 
\begin{bmatrix}
\int q_0(t) q_0^*(t) \, dt & \int q_0(t) q_1^*(t) \, dt & \cdots & 0 \\
\int q_1(t) q_0^*(t) \, dt & \int q_1(t) q_1^*(t) \, dt & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & \int q_{N-2}(t) q_{N-2}^*(t) \, dt & \int q_{N-2}(t) q_{N-1}^*(t) \, dt \\
0 & \cdots & \int q_{N-1}(t) q_{N-2}^*(t) \, dt & \int q_{N-1}(t) q_{N-1}^*(t) \, dt
\end{bmatrix}
\]

Here the vector \( y \) are the results of the matched filtering with dimensions \( 1 \times N \), \( R \) is the channel matrix with dimensions \( N \times N \), \( b \) is the vector which contains the original symbols and the noise is located in the noise vector \( n \) with dimensions \( N \times 1 \). This
way of defining $R$ means that if two symbols do not interfere with each other the entry in the matrix $R$ is zero. The number of non zero entries indicates the amount of ISI present. If $R$ does not contain any zero entries then the ISI span extends to all the symbols in one block.

$R = \begin{bmatrix} \vdots & \vdots & \vdots \\ \cdots & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad R = \begin{bmatrix} \vdots & \vdots & \vdots \\ \cdots & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad R = \begin{bmatrix} \vdots & \vdots & \vdots \\ \cdots & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix}$

Figure 3.6: Showing the 3 possible structures the channel matrix $R$

There are three possible structures for $R$ which are shown in Figure 3.6. This is dependent on the Doppler rates that are present in the channel. The first possible structure is the left one in Figure 3.6. The ISI span shrinks as the block length increases meaning that there are paths with Doppler rates smaller than 1 present in combination with a sufficient block length. The ISI is dependent on index as was earlier seen in Figure 3.5, the ISI span is shrinking for later symbols in the block. The second possible structure is a banded matrix which means that the non zero part of $R$ remains the same and does not change over the symbol index. This occurs when there is no Doppler present and or the Doppler parameters are such that the ISI does not change within the duration of 1 block. The third possible structure is shown as the right matrix in Figure 3.6. The ISI span increases for symbols later in the block. This can happen if there are paths present with a Doppler rates larger than 1 present in combination with a sufficient block length. These possible structure of $R$ limit the use of conventional techniques to exploit the structure of a banded matrix.

### 3.4 The equalizers

The equalization is done in order to remove the ISI. Since the matched filtering results are still distorted versions of the original symbols. If $b$ needs to be recovered from $y$ it can be easily seen that the solution is finding a matrix $W$ that satisfies the condition in (3.20).

$$Wy = W(Rb + n) = WRb + Wn = b \quad (3.20)$$

A logical first choice for $W$ is to make it equal to $R^{-1}$ so that the vector $b$ with the original symbols is recovered with an additional term $R^{-1}n$, see (3.21).

$$WR^{-1}y = R^{-1}(Rb + n) = Ib + R^{-1}n \quad (3.21)$$

These kind of equalizers are known as zero forcing equalizers(ZF-equalizers). The ZF equalizers set all the interference to zero by eliminating all the symbol interference.
However the remaining noise term $R^{-1}n$ may lead to an enhancement of the noise.

In [21] the following equalizer is presented for this kind of channel and it is based on the minimum mean square error (MMSE) equalizer. A MMSE equalizer takes the noise enhancement effect in to account by providing the best trade off between noise enhancement and removal of interference. The matrix $W$ is equal to $(R + \sigma^2 I)^{-1}$. Where $\sigma$ stands for the variance of the noise.

The matrix $R$ has dimensions of $N \times N$ and taking the inverse of a matrix can be quite computational. For example if 128 symbols form 1 block, a $128 \times 128$ matrix needs to be inverted, which requires a significant amount of computational power. In [21] three other types of equalizers based on the MMSE equalizer shown above but require less computational power. For now this equalizer is called the full block equalizer since it requires the inverse of the entire channel matrix $R$.

### 3.4.1 Partial block equalizer

The ISI span is dependent on the symbol index as seen in Figure 3.5. The ISI for symbol 2 comprises the symbols is 2,3,4 while for symbol 7 it is comprised of symbols 6,7,8,9. Let’s define the support of a symbol as the range of symbols to which the ISI span for a symbol extends. The support of a symbol is determined from the scales and delays as follows [21]:

$$\Psi_n = \left[\min_l \left(\frac{\alpha_l(n-1)}{T} + \tau_l\right), \max_l \left(\frac{\alpha_l n}{T} + \tau_l\right)\right]$$  \hspace{1cm} (3.22)

Meaning that if a symbol is within the support $\psi_n$ for symbol $n$ then this symbol is in the ISI span of symbol $n$. The idea is to find the support of each symbol and then use that information to select the necessary sub matrix of $R$. With this sub matrix, containing the information from the symbols in the ISI span for that specific symbol, equalization is performed. In this way, instead of inverting a large $N \times N$ matrix a number of $N$ smaller matrices needs to be inverted where the size of these sub matrices is dependent on the ISI span of the symbols.

$$Q_n = \left[q_k(t) \neq 0 \text{ for } t \in \Psi_n\right]$$  \hspace{1cm} (3.23)

$$\tilde{q}_k(t) = q_k(t)I_{t \in \Psi_n}$$  \hspace{1cm} (3.24)

The partial block equalizer is the first type of these equalizer and it performs the partial equalization by first finding the support of symbols for which symbol $n$ experiences interferences and selects the respective channel functions for symbol $n$. Lets name this set of respective channel functions $Q_n$ with $k$ the index for the support defined as $[k_{\text{min}}^n \leq k \leq k_{\text{max}}^n]$, see (3.23). The partial block equalizer selects the respective channel functions that are in $Q_n$ and with those channel functions constructs the sub matrix that encompasses the ISI-span for symbol $n$. For each symbol the set of channel functions that are present in the ISI changes and a selection is made according to (3.24).
\[ \tilde{R}_n(i-k_{\text{min}}+1, k-k_{\text{min}}+1) = \int \tilde{q}_i(t)\tilde{q}_k^*(t)dt \quad i, k \in Q_n \quad (3.25) \]

\[ \tilde{y}_k = \int_{-\infty}^{\infty} r(t)\tilde{q}_k^*(t)dt \quad k \in Q_n \quad (3.26) \]

\[ \tilde{y} = \tilde{R}b + \tilde{n} \quad (3.27) \]

\[ W = (\tilde{R} + \sigma^2 I)^{-1} \quad (3.28) \]

After this the normal procedure is followed only now the number of channel functions and equalized symbols is reduced to the size of the ISI span. This procedure is shown in (3.25) and (3.26) and (3.27). The noise has the same properties as in the full block equalizer but now with variance \( \sigma \tilde{R}_n \). Again a MMSE equalizer can be formed according to (3.28) and the original symbol \( n \) is recovered. Where \( b = [b_k_{\text{min}} \cdots b_k_{\text{max}}]^T \), \( \tilde{y} = [y_k_{\text{min}} \cdots y_k_{\text{max}}]^T \) and \( \tilde{n} = [n_k_{\text{min}} \cdots n_k_{\text{min}}]^T \) are now limited to the ISI span.

### 3.4.2 Truncated block equalizer

With the previous equalizer the simplification was made by choosing and truncating the respective channel functions. However in the truncated equalizer the selecting is done after the construction of \( R \) and \( y \).

\[ y'_n = [y(l_{\text{min}}^n) \cdots y(l_{\text{max}}^n)]^T \quad (3.29) \]

\[ b'_n = [b(l_{\text{min}}^n) \cdots b(l_{\text{max}}^n)]^T \quad (3.30) \]

\[ n'_n = [n_k_{\text{min}} \cdots n_k_{\text{min}}]^T \quad (3.31) \]

\[ R'_n = R(l_{\text{min}}^n : l_{\text{max}}^n) \quad (3.32) \]

\[ y'_n = R'_n b'_n + n'_n \quad (3.33) \]

\[ R'_{n} = R(l_{\text{min}}^n : l_{\text{max}}^n, k_{\text{min}}^n : k_{\text{max}}^n) \quad (3.34) \]
Again the idea is the same and it is about limiting the equalizer to the support of one symbol that is of interest. Again a smaller matrix needs be inverted at the cost of doing it $N$ times but it still less computational than inverting a very large matrix at once. The total procedure is shown in (3.29-3.34). With $k$ is the same as in the partial block equalizer. These are the matrices in case for the truncated block equalizer with the noise now having zero mean and variance $\sigma^2 R_n'$. Since $R_n'$ is not a square matrix but a wide matrix with dimension $(k^{max} - k^{min}) \times N$. So a slightly different approach is taken in order to construct the equalizer. Instead of taking the normal inverse the pseudo inverse is taken so the equalizer becomes as in (3.31).

$$W = (R_n')^H (R_n'R_n'^H) + \sigma^2 (R_n')^{-1}$$

(3.35)

This equalizer is again based on a MMSE equalizer and multiplying $y_n'$ with this matrix the original symbol $n$ is recovered. However this equalizer has the disadvantage that the matrix $R_n'$ is in nature a wide matrix and is not always full rank depending on the channel.

