EXPERIMENTAL INVESTIGATION OF THE CHARACTERISTICS
OF FLOW ABOUT CURVED CIRCULAR CYLINDERS

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by

JEAN SURRY
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SUMMARY

The flow and the associated forces on curved circular cylinders, at Reynolds Numbers between \(10^4\) and \(10^6\), were investigated. The shape and normal force distribution of a flexible curved cylinder were found. Flow visualisation tests and pressure surveys were made on rigid curved cylinders. A comparison was made with earlier results for infinite straight cylinders inclined to the flow. It is concluded that the curvature of a cylinder has a significant effect on the flow and pressure in the wake, and must be considered in the prediction of the aerodynamic forces on such a cylinder.
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NOTATION

(b, d, h) dimensions of dynamic transducer (Sec. 2.2.2)

C_d local drag coefficient = C_N sin\( \theta \)

C_D total drag coefficient = \( \frac{D}{qH_d} \)

C_N normal force coefficient = \( \frac{N}{qd} \)

C_p pressure coefficient = \( \frac{p-P_s}{q} \)

C_t tangential force coefficient = \( \frac{t}{qd} \)

C_T tension force coefficient = \( \frac{T}{qd} \)

\( \Delta C_T \) \( (C_{T1} - C_{T2}) \)

d diameter of cylinder

D aerodynamic drag on body

e_i/e_{o_i} signal voltage/excitation voltage of a bridge circuit

E Young's Modulus

H spanwise length of cylinder

k calibration constant

l axial coordinate of cylinder

M moment

N normal component of total aerodynamic force per unit length

p static pressure

P_s free stream static pressure

P_T free stream total pressure

P_a atmospheric pressure

q dynamic pressure = \( \frac{1}{2} \rho V^2 \)

R radius of curvature
Reynolds' number

tangential component of total aerodynamic force per unit length

axial tension

free stream velocity

weight per unit length

cartesian coordinates

Figure 2.4

total height of cable

section modulus = 1/6 bd^2

angles related to dynamic transducer (Fig. 2.4)

boundary layer thickness

strain

azimuthal angle in diametric plane (i.e. plane perpendicular to the cylinder axis)

azimuthal angle of separation

kinematic viscosity

density of air

angle of inclination of cylinder axis to stream (Fig. 1.1)

highest and lowest points of the cable in the wind

component normal to cylinder axis

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I INTRODUCTION AND REVIEW

1.1 The Present Status of the Towed Body Problem

A survey of the existing literature on the problem of the towed body was made following the publication of a study of the body dynamics (Ref. 1), and is summarised in Appendix A. From such a breakdown it was clear that the weak point was the role of the cable in the cable-body system. Firstly, the aerodynamic forces on such a cable, which will define the equilibrium shape, are not known precisely and secondly, the dynamics of the cable around the quasistatic shape have not been defined. On the second problem Phillips (Ref. 2) has done some initial theoretical work on the propagation of waves down the cable and Billing (Ref. 3) performed some experiments on such travelling waves, which cause longitudinal whipping of the body. The first problem is the concern of the present report.

1.2 Prior Work

In 1917 Relf and Powell (Ref. 4) measured the cross wind force (lift) and drag on rigid wires inclined to the wind. Their results (Fig. 1.1), which have been widely accepted and used (Ref. 5), show that for subcritical flows to a good approximation, for

\[ N \propto \sin^2 \theta \]

and, since

\[ V_n = V \sin \theta \]  

(Fig. 1.1)

\[ N \propto V_n^2 \]

Hence the flow in planes normal to the cylinder is simply a function of the local normal velocity. The tangential force coefficient is small (0 to 0.06) implying the tangential velocity component just results in a friction force.

Table 1 is a summary of the analytical expressions used by various authors to fit the results of Relf and Powell. Bursnall and Loftin (Ref. 15), doubting the validity of these results for supercritical turbulent flow, made a pressure survey on cylinders inclined to the wind. For subcritical Reynolds' Numbers their results are similar to the earlier ones (Fig. 1.1), but they observed that the pressure in the wake, to a small extent, was not simply a function of the normal velocity component. This may be explained in terms of the laminar flow before separation being predictably only a function of \( V_n \) (Ref. 16), but the turbulent wake is affected by the shearing of the external cross flow. This is discussed in greater detail in Section 3.1.

1.3 Postulation of Three Dimensionality of the Wake of a Curved Cylinder

All the analytical work on cable shapes is based on the aerodynamic tests of infinite straight cylinders. The validity of the
application of this data to curved cylinders can be challenged by the following argument. Consider a curved cylinder: as $\phi$ varies, so will the pressure distribution (Fig. 1.2). Assuming the $\sin^2 \phi$ relation, the wake pressures will be most negative when $\phi = 90^\circ$ (cylinder axis normal to flow) and will increase for decreasing $\phi$ (Fig. 1.1). There is no mechanism for sustaining such a pressure gradient in the wake, hence the fluid will tend to accelerate in the direction of the negative gradient. Thus we might predict axial motion in the normally two-dimensional wake with an associated pressure redistribution. Finally, we might suspect the normal force will no longer be simply proportional to $\sin^2 \phi$.

The following work was undertaken to investigate this suspicion in two parts. Section II finds the normal force distribution on a flexible curved cable and compares it with the generally accepted distribution. Section III investigates the flow in detail around a rigid curved cylinder, with flow visualisation, and a pressure survey; then, finally, an estimate is made of the normal force distribution.

II SHAPE AND NORMAL FORCE DISTRIBUTION OF FLEXIBLE CABLE

2.1 Equilibrium of Flexible Cable in Airstream

With reference to Fig. 2.1 and the definitions in the symbol table, the equations of force equilibrium of a flexible cable in a uniform wind are:

\[
N \Delta l = T \Delta \phi + w \Delta l \cos \phi + \frac{\Delta T \Delta \phi}{2}
\]

\[
t \Delta l = -(\Delta T + w \Delta l \sin \phi)
\]

Linearising, nondimensionalising by division by $(qd)$ and rearranging, the equations become, in the limit:

\[
C_N = C_T \frac{d\phi}{dl} + \left( \frac{w}{qd} \right)
\]

\[
C_t = - \left[ \frac{dC_T}{d\phi} + \left( \frac{w}{qd} \right) \sin \phi \right]
\]

If both $C_N(\phi)$ and $C_t(\phi)$ are known, these equations give the cable shape and tension distribution $C_T(\phi)$. Conversely, if the cable shape $\phi (l)$ and $C_T(\phi)$ are known, $C_N(\phi)$ and $C_t(\phi)$ can be found from equations (3) and (4) respectively.

It was not possible to find the $C_T$ distribution accurately, (Appendix B describes an interesting, but aborted attempt, using a photoelastic technique). Since $dl \sin \phi = dy$, and assuming $C_t$ is constant (discussed in Section 2.5), integration of equation (4) gives:
\[ C_{T1} - C_T = C_t (\ell - \ell_1) + \left(\frac{w}{qd}\right) y \]  

(5)

At the lower end of the cable

\[ C_{T1} - C_{T2} = \Delta C_T = C_t (\ell_2 - \ell_1) + \left(\frac{w}{qd}\right) y_F \]

then

\[ C_t = \left[ \Delta C_T - \left(\frac{w}{qd}\right) y_F \right] \left(\frac{1}{\ell_2 - \ell_1}\right) \]

(6)

Hence the tensions at the top and bottom of the cable were measured as functions of shape and wind speed. The distribution of \( C_T \) found from equations (5) and (6) was used in equation (3) to find the normal force distribution \( C_N \).

