Stellingen

1. De Nederlandse taal is een feilbaar instrument voor het uitdrukken van stellingen.

2. Het begrip lokalisatie uit de holografische interferometrie is onbelangrijk voor spikkelinterferometrie.

3. De term TV-holografie is een onjuiste aanduiding voor elektronische spikkelinterferometrie.

4. De interferometrische meetmethode zoals beschreven in dit proefschrift zal holografische interferometrie grotendeels overbodig maken.

5. Beeldbewerking is het vervormen van een projektie van de werkelijkheid.

6. De kundigheid der natuur omvat meer dan de natuurkunde.


8. Er wordt steeds meer geschreven en minder gelezen.

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Phase shifting speckle interferometry

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Chapter 1

General introduction

1.0 Introduction

This thesis describes the development of an interferometric measuring method and its application in the measurement of mechanical properties of a solid object, like shape, deformation and strain.

The possibility of using the wave character of light to measure macroscopic quantities has been recognized a long time ago. A very early observation of the interference of light was made by Isaac Newton: when two parallel smooth surfaces, that are close to each other, are illuminated by white light, colored rings become visible. Much later Young performed an experiment that gave a more direct proof of the wave character of light: two parallel slits are illuminated to produce a set of parallel interference lines on a screen. From that time onwards numerous optical configurations employing the interference phenomenon were developed by scientists and inventors like Michelson, Fizeau, Mach and Zehnder, Fabry and Pérot. Most of these classical interferometers appeared to be excellent tools for the optical industry to determine the quality of products like mirrors and lenses, leading to considerable manufacturing improvement. Specularly reflecting or smooth transparent objects could be compared interferometrically with other well known surfaces or objects. The urge for interferometric measurements on diffusely reflecting or opaque objects was relieved by the development of holography in the early 1960s and its interferometric applications in the late 1960s. In 1969 a related technique was discovered: speckle interferometry. Both techniques appeared capable of providing full field data on the surface displacement vectors or on the shape of the object under test.
Since that time, the science of optical interferometry has advanced enormously. Stimulations for this came from the availability of coherent laser light and the introduction of digital processing of the measured data by computers, allowing interferograms to be digitized and analyzed using image processing techniques. The first challenge was the accurate determination of the phase difference between the object beam and the reference beam of the interferometer, in each detector point. The second one was to relate this phase to the actual purpose of the measurement: the deformation, strain, shape or vibrational state of the object. Both problems have been tackled in the research described in this thesis.

Although the principle of speckle interferometry was established two decades ago, obtaining accurate quantitative results from the interference patterns remained extremely laborious until recently. This problem was solved in 1985, when the theory of phase measurement using several phase-shifted interference patterns was first applied to speckle interferometry. Investigating the full potential of this new measuring technique called for a combination of optical interferometry and digital image processing: digital phase shifting speckle interferometry. Optimizing the speckle interferometer and applying new image processing techniques led to a considerable improvement of the measuring accuracy. The availability of quantitative data allowed for digital processing to extract the desired information. During the development of the measuring system, the interaction with the field of mechanics became stronger: a numerical technique like the Finite Elements Method was imported from that field to verify measurements.

This chapter will present the basic principles of holographic and speckle interferometry (Sections 1.1 and 1.2), followed by two sections introducing several methods for producing quantitative data from interference patterns: single and multiple interferogram analysis (Sections 1.3 and 1.4). A discussion of the advantages and disadvantages of various techniques, in Section 1.5, precedes a section that summarizes fields in which speckle interferometry is being applied at present and new fields where it is expected to be useful (Section 1.6).
1.1 Holographic interferometry

One of the most important applications of holography is holographic interferometry. Holographic interferometry was accidentally discovered in the autumn of 1964 by Robert L. Powell and Karl A. Stetson, while they were trying to find out why some of the holograms they recorded were of poor quality. The principle of holographic interferometry is, that a wavefront emanating from an object can be made to interfere with a wavefront from the same object recorded at an earlier time. In this way dark areas in space are generated where the relative phase change amounts to an odd multiple of \( \pi \). Holography offered the possibility to apply the accurate measuring technique of interferometry to diffusely reflecting and opaque objects. In holographic interferometry the two interfering wavefronts are usually generated by the same object in two different states, while in classical interferometry one object is compared to another.

Three major techniques in holographic interferometry are: the double exposure, the real time and the time average techniques. A short description of these techniques will be given in the following lines. The \textit{double exposure} technique implies taking two holographic recordings of the object on the same photosensitive plate. During reconstruction two interfering images of the object will be formed. If the object has been deformed or rotated in between the two recordings, dark areas will occur where the wave functions of the image-forming beams have a phase difference of an odd multiple of \( \pi \). In the \textit{real time} technique a dynamic interference pattern is obtained by taking one holographic recording with the object in its initial state, and illuminating the object in its deformed state during reconstructing. In this way the interference will occur between the beam forming the recorded image and that forming the live image, resulting in an interference pattern that provides real time information of the object state. The \textit{time average} technique is based upon taking a holographic recording of a vibrating object. For a long exposure time, this can be considered as an infinite number of recordings of the object in different states. Bright areas will occur in nodes and in object points with a vibrational amplitude corresponding to a phase of a multiple of \( \pi \) at the detector. The intensity of this fringe pattern can be described by a
squared zero-order Bessel function, with its argument proportional to the vibrational amplitude. Another technique takes one holographic recording of the stationary object, and produces a dynamic interference pattern by illuminating the vibrating object and simultaneously reconstructing the beam of the stationary object. This variant produces a fringe pattern that can be described by a zero-order Bessel function to the first power.

Especially in mechanical engineering, these basic techniques have proved to be useful during the past two decades. Similar techniques can be applied if speckle interferometry, in stead of holographic interferometry, is used.

1.2 Speckle interferometry

In July 1969 John Leendertz showed how to perform interferometry on diffusely reflecting objects without the need of recording a hologram: speckle interferometry was born. Like holographic interferometry, speckle interferometry is a non-contact measuring technique that can be used to determine the displacement at points on the surface of a diffusely reflecting object with an accuracy in the order of a fraction of the wavelength of the light used in the interferometer. In contrast with holographic interferometry this technique does not rely upon the interference of two wavefronts emanating from the same object, but on the correlation between the intensity distributions of two speckle interference patterns. Subtraction and squaring of the intensities of these patterns (which can be performed photographically or electronically) produces a fringe pattern from which the phase change can be derived by determination of the fringe extrema or zero crossings followed by interpolation of the phase data. In case of electronic processing, the technique is referred to as electronic speckle pattern interferometry (ESPI), which nowadays is an established measuring technique. This technique produces fringe patterns that are comparable to those generated by holographic interferometry, but with speckle noise added. Due to this noise the analysis of these fringe patterns is more complicated.
Most techniques used in holographic interferometry (Section 1.2) can also be applied in speckle interferometry\textsuperscript{14,15}. For vibration analysis the time-average technique, with a recording of the \textit{stationary} object as a reference, can be used\textsuperscript{16}. Due to the squaring of the intensities during processing, the intensity of the resulting fringe pattern can be described by a squared Bessel function, analogous to the holographic time-average technique.

1.3 Single interferogram analysis

Two digital image processing algorithms that provide numerical phase data from a single interferogram will be discussed in this section. There exist several other ways to analyze single interferograms, but they usually require a skilful operator if accuracy is needed. A well known method in the optical workshop is the measurement of the deviation of the interferometric fringes from straight lines to estimate wavefront aberrations. Another is the determination of fringe distances and fringe directions. For qualitative analysis the interferogram is usually judged by an expert, looking for irregularities or discontinuities in the fringe pattern. The algorithms that will be reviewed below have in common that they produce an estimate of the phase difference between the two light beams that formed the interference.

The first algorithm will be referred to as intensity analysis, and is based upon finding the extremes in the intensity of the interferogram\textsuperscript{17,18,19,20,21,22,23,24}. Each line of extreme values marks points of equal phase. Two adjacent lines indicate a phase difference of $\pi$ if all extremes have been detected or a phase difference of $2\pi$ if only the maximums or the minimums in the intensity have been detected. In order to enable an accurate determination of the position of these lines, the digitized interferogram has to be normalized first. This implies corrections for spatial variations in the background intensity and in the intensity modulation (being the amplitude of the sinusoidal intensity variation), using existing image processing algorithms. Next, a threshold is introduced, yielding a binary fringe pattern image. Skeletonizing provides a line pattern that is an estimate of the positions of the intensity
extremes in the original interferogram. To determine the relative fringe order, knowledge about the direction of the slope of the phase is needed. In general, the operator will have to indicate the slope direction at relevant positions in the binary image and the phase values in each point of the lines in the image are then computed. Finally, a phase surface fit can be performed to determine the phase values for points in between those lines.

The second algorithm for single interferogram analysis is the Fourier-transform method\textsuperscript{25,26}. This method requires a spatial carrier frequency in the original interferogram, that has to be significantly higher than the spatial frequencies that correspond to the phase difference, the background intensity and the intensity modulation. Applying a Fast Fourier Transform to the digitized interferogram then yields three separate spectra. One, positioned close to the origin of the frequency domain, is related to the background intensity in the interferogram, and is usually neglected. Either of the other two spectra contains information about the intensity modulation and the phase. Translation of one of those spectra towards the origin, followed by application of an inverse Fourier transform and calculation of its complex logarithm, yields an imaginary part equal to the phase modulo $2\pi$. The phase can also be retrieved by taking the arctangent of the ratio of the imaginary and the real part which result from the inverse Fourier transform.

If either of these methods is applied to ESPI fringe patterns (see Section 1.2), additional image processing techniques will be indispensable to suppress the speckle noise in the original interferogram.

1.4 Phase shifting interferometry

The basic idea of the algorithms for single interferogram analysis discussed in the previous section is the removal of the disturbing variations in the background intensity and the intensity modulation for accurate phase determination. Another way to achieve this is offered by the phase shifting technique\textsuperscript{27,28,29,30,31}, also known as quasi-heterodyne interferometry\textsuperscript{32,33}, fringe-scanning interferometry\textsuperscript{34}, phase stepping
interferometry\textsuperscript{35} or electro-optic interferometry\textsuperscript{36}. The phase shifting method uses two or more interference patterns, from which a system of solvable equations in each point of the recorded image is deduced\textsuperscript{37}. This can be achieved by digitizing the intensities of those interference patterns, which differ because the phase difference between the two interfering light beams is shifted over a fixed amount between two subsequent recordings. If the phase shift is known accurately, at least three unknown variables remain (the phase, the background intensity and the intensity modulation); if not, the problem contains at least four unknown variables. In both cases the system of equations can be solved if the number of digitized interference patterns is at least equal to the number of variables. Thus, the phase modulo $2\pi$ can be determined.

Phase shifting can also be applied in speckle interferometry. In the case of application to ESPI fringe patterns (see Sec. 1.2), additional image processing will be needed to suppress noise\textsuperscript{38}. The phase shifting technique can be applied to speckle interference patterns directly too: \textit{digital phase shifting speckle interferometry}\textsuperscript{35,39,40}. In this case the technique has to be applied both before and after the deformation of the object. Subtraction of the calculated phase values then gives the phase change modulo $2\pi$ related to the deformation. Digital phase shifting speckle interferometry is based upon speckle phase correlation, whereas ESPI is based upon speckle intensity correlation.

1.5 Comparison of methods

The major difference between holographic and speckle interferometry is the way in which the fringe pattern is formed. In holographic interferometry usually a photosensitive plate is used to record a wavefront, which is reconstructed afterwards in order to enable interference of two wavefronts emanating from the same object. In speckle interferometry (ESPI) the fringe pattern generally arises by electronical processing of two subsequent video recordings. An advantage of holographic interferometry over ESPI is the low noise level of the resulting fringe pattern. A disadvantage is often the need for chemical developing and fixing of the
photosensitive layer and, in the case of the real time technique, the need for repositioning the hologram very accurately in the interferometer. Another disadvantage is the fringe localization phenomenon, which is relatively unimportant in speckle interferometry. In ESPI some electronical processing is needed to provide the fringe pattern. Considering the present state and availability of video digitizing and processing hardware, this is hardly a disadvantage anymore. In the case of vibration analysis using the time average technique, holographic interferometry provides results that are significantly better than those from speckle interferometry. This is due to the fact that the visibility of the fringes described by a squared Bessel function decreases rapidly in the presence of speckle noise.

Comparing single interferogram analysis to phase shifting interferometry, the main difference is that the latter technique entails a direct relation between the intensities in the interference patterns and the phase in each detector point. A second advantage over intensity analysis is that there is no need for a priori knowledge concerning the slope direction of the phase in the interference pattern. Also considering its higher accuracy, the benefits of phase shifting interferometry will in most cases outweigh the disadvantage of the need for some phase shifting device in the optical system. On the other hand, many single interferograms are already available and would gain value if they could be analyzed numerically. Furthermore, many institutions own interferometric systems that are not yet capable of employing the phase shifting technique. Concluding it can be stated that much research has still to be done concerning single interferogram intensity analysis. Considering the developments in the calculating speed of computer systems, the Fourier transform method certainly has a promising future. A disadvantage of this technique is the need for a spatial carrier frequency, making it unsuitable for many existing fringe patterns.

For accurate quantitative shape or deformation measurement, that needs to be fast and fully automated, phase shifting speckle interferometry is superior to holographic interferometry or ESPI, combined with single interferogram analysis.
1.6 Applications

The interferometric techniques described in the previous sections can be used for a variety of purposes, such as measurement of deformation\textsuperscript{42}, strain, shape and vibration\textsuperscript{43,44,45,46} of mechanical components as well as detection of defects\textsuperscript{47,48,49}, cracks, disbonds and inclusions\textsuperscript{50} in materials\textsuperscript{51,52,53}. Although some applications only require a qualitative measuring technique, a quantitative technique like digital phase shifting speckle interferometry offers advantages when high accuracy or numerical processing of the data is needed, e.g. for the determination of spatial derivatives of the displacement in the case of strain analysis.

Mechanical engineering is the major industrial area where holographic and speckle interferometry are used presently. Subdivisions of this field are non-destructive testing and mechanical design. The former benefits mainly from qualitative analysis, whereas in the latter quantitative analysis is often required. Phase shifting speckle interferometry, in particular, is expected to become a useful tool in both disciplines.

Interferometric techniques can also be useful in areas such as medical science\textsuperscript{43}, art preservation\textsuperscript{42}, acoustical noise reduction\textsuperscript{44} and gas and heat distribution and flow\textsuperscript{22}. Holographic interferometry is occasionally applied in those areas, mostly in an experimental stage. Speckle interferometry will become more important and will stimulate the application of interferometry in practice.

Apart from its possible scientific value, this investigation also endeavors to decrease the existing gap between mechanical and optical engineering, without suggesting that other areas where speckle interferometry has a potential, such as the medical science, do not deserve special attention as well.
1.7 References

Chapter 2

Phase shifting speckle interferometry

2.0 Introduction

The phase shifting technique for direct phase measurement has been used in combination with almost every type of interferometer\textsuperscript{1,2}. This method shifts the phase of one beam in the interferometer relative to the other beam, recording the interference pattern at several different phase shifts. From the recordings taken at appropriate intervals, the phase difference at the detector can be calculated. When applying this technique to speckle interferometry\textsuperscript{3,4,5,6} the result of a phase measurement is governed by the randomly distributed phase in the speckle field, which is usually of no interest. The useful feature of phase shifting speckle interferometry is the ability of accurate determination of the phase change in the speckle field. This thesis describes the application of this method to deformation and shape measurement on solid objects. The application to vibration measurement\textsuperscript{7,8} is not within the scope of this investigation.

In this chapter the general theory of the phase shifting technique will be presented (Section 2.1), followed by an overview of the relevant statistics associated with the speckle phenomenon and a discussion of the consequences for the application of the phase shifting technique (Section 2.2). The subsequent three sections give the theory of the determination of object displacement, shape and surface strain from measured phase data (Sections 2.3, 2.4 and 2.5). The final section contains a concise introduction in elementary image processing as well as a presentation of new image processing techniques, which have been developed especially for the analysis of phase shifted speckle interference patterns.
2.1 Theory

Although it is not essential that the phase shifts between subsequently recorded interference patterns are equal\(^9\), phase shifting interferometers are usually utilized in that way\(^1,10,11,12\). This section will describe both the general phase shifting technique as well as some derived techniques, that are based upon the assumption of equal phase shifts. The interference patterns can either be recorded while the phase is shifting or after the phase has been shifted to the desired value. In the latter case the method is usually referred to as the phase stepping technique. The first, more general, phase shifting technique will be described here. *Perfect coherence and equal polarization states of the object and reference beams in the interferometer will be assumed throughout this thesis.* Significant parameters are the values of the phase shifts and the number of recordings. The required minimum number of recordings is three if the intensity \(I(x,y)\) of the interference pattern in the detector plane can be described as

\[
I(x,y) = I_B(x,y) + I_M(x,y) \cos(\varphi(x,y)) \, ,
\]

which is the case if the object and reference beam in the interferometer are perfectly coherent and have the same polarization states. This expression contains three unknowns: the background intensity \(I_B(x,y)\), the modulation intensity \(I_M(x,y)\) and the phase difference between the interfering beams \(\varphi(x,y)\), in this thesis briefly referred to as the *phase*. Here, the *detector coordinate system* \(x,y,z\) was introduced. The \(x\)-axis and \(y\)-axis are situated in the detector plane, both perpendicular to the \(z\)-axis: the optical axis of the imaging system. If the phase shifts are also unknown, some restriction on their values as well as more recordings will be needed to enable the solution of the corresponding system of equations\(^3,13\). That case will not be discussed in this thesis. Assuming that an accurately known average phase shift \(\alpha_i\) has been introduced for the \(i\)-th recording, and that during each exposure the phase shifts linearly from \(\alpha_i - \Delta\) to \(\alpha_i + \Delta\), the expression for the measured intensity becomes\(^{14}\)
\[ I_i(x,y) = I_B(x,y) + I_M(x,y) \text{sinc}\left(\frac{1}{2} \Delta \right) \cos(\varphi(x,y) + \alpha_i). \] (2.2)

In Fig. 2.1 the intensity versus the phase and the integration intervals are shown. The factor \( \text{sinc}\left(\frac{1}{2} \Delta \right) \) implies that the measured modulation intensity will be less if phase shifting is used instead of phase stepping (for which \( \Delta \) equals zero).

![Diagram showing intensity versus phase with integration intervals labeled as \( \Delta \).](image)

**Fig. 2.1** The intensity versus the phase. The integration intervals \( \Delta \) are related to the exposure time of the detector.

Equation (2.2) can be rewritten as\(^{14}\)

\[ I_i(x,y) = c_0(x,y) + c_1(x,y) \cos(\alpha_i) + c_2(x,y) \sin(\alpha_i), \] (2.3)

where

\[
\begin{align*}
c_0(x,y) &= I_B(x,y), \\
c_1(x,y) &= I_M(x,y) \text{sinc}\left(\frac{1}{2} \Delta \right) \cos(\varphi(x,y)), \\
c_2(x,y) &= I_M(x,y) \text{sinc}\left(\frac{1}{2} \Delta \right) \sin(\varphi(x,y)).
\end{align*}
\] (2.4)
If at least three different recordings of the intensity are available, the system of equations (2.3) has a linear least squares estimate for the three unknowns $c_i(x,y)$, given by the solution of the matrix equation

$$A(\alpha_i) \cdot c(x,y) = B(x,y,\alpha_i).$$

(2.5)

Here the following matrices were introduced:

$$A(\alpha_i) = \begin{pmatrix} N & \sum \cos(\alpha_i) & \sum \sin(\alpha_i) \\ \sum \cos(\alpha_i) & \sum \cos^2(\alpha_i) & \sum \sin(\alpha_i) \cos(\alpha_i) \\ \sum \sin(\alpha_i) & \sum \sin(\alpha_i) \cos(\alpha_i) & \sum \sin^2(\alpha_i) \end{pmatrix},$$

(2.6)

$$c(x,y) = \begin{pmatrix} c_0(x,y) \\ c_1(x,y) \\ c_2(x,y) \end{pmatrix}$$

(2.7)

and

$$B(x,y,\alpha_i) = \begin{pmatrix} \sum I_i(x,y) \\ \sum I_i(x,y) \cos(\alpha_i) \\ \sum I_i(x,y) \sin(\alpha_i) \end{pmatrix};$$

(2.8)

all summations are taken over the number of recordings $N$. Equation (2.5) must be solved at each point $(x,y)$ in the interference pattern by calculating and inverting the matrix $A$ and calculating the matrix $B$ at each point $(x,y)$. For the matrix $A$ to be invertible, at least three values of $\alpha_i$ must be unequal (modulo $2\pi$). Once the values of $c_1(x,y)$ and $c_2(x,y)$ are known, the phase $\phi(x,y)$ can be determined from
\[ \varphi(x,y) = \tan^{-1}\left[ \frac{c_2(x,y)}{c_1(x,y)} \right]. \] (2.9)

Considering the signs of \( c_1(x,y) \) and \( c_2(x,y) \), the value of \( \varphi(x,y) \) can be determined modulo \( 2\pi \). As can be deduced from Eqs. (2.2) and (2.4), the measured modulation intensity \( I_M' \) is given by

\[ I_M'(x,y) = I_M(x,y) \text{sinc}(\frac{1}{2} \Delta) = \sqrt{c_1^2(x,y) + c_2^2(x,y)}. \] (2.10)

The corresponding “fringe visibility” \( V(x,y) \) in the measured interference patterns can be found by normalizing the measured modulation intensity with respect to the background intensity \( I_B(x,y) \). Using Eq. (2.4) this yields:

\[ V(x,y) = \frac{\sqrt{c_1^2(x,y) + c_2^2(x,y)}}{c_0(x,y)}. \] (2.11)

Because of the absence of interference fringes in the forms of speckle interferometry described in this thesis, the quantity \( V(x,y) \) is referred to as the normalized modulation intensity. In phase shifting speckle interferometry the ratio of the measured modulation intensity (Eq. (2.10)) and the noise in the measured intensity is usually the more important parameter with regard to the measuring accuracy (see Sec. 5.1).

