Dynamic Forces on a Bed Element in Open Channel Flow with a Backward-facing Step

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Contents

Abstract .................................................. 1

Notation .................................................. 1

List of Figures .......................................... 2

1. Introduction .......................................... 3

2. Analysis and Formulae ............................... 6

3. Experimental Equipment and Instruments .......... 11
   3.1. Open Channel Flume ............................ 11
   3.2. Laser Doppler Anemometer (LDA) Optical System 11
   3.3. Dynamometer .................................. 14

4. Experimental Procedure ............................. 15

5. Experimental Results ................................ 21
   5.1. Experimental Results with the Same Flow Conditions 21
   5.1. Experimental Results with various Flow Conditions 26
   5.3. Turbulence Characteristics ..................... 35

6. Conclusions .......................................... 53

Acknowledgements ....................................... 54

References .............................................. 55
Notation

\( A_d \) = projected area perpendicular to the flow direction.
\( A_l \) = projected area parallel to the flow direction.
\( C_d \) = drag force coefficient.
\( C_l \) = lift force coefficient.
\( C_e \) = kurtosis coefficient.
\( C_s \) = skewness coefficient.
\( D \) = mean particle diameter.
\( \overline{F_d} \) = mean drag force.
\( \overline{F_l} \) = mean lift force.
\( f'_{d} \) = fluctuating intensity of drag force.
\( f'_{l} \) = fluctuating intensity of lift force.
\( f_n \) = sample frequency.

\( G_{(n)} \) = function of spectral density.
\( N \) = sampling number.
\( n \) = turbulence frequency.

\( P_{(fl)} \) = function of probability density of lift force.
\( P_{(fl)} \) = probability distribution of lift force.
\( R_{(u,u)} \) = auto-correlation coefficient of \( u \).
\( R_{(u,fl)} \) = cross-correlation coefficient of \( u \) and \( f_l \).
\( r \) = correlation coefficient of regression analysis.
\( t \) = time.
\( \overline{U}, \overline{V} \) = time-average velocities.
\( u, v \) = fluctuating velocities.
\( u', v' \) = turbulent intensities.
\( u_x \) = shear velocity.
\( \mathcal{W}(i) \) = window function.
\( x \) = longitudinal coordinate.
\( y \) = vertical coordinate.
List of Figures

Fig. 1 Sketch of LDA system
Fig. 2 Sketch of force Dynamometer
Fig. 3 Sketch of experimental conditions
Fig. 4 Profiles of water surface
Fig. 5 (1) Relationship of mean forces and velocities
Fig. 5 (2) Relationship of mean forces and velocities
Fig. 6 (1) Distributions of mean velocities (Run 1)
Fig. 6 (2) Distributions of mean velocities (Run 2)
Fig. 7 Distributions of turbulent intensities
Fig. 8 Distributions of Reynolds stresses
Fig. 9 Variations of mean forces with the distance
Fig. 10 Distributions of $C_d$ and $C_l$ near the boundary
Fig. 11 Relationship of mean forces and Reynolds stresses
Fig. 12 Relationship of intensities of fluctuating forces and velocities
Fig. 13 (1) Relationship of mean forces and velocities
Fig. 13 (2) Relationship of mean forces and velocities
Fig. 13 (3) Relationship of mean forces and velocities
Fig. 14 Variations of $C_d$ with the distance
Fig. 15 Variations of $C'_d$, $C'_{lu}$ and $f'_d / \bar{F}_d$, $f'_l / \bar{F}_l$
with the distance from the step
Fig. 16 (1) Probability density distribution (u)
Fig. 16 (2) Probability density distribution (v)
Fig. 16 (3) Probability density distribution ($f'_d$)
Fig. 16 (4) Probability density distribution ($f'_l$)
Fig. 17 Variations of $C_s$ and $C_e$ of velocities along the vertical
Fig. 18 Probability distributions of lift force
Fig. 19 Variations of $C_s$ and $C_e$ with the distance
Fig. 20 Joint-probability density distributions
Fig. 21 Distributions of auto-correlation coefficients of velocities
Fig. 22 (1) Distributions of auto-correlation coefficients of forces
Fig. 22 (2) Distributions of auto-correlation coefficients of forces
Fig. 23 Distributions of cross-correlation coefficients
Fig. 24 (1) Energy spectral distributions (Run 1)
Fig. 24 (2) Energy spectral distributions (Run 2)
Fig. 25 Variations of $G_{50}$ and $G_{90}$ with the distance
1. Introduction

The study of the hydrodynamic forces acting on the bed elements, especially on those in the downstream of a hydraulic drop, is very important in understanding the mechanism of sediment transport. In order to determine the incipient motion of solid particles resting on a stream bed, e.g. their stability, these forces have to be studied.

The force on a particle in a stream consists of two components, viz. the drag force $F_d$, which is parallel to the flow direction, and the lift force $F_l$, which is perpendicular to the flow direction. In this respect, two different approaches can be used.

The first approach is to study the flow pattern around the particles and then to describe the forces by means of the statistical turbulent flow parameters. Nath and Yamamoto (20) applied potential flow theory to study the drag and lift forces acting on a cylinder. They predicted the existence of large positive lift forces when a cylinder rests on the boundary. Benedict and Christensen (4) used the solution of potential flow to predict the pressures and obtained a lift force coefficient $C_l = 0.207$ for Einstein's experimental conditions.

