2 Rolling and Roll Damping

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List of main symbols.

$O\xi\eta\zeta$	space fixed coordinate system with the origin in O (see fig.2)
$Gxyz$	Space fixed coordinate system with the origin in G (see fig.2)
a'_{ij}, b'_{ij}, c'_{ij}	local hydrodynamic coefficients per unit length with respect to $O\xi\eta\zeta$
a_{ij}, b_{ij}, c_{ij}	hydrodynamic coefficients for the ship as a whole with respect to $O\xi\eta\zeta$
g	acceleration of gravity
m	ship's mass
p_{ij}, q_{ij}, r_{ij}	hydrodynamic coefficients for the ship as a whole with respect to Gxyz
y	sway motion
B_f	moment arm for lift forces on fins
F_f	fins area (at one side of the ship)
I	mass moment of inertia
K_f	external moment produced by the fins
K_w	wave exciting moment about Gx
L	ship's length
L_f	lift force on a fin
N_w	wave exciting moment about Gz
V	forward speed
Y_w	transverse wave exciting force
α	fin angle of attack
δ	fin tilt
$\zeta_o = \overline{OG}$	vertical position of G below the waterline
μ	wave to course angle
η	transverse motion

ρ	specific mass of water
ξ_o	longitudinal position of G forward of $\frac{1}{2}L$
ϕ	rolling motion
ψ	yawing motion
$\frac{dC_L}{d\alpha}$	coefficient; slope of the liftcoefficient curve vs. α

Summary.

From a general discussion of the ship motion problem the assumptions are specified under which calculations can be made. The mathematical model describing the coupled roll, sway and yaw motions in regular waves of an arbitrary direction is subsequently derived.

To create higher roll damping the principles and the application of controlled fins and roll-damping tanks is discussed.

Finally an example of the theoretical approach illustrates the effect of the various phenomena involved. The effect of a roll-damping tank of the free surface type is approximately included.

1. Introduction.

Rolling and its associated phenomena is puzzling people already for a long time. Its annoying effects became fully apparent when the sails with their stabilizing influence disappeared from oceangoing ships. It is inconvenient for passengers and crew, it injures the economy of the ship and may even endanger its safety, it prevents a stable platform for shooting of war-ships or for scientific instruments or observations.

Observations made on board Dutch ships have shown that a speed reduction or a change of course was necessary in 2.9 percent of the cases. From these cases one quarter was caused by rolling, partly in combination with pitching. When only changes in course are considered rolling even accounts for 55 percent of the cases. This demonstrates at the least the importance of obtaining a better understanding of the events.

But also some other facts require to concentrate the attention on rolling. Newly introduced vessels as containerships and roll-on-roll-off ships demand minimal motions for the prevention of cargo damage with as little stowing as possible. Further fast and sharp ships sometimes seem to behave peculiarly in quartering seas. Existing experience, as far as present, fails to understand these phenomena. And last, but certainly not least, the stability and safety of many smaller ships is strongly endangered by the waves and by the imposed motions.

2. Formulation of the problem.

A ship at sea sails in general in the interface of two media and is thereby subject to the influence of both the water and the air environment. Since the density of water is roughly a thousand times as large as that of air it is clear that the

explicit influence of air may be neglected for most problems. Perhaps it is just the rolling motion which is an exception to this rule in some cases.
The water waves at the surface are very irregular and seemingly elusive for a description. Fortunately it has turned out to be possible to think of any sea as a composition of a lot of regular waves, each having its own length, height and direction. In this way statistical methods can be used to study the irregular sea and irregular ship motions. This concept implies linearity. That means that the contributions to a certain motion caused by each separate wave may be superposed, just as the waves themselves.
It also means that if the input-wave is increased twice, the output-motion is doubled as well. In the "modern era" of seakeeping research a lot of attention has been paid to this assumption of linearity and it has been proved to be surprisingly correct, anyway for engineering purposes. Again it is rolling, however, where departures from linearity may show to be important and where we have to be alert.

The general problem of a body moving in the surface of a liquid is still not accesssible for theoretical methods. It is necessary to make some further assumptions about the nature of the liquid. So we suppose the fluid to be ideal, that is homogeneous, inviscid and free of surface tension. Fortunately for water this idealization is not a serious limitation and it has been proved that this approach is strikingly valuable for ship motions in waves, just as the linearity already mentioned. As a matter of fact these assumptions are not fully unrelated. If viscosity, for instance, would be of great influence then linearity would also be impaired. So, after the statements

above, it will not be surprising to make once more a possible exception for rolling, where viscous effects are clearly perceptible in some cases, especially in roll damping.
Now we have reduced the problem to the harmonic oscillations of a rigid body, moving at forward speed in the surface of an ideal fluid. This problem can be solved by a combination of rigid body and classical fluid dynamics. But only in a strictly numerical sense and with the aid of an immense computer, which has only recently become available. This does not offer much possibilities for gaining experience and insight in actual problems and their improvement. Therefore analytical methods, if need be approximate methods, remain of great importance.
At the moment we look for a real engineer's solution, which exhibits all essential features correctly and is capable of producing quantitative information to an acceptable degree of accuracy. To this end we make one further step. We simplify the actual occurence from a three-dimensional to a series of two-dimensional problems for each separate cross section of the ship. This case is theoretically solvable. Next the results for the separate sections are combined by some form of strip theory, if necessary modified to include special effects such as forward speed, the presence of bilge keels, fins and the like.

You see that we had to go a long way to make the problem manageable. It is well possible that you are afraid that what is left does not bear much relation to reality any more and that results for practical applications obtained according to these lines are only of academic interest. But that is not true as will be discussed further on. It is very useful however, to have a clear understanding of the assumptions made and of their possible consequences.

3. The two-dimensional case.

From the approach outlined above it is clear that the two-dimensional case of the motions of infinitely long cylinders in beam waves is of primary importance. The three possible motions are swaying, heaving and rolling. It is easily understood that due to the symmetry of the ship's cross sections the vertical motion cannot influence the lateral motions. Since at the moment we are only concerned with the latter we shall restrict our attention to the case of coupled swaying and rolling.

Suppose an arbitrary cross-section to have a transverse translation and a rotation about the point O; see figure 1. The coordinate system $O\eta\zeta$ is geometrically fixed with the origin in the intersection of the waterline and the section's centre line. Newton's laws of dynamics present the equations of motion as:

$$\begin{aligned} m\ddot{\eta} &= F \\ I\ddot{\phi} &= M \end{aligned} \qquad (1)$$

The external force and moment depend on the fluid reactions to the motion of the body and on the forces produced by the incoming waves. In a general formulation they can be stated to be:

$$\left.\begin{aligned} F &= -a'_{yy}\ddot{\eta} - b'_{yy}\dot{\eta} - a'_{y\phi}\ddot{\phi} - b'_{y\phi}\dot{\phi} + Y'_w \\ M &= -a'_{\phi\phi}\ddot{\phi} - b'_{\phi\phi}\dot{\phi} - c'_{\phi\phi}\phi - a'_{\phi y}\ddot{\eta} - b'_{\phi y}\dot{\eta} - \overline{OG}.mg\phi + K'_w \end{aligned}\right\} \qquad (2)$$

where a, b and c are suitably chosen coefficients and the prime denotes that they are to be taken per unit length. Y'_w and K'_w are the transverse wave force and the wave moment, also per unit length.

Swaying is defined as a translation of the centre of gravity and rolling as a rotation about it. So the ultimate equations of motion have to be expressed in the coordinate system Gyz (figure 1).

This is obtained by transforming:

$$\begin{aligned} \eta &= y + \overline{OG}.\phi \\ M_O &= M_G - \overline{OG}.F \end{aligned} \qquad (3)$$

Substituting (3) into (2) and (1) the equations of motion become ultimately:

$$\left.\begin{aligned} &\{m+a'_{yy}\}\ddot{y} + b'_{yy}\dot{y} + \{a'_{y\phi} + \overline{OG}.a'_{yy}\}\ddot{\phi} + \{b'_{y\phi} + \overline{OG}.b'_{yy}\}\dot{\phi} = Y'_w \\ &\{I + a'_{\phi\phi} + \overline{OG}.a'_{\phi y} + \overline{OG}^2.a'_{yy} + \overline{OG}.a'_{y\phi}\}\ddot{\phi} + \{b'_{\phi\phi} + \\ &\overline{OG}.b'_{\phi y} + \overline{OG}^2.b'_{yy} + \overline{OG}.b'_{y\phi}\}\dot{\phi} + \{c'_{\phi\phi} + \overline{OG}.mg\}\phi + \\ &\{a'_{\phi y} + \overline{OG}.a'_{yy}\}\ddot{y} + \{b'_{\phi y} + \overline{OG}.b'_{yy}\}\dot{y} = K'_w + \overline{OG}.Y'_w, \end{aligned}\right\} (4)$$

Under the assumptions made in section 2 all of the quantities in the equations (4) can be computed theoretically. To investigate the correctness of these computations and of the formulation of the physical occurence in the equations (4)

a number of investigations have taken place [1], [2].
It has been shown that the calculated coefficients are certainly accurate enough to use in an engineer's solution. Only the roll damping is definitely influenced by viscous effects and has to be corrected. A comparison of the swaying and rolling motions computed by (4) and actually measured in beam waves also showed a good correspondence.

