Anticipatory Demand
Responsive Transport
A Tata Steel IJmuiden Case Study

M.J. van Engelen
Anticipatory Demand Responsive Transport
A Tata Steel IJmuiden Case Study

by

M.J. van Engelen

in partial fulfillment of the requirements for the degree of

Master of Science
in Transport, Infrastructure & Logistics

at the Delft University of Technology,
to be defended publicly on the 10th of November, 2016.

An electronic version of this thesis is available at http://repository.tudelft.nl/.
Abstract

Transport operators worldwide need to focus on how they can implement automated vehicles in their schedules in the coming years. Automated vehicles are particularly suited for use in demand-responsive transport, which is often modeled as a Dial-a-Ride problem. This report investigates how public transport demand forecasts can be used to improve the routing of vehicles in the Dial-a-Ride problem. In particular, two improvements to solutions of the Dial-a-Ride problem are investigated. The first, empty vehicle rerouting, deals with the relocation of vehicles that are currently idle, which has been studied before. The second, demand rerouting, is introduced in this thesis and uses demand forecasts to deal with the issue of which vehicles serve which passenger.

To analyze the performance of these additions to the Dial-a-Ride problem, the rerouting measures are modeled for various demand distribution patterns on the road network of the Tata Steel IJmuiden area in the Netherlands. A visualization tool is created to analyze the key performance indicators of the solution algorithm, with an insertion algorithm used to assign passengers to vehicles. Throughout this report it is assumed that deterministic distance and path matrices are available for the network. Stochastic travel times are not taken into account, but can be incorporated in the model.

The inclusion of empty vehicle rerouting reduces passenger rejections by up to 98% and can reduce passenger travel and waiting times by up to 5 and 16% respectively. This requires an increase of the vehicle distance driven per passenger by up to 25%. Demand rerouting reduces travel and waiting times by up to 6 and 30% respectively, with only a very minor increase in vehicle distance driven.

The main conclusion of this thesis is that the two rerouting measures can be combined and that this combines the strengths of both types of rerouting. Passenger rejections are all but eliminated, passenger travel times can be reduced by 10% and waiting times can be reduced by almost 50%. This comes at the cost of only a 25% increase of vehicle distance driven per passenger. A sensitivity analysis of the input parameters of the model shows that the advantages of the rerouting measures are effective in all cases. Demand forecasts can thus be a very strong tool in optimizing the solution to Dial-a-Ride problems.
## Contents

1 Problem Definition ................................................. 1
   Introduction ................................................. 3

1 Problem Statement and Thesis Setup ............................ 5
   1.1 Research Question and Scope ............................. 5
      1.1.1 Thesis Structure .................................... 6

II Theoretical Background ........................................ 7

2 The Dial-a-Ride Problem ......................................... 9
   2.1 Introduction ............................................. 9
   2.2 Variants of the Dial-a-Ride Problem ...................... 9
   2.3 Aspects of the Dial-a-Ride Problem ...................... 11
      2.3.1 Degree of Dynamism ................................ 11
      2.3.2 Stochastic Dynamic Dial-a-Ride Problems .......... 11
      2.3.3 Ride Sharing ........................................ 11
      2.3.4 Fleet Sizing ......................................... 12
   2.4 Mathematical Model for the Dial-a-Ride Problem ......... 12
      2.4.1 Model Extensions and Adaptations .................. 14
      2.4.2 Objective Functions ................................ 15
   2.5 Approximate Solution Methods ............................. 15
      2.5.1 Tabu Search .......................................... 15
      2.5.2 Genetic Algorithms ................................... 15
      2.5.3 Insertion Heuristics ................................ 16
      2.5.4 Variable Neighborhood Search ...................... 16

3 Automated Vehicles .............................................. 17

III Modelling Methodology ......................................... 19

4 Model Definition .................................................. 21
   4.1 Model Components ........................................ 21
   4.2 Graph Module ............................................. 22
   4.3 Trip Module ................................................ 22
   4.4 Vehicle Module ............................................ 22
      4.4.1 Vehicle parameters .................................. 22
      4.4.2 Empty Vehicle Rerouting ............................ 23
      4.4.3 En-Route Demand Rerouting ........................ 24
   4.5 Main Module ................................................ 25
      4.5.1 Insertion Heuristic .................................. 25
      4.5.2 Vehicle Selection .................................... 25
      4.5.3 Finding Feasible Insertions ......................... 26
   4.6 Model Assumptions ........................................ 27
   4.7 Pseudocode ................................................ 27

5 Simulation Setup ................................................ 29
   5.1 The Case Study Graph ..................................... 29
   5.2 Case Study Demand ........................................ 31
      5.2.1 Demand Distribution Patterns ....................... 31
   5.3 Fleet Specification ....................................... 32
      5.3.1 Empty Vehicle Rerouting ............................ 32
Problem Definition
Introduction

Worldwide, transport operators have access to an increasing amount of data about the travel habits of their customers. This data can be used to predict the demand for public transport in the future, in the form of probability distributions. One of the ways that this demand forecast could be used is for providing better demand-responsive transport services.

In theory, demand-responsive services should be able to handle a given demand with a lower amount of resources than line-based public transport, by offering a capacity that is closer to the amount of capacity required [37, 39]. Furthermore, demand-responsive services have the added advantage that they can more easily serve a larger number of stops than traditional line-based public transport. However, demand-responsive systems that are currently in operation are often more than three times as expensive as comparable traditional systems [2]. This is mainly because of the staff costs; demand-responsive services often have a larger number of vehicles, with a smaller capacity, than conventional public transport services. Staff costs can run up to 70% of total public transport costs [13] and public transport operators worldwide are often not profitable and depend heavily on subsidies [27].

One of the options transport operators are considering to cut costs is to implement autonomous vehicles, in addition to or as a replacement of the current manually driven vehicles. Although these vehicles are not yet available for this use, transport operators are already considering how these vehicles could be used to provide efficient and effective public transport in the future. Autonomous vehicles are especially well-suited as an alternative to traditional vehicles for demand-responsive public transport, since costs in demand-responsive public transport are very high mostly due to the large amount of personnel required in these systems.

The use of autonomous vehicles also allows for much more convenient rerouting of vehicles. With manually driven vehicles, it is difficult to adapt the current route of a vehicle too often as this causes inconvenience to the driver. Furthermore, autonomous vehicles can easily be driven to another location while empty, without much extra cost, while in traditional line-based systems, relocating an empty vehicle still requires a driver for that trip, without bringing in any revenue from passengers.

The problem to route the vehicles in demand-responsive transport as efficiently as possible is often modeled as a Dial-a-Ride Problem, which is an extension to the more intensively studied Vehicle Routing Problem. In this thesis, we explore the possibility to adapt the Dial-a-Ride Problem to allow the use of demand forecasts to include an additional type of rerouting, not used in the traditional algorithms used to solve the Dial-a-Ride Problem.
1

Problem Statement and Thesis Setup

In this chapter the problem as stated in the introduction is explained in more detail, and the scope of the research is defined. The main research question is then identified and the structure of this thesis is presented.

1.1. Research Question and Scope

As explained in the introduction, the main goal of this thesis is to examine the possibilities to incorporate demand forecasts in the routing and timing (from now on simply called routing) of vehicles. The routing of vehicles is modeled as a Dial-a-Ride Problem (DARP) with stochastic customers (see Chapter 2). In particular, it is assumed that a demand distribution for passengers is known for each origin-destination pair and that the number of vehicles is fixed based on the demand pattern studied. The main research question is:

"How can demand forecasts be used to improve the routing of vehicles in a Dial-a-Ride Problem?"

This improvement can come in the form of several indicators: total driving time, total driving distance, passenger waiting times, passenger travel times, number of rejected requests or costs.

To answer the research question, two different types of rerouting behavior will be considered:

**Rerouting Empty Vehicles** First, the rerouting of empty vehicles is considered. This could for example be used when the transport operator knows that trains arrive at a station with a certain pattern. This information can then be used to ensure that enough vehicles are present at the station when the train arrives. This process has been studied before, see for example [19, 33].

**En-Route Demand Rerouting of Non-Empty Vehicles** The second type of rerouting is the rerouting of vehicles while they are en-route, carrying passengers. Specifically, every time a new passenger request arrives, a choice needs to be made which vehicle will pick up this passenger. This choice could be influenced by using available demand forecasts. A vehicle with only a single empty seat might not be chosen to pick up a passenger if it is expected that more passengers will arrive shortly at the location of the first passenger. Instead, a vehicle with multiple free seats could be chosen to service the passenger, although in traditional algorithms this choice would not be made.

Tensen [53] presents a model where forecasts for dynamic requests are taken into account when creating the planning for the static requests, but she does not take these forecasts into account when planning dynamic requests.

To analyze these types of rerouting, a model first defined by Jaw et al. [30] will be used.
In total, 24 different scenarios will be analyzed: four variations of the rerouting possibilities of the model will be tested: No rerouting, only empty vehicle rerouting, only en-route demand rerouting and both types of rerouting. All of these possibilities will be modeled for two demand patterns: A demand pattern where the demand at one location dominates the demand at other locations and a demand pattern where the demand is spread out evenly across most stops. Finally for each of these demand patterns, three demand levels will be tested: a low, a medium and a high level of demand. See Figure 1.1 for an overview. These scenarios will be modeled on a map of the Tata Steel IJmuiden area in the Netherlands on maps from TomTom.

![Figure 1.1: Overview of all the model types that will be analyzed; each combination of routing type, demand type and demand level will be modeled.](image)

As mentioned in the introduction, automated vehicles would be ideally suited for this type of problem. However, the solution method used can just as easily be applied to a solution with manually driven vehicles, albeit with higher costs for the transport operator. In this thesis it is assumed that automated vehicles are available for the implementation.

### 1.1.1. Thesis Structure

Part II of this thesis introduces the DARP and some variations on it in Chapter 2 and gives a short introduction on automated vehicles in Chapter 3. Part III explains the algorithms and model that will be used in this thesis in detail in Chapter 4 and a full explanation of the model input and specification for the simulations in Chapter 5. Part IV presents the results of the simulation in Chapter 6 and concludes with a summary and discussion of the obtained results in Chapter 7.
Theoretical Background
The Dial-a-Ride Problem

2.1. Introduction

The Dial-a-Ride problem was introduced by Wilson and Colvin [58] and has received a large amount of attention in the previous decades. For an overview of research on the Dial-a-Ride problem, see Cordeau and Laporte [12]. Simply stated, the Dial-a-Ride problem consists of designing vehicle routes and schedules for passengers who specify pick-up and drop-off requests between origins and destinations [12]. Transport is supplied by a certain set of vehicles (possibly with varying sizes) that are based at some set of depots. Passengers will have a requested pick-up time, a requested drop-off time, or both of these. These times are then often converted into allowable time windows for pick-off or drop-off of each passenger. The goal of the DARP is to solve the Pick-up and Delivery Problem (PDP) such that the time windows are not violated, with a minimal cost for operating the vehicles. This cost can be dependent not just on the financial cost of operating the vehicles, but also for example on the time costs, the passenger convenience or the waiting time of passengers.

The first research into the DARP considered cases with only a single vehicle and up to fifty requests (see Desrosiers et al. [16], Psaraftis [42]), but quickly evolved to multiple-vehicle cases with several hundreds to thousands of requests in specific cases [18]. The most common examples where a DARP is solved are the door-to-door transport of elderly or disabled people (see, for example Madsen et al. [36], Toth and Vigo [54, 55] or Cordeau and Laporte [12] and Pillac et al. [41] for a recent overview) and taxi services [41, 46], but another problem that is often tackled as a DARP are ambulance services (see Gendreau et al. [21] and Schilde et al. [49]).

In this chapter, various types of DARPs are explained in Section 2.2 and a mathematical model for the DARP is given in Section 2.4. Possible adaptations to this standard DARP model are presented in Section 2.4.1 and Section 2.5 presents several solution methods for the DARP.

2.2. Variants of the Dial-a-Ride Problem

There are two main variants of the DARP, called the static and the dynamic variant [12].

**Static** In the static variant, all passenger requests are known ahead of time. This could for example be the case in elderly transport, where it is required that passengers book their transport a day in advance. An example of a static problem is shown in Figure 2.1. Figure 2.1a shows the depot where all vehicles start and end in gray, the pick-up locations in yellow and the drop-off locations in orange. Passenger $i$ is picked up at node $P_i$ and dropped off at point $D_i$. All these locations are known in advance and assuming that there are at least two vehicles available with enough capacity, and pick-up and drop-off times are not violated, one possible solution to the problem is shown in Figure 2.1b. Here the red arrows represent one vehicle and the blue arrows another.

**Dynamic** In the dynamic variant, some or all of the requests are revealed during the execution of the plan, requiring online decision making. This happens for example in ambulance services or
The Dial-a-Ride Problem

taxi services. Figure 2.2 shows an example of a dynamic problem. Assume that the pick-up and drop-off locations of four passenger are known, as in Figure 2.1 and the vehicles have started their routes. Figure 2.2a shows the current route segment of the vehicles as a dashed line. A new pick-up and drop-off request \((P_3\) and \(D_5)\) then comes in and needs to be incorporated in the route of one of the vehicles. A possible solution is shown in Figure 2.2b, where the black dotted lines show the originally planned route of the vehicle and the thicker lines show the new route segments.

The Dynamic Vehicle Routing Problem (DVRP) and Dynamic Dial-a-Ride Problem (DDARP) bear much closer relation to many real-life situations and as such they are currently investigated more extensively than the static variants. The research presented in this thesis is also all on the dynamic variant of the Dial-a-Ride problem.

Figure 2.1: Illustration of the static PDP (or DARP without time windows). Figure (a) shows pick-up (in yellow) and drop-off (in orange) locations for four passengers. Figure (b) shows possible routes for two vehicles in red and blue.

Figure 2.2: Illustration of the dynamic PDP (or DARP without time windows). Figure (a) shows vehicles en route when a new pick-up and drop-off request comes in. Figure (b) shows the adapted routes of the vehicles.
2.3. Aspects of the Dial-a-Ride Problem

2.3.1. Degree of Dynamism

So far, the static variant of the DARP has been distinguished from the dynamic variant of the DARP. However, the difference is only that at least some requests are not known ahead of time. There can still be a large variation in the number of requests that are dynamic. This is represented by the Degree of Dynamism, which is the percentage of dynamic requests out of the total number of requests. For the problem that will be tackled in this thesis the degree of dynamism is 100%, as all requests will come in during the day and passengers can not book ahead of time. Most current research focuses on a smaller degree of dynamism (often between 40% and 60%), but 100% dynamic systems have been studied before (see Ritzinger et al. [45] for an overview).

Although the degree of dynamism is 100%, information about the future is often still available in terms of probability distributions, which can be incorporated into the model. Various different ways of adding stochasticity to the DARP are presented in the following section.

2.3.2. Stochastic Dynamic Dial-a-Ride Problems

In stochastic DDARPs, some information about upcoming events is not known deterministically, but only as a stochastic variable with a known probability distribution. When such an event occurs, the routes of vehicles are updated according to the new information. There are three main overarching types of stochasticity that have been extensively studied in DDARPs, for an overview see Ritzinger et al. [45].

