Channel Equalization for Wavelet Packet Modulation

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THESIS

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Abstract

Wavelet Packet Modulation (WPM) is a multicarrier modulation technique like the well-established Orthogonal Frequency Division Multiplexing (OFDM). It has several advantages over OFDM, such as flexibility of operation and excellent localization in frequency. The Channel Equalization for WPM is new and an unexplored research area. It is also different from equalization for OFDM because WPM symbols overlap in time.

This thesis work presents a channel equalizer for WPM systems. The algorithm proposed is a pre-detection channel equalizer, which performs time-domain equalization. The equalizer applies the principle of peak distortion criterion where the maximum inter-symbol interference caused by the channel is minimized. The operation of the equalizer is tested under different realistic channels with different taps and characteristics. Furthermore, investigations to understand the impact of the wavelet family, length of the wavelet filters, and the number of equalizer taps on the performance of the equalizer are carried out. Results of the simulation studies illustrate notable performance gains including better Bit Error Rate (BER) performance and lower Inter symbol interference (ISI). Moreover, the different components of interference such as ISI(Inter Symbol Interference), ICI(Inter Carrier Interference) and ISCI(Inter Symbol Carrier Interference) are mathematically analyzed.
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Acronyms

AWGN: Additive White Gaussian Noise
BER: Bit Error Rate
DFE: Decision Feedback Equalizer
DFT: Discrete Fourier Transform
DVB: Digital Video Broadcasting
DWT: Discrete Wavelet Transform
ETSI: European Telecommunication Standard Institute
FDM: Frequency Division Multiplexing
FFT: Fast Fourier Transform
FIR: Finite Impulse Response
ICI: Inter Carrier Interference
IDFT: Inverse Discrete Fourier Transform
IDWT: Inverse Discrete Wavelet Transform
IFFT: Inverse Fast Fourier Transform
ISI: Inter Symbol Interference
ISCI: Inter Symbol Carrier Interference
MCM: Multi Carrier Modulation
MMSE: Minimum Mean Square Error
OFDM: Orthogonal Frequency Division Multiplexing
QPSK: Quadrature Phase Shift Keying
SNR: Signal to Noise Ratio SNIR: Signal to Noise Interference Ratio
WPT: Wavelet Packet Transform
ZFE: Zero Forcing Equalizer
Abstract

Acknowledgments

Acronyms

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In this chapter the motivation and objective behind the thesis work is presented.

Wireless Communication has shown tremendous growth in recent past and is expected to grow more in the years to come. The main motivation for such growth is the desire to achieve high data rates and high capacity; with higher data rates quality of communication increases and with higher capacity more users can be served and more services can be offered. In wireless systems, the wireless channel limits data rates and achievable capacity [1]. The wireless channel is the physical environment, as shown in the Figure 1.1. It distorts the transmitted data and makes the detection of the data difficult at the receiver. In order to correctly obtain the data at the receiver, a channel equalizer is used at the receiver which corrects for the distortions caused by the wireless channel.

Figure 1.1: Physical Environment
1.1 Motivation

In order to develop high data rate wireless systems, the technology made a shift from single carrier modulation systems to multi-carrier modulation (MCM) systems. In a MCM system a high data rate $R$ serial data stream is converted into parallel lower data rate $\frac{R}{n}$ data streams, where $n$ is the number of the parallel streams. By performing this operation different data streams are modulated over different frequencies called as sub-carriers. These sub-carriers in a MCM system are orthogonal, that is they occupy different orthogonal frequencies which do not interfere with each other [2]. This perspective of MCM’s birth because of the need of higher data rates is very true. But there is another reason also why MCM came into existence, it is related to the equalization of channel.

Consider a single carrier modulation system, in which the symbol time is given as $T_s$ and the wireless channel duration is given as $T_c$. Under this condition the wireless channel induces inter symbol interference (ISI)\(^1\), where the number of symbols affected $L$ can be given as [3]:

$$L = \left\lfloor \frac{T_c}{T_s} \right\rfloor$$  \hspace{1cm} (1.1)

From Eq. 1.1 it can be seen that $L$ increases with increase in $T_c$ and reducing $T_s$. Nothing much can be done about $T_c$ as it depends on the physical environment but if $T_s$ is increased then $L$ can be reduced. One of the methods to increase $T_s$ is through parallel transmission. This parallel transmission corresponds to MCM, thus it can be seen that MCM came into existence to reduce the deleterious effect of wireless channel. The most promising MCM till date is Orthogonal Frequency Division Multiplexing (OFDM), which is being deployed in broadcast as well as in cellular systems. OFDM is being explained briefly subsequently.

1.1.1 Orthogonal Frequency Division Multiplexing

OFDM is one of the promising technology for high data rate transmission. Such high rates can be achieved because of serial to parallel conversion of the data stream [4]. Figure 1.2 shows an OFDM transmitter.

\[\text{Figure 1.2: OFDM Transmitter}\]

In OFDM the sub-carriers are the basis functions, which are windowed sinusoids. It is a kind of frequency division multiplexing but because of the overlap of sub-carriers

\[^1\text{ISI is discussed in detail in later chapters}\]
in frequency domain, the spectral efficiency is high. Although the sub-carriers overlap, because of the orthogonality, they can be easily separated and detected at the receiver. Figure 1.3 shows the sub-carriers of OFDM.

![OFDM Sub-Carriers](image)

**Figure 1.3: OFDM Sub-Carriers**

OFDM is less sensitive to Inter Symbol Interference (ISI) as it introduces a guard band at the start of the OFDM symbol, the length of the guard band is taken to be larger than the maximum delay spread of the channel. One of the popular methods to achieve guard band is Cyclic Prefix (CP) in which the end part of the OFDM Symbol is copied at the start of the symbol, in this way CP helps to counter ISI as well as can be used to help in synchronization.

![OFDM Symbols and Cyclic Prefix](image)

**Figure 1.4: OFDM Symbols and Cyclic Prefix**

### 1.1.2 Wavelet Packet Modulation

An alternative to OFDM is Wavelet Packet Modulation. In this MCM the basis functions are Wavelet basis functions [5]. The sub-carriers are generated using filter banks, and are orthogonal because of the property that all the wavelets derived from a mother wavelet are orthogonal to each other [6]. WPM has a higher resolution than OFDM in frequency domain, as the waveforms of WPM are longer than that of OFDM in time domain. This property of WPM encourages it to be used in applications where frequency localization is desirable. OFDM is implemented using FFT whereas WPM is implemented using a tree structure of filters. This tree structure makes the system flexible, unlike in OFDM where the basis functions are pre-determined. If the channel
detected is bad then the number of sub-carriers can be reduced, and if the channel is good then the number of sub-carriers can be increased, but in case of OFDM the number of sub-carriers are fixed and cannot be changed on the fly. Thus owing to these two advantages, WPM can be considered as a potential alternative to OFDM [7].

1.2 Objective

The main objective of this thesis work, is to design and implement a channel equalizer for WPM systems. Since WPM technology is still under development, this field of channel equalization for WPM is not been dealt adequately yet. The main objectives of this thesis work are summarized below:

- To check the viability of channel equalization for WPM system.
- To find the issues related with channel equalization.
- To implement a channel equalizer and verify its performance results.

We know that WPM is an alternative to OFDM, thus it would be interesting to note that how channel is equalized in OFDM system and then to state the differences between equalization in an OFDM and WPM system.

1.2.1 Channel Equalization for OFDM

We have seen that in OFDM, a CP is appended at the start of every OFDM symbol, which helps in reducing the ISI. CP is the last part of the OFDM symbol which is prefixed at the start. Thus, because of presence of CP every OFDM symbol with CP is circular in nature. We also know that OFDM is implemented using FFT, therefore by the usage of properties of fourier transform the channel equalization for OFDM systems becomes extremely simple. We explain the process in detail below:

Let us consider a vector $\bar{a}$ of data symbols given as:

$$\bar{a} = [a_0 a_1 \cdots a_{n-1}]$$

(1.2)

At the output of the IDFT(See Figure 1.2), the OFDM signal $\bar{s}$ can be written as:

$$\bar{s} = F^H \bar{a}$$

(1.3)

where $F^H$ is the IDFT matrix. During the transmission the signal $\bar{s}$ will be convolved with the channel, if represented in matrix notations, the received signal $\bar{y}$ at the receiver can be written as:

$$\bar{y} = H_c \bar{s}$$

(1.4)

where $H_c$ is circular channel matrix. The circular channel matrix becomes circular because of the presence of CP in the signal $\bar{s}$. At the receiver the inverse of IDFT i.e. DFT is being performed, thus the output of DFT $\bar{z}$ can be written as:

$$\bar{z} = F^H \bar{y}$$

(1.5)
if substituting Eq. 1.4 into Eq. 1.5 we obtain

\[ \bar{z} = F H_c \tilde{s} \]  

(1.6)

further substituting Eq. 1.3 into Eq. 1.6 we get

\[ \bar{z} = F H_c F^H \tilde{a} \]  

(1.7)

where Eq. 1.7 can be written as

\[ \bar{z} = \Omega \tilde{a} \]  

(1.8)

\( \Omega \) is the diagonal matrix. Thus when original data vector \( \tilde{a} \) is to be obtained it corresponds to a mere division \[ \Box \], making channel equalization for OFDM systems a very simple process.

