A cost optimal design of a sea dike

Using probabilistic methods and flexibility in the distribution of the total failure probability over the various failure mechanisms

Philip van Loon
MSc Thesis
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of a sea dike

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Master Thesis
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Preface

This report is the product of my thesis project, the final part of my Master of Science study Hydraulic Engineering. This research has been carried out at the Rivers, Deltas and Coasts division of Royal HaskoningDHV, an independent, international engineering and project management consultancy. It was a great experience to be a graduate intern at Royal HaskoningDHV and I would like to thank Marjan den Braber and my supervisor Michel Tonneijck for providing me this opportunity. It gave me valuable insight in the work of an engineering consultant.

At the beginning of my research I had some difficulties with determining my thesis subject. This search ended in Singapore, where I had several valuable discussions. This not only provided me a subject that interested me, it also gave me the necessary energy for the final stage of my study. I would like to thank Mark van Zanten and Marten Hillen for making the trip to Singapore possible. I felt very welcome and it was a great experience.

I do want to thank all the colleagues at Royal Haskoning. Not only for helping me during my research with their knowledge and experience, but also for providing a fun and comfortable environment to work in. Special thanks go to my daily supervisor Matthijs Bos, who was always there to help me.

In addition, I would like to thank Michel Tonneijck and Mathijs van Ledden. Two experts in respectively dike design and flood risk. They helped me determining my subject and in finding a good balance between the fundamental and probabilistic elements of dike design. Their guidance and extensive feedback were extremely valuable and every time provided me new ideas to work with. I was privileged to have them in my graduation committee.

I would also like to thank Henk Jan Verhagen for being part of my graduation committee. Although we have spent limited time together during my research, your input was valuable.

Furthermore, I would like to thank my professor Bas Jonkman, who always ensured I stayed on track and through his adequate comments enhanced the focus of my research.

Last but not least, I would like to thank Dominique, my family and especially my parents for their unconditional support, being a second reader and for providing me the needed distraction.

I hope you will enjoy reading this report.

Philip van Loon
November 14, 2014
Abstract

In the current Dutch guidelines, the probability that an overtopping discharge occurs, that is greater than the permissible discharge, may not exceed a predetermined norm. Here a design water level is typically assumed, with associated wave climate from which a wave overtopping follows. The norm is expressed in exceedance frequencies on which the design water levels are based. For water levels equal to or lower than the design water level the probability of failure due to other failure mechanisms, such as macro-instability and armour layer instability, may not exceed 10% of this norm (TAW, 1998). In this design approach, the total failure space is dominated by wave overtopping, whereas the contribution of other failure mechanisms to the total failure space is an order of magnitude smaller.

New Dutch guidelines are being developed and in these new guidelines, in which more flexibility is allowed of the distribution of the failure space over the various failure mechanisms. This implies that more failure space is available than the aforementioned 10% for other failure mechanisms than overflow and overtopping. As a result, the dimensions of a dike design based on the new guidelines may differ from a design based on the current guidelines when the input variables are the same. Such a different design approach also provides opportunities for cost savings.

During this research, the consequences and cost savings opportunities of this new design approach are explored for sea dikes. Next, a method is developed for finding the cost optimal design of a sea dike when a probabilistic method is being applied and flexibility is allowed in the distribution of the total failure probability over the various failure mechanisms. This model takes into account:

- the failure mechanisms overtopping, armour layer stability and macro-stability of the inner slope;
- a number of input geometries that differ in crest height, outer slope angle, berm dimensions and \( d_{50} \);
- various correlations between water level and wave height and;
- the prices for the various elements of the dike section based on the investment costs.

The model has been evaluated with a case study and a sensitivity analysis has been carried out.

When using a probabilistic approach, but no flexibility in failure space distribution, a dike design has been obtained at lower cost (3%) and a lower failure probability (factor 2). It is stressed that this design still satisfies the maximum 10% for other failure mechanisms than overtopping. The difference is explained by some minor changes in the key dimensions. This comparison shows the potential of using a probabilistic approach instead of a deterministic approach.

By also introducing flexibility in the distribution of the total failure probability over the various failure mechanisms, a dike design has been obtained with significant lower costs (31 million Euro, 13%) and about the same total failure probability compared to the deterministic design. This is achieved by changing the key dimensions of the dike section in such a way that trade-off of failure space between the various failure mechanisms occurs.

It is recommended to further optimise this approach. It can be a valuable tool for future dike design. Not only a cost saving design of a sea dike may be obtained, it may also provide valuable insight into the failure of a sea dike.
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<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>Erosion area of armour layer in a cross-section</td>
<td>m²</td>
</tr>
<tr>
<td>Aᵢ</td>
<td>Area of segment</td>
<td>m²</td>
</tr>
<tr>
<td>B</td>
<td>Constant in overtopping equation</td>
<td>-</td>
</tr>
<tr>
<td>bᵢ</td>
<td>Width of segment</td>
<td>m</td>
</tr>
<tr>
<td>C</td>
<td>Cohesion clay</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Cᵢ</td>
<td>Cohesion at segment</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Cₚᵢ</td>
<td>Constant in Van der Meer equation</td>
<td>-</td>
</tr>
<tr>
<td>Cₛᵢ</td>
<td>Constant in Van der Meer equation</td>
<td>-</td>
</tr>
<tr>
<td>Cₛ</td>
<td>Cohesion sand</td>
<td>kN/m²</td>
</tr>
<tr>
<td>d</td>
<td>Decimation height</td>
<td>m</td>
</tr>
<tr>
<td>d₅₀</td>
<td>Nominal median block diameter</td>
<td>m</td>
</tr>
<tr>
<td>Dₙ₅₀</td>
<td>d₅₀, used as variable in model</td>
<td>m</td>
</tr>
<tr>
<td>E(S)</td>
<td>Present value of loss due to flooding</td>
<td>€</td>
</tr>
<tr>
<td>f</td>
<td>Probability density function</td>
<td>-</td>
</tr>
<tr>
<td>Fₐᵢ</td>
<td>Cumulative distribution function of the strength of a dike</td>
<td>-</td>
</tr>
<tr>
<td>fₐₛ</td>
<td>Joint probability density function of R and S</td>
<td>-</td>
</tr>
<tr>
<td>Fₛᵢ</td>
<td>Safety factor for macro-stability of the inner slope</td>
<td>-</td>
</tr>
<tr>
<td>fₛᵢ</td>
<td>Probability density function of the load S</td>
<td>-</td>
</tr>
<tr>
<td>Fₓᵢ</td>
<td>Cumulative distribution function of variable x</td>
<td>-</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>Gᵢ</td>
<td>Mass force of segment</td>
<td>kN</td>
</tr>
<tr>
<td>H</td>
<td>Water level</td>
<td>m</td>
</tr>
<tr>
<td>h</td>
<td>Dataset of water level H</td>
<td>m</td>
</tr>
<tr>
<td>H₉₀</td>
<td>Berm height, used as variable in model</td>
<td>m+CD</td>
</tr>
<tr>
<td>Hᵈᵢₗᵉᵉ</td>
<td>Crest height of the dike</td>
<td>m</td>
</tr>
<tr>
<td>Hᵈᵢₗᵉᵉ</td>
<td>Crest height, used as variable in model</td>
<td>m+CD</td>
</tr>
<tr>
<td>hᵢ</td>
<td>Height of segment</td>
<td>m</td>
</tr>
<tr>
<td>Hₛᵢ</td>
<td>Dataset of wave height Hₛ</td>
<td>m</td>
</tr>
<tr>
<td>Hₛᵢ</td>
<td>Wave height</td>
<td>m</td>
</tr>
<tr>
<td>Hₛₑᵣᵡ</td>
<td>New dataset for wave height Hₛ, derived from old dataset Hₛ</td>
<td>-</td>
</tr>
<tr>
<td>Mₕ₀</td>
<td>Median mass of armour rock</td>
<td>kg</td>
</tr>
<tr>
<td>N</td>
<td>Number of waves</td>
<td>-</td>
</tr>
<tr>
<td>Nₛᵢ</td>
<td>Number of Monte Carlo simulations</td>
<td>-</td>
</tr>
<tr>
<td>p</td>
<td>Given failure probability</td>
<td>-</td>
</tr>
<tr>
<td>P</td>
<td>Notional permeability coefficient</td>
<td>-</td>
</tr>
<tr>
<td>P(R ≤ S)</td>
<td>Probability that strength R is smaller than or equal to load S</td>
<td>-</td>
</tr>
<tr>
<td>P(S &lt; R)</td>
<td>Probability that the loads are smaller than the strength of a dike section</td>
<td>-</td>
</tr>
<tr>
<td>P(Z ≤ 0)</td>
<td>Probability that limit state function Z is smaller than or equal to zero</td>
<td>-</td>
</tr>
<tr>
<td>Pᵢ</td>
<td>Total failure probability</td>
<td>-</td>
</tr>
<tr>
<td>Pᵢₗₑᵣᵡ</td>
<td>Total failure probability derived by model</td>
<td>-</td>
</tr>
<tr>
<td>Pᵢᵣᵢ</td>
<td>Probability of failure for one element (failure mechanism) within a system</td>
<td>-</td>
</tr>
<tr>
<td>Pₚᵢₗₑᵣᵡ</td>
<td>Probability of failure of a system (dike section)</td>
<td>-</td>
</tr>
<tr>
<td>qₑᵣᵡ</td>
<td>Admissible overtopping rate</td>
<td>l/s/m</td>
</tr>
</tbody>
</table>
\( q_{\text{over}} \)  
Amount of overtopping  
\([\text{l/s/m}]\)

\( R \)  
Strength or resistance of a dike section  
\([-\text{]}\)

\( r \)  
Certain value of strength \( R \)  
\([-\text{]}\)

\( R_{c} \)  
Freeboard of the dike  
\([\text{m}]\)

\( r_{\text{circle}} \)  
Radius of crack circle  
\([\text{m}]\)

\( R_{d} \)  
Design value of the resistance (strength) of a dike section  
\([-\text{]}\)

\( R_{M} \)  
Resisting moments of single slices  
\([\text{kNm}]\)

\( S \)  
Load or solicitation of a dike section  
\([-\text{]}\)

\( s \)  
Certain value of load \( S \)  
\([-\text{]}\)

\( S_{a} \)  
Damage level \( A/(d_{50})^{2} \)  
\([-\text{]}\)

\( S_{d} \)  
Design value of the solicitation (load) on a dike section  
\([-\text{]}\)

\( S_{M} \)  
Driving moments of single slices  
\([\text{kNm}]\)

\( T \)  
Shear resistance in gap  
\([\text{kN/m}^{2}]\)

\( T_{m-1.0} \)  
Wave period  
\([\text{s}]\)

\( u_{i} \)  
Water pore pressure at segment  
\([\text{kN/m}^{2}]\)

\( V \)  
Coefficient of variation  
\([-\text{]}\)

\( W_{50} \)  
Mean weight  
\([\text{kg}]\)

\( W_{\text{berm}} \)  
Berm width, used as variable in model  
\([\text{m}]\)

\( Z \)  
Limit state function  
\([-\text{]}\)

\( \alpha \)  
Location parameter for the Gumbel distribution  
\([-\text{]}\)

\( \alpha \)  
Angle of the seaward slope of a structure  
\([\text{°}]\)

\( \alpha_{0} \)  
Parameter for calculating \( h_{\text{new}} \)  
\([-\text{]}\)

\( \alpha_{1} \)  
Parameter for calculating combined probability density function  
\([-\text{]}\)

\( \beta \)  
Scale parameter for the Gumbel distribution  
\([-\text{]}\)

\( \gamma_{b} \)  
Influence factor for berms  
\([-\text{]}\)

\( \gamma_{f} \)  
Influence factor for slope roughness  
\([-\text{]}\)

\( \gamma_{v} \)  
Influence factor for vertical wall  
\([-\text{]}\)

\( \gamma_{\beta} \)  
Influence factor for angle of wave attack  
\([-\text{]}\)

\( \gamma_{i} \)  
Volume weight of single soil slice  
\([\text{kN/m}^{3}]\)

\( \Delta \)  
Relative mass density \((\rho_{s} – \rho_{w}) / \rho_{w}\)  
\([-\text{]}\)

\( \varepsilon \)  
Autonomous dataset – the variance gives the spreading around a fully correlated situation  
\([-\text{]}\)

\( \vartheta \)  
Direction angle of segment  
\([\text{°}]\)

\( \mu \)  
Mean value of the distribution of a parameter  
\([-\text{]}\)

\( \xi_{0} \)  
Breaker parameter or surf similarity parameter  
\([-\text{]}\)

\( \rho \)  
Correlation factor  
\([-\text{]}\)

\( \rho_{c,d} \)  
Mass density of dry clay  
\([\text{kN/m}^{3}]\)

\( \rho_{c,w} \)  
Mass density of wet clay  
\([\text{kN/m}^{3}]\)

\( \rho_{h,h_{s}} \)  
Correlation between dataset \( h \) and dataset \( h_{s} \)  
\([-\text{]}\)

\( \rho_{c,d} \)  
Mass density of dry residual soil  
\([\text{kN/m}^{3}]\)

\( \rho_{s} \)  
Mass density of stone  
\([\text{kN/m}^{3}]\)

\( \rho_{s,d} \)  
Mass density of wet sand  
\([\text{kN/m}^{3}]\)

\( \rho_{s,w} \)  
Mass density of wet sand  
\([\text{kN/m}^{3}]\)

\( \rho_{w} \)  
Mass density of water  
\([\text{kN/m}^{3}]\)

\( \sigma \)  
Standard deviation of the distribution of a parameter  
\([-\text{]}\)

\( \varphi \)  
Internal friction angle  
\([\text{°}]\)

\( \varphi_{c} \)  
Friction angle clay  
\([\text{°}]\)

\( \varphi_{s} \)  
Friction angle sand  
\([\text{°}]\)
**List of abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>Chart Datum (reference level)</td>
</tr>
<tr>
<td>cdf</td>
<td>Cumulative distribution function</td>
</tr>
<tr>
<td>FoS</td>
<td>Partial factor of safety</td>
</tr>
<tr>
<td>pdf</td>
<td>Probability density function</td>
</tr>
<tr>
<td>SLR</td>
<td>Sea level rise</td>
</tr>
<tr>
<td>SWL</td>
<td>Still water level</td>
</tr>
</tbody>
</table>
1. Introduction

1.1. Background

“3.5 billion people live in cities and 80% of the world’s megacities are located along coasts or in river deltas. Water introduces both risks and opportunities into urban areas” (www.dutchcham.sg, 2014).

The Dutch, living near the sea, have been dealing with these risks and opportunities for a long time and have acquired international fame for their polders and protection works. In several dictionaries the definition of a polder is described as: “A piece of low-lying land reclaimed from the sea or a river and protected by dikes, especially in the Netherlands” (Oxford University Press, 2014).

Dikes do not only exist in the Netherlands; most countries have dikes in their river and coastal systems. It is estimated that there are several hundreds of thousands of kilometres of dikes in Europe and the USA alone. These dikes are maintained and improved and new dikes are being built (CIRIA, 2013).

1.2. Dike design

When a new dike is being designed, it must be ensured that the dike can resist the forces that act on the dike. Failure or breaching of a dike can occur due to twelve typical failure mechanisms (Figure 1.1). Each failure mechanism has its own limit state, which is determined by the relevant design criteria. A limit state is a condition of a structure beyond which it no longer complies to the relevant design criteria and, in the case of a dike section, failure or breaching occurs. The design of the structure should fulfil the demands of all the failure mechanisms.