All this concerned motions in an unrestricted fluid domain. The two-dimensional numerical solution for the hydrodynamic problem permits, however, to take due account of all special circumstances, such as shallow water, the proximity of a wall or the cross section of a canal, multiple hull configurations, etcetera. A computer programme for these special cases does not yet exist, but is in preparation. So if the whole methodology proves to be correct there is no essential limitation for the application to actual cases in practice.

4. The three-dimensional case.

For a ship three additional motions appear with respect to an infinitely long cylinder, namely surge, pitch and yaw.
Again due to symmetry the motions in arbitrary oblique waves can be split up into two groups:
surge, heave and pitch in the plane of symmetry and sway, roll and yaw perpendicular to it. The two groups are mutually independent as can be shown easily and we will only consider the latter.
To proceed from the foregoing to the three-dimensional case of ship's hulls the hydrodynamic forces on each section separately are added. This procedure is called strip theory.

An example of how it works is shown in figure 2. Let the transverse force on a section be F and the moment M.
For the whole ship then holds:

$$\left.\begin{aligned} &\text{transverse force} = \int_L F(\xi)d\xi \\ &\text{moment about } O\xi = \int_L M(\xi)d\xi \\ &\text{moment about } O\zeta = \int_L F(\xi).\xi d\xi \end{aligned}\right\}(5)$$

For a certain section a yaw motion of the ship ψ is equivalent to a local transverse motion $\xi\psi$ and the hydrodynamic forces on the section arising from yawing are supposed to be equal to those produced by a transverse motion of the section. To obtain the equations of motion again a transformation from $O\xi\eta\zeta$ to $Gxyz$ is necessary. Ultimately there appears the following set of equations for the coupled sway-roll-yaw motions in arbitrary regular waves:

$$\left.\begin{aligned} &(m+p_{yy})\ddot{y}+ q_{yy}\dot{y} + p_{y\phi}\ddot{\phi}+ q_{y\phi}\dot{\phi}+ p_{y\psi}\ddot{\psi}+ q_{y\psi}\dot{\psi} = Y_w \\ &(I_x+ p_{\phi\phi})\ddot{\phi}+ q_{\phi\phi}\dot{\phi} + r_{\phi\phi}\phi + (-I_{xz} + p_{\phi\psi})\ddot{\psi}+ \\ &q_{\phi\psi}\dot{\psi} + p_{\phi y}\ddot{y}+ q_{\phi y}\dot{y} = K_w \\ &(I_z+ p_{\psi\psi})\ddot{\psi} + q_{\psi\psi}\dot{\psi} + p_{\psi y}\ddot{y}+ q_{\psi y}\dot{y} +(-I_{xz}+ p_{\psi\phi})\ddot{\phi}+ q_{\psi\phi}\dot{\phi} = N_w \end{aligned}\right\}(6)$$

With p, q and r given by:

equation \ motion	y	ϕ	ψ
y	$p_{yy}=a_{yy}$ $q_{yy}=b_{yy}$ $r_{yy}=0$	$p_{y\phi}=a_{y\phi}+\zeta_o a_{yy}$ $q_{y\phi}=b_{y\phi}+\zeta_o b_{yy}$ $r_{y\phi}=0$	$p_{y\psi}=a_{y\psi}-\xi_o a_{yy}$ $q_{y\psi}=b_{y\psi}-\xi_o b_{yy}$ $r_{y\psi}=0$
ϕ	$p_{\phi y}=a_{\phi y}+\zeta_o a_{yy}$ $q_{\phi y}=b_{\phi y}+\zeta_o b_{yy}$ $r_{\phi y}=0$	$p_{\phi\phi}=a_{\phi\phi}+2\zeta_o a_{\phi y}+\zeta_o^2 a_{yy}$ $q_{\phi\phi}=b_{\phi\phi}+2\zeta_o b_{\phi y}+\zeta_o^2 b_{yy}$ $r_{\phi\phi}=c_{\phi\phi}+\rho g\nabla.\zeta_o$	$p_{\phi\psi}=a_{\phi\psi}-\xi_o a_{\phi y}+\zeta_o a_{y\psi}-\zeta_o\xi_o a_{yy}$ $q_{\phi\psi}=b_{\phi\psi}-\xi_o b_{\phi y}+\zeta_o b_{y\psi}-\zeta_o\xi_o b_{yy}$ $r_{\phi\psi}=0$
ψ	$p_{\psi y}=a_{\psi y}-\xi_o a_{yy}$ $q_{\psi y}=b_{\psi y}-\xi_o b_{yy}$ $r_{\psi y}=0$	$p_{\psi\phi}=a_{\psi\phi}-\xi_o a_{y\phi}+\zeta_o a_{\psi y}-\zeta_o\xi_o a_{yy}$ $q_{\psi\phi}=b_{\psi\phi}-\xi_o b_{y\phi}+\zeta_o b_{\psi y}-\zeta_o\xi_o b_{yy}$ $r_{\psi\phi}=0$	$p_{\psi\psi}=a_{\psi\psi}-2\xi_o a_{\psi y}+\xi_o^2 a_{yy}$ $q_{\psi\psi}=b_{\psi\psi}-2\xi_o b_{\psi y}+\xi_o^2 b_{yy}$ $r_{\psi\psi}=0$

(7)

The coefficients a, b and c for the whole ship in the geometrical coordinate system $O\xi\eta\zeta$ are found from the local values a', b' and c' by integration along the length in the form:

equation \ motion	y	ϕ	ψ
y	$a_{yy}=\int_L a'_{yy}(\xi)d\xi$ $b_{yy}=\int_L b'_{yy}(\xi)d\xi$ $c_{yy}=0$	$a_{y\phi}=\int_L a'_{y\phi}(\xi)d\xi$ $b_{y\phi}=\int_L b'_{y\phi}(\xi)d\xi$ $c_{y\phi}=0$	$a_{y\psi}=\int_L a'_{yy}(\xi).\xi d\xi$ $b_{y\psi}=\int_L b'_{yy}(\xi).\xi d\xi$ $c_{y\psi}=0$
ϕ	$a_{\phi y}=\int_L a'_{\phi y}(\xi)d\xi$ $b_{\phi y}=\int_L b'_{\phi y}(\xi)d\xi$ $c_{\phi y}=0$	$a_{\phi\phi}=\int_L a'_{\phi\phi}(\xi)d\xi$ $b_{\phi\phi}=\int_L b'_{\phi\phi}(\xi)d\xi$ $c_{\phi\phi}=\int_L c'_{\phi\phi}(\xi)d\xi$	$a_{\phi\psi}=\int_L a'_{\phi y}(\xi).\xi d\xi$ $b_{\phi\psi}=\int_L b'_{\phi y}(\xi).\xi d\xi$ $c_{\phi\psi}=0$
ψ	$a_{\psi y}=\int_L a'_{yy}(\xi).\xi d\xi$ $b_{\psi y}=\int_L b'_{yy}(\xi).\xi d\xi$ $c_{\psi y}=0$	$a_{\psi\phi}=\int_L a'_{y\phi}(\xi).\xi d\xi$ $b_{\psi\phi}=\int_L b'_{y\phi}(\xi).\xi d\xi$ $c_{\psi\phi}=0$	$a_{\psi\psi}=\int_L a'_{yy}(\xi).\xi^2 d\xi$ $b_{\psi\psi}=\int_L b'_{yy}(\xi).\xi^2 d\xi$ $c_{\psi\psi}=0$

(8)

The wave exciting forces and moments are also found by integrating the local two-dimensional values:

$$\left.\begin{aligned} Y_w &= \int_L Y'_w(\xi)d\xi \\ K_w &= \int_L \{K'_w(\xi) + \zeta_o Y'_w(\xi)\}\, d\xi = \int_L K'_w(\xi)d\xi + \zeta_o \int_L Y'_w(\xi)d\xi \\ N_w &= \int_L Y'_w(\xi).(\xi-\xi_o)d\xi = \int_L Y'_w(\xi).\xi\, d\xi - \xi_o \int_L Y'_w(\xi)d\xi \end{aligned}\right\} (9)$$

These equations are rather complicated in form, but they result by a logic and purely algebraic operation from the basic two dimensional case. They demonstrate that the coupling effects between the motions depend on the position of the centre of gravity, both in length and in height.
For a certain underwater shape the length position of G is fixed by the centre of buoyancy. But two identical hulls can have very different $\overline{GK}$'s and so exhibit quite a different performance due to different coupling effects. This influence is especially important for rolling. Hydrodynamic coupling effects due to the shape of the hull and the influence of the vertical position of the centre of gravity are mixed up. This fact causes that it is hard to judge about the results in a special case beforehand or to extrapolate to other conditions of loading from available information.
However, by solving the above equations each effect is given its correct value automatically.
Motion data in oblique waves are scarce. In Japan a number of experiments have been performed [3] with a shipmodel of the standard Todd Sixty Series, C_B = 0.70. It was run completely free at various forward speeds in waves of different directions. In [4] the same shipmodel has been investigated in beam waves at zero forward speed.

Some computations are made as well. In [5] is reported about calculations for some different ships.