The solution of Eq. (2.5) can be simplified considerably by choosing appropriate values for the phase shifts \( \alpha_i \). When \( N \) measurements of the intensity \( I_i(x,y) \) are performed, choosing phase shifts obeying

\[ \alpha_i = i \frac{2\pi}{N}, \quad (i = 1,\ldots,N), \] (2.12)

all off-diagonal terms of the matrix \( A \) in Eq. (2.6) vanish, and the phase \( \varphi(x,y) \) can be determined directly from
\[
\varphi(x,y) = \tan^{-1}\left[ \frac{\sum I_i(x,y) \sin(\alpha_i)}{\sum I_i(x,y) \cos(\alpha_i)} \right].
\] (2.13)

The most common algorithms to perform this method use N=3 and N=4, resulting in phase shifts \( \Delta \alpha \) between subsequent intensity recordings of \( \frac{2}{3} \pi \) and \( \frac{1}{2} \pi \) radians, respectively. The first algorithm, mostly referred to as the three-bucket technique, yields the phase \( \varphi(x,y) \) by performing three measurements with \( \alpha_1 = \frac{2}{3} \pi \), \( \alpha_2 = \frac{4}{3} \pi \) and \( \alpha_3 = 2 \pi \) radians:

\[
\varphi(x,y) = \tan^{-1}\left[ \sqrt{3} \frac{I_1(x,y) - I_2(x,y)}{2I_3(x,y) - I_1(x,y) - I_2(x,y)} \right].
\] (2.14)

The four-bucket technique (N=4) yields the phase \( \varphi(x,y) \) by performing four measurements with \( \alpha_1 = \frac{1}{3} \pi \), \( \alpha_2 = \pi \), \( \alpha_3 = \frac{3}{2} \pi \) and \( \alpha_4 = 2 \pi \) radians:

\[
\varphi(x,y) = \tan^{-1}\left[ \frac{I_1(x,y) - I_3(x,y)}{I_4(x,y) - I_2(x,y)} \right].
\] (2.15)

Various algorithms exist in which the phase shift does not obey Eq. (2.12). Two two-bucket, or single step\(^{15} \), techniques and one five bucket technique will be mentioned here. Since the three unknowns in Eq. (2.1) dictate a minimum of three intensity recordings, a single step technique requires one of the unknowns to be known in advance. If the background intensity \( I_B(x,y) \) can be determined and subtracted from two measured intensities, phase shifted over \( \frac{1}{2} \pi \) radians relative to each other, the phase \( \varphi(x,y) \) becomes, using Eq. (2.2):

\[
\varphi(x,y) = \tan^{-1}\left[ \frac{I_2(x,y) - I_B(x,y)}{I_1(x,y) - I_B(x,y)} \right].
\] (2.16)

As long as the background intensity remains constant this expression will yield the phase accurately. Another single step technique\(^{16} \) can be applied, if the phase change between two measurements is of interest, which is
generally the case for speckle interferometry. Here, in addition to the three unknowns in Eq. (2.1), the phase change $\psi(x,y)$ constitutes a fourth unknown (see Sec. 2.3). Thus, four intensity measurements (two before and two after the phase change) should be enough to calculate the phase change from:

$$
\psi(x,y) = \frac{1}{2} \pi + 2 \tan^{-1} \left[ \frac{I_{1,\text{after}}(x,y) - I_{2,\text{before}}(x,y)}{I_{2,\text{after}}(x,y) - I_{1,\text{before}}(x,y)} \right],
$$

(2.17)

where $I_2(x,y)$ is phase shifted over $\frac{1}{2} \pi$ radians relative to $I_1(x,y)$. This technique assumes both the background intensity and the modulation intensity to remain constant during the entire measurement, but they need not to be known. These single step techniques have a great potential for application in combination with a spatial phase shifting technique (see Sec. 2.7), because only two interference patterns have to be generated simultaneously. A five-bucket algorithm that is relatively insensitive to deviations from its ideal phase shift $\Delta \alpha = \frac{1}{2} \pi$ radians, is given by:

$$
\phi(x,y) = \tan^{-1} \left[ \frac{2(I_1(x,y) - I_3(x,y))}{I_5(x,y) - 2I_2(x,y) + I_4(x,y)} \right].
$$

(2.18)

When the phase shift cannot be adjusted accurately, this technique can offer a significant improvement to the measuring accuracy.

In the case of the four-bucket algorithm, four independent measurement results are available to solve a system of equations with three unknowns. This means that a fourth unknown, e.g. the phase shift, can be determined to verify its value. Assuming equal average phase shifts between subsequent intensity recordings, the relation between the actual phase shift $\Delta \alpha(x,y)$ and the measured intensities $I_i(x,y)$ becomes:

$$
\Delta \alpha(x,y) = \cos^{-1} \left[ \frac{I_1(x,y) - I_2(x,y) + I_3(x,y) - I_4(x,y)}{2 \left[ I_2(x,y) - I_3(x,y) \right]} \right].
$$

(2.19)
The value of the expression within square brackets should lie in the interval \([-1,1]\); if not, the measurement in that particular detector point cannot be used to calculate the phase shift \(\Delta \alpha(x,y)\). Using this expression, phase shifts ranging from 0 to \(\pi\) radians can be determined. In the case of the three-bucket method the phase shift \(\Delta \alpha(x,y)\) cannot be determined in each detector point. When this method is applied in speckle interferometry, however, the uniformly distributed speckle phase enables the phase shift, averaged over all detector points, to be estimated. This method is presented in Appendix C.

*The four-bucket technique (Eq. (2.15)) was used for all measurements presented in this thesis.* Compared to the three-bucket technique it offers a higher accuracy and the possibility of verifcation the value of the phase shift in each point \((x,y)\).

All techniques mentioned in this section result in knowledge of the phase values modulo \(2\pi\) radians on a rectangular grid defined by the detector array. To reconstruct a continuous phase map, all occurring \(2\pi\) phase discontinuities must be detected and removed. The most straightforward method to do this is to consider the phase difference between adjacent detector points (pixels). When the phase difference exceeds \(\pi\), amounts of \(2\pi\) are added or subtracted until the difference is less than \(\pi\). Techniques for the removal of \(2\pi\) phase discontinuities will be referred to as *phase unwrapping algorithms*. The algorithm mentioned above is only reliable if both the actual phase difference between two adjacent pixels and the peak-to-peak value of the noise of the phase are less than \(\pi\). The first condition is met by sampling the interference pattern sufficiently. The second condition is often not met in speckle interferometry, necessitating the development of more complex phase unwrapping algorithms. Phase unwrapping algorithms for phase shifting speckle interferometry will be discussed in Section 2.6.
2.2 Speckle statistics

In a speckle interferometer at least one of the interfering beams is a speckle field. Consequently, the statistical properties of the speckle phenomenon play an important role in the description of the characteristics of the speckle interferometer. This section will present the relevant statistics of a perfectly monochromatic and polarized speckle field.

The definition of the speckle size is the statistical average of the distance between adjacent regions of maximum and minimum intensity. The observed speckle size at the detector of an imaging geometry can be derived from second order statistics\textsuperscript{18}. For the case of a circular lens pupil of diameter $D$, the spatial autocorrelation function $R_1(r)$ for the intensity of the speckle field in the image-plane is expressed as

$$
R_1(r) = I_0^2 \left[ 1 + 4 \left( \frac{\pi Dr}{\lambda z_0} \right)^2 \right],
$$

(2.20)

where $r$ is the distance between two points in the speckle field at the detector, $I_0$ the average intensity, $\lambda$ the wavelength of the light and $z_0$ the distance between the lens pupil and the detector plane. The first minimum of the Bessel function yields the speckle size $d_s$:

$$
d_s = \frac{1.22 \lambda z_0}{D} = \frac{0.6 \lambda}{NA},
$$

(2.21)

where NA is the numerical aperture of the lens.

The phase in a speckle field is uniformly distributed, while the intensity obeys negative exponential statistics\textsuperscript{18}. The probability density $p(I)$ of the intensity $I$ can be expressed as
p(I) = \frac{1}{I_0} e^{-\frac{1}{I_0}},

(2.22)

where \( I_0 \) is the average intensity. This equation implies that the most probable intensity is zero: dark speckles occur relatively frequent. When measuring the intensity at a certain point in a speckle field, the finite dimensions of the detector will cause integration of the intensity over the effective detector area. This causes the measured probability density for the intensity to deviate from Eq. (2.22). If the parameter \( \mathcal{M} \), the number of speckles influencing the measured intensity, is introduced, an approximation for the probability density of the measured intensity is given by\(^{18}\)

\[
p(I) = \frac{1}{\Gamma(\mathcal{M}) \left(\frac{\mathcal{M}}{I_0}\right)^{\mathcal{M}}} \left(\frac{\mathcal{M}}{I_0}\right)^{\mathcal{M}-1} e^{-\frac{\mathcal{M}}{I_0}},
\]

(2.23)

where \( \Gamma(\mathcal{M}) \) is the gamma function. If the effective detector area is much larger than the correlation area of one speckle, \( \mathcal{M} \) is determined by the ratio of the first and the latter. If the effective detector area is much smaller than the speckle correlation area, \( \mathcal{M} \) approaches unity asymptotically. In speckle interferometry a common choice is to design the interferometer in such a way that the speckle size is equal to the effective pixel size (optical resolution) of the detector array. From the exact calculation of the relation between \( \mathcal{M} \) and the ratio of the effective detector area and the speckle correlation area follows that in this case \( \mathcal{M} = 2.142 \). The effect of other speckle size to pixel size ratios will be discussed in Sec. 3.2. The normalized probability density function \( P(I/I_0) = I_0 \, p(I/I_0) \), for several values of \( \mathcal{M} \), is shown in Fig. 2.2.
With regard to speckle interferometry the case of the coherent addition of a speckle field and a reference beam, with uniform intensity $I_R$ and uniform phase, is the most interesting. As mentioned earlier, a large modulation intensity $I_M$ is of prime importance to achieve accurate phase measurement using the phase shifting technique. The modulation intensity in the detector plane can be expressed as

$$I_M(x,y) = 2\sqrt{I_R I_S(x,y)}, \tag{2.24}$$

where $I_S(x,y)$ is the intensity in the speckle field, obeying the statistics described by Eq. (2.23). The spatially varying intensity of the speckle field will cause a modulation intensity to be observed at the detector that varies strongly from pixel to pixel. Maximizing the average modulation intensity implies finding the optimal values for the speckle size relative to the effective pixel size and for the ratio of $I_R$ and the average speckle intensity $I_0$. A limiting condition is that the resulting intensity values remain within the intensity range that can be measured by the detector. This subject will be discussed more extensively in Chapter 3.
2.3 Displacement measurement

The main benefit of phase shifting speckle interferometry is the ability to measure changes of the phase $\varphi(x,y)$, similar to holographic interferometry$^{19,20,21,22}$. The phase change $\psi(x,y)$ in the detector plane is defined as

$$\psi(x,y) = \varphi_a(x,y) - \varphi_b(x,y),$$  \hspace{1cm} (2.25)

where $\varphi_b(x,y)$ is the measured phase before, and $\varphi_a(x,y)$ the measured phase after the object has been rotated or deformed by some excitation technique. For each point at the surface of the object corresponding to a detector location $(x,y)$, the measured phase change $\psi(x,y)$ is directly related to the displacement vector $l(x,y)$ by$^{23}$:

$$\psi(x,y) = s(x,y) \cdot l(x,y),$$  \hspace{1cm} (2.26)

where the sensitivity vector $s(x,y)$ equals

$$s(x,y) = \frac{2\pi}{\lambda} \{i_i(x,y) + i_d(x,y)\},$$  \hspace{1cm} (2.27)

and $i_i(x,y)$ and $i_d(x,y)$ are unit vectors in the illumination respectively the detector direction. In general, the sensitivity vector depends on the position at the surface of the object and thus on the position in the image that contains the phase data. If the length of the sensitivity vector $s(x,y)$ is known, the displacement component along $s(x,y)$ can be calculated using Eq. (2.26). In order to determine all components of the displacement vector $l(x,y)$, at least three phase measurements $\psi_i(x,y)$ have to be performed. The corresponding sensitivity vectors $s_i(x,y)$ must be independent in each point in the image. In the case of three measurements, the sensitivity matrix $S(x,y)$ is defined as
\[ S(x,y) = \begin{pmatrix} s_{1u}(x,y) & s_{1v}(x,y) & s_{1w}(x,y) \\ s_{2u}(x,y) & s_{2v}(x,y) & s_{2w}(x,y) \\ s_{3u}(x,y) & s_{3v}(x,y) & s_{3w}(x,y) \end{pmatrix} \]  

(2.28)

where \( s_{ij}(x,y) \) is the component in the \( j \)-direction of the sensitivity vector of the \( i \)-th measurement. Here, an *object-space coordinate system* \( u,v,w \) has been introduced. The positive \( u \)- and \( v \)-directions coincide with the negative \( x \)- and \( y \)-directions, respectively. The \( w \)-axis is parallel to the \( z \)-axis and directed towards the detector. The elements of this matrix can be established from known object displacements\(^{24}\) or from the geometry of the interferometer. The method used to determine the sensitivity matrix is discussed in Appendix A. If the phase vector \( \Psi(x,y) \) is defined as a column matrix containing the phase values \( \psi_i(x,y) \), the matrix equation becomes

\[ \Psi(x,y) = S(x,y) I(x,y). \]  

(2.29)

The complete three-dimensional displacement vector \( I(x,y) \) can now be determined by inverting the matrix \( S(x,y) \) for each detector point:

\[ I(x,y) = S^{-1}(x,y) \Psi(x,y). \]  

(2.30)

Assuming that the absolute value of the phase change can be determined in each measurement, this equation provides a direct relation between the set of measured phase values \( \psi_i(x,y) \) at each detector point and the displacement vector at the corresponding point at the object surface. Since the phase values, obtained by the phase shifting technique discussed in Section 2.1, are only known modulo \( 2\pi \), the absolute phase value is generally unknown. Apart from novel techniques to circumvent this problem\(^{25}\), a practical solution is to provide for a point in the image with a well known displacement. Often a point with zero displacement can be determined and used as such.
If more than three phase measurements have been performed, a linear least squares estimate for the components of the displacement vector can be found by solving the following matrix equation

$$\mathbf{P}(x,y) \mathbf{I}(x,y) = \mathbf{Q}(x,y),$$  \hspace{1cm} (2.31)

where

$$\mathbf{P}(x,y) = \begin{pmatrix} \sum s_{iu}^2 & \sum s_{iu} s_{iv} & \sum s_{iu} s_{iw} \\ \sum s_{iu} s_{iv} & \sum s_{iv}^2 & \sum s_{iv} s_{iw} \\ \sum s_{iu} s_{iw} & \sum s_{iv} s_{iw} & \sum s_{iw}^2 \end{pmatrix}$$  \hspace{1cm} (2.32)

and

$$\mathbf{Q}(x,y) = \begin{pmatrix} \sum \psi_i(x,y) s_{iu} \\ \sum \psi_i(x,y) s_{iv} \\ \sum \psi_i(x,y) s_{iw} \end{pmatrix};$$  \hspace{1cm} (2.33)

all summations are taken over the number of measurements $N$ and the $(x,y)$ dependency of the sensitivity vector components has been omitted. Equation (2.31) must be solved at each point $(x,y)$ in the interference pattern by calculating the matrices $\mathbf{P}$ and $\mathbf{Q}$ and inverting $\mathbf{P}$.

2.4 Shape measurement

For a complete surface strain analysis (Sec. 2.5), the surface topology has to be known in order to determine the in-plane strain components from the measured three-dimensional displacement data. Several interferometric methods for surface shape measurement, also known as contouring techniques, have been developed in the past three decades\textsuperscript{26,27,28}. Besides the Moiré technique using fringe projection, most techniques have been
applied in holographic interferometry, such as the *immersion method*, the *two-wavelength method* and the *two-illumination-source method*. A modification of the two-illumination-source method has been used for object shape measurement and will be presented in this section. This method was chosen because of its possibility to use the same optical arrangement as was used to measure the displacement vector field.

The applied modification of the two-illumination-source method is based upon measuring the phase change $\psi(x,y)$ caused by a controlled displacement $\Delta r_1$ of the object illumination point source. From Fig. 2.3 it can be seen that the phase change is given by

$$\psi(x,y) = -\mathbf{k}_i(x,y) \cdot \Delta r_1 - \Delta \mathbf{k}_i(x,y) \cdot (\mathbf{r}(x,y) - \mathbf{r}_1 - \Delta \mathbf{r}_1), \quad (2.34)$$

where $\mathbf{k}_i(x,y)$ is the wave vector of the illumination, $\Delta \mathbf{k}_i(x,y)$ its change corresponding to $\Delta r_1$ (both at the position given by the vector $\mathbf{r}(x,y)$) and $\mathbf{r}_1$ the position of the illumination source.

![Fig. 2.3 The modified two-illumination-source method.](image)

The position vector $\mathbf{r}(x,y)$, with components $u$, $v$ and $w$, gives the position at the object surface corresponding to the point $(x,y)$ at the detector. The origin of the $u,v,w$ coordinate system is located on the $z$-axis of the imaging system at the point the system is focussed on. If $\Delta r_1$ is small with respect to the distance between the illumination source and the object, $\Delta \mathbf{k}_i(x,y)$ is practically orthogonal to $[\mathbf{r}(x,y) - \mathbf{r}_1 - \Delta \mathbf{r}_1]$. In that case, the relation between
the vector $\Delta \mathbf{r}_1$ and the measured phase change $\psi(x,y)$ can be approximated by

$$\psi(x,y) = -\mathbf{k}(x,y) \cdot \Delta \mathbf{r}_1,$$  \hspace{1cm} (2.35)

Using the relation $\mathbf{k}(x,y) = (2\pi/\lambda)|\mathbf{r}(x,y) - \mathbf{r}_1|/|\mathbf{r}(x,y) - \mathbf{r}_1|$, Eq. (2.35) can be rewritten as

$$\psi(x,y) = \frac{2\pi (\mathbf{r}(x,y) - \mathbf{r}_1) \cdot \Delta \mathbf{r}_1}{\lambda|\mathbf{r}(x,y) - \mathbf{r}_1|}.$$  \hspace{1cm} (2.36)

A useful method that was developed to establish the object shape from Eq. (2.36) will be described in the following lines. If a reference phase change measurement $\psi_r(x,y)$ of a flat surface perpendicular to the z-axis is performed, the calculated difference $\Delta \psi(x,y)$ of the phase changes $\psi(x,y)$ and $\psi_r(x,y)$ will contain information about the deviation from flatness of the object surface:

$$\Delta \psi(x,y) = \psi(x,y) - \psi_r(x,y).$$  \hspace{1cm} (2.37)

The object, of which the shape has to be determined, has to be positioned in the same area as the reference object surface. The imaging system is focussed on the reference surface and its intersection with the z-axis thus defines the origin of the u,v,w-system. If the object is positioned such that this origin is located upon its surface, the phase change difference $\Delta \psi(x,y)$ in that point equals zero.