### 1.2.2 Difference in Channel Equalization for OFDM and WPM

In the previous section we saw that channel equalization for OFDM system is a simple process, and is performed in the frequency domain. The approach for channel equalization for WPM cannot be same as that of OFDM. This is due to two reasons, firstly there is no CP or guard interval in WPM system as the waveforms in time domain are pretty long and they overlap with adjacent waveforms. Secondly because of the interference of waveforms there is an inherent ISI in the WPM system, which have to be dealt with as a system design constraint. Unlike OFDM, this inherent ISI may worsen the performance of the system under a wireless channel. Thus for these two reasons, the methods used for channel equalization in OFDM cannot be applied for WPM systems. Therefore, we need to design new methods to implement channel equalizers for WPM, which makes it interesting and difficult.

### 1.3 Scope

In the wireless radio group of Microwave Technology Systems and Radar at TU Delft, a detailed research on Wavelet Packet Modulation for radio communication is carried out. As a part of the program many issues related to the design of WPM radios such as synchronization, PAPR equalization are studied. One aspect of the study name channel equalization is worked in this thesis work.

### 1.4 Organization of the Report

The thesis report is organized as follows. In Chapter 2, wavelet theory is discussed. This chapter also describes how wavelet theory can be suitable for wireless communications in the form of Wavelet Packet Modulation. In Chapter 3, background information on channel equalization is presented. In Chapter 4, a survey of existing literature available on WPM channel equalization techniques is discussed. In Chapter 5, a mathematical analysis of the channel equalization is presented, discussing the contributions of the individual components of interference like ISI, ICI and ISCI. In Chapter 6, the channel
equalization by minimizing the peak distortion is presented. Its implementation and simulation results are also discussed. Lastly, in Chapter 7 summary and concluding remarks are provided.
This chapter focuses on the theoretical concepts of Wavelet Theory and how it can be used in wireless communications in the form of Wavelet Packet Modulation (WPM).

To start the discussion on wavelets, let us consider the space $L^2(\mathbb{R})$ of all square integrable function having domain $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. The condition for a function to belong to $L^2(\mathbb{R})$ is given in 2.1.

$$\|f\|^2 = \int_{-\infty}^{+\infty} |f(u)|^2 du < \infty \quad (2.1)$$

The reason for limiting the discussion to $L^2(\mathbb{R})$ space is that in real world the signals which we deal with have finite energy. Thus in this space let us consider a function $f(x)$ given as

$$f(x) = \sum_k \alpha_k \phi_k(x), k \in \mathbb{Z} \quad (2.2)$$

where $\alpha_k$ are the expansion coefficients, $\phi(x)$ is called as the scaling function and $x$ is the continuous variable. The scaling function $\phi_{r,s}(x)$ has two parameters associated with it, $r$ being the scaling parameter and $s$ being the shift parameter, with $\{r, s \in \mathbb{Z}\}$. Thus by varying $r$ and $s$ over their complete range, the entire space $L^2(\mathbb{R})$ can be covered. The mathematical definition of $\phi_{r,s}(x)$ is given as

$$\phi_{r,s}(x) = \mu^r \phi(\mu^r x - \beta s) \quad (2.3)$$

where $\mu$ is the fixed dilation step, but in practical scenario it is considered to be 2 as this results in octave bands, $\beta$ is the shift factor which is usually set to 1. Thus for practical purposes, the equation can be re-written as:

$$\phi_{r,s}(x) = 2^{\frac{r}{2}} \phi(2^r x - s) \quad (2.4)$$

In order to understand the effect and role of the scaling parameter and shift parameter let us consider an example of scaling function $\phi(x)$ being the Haar function. The Haar function is defined as

$$\phi(x) = \begin{cases} 1, & \text{if } 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases} \quad (2.5)$$

If we consider both $r$ and $s$ to be equal to 0, then we get the scaling function $\phi_{0,0}(x)$ and if the values of $r$ and $s$ are substituted in Eq. 2.4, then we see that $\phi_{0,0}(x)$ is equal to $\phi(x)$ that is nothing but the Haar function. Similarly we can obtain $\phi_{1,0}(x)$ which
Figure 2.1: Haar Function

is being shown in the Figure 2.1. If we have a closer look at the figure, we can find that we have haar function at different scales, the $\phi_{0,0}(x)$ is a course scale and $\phi_{1,0}(x)$ is a finer scale, thus if the value of $r$ would be increased further we can obtain finer versions of the haar function. This is called as Multiscale/Multiresolution [11], where a function is being considered at different scales. Now let us consider the case where we can express $\phi_{0,0}(x)$ in terms of $\phi_{1,0}(x)$, it can be seen that the coarser function can be expressed in terms of the scaled and shifted version of the finer function, which is given as

$$
\phi_{0,0}(x) = \frac{1}{\sqrt{2}}(\phi_{1,0}(x) + \phi_{1,1}(x))
$$

2.1 Multiresolution Analysis

We have seen in the previous section that scaling functions $\phi_{r,s}(x)$ for all the possible values of $r$ and $s$ cover the entire space of $L^2(\mathbb{R})$. Now let us consider a case, where we can fix the value of $r$ to be $r_0$, thus we can write scaling functions as $\phi_{r_0,s}(x)$ where only the shift parameter $s$ is varying. From the example of Haar function in the previous section it can be clear that with only one value of $r$ the entire $L^2(\mathbb{R})$ cannot be covered, it would cover only a specific subspace and let us call that subspace as $V_0$. Similarly we can consider scaling functions $\phi_{r_0+1,s}(x)$ and their corresponding subspace to be $V_1$. Thus it can be seen that subspace $V_0$ will be contained in the subspace $V_1$ and in general the subspace relationship can be given as

$$
V_{-\infty} \subset \ldots V_0 \subset V_1 \subset \ldots \subset V_{\infty}
$$

This subspace relationship is pictorially presented in the Figure 2.2.
Now if we have a function $g_1(x)$ belonging to the subspace of $V_1$, then it can be approximated using the scaling function $\phi_{r_{0+1,s}}(x)$, it would be interesting to note that if a function $g_0(x)$ belongs to the subspace $V_0$ then this function can also be approximated by $\phi_{r_{0+1,s}}(x)$, it is because subspace $V_0$ is contained in $V_1$. It means that a higher order scaling function can be used to approximate a lower order function. Thus in general the coarser(lower order) function in terms of the finer(higher order) function, can be written as

$$\phi(x) = \sum_n h_{\phi}(n) \sqrt{2} \phi(2x - n)$$  \hspace{1cm} (2.8)

where $h_{\phi}(n)$ are the coefficients for the finer functions and $n \in \mathbb{Z}$ is the shifting parameter.

### 2.2 Wavelet Functions

From the Multiresolution Analysis we have seen that a lower order scaling function can be expressed in terms of scaled and shifted version of higher order scaling function, that means we only used scaling functions. We can have another way to look at this Multiresolution Analysis with the help of scaling and wavelet functions. In Figure 2.2 we saw that the subspace $V_0$ was contained in the subspace $V_1$, thus we can write :

$$V_1 = V_0 \bigoplus W_0$$  \hspace{1cm} (2.9)

where $W_0$ is the difference subspace between $V_1$ and $V_0$. Similarly we can also have $W_1$ which is the difference subspace between $V_1$ and $V_2$ and can be written as :

$$V_2 = V_1 \bigoplus W_1$$  \hspace{1cm} (2.10)
If we substitute the value of $V_1$ in the above equation we can get $V_2$ as

$$V_2 = V_0 \bigoplus W_0 \bigoplus W_1$$  \hspace{1cm} (2.11)

thus in general then $n^{th}$ subspace can be expressed in terms of $V_0$ and $n-1$ difference subspaces, which is shown in the Figure 2.3.

$$V_n = V_0 \bigoplus W_0 \bigoplus W_1 \ldots \bigoplus W_{n-1}$$  \hspace{1cm} (2.12)

![Figure 2.3: Difference Subspaces](image)

As we know that vector subspaces can be spanned by scaling functions, similarly the difference subspaces can be spanned by wavelet functions, which are defined \cite{10} as :

$$\psi_{r,s}(x) = 2^r \psi(2^r x - s)$$  \hspace{1cm} (2.13)

Although the functional form of wavelet functions is similar to scaling functions, they possess important properties such as:

- Wavelet functions are oscillatory in nature
- The area under the wavelet function is zero
- The shifted versions of the wavelet functions are orthogonal to each other. This is the property which is of greatest interest in wireless communications.

Thus if one wants to approximate a function in the subspace $V_1$, then we can use the scaling function $\phi_{0,0}(x)$ of subspace $V_1$ and the wavelet function $\psi_{r,s}(x)$ which belongs to the difference subspace between the subspace $V_1$ and $V_0$.

Till now we have seen that what are scaling and wavelet functions, so in order to have a relationship between the scaling and wavelet functions we have all the necessary information. In order to start the discussion we can refer to Figure 2.3, we saw earlier
that a subspace can be expressed in terms of subspace $V_0$ and $n-1$ wavelet functions, now we would be interested in knowing a difference subspace in terms of a vector subspace. Let us consider the case if we wish to find $W_1$, then from Figure 2.3 it can be seen that we can find difference subspace $W_1$ using the vector subspace $V_2$ and not by vector subspaces $V_0$ and $V_1$, which means that a lower order wavelet function can expressed in terms of the scaled and shifted versions of the higher order scaling function. In general it can written as:

$$\psi(x) = \sum_n h_\psi(n) \sqrt{2} \phi(2x - n)$$

(2.14)

where $h_\psi(n)$ are the coefficients associated with wavelet function and $n \in \mathbb{Z}$ is the shifting parameter.