![Figure 1.1: Failure mechanisms (Tonneijck & Weijers, 2009)](image)

Uncertainty plays a key role in calculating the failure probability of dikes. In the current Dutch guidelines the uncertainties in the loads and strengths are expressed by characteristic values and safety factors. These characteristic values and safety factors are ideally chosen in such a way that the failure probability of a dike is small enough when it complies with the design criteria of the current guidelines.
1.2. Dike design

The precise strength properties are seldom known. It is, however, generally possible to assign probabilities to the possible loads and strengths, on the basis of statistics and expert judgement. By definition the failure probability of a sea dike is the total probability of all combinations of loads and strengths at which the sea dike will fail. This approach is particularly suitable in situations where the actual values for load and strength properties are uncertain, as this makes it difficult to choose a single set of input values.

In the current Dutch guidelines (TAW, 1998) it is checked if the design value of the load or solicitation ($S_d$) is smaller than the design value of the strength or resistance ($R_d$). New Dutch guidelines (Rijkswaterstaat, 2013) are being developed. In these guidelines the probability is calculated that the loads are larger than the strength. This probability should be smaller than a predetermined requirement (Figure 1.2). There are several methods developed to derive this failure probability, an often used fully probabilistic method is Monte Carlo sampling (Appendix C.).

![Figure 1.2: Relation between probability density functions and design values (VNK2, 2012).](image)

- Current guidelines (semi-probabilistic): $S_d < R_d$?
- New guidelines (fully probabilistic): $P(S > R) < \text{requirement}$?

In the current Dutch guidelines, the probability that an overtopping discharge occurs, that is greater than the permissible discharge, may not exceed a predetermined norm. Here a design water level is typically assumed, with associated wave climate from which a wave overtopping follows. The norm is expressed in exceedance frequencies on which the design water levels are based. For water levels equal to or lower than the design water level the probability of failure due to other failure mechanisms, such as macro-instability and armour layer instability, may not exceed 10% of the norm (TAW, 1998). In this design approach, the total failure space is dominated by wave overtopping, whereas the contribution of other failure mechanisms to the total failure space is an order of magnitude smaller.

So if a probabilistic evaluation shows that another limit state than overflow or overtopping is possible, the probability of this failure should be less than 10% of the total failure probability of
the considered area. In that case the dike section is considered to be “safe enough” (Tonneijck & Weijers, 2009).

Basically two requirements are being distinguished here:

1. The probability requirement: a dike section should fulfil a predetermined total failure probability. In this research the total failure probability is based on the exceedance probabilities of the hydraulic boundary conditions.

2. The 90/10 requirement: in the current Dutch guidelines the failure mechanism overtopping (for a sea dike) takes up at least 90% of the total failure probability. All the other failure mechanisms together may take up a maximum of 10% of the total failure probability. For a total failure probability of 1/10,000 per year for a sea dike section, this means that the failure probability for all the failure mechanisms together, with exception of overtopping, should fulfil a failure probability of 1/100,000 per year. While a sea dike section may have a failure probability for overtopping of 9/100,000 per year.

In the new guidelines, more flexibility is allowed of the distribution of the failure space over the various failure mechanisms. This implies that more failure space is available than the aforementioned 10% for other failure mechanisms than overflow and overtopping. As a result, the dimensions of a dike design based on the new guidelines may differ from a design based on the current guidelines, even if the same total failure probability on cross-sectional level is used for both (Figure 1.3). An economic advantage may be possible because of this difference in design.

![Figure 1.3: By changing the dimensions of the dike, the total failure probability is distributed differently over the various failure mechanisms (Bischiniotis, 2013).](image)

The first person that used the concept of economic optimisation of dikes was Van Dantzig in 1956 short after the major flooding disaster in the Netherlands in 1953. Van Dantzig schematised the flood defence to a single dike cross section with only the crest level as a variable and the only failure mechanism considered was overflow (Van Dantzig, 1956).

Bischiniotis already showed in his graduation research that, using a probabilistic approach with more flexibility in the distribution of the total failure probability over the failure mechanisms, a river dike could be designed more economical in terms of construction costs than the one produced by a semi-probabilistic approach and a fixed distribution over the failure mechanisms (Figure 1.4).
1.3. Objective and research questions

Bischiniotis focused in his research on river dikes with only the water level as load. For a sea dike, wave height also has a high contribution to the design and other failure mechanisms (e.g. armour layer stability) are more dominant.

Research about the cost optimal design using probabilistic methods has been performed for sea dikes (Voortman, 2003). The new element in this research is using flexibility in the distribution of the total failure probability over the various failure mechanisms in combination with probabilistic methods to come to a cost optimal design for a sea dike.

1.3. Objective and research questions

This study should give insight into the impact on dike design when using a fully probabilistic method and changing the distribution of the total failure probability over the various failure mechanisms for a sea dike. It should contribute to the discussion not only for the Netherlands but also internationally. This leads to the following objective:

“Find a cost optimal design of a sea dike by using probabilistic methods and flexibility in the distribution of the total failure probability over the various failure mechanisms. “

From the main objective several research questions arise (not exhaustive):

1. What is the difference with a design based on the current guidelines?
2. What variables characterise the cost optimal design and what is the effect of the exchange of variables on the distribution of the total failure probability over the various mechanisms for the cost optimal design?
3. What element that characterises the geometry of a dike section has the largest influence on the cost optimal design?
4. What is the effect of different soil parameters on the cost optimal design?
5. What is the effect of correlation between wave height and water level on the dike section?
6. What is the effect when a lower failure probability is chosen?

Questions 4 to 6 concern the sensitivity of the results.

1.4. Case introduction

For this research an existing project in Singapore is used as a case study. Pulau Tekong is an island located at the northeast of Singapore close to the border with Malaysia. It is the second largest of Singapore’s outlying islands with an area of 24.43 km². The island is surrounded by the

<table>
<thead>
<tr>
<th>UNIT</th>
<th>Optimal cross-section from probabilistic approach (Case 1)</th>
<th>Cross-section derived by semi-probabilistic approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hdike (m)</td>
<td>4</td>
<td>3.92</td>
</tr>
<tr>
<td>Gradient of inner slope</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>Wdike (m)</td>
<td>43.57</td>
<td>49.58</td>
</tr>
<tr>
<td>Wtherm (m)</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Ltherm (m)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Leakage length (m)</td>
<td>49.57</td>
<td>55.58</td>
</tr>
<tr>
<td>Total cost [€/m length *10^6]</td>
<td>7.15</td>
<td>7.96</td>
</tr>
</tbody>
</table>

Figure 1.4: Difference in the cross-sectional design using a probabilistic approach and a semi-probabilistic approach (Bischiniotis, 2013).
Johor Straits and the largest river discharging in these Straits is the Sungai Johor, which is located northeast of Pulau Tekong. Currently four reclamation areas are planned for the island (Figure 1.5).

The two areas B and D will be reclaimed by means of traditional landfill. The areas A and C will be developed as one polder and will be protected by means of a sea dike. This dike is the subject of the case study.

Several alternatives for the sea dike are developed by Royal HaskoningDHV. A simplified dike section, including the key dimensions and based on an alternative developed by Royal HaskoningDHV is presented in Figure 1.6. This dike section is used as the reference design for all the calculations and examples in this research.

Figure 1.5: Four reclamation areas around the island Pulau Tekong. The location of the dike is given by the red line (Royal HaskoningDHV & Surbana, 2014c).

Figure 1.6: Cross-sectional design, including the key dimensions that will serve as the reference geometry.

1.5. Report outline

This report is subdivided into six chapters and the outline of the report is given in Figure 1.7.

In the second and third chapter the model description is given. The input for the model is a dike geometry, for instance the reference geometry presented in the previous section. In chapter 2 a general description is given on how the model determines the failure probability of a dike section. The description of the model is completed in chapter 3 where other elements are introduced that are necessary for the calculation of a cost optimal design for a sea dike. The model is visualised in the following flow diagram:
The case study is presented in chapter 4 and a sensitivity analysis is presented in chapter 5. For the sensitivity analysis the results of the case study are used. The content of both chapters and the relation between them is visualised in the following flow diagram:
1. Introduction

Figure 1.9: Flow diagram of the case study and the sensitivity analysis.
1.5. Report outline
2. Theory behind the determination of the failure probability

2.1. Introduction

In this chapter all the ingredients are given for the determination of the failure probability of a sea dike section. The failure probability of a dike section depends on two elements: the joint probability density function of the loads that act on the dike section and the cumulative distribution function of the strength of the dike section (the combined fragility of all the selected failure mechanisms). The failure probability is calculated by taking the integral of both functions. The outline of this chapter is presented in Figure 2.1.

Figure 2.1: Outline of chapter 2.

2.2. Fragility curves – The failure probability given a specific load and dike section

The fragility of a dike section is defined as the failure probability given a specific load. In this research the failure probability of the dike section is derived from the failure probability of a combination of failure mechanisms. The load is a combination of water level and wave height.

2.2.1. Limit state function – Difference between load and strength

The failure probability of each failure mechanism is calculated by means of ultimate limit state functions, also called Z-functions. An ultimate limit state function describes the difference between load and strength:

\[ Z = R - S \] (2.1)
2.2. Fragility curves – The failure probability given a specific load and dike section

In this equation R is the strength (resistance) and S (solicitation) is the load. As long as a limit state function is greater than zero (Z>0), the strength is greater than the load and the dike section will not be in a state of failure. If Z≤0, the load is at least as great as the strength and the dike section will fail. In the following figure this has been made visible.

![Figure 2.2: Limit state function visible in the RS-plane (Bischiniotis, 2013)](image)

When Z=0 it means that the design of the dike section is critical. The design point is the point that is located on the border between the safe and unsafe area and this point has the largest probability density.

2.2.1. General description of the fragility curve

A general description of the failure probability of a dike section can be described as (Van der Meer et al., 2009):

\[
P_f = P(Z \leq 0) = P(R \leq S) = \int_{Z \leq 0} f_{RS}(r, s) dr ds
\]

(2.2)

In this \( f_{RS} \) is the joint probability density function of R and S. Assuming R and S to be independent the failure probability can be described by the following integral, where s is a certain value of the load S:

\[
P_f = \int_{s=\infty}^{s} \int_{r=\infty}^{r} f_S(s) f_R(r) dr ds
\]

(2.3)

This can be rewritten as:

\[
P_f = \int_{s=\infty}^{s} \left( \int_{r=\infty}^{r} f_S(s) f_R(r) dr \right) ds = \int_{s=\infty}^{s} f_S(s) F_R(s) ds
\]

(2.4)

In this equation, \( f_s(s) \) is the probability density function (pdf) for the random variable load S, and \( F_R(s) \) is the cumulative distribution function (cdf) which returns the failure probability given the value \( s \) of a certain load.
It has been noted that Equation 2.4 may only be derived from Equation 2.2 if the solicitation (S) and the resistance (R) are assumed to be independent. Because of this assumption Equation 2.4 can be divided into two parts:

1. \( f_S(s) \): probability density function of the load, what in this case is a combination of water level and wave height. This is further highlighted in section 2.3, and;
2. \( F_R(s) \): cumulative distribution function of the strength of the dike, the fragility curve.

As can be seen above both \( f_S \) and \( F_R \) are a function of the load \( S \) again. This is where the independency plays an important role. Because the load \( S \) (water level and wave height) is taken as deterministic values the two functions can be seen as independent. When both the fragility curve (cdf) of the strength and the joint probability density function of the water level and wave height is known the failure probability of the dike section can be derived by integrating both functions over the (deterministic) load \( s \). This process is described in section 2.4.

Because it is assumed that negative values of the water level and the wave height will not occur Equation 2.4 can also be written as:

\[
P_f = \int_{s=0}^{s=\infty} f_S(s)F_R(s)ds
\]

(2.5)

The cumulative distribution function of the strength of the dike section \( F_R \) gives the relation between the load and the failure probability given this load and is called the fragility curve of the dike section (Figure 2.3).

2.2.2. Construction of the fragility curves

The fragility curve of a dike section gives the failure probability as a function of the load. To get a better understanding of the construction of such a curve, the fragility curve for the failure mechanism overflow is used here to explain it. Although later in this report it can be seen that the failure mechanism overflow will not be used in the model, it is a very simple mechanism and therefore it is ideal to be used as an example.

Because failure of a dike section due to overflow only happens when the water level reaches a higher level than the crest height of the dike the limit state function for overflow only depends on the water level \( H \) and the crest height \( H_{dc} \):
2.2. Fragility curves – The failure probability given a specific load and dike section

\[ Z = H_{dc} - H \]  

(2.6)

For the construction of the fragility curve all parameters are taken into account and expressed as stochastic variables. For a dike section the load parameters are the hydraulic boundary conditions and the strength parameters are the dike characteristics. In the case of overflow only the water level will act as a load parameter and the only strength parameter is the height of the dike.

In this example the dike height is given a normal distribution with a mean (μ) and a standard deviation (σ). The water level is divided into very small steps within a certain range and each step is considered as a deterministic value. In the previous section it has already been noted that, for the calculation of the fragility curve, the water level should not follow a certain distribution but should be chosen deterministic.

For each step a high number of calculations of the limit state function are performed (Monte Carlo sampling). For each calculation a random value (given the normal distribution) of the dike height is taken. The number of times the dike section fails divided by the number of calculations for each step of the water level gives the failure probability of the dike section for that certain water level. The fragility curve can be plotted when a line is drawn to all the failure probabilities for each water level (Figure 2.4). In this figure the red circles are the steps taken and a high number of calculations have been performed to get each circle. For smaller steps the fragility curve will get more accurate, but because of the high number of calculations it is better to reduce the number of steps to decrease the calculation time.

In the figure above the fragility curve is relatively steep. This can be explained by the fact that there is only one strength parameter (H_{dc}) with a small standard deviation; there is not a lot of uncertainty in the failure of the dike section. For a limit state function with more parameters (with each their own distribution and uncertainty) a gentler slope can be expected.
When assigning a larger standard deviation to the crest height in the case of overflow and thus adding more uncertainty the fragility curve becomes less steep. Figure 2.5 gives the fragility curves for various standard deviations.

As can be seen in Figure 2.5 all the fragility curves intersect at the point with a water level of 6.1 metre and at a failure probability that is slightly above 50%. That it is slightly above 50% can be explained by the fact that when $Z=0$ failure will also take place by definition.

### 2.2.3. Fragility curves for two loads

In the case of overflow the only load is the water level. In reality the hydraulic boundary conditions for a dike section are combinations of water level and wave height. When constructing a fragility curve for both loads also the wave height should be divided into small steps within a certain range. For every combination of water level and wave height a high number of calculations need to be performed and the fragility curve can be obtained by interpolating between the calculated points. This results in the following fragility curve for overflow for a dike with a crest height of 6.1m+CD.
2.2. Fragility curves – The failure probability given a specific load and dike section

Since the limit state for overflow only depends on the water level the 3-dimensional fragility curve does not vary for different wave heights as can be seen in Figure 2.6. The limit state for the failure mechanism stability of the armour layer only depends on the wave height, as is explained later in the report. An example of the 3-dimensional fragility curve for the failure mechanism armour layer stability is given below.

Certain failure mechanisms depend both on water level and wave height. An example of such a failure mechanism is overtopping and below a possible 3-dimensional fragility curve for overtopping is given.
2. Theory behind the determination of the failure probability

2.2.4. Combined fragility curve for several failure mechanisms

Generally, the failure of a dike section does not depend on only one failure mechanism. To be able to calculate the total failure probability of a dike section the fragility curves of the various failure mechanisms need to be combined.