The second approach is to measure the forces directly. Einstein and El-Samni (15) glued hemispheres (diameter $D = 68.6$ mm) on the bottom of a flume in a hexagonal pattern. They measured the velocity distributions and got the shear velocity $u_\ast$. The drag force $F_d$ acting per unit area of the flume bed then could be obtained from $F_d = \rho u_\ast^2$. The lift force was measured directly and expressed by:

$$F_l = C_l \rho \frac{u_\ast^2}{2} \quad \text{(1)}$$

If $U$ was measured at $y = 0.15 D$ above the top plane of the spheres, the lift force coefficient $C_l$ had a constant value of 0.178. The pressure fluctuations were measured by a Trimount pressure cell. The fluctuation of $F_l$ was statistically distributed according to the normal error law; the standard deviation was $\sigma_F = 0.364 \bar{F}_l$. Chepil's (10) experiments (similar to those
of Einstein) were conducted in a wind tunnel duct. He obtained $C_l = 0.194$, which is similar to Einstein's value $C_l = 0.178$. Cheng and Clyde (9) attached hemispheres to the bed and embedded spheres on top of these hemispheres in the most densely packed arrangement. They found that the lift force and the drag force at incipient motion were related to the mean velocity. The probability densities of the fluctuating lift and drag force were normally distributed. Watters and Rao (21) used a strain gauge dynamometer to measure the dynamic forces. The instrumented sphere had a diameter of 95.3 mm and it was one of the bed particles. When the representative velocity was the shear velocity, the drag force coefficient $C_d$ is 2.2; when it was measured at $y = 0.15 D$, $C_d = 0.382$. Apperley (2) measured the forces acting on the bed material. He obtained that the ratio of the lift force to the drag force is 0.5.

Coleman (12,13) analysed the forces acting the bed particles, he attached spheres (D=12.7 mm) to the bed surface and placed some relatively isolated steel or plastic balls with the same diameter on the bed. The coefficient of the drag force is a function of $R_{ed}$ ($R_{ed} = UD/\nu$). The curve of $C_d$ vs. $R_{ed}$ is similar to the drag coefficient curve for a sphere in free fall. Coleman assumed the lift force as:

$$F_l = C_l \frac{\pi}{6} (\gamma_s - \gamma) D^3$$  \hspace{1cm} \text{(2)}

When $R_{ed} > 200$, $C_l = 0.3 - 0.5$. For neutrally buoyant particles ($\gamma_s = \gamma$), the lift force then is always equal to zero, so the form of Eq. (2) is incorrect.

Garde and Sethuraman (17) let glass, steel and plastic spheres (D = 9.2 to 25.3 mm) roll along a boundary and measured the drag forces. The experimental results showed that the coefficient of the drag force is larger than that of the same sphere in free fall. Aksoy (1) used a strain gauge to measure the lift force and the drag force. He obtained that the drag coefficients agree well with the curve of a sphere rolling down an inclined boundary. $F_l / F_d = 0.143$. For $Re > 6600$, $f_{l \text{max}} / F_l \approx 3$.

Willetts and Naddeh (22) and Willetts and Murray (23) measured the lift forces nearby a wall. They found that the lift force experienced by a sphere was a function of the size of the
sphere, its distance from the wall and the sphere Reynolds number. Large, outwards directed lift forces were found for distances between the sphere and the wall less than 0.02 D or between 0.1 — 0.2 D. Davies and Samad (14) measured the lift force on a particle bed. They found that the lift force changed from negative to positive when $Re_x = 5$ $(Re_x - u_x D/\nu)$ and suggested a physical connection between the sign of the lift force and the bedform.

As mentioned above, one obtained a lot of information on the mean forces acting on bed elements in various conditions, but very little on the turbulence characteristics. Einstein and El-Samni (15) as well as Cheng and Clyde (9) found that the probability density distributions of the forces were normally distributed. But Christensen (11) analysed Einstein's data and found that the probability density distributions of the velocities were normally distributed rather than those of the forces. The distribution of the instantaneous lift force observed by Apperley (2) indicated that there was a predominance of negative values, but that there were infrequent bursts of large positive lift forces. This result is different from those of Einstein and Cheng. In order to study the turbulent structure of open channel flow and its effect on hydraulic structures, the turbulent characteristics of forces and velocities must be quantitatively described.

All above results were based on uniform flow conditions. For a backward-facing step flow in an open channel, there are some experimental results about the structure of the flow field (19), but there are few results about the forces in this situation.

Systematical experiments to measure simultaneously the forces and the velocities downstream of a backward-facing step in an open channel flow were carried out. According to the statistic theory of turbulence, this report presents a program to calculate the turbulent characteristic factors for two-dimensional variables. Based on the experimental and calculating system, the mean forces and the fluctuating characteristics in the region downstream of a hydraulic project with a hydraulic drop as well as the relationship of them with the velocities could be obtained.
2. Analysis and Formulae

The forces acting on the bed elements are the lift force, the drag force, gravity forces (weight and buoyancy) as well as reaction force from the channel bed. For a particle fixed on the channel bed, the reaction force and the gravity forces can be ignored.

The drag force is caused by the pressure and skin friction. The lift force is caused by differences of both pressure and shear stress on the upper and lower parts of the element due to asymmetry of the flow. Based on a number of experimental results (1, 4, 9, 10, 15, 20, 21), the drag force \( F_d \) and the lift force \( F_l \) acting per unit area of the bed elements can be expressed:

\[
F_d = C_d \rho \frac{U^2}{2}
\]  

(3)

\[
F_l = C_l \rho \frac{U^2}{2}
\]  

(4)

The determination of the representative velocity \( U \) in these formulae is important. For instance, Einstein and El-Samni (15) found that in Eq. (4) \( C_l \) was only a constant if the velocity was measured at 0.15 \( D \) above the top plane of the bed particles. Some other authors also used this velocity (4, 10).