**Stochastic Demand** In the stochastic demand case, the location of customers is known in advance, but the actual number of customers is not known ahead of time. It is therefore possible that the vehicle that is chosen to serve this request actually does not have enough capacity to serve this request and that another vehicle needs to be sent. This is for example the case if a group of people reserve a taxi and a four-person taxi is sent, but the group actually consists of 6 people. Campbell and Thomas [8] give an overview of solution methods for this problem.

**Stochastic Travel Times** One well-studied adaptation of the DARP is the DARP with stochastic travel times. In reality, travel times can vary with time of day, traffic or the occurrence of accidents. This is the most commonly studied stochastic variant of the DARP. Research on this variant of the DARP started three decades ago with Hall [24] on stochastic travel times in the VRP. Taş et al. [52] give a recent overview of research in this field.

**Stochastic Customers** When stochastic customers are used in DARPs, not all customer requests (both time, origin and destination) are known in advance, but a probability function for their expected arrival rate is known. This is the variant that we will analyse in this thesis. Pavone et al. [40] give a recent overview of papers published in this field.

Research on DARPs with a combination of the above stochastic aspects has been performed as well, although much less extensively than on DARPs with a single stochastic aspect. The most common combination that is studied is that of the stochasticity of travel times and demand. Schilde [47] shows how some standard algorithms for solving the DARP can be adapted to include stochasticity.

Very recently, a fourth type of stochasticity has been introduced in VRPs and DARPs, namely an uncertainty in the location where the pick-up or drop-off has to take place. This issue arises when for example a package has to be delivered in person, but the recipient moves around during the day between an office and his house [44].

Incorporating stochasticity in the DARP increases the robustness of the results and can lead to large improvements compared to the deterministic counterparts, see Figure 2.3 [45]. This figure shows many papers published in the last years on dynamic and stochastic VRPs. For each paper, a dot is placed in the corresponding column, based on the type of stochasticity studied and the publication year. On the y-axis the improvement this paper claims over a deterministic counterpart is shown.

2.3.3. Ride Sharing

In many cases, ride sharing is allowed in the solution of DDARPs. Agatz et al. [1] give an overview of papers on dynamic ride-sharing. Ride-sharing in solutions to the DARP is also used by for example...
12

2. The Dial-a-Ride Problem

Lees-Miller et al. [34] and Santos and Xavier [46] in personal rapid transit and taxis respectively. In the DARP, it is commonly allowed for people to share a vehicle as long as capacity constraints are not violated. For the DDARP, in many cases passengers are only allowed to share a vehicle if they share either the same origin, the same destination or both, to save computation time. Furthermore, when a dynamic request comes in, it is mostly not possible for a vehicle on a trip to abort that trip and travel to the request immediately; it will always first have to finish its current trip (i.e. to the next drop-off or pick-up point). The main innovation in this thesis is the adaptation of the solution of the DDARP such that it does allow this type of trip abortion and to include demand forecasts in the decision making process for this type of rerouting.

2.3.4. Fleet Sizing

An important aspect of the DARP from the transport operator’s point of view is the size of the vehicle fleet. Most papers consider either an infinite number (or at least a sufficiently large number) of vehicles or do an analysis on the number of vehicles required for a given expected demand [43]. Many methods have been proposed for determining the required fleet size for a desired level of service, see Winter [59] for an overview. In general, most models consider a fixed fleet size and then see what level of service is possible with this fleet, but there are some papers in which optimizing the fleet size is the main objective [14, 28, 29, 52].

2.4. Mathematical Model for the Dial-a-Ride Problem

In this section a basic mathematical model is presented for the Dial-a-Ride problem, as outlined in Cordeau [11]. Furthermore, several possible adaptations to this model to incorporate various extra constraints are explored in Section 2.4.1.

Consider a complete (possibly directed) graph $G = (V, A)$, where $V$ is a set of vertices representing pick-up and drop-off locations, combined with a depot, and where $A$ is a set of arcs representing roads between these locations. If there are $n$ requests to be served, denote the set of pick-up locations as $P = \{1, \ldots, n\}$, the set of drop-off locations as $D = \{n + 1, \ldots, 2n\}$ and the origin and destination depots as 0 and $2n + 1$ respectively. We then write $V = P \cup D \cup \{0, 2n + 1\}$. With each request $r$, an origin node $r \in P$ and a destination node $n + r \in D$ are associated.

Let $K$ be a set of vehicles, where each vehicle $k \in K$ has a certain capacity $Q^k$ and a possible maximum
trip length $R_k$. This trip length can be necessary due to driver contracts or vehicle battery or fuel capacity. With each request, and thus with each node $i \in \mathcal{V}$, a certain load $q_i$ is associated (representing the number of passengers), such that $q_0 = q_{2n+1} = 0$ and $q_i = -q_{i+n}$ $\forall i \in \mathcal{P}$. Furthermore, it is possible to associate a non-negative service duration $s_i$ with each node $i \in \mathcal{V}$, representing for example boarding and disembarking times. With each node, a time window $[e_i, \ell_i]$ is associated, representing the earliest and latest time at which service may begin at this node.

With each arc $(i, j) \in \mathcal{A}$, we associate a travel cost $c_{ij}$ and a travel time $t_{ij}$. This travel cost can be based on various measures, as explained in Section 2.4.2.

A binary variable $x_{ij}$ is defined for each arc $(i, j) \in \mathcal{A}$ and each vehicle $k \in \mathcal{K}$ as $x_{ij} = 1$ if and only if vehicle $k$ travels from node $i$ to node $j$. Furthermore, let $B_k$ be the time when vehicle $k$ begins service at node $i$ and let $Q_k$ be the load of vehicle $k$ after serving a request at node $i$. We can then formulate the mathematical model for the DARP:

$$\begin{align*}
\min & \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} c_{ij} x_{ij}, \\
\text{subject to:} & \\
\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{V}} x_{ij} &= 1 \quad \forall i \in \mathcal{P}, \\
\sum_{j \in \mathcal{V}} x_{ij} - \sum_{j \in \mathcal{V}} x_{ni,j} &= 0 \quad \forall i \in \mathcal{P}, k \in \mathcal{K}, \\
\sum_{j \in \mathcal{V} \setminus \{2n+1\}} x_{ij} &= 1 \quad \forall k \in \mathcal{K}, \\
\sum_{i \in \mathcal{P} \cup \mathcal{D}} x_{2n+1} &= 1 \quad \forall k \in \mathcal{K}, \\
\sum_{j \in \mathcal{V} \setminus \{0\}} x_{ij} - \sum_{j \in \mathcal{V}} x_{ij} &= 0 \quad \forall i \in \mathcal{P} \cup \mathcal{D}, k \in \mathcal{K}, \\
B_k^i &\geq (B_k^i + s_i + t_{ij})x_{ij} \quad \forall i, j \in \mathcal{V}, k \in \mathcal{K}, \\
Q_k^j &\geq (Q_k^j + q_j)x_{ij} \quad \forall i, j \in \mathcal{V}, k \in \mathcal{K}, \\
B_k^i - B_k^i &\leq R_k \quad \forall k \in \mathcal{K}, \\
e_i &\leq B_k^i \leq \ell_i \quad \forall i \in \mathcal{V}, k \in \mathcal{K}, \\
\max(0, q_i) &\leq Q_k \leq \min(Q_k, Q_k + q_i) \quad \forall i \in \mathcal{V}, k \in \mathcal{K}, \\
x_{ij} &\in \{0, 1\} \quad \forall i, j \in \mathcal{V}, k \in \mathcal{K}.
\end{align*}$$

Here the objective function (2.1) minimizes the cost; how that cost function can be defined is described in Section 2.4.2. Constraint (2.2) ensures that all requests are served, while (2.3) ensures that if a vehicle serves request $i$ (as a pick-up) it will also serve request $i+n$ (as a drop-off of that passenger). Constraints (2.4) and (2.5) ensure that each vehicle starts and ends at a depot; (2.6) ensures that a vehicle that arrives at a request node also leaves that request node. Constraints (2.7) and (2.8) force consistency of the time and load variables, while (2.9) ensures that vehicles do not exceed their maximum ride time. Finally, Constraint (2.10) ensures that each passenger is picked up and dropped off within the allowed time windows and (2.11) ensures that the maximum capacity of the vehicles is not exceeded.

This formulation of the DARP is non-linear, but it can be linearised by rewriting Constraints (2.7) and (2.8) utilizing constants $M_{ij}$ and $W_{ij}$, which obey the following constraints:

$$\begin{align*}
M_{ij} &\geq \max(0, \ell_i + s_i + t_{ij} - e_j), \\
W_{ij} &\geq \min(Q_k, Q_k + q_i),
\end{align*}$$
and then rewriting Constraints (2.7) and (2.8) as

\[ \begin{align*}
B^k_i &\geq B^k_i + s_i + t_{ij} - M_{ij}(1 - x^k_{ij}) & \forall i, j \in \mathcal{V}, k \in \mathcal{K}, \\
Q^k_i &\geq Q^k_i + q_j - W^k_i(1 - x^k_{ij}) & \forall i, j \in \mathcal{V}, k \in \mathcal{K}.
\end{align*} \]  

(2.15) \hspace{1cm} \text{(2.16)}

### 2.4.1. Model Extensions and Adaptations

The model presented in Section 2.4 is one of the most general models of the DARP, but adaptations can be made to add extra constraints to the model or to remove certain constraints, depending on the problem being solved. The most common adaptations are introducing maximum ride times, introducing multiple depots or allowing different types of seats with different capacities (which can for example be necessary when introducing user group differentiation). Furthermore, many of the Constraints (2.2)-(2.12) are often made soft, also depending on the objective function (see Section 2.4.2). The constraints that are made soft are taken into account when evaluating the solution quality, but are not enforced as the other (hard) constraints are.

#### Maximum Ride Time

To introduce a maximum ride time \( L_i \) for each trip request \( i \), based on the travel time \( t_{i(j+1)} \), we can introduce a maximum detour factor \( \alpha \) and constant \( \beta \) and set \( L_i = \alpha t_{i(j+1)} + \beta \forall i \in \mathcal{P} \). If we let \( L^k_i = B^k_i - (B^k_i + s_i) \) denote the ride time of passenger \( i \) on vehicle \( k \), we introduce the following constraint:

\[ t_{i(j+1)} \leq L^k_i \leq L_i \quad \forall i \in \mathcal{P}, k \in \mathcal{K}. \]  

(2.17)

#### Multiple Depots

It is possible to introduce multiple depots, in which case Constraints (2.4) and (2.5) need to be adapted. Assume that there is a set of depots \( \Delta \) and that each vehicle \( k \) starts at a depot \( \delta^+_k \in \Delta \) and ends at a depot \( \delta^-_k \in \Delta \), then these constraints are changed to:

\[ \begin{align*}
\sum_{j \in \mathcal{P} \cup \delta^+_k} x_{ijk} & = 1 & \forall k \in \mathcal{K}, \\
\sum_{i \in \mathcal{D} \cup \delta^-_k} x_{ijk} & = 1 & \forall k \in \mathcal{K}.
\end{align*} \]  

(2.18) \hspace{1cm} (2.19)

#### Multiple Seating Types

Another option that is often presented is to have multiple types of seats in a vehicle. For example, in elderly transport, there might be several normal seats but only one or two wheelchair spots in a bus. Assume that at least some vehicles \( k \) have several seat types \( \Sigma \). We then redefine \( Q^k \) as \( (Q^{k,\sigma} \geq 0 \forall \sigma \in \Sigma) \), the maximum capacity of vehicle \( k \) for each seat type. Furthermore, we redefine \( q_i \) as \( (q^\sigma_i \geq 0 \forall \sigma \in \Sigma) \), the requested load per seat type at each node \( i \) and we redefine \( Q^k_i \) as \( (Q^{k,\sigma}_i \geq 0 \forall \sigma \in \Sigma) \), the load of vehicle \( k \) after serving node \( i \) for each seat type. We then rewrite Constraints (2.8) and (2.11) to:

\[ \begin{align*}
Q^{k,\sigma}_i &\geq (Q^{k,\sigma}_i + q^\sigma_i)x_{ijk} & \forall i, j \in N, k \in K, \sigma \in \Sigma, \\
\max(0, q^\sigma_i) &\leq Q^{k,\sigma}_i \leq \min(Q^{k,\sigma}_i, Q^k_i + q^\sigma_i) & \forall i \in N, k \in K, \sigma \in \Sigma.
\end{align*} \]  

(2.20) \hspace{1cm} (2.21)

Note that this assumes that there is no possibility to exchange one seat type for another, which in many real-life cases is possible. For example it might be possible to stow away two normal seats to make room for a wheelchair.
2.5. Approximate Solution Methods

Many approximate solution methods have been proposed for both the static and the dynamic variants of the VRP and the DARP. These solution methods try to find a solution to the problem as close to the optimum as possible, based on some restrictions. The disadvantage of the methods below is that one can never be sure that the optimal solution will actually be reached, but the advantage is that they can mostly find a solution in a reasonable amount of time compared to solution methods finding the optimum. A recent overview is found in Psaraftis et al. [43]. In this section, the main solution methods are presented.

2.5.1. Tabu Search

Tabu search has traditionally mostly been used for the static DARP, but has lately also been applied to partly dynamic problems. Berbeglia et al. [7] generate a list of possible solutions for the currently known requests and then, when a dynamic request is received, a tabu search is used to filter out the solutions that are not valid anymore, and a random feasible solution from the list is picked. This process is continued as more dynamic requests come in.

Kergosien et al. [31] apply a tabu search algorithm for a DDARP for the transportation of patients in a hospital. Here, each request requires a specific vehicle (a hospital bed), which only has a capacity of a single person.

2.5.2. Genetic Algorithms

Genetic algorithms have been used to model the DDARP with stochastic travel times by Taniguchi and Shimamoto [51]. More recently, genetic algorithms have been used to solve the DDARP with fuzzy time windows and travel times in [22, 23]. Vuong [56] utilized genetic algorithms to cluster customers for transport by a single taxi and subsequently to compute routes for each of these taxis. The difficulty
with genetic algorithms is that a classification of different possible solutions needs to be made such that the inheritance rules can somehow be applied. This classification is not as straightforward as it is for some other problems.

2.5.3. Insertion Heuristics

Insertion heuristics are often used alongside other solution methods. They are often used in case of dynamic problems, where solutions quickly need to be updated. For example, Li et al. [35] examined a case where a vehicle breaks down during a scheduled trip where other vehicles quickly need to be rerouted to serve the trip that the broken down vehicle was making and future trips that the broken down vehicle would have made. Beaudry et al. [4] analyzed patient assignment in a hospital, where it can also be necessary to find a new solution very quickly. The main benefit of using insertion heuristics is the speed with which they can be performed.

2.5.4. Variable Neighborhood Search

Variable Neighborhood Search (VNS) for the VRP and DARP was first introduced by Hansen and Mladenović [25]. With the rise of more dynamic DARPs, VNS has gained in popularity and many recent papers use some variant of the VNS to tackle the DDARP. For more applications of VNS see for example Schilde [47], Schilde et al. [48, 49], Xu et al. [60] and Khouadjia et al. [32]. Numerical results of VNS show that it can bring a large benefit to solution quality in the dynamic case [43].
Automated Vehicles

In this chapter, a short overview of the current state of automated vehicles is presented. There are two main standards for the classification of automated vehicles: those by the National Highway Traffic Safety Administration [38] and those by the Society of Automotive Engineers [50]. Here we present the classification from the Society of Automotive Engineers (SAE) as presented in their J2016 informational report, but the classification by the National Highway Traffic Safety Administration is similar.

