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Wave-Exciting Forces and Moments on an Ocean Platform

1. Seakeeping
2. Platform
3. Forces
4 Theory.

By

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ABSTRACT

The authors describe a new method of predicting the wave-exciting forces and moments acting on an ocean platform restrained in oblique seas. Procedures are based on strip theory.

The wave-exciting forces and moments on restrained cylinders have been investigated before, in beam seas as well as in oblique seas, but the cylinders were always Lewis cylindrical forms. In this study, the two-dimensional method developed by Frank is extended to the calculation of wave forces and moments, and the strip method devised by Grim is applied to obtain the three-dimensional forces and moments. The results of the numerical calculations for a Series 60 model are compared with the results of the latest experimental work by Lalangas. The results of an experiment conducted for the present study, on a rectangular light-drafted barge restrained in quartering seas, are compared with theoretical predictions. Theory and experiment are in generally good agreement in both cases.

INTRODUCTION

The vertical wave-exciting forces on Lewis cylindrical forms restrained in beam seas were calculated by Grim in 1960.[1] His method was later applied by Tamura[2] to the calculation of sway- and roll-exciting forces and moments on Lewis cylinders restrained in beam seas. Tasai then performed approximate calculations of the lateral wave-exciting forces and moments in oblique waves according to Watanabe's strip method.[3] Grim and Schenzle, in 1968, further refined the calculation of lateral exciting forces and moments on a ship restrained in oblique seas, by applying the strip method [crossflow hypothesis].[4]

References and illustrations at end of paper.

Grim's method is based on the assumption that the disturbance of an incident wave caused by the ship's body is represented by the potential used in describing the water flow around the body when the body is oscillating harmonically in the calm water surface. This potential, together with the incident wave potential, constitutes the potential that describes the flow around the body under restraint in waves. In the present study, we select for disturbance the potential used by Frank.[5] His method enables us to compute hydrodynamic forces and moments not only for the non-Lewis cylindrical form, but also for the widely varying configurations of ocean platforms of the semisubmersible type.

To determine the degree of reliability of the prediction method, theoretical values are compared with Lalangas' latest experimental results for a Series 60 model[6] and with additional experimental results for a rectangular barge obtained in the present study.

COORDINATE SYSTEMS

Let o-XYZ and o-xyz be the right-hand rectangular coordinate systems, as illustrated in Fig. 1. Coordinate planes o-XZ and o-xz lie on the calm water surface, and the Y- and y-axes point vertically upward.

Let the incidence of the wave be designated by μ and let the wave progress in the positive X-direction. Then the wave profile is

$$h = a \cos(\nu X - \omega t)$$

in the space coordinates, and

$$h = a \cos(\nu x \sin\mu + \nu z \cos\mu - \omega t) \qquad [1]$$

in the body coordinates,

where a = wave amplitude
ν = wave number $[\omega^2/g]$
ω = circular frequency of the wave.

Now, suppose two vertical control planes cut the body at z and z + dz, and observe the wave motion within the fictitiously confined domain. The wave equation [Eq. 1] can be interpreted by noting that the term $\nu x \sin\mu$ determines the wave form in the two-dimensional domain and that the term $\nu z \cos\mu$ represents the phase shift of the wave at x = 0 and z = z from the crest of the incident wave at the origin. Cutting the whole body by many vertical control planes such as are demonstrated above, we can apply the strip method, or the crossflow hypothesis. That is, the three-dimensional forces on the restrained body, induced by the oblique waves, can be determined approximately by summation of the two-dimensional elementary forces induced in the waves of the strip domain at the subdivided elementary sections over the length of the body.

WAVE POTENTIAL

Let us suppose that there is no obstacle in the strip domain defined by the two control planes. Then the potential of the flow is only that of the wave [Eq. 1] and is represented by

$$\varphi = \frac{ga}{\omega} e^{\nu y} \sin(\nu x \sin\mu + \nu z \cos\mu - \omega t)$$

or, conveniently, by the wave potential per unit amplitude of the incident wave,

$$\frac{1}{a}\varphi(x,y,z\ ;\ t,\mu) = \frac{g}{\omega} e^{\nu y} \sin(\nu x \sin\mu + \nu z \cos\mu - \omega t) \qquad [2]$$

The terms $\nu x \sin\mu$ and $\nu z \cos\mu$ have physical meanings interpreted as in the preceding section.

