SUSTAINABILITY AND URBAN DENSITY
A DECISION BASED DESIGN APPROACH

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Abstract
This paper concerns the definition, construction and application of a decision based design model which able the integration of the allocation of a variety of urban land uses with the distribution of different urban densities, in particular of residential urban areas. Urban planning is, among others things about the spatial distribution of human activities and their physical facilities like buildings, roads, green areas etc. in amount, place and time over a well-defined area. Today, sustainable urban development and sustainability in urban areas are important issues in urban planning. The aspects related to these issues have to be taken in account when developing urban areas. One of these aspects is urban density. Nowadays, it is generally assumed (and accepted) that urban density is related to sustainability. New urban planning approaches, loosely based around new urbanism, are successfully reducing environmental impacts by altering the built environment to create and preserve sustainable cities which support sustainable transport. Residents in compact urban neighborhoods drive fewer miles, and have significantly lower environmental impacts across a range of measures, compared with those living in sprawling suburbs.

Keywords: Sustainability, built environment, urban density, urban design and decision making.

URBAN DENSITY

The concept of density in urbanism is frequently used to describe the relationship between an area and the number of certain entities in that area. These entities might be people, dwellings, services, or floor space. However, the simple fact that density is used in, for instance, design requirements, plan descriptions and communication between parties, does not mean that it is used correctly or to its full potential. One of the problems of defining density in operational terms is the relatively weak relationship between density and building type. The same density can be obtained with radically different building types, and the same type can be used to obtain different densities. It is important to make a distinction between urban density used to describe a built environment (descriptive use); and urban density used as a norm in the process of planning and designing the city (prescriptive, or normative, use). Prior to the 20th century, density in cities was merely a result of the complex process of city development. Building techniques, legal constraints, traditions, the requirements for economic profitability, etcetera determined the possible resulting densities. However, no conscious use was made of density. As a matter of fact, density as a concept in urban analysis and planning probably did not exist until the second half of the 19th century. During this period, high densities in industrializing cities
were argued to be one of the major causes of fires, disease and social disorder. Mainly through critical publications in England and Germany, the awareness of the problem grew among legislators and urban planners. As a result, planning controls were developed that prescribed *maximum* allowable densities (M. Berghauser Pont and P. Haupt, 2010).

Urban density can be expressed in many ways. A widely used measurement is the number of dwelling units per unit area (acre, hectare). This measurement gives only information about the number of dwellings, not about their size or the way they are grouped. More general and precise ways of measuring density are measurements based on built area or gross floor area, respectively named ground space index and floor space index.

Using these measurements in urban design processes cannot guarantee a good or bad urban area because they are only measurements about the ratio of built and non-built spaces and give no information about the activities and functions within these spaces, neither about their distribution. Therefore, the outcome of an urban design model based on only density factors (as the amounts of built and non-built areas) gives no information about contents of these spaces. Consequently, these types of models are not applicable in a context of social oriented decision making, as in urban planning.

In this paper we present the integration of an urban function or urban activity based model with an urban density based model.

The first part will give an overview of the definitions, differences and similarities of both function based and density based models, as well as definitions of the combined model. The second part describes the application of the combined model in a number of tests, including a final conclusion.

**THE BASIC MULTI ACTOR URBAN PLANNING MODEL**

We begin with an example for which a mathematical urban planning model can be constructed (Van Loon, 1998).

**The decision-making problem of a housing association**

A housing association wants to build a number of blocks of residential property and facility units (shops, school, social and cultural centre, etc.) on a particular site. The site covers 14,000 m$^2$. The association hopes to complete the project within 16 months. A block (construction time 2 months) covers 1,000 m$^2$, while a facility unit (construction time 1 month) covers 2,000 m$^2$. A residential block costs 8.10$^6$ Euros, and a facility unit costs 5.10$^6$ Euros; the overall budget is 80.10$^6$ Euros. It is not necessary to cover the entire site. A survey has been conducted among the future residents. This has revealed that they value housing blocks and facilities at a ratio of 5:3. The aim is to ensure that the future residents are as pleased with their neighborhood as possible.