There are several ways to calculate the total failure probability of a system. The components of a system can have a parallel or a series formation and they can be fully dependent with each other, mutually exclusive (they cannot occur at the same time) or totally independent. When all the components (failure mechanisms) of the strength are stochastically independent it is possible to define the total strength of the system (dike section) as the lowest strength of all components. The probability distribution of the strength of the system can then be calculated using the theory of minima (CUR, 1997):

\[ P_{\text{system}} = 1 - \prod_{i=1}^{n} (1 - P_i) \]

The failure of the dike section \( P_{\text{system}} \) is equal to one minus the chance that no failure will occur. The chance that no failure will occur is equal to the product of the chance that no failure will occur for the separate failure mechanisms \( P_i \). As can be seen the chance that no failure occurs for the separate failure mechanisms is again equal to one minus the chance that failure occurs.

Earlier in the report it has been noted that for the construction of the fragility curves the water level should be taken as a deterministic value. All the failure mechanisms are correlated by the water level. When the water level is treated as a deterministic variable and not as a stochastic one, as it really is, the failure mechanisms are not correlated by the water level anymore.

When both the water level as the wave height are taken as deterministic variables, the failure mechanisms are not correlated anymore and they become stochastically independent. Now the theory of minima can be applied.

Applying the theory to the examples for overflow (Figure 2.6), armour layer stability (Figure 2.7) and overtopping (Figure 2.8), the above results in the following figure:
2.3. Joint probability – the likelihood of two events occurring together and at the same point in time

For a sea dike, violence of the waves plays a dominant role in combination with high water. This is also visible in Figure 2.9 where overflow only plays a role when there is zero wave height and a high water level (the ‘strange’ part in the right corner of Figure 2.9). It can be concluded that, before a sea dike fails due to overflow, it has already failed because of another failure mechanism. Therefore in the model the failure mechanism overflow is not taken along in the model.

2.3. Joint probability – the likelihood of two events occurring together and at the same point in time

A probability density function is a function of a continuous variable from which the integral over a region gives the probability that a random variable falls within the region. An example of a probability density curve is given in Figure 2.10.
2. Theory behind the determination of the failure probability

Two important properties of a pdf are given below:

- The area bounded by the curve of the density function and the x-axis is equal to 1.
- The probability that a random variable assumes a value between \( x_1 \) and \( x_2 \) is equal to the area under the density function bounded by \( x_1 \) and \( x_2 \).

2.3.1. Relation between the selected failure mechanisms and the various loads

In the report ‘A failure mechanism of sea dikes- inventory and sensitivity analysis for coastal sea dikes in Vietnam’ it has been shown that design wave height and design water level are the most important loading parameters in the design of a sea dike which provide 41% and 38% of influences, respectively, to the total failure probability. Because of the high occurrences of wave overtopping, good protection of the upper part of the outer slope, dike crest, and inner slope must be provided (Mai et al, 2009).

Below a relation diagram is given that relates some of the failure mechanisms to the various loads.

![Relation diagram between certain failure mechanisms and the various loads.](image)
2.3. Joint probability – the likelihood of two events occurring together and at the same point in time

2.3.2. Gumbel distribution

For yearly maxima of wave height, river discharges, high sea water levels, etc., an extreme value distribution is used. A shape parameter determines the behaviour of the tail of the distribution (the maxima). Depending on this shape parameter three types of extreme value distributions are distinguished (Gumbel, Fréchet or Weibull). For type I the shape parameter is zero and this type is therefore often used. This is the so called Gumbel distribution (Vrijling & Van Gelder, 2002):

\[
F_x(h) = e^{-e^{-\frac{h-\alpha}{\beta}}} = \exp\left(-e^{\frac{h-\alpha}{\beta}}\right)
\]

From this the probability density function can be derived by differentiating the above cumulative distribution function:

\[
f(h) = \frac{1}{\beta} \exp\left(-\frac{h-\alpha}{\beta}\right) \exp\left[-\exp\left(-\frac{h-\alpha}{\beta}\right)\right]
\]

In this function alpha (α) is the location parameter and beta (β) is the scale parameter. Where μ defines the top of the of the probability density curve for a normal distribution and σ the spreading, α defines the top for the Gumbel distribution and β the spreading. Both parameters can be estimated according to the following equation:

\[
\sigma = \frac{\beta \pi}{\sqrt{6}}
\]

\[
\mu = \alpha + 0.577 \beta
\]

When the mean (μ) and standard deviation (σ) are unknown, alpha and beta can be estimated using the decimation height (d) and a given water level (H) for a given failure probability (p).

\[
\alpha = \frac{H + d \log(-\log(1-p))}{\log(-\log(1-p)) - \log(-\log(1-p/10))}
\]

\[
\beta = \frac{d}{\log(-\log(1-p)) - \log(-\log(1-p/10))}
\]

An example of the Gumbel probability density function for varying location and scale parameters is shown below.
2. Theory behind the determination of the failure probability

In this research it is assumed that both the water level and the wave height fit the Gumbel distribution.

2.3.3. Joint probability density function and correlation between water level and wave height

In the case of a sea defence certain combinations of the water level and the wave height are important for the dimensions of the dike. In Singapore there is almost no correlation between the two loads, while in the Netherlands there is a high correlation. High correlation means that the probability is higher that larger waves occur during high water and lower waves occur during low water. With a low correlation high waves occur during both high and low water.

An explanation about the low correlation in Singapore is given in chapter 4. To be able to calculate the failure probability of the dike it is necessary to combine the distributions of the water level and the wave height so that all the possible combinations are known together with their probability of occurrence.

When two independent datasets of the water level \(h\) and wave height \(H_S\) are given it is possible to construct the combined probability density function for a given correlation. This can be done when both variances \(\sigma^2\) of the distributions are known. First it is necessary to calculate the parameter \(\alpha_1\) (Vrijling & Van Gelder, 2002).

\[
\alpha_1^2 = p_{Z,H_S}^2 \frac{\text{var}(H_S)}{\text{var}(h)} \tag{2.12}
\]
2.3. Joint probability – the likelihood of two events occurring together and at the same point in time

The total variance of $h$ can be divided in a (from $h$) explainable variance and the so called autonomous variance ($\text{var}(\varepsilon)$). This autonomous variance can be seen as the spreading around a fully correlated situation.

$$
\mu_\varepsilon = \mu_{H_h} - \alpha_s \mu_h \\
\text{var}(\varepsilon) = \text{var}(H_S) - \alpha_s^2 \text{var}(h)
$$

(2.13)

From a random dataset $h$ with a given correlation $\rho$, a new dataset $H_{S,\text{new}}$ can be computed.

$$
H_{S,\text{new}} = \alpha_0 + \alpha_s h + \varepsilon
$$

(2.14)

The influence of correlation is illustrated below for low correlation ($\rho=0.3$) and high correlation ($\rho=0.9$).

Figure 2.13: Influence of correlation.

When the new dataset $H_{S,\text{new}}$ is known it is possible using Equation (2.8) and (2.9) to construct the cumulative distribution function (cdf) and the probability density function (pdf). With both the cdf’s and pdf’s of dataset $x$ and the new dataset $y$ it is possible to create a combined probability density function using a so called copula (Intermezzo 2-1).
2. Theory behind the determination of the failure probability

Intermezzo 2-1: Copulas

Well-known statistical methods are available to derive the marginal distributions of all stochastic wave parameters. However marginal analysis is in itself insufficient to come to an accurate description of the long-term wave climate. Research effort of several groups around the world has led to a large number of methods to deal with the problem of bivariate statistical analysis in wave climate studies and other areas. Copulas became increasingly popular for analysing multivariate data when the marginal are known and heavy tailed. There are different copulas developed regarding to their ability to describe the data, their behaviour when used for extrapolation to extreme events, and the transparency of the methods. A judgement which model is preferred in actual practice is difficult to give (De Waal & Van Gelder, 2006).

A copula expresses a joint distribution $F(x,y)$ as a function of $F_x(x)$ and $F_y(y)$, the individual (or marginal) distribution functions for $X$ and $Y$. It allows a separate description of the individual distributions and their association. In this research the marginal functions are the cumulative distribution functions for the wave height and water level. Copulas work in the multivariate context also, but in this research the copula is used for the bivariate distribution. The bivariate copula is defined by a single parameter $\alpha$ based on the correlation coefficient $\rho$ (Venter, 2002).

An example of the separate cumulative distribution curve and an example of the separate probability density curve are given below (Figure 2.14).

![Cumulative distribution curves and probability density curves](image)

Figure 2.14: Cumulative distribution curves and probability density curves.

The joint probability density curve for a correlation of 0.3 created using a copula is given in Figure 2.15. The total probability is given by the area underneath the graph and is exactly 1.
2.4. Integration and calculation of the failure probability

The joint probability density curve for a correlation of 0.9 created using a copula is given in Figure 2.16. The total probability is given by the area underneath the graph and is again exactly 1.

In the contour plot of Figure 2.15 it is visible that high waves occur during both low and high water. In the contour plot of Figure 2.16 high waves are more concentrated around high water and low waves around low water.

2.4. Integration and calculation of the failure probability

When the cumulative distribution function of the strength of the dike (\( F_R \), visualised by the combined fragility curve) and the probability density of the load (\( f_S \), visualised by the combined probability density curve) is given it is possible to obtain the failure probability of the dike section.

2.4.1. Failure probability of the dike section

The failure probability of the dike section is calculated using Equation 2.5 as given in section 2.2:

\[
P_f = \int_{s=0}^{s=\infty} f_S(s) F_R(s) ds
\]
Both functions are integrated over the load within the limits $s=0$ to $s=\infty$. This is done by dividing both the combined fragility function and the combined probability density function in very small steps. The area beneath every step of the combined pdf is calculated and multiplied with the height of the fragility curve of that same step. Doing this for all the steps and summarizing this, results in the total failure probability ($P_f$) of the dike section.

In the model both the water level as the wave height are used but for better understanding only the water level is used in the figure below to visualise the calculation of the failure probability.

![Combined fragility curve and probability density function of the water level (Bischiniotis, 2013).](image)

In the case of a sea dike both the water level and the wave height have an important role and Figure 2.17 becomes 3-dimensional. Instead of small steps, both the combined fragility curve as the combined density curve should be divided into small squares (Figure 2.18) and the same approach as given above is used.
2.4. Integration and calculation of the failure probability

2.4.2. Contribution of each failure mechanism

Each failure mechanism contributes to the total failure probability of the dike section. The level of the contribution of each failure mechanism depends on the geometry of the dike section and the hydraulic boundary conditions. In Figure 2.19, a schematic visualisation of the previous section is given. The total failure probability is the overlapping part between the joint probability of the water level combined with the wave height and the failure probability of the various failure mechanisms. It is clearly visible that some of the failure mechanisms are overlapping, therefore the total failure probability is not just a summation of the failure probability of the separate failure mechanisms and the theory of minima should be applied (Equation (2.7)).
Also the contribution of the separate failure mechanisms to the total failure probability is determined in this research.
3. Probabilistic model for the cost optimal design of a sea dike

3.1. Introduction

In chapter 2 all the ingredients were given for the calculation of the total failure probability of a sea dike section. In this chapter the remaining elements for the calculation of the cost optimal design are given. The flow diagram given in Figure 1.8 is repeated below. It gives the relation between chapter 2 and 3, and all the ingredients for the calculation of the cost optimal design. For the calculation of the failure probability the same strategy is used as Bischiniotis did in his graduation report (Bischiniotis, 2013), although for a sea dike there are significant differences in the approach.

![Flow diagram for the determination of the cost optimal design](image)

Figure 3.1: Flow diagram for the determination of the cost optimal design.

3.2. Selected failure mechanisms and calculation of the limit states

Earlier in this report, an overview was presented of the twelve failure mechanisms of a dike failure (section 1.2). For the calculation of the cost optimal sea dike section, the most important failure mechanisms for a sea dike are selected and used in the model. They are given in Figure
3.2. Selected failure mechanisms and calculation of the limit states

3.2 by means of a fault tree. Although it has been concluded that overflow will not play a role in the failure of a sea dike, it has been used in several examples in this report. This is the reason that it is marked by a dashed line in the figure.

![Fault tree for the selected failure mechanisms.](image)

Piping and macro-instability of the outer slope are also important failure mechanisms for a sea dike, but are not treated further in this research. Macro-instability of the outer slope may only occur when the water level of the sea drops, leaving a saturated and weaker slope behind. It is assumed that the occurred damage will be repaired before the next high water occurs and therefore it is not a direct threat for the failure of a sea dike. Hence, only macro-instability of the inner slope is treated in this research.

Piping is not treated because, when a sea dike has been designed properly for the macro-instability of the inner slope, in most cases it will also fulfil the demands for the mechanism piping. The reason is that failure because of piping only occurs when there is a long lasting high water level on the outside of the dike. When the high water level has a short duration the so-called pipes cannot develop completely. This also means that piping does not have a significant influence on the dimensions of the sea dike.

In Intermezzo 3-1 the difference between failure and breaching is explained. In this research the limit states are based on failure of the dike section and not on breaching of the dike section.
3. Probabilistic model for the cost optimal design of a sea dike

Intermezzo 3-1: Failure versus breaching

There are two important properties of a dike; the height of a dike and the strength or stability of a dike. Where the height and the shape of the dike determine the resistance against overflow and overtopping, it must also be ensured that the dike can resist the forces that act on the dike. This resistance is called the strength of the dike and is expressed in terms of resistance against shearing, water tightness, etc. Later in this report the strength/stability is further illustrated using the so called limit states.

The height and the strength of a dike together determine the resistance against failing. Failing is a technical term, indicating that the dike fails to meet its technically specified safety requirements. Although failing should be avoided it does not yet mean that it will lead to a flooding of the hinterland. An important difference should be made between breaching and failure of a dike. Failure does not necessarily mean the complete loss of the water retaining function. Overflow and wave overtopping are seen as failure mechanisms as well. These failures may lead to loss of functionality and eventually lead to breaching, the actual collapse of the dike.

The failure mechanisms overtopping, macro-instability of the inner slope and instability of the armour layer will be treated in the next section and the limit state functions of these failure mechanisms will be described.

3.2.1. Overtopping

For overtopping the sea dike is considered to have failed functionally when the amount of overtopping \( q_{\text{over}} \) is higher than a predefined admissible overtopping rate \( q_{\text{adm}} \). The strength is thus the admissible overtopping rate and the load is the occurring overtopping. This result in the following limit state function:

\[
Z = q_{\text{adm}} - q_{\text{over}}
\]

For the occurring wave overtopping the following equations can be used (Pullen et al., 2007):

\[
\frac{q_{\text{over}}}{\sqrt{8gH_s^3}} = \begin{cases} 
0.067 \cdot \tan \alpha \cdot \xi_0 \cdot \exp \left( -4.3 \frac{R_c}{H_s} \cdot \frac{1}{\xi_0 \cdot \gamma_b \cdot \gamma_f \cdot \gamma_v \cdot \gamma_\beta} \right) & \text{for plunging waves} \\
0.2 \cdot \exp \left( -2.3 \frac{R_c}{H_s} \cdot \frac{1}{\gamma_f \cdot \gamma_\beta} \right) & \text{for surging waves}
\end{cases}
\]

In this equation \( \gamma_b, \gamma_f \) and \( \gamma_\beta \) are factors that influence the wave overtopping by respectively the outer berm width, the slope roughness and the angle of the wave attack. \( \xi_0 \) is the breaker parameter or the surf similarity parameter and is a dimensionless quantity that relates to the bank slope, wave period, wave height and wave length to distinguish between the types of breaking waves. In this research it is assumed that all waves are plunging and only the equation for plunging waves is used (Intermezzo 3-2).
3.2. Selected failure mechanisms and calculation of the limit states

Intermezzo 3-2: Breaker types

The surf similarity or breaker parameter $\xi_0$ is a dimensionless quantity that relates to the bank slope, wave period and wave height and length to distinguish between the types of breaking waves. The following types exist and are visualised on the right (Verhagen, 2014):

- spilling $\xi_0 < 0.5$
- plunging $0.5 < \xi_0 < 3$
- collapsing $\xi_0 = 3$
- surging $\xi_0 > 3$

The combination of the slope of the structure and the wave steepness gives the type of wave breaking. Waves on a gentle foreshore break as spilling waves. Plunging waves break with steep and overhanging fronts and the wave tongue will hit the structure or back washing water. The transition between plunging and surging waves is known as collapsing. Surging waves are considered not to be breaking, although there may still be some breaking. The majority of waves that reach sea dikes with a slope of 1:3 fall within the plunging region and calculations with the data provided for the Singapore case support this. In this research therefore it assumed that only plunging waves occur.