The wave potential is broken down into two component potentials, of the form

$$\frac{1}{a}\varphi_o = \frac{g}{\omega} e^{\nu y} \sin(\nu x \sin\mu) \cdot \cos(\nu z \cos\mu - \omega t) \qquad [3]$$

$$\frac{1}{a}\varphi_e = \frac{g}{\omega} e^{\nu y} \cos(\nu x \sin\mu) \cdot \sin(\nu z \cos\mu - \omega t) \qquad [4]$$

Since the potentials $[1/a]\phi_o$ and $[1/a]\phi_e$ are, respectively, the odd and even functions of x, the odd function $[1/a]\phi_o$ is applied to the asymmetric motion of water about the y-axis in the xy-plane, while the even function, $[1/a]\phi_e$, is applied to the symmetric motion of water about the y-axis. Therefore, the odd function will not be employed as the potential causing the sway- and roll-exciting force and moment on a cylinder section in the beam sea, while the even one will be used as the potential causing the heave-exciting on the section.

DISTURBANCE POTENTIAL

The non-existence of the sectional body in the strip domain, assumed in the preceding section, does not, in fact, hold; and this creates the disturbance to the wave potential flow represented by Eqs. 2, 3 and 4. The disturbance is mathematically expressed by a distribution of pulsating sources over the section surface. Frank[5] has reported on the source-distribution method as applied to the calculation of the hydrodynamic forces and moments on oscillating cylinders in, or below, the free surface of deep water. The potential used by Frank is employed as our disturbance potential. Since the disturbance is supposed to respond to the incident wave and the position and form of the sectional body, it is written

$$\Phi^{(m)}(x,y,z\ ;\ \mu,t) = R_e\left[\int_C Q^{(m)}(s) \cdot G(x,y\ ;\ \xi,\eta) e^{i(\nu z \cos\mu - \omega t)}\, ds\right]. \qquad [5]$$

In this equation, m designates the mode of excitation [m - 2,3,4 = sway, heave, roll]. The expression $Q[m][s]$ designates the unknown complex source intensities distributed along the section contour C. The expression depends on the mode of excitation, the geometry of the section, and the incident wave. The expression $G[x,y;\ \xi,\eta]$ is the pulsating source potential of unit intensity at the point ξ,η in the lower-half xy-plane of Fig. 2. The term $e^{i\nu z \cos\mu}$

represents the position of the strip section z where the disturbance occurs in response to the oblique incident wave of wave number ν and incidence μ. This term takes into account the phase difference of maxima of the disturbance pressure and the force acting on the section contour from the crest of the incident wave at the coordinate origin.

Since $Q^{[m]}$ and G are the complex source intensity and function, respectively, let them be

$$Q^{(m)} = Q_r^{(m)} + iQ_i^{(m)}$$

$$G = G_r - iG_i ,$$

where $i = \sqrt{-1}$ and where $Q_r^{[m]}$ and $Q_i^{[m]}$ are real and imaginary parts of $Q^{[m]}$, and G_r and $-G_i$ are real and imaginary parts of G. Eq. 5 is then changed to

$$\phi^{(m)} = \int_c (Q_r^{(m)} G_r + Q_i^{(m)} G_i)\, ds \cdot \cos(\nu z \cos\mu - \omega t) - \int_c (-Q_r^{(m)} G_i + Q_i^{(m)} G_r)\, ds \cdot \sin(\nu z \cos\mu - \omega t) \quad \text{. . . . [6]}$$

BOUNDARY CONDITION

The ultimately required potential which describes the water flow around the restrained body in the strip region is therefore obtained by superimposing the wave potential [Eq. 2] and the corresponding disturbance potential [Eq. 6]. Specifically, the potentials are written according to the mode of excitation [i.e., sway, heave and roll], as

$$\Phi_S = \phi^{(2)} + \frac{1}{a}\varphi_o$$

$$\Phi_H = \phi^{(3)} + \frac{1}{a}\varphi_e$$

$$\Phi_R = \phi^{(4)} + \frac{1}{a}\varphi_o \quad \text{. . . . . [7]}$$

The potentials Φ_H, Φ_S and Φ_R satisfy the following four conditions, out of the five required.[7]

1. The continuity of the liquid in the whole domain
2. The linearized free-surface condition
3. The radiation condition
4. The deepwater condition.

The fifth condition to be satisfied is the kinematical boundary condition on the body surface [no flow through the surface of the restrained body]. For the three modes of excitation, we write

$$\frac{\partial}{\partial n}(\Phi_H) = 0 \;;\; \frac{\partial}{\partial n}(\Phi_S) = 0 \;;\; \frac{\partial}{\partial n}(\Phi_R) = 0 , \quad \text{. . . . . [8]}$$

where $\partial/\partial n$ designates the normal derivative of the function at a point on the body surface. The equations are reduced to simultaneous linear algebraic equation systems [see Appendix for explicit expressions].