### 2.3 Wavelet Series

Wavelet Series expresses a continuous function $f(x)$ belonging to $L^2(\mathbb{R})$, where $x$ is a continuous variable in terms of scaling and wavelet functions. In other words it can be said that wavelet series helps us to approximate a function $f(x)$ from the known scaling and wavelet functions. The wavelet series is given as:

$$f(x) = \sum_s a_{r_0,s} \phi_{r_0,s}(x) + \sum_{r=r_0}^{\infty} \sum_s b_{r,s} \psi_{r,s}(x)$$

(2.15)

where:

- $\phi_{r_0,s}(x)$ is scaling function at a fixed scale of $r_0$, the shifted versions of the scaling function at this scale would be orthogonal to each other.
- $\psi_{r,s}(x)$ is wavelet function at different scales and shifts, with $r \geq r_0$.

$$a_{r_0,s} = \int f(x) \phi_{r_0,s}(x) dx$$

(2.16)

$$b_{r,s} = \int f(x) \psi_{r,s}(x) dx$$

(2.17)

This implies that to approximate a function $f(x)$, we just need to calculate scaling function coefficients $a_{r_0,s}$ and wavelet function coefficients $b_{r,s}$ as all other functions are known to us.

### 2.4 Discrete Wavelet Transform

We know that in real life, we use discrete signals rather than continuous signals, so now we would consider a sampled version $s(n)$ of the continuous signal $f(x)$, where $n = 0, 1, 2 \ldots M-1$. Thus to approximate the signal $s(n)$ using the scaling function coefficients $W_\phi$ and wavelet function coefficients $W_\psi$, where these coefficients can be written in discrete domain as:

$$W_\phi(j_0,k) = \frac{1}{\sqrt{M}} \sum_n s(n) \phi_{j_0,k}(n)$$

(2.18)
\[ W_\psi(j, k) = \frac{1}{\sqrt{M}} \sum_n s(n)\psi_{j,k}(n) \] (2.19)

where \( j \geq j_0 \). The above equations are in accordance with Eq. 2.16 and Eq. 2.17, just that Eq. 2.16 and Eq. 2.17 are in continuous domain, Eq. 2.18 and Eq. 2.19 are called as Discrete Wavelet Transform (DWT). The inverse DWT is given as:

\[ s(n) = \frac{1}{\sqrt{M}} \sum_k W_\phi(j_0, k)\phi_{j_0,k}(n) + \frac{1}{\sqrt{M}} \sum_{j=j_0}^\infty \sum_k W_\psi(j, k)\psi_{j,k}(n) \] (2.20)

Usually \( M \) is taken to be a power of 2, like \( 2^J \) where \( J \) is the scale of the wavelet functions and in this case \( j = 0,1,\ldots,J-1 \). Thus we have defined the DWT and inverse DWT, now we are interested in knowing an efficient way of calculating the DWT. In order to have a detailed idea of implementation of DWT, below is given a complete calculation and its interpretation. From Eq. 2.14 \( \psi_{j,k}(n) \) in discrete domain can be written as:

\[ \psi_{j,k}(n) = 2^j \psi(2^j n - k) \] (2.21)

Now we would substitute Eq. 2.21 in Eq. 2.19 and get

\[ W_\psi(j, k) = \frac{1}{\sqrt{M}} \sum_n s(n)2^j \psi(2^j n - k) \] (2.22)

From Eq. 2.14 we can write \( \psi(n) \) as:

\[ \psi(n) = \sum_p h_\psi(p)\sqrt{2}\phi(2n - p) \] (2.23)

we would like to scale \( \psi(n) \) with \( 2^j \) and shift it by \( k \), to get \( \psi(2^j n - k) \) as:

\[ \psi(2^j n - k) = \sum_p h_\psi(p)\sqrt{2}\phi(2^{j+1}n - 2k - p) \] (2.24)

changing the variable

\[ p = v - 2k \] (2.25)

and we get

\[ \psi(2^j n - k) = \sum_v h_\psi(v - 2k)\sqrt{2}\phi(2^{j+1}n - v) \] (2.26)

now we would substitute Eq. 2.26 into Eq. 2.22 to get the wavelet function coefficients \( W_\psi(j, k) \) in terms of the scaling functions \( \phi_{j,k}(n) \)

\[ W_\psi(j, k) = \sum_v h_\psi(v - 2k)[\frac{1}{\sqrt{M}} \sum_n s(n)2^{j+1}\phi(2^{j+1}n - v)] \] (2.27)

if we carefully watch the term in [ ] it is nothing but \( W_\phi(j + 1, v) \), thus we can write

\[ W_\psi(j, k) = \sum_v h_\psi(v - 2k)W_\phi(j + 1, v) \] (2.28)
and similarly
\[
W_\phi(j,k) = \sum_v h_\phi(v - 2k)W_\phi(j + 1,v)
\] (2.29)

In Eq. 2.28 and Eq. 2.29 it can be seen that the wavelet function coefficients and scaling function coefficients are nothing but convolution of a filter and scaling function coefficients. It means if the lower order scaling or wavelet functions coefficients are to be calculated then we can just convolve the higher order scaling function coefficients to a specific filter. Now let us see what defines these filters, in order to find \(W_\psi(j,k)\) the filter coefficients \(h_\psi\) should be a high pass filter, it is because it belongs to the difference subspace, which was defined in the previous section. The difference subspace was a difference of two vector subspaces, and since approximating a function from these vector subspaces is like doing a running average which is nothing but low pass filtering, and difference of two low pass filters will be a high pass filter. Thus \(h_\psi\) will be coefficients of a highpass filter ans similarly we can also say that \(h_\phi\) will be coefficients of a lowpass filter. The implementation shown in the Figure 2.4 is an effective method to find the Discrete Wavelet Transform. In DWT only the low pas filter outputs are carried forward to the next stage of the tree and the high pass results are neglected. Now this idea of highpass and lowpass filters which are quadrature mirror filters(QMF)

\[\begin{align*}
W_\phi(j,k) & \quad \xrightarrow{LPF} \quad W_\psi(j,k) \\
W_\psi(j,k) & \quad \xrightarrow{HPF} \quad W_\phi(j,k)
\end{align*}\]

Figure 2.4: Lowpass and Highpass filters

can be further extended to filter banks which can used to find the wavelet and scaling function coefficients at all the scales \(J\). The quadrature mirror filters and filter banks are dealt in detail in the following sections.

### 2.5 Filter Banks

Filter Banks are generally categorized as two types analysis filter banks and synthesis filter banks. An analysis filter bank consists of a set of filters with system functions \(H_k(z)\), arranged in a parallel bank. The frequency response characteristics of this filter bank splits the signal into a corresponding number of sub-bands [12]. The synthesis
filter bank consists of a set of filters with system functions $G_k(z)$, the outputs the filters are summed to form the synthesized signal. As we saw in the previous section we use low-pass and high-pass filters to realize the discrete wavelet transform at a particular scale, similarly we can use a filter bank to realize the IDWT and DWT at all the scales in a tree structure. In our application of Wavelet Packets we also use the filter banks which would be seen later.

2.6 Quadrature Mirror Filters

The basic building block in applications of quadrature mirror filters (QMF) is a two channel QMF bank [12] comprising of a LPF and HPF, which also employs two decimators in the analysis section and two interpolators in the synthesis section.
A desired frequency response of QMF is shown in the figure below, but the main issue in the QMF is aliasing resulting from decimation in the analysis section of QMF, which is to be eliminated. Thus in order to eliminate the aliasing effect two conditions need to be satisfied, the low pass filter of the synthesis section should be:

\[ G_0(\omega) = 2H(\omega) \]  \hspace{1cm} (2.30)

and the high pass filter of the synthesis section should be:

\[ G_1(\omega) = -2H(\omega - \pi) \]  \hspace{1cm} (2.31)

where \( H(\omega) \) is the frequency response of a low pass filter. The factor 2 corresponds to the interpolation factor used to normalize the overall frequency response of the QMF.
The low pass filter $H_0(\omega)$ of the analysis section and high pass filter $H_1(\omega)$ of the analysis section are given as below:

$$H_0(\omega) = H(\omega)$$  \hspace{1cm} (2.32)

$$H_1(\omega) = H(\omega - \pi)$$  \hspace{1cm} (2.33)

2.7 Wavelet Packets

In DWT’s implementation the iterations over a tree structure are performed only on low pass branches which result in high frequencies having wide bandwidths and low frequencies having narrow bandwidths \cite{13}(refer Figure 2.8).

![Figure 2.8: Frequency Response of 3-stage DWT](image-url)

To avoid this, wavelet packets were proposed in which we iterate both on the low pass and high pass filters. As a result we get equally spaced frequencies as shown in the Figure 2.9.

![Figure 2.9: Frequency Response of 2-stage WPT](image-url)

An example of 2-stage WPT implementation using filter banks is shown in the Figures 2.10 and 2.11. Figure 2.10 shows the analysis operation and Figure 2.11 represents the synthesis process.

2.8 Multicarrier Modulation using Wavelet Packet Modulation

In wireless communications the above mentioned Wavelet Packets are being used. Each frequency band as shown in the Figure 2.9 acts as carrier and data symbols are modu-
lated on each of these carriers. The complete frequency band is divided into number of sub-carriers, each sub-carrier is orthogonal to each other because of the property from wavelet theory. The property says that all the children wavelets obtained from a parent
wavelet are orthogonal to each other.

![Diagram](image)

Figure 2.12: Three level Synthesis Section at WPM Transmitter

The orthonormal wavelets can be given as:

\[
f_k(n) = \prod_{p=1}^{P} \sqrt{2} t_{k,p}(\frac{n}{2^p - 1})
\]  

(2.34)

where \( \prod \) represents the convolution operation, \( P \) is the number of levels in the tree structure, \( k \epsilon \{0,1,2,3\ldots,2^P - 1\} \) and \( t_{k,p} \epsilon \{f_l(n), f_h(n)\} \) is the filter response corresponding to the \( k^{th} \) sub-channel at \( p^{th} \) level and \( f_l(n), f_h(n) \) are impulse responses of the low pass and high pass filters respectively.