5. Generalization, improvement and limitations of the procedure.

In the two-dimensional case forward speed of the vessel cannot enter the picture. The three-dimensional extension discussed in section 4 is a mere integration of the two-dimensional forces along the length and is likewise only valid for zero speed. But for realistic applications it is absolutely necessary to take due account of the speed of advance. This problem is the most important at the moment. Strip theory helps us to pass from two to three dimensions. Now we have to modify it or to add separate considerations to it to incorporate the effects of forward speed on the various coefficients in the equations (6). At the moment an investigation of this point is in progress.
Other circumstances which have to be looked at are the following. The presence of appendages as stern tubes, keels, fins or skegs may arise special effects as hydrodynamic lift or flow separation. These influences have to be included separately, for they cannot be dealt with in the two-dimensional hydrodynamic approach and subsequent strip theory. The viscous contribution to the damping of hull and appendages in rolling, which is known to exist, must be made accessible to computation. Work in this field is also in progress. Another point is the influence of the rudder. It is well known that applying the rudder and the way in which it is done by automatic pilot or helmsman produces yawing and rolling angles in addition to those generated by the waves. So perhaps the three motion degrees of freedom will have to be completed by the rudder, thus

forming a system of coupled roll, sway, yaw and rudder motions. While including the forward speed is a necessity several of the comments made above must be considered as improvements on the mathematical model or as special features of special applications.

Finally we must remain aware of the general assumptions made in section 2. Of these assumptions linearity is open to discussion for rolling. Until now it has been supposed unconditionally. When the non-linearities become strong the calculations may fail to be accurate enough. But as far as experience goes it is to be expected that a great variety of practical applications can be handled sufficiently by the linear formulation. An exception has to be made for very special phenomena as the instability in longitudinal regular waves, which, without any moment, may lead to severe rolling. These aspects can never be covered by linear theory and must be approached entirely separately.

6. Roll-damping devices.

6.1. General discussion.

In the course of time many special means of damping the annoying rolling motion have been considered and have been applied to ships. They are of the external type, that is: working through the action of the water surrounding the ship, or of the internal type, so installed in or on the ship. They can be distinguished in passive and active means, according to the way in which they perform. A summary of the various possibilities is presented in table 1.

Table 1: Roll-damping devices.

position	description	passive or active	based on
external	bilge keels	P	energy loss by flow separation
	fixed fins	P	hydrodynamic lift
	controlled fins	A	hydrodynamic lift
internal	gyroscope	P/A	gyroscopic moment
	moving weight	P/A	gravity moment due to displacement of weight
	roll-damping tanks	P/A	gravity moment due to displacement of weight

Of each system many modifications exist. All of them can be placed within this scheme, however, and it is not very useful to discuss details on this occasion.

Some very general facts may be stated as a direct consequence of this classification. Well-designed active means will be more effective than the corresponding passive ones. However, they require a sensing unit, a control system and a motion mechanism and they will consequently be more complicated, more vulnerable, more expensive and will need more maintenance. Further any device of the external type may be accompanied by operational objections, while internal means will create the loss of space.

From the possibilities mentioned in table 1 the gyroscope has been used 40 to 50 years ago, but nowadays its application is restricted to very special objects. The moving weight system is technically rather difficult, while its principle does not differ from the tank systems. In one case the transfer of weight is due to the displacement of a solid piece of material, in the other case to the transfer of fluid from

one side of the ship to the other. When it appears possible to overcome the technical difficulties it is a very promising installation, however. Since the specific weight can be much higher than that of water and its track in the ship is fixed the whole installation can be very compact in principle. Actually some proposals have been made to realize this system for small ships.
We will discuss controlled fins, which form the majority of the external systems and roll-damping tanks, which is the only practical internal system in use at the moment, a little further.
Before doing so I like to make some comments on bilge keels. It is the oldest and simplest attempt to increase roll damping. Nowadays one sometimes gets the impression that this method is out of date and very ineffective. In my opinion this is not true. We still have little understanding of their precise action and of their effect on the complicated and inter-related whole of motions. Undoubtedly unfavourable experiences are attributed to the ineffectiveness of bilge keels where, in fact, they originate from a combination of entirely different causes. Only when our knowledge in this matter has been improved much we will be able to say where bilge keels are important and where not and how much their effect will be. Until then we are dependent on "general practice" and on model tests, but we will be wise not to abandon them rashly.

6.2. Controlled fins.
The action of controlled fins is illustrated in fig. 3. At each side of the midship part of the vessel a hydrofoil is installed, wich are rotated in opposite directions.

The lift force on the hydrofoil is expressed by:

$$L_f = \frac{dC_L}{d\alpha}\,\alpha\,.\,\tfrac{1}{2}\rho V^2.F_f \qquad (10)$$

as usual. And it is easily seen that the fins produce a moment:

$$K_f = -L_f.B_f \qquad (11)$$

In figure 4 the fin angle of attack is seen to be:

$$\alpha = \frac{\tfrac{1}{2}B_f}{V}\,\dot{\phi} + \delta \qquad (12)$$

if we neglect the influence of other shipmotions.
The first term in (12) is generated by the ship's rolling itself and will also be present in passive installations. The angle δ is the fin tilt produced by a motion mechanism. It is ordered by a control system. This system may differ from firm to firm and from one application to the other.
It will always include an order that δ is proportional to the roll velocity $\dot{\phi}$. Further it may act on roll angle ϕ, roll acceleration $\ddot{\phi}$, sway velocity $\dot{y}$, sway acceleration $\ddot{y}$ and even on yaw signals. Suppose, just by way of illustration, that the fin tilt will be proportional to roll velocity $\dot{\phi}$ and roll acceleration $\ddot{\phi}$. So

$$\delta = C_1\dot{\phi} + C_2\ddot{\phi} \qquad (13)$$

Substituting the above relations in the fin moment it is easily derived that:

$$K_f = -\frac{dC_L}{d\alpha}\,.\,\tfrac{1}{2}\rho V^2.F_fB_f\left(\frac{\tfrac{1}{2}B_f}{V} + C_1\right)\dot{\phi} - \frac{dC_L}{d\alpha}\,.\,\tfrac{1}{2}\rho V^2.F_fB_f.C_2\ddot{\phi} \qquad (14)$$

By adding this external moment to the roll equation in (6) the influence of the fins is accounted for. The equations can be solved again and the rolling with the fins in action is found.

The hydrodynamic lift force is proportional with the square of the forward velocity. So this large speed dependence obviously restricts their application to fast ships. Naturally large variations in draught may also hamper their use, for they must remain sufficiently submerged. But their effectiveness is practically unsurpassed. A well-designed installation can nearly eliminate the rolling, while reductions of the order of 90 per cent must be considered as normal.

As you see their principal features are easily understood. The technical realisation, however, is more difficult. Therefore some firms have specialized in this field and several technically advanced systems are obtainable. A good paper elaborating on fins has been given by Conolly [6] ; interested readers will find a lot of valuable information there.

6.3. Roll-damping tanks.

There are two different types of roll-damping tanks: the U-tanks introduced by Frahm in 1911 [7,8] and the free surface tanks introduced by Watts in 1883 [9,10]

When a tanksystem is activated it is always of the U-tube type. The majority of the tanksystems act passively, however. In that way their specific features also show to full advantage. Therefore we will restrict the discussion to passive tanks.

The water motion in the U-type of tank is an oscillating column of water. It resembles the motion of a pendulum and can be described analogously.

The moment is reasonably accessible to calculation and thus the tank effect can also be computed [11] . In a free surface tank the water motion is entirely different. The transfer of mass from one side to the other takes place by a shallow water wave phenomenon. Until now calculations [12] have not been able to account for the exerted moment, although trends are confirmed fully. Therefore a systematic series of experiments was necessary to provide for the tank moments necessary to include in the equations of motion [13] . With these data the equations can be solved again and the rolling with a tank in action can be compared with the rolling of the ship alone.

For both types of tanks the investigations until now have been restricted to the hypothetical condition of pure rolling in beam waves. This restriction was necessary for a number of reasons. In the first place too little was known about the behaviour of the ship itself in waves. The hydrodynamic forces associated with the rolling motion and the coupling effects with swaying and yawing were fully unknown. The knowledge about the ship even formed a greater limitation to the design and understanding of tanks than the knowledge about the tanks. This situation is now gradually being improved and we may expect a still better understanding and prediction of tank applications in the future. In the second place the action of roll damping tanks is clearly nonlinear. This makes it difficult out of principle to add the exerted tank moment to the essentially linear equations of motion for the ship. It also causes that tank moments due to the rolling and to the swaying of the ship cannot be superposed. These facts are known to be true for the free surface tank. For the U-tanks little information on these points is available.

But presumably the above facts will not be much different. It is as yet unknown to which extent a linearization of the tank moments will influence the results of the calculations. Only linearized the additional moments can be included in the equations (6). For the U-tank the same holds. Only then the mathematical formulation for the ship and for the U-tank can be combined to form a system of 4 equations for 4 degrees of freedom.

The effect of tanks on the rolling of the ships is a reduction of about 50-75% in regular waves and 40-60% in irregular waves. These figures are averages. They apply to well-designed tanks of normal dimensions in ordinary applications. Of course, lower or higher effects are possible in special cases. In general they cannot compete with active fins. But they also act at zero forward speed, are simple, cheap and invulnerable. Which type of tank has to be preferred in a certain situation, or which modification of that type, is hard to say. Each has its own characteristics, its pros and cons. But certainly no system is superior over all other systems in any situation concerned. If an experienced designer gets the opportunity to adjust his design, if necessary, to the case in consideration any system or modification will act satisfactorily. For comparative studies about various roll damping tanks reference is made to [14] and [15].