HYDRODYNAMIC PRESSURE AND FORCE

The linear hydrodynamic pressures induced at a point on the body surface by the incident wave of unit amplitude are

$$\frac{p_S}{a} = -\rho \frac{\partial \Phi_S}{\partial t} \quad \text{[for sway]}$$

$$\frac{p_H}{a} = -\rho \frac{\partial \Phi_H}{\partial t} \quad \text{[for heave]}$$

$$\frac{p_R}{a} = -\rho \frac{\partial \Phi_R}{\partial t} \quad \text{[for roll]. . . [9]}$$

The integration of these pressures over the submerged surface of the body gives us the wave-exciting forces and moments on the body per unit amplitude of the incident wave. The following symbols are introduced for the forces and moments.

F_z = surge-exciting force
F_x = sway-exciting force
F_y = heave-exciting force
M_z = roll-exciting moment about the origin
M_x = pitch-exciting moment about the origin
M_y = yaw-exciting moment about the origin

We formulate them as

$$\frac{F_y}{a} = \int_{(L)}\int_{(C)} \frac{p_H}{a}\, dx\, dz$$

$$\frac{F_x}{a} = \int_{(L)}\int_{(C)} -\frac{p_S}{a}\, dy\, dz$$

$$\frac{M_y}{a} = \int_{(L)}\int_{(C)} -\frac{p_S}{a}\, z\, dy\, dz$$

$$\frac{M_x}{a} = -\int_{(L)}\int_{(C)} \frac{p_H}{a}\, z\, dx\, dz$$

$$\frac{M_z}{a} = \int_{(L)}\int_{(C)} \frac{p_R}{a} (x\, dx + y\, dy)\, dz, \quad [10]$$

where [L] and [C] mean that the integration is executed over the length of the body and over the contour of a section, respectively. As to the surge-exciting force F_z/a, it may, if the restrained body is geometrically symmetrical about the x-axis, be calculated by taking the strip in the direction parallel to the z-axis. The positive signs of the forces and moments take the positive directions of the coordinate systems [Fig. 1].

EXPERIMENTS AND CALCULATIONS

To establish the applicability of the theoretical method to the calculation of wave forces and moments on ocean platforms floating in oblique seas, two very different models were chosen: the Series 60 model of $C_B = 0.6$ and a light-drafted rectangular barge model. The model particulars are given in Table 1.

The experiment with the Series 60 model was reported by Lalangas,[6] but the rectangular barge model was tested under the present study program in Davidson Laboratory's Tank 2. The forces and moments were measured by a three-component dynamometer at a 45-degree inclination to the longitudinal axis of the barge, which corresponds to $\mu = 135$ and 225 degrees, according to our definition of μ. At $\mu = 135$ degrees, the heave-, surge- and pitch-exciting forces and moments were measured; and at $\mu = 225$ degrees, the sway- and roll-exciting forces and moments were measured. This was done by rotating the model 90 degrees while the dynamometer remained fixed. The origin of the body coordinates lay in the calm-water level, and the moments were measured about the origin. The wave recorder was located 40.1 in. ahead of the origin [Fig. 3]. The phase difference of the wave force and moment from the crest of the incident wave at the origin was determined by correcting the location of the wave recorder.

The numerical calculation of the exciting forces and moments for the two models was carried out on the IBM 360 computer at Stevens Institute of Technology, and the results were compared with experimental results [see Figs. 4 to 18]. It was found that the roll-exciting moment is largely dependent upon the number of sources and the position of the moment center. In calculating the surge-exciting force on the barge model, an attempt was made to take longitudinal strips, and the side force in the direction of the z-axis was taken as the surge force [Fig. 3]. End effects were not corrected in any of the calculations.

CONCLUSIONS

1. The agreement between theoretical and experimental results is very good for the ship model. This agreement is attributable to the slenderness of the body, as expected.

2. The agreement between theoretical and experimental results is fairly good for the barge. The agreement is attributable to the large end effects.

3. The agreement between theoretical and experimental results for both ship model and barge model is generally better for the vertical forces than for the lateral forces.

ACKNOWLEDGMENTS

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NOMENCLATURE

- A_W = water-plane area
- a = wave amplitude
- B = breadth of model
- F = force
- G = source potential
- g = gravitational constant
- h = wave elevation
- L = length of model
- M = moment
- m = number indicating mode of excitation
- o = origin of the body coordinate system
- p = hydrodynamic pressure
- Q = source intensity
- s = contour length
- t = time
- X,Y,Z = space coordinates
- x,y,z = body coordinates
- ϵ = phase difference
- λ = wave length
- μ = incident angle
- ν = wave number
- ω = circular frequency of the wave
- ξ = x-coordinate of position of source
- η = y-coordinate of position of source

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APPENDIX

The Kinematical Boundary Conditions

The kinematical boundary conditions of Eq. 8 are presented below as formulated by Frank,[5] with Frank's symbols retained.