2.2 Experimental Technique

Figure 2.2 illustrates the basic test set-up; a length of Tygon tubing was suspended between the roof and floor of the UTIAS Wind Tunnel, such that the "cable" curve lay in the plane defined by the vertical and wind vectors. The ends were free to pivot in the cable plane. Two samples were used as in Table 2, to test the effect of diameter and weight per unit length.

2.2.1 Cable Shape Measurement

A one-inch grid was drawn on a false wall. The cable plane was parallel to and 2.4 inches in front of this wall. Tuft surveys indicated the direction of flow in the cable plane to be unaffected by the boundary layer on the wall. Since a typical \( Re \) is \( 2.2 \times 10^6 \) the boundary layer will be turbulent with a displacement thickness of 0.1 inches (Ref. 17) at the furthest point of the cable downstream. This is small compared to the cable distance away from the board, though any displacement effect the boundary layer will have is only to tend to alter the plane of flight of the cable slightly.

A 4" x 5" view camera was mounted rigidly with its axis approximately perpendicular to the grid, outside the test section. With flood lighting, four high speed exposures were superimposed for each test run. Figure 2.2 is a typical photograph. An arithmetic mean of the four images was used to give the shape of the cable, which tended to oscillate ten diameters or so at 20-30 cps. A node can be seen about 2/3 from the top.

The photographs were enlarged to a practical working size to reduce the read-off errors. The cable coordinates were read by measuring in arbitrary units from the closest grid lines and dividing by the local grid width measured in the same units. In this manner, all distortion due to the camera, or reproduction was eliminated. Parallax corrections were
made, since the cable was 2.4 inches in front of the grid. This was a 4.2% linear correction applied to the vertical and horizontal coordinates measured from the vanishing point (Fig. 2.3a). The latter point was found by joining lines, on the photograph, in the test section of the tunnel that were known to be perpendicular to the grid (Fig. 2.3b).

2.2.2 Cable Tension Measurement

The tension in the cable was measured by two strain gauge transducers, mounted above and below the tunnel. Figure 2.4 shows the salient features. The brass "U" (Fig. 2.4b) rotates on needle points to ensure measurement of the axial force in the cable. The line of action of the tension, T, is always perpendicular to the side arms and passes through the pivot line (to within a degree, as discussed below). The side arm is rigid and the moment of the tension simply acts about the corner, A, then

\[ M = Th \]

This moment causes a curvature and hence strain in the thin cross member

\[ \varepsilon = \frac{M}{ZE} = \frac{h}{ZE} \]

Figure 2.4c shows the circuit in which the error signal from the balanced bridge is proportional to the tension

since \[ e_i = e_{o_i} \frac{k \varepsilon}{2} \]

where \( k \) is the sensitivity factor of the strain gauge

then \[ \frac{e_i}{e_{o_i}} \propto T \]

With the two strain gauges mounted spatially close together and wired as shown, they gave a temperature compensated signal: any temperature fluctuation in the test region causes equal change in resistance of both gauges, hence the cancelling effect. It was found necessary to have high temperature stability in the resistors completing the bridge to avoid spurious signals.

The only force keeping the "U" from aligning with the load is its own weight. The "U" was counterbalanced with a pointer such that unloaded the "U" balanced horizontally, i.e., the cg. was in the pivot side of the "U", below the pivot line. From Fig. 2.4b, for equilibrium

\[ Fy' = Wx', \]

and

\[ y' = b (\alpha - \beta) \]

\[ x' = h' \sin \beta \]

then

\[ (\alpha - \beta) = \frac{W}{F} \frac{h'}{b} \]

For the worst test case the error in alignment would be 1° or 0.2% in the moment M.
The transducers were calibrated by suspending known weights over a small free pulley. This was done at various angles off vertical to test the above discussed misalignment effect. The sensitivity for loads up to 7 oz. (0.337 and 0.374 mv/v/oz), the repeatability (1/2% zero error for 4 oz loads), and the linearity (1/2% FSD) were good. The calibration appeared independent of the angle of application of the load, though the angle of the misalignment was only 1/2 - 1° for 1 oz loads.

The transducers had two inherent natural frequencies: a pendulum mode of 1.3 cps and a bending mode about 10 cps, though the latter was unexpectedly independent of the load.

2.2.3 Tunnel Dynamic Pressure Measurement

A standard strain gauge differential pressure transducer was used to measure the difference between the total head and the test section static pressure. A calibration was made with known pressures. The d.c. signals were amplified a thousandfold and carried to the UTIAS Analogue Computer for "on-line" reduction.

2.3 Data Reduction

2.3.1 Analogue Computer - "On-Line"

Figure 2.5 outlines the circuitry. Principally the logic is

\[
\left(\frac{e_1}{e_0}\right)^{k_1} = T_1 \quad ; \quad \left(\frac{e_2}{e_02}\right)^{k_2} = T_2 \quad ; \quad \left(\frac{e_q}{e_0q}\right)^{k_q} = q
\]

The continuous division of these quantities by the excitation voltages and q, removed any effects due to drift from the required information. Simulated filter circuits were used to reduce the noise level (due to the high amplification and 60 cps picked up "en route" to the computer) in both q and CT. The signals q, CT, and \(\Delta CT\) drove three galvanometers of a multichannel recorder. Hence the final output was continuous smoothed data.

2.3.2 Digital Computer - Data Smoothing and Analysis

The University of Toronto I.B.M. 7090 Computer was used to smooth and apply parallax corrections to the raw data from the photographs, and then to generate the \(C_N(\theta)\) distribution, using the appropriate \(CT_1, \Delta CT\) and q. Table 3 shows the particular cases analysed. Fig. 2.6 is a schematic of the program used.

The raw data on the cable shape was corrected for parallax, giving spatially correct co-ordinates \((x(y), y)\). The curve length was found from
\[ \ell = \int_{0}^{y} \left[ 1 + \left( \frac{dx}{dy} \right)^2 \right]^{1/2} dy \]

which was integrated stepwise along the cable (34 steps). The cable angle was then calculated from

\[ \phi = \cot^{-1} \frac{dx}{dy} \]

and the curvature from

\[ \frac{d\phi}{d\ell} = -\frac{d^2x/dy^2}{\left[ 1 + (dx/dy)^2 \right]^{3/2}} \]

Note that \( \phi \) is a function of the first derivative, while \( d\phi/d\ell \) is essentially a function of the second derivative of the data. Since the weight term of equation 5 was small \( C_N(\phi) \) was very sensitive to \( d\phi/d\ell \). Hence it was essential to obtain the most accurate second derivative possible from the photographic data. Several techniques were tried:

(a) A simple iterative hand calculation. The first derivative was found from calculating the \( \Delta x/\Delta y \) for the 34 points taken from the photographs. These values were plotted and a smooth curve was fitted by eye. Then the second derivative was found from \( \Delta (\Delta x/\Delta y) \) smoothed. Again smoothed graphically, these values were used to calculate \( C_N(\phi) \). This procedure indicated the seriousness of the problem of finding \( (d^2x/dy^2) \) accurately and suggested more sophisticated methods were required.