This problem can be represented mathematically in a LP model. $X_1$ is the number of blocks of residential property and $X_2$ is the number of facility units. Two decision-makers are involved in this problem: the housing association and the future residents. The housing association decides what site area is to be built on, how long the building work will take, how much it will cost and sets out the timetable for the project. The future residents decide on their opinion of the houses and facilities. These give us the decision variables. The input variables are the total budget (80.10$^6$ Euros maximum) and the land available (14,000 m$^2$ maximum). These have been determined by the local authority within the constraints of its overall urban plan and the regulations governing its housing budget. The future residents want to see their views taken into account to the greatest possible extent, so $5X_1 + 3X_2$ must be maximized. The housing association wants to complete the project within 16 months and
stick to its decisions regarding construction costs, construction time and site area. These are
the goals, and they can be represented as follows:

\[
\text{MAX! } 5X_1 + 3X_2 \quad \text{(appreciation)}
\]

Sub:

\[
\begin{align*}
1,000X_1 + 2,000X_2 & \leq 14,000 \quad \text{(site area)} \\
2X_1 + X_2 & \leq 16 \quad \text{(construction time)} \\
8 \times 10^6 X_1 + 5 \times 10^6 X_2 & \leq 80 \times 10^6 \quad \text{(budget)} \\
X_1 & \geq 0 \\
X_2 & \geq 0
\end{align*}
\]

The simplex algorithm (a mathematical procedure which allows an LP model to be solved
with 2 or more unknown variables) can be used to find the mathematical solution to this
problem. Since the example has only two unknown variables, it can be solved using a simple
drawing. This can be explained quite simply and allows the mathematical solution to be
presented graphically. The problem facing the housing association is represented in Figure 1

\[\text{Figure 1: The solution space (shaded)}\]

The maximum value of the linear equation \(5X_1 + 3X_2\) (the objective function) must be found
within the shaded area. Consider the group of parallel lines \(5X_1 + 3X_2 = c\). The highest
possible value of \(c\) has to be obtained, within the constraints. This can be achieved when \(X_1 = 6\) and \(X_2 = 4\), because \(c = 42\). The best outcome is achieved with 6 housing blocks and 4
facility units (Figure 2).
This modeling of the decision problem faced by the housing association is represented in diagrammatic form in Figure 3.

The housing association and the future residents will undoubtedly continue negotiating their decisions and goals after this ‘initial’ solution has been found. Such negotiation is useful in order, for instance, to establish whether a change in the construction costs might better suit the preferences of the residents. Other, cheaper building materials could lower the costs, which might lead to a better distribution of houses and facilities.

Figure 3: The decision model for the problem faced by the housing association

The general structure of this model is as follows. This model is to select the values for the decision variables $x_1, x_2, \ldots, x_n$ so as to:

Maximize $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$$
\begin{align*}
\text{Maximise} \quad & Z = \sum_{j=1}^{n} c_j x_j \\
\text{subject to} \quad & \sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad \text{for } i = 1, 2, \ldots, m \\
\text{and} \quad & x_j \geq 0 \quad \text{for } j = 1, 2, \ldots, n.
\end{align*}

This is adopted as the \textit{standard form} for the linear programming problem. Any situation whose mathematical formulation fits this model is a linear programming model.

The function $Z$ being maximised, $c_1x_1 + c_2x_2 + \ldots + c_nx_n$, is called the \textit{objective function}. The \textit{decision variables} – the $x_j$ – are sometimes referred to as the \textit{uncontrolled} or \textit{endogenous} variables. The input variables – the $a_{ij}$, $b_i$, and $c_j$ – may be referred to as \textit{parameters} of the model or as the \textit{controlled} or \textit{exogenous variables}.

The restrictions are referred to as \textit{constraints}. The first $m$ constraints $b_1$, $b_2$, $\ldots$, $b_m$ (those with a function $a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n$ representing the total usage of resource $i$, on the left) are called functional constraints. The $x_j > 0$ restrictions are called non-negativity constraints. In this paper the non-negativity constraints of the variables $x_j$ and $x_{ij}$ will be implicitly assumed.

\section*{Allocation of Urban Activities to Space}

\subsection*{First extension of the basic model}

In urban planning not only the quantities of and the preferences for the resources (like land to be used, buildings, infrastructure) to be allocated play a role, but also the location of the resources in the urban space. (R. Binnekamp, e.a. 2006). With an extension of the basic multi-actor urban planning model – the linear programming model with negotiable constraints – we are able to model the allocation of urban activities to space (urban land use). In urban design and planning, a dominant spatial dimension of resources is the position of resources in two- and three-dimensional space. This position is commonly expressed in floor plans, land use plans, and three-dimensional models of buildings and their urban environments. In terms of allocation of resources, a floor plan is a proposal for allocation of architectural spaces to accommodate human activities such as living, shopping, eating, and office work: Which spatial layout of the resources fits the activities to be accommodated best, in accordance with stakeholders' wishes, goals, and constraints, and with the architectural style chosen?
If we define the activities as demand \((d)\) and the resources as supply \((s)\) we can represent this problem (which is called in Operations Research literature the transportation problem or the distribution problem) in an LP model as follows:

\[
\text{Minimise } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

subject to

\[
\sum_{j=1}^{n} x_{ij} \geq d_i \quad \text{for } i = 1, 2, \ldots, m
\]

\[
\sum_{i=1}^{m} x_{ij} \leq s_j \quad \text{for } j = 1, 2, \ldots, n
\]