The normal velocity components of the wave potentials [Eqs. 3 and 4] are, explicitly,

$$\frac{1}{a}\frac{\partial\varphi_e}{\partial n} = -\omega e^{\nu y_i}\left[\sin\mu \cdot \sin(\nu x_i \sin\mu)\sin\alpha_i + \cos(\nu x_i \sin\mu)\cos\alpha_i\right] \cdot \sin(\nu z\cos\mu - \omega t)$$

$$\frac{1}{a}\frac{\partial\varphi_o}{\partial n} = \omega e^{\nu y_i}\left[\sin\mu \cdot \cos(\nu x_i \sin\mu)\sin\alpha_i - \sin(\nu x_i \sin\mu)\cos\alpha_i\right] \cdot \cos(\nu z\cos\mu - \omega t)$$

where α_i is the slope of the contour at a point x_i, y_i [Fig. 2]. Thus the kinematical conditions at the point x_i, y_i on the body surface are, for sway,

$$\sum_{j=1}^{N} Q_j^{(2)} I_{ij}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} J_{ij}^{(2)}$$

$$= -\omega e^{\nu y_i}\left[\sin\mu \cdot \cos(\nu x_i \cdot \sin\mu)\sin\alpha_i - \sin(\nu x_i \cdot \sin\mu)\cos\alpha_i\right]$$

$$-\sum_{j=1}^{N} Q_j^{(2)} J_{ij}^{(2)} + \sum_{j=1}^{N} Q_{N+j}^{(2)} I_{ij}^{(2)} = 0,$$

and are, for heave,

$$\sum_{j=1}^{N} Q_j^{(3)} I_{ij}^{(3)} + \sum_{j=1}^{N} Q_{N+j}^{(3)} J_{ij}^{(3)} = 0$$

$$-\sum_{j=1}^{N} Q_j^{(3)} J_{ij}^{(3)} + \sum_{j=1}^{N} Q_{N+j}^{(3)} I_{ij}^{(3)}$$

$$= -\omega e^{\nu y_i}\left[\sin\mu \cdot \sin(\nu x_i \cdot \sin\mu)\sin\alpha_i + \cos(\nu x_i \cdot \sin\mu)\cos\alpha_i\right]$$

where $I_{ij}^{[m]}$ and $J_{ij}^{[m]}$ are the "influence coefficients" given in the Appendix of Frank's report. For roll, the formula is the same as for sway, where only the mode number takes 4.

If N sources are taken [i.e., i = 1,2,... N], we obtain a [2N]·[2N] equation system with 2N unknowns, the real and imaginary parts of the unknown complex source intensities Q.

The Hydrodynamic Forces and Moments

The hydrodynamic forces and moments on a section of the body are the pressure integrals over the section surface. They are expressed as noted below.

For heave,

$$\int_{(C)} \frac{p_H}{a}\cdot dx = \frac{P_{Hc}}{a}\cos(\nu z\cos\mu - \omega t) + \frac{P_{Hs}}{a}\sin(\nu z\cos\mu - \omega t).$$

For sway,

$$\int_{(C)} \frac{p_S}{a}\cdot dy = \frac{P_{Sc}}{a}\cos(\nu z\cos\mu - \omega t) + \frac{P_{Ss}}{a}\sin(\nu z\cos\mu - \omega t).$$

For roll,

$$\int_{(C)} \frac{p_R}{a}\cdot(x\,dx + y\,dy) = \frac{P_{Rc}}{a}\cos(\nu z\cos\mu - \omega t) + \frac{P_{Rs}}{a}\sin(\nu z\cos\mu - \omega t).$$