In order to understand the application of Wavelet Packets for multicarrier modulation let us consider the example of three stage synthesis and analysis sections of QMFs which are used in the transmitter and receiver respectively. The number of sub-carriers are defined as \( 2^m \) where \( m \) is the number of levels, thus in such a transceiver the number of sub-carriers are 8 in number. At the transmitter side the synthesis section of QMF is used, where the data bits are modulated on the sub-carriers and transmitted as shown in the Figure 2.12. Similarly at the receiver side the analysis section of QMF is being used and the data bits on each sub-carrier are being reconstructed as shown in the Figure 2.13. The time domain waveforms for a three level WPM transceiver are shown in the Figure 2.14 and the subsequent sub-carriers in the frequency domain are shown in the Figure 2.15.

### 2.9 Summary

In this chapter, we have seen concept of wavelet theory in detail starting from the multiresolution analysis to wavelet packets. The applicability of wavelet theory in wireless communications, in the form of Wavelet Packet Modulation is also being discussed.
Figure 2.13: Three level Analysis Section at WPM Receiver

Figure 2.14: Waveforms in Time Domain
Figure 2.15: Sub-Carriers in Frequency Domain
In this chapter channel equalization as a concept is being presented, and also channel equalization techniques are discussed.

3.1 Concept

A channel equalizer is an integral part of receivers in contemporary wireless communication systems. The wireless channel corrupts the transmitted signal and the equalizer is used to counteract those irregularities and bring the signal as close to the one which was transmitted. The multipath effects of a wireless channel introduce Inter Symbol Interference (ISI), which is a major irregularity and the equalizer should remove this ISI to get the uncorrupted signal.

As the field of wireless communication is inching towards high data rates, there would be a significant need for equalization as the higher data rate applications are more sensitive to channel delay spread [14]. Even a mildly dispersive channel creates sufficient distortion to significantly limit the link throughput. The mitigation of such interference is thus critical to the performance of the system and the design of a suitable equalization method is required. Apart from ISI mitigation, equalizer should also reduce the noise, since both the signal and the noise pass through the equalizer, which can increase the noise power. Thus the equalizer design should take care of Noise Enhancement [14]. In practice we deal with time variant wireless channels, in such a case the equalizer should be able to adapt to the new channel and update its coefficients, it is called as Adaptive Equalization. Thus the main purpose an equalizer is to mitigate the ISI.

3.1.1 Inter Symbol Interference

The wireless channel acts like a low-pass filter, spreading or smearing the signal. Mathematically the operation can be defined as convolution of the data signal with the channel impulse response.

\[
r(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau
\]

(3.1)

where \( r(t) \) is the received signal, \( h(t) \) is the channel impulse response and \( x(t) \) is the transmitted signal. The \( x(t) \) can be defined as:

\[
x(t) = \begin{cases} 
0, & \text{for } t \neq kT \\
X_k, & \text{for } t = kT 
\end{cases}
\]

(3.2)
where $T$ represents the symbol period. This means that the only significant values of the variable of integration in the above integral are those for which $\tau = kT$. Any other value of $\tau$ amounts to multiplication by 0. Therefore $r(t)$ can be written as:

$$r(t) = \sum_{k=-\infty}^{\infty} X_k h(t - kT)$$  \hspace{1cm} (3.3)

The Eq. 3.3 is a convolution sum and it represents that the received signal as the sum of the scaled and shifted versions of channel impulse response. Now let us consider the case where the received signal $r(t)$ is sampled,

$$r(nT) = \sum_{k=-\infty}^{\infty} X_k h(nt - kT)$$  \hspace{1cm} (3.4)

which can be re-written as:

$$r(nT) = X_n h(0) + \sum_{k\neq n} X_k h(nt - kT)$$  \hspace{1cm} (3.5)

In practical scenario there will also be an arbitrary phase offset, which can be shown as :

$$r(t_0 + nT) = X_n h(0) + \sum_{k\neq n} X_k h(t_0 + nt - kT)$$  \hspace{1cm} (3.6)

In the equation above, the first term is the component of $r(t)$ due to the $N'th$ symbol. It is multiplied by the center tap of the channel-impulse response. The other product terms in the summation are ISI terms [15]. The input symbols in the neighborhood of the Nth symbol are scaled by the appropriate samples in the tails of the channel impulse response. Thus the main purpose of an equalizer is to eliminate these ISI terms.

### 3.1.2 Designing a Channel Equalizer

In order to design the a channel equalizer, we have to consider the complete transceiver system, as shown in the Figure 3.1. In the figure each of the each of the blocks is represented by its transfer functions. The transmitter is defined by $T(z)$, the channel by $H(z)$. At the receiver side, $R(z)$ and $E(z)$ give the transfer functions of demodulator and equalizer respectively. The position of the equalizer can also vary, depending on post/pre detection equalization. In this case a post detection equalization scheme is being shown.

![Figure 3.1: Transceiver System Functions](image)
The complete transfer function of the transceiver can be given by $\Psi(z)$, which is given as [16]:

$$
\Psi(z) = T(z)H(z)R(z)E(z)
$$

Clearly from the equation Eq. 3.7, if perfect equalization is achieved then $\Psi(z)$ will be an identity matrix or equal to 1. We also know that because of the multipath, ISI happens, thus in order to mitigate the effect of ISI perfectly, it is required that the equalizer $E(z)$ has an order in $z$ equal to $R(z)H(z)T(z)$.

### 3.2 Channel Equalization Techniques

In literature, there are optimal and sub-optimal channel equalizers. Optimal equalizers are the one which give the best performance but are very complex and difficult to implement on hardware. Sub-optimal equalizers are the one which are used in practice, even though they are not perfect but they are realizable. Thus in the following sections we will discuss the different kind of sub-optimal equalizers which can be used in equalization for WPM. The equalizers can be classified basically into two categories Linear and Non-Linear (see Figure 3.2) [14]. In linear equalizers a linear filter is designed in order to reduce noise and ISI according to some optimal criterion and a linear combination of the taps of the equalizer are used to produce the output. Non-linear equalizers are designed for severe ISI conditions. In both the categories there are different algorithms which are used for implementation. For example Zero Forcing Equalizer (ZFE) and Minimum Mean Square Error (MMSE) fall into the category of linear equalizers and Decision Feedback Equalizers (DFE) falls into the category of non-linear equalizers. In the following sections we shall delve more on the equalizers.

![Figure 3.2: Types of Equalizers](image-url)
3.2.1 Minimization of Peak Distortion or Zero Forcing Equalizer

The peak distortion is defined as the worst case ISI at the output of equalizer [17]. This terminology has been derived from older days when the signal was seen on the oscilloscope, there was a an eye like pattern, if the eye was completely open then no ISI was considered and if the eye was distorted then ISI was there. Thus from the maximum of this distortion the term peak distortion is being derived [18]. It implies that if peak distortion is being minimized then in a way the signal is being equalized as ISI effect is being nullified. In order to design an efficient equalizer to remove (or minimize) the channel distortion, we use a performance metric called the peak distortion criterion. Representing the impulse response of the channel with \(c_n\) and that of the equalizer with \(e_n\), we can define a single equivalent filter \(q_n\) as the convolution between \(c_n\) and \(e_n\), as,

\[
q_n = \sum_{j=-\infty}^{\infty} e_j c_{n-j}. \tag{3.8}
\]

Under these circumstances, two scenarios can be defined to understand the nature of the equalization process,

- when the equalizer has infinite number of taps, and
- when the equalizer has a finite number of taps.

3.2.1.1 Equalizer with Infinite Taps

With an equalizer of infinite taps, the output at the \(k\)th sampling instance can be expressed as [17]:

\[
\hat{I}_k = q_0 I_k + \sum_{n \neq k} I_n q_{k-n} + \sum_{j=-\infty}^{\infty} e_j \eta_{k-j}. \tag{3.9}
\]

In (3.9) the first term represents the desired symbol scaled by a factor \(q_0\), the second term is the ISI and the third term is AWGN. The peak value of this distortion \(\Omega(e)\), is given by [17]:

\[
\Omega(e) = \sum_{n=-\infty,n \neq 0}^{\infty} |q_n| = \sum_{n=-\infty,n \neq 0}^{\infty} | \sum_{j=-\infty}^{\infty} e_j c_{n-j} | \tag{3.10}
\]

Indeed, \(\Omega(e)\) is a function of the equalizer tap weights. For an equalizer with infinite taps, it is possible to select the tap weights such that \(\Omega(e) = 0\), i.e., the ISI can be completely eliminated. Under these circumstances, the tap weights can be determined as,

\[
q_n = \sum_{j=-\infty}^{\infty} e_j c_{n-j} = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \tag{3.11}
\]

In the frequency domain (3.11) can be written as:

\[
Q(f) = E(f)C(f) = 1 \tag{3.12}
\]
or,

\[ E(f) = \frac{1}{C(f)} \quad (3.13) \]

From Eq.(3.13) it can be inferred that in order to completely eliminate the ISI, the equalizer should be an inverse of the channel filter. For this reason the peak distortion criterion is also referred to as zero-forcing (ZF) equalization.