In this model \(x_{ij}\) is the representation of an activity \(i\) in space \(j\). \(c_{ij}\) is the representation of the cost (expressed in money, energy, appreciation, and the like) of the realisation of activity \(i\) in space \(j\). This representation can be explained with two aspects of the relationship between activities and spaces as follows: Since in buildings and urban areas human activities are not fixed to one unique space – or in other words activities are spread out over more spaces, like rooms, auditoria, corridors, zones, areas – a design expresses, among a lot of other things, a spatial pattern of different architectural and urban spaces to fit a set of different activities allocated to the designed spaces. In the remainder of this paper the index \(i\) refers to an activity, \(j\) to a space or zone, \(k\) to a lot, \(m\) to number of activities, \(n\) to number of zones and \(p_j\) to the number of lot within zone \(j\).

The second aspect concerns the fact that most of the urban spaces are suited for more than one activity, but of course not all. This means that the designer can propose alternative arrangements of the activities required, for a given spatial arrangement of spaces. Also the other way around: for a given spatial arrangement of activities, alternative layouts of urban spaces may be proposed. By changing the input values of \(c_{ij}\), a representation of the design process on both aspects becomes available. With this mechanism, a designer can represent his pattern of possible activities in such a way that he can see how well this pattern fits the activities required.

While urban spaces may be suited for more than one activity, they are not necessarily suited for all activities due to technical constraints such as daylight, noise hindrance, permitted location in the building, or conceptual constraints such as structure of spaces and patterns of connections.

The model for this design problem (the limited distribution problem) can be formulated as follows:

\[
\text{Minimise } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

subject to

\[
\sum_{j=1}^{n} a_{ij} x_{ij} \geq d_i \quad \text{for } i = 1, 2, \ldots, m
\]

\[
\sum_{i=1}^{m} a_{ij} x_{ij} \leq s_j \quad \text{for } j = 1, 2, \ldots, n
\]

and
Due to the LP problem solving algorithm, $x_{ij}$ will be zero if $a_{ij} = 0$, and $x_{ij}$ will get a value greater than or equal to zero if $a_{ij} = 1$. This means that if the designer decides that space $s_j$ is not suited or otherwise not appropriate for activity $i$, he sets $a_{ij} = 0$ and automatically $x_{ij}$ becomes 0. In other words, using the zero and one value of $a_{ij}$, the designer uses the model to calculate the best allocation of activities to the designed pattern of spaces.

In the representation of the space allocation described above, it is assumed that the total demanded space for activities equals the total supplied space for the activities. In the beginning of a design process this is often not the case. In architectural design and urban planning, demand and supply are independent of each other. They are not fixed at the start of a design process. Designers propose spatial arrangements of spaces based on their ideas, style, and concepts. Of course, these proposals are not that far from the required spaces, but they are not equal. So, a design can give ideas for activities one was not thinking of. Similarly, a designer can discover that he does not yet have space for an activity which certainly should be in the building. The designers have to find the best fit. With two extensions to the above model, it is possible to cope with this design question.

Minimise $\quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to

\[ \sum_{i=1}^{m} a_{ij} x_{ij} - S_j = 0 \quad \text{for } j = 1, 2, \ldots, n \]

\[ \sum_{j=1}^{n} a_{ij} x_{ij} - D_i = 0 \quad \text{for } i = 1, 2, \ldots, m \]

\[ D_i \geq d_{\text{min}} \quad \text{for } i = 1, 2, \ldots, m \]

\[ D_i \leq d_{\text{max}} \quad \text{for } i = 1, 2, \ldots, m \]

\[ S_j \geq s_{\text{min}} \quad \text{for } j = 1, 2, \ldots, n \]

\[ S_j \leq s_{\text{max}} \quad \text{for } j = 1, 2, \ldots, n \]

and

\[ D_i \geq 0 \quad \text{for } i = 1, 2, \ldots, m \]

\[ S_j \geq 0 \quad \text{for } j = 1, 2, \ldots, n \]

\[ a_{ij} = \{0,1\} \quad \text{for } i = 1, 2, \ldots, m \quad \text{for } j = 1, 2, \ldots, n \]

where
$S_j$ Allocated space in zone $j$.
$D_j$ Allocated space for activity
$s_{\text{min}}_j$ Minimum available space in zone $j$.
$s_{\text{max}}_j$ Maximum available space in zone $j$.
$d_{\text{min}}_i$ Minimum demand for activity $i$.
$d_{\text{max}}_i$ Maximum demand for activity $i$.