The SAE divides vehicles into six classes, ranging from 'No Automation' to 'Full Automation'. In the first three levels the human driver monitors the driving environment, while in the latter three levels the automated driving system monitors the driving environment. See Table 3.1 for an overview of which tasks are performed by the driver and which are performed by the automated driving system in each level.

Current automated road vehicles that are in operation mostly fall into categories 3 and 4. There are systems that could be said to fall into category 5, but they often use a separate infrastructure. One current pilot project that falls in category 5 are the WEpods [57]. Many companies worldwide are currently testing fully automated (level 5) vehicles, see CB insights [9] for a recent overview. The system proposed in this thesis is not dependent on what type of automated vehicle is used (it is even possible to use non-automated vehicles), but the convenience and cost gains are highest if the vehicle falls in category 4 or 5. In the rest of this thesis it is assumed that all vehicles are level 5 automated vehicles.

Although the algorithms presented in Chapter 2 do not require that the vehicles are fully automated, using automated vehicles brings several benefits:

**Rerouting** Due to the scale of the number of requests coming in, vehicles will often have to suddenly reroute to a new location. This is very annoying for drivers, but it is also possible that a driver will miss a rerouting task, thus requiring a new solution to be found to the DARP. With automated vehicles, continuous rerouting is not an issue as long as the vehicle remains connected to the main routing system.

**Rostering** When drivers are employed for the operation of the vehicles, there are often extra time windows requirements, due to labor regulations. These extra constraints are unnecessary when automated vehicles are used.

**Depot Constraint** In many cases in the DARP, it is required that vehicles start and end at some depot, as represented by Constraints (2.4) and (2.5), since the drivers of the vehicles will access their vehicles at the depot. With automated vehicles, it is much easier to use multiple, smaller depots. It is even possible to leave vehicles at stops during nights or weekends for example, although this might be undesirable in some areas.

**System Capacity** As automated vehicles can drive closer together and, with proper routing, should be able to avoid conflicts, the total capacity of passengers that can be transported can be increased. However, because this capacity is not likely to be reached in this case study, this will not be analysed further.
Cost In general, it is thought that using automated vehicles can reduce the cost of operating public transport, as driver salaries are one of the main costs [13]. Furthermore, these vehicles are often more fuel-efficient or are electrical, bringing a further reduction in cost compared to the current vehicles. This could potentially lead to lower ticket prices and thus an increase in public transport use. However, an initial investment to purchase the vehicles has to be made.

Table 3.1: Classification of vehicles by their level of automation [50].
III

Modelling Methodology
In this chapter the structure of the model that is used for the simulations will be explained. Chapter 5 shows the way in which this model will be used for the simulation. Section 4.1 begins with a summary of the components of the model and Sections 4.2 – 4.5 explain these components in more detail. In particular, Section 4.5 explains the insertion heuristic that is used for choosing which vehicle serves which request. Section 4.6 lists some of the assumptions used in this model and Section 4.7 presents pseudocode for this model.

4.1. Model Components

The model consists of three main components, which each consist of several subcomponents (see Figure 4.1). These components are connected through a main module, which will run the simulations and contains all the fixed data (see Section 4.6). These components are explained in more detail in Sections 4.2 – 4.5.

Graph The map that represents the location where the vehicles will operate is described by a graph in the model. This graph consists of

- **Nodes** Nodes in the network have several attributes:
  - **ID** Each node has an ID.
  - **Coordinates** Each node has coordinates, giving its location.
  - **Type** Each node has a type associated with it: intersection, stop or depot.
  - **Arcs** Each node has a list of arcs that are incident to it.

- **Arches** Each road in the network is represented by an arc with several attributes:
  - **ID** An ID defining that arc.
  - **Start and End Node** The nodes that form the endpoints of the arc.
  - **Length** The length of the arc, both in distance and time.
  - **Polyline** Each arc consists of a polyline, which is a sequence of coordinates \([(x_0, y_0), \ldots, (x_n, y_n)]\) that is used to determine the exact shape of the road. This is used for visualization.

- **Paths** A path is a sequence of arcs.

- **Path Matrix** The path matrix consists of the shortest (time-wise) path for each pair of nodes in the network with the associated travel time, calculated by for example the Dijkstra algorithm.

- **Trips (Passengers)** Trips represent passengers who want to be transported from one location to another. Each trip consists of several aspects:
  - **OD Pair** Each trip has an origin and a destination, which are both nodes in the graph.
**Maximum Travel Time (MTT)** The maximum travel time is the maximum time the passenger is willing to spend on his trip, from the time of his request to the time he is dropped off.

**Earliest Pick-up Time (EPT)** Each trip has an earliest pick-up time.

**Latest Pick-up Time (LPT, optional)** A trip can have a latest pick-up time, based on an optional maximum waiting time.

**Latest Drop-off Time (LDT)** From the earliest pick-up time and the maximum travel time of each passenger, a latest drop-off time is determined.

**Volume** Each trip has an optional volume; i.e. one trip could encompass multiple passengers.

**Status** Each trip has a status, representing whether the trip is already in a vehicle or waiting for a vehicle. If it is in a vehicle it lists what vehicle the passenger is in.

**Vehicles** Vehicles represent the vehicles that serve the trip requests. They consist of:

- **Route** Each vehicle has a route and this route consists of a sequence of nodes and arcs. For each node, the time it will reach this node is known as well as what trips (if any) are served there. Furthermore, several quantities related to the insertion algorithm are associated with each node, see Section 4.5.3 for a more in-depth explanation.

- **Time/Distance Driven** The distance driven since the vehicle was last charged/refueled or since a new driver has started operating the vehicle.

- **Starting Depot** Each vehicle starts the day at a given depot.

- **Ending Depot (optional)** It is possible to force vehicles to end the day at a given depot.

- **Capacity/Occupancy** For each vehicle, the maximum capacity and the current occupancy are given (thus, the number of empty seats is known).

### 4.2. Graph Module

The graph module is mostly self-explanatory. The path matrix for the graph is computed once, before the simulation, and is thus assumed to be deterministic. Each node in the graph is either an intersection, where vehicles can change direction; a stop, where vehicles can drop-off and pick-up passengers; or a depot, where vehicles start (and possibly end) their route. All stops and depots are also intersections and a depot can also be a stop.

### 4.3. Trip Module

Each passenger has a maximum travel time he is willing to spend on his trip. This maximum travel time can further be split into a maximum waiting time and a maximum in-vehicle time. These times can for example be based on the direct travel time. Based on the earliest pick-up time (which is the request time for each trip as the model has a 100% degree of dynamism) and the maximum travel time, a latest drop-off time can be determined for each passenger. If a maximum waiting time is set as well, a latest pick-up time can be determined. If it is not possible for a vehicle to pick up the passenger before his latest pick-up time or to drop off the passenger before the latest drop-off time for that passenger, the request will be rejected and the passenger will not be serviced at all.

### 4.4. Vehicle Module

In this section an explanation of the attributes of the vehicles is given as well as a detailed explanation of how the rerouting of the vehicles will be performed in the model for both the empty vehicle rerouting and the en-route demand rerouting. Unless noted otherwise, notation is as in Chapter 2.

#### 4.4.1. Vehicle parameters

For each vehicle, several characteristics are required as input of the model:
4.4. Vehicle Module

**Figure 4.1**: Overview of the structure of the model and the main interactions between parts within the model.

**Maximum Vehicle speed** The vehicle speed is used to determine the times required to traverse each arc in the network.

**Maximum Operating Distance/Time** The maximum operating distance and/or time are used to determine when a vehicle will need to be recharged/refueled.

**Battery Recharge Time/Refuelling Time** The recharge/refuelling time determines the time a vehicle needs to spend at a depot to be recharged or refuelled.

**Capacity** The capacity of the vehicles needs to be specified in advance. It is possible to differentiate between various types of seats, see also Section 2.4.1.

4.4.2. Empty Vehicle Rerouting

For empty vehicle rerouting, the Dynamic Transportation Problem algorithm as it is presented by Lees-Miller [33] is used. If $\mathcal{S}$ is the set of all stops, then for each stop $i \in \mathcal{S}$ in the network graph and for each time $t \in T$, a target number of inbound vehicles $\theta^i_t$ is computed, based on the demand at that node. Here $T = \{0, 1, \ldots, t_{\text{max}}\}$ is a set of times in the simulation, with a certain interval, where $t_{\text{max}}$ is the total simulation time. The number of inbound vehicles encompasses both the number of...
empty vehicles that are driving towards stop \( i \) as well as the number of idle vehicles already at \( i \) at a specific time. At each time instant \( t \) the target number of vehicles for node \( i \) can be computed by computing the number of passengers that are expected to arrive at that node within \( x \) seconds from the probability distribution; let this number of passengers be \( \pi_i^{tx} \). Here \( \mathbf{P}_j^1(1) \) represents the probability that exactly 1 trip will arrive at stop \( j \) at time instant \( j \). Higher order terms (i.e. for \( \mathbf{P}_j^1(2) \) etc.) can be included if demand is high enough that it is likely that more than 1 trip arrives at a time instant. Assume that the capacity of each vehicle \( k \) is equal to \( Q \) (i.e. uniform for all vehicles). Then we can compute \( \theta_i^t \) by:

\[
\theta_i^t = \left[ \frac{\pi_i^{tx}}{Q} \right] \quad \forall i \in S, t \in T. \quad (4.1)
\]

Now let \( K_i^t \) be the number of vehicles inbound to or idle at stop \( i \) at time \( t \) and let \( \lambda_i^t \) be the number of vehicles idle at stop \( i \) at time \( t \). Note that \( K_i^t \geq \lambda_i^t \geq 0 \). Then define:

\[
u_i^t = \min(K_i^t - \theta_i^t, \lambda_i^t) \quad \forall i \in S, t \in T, \quad (4.2)
\]

as the *surplus* of vehicles at stop \( i \) at time \( t \). If \( u_i^t > 0 \) then there is a surplus of inbound vehicles at stop \( i \), if \( u_i^t < 0 \) there is a deficit of inbound vehicles at stop \( i \). The \( \lambda_i^t \) is required in Equation (4.2) as it is never possible to move vehicles away from a node before they are actually idle there. Finally, as the surpluses and deficits are not necessarily balanced, a dummy node \( \rho \) is introduced for which \( u_i^\rho = - \sum_{i \in S} u_i^t \). Now let \( S' = S \cup \rho \) and partition \( S' \) into sets \( S^+ = \{ i \in S' \mid u_i^t \geq 0 \} \) and \( S^- = \{ i \in S' \mid u_i^t < 0 \} \) for a given time \( t \). Then, denote \( v_{ij} \) as the number of vehicles that moves from \( i \) to \( j \) \( \forall i \in S \) and the time to travel between \( i \) and \( j \) as \( t_{ij} \), where \( t_{ii} = t_{ij} = 0, \forall i \in S \). Then we want to solve:

\[
\min \sum_{i \in S^+} \sum_{j \in S^-} t_{ij} v_{ij}, \quad (4.3)
\]

s.t

\[
\sum_{j \in S^-} v_{ij} = u_i \quad \forall i \in S^+, \quad (4.4)
\]

\[
\sum_{i \in S^+} v_{ij} = -u_j \quad \forall j \in S^-, \quad (4.5)
\]

\[
v_{ij} \geq 0 \quad \forall i \in S^+, j \in S^- . \quad (4.6)
\]

Any vehicle that is sent to the dummy node \( \rho \) is then simply not moved at all. As it is not necessary to continuously check whether it is required to move some empty vehicles, this computation step can be performed at a predetermined interval.

### 4.4.3. En-Route Demand Rerouting

With en-route rerouting, it is meant that vehicles can reroute while they are on a route segment (i.e. en-route between two nodes in the model). An example is shown in Figure 4.2. Here, the dotted red line between \( D_1 \) and \( P_2 \) shows the route segment of a vehicle when a request \( (P_3, D_2) \) comes in. If there is an intersection at point \( I \), it is allowed for the vehicle to head to \( P_3 \) from \( I \), before going to \( P_2 \), as opposed to Figure 2.2. In Figure 4.2(b) the dotted line represents the original route of the vehicle. Note that this type of rerouting is not used yet in literature, but is possible in our case as the full network is saved as a graph and all distances and times are computed in advance of the simulation (i.e. are considered deterministic). The main goal of this thesis is to identify to what extent this type of rerouting can be used in a combination with demand forecasts to improve the solution to the DARP. The process to include demand forecasts is described here and this type of rerouting is from now on called en-route demand rerouting or simply demand rerouting.

When a request \( i \) comes in, first a selection of vehicles that could possibly serve this request is made (for more details see Section 4.5.2). Each of the vehicles \( k \) in this selection has a certain capacity \( Q^k \) and a certain number of occupied seats \( O^k_i \) after serving a request at node \( j \) on its route; thus it has \( E^k = Q^k - O^k_i \) empty seats at that point. Assume that for vehicle \( k \) it takes \( T_k \) seconds to serve a request \( i \) after serving request \( j \). Then, calculate the number of requests that are expected to occur at
the node where request $i$ occurs within $T_e$ seconds of $i$ occurring, using the demand forecast. Denote this number of requests $R_i^k$. Then, if $E_k < R_i^k$ do not allow the vehicle to serve request $i$, unless no other vehicle is available. Based on the risk-taking preference of the transport operator and the trust he places in the demand forecast, the above can be changed to $E_k < \epsilon R_i^k$, where $\epsilon$ is a positive constant. If $\epsilon < 1$ this indicates that even if a vehicle has less empty seats than the expected number of passengers, it might be allowed to serve the trip. A transport operator could choose $\epsilon < 1$ if the demand forecast is not very accurate. If $\epsilon > 1$ it means that a vehicle needs more empty seats than the expected number of passengers, which could be desirable if the transport operator wants to err on the safe side. Note that if $\epsilon = 0$ the model returns to the case without demand rerouting.

4.5. Main Module

4.5.1. Insertion Heuristic

For the computation of which vehicle should serve an incoming request, an insertion heuristic is used, as presented by Jaw et al. [30]. The choice for an insertion heuristics is made because of the large scale of the simulation.

Whenever a request comes in, a candidate set of vehicles is created from all vehicles that could reasonably serve this request (see Section 4.5.2). Then, for each vehicle in the candidate set, all the feasible ways in which this customer can be inserted into the vehicle’s schedule are computed (see Section 4.5.3). For each insertion, the value of an objective function is computed (Section 5.3.4) and then finally the customer is either inserted into the position where the value of the objective function is lowest or the customer is rejected (if the value of the objective function for not serving the customer is lower than the lowest insertion cost).

4.5.2. Vehicle Selection

The selection of vehicles can be made in several ways. Vehicles can be selected based on the distance (either length- or timewise) to the incoming trip request. Furthermore, vehicles that will already visit
the node where the request originates as part of its schedule (either for a pick-up or drop-off) could also be included in the selection. Another option would be to include vehicles that are currently idle (i.e. not carrying any passengers) or that will become idle within a short time.