In this report a statistical theory of turbulence is applied to deal with the force and velocity signals (18). Assuming that the steady turbulence is a stationary stochastic process which satisfies the ergodic hypothesis, the statistical characteristic factors may be substituted by time-average ones. The fluctuating values of these four variables can be defined as:

\[
u_{(i)} - \bar{U}
\]  

(5)

\[
u_{(i)} - \bar{V}
\]  

(6)

\[
u_{(i)} - \bar{F}_l
\]  

(7)

\[
u_{(i)} - \bar{F}_d
\]  

(8)

where the upper-bar indicates the time-average value, and the small letter fluctuating value. In the following, only the
formulae representing the turbulent factors of the longitudinal velocity $U_{(i)}$ are given; the others are similar to these ones. The time-average velocity \( \bar{U} \) and the turbulent intensity $u'$ are:

\[
\bar{U} = \frac{1}{N} \sum_{i=1}^{N} U_{(i)} \tag*{(9)}
\]

\[
u' = \left( \frac{1}{N} \sum_{i=1}^{N} [u_{(i)}]^2 \right)^{1/2} \tag*{(10)}
\]

in which $N$ is the sampling number. In this report, $N = 16384$.

The time-average value of a variable is a basic factor for the engineering design. For instance, $F_d$ represents the drag force acting on the bed elements averaged over a long time. The turbulence intensity of a variable represents the turbulence level deviating from the mean value. With respect of the incipient motion of the bed elements, the fluctuating forces superimposed on their average values make the bed elements start moving. For the same time-average forces, it holds that the stronger the turbulent intensities of the forces, the more easily the bed elements start moving.

In order to investigate the turbulence characteristics of a variable in greater detail, some more turbulent factors must be calculated, such as the probability density distribution, the correlation coefficient and the spectral density distribution.

The probability density distribution of a variable represents the frequencies with which various turbulent components occur. If the forces exceed the critical forces of incipient motion, the bed elements will start moving. From the probability density distributions of the forces, we may know how long these forces exceed the critical forces during the measuring period.

Let $Z_{(i)} = u_{(i)} / u'$ be the dimensionless fluctuating velocity and divide the difference of the maximum $Z_{\text{max}}$ and the minimum $Z_{\text{min}}$ into $M$ equal parts:

\[
\Delta Z = \frac{Z_{\text{max}} - Z_{\text{min}}}{M} \tag*{(11)}
\]

Then, a new series, arranged according to increasing order of magnitude, is obtained:

. 7 .
\[ T(k) = z_{\min} + k \Delta Z, \quad (k = 0, 1, \ldots, M) \]  \quad (12)

If the number of values of \( z(i) \) in the range larger than the middle value of \( T(k-1) \) and \( T(k) \) smaller than that of \( T(k) \) and \( T(k+1) \) is defined as \( N_k \), the probability density function can be defined as:

\[ p(T(k)) = \lim_{N \to \infty} \frac{N_k}{N} \]  \quad (13)

The probability distribution is:

\[ P(T(j)) = \sum_{k=0}^{j} \Delta Z \cdot p(T(k)) \]  \quad (14)

in which \( j = 0, 1, \ldots, M \).

The characteristics of the probability density distributions can be completely described by their first through fourth moments. The first moment is the mean value. The second moment represents the deviation level; it can be changed into the turbulent intensity. The third one can be used to calculate the skewness coefficient \( \mathcal{C}_s \) and the fourth one to calculate the kurtosis coefficient \( \mathcal{C}_e \) of the probability density distribution:

\[ \mathcal{C}_s = \Delta Z \sum_{k=0}^{M} [T(k)^3 \cdot p(T(k))] \]  \quad (15)

\[ \mathcal{C}_e = \Delta Z \sum_{k=0}^{M} [T(k)^4 \cdot p(T(k))] \]  \quad (16)

For a normal distribution, \( \mathcal{C}_s = 0, \mathcal{C}_e = 3 \).

The correlation coefficient indicates the tendency of the random variable varying with the delayed time or distance. For a certain correlation level, it holds that the longer the delayed time, the more slowly the variable has changed.

The autocorrelation coefficient \( R(u,u) \) of the fluctuating velocity is defined as:

\[ R(u,u) = \frac{1}{u^2} \frac{1}{N-t} \sum_{i=1}^{N-t} u(i) \cdot u(i+t) \]  \quad (17)

The spectral energy function represents the distribution of the total turbulent energy along the frequencies. For an engineering project, it is interesting to know the main frequency of the forces. Assuming that the stationary stochastic
process $u(t)$ is composed of turbulence components with various frequencies, $u(t)$ can be expressed by Fourier series (3):

$$u(t) = \int_{0}^{\infty} a(n) \exp(-j2\pi nt) \, dn$$

(18)

where $n =$ turbulent frequency; $a(n)$ is the Fourier transform of $u(t)$:

$$a(n) = \int_{0}^{\infty} u(t) \exp(j2\pi nt) \, dt$$

(19)

The spectral density is defined as:

$$G(n) = \frac{1}{u^2} \frac{2}{N \Delta t} |a(n)|^2$$

(20)

where $\Delta t$ is the sampling interval. Substituting Eq. (19) into Eq. (20) and writing in disperse form:

$$G(n) = \frac{1}{u^2} \frac{1}{0.875} \frac{2}{N f_n} \times$$

$$\times |\sum_{i=1}^{N-1} u(i) W(i) \exp(j2\pi in/N)|^2$$

(21)

in which $f_n$ is the sampling frequency ($f_n = 1/\Delta t$). $W(i)$ is a cosine slope window function which eliminates the spectral leakage (6):

$$W(i) = \begin{cases} 
\frac{1}{2} \left[1 - \cos \frac{\pi(i-1)}{0.1N} \right], & (1 \leq i \leq 0.1N) \\
1, & (0.1N < i < 0.9N) \\
\frac{1}{2} \left[1 - \cos \frac{\pi(N-i)}{0.1N} \right], & (0.9N \leq i \leq N) 
\end{cases}$$

(22)

The coefficient 0.875 in Eq. (21) is a correction constant due to introducing Eq. (22). The spectral density $G(n)$ in Eq. (21) is directly calculated by fast Fourier transform technique (8). Finally, the energy spectrum Eq. (21) is subjected to a smoothing procedure.