(b) Reference 18 proposed curve fitting by a "moving arc". A low order polynomial is fitted to a fraction of the total number of data points; the least squares best fit to the central point of this group is calculated, including any required derivatives at this point; then the attention is shifted one point over, and a new fit of the polynomial is made. A simple looping program was written that performed the local fit and also shifted the fit origin one interval on the \( y \)-axis. A cubic fitted to seven points was tried.

The least squares curve differed from the data points by one part in the fourth significant figure, which was 67% of the estimated error in the data (discussed in section 2.4). Smooth values of \( dx/dy \) were generated (Figure 2.7) but the \( (d^2x/dy^2) \) values tended to be erratic (Figure 2.8). Theoretically the best information available from the data is given by the curve that has a mean deviation from the data points equal to the estimated experimental error in the data. The "cubic to seven points" was not smoothing the data sufficiently, hence the values of \( (d^2x/dy^2) \) contained extraneous information.

(c) A second smoothing was performed, by repeating the technique above, on the derivatives found by the "cubic to seven points". The derivatives
only changed by four parts in the fourth significant figure, but the second derivatives were even more erratic (Figure 2.8). It was implicitly assumed, that \( C_N(\theta) \), hence \( (d^2x/dy^2) \), most probably was monotonic, since no aerodynamic mechanism was apparent that could sustain steady non-monotonic values. Again the smoothing was insufficient to improve the derivatives of the fitted curve.

(d) The cubic was then fitted to eleven points at a time. The mean deviation of the data points from this least squares fit was equal to the estimated error in the data. The second derivative was quite smooth, as seen in Figure 2.8. This "moving arc smoothing" technique has inherently poorer values for the first and last few points, which, for the second derivatives of this curve, required the fit of a linear polynomial to the end five points. This is quite apparent in the figure, and limits the range of useful \( (d^2x/dy^2) \).

(e) The traditional technique of fitting a high order polynomial to the entire set of data was then tried. A simple program was written to fit a ninth order polynomial to the data, using the least squares library program of the IBM 7090 computer. The mean deviation of the data points from this curve was 93% of the estimated error in the data. Figure 2.8 shows the smooth second derivatives, which contain no obvious end effects. It might also be noted that this curve tends to be the mean of all the curves in Figure 2.8.

Since the ninth order fit was the most satisfactory, this was used finally to find \( \phi \), \( d\phi/d\ell \), and \( C_N(\theta) \). The last four points probably have little significance in terms of a second derivative since the fast moving portion tended to be blurred in the photo, hence the data points had higher inherent error.

2.4 Results

Figure 2.9 shows the tension co-efficients at the top and bottom of the cable, \( C_T1 \) and \( C_T2 \), as functions of \( q \), for the two test cases. \( C_T1 \) is constant in this \( q \) range with a mean deviation of 1/2%. \( C_T2 \) tends to increase with speed, hence the difference, \( \Delta C_T \), tends to decrease with speed. The tension decreased by 15% of its value at the top. Seventy percent of this decrease is due to the weight term \( (w/qd)yF \). Table 3 shows the measured values of \( C_{T1}, \Delta C_T \) and \( q \) for the particular test cases analysed.

The procedure described in Sec. 2.3 was followed to find the \( C_N(\theta) \) distributions for these cases. Figure 2.10 shows the five solutions. Also plotted are the results assumed from infinite straight inclined cylinders, ie. the \( \sin^2\theta \) relation. These predicted values are directly proportional to the peak value assumed, ie. \( C_{D0} \) as generally defined for a cylinder. \( C_{D0} \) is a function of \( Re \) as seen in Reference 5, and for the \( Re \) range tested, \( C_{D0} \) varies from 1.0 to 1.2. Reference
15 proposed that for inclined cylinders the Re should be based on the "streamwise diameter" of the inclined cylinder (i.e. \( d / \sin \theta \)) which raises the mean test CD from 1.1 to 1.2. In Figure 2.10, CD is assumed to be 1.2 but the uncertainty should be remembered.

The results for cases 1 and 2, which had essentially the same configuration and Re, indicate the repeatability of the solution. The maximum difference in CN is 8.3% which falls within the possible 10% range estimated in the following section. Though the quantitative values may have this order of error, the curves reveal definite trends away from the previously predicted \( \sin^2 \theta \) distribution. At the lower angles of inclination the calculated values are well approximated by the inclined cylinder. Above 40 degrees the agreement is poorer. The peak CN does not occur as predicted, at the portion of the cable normal to the flow, but at lower angles of inclination, with a substantial decrease at 90°. The rapid rise appearing in the last points after 90° is probably a significant trend but cannot be assumed quantitatively accurate, for the reason discussed in the last section. As the Re increases the peak remains about the same value but moves to lower angles, i.e. moves up the curved cable.

The mean radius of curvature of the cable was about 2.5 ft. for each case. \( R/d \), the nondimensional radius of curvature, is listed in Table 3 for each case.

2.5 Errors

There are many sources of error, and difficulty was encountered in interpreting particular errors. A final estimate is made below of 5% as the probable error in CN, and less than 0.5 degrees in \( \phi \). Table 4 lists the major sources of error. No further comment is required for the first six mentioned (see the relevant sections).

The error in \( C_T \) is due to errors in \( C_{T1} \), w/qd, the calculated constant \( C_t \) (Eqs. 5, 6) and the assumption of a constant tangential force coefficient \( C_t \). Equation 2.2c can be rewritten as:

\[
\frac{C_T}{C_{T1}} = 1 - \left[ \frac{C_t}{C_{T1}} + \left( \frac{w}{qd} \right) y \frac{1}{C_{T1}} \right]
\]

The largest value the bracketed term can have is \( \Delta C_T / C_{T1} \), which was about 15%. The maximum weight term was calculated and found to be 67% of \( \Delta C_T / C_{T1} \), hence the tangential component correction is only 5% of \( C_{T1} \). \( C_t \) was calculated from equation 6, which contains the term \( \Delta C_T \). The error in \( C_T \) was about 10% resulting in a 20% error in \( C_t \), since the weight term in equation 2.3 was approximately \( \Delta C_T / 2 \). But the 20% error in \( C_t \) only gives 1% in \( C_T \). Also, since the tangential component term is so small, the assumption of constant \( C_t \) is not serious.
Further indication that this is a fair assumption is seen from Figure 1.2; for straight cylinders 20° < Φ < 90° a mean value of C_t can be found and the probable error is only 10% of the mean. Considering the tabulated errors in C_T1 and w/qd, the maximum error in C_T was 3.5%. The probable error in C_T is then .6 x 3.5% = 2.1%. Without using the theory of least squares fitting to find the effect of the standard deviation of the data on the first and second derivatives, a method of estimating the probable errors was found. It was shown in Section 2.4 that the cubic fit to seven points duplicate the original points too closely; i.e. the standard deviation of the fit was less than the error in the points. The derivatives found with this technique were assumed to be those resulting from the raw data. Then the mean deviation of these results, from the assumed "best fit" ninth order fit results, was calculated. In this way an estimate of the probable error in the final derivatives was found. This estimate would be rather conservative since the probable error should be much reduced from the original by the smoothing. It was found that the probable error in dx/dy was 0.7% and in d^2 s/dy^2 was 2.8%. Thus, at the smallest angle (Φ = 20°) the error is about one degree, but at 90°, the error is zero; a mean of 1/2 degree might be assumed over the test range. Since dΦ/dl = d^2 x/dy^2 the error in dΦ/dl is 2.8%.