### 4.5.3. Finding Feasible Insertions

To find feasible insertions, so-called schedule blocks for each vehicle are introduced. A schedule block is a period of a vehicle’s route in between two idle periods. A schedule block always begins with a vehicle starting from an idle period (for example from a depot) and ends either when it reaches the end of its route or when the vehicle returns to a depot. The schedule of a vehicle consists of a sequence of nodes with associated arrival and departure times as well as which trips (if any) it serves at each node. A schedule of a vehicle can thus be represented by a sequence of pick-up nodes, drop-off nodes and through nodes. For example \((p_1, n_1, n_2, p_2, n_3, p_3, n_4, d_2, n_5, d_6, d_1)\), where \(p_i\) and \(d_i\) represents the pick-up and drop-off of passenger \(i\) respectively and \(n_j\) represent some through nodes (either intersections or stops where the vehicle does not serve a trip). Given a route of \(q\) nodes, a new pick-up and drop-off request can be inserted into this route in \((q + 1)(q + 2)/2\) ways, as the pick-up must precede the drop-off of the passenger. Some of these insertions might however be infeasible due to one or more of the constraints imposed on the model. To improve the searching speed for feasible insertion, not all constraints are immediately checked, but instead only the time windows are checked. That is, it is tested whether an insertion of a pick-up and drop-off will violate any of the maximum travel times (see Section 4.3) of one of the passengers in the vehicle or of the new passenger. For each trip \(i\) define:

\[
\begin{align*}
\text{EDT}_i &= \text{EPT}_i + \text{DRT}_i, \\
\text{LDT}_i &= \text{EPT}_i + \text{MRT}_i.
\end{align*}
\]

Here, \(\text{EPT}_i\) denotes the earliest pick-up time (i.e. the request time), \(\text{DRT}_i\) denotes the direct ride time and \(\text{MRT}_i\) denotes the maximum ride time for passenger \(i\). \(\text{EDT}_i\) denotes the earliest drop-off time and \(\text{LDT}_i\) denotes the latest drop-off time for passenger \(i\). These times are used for a first check on whether an insertion might be feasible. The actual pick-up time (\(\text{APT}_i\)) and actual drop-off time (\(\text{ADT}_i\)) of each passenger \(i\) now have to satisfy:

\[
\begin{align*}
\text{EPT}_i &\leq \text{APT}_i \leq \text{LPT}_i, \\
\text{EDT}_i &\leq \text{ADT}_i \leq \text{LDT}_i.
\end{align*}
\]

For each node \(\gamma\) in a schedule block, the information whether it is a pick-up, drop-off or through node is then discarded. The nodes are then numbered from earliest to latest and for each node the actual time \(\text{AT}_\gamma\) and latest time \(\text{LT}_\gamma\) are determined. For pick-up nodes thus \(\text{AT}_\gamma = \text{APT}_\gamma\) and \(\text{LT}_\gamma = \text{LPT}_\gamma\). For drop-off nodes \(\text{AT}_\gamma = \text{ADT}_\gamma\) and \(\text{LT}_\gamma = \text{LDT}_\gamma\). For through nodes set \(\text{LT}_\gamma = \infty\). Assume that there are a total of \(d\) nodes in the schedule block. Now, a statistic is defined for each stop \(\gamma\) in a schedule block and is updated throughout the simulation. This statistic defines by what amount of time the actual arrival time at each stop can be delayed. Let \(\text{SKT}\) denote the length of the slack period immediately following the schedule block and set:

\[
\text{ADOWN}_\gamma = \min\{\min_{\gamma \neq \gamma} (\text{LT}_\gamma - \text{AT}_\gamma), \text{SKT}\}.
\]

\(\text{ADOWN}_\gamma\) represents the time by which every node following but not including node \(\gamma - 1\) can be delayed. This statistic can then be used to determine whether an insertion violates any time windows. As an example, assume that we try to insert a new pick-up after the \(4^{th}\) node in a schedule block. Then the constraint is given by \(\text{DETOUR} \leq \text{ADOWN}_4\), where \(\text{DETOUR}\) is the amount of extra time spent by the vehicle to pick-up the request. The drop-off time can then be inserted and checked in the same way. Jaw et al. [30] introduce three other statistics, called \(\text{BUP}\), \(\text{BDOWN}\) and \(\text{AUP}\), which check whether nodes can be advanced and whether nodes preceding a certain node can be delayed. However, this is only necessary in the case that static requests would be part of the model. There is no restriction on the pick-up time window (except based on the constraint for total trip time) since requests are immediately available for pick-up when they come in. Thus it is never necessary to delay any nodes before node \(\gamma + 1\) or to advance any nodes at all.
Note that in general, due to the high number of trip requests, \( SKT_i \) is most often determined by the
time between the current latest drop-off in a vehicle route and the time at which the vehicle will need
to be refuelled/recharged.
After the time window constraints are checked using \( ADOWN_i \), the other constraints can be checked
for any remaining feasible insertions and finally the value of the objective function is calculated to
determine the best position for the insertion.
The operation to update the \( ADOWN_i \) values can be performed in order \( O(d) \), where \( O \) represents
the Bachmann-Landau big \( O \) notation \( [26] \). Recall that \( d \) is the number of nodes in a schedule block.
This update only needs to be performed once for every insertion. The actual check on \( ADOWN \) can
then be performed in order \( O(1) \). This check possibly removes the need to do several other order \( O(d) \)
operations to check the other constraints, which would otherwise have had to be checked for each
insertion position.

4.6. Model Assumptions

For the model, it is assumed that the locations of the stops, intersections and possible depots are
known beforehand and that a full distance matrix is known for each pair of nodes in the network.
Furthermore, a deterministic time matrix is available as well, as no stochastic travel times are used. It
is assumed that there is enough capacity for vehicles everywhere on the network, i.e. there is no limit
on the number of vehicles that can stay at a stop or that can drive on a certain road. For each pair of
nodes in the network, a list of all nodes lying on the shortest path between these nodes is set. The
starting location of each vehicle is known and an optional ending location for each vehicle is known.
The demand forecast for each OD pair is available in the form of a probability distribution.
For each vehicle, the maximum service duration (or operating distance), maximum capacity and max-
imum speed are fixed.

4.7. Pseudocode

Algorithm 1 represents the main model structure, incorporating empty vehicle rerouting. The en-route
demand rerouting can be implemented in the constraints on Line 13. The model requires input (the
input for the simulation used in this thesis is presented in Chapter 5) and outputs the vehicle schedules
that were used during the simulation as well as the performance indicators of interest, see Section 5.5.
Before the simulation starts, the path matrix is created and empty vehicle routes are initialized.
During the simulation, vehicle positions are updated continuously (Line 3) and passengers are dropped
off and picked up when required (Lines 5-6). Empty vehicle rerouting is performed every \( y \) min-
utes (Line 8). Then, from the demand forecast distributions, new requests are generated (Line 9) and
the insertion algorithm from Section 4.5.1 is performed (Lines 10-18) for each trip request. Finally, for
all vehicles that changed their route (either due to empty vehicle rerouting or due to added trips) it is
checked whether they are low on battery or fuel after finalizing their current route; if so they are sent
to a depot (Lines 19-21).
Algorithm 1: Main model overview

**Input:** Demand Distributions for each node

**Input:** Vehicle Specifications

**Input:** Graph for Simulation Location

**Input:** Objective Function

**Output:** Complete vehicle schedules

**Output:** Model Performance Indicators

**Initialize:** Path Matrix

**Initialize:** Empty Vehicle Route

1. Start model timer $t$
2. while $t < \text{simulation time}$ do
   3. Update Vehicle Positions
   4. if Vehicle $v$ passed a stop then
   5. Drop Off Passengers
   6. Pick Up Passengers
   7. if $t \mod (y \text{ minutes}) = 0$ then
   8. Reroute Empty Vehicles
   9. Trip Generation

for all Generated Trips $r$ do

10. Find Vehicle Set $K_r$
11. Insertion Of Trip
12. Check Constraints
13. Find Cheapest Insertion Cost $c$
14. if $c < \text{Rejection Cost}$ then
15. Insert $r$
16. else
17. Reject $r$

for all Vehicles $v$ with route changes do

19. if Battery Low After Route $v$ then
20. Insert Drive To Depot
5

Simulation Setup

In this chapter, the input parameters for the model from the previous chapter are presented and the scenarios that will be modeled are described. Section 5.1 presents the graph that will be used in all the scenarios. Section 5.2 lists the scenarios that will be analyzed. Section 5.2.1 defines the two types of demand that will be used, as well as the levels of demand and describes how trips are generated. Section 5.3 shows the vehicles that will be used in the simulation. Section 5.3.4 sets the objective function used in the insertion algorithm and for evaluation of the scenarios. Section 5.4 lists the constraints used in the insertion algorithm. Finally, Section 5.5 lists the key performance indicators that will be used in analyzing the scenario results.

5.1. The Case Study Graph

The network that is used in this thesis to test the different models is the Tata Steel IJmuiden area in the Netherlands. This business area is 300 hectares and has a road network that is used both by vehicles transporting goods between the different factories, a small amount of private transport by employees, as well as public transport to move personnel between buildings. A map of the area from TomTom data is presented in Figure 5.1. This area was chosen because it is an area that is ideally suited to be served by automated vehicles, since there is not nearly as much traffic on the roads as on normal public roadways in cities for example. Therefore, it is most likely easier to implement a system with automated vehicles in an industrial area like Tata Steel IJmuiden than in a city center. Furthermore, the scale of the area is small enough that the simulations can still run in reasonable time.

The area consists of 712 roads, 36 stops, 327 intersections and 6 depots. All roads visible in Figure 5.1 that are not colored black or gray are not included in the simulation. This could be due to them not belonging to Tata Steel IJmuiden, or being dirt roads, or other reasons. It has a total daily passenger demand of around 1000 passengers, see also Section 5.2. For each OD pair, travel times and distances are computed before the simulation starts and it is assumed that travel times are deterministic. It is possible to incorporate stochastic travel times [43], but since the vehicles will drive on infrastructure that is not heavily used this is not done in this thesis.

The location of stops and depots was decided based on the location of the buildings at Tata Steel IJmuiden, but does not represent the current situation of public transport there. The demand case with one dominating location (see Section 5.2) is the one that is most accurate for the Tata Steel IJmuiden area, as there is one main entrance where most employees will arrive in the morning, either by public transport or by car (indicated in Figure 5.1). However, the other demand type scenarios will also be tested on the graph based on the Tata Steel IJmuiden road network, as this simplifies computations and comparisons.
Figure 5.1: Map of the Tata Steel IJmuiden area in IJmuiden (TomTom), with overlay. The inset (Google Maps) shows the location of the Tata Steel IJmuiden area in relation to Amsterdam. The Black lines in the main figure are the roads used in the simulation, gray lines are one-directional roads. The white nodes are intersections, the red nodes are the public transport stops used in the simulation. The yellow nodes are the vehicle depots and the large yellow node is the entrance (which is also a depot).
5.2. Case Study Demand

As stated in Part I the goal of this thesis is to test two ways of using demand forecasts to improve rerouting of vehicles during the morning peak. The scenarios that are tested are shown in Figure 5.2. In total, 24 scenarios will be tested. Four variations of the rerouting possibilities of the model will be tested: No rerouting, only empty vehicle rerouting, only en-route demand rerouting and both types of rerouting. In cases where empty vehicle rerouting is used it is assumed that vehicles start the empty vehicle rerouting 10 minutes before the first passengers arrive, as this is enough time for this road network.

All of the rerouting possibilities will be modeled for two demand patterns: First, a demand pattern where the demand at one location dominates the demand at other locations and secondly a demand pattern where the demand is spread out evenly across most stops. For each of these demand patterns, three demand levels will be tested: a low, a medium and a high level of demand.

![Figure 5.2: Overview of all the model types that will be analyzed; each combination of routing type, demand type and demand level will be modeled.](image)

In the model it is always assumed that the degree of dynamism is 100%, i.e. all trips are dynamic. Furthermore, stochastic customers will be used, which means that at the start of the model run a probability distribution for the expected arrival of passengers is known for each stop, but not the precise time or destination of these passengers. A Poisson distribution is used to characterize the demand of every OD pair, as this is a common method in DARPs [6, 17]. The Poisson distribution is set up in such a way that over the 3 hours of simulation time, on average the number of passengers required by the demand level are generated. In all scenarios we assume that the trip volume is always 1, i.e. each trip request consists of a single passenger.

5.2.1. Demand Distribution Patterns

For the medium demand level we assume the typical number of passengers using public transport to travel to work, which in the Netherlands is 11.4% [10]. Around 10,000 people are employed at Tata Steel IJmuiden, so an expected public transport demand of 1,000 passengers is used for the medium demand case, for both types of demand. This number is the number of passengers that arrives during the morning peak (7 AM - 10 AM) that is modeled. For the low demand case half of this number is used and for the high demand case this number is doubled; see Table 5.1.

For the demand type where one location dominates demand, 40% of all demand originates at the entrance to Tata Steel IJmuiden in the morning peak. The destination of these trips is spread out randomly across all stops in the network. The rest of the trips then have random origins and destinations. For the demand type where demand is spread out over all stops, we assume that demand is spread out randomly amongst all stops, for both the origin and destination of trips, see Table 5.1.
### Table 5.1: Morning peak demand distribution and levels for each Demand Type.

<table>
<thead>
<tr>
<th>Demand Type</th>
<th>Morning Peak Demand</th>
<th>Demand level</th>
<th># Requests</th>
<th># Vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Location</td>
<td>40% of pick-ups at entrance, drop-offs randomized.</td>
<td>Low</td>
<td>500</td>
<td>30</td>
</tr>
<tr>
<td>Dominating Demand</td>
<td></td>
<td>Medium</td>
<td>1000</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High</td>
<td>2000</td>
<td>100</td>
</tr>
<tr>
<td>Evenly Distributed Demand</td>
<td>Pick-ups &amp; drop-offs randomized based on initial spread.</td>
<td>Low</td>
<td>500</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Medium</td>
<td>1000</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>High</td>
<td>2000</td>
<td>100</td>
</tr>
</tbody>
</table>

### 5.3. Fleet Specification

The vehicle in the simulation is based on the WEpod [57], but each vehicle in the simulation will have a capacity of 5 passengers instead of 10 as this allows more vehicles to be purchased. That is, more vehicles are needed to have the same capacity. This will allow the vehicles to be more spread out over the varying stops throughout the network, leading to less rejected passengers. An analysis on the impact of different types of vehicles on the algorithm performance is given in Section 6.7.2. Although it is possible to distinguish between standing and seating spaces, this is not something that will be done, as it is assumed passengers have short trip times in the case study and thus little preference. Acceleration and deceleration are not taken into account and it is assumed the vehicles always travel at 40km/h. The vehicles can drive a maximum distance of 100 kilometers before their battery has to be charged or they have to be refueled. After 100 kilometers, the vehicle has to visit a depot, where it can be recharged or refueled in 20 minutes, see also Section 5.3.2. Vehicles spend 20 seconds at each stop where they drop off or pick up passengers. An overview of the vehicle characteristics is shown in Table 5.2.