### 3.4.3 The path combining equalizer

In the path combining equalizer complexity is further reduced by considering a set of $p$ correlators matched to each path for a particular symbols. The outputs of these path filters would not be added up but combined optimally. The principle is as follows

$$\hat{y}_n(l) = \int v^l_n(t) r^+(t) dt$$

(3.36)

For each path matched filtering is performed for each symbol. The $R$ matrix changes to include each path that cause interference to the current path and for which data symbols these paths originate from. The matched filter results of symbol at time are optimally combined via (3.36) where $\hat{n}_n \sim CN(0, \sigma^2 \hat{R}_n)$ and $\otimes$ is the noise contribution for symbol $n$ and the Kronecker product respectively.

$$\hat{R}_n(l, c) = \int v^{l}_m(t) v^{k}_n(t) dt$$ \hspace{1cm} l, k = 1, \ldots, p
\hspace{1cm} n \in \Psi_n
\hspace{1cm} c = k + n(p - 1) - k_{min} + 1
\hspace{1cm} \hat{R}_n = \hat{R}_n(I_{N_n} \otimes 1_p)
\hspace{1cm} \hat{y}_n = \hat{R}_n b + \hat{n}_n$$

(3.37) \hspace{1cm} (3.38) \hspace{1cm} (3.39)
\[ \hat{R}_n^p = \hat{R}_n(1 : p, n(p - 1) + 1 : np) \]  

(3.40)

\[ W_n^p = \hat{R}_n^H \left( \hat{R}_n \hat{R}_n^H + \sigma^2 \hat{R}_n^p \right)^{-1} \]  

(3.41)

The total structure of the path combining equalizer is shown throughout (3.36-3.40) and the final equalizers used to obtain the original symbol is shown in (3.41). The path combining equalizer has the least complexity of the later ones, it only uses the matched filter results that are associated with symbol \( n \). It achieves good performance since it introduces degrees of freedom for ISI suppression by using an equal amount of results to the number of paths present filters per symbols. However it can occur that the sub matrices for certain paths do not have full rank. These are the four possible variation to the equalizer method introduced earlier and their performance shall be evaluated.

3.5 The performance of the equalizers

The performance of the equalizers is evaluated using MATLAB simulations. All four of the earlier mentioned equalizers are tested in order to evaluate the bit error rate (BER) performance and under which circumstances the equalizers perform best. The BER found via the simulations is the un-coded error BER and there is still room for improvement by adding an error correcting code. Although the BER was also evaluated in [21] this was done theoretically, the BER found in this report are based on simulations.

The general settings that remain constant in all the scenarios is as follows, the used modulation scheme is 8-PSK, a resolution of 100 samples per symbol, a block length of 64 symbols, the channel coefficients are assumed Rayleigh fading and the noise is modeled as additive white Gaussian noise (AWGN) with a variance equal to 1. The sampling frequency was chosen as 44.1 kHz with a carrier of 6 kHz, with a 100 samples per symbols results in a symbol period of 2.3 milliseconds.

For every simulation a new set of channel coefficients, delays and Doppler rates are randomly chosen from a Rayleigh distribution, the range of the delay spread and the range of the the Doppler spread of the channel respectively. The corresponding BER is plotted against the SNR and for each SNR value 1000 simulations were executed.
Figure 3.7: The BER performance for the different equalizers, with 3 paths, a Doppler spread of 0.99-1.01 compression and/or dilation and a delay spread of 2.3 ms

Figure 3.8: The BER performance for the different equalizers, with 10 paths, a Doppler spread of 0.99-1.01 sample compression and/or dilation and a delay spread of 2.3 ms

In Figures 3.7-3.9 the number of paths is changed from three paths in Figure 3.7,
to ten paths in Figure 3.8 and to an amount of 50 paths in Figure 3.9. It can be seen that with a larger number of paths the same BER is achieved for a lower SNR. For example the $10^{-3}$ BER level is achieved earlier in Figure 3.9 than in Figure 3.7 where Figure 3.8 is in between. In Figure 3.9 the $10^{-3}$ level is already achieved for approximately 2 dB SNR where as in Figure 3.8 this level is achieved at a SNR of approximately 5 dB and for 3 paths it achieved around approximately at 8 dB SNR. The greater the number of paths the channel has the better the performance bit error wise becomes. This makes sense since the symbols received on all the paths are added coherently and due to the noise being uncorrelated this results in a decrease of the noise influence. The disadvantage of a high number of paths is that the equalizer becomes more complex since more information requires to be taken into account.
Figure 3.10: The BER performance for the different equalizers, with 10 paths, a Doppler spread of 0.95-1.05 compression and/or dilation and a delay spread of 2.3 ms

Figure 3.11: The BER performance for the different equalizers, with 10 paths, a Doppler spread of 0.95-1.05 compression and/or dilation and a delay spread of 7 ms

In Figures 3.10 and Figure 3.11 the performance is evaluated with a number of 10
paths, a Doppler spread of 0.95-1.05 compression or dilation and a delay spread of 2.3 and 7 milliseconds respectively. As can be seen that tripling the delay spread results in a performance loss of 2 dB at $10^{-5}$. A large delay spread has the consequence that ISI span of symbols becomes larger and more information for each symbol needs to be taken into account. For the partial and truncated equalizers this will result in extra computations because the sub matrices covering the ISI span which need to be inverted become larger will result in also more complexity.

In Figure 3.12 the delay spread is again 100 samples and the Doppler spread is enlarged to 20 sample compression and/or dilation. It can be seen from Figures 3.8, Figure 3.10 and Figure 3.11 where the only difference is the Doppler spread, that the Doppler spread is not a very influencing factor on the BER. For example the $10^{-3}$ level is around 5 dB in all these Figures. However it has effect on the structure of $\mathbf{R}$ and may also lead to more complexity for the partial and truncated block equalizer by increasing the ISI span.

In general the full block equalizer shows the best performance in all cases followed closely the partial block and the truncated block equalizer and finally the path combining equalizer. The path combining equalizers shows the worst performance but also offers the lowest complexity, it has an error floor which varies around $10^{-1}$ to $10^{-3}$ depending on the situation. The full block shows the best performance but is also the most complex since it uses all information, followed closely by the partial block equalizer which shows almost the same BER but is less computational. The truncated equalizer shows almost the same performance but also like the path combining equalizer can have an error floor of around $10^{-5}$ as can be seen in Figure 3.13. This is due to the
wide nature of the sub matrix used which may not always be full rank due to the large amount of zero columns. At the point in where the noise is not the main contributor to the probability of an error for some scenarios this effect starts to become visible such as in Figure 3.13. However the remaining error floor is quite low and may be easily compensated using an error correcting code.

3.6 Conclusion

In this chapter the entire system model is introduced, the transmitter, receiver and the channel effects on the received signal were all shown. The channel is modeled as a multi-scale/multi-lag channel. Further the equalization at the receiver was shown and four possible equalizer structures were presented based on the standard MMSE equalizer. First the so called full block equalizers is shown followed by three less computational options for the equalizer. The performance of these equalizers was tested which showed that full block equalizer performed best under all circumstances closely followed by the partial block equalizer and the truncated block equalizer and path combining equalizer. It was shown that the number of paths may increase the performance at the expense of increased complexity. The truncated and/or partial equalizer provide the best performance for least complexity. However these equalizers have been developed with perfect knowledge of the channel and in practice this is often not available.
In this chapter a method for estimating the channel is explained. In the previous chapter it was assumed that when the equalizers are constructed there was full channel knowledge available. However in practice there is often no channel knowledge available and it is necessary to estimate the channel since the construction of the equalizers requires channel knowledge. The necessary channel information that needs to be obtained are the delays, Doppler rates and channel gains. The channel estimation is done by using a known preamble. From the received version of this preamble the necessary channel parameters shall be retrieved.

4.1 The new transmission scheme

In the previous transmission scheme a block of \( N \) symbols is transmitted over the channel. It was assumed that within the duration of one block the channel parameters remain constant meaning that the Doppler rates and channel gains do not change within the duration of one block. However to be able to still model the time-varying nature of the UAC it is assumed that for a subsequent block these mentioned channel conditions have changed.