Finally, the error in C_N is principally due to the errors in C_T and dΦ/dl. Then the probable error in C_N is (2.8 + 2.1)% = 5%.

A comment should be made on the assumption of a completely flexible cable. The bending moment due to curving the cable into the shape x(y) requires a load distribution w:

\[ w = EI \frac{d^4 x}{dy^4} \]

A simple extension measurement of E was made and found to be 7.6 x 10^2 psi for the Tygon. \( I = \frac{\pi d^4}{64} = 1.2 \times 10^{-5} \text{ in}^4 \)

Hence

\[ w = 6.3 \times 10^{-5} \frac{d^4 x}{dy^4} \text{ (lb/ft)} \]

Now in order that the bending moment be negligible for this case, \( w \ll N \). The smallest N was 1.8 x 10^{-2} lb/ft

Thus

\[ w = 6.3 \times 10^{-5} \frac{d^4 x}{dy^4} \ll 1.8 \times 10^{-2} \text{ lb/ft} \]

\[ \frac{d^4 x}{dy^4} \ll 300 \text{ /ft}^3 \]

From the ninth order polynomial \( d^4 x/dy^4 \) was of the order of \( 10^{-1}/\text{ft}^3 \), hence the stiffness of the cable is negligible.
III VISUALISATION AND PRESSURE SURVEY OF FLOW AROUND A CIRCULAR CYLINDER

3.1 Infinite Straight Cylinder Inclined to Flow

3.1.1 Technique of Flow Visualisation

Figure 3.1 illustrates the technique used to find flow patterns on a 1.4 in. diameter circular cylinder inclined to the wind at 60°. A suspension of lamp black in kerosene was painted over the entire exposed surface. The kerosene evaporated in the wind in about 10 minutes leaving the lamp black pattern on the cylinder surface. Hence, any flow interpretation must be on long term action. The patterns are the result of the streamlines near the bottom of the boundary layer. Since at the separation line,

\[ \text{Re}_x = \frac{Vx}{\nu} = \frac{100 \times 1/12}{1.56 \times 10^{-4}} = 6 \times 10^4 \]

which from Ref. 19, implies a laminar boundary layer whose thickness is of the order

\[ \delta_x = \frac{5x}{(\text{Re}_x)^{1/2}} = \frac{5}{(6 \times 10^4)^{1/2}} \text{ inches} \]

\[ \cong 0.020 \text{ inches} \]

The carbon particles are of the order of 0.0001 inches. Hence the pattern is a mean picture of the lower portion of the boundary layer.

Figure 3.2 is four views of the cylinder showing the whole circumference consecutively. The two flow regimes are readily identified as a laminar boundary layer and a turbulent wake region.

3.1.2 Laminar Flow Region

The flow pattern in the laminar region of Fig. 3.3a, is quite predictable from basic laminar flow theory (Ref. 19). The front dividing line, \( \theta = 0 \), is defined by the streamlines lying parallel to the cylinder axis. This is not a stagnation line as for the perpendicular cylinder since only the normal velocity component is zero at the surface, while the tangential component is unaffected (\( V \cos \theta \)). As \( \theta \) increases initially, the streamlines are bent toward the diametric plane; the flow tends to accelerate in the direction of the most negative gradient which is in this plane (Fig. 3.11b - "predicted curve"). Then, as \( \theta \) increases further (\( \theta > 70^\circ \)), the diametric pressure gradient becomes positive and the flow bends away from the normal again. The flow loses energy to friction in the boundary layer and hence slows down. Since the gradient normal to the streamline

\[ \frac{\partial \rho}{\partial n} = -\frac{\rho V^2}{R} \]

where \( R \) is the radius of curvature of streamline.
then \( R \propto V^2 \) for constant \( \partial p/\partial n \)

If \( (\partial p/\partial n) \) is assumed symmetrical about the peak at \( \theta = 70^\circ \), the flow makes a sharper turn for \( \theta > 70^\circ \) than for \( \theta < 70^\circ \) due to the reduced velocity. At the separation line, the diametric velocity component close to the surface must be zero (i.e. the particles here have no momentum in this direction to overcome the pressure gradient) (Fig. 3.3b). The remaining velocity component is axial, though it is also reduced by frictional losses (the no-slip condition gives zero total velocity exactly on the surface in the laminar sublayer). Hence for yawed cylinders, the flow must be parallel to the separation line at the surface. The cross component is not necessarily zero higher in the boundary layer, and hence, the streamlines form a three-dimensional fan as shown in Fig. 3.3c.

3.1.3 Separation Line

Typical of laminar boundary layers, the separation line is very well defined and stable. It occurs at \( \theta_s = 75^\circ \left(\theta^0\right) \). Reference 6 shows that the separation angle is independent of the inclination angle for subcritical laminar separation and occurs at \( 75^\circ < \theta < 80^\circ \).

3.1.4 Wake

In the wake, immediately behind the separation line, the flow appears axial in the direction of the free stream tangential velocity component. This was confirmed by tuft tests. As \( \theta \) tends to \( 180^\circ \), the flow along the surface tends away from the centre. This wake flow is a manifestation of the conclusion in Ref. 15, that the wake pressure is neither uniform nor simply a function of the normal velocity component. At these Reynolds' numbers the wake is turbulent with small stochastic momentum components. As seen earlier, at separation only the tangential momentum remains in the surface fluid. This exerts a shearing stress on the wake behind the separation. Hence the wake will have a mean axial flow component which decreases towards the centre of the wake, since the influence of the external flow decreases as the wake is penetrated deeper.

3.1.5 End Effects

The problem arises as to whether this wake flow observed is due to the ends. The length to diameter ratio tested was 20, which should give unaffected pressure distributions over the central portion of a cylinder perpendicular to the flow (Ref. 5). As for wake flows due to end vortices, the roof and floor acted as infinite end plates. A 0.3 in. gap was left around the cylinder in the floor. Since test section static pressure is only 10.5% of tunnel q greater than atmospheric, and the wake pressure is about 100% of q less than static, flow through the gap will tend to be into the wake from outside. Since the wake flow observed was in the opposite direction, the above could not have a major effect on the wake.
3.1.6 Conclusion on Wake Flow Behind Straight Cylinders

From the above arguments, we may conclude that the wake flow can indeed be three-dimensional as observed in Section 1.3, although in this case it is apparently due to shear from the neighbouring airstream.

3.2 Flow Visualisation on a Curved Circular Cylinder

3.2.1 Technique

Figure 3.5 illustrates the test set-up: The curved cylinder used was a piece of hollow polyethylene tubing of one inch O.D. Figure 3.4 shows the variation of the angle of attack, along the cylinder axis, \( \ell \). The slope of the curve gives the radius of curvature of the cylinder, since

\[
R = \frac{d\ell}{d\phi}
\]

The mean radius is 18 inches, i.e. \( R/d = 18 \). The cylinder ends were cut and mounted flush with the floor and roof of the test section. Two forms were tested - (a) bowed away from the wind and (b) bowed into the wind. As shown in Fig. 3.5, tufts were spot glued along the inner and outermost curves \( (\theta = 0, 180^\circ) \). For more detailed flow patterns the lamp black and kerosene was used again (Figs. 3.6 and 3.8). The test Re, even accounting for angle of inclination (Ref. 15.) was subcritical, as for the tests in Sec. 3.1.