7. Illustration.

In figure 5 various computations for the rolling of a destroyer type hull in quartering waves are shown. In the first place the rolling has been calculated by the equations (6), so fully coupled with swaying and yawing. Next the yaw-terms in

the first and second and the whole third equation of (6) have been neglected. Finally the sway-terms were dropped as well, after which pure rolling remained.

It can be seen that the direct coupling effects are not so large. At least in this case, for it is questionable whether this holds true in other cases. But anyway the pure rolling has been computed with the correct hydrodynamic coefficients $p_{\phi\phi}$, $q_{\phi\phi}$, $r_{\phi\phi}$ and with the correct wave exiting moment K_w. That means that a part of the coupling influences has been included by taking due account of the vertical position of the centre of gravity; compare (7) and (9). If the calculation of pure rolling is made (by lack of any better information) with a radius of gyration of 0.38 B, with a nondimensional damping coefficient of 0.10, with only the relative position of the centre of gravity and the metacentre in the metacentric height $\overline{GM}$ and with the Froude-Krylov moment as wave exciting moment the curve 2 results. The difference with the complete computation is large. So in this case the largest error is not made by the restriction to pure rolling but by the value of the hydrodynamic coefficients and of the wave moment. If, however, we have to determine the correct values for rolling we need the coefficients for swaying and the values for yawing will automatically result. Since the whole computational procedure has to be performed by a computer it requires very little extra effort to solve the complete set of equations instead of rolling alone. A real simplification and saving of time is hardly possible.

Another point to note is that the direct coupling effects may show to have little influence on the amplitude, but they can change the mutual phase relations between the motions

to a greater extent. This can certainly be of importance for the absolute or relative motion of specific points of the ship and for an effective action of a control system.
Finally two other curves have been drawn in figure 5. They denote the rolling when a roll damping tank of the free surface type has been installed. The waterdepth has been chosen as $h/b = 0.04$ and the tank length $l = 0.25$ b. One curve belongs to the complete calculation, the other to the rough estimate. The tank moments have been linearized in both cases with respect to their values at $\phi_a = 0.10$ and were determined from the systematic experimental data in [13]. The tank has been chosen rather arbitrarily and will most probably not be the best obtainable.

I hope that I have been able to give you an impression of the difficulties involved in the problem of determining the ship's behaviour at sea, especially with regard to rolling. But also of the possibilities which are present at the moment. The investigation will continue for a long time. But in my opinion we have reached the point that we may consider the very cautious application to practical problems. That requires a good cooperation between theoretical and practical people. A task which is worth while, for in our field all of us ultimately work to use the knowledge we have obtained in actual applications.

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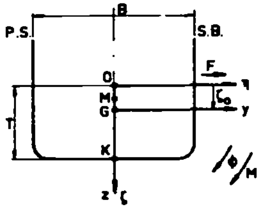


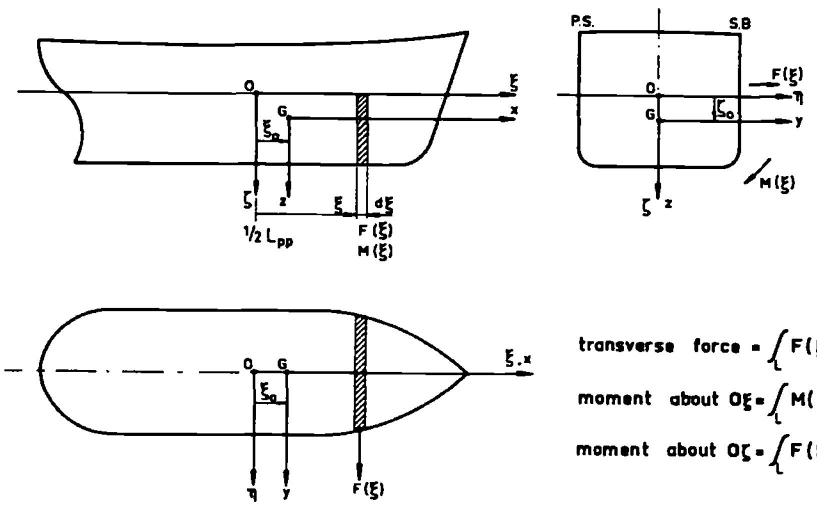
$$m\ddot{\eta} = F$$

$$I\ddot{\phi} = M$$

$$F = -a'_{yy}\ddot{\eta} - b'_{yy}\dot{\eta} - a'_{y\phi}\ddot{\phi} - b'_{y\phi}\dot{\phi} + Y'_w$$

$$M = -a'_{\phi\phi}\ddot{\phi} - b'_{\phi\phi}\dot{\phi} - c'_{\phi\phi}\phi - a'_{\phi y}\ddot{\eta} - b'_{\phi y}\dot{\eta} - \overline{OG}\cdot mg\phi + K'_w$$

Fig. 1 Definition of symbols

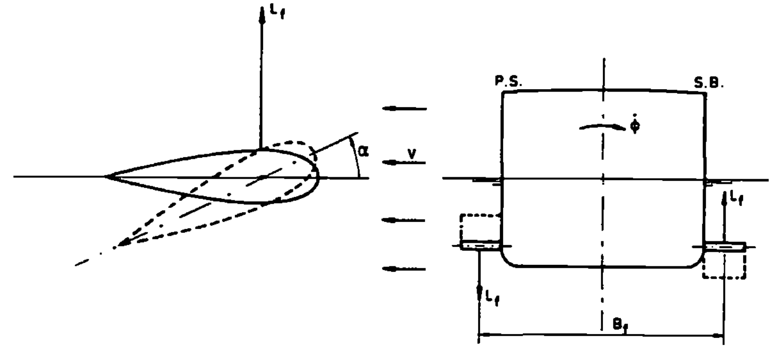


transverse force $= \int_L F(\xi)\,d\xi$

moment about $O\xi = \int_L M(\xi)\,d\xi$

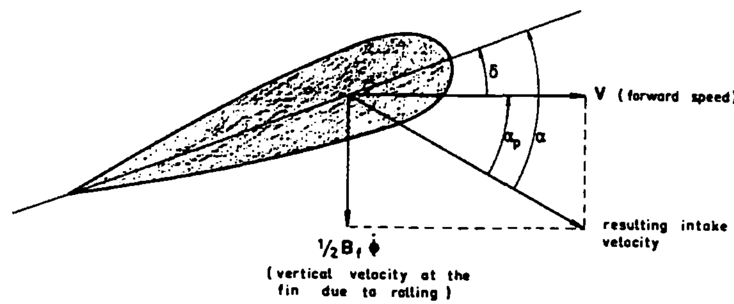
moment about $O\zeta = \int_L F(\xi)\cdot\xi\,d\xi$

Fig. 2 Illustration of strip theory procedure



$$L_f = \frac{dC_L}{d\alpha}\,\alpha\cdot\tfrac{1}{2}\rho V^2\cdot F_f \qquad K_f = -L_f\cdot B_f$$