![Figure 4.1: The new transmission scheme with channel estimation](image)

Thus before each block of data symbols, channel estimation is redone in order to obtain the necessary channel knowledge to perform equalization on the subsequent data symbols. The used transmission scheme as shown in Figure 3.1 is changed to that of Figure 4.1. The extra guard time is added so that the preamble does not interfere with the block of data symbols, at the expense of reduced efficiency. If the channel parameters change slowly and remain constant over several blocks the preamble does not need to be inserted after every block but can be inserted after every two, three or four blocks for example. In this chapter a worst case scenario is assumed and channel estimation is necessary on a per block basis.
4.2 The requirements of the preamble

The preamble should be chosen such that the necessary channel information is retrieved from the received signal. In other words the preamble should make it possible to find the delays, channel gains and Doppler rates. A preamble normally used for channel estimation and/or synchronization is a so called pseudo random noise (PN) sequence. Ideally a PN sequence \( p(t) \) should have an autocorrelation function as denoted in (4.1) which is a delta function [1].

\[
\int_{-\infty}^{\infty} p(t)p(t - \tau) \, dt = \delta(\tau) \tag{4.1}
\]

Where \( \tau \) is the shift in time. If there is multipath present the received signal shall be a collection of delayed versions of the PN sequence. Because the PN sequence has the property denoted in (4.1) it is possible to find the respective arrival times of each path by shifting the known PN sequence over the received signal and performing matched filtering. In the case of no Doppler when the preamble is transmitted the received signal \( r(t) \) from (3.11) is matched filtered with shifted versions of the preamble, it is possible to find the delays of each path, see (4.2).

\[
\int r(t)p^*(t - \tau_l) \, dt = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l) \tag{4.2}
\]

Where \( \tau_l \) is the delay time for path \( l \), \( h_l \) is the channel gain for path \( l \) and \( L \) is the total number of paths present [1]. Ideally there will only be non zero values on the arrival times of the multi path components located at time \( \tau_l \) from the arrival time of the first path. Besides estimating the delays it is also still necessary to estimate the Doppler rates since the equalizers are constructed knowing the Doppler rates of each path. It is preferable that these parameters can be found using the same preamble, so that there is no need to send two preambles which would make the communication system even more inefficient. If the same sequence should be able to find both the delays and Doppler rates of each path (4.1) is changed to (4.3).

\[
\int_{-\infty}^{\infty} p(\alpha t)p(\alpha_i(t - \tau)) \, dt = \delta(\tau)\delta(\alpha - \alpha_i), \quad \alpha_i, \alpha \in [\alpha_{\min}, \alpha_{\max}] \tag{4.3}
\]

This way a sequence that full fills the requirement that only non shifted versions of the same scale have non zero autocorrelation is obtained. In (4.3) \( \tau \) is again the delay and the Doppler rates are denoted as \( \alpha \) and \( \alpha_i \) where both of them are within the Doppler spread of the channel denoted as \( \alpha_{\min} \) and \( \alpha_{\max} \). The channel estimation now becomes two-dimensional, estimating the delays and the Doppler rates while using the same sequence. There are numerous possible PN sequences that exist such as binary sequences which consist of 0 and 1’s and m-sequences. Unfortunately sequences which full fill both requirements perfectly, as in (4.3), do not exist. Even in practice sequences that have an autocorrelation as in (4.1) do also not exist.
4.3 Constructing the preamble

In this part the structure of the preamble for the channel estimation is introduced. It is later evaluated if the sequence full fills the requirements as in (4.3) or at least performs good enough in order to obtain the necessary channel information for the equalizers. In [35] an attempt is made to find a sequence that complies to (4.3). The preamble is based on DSSS where the spreading code is chosen such that the receiver is capable of leveraging the mobility using diversity reception. A Rake receiver is designed so that each finger matches a certain Doppler rate and delay. Although their purpose is different, the sequences mentioned in [35] should full fill the same requirements and could still be of interest to the channel estimation problem in this report. The entire transmitted signal is a collection of DSSS spreading codes \( x(t) \) multiplied with their respective data symbols.

\[
x(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} c_k p(t - kT_p)
\] (4.4)

One of these spreading codes shall be used to construct a preamble. The spreading code \( x(t) \) in (4.4) consist of a chip sequence with on every chip a pulse \( p(t) \) linearly modulated onto the chip code. The number of chips is represented by \( N_c \), \( c_k \) represents the chip value and the pulse duration is \( T_p \). Several additions and changes to the spreading code are made compared to [35]. Multiple preambles shall be tested in order to evaluate their performance bit error wise and to see what kind of preamble provides the best results. The next part shall introduce several variations to the sequences.

4.3.1 The chip code

The first problem is the design chip code which should preferably have the property as in (4.1). The chip code determines the values of \( c_k \). In this report two chip codes are used. The first one is a ternary zero correlation zone (ZCZ) sequence [39], which consists of -1,0 and 1. A zero correlation zone (ZCZ) sequence set has the periodic and/or aperiodic correlation values equal to zero for a contiguous set of delays starting with a single delay. Thus, it can significantly alleviate the multipath interference and multiple access interference. The second chip code is a random sequence of BPSK symbols which which consists of -1 and 1.

The chip codes and the autocorrelation of both chip codes are shown in Figure 4.2 and 4.3 respectively where it can be seen that both of the chip codes have low autocorrelation values but not perfectly as in (4.1) or (4.3). Due to their low autocorrelation values they remain of interest to use to construct the preamble. The content of the sequence is not that important, the most important property is that the autocorrelation is low since so that the arrival times of the multipath components can be found by matched filtering. It may be possible that there are more options available for the chip code.
Figure 4.2: The two chip codes: left, chip code with random BPSK symbols and right, a ternary ZCZ code, with 100 chips and modulated on the chip a rectangular pulse.

Figure 4.3: The autocorrelation of the two chip codes: left, for a random BPSK sequence and right, for ternary ZCZ code

4.3.2 The second derivative of a Gaussian chip

The second part in creating the preamble is to linearly modulate the function $p(t)$ onto the chip code. It is necessary to keep in mind that it is preferable that the preamble used for the channel estimation is within the constraints of the available bandwidth of the system. In [35] a second derivative of a Gaussian chip pulse is chosen but no specific reason is mentioned.

$$p(t) = \frac{\sqrt{\frac{\sqrt{32}}{3}}}{\sqrt{3}} [1 - 2(\pi f_0 t)^2] e^{-(\pi f_0 t)^2}$$ (4.5)
\[ P(f) = \frac{\sqrt{f_0} \sqrt{\frac{32}{3}}}{\sqrt{\frac{\pi f_0^2}{f}}} \left( \frac{f}{f_0} \right)^2 e^{-\frac{f^2}{f_0^2}} \]  

(4.6)

There are alternatives that can be used such as other types of wavelets or a raised cosine since it is already available within the communication system. The second derivative of the Gaussian chip pulse is given as in (4.5) and (4.6) and the waveform and its spectrum are shown in Figure 4.4 respectively.

![Figure 4.4: Left the time domain representation of the second derivative of a Gaussian chip pulse and right the frequency domain representation, also known as ‘Mexican hat wavelet’.

4.3.3 Other random sequences

Another idea is to not use a pulse \( p(t) \) but another chip sequence that is modulated onto the chip values \( c_k \) which would create a long chip sequence. The longer chip sequence consists of the interleaving of two separate chip codes. This secondary chip sequence is called \( p[n] \) and again the same requirements of low autocorrelation apply.

In [33] and [34] very low autocorrelation sequences are deducted. These sequences are called constant amplitude zero autocorrelation (CAZAC) sequences. These sequences have low autocorrelation with the constraint that the absolute value of all the values in the sequence is equal to 1. Instead of having pulses modulated on the chips these CAZAC sequences shall be used.

\[ p[n] = e^{j\pi \frac{n(n-1)}{N}}, \ n = 1, ..., N \]  

(4.7)

\[ p[n] = e^{2j\pi \frac{(m-1)(p-1)}{K}}, m, p = 1, ..., K, \ N = K^2 \]  

(4.8)
\[ p[n] = e^{j\frac{2\pi}{N}(n-1)(\frac{n-1}{2}-N)} , \quad n = 1, \ldots, N \] (4.9)

The CAZAC sequences of possible use are the Golomb, Franks, P4 and the cyclic algorithm new (CAN) sequence. Where the first three are mentioned in [33] and the (CAN) sequence is derived in [34]. The Golomb, Franks and P4 are shown in (4.7),(4.8) and (4.9) respectively and the algorithm used to construct a CAN sequence is located in Appendix A.1. It is noted here that the Golomb, Frank, P4 and CAN sequences are complex but only the real part is transmitted. In the next section it will be explained on how the delays and Doppler rates of each path can be found from the preamble.