**GROUND SPACE VERSUS FLOOR SPACE**

**Second extension of the basic urban planning model**

The above described model deals with the distribution of a number of activities (demand) to a number of available urban areas (supply) both expressed as surfaces. Within an urban context the available space is commonly assumed to be ground space. Consequently, the demand of space is also assumed to be ground space. However, in many cases the supply of as well the demand for (urban) space is expressed as floor space, for instance the supply of space for schools is expressed in floor space, which can be distributed to one or more stories.

In order to facilitate this floor space in the described model, a new object is introduced, called a *lot*. A lot can be considered as a universal object for describing all built.

Related to this model, the most important spatial properties of a lot are:

- the base area of the lot (ground space),
- the area of the lot to be built also called the footprint of the building (a footprint is the amount of space on a surface that something needs),
- the floor area, which can be distributed to several stories.

Between these properties a number of relations can be defined, expressed as ratios, called the Ground Space Index and Footprint Ratio.

The values of these ratios can be defined within a margin between a given minimum and maximum value.

**Ground Space Index**

The Ground Space Index defines the relation between the built and non-built area of the lot, expressed as a ratio.

\[
\frac{A_{\text{BLT}}}{A_{\text{LOT}}} \geq p_{\text{min\_a\_blt}} \cdot \frac{A_{\text{LOT}}}{A_{\text{BLT}}} \leq p_{\text{max\_a\_blt}} \cdot \frac{A_{\text{LOT}}}{A_{\text{BLT}}}
\]

where
- $A_{\text{LOT}}$ Total area of the lot.
- $A_{\text{BLT}}$ Total amount of built area of the lot (at ground level).
- $p_{\text{min\_a\_blt}}$ Minimum percentage of built area of the lot.
- $p_{\text{max\_a\_blt}}$ Maximum percentage of built area of the lot.

**Footprint Ratio**

The Footprint Ratio defines the relation between the area of the footprint built and total floor area, expressed as a ratio. Or, the percentage of the floor area at ground level.

\[
\frac{A_{\text{BLT}}}{A_{\text{GFA}}} \geq p_{\text{min\_a\_ftp}} \cdot \frac{A_{\text{GFA}}}{A_{\text{BLT}}} \leq p_{\text{max\_a\_ftp}} \cdot \frac{A_{\text{GFA}}}{A_{\text{BLT}}}
\]
where

\( A_{BLT} \)  Total amount of built area of the lot (at ground level).
\( A_{GFA} \)  Total amount of floor area.

\( p_{\text{min}_a_{ftp}} \)  Minimum percentage of floor area at ground level.
\( p_{\text{max}_a_{ftp}} \)  Maximum percentage of floor area at ground level.

The basic planning model can now be extended using these two constraints, first on the supply site, second on the demand site.

Supply of urban space

In order to be able to allocate the activities in several buildings, one or more lots are added to each resource or space (called zone), each lot having its own properties.

\[
S_j = S_{TOT\_LOT_j} \quad \text{for } j = 1, \ldots, n
\]
\[
S_{TOT\_LOT_j} = \sum_k S_{LOT_{jk}} \quad \text{for } j = 1, \ldots, n
\]
\[
S_{TOT\_BLT_j} = \sum_k S_{BLT_{jk}} \quad \text{for } j = 1, \ldots, n
\]
\[
S_{TOT\_GFA_j} = \sum_k S_{GFA_{jk}} \quad \text{for } j = 1, \ldots, n
\]