Both measurements have to be performed with an identical change $\Delta \mathbf{r}_1$ of the illumination source position. Since the surface of the flat reference object is positioned at $w=0$, the use of a Taylor-series expansion to describe the relation between $\Delta \psi(x,y)$ and the object shape function $w(u,v)$ is possible. To simplify the determination of the object shape, in view of shortening the calculation time, two assumptions have been made:
1. The dimensions of the object are much smaller than the object-detector distance. In that case the coordinates $u$ and $v$ of each point at the object surface are approximately linearly related to $x$ and $y$, respectively, independent of the object shape. The object shape is then also described by $w(x,y)$.

2. The object dimensions are much smaller than the distance between the object and the illumination source. In that case the first term of the Taylor-series expansion will give a sufficiently accurate approximation of the object shape.

Then the relation between $w(x,y)$ and the phase change difference $\Delta \psi(x,y)$, using Eqs. (2.36) and (2.37), was found to become

$$\Delta \psi(x,y) = f(x,y) \left[ \frac{w(x,y)}{d_i(x,y)} \right],$$

(2.38)

where

$$f(x,y) = \frac{2\pi}{\lambda} \left[ \frac{\Delta u_i \left( \frac{x}{M} - u_i \right) + \Delta v_i \left( \frac{y}{M} - v_i \right)}{\Delta w_i + 2w_i d_i^2(x,y)} \right].$$

(2.39)

In these equations, $d_i(x,y)$ is the distance between the illumination source and the points at the flat reference object corresponding to $(x,y)$:

$$d_i(x,y) = \sqrt{\left( \frac{x}{M} - u_i \right)^2 + \left( \frac{y}{M} - v_i \right)^2 + w_i^2},$$

(2.40)

where $M$ is the magnification factor of the imaging system, that is focussed on the reference surface.

If the assumptions made in this section are not legitimate or if a higher accuracy is desired, a more elaborate theory will be needed to describe the
relation between the object shape and the measured phase change difference. The exact object shape can be derived using the two-illumination-source method, but that requires time consuming calculations. This subject will be considered more extensively in Appendix B. When determining the local orientation of the object surface for the purpose of strain analysis, the first order approximation is usually sufficient.

2.5 Surface strain measurement

When stress is applied to a solid object, a three-dimensional strain pattern will occur throughout its volume. Various interferometric methods have been applied to the measurement of strain\textsuperscript{30,31,32,33,34,35,36,37,38,39,40}. If the object is opaque, an optical method like speckle interferometry can only give information about the surface. The surface strain or in-plane strain components can be determined if the three-dimensional displacement vector and the surface normal vector are known at each point of the object surface\textsuperscript{41}.

The local orientation of the object surface is described by the surface normal, that can be determined by taking the derivatives \( m_u(x,y) \) and \( m_v(x,y) \) of \( w(x,y) \) with respect to the coordinates \( u \), respectively \( v \):

\[
\begin{align*}
  m_u(x,y) &= M \frac{\partial w(x,y)}{\partial x}, \\
  m_v(x,y) &= M \frac{\partial w(x,y)}{\partial y}.
\end{align*}
\]

(2.41)

To each point at the object surface a local coordinate system \( u',v',w' \) can now be assigned. The unit vector \( i_w \) corresponds to the surface normal and the unit vectors \( i_u \) and \( i_v \) are taken along the projections of \( i_u \) and \( i_v \), respectively, on the local plane described by the surface normal. In the \( u,v,w \)-system these unit vectors can now be expressed as:
\[ i_w(x,y) = \frac{1}{\sqrt{m_u^2(x,y) + m_v^2(x,y) + 1}} \begin{bmatrix} -m_u(x,y) \\ -m_v(x,y) \\ 1 \end{bmatrix}, \]

\[ i_u(x,y) = \frac{1}{\sqrt{m_u^2(x,y) + 1}} \begin{bmatrix} 1 \\ 0 \\ m_u(x,y) \end{bmatrix}, \] 

\[ i_v(x,y) = \frac{1}{\sqrt{m_v^2(x,y) + 1}} \begin{bmatrix} 0 \\ 1 \\ m_v(x,y) \end{bmatrix}. \] 

(2.42)

This yields a transformation matrix \( \mathbf{T}(x,y) \) describing the transformation from the \( u,v,w \)-system to the local \( u',v',w' \)-system:

\[ \mathbf{T}(x,y) = \begin{bmatrix} i_{u'u}(x,y) & i_{u'v}(x,y) & i_{u'w}(x,y) \\ i_{v'u}(x,y) & i_{v'v}(x,y) & i_{v'w}(x,y) \\ i_{w'u}(x,y) & i_{w'v}(x,y) & i_{w'w}(x,y) \end{bmatrix}, \] 

(2.43)

in which \( i_{kl} \) indicates the \( l \)-component in the \( u,v,w \)-system of the unit vectors \( \mathbf{i}_k \), described by Eq. (2.42). This matrix will be used to transform the measured three-dimensional displacement vector data to the local coordinate system at each point on the surface of the object:

\[ \mathbf{I}'(x,y) = \mathbf{T}(x,y) \mathbf{I}(x,y). \] 

(2.44)

If several displacement measurements of the same object have to be performed, it is convenient to combine Eqs. (2.30) and (2.44) in order to minimize the number of multiplications:

\[ \mathbf{I}'(x,y) = \mathbf{T} \mathbf{S}^{-1}(x,y) \mathbf{\Psi}(x,y), \] 

(2.45)
where the matrix $\mathbf{T} \mathbf{S}^{-1}(x,y)$ is the product of the transformation matrix $\mathbf{T}(x,y)$ and the inverse of the sensitivity matrix $\mathbf{S}^{-1}(x,y)$.

The derivatives that can directly be obtained from $\mathbf{l}'(x,y)$ by two-dimensional image processing techniques are $dl'(x,y)/dx$ and $dl'(x,y)/dy$. They can be used to calculate the in-plane strain components by relating the small steps $dx$ and $dy$ to $du'$ and $dv'$, respectively. Using the transformation matrix $\mathbf{T}(x,y)$ this yields ($dw = dz/M = 0$):

\begin{align}
    du'(x,y) &= \frac{i_{uu}(x,y)}{M} \, dx, \\
    dv'(x,y) &= \frac{i_{vv}(x,y)}{M} \, dy. \\
\end{align}

(2.46)

The local in-plane strain components are the normal strains $e_{xx}(x,y)$ and $e_{yy}(x,y)$ and the shear strains $e_{xy}(x,y) = e_{yx}(x,y)$. Using their definition and Eq. (2.46), they satisfy

\begin{align}
    e_{xx}(x,y) &= \frac{dl_u(x,y)}{du'(x,y)} = \frac{M}{i_{uu}(x,y)} \frac{dl_u(x,y)}{dx}, \quad (2.47) \\
    e_{yy}(x,y) &= \frac{dl_v(x,y)}{dv'(x,y)} = \frac{M}{i_{vv}(x,y)} \frac{dl_v(x,y)}{dy}, \quad (2.48) \\
    e_{xy}(x,y) &= \frac{1}{2} \left[ \frac{dl_u(x,y)}{dv'(x,y)} + \frac{dl_v(x,y)}{du'(x,y)} \right] \\
    &= \frac{1}{2} \left[ \frac{M}{i_{vv}(x,y)} \frac{dl_u(x,y)}{dy} + \frac{M}{i_{uu}(x,y)} \frac{dl_v(x,y)}{dx} \right]. \quad (2.49)
\end{align}

For the sake of completeness, the expression for the rotation in each object surface point will be given here:
\[ \phi(x,y) = \frac{1}{2} \left[ \frac{M}{i_x(x,y)} \frac{dl_u(x,y)}{dy} - \frac{M}{i_y(x,y)} \frac{dl_v(x,y)}{dx} \right]. \]

(2.50)

The calculation of the strain components involves the determination of the first spatial derivatives of the local in-plane displacement components. This can be carried out by fitting a polynomial in \(x\) and \(y\) to a subset of the displacement data and analytical differentiation. A least squares fit of a plane to a set of neighboring data points was used in each data point, yielding the derivatives with respect to \(x\) and \(y\) for one displacement component at the same time. A detailed discussion of this method will be presented in the next section.

2.6 Image processing techniques

Digital image processing is a relatively new discipline, that has evolved rapidly over the past decade, mainly thanks to the availability of fast computers. Image processing can be subdivided into the recording, digitization, processing, measurement and presentation of images. The recording is performed by some sensor that usually transduces the physical quantity of interest to a voltage (video signal). To allow a computer to digest the recorded image, it has to be digitized into an array of numbers. The digitization involves the spatial quantization (mostly a rectangular sampling grid is used), leading to a finite number of samples, as well as the intensity quantization (mostly uniform sampling is used), leading to a finite number of gray values. The combination of position and gray value is defined as a pixel. The processing of images can be a complex arrangement of image enhancement, algebraic operations, restoration, segmentation and morphological operations. Image measurement concerns the determination of the properties of objects in the image, e.g. the position in the image, the surface area or the average gray value. Finally, the results have to be presented, leading to a digestible description of the image.

In this thesis, the two-dimensional data is presented mostly by gray value images. The gray values in these images can refer to various physical
quantities, such as intensity, phase, displacement or strain. In general
the scaling factor between the gray values and the actual data can be
chosen freely. To enhance the contrast, the gray value range (from black to
white) can be made much smaller than the total range of the data.
However, this causes the gray value image to display discontinuities in the
data that do not exist. Thus, care should be taken when interpreting the
gray value images in Chapters 3 and 4.

A sequence of image processing algorithms for the analysis of interference
patterns, generated by a phase shifting speckle interferometer, is discussed in this section. The goal is the accurate determination
of displacement and strain components at the surface of an object. Digital
image processing algorithms have been developed for phase calculation,
phase unwrapping, phase restoration and phase fitting.

**Phase calculation**

The calculation of $\varphi(x,y)$ from Eq. (2.13) is performed by determination of
the numerator and denominator from the measured intensities and applying
a 2D look-up table (Fig. 2.4), that renders the phase value modulo $2\pi$.
Characteristic for speckle interferometry is the loss of accuracy in points
with low modulation caused by dark speckles or dark areas. Consequently,
the resulting image contains invalid pixels, i.e. pixels with an inaccurate
phase value. These pixels disturb subsequent processing of the data, if
they are not excluded from the calculations. They are identified by
considering the expression for the measured modulation intensity $I_M'(x,y)$,
given by Eq. (2.10). The following account will be restricted to the four-
bucket method, in which case $I_M'(x,y)$ follows from solving Eq. (2.5) and
applying Eq. (2.10):

$$I_M'(x,y) = \frac{1}{2} \sqrt{(I_1(x,y) - I_3(x,y))^2 + (I_4(x,y) - I_2(x,y))^2}.$$  

(2.51)

Pixels with a modulation intensity $I_M'(x,y) = M_0$ lie on a circle in the 2D
look-up table with its center at the origin of the $(I_4-I_2),(I_1-I_3)$-plane and a
radius $2M_0$. Hence, pixels with a modulation intensity below a certain threshold $M_T$ can be detected using a circular area in the 2D look-up table.

![Diagram](image)

Fig. 2.4 2D look-up table for calculating the phase modulo $2\pi$. The origin is located at the center of the circular area; the distance from the origin is $2I_M'$ (Eq. (2.51)).

with a radius $2M_T$ (modulation intensity threshold, Fig. 2.4). This identification yields a binary image $B_M(x,y)$ containing a mask that will be used during further processing of the data. The minimal size of the circular area depends on the amount of noise in the intensity data. Points with zero modulation should always be detected. To achieve this, a practical choice for $M_T$ is $4\sigma_I$, where $\sigma_I$ is the standard deviation of the intensity noise.

Pixels having the maximum gray value 255 in at least one of the four interference patterns are assumed to be related to a saturated detector element. These invalid pixels can also be registered in the binary image $B_M(x,y)$ using basic image processing techniques. This mask is used and processed during phase unwrapping and phase restoration to identify object and background areas.

**Phase unwrapping**

Because the phase $\varphi(x,y)$ is determined modulo $2\pi$, artificial $2\pi$-steps and $4\pi$-steps may occur in the phase change $\varphi(x,y)$. The removal of these steps, called phase unwrapping, is necessary to make the phase data continuous. A simple algorithm to unwrap the phase data is based upon adding offsets
(multiples of $2\pi$) to the pixel values. The offset changes each time a $2\pi$-step is detected. A $2\pi$-step is detected if the absolute value of the difference between a pixel value and the previous pixel value exceeds $\pi$. Starting at the top left pixel of the image, the offset is set to zero. Then the first column is scanned to determine the offsets for the first pixels of the rows. Finally for each row the offsets are calculated starting with the offset in the first column. The unwrapping procedure can also be started at another location, e.g. the center of the image.

The scanning method mentioned above is successfully used in classical interferometry. This method is not suitable for speckle interferometry, because the $2\pi$-steps can be hidden in speckle noise, causing steps to be neglected or offsets to be added at the wrong points. A further restriction on this method is that all pixels in the input image have to be valid data pixels. If invalid pixels occur, these have to be corrected before phase unwrapping.

The computation of a continuous phase map can be improved by excluding the invalid pixels from the unwrap procedure. An algorithm based upon a pixel queue was developed to scan the data around invalid pixels$^{45}$ (Fig. 2.5). A pixel queue is a one-dimensional data structure. The pixel addresses of new valid neighbors of the currently processed pixel are added at the input side of the queue. The address of the next pixel to be processed is taken from the output side, and all pixel addresses in the queue shift one position towards the output. This procedure continues until the pixel queue is empty.

![Diagram](image)

Fig. 2.5 The pixel queue based phase unwrapping algorithm propagates in a diamond shape around marked pixels.
If disconnected areas occur in the image, i.e. not all areas are interconnected by a path of valid pixels, each area has to be processed separately by choosing a start location within that area. Because only pixels that are 4-connected to (share an edge with) the current pixel are put on the pixel queue the processing propagates with a diamond shape through the image. If the 8-connected neighbors (pixels that share at least one corner with the current pixel) are taken into account a square propagation occurs. Because failure in the detection of $2\pi$-steps appears to occur mostly at the corners of the diamond shaped propagation, new algorithms have been developed that provide a circular propagation. The adjustment of a pixel is carried out by adding or subtracting $2\pi$ until the difference with the average of the phase values of the unwrapped pixels in a $3 \times 3$ neighborhood is less than $\pi$. During the phase unwrapping, the binary mask $B_M(x,y)$ is used to identify invalid pixels. After phase unwrapping, $B_M(x,y)$ is put to "zero" if the phase is unwrapped at $(x,y)$ and to "one" at the remaining locations. The algorithm was extended by adding the possibility to set a threshold for $2\pi$-step detection different from $\pi$. Tolerances are introduced through symmetric intervals around the phase value, where 50% tolerance corresponds to an interval of width $2\pi$, as was used in the original algorithm. If the difference between the phase of the pixel under investigation and the average phase of the unwrapped pixels in a $3 \times 3$ neighborhood is considered and, for instance, a 25% tolerance is chosen, the cases that can occur are given in Table 2.1.

Table 2.1  Action of the phase unwrapping algorithm for the three cases that can occur if the tolerance is set to 25%.

<table>
<thead>
<tr>
<th>phase difference</th>
<th>action</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[-\frac{\pi}{4},\frac{\pi}{4}]$</td>
<td>phase value remains the same</td>
</tr>
<tr>
<td>$[\frac{\pi}{2},\frac{3\pi}{2}] \mod 2\pi$</td>
<td>pixel is marked invalid in $B_M(x,y)$</td>
</tr>
<tr>
<td>$[\frac{\pi}{2},\frac{5\pi}{2}] \mod 2\pi$</td>
<td>phase value is corrected</td>
</tr>
</tbody>
</table>
This technique yields more accurate results, because noisy pixels (e.g. caused by phase decorrelation, see Sec. 5.2) are excluded from the resulting continuous phase map. A second advantage is the increased robustness of the unwrapping procedure against phase decorrelation noise: a significantly larger object deformation, rotation or translation can be tolerated, without disturbing the unwrapping procedure.

**Phase restoration**

Two methods are used to restore the phase value of invalid pixels. The first one substitutes the value of an invalid pixel by the value of an arbitrary pixel from the set of valid pixels in the $3 \times 3$ neighborhood. This restoration technique can be performed before phase unwrapping. Averaging neighboring pixels is not possible because $2\pi$-steps may be present. If no valid pixel is available in the neighborhood, the pixel remains invalid. A repeated application of this technique would eventually correct all invalid pixels, but the systematic error in the output data becomes larger with each application.

The pixel queue based phase unwrapping algorithm, mentioned in this section, allows invalid pixels to occur in the input. After phase unwrapping, such pixels can be replaced by the average of the valid pixels in their $3 \times 3$ neighborhood. For each pixel that is restored, the pertinent location in $B_M(x,y)$ is set to zero. This corresponds to an 8-connected binary erosion of the mask. This phase restoration is not necessary if further processing steps make use of the mask $B_M(x,y)$ to identify invalid pixels. As with the first of the discussed restoration techniques, this method can be applied a number of times to restore the data. This causes small clusters of invalid pixels (dark speckles) to disappear and large clusters (dark areas) to be eroded. The same number of 8-connected binary dilations can be applied to $B_M(x,y)$ to restore the (eroded) background, yielding a mask that separates the object(s) from the background (Fig. 2.6).
Phase fitting

Although "phase fitting" refers to phase data, the image processing algorithm described in the following lines can be applied to any physical quantity (e.g. intensity or displacement). Because of the occurrence of invalid pixels, a convolution filter is inappropriate to determine the first derivatives of the phase with respect to the x- and y-coordinates. A linear least squares (LLS) fit of a plane to the set of valid pixels in a rectangular neighborhood of each valid pixel was used, yielding the derivatives with respect to both coordinates simultaneously. This enabled the measurement of the in-plane strain near invalid areas. As will be shown, this method can also be applied successfully as a smoothing filter.

For each point (x,y) in the image a rectangular neighborhood is considered (Fig. 2.7). This neighborhood contains a set of valid pixels \((x+m,y+n,g(x+m,y+n))\), where \((m,n)\) indicates the position of a data point relative to \((x,y)\) and \(g(x+m,y+n)\) is the gray value at that position. A LLS fit of a plane at location \((x,y)\) is performed by minimizing the following expression:

\[
S(x,y) = \sum (a(x,y)m + b(x,y)n + c(x,y) - g(x+m,y+n))^2, \tag{2.52}
\]
where the summation is performed over all valid pixels in the rectangular neighborhood. The values of $a(x,y)$ and $b(x,y)$ are estimates of the first derivatives with respect to $x$ and $y$ at the location $(x,y)$. The computation of $c(x,y)$ provides a smoothing filter, because it yields an estimate of the gray value at the center of the rectangular neighborhood and thus at the location $(x,y)$.

If no invalid pixels are present, the computation of $a(x,y)$, $b(x,y)$ and $c(x,y)$ is equivalent to the application of the proper convolution filters. If invalid pixels occur, the described fitting procedure is more accurate than the application of convolution filters, because the inaccurate values of those pixels are excluded from this procedure. Near the object edges a systematic error can occur since most valid data points are located on one side of the center of the rectangular neighborhood. This error can be reduced by performing a LLS fit of a function with second or higher order terms to the data, but then the noise reduction will be less (see Sec. 5.4).

Because of the occurrence of invalid data points, all summations involved in the solution of Eq. (2.52) have to be calculated each time the rectangular neighborhood has been shifted in order to apply the LLS fit to the next pixel.
To reduce computation time, the summations are computed using a special updating mechanism\textsuperscript{45}.

Application of the gradient filter detailed in the above paragraphs enables the accurate determination of the in-plane strain components at each point of the object surface.

### 2.7 Phase shifting techniques

The various possible techniques for shifting the phase can be divided into two methods: temporal phase shifting and spatial phase shifting\textsuperscript{1}. The former method has been applied in the experimental system described in this thesis.