Integrating heave, sway forces and roll moment over the length of the body [Eq. 10], we obtain the total forces and moments. Heave- and roll-exciting moment, for example, become

$$\frac{F_y}{a} = \int_{(L)} \frac{p_{Hc}}{a} \cos\,(\nu z \cos\mu - \omega t)\,dz + \int_{(L)} \frac{p_{Hs}}{a} \sin\,(\nu z \cos\mu - \omega t)\,dz$$

$$= \frac{F_{yc}}{a} \cos\omega t + \frac{F_{ys}}{a} \sin\omega t$$

$$\frac{M_z}{a} = \int_{(L)} \frac{p_{Rc}}{a} \cos\,(\nu z \cos\mu - \omega t)\,dz \quad \int_{(L)} \frac{p_{Rs}}{a} \sin\,(\nu z \cos\mu - \omega t)\,dz$$

$$= \frac{M_{zc}}{a} \cos\omega t + \frac{M_{zs}}{a} \sin\omega t\,.$$

Each force and moment has two components which determine the magnitude and the phase difference; that is,

$$|F_y| = (F_{yc}^2 + F_{ys}^2)^{\frac{1}{2}}\;;$$

$$|M_z| = (M_{zc}^2 + M_{zs}^2)^{\frac{1}{2}}$$

$$\varepsilon_{fyh} = \tan^{-1}\left(\frac{F_{ys}}{F_{yc}}\right)\;;$$

$$\varepsilon_{mzh} = \tan^{-1}\left(\frac{M_{zs}}{M_{zc}}\right).$$

Table 1

	Series 60 Model	Barge Model
LBP (L)	5.00 ft	15 in.
Breadth (B)	0.667 ft	10 in.
Draft (H)	0.267 ft	1 in.
Displacement (FW)	33.27 lb	5.41 lb
LCG (abaft midship section)	0.075 ft	0
VCG (below waterline)	0.022 ft	0
Rudder area	0.030 ft^2	Nil
Waterplane area	2.355 ft^2	150 $in.^2$
Load waterline coefficient	0.706	1.0
Section coefficient	-	0.988 throughout

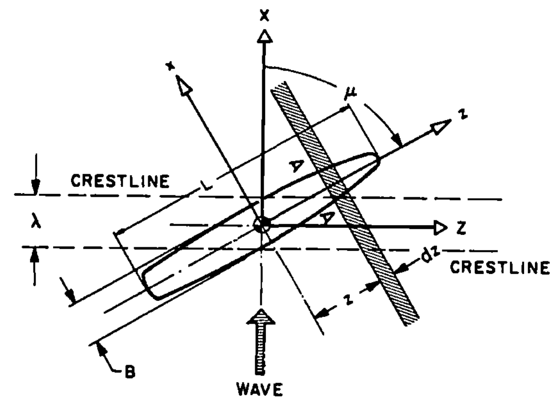


FIG. I. COORDINATE SYSTEMS

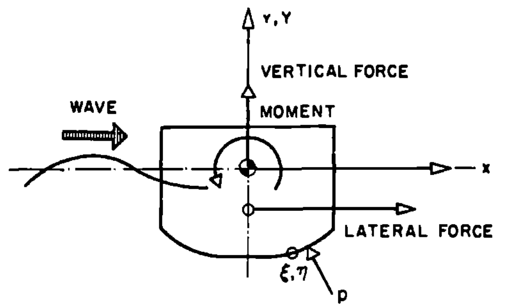


FIG. 2. STRIP SECTION A-A

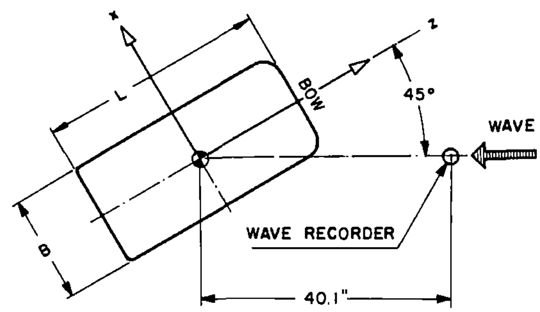


FIG. 3. BARGE IN AN OBLIQUE SEA

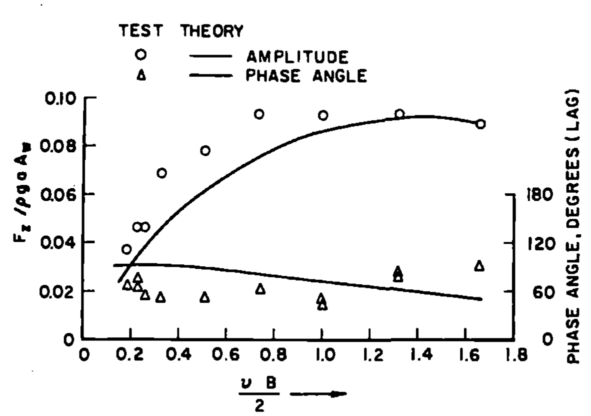


FIG. 4. SURGE-EXCITING FORCE FOR BARGE
(μ=135°)