### 3.2.1.2 Equalizer with Finite Taps

Thus far, we have considered an equalizer of infinite length. Let us now consider an equalizer of finite length, say, \(2M+1\). Since \(e_j = 0\) for \(|j| > M\), the convolution of \(c_n\) with \(e_n\) is zero outside the range \(-M \leq n \leq M + L_c - 1\). That is, \(q_n = 0\) for \(n \neq M\) and \(n \neq M + L_c - 1\), where \(L_c\) is the channel length. With \(q_0\) normalized to unity, the peak distortion becomes

\[
\Omega(e) = \sum_{n=-M}^{M+L_c-1} |q_n| = \sum_{n=-M}^{M+L_c-1} |\sum_{j} e_j c_{n-j}|. \quad (3.14)
\]

Although the equalizer has \(2M + 1\) adjustable parameters, there are \(2M + L_c\) non-zero values in \(q_n\). Therefore, it is impossible to completely eliminate the ISI and there will always remain residual interference even when the optimum coefficients are used. The peak distortion criterion given in Eq.(3.14) has been shown to be a convex function of the equalizer coefficients. The general solution of the peak distortion criterion can be obtained by the method of steepest descent.

This kind of equalizer is prone to noise enhancement, we can write:

\[
R(z)E(z) = S(z) + \tilde{N}(z) \quad (3.15)
\]

where \(S(z)\) \(R(z)\) are the transmitted and received signals respectively and \(\tilde{N}(z)\) is the noise at the output of the equalizer. We know \(R(z)\) can be written as:

\[
R(z) = S(z)C(z) + N(z) \quad (3.16)
\]

If we substitute 3.16 into 3.15 we get:

\[
[S(z)C(z) + N(z)]E(z) = S(z) + \tilde{N}(z) \quad (3.17)
\]

if the channel response is attenuated for some frequencies then the noise at the output of the equalizer \(\tilde{N}(z)\) will be very high, resulting in Noise Enhancement. Although implementation of ZFE is simple but it has severe disadvantage of Noise Enhancement. The equalizer is being implemented using the transversal filter [19], as shown in the Figure 3.3. The center tap of this is being set to 1 and rest all the taps are being set to zero. The main assumption in the implementation of this equalizer is that the frequency resolution is high enough to avoid any kind of ICI and ISCI, thus this equalizer only tends to eliminate the ISI.
3.2.2 Minimum Mean Square Error Equalizer

The Minimum Mean Square Error (MMSE) equalizer is a more general approach. Instead of solving a set of N simultaneous equations as was done in the ZFE, the coefficients are gradually adjusted to converge to a filter that minimizes the error between the equalized signal and the reference signal.

\[
\min\left( E\left[ S_k - \hat{S}_k \right]^2 \right) \quad (3.18)
\]

where \( X_k \) is the reference signal and \( \hat{S}_k \) is the equalized signal. The filter convergence is based on approximations to a gradient calculation of the quadratic equation representing the mean square error. The beauty of the approach is that the only parameter to be adjusted is the adaptation step size. Through an iterative process, all filter tap weights are adjusted during each sample period in the training sequence. Eventually, the filter will reach a configuration that minimizes the mean square error between the equalized signal and the stored reference. As might be expected, the choice of step size involves a tradeoff between rapid convergence and residual steady-state error. A too-large setting for step size can result in a system that converges rapidly on start-up, but then chops around the optimal coefficient settings at steady state. The MMSE equalizer can also be shown to have better noise performance than the ZFE, which can be seen from the following equation as the Noise factor comes in the denominator part.

\[
C_{MMSE}(z) = \frac{1}{C(z) + N(z)} \quad (3.19)
\]

Heuristically, the ZFE calculates coefficients based upon the received samples of one training signal. Since the captured data will always contain some noise, the calculated coefficients will be noisy noise in / noise out. On the other hand, the MMSE algorithm
gradually adapts a filter based on many cycles of the training signal. If the noise is zero mean and is averaged over time, its effect will be minimized. Noise integrates to 0.

In a MMSE equalizer the algorithm used to update the tap weights might be LMS or RLS, depending upon the application. But in general LMS algorithm is used the most because it is less complex than RLS and can be simply implemented using a transversal filter structure.

### 3.2.3 Decision Feedback Equalizer

Decision Feedback Equalizer (DFE) falls in the category of Non-Linear Equalizers. DFE is based on the principle that once you have determined the value of the current transmitted symbol, you can exactly remove the ISI contribution of that symbol to future received symbols. The nonlinear feature is due to the decision device, which attempts to determine which symbol of a set of discrete levels was actually transmitted. Once the current symbol has been decided, the filter structure can calculate the ISI effect it would tend to have on subsequent received symbols and compensate the input to the decision device for the next samples. This postcursor ISI removal is accomplished by the use of a feedback filter structure. In addition to the one transversal filter called as feed forward, there is a second adaptive filter structure fed by the output of the decision device. This second filter is the feedback stage that cancels the postcursor ISI. Its inputs are the symbol decisions, and the tap weights converge through the LMS process to resemble the tail of the channel impulse response (taps beyond the center tap). For highly frequency selective channels having deep spectral nulls, the DFEs perform better than the linear equalizers.

![Decision Feedback Equalizer](image_url)

**Figure 3.4: Decision Feedback Equalizer**

### 3.2.4 Decision Directed Equalization

Another Non-Linear Equalizer is Decision Directed Equalizer (DDE). Most of the times with data communication systems, one can take advantage of prior knowledge of the transmit signal characteristics to deduce a more accurate representation of the transmit signal than can be afforded by the linear filter. It is possible to devise a decision device (a predictor or a slicer) that estimates what symbol value was most likely transmitted, based on the linear filter continuous output. For example, in the case of the bipolar sequence transmission scheme, a very simple decision device could replace all positive
values with a positive 1 and all negative values with a negative 1. The difference between the decision device input and output forms an error term which can then be minimized to adapt the filter coefficients. This is true because a perfectly adapted filter would produce the actual transmitted symbol values, and, therefore, the slicer error term would go to 0. In practice, the error is never 0, but if the adapted filter is near ideal, the decisions are perfect. In this case, the slicer is effectively throwing away received noise with each decision made. Coefficients can be updated in a manner similar to that employed by the LMS equalizer. There is however, one important thing that the error term is computed as the difference between the input and the output of the decision device, as opposed to the MMSE error term, which was based on a reference training signal. This means that the decision-directed equalizers do not require a training sequence.

3.2.5 Maximum Likelihood Sequence Estimation Equalizer

Maximum Likelihood Sequence Estimation (MLSE) avoids the problem of noise enhancement because it doesn’t use any equalizing filter, instead it estimates the sequence of transmitted symbols. The MLSE algorithm chooses the input sequence that maximises the likelihood of the received signal. Performance wise the MLSE is the optimal and all the equalizers look at the MLSE as a benchmark but complexity wise it is expensive.

3.3 Summary

In this chapter, the concept of channel equalization and ISI was discussed. Different kinds of channel equalization techniques such as Zero-forcing, MMSE, Decision Feedback Equalizer, Decision Directed Equalizers and MLSE were also presented.
In this chapter we discuss existing and proposed WPM channel equalization techniques and architectures.

The WPM is a system under development and has not been studied extensively, especially, the equalization at the receiver to mitigate interferences induced by the radio channel. A radio channel can be frequency-selective or dispersive and could lead to loss of orthogonality between the sub-carriers causing disturbances such as the ISI and ICI. Channel equalization is a simple technique to mitigate the detrimental effects of the communication channel between transmitter and receiver.

4.1 Overlap of Symbol in WPM

The channel equalization for WPM systems is unique because the WPM symbols overlap in time. Hence both ISI and ISCI occur and have to be factored in the design of the equalizer. It can be shown that for a WPM system with M sub-carriers and wavelet filter length of $L_{wl}$, the number of WPM symbols that overlap can be given as [20]:

$$L_w = (M - 1)(L_{wl} - 1) + 1 \quad (4.1)$$

The effect of this overlap causes Inter Symbol Carrier Interference (ISCI) in addition to the usual Inter Carrier Interference (ICI). Hence ISI, ICI and ISCI all have to be factored in the design of the equalizer. Figure 4.1 demonstrates the overlap of symbols in time domain. It shows two WPM symbols and it can be clearly seen that length of one is very long to overlap with the other symbol. In the Figure 4.1 overlap with only one symbol is shown but it can extend to multiple symbols.

The Figure 4.2 shows a two path channel and its effect on the sub-carriers in the frequency domain can be seen in the Figure 4.3. It is clearly seen that the sub-carriers are distorted under the effect of channel. In the following sections we shall present few of the existing techniques available in literature of channel equalization.

4.2 Distributed Equalizer Architecture

This work [16] largely presents the channel equalization methods used in other multi carrier modulation (MCM) like Orthogonal Frequency Domain Multiplexing (OFDM). Two main types of equalization techniques are discussed Equalization with redundancy and Equalization without redundancy. Equalization with redundancy mainly means
that we transmit some known information which helps us to mitigate Inter Symbol Interference (ISI) at the receiver side. For example in case of OFDM we add a cyclic prefix to mitigate ISI and it also has numerous advantages like it also helps in synchronization, but there is a major drawback of equalization with redundancy is the loss of bandwidth because of the redundant information.