3.2.2 Description of Flow Around a Cylinder Bowed Away From the Wind

In Figs. 3.5a the tufts at the front lie on the cylinder surface quite steadily. Since the tufts were of the same length as the cylinder diameter they indicate a mean flow direction. The tufts are seen to be between the freestream and the diametric directions. In the wake the tufts lie steadily along the axis toward the cylinder section which is normal to the flow. At the normal section the tufts in the wake appear unsteady with a mean downstream, horizontal, direction. Hence we may conclude the flow is laminar at the front and the wake has a strong flow component toward the normal section where it separates and moves downstream (Fig. 3.7a).

Figure 3.6 confirms and further elucidates the above flow picture. The flow seen from the detailed pattern, is laminar at the front and shows a sharp separation line. (The dark line along the cylinder seen in this region is due to a scratch in the surface.) As for the straight cylinder the streamlines are parallel to the cylinder axis at \( \theta = 0^\circ \), forming a dividing line. It is clear that when \( \phi = 90^\circ \) the flow has no tangential component thus the streamlines must be perpendicular to the
dividing line, as seen. Again, the flow bends towards the local diametric plane as $\theta$ increases. The separation line is sharply defined but the angle of separation, $\theta_s$ varies with $\phi$.

The most striking feature of this test was the wake region: a steady asymmetric cell structure, which was quite reproducible, lay in between the azimuthal angles $135^\circ$ to $225^\circ$ (Figs. 3.6 and 3.7b). The basic flow direction in the cellular region was parallel to the cylinder axis towards the normal section. In the latter region eddies formed, producing a narrow reverse flow channel behind the separation line, culminating in four stagnation areas. Here, also, the flow detached and bled off into the downstream wake, as shown by still wet kerosene droplets (seen as a grey stippled region in the wake picture of Fig. 3.6). Figure 3.7b is a schematic of the cell structure. The cell lengths increase ($L/D = 1.5$ to $2.0$) with decreasing inclination. Similar cells appeared in the top half of the bowed cylinder, though less well defined.

3.2.3 Description of the Flow Around a Cylinder Bowed Into the Wind

In Fig. 3.5b the tufts indicate laminar flow of predictable form ahead of separation. The wake was turbulent over most of the curve but began to show a directed flow as $\phi$ decreases below $45^\circ$. Here the flow is away from the normal section. The overall streamline pattern might be as shown in Fig. 3.9.

Figure 3.8 shows the more detailed flow. Once again the laminar boundary layer flow was parallel to the cylinder axis, forming the front dividing line. The flow bent towards the diametric plane. For inclinations greater than $60^\circ$ the streamlines entered the separation line approximately normal, presumably because the remaining tangential velocity component at this point was too small to turn the flow. For $\phi$ less than $60^\circ$ the streamlines turned parallel to the axis before separating. The separation line ceased to be sharply defined for $\phi$ less than $30^\circ$. The laminar flow tended to remain attached and turned into the wake region with an unsteady transition point at $\theta = 90^\circ$. Of course, in the limit as $\phi$ tends to zero, there should be no separation, simply a transition from laminar to turbulent boundary layer. The flow pattern in Fig. 3.8 for shallow angles is an intermediate stage between the two regimes.

In this case, no cell structure was apparent, but once again the wake has a central and two side regions. In the range $80^\circ < \theta < 120^\circ$, the wake on the cylinder surface appeared almost stagnant but flow tended to be around the diametric plane. The wake then tended to turn and flow parallel to the axis at $\theta = 180^\circ$. The central wake appeared to "gather strength" as it moved away from the normal section, as also seen by the tufts.
3.3 Pressure Distribution Around a Curved Circular Cylinder

To add to the flow visualisation pictures shown above, a pressure survey was made on the cylinder both bowed into and away from the wind.

3.3.1 Method

Pressure taps were imbedded in the surface of the 1"D polyethylene tube at several locations. 1/8" OD Tygon tubing with a small steel collar was cut flush with the outer surface and fed down the centre of the tube. The tubes passed through the floor and ceiling of the tunnel such that the cylinder remained as shown in Fig. 3.5. Figure 3.4 shows the location of the thirty seven pressure taps. There were two taps across the diametric plane at $\theta = 0^0$ and $180^0$ at each station. Also, at the planes $\theta = 83^0$ and $45^0$, there were taps every $30^0$ around the circumference. Both configurations of the cylinder were tested.

The pressure leads were connected to a 42 tap scanivalve. The rate of scanning was about one second per tap, but a switch was included to stop the scanning at any chosen tap and allow a longer record at the particular station to be taken. The scanivalve contained a strain-gauge differential pressure transducer. The output signal was amplified by a d.c. differential amplifier and filtered in the analogue computer. A variable filter time constant was used, but generally only 60 cps noise and higher frequencies were removed. The result was plotted against a time base on the x-y plotter. A typical record is shown in Fig. 3.10. $C_p$ was calculated by dividing the pressure differences by tunnel dynamic pressure.

3.3.2 Results of Pressure Measurements

Figures 3.11a and 3.12a are plots of $C_p$ vs. the angle of inclination to the flow, $\theta$ for the curves bowed away from and into the wind respectively. The pressures along the front dividing line and centre of the wake are both plotted. Three speeds were tested to find any dependence in $Re$, which was subcritical at all speeds. No significant difference appeared between $Re = 3 \times 10^4$ and $7 \times 10^4$. Also plotted are the results from Ref. 15 for $\theta = 0^0$ and $180^0$, at $Re = 2 \times 10^5$. These points are for $\infty$ straight cylinders inclined at the particular $\theta$.

In both 3.11a and 3.12a the pressure distribution along the front dividing line is almost the same as for the straight cylinders. The apparent slight asymmetry in the curved cylinder results may be due to error in the angle measured (1-2 degrees), or may be inherent in the vertical asymmetry of the curve in the tunnel.

The measured wake pressures on the curved cylinders are quite different from those for the straight cylinders. For the curve
away from the wind the wake pressure varies both more and less negative than the predicted value. For the curve into the wind, the pressures are more negative throughout the wake.

Figures 3.11b, c and 3.12b, c show the diametric pressure distribution for $\phi = 45^\circ$ and $83^\circ$ for the two cases and the infinite inclined case. It becomes apparent that ahead of the separation point the pressure is fairly well predicted by the $\sin^2\phi$ relation but at and after separation the pressure has no apparent relation to that predicted. The pressure in the wake is approximately constant in a diametric plane, at the value shown in Figs. 3.11a, 3.12a. The separation point is not sharply defined on these plots due to the difficulty of inserting the pressure taps closer together.

3.3.3 Forces on Curved Cylinder

From the $C_p(\theta, \phi)$ distribution found above, the normal force distribution $CN(\theta, R/d)$ can be found. $CN$ is considered a function of the relative curvature $R/d$.