Temporal phase shifting comprises techniques that produce the phase shifted interference patterns successively in time. In general this means that one detector will perform several subsequent recordings that have to be stored in order to enable processing of the data. Five techniques will be mentioned here. The most straightforward one employs the motion of a mirror or of the far end of an optical fiber placed in one of the interferometer arms\textsuperscript{38}. This motion is usually performed by a piezo-electric translator (PZT) and linear in time, although the use of a sinusoidal motion has been reported too\textsuperscript{46}. Another technique employing an optical fiber, is based upon a fiber wrapped around a piezo-electric element\textsuperscript{47}. Expansion of the element causes an elongation of the fiber and thus of the optical path length. A third possibility is the use of an electro-optic modulator in one of the interferometer arms. Here, a drawback is the non-uniform operation over the aperture of the modulator. A fourth technique makes use of the polarization state of the interfering beams\textsuperscript{48,49,50,51}. The interferometer is designed in such a way that the two beams are circularly polarized, having opposite rotation directions. After recombination, a polarizer will make the interference pattern observable; the phase shift equals twice its angle. Finally, a tunable laser source can be used: the optical path difference between the arms of the interferometer causes the phase to shift as the frequency changes. A disadvantage is the dependence of the phase shift on
the optical path difference, making accurate measurements on three-dimensional objects impossible.

Spatial phase shifting comprises techniques that produce several phase shifted interference patterns simultaneously. This means that several detectors or several areas of one detector record the interference patterns at the same time, thus enabling instantaneous processing of the data. The major advantage of this method is the possibility to cope with transient phenomena by use of a pulsed laser source. A drawback with respect to temporal phase shifting, especially in the case of speckle interferometry, is the requirement that the detectors or detector areas have to be aligned with sub-pixel accuracy to ensure sufficient correlation between the recorded interference patterns. The most well known technique employs a diffraction grating, where the zeroth and first order diffractions constitute the three phase shifted interference patterns\textsuperscript{52,53,54,55}. In this case, one detector array can be used if a lower sampling density of the data is admissible. The technique using the polarization state of the interfering beams, mentioned in the previous paragraph, can be used for spatial phase shifting by amplitude splitting the recombed beams in several parts and locating polarizers in front of the detectors. Thus, several phase shifted interference patterns can be generated simultaneously. If a beam splitting prism, similar to those used for color video cameras, is applied, three separate detectors will be needed. During the research reported in this thesis, a preliminary study was made of a modified version of the latter technique, using a Wollaston prism. When the Wollaston prism is considered as an amplitude splitting device combined with two orthogonal polarizers in each beam, this technique is basically similar to the previously discussed one, although now two instead of three interference patterns are generated. If this prism is located at the output of an interferometer, in which one beam is circularly and the other linearly polarized (the latter having a polarization angle of $\frac{1}{4}\pi$ radians with respect to both polarization directions behind the Wollaston prism), two interference patterns, phase shifted by $\frac{1}{2}\pi$ radians, will be generated. The position of the detector should allow the two images to be observed separately, within the same detector area. The first measurements, using a single step technique as discussed in Section 2.1, confirmed the applicability of this spatial phase shifting
method. Especially in the case of speckle interferometry, the angle between the two split beams should be small. This is due to the fact that the difference in image distortion, caused by the Wollaston prism, must be negligible with respect to the pixel size in order to minimize the speckle decorrelation between the two speckle patterns. A drawback of this method is that this causes the lens pupil-to-detector distance to become large, leading to a very small field of view. Further research will be needed to solve this problem.

In the interferometer used in the experimental system reported on in this thesis, an optical fiber attached to a PZT was used to shift the phase. The flexibility of using an optical fiber and the accuracy of a PZT were the main reasons for this choice. At present research is being performed on a spatial phase shifting speckle interferometer using three separate detector arrays.

2.8 References


Chapter 3

Experimental system

3.0 Introduction

The central part of the experimental system is a speckle interferometer. From the numerous possible arrangements\textsuperscript{1,2,3,4,5,6}, such as in-plane displacement sensitive or shearing speckle interferometers, a general version with a variable object illumination direction was chosen for the experiments described in this thesis. The ability to measure a specified displacement component, its derivative or the object shape, is mainly included in the developed software that processes the data derived from interference patterns according to the theory presented in Chapter 2. This feature permits the experimental system to be a multi-purpose measuring device. It has the additional advantage that changes in the properties of the device can easily be obtained by modifying the software.

One of the major problems in speckle interferometry is, that only a small fraction of the light scattered by the object is captured by the imaging system. This fact demands a high laser power, highly reflecting object surfaces or methods to amplify the intensity of the interference images, such as an image intensifier. For the experiments described in this thesis a high laser power combined with intensity averaging was chosen, to enable measurements on arbitrary objects without the necessity of applying of special highly reflecting surface coating.

In this chapter the general form of a speckle interferometer as well as the specific phase shifting speckle interferometer used for the experiments will be presented (Sec. 3.1). In Section 3.2 the optimization of the interferometer will be discussed, followed by two sections that describe the computer system and the overall system characteristics (Secs. 3.3 and 3.4). An example of a phase change measurement is given in Section 3.5.
3.1 Optical arrangement

The optical arrangement for a general speckle interferometer is shown in Fig. 3.1. In a speckle interferometer the object is imaged onto a detector that registers the intensity of the interference pattern. Thus, each pixel is related to a small object surface area ("point") and the measured phase change to the displacement of that point. The numerical aperture has to be chosen small enough to produce speckles that are resolved by the detector (see Sec. 2.2). This causes the problem that only a very small fraction of the light reflected by the object is collected by the imaging system.

![Diagram of speckle interferometer](image)

**Fig. 3.1** General arrangement of a speckle interferometer.

The reference and object beams are recombined in front of the detector. The phase of the reference beam should be practically constant over the area of each speckle at the detector, to avoid the occurrence of interference fringes within the speckles that cannot be resolved by the imaging system. This criterion is met by locating the reference beam source at the center of the aperture of the imaging system, as will be shown in the following account.
Fig. 3.2  A schematic representation of the combination of the object and reference beams in front of the detector.

If the object beam at the detector is approximated by the superposition of a spherical wave and a random speckle field, the phase difference $\varphi'(x,y)$ at the detector between this spherical wave and the reference beam can be considered separately\. If the deviations $\Delta z_R$ and $\Delta x_R$ of the position of the reference beam source from the center of the aperture are small with respect to the aperture-detector distance $z_0$, the phase difference along the x-axis can be approximated by:

$$\varphi'(x) = \frac{2\pi}{\lambda} \left[ \Delta z_R + \frac{x^2 \Delta z_R}{2z_0^2} + \frac{x \Delta x_R}{z_0} \right],$$

(3.1)

where $x$ is the position at the detector (Fig. 3.2). Since the speckle size is usually in the order of the size $x_P$ of one detector element (Sec. 2.2), the change of the phase $\varphi'(x)$ over one element should be much smaller than $2\pi$. Thus, the third term of Eq. (3.1) implies that $\Delta x_R < (\lambda z_0)/x_P$. Assuming a total detector size of $512x_P$, the second term of Eq. (3.1) renders the condition $\Delta z_R < (\lambda z_0^2)/(256x_P^2)$, which is much less restrictive than the
previous condition. This means that the reference beam source should be accurately centered in the diaphragm of the imaging lens, while an axial deviation from the ideal location is relatively unimportant. For the practical case of a visible laser source, using an aperture-detector distance of 100 mm and a detector element size of 10 μm, the first condition becomes $Δx_R << 5$ mm. A positioning resolution of 0.1 mm will then be sufficient.

The positioning of the reference beam source can be realized by coupling the reference beam into an optical fiber and locating its far end in the center of the diaphragm. In the interferometer described in this thesis, the far end of the fiber was positioned in the virtual center of the diaphragm by application of a beam combiner to enable the use of very small diaphragm diameters. An often applied alternative method, using a lens and a beam splitter, is illustrated by the arrangement shown in Fig. 3.1. A technique employing a mirror-pinhole combination has been reported too\textsuperscript{11}.

![Fig. 3.3](image)

Fig. 3.3 Outline of the experimental system.

The experimental system consists of a speckle interferometer coupled to a VME microcomputer for digitizing and processing the interference patterns and for shifting the phase (Fig. 3.3). In the optical arrangement, illustrated in Fig. 3.4, an Argon ion laser is used at a wavelength of 514 nm single line, usually at an output power of approximately 500 mW. The
output of the laser is coupled into a monomode optical fiber that feeds the interferometer, referred to as source fiber. This enables the movement of the interferometer independent of the laser, which is useful when using a high power laser with large dimensions. The interferometer is equipped with an object illumination section and a small CCD video camera that functions as a two-dimensional detector array of $604 \times 575$ pixels with the dimensions $10 \ \mu m \times 7.8 \ \mu m$.

![Diagram of optical arrangement](image)

**Fig. 3.4** Outline of the optical arrangement.

The object illumination direction can be changed by rotating the illumination arm around the optical axis of the imaging system (Fig. 3.5). This is performed without the use of mirrors by fixing the laser source fiber directly to the rotatable arm of the illumination section. One of the reflections of the lens which is used to diminish the divergence of the illuminating beam, is coupled into an optical fiber, referred to as the reference beam fiber. The position of the far end of this fiber determines the source of the reference beam. Phase shifting is achieved by attaching both the source fiber and the reference beam fiber to a piezo-electric translator (PZT), which could be conveniently located in the rotatable illumination arm. The PZT has a traveling range of 2 $\mu m$. A translation $\Delta z_{PZT}$ of the
PZT causes the phase of the reference beam to change twice the amount of the phase of the object beam of the interferometer, resulting in a relative phase shift that equals \((2\pi/\lambda)\Delta\phi_{PZT}\). To achieve phase shifting, a high voltage ramp is applied to the PZT. The triggering of the voltage ramp is controlled by the microcomputer to ensure that all interference patterns are recorded within one ramp period. A digital-to-analog converter is used to generate the trigger signal for the high voltage ramp generator.

![Diagram of interferometer](image)

Fig. 3.5  The spatial configuration of the interferometer with rotatable illumination arm (compare Fig. 3.4).

The entire optical arrangement was located on a vibration isolated table (air-damped) and shielded from air turbulence by a polystyrene housing.

The use of a semi-conductor laser source in a speckle interferometer has been reported\(^\text{12}\), but it can not yet compete with the combination of high output power and long coherence length of the Argon ion laser. The compactness of such a laser makes it a promising candidate to replace the bulky Argon ion laser system in the near future. The use of a pulsed laser source for the study of transient events has been reported as well\(^\text{13}\).
3.2 Optimization of the interferometer

The parameters that play an essential role in the behavior of the speckle interferometer are the focal length and the numerical aperture of the imaging lens as well as the intensity ratio of the reference and object beams at the detector\textsuperscript{11,14}. The aim of the optimization of the phase shifting interferometer is to minimize the r.m.s phase error in the measurements. To achieve this, the measured average intensity modulation relative to the intensity noise should be made sufficiently large\textsuperscript{14}, taking care to keep the fraction of unacceptable data points (invalid pixels, see Sec. 2.6) limited\textsuperscript{15}. Naturally, the wavelength and the coherence length of the laser source and the characteristics of the detector and the frame grabber are important as well, but it is usually not possible to choose them freely.

Focal length

The choice for the focal length of the imaging lens is restricted by the size of the object region that is to be investigated, by the actual object-detector distance and by the amount of space needed in between the lens and the detector. The latter is a construction problem, because the object and reference beams should be combined between the lens and the detector and a diaphragm, and possibly a polarizer, should be located there. This poses a lower limit to the lens-detector distance and thus a lower limit to the focal length of the lens, leading to an upper limit to the field of view. If it is impossible, e.g. in the case of a large object, to make the object-lens distance large enough, a multiple-lens system may be used for optical demagnification. It should be noted, however, that the focal length as well as the quality of the lens affect the performance of the speckle interferometer, in particular with respect to the phenomenon of speckle decorrelation\textsuperscript{16} (see Sec. 5.2).

Numerical aperture

The numerical aperture of the lens can be set by adjusting the diameter of the lens diaphragm. As was shown in Section 2.2, the speckle size is inversely proportional to this numerical aperture. The power in the portion of the light scattered by the object and collected by the imaging
system is proportional to the squared diaphragm diameter. The diaphragm diameter has to be made small enough to produce speckles that are resolvable by the detector. On the other hand the system has to collect as much light as possible, especially in the case of large object surfaces with low reflectivity. Thus, depending on the object surface, the diameter of the lens diaphragm can be optimized to assure a sufficiently large average modulation intensity. Since an often occurring practical problem is the low object light level at the detector, measurements were performed to find the optimum interferometer setting for this case. Fig. 3.6 shows the results of measurements of the r.m.s. phase error for three illuminating power values. The phase error was calculated by performing one phase change measurement, subtracting the average phase change from the measured values in each pixel and calculating the r.m.s. value divided by √2. For each measurement the illuminating power was kept constant while the diaphragm diameter was varied. The reference beam intensity was set at half the detector saturation intensity for each measurement. The number of saturated pixels was limited to one percent of the total pixel number.

![Graph](image.png)

**Fig. 3.6** The measured r.m.s. phase error versus the numerical aperture NA for various values of the illuminating power, for a focal length of 100 mm. \( \langle I_p \rangle \) is the average intensity in the pupil plane. \( I_o \) is the intensity in the pupil plane that corresponds to the saturation level of the detector for NA = 0.10. Fourth order polynomials were fit to the data.
Fig. 3.6 shows that for lower light levels the optimal value of the numerical aperture becomes larger. Adjusting the diaphragm diameter, the optimal value of the speckle size and of the reference-to-object beam ratio $r_1$ can be obtained, given the available light power. This figure indicates that a very precise adjustment of the diaphragm diameter is not necessary, since the curves show broad minima. If a phase change measurement is performed, the speckle phase decorrelation error usually becomes more important than the phase error shown by Fig. 3.6 (see Secs. 3.4 and 5.2). The magnitude of the decorrelation error is also related the numerical aperture, as will be shown experimentally in Section 3.4 and will be discussed theoretically in Section 5.2.

**Intensity ratio**

The average intensity ratio $r_1$ of the reference and object beams can be optimized by adjusting the attenuation of those beams. Theoretical analysis\textsuperscript{11} indicates that the maximum intensity modulation is obtained for $r_1 = 2$, but the intensity modulation stays close to its optimal value for ratios much larger than 2. Phase measurements have been performed with $r_1 > 1000$. Adjustment of the numerical aperture, as discussed in the previous paragraph, also changes $r_1$. Separate optimization of the diaphragm diameter and the ratio $r_1$, is only useful if ample light power is available, since for low object beam intensities the reference beam intensity must always be made as high as possible in order to obtain the maximum intensity modulation. If ample light power is available, an attenuation of the object illumination intensity may be needed to minimize the phase error.

### 3.3 Digitizing and processing system

The CCD camera is coupled *pixel-synchronously*\textsuperscript{17} to a frame grabber (Datacube MaxScan) that digitizes the interference patterns to $512 \times 512$ arrays of 8 bit pixels. These can be stored on a 2 Mb image memory board (Datacube ROI-Store) or can be used for calculations directly, by leading the digital output of the framegrabber through pipeline processors (Datacube MaxSP and MaxMux) that can process 16 bit data at video speed
(10 MHz). The hardware is a VME-bus system, controlled by a MC68010-based mother board (Ironics) (see Fig. 3.7). The microcomputer system, having 5 Mb of RAM, operates under UNIX V.2.

![Diagram of digitizing and processing hardware]

Fig. 3.7 Outline of the digitizing and processing hardware.

The application of the special purpose hardware allows digitizing of four phase shifted interference patterns and calculation of the phase change modulo 2π on a 512 × 512 grid in approximately 240 ms. This equals six video frame times: the first four are needed to record the interference patterns and simultaneously calculate the phase, the fifth to subtract the reference phase and the sixth is a time interval needed to bring the PZT back in its starting position and start the next voltage ramp. Thus a refresh rate of 4 Hz is achieved, providing an almost real-time image of the phase change, e.g. for object load adjustment or for monitoring of slowly changing phenomena.

The image processing system software was the TCL-Image software package. This package provides an image processing environment in which numerous basic operations can be performed with simple com
mands. The variables, to be declared by the user, are arrays of arbitrary dimensions. Sophisticated command files can easily be generated, yielding the possibility of creating complex commands. New algorithms (programmed in C) can be added to the package, as was the case for most of the algorithms described in Section 2.6. Finally, an entire measurement cycle, including variation of the illumination direction and deformation of the object, is contained in one single command file with several arguments. These arguments may be: the illumination source locations, the number of averagings, filter sizes, the number of binary iterations and the start location in the image for the phase unwrapping algorithm. This measuring command file can be altered easily to provide special options such as graphical output or repetitions of the entire measurement.

For time consuming floating point operations, such as the LLS filter for the determination of the spatial derivatives (see Sec. 2.6), the data was transferred from the VME microcomputer to a SUN workstation, using Ethernet.

Finally, a small measuring system has been developed based upon a Macintosh IIx microcomputer. This appeared to be a fast and easy-to-use system compared to the VME microcomputer, but lacking the image processing hardware possibilities of the latter. All measurements presented in this thesis were performed with the Macintosh IIx system.

3.4 System characteristics

The characteristics of the experimental system have been investigated. This section presents an overview of the results.

Intensity noise

The noise in the measured intensity can be subdivided depending on the character of its sources. The first category is the noise due to the devices: amplitude noise of the laser and electronic noise in the detector and digitizing system (see Chapter 5). Measurements showed a signal-to-noise ratio (SNR) of the intensity of 200, where the SNR is defined as the maximum
signal divided by the r.m.s. signal noise. The r.m.s. signal noise was measured by subtracting two subsequently recorded images of maximum intensity, calculating the r.m.s. value (intensity repeatability) over all pixels, and dividing the result by $\sqrt{2}$. The second category is the intensity noise resulting from the use of an interferometric system: noise due to changes in the optical path difference between the arms of the interferometer, the causes of which will be discussed more extensively in Chapter 5. The measured SNR for the speckle interferometer, when optimally shielded, was typically 175 (which includes both categories).

**Intensity versus phase**

When shifting the phase in one of the interferometer arms, the intensity will vary sinusoidally at each pixel. Both the modulation intensity and the phase vary strongly from pixel to pixel. Fig. 3.8 shows the measured intensity for three randomly chosen pixels as a function of the voltage applied to the PZT. Without shielding (see Sec. 3.1) of the interferometer from the environment, the results deviated much more from a sinusoidal curve.

![Graph](image)

**Fig. 3.8** The measured intensity (coded in gray values) versus the PZT voltage, measured at three different pixels.
Phase shifting accuracy

The phase shifting accuracy is determined by the accuracy of the motion of the PZT and by unwanted changes in the optical path difference in between the recordings (see Chapter 5). The non-linearity of the PZT and the high voltage ramp generator is specified to be within 1% over 80% of the total traveling range. Since only 25% of the traveling range is used in the case of the four-bucket method, this effect is not a dominant phase shift error source (see Sec. 5.1). Noise in the PZT position and fluctuations of the optical path difference, will cause an error in the average phase shifts for each recording. An indication of the magnitude of this effect, for the four-bucket algorithm, is obtained from a series of measurements of the phase shift $\Delta \alpha$, averaged over all pixels. The phase shift was calculated from the recorded interference patterns using Eq. (2.19). The measurements showed an r.m.s. noise in the average phase shift $\Delta \alpha$ of 0.007 radians.

![Graph](image)

Fig. 3.9 The probability density of the measured modulation intensity.

Modulation intensity

The probability density of the measured modulation intensity $p(I_M')$ depends on several parameters (see Section 2.2). For $r_1 = 2$ and $M = 2$, $p(I_M')$ was determined from measurements, using Eq. (2.51) to determine the modulation intensity in each pixel. Fig. 3.9 shows the result, where
the measured modulation intensity was normalized with respect to its average value.

Phase stability

The unwanted changes in the optical path difference between the object and reference beams cause the phase to be unstable in time. Fig. 3.10 shows a phase measurement over a time span of one hour. The measurement was performed at a (chosen) pixel that showed a high intensity modulation. From Fig. 3.9 the average phase drift rate is calculated as 0.0035 rad/s.

![Graph showing phase vs. time](image)

Fig. 3.10 Measurement of the phase versus time at one pixel.