\[
S_{LOT_{jk}} \geq p_{\text{min}_s\_lot_{jk}} \cdot S_j \quad \text{for } j = 1, \ldots, n \quad k = 1, \ldots, p_j
\]
\[
S_{LOT_{jk}} \leq p_{\text{max}_s\_lot_{jk}} \cdot S_j \quad \text{for } j = 1, \ldots, n \quad k = 1, \ldots, p_j
\]
\[
S_{BLT_{jk}} \geq p_{\text{min}_s\_blt_{jk}} \cdot S_{LOT_{jk}} \quad \text{for } j = 1, \ldots, n \quad k = 1, \ldots, p_j
\]
\[
S_{BLT_{jk}} \leq p_{\text{max}_s\_blt_{jk}} \cdot S_{LOT_{jk}} \quad \text{for } j = 1, \ldots, n \quad k = 1, \ldots, p_j
\]
\[
S_{BLT_{jk}} \geq p_{\text{min}_s\_ftp_{jk}} \cdot S_{GFA_{jk}} \quad \text{for } j = 1, \ldots, n \quad k = 1, \ldots, p_j
\]
\[
S_{BLT_{jk}} \leq p_{\text{max}_s\_ftp_{jk}} \cdot S_{GFA_{jk}} \quad \text{for } j = 1, \ldots, n \quad k = 1, \ldots, p_j
\]

where

\( S_j \)  Total available area in zone \( j \).
\( S_{TOT\_LOT_j} \)  Total available lot area in zone \( j \).
\( S_{TOT\_BLT_j} \)  Total built area within all lots in zone \( j \).
\( S_{TOT\_GFA_j} \)  Total floor area within all lots in zone \( j \).

\( S_{LOT_{jk}} \)  Total available lot area of lot \( k \) in zone \( j \).
\( S_{BLT_{jk}} \)  Total built area within lot \( k \) in zone \( j \).
\( S_{GFA_{jk}} \)  Total floor area within lot \( k \) in zone \( j \).

\( n \)  Total number of zones
\( j \)  Index of a zone
Number of lots within zone \( j \).

Index of a lot within a zone \( k \).

Minimum percentage of lot area in zone \( j \).

Maximum percentage of lot area in zone \( j \).

Minimum percentage of built area related to lot area within lot \( k \) in zone \( j \).

Maximum percentage of built area related to lot area within lot \( k \) in zone \( j \).

Minimum percentage of footprint area related to floor area within lot \( k \) in zone \( j \).

Maximum percentage of footprint area related to floor area within lot \( k \) in zone \( j \).

**Demand of urban space**

The demand for space for activities is now expressed in floor area instead of ground space.

\[
\begin{align*}
D_{\text{GFA}_i} &\geq d_{\text{min}\_\text{gfa}_i} & \text{for } i = 1, \ldots, m \\
D_{\text{GFA}_i} &\leq d_{\text{max}\_\text{gfa}_i} & \text{for } i = 1, \ldots, m
\end{align*}
\]

where

\( D_{\text{GFA}_i} \) Total allocated floor area of activity \( i \).

\( d_{\text{min}\_\text{gfa}_i} \) Minimum demand for floor area activity \( i \).

\( d_{\text{max}\_\text{gfa}_i} \) Maximum demand for floor area activity \( i \).

**Allocation of demand to supply**

The allocation in this extended model concerns the distribution of floor space for activities to the lots of each zone.

\[
\begin{align*}
\sum_{i=1}^{m} a_{ijk} x_{\text{gfa}_{ijk}} - S_{\text{GFA}_{jk}} &= 0 & \text{for } j = 1, \ldots, n \ k = 1, \ldots, p_j \\
\sum_{i=1}^{n} a_{ijk} x_{\text{gfa}_{ijk}} - D_{\text{GFA}_i} &= 0 & \text{for } i = 1, \ldots, m \ k = 1, \ldots, p_j \\
a_{ijk} &= \{0,1\} & \text{for } i = 1, 2, \ldots, m \ j = 1, 2, \ldots, n \ k = 1, \ldots, p_j
\end{align*}
\]

where

\( S_{\text{GFA}_{jk}} \) Total supply of floor area within lot \( k \) in zone \( j \).

\( D_{\text{GFA}_i} \) Total allocated floor area of activity \( i \).

\( a_{ijk} \) Indication whether activity \( i \) may be allocated to lot \( k \) of zone \( j \).

\( x_{\text{gfa}_{ijk}} \) Allocated floor area for activity \( i \) in lot \( k \) of zone \( j \).