Since

$$CN(\theta, R/d) = \int_0^{2\pi} \frac{p d/2 \cos \theta d \theta}{1/2 \rho V^2 d}$$

$$= \int_0^{\pi} C_p(\theta, \phi, R/d) \cos \theta d \theta$$

$$= \int_0^{\theta_s} C_p(\theta, \phi, R/d) \cos \theta d \theta + \int_{\theta_s}^{\pi} C_p(\theta, \phi, R/d) \cos \theta d \theta$$

(7)

It was shown in section 3.3.2 that

$$C_p(\theta, \phi, R/d) \approx C_p(\theta, \phi, \infty) \quad 0<\theta<\theta_s$$

Also, in the wake region, $C_p$ is independent of $\theta$, ie.

$$C_p(\theta, \phi, R/d) \approx C_{pw}(\phi, R/d) \quad \theta_s<\theta<\pi$$

Then eq. 7 becomes

$$CN(\theta, R/d) = \int_0^{\theta_s} [C_p(\theta, \phi, \infty) \cos \theta] d \theta - C_{pw}(\phi, R/d) \sin \theta_s$$

(8)

and, assuming $\theta_s$ is independent of $R/d$, we have

$$CN(\theta, \infty) = \int_0^{\theta_s} [C_p(\theta, \phi, \infty) \cos \theta d \theta - C_{pw}(\phi, \infty) \sin \theta_s$$

(9)

If the integration term in eq. 8 is substituted for from eq. 9, eq. 8 becomes
Thus $C_N$ for the curved cylinder is the value predicted from the straight cylinders ($R/d = \infty$) and an additional term $\Delta C_N$, due to the altered wake pressure. Using the values of $C_{p_w}(\phi, \infty)$ and $C_{p_w}(\phi, R/d)$ in Figs. 3.11a, 3.12a, $\theta_s = 75^\circ$ and $C_N(\phi, \infty)$ as in Fig. 2.8, $C_N(\phi, R/d)$ was plotted against $\phi$ for both test cases, Fig. 3.13.

The normal force is seen to have a peak value at an inclination of about $60^\circ$, for the rigid cylinder bowed away from the wind, and the distribution is very different from the "$\sin^2 \phi$" shape. The results shown in this figure can be directly compared with those for the flexible cable in Fig. 2.10. It appears that as $R/d$ decreases, or non-dimensional curvature increases, the peak moves away from the portion of the cylinder normal to the flow, and the distribution ceases to resemble the results for infinite straight inclined cylinders. For the curve bowed into the wind the normal forces are greatly increased over the predicted peak for a large portion of the cylinder.

The results reflect the shape of the $C_{p_w}(\phi, R/d)$ curves, and show the strong influence the wake pressure has on the normal force. This result can be predicted by a simple theory. Since the ideal distribution describes the real case well for $\theta < \theta_s (= 75^\circ)$, if the ideal, inviscid flow, pressure distribution is integrated to give a drag or normal force on a portion of the cylinder $-\theta$ to $+\theta$,

$$C_N(\theta) = \int_{-\theta}^{\theta} C_p(\theta) \cos \theta d\theta$$

$$= \int_{-\theta}^{\theta} (1-4\sin^2 \theta) \cos \theta d\theta$$

$$= \sin \theta - 4/3 \sin^3 \theta$$

If $\theta = 60^\circ$ the normal force on the front portion $-60^\circ$ to $+60^\circ$ is zero. The separation in the real case occurs at $75^\circ$.

$$C_N(-75^\circ \rightarrow 75^\circ) = .966 - 1.20 = -.23$$

ie. the normal force on the laminar flow portion is a small thrust. Hence the constant pressure wake is the major contribution to the total normal force on the cylinder, ie.

$$C_N \approx C_{p_w} \sin \theta_s$$

The final conclusion is that $C_N(\phi, R/d)$ can be estimated to a good approximation by $C_{p_w}(\phi, R/d)$, but the latter cannot be predicted from straight cylinder cross flow theory, since it is a function of the curvature, $R/d$. 

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The total drag of the curved cylinder was found as follows. The local drag coefficient

\[ C_d(\phi, R/d) = C_N(\phi, R/d) \sin \phi \]

is shown in Fig. 3.14 for the two test cases ("R/d = 18") along with the predicted result ("R/d = \infty"), \( C_d(\phi, \infty) \propto \sin^3 \phi \). Again the test results are quite different from the predicted. Integration of these curves gives the total drag coefficient

\[ C_D = \frac{D}{qdH} = \frac{1}{H} \int_0^\phi C_d d\phi \frac{d\phi}{\sin \phi} \]

It was found that \( C_D \) for the cylinder bowed away from the wind was only 5% lower than predicted, while that for the curve into the wind \( C_D \) was 80% higher. Thus even though the \( \phi \) distribution in the former case was severely different from the predicted, the overall drag remained the same; while in the latter case, the overall drag increased considerably.

The reference area for nondimensionalising the drag on the cylinder has been taken as the frontal area, \( Hd \), where \( H \) is the height of the curved cylinder (the test section height in this case). Thus the results can be compared with that for straight cylinder perpendicular to the flow, \( C_{Do} = 1.2 \). The drag of the cylinder bowed away from the wind is 30% less than the corresponding vertical cylinder, while the drag of the cylinder bowed into the wind is 38% more. (This is precisely the reverse of the drag forces on hollow half spheres and corresponding circular plates, [Ref. 5], since the aerodynamic properties at separation and in the wake are quite different.)

3.4 Discussion of the Flow on a Curved Circular Cylinder

In this section an attempt is made to discuss the results of Sections 3.1, 3.2 and 3.3 in terms of known flow phenomena. The major interest has been the wake of the cylinder. The specific flow pattern in the wake is not essential for describing the external potential flow; hence for describing the pressure field in the external flow, the wake might be considered as an afterbody streamlining the cylinder, and a source or sink as required. For the infinite straight cylinder the wake can be considered as simply an afterbody, and axial flows within the wake do not directly affect the external pressure distribution. For the curve bowed away from the wind the three-dimensional wake flows act as a source adding fluid to the external flow at the normal portion. For the curve bowed into the wind, the sink action of the wake flow entrains fluid from the surroundings at the normal portion.

The flows within the wake cannot be described in detail without further flow visualisation. However, work reported in Ref. 20
on cones at an angle of attack, demonstrates the presence of subsidiary vortices, with corresponding separation and attachment points in the wake, Fig. 3.15. This report also comments on work by Grissom (Ref. 21) pointing to the appearance of steady asymmetric flows at high angles of incidence. This could be the result of very slight asymmetry in the mounting, as seen by Maltby (Ref. 22) who investigated the separated flow behind low AR flat plates. At high angles of attack and very slight angles of yaw, the wake vortices formed asymmetric "side-to-side" cell patterns. In the light of these reports the steady complex wake flows observed in this work appear to be peculiar to wake flows inclined to the main stream.

In the introduction, Section 1.3, a mechanism was suggested by which the wake would become three dimensional with an axial flow component. There it was predicted that the wake flow would be toward the normal portion of any curved cylinder to reduce the pressure gradients. This is the result observed in the case of the cylinder bowed away from the wind. The predicted wake pressure gradients have been greatly modified by this flow; the flow accelerates in the direction of the negative pressure gradients at all points (Fig. 3.11a). However, for the opposite curvature the flow, though slower, tended away from the normal portion. An alternate mechanism must be postulated. In both cases, the wake flow was in the direction of the tangential component of the free stream velocity. Since the velocities in the wake are low, there is a shear action on the wake from the main stream, which would tend to transmit this tangential velocity component into the wake. The two mechanisms, the pressure gradients and the shearing, are probably both significant since when they acted in the same direction, the wake velocities were higher than when they opposed (Fig. 3.5).