Besides a factor ($\gamma_v$) for a vertical wall on top of the dike, there are still two unknowns in the above equations, namely the freeboard of the dike ($R_c$) and the discharge ($q$). This means that the overtopping discharge can be calculated given a certain crest height (Figure 3.3) or the required crest height can be calculated given a maximum wave overtopping.

![Figure 3.3: Free crest height for wave overtopping (Pullen et al., 2007).](image)

The failure probability of the failure mechanism wave overtopping not only depends on the significant wave height. As can be seen in Figure 3.3 the freeboard of the dike ($R_c$) also depends on the dike height and the still water level (SWL) or in the model given by $H$ and can be calculated as given in Equation(2.6):
3. Probabilistic model for the cost optimal design of a sea dike

\[ R_c = H_{dc} - H \]

To give an impression of the amount of overtopping, below a classification of the mean overtopping discharge is given that is indicative for erosion of the inner slope (Pullen et al., 2007):

- \( q_{\text{over}} < 0.1 \text{l/s per m} \): Insignificant with respect to erosion of crest and landside slope of the dike section.
- \( q_{\text{over}} = 1 \text{l/s per m} \): Crest and rear side need at least a good grass cover.
- \( q_{\text{over}} = 10 \text{l/s per m} \): Significant overtopping for dikes and embankments. Some overtopping for rubble mound breakwaters.
- \( q_{\text{over}} = 100 \text{l/s per m} \): Crest and inner slopes of dikes have to be protected by asphalt or concrete. For rubble mound breakwaters transmitted waves may be generated.

The allowable overtopping discharge does not only depend on the resistance of the inner slope. A restriction may be the amount of water that is allowed to flow into the polder. For \( 1 \text{l/s/m} \) for a dike ring of 10 km it means that 10,000 litres per second enters the polder. A storm that lasts for 10 hours (36,000 seconds) means that 360,000 m\(^3\) of water enters the polder, which is equivalent to a water level of 3 cm in the polder. Because of the relative small polder area, for \( 10 \text{l/s/m} \) this level can reach up to 30 cm of water in the polder.

3.2.2. Stability of the armour layer

The limit state function for the stability of the armour layer is based on the Van der Meer equations. Although only plunging waves are assumed in this research, for the sake of completeness also the equations for surging waves are given (CIRIA et al., 2007).

\[
\frac{H_S}{\Delta d_{n50}} = c_{pl} P^{0.18} \left( \frac{S_A}{\sqrt{N}} \right)^{0.2} \xi_0^{-0.5} \quad \text{for plunging waves} \tag{3.3}
\]

\[
\frac{H_S}{\Delta d_{n50}} = c_s P^{0.13} \left( \frac{S_A}{\sqrt{N}} \right)^{0.2} \xi_0^P \sqrt{\cot \alpha} \quad \text{for surging waves}
\]

In these equations the following parameters are present:

- \( H_S \): significant wave height [m]
- \( \Delta \): relative mass density \( (\rho_s - \rho_w) / \rho_w \) where \( \rho_s \) is mass density of stone and \( \rho_w \) is mass density of water [-]
- \( \xi_0 \): breaker parameter [-]
- \( d_{n50} \): nominal median block diameter [m], or equivalent cube size, derived from the median mass \( M_{50} \), \( d_{n50} = (M_{50} / \rho_s)^{1/3} \)
- \( c_{pl} \): constant [-]
- \( c_s \): constant [-]
- \( P \): notional permeability coefficient [-]
- \( S_A \): damage level \( A/(d_{n50})^2 \) [-], where \( A \) = erosion area in a cross-section
- \( N \): number of waves [-]
- \( \alpha \): angle of the seaward slope of a structure [%]
3.2. Selected failure mechanisms and calculation of the limit states

In general the damage to the armour layer is difficult to compare between dikes with different armour layers. A possibility is to describe the damage by the erosion area around still water level. When this erosion area is related to the size of the rocks, a dimensionless damage level ($S_A$) is presented which is independent of the size (slope angle and height) of the dike.

The limit state function for the failure of the armoured layer can be derived from the Van der Meer equations.

$$ Z = c_p P^{0.18} \left( \frac{S_A}{N} \right)^{0.2} \xi_0^{-0.5} \Delta d_{n50} - H_S \quad \text{for plunging waves} $$

$$ Z = c_s P^{-0.13} \left( \frac{S_A}{N} \right)^{0.2} \xi_0 \cot \alpha \Delta d_{n50} - H_S \quad \text{for surging waves} $$

The limit state function for the failure of the armoured layer can be derived from the Van der Meer equations.

$$ Z = c_p P^{0.18} \left( \frac{S_A}{N} \right)^{0.2} \xi_0^{-0.5} \Delta d_{n50} - H_S \quad \text{for plunging waves} $$

The armoured layer often fails under storm conditions and the failing is mainly caused by wave actions. Therefore the failure probability of the armoured layer only depends on the significant wave height and the corresponding wave period.

3.2.3. Macro-stability

The macro-stability of the soil structure is defined as: resistance against shearing of large sections of the soil structure along straight or curved slip planes. Often this results in excessive deformations (Figure 3.4).

![Figure 3.4: Shearing of the inner slope (Tonneijck & Weijers, 2009).](image)

The limit state function for this failure mechanism is defined as:

$$ Z = F_s - 1 $$

In this limit state function $F_s$ is a safety factor defined by:

$$ F_s = \frac{\sum R_M}{\sum S_M} $$

In which:

- $\sum R_M$ = sum of the resisting moments of single slices [kNm]
- $\sum S_M$ = sum of the driving moments of single slices [kNm]
The calculations for finding the resisting and driving moments can be done either by slip plane calculations or by finite elements methods. From the results of the slip plane calculations using a number of slip planes it is possible to determine the most dangerous slip plane; the plane with the lowest safety factor.

There are several methods to perform these calculations. The Bishop method is often used in the Netherlands. In this method, the soil mass is divided into a number of vertical slices and the equilibrium of each one of these slices is considered. In Figure 3.5 the resisting and driving moments for one circular slip plane are given.

![Figure 3.5: Driving (SM) and resisting (RM) moments of a circular slip plane (Allsop et al., 2007).](image)

The moments can be calculated using the equations given below:

<table>
<thead>
<tr>
<th>Loading equations:</th>
<th>Resistance (strength) equations:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driving mass forces calculated from:</td>
<td>Shear resistance calculated from:</td>
</tr>
<tr>
<td>$\sum S_M = r_{\text{circle}} \sum G_i \cdot \sin \theta_i$</td>
<td>$\sum R_M = r_{\text{circle}} \sum T_i = r_{\text{circle}} \sum \frac{(G_i - u_i \cdot b_i) \cdot \tan \phi_i + c_i \cdot b_i}{\cos \theta_i + \frac{1}{F_s} \cdot \tan \phi_i \cdot \sin \theta_i}$</td>
</tr>
<tr>
<td>With:</td>
<td>In order to calculate $F_s$ an iterative procedure is needed since $R_M = R_{M}(F_s)$.</td>
</tr>
<tr>
<td>Weight of single segment</td>
<td></td>
</tr>
<tr>
<td>$G_i = \gamma_i \cdot A_i$</td>
<td></td>
</tr>
<tr>
<td>Length of single segment</td>
<td></td>
</tr>
<tr>
<td>$l_i = \frac{b_i}{\cos \theta_i}$</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 3.6: Loading and resisting equations for the Bishop method (Allsop et al., 2007).](image)

The parameters used in the equations are the following:

- $G_i$ = mass force of segment [kN]
- $T$ = shear resistance in gap [kN/m²]
- $F_s$ = safety factor [-]
- $b_i$ = width of segment [m]
- $h_i$ = height of segment [m]
- $\gamma_i$ = volume weight of single soil slice [kN/m³]
- $A_i$ = area of segment [m²]
- $u_i$ = water pore pressure at segment [kN/m²]
3.3. Variables and reference geometry

\[ c_i \] = cohesion at segment [kN/m²]

\[ r_{\text{circle}} \] = radius of crack circle [m]

\[ \varphi \] = internal friction angle [%]

\[ \vartheta \] = direction angle of segment [%]

In Figure 3.7 the active and passive zone of a random slip circle is illustrated. The weight of the soil in the active zone has a significant influence on the driving moments while the weight of the soil in the passive zone enhances the resistance against sliding. One can imagine that a berm on the inner side of the dike section increases the weight of the passive zone and therefore has a positive influence on the macro-stability of the inner slope.

![Figure 3.7: Active and passive zone of a random slip circle (Tonneijck & Weijers, 2009).](image)

Because of the high number of computations several software packages have been developed to do this efficiently. In this research the package D-Geo Stability is used. In Appendix D. more information about D-Geo Stability and about the performed calculations is given.

3.3. Variables and reference geometry

The stability/strength of a sea dike depends on the geometry and therefore on the various elements in the cross section of the sea dike. Below a figure is presented in which the elements are coupled to the selected failure mechanisms (Tonneijck & Weijers, 2009). In this figure it is shown which elements are used as a variable. In the next section it is explained why certain elements are used as a variable and other elements are not.
3.3.1. Elements used in this research

In Figure 3.8 it is visible that some elements are used as a variable while others are used as a constant or are not taken into account in the calculations. Some elements even have a double function and are taken as a variable for one failure mechanism and as a constant or not even into account for the other failure mechanism.

A variable is deterministically changed according to manual input. The geometry and the failure probability of the dike section depend on these variables, as well as the costs of the construction of the dike. For every combination of variables a geometry, failure probability and cost estimate is calculated.

Crest:
In the previous section it has already been noted that the height of a dike is an important parameter for the amount of overtopping that will occur. A higher dike will have a lower amount of overtopping. Alternatively a higher dike can have a negative influence on the stability of the inner slope; the weight of a single segment increases, hence the sum of the driving moments for a single segment increases (section 3.2). Therefore the height of the dike can be an important variable in finding the cost optimal design of a sea dike.

Armour layer and outer slope angle:
The failure mechanisms stability of the outer slope and macro-instability of the inner slope are both influenced by the dimensions of the chosen armour layer and the angle of the outer slope (section 3.2). Both elements are used as a variable in this research. Depending on the type of revetment the parameter \( \gamma_f \) (factor for slope roughness) changes and is taken as a constant in the calculations for overtopping. In this model only a stone revetment is used. The difference in roughness, because of larger armour stones, is difficult to estimate. Therefore the \( d_{50} \) is taken as a variable for the armour layer stability, but the parameter \( \gamma_f \) remains constant for the failure mechanism overtopping.

**Figure 3.8:** Selected failure mechanisms coupled to the dike elements that have influence on them.
3.3. Variables and reference geometry

Berm on the inside of the dike and inner slope:
Both the inner slope angle and a berm on the inside of the dike have an influence on the failure mechanism macro-instability of the inner slope. Based on expert opinions it is assumed that the influence of a berm is larger than the influence of the inner slope angle and that it has a larger effect on the geometry of the dike section (section 3.2.3). Therefore the dimensions of the berm are used as a variable and the angle of the inner slope of the dike is kept constant. In this research it is assumed that the width and height of the berm are in proportion. Instead of to variables (width and height) only one variable is used for the berm. The inner slope angle only has an influence on the failure mechanism overtopping when looked at erosion of the inner slope. In this research it is assumed that failure because of overtopping will only occur when the critical overtopping rate has been reached (Intermezzo 3-1); the angle of the inner slope will not be used in the calculations for overtopping.

Dike core and berm ditch:
Soil properties have influence on the macro-stability of the inner slope as explained in section 3.2. They are used as a constant for the calculations in D-Geo Stability. In Figure 3.8 it can be seen that the dike core (soil properties) is also related to the failure mechanism overtopping. Infiltration of water into the dike core can occur due to overtopping. This infiltrated water can weaken the dike core and threaten the stability of the dike. In this research it is assumed that no infiltration of water occurs due to overtopping. The ditch can have a positive or negative influence on the failure mechanism macro-instability of the inner slope (section 3.2). It is used as a constant in the D-Geo Stability calculations.

A summary of this section is given in Table 3.1.

<table>
<thead>
<tr>
<th>Element</th>
<th>Overtopping</th>
<th>Armour layer stability</th>
<th>Macro-instability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crest height</td>
<td>Variable</td>
<td>-</td>
<td>Variable</td>
</tr>
<tr>
<td>Crest width</td>
<td>-</td>
<td>-</td>
<td>Constant</td>
</tr>
<tr>
<td>Armour layer</td>
<td>Constant</td>
<td>Variable</td>
<td>-</td>
</tr>
<tr>
<td>Outer slope angle</td>
<td>Variable</td>
<td>Variable</td>
<td>Variable</td>
</tr>
<tr>
<td>Inner slope angle</td>
<td>-</td>
<td>-</td>
<td>Constant</td>
</tr>
<tr>
<td>Inner berm</td>
<td>-</td>
<td>-</td>
<td>Variable</td>
</tr>
<tr>
<td>Berm ditch</td>
<td>-</td>
<td>-</td>
<td>Constant</td>
</tr>
<tr>
<td>Dike core</td>
<td>-</td>
<td>-</td>
<td>Constant</td>
</tr>
</tbody>
</table>

Table 3.1: Use of elements in the calculations of the various failure mechanisms.

Ideally, the armour layer for overtopping and the inner slope angle for macro-stability are taken as variables for deriving the cost optimal design. For simplification purposes they are both assumed as a constant in this research based on the reasoning given earlier in this section.

3.3.2. Reference geometry
For the cost optimisation of a sea dike it is necessary to specify a certain reference geometry in which the chosen variables that have influence on the dike geometry and the failure probability will change. The reference geometry is based on alternative 2 (Figure 3.9) that has been developed for the island Pulau Tekong.
This variant is the basis for the optimisation process. To decrease the difficulty and computation time of the calculations variant 2 has been simplified and this results in the following geometry that serves as the reference geometry for the optimisation of the dike section (Figure 3.10).

In section 4.3 the exact dimensions of the reference dike section are calculated and explained. In this stage of the report the reference dike section is only used to show what variables are changed to come to the most cost efficient dike section.

It is important to emphasise that only for one location of the dike ring, which will protect the polder at Pulau Tekong, the cost optimal design is calculated in this research. Only the boundary conditions for that location are used. On other locations along the dike ring another cross section might be economically more attractive.

### 3.4. Cost determination of the design

In the report ‘Risk-based design of large-scale flood defence systems’ Voortman describes the process of selecting the cost optimal design as follows:

“Additional geometric requirements may be set, but the combination of all constraints will generally still leave a large number of geometries to be acceptable. For a final choice it is useful to analyse the costs of every alternative geometry within the solution space. From the viewpoint of rational decision-making the costs should be minimised, provided the design satisfies all requirements. Combination of the investment with a reliability requirement leads to the cheapest design that just suffices the reliability requirement. This is denoted the optimal design.
3.4. Cost determination of the design

Figure 3.11 shows the investment contours, the probability constraint and the corresponding optimal design for a fictitious sea dike" (Voortman, 2003).