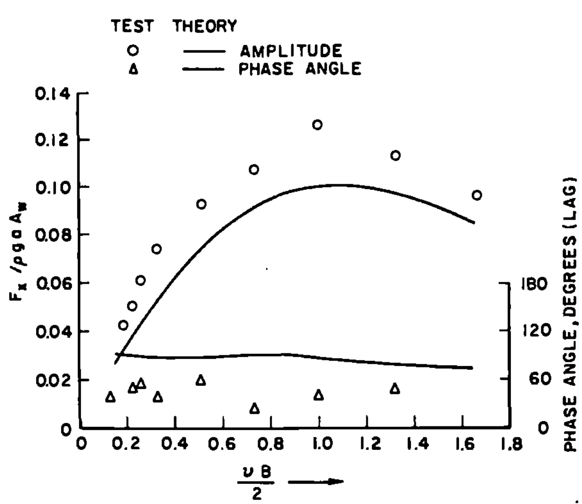


FIG. 5. SWAY-EXCITING FORCE FOR BARGE
(μ=225°)

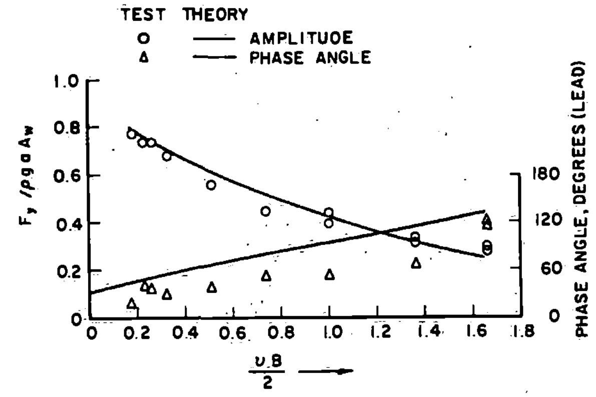


FIG. 6. HEAVE-EXCITING FORCE FOR BARGE
(μ=135°)

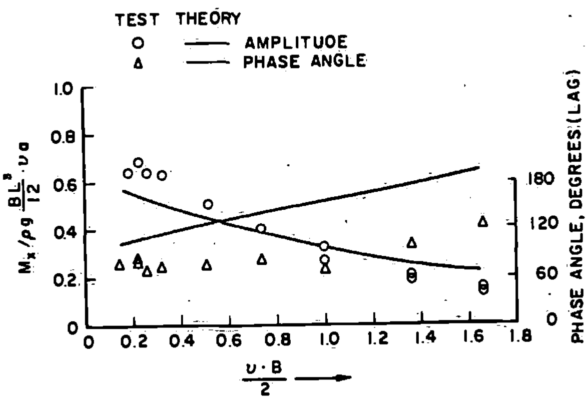


FIG. 8. PITCH-EXCITING MOMENT FOR BARGE
(μ=135°)

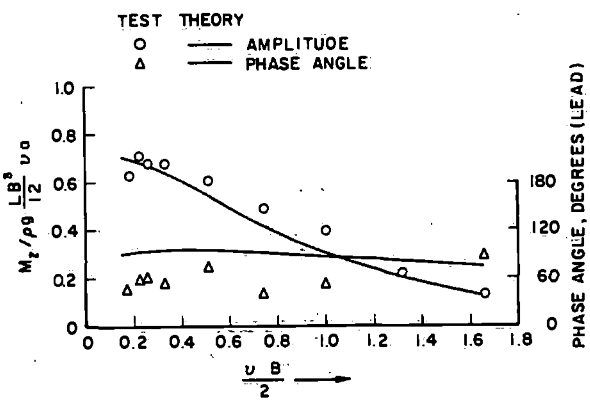


FIG. 7. ROLL-EXCITING MOMENT FOR BARGE
(μ=225°)