The cross correlation coefficient represents the measure of dependence between two variables. If they are independent, the cross correlation coefficient is zero. The cross correlation coefficient of $f_d(i)$ and $u_j(i)$ is defined as:

.9.
\[ R(u,f_{d}) = \frac{1}{f_{d}} \frac{1}{N} \sum_{i=1}^{N} f_{d}(i) u(i) \quad \cdots \cdots \cdots (23) \]

For a certain flow condition, the velocity \( u(i) \) can be measured at a position above the channel bed and \( R(u,f_{d}) \) calculated. Then we can take the velocity which has the largest \( R(u,f_{d}) \) value as the representative velocity in Eq. (3). By the same way, we can define:

\[ R(u,f_{1}) = \frac{1}{f_{1}} \frac{1}{N} \sum_{i=1}^{N} f_{1}(i) u(i) \quad \cdots \cdots \cdots (24) \]

The Reynolds stress is:

\[ -\rho \overline{uv} = -\rho \frac{N}{N} \sum_{i=1}^{N} u(i) v(i) \quad \cdots \cdots \cdots (25) \]

Therefore, the distribution of the Reynolds stress along the vertical can be obtained.

The joint probability density distribution of two variables indicates the probability that the two variables simultaneously appear at a certain condition. The joint probability density distribution of \( u \) and \( v \) can be written as:

\[ P(u,v)(u=u_{i}, v=v_{j}) = \lim_{N \to \infty} \frac{N_{i,j}}{N} \quad \cdots \cdots \cdots (26) \]

in which \( N_{i,j} \) is the event in which \( u = u_{i}, v = v_{j} \) simultaneously appear. Analogously, the joint probability density distributions of \( f_{1} \) and \( u \) as well as of \( f_{d} \) and \( u \) can be written as:

\[ P(u,f_{1})(u=u_{i}, f_{1}=f_{1k}) = \lim_{N \to \infty} \frac{N_{i,k}}{N} \quad \cdots \cdots \cdots (27) \]

\[ P(u,f_{d})(u=u_{i}, f_{d}=f_{dm}) = \lim_{N \to \infty} \frac{N_{i,m}}{N} \quad \cdots \cdots \cdots (28) \]

Based on these formulae, a Fortran program was prepared. When the signals of the forces and velocities are available, all above parameters can be calculated by this program.
3. Experimental Equipment and Instruments

The experiments were carried out in the Laboratory of Fluid Mechanics of the Department of Civil Engineering, Delft University of Technology. A laser doppler anemometer was used to measure the velocities in an open channel flow with a backward-facing step and a dynamometer to measure simultaneously the forces acting on a bed element. The signals of the velocities and the forces were analysed by an apple II computer or the central IBM-computer of the Delft University of Technology.

3.1. Open Channel Flume

The experiments were conducted in a tilting flume; its main dimensions are: length = 14.1 m, width = 0.4 m, depth = 0.3 m. A backward-facing step with a height of 0.07 m was set up 7.5 m from the entrance. The channel bed of the upstream part is a concrete plate and hydraulically smooth. That of the downstream part is covered by gravel with a mean diameter D of 33.3 mm. In order to get a uniform flow condition, the bed slope of 0.1 % is used. The water is supplied by a constant-head tank. The flume has a gate at the entrance and an end weir, both of them being used to control the flow depth.

3.2. Laser Doppler Anemometer (LDA) Optical System

A heterodyne LDA system (Model 105-I) was used to measure the velocities. A 5mW He-Ne Laser with a wavelength $\lambda_0 = 6.328 \times 10^{-4}$ mm and a beam diameter $d_0 = 1$ mm at the measuring point was used to produce three beams; one is the illuminating beam (I) with high intensity and the two others are the reference beams ($R_1, R_2$) with low intensity. The half angle $\phi$ between I and $R_1$ or I and $R_2$ is $2.736^\circ$. The optical arrangement and the coordinate system are shown in Fig. 1. The measuring volume has the dimensions $\Delta x = \Delta y = 0.145$ mm, $\Delta z = 4.05$ mm. The frequency shift is 819.2 kHz. The laser and the photodetectors (PD) are at the two sides of the flume, as shown in Fig. 1. They are placed on a frame which can be moved up and down with an accuracy of 0.1 mm.
Fig. 1 Sketch of LDA system

Fig. 2 Sketch of force Dynamometer
The light of the illuminating beam scattered by particles moving through the measuring volume is mixed with the light of the reference beams; then the reference beams are directly projected on the photodetectors. The signals from the photodetectors are transferred to a frequency tracker, then through a low-pass filter of 60 Hz. The output signals from the filter are digitized by means of a 12 bits A/D converter with a sampling frequency $f_n$ of 125 Hz; the new digital data are putted into an Apple II computer in order to calculate the mean values, turbulent intensities and Reynolds stresses. This was the procedure for most of the experiments. The output signals from a few experiments are digitized by a 14 bits A/D converter ($f_n = 125$ Hz) and the digital data are putted into a Data Acquisition System (DAS) operated by a HP1000 computer. To calculate the turbulence characteristics, the data information on magnetic tape is then transferred to the central IBM-computer of the Delft University of Technology.