IV CONCLUSIONS

The following picture of the flow around a curved cylinder in subcritical flow has emerged from this work. The flow on the front of the surface is laminar, which can be reasonably predicted from the cross flow theory for infinite inclined cylinders. The separation line occurs close to the predicted position but is sensitive to the wake pressures. The wake pressure is approximately constant about any diameter and varies with the local angle of inclination but is not predictable from existing theories. The curvature seriously affects the wake, sometimes producing quite complex structures, and must be considered in predictions of wake pressures.

The local normal force coefficient is very sensitive to the wake pressure, hence for cylinders of appreciable curvature $C_N(\theta)$ is not readily predictable. However for large ratios of radius of curvature to diameter, the departure from two dimensionality due to the alteration of $C_{PW}$ and wake flows becomes small, hence $C_N$ to a good approximation is that given by the "cross flow" approximation.
<table>
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<tr>
<th>Reference</th>
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**Legend:**
- **T** - theoretical work
- **E** - experimental work
- **()** - cursory examination
APPENDIX B

A Photoelastic Technique for Measuring Force Gradients in Flexible Tygon Plastic

The following describes an aborted method of measuring the tension gradient in the "cable". It is presented as a promising technique for measuring forces and their gradients in inherently flexible test objects.

It was found that commercial Tygon plastic exhibits a high degree of birefringence. That is, the index of refraction is different along the two secondary principal stress axes (Ref. 33). Orthogonal components of polarised polychromatic light parallel to these axes, then, travel at different speeds, and acquire a phase retardation, $\delta$, relative to each other.

$$\delta = k'i(t(\sigma_1 - \sigma_2))$$

where the secondary principal stresses $\sigma_1$ and $\sigma_2$ are constant throughout the light path $t$, and where $k'$ is the stress-optic coefficient.

For the case of the tensile loading in the flexible tube $\sigma_2 = 0$, then $\delta = k'i \sigma_1 = k \sigma_1$. The calibration factor $k$ can be found experimentally by measuring the phase retardation associated with known tensile stresses, with a polaroid analyser, and the goniometric compensation method described in Reference 33. With the calibration factor for this plastic the colour-strain conversion chart in Reference 33 can be used to give the strains, and by Hooke's Law, the tensile stresses, associated with the colours observed in the material.

Figure B1 shows the simple test set-up required. Any transparent medium (e.g., the window of a wind tunnel test section) can be across the light path between the polaroids and the sample, provided a pre-calibration is made to find any strain patterns in this medium, which can be neglected in later analyses. The technique proposed was to observe or photograph the "cable" under test and to match the colours with the calibration chart to find the tension distribution.

Tygon was found to have a permanent stress pattern due to prestressing in manufacture, which could be released by heating in boiling water for one or more minutes. The calibration was quite repeatable for any one sample, but varied with the history of severe conditions on each sample. A typical calibration factor was 1/170 fringes per (oz/in$^2$) for 1/8"D. solid Tygon. The calibration factor for thicker tubing was higher since the light path $t$ increases with diameter. This path length was doubled by observing the light reflected from the rear surface, which was aluminiumised for this reason, rather than directly transmitted light.

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This technique was not used because the change in stress over the entire length of the cable in the wind tunnel tests was about 100 oz/in². This would barely give one fringe shift, or four colours, hence the measurement of $dT/d\ell$ would have been insensitive. However the advantages are clear when larger gradients exist: simple apparatus, measurement which does not interfere with the phenomenon in question, and Tygon is flexible, unlike many birefringent plastics used.
### TABLE 1

**Summary of Prior Work on Forces on Inclined Cylinders**

<table>
<thead>
<tr>
<th>Ref. No.</th>
<th>Author</th>
<th>Expression for $C_N$</th>
<th>Expression for $C_t$</th>
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<tr>
<td>8</td>
<td>Phillips, W. H.</td>
<td>$C_{N_0} \sin^2 \theta$</td>
<td>--</td>
<td>Ref. 7</td>
</tr>
<tr>
<td>9</td>
<td>Neumark, S.</td>
<td>$C_{N_0} \sin^2 \theta$</td>
<td>--</td>
<td>Ref. 6</td>
</tr>
<tr>
<td>10</td>
<td>Pode, L.</td>
<td>$C_{N_0} \sin^2 \theta$</td>
<td>$C_t\cos \theta$</td>
<td>Ref. 4</td>
</tr>
<tr>
<td>11</td>
<td>Whicker, L. F.</td>
<td>$A_1 \sin \theta + A_2 \sin^2 \theta$</td>
<td>$B_1 \cos \theta + B_2 \cos^2 \theta$</td>
<td>Fit to Ref. 4</td>
</tr>
<tr>
<td>12</td>
<td>Reber, R. K.</td>
<td>?</td>
<td>$C_t\cos \theta$</td>
<td>Fit to Ref. 4</td>
</tr>
<tr>
<td>13</td>
<td>Quick, S. L.</td>
<td>$A_1 \sin \theta + A_2 \sin^2 \theta$</td>
<td>$C_t\cos \theta$</td>
<td>Fit to Ref. 4</td>
</tr>
<tr>
<td>14</td>
<td>Mustert</td>
<td>$C_{N_0} \sin^2 \theta$</td>
<td>$C_t\cos^2 \theta$</td>
<td>Experiment</td>
</tr>
</tbody>
</table>

### TABLE 2

**Data on Cable Samples**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Length inches</th>
<th>Weight gm</th>
<th>$w$ lb/ft</th>
<th>$d$ ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8&quot; solid</td>
<td>51.2</td>
<td>13.5</td>
<td>$6.99 \times 10^{-3}$</td>
<td>.0104</td>
</tr>
<tr>
<td>1/4&quot; hollow</td>
<td>51.25</td>
<td>43.5</td>
<td>$22.5 \times 10^{-3}$</td>
<td>.0208</td>
</tr>
</tbody>
</table>
TABLE 3

Data on Test Cases to Find $C_N(\theta)$

<table>
<thead>
<tr>
<th>Case</th>
<th>Sample</th>
<th>$C_{T1}$ ft</th>
<th>$\Delta C_T$ ft</th>
<th>q psf</th>
<th>Re</th>
<th>R/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>1/8&quot; solid</td>
<td>1.48</td>
<td>0.26</td>
<td>9.14</td>
<td>$5.8 \times 10^3$</td>
<td>210</td>
</tr>
<tr>
<td>4.2</td>
<td>1/8&quot; solid</td>
<td>1.46</td>
<td>0.25</td>
<td>9.18</td>
<td>$5.8 \times 10^3$</td>
<td>250</td>
</tr>
<tr>
<td>4.6</td>
<td>1/8&quot; solid</td>
<td>1.46</td>
<td>0.14</td>
<td>14.7</td>
<td>$7.4 \times 10^3$</td>
<td>220</td>
</tr>
<tr>
<td>5.2</td>
<td>1/4&quot; hollow</td>
<td>1.65</td>
<td>0.35</td>
<td>9.11</td>
<td>$1.16 \times 10^4$</td>
<td>120</td>
</tr>
<tr>
<td>5.5</td>
<td>1/4&quot; hollow</td>
<td>1.64</td>
<td>0.22</td>
<td>11.86</td>
<td>$1.33 \times 10^4$</td>
<td>140</td>
</tr>
</tbody>
</table>