Fig. 3 Schematic action of controlled fins



$$\alpha = \frac{\tfrac{1}{2}B_f}{V}\dot{\phi} + \delta$$

α_p = passive angle of attack $= \dfrac{\tfrac{1}{2}B\dot{\phi}}{V}$

δ = fin tilt

Fig. 4 Flow diagram at the fin

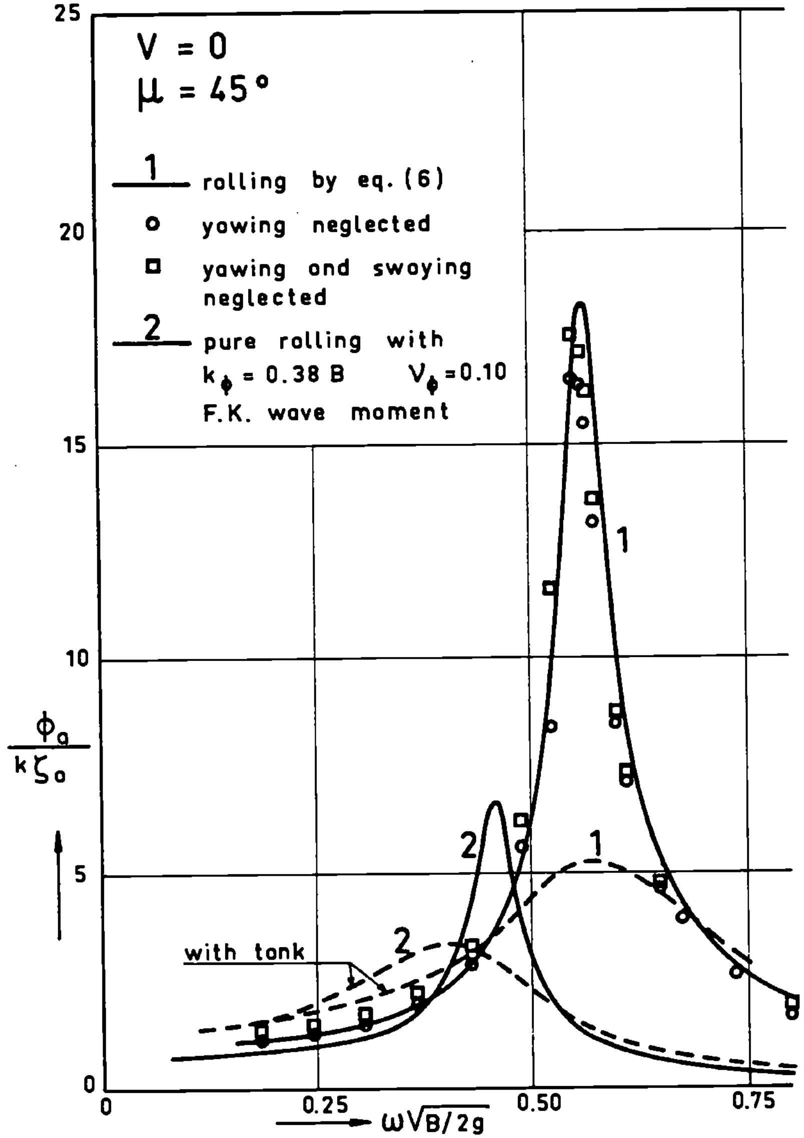


Fig. 5 Computed rolling for destroyer type hull in quartering waves

3 Simulation of Ship Manoeuvring Qualities

by IR. J. B. VAN DEN BRUG

Summary

Simulation of ship manoeuvring qualities by means of free-running models and a manoeuvring simulator is discussed briefly.
Some aspects of the influence of the time-scale effect in model manoeuvring are mentioned.
It is shown how a manoeuvring simulator can be used in ship design.

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1. Introduction
2. Simulation techniques
   2.1. Simulation by means of free-running models
   2.2. Simulation by means of a manoeuvring simulator
3. The influence of the time-scale effect in model manoeuvring
4. The use of simulation techniques in ship design
5. Conclusions
   References

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TECHNISCHE HOGESCHOOL DELFT

ROLLING AND ROLL DAMPING.

by

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(Voordracht vakantie-leergang K.I.V.I.)

mei 1969.

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Summary.

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List of main symbols.

$\mathcal{O}\xi\eta\zeta$	space fixed coordinate system with the origin in $\mathcal{O}$ (see fig.2).
Gxyz	space fixed coordinate system with the origin in G (see fig.2)
a'_{ij}, b'_{ij}, c'_{ij}	local hydrodynamic coefficients per unit length with respect to $\mathcal{O}\xi\eta\zeta$
a_{ij}, b_{ij}, c_{ij}	hydrodynamic coefficients for the ship as a whole with respect to $\mathcal{O}\xi\eta\zeta$
g	acceleration of gravity
m	ship's mass
p_{ij}, q_{ij}, r_{ij}	hydrodynamic coefficients for the ship as a whole with respect to Gxyz
y	sway motion
B_f	moment arm for lift forces on fins
F_f	fin area (at one side of the ship)
I	mass moment of inertia
K_f	external moment produced by the fins
K_w	wave exciting moment about Gx
L	ship's length
L_f	lift force on a fin
N_w	wave exciting moment about Gz
V	forward speed
Y_w	transverse wave exciting force
α	fin angle of attack
δ	fin tilt
$\zeta_o = \overline{\mathcal{O}G}$	vertical position of G below the waterline
μ	wave to course angle
η	transverse motion
ρ	specific mass of water
ξ_o	longitudinal position of G forward of $\frac{1}{2}L$
ϕ	rolling motion
ψ	yawing motion
$\frac{dC_L}{d\alpha}$	coefficient; slope of the liftcoefficient curve vs. α

Summary.

From a general discussion of the ship motion problem the assumptions are specified under which calculations can be made. The mathematical model describing the coupled roll, sway and yaw motions in regular waves of an arbitrary direction is subsequently derived.

To create higher roll damping the principles and the application of controlled fins and roll-damping tanks is discussed.

Finally an example of the theoretical approach illustrates the effect of the various phenomena involved. The effect of a roll-damping tank of the free surface type is approximately included.

1. Introduction.

Rolling and its associated phenomena is puzzling people already for a long time. Its annoying effects became fully apparent when the sails with their stabilizing influence disappeared from oceangoing ships. It is inconvenient for passengers and crew, it injures the economy of the ship and may even endanger its safety, it prevents a stable platform for shooting of war-ships or for scientific instruments or observations.

Observations made on board Dutch ships have shown that a speed reduction or a change of course was necessary in 2.9 percent of the cases. From these cases one quarter was caused by rolling, partly in combination with pitching. When only changes in course are considered rolling even accounts for 55 per cent of the cases. This demonstrates at the least the importance of obtaining a better understanding of the events.

But also some other facts require to concentrate the attention on rolling. Newly introduced vessels as containerships and roll-on-roll-off ships demand minimal motions for the prevention of cargo damage with as little stowing as possible. Further fast and sharp ships sometimes seem to behave peculiarly in quartering seas. Existing experience, as far as present, fails to understand these phenomena. And last, but certainly not least, the stability and safety of many smaller ships is strongly endangered by the waves and by the imposed motions.

2. Formulation of the problem.

A ship at sea sails in general in the interface ~~plane of contact~~ of two media and is thereby subject to the influence of both the water and the air environment. Since the density of water is roughly a thousand times as large as that of air it is clear that the explicit influence of air may be neglected for most problems. Perhaps it is just the rolling motion which is an exception to this rule in some cases.

The water waves at the surface are very irregular and seemingly elusive for a description. Fortunately it has turned out to be possible to think of any sea as a composition of a lot of regular waves, each having its own length, height and direction. In this way statistical methods can be used to study the irregular sea and irregular ship motions. This concept implies linearity. That means that the contributions to a certain motion caused by each separate wave may be superposed, just as the waves themselves. It also means that if the input-wave is increased twice, the output-motion is doubled as well. In the "modern era" of seakeeping research a lot of attention has been paid to this assumption of linearity and it has been proved to be surprisingly correct, anyway for engineering purposes. Again it is rolling, however, where departures from linearity may show to be important and where we have to be alert.

The general problem of a body moving in the surface of a liquid is still not accessible for theoretical methods. It is necessary to make some further assumptions about the nature of the liquid. So we suppose the fluid to be ideal, that is homogeneous, inviscid and free of surface tension. Fortunately for water this idealization is not a serious limitation and it has been proved that this approach is strikingly valuable for ship motions in waves, just as the linearity already mentioned. As a matter of fact these assumptions are not fully unrelated. If viscosity, for instance, would be of great influence then linearity would also be impaired. So, after the statements above, it will not be surprising to make once more a possible exception for rolling, where viscous effects are clearly perceptible in some cases, especially in roll damping.

Now we have reduced the problem to the harmonic oscillations of a rigid body, moving at forward speed in the surface of an ideal fluid. This problem can be solved by a combination of rigid body and classical fluid dynamics. But only in a strictly numerical sense and with the aid of an immense computer, which has only recently become available. This does not offer much possibilities for gaining experience and insight in actual problems and their improvement. Therefore analytical methods, if need be approximate methods, remain of great importance.

At the moment we look for a real engineer's solution, which exhibits all essential features correctly and is capable of producing quantitative information to an acceptable degree of accuracy. To this end we make one further step. We simplify the actual occurrence from a three-dimensional to a series of two-dimensional problems for each separate cross section of the ship. This case is theoretically solvable. Next the results for the separate sections are combined by some form of strip theory, if necessary modified to include special effects such as forward speed, the presence of bilge keels, fins and the like.

You see that we had to go a long way to make the problem manageable. It is well possible that you are afraid that what is left does not bear much relation to reality any more and that results for practical applications obtained according to these lines are only of academic interest. But that is not true as will be ~~shown~~ discussed further on. It is very useful, however, to have a clear understanding of the assumptions made and of their possible consequences.

3. The two-dimensional case.

From the approach outlined above it is clear that the two-dimensional case of the motions of infinitely long cylinders in beam waves is of primary importance. The three possible motions are swaying, heaving and rolling. It is easily understood that due to the symmetry of the ship's cross sections the vertical motion cannot influence the lateral motions. Since at the moment we are only concerned with the latter we shall restrict our attention to the case of coupled swaying and rolling.

Suppose an arbitrary cross-section to have a transverse translation and a rotation about the point σ; see Figure 1. The coordinate system $\sigma\eta\zeta$ is geometrically fixed with the origin in the intersection of the waterline and the section's centre line. Newton's laws of dynamics present the equations of motion as:

$$\begin{aligned} m\ddot{\eta} &= F \\ I\ddot{\phi} &= M \end{aligned} \tag{1}$$

The external force and moment depend on the fluid reactions to the motion of the body and on the forces produced by the incoming waves. In a general formulation they can be stated to be:

$$\begin{aligned} F &= -a'_{yy}\ddot{\eta} - b'_{yy}\dot{\eta} - a'_{y\phi}\ddot{\phi} - b'_{y\phi}\dot{\phi} + Y'_{w} \\ M &= -a'_{\phi\phi}\ddot{\phi} - b'_{\phi\phi}\dot{\phi} - c'_{\phi\phi}\phi - a'_{\phi y}\ddot{\eta} - b'_{\phi y}\dot{\eta} - \overline{OG}.mg\phi + K'_{w} \end{aligned} \tag{2}$$

where a, b and c are suitably chosen coefficients and the prime denotes that they are to be taken per unit length. Y'_w and K'_w are the transverse wave force and the wave moment, also per unit length.