The plane of $R_1$ and $R_2$ is parallel to the flume bed and the bisection of the angle between them is perpendicular to the flow direction, the angle $\psi$ of the beam configuration is $45^\circ$, so two velocity components perpendicular to each other can be obtained:

$$U_1 = \frac{\lambda_0}{2 \sin \phi} \ k \ V_1$$  \hspace{1cm} (29) \hspace{1cm} $$

$$U_2 = \frac{\lambda_0}{2 \sin \phi} \ k \ V_2$$  \hspace{1cm} (30) \hspace{1cm} $$

where: $\lambda_0$ = laser wavelength, $\lambda_0 = 6.328 \times 10^{-4}$ (mm);
$\phi$ = angle between $R_1$ and $I$ or $R_2$ and $I$;
$k$ = conversion factor of the tracker, $k = 2 \times 10^{-7}$ (hz/volt);
$V_1$ = voltage measured in the plane of $R_1$ and $I$;
$V_2$ = voltage measured in the plane of $R_2$ and $I$.

The longitudinal velocity $U$ and the vertical velocity $V$ are:

$$U = 0.707 \ (U_1 + U_2)$$  \hspace{1cm} (31) \hspace{1cm} $$

$$V = 0.707 \ (U_1 - U_2)$$  \hspace{1cm} (32) \hspace{1cm} $$
3.3. Dynamometer

A dynamometer measuring the forces acting on a bed element was developed (5), (see Fig. 2). It is composed of two couples of spring gauges and electric capacity detectors (Model Tr 10 D). A test gravel is attached on its top. This gravel has a natural form and its main dimensions are 37 mm in the direction of the flow, 33 mm high and 30 mm wide. The mean diameter \( D \) is 33.3 mm. The projected area \( A_1 \) parallel to the flow direction is 10.8 cm\(^2\), the projected area \( A_d \) perpendicular to the flow direction is 7.5 cm\(^2\). The dynamometer is placed at 1.6 m downstream from the step. The level of the test gravel is equal to that of the bed gravel.

The drag force detector responds to the deformation of the drag spring gauge caused by the drag force acting on the test gravel. The lift force detector responds to the deformation of the lift spring gauge caused by the lift force acting on the test gravel; in addition, the drag force has an effect of 4.0 % on the lift detector. The drag force and the lift force measured by the dynamometer have a linear relationship with the output voltages, namely:

\[
F_d = \beta_d V_d
\]

\[
F_l = \beta_l V_l - 0.04 F_d
\]

in which \( \beta_l, \beta_d \) are the respective calibration coefficients of the lift detector and the drag detector; \( V_l \) and \( V_d \) are the voltages delivered by the lift and drag detectors, respectively.

The output signal of the detectors are transferred to a HBM amplifier, then to a low-pass filter of 25 Hz, which was determined by a preliminary experiment, and an A/D converter \( f_n = 125 \text{ Hz} \). The data information is treated in the same way as in the case of the velocities.

Now, the series of samples of the disperse values of \( F_l(i), F_d(i), U(i) \) and \( V(i) \) (\( i = 1, 2, \ldots, N \)) can be obtained by measuring the forces \( F_l, F_d \) and the velocities \( U \) and \( V \).
4. Experimental Procedure

It is necessary to change the position of the instruments in order to make measurements in various sections after the step. It is easy to move the LDA system, but it is very difficult to move the dynamometer. If the flow in the upstream part and after the rapid distortion region downstream of the step is an approximately uniform flow, the same measurements can be done by changing the relative distance between the instruments and the step, namely, by fixing the instruments in their position but extending the upstream bed with concrete plates, as shown in Fig. 3. The original situation is No. A. The distance of the step to the measuring gravel is 1.6 m. Then, a concrete plate (0.3 m long) is added to the upstream bed (No. B); the distance then is 1.3 m, and so on. For situation No. F (the distance is 0.1 m) the dynamometer is always in the recirculation zone. The velocities are measured at the vertical line above the measuring gravel. For each experiment with a certain discharge Q, the flow depth \( h_1 \) of the upstream part and \( h_2 \) of the downstream part are measured, and then, the gate at the entrance of the flume and the end weir are adjusted in order to get a uniform flow condition as far as possible.

The experiments consist of two parts. One part deals with keeping the same flow conditions, for example, \( Q, h_1, h_2 \), and with measuring the velocities and forces sequentially at each section (from No. A to No. F). There are two runs in this experiment. The experimental conditions are listed in Table 1.

<table>
<thead>
<tr>
<th>Run</th>
<th>( Q )</th>
<th>( h_1 )</th>
<th>( h_2 )</th>
<th>( U_1 )</th>
<th>( U_2 )</th>
<th>( F_{r1} )</th>
<th>( F_{r2} )</th>
<th>( R_e ) x10^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.1</td>
<td>8.3</td>
<td>16.6</td>
<td>.606</td>
<td>.303</td>
<td>.672</td>
<td>.237</td>
<td>5.03</td>
</tr>
<tr>
<td>2</td>
<td>36.7</td>
<td>11.8</td>
<td>20.2</td>
<td>.777</td>
<td>.454</td>
<td>.722</td>
<td>.323</td>
<td>9.17</td>
</tr>
</tbody>
</table>

* Subscript 1 indicates upstream, 2 downstream *
Fig. 3 Sketch of experimental conditions

Fig. 4 Profiles of water surface
$U_1$ is the section-averaged velocity in the upstream part of the flume; $F_{r1} = U_1 / \sqrt{gh_1}$ is the corresponding Froude number, $Re = U_1 h_1 / \nu = U_2 h_2 / \nu$ is the corresponding Reynolds number.