TABLE 4

Major Sources of Error in Experimental Work

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Major Source of Error</th>
<th>Error</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{1, 2}$</td>
<td>Electronic Measurement</td>
<td>1/2 - 1%</td>
<td>Principally due to zero errors</td>
</tr>
<tr>
<td>q</td>
<td>&quot;</td>
<td>1/2 - 1%</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Mechanical Measurement</td>
<td>1 or 2%</td>
<td>Larger error</td>
</tr>
<tr>
<td>w</td>
<td>&quot;</td>
<td>.3 or .7%</td>
<td>for thinner cable</td>
</tr>
<tr>
<td>(x, y)</td>
<td>&quot;</td>
<td>.014&quot;</td>
<td>Error in 1 inch grid</td>
</tr>
<tr>
<td>$C_{T1, 2}$</td>
<td>$C_{T1, 2} = T/qd$</td>
<td>mean 2.5%</td>
<td>--</td>
</tr>
<tr>
<td>$C_T$</td>
<td>$C_{T1, C_t, w/qd}$</td>
<td>3.5%</td>
<td>See text</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Data Reading</td>
<td>mean 0.5°</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>$d\theta/d\ell$</td>
<td>&quot;</td>
<td>2.8%</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>$C_N$</td>
<td>$C_T, d\theta/d\ell$</td>
<td>5%</td>
<td>&quot; &quot;</td>
</tr>
</tbody>
</table>
FIGURE 1.1 $C_N(\phi)$ DISTRIBUTION FOR INCLINED CYLINDERS $\phi$ (degrees)
FIGURE 1.2 SCHEMATIC TO SHOW THE FLOWS INDUCED BY THE PRESSURE GRADIENTS ON A CURVED CYLINDER
FIGURE 2.1 FORCES ON A FLEXIBLE CABLE IN AN AIRSTREAM
FIGURE 2.2 TEST ARRANGEMENT IN THE UTIAS WIND TUNNEL
FIGURE 2.3 DIAGRAMS TO SHOW THE PARALLAX CORRECTION REQUIRED FOR THE PHOTOGRAPHIC COORDINATES
FIGURE 2.4 DETAILS OF THE DYNAMIC FORCE TRANSDUCER
FIGURE 2.5 ANALOGUE COMPUTER "ON-LINE" DATA REDUCTION

FIGURE 2.6 FLOW DIAGRAM FOR CALCULATING $C_N(\phi)$
FIGURE 2.7 FIRST DERIVATIVE FROM LEAST SQUARES SMOOTHING OF DATA
FIGURE 2.8  SECOND DERIVATIVE FROM FOUR METHODS OF LEAST SQUARES SMOOTHING OF DATA

LEGEND

• 3rd Order Fit to 7 Points
△ " " " " '7" and Additional Smoothing
□ " " " " 11 Points
○ 9th Order Fit to 34 Points
FIGURE 2.9 VARIATION OF $C_{T1}$ AND $C_{T2}$ WITH DYNAMIC PRESSURE

LEGEND
- $1/4''$ Hollow 'Cable'
- $1/8''$ Solid 'Cable'
FIGURE 2.10 $C_N(\phi)$ DISTRIBUTION ON THE FLEXIBLE CURVED CYLINDER

LEGEND

<table>
<thead>
<tr>
<th>$d$</th>
<th>$a$</th>
<th>$Re$</th>
<th>$R/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8</td>
<td>9.14</td>
<td>$5.8 \times 10^3$</td>
<td>210</td>
</tr>
<tr>
<td>1/8</td>
<td>9.18</td>
<td>$5.8 \times 10^3$</td>
<td>250</td>
</tr>
<tr>
<td>1/8</td>
<td>14.7</td>
<td>$7.4 \times 10^3$</td>
<td>220</td>
</tr>
<tr>
<td>1/4</td>
<td>9.11</td>
<td>$1.2 \times 10^4$</td>
<td>120</td>
</tr>
<tr>
<td>1/4</td>
<td>11.9</td>
<td>$1.3 \times 10^4$</td>
<td>140</td>
</tr>
</tbody>
</table>

$C_N$ $\phi$ (radians)
FIGURE 3.1 CYLINDER INCLINED TO THE WIND FOR FLOW VISUALISATION TESTS
FIGURE 3.2  FOUR CONSECUTIVE VIEWS OF A TYPICAL FLOW PATTERN AROUND A STRAIGHT CYLINDER INCLINED TO THE WIND
FIGURE 3.3 SKETCHES ILLUSTRATING THE FLOW ON AN INCLINED CYLINDER

a. Flow in Laminar Boundary Layer and Wake Region

b. Pressure Distribution

c. Fan of Streamlines at Different Heights in the Boundary Layer at Separation

WIND
FIGURE 3.4  CURVATURE OF THE RIGID CYLINDER

LEGEND
- Measured Pressure Tap Stations
- Best Fitted Curve for Cylinder
FIGURE 3.5  FLOW VISUALISATION BY TUFTS ON A CURVED CYLINDER

a. Bowed Away From the Wind

b. Bowed Into the Wind
FIGURE 3.6 THREE VIEWS OF THE FLOW PATTERN AROUND A CURVED CYLINDER BOWED AWAY FROM THE WIND
FIGURE 3.7 SKETCHES ILLUSTRATING THE FLOW ON A CURVED CYLINDER BOWED AWAY FROM THE WIND
FIGURE 3.8 THREE VIEWS OF THE FLOW PATTERN AROUND A CURVED CYLINDER BOWED INTO THE WIND
FIGURE 3.9 SKETCH ILLUSTRATING THE FLOW ON A CURVED CYLINDER BOWED INTO THE WIND
FIGURE 3.10 TYPICAL RESULTS FROM THE SCANIVALVE
FIGURE 3.11(a) PRESSURE COEFFICIENT DISTRIBUTION ALONG THE FRONT AND REAR OF A CURVED CYLINDER BOWED AWAY FROM THE WIND
FIGURE 3.11(b) PRESSURE COEFFICIENT AT $\phi = 45^\circ$ AROUND THE DIAMETRIC PLANE OF A CURVED CYLINDER BOWED AWAY FROM THE WIND
FIGURE 3.11(c) PRESSURE COEFFICIENT AT $\phi = 90^\circ$ AROUND THE DIAMETRIC PLANE OF A CURVED CYLINDER BOWED AWAY FROM THE WIND
Figure 3.12(a) Pressure Coefficient Distribution Along the Front and Rear of a Curved Cylinder Bowed into the Wind
FIGURE 3.12(b) PRESSURE COEFFICIENT AT $\phi = 45^\circ$ AROUND THE DIAMETRIC PLANE OF A CURVED CYLINDER BOWED INTO THE WIND
FIGURE 3.12(c) PRESSURE COEFFICIENT AT $\phi = 90^\circ$ AROUND THE DIAMETRIC PLANE OF A CURVED CYLINDER BOWED INTO THE WIND
FIGURE 3.13  \( C_N(\phi) \) DISTRIBUTION ON A RIGID CURVED CYLINDER
FIGURE 3.14 LOCAL DRAG COEFFICIENT DISTRIBUTION ON A RIGID CURVED CYLINDER
FIGURE 3.15  SKETCH OF THE MOTION IN THE WAKE OF A CURVED CYLINDER

REF. 20
LEGEND

- plane polarised light
- circularly polarised light
- elliptically polarised light

FIGURE B.1 TEST ARRANGEMENT FOR PHOTOELASTIC TECHNIQUE TO MEASURE AXIAL STRESS IN PLASTIC "CABLE"