In the figure above:

- Only the variables crest height and outer slope angle are considered. In this research also the berm dimensions and the armour layer are taken into account;
- the costs become larger for a higher crest height and gentler slope angles;
- the probability constraint divides the cross-sections that comply with the design failure probability (acceptable region, right of the thick black line) and the cross-sections that do not comply with the design failure probability (unacceptable region, left of the thick black line). Cross-sections that are on the thick black line just fulfil the technical requirements.
- the thin black lines represents cross-sections with the same direct cost (contours of direct costs);
- The black dot represents the cross-sections that just complies with the technical requirements and consist of the lowest direct costs within the acceptable region.

From the viewpoint of rational decision-making the costs should be minimised, provided the design satisfies all requirements. In the model the failure probability is calculated for all geometries that vary based on the input variables. Every design with a lower failure probability then the required failure probability satisfies. The final step is to find the design with the lowest costs.
3.4.1. Input for cost

The costs are based on the investment costs of the dike section and all costs are taken per metre length of the dike. The costs are split into four parts (Figure 3.12):

1. The soil for the main dike [€/m²]
2. The soil for the berm including compaction costs for the berm [€/m²]
3. The footprint of the dike section based on the opportunity costs of land [€/m]
4. The stones for the armoured layer based on d₅₀ [€/m]

Figure 3.12: Costs are based on four part; 1dike area, 2 berm area, 3 footprint, 4 armour layer.

The costs for the footprint of the dike are based on the opportunity cost of the land. Every metre that the sea dike is shorter results in an extra metre that can be used for other purposes. In highly populated areas, for example Singapore, the opportunity costs of land is higher than in less dense populated areas.

The costs of the revetment are difficult to define. Normally a revetment consist of several layers of stones with different sizes that act as a filter so that the smaller dike core material below will not wash out. For the same reason also a geotextile is often used. A geotextile is water-permeable, but impermeable for soil. In the model only the layer closest to the sea, the largest stones, is considered.

The costs of stones depend on many variables; the availability, experience and equipment of a quarry, the distance from the quarry to the sea dike, the type of stone, the layer thickness, etc. Usually stone sizes are divided in rock grading classes (Figure 3.13) and the costs are based on these grading classes.
3.4. Cost determination of the design

In this model the costs of the revetment are based on the \( d_{50} \) of the outer layer. The values of this variable do not vary a lot in the model. The consequence is that the costs do not vary much either. The \( d_{50} \) is an important design variable and therefore it is necessary to assign a value to the costs of the revetment. In Figure 3.13 it is given that the layer thickness is 1.5 times the \( d_{50} \).

In this research it is assumed that the costs of the revetment are:

\[
\text{Costs revetment} = 1.5 \cdot d_{50} \cdot \text{reference value} \cdot \text{length outer slope}
\]  

(3.7)

The reference value is a constant value for every m\(^3\) of the revetment and independent from the stone size of the revetment. It is assumed that the revetment covers the entire outer slope. Therefore the reference value should be multiplied with the length of the outer slope. From the equation it follows that a smaller \( d_{50} \) directly leads to lower costs for the revetment.

The costs for the installation of the sand key are not taken into account. The sand key was already installed before the various alternatives for the sea dike at Pulau Tekong were developed.

3.4.2. Changes of the geometry because of the variables

In this research four elements are taken as a variable; dike height, outer slope angle, inner berm dimensions and the stone size of the revetment. In Figure 3.14, Figure 3.15 and Figure 3.16 the impact of the first three variables on the geometry are made visible.
From the above figure it can be concluded that a larger dike height directly means a larger cross-sectional dike area and a larger footprint. Although the berm area remains the same, a higher crest height can have a negative influence on the macro-stability of the dike section (section 3.3.1).

*Figure 3.15: Influence of change of outer slope angle on the geometry.*

Also the slope angle has an influence on the cross-sectional dike area and on the footprint, while the berm area remains the same again.

*Figure 3.16: Influence of change in berm dimensions on the geometry.*

In this case the cross-sectional berm area changes. A change in berm dimensions also has an effect on the footprint of the sea dike. The number of input variables does have a significant effect on the number of input geometries for the model, as is explained later on in the report. To limit the complexity and the computation time of the model, the berm height and berm width changes with a certain ratio. In this way the berm can be seen as one variable.

The dimensions of the inner berm depend on several elements (e.g. stability of the inner slope, user requirements and seepage length. A larger seepage length decreases the amount of seepage and the possibility of failure because of piping and uplift). Usually the berm width is larger than the berm height. Therefore in the model the width of the berm is assumed twice the height of the berm.

The influence of larger $d_{50}$ does not have direct influence on the geometry of the dike section. In the previous section the influence of the $d_{50}$ on the costs of the dike section is explained.
4. Case study

4.1. Framework

According to the population white paper the population of Singapore will grow and reach between 6.5 and 6.9 million citizens in 2030 (National population and talent division, 2013). In order to cope with the growing population the ministry of national development has set up a land use plan. The aim is to provide a high quality living environment for all Singaporeans to live and work in. Currently Singapore exists of 71,000 ha and to provide enough space, another 5,000 ha is required (Ministry of national development, 2013). Part of this enlargement is the land reclamation at Pulau Tekong, an island on the northeast of the mainland of Singapore (Figure 4.1). Up to now all land reclamation in Singapore was done by means of a landfill, raising the entire area above sea level. Most of the sand needed for the landfill came from the neighbouring countries. But as they stopped the export of dredged sand to Singapore, the sand nowadays has to come from countries further away (e.g. Cambodia) which makes it more expensive.

4.1.1. Project area

Pulau Tekong is located at the northeast of Singapore close to the border with Malaysia. It is the second largest of Singapore’s outlying islands with an area of 24.43 km². The island is
surrounded by the Johor Straits and the largest river discharging in these Straits is the Sungai Johor, which is located at the northeast of Pulau Tekong.

Currently the island is divided in four reclamation areas (Figure 4.2). The two areas B and D will be reclaimed by means of traditional landfill. The areas A and C will be developed as one polder and the sea dike for this polder is investigated in this research.

![Figure 4.2: Four reclamation areas around the island Pulau Tekong. The location of the dike is given by the red line (Royal HaskoningDHV & Surbana, 2014c).](image)

The polder has a surface area of plus minus 1,000 ha and the length of the ring dike will be approximately 10,000 m. In Figure 4.3 an artist impression of Pulau Tekong is given with the areas A, B and C.

![Figure 4.3: Artist impression of Pulau Tekong including the new areas (Royal HaskoningDHV & Surbana, 2014c).](image)
4. Case study

4.1.2. Dike alternatives

In the preliminary phase a set of variants for the polder design and a set of alternatives for the sea dike design are evaluated. The goal is to have one variant and one alternative for the final phase of the project. The preliminary phase is from January till April 2014 and there are already some variants presented for the polder and for the sea dike (Royal HaskoningDHV & Surbana, 2014c).

The polder will be designed with a polder level of -1m+CD and -2m+CD. CD is an abbreviation for Chart Datum, which is used as reference level in this report. The polder has a surface area of plus minus 1,000 ha and the length of the ring dike will be approximately 10,000 m. Although there is some interaction with the polder variants, this research will mainly focus on the design of the sea dike.

Dike alternatives

“The primary function of a dike is to ensure safety, in other words to protect the hinterland from inundation in the event of high water levels” (Tonneijck & Weijers, 2009). The first key driver for the design of the sea dike for the polder at Pulau Tekong follows from this and is the flood protection level. The three key drivers for the design are given below (Royal HaskoningDHV & Surbana, 2014c):

1. Flood protection level;
2. Seepage of saline water under and through the dike;
3. Phasing of the project

Although all three key drivers are important, this research mainly focuses on the first key driver. Several alternatives are given in the conceptual design. The design will probably be adjusted to the local situation at different locations at Pulau Tekong. The alternatives that fit the requirements are given below:

![Figure 4.4: Alternative 1 Traditional land reclamation through landfill (Royal HaskoningDHV, 2014).](image-url)
4.1. Framework

There are another two alternatives that are presented in the next section.

**Over-dimensioned dike**

“The starting point for all safety approaches is the risk involved. In the case of a sea dike, the risk is the probability of flooding combined with the corresponding consequences. This can be expressed as the probability an inundation will occur multiplied by a certain consequence (a measure of the loss of money and/or human life). The measure for the risk is the average loss of
money or human life per year. The definition of a certain accepted risk implies that the more severe the consequences, the smaller the probability should be. It is not possible to totally preclude risk, because the probability “zero” is impossible, given the lack of an upper limit to natural phenomena.” (Tonneijck & Weijers, 2009).

The quote given above is visualised in Figure 4.8. In this figure $C_{\text{const}}$ are the construction costs of the dike section and $E(S)$ is the present value of the loss due to flooding (the consequence) multiplied with the probability of failure. Adding up $C_{\text{const}}$ and $E(S)$ results in a curve with a lowest point. At this point the costs are lowest and the cost optimal dike design is found.

![Figure 4.8: Optimal design considering the construction costs and the consequence due to flooding. The optimal design is there where the total costs ($C_t$) are minimal (Henny, 2012).](image)

The first thirty years after the development of the polder the reclaimed land will not be densely populated. In the second stage the land will be used for urbanisation, which implies a direct rise of the consequences of an inundation. In Figure 4.8 the dashed line ($E(S)$) moves up. Close to the origin the dashed line will move up more than further from the origin, with larger dike dimensions. As a result the black line ($C_{\text{const}} + E(S)$) also changes and a new cost optimal design is found, which in this case will be larger.

As mentioned above the only way Singapore did land reclamation was by means of landfill and the polder at Pulau Tekong therefore will be a pilot polder. One way to ensure that the dike will provide the required strength and even give a comparable safety profile as landfill is to construct a larger dike. In Figure 4.8 it is visible that the black line on the right of the optimal design is relative gentle. This means by increasing the dike dimensions for relative low costs, relative high extra safety can be bought. Especially at Pulau Tekong this is the case, where compared to the Dutch situation gentle hydraulic boundary conditions are observed. Two alternatives for the sea dike at Pulau Tekong are given below:
4.1. Framework

Figure 4.9: Alternative 5 Sealed dike: Over-dimensioned dike with sealing on the outer slope (Royal HaskoningDHV, 2014).

Figure 4.10: Alternative 5 Sealed dike: Over-dimensioned dike with clay key on the inner slope (Royal HaskoningDHV, 2014).

Although this so-called over-dimensioned dike implies higher costs than needed an additional economic benefit of multiple uses in an urban situation can outweigh this. The additional costs are then compensated. An extreme example is the so-called over-dimensioned dike along the Arakawa River in Tokyo.

Figure 4.11: Design of a super levee along the Arakawa River (Nakamura et al., 2013).
4.1.3. Selected dike alternative

In section 3.3.2 it has already been noted that alternative 2 is being used for this research. In Singapore alternative 3 is preferred to the other alternatives. Alternative 3 has a cement-bentonite wall which blocks the groundwater flow and therefore has a positive effect on seepage control. For the polder at Pulau Tekong strict requirements are given for the amount of salt water flowing into the polder (seepage). These requirements do not apply for the stability of the dike itself, but are requirements for the water management system in the polder. The seepage water can also be pumped out of the polder. For the stability of the dike it is, based on the reasoning given in section 3.2, assumed that if alternative 2 (including a berm for stability reasons) fulfils the stability requirement, alternative 3 will also. Therefore alternative 2 is used in this research.

4.2. Hydraulic boundary conditions as presented by Royal HaskoningDHV

The hydraulic boundary conditions are determined in a study of Royal HaskoningDHV (Royal HaskoningDHV & Surbana, 2014). The extreme hydraulic loads are used as input for the detailed design of the sea dike. The part of the sea dike for which the hydraulic loads are determined is given in Figure 4.2 by means of a red line.

The extreme hydraulic load on a sea dike consists of a combination of extreme water level and wave conditions, which are mainly a result of tide and wind. Secondly they are influenced by the offshore wave conditions penetrating to the project area and the local flow conditions.

There are four elements that determine the hydraulic boundary conditions that are distinguished by Royal HaskoningDHV:

1. Extreme water level statistics;
2. Surge – wave combinations for the determination of the crest level;
3. Wave characteristics for the revetment;
4. Climate change scenario.

For deriving the extreme water level statistics, Royal HaskoningDHV has applied Annual Maxima in combination with a Gumbel distribution and least-square fitting procedure. In Table 4.1 the derived extreme water levels are presented for the all-year maxima.

For the outer slope erosion different hydraulic boundary conditions are necessary. Table 4.1 also presents the recommended wave characteristics for the determination of the required revetment. For a failure probability of 1/10,000 per year the return period is 10,000 years.

<table>
<thead>
<tr>
<th>Return period (years)</th>
<th>Water level (m+CD)</th>
<th>H₅ (m)</th>
<th>Tₘ₋₁₀ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>3.48</td>
<td>0.8</td>
<td>3.1</td>
</tr>
<tr>
<td>100</td>
<td>3.67</td>
<td>0.9</td>
<td>3.3</td>
</tr>
<tr>
<td>1,000</td>
<td>3.86</td>
<td>1.1</td>
<td>3.5</td>
</tr>
<tr>
<td>10,000</td>
<td>4.05</td>
<td>1.3</td>
<td>3.6</td>
</tr>
</tbody>
</table>

*Table 4.1: Extreme water levels and design points for the revetment* (Royal HaskoningDHV & Surbana, 2014b).
To determine the crest level, the combination of surge and waves is important. Royal HaskoningDHV has evaluated the correlation between surge and wave by plotting the (instantaneous) local surge versus the local wind speed (Figure 4.12). It is noted that this is an instantaneous comparison whereas the surge duration also plays a role. Also, all surge events are presented in the figure below, whereas the more extreme events are only important for the extreme conditions. Nevertheless, there seems to be little correlation between the local surge and the local wind speed. Only the cloud on the right might indicate some correlation, but it has been investigated and this is not the case.

Because of the low correlation between the surge and waves, it is assumed that there is no correlation. For the determination of the hydraulic boundary conditions Royal HaskoningDHV followed a probabilistic approach for the separate loads. Combinations of both loads resulted in the hydraulic boundary conditions (Table 4.2). To find for instance the 1/10,000 per year conditions several combinations were formed; the 1/1 per year wave height together with the 1/10,000 per year water level, the 1/10 per year wave with the 1/1,000 per year water level, etc. The combination that resulted in the highest load on the sea dike was chosen. In this research not only one combination is used, but, because a joint probability is derived, all combinations are taken into account.

Some uncertain aspects were excluded from the intermediate results of the probabilistic approach, like the assumption of no correlation between surge and waves. The recommended hydraulic boundary conditions are also uncertain because of the limited measurement records of water levels, wind and waves. This is especially true for larger return periods (say 1000 years and beyond). The effects of these uncertainties are evaluated and the result is that an extra allowance on the crest height is recommended to cover the uncertainties. The extra allowance is presented in Table 4.2.

<table>
<thead>
<tr>
<th>Return period (years)</th>
<th>Water level (m+CD)</th>
<th>$H_s$ (m)</th>
<th>$T_{1,0}$ (s)</th>
<th>Crest level allowance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3.17</td>
<td>0.8</td>
<td>3.1</td>
<td>0.2</td>
</tr>
<tr>
<td>100</td>
<td>3.17</td>
<td>0.9</td>
<td>3.3</td>
<td>0.3</td>
</tr>
<tr>
<td>1,000</td>
<td>3.17</td>
<td>1.1</td>
<td>3.5</td>
<td>0.3</td>
</tr>
<tr>
<td>10,000</td>
<td>3.17</td>
<td>1.3</td>
<td>3.6</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 4.2: Hydraulic boundary conditions for the various locations along the flood defence (Royal HaskoningDHV & Surbana, 2014b).