Phase decorrelation

One of the most important errors in the phase measurement using speckle interferometry is due to speckle phase decorrelation (see Sect. 5.2). This effect causes the phase error in a phase change measurement to be dependent on the displacements of the object surface points.
The resulting r.m.s. phase error was measured by performing a series of phase change measurements with increasing out-of-plane rotation (tilt) and a series with increasing in-plane displacement of the flat object surface. Both series of measurements were performed for various values of the diaphragm diameter. The measured tilt and shift in the phase change were removed by performing a linear least squares fit of a plane phase surface to all data, and subtracting the result from the data. The r.m.s. value of the phase with respect to this plane phase surface can then be calculated over all pixels. Fig. 3.11 displays the result for out-of-plane object rotation. The phase change repeatability is found as the error value for vanishing rotation. The increment in the error for larger displacements results from phase decorrelation only. It can be deduced that for object tilts above $10^{-5}$ radians, the phase decorrelation error is dominant over the phase change repeatability. The phase error increases with decreasing diaphragm diameter size. This effect will be discussed in Section 5.2.
Fig. 3.12 Measurement of the r.m.s. phase error versus the in-plane object translation for various values of the diaphragm diameter. The focal length of the imaging lens was 100 mm, the optical magnification 0.2.

Fig. 3.12 shows the r.m.s. phase change error versus the in-plane displacement for various diaphragm diameter sizes. Compared to the results shown in Fig. 3.11, this phase decorrelation error shows a less steep increase as a function of the displacement for small displacement values. In this case the error decreases with decreasing diaphragm size. The phase decorrelation error is dominant over the phase change repeatability for in-plane object translations above 1 μm.

An averaging technique to diminish the speckle decorrelation error for the ESPI technique (see Sec. 1.2) has been reported\textsuperscript{18}.

**Measurement timing**

The measuring time of a complete surface strain measurement will be discussed in the following lines. All figures refer to $512 \times 512$ pixels images and a SUN3 or Macintosh II computer with floating point coprocessor. The time needed for a measurement of the phase change modulo $2\pi$ is 240 ms if the VME bus system (see Sec. 3.3), and approximately 10 s if the Macintosh IIX system is used. The subsequent image processing steps take up more time, depending on the sizes of the
images and filters. A complete surface strain measurement can be subdivided into a preparation stage and a measuring stage. The preparation stage comprises the following items (typical figures for the time needed are given in brackets):

- Determination of the inverse sensitivity matrix (45 s)
- Phase change measurement for shape determination (60 s)
- Calculation of the shape from the phase change data (100 s)
- Calculation of the transformation matrix (600 s)

Here, the inverse sensitivity matrix was determined in $32 \times 32$ points. The first item is independent of the object and has to be performed only once for a given set of sensitivity vectors. The measuring stage comprises:

- Measurement of the three phase change images (200 s)
- Determination of the displacement vector data (150 s)
- Transformation of the displacement vector data (100 s)
- Calculation of the surface strain components (600 s)

Calculation of the transformation matrix and of the strain components are the most time consuming. This is caused by the use of large filters ($25 \times 25$) to accurately determine the derivatives at each pixel in the displacement image.

**Measuring range**

As was reported in the above account, the minimum measured phase change noise amounts to 0.1 radians r.m.s. (see Fig. 3.6). For the case of an out-of-plane sensitive speckle interferometer, this corresponds to a displacement noise of 4 nm r.m.s., without the application of any filtering or fitting algorithms. The application of a $25 \times 25$ smoothing filter leads to a displacement noise value of about 0.2 nm r.m.s.

For the measurements on the T-shaped object, presented in Secs. 4.1 and 4.3, an area of 3.5 mm$^2$ around each object point (corresponding to $45 \times 45$ pixels) was used to calculate the derivatives. In this case, the repeatability of the measured strain at each pixel, when using a $45 \times 45$ filter to
calculate the derivatives, amounts to approximately 0.3 μstrain r.m.s. If
the systematic errors can be made small enough (see Sec. 5.3), the
estimated measuring inaccuracy approaches the repeatability divided by
\( \sqrt{2} \) for very small loads, because the speckle decorrelation effect then
becomes negligible (see Figs. 3.11 and 3.12).

The upper limit of the measuring range of the surface strain depends on
the size of the object region that is imaged onto the detector, the maximum
number of phase fringes that can still be successfully unwrapped and the
direction of the sensitivity vectors during the phase change measure-
ments. If the angles between the sensitivity vectors and the object surface
normal(s) are made smaller, the measurement will become less sensitive
for in-plane displacements. This means that, as long as the speckle phase
decorrelation effect stays small, the in-plane displacement measuring
range, and thus the surface strain measuring range, is shifted upward
(the upper and lower limits increase by the same factor). Given the direc-
tions of the sensitivity vectors, the upper limit of the surface strain
measuring range is proportional to the maximum acceptable phase
change difference between two pixels. The limits of the surface strain
measuring range are proportional to the optical magnification \( M \) of the
imaging system, as can be deduced from Eqs. (2.47)-(2.49). In our
experimental system using the new phase unwrapping algorithm, over 50
phase fringes across the image could be unwrapped. For a 100 × 100 mm\(^2\)
object region this implies an upper limit of at least 200 μstrain.

3.5 Phase change measurement

To picture the measuring process of the phase change due to a deforma-
tion, rotation or translation of the object, an example is presented in this
section. A solid, flat object was used, of which the out-of-plane rotation
could be adjusted. An artificial circular hole was digitally put in the object
to illustrate the ability of the algorithms to cope with shadows, object holes
and edges (in general: invalid areas). The basis of the measurement is the
digitizing of two sets of four phase shifted interference patterns corre-
sponding to the unrotated and the rotated states. Fig. 3.13 displays one of
those speckle interference patterns. From each of those two sets of four interference patterns the phase is calculated, using the four-bucket algorithm described in Section 2.1. The invalid pixels are determined as described in Section 2.6. Fig. 3.14 displays the speckle phase modulo $2\pi$ radians and the corresponding mask, containing the invalid pixels. Because the optical arrangement was adjusted to give relatively large speckles (Fig. 3.13), the speckle structure can be recognized in the phase image of Fig. 3.14. For smaller speckles the left hand image of Fig. 3.14 would show a much finer pattern, comparable to uniformly distributed noise.

![Image of phase and mask](image)

**Fig. 3.13** One out of a set of four phase shifted speckle interference patterns.

![Calculated speckle phase and mask](image)

**Fig. 3.14** Calculated speckle phase modulo $2\pi$ coded in gray values (left, invalid pixels white) and the corresponding mask (right, invalid pixels shown black). The gray value range corresponds to a phase range of $2\pi$ radians.
Subtraction of the speckle phase data, calculated from both sets, yields the phase change. Performing a logical OR on the masks related to the phase measurements before and after deformation, yields a new mask. Invalid pixels are shown white in the phase change image, shown in Fig. 3.15. To picture the large amount of $2\pi$-steps that occur in a speckle phase change measurement, the data in this figure was rescaled: the gray value range corresponds to a $4\pi$ phase range here. The new mask can be used by the phase unwrapping algorithm. After unwrapping the phase, a continuous phase map is obtained and the mask is updated, adding pixels of which the phase deviates too much from the neighboring values (see Sec. 2.6). This updated mask is used to restore the phase change data by bilinear interpolation. Fig. 3.16

Fig. 3.15 The phase change coded in gray values.

Fig. 3.16 The unwrapped (continuous) phase change (left) and the restored continuous phase change (right), both coded in gray values.
presents the resulting unwrapped phase change and its restored version. This figure illustrates the removal of small clusters of invalid pixels, while the large cluster (background, here the object hole) remains invalid.

If a high accuracy phase change measurement is required, the LLS filtering algorithm (see Sec. 2.6) can be applied to the restored phase change of Fig. 3.16. The results for two different filter sizes are shown in Fig. 3.17. The benefit of using this algorithm, compared to a general uniform filtering algorithm (averaging), is observable near the object hole, where no systematic deviation from the expected phase change values is visible. To emphasize the continuity of the calculated displacement data, Fig. 3.18 shows the right hand result of Fig. 3.17 for different scaling factors. Three-dimensional plots of the restored phase change of Fig. 3.16 and its filtered edition (filter size $25 \times 25$) of Fig. 3.17, are presented in Fig. 3.19. The phase noise reduction (expected factor: 25, i.e the square root

![Fig. 3.17](image1.png)![Fig. 3.17](image2.png)

Fig. 3.17 Filtered phase change data using filter sizes $5 \times 5$ (left) and $25 \times 25$ (right).

of the number of pixels inside the filter window) is clearly seen, while the edges around the hole in the object remain equally sharp. If further calculations using the acquired data are needed (e.g. to determine the strain components), it is time saving to postpone the filtering until the final result has been obtained.
Fig. 3.18  Filtered phase change shown in the right hand image of Fig. 3.17 using different scaling factors

Fig. 3.19  Restored phase change (above) and the filtered phase change (filter size 25 X 25; below).
3.6 References

4.0 Introduction

Phase shifting speckle interferometry as described in this thesis can be applied to engineering metrology and non-destructive testing\textsuperscript{1-8}. In order to evaluate the experimental system detailed in Chapter 3, several test objects were used. Shape, deformation and strain measurements were performed on them, according to the theory described in Chapter 2. Several results have been compared to predictions from the Finite Element Method\textsuperscript{9}.

When performing strain analysis, an important choice has to be made with respect to the excitation technique. Applying a force at one point of the object is the most straightforward technique and was used to study the surface strain of a simple T-shaped object (cantilever beam). For a hollow object, e.g. a vessel or a bottle, internal pressure loading is an obvious technique. Another method is heating or cooling of the object, either the entire surface or just a particular area.

One object used for studying the characteristics of the experimental system is a T-shaped aluminium plate (Fig. 4.1). Use of a flat object enabled the measurement of the surface strain components without measuring the object shape. The load establishing the deformation is set by attaching a specified weight to the tip of the object arm.

An solid copper cube was used to illustrate the application of the shape measurement. The determination of all surface strain components and the complete three-dimensional displacement will be demonstrated with a pressurized glass bottle as the object.
This chapter will present the measurement results for the surface displacement vector field of loaded objects (Sec. 4.1). In Section 4.2 an example of object shape detection and an application of this technique in surface strain measurement is shown and discussed. The results for all surface strain components of a loaded object are presented in Section 4.3. Section 4.4 offers a comparison of the results of the surface strain measurement on the T-shaped object with its Finite Element Method predictions and Section 4.5 an outlook towards the combination of phase shifting speckle interferometry and the Finite Element Method.

4.1 Displacement vector measurement at a loaded object

The first measurements presented in this section were performed on the T-shaped aluminum test object. The deformation was introduced by loading the T-shaped object with a weight of 0.5 kg at the point indicated in Fig. 4.1. Three phase change measurements, corresponding to three different illumination directions, were performed to determine the displacement
vector field, according to the theory presented in Section 2.3. The three orientation angles of the rotatable illumination arm, corresponding to three independent sensitivity vectors, were -20, 90 and 200 degrees relative

![Graph showing phase versus position for different angles.](image)

**Fig. 4.2** Measured phase (unfiltered) versus position along the central horizontal line within the region of interest (Fig. 4.1), for three different angles of the illumination arm.

![Graph showing displacement components versus position.](image)

**Fig. 4.3** Calculated object displacement components (unfiltered) $l_u$, $l_v$, and $l_w$ versus position along the central horizontal line within the region of interest.
to the positive x-axis. Fig. 4.2 gives the measured phase change values along the central horizontal line within the region of interest indicated in Fig. 4.1. The corresponding object displacement values are plotted in Fig. 4.3.

The complete results of the phase change measurements are presented in Fig. 4.4 for each illumination direction, where the phase change values are coded in gray values: the range from black to white corresponds to a phase change of $2\pi$ radians. The background (invalid pixels contained in the final mask; see Sec. 3.5) has been marked white. These three images represent the (continuous) phase change along the direction of the corresponding sensitivity vectors.

In order to calculate orthogonal displacement vector components, as shown in Fig. 4.3, the sensitivity matrices are determined.

**Fig. 4.4** Three phase change measurements coded in gray values; the images correspond to illumination arm angles of -20 (top), 90 (center) and 200 degrees (bottom). The gray value range is $2\pi$ radians.
Measurement of the three illumination source positions, the imaged object area dimensions and the distance between the object and the lens pupil, yields the input parameters for the automated calculation of the sensitivity matrix corresponding to each pixel (see Appendix A).

The phase change data, shown in Fig. 4.4, is first transformed by assuming zero phase change in the same (chosen) pixel in all three images. In this case the calculated phase change was set to zero at a pixel located in the body of the object close to the axis of the object arm. From the phase change data the displacement vector components have been calculated by determining the inverse sensitivity matrix, followed by application of Eq. (2.30) in all pixels. The obtained values will only be useful if the pertinent pixel is a valid one in all three phase change measurements.

Fig. 4.5 The displacement vector components coded in gray values: $I_u(x,y)$ (top), $I_v(x,y)$ (center) and $I_w(x,y)$ (bottom). The gray value ranges correspond to 500 nm displacement.
To this aim the three masks, corresponding to the three phase change measurements, are logically OR-ed. The results of the calculation of the displacement vector components are shown in Fig. 4.5, in which the images were scaled to make the gray value range equal to a displacement range of 500 nm. The somewhat higher noise level of the in-plane displacement components \( l_u(x,y) \) and \( l_v(x,y) \), as compared to the out-of-plane component \( l_w(x,y) \), is caused by the small \( u \)- and \( v \)-components of the sensitivity vectors. This can be improved by elongating the object illumination arm or by decreasing the object-to-detector distance. The former solution implies an undesirable reduction of the mechanical stability of the interferometer, whereas the latter implies the necessity of increasing the optical field of view (see Sec. 3.2). A third possibility to reduce the noise level is the application of digital image processing, as will be shown in the next paragraph. The result of the out-of-plane displacement measurement would be expected to exhibit zero displacement at the axis of the object arm (neutral line) and negative and positive displacements in the upper and lower parts of the arm, respectively. The slight deviation of the neutral line in the result for \( l_w(x,y) \) (Fig. 4.3 and bottom picture of Fig. 4.5) can be explained by torsion of the arm and out-of-plane displacement of the arm tip, caused by non-axial loading.

![Fig. 4.6](image-url)  
**Fig. 4.6** The filtered displacement components \( l_u(x,y) \) and \( l_v(x,y) \) (from Fig. 4.5; 25 × 25 LLS filter applied). The gray value range is 250 nm.
To reduce the noise, the LLS algorithm discussed in Sec. 2.6 can be applied to the data of Fig. 4.5. The results are shown in Fig. 4.6 for the x- and y-components of the displacement, since only those will be used for the surface strain calculations presented in Section 4.3. These components of the in-plane displacement vector can be presented by a vector plot, shown in Fig. 4.7. In this figure, the rotation of the object material can be observed.

Fig. 4.7 In-plane displacement vector plots for two different scaling factors.

For the example of Section 4.3 concerning the determination of all surface strain components of a three-dimensional object, the surface displacement vector field of a deformed glass bottle was measured. The bottle was repressurized for each phase change measurement. Three measurements were performed for three different illumination directions. Measurements on both the entire bottle as well as on part of it were performed. The latter permitted a closer view which yielded more details. The top left image of Fig. 4.8 displays the result of a phase change measurement of the entire bottle. Due to low reflection, and consequently a large fraction of invalid pixels, the edges of the object have become notched in the results of the bottle measurements.

This section will further focus on the close view measurements, the results of which will be used in Section 4.3.
At the right hand side of Fig. 4.8, images of the surface displacement components are shown, as they were calculated from three phase change measurements using the same procedure as described for the T-shaped object measurements. The applied pressure was $10^5$ N/m$^2$.

Fig. 4.8 Typical phase change measurement of the entire bottle (top left). The right hand side shows the displacement vector components of part of it: $l_1(x,y)$ (top), $l_2(x,y)$ (center) and $l_4(x,y)$ (bottom). The LLS filter size was $25 \times 25$. The gray value ranges correspond to 250 nm displacement.
For the calculation of the surface strain components, the displacement components relative to the local coordinate system in each object surface point have to be determined. As described in Section 2.5, this can be carried out by using the measured shape function \( w(x,y) \) to calculate the unit vectors of the local coordinate system relative to the object coordinate system \( u,v,w \). These unit vectors then define the transformation matrix \( T(x,y) \) according to Eq. (2.43). The result of the shape function measurement on the bottle, as presented in the next section, was used to determine \( T(x,y) \). The displacement components in the local coordinate system, shown in Fig. 4.9, were calculated by transforming the displacement data from Fig. 4.8 with the computed transformation matrix.

**Fig. 4.9** The displacement vector components in the local coordinate system: \( l_u'(x,y) \) (top), \( l_v'(x,y) \) (center) and \( l_w'(x,y) \) (bottom). The gray value ranges correspond to 500 nm displacement.
4.2 Shape measurement at an object

As described in Section 2.4, measurement of the object shape requires the illumination source to be displaced. For the measurements presented in this section, this was performed by locating a planar glass plate in front of the illumination arm, and rotating this plate over a fixed angle. The displacement components of the illumination source have to be known in order to calculate the object shape (Eqs. (2.38)-(2.40)). This is achieved by tilting the flat object, used for the reference phase change measurement (see Sec. 2.4), over a known angle and determining the illumination source displacement for which the calculated shape corresponds best with the applied tilt.

The first measurements presented in this section were performed on a solid cube, having the dimensions $20 \times 20 \times 20$ mm$^3$. Fig. 4.10 shows the result of a reference and an object phase change measurement and their difference $\Delta \psi(x,y)$.

Fig. 4.10 The reference phase change (top), object phase change (center) and their difference (bottom). The gray value ranges equal $2\pi$ radians.
The object shape function \( w(x,y) \) was calculated from the phase change difference \( \Delta \psi(x,y) \) using the Eqs. (2.38)-(2.40). Fig. 4.11 displays the measured shape function, coded in gray values. Fig. 4.12 shows two three-dimensional plots of part of the object shape: the original measurement and the filtered data. The application of the LLS filtering algorithm smooths the edges of the cube, as can be observed from Fig. 4.12.

Fig. 4.11 The calculated object shape function \( w(x,y) \) coded in gray values (filter size \( 9 \times 9 \)). The gray value range is 2.5 mm.

Fig. 4.12 The object shape without (left) and with (right; filter size \( 9 \times 9 \)) the application of the LLS filtering algorithm.
Apart from its intrinsic value, object shape measurement can also be used for surface strain measurement on three-dimensional objects. In that case, the aim of the measurement is the determination of the local orientation of the object surface, needed to calculate the matrix $T(x,y)$ for transformation of the displacement vector components from the object coordinate system $u,v,w$ to the local coordinate system $u',v',w'$ (see Sec. 2.5). The local surface orientation can be determined by taking the spatial derivatives of the object shape function $w(x,y)$ (Eq. (2.41)) and calculating the unit vectors of the local coordinate system (Eq. (2.42)). Determination of the derivatives of $w(x,y)$ is performed by applying the LLS fitting algorithm, that fits a plane surface to the neighboring data of each pixel and outputs the (analytical) derivatives of that surface (see Sec. 2.6). An indication of both calculated derivatives of the cube shape function (see Fig. 4.11) is given in Fig. 4.13. The deviation of the expected result (a constant value for each cube face) is mainly caused by the quantization into a restricted number of gray values for display purposes. The smooth transition of the derivatives values at the cube edges is introduced by the LLS filter (see Sec. 5.4).

Fig. 4.13 A qualitative display (the data was contrast stretched) of the spatial derivatives of the shape function $m_u(x,y)$ (left) and $m_v(x,y)$ (right). A $25 \times 25$ filter was used in the LLS algorithm.
The shape function of the part of the glass bottle for which displacement measurements were presented in Section 4.1, was calculated following the same procedure. The results were used for the determination of the transformation matrix $T(x,y)$. Fig. 4.14 presents a three-dimensional plot of the measured shape.

![Three-dimensional plot of the measured shape](image)

**Fig. 4.14** Shape of part of the glass bottle (25 × 25 LLS filter applied).

### 4.3 Surface strain measurement at a loaded object

The surface strain measurements have been performed both on flat and three-dimensional objects. In the latter case the strain analysis is much more complicated, since the object shape must be determined and coordinate system transformations have to be performed.
Fig. 4.15  The normal strain $e_{xx}(x,y)$ (left) and $e_{yy}(x,y)$ (right), calculated using a $35 \times 35$ filter. The gray value range corresponds to 25 µstrain.

In the case of the flat T-shaped object, the calculation of the strain components is straightforward. The LLS fitting algorithm can be used to determine the $u$- and $v$-derivatives of both the $u$- and $v$-components of the displacement. Fig. 4.15 shows the normal surface strain components $e_{xx}(x,y)$ and $e_{yy}(x,y)$. Fig. 4.16 displays the shear strain component $e_{xy}(x,y)$. Fig. 4.17 presents a plot of the shear strain values along a vertical line over the object body, about 0.1 mm from the edge.