\( i, j, k, m, n, p_j \) As above

The described model only allocates activities to floor areas within lots and does not provide in the allocation of "non-built" activities, like pavement or parks. This can be solved by considering one lot as being open space. Its floor area is considered as non-built space and the values of both its Ground Space Index and Footprint Index are fixed to one. Also, the floor areas of the activities which will be allocated to these non-built lots are considered to be open space.
URBAN DENSITY AND MULTI ACTOR DECISION MAKING

Third extension of the basic planning model
The third extension of the basic planning model is urban density. As stated in the introduction, the concept of density in urbanism is frequently used to describe the relationship between an area and the number of certain entities in that area, for instance people, dwellings or floor space. This means that density is a feature of a set of objects, in case the urban system. And features are always expressions of relations between objects.
In order to be widely applicable urban entities are chosen that do not depend on the specific urban use of space, like dwellings, offices, shops. Therefore, entities based on built space and non-built space are used as entities of densities.

Building Intensity (FSI)
FSI reflects the building intensity of a base land area independently of the programmatic composition and is calculated as follows:

\[ FSI = \frac{A_{GFA}}{A_{BAS}} \]

where
\( A_{GFA} \) Total gross floor area.
\( A_{BAS} \) Total base land area.

Coverage (GSI)
GSI, or coverage, demonstrates the relationship between built and non-built space of a base land area and is calculated as follows:

\[ GSI = \frac{A_{BLT}}{A_{BAS}} \]

where
\( A_{BLT} \) Total built area (footprint).
\( A_{BAS} \) Total base land area.

The formula for \( GSI \) can be rewritten as.

\[ A_{BLT} = GSI \cdot A_{BAS} \]

In this form it can be added to the LP-model if the value of \( GSI \) is fixed and known in advance. Both \( A_{BLT} \) and \( A_{BAS} \) are assumed to be endogenous variables. Unfortunately, in case the variable \( GSI \) is introduced as an endogenous variable the LP-model becomes non-linear because of the multiplication of two endogenous variables within this constraint. However, in case \( GSI \) is only used in this constraint and is bounded between a known minimum value \( gsi_{min} \) and known maximum value \( gsi_{max} \), the variable \( GSI \) can be eliminated from the LP-model without violating the boundary constraints by replacing the constraint by two other ones.
The endogenous variable $GSI$ has now been eliminated from the LP-model. Its actual value can be calculated afterwards, i.e. after (successfully) solving the LP-model. In a similar way the endogenous variable $FSI$ can be eliminated from the LP-model and be replaced by two constraints defining a minimum value $f_{si_{\text{min}}}$ and maximum value $f_{si_{\text{max}}}$.

$$A_{GFA} \geq f_{si_{\text{min}}} \cdot A_{BAS}$$
$$A_{GFA} \leq f_{si_{\text{max}}} \cdot A_{BAS}$$

These basic constraints can easily be applied to both the supply and demand variables in the LP-model. For instance, the overall $FSI$ and overall $GSI$ of a particular zone $j$ can be defined as follows (the first lot having index 1 is considered to be a non-built lot):

$$\sum_{k=2}^{p_j} S_{BLT_{jk}} \geq gsi_{\text{min}_j} \cdot S_{TOT\_LOT_j}$$
$$\sum_{k=2}^{p_j} S_{BLT_{jk}} \leq gsi_{\text{max}_j} \cdot S_{TOT\_LOT_j}$$

$$\sum_{k=2}^{p_j} S_{GFA_{jk}} \geq f_{si_{\text{min}_j}} \cdot S_{TOT\_LOT_j}$$
$$\sum_{k=2}^{p_j} S_{GFA_{jk}} \leq f_{si_{\text{max}_j}} \cdot S_{TOT\_LOT_j}$$

**EXAMPLE.**

The described model is tested in a case: the development of a residential area of 100 hectares including space for green areas, pavement and basic facilities like schools, shops, medical and cultural centres.

The municipality of the city to which this area belongs, wants to achieve a dwelling density between 45 and 70 dwellings per hectare. The dwellings to be built are divided into twelve possible dwelling types. For each dwelling type (average) values are given for the size of its parcel, the size of the footprint (built area of the parcel), the dwelling size and the number of dwellings per parcel. In order to create a variety of dwellings for each dwelling type minimum and maximum percentages are given according to the housing needs.

Three different models have been applied. In all models the total number of dwellings has been maximised. As an output, the GSI and FSI of the area for only housing are calculated. The area for housing comprises the area of the lots, the space needed to access these houses, green area nearby the housing blocks, small parks (e.g. childrens playground) at a short distance of the housing blocks and parking lots for the inhabitants and their visitors.
Area needed for other facilities like schools, shops, large parks, water are not included in this calculation.

First, the municipality wanted to determine the maximum number of dwellings that can be built without dividing the area into subareas. The result is 6,219 dwellings. The GSI of the whole area for housing is about 0.62; the floor space index is about 1.1.