Finally the climate change effects need to be included in the hydraulic boundary conditions. The planning horizon for the sea dike taken into account is 100 years. An indication for this period is a sea level rise of 0.85, although this is much higher than historical sea level rise, which is around 0.20m per century (Royal HaskoningDHV & Surbana, 2014b). In this research sea level rise is not
taken into account for the cost optimal design. The effect of sea level rise on the cost optimal design is given in Appendix H.

4.3. Cross sectional design of the dike based on the current Dutch guidelines

In this research the current Dutch guidelines are used as defined in the report 'Fundamentals on Water Defences' (TAW, 1998). The design of a sea dike always starts with the requirements based on the water retaining function. The dike should be able to withstand the water level and wave attack that comes with the chosen design level. The corresponding hydraulic boundary conditions are important parameters for the design of the dike. First the crest height of the dike is determined. A return period of 10,000 years is chosen for the failure mechanism overtopping. As mentioned in section 1.2 the probability of failure of all other failure mechanisms should be less than 10% of the return period to be considered “safe enough”.

4.3.1. Crest height

Based on the failure mechanism overflow the height of the crest should be higher than the maximum water level. This means that the crest level should be at least 4.05 m+CD (Table 4.1).

For the sea dike, the overtopping criteria are often governing so the height of the crest level can be determined using Equation (3.2).

For the sea dike geometry, a constant slope (1:3) with a smooth surface is applied. This is consistent with the current situation of the already existing sea dike at Pulau Tekong. As allowable overtopping \(q_{\text{over}}\) 1 l/s per metre is taken. The input for the calculation of the freeboard \((R_c)\), based on the most governing location along the sea dike at Pulau Tekong, is:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overtopping rate</td>
<td>(q_{\text{over}}) 1 l/s per metre</td>
</tr>
<tr>
<td>Slope angle</td>
<td>(\tan \alpha) 0.33 (slope 1/3)</td>
</tr>
<tr>
<td>Significant wave height</td>
<td>(H_s) 1.3 m</td>
</tr>
<tr>
<td>Gravitational constant</td>
<td>(g) 9.81 m/s(^2)</td>
</tr>
<tr>
<td>Wave period</td>
<td>(T_{m-1,0}) 3.6 s</td>
</tr>
<tr>
<td>Angle of wave attack factor</td>
<td>(Y_{\beta}) 0.97 (incoming wave angle of 9(^\circ))</td>
</tr>
<tr>
<td>Slope roughness factor</td>
<td>(\gamma_f) 1</td>
</tr>
<tr>
<td>Berm factor</td>
<td>(\gamma_b) 1</td>
</tr>
<tr>
<td>Vertical wall factor</td>
<td>(\gamma_v) 1</td>
</tr>
</tbody>
</table>

Table 4.3: Input for the calculation of the freeboard \((R_c)\).

This gives a freeboard of 2.53 metres. The calculated freeboard together with the water level (3.17 m+CD) and the extra allowance for uncertainties (0.4 metre) determine the crest level. The minimum crest level is set on 6.1 m+CD.

4.3.2. Stone size of the revetment

The failure probability of the armour layer should be less than 10% of the return period. This means that the revetment should be able to resist waves with at maximum a return period of 1:100,000. The significant wave height and wave period for this return period are derived with linear extrapolation from Table 4.1. For the required stone size for the revetment Equation 3.3 can be used. The following input is used:
4.3. Cross sectional design of the dike based on the current Dutch guidelines

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant wave height</td>
<td>$H_S$</td>
</tr>
<tr>
<td>Wave period</td>
<td>$T_{m,1,0}$</td>
</tr>
<tr>
<td>Mass density rock</td>
<td>$\rho_s$</td>
</tr>
<tr>
<td>Mass density water</td>
<td>$\rho_w$</td>
</tr>
<tr>
<td>Damage factor</td>
<td>$S$</td>
</tr>
<tr>
<td>Number of waves (storm duration)</td>
<td>$N$</td>
</tr>
<tr>
<td>Constant factor</td>
<td>$C_{pl}$</td>
</tr>
<tr>
<td>Constant factor</td>
<td>$C_s$</td>
</tr>
<tr>
<td>Notional permeability factor</td>
<td>$P$</td>
</tr>
<tr>
<td>Slope angle</td>
<td>$\tan \alpha$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slope</th>
<th>Initial Damage (needs no repair)</th>
<th>Intermediate Damage (needs repair)</th>
<th>Failure (core exposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1:1.5$</td>
<td>2</td>
<td>3 – 5</td>
<td>8</td>
</tr>
<tr>
<td>$1:2$</td>
<td>2</td>
<td>4 – 6</td>
<td>8</td>
</tr>
<tr>
<td>$1:3$</td>
<td>2</td>
<td>6 – 9</td>
<td>12</td>
</tr>
<tr>
<td>$1:4$</td>
<td>3</td>
<td>8 – 12</td>
<td>17</td>
</tr>
<tr>
<td>$1:6$</td>
<td>3</td>
<td>8 – 12</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 4.4: Input for the dimensions of the required stone size.

The damage factor ($S$) represents the number of stones removed from the cross-section. Critical values for $S$ are given in Figure 4.13.

The number of waves ($N$) is based on a storm duration of six hours and a wave period of 4 seconds. The armour layer is assumed not permeable ($P=0.1$). In reality the armour layer consists of three layers of filtering and cemented armouring (Figure 4.14). Especially because of the cemented armouring the armour layer is assumed not permeable.

Figure 4.13: Classification of damage levels $S$, for quarry stone (Verhagen, D’Angremond, & Van Roode, 2009).

Figure 4.14: Cemented armouring of the armour layer on the existing sea dike of Pulau Tekong.
With this input the required stone size \( (d_{50}) \) becomes 0.51 metre and the mean weight \( (W_{50}) \) 345 kg.

4.3.3. Crest width and berm dimensions

Based on the user requirements for the sea dike a crest width of 9 metre is assumed. The inner slope of the dike and the berm is assumed 1:3 (Royal HaskoningDHV & Surbana, 2014). In this research a berm width of 8 metre and height of 3m+CD is assumed based on Figure 3.9. Considering a polder level of -1m+CD the actual berm height is 4 metres.

This cross section has been tested in D-Geo Stability as explained in Appendix D. The partial factor of safety \( (FoS) \) that has been used in the calculation is 1.38 for the soil strength parameters (based on Eurocode). The soil parameters that are used in the calculation are given below (Royal HaskoningDHV & Surbana, 2014):

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>Bulk density above water ( (\text{kN/m}^3) )</th>
<th>Bulk density below water ( (\text{kN/m}^3) )</th>
<th>Angle of internal friction ( (\text{Degree}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>-</td>
<td>10.05</td>
<td>-</td>
</tr>
<tr>
<td>Berm material</td>
<td>17.64</td>
<td>19.85</td>
<td>31.37</td>
</tr>
<tr>
<td>Sand (dike material)</td>
<td>17.64</td>
<td>19.85</td>
<td>31.37</td>
</tr>
<tr>
<td>Sand (sand key)</td>
<td>17.64</td>
<td>19.85</td>
<td>31.37</td>
</tr>
<tr>
<td>Marine clay</td>
<td>-</td>
<td>16.50</td>
<td>-</td>
</tr>
<tr>
<td>Residual soil</td>
<td>-</td>
<td>18.00</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 4.5: Soil parameters*

The recommended required safety factor \( (F_s) \) is 1.25. For a design water level of 4.05m+CD (Table 4.1) on the outer side of the dike the cross section proved to satisfy the requirements for the macro-stability of the inner slope. The minimum calculated safety factor is 2.13 and the critical slip circle is given below.

*Figure 4.15: Critical slip circle reference cross section as given in D-Geo Stability.*
4.3. Cross sectional design of the dike based on the current Dutch guidelines

4.3.4. Dimensions of the cross section

The final cross section based on the current guidelines is given in Figure 4.16. In this figure it is also visible that for the polder level is located on -1m+CD.

![Cross-sectional design based on the current guidelines.](image)

The most important dimensions are summarized in Table 4.6.

<table>
<thead>
<tr>
<th>Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polder level (m+CD)</td>
<td>-1.0</td>
</tr>
<tr>
<td>Crest height (m+CD)</td>
<td>6.1</td>
</tr>
<tr>
<td>Crest width (m)</td>
<td>9.0</td>
</tr>
<tr>
<td>Outer slope angle (tanα)</td>
<td>1:3</td>
</tr>
<tr>
<td>Inner slope angle (tanα)</td>
<td>1:3</td>
</tr>
<tr>
<td>Berm height (m+CD)</td>
<td>3.0</td>
</tr>
<tr>
<td>Berm width (m)</td>
<td>8.0</td>
</tr>
<tr>
<td>Stone diameter of the revetment (m)</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 4.6: Dimensions of the reference cross section

4.3.5. Cost of the design

As described in section 3.4.1 the investment costs are determined based on four parts of the dike section. In the table below the four parts of the dike section are given together with the costs determined by Royal HaskoningDHV. The cost of the cross-sectional design based on the current guidelines is given in Table 4.7.

<table>
<thead>
<tr>
<th></th>
<th>Costs per unit</th>
<th>Area/Length</th>
<th>Costs (€)</th>
<th>Costs (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dike area</td>
<td>27,50 (€/m²)</td>
<td>215 m²</td>
<td>€ 5,916</td>
<td>25 %</td>
</tr>
<tr>
<td>Berm area and</td>
<td>16,50 (€/m³)</td>
<td>32 m²</td>
<td>€ 528</td>
<td>2 %</td>
</tr>
<tr>
<td>compaction</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Footprint</td>
<td>216 (€/m)</td>
<td>59.6 m</td>
<td>€ 12,874</td>
<td>55 %</td>
</tr>
<tr>
<td>Revetment</td>
<td>1.5 * Dn50 * 250 (€/m)</td>
<td>22.5 m</td>
<td>€ 4,294</td>
<td>18 %</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>-</td>
<td>€ 23,612</td>
<td>100 %</td>
</tr>
</tbody>
</table>

Table 4.7: Cost of the cross-sectional design based on the current guidelines.

From Table 4.7 it follows that the cross-sectional design costs € 23,612 per metre length. The total cost of the sea dike for the polder at Pulau Tekong is:

€ 23,612 * 10,000 m = € 236,120,000.-

Although the investment costs of the berm area are relatively small (2%) it does not reflect the effect of the berm on the total costs. An increase of the berm increases also the investments costs of the footprint, which is 55% of the total costs. An increase of dike height or a gentler slope angle also increases the footprint. It can be expected that the cost optimal dike section is the dike section with the smallest footprint.
It has to be noted that the cost optimisation is based on the direct investment cost. Maintenance costs, settling of the sub layer and sea level rise are not taken into account. There is uncertainty in the amount of sea level rise and settlement. The dike is designed to function until the year 2100. During the lifetime of the sea dike it might be that the actual sea level rise and settling is less than expected. Therefore it might be profitable not to take the full expected sea level rise and settlement for the entire lifetime of the sea dike into account. After some years the actual sea level rise and settling can be measured and the dike section can be adjusted.
4.4. Cross-sectional design using a probabilistic approach

In the previous section the cross-sectional design based on the current guidelines is derived. In this section the cost optimal design is derived based on the model that has been described in chapter 2 and 3. First the input variables and parameters are given before the results are presented. Based on the results several conclusions are given. One of the conclusions is about the costs of the optimal design. In a separate section the costs are highlighted.

4.4.1. Variables and cross-sections

For the following variables cross-sections have been produced by Matlab:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum value</th>
<th>Step size</th>
<th>Maximum value</th>
<th># of values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dike height (m)</td>
<td>5.6</td>
<td>0.2</td>
<td>6.8</td>
<td>7</td>
</tr>
<tr>
<td>Outer slope angle (tanα)</td>
<td>0.20</td>
<td>0.04</td>
<td>0.40</td>
<td>6</td>
</tr>
<tr>
<td>Dn50 (m)</td>
<td>0.46</td>
<td>0.02</td>
<td>0.58</td>
<td>7</td>
</tr>
<tr>
<td>Berm (m)</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

*Table 4.8: Input for the variables.*

The number of cross-sections is a multiplication of the number of values of each variable and is in this case 1470. In Figure 4.17 various cross-sections are given that are used for the calculation of the failure probability and the cost. These cross-sections are retrieved from D-Geo Stability. The variable Dn50 is not given in these cross-sections because it is not taken into account for the failure probability of macro-stability.

In Figure 4.17 four different cross-sections are presented. These cross-sections are presented to give an impression what happens when one of the variables changes. The width height ratio does not represent the reality. In reality the width is in the order of seven or eight times the height.

![Cross-sections](image)

*Figure 4.17: Examples of cross-sections used for calculation of failure probability and cost.*

It is important to emphasize that for every combination of variables a new cross-section is generated and new fragility curves and failure probabilities are calculated.

4.4.2. Case specific input parameters

Input for the hydraulic boundary conditions:
It is assumed that there is zero correlation between the water level and wave height (section 4.2).

It has to be emphasized that a large number of samples is required in order to estimate the required probabilities with the desired accuracy. An often used method to estimate the number of Monte Carlo simulations based on the failure probability is (Vrijling & Van Gelder, 2002):

$$N_s > 400 \left( \frac{1}{P_f} - 1 \right)$$

(4.1)

For small probabilities of failure, the required number of simulations becomes very large and results in $N_s = 3,999,600$ for a failure probability of $1/10,000$ per year. The number of simulations has a large effect on the computation time. To reduce the computation time of the model a smaller number of simulations is chosen ($N_s=10,000$). The results of the calculations are solely used to investigate the effects of this model. When the model is used for the design of a dike in reality, a larger number of simulations are recommended. Another possibility is to use importance sampling, which is a more efficient approach in which the emphasis is put on the values of a parameter that have more impact on this parameter. In Appendix F the influence of a higher number of simulations is checked and no significant differences are observed.

The input parameters and distributions for the various failure mechanisms are given in Table 4.10 (overtopping), Table 4.11 (armour layer stability) and Table 4.12 (macro-stability).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Unit</th>
<th>Distribution</th>
<th>Mean value</th>
<th>Standard dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_b$</td>
<td>Influence factor for berms</td>
<td>-</td>
<td>Deterministic</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$Y_t$</td>
<td>Influence factor for slope roughness</td>
<td>-</td>
<td>Deterministic</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$Y_v$</td>
<td>Influence factor for vertical wall</td>
<td>-</td>
<td>Deterministic</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$Y_{beta}$</td>
<td>Influence factor for wave angle</td>
<td>-</td>
<td>Deterministic</td>
<td>0.97</td>
<td>-</td>
</tr>
<tr>
<td>$q_{crit}$</td>
<td>Critical overtopping discharge</td>
<td>l/s/m</td>
<td>Lognormal</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>$B$</td>
<td>Constant in overtopping equation</td>
<td>-</td>
<td>Lognormal</td>
<td>4.75</td>
<td>0.5</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
<td>m/s$^2$</td>
<td>Deterministic</td>
<td>9.81</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.10: Input for the calculation of the fragility curve of overtopping.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Unit</th>
<th>Distribution</th>
<th>Mean value</th>
<th>Standard dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{pl}$</td>
<td>Constant in vd Meer equation</td>
<td>-</td>
<td>Lognormal</td>
<td>6.2</td>
<td>0.04</td>
</tr>
<tr>
<td>$S_A$</td>
<td>Damage level</td>
<td>-</td>
<td>Deterministic</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of waves</td>
<td>-</td>
<td>Deterministic</td>
<td>5600</td>
<td>-</td>
</tr>
<tr>
<td>$P_w$</td>
<td>Density of water</td>
<td>kg/m$^3$</td>
<td>Deterministic</td>
<td>1005</td>
<td>-</td>
</tr>
<tr>
<td>$P_{stone}$</td>
<td>Density of stone</td>
<td>kg/m$^3$</td>
<td>Deterministic</td>
<td>2650</td>
<td>-</td>
</tr>
<tr>
<td>$P$</td>
<td>Permeability of revetment</td>
<td>-</td>
<td>Deterministic</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
<td>m/s$^2$</td>
<td>Deterministic</td>
<td>9.81</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.11: Input for the calculation of the fragility curve of armour layer stability.
4.5. Cost optimal design

For probabilistic calculations, instead of the coefficient 4.3 in Equation 1.16, the coefficient 4.75 should be used. The reliability of Equation 1.16 for overtopping, is described by this parameter with a standard deviation of 0.5 (Pullen et al., 2007).