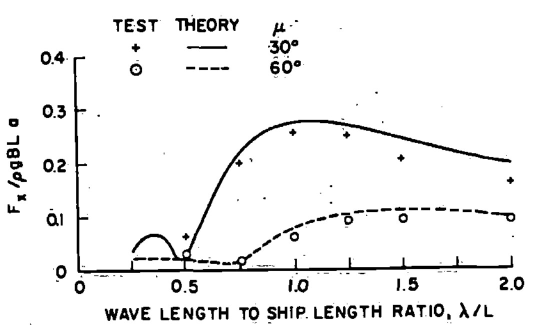


FIG. 9. AMPLITUOE OF SWAY-EXCITING FORCE FOR SERIES 60 MOOEL

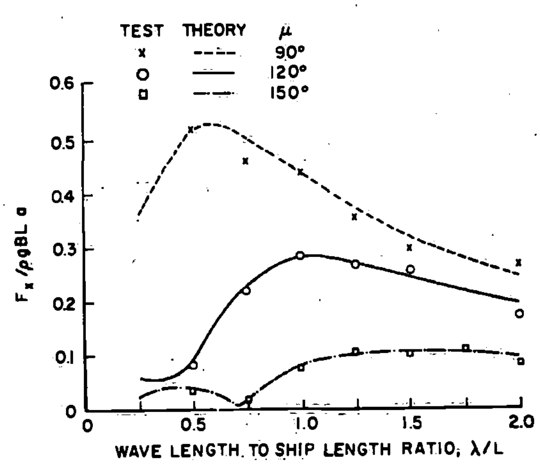
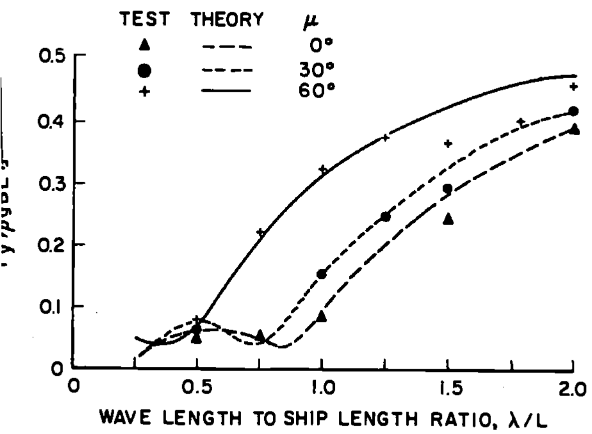


FIG. 10. AMPLITUOE OF SWAY-EXCITING FORCE FOR SERIES 60 MOOEL



'IG. 11. AMPLITUDE OF HEAVE-EXCITING FORCE FOR SERIES 60 MODEL

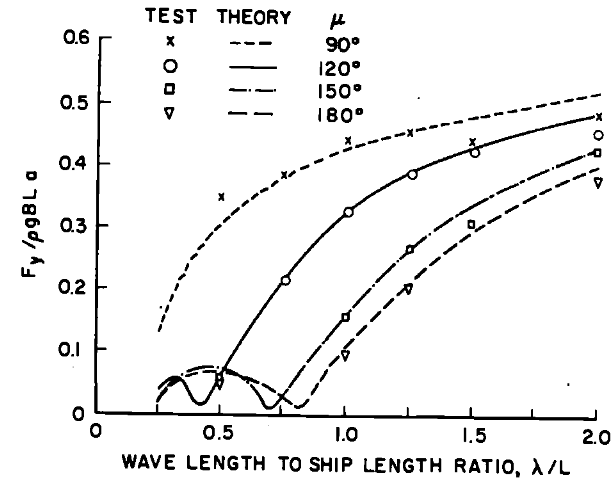
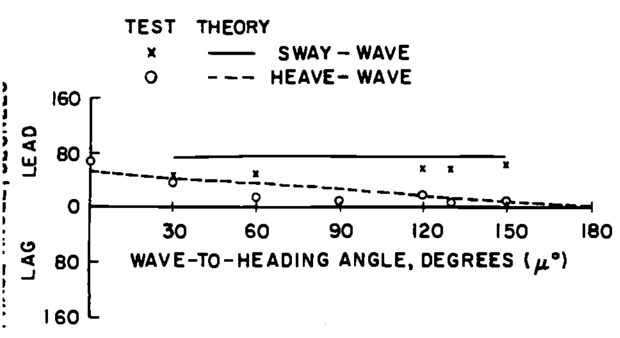