### Table 5.2: Information about the vehicles that are used in each of the simulations.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>5 seats</td>
</tr>
<tr>
<td>Commercial Speed</td>
<td>40km/h</td>
</tr>
<tr>
<td>Maximum Operating Distance</td>
<td>100km</td>
</tr>
<tr>
<td>Battery Recharge Time</td>
<td>20 minutes</td>
</tr>
<tr>
<td>Service Time</td>
<td>20 seconds</td>
</tr>
</tbody>
</table>

The fleet size for each demand case is shown in Table 5.1. This fleet size was based on initial simulations, where the number of rejected passengers could not exceed 10% of the total number of passengers over all rerouting types with that demand level and was then rounded up.

### 5.3.1. Empty Vehicle Rerouting

As the travel times in the case study area are relatively short, a look-ahead time $x$ of 5 minutes is used for empty vehicle rerouting (see Section 4.4). The empty vehicle rerouting is performed every 2 minutes of modeling time, starting from 10 minutes before the first trip request arrives.

### 5.3.2. Depots

Since each vehicle has to be charged after driving a maximum of 100 km, several depots need to be placed where vehicles can be recharged. Although no locations for this charging currently exist, we assume that six depots are spread out throughout the Tata Steel IJmuiden area as shown in Figure 5.1. Each vehicle can be charged in 20 minutes and it is assumed that this includes potential waiting times for other vehicles (i.e. the capacity of the depots is infinite). Vehicles are always charged up to 100% battery or fuel capacity and it is thus not possible to partly charge a vehicle. Furthermore, at the start of each day each vehicle starts at one of the 6 depots in such a way that an approximately equal number of vehicles is situated at each depot.
5.3.3. Selecting Relevant Vehicles for Insertion

The set of vehicles $K_r$ (see Section 4.7) on which the insertion algorithm is run when a trip request comes in is composed of two types of vehicles:

1. Vehicles that are within 2 kilometers from driving distance to the pick-up location of the trip request.
2. Vehicle that will already visit the pick-up location of the trip request at some point along their route.

5.3.4. Objective Function

The chosen objective function is a function of three variables: the passenger travel time, the vehicle travel time and the number of rejected requests. Although rejected requests are possible, they are penalized heavily as it is bad for both the passenger and the transport operators, especially in an environment where line-based services were traditionally operating. The passenger travel time is the factor that customers experience directly. The vehicle travel time is included for the operator and is used as a proxy for the costs induced to the operator. The objective function that is used is given by:

$$\min_{\text{insertions}} \left[ \left( \sum_{\text{vehicles}} 3 \cdot \text{Travel Time}_{\text{veh}} \right) + \left( \sum_{\text{passengers}} \text{Travel Time}_{\text{Pass}} \right) + \left( 40 \cdot \# \text{ rejected requests} \right) \right]. \quad (5.1)$$

Thus, for example, a vehicle with 6 passengers being given an extra 2 minutes of travel time gives a value of 18; the same holds for an empty vehicle given an extra 6 minutes of travel time. A rejected passenger is equal to either 10 minutes of empty vehicle travel time or 30 minutes of passenger travel time. Note that this objective function is not the only objective function that could be used, see Section 2.4.2. An analysis of the impact of changes to the objective function on the effectiveness of the algorithm is presented in Section 6.7.1.

5.4. Constraints on Insertion

The constraints that will be tested in the insertion algorithm (see Section 4.5.1) are the following:

**Vehicle Capacity**
This constraint ensures that the vehicle will never exceed its capacity between picking up and dropping off the requested trip.

**En-Route Demand Rerouting**
This constraint ensures that the en-route demand rerouting is used if the scenario requires it. Here, a factor $\epsilon = 1$ is used, meaning a vehicle will not serve a request if the total number of incoming requests at the request node exceeds its remaining capacity (see Section 4.4.3).

**Passenger Time Window Constraint**
This constraint ensures that the passenger that is picked up is dropped off on time to his destination. For simplification only the maximum travel time is taken into account, not a maximum waiting time. The maximum travel time is

$$\max \text{ total travel time} = \max(\text{direct travel time} + 5 \text{ minutes}, 1.5 \times \text{direct travel time}). \quad (5.2)$$

The maximum travel time used here is just an assumption. Based on the preference of the operator, the maximum total travel time can be increased or decreased.

**Vehicle Battery**
The final constraint ensures that vehicles are rerouted to a depot when their battery or fuel tank is almost empty. In all scenarios a vehicle is rerouted to a depot once its battery charge is down to 10%.

5.5. Key Performance Indicators

The efficiency of the model can be measured with several key performance indicators. The effect of the rerouting on these indicators is also dependent on the objective function that is used. An objective function that penalizes total distance driven by a large amount might mean rerouting is used less often,
while an objective function that penalizes vehicle idle time might mean rerouting is used more often. For more information on the objective function see Section 5.3.4.

The key performance indicators that the model will calculate are based on both the operator costs and passenger costs:

**Simulation Time** This is included to test whether the simulation time increases by a prohibitive amount when including either of the rerouting types, because it is important that the simulation could also be run in real-time if it were to be implemented by a transport operator.

**Percentage of Passengers Rejected** The percentage of passengers rejected is of course based on the number of vehicles and the cost of a rejection in the objective function, but it is still useful to analyze what effect rerouting has on the percentage of rejections.

**Average Distance Driven per Vehicle** Related to the time driven, the average distance driven by vehicles is an indicator of the efficiency of the algorithm. Especially with automated vehicles, the main costs for the operator are the vehicle operating costs (as there are no personnel costs).

**Average Vehicle Distance per Passenger** The average distance driven per vehicle to service one passenger is a good indicator for the operator to analyze how much he could charge for tickets for example.

**Average Passenger Waiting Time** From the passengers’ perspective there are two important indicators. The first is the passenger waiting time. An increase in waiting time is more important to passengers than an equal increase in in-vehicle time according to for example de Dios Ortúzar and Willumsen [15].

**Average Passenger Travel Time** In-vehicle time is also important to passengers and if rerouting causes longer travel times this might be an issue. The passenger travel time also gives full information on the amount of time spent on rerouting as this is simply the passenger travel time minus the direct travel time.

**Objective Function** Finally, the objective function that is used during the insertion algorithm is also used as a performance indicator at the end of the simulation run, where the overall value of the objective function is computed.
IV

Results and Discussion
6

Results

In this chapter the results of the scenarios from the previous chapter are provided and discussed. The results are first analyzed per demand type in Sections 6.1 to 6.3. In each of these sections the results are split for the evenly distributed demand type and for the dominating demand type. Each of the scenarios was run a total of 11 times because there was little variation between the simulation runs as the simulated time was 3 hours. The key performance indicators presented in the tables in this chapter are the averages of the key performance indicators from these 11 simulation runs per scenario. A complete comparison of all the scenarios is given in Section 6.4 and the detour and waiting time distributions of scenarios are analyzed in Section 6.5. An analysis of the effect of allowing vehicles to reroute at every node and not just at stops is made in Section 6.6. A sensitivity analysis of the results is presented in Section 6.7. All simulations were run on Delphi 10 Seattle on an Intel Xeon E5-1620 CPU @3.60GHz with 16GB of RAM using Windows 7. In Delphi, a simulation tool

Figure 6.1: Screen capture from the visualization tool used to generate the results in this chapter.
was built, which generates the results but can also present a visual representation of the routing of the vehicles. Figure 6.1 shows a still from a run of the visualization. In this figure, the blue lines represent the arcs of the graphs and the grey vertices are the nodes. The black and white dots, which are connected by gray lines, represent origin-destination pairs. The yellow sequences of dots represent the vehicles and their current and previous position. The numbers under the vehicles are the vehicle ID and between brackets the number of passengers currently in the vehicle.

### 6.1. Low Demand Case Results

#### 6.1.1. Evenly Distributed Demand

Table 6.1 presents the key performance indicators for the evenly distributed low demand scenarios. It is clear that both types of rerouting do not increase the simulation time in the low demand case, this is because only 30 vehicles are used and the demand is spread out over many stops. Because the demand is so low at each of these stops, the empty vehicle rerouting algorithm does not prefer any of the stops over any of the other stops, thus keeping empty vehicle rerouting to a minimum. Using only empty vehicle rerouting with such low demand has almost no impact. This is most likely because with only 500 trips in 3 hours and a randomly spread out demand, the demand forecast used to predict where vehicles are needed is not very helpful. Using demand rerouting also has a negligible impact. Neither types of rerouting have much impact on the percentage of rejected passengers, again because the demand forecast simply is not that useful with only 500 trips in 3 hours spread over many stops. The resulting value of the objective function is similar for all scenarios, although slightly higher when using both types of rerouting, this is a consequence of the vehicle distance factor in the objective function.

<table>
<thead>
<tr>
<th>Empty Vehicle Rerouting</th>
<th>Demand Rerouting</th>
<th>Simulation Time (s)</th>
<th>% of Passengers Rejected</th>
<th>Average Passenger Travel Time (s)</th>
<th>Average Waiting Time (s)</th>
<th>Average Vehicle Distance (km)</th>
<th>Average Veh. Dist./Passenger (km)</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>no</td>
<td>6</td>
<td>7%</td>
<td>514</td>
<td>133</td>
<td>36</td>
<td>2.36</td>
<td>10102</td>
</tr>
<tr>
<td>yes</td>
<td>no</td>
<td>6</td>
<td>6%</td>
<td>514</td>
<td>133</td>
<td>36</td>
<td>2.36</td>
<td>10093</td>
</tr>
<tr>
<td>no</td>
<td>yes</td>
<td>6</td>
<td>7%</td>
<td>510</td>
<td>134</td>
<td>36</td>
<td>2.32</td>
<td>10161</td>
</tr>
<tr>
<td>yes</td>
<td>yes</td>
<td>6</td>
<td>7%</td>
<td>515</td>
<td>130</td>
<td>37</td>
<td>2.40</td>
<td>10370</td>
</tr>
</tbody>
</table>

#### 6.1.2. Dominating Demand

The key performance indicators for the dominating low demand scenarios are presented in Table 6.2. Here, the impact of using empty vehicle rerouting is very pronounced compared to the evenly distributed demand case. The percentage of passengers being rejected drops significantly. The percentage of rejected passengers is high (25%) without empty vehicle rerouting, because 40% of the demand comes from one location and empty vehicles are not routed there to pick up these passengers. Once a passenger request comes in, vehicles need to travel from far away to pick up the passenger and it becomes impossible to service the request within the required time window. Vehicle distance driven of course becomes larger with empty vehicle rerouting, because empty vehicles drive around as well. However, this effect also partly comes from the fact that more passengers are being moved. Thus we see a 53% increase in average vehicle distance, but only a 17% increase in vehicle distance per passenger. Passenger travel and waiting times increase slightly when using empty vehicle rerouting, most likely due to empty vehicle rerouting. This causes some passengers that would be rejected without empty vehicle rerouting to now have a vehicle close enough to service the passenger, but still quite far away. The value of the objective function decreases by around 5% when using just empty vehicle rerouting. Average passenger waiting time and travel time decrease slightly when using demand
rerouting only, without increasing vehicle distance. However this leads to a similar value of the objective function. Combining both types of rerouting leads to the lowest value of the objective function, trading off vehicle distance for passenger rejections compared to the no rerouting case.

Table 6.2: Key performance indicators for dominating low demand scenarios.

<table>
<thead>
<tr>
<th>Empty Vehicle Rerouting</th>
<th>no</th>
<th>yes</th>
<th>no</th>
<th>yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Rerouting</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Simulation Time (s)</td>
<td>5</td>
<td>14</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>% of Passengers Rejected</td>
<td>25%</td>
<td>5,4%</td>
<td>25%</td>
<td>5,2%</td>
</tr>
<tr>
<td>Average Passenger Travel Time (s)</td>
<td>528</td>
<td>536</td>
<td>521</td>
<td>524</td>
</tr>
<tr>
<td>Average Waiting Time (s)</td>
<td>122</td>
<td>139</td>
<td>114</td>
<td>123</td>
</tr>
<tr>
<td>Average Vehicle Distance (km)</td>
<td>30</td>
<td>46</td>
<td>30</td>
<td>46</td>
</tr>
<tr>
<td>Average Veh. Dist./ Passenger (km)</td>
<td>2,42</td>
<td>2,83</td>
<td>2,42</td>
<td>2,91</td>
</tr>
<tr>
<td>Objective Function</td>
<td>12355</td>
<td>11693</td>
<td>12336</td>
<td>11350</td>
</tr>
</tbody>
</table>

6.2. Medium Demand Case Results

6.2.1. Evenly Distributed Demand

Table 6.3 lists the key performance indicators for the evenly distributed medium demand scenarios. Because there is now a higher demand, the demand forecast for rerouting becomes more accurate and can make a distinction between stops, due to random deviations in the demand between stops. The increase in demand also leads to a higher variability in the demand. A clear impact on travel and waiting times now becomes apparent in both the empty and demand rerouting scenarios. Empty vehicle rerouting leads to a reduction in rejected passengers, as is to be expected, but also leads to much lower travel and waiting times, even while serving more passengers. Note that the passenger travel time is the time from the request time to the drop-off time and thus includes the waiting time, which is responsible for a large part of the reduction in travel time. The value of the objective function is higher than without rerouting, due to the large cost of vehicle time driven in the objective function. Considering that each of the 60 vehicles drives 8 km more on average, which takes about 12 minutes. This leads to a $3 \cdot 60 \cdot 12 = 2160$ increase in the objective function. The decrease in rejected passengers (25 passengers), only leads to a $25 \cdot 40 = 1000$ reduction of the objective function. Empty vehicle rerouting does almost double the simulation time.

When using demand rerouting, passenger travel and waiting times decrease less than when using empty vehicle rerouting and there is only a slight reduction in rejected passengers. However, there is no increase in vehicle distance driven. Again, note that the average passenger travel time includes the waiting time. A reduction in passenger waiting time using demand rerouting is expected, and this reduction leads to the decrease in passenger travel time. Combining the two types of rerouting amplifies their beneficial effects, leading to an even larger decrease in passenger travel and waiting times and a large reduction in the number of rejected passengers. However, the increase vehicle distance again leads to a higher value of the objective function.

Table 6.3: Key performance indicators for evenly distributed medium demand scenarios.

<table>
<thead>
<tr>
<th>Empty Vehicle Rerouting</th>
<th>no</th>
<th>yes</th>
<th>no</th>
<th>yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Rerouting</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Simulation Time (s)</td>
<td>19</td>
<td>32</td>
<td>21</td>
<td>32</td>
</tr>
<tr>
<td>% of Passengers Rejected</td>
<td>4%</td>
<td>1,4%</td>
<td>3,4%</td>
<td>1,3%</td>
</tr>
<tr>
<td>Average Passenger Travel Time (s)</td>
<td>527</td>
<td>485</td>
<td>512</td>
<td>470</td>
</tr>
<tr>
<td>Average Waiting Time (s)</td>
<td>134</td>
<td>109</td>
<td>119</td>
<td>92</td>
</tr>
<tr>
<td>Average Vehicle Distance (km)</td>
<td>34</td>
<td>42</td>
<td>34</td>
<td>43</td>
</tr>
<tr>
<td>Average Veh. Dist./ Passenger (km)</td>
<td>2,10</td>
<td>2,57</td>
<td>2,10</td>
<td>2,62</td>
</tr>
<tr>
<td>Objective Function</td>
<td>19033</td>
<td>19959</td>
<td>18782</td>
<td>19755</td>
</tr>
</tbody>
</table>
6.2.2. Dominating Demand

For the dominating medium demand scenarios, the results are presented in Table 6.4. In the dominating low demand scenarios there was almost no change in passenger travel and waiting times. In the dominating medium demand scenarios however, there is a clear effect. This is due to the fact that demand forecasts are now much more useful. Average travel times and waiting times are not affected as heavily as in the evenly distributed medium demand scenario. This is mostly the case because in evenly distributed scenarios, travel times and waiting times without empty vehicles rerouting are very high, but rejections are pretty low. The reason for this is that a vehicle is often relatively close to each stop due to the random nature of trip generation and the placement of the depots. In the dominating scenario, the travel times without rerouting are already slightly lower, but this is because many passengers are rejected instead of served. Thus the passengers who would have the longest travel and waiting times are rejected, due to the way the objective function is set up. Based on the objective function, these results might change, see also Section 6.7.1.