Fig. 4.16  The shear strain component $e_{xy}(x,y)$, calculated using a $35 \times 35$ filter. The gray value range corresponds to 10 µstrain.
Fig. 4.17 Calculated shear strain values versus position along a vertical line about 0.1 mm from the object edge.

Calculation of the surface strain components has been performed on the part of the pressurized glass bottle, a displacement vector and a shape measurement of which were presented in Sections 4.2 and 4.3, respectively. The resulting surface strain components are presented in Fig. 4.18. From this figure can be concluded that the surface strain distribution is not as smooth as would be expected for perfectly homogeneous glass of constant thickness, since this figure barely reveals any symmetry.
Fig. 4.18  The surface strain components of the pressurized bottle, calculated using a $35 \times 35$ filter: $e_{xx}(x,y)$ (top left), $e_{yy}(x,y)$ (top right), $e_{xy}(x,y)$ (bottom left) and the rotation (bottom right). The strain gray value ranges corresponds to 10 μstrain, the rotation gray value range to 10 μrad.
4.4 Comparison with FEM predictions

Displacement and strain measurement results for the T-shaped object were compared to Finite Element Method predictions, using the ANSYS43 software package. The object was simulated by a mesh of two-dimensional elements as shown in Fig. 4.19. Young's modulus was set to $7.0 \times 10^{10}$ Nm$^{-2}$ and Poisson's ratio to 0.3. The predicted and measured x-components of the displacement are displayed in Fig. 4.20, using a contour plot. The predicted and measured normal strains in the x-direction and the corresponding shear strains are shown in Fig. 4.21. All results show good qualitative agreement with the measurement results. The predicted displacement

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Fig. 4.19 The element mesh used for the FEM calculations.

Fig. 4.20 The predicted (right) and measured (left) in-plane displacement component $l_x(x,y)$. The contour spacing corresponds to 100 nm.
and strain values showed a systematic deviation of approximately 10% from the measured ones. This can be explained from the occurrence of systematic measurement errors (Sec. 5.3) and from the fact that Young's modulus of the used material was not accurately known. The FEM results in Figs. 4.20 and 4.21 were rescaled to correct for that deviation. The maximum discrepancy, after rescaling, between the predicted and the measured values can be estimated from these figures. For the displacement data this yields approximately 50 nm, on a total range of 1 μm. For the strain data the discrepancy is in the order of 5 μstrain, on a total range

Fig. 4.21 The predicted (right) and measured (left) surface strain components $e_{xx}(x,y)$ (top) and $e_{xy}(x,y)$ (bottom). The contour spacing corresponds to 10 μstrain.
of 80 μstrain. The mesh element size was chosen to be in the same order of magnitude as the filter sizes used in the determination of the measured results. This entails that fine details are not revealed in the measurement results, nor in the predictions. An improvement of the resolution would increase the computation time for the FEM predictions (more elements) and the noise in the measurement results (smaller filters).

In conclusion, the comparison presented in this section confirms the ability of phase shifting speckle interferometry to measure displacement vector components and surface strain values.

4.5 Combination of speckle interferometry and FEM

Phase shifting speckle interferometry is an excellent technique to determine the displacement and strain at the object surface. The Finite Element Method is also capable of calculating displacement and strain in the interior of a solid object or at invisible locations of a complex mechanical structure. However, FEM predictions are reliable only if the proper description of the object structure, the correct boundary conditions and the actual material parameters are given. Unexpected structure defects or inhomogeneous material cause the predictions to deviate from the actual object state. For the description of the actual object structure, interferometric shape measurement may be useful. Boundary conditions, such as the surface displacement, can be measured by phase shifting speckle interferometry. In that case, the FEM calculations are used to extrapolate the measured surface displacement data into the object interior. Thus, the techniques described in this thesis can be used to generate an important part of the input data for a FEM calculation.

A second possible feature of the combination of speckle interferometry and the FEM is the use of the latter to generate information needed to perform a successful interferometric measurement. Especially in the case of complex mechanical structures, the determination of the magnitude and the location of the deforming loads is not straightforward. The aim is to specify such loads that, at the object areas of interest, the phase change can be
measured accurately. To achieve this, the phase change must be large with respect to the sensitivity of the measuring system and small enough to prevent the phase decorrelation from disturbing the measurement (the phase unwrapping algorithm can handle a limited number of phase fringes across the image: see Sec. 3.4). A simplified description of the object structure, in which only a limited number of mesh elements are involved, will be sufficient to investigate several loading schemes, leading to a prescription for the execution of the measurement.

To facilitate the interaction between FEM and phase shifting speckle interferometry, both techniques should be joined into a single software package. This would make both the exchange of data and the comparison of results practical.

4.6 References

Chapter 5

Accuracy considerations

5.0 Introduction

The practical limitation on the error of an interferometric measurement is usually much higher than its fundamental limit\textsuperscript{1,2}. The accuracy of a shape, displacement or strain measurement using phase shifting speckle interferometry is determined by numerous factors, the most important of which will be discussed in this chapter. As was expected, the experimental measuring system appeared to be highly sensitive to undesired optical path length fluctuations. Consequently, the optical arrangement was made rigid and, as far as possible, shielded mechanically and aerodynamically from the environment. Optical shielding is only necessary if the ambient light level at the detector cannot be neglected with respect to the background intensity $I_B(x,y)$ of the interference pattern. Furthermore, the measuring accuracy will be determined by the noise and stability of the various components of the measuring system. In most cases, the main source of phase change errors is the decorrelation of the speckle phase caused by rotation and translation of the object surface. Finally, the necessary adjustments and settings performed by the system operator always introduce inaccuracies, leading to systematic errors in the result of a measurement. In the experimental results presented in Chapter 4, the accuracy of the measurements is mainly determined by speckle decorrelation and systematic errors.

In Table 5.1 the order of magnitude of the random and systematic errors for displacement, shape and deformation measurement are given. These values refer to the experimental system and image processing techniques that were described in this thesis. They were deduced from the experimental results by determining the reproducibility and by comparison to
FEM predictions. The values in Tables 5.1 and 5.2 correspond to an object deformation or shape that causes several phase fringes to occur across the image. In particular, the error values for shape and strain measurement depend heavily on the object dimensions. The listed values are valid for object dimensions in the order of 0.1 m.

Table 5.1 Order of magnitude of random and systematic errors in displacement, shape and strain measurement for the experimental system presented in this thesis.

<table>
<thead>
<tr>
<th></th>
<th>displacement</th>
<th>shape</th>
<th>strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>random</td>
<td>1 nm</td>
<td>100 μm</td>
<td>1 μstrain</td>
</tr>
<tr>
<td>systematic</td>
<td>2 %</td>
<td>5 %</td>
<td>5 %</td>
</tr>
</tbody>
</table>

If a high accuracy is required, the errors can be decreased by additional averaging techniques and by use of superior detectors, electronics and positioning equipment. For that case, estimates of the expected error lower limits are given in Table 5.2.

Table 5.2 Estimates of the feasible order of magnitude of random and systematic errors in displacement, shape and strain measurements.

<table>
<thead>
<tr>
<th></th>
<th>displacement</th>
<th>shape</th>
<th>strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>random</td>
<td>0.2 nm</td>
<td>10 μm</td>
<td>0.2 μstrain</td>
</tr>
<tr>
<td>systematic</td>
<td>0.2 %</td>
<td>0.2 %</td>
<td>0.5 %</td>
</tr>
</tbody>
</table>

The sources of the errors in Table 5.1 will be discussed in this chapter. Section 5.1 presents a general description of the accuracy of phase shifting interferometry, followed by three sections discussing more specifically the speckle phase decorrelation phenomenon, the occurrence of systematic errors and the influence of the applied image processing techniques on the measuring accuracy (Secs. 5.2, 5.3 and 5.4).
5.1 Accuracy of phase shifting interferometry

All phase shifting algorithms described in Sec. 2.1 assume the background intensity $I_B(x,y)$ and the modulation intensity $I_M(x,y)$ to be constant during the measurement of one set of phase shifted interference patterns. Furthermore, they assume the phase shifts $\alpha_i$ to be equally spaced and well-known. Thus, assuming a linear characteristic of the detector (which is the case for CCD cameras), the accuracy of a single phase measurement using the phase shifting technique is determined by the intensity noise and by the accuracy of the phase shifts. As discussed in Section 3.4, the noise in the measured intensity can be subdivided into two categories: noise due to the devices used in the experimental system and intensity noise due to unwanted changes in the optical path difference between the arms of the interferometer (caused by vibration and changes in the environmental conditions). Assuming the general case of uniform optical path difference changes, the latter effect will be considered as an error in the phase shifts $\alpha_i$.

**Intensity noise**

If the phase shifts have the correct values, the standard deviation of the phase noise $\sigma_\theta(x,y)$ in each pixel is determined by the ratio of the standard deviation of the intensity noise $\sigma_I$ and the measured modulation intensity $I_M'(x,y)$. This can be seen by rewriting Eq. (2.15) as $\varphi = \tan^{-1}[N_I/D_I]$, where $N_I$ is the numerator and $D_I$ the denominator. In the case of the four-bucket method the standard deviation of the noise in both $N_I$ and $D_I$ equals $\sqrt{2}\sigma_I$, and $\sigma_\theta(x,y)$ satisfies

$$
\sigma_\theta(x,y) = \sqrt{\left[\frac{D_I}{N_I^2 + D_I^2}\right]^2 + \left[\frac{N_I}{N_I^2 + D_I^2}\right]^2} \sqrt{2}\sigma_I.
$$

(5.1)

If Eq. (2.51) is rewritten as $4I_M' = N_I^2 + D_I^2$, Eq. (5.1) becomes

$$
\sigma_\theta(x,y) = \frac{\sigma_I}{\sqrt{2}I_M'(x,y)}.
$$

(5.2)
This relation implies that the modulation intensity has to be maximized and the intensity noise has to be minimized in order to achieve low phase noise. The intensity noise standard deviation was measured as 1.25, expressed in gray values (SNR=200, 8-bit AD-converter, see Secs. 3.3 and 3.4). If ample light power is available the average modulation intensity can be up to 35 gray values. This yields a lower limit for the r.m.s. phase noise $\sigma_\phi$ of 0.03 radians.

The intensity noise is due to the amplitude noise in the laser, electronic noise in the detector, a digitizing error in the frame grabber and a quantization error. All of these are determined by the quality of the components in the measuring system, and defined by their specifications. For speckle interferometry, video jitter can also be important. Since the interference pattern exhibits high spatial frequencies, this jitter in the digitizing system will cause intensity fluctuations. Pixel-synchronous coupling of the CCD camera and the frame grabber has been applied to minimize this effect. The remaining fluctuations due to video jitter are part of the measured intensity noise as mentioned in the previous paragraph.

**Intensity averaging method**

The electronic noise of the detector was the major intensity noise source in the experimental system. Cooling of the detector is not suitable, because of the corresponding longer exposure times, prolonging the total time needed to record four phase shifted interference patterns. This could lead to unacceptable errors in the phase shift between subsequent recordings, caused by drift in the phase. The same holds for digitizing each interference pattern many times and applying digital averaging. To prevent these excessive phase shift errors, complete sets of phase shifted interference patterns can be recorded many times and the corresponding intensities can be summed. This essentially means that both the numerator and the denominator in Eq. (2.9) are averaged, assuming that the phase shift $\Delta \alpha$ keeps its proper value. The phase drift now influences the phase for all intensity recordings within each set in approximately the same way, leading to a modification of Eq. (2.9), using Eq. (2.4):
\[
\varphi(x) - \varphi_0 = \tan^{-1}\left[ \frac{\sum c_{2,i}(x)}{\sum c_{1,i}(x)} \right] = \tan^{-1}\left[ \frac{\sum \sin(\varphi(x) + \varphi_i)}{\sum \cos(\varphi(x) + \varphi_i)} \right],
\]

(5.3)

where the subscripts \(i\) refer to the measurement of the \(i\)-th set of interference patterns, \(\varphi_i\) is the random phase drift value for the \(i\)-th measurement, \(\varphi_0\) is a constant phase value and the summations are taken over all \(i\)’s. Proof of Eq. (5.3) is given in Appendix D. In the case of the four-bucket method (Eq. (2.15)), the numerator in Eq. (5.3) becomes \((\Sigma I)_1 - (\Sigma I)_3\) and the denominator \((\Sigma I)_4 - (\Sigma I)_2\), where e.g. \((\Sigma I)_1\) denotes the sum of all \(I_1\)’s of the various sets. Equation (5.3) shows that this method of intensity averaging, regardless of the phase drift, will accurately yield the phase \(\varphi(x)\) relative to an unknown constant phase \(\varphi_0\). The \(x\)-dependent part of \(\varphi(x)\) is unaffected by the phase drift values \(\varphi_i\) (they only affect the value of \(\varphi_0\)). However, the drift of the phase in time lowers the measured modulation intensity, and thus affects the measuring accuracy unfavorably. If the phase drift extends over more than \(\frac{1}{2}\pi\) radians, this effect will become considerable. To solve this problem, a special intensity averaging technique was developed. Its algorithm is based upon the fact that when a phase drift in, for instance, the interval \([\frac{1}{2}\pi, \frac{3}{2}\pi]\) radians has occurred during the recording of the sets of interference patterns, it is convenient to add the following intensities \(I_j(x,y)\) to the sums of all previously recorded intensities \(I_{j+1}(x,y)\). This is only applicable in the case of evenly spaced phase steps over \(2\pi\) radians (see Eq. (2.12)), because \(I_N(x,y)\) can then be added to the sum of the previously recorded intensities \(I_1(x,y)\). This technique causes the modulation intensity to remain high, even for large phase drift values. The implementation of this averaging technique is carried out along the following lines. The differences between the intensities in a subset of the recorded interference pattern \(I_1\) and the corresponding intensities in the four initially recorded interference patterns \(I_{i,0}\) are calculated. The pattern \(I_{i,0}\) that yields the lowest r.m.s.
Table 5.3  Intensity sum to which $I_i$ must be added when using the special intensity averaging technique. $I_{i,0}$ denotes the initially measured interference patterns and $(\Sigma I)_i$ the i-th sum of interference patterns. If no phase drift occurs, $(\Sigma I)_i$ denotes the sum of all intensities $I_i$. Otherwise one or more steps in the indexes of the summed intensities are applied. The $(x,y)$ dependences were omitted.

<table>
<thead>
<tr>
<th>measured intensity $\rightarrow$</th>
<th>$I_1$</th>
<th>$I_2$</th>
<th>$I_3$</th>
<th>$I_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{1,0}$</td>
<td>$(\Sigma I)_1$</td>
<td>$(\Sigma I)_2$</td>
<td>$(\Sigma I)_3$</td>
<td>$(\Sigma I)_4$</td>
</tr>
<tr>
<td>$I_{2,0}$</td>
<td>$(\Sigma I)_2$</td>
<td>$(\Sigma I)_3$</td>
<td>$(\Sigma I)_4$</td>
<td>$(\Sigma I)_1$</td>
</tr>
<tr>
<td>$I_{3,0}$</td>
<td>$(\Sigma I)_3$</td>
<td>$(\Sigma I)_4$</td>
<td>$(\Sigma I)_1$</td>
<td>$(\Sigma I)_2$</td>
</tr>
<tr>
<td>$I_{4,0}$</td>
<td>$(\Sigma I)_4$</td>
<td>$(\Sigma I)_1$</td>
<td>$(\Sigma I)_2$</td>
<td>$(\Sigma I)_3$</td>
</tr>
</tbody>
</table>

value of the difference determines the sum of intensities to which $I_1$ will be added. Table 5.3 gives the sums of intensities to which the measured intensities must be added for the four cases that can occur. The resulting numerator and denominator (see Eq. (5.3)) can be divided by an arbitrary number without changing the phase $\varphi(x,y)$ calculated from them. Two practical procedures can be used:

1. Division by the number of averagings: the intensity noise is reduced while the modulation intensity stays the same.
2. Division by the square root of the number of averagings: the measured intensity noise stays the same while the modulation intensity is amplified.

When using the two-dimensional look-up table, described in Section 2.6, the second procedure is used since the size of the circular area in that table is related to the measured intensity noise. Another averaging procedure has been reported previously by Creath\(^3\).

Modulation intensity

The remaining problem is to maximize the measured modulation intensity $I_{M'}(x,y)$ for a single measurement of four phase shifted interference pat-
terns. The values of $I_M'(x,y)$ in Eq. (5.2) are related to the values of the intensity of the speckle field $I_S(x,y)$. If a constant intensity $I_R$ of the reference beam and a point-shaped detector are assumed, this relation is derived from Eqs. (2.10) and (2.24) as

$$I_M'(x,y) = \text{sinc}[\frac{\Delta}{2}] \frac{2\sqrt{I_R I_S(x,y)}}{\sqrt{I_S(x,y)}}.$$  \hspace{1cm} (5.4)

In this case the probability density of the modulation intensity is determined by the probability density of the speckle intensity (see Sec. 2.2) through Eq. (5.4). If detectors with larger dimensions are considered, the description becomes more complicated. Then several speckles, having different phases and modulation intensities, can influence a single detector element. This means that, statistically, $I_M'(x,y)$ will be less than the value following from Eq. (5.4). For a large number of speckles within a detector element area, the average measured modulation intensity will be inversely proportional to the square root of that number.

In order to maximize $I_M'(x,y)$, the average measured modulation intensity is considered as a function of two parameters: the numerical aperture NA and the available power of the light scattered by the object. Given the available intensity at the pupil, the optimal value of NA can be determined. Fig. 5.1 shows the measured results for three values of the available light power. For all measurements the fraction of saturated pixels was smaller than 1%. This figure should be compared to Fig. 3.6, that shows the measured phase error as a function of the same parameters. The curves in these figures are clearly correlated, as would be expected from Eq. (5.2). From both figures can be concluded that for lower available laser power the lens pupil size has to be increased. This is caused by the fact that the decrease in modulation intensity caused by the increase of the number of speckles influencing a single pixel, is less than the increase in the modulation intensity due to the increase in the average intensity of the speckle field.
Fig. 5.1  The measured average intensity modulation, expressed in gray values, versus the numerical aperture NA for various values of the illuminating power. $<I_p>$ is the average intensity in the pupil plane. $I_o$ is the intensity in the pupil plane that corresponds to the saturation level of the detector for NA = 0.10.

Phase shift errors

The error in the phase shifts can be subdivided in a linear error, a non-linear error and random errors\textsuperscript{4,5,6}. The linear error is caused by a constant phase drift rate and by the inaccuracy in the adjustment of the high voltage ramp that drives the PZT for the phase shifts. The non-linear error is caused by the non-linearities of the piezo-electric element and by the non-linear part of the phase drift rate. Finally the random errors are caused by vibration, air turbulence, temperature fluctuations, laser frequency noise and noise in the PZT position.

The linear error was determined by calculating the average phase shift $\Delta\alpha$ from the recorded interference patterns (see Sec. 3.4). In the case of the four-bucket algorithms described in Section 2.2, the r.m.s. phase error approximately equals $(2\sqrt{2})^{-1}$ times the deviation of $\Delta\alpha$ from its appropriate value of $\frac{1}{4}\pi$ radians (linear approximation used\textsuperscript{6}). Measurements indicated an r.m.s. noise in the average phase shift $\Delta\alpha$ of 0.007 radians (see Sec. 3.4). Phase shift errors up to 0.03 radians were observed during the
measurements described in Chapter 4, yielding an r.m.s. phase error of approximately 0.01 radians. In the experimental system four phase-shifted interference patterns are digitized within 160 ms (four video frame times) to minimize the influence of fluctuations of the optical path difference in the interferometer. Shielding the optical arrangement from the environment (see Sec. 3.1) appeared to improve the results of the measurements considerably. Without shielding the r.m.s phase shift error was typically an order of magnitude larger.

The non-linear error in the PZT position is specified to be within 1%. In the case of the four-bucket algorithm described in Section 2.2, the r.m.s. phase error approximately equals 0.01 times the non-linearity percentage\(^7\), yielding an upper limit for the r.m.s. phase error of 0.01 radians.

Like all phase shift errors, the magnitude of the random phase shift errors is largely dependent on the environmental conditions and the effectiveness of the shielding from the environment of the interferometer. From the calculated influence\(^6\) on the measured phase of the deviation of one of the phase shifts \(\alpha_i\) can be deduced that the phase error standard deviation equals \(1/\sqrt{2}\) times the random phase shift error standard deviation in the case of the four-bucket method. This error is part of the intensity noise measured as described in Section 3.4. The random phase shift errors have not been measured. If they are estimated to be in the same order of magnitude as the r.m.s noise in the average phase shift \(\Delta\alpha\) (see Sec. 3.4), the resulting phase error standard deviation is in the order of 0.01 radians.