Swaying is defined as a translation of the centre of gravity and rolling as a rotation about it. So the ultimate equations of motion have to be expressed in the coordinate system Gyz (figure 1).

This is obtained by transforming:

$$\begin{aligned} \eta &= y + \overline{OG}.\phi \\ M_{\sigma} &= M_G - \overline{OG}.F \end{aligned} \tag{3}$$

Substituting (3) into (2) and (1) the equations of motion become ultimately:

$$\left.\begin{aligned} &\{m+a'_{yy}\}\ddot{y} + b'_{yy}\dot{y} + \{a'_{y\phi} + \overline{OG}.a'_{yy}\}\ddot{\phi} + \{b'_{y\phi}+\overline{OG}.b'_{yy}\}\dot{\phi} = Y'_{w} \\ &\{I + a'_{\phi\phi}+\overline{OG}.a'_{\phi y}+\overline{OG}^2.a'_{yy}+\overline{OG}.a'_{y\phi}\}\ddot{\phi}+\{b'_{\phi\phi}+\overline{OG}.b'_{\phi y} + \overline{OG}^2.b'_{yy}+\overline{OG}.b'_{y\phi}\}\dot{\phi} \\ &+\{c'_{\phi\phi}+\overline{OG}.mg\}\phi+\{a'_{\phi y}+\overline{OG}.a'_{yy}\}\ddot{y}+\{b'_{\phi y}+\overline{OG}.b'_{yy}\}\dot{y} = K'_{w}+\overline{OG}.Y'_{w} \end{aligned}\right\} \tag{4}$$

Under the assumptions made in section 2 all of the quantities in the equations (4) can be computed theoretically. To investigate the correctness of these computations and of the formulation of the physical occurrence in the equations (4) a number of investigations have taken place [1], [2]. It has been shown that the calculated coefficients are certainly accurate enough to use in an engineer's solution. Only the roll damping is definitely influenced by viscous effects and has to be corrected. A comparison of the swaying and rolling motions computed by (4) and actually measured in beam waves also showed a good correspondence.

All this concerned motions in an unrestricted fluid domain. The two-dimensional numerical solution for the hydrodynamic problem permits, however, to take due account of all special circumstances, such as shallow water, the proximity of a wall or the cross section of a canal, multiple hull configurations, etcetera. A computer programme for these special cases does not yet exist, but is in preparation. So if the whole methodology proves to be correct there is no essential limitation for the application to actual cases in practice.

4. The three-dimensional case.

For a ship three additional motions appear with respect to an infinitely long cylinder, namely surge, pitch and yaw. Again due to symmetry the motions in arbitrary oblique waves can be split up into two groups: surge, heave and pitch in the plane of symmetry and sway, roll and yaw perpendicular to it. The two groups are mutually independent as can be shown easily and we will only consider the latter.

To proceed from the foregoing to the three-dimensional case of ship's hulls the hydrodynamic forces on each section separately are added. This procedure is called strip theory. An example of how it works is shown in figure 2. Let the transverse force on a section be F and the moment M. For the whole ship then holds:

$$\left.\begin{aligned} \text{transverse force} &= \int_L F(\xi)d\xi \\ \text{moment about } O\xi &= \int_L M(\xi)d\xi \\ \text{moment about } O\zeta &= \int_L F(\xi).\xi d\xi \end{aligned}\right\} \quad (5)$$

For a certain section a yaw motion of the ship ψ is equivalent to a local transverse motion $\xi\psi$ and the hydrodynamic forces on the section arising from yawing are supposed to be equal to those produced by a transverse motion of the section. To obtain the equations of motion again a transformation from $O\xi\eta\zeta$ to $Gxyz$ is necessary. Ultimately there appears the following set of equations for the coupled sway-roll-yaw motions in arbitrary regular waves:

$$\left.\begin{aligned}
(m+p_{yy})\ddot{y} + q_{yy}\dot{y} + p_{y\phi}\ddot{\phi} + q_{y\phi}\dot{\phi} + p_{y\psi}\ddot{\psi} + q_{y\psi}\dot{\psi} &= Y_w \\
(I_x+p_{\phi\phi})\ddot{\phi} + q_{\phi\phi}\dot{\phi} + r_{\phi\phi}\phi + (-I_{xz}+p_{\phi\psi})\ddot{\psi} + q_{\phi\psi}\dot{\psi} + p_{\phi y}\ddot{y} + q_{\phi y}\dot{y} &= K_w \\
(I_z+p_{\psi\psi})\ddot{\psi} + q_{\psi\psi}\dot{\psi} + p_{\psi y}\ddot{y} + q_{\psi y}\dot{y} + (-I_{xz}+p_{\psi\phi})\ddot{\phi} + q_{\psi\phi}\dot{\phi} &= N_w
\end{aligned}\right\} \quad (6)$$

With p, q and r given by:

equation \ motion	y	ϕ	ψ
y	$p_{yy} = a_{yy}$ $q_{yy} = b_{yy}$ $r_{yy} = 0$	$p_{y\phi} = a_{y\phi} + \zeta a_{yy}$ $q_{y\phi} = b_{y\phi} + \zeta_0 b_{yy}$ $r_{y\phi} = 0$	$p_{y\psi} = a_{y\psi} - \xi a_{yy}$ $q_{y\psi} = b_{y\psi} - \xi_0 b_{yy}$ $r_{y\psi} = 0$
ϕ	$p_{\phi y} = a_{\phi y} + \zeta_0 a_{yy}$ $q_{\phi y} = b_{\phi y} + \zeta_0 b_{yy}$ $r_{\phi y} = 0$	$p_{\phi\phi} = a_{\phi\phi} + 2\zeta_0 a_{\phi y} + \zeta_0^2 a_{yy}$ $q_{\phi\phi} = b_{\phi\phi} + 2\zeta_0 b_{\phi y} + \zeta_0^2 b_{yy}$ $r_{\phi\phi} = c_{\phi\phi} + \rho g \nabla . \zeta_0$	$p_{\phi\psi} = a_{\phi\psi} - \xi_0 a_{\phi y} + \zeta_0 a_{y\psi} - \zeta_0 \xi_0 a_{yy}$ $q_{\phi\psi} = b_{\phi\psi} - \xi_0 b_{\phi y} + \zeta_0 b_{y\psi} - \zeta_0 \xi_0 b_{yy}$ $r_{\phi\psi} = 0$
ψ	$p_{\psi y} = a_{\psi y} - \xi_0 a_{yy}$ $q_{\psi y} = b_{\psi y} - \xi_0 b_{yy}$ $r_{\psi y} = 0$	$p_{\psi\phi} = a_{\psi\phi} - \xi_0 a_{y\phi} + \zeta_0 a_{\psi y} - \zeta_0 \xi_0 a_{yy}$ $q_{\psi\phi} = b_{\psi\phi} - \xi_0 b_{y\phi} + \zeta_0 b_{\psi y} - \zeta_0 \xi_0 b_{yy}$ $r_{\psi\phi} = 0$	$p_{\psi\psi} = a_{\psi\psi} - 2\xi_0 a_{\psi y} + \xi_0^2 a_{yy}$ $q_{\psi\psi} = b_{\psi\psi} - 2\xi_0 b_{\psi y} + \xi_0^2 b_{yy}$ $r_{\psi\psi} = 0$

(7)

The coefficients a, b and c for the whole ship in the geometrical coordinate system $O\xi\eta\zeta$ are found from the local values a', b' and c' by integration along the length in the form:

equation \ motion	y	ϕ	ψ
y	$a_{yy} = \int_L a'_{yy}(\xi)\,d\xi$ $b_{yy} = \int_L b'_{yy}(\xi)\,d\xi$ $c_{yy} = 0$	$a_{y\phi} = \int_L a'_{y\phi}(\xi)\,d\xi$ $b_{y\phi} = \int_L b'_{y\phi}(\xi)\,d\xi$ $c_{y\phi} = 0$	$a_{y\psi} = \int_L a'_{yy}(\xi)\cdot\xi\,d\xi$ $b_{y\psi} = \int_L b'_{yy}(\xi)\cdot\xi\,d\xi$ $c_{y\psi} = 0$
ϕ	$a_{\phi y} = \int_L a'_{\phi y}(\xi)\,d\xi$ $b_{\phi y} = \int_L b'_{\phi y}(\xi)\,d\xi$ $c_{\phi y} = 0$	$a_{\phi\phi} = \int_L a'_{\phi\phi}(\xi)\,d\xi$ $b_{\phi\phi} = \int_L b'_{\phi\phi}(\xi)\,d\xi$ $c_{\phi\phi} = \int_L c'_{\phi\phi}(\xi)\,d\xi$	$a_{\phi\psi} = \int_L a'_{\phi y}(\xi)\cdot\xi\,d\xi$ $b_{\phi\psi} = \int_L b'_{\phi y}(\xi)\cdot\xi\,d\xi$ $c_{\phi\psi} = 0$
ψ	$a_{\psi y} = \int_L a'_{yy}(\xi)\cdot\xi\,d\xi$ $b_{\psi y} = \int_L b'_{yy}(\xi)\cdot\xi\,d\xi$ $c_{\psi y} = 0$	$a_{\psi\phi} = \int_L a'_{y\phi}(\xi)\cdot\xi\,d\xi$ $b_{\psi\phi} = \int_L b'_{y\phi}(\xi)\cdot\xi\,d\xi$ $c_{\psi\phi} = 0$	$a_{\psi\psi} = \int_L a'_{yy}(\xi)\cdot\xi^2\,d\xi$ $b_{\psi\psi} = \int_L b'_{yy}(\xi)\cdot\xi^2\,d\xi$ $c_{\psi\psi} = 0$