Profiles of the water surface of the flow are shown in Fig. 4. It can be seen that the profiles of the flow surfaces corresponding with experimental conditions No. A and No. F do not change in the upstream part and after the rapid distortion region of the downstream part. This result shows that the experimental design is reasonable.

Based on these experiments, the mean values and turbulence characteristics (inclusive of all the information of Eq. (9) through Eq. (28)) of the forces and the velocities can be obtained. Furthermore, the variation of the forces with the flow characteristics at different sections can be investigated.

The other part of the experiments is to make measurements at the same section but with various flow conditions. Its purpose is to investigate the relationship between the forces and the representative velocity. Einstein (15) used the velocity measured at 0.15 D above the top of the gravel as the representative velocity. For condition No. A, 28 sets of experiments were conducted. For each experiment, the forces were measured and also the velocities (at $y = 0.09 \cdots 0.9$ D, i.e. at $y = 0.3$, 0.5, 1.0, 2.0, 3.0 cm). Relationships between the mean forces and the square of the mean velocities are shown in Fig. 5. It can be seen from these figures that these relationships are linear. So the method of least-squares can be used to get a regression equation. For example, let $X = \rho U^2 / 2$:

$$F_d = a + C_d X$$  \hspace{1cm} (35)

then

$$C_d = \frac{\Sigma (X - \bar{X})(F_d - \bar{F}_d)}{\Sigma (X^2 - \bar{X}^2)}$$  \hspace{1cm} (36)

The correlation coefficient of the regression analysis is defined as:

$$r = \frac{\Sigma X F_d}{(\Sigma X^2 \Sigma F_d^2)^{1/2}}$$  \hspace{1cm} (37)
Fig. 5 (1) Relationship of mean forces and velocities
Fig. 5 (2) Relationship of mean forces and velocities
The regression analysis reveals that the best correlation coefficient occurs at $y = 0.5$ cm (0.15 D above the top of the bed elements): for the mean drag force and the longitudinal velocity the correlation coefficient $r = 0.993$, for the mean lift force and the longitudinal velocity, $r = 0.903$. This result proves Einstein's conclusion. Therefore, for other experiments (No. B through No. F), only the velocities at $y = 0.5$ cm are measured. According to these experiments, the variation of the lift force and drag force coefficients with the distance from the step can be achieved.
5. Experimental Results

5.1. Experimental Results with the Same Flow Conditions

The experimental results of Run 1 and Run 2 clearly reveal the structure of the flow field in the regions of rapid distortion and transition. The distributions of mean velocities, turbulent intensities and Reynolds stresses are shown in Fig. 6 through Fig. 8. These figures indicate the varying process of flow characteristics from a uniform flow above the step to another uniform flow after the measuring region. From the distributions of the mean velocity $\bar{U}$, it can be seen that the velocity above the step is a log-law distribution. When the flow separates from the step, the boundary-layer flow becomes a separated flow and forms a strong mixing layer in which the velocity gradient is very large. And then, the velocity gradient gradually decreases. There is a recirculation region near the step. The reattachment length is about eight times the step height. For the distributions of the vertical mean velocity $\bar{V}$, there is an upward flow tendency because of the rising of the water surface in the measuring region. Only in the middle of the recirculation region, the vertical velocity is negative.

The distributions of the turbulent intensities of the longitudinal and vertical velocities have the same forms, only the former is larger than the latter. In Fig. 7, when the flow from the normal boundary-layer becomes a separated flow, strong turbulence takes place at the interface of the main flow and the recirculation flow. The turbulent intensities approach the maximum values at No. E. After this section, the turbulence decreases with the distance from the step. The varying tendency of the Reynolds stresses is similar to that of the turbulent intensity. They reach their maxima near the step caused by the high velocity gradient and strong mixing. In Fig. 8, the Reynolds stress of Run 1 just above the step is a straight-line distribution. If extending this straight line to the bed level, the Reynolds stress is $0.50 \text{ N/m}^2$. At this point, the bed shear stress is $0.587 \text{ N/m}^2$. Therefore, in the main flow region, the
Fig. 6 (1) Distributions of mean velocities (Run 1)
Fig. 6 (2) Distributions of mean velocities (Run 2)
Fig. 7  Distributions of turbulent intensities
Fig. 8 Distributions of Reynolds stresses
shear stress almost mainly consists of the Reynolds stress. This is a typical boundary-layer flow.

The structures of the flow field as presented in this report are consistent with those obtained by Nakagawa and Nezu (19).

The variation of the forces with the distance is shown in Fig. 9. At situations No. A and No. B (1.3 – 1.6 m from the step), the mean forces and the intensities of the fluctuations obviously do not change. The intensities of the fluctuations have the same order of magnitude as the mean values. This is very important because the incipient motion of the gravel depends on the instantaneous forces (mean force plus fluctuating force). At situation No. E, the forces approach the maximum values. In this section, the intensities of the fluctuations are the strongest. It is interesting to note that the lift force is always positive. Many researchers obtained this result in a flow with the roughness elements on the channel bed (1, 15, 23, 22). This can be explained by the asymmetrical flow around the measuring gravel. At the lower part of the gravel, the velocity is smaller and the pressure is larger. But in the upper part, the velocity is higher and the pressure is smaller. So the resultant of the forces acting on the gravel is in the upward direction. The drag force becomes negative at situation No. E due to the effect of the recirculation flow.