Empty vehicle rerouting leads to a very large decrease in passenger rejections, even larger than in the low dominating demand scenario. This is due to two factors: firstly, there are simply more vehicles to reroute, reducing the probability that a passenger is still too far away from all the vehicles. Secondly, the demand level is now higher, which means that there are simply more stops requesting a vehicle. In the low demand case it is often the case that for many stops, the required number of vehicles is zero, because no trips are expected, leading to many vehicles standing at the depots or their last drop-off location. Contrary to the evenly distributed medium demand scenarios, the use of empty vehicle rerouting now leads to a decrease in the value of the objective function as the number of rejected passengers is so much lower that it makes up for the increase in vehicle distance driven.

Demand rerouting leads to a large decrease in passenger waiting times, and an accompanying decrease in passenger travel times. The travel times are reduced less than the waiting times though, indicating that there is a small increase in in-vehicle travel times, see also Section 6.5. Combining the two types of demand rerouting again combines the strengths and weaknesses of both approaches. Travel and waiting times are lowest out of all scenarios and there is a small number of rejected passengers. However, both average vehicle distance and average vehicle distance per passenger are highest. The value of the objective function is similar to the one using only empty vehicle rerouting.

Table 6.4: Key performance indicators for dominating medium demand scenarios.

<table>
<thead>
<tr>
<th>Empty Vehicle Rerouting</th>
<th>no</th>
<th>yes</th>
<th>no</th>
<th>yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Rerouting</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Simulation Time (s)</td>
<td>16</td>
<td>43</td>
<td>17</td>
<td>47</td>
</tr>
<tr>
<td>% of Passengers Rejected</td>
<td>23%</td>
<td>3%</td>
<td>21%</td>
<td>3%</td>
</tr>
<tr>
<td>Average Passenger Travel Time (s)</td>
<td>541</td>
<td>546</td>
<td>525</td>
<td>520</td>
</tr>
<tr>
<td>Average Waiting Time (s)</td>
<td>129</td>
<td>136</td>
<td>103</td>
<td>100</td>
</tr>
<tr>
<td>Average Vehicle Distance (km)</td>
<td>27</td>
<td>40</td>
<td>29</td>
<td>41</td>
</tr>
<tr>
<td>Average Veh. Dist./ Passenger (km)</td>
<td>2,14</td>
<td>2,48</td>
<td>2,19</td>
<td>2,56</td>
</tr>
<tr>
<td>Objective Function</td>
<td>23150</td>
<td>20816</td>
<td>23124</td>
<td>20755</td>
</tr>
</tbody>
</table>

6.3. High Demand Case Results

6.3.1. Evenly Distributed Demand

The key performance indicators for the evenly distributed high demand scenarios are presented in Table 6.5. Similar results as for the evenly distributed medium demand scenarios are obtained. Travel time and waiting time are down significantly, while vehicle distance per passenger is increased. Travel times and waiting times are down much more with demand rerouting than in the medium demand scenarios. This is because the demand forecast is simply much more accurate for the high demand scenarios, especially in the evenly distributed demand cases. This leads to a better estimate of how many passengers will be arriving and thus the algorithm can send a vehicle with enough empty seats more often. Simulation times again almost double when using empty vehicle rerouting, while the
6.4. Overall Scenario Analysis

Simulation time increase for demand rerouting is very low. The value of the objective function is much higher when using empty vehicle rerouting, this is similar to the evenly distributed low demand case. The distance vehicles have to travel outweighs the minor reduction in rejected passengers in the particular objective function being used.

Table 6.5: Key performance indicators for evenly distributed high demand scenarios.

<table>
<thead>
<tr>
<th>Empty Vehicle Rerouting</th>
<th>no</th>
<th>yes</th>
<th>no</th>
<th>yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Rerouting</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Simulation Time (s)</td>
<td>60</td>
<td>102</td>
<td>65</td>
<td>113</td>
</tr>
<tr>
<td>% of Passengers Rejected</td>
<td>2,7%</td>
<td>1,1%</td>
<td>2,8%</td>
<td>1,2%</td>
</tr>
<tr>
<td>Average Passenger Travel Time (s)</td>
<td>548</td>
<td>521</td>
<td>515</td>
<td>490</td>
</tr>
<tr>
<td>Average Waiting Time (s)</td>
<td>147</td>
<td>129</td>
<td>103</td>
<td>92</td>
</tr>
<tr>
<td>Average Vehicle Distance (km)</td>
<td>36</td>
<td>45</td>
<td>36</td>
<td>46</td>
</tr>
<tr>
<td>Average Veh. Dist./ Passenger (km)</td>
<td>1,84</td>
<td>2,26</td>
<td>1,89</td>
<td>2,33</td>
</tr>
<tr>
<td>Objective Function</td>
<td>35843</td>
<td>38210</td>
<td>35581</td>
<td>37882</td>
</tr>
</tbody>
</table>

6.3.2. Dominating Demand

Table 6.6 shows the key performance indicators for the dominating high demand scenarios. The results of adding rerouting for the dominating high demand are much better than for the dominating medium and low demand. Passenger travel times and waiting times are down significantly for both empty vehicle and demand rerouting.

The percentage of rejected passengers is almost 0 when empty vehicle rerouting is used. This is for similar reasons that the number of rejected passengers decreased so much in the medium dominating demand scenario with empty vehicle rerouting. There are simply a lot of vehicles everywhere on the network when using empty vehicle rerouting. Using just demand rerouting gives a high decrease to travel and waiting times at a more modest increase to average vehicle distance per passenger than empty vehicle rerouting. Because there is such a large reduction in rejected passengers compared to the evenly distributed high demand scenario, there is now also a decrease in the value of the objective function when using empty vehicle rerouting.

Table 6.6: Key performance indicators for dominating high demand scenarios.

<table>
<thead>
<tr>
<th>Empty Vehicle Rerouting</th>
<th>no</th>
<th>yes</th>
<th>no</th>
<th>yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Rerouting</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Simulation Time (s)</td>
<td>47</td>
<td>120</td>
<td>53</td>
<td>133</td>
</tr>
<tr>
<td>% of Passengers Rejected</td>
<td>24%</td>
<td>0,5%</td>
<td>22%</td>
<td>0,5%</td>
</tr>
<tr>
<td>Average Passenger Travel Time (s)</td>
<td>553</td>
<td>534</td>
<td>528</td>
<td>496</td>
</tr>
<tr>
<td>Average Waiting Time (s)</td>
<td>139</td>
<td>117</td>
<td>97</td>
<td>75</td>
</tr>
<tr>
<td>Average Vehicle Distance (km)</td>
<td>28</td>
<td>45</td>
<td>31</td>
<td>47</td>
</tr>
<tr>
<td>Average Veh. Dist./ Passenger (km)</td>
<td>1,87</td>
<td>2,28</td>
<td>1,99</td>
<td>2,37</td>
</tr>
<tr>
<td>Objective Function</td>
<td>45510</td>
<td>38502</td>
<td>45300</td>
<td>38028</td>
</tr>
</tbody>
</table>

6.4. Overall Scenario Analysis

In this section, the results of the scenarios will be compared between different demand levels. Table 6.7 shows the key performance indicators for all scenarios combined. Table 6.8 shows the percentual change for each of the rerouting scenarios compared to the no rerouting scenario with that demand type and level. This allows for an easier comparison between different demand levels and types. For each of the key performance indicators we can observe the following:

Simulation Time The simulation time increases particularly for the empty vehicle rerouting, but also slightly for the demand rerouting in all cases. For dominating demand, this increase is much
higher. This is because it is almost always required to reroute around 40% of vehicles to the entrance. When using both types of rerouting, increases of around 150% are to be expected. However, this simulation time is definitely not prohibitive if such rerouting would be implemented in real systems; total simulation time for a 3-hour period is still only around 2 minutes in the high demand case.

% of Passengers Rejected The percentage of passengers that is rejected is especially influenced by empty vehicle rerouting while demand rerouting has little to no effect. This is partly because of the scenario setup, where vehicles are placed at predetermined depots. Furthermore, when vehicles finish a trip, they stay at that location without rerouting, and this location might be far away from most other requests. The benefit from empty vehicle rerouting is slightly higher in the dominating demand scenarios. Here, it can reduce the number of rejections by 80-100%. In the evenly distributed demand case this reduces to 50-60%, with the exception of the evenly distributed low demand case, where the demand forecast is simply not accurate enough.

Average Passenger Travel Time Demand rerouting has more effect on average passenger travel times than empty vehicle rerouting, especially in the dominating demand scenarios. This is because demand rerouting is used if there are multiple trips expected at a single node, something that happens much more often in the dominating demand scenarios. Note that travel time here is the time from the moment a trip request comes in to the drop-off time of that request and thus includes waiting time. A combination of the two types of rerouting leads to even better results. Again disregarding the low demand case, we see reductions of 15-20% in the evenly distributed demand scenarios and 5-10% in the dominating demand scenarios.

Average Passenger Waiting Time Demand rerouting has a very large effect on passenger waiting times for almost all demand types and levels and especially for the dominating demand scenarios. Empty vehicle rerouting also has some effect on passenger waiting times, as is to be expected. A combination of the two rerouting types leads to even larger reductions, with waiting time reductions of 20-50% for the dominating demand scenarios and 50-60% for the evenly distributed demand scenarios, not taking the low demand scenarios into account.

Average Vehicle Distance Empty vehicle rerouting causes a large increase in average vehicle distance, as is to be expected. Demand rerouting also slightly increases the average vehicle distance, which can be explained by the fact that sometimes a vehicle with more vacant seats that is slightly further away can be chosen to serve a request, while it is also possible that extra rerouting was not necessary if the anticipated demand does not realize in actual demand. In the dominating demand scenarios, the average vehicle distance increase is highest, ranging from a 50-60% increase with empty vehicle rerouting. This is because vehicles need to drive to the entrance of the Tata Steel area from all over the network. In the evenly distributed demand scenarios, average vehicle distance increases by up to 30% in the high demand scenario.

Average Vehicle Distance/Passenger Although average vehicle distance increases, especially for empty vehicle rerouting, this also leads to a reduction in passenger rejections. Thus, the average vehicle distance per passenger increases less than the average vehicle distance. In most scenarios we see increases ranging from 20-35%, much lower than the 40-60% increase in average vehicle distance.

Several conclusions can be drawn from the observations made above and Tables 6.7 and 6.8. First of all, empty vehicle rerouting requires quite a bit of extra simulation time, but reduces the number of rejected passengers tremendously. The trade-off for this is that vehicles need to drive slightly further for each passenger serviced. Travel time and waiting time for passengers are reduced as well. Demand rerouting, the main new idea in this thesis, also has a large effect. Passenger waiting times can be reduced by a large amount, and passenger travel times are reduced slightly as well. There is much less of a trade off than for empty vehicle rerouting; only a slight increase in simulation time and a 0-5% increase in distance driven per vehicle. If both types of rerouting are implemented, the effects of both are amplified. Simulation time is increased about threefold, but as mentioned before, this should be no issue in any real life application as trip insertion time is still low enough (at least a 10th of a second per insertion for the high demand scenarios). The number of rejected passengers can be reduced by up to 98% and passenger waiting and travel times can be shortened significantly, even more than with just empty or just demand
rerouting. However, vehicle distance per passenger is also slightly higher than when just implementing empty vehicle rerouting or just demand rerouting. It is interesting that the inclusion of both types of rerouting almost adds up the benefits of using only one of the two types of rerouting. It seems that, although it would be possible that the demand rerouting takes over some trips that would normally be served by a rerouted empty vehicle, this does not actually happen much.

The value of the objective function is affected mostly by empty vehicle rerouting. Demand rerouting has little influence on the value of the objective function, as it only really affects passengers travel times, and the value of passenger travel time is low in the objective function. For empty vehicle rerouting, there is always an increase in the objective function in evenly distributed scenarios, and a decrease in dominating scenarios. This is because, in the evenly distributed scenarios, there is in the absolute sense a much lower reduction in the number of rejected passengers, because even without empty vehicle rerouting there is a relatively low number of rejections. Compare this to the dominating scenarios, where there is a very large number of rejected passengers without empty vehicle rerouting and thus a much larger reduction (in the absolute sense) when using empty vehicle rerouting. Thus, in the evenly distributed scenario, the small reduction in rejections does not outweigh the extra time vehicles spend driving, while in the dominating case, the large reduction in rejections does outweigh this extra time spent. Although travel and waiting times are reduced significantly when implementing rerouting, it is possible that the distribution of the detour (in vehicle time minus direct travel time) or waiting times is undesirable. For example, if instead of most passengers having a medium waiting time, there are now a few passengers with very long waiting times and some passengers with very short waiting times, this might not be desirable for the transport operator.
Table 6.8: Percentual change for rerouting scenarios compared to the scenario without any type of rerouting with the same demand type and level.

<table>
<thead>
<tr>
<th>Demand Type</th>
<th>Empty Vehicle Rerouting</th>
<th>Demand Rerouting</th>
<th>Simulation Time (s)</th>
<th>% of Pass. Rejected</th>
<th>Av. Pass. Travel Time (s)</th>
<th>Av. Waiting Time (s)</th>
<th>Av. Veh. Dist. (km)</th>
<th>Av Veh. Dist./ Pass. (km)</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Demand</td>
<td>yes</td>
<td>no</td>
<td>+0%</td>
<td>-4%</td>
<td>+0%</td>
<td>+0%</td>
<td>+0%</td>
<td>+0%</td>
<td>+0%</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>yes</td>
<td>-1%</td>
<td>+4%</td>
<td>-1%</td>
<td>+1%</td>
<td>+1%</td>
<td>-2%</td>
<td>+1%</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>yes</td>
<td>-1%</td>
<td>+8%</td>
<td>+0%</td>
<td>-2%</td>
<td>+2%</td>
<td>+2%</td>
<td>+3%</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>yes</td>
<td>+187%</td>
<td>+2%</td>
<td>+198%</td>
<td>+78%</td>
<td>+1%</td>
<td>-79%</td>
<td>+13%</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>yes</td>
<td>+2%</td>
<td>-1%</td>
<td>-1%</td>
<td>+13%</td>
<td>-7%</td>
<td>+1%</td>
<td>+1%</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>yes</td>
<td>+51%</td>
<td>-1%</td>
<td>+51%</td>
<td>+51%</td>
<td>+1%</td>
<td>+20%</td>
<td>+20%</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>yes</td>
<td>+17%</td>
<td>+0%</td>
<td>+20%</td>
<td>-5%</td>
<td>+0%</td>
<td>-8%</td>
<td>-8%</td>
</tr>
</tbody>
</table>

Legend:

- **Much better performance**
- **Better performance**
- **Equal performance**
- **Slightly worse performance**
- **Worse performance**
- **Much worse performance**

6.5. Detour and Waiting Time Distribution

Detour and waiting time distributions were only made for the high demand scenarios. For the low demand scenarios, the demand forecast seems to be too inaccurate to show the full impact of the rerouting measures. The medium and high demand results are similar enough that only one of these two will be analyzed further. The analysis in this section will be split into an analysis of detour and waiting time distributions for the evenly distributed high demand scenarios and the dominating high demand scenarios.