(8)

The wave exciting forces and moments are also found by integrating the local two-dimensional values:

$$\left.\begin{aligned} Y_w &= \int_L Y'_w(\xi)\,d\xi \\ K_w &= \int_L \left\{K'_w(\xi) + \zeta_o Y'_w(\xi)\right\}d\xi = \int_L K'_w(\xi)\,d\xi + \zeta_o \int_L Y'_w(\xi)\,d\xi \\ N_w &= \int_L Y'_w(\xi).(\xi-\xi_o)\,d\xi = \int_L Y'_w(\xi).\xi\,d\xi - \xi_o \int_L Y'_w(\xi)\,d\xi \end{aligned}\right\} \quad (9)$$

These equations are rather complicated in form, but they result by a logic and purely algebraic operation from the basic two dimensional case. They demonstrate that the coupling effects between the motions depend on the position of the centre of gravity, both in length and in height. For a certain underwater shape the length position of G is fixed by the centre of buoyancy. But two identical hulls can have very different $\overline{GK}$'s and so exhibit quite a different performance due to different coupling effects. This influence is especially important for rolling. Hydrodynamic coupling effects due to the shape of the hull and the influence of the vertical position of the centre of gravity are mixed up. This fact causes that it is hard to judge about the results in a special case beforehand or to extrapolate to other conditions of loading from available information.

However, by solving the above equations each effect is given its correct value automatically.

Motion data in oblique waves are scarce. In Japan a number of experiments have been performed [3] with a shipmodel of the standard Todd Sixty Series, $C_B = 0.70$. It was run completely free at various forward speeds in waves of different directions. In [4] the same shipmodel has been investigated in beam waves at zero forward speed. Some computations are made as well. In [5] is reported about calculations for some different ships.

5. Generalization, improvement and limitations of the procedure.

In the two-dimensional case forward speed of the vessel cannot enter the picture. The three-dimensional extension discussed in section 4 is a mere integration of the two-dimensional forces along the length and is likewise only valid for zero speed. But for realistic applications it is absolutely necessary to take due account of the speed of advance. This problem is the most important at the moment. ~~Pure~~ Strip theory ~~only~~ helps us to pass from two to three dimensions. Now we have to modify it or to add separate considerations to it to incorporate the effects of forward speed on the various coefficients in the equations (6). At the moment an investigation of this point is in progress.

Other circumstances which have to be looked at are the following. The presence of appendages as stern tubes, keels, fins or skegs may arise special effects as hydrodynamic lift or flow separation. These influences have to be included separately, for they cannot be dealt with in the two-dimensional hydrodynamic approach and subsequent strip theory. The viscous contribution to the damping of hull and appendages in rolling, which is known to exist, must be made accessible to computation. Work in this field is also in progress. Another point is the influence of the rudder. It is well known that applying the rudder and the way in which it is done by automatic pilot or helmsman produces yawing and rolling angles in addition to those generated by the waves. So perhaps the three motion degrees of freedom will have to be completed by the rudder, thus forming a system of coupled roll, sway, yaw and rudder motions. While including the forward speed is a necessity several of the comments made above must be considered as improvements on the mathematical model or as special features of special applications.

Finally we must remain aware of the general assumptions made in section 2. Of these assumptions linearity is open to discussion for rolling. Until now it has been supposed unconditionally. When the non-linearities become strong the calculations may fail to be accurate enough. But as far as experience goes it is to be expected that a great variety of practical applications can be handled sufficiently by the linear formulation. An exception has to be made for very special phenomena as the instability in longitudinal regular waves, which, without any moment, may lead to severe rolling. These aspects can never be covered by linear theory and must be approached entirely separately.

6. Roll-damping devices.

6.1. General discussion.

In the course of time many special means of damping the annoying rolling motion have been considered and have been applied to ships. They are of the external type, that is: working through the action of the water surrounding the ship, or of the internal type, so installed in or on the ship. They can be distinguished in passive and ~~or~~ active means, according to the way in which they perform. A summary of the various possibilities is presented in table 1.

Table 1: Roll-damping devices.

position	description	passive or active	based on
external	bilge keels	P	energy loss by flow separation
	fixed fins	P	hydrodynamic lift
	controlled fins	A	hydrodynamic lift
internal	gyroscope	P/A	gyroscopic moment
	moving weight	P/A	gravity moment due to displacement of weight
	roll-damping tanks	P/A	gravity moment due to displacement of weight

Of each system many modifications exist. All of them can be placed within this scheme, however, and it is not very useful to discuss details on this occasion.

Some very general facts may be stated as a direct consequence of this classification. Well-designed active means will be more effective than the corresponding passive ones. However, they require a sensing unit, a control system and a motion mechanism and they will consequently be more complicated, more vulnerable, more expensive and will need more maintenance. Further any device of the external type may be accompanied by operational objections, while internal means will create the loss of space.

From the possibilities mentioned in table 1 the gyroscope has been used 40 to 50 years ago, but has fallen out of use nowadays. The moving weight system is technically rather difficult, while its principle does not differ from the tank systems. In one case the transfer of weight is due to the displacement of a solid piece of material, in the other case to the transfer of fluid ~~water~~ from one side of the ship to the other. We will discuss controlled fins, which form the majority of the external systems and roll-damping tanks, which is the only practical internal system in use at the moment, a little further.

Before doing so I like to make some comments on bilge keels. It is the oldest and simplest attempt to increase roll damping. Nowadays one sometimes gets the impression that this method is out of date and very ineffective. In my opinion this is not true. We still have little understanding of their precise action and of their effect on the complicated and inter-related whole of motions. Undoubtedly unfavourable experiences are attributed to the ineffectiveness of bilge keels where, in fact, they originate from a combination of entirely different causes. Only when our knowledge in this matter has been improved much we will be able to say where bilge keels are important and where not and how much their effect will be. Until then we are dependent on "general practice" and on model tests, but we will be wise not to abandon them rashly.

6.2. Controlled fins.

The action of controlled fins is illustrated in fig.3. At each side of the midship part of the vessel a hydrofoil is installed, which are rotated in opposite directions. The lift force on the hydrofoil is expressed by:

$$L_f = \frac{dC_L}{d\alpha}\alpha \cdot \tfrac{1}{2}\rho V^2 . F_f \qquad (10)$$

as usual. And it is easily seen that the fins produce a moment:

$$K_f = -L_f . B_f \qquad (11)$$

In fig.4 the fin angle of attack is seen to be:

$$\alpha = \frac{\tfrac{1}{2}B_f}{V}\dot{\phi} + \delta \qquad (12)$$

if we neglect the influence of other shipmotions.

The first term in (12) is generated by the ship's rolling itself and will also be present in passive installations. The angle δ is the fin tilt produced by a motion mechanism. It is ordered by a control system. This system may differ from firm to firm and from one application to the other. It will always include an order that δ is proportional to the roll velocity $\dot{\phi}$. Further it may act on roll angle ϕ, roll acceleration $\ddot{\phi}$, sway velocity $\dot{y}$, sway acceleration $\ddot{y}$ and even on yaw signals. Suppose, just by way of illustration, that the fin tilt will be proportional to roll velocity $\dot{\phi}$ and roll acceleration $\ddot{\phi}$. So

$$\delta = C_1\dot{\phi} + C_2\ddot{\phi} \qquad (13)$$

Substituting the above relations in the fin moment it is easily derived that:

$$K_f = -\frac{dC_L}{d\alpha} \cdot \tfrac{1}{2}\rho V^2 . F_f B_f \left(\frac{\tfrac{1}{2}B_f}{V} + C_1\right)\dot{\phi} - \frac{dC_L}{d\alpha} . \tfrac{1}{2}\rho V^2 . F_f B_f . C_2 \ddot{\phi} \qquad (14)$$

By adding this external moment to the roll equation in (6) the influence of the fins is accounted for. The equations can be solved again and the rolling with the fins in action is found.

- The hydrodynamic lift force is proportional with the square of the forward velocity. So this large speed dependence obviously restricts their application to fast ships. Naturally large variations in draught may also hamper their use, for they must remain sufficiently submerged. But their effectiveness is practically unsurpassed. A well-designed installation can nearly eliminate the rolling, while reductions of the order of 90 per cent must be considered as normal.
- As you see their principal features are easily understood. The technical realisation, however, is more difficult. Therefore some firms have specialized in this field and several technically advanced systems are obtainable. A good paper elaborating on fins has been given by Conolly [6]; interested readers will find a lot of valuable information there.

6.3. Roll-damping tanks.

There are two different types of roll-damping tanks: the U-tanks and the free surface tanks. When a tanksystem is activated it is always of the U-tube type. The majority of the tanksystems act passively, however. In that way their specific features also show to full advantage. Therefore we will restrict the discussion to passive tanks.