The errors described in this section for the case of the four-bucket method, yield an r.m.s. noise in the measured phase of approximately 0.02 radians.

5.2 Speckle phase decorrelation

Displacement of the object surface points causes different scattering contributions to be collected by the optical system and a different area to be imaged onto a particular detector element. This leads to decorrelation of the speckle intensity and phase. Decorrelation caused by a change of the
laser wavelength will not be discussed in this thesis. Since phase shifting interferometry is a phase measuring method, the phase decorrelation is of prime interest. In this section two separate cases will be considered: the phase change error caused by an out-of-plane rotation of the object surface and the error resulting from an in-plane translation of the surface. For both cases theoretical predictions will be given and compared with the experimentally obtained data presented in Section 3.4.

In order to describe the speckle phase decorrelation phenomenon, the complex correlation coefficient \( \mu_{ab} \), specifying the correlation between the speckle pattern before and after the deformation, is introduced. Assuming no defocus and small object displacements, \( \mu_{ab} \) can be expressed as

\[
\mu_{ab} = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(p) P(p-A_p) e^{\frac{ik \cdot p \cdot A_d}{r_0}} d^2p}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P^2(p) d^2p},
\]

(5.5)

where \( p \) is the position vector in the pupil plane having the components \( p_x \) and \( p_y \) and \( P(p) \) is the pupil function of the imaging system. The vectors \( A_d \) and \( A_p \) describe the speckle displacement in the detector and pupil planes, respectively. The relation between the r.m.s. value of the phase change error \( \sigma_\psi \) and the correlation coefficient \( \mu_{ab} \) is given by

\[
\sigma_\psi = \sqrt{\frac{\pi^2}{3} - \pi \sin^{-1}|\mu_{ab}| + (\sin^{-1}|\mu_{ab}|)^2 - \sum_{n=1}^{\infty} \frac{\mu_{ab}^{2n}}{2n^2}}.
\]

(5.6)

An accurate approximation of this relation has been found as

\[
\sigma_\psi = \frac{\pi}{\sqrt{3}} (1 - |\mu_{ab}|)^{0.42}.
\]

(5.7)
This expression was used for the following calculations in this section.

In the next account the illumination and viewing directions are assumed to be parallel to the axis of the imaging system. The pupil function \( P(p) \) equals unity inside and zero outside the circular aperture, which means that the numerator in Eq. (5.5) reduces to the aperture area. Two cases will be considered here: out-of-plane object rotation and in-plane object translation.

**Out-of-plane rotation**

In the case of out-of-plane rotation only, the transverse components of \( A_d \) and \( A_p \) equal

\[
A_{dx} = 0, \quad A_{dy} = 0,
\]

\[
A_{px} = \frac{2z_0 \Omega_y}{M}, \quad A_{py} = \frac{2z_0 \Omega_x}{M},
\]

where \( \Omega_x \) and \( \Omega_y \) are the \( x \)- and \( y \)-components of the rotation vector. If \( \Omega_y = 0 \), Eq. (5.5) yields, without loss of generality

\[
|\mu_{ab}| = \frac{2}{\pi} \left[ \cos^{-1}\left( \frac{\Omega_x}{M \ NA} \right) - \frac{\Omega_x}{M \ NA} \sqrt{1 - \left( \frac{\Omega_x}{M \ NA} \right)^2} \right].
\]

This expression indicates that, for a fixed magnification \( M \), this speckle phase decorrelation error is heavily dependent on the numerical aperture of the imaging system \( NA \). Fig 5.2 shows the calculated r.m.s. phase error versus the out-of-plane rotation for various diaphragm diameters of the imaging system, using the Eqs. (5.7) and (5.9). This figure shows good qualitative agreement with the measured values of Fig. 3.10. The fact that the measured phase errors are smaller than those calculated, can be explained by the difference in focal length and by considering the phase unwrapping algorithm (see Sec. 2.6): large phase decorrelation errors are treated as \( 2\pi \)-steps and are thus corrected.
Fig. 5.2  The r.m.s. phase error versus the out-of-plane rotation for various diaphragm diameters of the imaging system (focal length is 80 mm, M = 0.2).

**In-plane translation**

In the case of in-plane translation only, the transverse components of $A_d$ and $A_p$ equal

$$
A_{dx} = -M l_x, \quad A_{dy} = -M l_y, \\
A_{px} = l_x, \quad A_{py} = l_y,
$$

(5.10)

To simplify calculation, a square aperture of dimensions D is assumed. If $l_y = 0$, Eq. (5.5) reduces without loss of generality to

$$
\mu_{ab} = \frac{1}{D} \int_{-\infty}^{\infty} P(p_x) P(p_x-l_x) e^{-ikMp_xl_x} dp_x,
$$

(5.11)

from which $|\mu_{ab}|$ can be approximated if $l_x << D$ ($|\text{Im}(\mu_{ab})| << |\text{Re}(\mu_{ab})|$):
\[ |\mu_{ab}| = \frac{1}{D} \int_{D + l_x}^{D} \cos \left( \frac{k M p x l_x}{z_0} \right) dp_x \]

\[ = \frac{1}{2k M l_x NA} \left[ \sin \left( k M l_x NA \right) + \sin \left( k M l_x \left[ NA - \frac{l_x}{z_0} \right] \right) \right]. \tag{5.12} \]

Thus, for a fixed magnification $M$, this speckle phase decorrelation error is again heavily dependent on the numerical aperture of the imaging system $NA$. In contrast with the expression for the out-of-plane rotation case (Eq. (5.9)), however, the lens-to-detector distance $z_0$ also influences the decorrelation error. This contradicts other theoretical results\textsuperscript{11}. For most practical cases, $z_0$ can be approximated by the focal length of the imaging lens. Fig. 5.3 shows the calculated r.m.s. phase change error versus the in-plane displacement for various diaphragm diameters of the imaging system, obtained with Eqs. (5.7) and (5.12). This figure also shows a good qualitative agreement with the measured values of Fig. 3.11. Again, the difference in focal length and the phase unwrapping algorithm are responsible for the fact that the measured phase errors are smaller than those calculated.

Fig. 5.3  The r.m.s. phase error versus the the in-plane displacement for various diaphragm diameters of the imaging system (focal length is 80 mm, $M = 0.2$).
Because the change in the light contributions collected by the imaging system has a randomly distributed phase, the phase error caused by speckle decorrelation is randomly distributed, too, and therefore can be reduced by applying a smoothing filter like the LLS filter (see Sec. 2.6).

It should be noted, that the quality of the lens affects the performance of the speckle interferometer also, in particular with respect to the phenomenon of speckle decorrelation\(^1\).

5.3 Systematic errors

The systematic phase errors that are discussed in literature\(^2\) for the general case of phase shifting interferometry, are not systematic in the case of a speckle interferometer. Due to the random speckle phase those phase related errors become randomly distributed errors. Nevertheless, several systematic errors can be introduced during the measuring procedure. The systematic errors introduced by the image processing algorithms will be discussed in the next section.

A systematic error such as the inaccuracy in the determination of the sensitivity vectors from the geometry of the interferometer will affect the calculated displacement vector components directly through Eq. (2.30). Without the use of sophisticated distance and orientation measuring instruments, the geometrical information needed to establish the sensitivity vectors (see Appendix A) can be obtained with a resolution of approximately 1 mm. To gain insight in the expected magnitude of the corresponding errors in those vectors, Eq. (A.1) is considered. Assuming object dimensions that are very small relative to the illumination distance, an illumination source position of e.g. (250,250,1000) in (relative to the center of the object) mm's and a viewing distance of 1000 mm, the largest errors will occur in the components \(s_u(x,y)\) and \(s_v(x,y)\). Those errors can be estimated, using Eq. (A.1), as 0.5 % of the sensitivity vector components, in linear approximation leading to the same relative error in the displacement vector components. A small error (e.g. 1 %) in the determination of the optical magnification \(M\) will cause a minor
systematic error in the sensitivity vectors. If the assumption that the object dimensions are much smaller than the illuminating distance is not valid, object shape information will have to be included in the determination of the sensitivity vectors. Another option is the use of known object rotations for that determination.\(^{13}\)

The systematic error mentioned in the previous paragraph will affect the shape function, defined in Eqs. (2.38)-(2.40), in approximately the same way. In addition, as can be deduced from those equations, an error in the determination of the illumination source displacement vector $\Delta r_i$, used to calculate the object shape, will cause the same relative error in the shape function. For the measurements presented in Section 4.2, the components of $\Delta r_i$ were determined by calibration of the object shape measurement against a known shape (flat object tilt, see Sec. 4.2). Repeated execution of this calibration showed that the largest component is $\Delta u_i = 105 \mu m$ with a standard deviation of $2 \mu m$. This can yield a relative error in the shape function as large as 6\% (three times the standard deviation), if the calibration is performed only once. If accurate shape measurement is required, the use of a better controlled displacement mechanism, e.g. a PZT, is recommended.

Apart from the sensitivity matrix, an error in the determination of the optical magnification $M$ also influences the transformation matrix $T(x,y)$ (Eq. (2.41)) and the calculated strain values (Eqs. (2.47)-(2.49)). The latter values are proportional to $M$, again leading to a relative error in the order of 1\%. As can be deduced from Eqs. (2.41)-(2.43), four elements of $T(x,y)$ are practically proportional to $M$. This yields a maximum error (due to this effect) in the components of $I'(x,y)$ of 1\%, relative to the components of $I(x,y)$. The main systematic error in $T(x,y)$, however, will be caused by the error in the spatial derivatives of the shape function $w(x,y)$ (see Eq. (2.41)). Those errors can become extremely large, especially in the vicinity of object corners (see Secs. 4.2 and 5.4).
5.4 Image processing errors

The main errors introduced by the image processing algorithms described in this thesis will be concisely discussed in this section. A more elaborate treatment of these errors will be presented in a separate thesis by Vrooman\textsuperscript{14}.

The phase unwrapping algorithm can introduce large erroneous pixel clusters in the phase data if a wrong decision is made in the $2\pi$-step detection at a single pixel. Such a cluster, usually with a phase of $2\pi$ above or below the surrounding phase values, arises from the interdependence of neighboring pixel values with respect to $2\pi$-step detection: once an error is introduced at one pixel it can be passed on to neighboring pixel values. If one or more of those areas occur in the unwrapped phase, it cannot be used for further processing since the phase change data is not completely continuous: the phase unwrapping failed. Failure of the phase unwrapping algorithm is due to large phase decorrelation noise and to steep phase change slopes (high phase fringe density). The extended algorithm, presented in Section 2.6, appeared to be able of unwrapping up to 50 phase fringes, corresponding to a phase change of $100\pi$, across a $512 \times 512$ image.

Two phase restoration algorithms were discussed in Section 2.6. The first one simply replaces the value of invalid pixels by the value of one valid neighbor. This can introduce a small systematic phase error, its magnitude depending on the local slope of the phase surface. In the case of 10 phase fringes across a $512 \times 512$ image, a maximum systematic error of approximately 0.1 radians can occur in the restored pixels. The second algorithm uses the average of the neighboring valid, unwrapped pixels to correct the invalid pixels. In this case, the only systematic phase errors occur at the object edges and are in the same order of magnitude as for the previous algorithm.

The LLS algorithm, used for smoothing and calculation of derivatives, introduces errors equivalent to those introduced by the application of standard convolution filters if no invalid pixels are present. Since a first order (flat) surface is fitted to the data, higher orders in the data will cause
the output of the algorithm to deviate from the proper values. This entails that variations with a high spatial frequency in the data will be attenuated. The attenuation increases with the filter size and the spatial frequency. The noise reduction, the actual purpose of the algorithm, is proportional to the square root of the number of pixels inside the filter window. Consequently, for the case of square windows, the noise reduction is proportional to the filter size. The special feature of the LLS algorithm is its capability of handling invalid pixels. Standard convolution filters would introduce excessive errors in the vicinity of invalid pixels, e.g. the object edge. The LLS algorithm however, performs almost equally well near object edges: the error is just slightly increased by the fact that less valid pixels are available within the filter window. For a straight object edge for example, only half the number of pixels is valid, leading to an increase of the error by a factor \sqrt{2}. The inaccuracy in the derivatives calculated by the algorithm, however, can increase significantly at the object edges. While for a curved second order surface the calculated values are correct if no invalid pixels are present, a systematic deviation occurs at the object edges (see Fig. 5.4). In that case the calculated strain values are less reliable near those edges.

![Diagram](image)

**Fig. 5.4** Schematic illustration of the error introduced at object edges when the LLS algorithm is used to calculate the derivative at a pixel touching the object edge. The spatial derivative is determined by the slope of the fitted surface.
5.5 References

Chapter 6

Conclusion

A phase shifting speckle interferometer combined with new image processing techniques has been developed, tested and successfully applied to the measurement of the three-dimensional displacement vector field and surface strain components of deformed solid objects. An existing shape measuring method was extended to enable fast and accurate determination of the object shape, using the same experimental system.

The speckle interferometer has been designed such that the illumination direction could easily be altered in order to change the sensitivity vector. Optical fibers are applied to enable the use of a large Argon ion laser source, mechanically disconnected from the interferometer. The optimization of the interferometer unexpectedly demonstrated that the speckles have to be made smaller than the optical resolution of the detector in the case of low object beam intensity.

The application of special purpose hardware in a VME-bus computer system allows digitizing of four phase shifted interference patterns and calculation of the phase change modulo $2\pi$ on a $512 \times 512$ grid in approximately 240 ms. The first step towards a more compact, transportable apparatus was set by developing a Macintosh II based measuring system.

The developed image processing algorithms for phase-shifted speckle interference patterns have proven to be very robust with respect to both temporal and decorrelation noise. This robustness is mainly due to the pixel queue based phase unwrapping algorithm and its extension that introduces a restricted tolerance for $2\pi$-step detection. The ability to distinguish the object from the background has improved the flexibility of the processing of phase-shifted speckle interference patterns with regard to
the restrictions on the shape of the object to be investigated. The feature of the developed LLS fitting algorithm to handle invalid areas in the data, has introduced the possibility to apply arbitrary filter window sizes without blurring the object edges. The special intensity averaging procedure enabled successful phase change measurements, even in the case of extremely low average object beam intensities.

The theoretical and experimental analysis of the phase change decorrelation error shows that for in-plane object displacements the speckle size should be made as large as possible, while for out-of-plane object rotation as small as possible. The analysis also indicates that, besides the case of extremely small displacements, the speckle phase decorrelation error is dominant. This error can be reduced significantly by application of the LLS fitting filter, leading to a displacement sensitivity in the order of 1 nm. For objects with dimensions around 0.1 m this yields a strain sensitivity of approximately 1 μstrain. The sensitivity of a shape measurement is in the order of 100 μm.

The comparison of Finite Element Method predictions with the measured results showed good qualitative agreement for both displacement and strain. A systematic deviation of 10 % was observed. Random discrepancies of up to 15 % of the total range for both the displacement components and the surface strain were deduced from the results.

Much work remains to be done; some suggestions for further research are:

1. Adaptation of the described measuring methods to larger engineering constructions.
2. Development of a spatial phase stepping speckle interferometer, enabling the use of a pulsed laser source.
3. Investigation of the full potential of the two-bucket or single step technique for speckle interferometry.
4. Replacement of the Argon ion laser source by a convenient compact semi-conductor laser.
5. Expansion of the applicability of the described measuring methods for industrial non-destructive testing.
A Determination of the sensitivity matrix

For calculation of the sensitivity vectors the assumption is made that the object dimensions are much smaller than the distance between the object and the illumination source. In that case the object surface can be taken approximately flat, because the object shape does barely influence the sensitivity vector. The \( u,v,w \) coordinate system used in Section 2.4 will be applied here. The sensitivity vector components defined by Eq. (2.27) can be written as

\[
\begin{align*}
  s_u(x,y) &= \frac{2\pi}{\lambda} \left[ \frac{(u_i - x)}{d_i(x,y)} - \frac{(x)}{d_v(x,y)} \right], \\
  s_v(x,y) &= \frac{2\pi}{\lambda} \left[ \frac{(v_i - y)}{d_i(x,y)} - \frac{(y)}{d_v(x,y)} \right], \\
  s_w(x,y) &= \frac{2\pi}{\lambda} \left[ \frac{w_i}{d_i(x,y)} + \frac{w_v}{d_v(x,y)} \right],
\end{align*}
\]

(A.1)

where \((0,0,w_v)\) indicates the position of the center of the imaging lens, \(d_i(x,y)\) is defined by Eq. (2.40) and the distance \(d_v(x,y)\) between the center of the imaging lens and the object surface is

\[
  d_v(x,y) = \sqrt{\frac{x^2}{M^2} + \frac{y^2}{M^2} + w_v^2}.
\]

(A.2)
Using the Eqs. (A.1) and (A.2), the sensitivity vector components corresponding to each detector point (x,y) can be estimated. The sensitivity matrix can now be determined by evaluating these expressions for each of the three illumination source positions. The sensitivity vectors, and consequently the inverse sensitivity matrix, are continuous functions of the coordinates (x,y). Therefore, it is not necessary to determine the inverse sensitivity matrix elements at each detector point by direct calculation: sampling of the data and interpolation can be applied. A sampling interval of 8 pixels was chosen for the measurements presented in Chapter 4. In the case of 512 × 512 images, accurate determination of the inverse sensitivity matrix on a 32 × 32 square grid, followed by a second order interpolation technique, will usually yield a sufficiently accurate value in each pixel.

B Calculation of the object shape using the modified two-illumination-source method

From Eq. (2.36) the exact object shape w(u,v) can be determined. The intersection of the object surface with the z-axis defines the origin of the u,v,w-system. The imaging system, having a magnification factor M, focuses onto the detector plane. Eq. (2.36) can be rewritten as:

\[
\psi(x,y) = \frac{2\pi}{\lambda} \left[ \frac{(x/M + \beta_u w - u_i) \Delta u_i + (y/M + \beta_v w - v_i) \Delta v_i + (w - w_i) \Delta w_i}{\sqrt{\left(\frac{x}{M} + \beta_u w - u_i\right)^2 + \left(\frac{y}{M} + \beta_v w - v_i\right)^2 + (w - w_i)^2}} \right],
\]

(B.1)

where \(\beta_u\) and \(\beta_v\) are the angles between the viewing direction for each object surface point and the v,w- and u,w-plane, respectively. The (x,y) dependences of \(\beta_u\), \(\beta_v\) and w have been omitted from Eq. (B.1). Determining \(w(x,y)\) from this equation yields a corresponding object surface point in the u,v,w-space for each detector point (x,y), in accordance with: \((x/M + \beta_u(x,y) w(x,y), y/M + \beta_v(x,y) w(x,y), w(x,y))\).
The absolute phase change $\psi(x,y)$ cannot be determined from a single measurement, because a multiple of $2\pi$ can always be added or subtracted. Besides, in an actual measurement the phase will always exhibit a certain amount of drift $\psi_d$, that is constant over the image. From the definition that $w(0,0) = 0$, the value of phase offset $\psi_o = \psi_d + 2\pi N$ can be determined. Introducing $\psi_m(x,y)$ as the measured phase change, Eq. (B.1) can be written as

$$
\psi_m(x,y) + \psi_o = \frac{2\pi}{\lambda} \left[ \frac{a_0(x,y) w(x,y) + a_1(x,y)}{\sqrt{a_2(x,y) w^2(x,y) + a_3(x,y) w(x,y) + a_4(x,y)}} \right],
$$

(B.2)
in which

$$
a_0(x,y) = \beta_u(x,y) \Delta u_i + \beta_v(x,y) \Delta v_i + \Delta w_i,
$$

$$
a_1(x,y) = \left( \frac{x}{M} - u_i \right) \Delta u_i + \left( \frac{y}{M} - v_i \right) \Delta v_i - w_i \Delta w_i,
$$

$$
a_2(x,y) = \beta_u^2(x,y) + \beta_v^2(x,y) + 1,
$$

(B.3)

$$
a_3(x,y) = 2(\frac{x}{M} - u_i)\beta_u(x,y) + 2(\frac{y}{M} - v_i)\beta_v(x,y) - 2w_i,
$$

$$
a_4(x,y) = \frac{x^2 + y^2}{M^2} - 2 \left[ \frac{u_i x + v_i y}{M} \right] + u_i^2 + v_i^2 + w_i^2.
$$

Using Eq. (B.2), the phase offset $\psi_o$ can be determined from:

$$
\psi_o = \frac{a_1(0,0)}{\sqrt{a_4(0,0)}} - \psi_m(0,0).
$$

(B.4)

Now, Eq. (B.2) can be solved for each point $(x,y)$.  