6.5.1. Evenly Distributed High Demand Scenarios

Figure 6.2 shows the cumulative detour and waiting time distributions for passengers for evenly distributed high demand scenarios. When using only demand rerouting, waiting times clearly shift from...
longer waiting times to shorter waiting times, while detour times become slightly longer, with an increase in medium detour times, but no real increase in very long detour times. When using only empty vehicle rerouting, waiting times shift slightly lower, while detour times also become significantly lower, with much more very short detour times instead of medium detour times. When using both types of rerouting, waiting times decrease most, and there is no increase in very long waiting times; detour times are similar to using just empty vehicle rerouting.

6.5.2. Dominating High Demand Scenarios

Figure 6.4 shows the cumulative detour and waiting time distributions for dominating high demand scenarios. For the dominating demand scenarios, the results are similar to the evenly distributed demand scenarios. However, demand rerouting does not reduce the amount of medium waiting times as much, although the decrease in average waiting time is the same as in the evenly distributed demand scenario. Average detour times are slightly longer than without any rerouting, but this is compensated by shorter waiting times. Incorporating demand rerouting leads to a much larger decrease in waiting times than in the evenly distributed demand case, because in the dominating scenarios it is much more often the case that there is an expected demand of many passengers at a single node (the entrance). Decreases in detour times are similar to the evenly distributed demand case.

Figure 6.5 shows the distribution of the waiting and detour times over the network. Compared to the evenly distributed demand case there are much longer detour times near the entrance. This is because most of the passengers picked up there will be rerouted to the entrance to pick up extra passengers. This effect is reduced by incorporating rerouting where there are no locations with a high increase in detour times. Very long waiting times exist without any rerouting in the dominating demand case, especially for the nodes closest to the entrance. This effect is removed almost entirely by incorporating both types of rerouting. Especially demand rerouting has a large impact here (see Table 6.8) because

![Graph showing detour and waiting time distributions](image)

Figure 6.2: Waiting and detour time distributions for trips for evenly distributed high demand scenarios.

Besides the distribution of the length of waiting and detour times, it is also possible that introducing rerouting changes the location in the network where most detours happen. It is possible that there are for example a few locations where passengers will have very long waiting or detour times, which is perhaps unacceptable for the transport operator. Figure 6.3 shows the average waiting and detour times for each stop in the network for the no rerouting scenario and the scenario with both types of rerouting. Larger nodes in these figures indicate a higher average detour or waiting time. Decreases in detour times when using rerouting are spread evenly across the network, and there is not a single location with very long waiting times. Waiting times are reduced in a similar manner, with an increase in average waiting time at only 2 nodes, and this increase is very slight.

In conclusion, rerouting in the evenly distributed demand case has no negative effects on the spread of waiting and detour times for passengers. Only about 12% of passengers have no detour time at all, indicating that there is a very large amount of ride-sharing in the evenly distributed demand case.
Evenly Distributed Demand

Evenly Distributed Demand
No Rerouting

Empty Vehicle and Demand Rerouting

Figure 6.3: Waiting and detour time location distribution for trips for evenly distributed high demand scenarios with no rerouting and with both types of rerouting.

it ensures that empty vehicles will serve the requests at the entrance, which leaves vehicles carrying passengers to pick up the remaining requests close to the entrance.

As in the evenly distributed demand case there is no problem with introducing rerouting as far as the spread of waiting times and detour times is concerned. As in the evenly distributed demand case, only about 12% of the passengers have no detour time. There is thus again a large percentage of ride-sharing passengers.
6.5. Detour and Waiting Time Distribution

Figure 6.4: Detour and waiting time distributions for trips for dominating high demand scenarios.

Figure 6.5: Waiting and detour time location distribution for trips for dominating high demand scenarios with no rerouting and with both types of rerouting.
6.6. Effect of En-Route (non-Demand) Rerouting

In Section 4.4.3, en-route demand rerouting is explained. This type of rerouting is a combination of en-route rerouting with the demand forecast. However, even using just en-route rerouting (without demand forecasts) might already have an effect on the solution quality. En-route rerouting allows vehicles to reroute at every node, not just at stops where the vehicle drops off or picks up a passenger. In this section the key performance indicators of en-route rerouting are compared with the key performance indicators of only allowing rerouting at stops. This section is split in two parts, the first describes the results for the evenly distributed high demand scenario, the second for the dominating high demand scenario.

6.6.1. Evenly Distributed Demand

Table 6.9 presents the key performance indicators for the evenly distributed high demand scenario with and without demand rerouting. The simulation time increases when using en-route rerouting, which is expected as there are many more insertion positions to check for the algorithm. The percentage of rejected passengers slightly decreases. There are quite large increases in passenger travel and waiting times, perhaps unexpectedly. There is however a large decrease in average vehicle distance driven and vehicle distance driven per passenger.

The reason why en-route rerouting has such a negative effect on the travel time is that there is much more ride-sharing (as can be seen from the average vehicle distance per passenger). The network of Tata Steel IJmuiden has a high density of stops and is quite small. Thus, vehicles will often be able to reroute quickly even without en-route rerouting, reducing the effectiveness of en-route rerouting with respect to waiting times. Due to the spread of passengers, it is apparently sometimes even better to wait slightly longer to reroute. More trip requests are made during this time and these new trips can be inserted more efficiently. In a larger network with higher stop spacing, the effect of en-route rerouting on waiting time might be positive.

<table>
<thead>
<tr>
<th>En-Route Rerouting</th>
<th>No</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Time (s)</td>
<td>14</td>
<td>60</td>
</tr>
<tr>
<td>% of Passengers Rejected</td>
<td>2,8%</td>
<td>2,7%</td>
</tr>
<tr>
<td>Average Passenger Travel Time (s)</td>
<td>461</td>
<td>548</td>
</tr>
<tr>
<td>Average Waiting Time (s)</td>
<td>104</td>
<td>147</td>
</tr>
<tr>
<td>Average Vehicle Distance (km)</td>
<td>45</td>
<td>36</td>
</tr>
<tr>
<td>Average Veh. Dist./ Passenger (km)</td>
<td>2,38</td>
<td>1,84</td>
</tr>
<tr>
<td>Objective Function</td>
<td>37316</td>
<td>358430</td>
</tr>
</tbody>
</table>

6.6.2. Dominating Demand

The key performance indicators for the dominating demand scenario with and without demand rerouting are presented in Table 6.10. The results are similar to the evenly distributed case for most indicators, with a 10% increase in passenger travel time, and a 20% increase in passenger waiting times. The average vehicle distance per passenger is reduced by almost 20%, slightly less than in the evenly distributed demand case.

As mentioned in the evenly distributed demand, the reason for this is that the Tata Steel IJmuiden network has a high density of stops. It is likely that the effect of en-route (non-demand) rerouting would be more positive with respect to travel and waiting times when using a network where the stops on vehicle routes are spaced further apart.
Table 6.10: Key performance indicators for comparison between rerouting everywhere and rerouting only at stops for the dominating demand scenario.

<table>
<thead>
<tr>
<th>En-Route Rerouting</th>
<th>No</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Time (s)</td>
<td>14</td>
<td>47</td>
</tr>
<tr>
<td>% of Passengers Rejected</td>
<td>18%</td>
<td>24%</td>
</tr>
<tr>
<td>Average Passenger Travel Time (s)</td>
<td>499</td>
<td>553</td>
</tr>
<tr>
<td>Average Waiting Time (s)</td>
<td>117</td>
<td>139</td>
</tr>
<tr>
<td>Average Vehicle Distance (km)</td>
<td>37</td>
<td>28</td>
</tr>
<tr>
<td>Average Veh. Dist./Passenger (km)</td>
<td>2.28</td>
<td>1.87</td>
</tr>
<tr>
<td>Objective Function</td>
<td>44529</td>
<td>45510</td>
</tr>
</tbody>
</table>

6.7. Sensitivity Analysis

In this section, a sensitivity analysis of some of the input parameters of the model is performed. In particular, in Section 6.7.1, several variations of the objective function are used to test the results. Section 6.7.2 analyzes the results when vehicle capacity and fleet size are changed. Section 6.7.3 deals with the variation of the percentage of trips that originate at the entrance in the dominating scenarios, and finally Section 6.7.4 analyzes the impact of changes in the maximum travel time for trips. For all these analyses, only the dominating high demand scenarios are analyzed. The results of these scenarios are deemed similar enough to the evenly distributed demand and medium demand types that this gives a sufficient image of the sensitivity of the model. The results of the inclusion of both types of rerouting against no rerouting is tested.

6.7.1. Objective Function

As mentioned in Section 5.3.4 the objective function that is used in the scenarios is not fixed and can be changed based on the operator’s preferences. For the sensitivity analysis, the weights $\phi, \pi, \varphi$ in the objective function below are changed:

$$\min\left[ \sum_{vehicles} \phi \cdot Travel\ Time_{veh} + \sum_{passengers} \pi \cdot Travel\ Time_{Pass} + \varphi \cdot \#\ rejected\ requests \right].$$

In the results presented so far the values $\phi = 3, \pi = 1, \varphi = 40$ have been used. The impact of changes of these parameters on the value of the original objective function are tested. Each of the values will be varied independently. The range of values that will be tested for each indicator are shown in Table 6.11. For each of the variations, the value of the original objective function (Equation (5.1)) will be calculated as well as any other relevant key performance indicators.

Table 6.11: Values of objective function parameter for sensitivity analysis. The original value is in bold.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>100</th>
</tr>
</thead>
</table>

**Vehicle Travel Time Weight Variation**

Figure 6.6a shows the value of the original objective function for the values for $\phi$ in Table 6.11 without rerouting. As the cost for vehicle time driven increases, there is an increase in the value of the objective function. This is because the number of rejections goes up as $\phi$ increase. Although the vehicle distance driven decreases, it does not decrease enough to compensate for the number of rejections.
See Figure 6.6b for the rejections and vehicle distance for each value of $\phi$. The number of rejections more than triples when $\phi = 20$ compared to when $\phi = 3$, meaning that nearly all trips are rejected. When the value of $\phi$ is between 1 and 4, the number of rejections changes only slightly, although the average vehicle distance driven changes more. There is a definitive impact when changing the value of $\phi$ to be much larger.

**No Rerouting**

![Graph](image1)

(a) Value of the original objective function for various $\phi$.  
(b) Number of rejections and average vehicle distance driven for various $\phi$.  

Figure 6.6

Figure 6.7a presents the value of the original objective function for the various values of $\phi$ with both types of rerouting. The value of the objective function starts off smaller and remains smaller than in the no rerouting cases for all $\phi$, except when $\phi = 20$, where in both cases almost all passengers are rejected. From Figure 6.7b it is apparent that the number of rejections when using both types of rerouting stays smaller than without rerouting, while the average vehicle distance driven is also larger. Concluding, rerouting has an effect for most values of $\phi$, except when this value becomes very high and almost all passengers will be rejected, in which case it performs almost identically to the no rerouting case. When rerouting is implemented in a real life case, it is unlikely that so many passengers will be rejected and likely that a reasonable value for $\phi$ will be chosen, if this objective function is used. The value of $\phi$ has an impact on the value of the objective function, but rerouting still performs well on reducing rejections for most values of $\phi$.

**Empty and Demand Rerouting**

![Graph](image2)

(a) Value of the original objective function for various $\phi$.  
(b) Number of rejections and average vehicle distance driven for various $\phi$.  

Figure 6.7
6.7. Sensitivity Analysis

**Passenger Travel Time Weight Variation**

In Figure 6.8 the value of the original objective function is shown for various values of $\pi$ both for the scenario with no rerouting and with both types of rerouting. It is clear that the objective function value does not vary nearly as much as when varying $\phi$. Furthermore, the improvement when using both types of rerouting is similar for the various values of $\pi$, although the improvement does diminish when $\pi$ becomes larger, as more passengers become rejected. The impact is smaller than the impact of varying $\phi$ as the duration of trips is already limited by the maximum travel time in Equation (5.2).

**Passenger Rejection Weight Variation**

Figure 6.9 shows the impact of varying $\phi$ on the objective function value. As with the variation of $\pi$, there is only a small impact when changing $\phi$. If $\phi$ becomes very small, it will always be more beneficial (according to the objective function) to reject passengers, and thus the value of the original objective function will increase. This is the cause of the apparent jump in the objective function value as $\phi$ goes from 20 to 30. The average travel time of passengers is about 500 seconds and the average vehicle time per passenger is about 210 seconds in the scenario with both types of rerouting. Without any rejections this leads to an objective function value of $\frac{210}{60} \times 3 + \frac{500}{60} = 18.9$. This is quite close to 20, which means that for many passengers the insertion algorithm will already reject them. However, it is unrealistic to set the value of $\phi$ so low, as in any real life case an effort will most likely be made not to reject too many passengers. The improvements that are made when using empty and demand rerouting are similar for all values of $\phi$.
6.7.2. Vehicle Capacity and Fleet Size

As mentioned in Section 5.3 there is always a trade-off between vehicle capacity and fleet size. In all scenarios that were analyzed a fleet size of 100 vehicles each with a capacity of 5 passengers were used. However, a change in fleet size or vehicle capacity could have an impact on the efficiency of the algorithm. In this section, an analysis of the key performance indicators is made for several fleet sizes and vehicle capacities. The total capacity of the system will be kept constant (or as close to constant as possible). Table 6.12 shows the various fleet sizes and vehicle capacities that will be analyzed, with the original value in bold.

Table 6.12: Fleet size and vehicle capacity variations. Original values are in bold.

<table>
<thead>
<tr>
<th>Fleet Size</th>
<th>166</th>
<th>125</th>
<th>100</th>
<th>83</th>
<th>71</th>
<th>62</th>
<th>55</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle capacity</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 6.10 shows the key performance indicators for these variations. On the x-axis, only the value of the vehicle capacity is shown as this is a more linear effect. The average vehicle distance per passenger is not shown as there is a large change in the number of vehicle between scenarios. Neither is the simulation time, which is slightly shorter the fewer vehicles are implemented, as fewer insertion checks need to be performed by the algorithm.