- The water motion in the U-type of tank is an oscillating column of water. It resembles the motion of a pendulum and can be described analogously. The moment is reasonably accessible to calculation and thus the tank effect can also be computed [11]. In a free surface tank the water motion is entirely different. The transfer of mass from one side to the other takes place by a shallow water wave phenomenon. Until now calculations, [12] have not been able to account for the exerted moment, although trends are confirmed fully. Therefore a systematic series of experiments was necessary to provide for the tank moments necessary to include in the equations of motion [13]. With these data the equations can be solved again and the rolling with a tank in action can be compared with the rolling of the ship alone.
- For both types of tanks the investigations until now have been restricted to

the hypothetical condition of pure rolling in beam waves. This restriction was necessary for a number of reasons. In the first place too little was known about the behaviour of the ship itself in waves. The hydrodynamic forces associated with the rolling motion and the coupling effects with swaying and yawing were fully unknown. The knowledge about the ship even formed a greater limitation to the design and understanding of tanks than the knowledge about the tanks. This situation is now gradually being improved and we may expect a still better understanding and prediction of tank applications in the future. In the second place the action of roll damping tanks is clearly nonlinear. This makes it difficult out of principle to add the exerted tank moment to the essentially linear equations of motion for the ship. It also causes that tank moments due to the rolling and to the swaying of the ship can not be superposed. These facts are known to be true for the free surface tank. For the U-tanks little information on these points is available. But presumably the above facts will not be much different. It is as yet unknown to which extent a linearization of the tank moments will influence the results of the calculations. Only linearized the additional moments can be included in the equations (6). For the U-tank the same holds. Only then the mathematical formulation for the ship and for the U-tank can be combined to form a system of 4 equations for 4 degrees of freedom.

The effect of tanks on the rolling of the ships is a reduction of about 50-75% in regular waves and 40-60% in irregular waves. These figures are averages. They apply to well-designed tanks of normal dimensions in ordinary applications. Of course, lower or higher effects are possible in special cases. In general they cannot compete with active fins. But they also act at zero forward speed, are simple, cheap and invulnerable. Which type of tank has to be preferred in a certain situation, or which modification of that type, is hard to say. Each has its own characteristics, its pros and cons. But certainly no system is superior over all other systems in any situation concerned. If an experienced designer gets the opportunity to adjust his design, if necessary, to the case in consideration any system or modification will act satisfactorily. For comparative studies about various roll damping tanks reference is made to [14] and [15].

7. Illustration.

In figure 5 various computations for the rolling of a destroyer type hull in quartering waves are shown. In the first place the rolling has been calculated by the equations (6), so fully coupled with swaying and yawing. Next the yaw-terms in the first and second and the whole third equation of (6) have been neglected. Finally the sway-terms were dropped as well, after which pure rolling remained.

It can be seen that the direct coupling effects are not so large. At least in this case, for it is questionable whether this holds true in other cases. But anyway the pure rolling has been computed with the correct hydrodynamic coefficients $p_{\phi\phi}$, $q_{\phi\phi}$, $r_{\phi\phi}$ and with the correct wave exciting moment K_w. That means that a part of the coupling influences has been included by taking due account of the vertical position of the centre of gravity; compare (7) and (9). If the calculation of pure rolling is made (by lack of any better information) with a radius of gyration of 0.38 B, with a nondimensional damping coefficient of 0.10, with only the relative position of the centre of gravity and the metacentre in the metacentric height $\overline{GM}$ and with the Froude-Krylov moment as wave exciting moment the curve 2 results. The difference with the complete computation is large. So in this case the largest error is not made by the restriction to pure rolling but by the value of the hydrodynamic coefficients and of the wave moment. If, however, we have to determine the correct values for rolling we need the coefficients for swaying and the values for yawing will automatically result. Since the whole computational procedure has to be performed by a computer it requires very little extra effort to solve the complete set of equations instead of rolling alone. A real simplification and saving of time is hardly possible.

Another point to note is that the direct coupling effects may show to have little influence on the amplitude, but they can change the mutual phase relations between the motions to a greater extent. This can certainly be of importance for the absolute or relative motion of specific points of the ship and for an effective action of a control system.

Finally two other curves have been drawn in figure 5. They denote the rolling when a roll damping tank of the free surface type has been installed. The waterdepth has been chosen as $^h/_b$ = 0.04 and the tank length l = 0.25 b. One curve belongs to the complete calculation, the other to the rough estimate. The tank moments have been linearized in both cases with respect to their value at ϕ_a = 0.10 and were determined from the systematic experimental data in [13]. The tank has been chosen rather arbitrarily and will most probably not be the best obtainable.

I hope that I have been able to give you an impression of the difficulties involved in the problem of determining the ship's behaviour at sea, especially with regard to rolling. But also of the possibilities which are present at the moment. The investigation will continue for a long time. But in my opinion we have reached the point that we may consider the very cautions application to practical problems. That requires a good cooperation between theoretical and practical people. A task which is worth while, for in our field all of us ultimately work to use the knowledge we have obtained in actual applications.

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nowadays its application is restricted to very special objects.

When it appears possible to overcome the technical difficulties it is a very promising installation, however. Since the specific weight can be much higher than that of water and its track in the ship is fixed the whole installation can be very compact in principle. Actually some proposals have been made to realize this sytem for small ships.

introduced by Frahm in 1911 [7,8].

introduced by Watts in 1883 [9,10] .

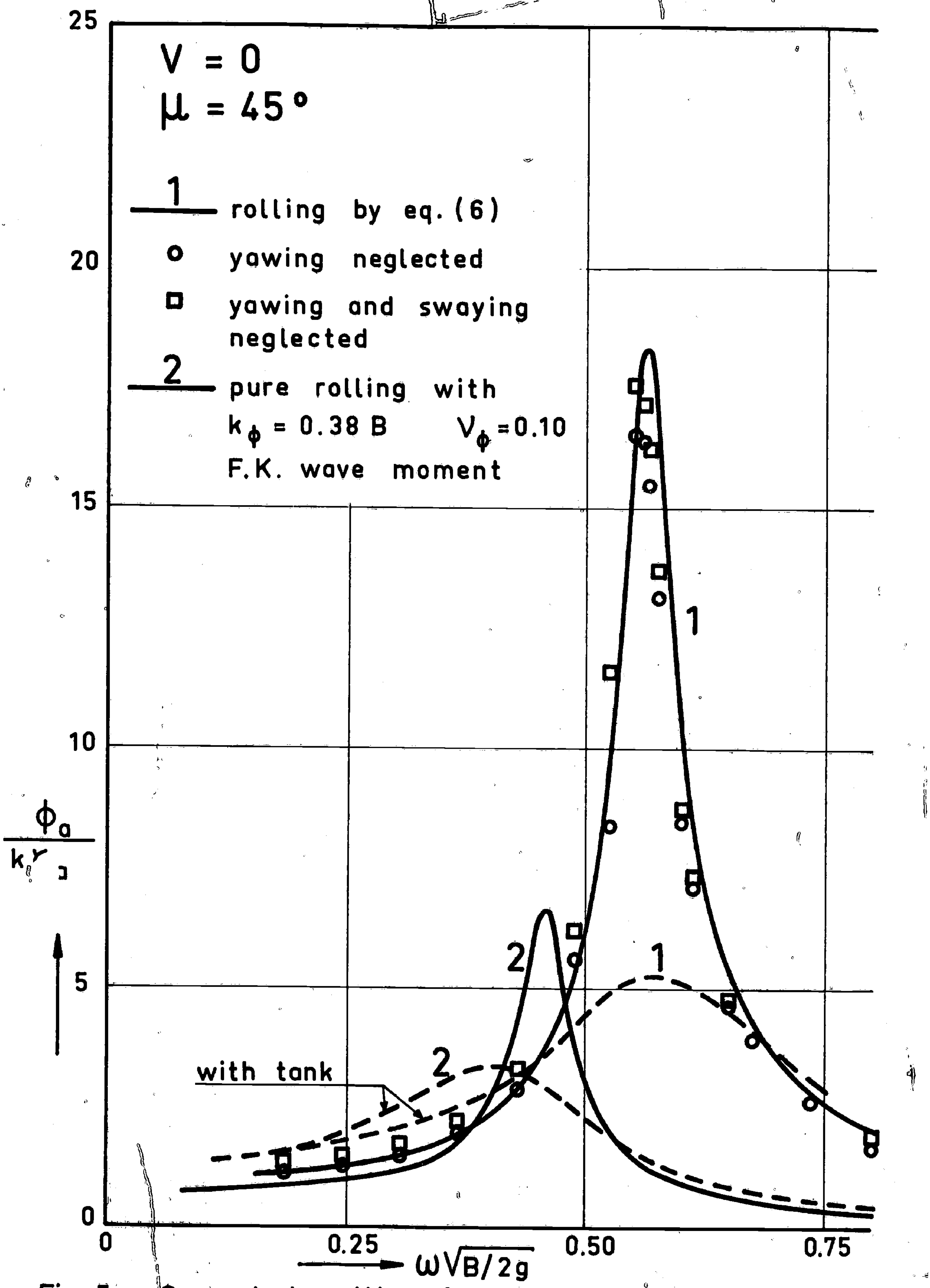

Fig. 5 Computed rolling for destroyer type hull in quartering waves