FIG. 12. AMPLITUDE OF HEAVE-EXCITING FORCE FOR SERIES 60 MODEL



G. 13. SWAY-WAVE AND HEAVE-WAVE PHASE ANGLES, 1.0 L WAVES FOR SERIES 60 MODEL

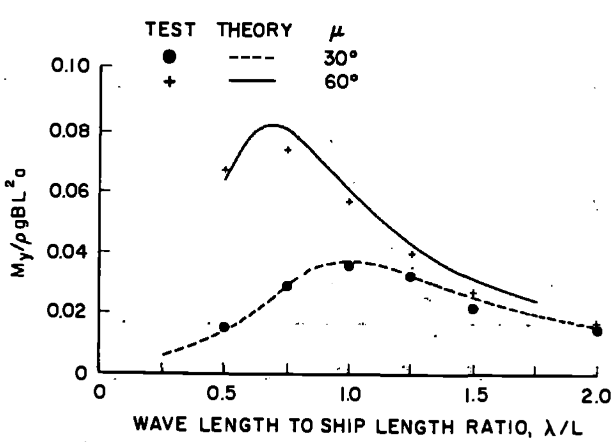


FIG. 14. AMPLITUDE OF YAW-EXCITING MOMENT FOR SERIES 60 MODEL

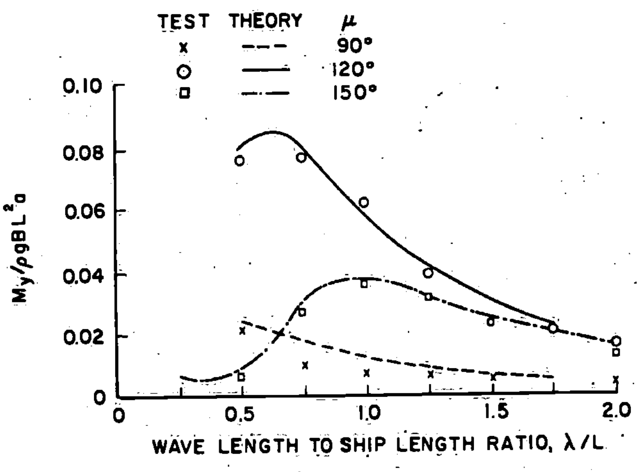


FIG. 15. AMPLITUDE OF YAW-EXCITING MOMENT FOR SERIES 60 MODEL

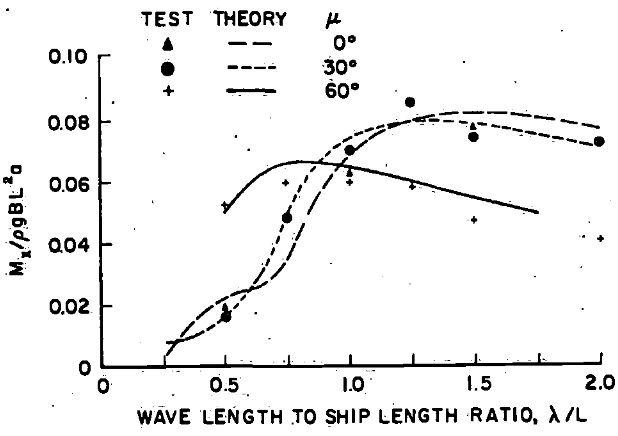


FIG. 16. AMPLITUDE OF PITCH-EXCITING MOMENT FOR SERIES 60 MODEL

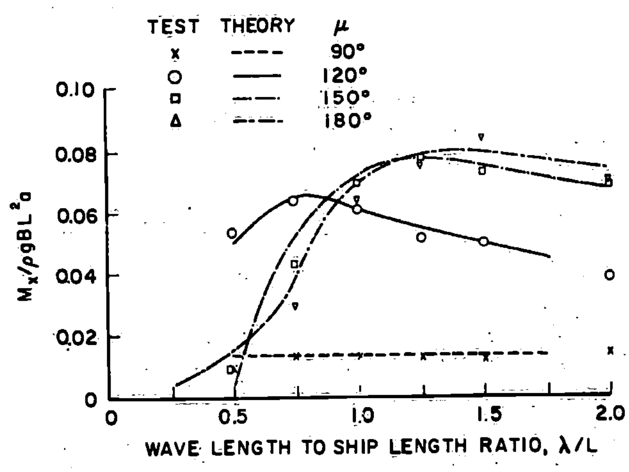


FIG. 17. AMPLITUDE OF PITCH-EXCITING MOMENT FOR SERIES 60 MODEL

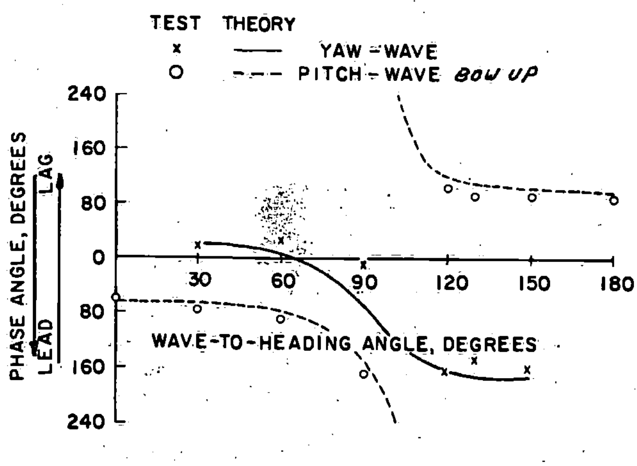


FIG. 18. YAW-WAVE AND PITCH-WAVE PHASE ANGLES, 1.0 L WAVES FOR SERIES 60 MODEL