Equalization without redundancy is also called as post-detection equalization, from the available literature it can be concluded that although this approach does not give optimum equalization performance, but with this approach we need not to make much modifications to the transceiver architecture and thus the less complex nature of WPM.
can be retained with less optimum equalization. Thus with this reasoning he develops an equalizer architecture called as Distributed Equalizer Architecture. The uniqueness of WPM inspired the idea of distributed equalizer architecture. Since WPM is organized as consecutive independent stages allows us to access the signal at multiple rates. The basic module of this WPM is the Wavelet packet Transform (WPT) block comprising of a Quadrature Mirror Filter (QMF) pair. The idea is to apply the equalizer individually on each of these basic blocks. Figure 4.4 shows an example of two stage distributed equalizer structure. The position of the equalizer can also be before the down sampler. The major difference in these two approaches is that in the case as shown as in Figure 4.4 equalizer utilizes full band, therefore it is sampled spaced equalizer, where as if the equalizer is placed before the down samplers it is a fractionally spaced equalizer as each work on half bandwidth. Each of these equalizer blocks is shown in the Figure 4.5. This equalizer consists for four filters. The role of $e_{11}$ and $e_{22}$ is to remove ISI on each branch of the basic block and the remaining to filters are responsible for mitigating the inter branch interference (ISCI). The usage of all the filters depends upon the channel conditions, if the channel conditions are not so bad we can only go with $e_{11}$ and $e_{22}$ and
it might work well, although the performance would be sub-optimal but it will be less complex. The case when all the four filters are used is called as per elementary block and when only two filters are used just to remove ISI and not ISCI it is called as per branch.

For simulation purposes, some details are being mentioned, firstly the tap updating algorithm used by him is LMS, that is an adaptive MMSE equalizer, secondly for training sequence he uses a pseudo-random sequence (prbs) but its details are not mentioned. Thirdly the performance metric used is the Signal to Noise Interference Ratio (SNIR), but no mention of how it is being obtained.

### 4.3 Minimum Square Variance Equalizer

In this reference [21], a basic assumption on the width of sub-carriers is being made. The assumption says that the number of sub-carriers should be narrow enough, so that the effect of channel can be only introducing the delays and no other distortions. As shown in the Figure 4.6, $H_i(z)$ is the cumulative filter coefficient for the $i^{th}$ sub-carrier.
Now the received signal on this branch is being delayed for some interval of time, and then between them which ever has least variance, will be taken to be as the equalized signal. This is a simple equalizer and will work only in simple channels also. Its initial assumption that the sub-carrier width should be very narrow, means that there are large number of sub-carriers and if this is true then the channel may not only introduce but also some more distortions. The system is also very complex as data on each sub-carrier is being delayed for some time, thus with this if the number of sub-carriers are large enough, then the system will be really complex and its implementation will be an issue.

4.4 Proposed Architecture and Simulations

In the previous sections, we have seen the existing architectures in the literature. Accordingly a new architecture was proposed and few simulations were performed to compare its performance with the existing architectures. For these simulations the WPM transmitter considered is shown in the Figure 4.7. In this structure a pseudo random binary sequence (prbs) is used as the training sequence and the simulation parameters are being shown in the Table 4.1.

![WPM Transmitter Structure](image)

**Figure 4.7: WPM Transmitter Structure**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Carriers</td>
<td>128</td>
</tr>
<tr>
<td>Wireless Channel</td>
<td>Two path channel model</td>
</tr>
<tr>
<td>Equalizer</td>
<td>Linear MMSE Equalizer</td>
</tr>
<tr>
<td>Wavelet</td>
<td>db10</td>
</tr>
<tr>
<td>Data Modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Training Sequence</td>
<td>PRBS</td>
</tr>
</tbody>
</table>

**Table 4.1: Simulation Parameters**

The Figure 4.8 shows two architectures, the architecture-1 is taken from [16], where equalizer is in the form of distributed equalizer architecture. The architecture-2 is simple post-detection equalizer. In architecture-1 the equalizer is at every level as
shown in the Figure 4.4. The output of the 2-path channel (see Figure ??) is fed to prbs removal block from where the prbs sequence is fed to equalizer to get the coefficients and using these coefficients the data is being equalized, either as in architecture-1 or in architecture-2. After getting the data bits on receiver side, error calculation is done to obtain the bit error rate (BER). The coefficients obtained from the MMSE equalizer are not adaptive, and it should not be an issue here as the channel considered is time invariant. The coefficients are defined as:

\[
c = \frac{R_{ya}}{R_{yy}}
\]  

(4.2)

Where \( R_{ya} \) is the cross-correlation between the received and desired training signal and \( R_{yy} \) is the auto-correlation of the received signal. It can be seen from Figure 4.9 that both the architectures have almost similar performance. But complexity wise the performance of architecture-2 is better, as it will be less complex for same performance. The coefficients shown in the figure correspond to equalizer taps.

4.5 Summary

In this chapter, we have seen existing equalizer strategies such as Distributed Equalizer Architecture and Minimum Square Variance Equalizer. We also proposed a receiver architecture and compared the performance of it with the existing one.
Figure 4.9: Performance of Receiver Architectures
In this chapter we mathematically analyze the WPM received signal and derive the expressions for power of ISI, ICI and ISCI components of the signal.

5.1 Analysis of Received Signal

Let us consider the WPM transmitted signal $x(n)$ to be given as:

$$x(n) = \sum_{s} \sum_{m=0}^{M-1} a_{s,m} H_m(n - m)$$ (5.1)

$a_{s,m}$ is the M-ary data symbol, per WPM symbol, per carrier
$s$ is the total number of symbols
$M$ is the total number of carriers
$H_m$ is per carrier cumulative filter coefficients

Considering a two path channel, then the received signal $y(n)$ at the output of channel can be written as:

$$y(n) = x(n) + x(n - \tau)e^{j\theta}$$ (5.2)

where $\tau$ and $\theta$ is the delay and phase associated with the second path. Substituting the value of $x(n)$ from 5.1 into 5.2 we obtain

$$y(n) = \sum_{s} \sum_{m=0}^{M-1} a_{s,m} H_m(n - m) + \sum_{s} \sum_{m=0}^{M-1} a_{s,m} H_m(n - \tau - m)e^{j\theta}$$ (5.3)

At the receiver the desired signal $\hat{a}_{s,m}$ can be written as:

$$\hat{a}_{s,m} = y(n) * F_m(n)$$ (5.4)

where $F_m(n)$ is the per carrier cumulative filter coefficient at the receiver and $*$ denotes convolution sign. From the theory of Quadrature Mirror Filters(QMF) we know that

$$F_m(n) = H_m(-n)$$ (5.5)

thus we replace $F_m(n)$ from 5.5 in 5.4 and we get

$$\hat{a}_{s,m} = y(n) * H_m(-n)$$ (5.6)
We can now substitute the \( y(n) \) from 5.3 in to 5.6 and we obtain

\[
\hat{a}_{s,m} = \sum_{s} \sum_{m=0}^{M-1} a_{s,m} H_{m}(n-m) * H_{m}(-n) + \sum_{s} \sum_{m=0}^{M-1} a_{s,m} H_{m}(n-\tau-m) e^{j\theta} * H_{m}(-n) \quad (5.7)
\]

Now let us consider only one symbol, thus \( s=1 \)

\[
\hat{a}_{m} = \sum_{m=0}^{M-1} a_{m} H_{m}(n-m) * H_{m}(-n) + \sum_{m=0}^{M-1} a_{m} H_{m}(n-\tau-m) e^{j\theta} * H_{m}(-n) \quad (5.8)
\]

The equation 5.8 can be manipulated for \( p^{th} \) sub-carrier as:

\[
\hat{a}_{p} = a_{p} H_{p}(n-p) * H_{p}(-n) + \sum_{m=0}^{M-1} a_{m} H_{m}(n-m) * H_{m}(-n) + \sum_{m=0}^{M-1} a_{m} H_{m}(n-\tau-m) e^{j\theta} * H_{m}(-n) + a_{p} H_{p}(n-\tau-p) e^{j\theta} * H_{p}(-n) \quad (5.9)
\]

The equation 5.9 has four different parts, which can be represented as:

\[
\begin{align*}
\text{Desired Signal} : & \quad a_{p} H_{p}(n-p) * H_{p}(-n) \\
\text{ISI} : & \quad a_{p} H_{p}(n-\tau-p) e^{j\theta} * H_{p}(-n) \\
\text{ICI} : & \quad \sum_{m=0}^{M-1} a_{m} H_{m}(n-m) * H_{m}(-n) \\
\text{ISCI} : & \quad \sum_{m=0}^{M-1} a_{m} H_{m}(n-\tau-m) e^{j\theta} * H_{m}(-n)
\end{align*}
\quad (5.10)
\]

The purpose of the equalizer here is to eliminate the parts due to ISI, ICI and ISCI. The equalizer by minimizing the peak distortion makes all the contributions due to the ISI, ICI and ISCI equal to zero and only gives the desired signal at the output. In literature, there are optimal and sub-optimal channel equalizers.

Optimal equalizers are the one which give the best performance but are very complex and difficult to implement on hardware. Sub-optimal equalizers are the one which are used in practice, even though they are not perfect but they are realizable. In the next chapter we discuss such a sub-optimal equalizer which aims to remove ISI only, under the assumption that since the frequency resolution is higher, thus it results into negligible ICI and ISCI.

We have seen that the WPM received signal can be broken into several components, now we can evaluate the contributions from each of the components ISI, ICI and ISCI for a two path channel.