4.4 Delay and Doppler rate estimation

In this part the method of finding the Doppler rates and delays of each individual multipath component is explained. At the receiver the preamble is received and from the distorted version the necessary channel information is obtained. This will be done in two steps, the first step is matched filtering the received signal and the second step is the detection of the paths from the matched filtering results.

4.4.1 The matched filtering

From the received signal the necessary channel information can be obtained via the following way. The first step is the matched filtering of the received signal with the shifted and scaled versions of the original preamble, it is shown below in (4.10)

\[ y[\tau; \alpha] = \int r(t)p^*(\alpha(t - \tau))dt \] (4.10)

Here \( y[\tau; \alpha] \) contains the matched filtering results for each shift and scale. The search for the delays and the Doppler rates can be limited to within the delay and Doppler spreads, respectively denoted as \( \tau \in [\tau_{\text{min}}, \tau_{\text{max}}] \) and \( \alpha \in [\alpha_{\text{min}}, \alpha_{\text{max}}] \), of the channel in order to limit computations.

From the procedure of the matched filtering mentioned above it may be hard to understand how it is possible to obtain the Doppler rates and the delays. To demonstrate how the matched filtering results look like and how it is possible that from these results the channel can be estimated, a convenient scenario is generated. For the preamble a BPSK chip code with 100 chips was chosen and each chip was modulated with a CAN sequence of length 100. The total duration of this preamble would be approximately equal to 220 milliseconds for a sampling frequency of 44.1 kHz. The autocorrelation of the preamble is shown in Figure 4.5 and shows good autocorrelation properties.

The channel conditions are as follows. The channel consists of 7 paths with a Doppler spread of between 0.9997 and 1.0003. The first path has a Doppler rate equal
to 0.9997, second 0.9998 etc and the last path has a Doppler rate equal to 1.0003, so that all the 7 paths have an unique Doppler rate. The delays are chosen 1-7 where the first path arrives at 1 millisecond the second at 2 milliseconds etc, the sampling frequency is 44.1 kHz since this is the standard frequency of audio systems. The 7 channel gains are equal to 1 in order to demonstrate the matched filtering results. The absolute value of the matched filtering results is shown in Figure 4.6.

Figure 4.5: The autocorrelation function of the preamble used for Figure 4.6.

Figure 4.6: Example of the matched filtering results with a preamble of 100 BPSK chips in combination with a CAN sequence. Each color represents a scaled version of the original preamble, the black horizontal line is the threshold for detection.
From Figure 4.6 clearly 7 peaks can be seen which represent the exact positions of the arrival times of the paths. Although it cannot be seen from the figure the recovered delays match exactly the preset delays. The color of each peak corresponds to a certain scaled version and these are also all correct. All the Doppler rates and delays are found exactly in this scenario. Hence it is possible to find the delays and the Doppler rates of the multipath components since one multipath component is represented by a peak in the matched filtering results. The dashed black line is the detection threshold which selects the 7 peaks from the rest of the matched filtering results this shall be discussed in more detail later on.

Figure 4.7: Matched filtering results plotted in the scale/delay domain with same preamble used as before.

Another method of showing the matched filtering results is in a 3-dimensional way where the matched filtering results are plotted against the scaling and the delays, the . This is shown in Figure 4.7. Where the preamble was shortened to 25 chips and used on an environment with 50 paths a delay spread of 200 milliseconds and a Doppler spread of 0.0025 which corresponds to Doppler rates 0.9975 to 1.0025. As can be seen again, there are peaks in the delay-scale domain which corresponds to the multipath components. The channel gains were randomly chosen and the height of the peak is mainly influenced by the channel gains. This example again shows that it is possible to estimate the delays and Doppler rates of the channel.

It has to be mentioned that the residual term given in (3.12) offers extra orthogonality between the different scaled versions. This term is also dependent on the Doppler rate and is different per scaling. Hence for larger differences in Doppler rate the orthogonality between different scaled versions increases and larger Doppler rates might
then be even beneficial for orthogonality between the different scaled versions.

### 4.4.2 Detection of paths from the matched filtering

It is still necessary to separate the delays and Doppler rates from the matched filtering results. In Figure 4.5 it can be clearly seen, by visual inspection, that there are peaks corresponding to paths but the delays and Doppler rates still need to be retrieved. The simplest way is setting a threshold as was done in Figure 4.5. In Figure 4.5 a threshold of magnitude 1500 would easily separate the 7 paths out of the matched filtering results. However the scenario used for Figure 4.5 was very convenient. All the 7 paths had sufficient power which was the reason for setting the channel gains equal to 1, so that each path contains enough power to be detected. Such a convenient situation is highly unlikely in the real world.

The possible amount of channel taps can be in the hundreds so the 7 used in this example is a very low number. Fortunately the UAC is sparse in nature meaning that although there may be a lot of channel taps present only a few of those channel taps are necessary in order to be able to recover the transmitted symbols. Only estimating these paths would still result in a reasonable performance.

In Figures 4.6 and 4.7 it can be seen that the preamble is not perfect since besides the matched filtering results of the paths there are more non-zero values and small peaks. These values originate from cross correlation between different scaled and shifted versions. If paths do not contain enough power then the matched filtering result for that path will disappear in these cross correlation values. Hence the data from this path is lost but due to the assumption of sparsity of the UAC this does not necessarily need to lead to a severe degradation in performance.

The height of the threshold is a trade off between how many paths are detected and how many false detections are made. False detection happen if peaks not originating from a multipath are still detected and interpreted as a path. If the threshold is set low then all paths are detected but with the possibility that there are numerous false detections. While a high threshold may not detect all paths but has the advantage that the probability of having false detections is very low. The decision for the height of the threshold is not straightforward since it also depends on the used preamble and the channel conditions. A good threshold for one channel is not always necessarily the best choice for another channel. For the example in Figure 4.6 a good threshold height was 1500 but now the same threshold is used for a channel with 40 channel taps, a delays spread of 30 ms in with the remaining parameters all staying the same. The matched filtering results are shown in Figure 4.7. As can be seen, the same threshold of 1500 is not optimal since there a few smaller peaks that reach the threshold level. It is too low and for better detection the height of the threshold should be slightly raised. Hence it is difficult to define a universal threshold for all preamble and channel conditions. Nevertheless a good coarse way of defining a threshold is by choosing a ratio. This ratio is defined as the ratio between the result of the matched filtering compared to the maximum matched filtering result and/or the maximum value of the autocorrelation function. The matched filtering result should at least be higher than this ratio. If the maximum matched filtering results is 100 and with ratio equal to 4, this means that matched filtering results larger than 25 are denoted as paths. From empirical results a
good choice for the threshold seems to be taking this ratio around to be a third, fourth or fifth of the maximum result. The threshold shall be different per preamble and is a good rough estimate for a threshold that can be used.

### 4.5 The channel gains

From the procedure above it is possible to find the delays and Doppler rates of multipath components if they have enough power, with this data the channel responses $q_n$ in (3.14) can be constructed partially. The only channel information still not known are the channel gains $h_l$ for each path. So $q_n$ can only be constructed by using the known delays and scale of each path. However it is important for equalization since the equalizers are also constructed with knowledge of the channel gains. There are two possible methods introduced for dealing with the channel gains. The first one is done by compensating for the channel gains and the second method is estimating the channel gains.

#### 4.5.1 Compensating for the channel gains

The explanation is given below with the following assumptions, the parameters $\tau_l$ and $\alpha_l$ are known, only one symbol is sent so there is no ISI and the number of paths equals
two, this way the true channel matrix is just reduced to one value, \( \mathbf{R}(1,1) \). We start from recalling that

\[
\mathbf{R}(i,j) = \int q_i(t)q_j^*(t) \, dt
\]  

(4.11)

Since there is only one symbol \( q_i = q_j = q_0 \) because \( q_i = 0 \) for \( i \neq 1 \) the channel matrix \( \mathbf{R} \) is equal to \( \mathbf{R}(1,1) \). The received signal for the symbols is as follows:

\[
q_0(t) = \sum_{l=0}^{L-1} v_n^i(t) = \sum_{l=0}^{L-1} v_n^0(t) = \sum_{l=0}^{L-1} \left[ h_l \sqrt{\alpha_l} p \left( \alpha_l(t - \tau_l) - \frac{T}{\alpha_l} \right) \times e^{2j\pi f_c ((\alpha_l-1)t - \alpha_l \tau_l)} \right]
\]  

(4.12)

Writing this for \( L = 2 \) and \( N = 1 \) we get:

\[
q_0(t) = h_0 \sqrt{\alpha_0} p \left( \alpha_0(t - \tau_0) - \frac{T}{\alpha_0} \right) \times e^{2j\pi f_c ((\alpha_0-1)t - \alpha_0 \tau_0)} \\
+ h_1 \sqrt{\alpha_1} p \left( \alpha_1(t - \tau_1) - \frac{T}{\alpha_1} \right) \times e^{2j\pi f_c ((\alpha_1-1)t - \alpha_1 \tau_1)}
\]  