Table 1 outlines the results of the dwelling distribution of each model as well as the dwelling density, the total GSI and total FSI of the area for housing.

<table>
<thead>
<tr>
<th>Dwelling type</th>
<th>Basic (no zoning)</th>
<th>Zoning without density constraints</th>
<th>Zoning with Density constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>%</td>
<td>Number</td>
</tr>
<tr>
<td>Terrace houses</td>
<td>622</td>
<td>10%</td>
<td>536</td>
</tr>
<tr>
<td>Apartments youngsters</td>
<td>933</td>
<td>15%</td>
<td>803</td>
</tr>
<tr>
<td>Apartments elderly</td>
<td>933</td>
<td>15%</td>
<td>640</td>
</tr>
<tr>
<td>Terrace houses small</td>
<td>311</td>
<td>5%</td>
<td>268</td>
</tr>
<tr>
<td>Terrace houses</td>
<td>311</td>
<td>5%</td>
<td>431</td>
</tr>
<tr>
<td>Semi-detached medium</td>
<td>0</td>
<td>0%</td>
<td>0</td>
</tr>
<tr>
<td>Semi-detached family</td>
<td>311</td>
<td>5%</td>
<td>268</td>
</tr>
<tr>
<td>Apartments seniors</td>
<td>933</td>
<td>15%</td>
<td>803</td>
</tr>
<tr>
<td>Semi-detached family</td>
<td>311</td>
<td>5%</td>
<td>268</td>
</tr>
<tr>
<td>Apartments</td>
<td>933</td>
<td>15%</td>
<td>803</td>
</tr>
<tr>
<td>Large Apartments</td>
<td>311</td>
<td>5%</td>
<td>268</td>
</tr>
<tr>
<td>Villas</td>
<td>311</td>
<td>5%</td>
<td>268</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6219</strong></td>
<td></td>
<td><strong>5356</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density (Calculated)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>GSI</td>
<td>0.6265</td>
<td>0.6277</td>
<td>0.6639</td>
</tr>
<tr>
<td>FSI</td>
<td>1.1136</td>
<td>1.1257</td>
<td>1.0495</td>
</tr>
<tr>
<td>Dwellings per hectare</td>
<td>62</td>
<td>54</td>
<td>47</td>
</tr>
</tbody>
</table>

In order to achieve a more the whole area has been divided into six subareas called zones. For each zone, one can define which dwelling types *may* be built in that zone. The two central zones, numbered 4 and 5, are reserved for other activities (education, leisure, shopping) primarily meant for the inhabitants of this area. Table 2 shows to which zone a dwelling type may be allocated.

<table>
<thead>
<tr>
<th>Possible allocation of dwelling types</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
<th>Zone 4</th>
<th>Zone 5</th>
<th>Zone 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dwelling type</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terrace houses</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apartments youngsters</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apartments elderly</td>
<td>*</td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terrace houses small</td>
<td>*</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terrace houses medium</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-detached medium</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-detached family</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Apartments seniors * * *  
Semi-detached family * 
Apartments * 
Large Apartments * 
Villas * 

Taking into account these allocation constraints, the total number of dwellings that can be built, reduces to 5356 (see table 3). In comparison with the first model, both the FSI and GSI (for housing) of the whole area do not change, but there are significant differences between the zones. The GSI varies between 0.55 and 0.73, the FSI between 0.90 and 2.12.