The standard deviation of the friction angles is based on a coefficient of variation $V$. A coefficient of variation is a statistical measure of the dispersion of data points in a data series around the mean. It is calculated as follows:

$$ V = \frac{\sigma}{\mu} \quad (4.2) $$

The standard deviation of the friction angles of both sand and clay is based on $V=0.1$. For the cohesion a coefficient of variation $V=0.2$ is used (Vrouwenvelder et al., 2013).

It is noted before that in D-Geo Stability for probabilistic calculation the limit value stability factor is used instead of the factor of safety. In Table 4.12 also a value for the standard deviation of the limit value safety factor is given. This value expresses the uncertainty caused by the Bishop model assumptions.

### Table 4.12: Input for the calculation of the fragility curve of macro-stability in D-Geo Stability.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Soil layer</th>
<th>Unit</th>
<th>Distribution</th>
<th>Mean value</th>
<th>Standard dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{w}$</td>
<td>Density of water</td>
<td>-</td>
<td>kN/m$^3$</td>
<td>Deterministic</td>
<td>10.05</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{s,d}$</td>
<td>Density of dry sand</td>
<td>Dike area, berm area, sand key</td>
<td>kN/m$^3$</td>
<td>Deterministic</td>
<td>17.64</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{s,w}$</td>
<td>Density of wet sand</td>
<td>Dike area, berm area, sand key</td>
<td>kN/m$^3$</td>
<td>Deterministic</td>
<td>19.85</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Friction angle sand</td>
<td>Dike area, berm area, sand key</td>
<td>°</td>
<td>Lognormal</td>
<td>31.33</td>
<td>3.13</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Cohesion sand</td>
<td>Dike area, berm area, sand key</td>
<td>kN/m$^2$</td>
<td>Lognormal</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_{c,d}$</td>
<td>Density of dry clay</td>
<td>Marine clay</td>
<td>kN/m$^3$</td>
<td>Deterministic</td>
<td>16.50</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{c,w}$</td>
<td>Density of wet clay</td>
<td>Marine clay</td>
<td>kN/m$^3$</td>
<td>Deterministic</td>
<td>16.50</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>Friction angle clay</td>
<td>Marine clay, residual soil</td>
<td>°</td>
<td>Lognormal</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>$C_c$</td>
<td>Cohesion clay</td>
<td>Marine clay, residual soil</td>
<td>kN/m$^2$</td>
<td>Lognormal</td>
<td>8</td>
<td>1.6</td>
</tr>
<tr>
<td>$\rho_{r,d}$</td>
<td>Density of dry residual soil</td>
<td>residual soil</td>
<td>kN/m$^3$</td>
<td>Deterministic</td>
<td>18.00</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{r,w}$</td>
<td>Density of wet residual soil</td>
<td>residual soil</td>
<td>kN/m$^3$</td>
<td>Deterministic</td>
<td>18.00</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>Limit value stability factor</td>
<td>-</td>
<td>-</td>
<td>Lognormal</td>
<td>1</td>
<td>0.08</td>
</tr>
</tbody>
</table>

For probabilistic design, from the 1470 different input geometries in the model, a total of 1010 geometries fulfil the probability requirement of 1/10,000 per year. From these geometries, 221 geometries have lower investment costs than the reference design. The top five cost optimal designs are given in this section.
4.5.1. Optimal design and conclusions based on the optimal design

In the table below the top five cost optimal designs are presented.

<table>
<thead>
<tr>
<th></th>
<th>Reference design</th>
<th>Cost optimal design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st</td>
</tr>
<tr>
<td>$P_{f,model}$</td>
<td>-</td>
<td>9.79E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 23,612</td>
<td>€ 20,547</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 236.1</td>
<td>€ 205.5</td>
</tr>
<tr>
<td>H dike (m+CD)</td>
<td>6.1</td>
<td>5.8</td>
</tr>
<tr>
<td>Outer slope (tanα)</td>
<td>0.33</td>
<td>0.36</td>
</tr>
<tr>
<td>Dn50 (m)</td>
<td>0.51</td>
<td>0.50</td>
</tr>
<tr>
<td>H berm (m+CD)</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>W berm (m)</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 4.13: Characteristic values for the reference geometry and the top 5 cost optimal designs.

In the table above first the costs per metre dike section are given, which are derived as explained in section 3.4.1. The total costs are the costs for the entire dike with a length of 10,000 metres. In the table no failure probability is given for the reference design. The failure probability in the table is the failure probability that is determined in the model. The reference design is determined on hydraulic boundary conditions that are based on a total failure probability of 1/10,000 per year. The actual total failure probability may differ from this value.

From the above table several observations can be drawn:

- Using the probabilistic method and flexibility in the distribution of the total failure probability over the various failure mechanisms, a cost saving design can be achieved; the cost optimal design has 30 million euros (13%) less investment costs for the total dike section.
- All of the top five cost optimal designs have a lower crest height of which three in a combination with a steeper slope. The lower crest height can be explained by the fact that in the cost optimal design no allowance is added for uncertainties.
- The five designs all have a much smaller berm.
- The cost optimal design has for every variable a smaller value (although the value for the slope is larger, it means that there is a steeper slope and in fact less investment costs).
- According to Table 4.13 there is interaction between the variables dike height, slope angle and the $d_{n50}$. There seems to be no interaction with the berm height and width.

4.5.2. Graphical presentation of the results

In this section a graphical presentation of the results and some noteworthy figures are given. First both cross-sections of the deterministic and the probabilistic dike designs are given in one figure.
4.5. Cost optimal design

![Image of deterministic and cost optimal probabilistic design given in one figure.](image)

Given the fact that the resistance against failing of the dike section is determined by the height and strength of the dike section, it is concluded from this figure that the cost optimal dike section is less robust than the reference design.

This conclusion is confirmed with the developed model when using the reference design as input.

<table>
<thead>
<tr>
<th></th>
<th>Reference design</th>
<th>Cost optimal design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deterministic</td>
<td>Probabilistic</td>
</tr>
<tr>
<td>$P_{f,\text{model}}$</td>
<td>-</td>
<td>1.11e-05</td>
</tr>
<tr>
<td>$H_{\text{dike}}$ (m+CD)</td>
<td>6.1</td>
<td>6.1</td>
</tr>
<tr>
<td>Outer slope (tan$\alpha$)</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>$D_{\text{n50}}$ (m)</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>$H_{\text{berm}}$ (m+CD)</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$W_{\text{berm}}$ (m)</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

*Table 4.14: Deterministic and probabilistic evaluation of the reference design compared to the cost optimal design.*

Using a deterministic approach clearly results in an “over-dimensioned” dike design. Uncertainties in the deterministic design are compensated by safety factors and extra allowances, also other failure mechanisms and user requirements are taken into account. In a probabilistic evaluation of the reference design 94% of the failure space is taken by the armour layer stability and 6% by overtopping. The contribution of macro-stability is almost zero.

Considering Figure 3.11 it is now possible to assign a location to the reference design and the cost optimal design that is found in this research. This is done in Figure 4.19. The axes represent the increasing dimensions of the dike section because of the increase in crest height, berm dimensions, $d_{n50}$ and slope angle. The small black dots represent the input geometries for the model. The dot in the upper right corner represents the largest input geometry and the dot in the lower left corner the smallest input geometry. The dot closest to the optimal design is the cost optimal design found in this research. The optimal design is found when taking infinitely small steps for the variables. Unfortunately this result in an infinitely long computation time and the optimal design can only be approached.
In Figure 4.19 a red dot and a green dot are visible. Both dots represent dike sections that are used in the research and as can be observed, these dike sections are in the unacceptable region of the probability constraint or requirement. The red dot is placed further to the left compared to the green dot. This means that the dike section that belongs to the red dot has smaller dimensions than the dike section that belongs to the green dot. The green dot is placed lower in the figure compared to the cost optimal design found in the research. This means that the dike section that belongs to the green dot has smaller dimensions than the cost optimal design. In Table 4.15 the dike sections that belong to the red and green dot are given and also the cost optimal design is given. In the table it is visible that the dike sections that belong to the red and green dot do not fulfil the probability requirement. By increasing the dike dimensions, in this case the height, one can simply go from the red dot to the green dot. By further increasing the dike dimensions, in this case the berm dimensions, the cost optimal design is found.

<table>
<thead>
<tr>
<th></th>
<th>Red dot</th>
<th>Green dot</th>
<th>Cost optimal design</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{f_{\text{model}}}$</td>
<td>5.34E-04</td>
<td>2.60E-04</td>
<td>9.79E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 19,393</td>
<td>€ 20,016</td>
<td>€ 20,547</td>
</tr>
<tr>
<td>Total cost (€×10⁶)</td>
<td>€ 193.9</td>
<td>€ 207.0</td>
<td>€ 205.5</td>
</tr>
<tr>
<td>$H_{\text{dike}}$ (m+CD)</td>
<td>5.6</td>
<td>5.8</td>
<td>5.8</td>
</tr>
<tr>
<td>Outer slope (tana)</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>$D_{n50}$ (m)</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>$H_{\text{berm}}$ (m+CD)</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$W_{\text{berm}}$ (m)</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

*Table 4.15: Three dike sections that are given to explain Figure 4.19.*
4.5. Cost optimal design

In the previous section it was concluded that there was interaction between the variables dike height, slope angle and the stone size of the revetment. It seemed that there was no interaction with the berm dimensions. In Figure 4.20 the contribution of each failure mechanism to the total failure probability is given for the five most cost optimal geometries.

![Figure 4.20: Contribution of each failure mechanism to the total failure probability.](image)

In this figure it is visible that both overtopping and armour layer stability takes up most of the available failure space. The contribution of overtopping varies between 29 and 72 percent and the armour layer stability varies between 10 and 50 percent; the influence of a higher crest and a larger $d_{50}$ on the distribution of the failure probability is significant. Both failure mechanisms depend on the slope angle. The failure space of overtopping also depends on the crest height, while the failure space for armour layer stability also depends on the stone size of the revetment.

A maximum of 24% is reserved for the failure mechanism macro-stability and a minimum of 13%. This, in combination with the fact that relative small berm dimensions are found, leads to the conclusion that the dike area (without the berm) is already very stable. In section 3.3.1 it is noted that the inner slope angle also has influence on the failure mechanism macro-instability of the inner slope and it is assumed that the influence of a berm is larger than the influence of the inner slope angle. Therefore the dimensions of the berm are used as a variable and the angle of the inner slope of the dike is kept constant. Based on the results it is concluded that an inner slope of 1:3 already provides a large part of the inner slope stability. In Appendix G. the effect of a steeper inner slope on the cost optimal design is presented.

The contribution of macro-stability to the failure space depends on the variables crest height and the dimensions of the inner berm. In Table 4.13 it is given that designs 1 and 2 have the same crest height and berm dimensions, nevertheless the contribution of the failure mechanism macro-stability varies considerably and it can be concluded that the berm also interacts with the other variables.

One possible reason why the berm dimensions do not vary in the five most optimal designs is the large investment costs for the purchase of land. It is illustrated in Figure 4.21 that for all five designs almost 60% of the investment costs are determined by the footprint and thus the value of the land.
In section 4.3.5 the expectation was already raised that the cost optimal design is the design with the smallest footprint. The same can be concluded from the above figures. The berm width has a large influence on the cost of the footprint and the top five optimal designs all have a relative small berm.

Although the five cost optimal designs all have a failure probability within a small range from the design failure probability, there are designs closer to the design failure probability that also fulfill the requirements (Figure 4.19). In the figure below the five designs closest to the failure probability are given.

<table>
<thead>
<tr>
<th>Reference design</th>
<th>Designs closest to the design failure probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st</td>
</tr>
<tr>
<td>$P_{f,model}$</td>
<td>1.11E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 23,612</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 236.1</td>
</tr>
<tr>
<td>Hdike (m+CD)</td>
<td>6.1</td>
</tr>
<tr>
<td>Outer slope (tana)</td>
<td>0.33</td>
</tr>
<tr>
<td>Dn50 (m)</td>
<td>0.51</td>
</tr>
<tr>
<td>Hberm (m+CD)</td>
<td>3</td>
</tr>
<tr>
<td>Wberm (m)</td>
<td>8</td>
</tr>
</tbody>
</table>

When comparing this table with Table 4.13 it is concluded that a lower crest, a steep outer slope and larger berm dimensions are more cost efficient and the expectation that the berm width has a large influence on the cost of the dike section is proven wrong. Comparing the design closest to the design failure probability with the reference design and the cost optimal design, it appears that for less investment costs a stronger dike can be bought.

According to Table 4.16 it is expected that almost all the available failure space is taken by macro-stability (small berm dimensions, high crest level and a relative flat slope). In Figure 4.22 the distribution of the failure probability of the various failure mechanisms over the total failure probability is given. In this figure the expectation is confirmed.
4.5. Cost optimal design

4.5.3. Conclusions

The following conclusions are based on the results from the model:

- The reference design is not optimal for a total failure probability of 1/10,000 per year; it is stronger than the cost optimal design (almost 1/100,000 per year);
- Using a probabilistic method and flexibility in the distribution of the total failure probability results in a cost saving design;
- There is interaction between the variables and it is possible to exchange failure space between the failure mechanisms. High interaction is observed between armour layer stability and overtopping and lower interaction with macro-stability;
- The dike area (without a berm) is, from a geotechnical perspective, very stable. An inner slope of 1:3 already provides a large part of the inner slope stability;
- A lower crest, a steep outer slope and larger berm dimensions are cost efficient. The berm width does not have a large influence on the cost of the dike section.

Certain assumptions are made to retrieve the results. These assumptions can have an influence on the results and are listed below.

- In the crest height of the deterministic design an extra allowance for uncertainties is taken into account. Although most uncertainties in the probabilistic model are given by the stochastic distributions, the uncertainty of the correlation between wave height and water level is not taken into account. A correlation of zero is assumed.
- In section 4.1.2 it is given that the second key driver for the dike alternatives is seepage of saline water. Two of the alternatives include an extra seepage measure. The reference geometry does not include a direct seepage measure. A larger berm has an influence on seepage. The seepage length becomes longer and less seepage occurs.
- The failure mechanism piping is not taken into account in this model. The larger berm of the deterministic design is based on the reference geometry designed for Pulau Tekong. Piping can be a reason that a larger berm is chosen.
4.6. Cost optimal design that satisfies the requirements of the current Dutch guidelines.

In the previous section it is concluded that the reference design is not designed cost optimal based on the three failure mechanisms. User requirements, allowances for uncertainties, seepage and other failure mechanisms can also have a large influence. This conclusion is confirmed with the model. According to the model, when using the reference design as input, it has a failure probability of almost 1/100,000 (1.11e-05). With the model it is possible to select a design that is cost optimal based on the three selected failure mechanisms and fulfils the 90/10 requirement of the current Dutch guidelines.