With a larger number of vehicles, the value of the objective function slightly increases, while it also increases with a smaller number of vehicles, compared to the standard scenario with 100 vehicles. The value of the objective function goes up when more vehicles are added, because the total vehicle distance driven is increased significantly. When using smaller vehicles, the vehicle distance might be less important from an operator perspective however, as smaller vehicles use less fuel or power. However, for this analysis the original objective function was used. When using a smaller number of vehicles, the benefit of rerouting is reduced compared to no rerouting, because there are many more rejected passengers with fewer vehicles. Passenger travel times go down with more vehicles, as there is less ride-sharing. Waiting times slightly increase when using a small number of vehicles, although only slightly if rerouting is used. In the case
with vehicles with a capacity of 3 passengers, the waiting time increases tremendously. This is because there is almost no ride sharing, so passengers have to wait for a vehicle to pick them up from one of the stops that has a vehicle. Without rerouting, this stop is often very far away. When reducing the number of vehicles, the travel times increases, as there is more ride-sharing due to the larger vehicle capacity. Rejections also steadily rise when using fewer but larger vehicles. Besides a larger value of the objective function, the only indicator where a larger fleet with smaller vehicle capacities performs worse than the original vehicle fleet is the total vehicle distance traveled. This is not surprising as there are more vehicles and, as mentioned before, the value of the vehicle travel time in the objective function might be different if a different vehicle size is used.

Overall, a change in fleet size and vehicle capacity has a clear impact on the effectiveness of the rerouting measures. The best value of the objective function is achieved when the original fleet size and vehicle capacity are used. The reduction in objective function value decreases as larger vehicles are used. The percentage of rejected passengers is similar for a vehicle capacity of 5 or less, but then steadily rises as larger and larger vehicles are used, and the benefit of rerouting all but disappears when 50 vehicles with a capacity of 10 passengers are used. Average passenger travel times increase as the vehicle size gets larger and fewer vehicles are used, but passenger waiting time stays similar. The benefit in travel times decreases with larger vehicles, but the benefit in waiting time stays relatively constant. Total vehicle distance increases the more vehicles are used.

As mentioned before, one of the main benefits of using automated vehicles is that more but smaller vehicles can be used as there are no personnel costs. These results show that using smaller vehicles indeed bring a benefit to the percentage of rejected passengers and passenger travel times, at the cost of increasing the total travel distance. When vehicle size becomes too large, the effect of rerouting diminishes, indicating that a smaller vehicle size is ideal, at least on this network.

### 6.7.3. Dominating Demand Trip Distribution

The percentage of passengers that arrive at the entrance is also an input parameter that can be varied. Figure 6.11 shows the key performance indicators for the variation of this percentage from 0 (similar to evenly distributed demand) to 100%.

The objective function value increases significantly in the case without rerouting as the percentage of passengers arriving at the entrance increases. This stems entirely from the increase in the number of rejected passengers. Without rerouting, the vehicles will not go to the entrance to pick up all passengers arriving there, leading to a large number of rejections. The decrease in passenger waiting times for the no rerouting scenario is therefore meaningless, it simply means that the only passengers that are picked up are those when a vehicle was coincidentally close to the entrance.

When using rerouting, the value of the objective function almost does not change and neither does the percentage of rejected passengers. These values were already similar for the original scenarios analyzed (evenly distributed demand and 40% demand at entrance), see Table 6.8. However, there are large changes in travel and waiting times. The waiting time increases to an average of 250 seconds when 100% of the demand originates at the entrance, which directly leads to the increase in passenger travel times as well. The increase in waiting time occurs because all vehicles always need to travel back to the entrance, no matter where they dropped off their passengers. When a lower percentage of trips originates at the entrance, the vehicles can pick up a passenger close to the drop off location of their previous passenger, but this is not the case when the demand at the entrance becomes very high. However, the more demand originates from one location, the higher the percentage of ride sharing is, which in turn leads to a smaller value of the average vehicle distance per passenger. This decrease is the reason that the objective function value is similar for all percentages of demand originating at the entrance.

In conclusion, there is a clear effect on the key performance indicators when the percentage of demand originating from one location changes. The increase in passenger waiting times could however be alleviated to some extent by using more vehicles, but this will require an investment, and will lead to an increase in vehicle distance driven. However, considering the number of rejections without using rerouting, there is a definite benefit in using rerouting regardless of the percentage of trips originating at a single location.
Figure 6.11: Key performance indicators for variations in the percentage of passengers originating at the entrance for dominating high demand scenarios.
6.7.4. Maximum Travel Time

The final input parameter that is analyzed in this chapter is the maximum travel time that passengers are willing to spend on their trip. This was set to the maximum of either 1.5 times their direct travel time or their direct travel time plus 10 minutes. Since the average travel time was around 500 seconds in most scenarios, the maximum was nearly always the direct travel time plus 10 minutes. In this sensitivity analysis, a flat addition to the travel time is thus the only thing that is varied. That is, the maximum travel time used is:

$$\text{max. Travel Time} = \text{Direct Travel Time} + \text{Extra Travel Time}.$$  \hfill (6.2)

In this analysis the Extra Travel Time will be varied between 100 and 1000 seconds (where the original scenarios all used 600 seconds). The resulting key performance indicators are shown in Figure 6.12. As the maximum travel time of passengers increases, there is a sharp reduction in the percentage of rejected passengers, which is to be expected. Even with a maximum extra travel time of over 15 minutes, 20% of all passengers are rejected without rerouting. A reduction in the number of rejected passengers when using rerouting is apparent at any maximum extra travel time. The average travel time and waiting time is shorter without rerouting when using very short maximum extra travel times, but this is because any passenger that would have a long waiting or travel time is rejected. The longer the maximum extra travel time, the larger the improvement in average passenger travel time and waiting time becomes when using rerouting. This is because without rerouting, but with a very long maximum extra travel time, vehicles can pick up passengers even if they are very far away. Average vehicle distance per passenger reduces for the rerouting case when a longer travel time is available as this allows for more ride sharing. Without rerouting, the vehicle distance per passenger is short with a short maximum extra travel time because of the many rejections. The combination of the above leads to a larger improvement in the objective function value when the maximum extra travel time increases, mostly because the average passenger travel times deviate more.

Regardless of the maximum extra travel time, there are still clear benefits to using rerouting, especially in the reduction in the percentage of rejected passengers. Assuming that the transport operator does not want a very large number of rejections, rerouting also leads to a reduction in average passenger travel time and waiting time. Again, the only trade-off being made is in the average vehicle distance driven.
Figure 6.12: Key performance indicators for variations in the maximum travel time of passengers for dominating high demand scenarios with no rerouting and with empty and demand rerouting.
Conclusion & Discussion

In this chapter, the answer to the research question is given and the main results from the simulations in this report are summarized in Section 7.1. Some implications of the results for transport operators, as well as some guidelines for implementing the algorithms in this report, are presented in Section 7.2. Finally, the limitations of the analysis in this report as well as some pointers for further research can be found in Section 7.3.

7.1. Key Findings and Contributions

At the start of this report the research question was phrased:

“How can demand forecasts be used to improve the routing of vehicles in a Dial-a-Ride Problem?”

In this report it is shown that demand forecasts can be used to improve the routing of vehicles in the Dial-a-Ride problem in two main ways. First of all, empty vehicles can be rerouted to public transport stops where demand is anticipated; this type of rerouting has already been studied in other papers. Secondly, if a new request comes in, the choice of which vehicle should pick up this request could be changed by analyzing the demand forecast at the stop where the request is located; this type of rerouting is called demand rerouting in this report.

The effectiveness of these rerouting measures was tested for several different scenarios on the road network of the Tata Steel IJmuiden area. In particular, the rerouting measures were tested in a scenario where the demand is spread out over all the stops and in a scenario where the demand is mostly located at a single stop (the entrance to the area). The main findings of this report for the two types of rerouting are the following:

Empty Vehicle Rerouting  Empty vehicle rerouting can lead to a large reduction in the number of rejected passengers of up to 98%. Furthermore it can reduce passenger travel and waiting times by up to 5 and 16% respectively. The main trade-off is that the total distance driven by all the vehicles increases by up to 60%.

Demand Rerouting  Demand rerouting has a less pronounced effect on rejections, reducing them by up to 7%. However, average passenger travel and waiting times can be reduced by up to 6 and 30% respectively, while only requiring an increase of up to 9% in the total distance driven by all vehicles.

Both Types of Rerouting  Combining the two types of rerouting combines the benefits of both. The number of rejected passengers can be reduced by 98%, while passenger travel and waiting times are down 11 and 46% respectively. This required an increase in the vehicle distance driven of up to 66%. However, because of the large reduction in rejected passengers, this translates to only a 25% increase in the vehicle distance driven per passenger.
In conclusion, when demand forecasts are used for empty vehicle and demand rerouting, there are almost no rejected passengers. Furthermore, passenger travel and waiting times can be reduced significantly. This comes at a cost to vehicle distance driven per passenger. However, when automated vehicles are used, this increase in vehicle distance driven is not as significant, as costs for distance driven are much lower than when using non-automated vehicles.

A sensitivity analysis of these results shows that the effects of using both types of rerouting are apparent in all but the most extreme cases (where for example more than 80% of the demand is rejected).

The main conclusion of this report is thus that demand forecasts can be used to reliably improve the routing in a Dial-a-Ride problem. Assuming that transport operators do not want to reject too many passengers, empty vehicle rerouting should always be included in algorithms to solve the Dial-a-Ride problem. However, the addition of demand rerouting to empty vehicle rerouting leads to an even larger reduction in the number of rejected passengers, as well as significant additional reductions in passenger travel and waiting times. The inclusion of demand rerouting only increases the vehicle distance driven marginally, compared to using just empty vehicle rerouting.

### 7.2. Implications for Transport Operators

Many transport operators worldwide are currently in the preliminary research phase when it comes to implementing automated vehicles in their operating schedules. Transport operators in many countries are also gathering large amounts of data about the transport behaviour of their customers; however, this data is often not used extensively yet. Furthermore, transport operators are facing competition by regular car traffic and by new companies offering ride-sharing taxis. Because of this competition, transport operators are forced to think about how they too can implement demand-responsive transport, as it often better serves the transport demand and is more suitable for dealing with fluctuations in transportation demand.

This report shows that the use of demand forecasts can improve demand-responsive transport significantly. Although these demand forecasts are mostly not yet available to transport operators, there is a large amount of customer data available to many operators. Transport operators should focus on creating accurate demand forecasts from this data, which could then be used for demand-responsive transport services. Especially the reduction in passenger travel time and waiting time, combined with a reduction in passenger rejections, could lead to a better position for transport operators to compete with current ride-sharing services.

Thus, the advice to transport operators is to investigate the possibilities of using customer data to generate demand forecasts, which can then be used to plan demand-responsive transport in applicable areas. Further research will have to point out whether demand forecasts can also be used in larger scale demand-responsive transport, but this research gives no indication that this would not be the case. Regardless, transport operators should take the first steps of implementing demand-responsive transport with demand forecasts, if only to gain more experience to later implement the system in larger-scale systems.

Note that the objective function used in this report does not explicitly take the operator costs into account, only in the form of the vehicle distance traveled. Investment costs, for example, are not taken into account. This is something that should be analyzed in further research.

### 7.3. Study Limitations and Further Research

Several assumptions for the model are listed in Section 4.6. One of the main limitations of the model is that a full distance and path matrix are required for the network. In larger networks (for example on a city level) it is perhaps not possible to store a complete distance and path matrix in memory. In these cases, a distance and path calculation needs to be performed at every insertion step, significantly increasing computation times. Furthermore, the requirement of a distance and path matrix also removes the possibility of including stochastic travel times.

The absence of stochastic travel times can be limiting in the case of a larger network, especially where the automated vehicles share their infrastructure with other traffic, or on a network with traffic lights. Further scenario analyses should be made when a deterministic path matrix is not available and
when stochastic travel times are included. It is expected that rerouting will still improve the quality of the solution, but it is unclear whether it will improve by the same amount as in the deterministic scenario.

In this research, only the morning peak was modeled, but the rerouting can of course also be implemented during the evening peak or during the rest of the day. In the evening peak there is not an origin with a high share of the demand, but a destination with a high share of the demand. This could change the efficiency of the rerouting as the demand forecast is only used for determining where trips might originate. Simulations for the evening peak might give results more similar to the evenly distributed demand scenarios than to the dominating demand scenarios.

The scalability of the algorithm should also be tested, even when using deterministic travel times. Simulation times for the network and demand level in this research were not prohibitively long, but this could become the case if much larger networks are used, more vehicles are used, or the demand level is much higher. The insertion algorithm can of course always be run for a limited time, but this will have an effect on the solution quality.

Another limitation that will have an impact on the efficiency of the algorithm, especially on empty vehicle rerouting, is the fact that there is no capacity restriction at stops. Thus, an unlimited number of vehicles can be sent to a stop, which is used especially in the dominating demand scenarios (where up to 50 vehicles can be present at the entrance). If there is a capacity restriction at (some of the) stops, the empty vehicle rerouting algorithm would have to be changed. Capacity restrictions at stops could be implemented and a limit on the number of required vehicles at each stop should then be set.

The final limitation in this work is that the demand pattern is generated based on a known demand distribution and thus the demand forecast is very accurate. In real life cases, the demand forecast will have a larger variability, especially in cases where there is a low demand at some of the stops in the network. Further research should test the rerouting algorithms on real demand patterns, with more variability in the demand generation as well. Furthermore, it is important to test the quality of the rerouting algorithms when the demand forecast is not completely accurate, as will mostly be the case in real applications. In this research only two demand patterns were tested, a completely evenly distributed demand pattern and a demand pattern where the demand at one location dominates the demand at other locations. However, in real-life applications there will often be a much more distributed demand, where there are perhaps multiple locations with a high demand and multiple locations with lower demands.

Aside from the mentioned limitations, there are several other improvements that could possibly be made to the quality of the solutions of the algorithm. First of all, it might be beneficial to use vehicles with varying sizes. Especially in the case where there are a few locations where a lot of demand originates, sending larger vehicles to those locations while using smaller vehicles for the other locations could be beneficial. This would of course require adaptations to the empty vehicle rerouting and demand rerouting algorithms.

Another adaptation to the current model, which uses only demand-responsive transport, is the combination of line-based transport and demand-responsive transport. It might be that using a line-based service for the stops with highest demand while using demand-responsive transport as a feeder service for the stops with low demand is more efficient than using only demand-responsive transport.
Acknowledgement

This thesis is the conclusion of my master in Transport, Infrastructure and Logistics at the TU Delft. This thesis was conducted at Connexxion, a subsidiary of Transdev from March until September 2016.

I would like to thank Henk Post at Connexxion for offering me a spot there and for making me interested in this field of research. Furthermore, he provided insight in the algorithms used in solving the Dial-a-Ride Problem. In particular, his recommendation of the paper by Jaw, Odoni, Psaraftis and Wilson proved invaluable in my research.

At the TU Delft I would like to thank Oded Cats for the many papers he provided that helped me define the direction of my research and also for the meetings that helped me push my research further in the right direction. I want to thank Karen Aardal for being the chair of my graduation committee after having supervised my internship as well and Martijn Warnier for his comments on the many draft versions of this report.

I would like to thank everybody at Connexxion for their help with my struggles with Delphi, always finding time to help me sort out an error that was mostly entirely my own fault.

I would finally like to thank my family and girlfriend for their support during all my time spent in university and my father for getting me inspired to work in mathematics.

M.J. van Engelen
Den Haag, September 2016