(4.13)

Substituting this in the previous equation and noticing that, \( h_l, \tau_l, \alpha_l \) and \( p(t) \) are real, we obtain

\[
\mathbf{R}(1,1) = \int \left( h_0 \sqrt{\alpha_0} p \left( \alpha_0(t - \tau_0) - \frac{T}{\alpha_0} \right) \times e^{2j\pi f_c ((\alpha_0-1)t - \alpha_0 \tau_0)} \\
+ h_1 \sqrt{\alpha_1} p \left( \alpha_1(t - \tau_1) - \frac{T}{\alpha_1} \right) \times e^{2j\pi f_c ((\alpha_1-1)t - \alpha_1 \tau_1)} \right) \\
\times \left( h_0 \sqrt{\alpha_0} p \left( \alpha_0(t - \tau_0) - \frac{T}{\alpha_0} \right) \times e^{-2j\pi f_c ((\alpha_0-1)t - \alpha_0 \tau_0)} \\
+ h_1 \sqrt{\alpha_1} p \left( \alpha_1(t - \tau_1) - \frac{T}{\alpha_1} \right) \times e^{-2j\pi f_c ((\alpha_1-1)t - \alpha_1 \tau_1)} \right) \, dt
\]  

(4.14)

Defining the following than this will lead to:

\[
e^{2j\pi f_c ((\alpha_0-1)t - \alpha_0 \tau_0)} = e^{m_1}
\]  

(4.15)

\[
e^{2j\pi f_c ((\alpha_1-1)t - \alpha_1 \tau_1)} = e^{m_2}
\]  

(4.16)
\[
\mathbf{R}(1, 1) = \int h_0^2 \alpha_0 p^2 \left( \alpha_0(t - \tau_0) - \frac{T}{\alpha_0} \right) dt \\
+ \int h_0 h_1 \sqrt{\alpha_0 \alpha_1} p(\alpha_0(t - \tau_0) - \frac{T}{\alpha_0}) p(\alpha_1(t - \tau_1) - \frac{T}{\alpha_1}) e^{m_1 - m_2} dt \\
+ \int h_0 h_1 \sqrt{\alpha_0 \alpha_1} p(\alpha_0(t - \tau_0) - \frac{T}{\alpha_0}) p(\alpha_1(t - \tau_1) - \frac{T}{\alpha_1}) e^{m_2 - m_1} dt \\
+ \int h_1^2 \alpha_1 p^2 \left( \alpha_1(t - \tau_1) - \frac{T}{\alpha_1} \right) dt
\]

This also can be written as:

\[
\mathbf{R}(1, 1) = h_0^2 \int \alpha_0 p^2 \left( \alpha_0(t - \tau_0) - \frac{T}{\alpha_0} \right) dt \\
+ h_0 h_1 \int \sqrt{\alpha_0 \alpha_1} p(\alpha_0(t - \tau_0) - \frac{T}{\alpha_0}) p(\alpha_1(t - \tau_1) - \frac{T}{\alpha_1}) e^{m_1 - m_2} dt \\
+ h_0 h_1 \int \sqrt{\alpha_0 \alpha_1} p(\alpha_0(t - \tau_0) - \frac{T}{\alpha_0}) p(\alpha_1(t - \tau_1) - \frac{T}{\alpha_1}) e^{m_2 - m_1} dt \\
+ h_1^2 \int \alpha_1 p^2 \left( \alpha_1(t - \tau_1) - \frac{T}{\alpha_1} \right) dt
\]

Now let’s make the following substitutions:

\[
A = h_0^2 \int \alpha_0 p^2 \left( \alpha_0(t - \tau_0) - \frac{T}{\alpha_0} \right) dt 
\]

\[
B = h_0 h_1 \int \sqrt{\alpha_0 \alpha_1} p(\alpha_0(t - \tau_0) - \frac{T}{\alpha_0}) p(\alpha_1(t - \tau_1) - \frac{T}{\alpha_1}) e^{m_1 - m_2} dt 
\]

\[
C = h_0 h_1 \int \sqrt{\alpha_0 \alpha_1} p(\alpha_0(t - \tau_0) - \frac{T}{\alpha_0}) p(\alpha_1(t - \tau_1) - \frac{T}{\alpha_1}) e^{m_2 - m_1} dt 
\]

\[
D = h_1^2 \int \alpha_1 p^2 \left( \alpha_1(t - \tau_1) - \frac{T}{\alpha_1} \right) dt
\]

Then using these substitutions it can be written as:

\[
\mathbf{R}(1, 1) = h_0^2 A + h_0 h_1 (B + C) + h_1^2 D
\]

This equation represents the true value of \( \mathbf{R}(1, 1) \) value of the channel matrix. Remember that the noiseless data model after matched filtering is:

\[
y = \mathbf{R} \mathbf{b}
\]
Which has the following characteristics:

\[ y = y_0 = \int g_0^*(t)r(t) \, dt = b_0 \int g_0^* g_0(t) \, dt R = R(1, 1) = h_0^2 A + h_0 h_1 (B + C) + h_1^2 D \]  

(4.25)

If \( b \) needs to be estimated we can multiply \( y \) with the inverse of \( R \).

\[ b_0 = R^{-1} y = b_0 R(1, 1)^{-1} y_0 = b_0 \frac{y}{h_0^2 A + h_0 h_1 (B + C) + h_1^2 D} \]  

(4.26)

Since the matched filtering result \( y \) is equal to equal to \( R(1, 1) \) multiplied with the symbol value \( b_0 \), (4.26) becomes:

\[ b_0 = \frac{y}{h_0^2 A + h_0 h_1 (B + C) + h_1^2 D} = \frac{b_0 (h_0^2 A + h_0 h_1 (B + C) + h_1^2 D)}{h_0^2 A + h_0 h_1 (B + C) + h_1^2 D} = b_0 \]  

(4.27)

The idea is as follows, \( q_n(t) \) is reconstructed using the delay and Doppler rates found. While reconstructing the channel gains \( h_i \)'s are unknown to the receiver in order to reconstruct \( q_n(t) \). For now it is assumed that there is no fading and the channel gains are set to 1 in the receiver. For our example earlier this result in:

\[ \tilde{R} = \tilde{R}(1, 1) = h_1 A + h_2 B + h_1 C + h_2 D \]  

(4.28)

This \( \tilde{R} \) is our best estimate of the real channel matrix \( R \) substituting this in (4.27):

\[ \tilde{b}_0 = \tilde{R}^{-1}(1, 1) y_0 = \frac{y}{A + B + C + D} = \frac{b_0 (h_0^2 A + h_0 h_1 (B + C) + h_1^2 D)}{h_1 A + h_2 B + h_1 C + h_2 D} \]  

(4.29)

This way the original symbol \( b_0 \) is not retrieved since the \( \tilde{R}(1, 1) \) is not equal to the real \( R(1, 1) \). This is because it was assumed that all \( h_i \) were equal to 1 in the receiver, which in reality is not the case. The result of the division in (4.29) is just a scalar, in (4.27) this scalar was equal to 1 and the original transmitted symbol was recovered. In this case the original symbol \( b_0 \) may still be recovered by finding a scalar \( c \) so that:

\[ b_0 = \tilde{b}_0 c = b_0 c = \frac{b_0}{b_0} \]  

(4.30)

Unfortunately is \( b_0 \) in general unknown but by adding a pilot symbol at the start of the block the scalar \( c \) can be found. With \( c \) found the original symbols \( b_0 \) is recovered. Then it is possible to to decode the entire block so that the original symbols are recovered. This way the channel coefficients do not need to known.

This method is based on the assumption that for every symbol in one block the constant \( c \) is the same. Hence only inserting one pilot symbol at the start is sufficient to find the
scalar and decode the entire block. This requires that the respective channel functions
in one block are the same for each symbol. In the case of Doppler this is certainly not
true and the channel functions of the symbols change over the block length. However
it shall be assumed that this change is small and the effect is limited. Furthermore the
noise was not included and just as in the case of the ZF equalizers noise enhancement
may be possible. Hence this method will probably be far from an optimal solution in
dealing with the channel gains.