Table 3: Allocation of dwellings to zones without density constraints

<table>
<thead>
<tr>
<th>Dwelling type</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
<th>Zone 4</th>
<th>Zone 5</th>
<th>Zone 6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terrace houses</td>
<td>536</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>536</td>
</tr>
<tr>
<td>Apartments youngsters</td>
<td></td>
<td>803</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>803</td>
</tr>
<tr>
<td>Apartments elderly</td>
<td></td>
<td></td>
<td>640</td>
<td></td>
<td></td>
<td></td>
<td>640</td>
</tr>
<tr>
<td>Terrace houses small</td>
<td></td>
<td></td>
<td></td>
<td>268</td>
<td></td>
<td></td>
<td>268</td>
</tr>
<tr>
<td>Terrace houses medium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>431</td>
<td></td>
<td>431</td>
</tr>
<tr>
<td>Semi-detached medium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>268</td>
</tr>
<tr>
<td>Semi-detached family</td>
<td>268</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>268</td>
</tr>
<tr>
<td>Apartments seniors</td>
<td>413</td>
<td>390</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>803</td>
</tr>
<tr>
<td>Semi-detached family</td>
<td></td>
<td>268</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>268</td>
</tr>
<tr>
<td>Apartments</td>
<td></td>
<td></td>
<td></td>
<td>803</td>
<td></td>
<td></td>
<td>803</td>
</tr>
<tr>
<td>Large Apartments</td>
<td>268</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>268</td>
</tr>
<tr>
<td>Villas</td>
<td>268</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>268</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1648</strong></td>
<td><strong>803</strong></td>
<td><strong>1194</strong></td>
<td><strong>-</strong></td>
<td><strong>-</strong></td>
<td><strong>-</strong></td>
<td><strong>1711</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density (Calculated)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>GSI</td>
<td>0.6413</td>
<td>0.5526</td>
<td>0.6654</td>
<td></td>
<td></td>
<td>0.7348</td>
<td>0.6277</td>
</tr>
<tr>
<td>FSI</td>
<td>1.0175</td>
<td>0.9002</td>
<td>2.1287</td>
<td></td>
<td></td>
<td>1.2133</td>
<td>1.1257</td>
</tr>
<tr>
<td>Dwellings per hectare</td>
<td>55</td>
<td>32</td>
<td>92</td>
<td></td>
<td></td>
<td>101</td>
<td>54</td>
</tr>
</tbody>
</table>

The municipality does not want the FSI to exceed 2.0, except the one of zone 6. This zone is considered as an extension of the neighbouring residential area. Therefore, its GSI and FSI must correspond to the ones of this neighbouring residential area. Consequently, the built area of zone 6 is set to at least 70% and its FSI is set to a maximum value of 1.0. For similar reasons, the GSI in zone 1 may not be less than 0.67.

These extra requirements cause the total number of dwellings that can be built decreases to 4678 (see table 4). Because the FSI in zone 3 may not exceed 2.0, it is not possible to built dwellings in this zone (in favour of other facilities). In order to agree the additional constraints more large dwellings will be realised in this new residential area.

Table 4: Allocation of dwellings to zones with density constraints

<table>
<thead>
<tr>
<th>Dwelling type</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
<th>Zone 4</th>
<th>Zone 5</th>
<th>Zone 6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terrace houses</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apartments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large Apartments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Villas</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terrace houses</td>
<td>468</td>
<td>468</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apartments youngsters</td>
<td>151</td>
<td>550</td>
<td>702</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apartments elderly</td>
<td>521</td>
<td>181</td>
<td>702</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terrace houses small</td>
<td>0</td>
<td>357</td>
<td>357</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terrace houses medium</td>
<td>468</td>
<td>468</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-detached medium</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-detached family</td>
<td>265</td>
<td>265</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apartments seniors</td>
<td>702</td>
<td>702</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semi-detached family</td>
<td>234</td>
<td>234</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apartments Large Apartments</td>
<td>547</td>
<td>547</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Villas</td>
<td>234</td>
<td>234</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2575</td>
<td>1015</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1089</td>
<td>4678</td>
</tr>
</tbody>
</table>

**Density (Calculated)**

| GSI | 0.6700 | 0.5381 | 0.7000 | 0.6639 |
| FSI | 1.1830 | 1.0630 | 1.0000 | 1.0495 |
| Dwellings per hectare | 86 | 41 | 78 | 47 |

**Input Data**

| Min GSI | 0.67 | 0.70 |
| Min FSI | | |
| Max FSI | 2.00 | 2.00 | 2.00 | 1.00 |

**CONCLUSIONS**

In spite of the practical advantages of the concept of urban density in urban planning, critics have argued – especially since the revolt in the 1970s against the quantitative methods of modernist planning – that the use of density for anything but statistical purposes is questionable, as it is perceived as a too elastic concept that poorly reflects the spatial properties of an urban area. Professionals, as well as researchers, hold the opinion that measured density and other physical properties are independent of each other: Very different physical layouts can have similar measured densities. (M. Berghauser Pont and P. Haupt, 2010)

Often people confuse density with building type and assume, for example, that detached houses are lower density than attached housing types. While this is generally true it is not always the case.

Besides the argued lack of relationship between density and form, density is also considered with suspicion because of the confusion regarding the definition of plan boundaries and the scale at which these are measured. Although it is common to distinguish between net and gross density, the definitions vary from place to place parcel density, net-net density, net and gross residential density, general density and community density are some of the units of measure used.
Since the urban planning model we described, represent building types as well as density parameters it is possible to decide on both aspect of urban areas in one

**LITERATURE**