5.2. Experimental Results with Various Flow Conditions

As mentioned above, 28 sets of experiments were conducted for situation No. A. The representative velocity was determined at y = 0.15 D. Based on these results, the drag force coefficient \(C_d\) (or the lift force coefficient \(C_l\)) was calculated. It equals the gradient of the straight line in Fig. 5. Fig. 10 shows the distributions of \(C_d\) and \(C_l\) in the boundary region. The coefficients increase with smaller distance to the channel bed, because the velocity decreases with smaller distance. At \(y = 0.15 D\), \(C_l = 0.151\); this is comparable with Einstein's measurement (15); \(C_d = 0.359\), this is similar to the results obtained by Watters and Rao (21); the ratio of the lift force to the drag force is 0.421, which is consistent to Apperley's observation (2).

. 26 .
Fig. 9 Variations of mean forces with the distance

Fig. 10 Distributions of $C_d$ and $C_l$ near the boundary
The mean forces and Reynolds stresses have a linear relationship as shown in Fig. 11, for the drag force \( r = 0.803 \), for the lift force \( r = 0.793 \).

According to the experimental results, the measured fluctuating intensities of the forces can be expressed as:

\[
f'_d = C'_d \frac{\rho u'^2}{2} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quito
Fig. 11 Relationship of mean forces and Reynolds stresses
Fig. 12 Relationship of intensities of fluctuating forces and velocities
Fig. 13 (1) Relationship of mean forces and velocities
Fig. 13 (2) Relationship of mean forces and velocities
Fig. 13 (3) Relationship of mean forces and velocities
Fig. 14 Variations of $C_d$ with the distance

Fig. 15 Variations of $C_{1u}$, $C_{1l}$ and $f_d/\overline{F_d}$, $f_l/\overline{F_l}$ with the distance from the step
are obviously changed. But outside this region, they almost remain constant.

5.3. Turbulence Characteristics

Probability Density Distribution. The distributions of the probability density of velocities and forces are shown in Fig. 16. The skewness coefficient $C_s$ and the kurtosis coefficient $C_e$ are included in these figures. The variations of $C_s$ and $C_e$ for the velocities along the vertical are shown in Fig. 17. These figures indicate that the probability density distributions of the velocities in the upper flow region for situation No. A are near the normal distribution. But those in the lower flow region deviate from the normal distribution. The probability density distributions of the forces are not normally distributed. For situation No. A, $C_s$ is 4.47 for the drag force and 4.84 for the lift force. This means that there is more scatter than in case of a normal distribution.

Fig. 18 is the probability distribution of the lift force. It can be seen that when the probability $P(f_1) = 84\%$, $f_1 / f'_1 = 1$. This indicates that there is a possibility of 16\% that the instantaneous lift force is larger than the fluctuating intensity. As mentioned above, the fluctuating intensity of the force has the same order of magnitude as the mean force. In other words, the instantaneous force acting on the bed elements is two times the mean force with a possibility of 16\%. This is very important for the stability of the bed elements or the incipient motion of them.

As mentioned in section 1, Einstein and Cheng found that the probability density distribution of the lift force was normally distributed. Their conclusion had some error caused by the calculation method. They recorded the signals by an oscillograph and measured the period in which every turbulent level occurred. By this way, it is difficult to obtain the period of the lowest and highest turbulent level accurately. In fact, they only obtained the distribution of 1\% through 97\%. In this range, Fig. 18 is also a straight line. Namely, it is consistent with Einstein's data. But out of this range, the data deviate from the
Run 1
No. A

<table>
<thead>
<tr>
<th>y (cm)</th>
<th>C_b</th>
<th>C_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.0</td>
<td>-0.062</td>
<td>2.669</td>
</tr>
<tr>
<td>8.0</td>
<td>0.290</td>
<td>3.139</td>
</tr>
<tr>
<td>3.0</td>
<td>0.395</td>
<td>3.073</td>
</tr>
<tr>
<td>1.0</td>
<td>0.374</td>
<td>3.083</td>
</tr>
<tr>
<td>0.5</td>
<td>0.478</td>
<td>3.278</td>
</tr>
<tr>
<td>0.1</td>
<td>0.482</td>
<td>4.307</td>
</tr>
</tbody>
</table>

Fig. 16 (1) Probability density distribution (u)
Fig. 16 (2) Probability density distribution ($v$)
Fig. 16 (3) Probability density distribution ($f_d$)
Run 1
$y = 0.5$ cm

<table>
<thead>
<tr>
<th>No.</th>
<th>$C_s$</th>
<th>$C_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.021</td>
<td>4.837</td>
</tr>
<tr>
<td>B</td>
<td>0.089</td>
<td>4.939</td>
</tr>
<tr>
<td>C</td>
<td>0.245</td>
<td>5.464</td>
</tr>
<tr>
<td>D</td>
<td>0.380</td>
<td>4.493</td>
</tr>
<tr>
<td>E</td>
<td>0.451</td>
<td>3.803</td>
</tr>
<tr>
<td>F</td>
<td>0.370</td>
<td>3.298</td>
</tr>
</tbody>
</table>

Fig. 16 (4) Probability density distribution ($f_1$)
Fig. 17 Variations of $C_s$ and $C_e$ of velocities along the vertical
Fig. 18  Probability distribution of lift force
straight line. Therefore, the probability density distribution of
the lift force deviates from the normal distribution.