4.5.2 Estimating the channel gains

Instead of compensating for the channel gains another option is to estimate the channel
gains which shall then be used in constructing the equalizers. Only the channel gains
of the paths that were found via the preamble are estimated including those of the false
detections which in reality are not present in the received s ignal. Paths that were not
found during the preamble are lost and ignored but are of course still present in the
real received signal.

\[
\begin{bmatrix}
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & s_0^2 & 0 & 0 & 0 \\
0 & \vdots & 0 & 0 & 0 \\
s_0^1 & s_{N_2-1}^2 & 0 & 0 & 0 \\
\vdots & 0 & s_0^3 & \cdots & 0 \\
s_1^{N_1-1} & 0 & \vdots & 0 & 0 \\
0 & \vdots & s_{N_3-1}^3 & s_{N_3}^L & \vdots \\
0 & 0 & 0 & \vdots & 0 \\
0 & 0 & 0 & \cdots & s_{N_L-1}^L
\end{bmatrix}
= \begin{bmatrix}
h_0 \\
\vdots \\
h_{L_{\text{est}}-1}
\end{bmatrix} = Sh
\]

(4.31)

The received signal is a collection of scaled and delayed versions of the preamble
hence the received signal can then be written as a collection of scaled and delayed
versions of the preamble as shown in (4.31). Which are the sampled versions of the
received signal and scaled versions, sampled at a rate equal to the sampling frequency.
The matrix \( S \) consists of the scaled and shifted versions of the preamble which were
found via the channel estimation. It has dimensions \( N \times L_{\text{est}} \),where \( N \) is the length
of the received signal, \( L_{\text{est}} \) the number of paths found in the channel estimation and
\( N_i \) stands for the length of the individual scaled versions. The to be estimated channel
coefficients are denoted in vector \( h \) with dimensions \( L_{\text{est}} \times 1 \). The vector \( r \) is the received
signal with dimensions \( N \times 1 \). The starting point and end point in the columns of \( S \)
are dependent on the delays found for each scaled version. For example if the first path
found has a delay of 50 samples it is located in the first column at the 50’th index and
the end point is the length of the scaled version \( N_i \) plus the delay.

It can be seen that if \( h \) needs to be retrieved from (4.32) that applying a least
squares solution should work since the system in (4.31) is overdetermined.
The estimated channel coefficients \( \hat{h} \) found using the least squares solution (4.32) are the best match to reconstruct the received signal.

Further the least squares approach has an advantage if the amount of false detections is not too high. Namely that estimated channel gains for these false detections have a small value. Since they are not truly there in the received signal the least squares solution puts the channel coefficients belonging to these false detections close to zero. This will limit the influence that these false detections have in constructing the equalizers which is beneficial. To even further limit their influence, the estimated channel coefficients having a low value after the initial estimates can be removed completely. However this only is valid if the number of false paths is in the order of the number of real detected paths. For a large amount of amount of false detections it is likely that there exist other solutions with the false detections to reconstruct the received signal. Then this advantage disappears and false detection shall degraded the performance.

The final estimated channel coefficients are obtained after removal of the false detections based on the size of their estimated channel gains. The decision of how high the threshold should be is again dependent on the trade off between the removal of false paths and maintaining paths with low energy. The choice for the threshold is based on the earlier set threshold. If a channel gain is estimated that could never be found in the first place because the matched filtering result would be below the original threshold it is likely that it is a false detection. Hence this method is chosen as whether to decide a low estimated channel gain is a path.

This method has however the disadvantage that it is sensitive to synchronization errors. The placement of the scaled versions within the matrix is very important. Hence this method only works well if the estimated delays are almost equal to the true delays. If this is not the case the performance degrades significantly. Also the sampling rate is important which should be the same for the received signal and reconstructed channel functions.

### 4.6 The choice of the wideband pulse

In the earlier used example the choice for \( p[n] \) was a CAN sequence but the choice is not limited to a CAN sequence although some choices are of course better than others. The reason for choosing a second derivative Gaussian chip is based on [35], the choice for the raised cosine is made because it is already available. The CAZAC sequences are chosen for their very low autocorrelation properties as was desirable. A good choice for \( p(t) \) is dependent on two properties, the first property is the cross correlation values between the different scaled and shifted versions of the preamble and the second property is the resolution and accuracy.

If the cross correlation values are high there will be a higher probability for false detections when using the same threshold. For the same probability for making
false detections the threshold should be higher. This has the consequence that more paths with low power are ignored since they cannot be distinguished from the cross correlation values. Therefore it is preferable to select \( p(t) \) or \( p[n] \) such that these cross correlation values are low.
The second important property is the resolution, with the resolution it is meant how good the preamble can distinguish two paths if the difference between arrival times and/or if the difference in scaling is small. Will it then still be possible to distinguish these two paths separately or shall they be merged into one detection. Another property which is linked to resolution is the accuracy. How close are the estimated arrival times and Doppler rates to the real delays and Doppler rates or is it possible that the result may be slightly off.

Another aspect concerning the resolution is the minimum speed that can be found using the preamble. An object moves with a certain speed and causes a certain amount of compression and or dilation. The preamble should be designed that if this amount has influence on the data symbols that this amount can be found. Hence to what kind of accuracy should the estimated Doppler rate be to the real one. If a Doppler rate resolution of $10^{-4}$ is necessary the preamble should at least be long enough that this amount of compression or dilation can occur so it can be detected. Hence longer sequences show better resolution and or oversampled version of shorter preambles can also show high resolution. It is impossible for an infinite resolution for detecting the speeds since it would require an infinite amount of time and samples. But the estimated Doppler rates found should be close enough to the real Doppler rates so that the equalizers still work. Also block length plays a role since a longer block of symbols means that small Doppler rates have impact while for shorter blocks there effect is not seen. In practice it shall be quite difficult to obtain Doppler rates perfectly. In our report it is assumed that the Doppler rates found always have sufficient resolution so that equalization can be performed. In Figures 4.9-4.11, for the same scenario as used in generating Figure 4.5, the absolute matched filtering results are shown for preambles constructed with a BPSK chip code in combination with a normalized root raised cosine, a second derivative of a Gaussian chip and a CAN sequence respectively. From these figures it can be seen that the matched filter results are dependent on the choice.
of $p(t)$. The raised cosine has the largest cross correlation values compared to the maximum values and also has the 'widest' peaks. Hence in terms of cross correlation values, accuracy and resolution it probably performs the worst the three preambles. The best sequence to use is for low cross correlation values, resolution and accuracy is probably the CAN sequence since the resulting peak is very narrow so there is not much deviation from the true arrival time and or Doppler rate that can occur.

4.7 The performance of the equalizers in combination with channel estimation

The BER performance of the equalizers in combination with the channel estimation shall be evaluated. The preambles will be constructed using the earlier mentioned chip codes, BPSK and ternary ZCZ codes, and for $p(t)$ a raised cosine, second derivative of a Gaussian chip and of the CAZAC sequences the CAN sequence is chosen. With these preambles and using the transmission scheme in Figure 4.1 the BER of the receiver is tested using Matlab simulations. From the possible equalizers in Chapter 3 only the full block equalizer shall be tested since this equalizer has the highest performance and the main focus in this report is the lowest achievable BER. There will be per preamble three BER curves, the uncompensated which just reconstructs the channel with the found delays and scaling and assumes the channel gains are all equal to 1, the compensated BER curve which compensates for the channel gains as seen in Chapter 4.5.1 and the final BER curve is where the channel gains estimated via the least squares approach as seen in Chapter 4.5.2.

In order to test the BER performance a scenario is used with a number of 15 paths. The delay spread is 10 milliseconds with a sample frequency of 44.1 kHz. The 10 milliseconds delay spread corresponds with the average delay spread of the UAC mentioned in [17]. The Doppler spread is chosen as $0.998 \sim 1.002$ compression or dilation. The channel coefficients are randomly chosen where it is assumed they are Rayleigh fading. For every run a whole new channel is created and a preamble is inserted before each block of data symbols. The noise is presented as white Gaussian with mean zero and a variance equal to 1. The sequence is constructed using a preamble consisting of 25 chips and on each chip 100 samples for $p(t)$. The channel information is then retrieved via the matched filtering procedure and detection. After the channel estimation a block with a length of 64 symbols is transmitted, where the symbols are chosen from a QPSK alphabet. This block will then be equalized using the equalizers constructed with the information retrieved from the preamble. Per SNR a number of 500 runs are executed.
Figure 4.12: The BER curves of all the introduced preambles using a BPSK chip code.