C Estimation of the phase step in a 3-bucket method

Suppose that three phase shifted interference patterns \( I_i \) have been measured:

\[
I_0(x,y) = I_B(x,y) + I_M(x,y) \cos(\phi(x,y) - \Delta \alpha),
\]

\[
I_1(x,y) = I_B(x,y) + I_M(x,y) \cos(\phi(x,y)),
\]

\[
I_2(x,y) = I_B(x,y) + I_M(x,y) \cos(\phi(x,y) + \Delta \alpha).
\]  

(C.1)

To estimate the phase shift \( \Delta \alpha \), we now calculate the quantities \( T_1(x,y) \) and \( T_2(x,y) \) as

\[
T_1(x,y) = I_2(x,y) - I_0(x,y),
\]

\[
T_2(x,y) = I_2(x,y) - 2I_1(x,y) + I_0(x,y).
\]  

(C.2)

Using (C.1), the Eqs. (C.2) can be rewritten as

\[
T_1(x,y) = -2I_M(x,y) \sin(\phi(x,y)) \sin(\Delta \alpha),
\]

\[
T_2(x,y) = 2I_M(x,y) [\cos(\phi(x,y)) \cos(\Delta \alpha) - \cos(\phi(x,y))].
\]  

(C.3)

If the phase \( \phi(x,y) \) is uniformly distributed, e.g. in the case of a speckle field, averaging over the detector of \(|T_1(x,y)|\) can be replaced by an integration over all possible values of \( \phi \), yielding

\[
\langle |T_1| \rangle = \frac{1}{2\pi} \int_0^{2\pi} |T_1| \, d\phi
\]
= -\frac{2}{\pi} \langle I_M \rangle \sin(\Delta \alpha) \int_{\pi}^{2\pi} \sin(\varphi) \, d\varphi

= \frac{4}{\pi} \langle I_M \rangle \sin(\Delta \alpha) .

(C.4)

Similarly, for $T_2(x,y)$ follows:

$$
\langle |T_2| \rangle = \frac{4}{\pi} \langle I_M \rangle (1 - \cos(\Delta \alpha)).
$$

(C.5)

Both expressions are valid for $0 < \Delta \alpha < \pi$ radians and if $I_M(x,y)$ is not related to $\varphi(x,y)$. The latter condition is valid in the case of speckle interferometry, since no correlation between the intensity and the phase in a speckle field exists (see Sec. 2.2). The $(x,y)$ dependences have been omitted, since the integrations in the Eqs. (C.4) and (C.5) are essentially performed over all detector points. Defining the ratio $r_T$ as

$$r_T = \frac{\langle |T_1| \rangle}{\langle |T_2| \rangle} = \frac{\sin(\Delta \alpha)}{1 - \cos(\Delta \alpha)},
$$

(C.6)

the phase shift $\Delta \alpha$ can be expressed as:

$$
\Delta \alpha = \arccos \left[ \frac{r_T^2 - 1}{r_T^2 + 1} \right].
$$

(C.7)
D  Proof of the intensity averaging expression (5.3)

The proof of Eq. (5.3),

\[ \varphi(x) = \varphi_0 + \tan^{-1} \left[ \frac{\sum \sin(\varphi(x) + \varphi_i)}{\sum \cos(\varphi(x) + \varphi_i)} \right], \]  

(D.1)

can be given by rewriting (using basic trigonometric relations) this equation as:

\[ \varphi(x) = \varphi_0 + \tan^{-1} \left[ \frac{C \sin(\varphi(x)) + E \cos(\varphi(x))}{C \cos(\varphi(x)) - E \sin(\varphi(x))} \right], \]  

(D.2)

where

\[ C = \sum \cos(\varphi_i), \]

\[ E = \sum \sin(\varphi_i). \]  

(D.3)

Now, Eq. (D.2) can be transformed into

\[ \varphi(x) = \varphi_0 + \tan^{-1} \left[ \frac{\sqrt{C^2 + E^2} \sin(\varphi(x)) - \tan^{-1} \left[ \frac{E}{C} + \frac{1}{2} \pi \right]}{\sqrt{C^2 + E^2} \cos(\varphi(x)) - \tan^{-1} \left[ \frac{E}{C} + \frac{1}{2} \pi \right]} \right], \]  

(D.4)

which is an identity provided that the constant phase \( \varphi_0 \) satisfies

\[ \varphi_0 = \tan^{-1} \left[ \frac{\sum \cos(\varphi_i)}{\sum \sin(\varphi_i)} \right] - \frac{1}{2} \pi. \]  

(D.5)
Consequently Eq. (5.2) holds, with a value of $\phi_0$ determined by the phase drift values $\phi_i$ only.
## List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$3 \times 3$ matrix</td>
<td></td>
</tr>
<tr>
<td>$A_d$</td>
<td>detector plane speckle displacement vector</td>
<td>m</td>
</tr>
<tr>
<td>$A_p$</td>
<td>pupil plane speckle displacement vector</td>
<td>m</td>
</tr>
<tr>
<td>a</td>
<td>parameter</td>
<td></td>
</tr>
<tr>
<td>$a_0,a_3$</td>
<td>parameter</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$a_1,a_4$</td>
<td>parameter</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>parameter</td>
<td>m$^2$</td>
</tr>
<tr>
<td>B</td>
<td>$1 \times 3$ matrix</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>parameter</td>
<td></td>
</tr>
<tr>
<td>$B_M$</td>
<td>binary mask</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>parameter</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>C</td>
<td>parameter</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>intensity symbol, $i = 0, 1, 2$</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>D</td>
<td>diameter of lens pupil</td>
<td>m</td>
</tr>
<tr>
<td>$D_i$</td>
<td>intensity denominator</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$d_i$</td>
<td>distance illumination source to object surface</td>
<td>m</td>
</tr>
<tr>
<td>$d_s$</td>
<td>speckle size</td>
<td>m</td>
</tr>
<tr>
<td>$d_v$</td>
<td>distance center of imaging lens to object surface</td>
<td>m</td>
</tr>
<tr>
<td>E</td>
<td>parameter</td>
<td></td>
</tr>
<tr>
<td>$e_{xx}$</td>
<td>normal strain in $u'$-direction</td>
<td></td>
</tr>
<tr>
<td>$e_{xy}$</td>
<td>shear strain</td>
<td></td>
</tr>
<tr>
<td>$e_{yy}$</td>
<td>normal strain in $v'$-direction</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>phase symbol</td>
<td>rad</td>
</tr>
<tr>
<td>g</td>
<td>gray value</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>intensity of interference pattern</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$I_0$</td>
<td>average intensity of speckle pattern</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$I_B$</td>
<td>background intensity of interference pattern</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$i_d$</td>
<td>unit vector in viewing direction</td>
<td></td>
</tr>
</tbody>
</table>
$I_i$ intensity of i-th recording of interference pattern \( W \text{ m}^{-2} \)

$i_i$ unit vector in illumination direction \( \text{m}^{-1} \)

$I_{M}$ modulation intensity of interference pattern \( W \text{ m}^{-2} \)

$I_{M'}$ measured modulation intensity of interference pattern \( W \text{ m}^{-2} \)

$I_R$ uniform reference beam intensity \( W \text{ m}^{-2} \)

$I_S$ intensity in speckle field \( W \text{ m}^{-2} \)

$k_i$ wave vector of illumination beam \( \text{m}^{-1} \)

$l$ displacement vector \( \text{m} \)

$l'$ displacement vector in local coordinate system \( \text{m} \)

$M$ magnification factor of imaging system

$\mathcal{M}$ number of speckles influencing the measured intensity

$m_u$ object surface slope in u-direction

$m_v$ object surface slope in v-direction

$N$ number of recordings

$N_i$ intensity numerator \( W \text{ m}^{-2} \)

$NA$ numerical aperture of lens

$N_{it}$ number of iterations

$P$ pupil function

$P$ \( 3 \times 3 \) matrix \( \text{rad}^2 \text{ m}^{-2} \)

$p$ probability density \( \text{m}^2 \text{ W}^{-1} \)

$p_x$ pupil plane position vector x-component \( \text{m} \)

$p_y$ pupil plane position vector y-component \( \text{m} \)

$Q$ \( 1 \times 3 \) matrix \( \text{rad}^2 \text{ m}^{-1} \)

$r$ distance between two points in speckle field at detector \( \text{m} \)

$r$ position at object surface \( \text{m} \)

$R_I$ spatial autocorrelation function for the intensity \( \text{W}^2 \text{ m}^{-4} \)

$r_i$ position of object illumination source \( \text{m} \)

$r_{T}$ ratio of average absolute values of $T_1$ and $T_2$

$S$ sum of squares for LLS fit

$s$ sensitivity vector \( \text{rad} \text{ m}^{-1} \)

$T$ transformation matrix $u,v,w-$ to $u',v',w'$-system

$T_i$ intensity quantities, $i = 1, 2$ \( W \text{ m}^{-2} \)

$\nu$ normalized modulation intensity (fringe visibility)

$x_P$ size of detector element \( \text{m} \)

$z_0$ lens pupil to detector plane distance \( \text{m} \)
### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>average phase shift for i-th recording</td>
<td>rad</td>
</tr>
<tr>
<td>$\beta_u$</td>
<td>angle between viewing direction and v,w-plane</td>
<td>rad</td>
</tr>
<tr>
<td>$\beta_v$</td>
<td>angle between viewing direction and u,w-plane</td>
<td>rad</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>phase shift during recording (integration interval)</td>
<td>rad</td>
</tr>
<tr>
<td>$\Delta \alpha$</td>
<td>phase shift between subsequent recordings</td>
<td>rad</td>
</tr>
<tr>
<td>$\Delta \mathbf{k}$</td>
<td>change of illumination wave vector corresponding to $\Delta \mathbf{r}_i$</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>$\Delta \mathbf{r}_i$</td>
<td>displacement of object illumination source</td>
<td>m</td>
</tr>
<tr>
<td>$\Delta x_R$</td>
<td>deviation of ideal reference beam source position</td>
<td>m</td>
</tr>
<tr>
<td>$\Delta \psi$</td>
<td>difference phase change and reference phase change</td>
<td>rad</td>
</tr>
<tr>
<td>$\Delta z_{PZT}$</td>
<td>translation of piezo-electric translator</td>
<td>m</td>
</tr>
<tr>
<td>$\Delta z_R$</td>
<td>deviation of ideal reference beam source position</td>
<td>m</td>
</tr>
<tr>
<td>$\phi$</td>
<td>phase difference between interfering beams at detector</td>
<td>rad</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>constant phase value</td>
<td>rad</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>phase drift i-th measurement</td>
<td>rad</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>phase difference spherical wave and reference beam</td>
<td>rad</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength of light used in interferometer</td>
<td>m</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>standard deviation of intensity noise</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>standard deviation of phase noise</td>
<td>rad</td>
</tr>
<tr>
<td>$\sigma_\psi$</td>
<td>standard deviation of phase change noise</td>
<td>rad</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>column matrix containing phase change values</td>
<td>rad</td>
</tr>
<tr>
<td>$\psi$</td>
<td>phase change at the detector</td>
<td>rad</td>
</tr>
<tr>
<td>$\psi_d$</td>
<td>phase drift</td>
<td>rad</td>
</tr>
<tr>
<td>$\psi_m$</td>
<td>measured phase change</td>
<td>rad</td>
</tr>
<tr>
<td>$\psi_o$</td>
<td>phase off-set</td>
<td>rad</td>
</tr>
<tr>
<td>$\psi_r$</td>
<td>reference phase change measurement</td>
<td>rad</td>
</tr>
<tr>
<td>$\mu_{ab}$</td>
<td>correlation coefficient of speckle patterns</td>
<td></td>
</tr>
<tr>
<td>$\Omega_x$</td>
<td>object rotation vector x-component</td>
<td></td>
</tr>
<tr>
<td>$\Omega_y$</td>
<td>object rotation vector y-component</td>
<td></td>
</tr>
<tr>
<td>$(m,n)$</td>
<td>position in image relative to (x,y)</td>
<td></td>
</tr>
<tr>
<td>$(u',v',w')$</td>
<td>local coordinate system at object surface</td>
<td></td>
</tr>
<tr>
<td>$(u,v,w)$</td>
<td>coordinate system in object space</td>
<td></td>
</tr>
<tr>
<td>$(u_i,v_i,w_i)$</td>
<td>position of object illumination source</td>
<td></td>
</tr>
<tr>
<td>$(x,y)$</td>
<td>position in detector plane or image</td>
<td></td>
</tr>
<tr>
<td>$(x,y,z)$</td>
<td>coordinate system at detector</td>
<td></td>
</tr>
</tbody>
</table>
Author Index


This thesis describes the development of an interferometric measuring method and its application in the determination of mechanical properties of a solid object, like shape, deformation and strain.

Chapter 1 presents the basic principles of holographic and speckle interferometry and several methods for producing quantitative data from interference patterns. Single and multiple interferogram analysis are described. A discussion of the advantages and disadvantages of the various techniques precedes a summary of fields in which speckle interferometry is being applied at present and new fields where it is expected to be useful.

Chapter 2 elaborates on the general theory of the phase shifting technique, followed by an overview of the statistics associated with the speckle phenomenon. A theoretical description is given of the determination of the displacement vector field and surface strain of a deformed object from measured phase data. A novel object shape measuring technique is described, followed by a concise introduction in image processing and a presentation of new image processing techniques, which have been developed especially for the analysis of phase shifted speckle interference patterns.

Chapter 3 describes the phase shifting speckle interferometer used for the experiments. The optimization of the interferometer will be discussed, followed by a description of the computer system, the measuring system characteristics and a phase change measurement.

Chapter 4 presents the measurement results for the displacement vector field and surface strain of loaded objects. An example of object shape detection and an application of this technique in surface strain measurement is
shown and discussed. The results of the measurements on a T-shaped object are compared with Finite Element Method predictions and an outlook is given towards the combination of phase shifting speckle interferometry and the Finite Element Method.

Chapter 5 gives a general description of the accuracy of phase shifting interferometry, followed by a discussion of the speckle phase decorrelation phenomenon, the occurrence of systematic errors and the influence of the applied image processing techniques on the measuring accuracy.

The speckle interferometer has been designed such that the illumination direction could easily be altered in order to change the sensitivity vector. Optical fibers are applied to enable the use of a large Argon ion laser source, mechanically disconnected from the interferometer. The application of special-purpose hardware in a VME-bus computer system allows for calculation of the phase change modulo $2\pi$ on a $512 \times 512$ grid in approximately $240$ ms. A more compact measuring system was developed based upon a Macintosh II computer.

The developed image processing algorithms proved to be very robust with respect to temporal and decorrelation noise. The ability to distinguish the object from the background has minimized the restrictions on the shape of the object. The developed filtering algorithm allows for the application of arbitrary filter window sizes without blurring the object edges. A special averaging procedure enabled successful phase measurements in the case of extremely low object beam intensities.

It has been demonstrated that, to optimize the phase measuring accuracy, the speckles have to be made smaller than the optical resolution of the detector in the case of low object beam intensity. The analysis of the phase change decorrelation error shows that this generally is the dominant inaccuracy. Minimizing is achieved by making the speckle size small for in-plane object displacements and large for out-of-plane object rotation. Further reduction can be obtained by filtering, leading to a displacement sensitivity in the order of $1$ nm. For objects of dimensions about $0.1$ m this
yields a strain sensitivity of approximately 1 μstrain. The sensitivity of a shape measurement is around 100 μm.

The comparison of Finite Element Method predictions with the measured results showed good qualitative agreement for both displacement and strain. A systematic deviation of 10% was observed. Random discrepancies of up to 15% of the total range for both the displacement components and the surface strain were deduced from the results.
Dit proefschrift beschrijft de ontwikkeling van een interferometrische meetmethode en de toepassing ervan op het bepalen van mechanische eigenschappen van een massief voorwerp, zoals vorm, vervorming and rek.

Hoofdstuk 1 presenteert de grondbeginselen van holografische en spikkel-interferometrie en van enkele methoden voor kwantitatieve analyse van interferentiepatronen. De analyse van één enkel interferogram en die van meerdere interferogrammen wordt beschreven. Een bespreking van de voor- en nadelen van de verschillende technieken wordt gevolgd door een opsomming van de vakgebieden waar spikkelinterferometrie nu wordt toegepast en van nieuwe vakgebieden waar deze techniek nuttig zou kunnen worden.

Hoofdstuk 2 introduceert the algemene theorie van de fase-geschoven techniek, gevolgd door een overzicht van de statistiek verbonden met het spikkel fenomeen. Een beschrijving van de theorie voor het bepalen van het verplaatsingsvektorveld en de oppervlakterek van een vervormd voorwerp uit de gemeten fase data wordt gegeven. Een nieuwe meettechniek voor de vorm van een voorwerp wordt beschreven, gevolgd door een korte inleiding in de beeldbewerking en een presentatie van de beeldbewerkingstechnieken die speciaal voor de analyse van fase-geschoven spikkelinterferentiepatronen ontwikkeld zijn.

Hoofdstuk 3 beschrijft de fase-geschoven spikkelinterferometer die gebruikt is in de experimenten. De optimalisatie van de interferometer zal besproken worden, gevolgd door een beschrijving van het computersysteem, van de eigenschappen van het meetsysteem en van een meting van de faseverandering.

Hoofdstuk 5 geeft een algemene beschrijving van de nauwkeurigheid van fase-geschoven interferometrie, gevolgd door een bespreking van het verschijnsel van dekorrelatie van de spikkel fase, de voorkomende systematische fouten en de invloed van de toegepaste beeldbewerkingstechnieken op de meetnauwkeurigheid.

De spikkelinterferometer is zodanig ontworpen, dat de belichtingsrichting eenvoudig gewijzigd kan worden, teneinde de gevoeligheidsvektor te kunnen veranderen. Optische fibers zijn toegepast om het gebruik van een Argon-ion laser, mechanisch ontkoppeld van de interferometer, mogelijk te maken. De toepassing van special-purpose hardware in een VME-bus computersysteem maakt de berekening van de fase modulo $2\pi$ op een $512 \times 512$ grid in 240 ms mogelijk. Een kompakter meetsysteem is ont- wikkeld, gebaseerd op een Macintosh II computer.

De ontwikkelde beeldbewerkingsalgoritmen bleken zeer robuust te zijn met betrekking tot zowel tijdsafhankelijke als dekorrelatie. De mogelijk- heid om het voorwerp van de achtergrond te onderscheiden heeft de aan het voorwerp te stellen beperkingen geminimaliseerd. Het ontwikkelde filter algoritme staat willkeurige filter afmetingen toe zonder de randen van het voorwerp onscherp te maken. Een speciale middelingsprocedure maakt succesvolle fase metingen bij extreem lage intensiteit van de voor- werpsbundel mogelijk.
Aangetoond wordt dat, ten einde de meetnauwkeurigheid van de fase te optimaliseren, de spikkels kleiner gemaakt moeten worden dan de optische resolutie van de detektor voor het geval van lage intensiteit van de voorwerpsbundel. De analyse van de dekorrelatiefout van de faseverandering laat zien dat dit de dominante fout is. Minimalisatie wordt bereikt door de spikkels klein te maken in het geval dat de verplaatsingen voornamelijk in het vlak liggen, en groot te maken als voornamelijk rotatie uit het vlak optreed. Verdere reductie kan worden bereikt door filteren, wat leidt tot een verplaatsingsgevoeligheid van de orde van 1 nm. Voor voorwerpen met afmetingen van 0.1 m leidt dit tot een rekgevoeligheid van ongeveer 1 μstrain. De gevoeligheid van een vormmeting is circa 100 μm.

Een vergelijking van voorspellingen die verkregen zijn met behulp van de Eindige Elementen Methode en de meetresultaten laat een goede kwalitatieve overeenkomst zien voor zowel verplaatsing als rek. Een systematische afwijking van 10 % is waargenomen. Variaties tot 15 % van het totale bereik voor zowel de verplaatsingskomponenten als de oppervlakte-rek zijn afgeleid uit de resultaten.
Of all persons that contributed to the research described in this thesis, Henri A. Vrooman should be mentioned first here. Most of the image processing algorithms emerged from the many discussions we had. He was responsible for the implementation of both hardware and software.

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