The variations of $C_s$ and $C_e$ of the forces and the
velocities are shown in Fig. 19. They do not changed outside the
recirculation region but change obviously in this region. Fig. 20
is the joint-probability density distributions (JPDD) of the
longitudinal velocity $u$ with the vertical velocity $v$, the
lift force $f_l$, and the drag force $f_d$. This figure reveals that
when $u$ is positive, the probability density distributions of
$v$ and $f_l$ deviate in the direction of the negative axis. Those
of $f_d$ deviate in the direction of the positive axis. When $u$
is negative, the probability density distributions of $v$ and $f_l$
deviate in the direction of the positive axis. Those of $f_d$
deviate in the direction of the negative axis. These results
indicate that the turbulent velocity $u$ and fluctuating drag
force $f_d$ as well as $v$ and $f_l$ have the same direction. $u$ and
$v$ as well as $u$ and $f_l$ have the opposite direction.

Correlation Coefficient. Fig. 21 presents the variations
of the auto-correlation coefficients of the velocities with time
and height. When the delay time is about 1.0 second for the
longitudinal velocity and 0.3 seconds for the vertical
velocity, the auto-correlation coefficients approach to zero. The
isographs of $R_{(u,u)}$ and $R_{(v,v)}$ are drawn in this figure. These
isographs represent the dimensions of the eddies. For the same
value of $R_{(u,u)}$ or $R_{(v,v)}$, it holds that the longer the time,
the bigger the eddy sizes. It can be seen that the largest eddy
appears at $y = 3.0$ cm. Fig. 22 shows the variations of the auto-
correlation coefficients of the forces against time and height.
When the delay time is about 0.3 second, the auto-correlation
coefficients approach to zero. For the same coefficient values
(0.5), situation No. F has the longest delay time. For the
same delay time, the correlation coefficients of Run 1 are
larger than those of Run 2.

Fig. 23 presents the cross-correlation coefficients of $u$ and
$f_d$ as well as of $u$ and $f_l$. Two points are of interest in this
figure. One is that the largest cross-correlation coefficients
appear at $y = 0.5$ cm (0.15 D). This result proves that the
representative velocity measured at 0.15 D is reasonable. The
Fig. 19 Variations of $C_s$ and $C_e$ with the distance
Fig. 20 Joint-probability density distributions
Fig. 21 Distributions of auto-correlation coefficients of velocities
Fig. 22 (1) Distributions of auto-correlation coefficients of
Fig. 22 (2) Distributions of auto-correlation coefficients of forces
Fig. 23 Distributions of cross-correlation coefficients
other is that the largest coefficients (at \( y = 0.15D \)) appear at \( t \approx 0.1 \) seconds (not at \( t = 0 \)). This means that an eddy first passes through the measuring volume and then approaches the measuring gravel. The travel time is about 0.1 seconds.

Energy Spectrum. The energy spectral distributions are shown in Fig. 24. It can be seen from these figures that the energy is concentrated in the low-frequency range. The highest frequency is less than 60 Hz for the velocities and less than 15 Hz for the forces. There is no obvious difference between Run 1 and Run 2. When the frequency is larger than 3 Hz, the energy spectrum of the forces are almost distributed according to a \(-4\) power. If the characteristic factors \( G_{50} \) and \( G_{90} \) of the energy spectral distributions are defined as:

\[
\int_{0}^{G_{50}} G_{(n)} \, dn = 50 \% \tag{41}
\]

\[
\int_{0}^{G_{90}} G_{(n)} \, dn = 90 \% \tag{42}
\]

Fig. 25 represents the variations of \( G_{50} \) and \( G_{90} \) along the distance from the step. The smallest frequency appears at the situation No. D and No. E. \( G_{50} \) is equal to or less than 1.0 Hz for the forces and the longitudinal velocity. \( G_{90} \) is equal to or less than 3.0 Hz for the same quantities. The vertical velocity has a higher frequency than the longitudinal velocity.
Fig. 24 (1) Energy spectral distributions (Run 1)
Fig. 24 (2) Energy spectral distributions (Run 2)
Fig. 25. Variations of $G_{50}$ and $G_{90}$ with the distance.
6. Conclusions

The instantaneous forces on a bed element and velocities were measured in an open channel with a backward-facing step; a program to deal with the signals was developed. The structure of the flow field obtained in this report is consistent with the typical results measured in a backward-facing step flow (19).

The mean forces and the square of the longitudinal mean velocity show a fine linear relationship if the velocity is measured at 0.15 D above the top of the measuring gravel. For situation No. A, the lift force coefficient $C_l$ is 0.151. The drag force coefficient $C_d$ for situation No. A is 0.359 and increases with the distance from the step. The ratio of the lift force coefficient to the drag force coefficient is 0.421. The forces acting on the bed elements in the recirculation region is a little larger than those outside this region.

The fluctuating intensities of the forces have the same order of magnitude as the mean forces and show a fine linear relationship with the square of the turbulent intensities of the velocities. The probability density distributions of the fluctuating forces are not normally distributed. There is a possibility of 16% that the instantaneous forces exceed the mean forces with a factor two. The auto-correlation coefficients of the velocities show that the largest eddy sizes occur at $y = 3.0 \text{ cm}$. The cross-correlation coefficients of the forces and longitudinal velocity show that the largest coefficients appear at $y = 0.5 \text{ cm}$.

The energy spectral distributions indicate that the energy of 90% of the forces and the longitudinal velocity is concentrated within 3.0 Hz. In the high-frequency range, the energy spectrums of the forces are distributed according to a $-4$ power.
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