Figure 4.13: The BER curves of all the introduced preambles using a ternary ZCZ chip code.
The resulting BER curves are shown in Figures 4.12 and 4.13 where the first figure are the results for the preamble using a BPSK chip code and the second figure using a ternary ZCZ code. As can be seen from both plots that the uncompensated version clearly performs the worst and is for all preambles approximately the same. The second best results are shown using the compensated version but this approach still offers poor results and is for most preambles also the same. The best method is estimating the channel coefficients via the least squares method. Where especially the CAN and the second derivative of a Gaussian chip show a good performance bit error wise. Hence the focus later shall be on the least squares approach. Further it is interesting to research which chip code offers the best performance so the results from Figures 4.12 and 4.13 are combined in Figure 4.14. As can be seen that the second derivative of a Gaussian chip in combination with a ternary ZCZ sequence shows worse performance than in the case of a second derivative of a Gaussian chip in combination with a BPSK chip code. But for the preamble with a raised cosine and least squares approach the ternary ZCZ code show the best better result. In general using a CAN sequence with the least squares approach shows the best performance with almost an equal BER for a BPSK or ternary ZCZ chip code. Hence the best choice for a low BER would be using a CAN sequence in combination with a BPSK or ternary ZCZ chip code as was also predicted earlier from the matched filtering results.
Figure 4.15: The BER curves of the preambles based on a BPSK chip code for three different Doppler spreads, the upper right figure shows the preamble constructed using a raised cosine, upper left figure a second derivative of a Gaussian chip and the bottom the preamble constructed using a CAN sequence.

Next the effect of the Doppler spread and the delay spread on the BER performance is researched. For every every preamble using a BPSK chip code the same scenario is repeated with only the Doppler spread changed to $0.9996 \sim 1.0004$ and $0.99 \sim 1.01$ dilation and compression. As can be seen in Figures 4.15 that a larger Doppler spreads results in a better BER performance. This is due to the large differences in scaling between the paths which result in extra orthogonality originating from the term mentioned in (3.12). This means that the chance for false detection goes down since the cross correlation values are lower and the detection is more accurate. When the difference in scaling is small as in the case for a Doppler spread of 1 sample the performance of the channel estimation degrades since the term in (3.12) does not offer the extra orthogonality anymore.
Further the effect of the delay spread of the channel is researched. The delay spread was changed to 2.3 milliseconds instead of 10 milliseconds. As can be seen that for all preambles using a BPSK chip code a larger delay spread results in a slightly better performance. A larger delay spread causes the arrival times of paths the be more apart from each other which causes less cross correlation. Although the increase in performance is less than that if the Doppler spread would be larger.

From Figures 4.12 and 4.13 a BPSK chip code in combination with a CAN sequence is shown to offer the lowest BER. The preamble used consisted of 25 chips with on each chip a CAN sequence of length 100. Maybe it is possible by adding chips or removing chips to get better results or a shorter preamble which would results in a better throughput. In Figure 4.19 the number of chips was changed and as can be seen that the number of chips does not seem to be important. Even for 1 chip the performance is almost similar but it would increase the efficiency of the system. Also the number of samples does not seem to have much influence as shown in 4.20. Hence the length of the preamble, if the CAN sequence is used, can be shortened so that a higher bit rate can be achieved.

The assumption was made that the channel could be considered sparse and that only a few paths would be sufficient in obtaining the original symbols. To verify this the number of paths was increased to 100. The extra 85 paths can have a maximum
Figure 4.17: Left the time domain representation of a second derivative Gaussian chip and right the frequency domain representation of a second derivative Gaussian chip, also known as 'Mexican hat wavelet'.

Figure 4.18: The BER curves of the preambles based on the BPSK chip code and CAN sequence in the case of 15 paths and 100 paths for which most are too small to detect.

equal to just below the threshold so they cannot be detected in theory. In theory only a maximum of 15 paths can be detected and used for channel estimation and equalization. The other 85 paths cannot be detected but do influence the received signal and equalization process. However according to our assumptions this should not be a major problem. The resulting BER curve is plotted against the original BER curve.
for the preamble constructed with a BPSK chip code and CAN sequence. This shows that the assumption is justified since the 85 extra path do not have a large influence and the BER curve is actually slightly better. Probably because some of these extra paths were detected occasionally which resulted in a better BER performance as was also seen in the simulations of Chapter 3.

![BER curves](image)

Figure 4.19: The BER curves of the preambles based on the CAN sequence compared to the BER if there was full channel knowledge available.

The final interesting thing to evaluate is to compare the preamble which showed the best results with the original equalizer which has full channel knowledge. This is done in Figure 4.19 and as can be seen that when a CAN sequence is used regardless the chip code, the performance is almost as the same as the case in which there was full channel knowledge available. This shows that via it is possible to estimate multi-scale/multi-lag channels using one of the preambles introduced here so that the equalizers almost show optimal performance as if full channel knowledge was available.

However there are some notes that need to be mentioned, the CAN sequence is designed that the algorithm returns exactly a number of points that have low autocorrelation properties. A re-sampled version of such a CAN sequence does not necessarily need to have the same low autocorrelation property anymore. However in the simulations it shows that with the re-sampled versions of the CAN sequence still very good results are obtained. But the reasoning behind the original choice, the low autocorrelation values, is not necessarily true for the re-sampled version of the CAN sequence. It just happens to be that the re-sampled versions work quite well. A second note is that the CAN sequence is noise like and is white, which means that the necessary bandwidth for a CAN sequence is at the least very large. Unfortunately the available bandwidth in underwater acoustic communications is limited and therefor for practical uses a CAN sequence realizable. The second derivative of the Gaussian chip is maybe
a better solution although still has a high bandwidth it is already lower than the one of the CAN sequence. The pulse with the lowest bandwidth is the raised cosine but unfortunately shows poor performance compare to the others. So for practical use there is still need to find a sequence which has low bandwidth and can be used for channel estimation.

Apparently the bandwidth is an important factor and a higher bandwidth is preferable. Since the Gaussian chip had a higher bandwidth than the raised cosine a good comparison of the effect of the pulse shapes is not made. Hence it is interesting to compare these two pulses if they would have the same bandwidth. To test the influence of the pulse shape a comparison is made between the raised cosine and the Gaussian chip if they have the same bandwidth with a BPSK chip code. The channel is the same as it was before and only the least squares method is shown.

![Figure 4.20: The BER curves of the preambles based on the the raised cosine and Gaussian for different bandwidth.](image)

By increasing the bandwidth of the raised cosine the BER performance has increased compared to the earlier simulations. The bandwidth for the second derivative of the Gaussian chip was lowered and so did the BER performance compared to the earlier simulations. Hence it seems that the earlier simulations were not really fair since the second derivative of the Gaussian chip had a far larger bandwidth. As can be seen that the BER under the same bandwidth availability is worse for the second derivative of the Gaussian chip compared to the raised cosine. Which is in contradiction to our previous results which clearly showed that the raised cosine performed worse under all conditions. Hence it seems that the available bandwidth is a very important factor. The preamble which had the largest bandwidth performed the best. Another observation is that the pulse shape can influence the results but unfortunately due to time limit this aspect of pulse shape is not further researched. Hence for the future it may be of interest to search for other types of pulse which perform better even for a low bandwidth.
4.8 Conclusion

In this chapter the channel estimation was discussed and evaluated by constructing a preamble and placing it in front of a block of symbols according to the used transmission scheme. Via the received version of the preamble through matched filtering and the detection results belonging to paths out of the matched filter results. From these results the channel coefficients are compensated or estimated using a least squares approach which had the extra advantage of being able to remove some of the false detections. Concluded by testing the preamble and channel estimation in combination with the full block equalizer mentioned in Chapter 3 to obtain the lowest achievable BER. The BER performance using different preambles was evaluated where it showed that for the preamble constructed with a CAN sequence almost optimal BER can be achieved as if there was full channel knowledge available.
Conclusions

In this thesis a receiver for acoustic underwater communications was introduced which was able to establish reliable communications by removing ISI and by compensating for Doppler effects. We considered a multi-scale/multi-lag channel applied with the wideband model used to represent the Doppler effects on the acoustic signals. This work comprised two major parts, introducing and testing an already existing equalizer that were specially designed for a multi-scale/multi-lag channel, and channel estimation which was able to recover the Doppler rate and delay of each multipath component if it has sufficient power.

First the components of the communication system were introduced including the transmitter and receiver followed by the channel model used to model the multi-scale/multi-lag channel. A mathematical description for the received signal was derived. The equalizers were based with taking into account the communication system and channel model. These equalizers were evaluated for their performance, bit error wise. Although the equalizers already existed the performance was only tested theoretically and not evaluated using simulations including data symbols but which was done in this report. However these equalizers were constructed assuming full channel knowledge which is not available in general.

Therefor the second part consisted of designing channel estimation targeted towards multi-scale/multi-lag channel. The chosen approach was using a known preamble which was inserted before each block of data symbols. It was shown that the preamble should have the property that scaled and shifted versions have no autocorrelation. The preamble constructed was based on DSSS spreading code adapted to suit the needs for the channel estimation in this report. Several other adaptions of the preambles were done in order to find which kind of preamble can optimally estimate the delays and Doppler rates. The channel coefficients belonging to these estimated paths were compensated for or estimated via a least squares. For the compensating a known pilot symbol was inserted in the block of data symbols.