### 5.2 Calculation of Power of ISI

From the equation 5.10 we can represent the ISI at the \( p^{th} \) sub-carrier as \( ISI_{p}(n) \), where \( n \) is the instant of time. Therefore we can write:

\[
ISI_{p}(n) = a_{p} H_{p}(n-\tau-p) e^{j\theta} * H_{p}(-n) \quad (5.11)
\]
Now we can write the convolution between the cumulative filter coefficients in detail as:

\[ ISI_p(n) = \sum_{k=-\infty}^{\infty} a_p H_p(n - \tau - p)e^{j\theta}H_p(-n - k) \]  \hspace{1cm} (5.12)

Now let us consider that detection of signal happens at 0\textsuperscript{th} time instant for the \( p \)\textsuperscript{th} sub-carrier, then we can write:

\[ ISI_p(0) = \sum_{k=-\infty}^{\infty} a_p H_p(-\tau - p)e^{j\theta}H_p(-k) \]  \hspace{1cm} (5.13)

The power of ISI is given by \( \sigma_{ISI_p}^2 = \mathbb{E}[ISI_p(0)ISI_p(0)^\ast] \), where \( \mathbb{E}[\cdot] \) is the expected value. We know that if the data transmitted is M-ary modulated and considering the power of transmitted data is normalized to one \( [22] \), we can write:

\[ \mathbb{E}[d_k(m)d_j(n)^\ast] = \delta(k - j)\delta(m - n) \]  \hspace{1cm} (5.14)

Thus, from 5.14 we can also write

\[ \sigma_{ISI_p}^2 \quad = \quad \mathbb{E}[a_p e^{j\theta} \ldots H_p(-\tau - p)H_p(2) + H_p(-\tau - p)H_p(1) \] 
\[ + H_p(-\tau - p)H_p(0) + H_p(-\tau - p)H_p(-1) \ldots] \times \] 
\[ a_p^\ast e^{-j\theta} \ldots H_p^\ast(-\tau - p)H_p^\ast(2) + H_p^\ast(-\tau - p)H_p^\ast(1) \] 
\[ + H_p^\ast(-\tau - p)H_p^\ast(0) + H_p^\ast(-\tau - p)H_p^\ast(-1) \ldots] \]  \hspace{1cm} (5.15)

Further the above equation can be written as:

\[ \sigma_{ISI_p}^2 \quad = \quad \mathbb{E}[|a_p H_p(-\tau - p)|^2 \ldots |H_p(2)|^2 + |H_p(1)|^2 + |H_p(0)|^2 \ldots] \]  \hspace{1cm} (5.16)

Thus the power of inter symbol interference, for a two path channel can be given as:

\[ \sigma_{ISI_p}^2 = \mathbb{E}[|a_p H_p(-\tau - p)|^2 \sum_{k=-\infty}^{\infty} |H_p(-k)|^2] \]  \hspace{1cm} (5.17)

\section*{5.3 Calculation of Power of ICI}

As seen in previous section, similarly the power of ICI can also be calculated. Let us denote the ICI for the \( p \)\textsuperscript{th} sub-carrier as \( ICI_p(n) \) and it can be written as:

\[ ICI_p(n) = \sum_{m=0}^{M-1} a_m H_m(n - m) \ast H_m(-n) \]  \hspace{1cm} (5.18)

We can now write the convolution sum between cumulative filter coefficients, and the equation 5.18 becomes:

\[ ICI_p(n) = \sum_{m=0}^{M-1} a_m \sum_{k=-\infty}^{\infty} H_m(n - m)H_m(-n - k) \]  \hspace{1cm} (5.19)
If the detection is being considered at the 0\textsuperscript{th} time instant, then we can write:

\[ ICI_p(0) = \sum_{m=0}^{M-1} a_m \sum_{k=-\infty}^{\infty} H_m(-m)H_m(-k) \] (5.20)

We know that the power of ICI is given by \( \sigma_{ICI_p}^2 = \mathbb{E}[ICI_p(0)ICI_p(0)^*] \). Thus, we can write

\[ \sigma_{ICI_p}^2 = \mathbb{E}\left[ \sum_{m=0}^{M-1} a_m \sum_{k=-\infty}^{\infty} H_m(-m)H_m(-k) \times \sum_{m=0}^{M-1} a_m^* \sum_{k=-\infty}^{\infty} H_m^*(-m)H_m^*(-k) \right] \] (5.21)

We can write equation 5.21 as:

\[ \sigma_{ICI_p}^2 = \mathbb{E}\left[ \sum_{m=0}^{M-1} |a_m|^2 \sum_{k=-\infty}^{\infty} |H_m(-m)|^2 |H_m(-k)|^2 \right] \] (5.22)

Thus the power of inter carrier interference is given by equation 5.22.

### 5.4 Calculation of Power of ISCI

In other MCM systems ISCI does not exist, but in WPM it does. Since the length of waveforms in time domain is pretty longer and they interfere with each other, it is an inherent ISI in the system. This inherent ISI results in ISCI, which may be considered to be negligible because of the fact that WPM waveforms have very good frequency resolution. Similar to ISI and ICI we can denote ISCI for the \( p \)\textsuperscript{th} sub-carrier as \( ISCI_p(n) \) and is given as:

\[ ISCI_p(n) = \sum_{m=0}^{M-1} a_m H_m(n - \tau - m)e^{j\theta} * H_m(-n) \] (5.23)

Similar to the two previous cases, we can again express equation 5.23 with the convolution sum as:

\[ ISCI_p(n) = \sum_{m=0}^{M-1} a_m \sum_{k=-\infty}^{\infty} H_m(n - \tau - m)e^{j\theta} H_m(-n - k) \] (5.24)

For the 0\textsuperscript{th} instant of time, the ISCI can be given as:

\[ ISCI_p(0) = \sum_{m=0}^{M-1} a_m \sum_{k=-\infty}^{\infty} H_m(-\tau - m)e^{j\theta} H_m(-k) \] (5.25)
The power of ISCI can be written as $\sigma^2_{ISCI_p} = E[ISCI_p(0)ISCI_p(0)^*]$. Thus, we can write

$$\sigma^2_{ISCI_p} = E\left[\sum_{m=0}^{M-1} \sum_{k=-\infty}^{\infty} a_m H_m(-\tau-m)e^{j\theta}H_m(-k) \times \sum_{m=0}^{M-1} a_m^* \sum_{k=-\infty}^{\infty} H_m^*(-\tau-m)e^{-j\theta}H_m^*(-k)\right]$$

(5.26)

Thus finally, the power due to the ISCI can be given as:

$$\sigma^2_{ISCI_p} = E\left[\sum_{m=0}^{M-1} |a_m|^2 \sum_{k=-\infty}^{\infty} |H_m(-\tau-m)|^2 |H_m(-k)|^2\right]$$

(5.27)

### 5.5 Summary

In this chapter, the mathematical analysis of the received signal and different components of interference is presented. The contribution of each of the component is being calculated in terms of the power.
In this chapter the implementation of Channel Equalization for Wavelet Packet Modulation, by minimizing the Peak Distortion is being presented.

6.1 WPM Transmitter and Receiver

The Figure 6.1 shows the block diagram of Wavelet Packet Modulation Transmitter. In this scheme channel encoding is not shown, but in real systems it would be included in the system. The Serial Bit Stream is uncoded bit stream, which is the input to the system as shown in the Figure 6.1. This serial bit stream is made into Parallel Bit Symbols in the Serial to Parallel block. This block forms the basis of all the multicarrier modulation(MCM) schemes, where the higher data rate $R$ is being reduced to $\frac{R}{n}$, where $n$ is the number of the parallel streams.

These Parallel Bit Symbols are then fed to Constellation Mapping block where every data symbol is being assigned an analog value having an Inphase and Quadrature phase component. For example if the Constellation Mapping considered is Quadrature Phase Shift Keying(QPSK) then every data symbol would be made of two bits and its Inphase and Quadrature phase components are shown in the Figure 6.2. The values of every bit symbol for QPSK are given in Table 6.1. These constellation mapped values act as input to the WPM Modulator, where symbols are modulated and finally a WPM signal is generated.

<table>
<thead>
<tr>
<th>Bit Symbol</th>
<th>Analog Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>1-1j</td>
</tr>
<tr>
<td>01</td>
<td>1+1j</td>
</tr>
<tr>
<td>10</td>
<td>-1+1j</td>
</tr>
<tr>
<td>11</td>
<td>-1-1j</td>
</tr>
</tbody>
</table>

Table 6.1: QPSK Constellation Values
At the WPM Receiver the reverse operation takes place, to correctly obtain the transmitted data an Equalizer block is also being included as shown in the Figure 6.3. In this system it is being assumed that the received data is perfectly synchronized, otherwise in real systems some synchronization blocks will also be included in the block diagram. The perfectly synchronized data is fed to the Equalizer with certain number of taps.

The Equalizer used here minimizes the peak distortion and thus helps in removing the ISI. In the Eq. 5.10, we have seen different components of interference in the received signal. This equalizer suppresses the ISI component only, under the assumption that the frequency resolution for WPM system is pretty high and thus ICI and ISCI will not contribute much to the distortion caused by the channel. Then the equalized signal is send to the WPM Demodulator where the signal is reconstructed and further constellation demapping happens for that signal. As the last stage the Parallel to Serial block converts the parallel bit symbols to a serial bit stream and thus we achieve the data which was transmitted.

### Figure 6.3: Block Diagram of WPM Receiver

#### 6.2 Simulation Setup

In this section we evaluate the performance of WPM system with the proposed channel equalizer (see section 3.2.1) and present results of the studies. The investigations were carried out with computer simulations. The performance metric chosen is the uncoded Bit Error Rate (BER). The WPM system is realized using a filter bank structure with 7 levels of decomposition which corresponds to 128 sub-carriers. The data modulation scheme used is Quadratic Phase Shift Keying (QPSK). The wavelet of choice is Daubechies 20 (denoted db20) which is of length 40. These simulation parameters will
be used throughout the experiments unless stated otherwise.