Finally these two parts were combined in order to test whether which preamble delivers the best channel estimates combined for the construction of the equalizers. Several sequences and channel scenario’s were simulated using Matlab. The preamble constructed with a CAN sequence in combination with the full block equalizer is shown to provide near optimal BER, where no error encoding was used. The length of the chip code and the length of the CAN sequence had little influence on the performance. Hence a CAN sequence as chip code with no pulse modulated is the best choice for the bit error wise.
5.1 Possible Future work

The receiver introduced here shows good results, considering that the underwater sound channel is a very challenging environment, but there are still issues that can be improved and/or other aspects that need to be noted. Some

1. The receiver introduced here has been solely tested using MATLAB simulations. How this receiver works in practice is unknown and future research can maybe include designing a real world application and test the receiver in practice.

2. Another aspect is on this receiver is that it is quite computational and inefficient due to the long guard times and the guard time between preamble and symbols. Also due to the limited bandwidth underwater the CAN preamble is not suitable in practice and it may be interesting to search for certain pulse shapes which have low bandwidth but can provide good results.

3. Another possibility of dealing with Doppler effects it is possible that they can be ignored by using a transmission scheme that is Doppler invariant. Or where Doppler shifts are somehow always converted for to 1 single shift for all paths and subcarriers. This way compensating for the Doppler might be easier and less computational.
A.1 The CAN algorithm

In [34] cyclic algorithms are derived that minimize the integrated sidelobe level (ISL) of unimodular sequences. The sequence is denoted as \( x \) with length \( N \) and for each entry in \( x \) the modulus is equal to 1. The ISL is related to the auto correlation function \( r_k \):

\[
ISL = \sum_{k=1}^{N-1} |r_k|^2 \tag{A.1}
\]

A high ISL means that the sidelobes of the auto correlation function are high and it is preferred that the ISL is as small as possible. The CAN algorithm works by minimizing the ISL constraint in (A.1) under the constraint \(|x| = 1\) via the following way. Let’s introduce the following matrices and vectors \( A, v, f \) and \( z \) as follows:

\[
z = [x_1 \cdots x_N \ 0 \cdots 0]^T \tag{A.2}
\]

\[
a_p^* = [e^{-j\omega p} \cdots e^{-j2N\omega p}] \tag{A.3}
\]

\[
A^* = \frac{1}{\sqrt{2N}} \begin{bmatrix} a_1^* \\ \vdots \\ a_{2N}^* \end{bmatrix} \tag{A.4}
\]

\[
f = A^* z \tag{A.5}
\]

\[
\phi_p = arg(f_p), \ p = 1, \cdots, 2N \tag{A.6}
\]

\[
v = \frac{1}{\sqrt{2}} [e^{j\phi_1} \cdots e^{j\phi_{2N}}]^T \tag{A.7}
\]

The vector \( z \) is a vector of length \( 2N \) which contains the sequence \( x \) and is zero-padded with another \( N \) zeros. The matrix \( A \) is the \( 2N \times 2N \) unitary FFT matrix.
made from the elements of (A.3).

The criterion of minimizing the ISL is given as follows:

$$||A^*z - v||^2$$  \hspace{1cm} (A.8)

The cyclic local minimization of the ISL criterion as in (A.1) can then be summarized as follows:

1. Set \(x\) to some initial values which can be randomly generated or given by an already existing sequence.

2. Compute the \(\phi_p\) that minimizes the constraint as in (A.8) via (A.5) and (A.6) fixed at their most recent values.

3. Compute the new sequence \(x\) via (A.6) and (A.7) with the constraint \(|x| = 1\) for \(\phi_p\) fixed at their most recent values.

4. Repeat steps 2 and 3 until a until a prespecified stop criteria is reached \(||x^{(i)} - x^{(i+1)}|| < \epsilon\). Where \(x^{(i)}\) is the sequence obtained in iteration \(i\) and \(\epsilon\) is a predefined threshold such as \(10^{-3}\).

A.2 The algorithm for creating a ternary ZCZ sequence

In [39] an algorithm is proposed as to construct ternary ZCZ sequences. In this part of the appendix the algorithm used in order to construct the ternary sequences shall shortly be explained, for more details and examples see [39].

The algorithm can be summarized into the following steps:

1. The first step is to construct the seed matrix \(\Delta^{(0)}\).

   Starting with a binary or ternary complementary pair \([c_1, c_2]\), the seed matrix \(\Delta^{(0)}\) is then constructed as follows:

   \[
   \Delta^{(0)} = \begin{bmatrix} c_1 & \tilde{c}_2 \\ c_2 & -\tilde{c}_1 \end{bmatrix}
   \]  \hspace{1cm} (A.9)

   Where \(\tilde{c}_2\) denotes the reverse of the sequence \(c_2\) and \(-\tilde{c}_1\) denotes the reversed sequence whose \(i\)'th element is the negation of \(i\)'th element in sequence \(\tilde{c}_1\). Note that \(\Delta^{(0)}\) can be partitioned into two mutually orthogonal complementary sets \([c_1, c_2]\) and \([\tilde{c}_2, -\tilde{c}_1]\).

2. Recursively construct larger mutually orthogonal complementary sets based on the seed matrix \(\Delta^{(0)}\).

   Let \(\Delta^{(p)}\) denote the matrix of sequence with \(M^{(p)}\) rows and each row \(M^{(p)}\) sequences
of length \(N^{(p)}\). The result of the recursive procedure is a new matrix \(\Delta^{(p+1)}\) with \(2M^{(p)}\) rows and now each row containing \(2M^{(p)}\) sequences of length \(2N^{(p)}\):

\[
\Delta^{(p+1)} = \begin{bmatrix}
\Delta^{(p)} \otimes \Delta^{(p)} & -\Delta^{(p)} \otimes \Delta^{(p)} \\
-\Delta^{(p)} \otimes \Delta^{(p)} & \Delta^{(p)} \otimes \Delta^{(p)}
\end{bmatrix}
\]

(A.10)

The two matrices of sequences are interleaved by interleaving their corresponding sequences among each others. Where \(-\Delta^{(p)}\) denoted the matrix whose \(i, j\)’th entry is the negation of the \(i, j\)’th entry of \(\Delta^{(p)}\) and \(\otimes\) denotes interleaving as follows, \(a = [a_1, a_2, a_3, a_4]\) and \(b = [b_1, b_2, b_3, b_4]\) then \(a \otimes b = [a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4]\).

3. The final step is to construct a ternary ZCZ sequence set by reorganizing the mutually orthogonal set matrix \(\Delta^{(p)}\) as follows:

\[
\Delta^{(p)} = \begin{bmatrix}
\Delta^{(p-1)} \otimes \Delta^{(p-1)} & -\Delta^{(p-1)} \otimes \Delta^{(p-1)} \\
-\Delta^{(p-1)} \otimes \Delta^{(p-1)} & \Delta^{(p-1)} \otimes \Delta^{(p-1)}
\end{bmatrix}
\]

(A.11)

Here \(M = 2^{p+1}\) is the number of complementary sets in the mutually orthogonal complementary set matrix \(\Delta^{(p)}\) and \(\Delta^{(p)}_{i,j}, i = 1, 2...M, j = 1, 2...M\) are the complementary sequences with equal sequence length. By using the reorganized mutually orthogonal complementary set matrix \(\Delta^{(p)}\) it is possible to construct the ternary ZCZ sequence set \(T^{(p)}_{ZCZ}\) containing \(M = 2^{p+1}\) ternary ZCZ sequences in matrix form:

\[
T^{(p)}_{ZCZ} = \begin{bmatrix}
\Delta^{(p)}_{1,1} \circ Z_M \circ \Delta^{(p)}_{2,1} \circ Z_M & \cdots & \circ \Delta^{(p)}_{M,1} \circ Z_M \\
\Delta^{(p)}_{1,2} \circ Z_M \circ \Delta^{(p)}_{2,2} \circ Z_M & \cdots & \circ \Delta^{(p)}_{M,2} \circ Z_M \\
\vdots & \ddots & \ddots \\
\Delta^{(p)}_{1,M} \circ Z_M \circ \Delta^{(p)}_{2,M} \circ Z_M & \cdots & \circ \Delta^{(p)}_{M,M} \circ Z_M
\end{bmatrix}
\]

(A.12)

Where \(Z_M\) denotes the sequence with \(M\) zero elements and the notation \(a \circ b\) means concatenation of two sequences. Thus the \(j\)’th row of the matrix \(T^{(p)}_{ZCZ}\) is a ternary ZCZ sequence with length \(N^{(p)} = 4^{p+1}N^{(0)}\) where \(N^{(0)}\) is the length of the seed complementary pair in the first step.