Two channels were considered for the experiments. The first channel is proposed by Proakis [17] whose impulse response is given by: 
\[ h_{chn1} = [0.04, -0.005, 0.07, -0.21, -0.5, 0.72, 0.36, 0, 0.21, 0.03, 0.07], \]
as shown in the Figure 6.4, we call this channel as Channel-1. The channel length is 11-taps and does not have high order frequency selectivity or nulls as can be seen in the Figure 6.5. The second channel considered is a 15 tap channel, with high order of frequency selectivity. This channel is proposed by European Telecommunications Standards Institute (ETSI) for Digital Video Broadcasting [23], we name this channel as Channel-2. The channel impulse response and channel frequency response of this channel are plotted in Figures 6.6 and 6.7 respectively.
6.3 Simulation Results

In this section, the simulation results showing the performance of the equalizer are being presented. The results presented here are for the two channels discussed in the previous section, being shown in the next two sub-sections.

6.3.1 Equalizer Performance under Channel-1

The performance of the equalizer can be verified by comparing the constellation diagram of the received signal and the equalized signal, shown in the Figure 6.8. The number of taps of equalizer for this simulation are 15 and at an received SNR of 21dB.

Fig. 6.9 shows the performance of the equalizer with different number of taps. Accept the case with an equalizer of 5 taps, the performance of the system for other
scenarios is good and comparable. In order to minimize the complexity, for the channel under consideration, an equalizer of 12 taps would be adequate.

Different wavelet families were also considered, to verify the effect of channel on each of them and how well they can be equalized. In this case a 15-Tap equalizer is being used and the results are shown in the Figure 6.10. It can be seen from the plots that there are no tangible differences in the performances of equalizer operating with different wavelets. The difference between other families is less because the channel was lenient and the distortions caused by it are not significant thus easily equalized.

Figure 6.8: Constellation Diagram of Received and Equalized Signal under Channel-1

Figure 6.9: Equalizer Performance with different taps of the Equalizer under Channel-1
only exception is the bi-orthogonal wavelet, bior3.5, which performs worst because the sub-carriers generated by this wavelet are not orthogonal. Table 6.2 gives the details of different wavelets.

In Fig. 6.11 the BER plots for the WPM system with the Daubechies family of wavelets of different lengths is plotted. The number of taps of the equalizer is fixed at 15. We can observe from the plots that there is no perceivable difference in the performances of the system. Since the channel in this case was lenient thus even though the length of the waveforms is different they do not contribute to much ISI and equalization is simple.

The performance of the equalizer for WPM was also compared with frequency domain equalization in OFDM. The number of OFDM sub-carriers considered is 128 and the length of cyclic prefix considered is to be $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{32}$ of the number of sub-carriers which correspond to length of 32, 16, 8 and 4. The equalization scheme for OFDM is frequency domain equalizer where a one-tap equalizer in the frequency domain is used, which is a simple division of data by the estimated channel frequency response. It can be seen from Figure 6.12 that when comparing WPM and OFDM systems, OFDM with frequency domain equalization outperforms WPM, this because of the simple reason

![Figure 6.10: Equalizer Performance for different Wavelet Families under Channel-1](image)

Table 6.2: Wavelet Specifications.

<table>
<thead>
<tr>
<th>Name</th>
<th>Short form</th>
<th>Orthonormal</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haar</td>
<td>Haar</td>
<td>Yes</td>
<td>2</td>
</tr>
<tr>
<td>Daubechies</td>
<td>db10</td>
<td>Yes</td>
<td>20</td>
</tr>
<tr>
<td>symlet</td>
<td>sym10</td>
<td>Yes</td>
<td>20</td>
</tr>
<tr>
<td>Coiflet</td>
<td>coif4</td>
<td>Yes</td>
<td>24</td>
</tr>
<tr>
<td>Discrete Meyer</td>
<td>dmey</td>
<td>Yes</td>
<td>102</td>
</tr>
<tr>
<td>Bi-Orthogonal</td>
<td>bior3.5</td>
<td>No</td>
<td>(5,3)</td>
</tr>
</tbody>
</table>
that in OFDM systems adding a CP helps extremely in equalization.

6.3.2 Equalizer Performance under Channel-2

Similar to the Channel-1, the performance of the equalizer was being verified for Channel-2 by comparing the constellation diagram of the received signal and the equalized signal, shown in the Figure 6.13. The number of taps of equalizer for this simulation are 200 and at an received SNR of 30dB.

Fig. 6.14 shows the performance of the equalizer with different number of taps. It
can be seen that more the number of taps of equalizer, better the performance, thus the equalizer with 200 taps performs the best. The Channel-2 was a rough channel as can be seen from its channel frequency response (see Figure 6.7) thus the number of taps of the equalizer have drastically increased from the case of Channel-1.

Different wavelet families were also considered, to verify the effect of channel on each of them and how well they can be equalized. In this case a 200-Tap equalizer was being used and the results are shown in the Figure 6.15. From the plot the effect of the length of the waveform can be clearly seen, the waveforms with longer length
like $dmey$ perform poorly in comparison to the waveforms of smaller lengths like $haar$. It is because smaller the length of the waveform, lesser the ISI and ISCI thus better equalization and performance results.

In Fig. 6.16 the BER plots for the WPM system with the Daubechies family of wavelets of different lengths is plotted. The number of taps of the equalizer is fixed at 200. The above mentioned reasoning about the length of the equalizers applies here as well, it can be seen that performance of equalizer for $haar$ is better than that of $db45$. The performance of the equalizer for WPM was also compared with frequency
domain equalization in OFDM. The number of OFDM sub-carriers considered is 128 and the length of cyclic prefix considered are the same as in the case of Channel-1. It can be seen from Fig. 6.17 that when comparing WPM and OFDM systems, in this case since the channel was rough in comparison with the Channel-1 the WPM equalization is better than that of equalization in OFDM. Thus it means that if the channel is rough then having a CP also does not help then better equalization methods are necessary.

We can also analyze the eye diagram of the data at different stages of the transceiver chain. In Figure 6.18 the QPSK modulated data at the transmitter is shown. Since there is no ISI the eye is completely open. The information bits are modulated by the WPM and then transmitted to the radio channel to be received by the WPM receiver. Figure 6.19 shows the eye diagram of the data received at the receiver. The received data has inherent overlap due to the WPM modulation as well as ISI induced by the channel; therefore the eye is closed. The received data is then equalized and demodulated by the WPM demodulator. In Figure 6.20 the eye pattern at the output of the WPM demodulator is shown. It is observable from the eye diagram that the equalizer has mitigated the ISI considerably.

6.4 Summary

In this chapter, a channel equalizer for WPM, based on minimization of peak distortion is presented and its results are discussed. The simulations were performed for two channel conditions and under different parameters such as equalizer taps, wavelet families and comparison with OFDM.
Figure 6.18: Eye diagram of QPSK modulated data at the transmitter

Figure 6.19: Eye diagram of the Received Data
Figure 6.20: Eye diagram of the QPSK Symbols after equalization at the receiver
7.1 Summary of Work

This thesis work focused on the design and implementation of the channel equalizer for Wavelet Packet Modulation. Wavelet Packet Modulation was studied and a simulation model for it was implemented. For the design of a WPM receiver, channel equalization is an important aspect to be dealt with. The channel distorts the transmitted data and introduces ISI, ICI and ISCI in the signal, the main purpose of the equalizer is to remove these distortions and to obtain the data in its correct form.

A mathematical analysis of the WPM received signal was performed, which decomposes the signal into different interference components due to the ISI, ICI and ISCI. Furthermore the interference due to each component was studied in terms of power. A time-domain equalizer based on minimization of peak distortion was designed and implemented. The simulations for this equalizer were performed for two wireless channels and under different parameters like equalizer taps, different wavelets etc. The performance metric used for the equalizer was uncoded BER, and accordingly the results are being presented and analyzed.

7.2 Conclusions of Study

The major conclusions of the study can be summarized as follows:

- The channel equalization for WPM is different from its counterpart of OFDM. In OFDM we have a guard interval which helps in channel equalization, unlike the case of WPM but it has been shown that the performance of WPM is comparable with that of OFDM. Thus, from the perspective of channel equalization, both the MCMs performed similarly.

- The WPM system has an inherent ISI, because of the overlap of the waveforms in the time domain, this is the reason it is called as overlapped transform. Since the waveforms are longer than the waveforms in OFDM, thus WPM has better frequency resolution than OFDM. This would mean that because of this property, there would be less ICI, thus less loss in orthogonality, therefore the channel equalization becomes simple.

- The length of the waveform also determines the amount of ISI and ISCI. By looking at the results we can conclude that longer waveforms are difficult to equalize
as they introduce large ISI and ISCI in comparison with shorter waveforms. For example haar wavelet will perform better than db20 in severe channel conditions. Thus, we need to have an appropriate waveform so that we have less ISI and also we can utilize high frequency resolution.

- The existing waveforms are suitable for channel equalization, but their usage depends upon the channel conditions and applications. For instance, if the applications are related with frequency localization then we can have longer waveforms and if the application is related with data transmission then we can have shorter waveforms.

- The calculation of power of interferences such as ISI, ICI and ISCI helps to understand the contributions of each component in the signal.

- The channel equalizer based on minimization of peak distortion, performs well under difficult channel conditions like the ETSI channel, which is highly frequency selective. The performance of this equalizer is also comparable to the equalization of OFDM.

### 7.3 Future Work

A few of the possible areas of further studies are:

- Design of a more robust equalizer such as MMSE which does not has issues of Noise Enhancement.

- Design of a equalizer that counteracts for ISI, ICI and ISCI individually.

- Design of new waveform which has an optimum length to suit channel equalization

- Tree pruning to avoid harmful channel.
Bibliography


[23] *DVB Standards and Specifications*, European Telecommunications Standards Institute, 10 August 2010.