4.6.1. Cost optimal design that fulfils the 90/10 requirement of the current Dutch guidelines and trade-off between variables

With the model it is possible to select a design that is cost optimal based on the three selected failure mechanisms and that fulfils the 90/10 requirement of the current Dutch guidelines.

One of the 1470 input geometries used in the calculations fulfils this requirement and has a lower failure probability than the required 1/10,000 per year. This geometry is given in Table 4.17.

<table>
<thead>
<tr>
<th>P₇,model</th>
<th>Cost optimal design found by model</th>
<th>Design that exactly fulfils the 90/10 requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.11E-05</td>
<td>9.79E-05</td>
<td>4.93E-06</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 23,612</td>
<td>€ 20,547</td>
</tr>
<tr>
<td>Total cost (€*10⁶)</td>
<td>€ 236.1</td>
<td>€ 205.5</td>
</tr>
<tr>
<td>Hdiike (m+CD)</td>
<td>6.1</td>
<td>5.8</td>
</tr>
<tr>
<td>Outer slope (tanα)</td>
<td>0.33</td>
<td>0.36</td>
</tr>
<tr>
<td>Dn50 (m)</td>
<td>0.51</td>
<td>0.50</td>
</tr>
<tr>
<td>Hberm (m+CD)</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Wberm (m)</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Overtopping</td>
<td>94 %</td>
<td>53 %</td>
</tr>
<tr>
<td>Armour layer</td>
<td>6 %</td>
<td>34 %</td>
</tr>
<tr>
<td>Macro-stability</td>
<td>0 %</td>
<td>13 %</td>
</tr>
</tbody>
</table>

Table 4.17: Design found by the model that exactly fulfils the 90/10 requirement of the current Dutch guidelines. To compare, the reference design and the cost optimal design are also given.

Based on these results several conclusions are given below:

- Although the reference design is calculated based on the hydraulic boundary conditions that belong to a total failure probability of 1/10,000 per year, according to the model the total failure probability is lower.
- One design is found by the model that exactly matches the 90/10 requirement of the current guidelines.
- The design found by the model fulfils the requirement of the current guidelines, has less investment costs and has a lower total failure probability. It follows that with less investment costs (3%) a stronger design is found when the model is used.
4.6. Cost optimal design that satisfies the requirements of the current Dutch guidelines.

- Relaxing in the requirement of the current Dutch guidelines, results in a cost saving design. Compared to the reference design it is 31 million euro (13%) cheaper. Compared to the design found by model that exactly fulfils the 90/10 requirement of the current Dutch guidelines it is 24 million euro (10%) cheaper.

Although only one design exactly fulfils the 90/10 requirement, there are several designs that have a maximum contribution to the total failure probability of 10% for other failure mechanisms than overtopping. In Table 4.18 the cost optimal design is given that has a maximum contribution of 10% for the other failure mechanisms than overtopping.

<table>
<thead>
<tr>
<th></th>
<th>Design that exactly fulfils 90/10 requirement</th>
<th>Cost optimal with max. 10% for armour layer and macro-stability combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_f$,model</td>
<td>4.93E-06</td>
<td>7.07E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 22,963</td>
<td>€ 21,596</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 229.6</td>
<td>€ 216.0</td>
</tr>
<tr>
<td>$H_{dike}$ (m+CD)</td>
<td>5.8</td>
<td>5.8</td>
</tr>
<tr>
<td>Outer slope (tan$\alpha$)</td>
<td>0.32</td>
<td>0.36</td>
</tr>
<tr>
<td>Dn50 (m)</td>
<td>0.58</td>
<td>0.56</td>
</tr>
<tr>
<td>$H_{berm}$ (m+CD)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Wberm (m)</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Overtopping</td>
<td>90 %</td>
<td>96 %</td>
</tr>
<tr>
<td>Armour layer</td>
<td>3 %</td>
<td>4 %</td>
</tr>
<tr>
<td>Macro-stability</td>
<td>7 %</td>
<td>1 %</td>
</tr>
</tbody>
</table>

Table 4.18: The design found by the model that exactly fulfils the 90/10 requirement further optimised.

From Table 4.18 it is concluded that the model is not only able to provide a cheaper design; it may also provide a cheaper design when the maximum contribution of certain failure mechanisms is predetermined.

4.6.2. Optimal design when other user requirements are taken into account

For several reasons not only the contribution of a failure mechanism may be limited, but also the maximum value of a variable is limited. A user requirement might be that the crest of the sea dike should be as low as possible. For instance when luxury villas are being built behind the dike and a view over the sea is desired. In Table 4.19, a design with a lower crest height is given. This design still fulfils the 90/10 requirement of the current guidelines.
It follows that the design with a lower crest still fulfils the probability and the 90/10 requirement and has lower investment costs. The total failure probability of the dike section with a lower crest height is still a factor four lower than the required total failure probability and it might be that the design stays within the limits as other variables are also lowered. In Table 4.20 the result is given.

- The dike section with a crest of 5.6m+CD can be cost optimised by taking a lower $d_{50}$ or by taking a smaller berm. A steeper slope results in a dike section that does not fulfil the safety requirement.
- Cost optimising the dike section by decreasing the berm dimensions results in a dike section that does not fulfil the 90/10 requirement of the current Dutch guidelines.
4.6. Cost optimal design that satisfies the requirements of the current Dutch guidelines.

- The only way to optimise the dike section and still fulfil the safety requirement and the 90/10 requirement is by decreasing the stone size of the armour layer. Decreasing the stone size results in a direct trade-off of the available total failure probability between overtopping and armour layer stability. The contribution of overtopping decreases with 6% and the contribution of the armour layer stability increases with 6%.

Using the results of the model, the dike section can even be further optimised by using this trade-off of the available failure probability between the various failure mechanisms. This is done by manually selecting dike sections with different values for the variables. An example is given in Table 4.21, where a design is chosen with respectively larger berm dimensions and a gentler slope in order to further decrease the stone size of the armour layer. A smaller stone size may be preferred when, for example, a quarry cannot deliver the required stone size.

<table>
<thead>
<tr>
<th></th>
<th>Design with lower crest and lower Dn50</th>
<th>Design with lower crest and lower Dn50 compensated by larger berm dimensions</th>
<th>Design with lower crest and lower Dn50 compensated by a gentler slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_f$,model</td>
<td>2.67E-05</td>
<td>2.62E-05</td>
<td>2.01E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 21,956</td>
<td>€ 22,457</td>
<td>€ 22,457</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 219.6</td>
<td>€ 224.8</td>
<td>€ 224.8</td>
</tr>
<tr>
<td>H_dike (m+CD)</td>
<td>5.6</td>
<td>5.6</td>
<td>5.6</td>
</tr>
<tr>
<td>Outer slope (tana)</td>
<td>0.32</td>
<td>0.32</td>
<td>0.28</td>
</tr>
<tr>
<td>Dn50 (m)</td>
<td>0.54</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>H_berm (m+CD)</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>W_berm (m)</td>
<td>6</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Overtopping</td>
<td>91 %</td>
<td>91 %</td>
<td>49 %</td>
</tr>
<tr>
<td>Armour layer</td>
<td>7 %</td>
<td>9 %</td>
<td>28 %</td>
</tr>
<tr>
<td>Macro-stability</td>
<td>2 %</td>
<td>0 %</td>
<td>23 %</td>
</tr>
</tbody>
</table>

*Table 4.21: The trade-off between variables in order to fulfil specific user requirements.*

Based on the table above it is concluded that it is possible to fulfil the demands of specific user requirements by changing the contribution of the failure mechanisms. This is achieved by the trade-off between the failure mechanisms by changing the variables. Not only can the stone size of the armour layer be decreased by this trade-off between the failure mechanisms, it may also be possible to obtain a steeper slope or smaller berm dimensions.

It has to be noted that his trade-off may be more evident with higher boundary conditions and with a higher decimation height of the water level and wave height. This is the case in the Netherlands and it is therefore highly recommended to use the model for optimising a sea dike in the Netherlands.

4.6.3. Summary and main conclusions

A short summary of the steps taken in this section is given below:
Based on these steps the most important conclusions are summarised below:

- Compared to the reference design and using a probabilistic approach, a cost saving design for the sea dike at Pulau Tekong is found that exactly fulfils the requirement of the current Dutch guidelines. The design not only has less investment costs, it also has a lower total failure probability. It follows that with less investment costs (3%) a stronger design is found when the model is used.

- Relaxing in this 90/10 requirement of the current Dutch guidelines, cheaper designs can be found. Compared to the reference design, 31 million euro (13%) can be saved. Compared to the design found by model that exactly fulfils the 90/10 requirement of the current Dutch guidelines, 24 million euro (10%) can be saved.

- The design with a lower crest still fulfils the probability and the 90/10 requirement and also has lower investment costs. The dike section with a crest of 5.6m+CD can be further cost optimised by taking a lower $d_{50}$ or by taking a smaller berm. A steeper slope results in a dike section that does not fulfil the probability requirement anymore.

- It is possible to fulfil the demands of specific user requirements by changing the contribution of the failure mechanisms. This is achieved by the trade-off between the selected variables. An example is given where a smaller stone size of the armour layer is compensated by a larger berm.
4.6. Cost optimal design that satisfies the requirements of the current Dutch guidelines.
5. **Sensitivity analysis**

5.1. **Introduction**

In this chapter first the influence of the each variable on the cost and the failure probability of the cost optimal design are investigated. The cost optimal design is based on the case study Pulau Tekong and several aspects that influence the design are made based on assumptions that are typical for the case in Singapore. The sensitivity of the cost optimal design on these aspects is investigated by replacing them by aspects that are typical Dutch. The questions given below, which relate to the situation in the Netherlands, are answered in this chapter.

- **What is the effect of the sand key on the cost optimal design?** In Singapore a sand key is built below the dike section. This sand key provides extra stability. On the other side this sand key has a negative influence on seepage. In the Netherlands often a different sub layer is found.

- **What is the effect of correlation between wave height and water level on the dike section?** In Singapore it is assumed that there is no correlation between water level and wave height and extra dike height is added to the design to cope with this uncertainty. For the Dutch situation a higher correlation is expected.

- **What is the effect when a lower failure probability is chosen?** Return periods with a maximum of 10,000 years are often used for sea dikes in the Netherlands. The new standards can lead to even lower failure probabilities. Not only in the Netherlands lower failure probabilities are considered, also in Singapore this is the case as is explained later on in this report.

In section 4.1.2 the three key drivers for the design of the dike section are noted and they are repeated below:

- Flood protection level;
- Seepage of saline water under and through the dike;
- Phasing of the project

The answers on the questions are given while keeping these key drivers in mind.

5.2. **Influence of each variable on the cost optimal design**

In the previous chapter it was concluded that the footprint of the dike section has a large influence on the cost optimal design. In the next section it is checked if this statement is true. The influence of the various variables on the cost of the design is investigated. Before selecting the cost optimal design, the model selects all the designs that fulfil the probability requirement. In section 5.2.2 the influence of the variables on the failure probability is investigated.

5.2.1. **Influence of each variable on the cost of the design**

There are two reasons why in all the designs the footprint is responsible for almost 60 percent of the costs:

1. The value for land expropriation is relative high in Singapore. For every metre extra width, of the dike section the investment costs are raised with 216 euro. In the Netherlands often a value of 100 euro is used in calculations (Bischiniotis, 2013).
5.2. Influence of each variable on the cost optimal design

2. The footprint is calculated with the following equation:

\[ \text{Footprint} = W_{dike} + \frac{H_{dike} + 1}{\text{OuterAngle}} + \frac{H_{dike} + 1}{\text{InnerAngle}} + W_{berm} \]

In this equation \( H_{dike} \) is increased with 1 metre. \( H_{dike} \) is the height of the dike measured from Chart Datum. The polder level is \(-1m+CD\). To retrieve the actual height of the dike, \( H_{dike} \) should be increased with 1 metre. The equation is a summation of four terms. Only the term \( W_{dike} \) is constant. The other three terms vary for every design and increase or decrease the footprint.

A wider berm automatically implies a larger footprint. Decreasing the berm width implies that the contribution of the failure mechanism macro-stability to the total failure probability increases. The available failure space for the other failure mechanisms, overtopping and armour layer stability, decreases. This decrease needs to be compensated by a higher resistance to overtopping and armour layer instability. This can be achieved by increasing the variables dike height, outer slope angle, the stone size or a combination of them. In most cases either a higher crest or a gentler outer slope is the result. Both of these changes increase the footprint of the dike section again and it can be concluded that a significant decrease of the footprint is not realistic. In the following figure this is illustrated.

**Distribution of the failure probability over the various failure mechanisms**

![Distribution of the failure probability over the various failure mechanisms](image)

*Figure 5.1: Possible reasons for a change in the distribution of the failure probability over the various failure mechanisms.*

The cost for the footprint depends on the crest height, slope angle and the berm width. The cost for the dike area depends on the outer slope angle and the dike height. In Figure 5.2 the influence of each variable on the cost optimal design is presented in a graph. In this figure the increase or decrease of the total costs are given when each variable changes per step and all the other variables are kept constant. The variables and steps in these graphs are given in Table 5.1. Also the minimum and maximum are given that are used in the costs investigation. These values
are the same values that are used to find the cost optimal design of the sea dike at Pulau Tekong. The values at step 2 represent the cost optimal design.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hdike (m)</strong></td>
<td>-</td>
<td>5.6</td>
<td>5.8</td>
<td>6.0</td>
<td>6.2</td>
<td>6.4</td>
<td>6.6</td>
<td>6.8</td>
</tr>
<tr>
<td><strong>Outer slope (tan α)</strong></td>
<td>-</td>
<td>0.40</td>
<td>0.36</td>
<td>0.32</td>
<td>0.28</td>
<td>0.24</td>
<td>0.20</td>
<td>-</td>
</tr>
<tr>
<td><strong>Hberm (m)</strong></td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Dn50 (m)</strong></td>
<td>0.46</td>
<td>0.48</td>
<td>0.50</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 5.1: Minimum, maximum and step size for each variable for the costs investigation.*

In the figure above the cost optimal design is given at step 2. It is already visible that a change in slope angle means the largest decrease or increase in costs. In Figure 5.3 the additional costs are further specified. The additional costs are the increase of the total costs when one variable changes and all the other variables are kept constant. In the left figure the increase compared to the cost optimal design is given, while in the right figure the increase compared to the previous step is given.
5.2. Influence of each variable on the cost optimal design

Additional costs

<table>
<thead>
<tr>
<th>cumulative</th>
<th>per step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>€6000</td>
<td>€2000</td>
</tr>
<tr>
<td>€5000</td>
<td>€1800</td>
</tr>
<tr>
<td>€4000</td>
<td>€1400</td>
</tr>
<tr>
<td>€3000</td>
<td>€1200</td>
</tr>
<tr>
<td>€2000</td>
<td>€1000</td>
</tr>
<tr>
<td>€1000</td>
<td>€800</td>
</tr>
<tr>
<td>€0</td>
<td>€600</td>
</tr>
</tbody>
</table>

Figure 5.3: Additional costs per step and cumulative for changes per variable, while the other variables are kept constant. The steps are taken within the area of interest of each variable and are specified in Table 5.1.

Several observations that follow from Figure 5.2 and Figure 5.3 are:

- It can be clearly observed that a gentler slope means the largest increase in costs.
- In the left figure it is visible that additional costs for a change in the dike height and the stone size are constant (straight line). The additional costs for a change in berm dimensions and slope angle increase for every step. This observation is confirmed in the right figure.
- A constant increase is observed for the berm dimensions, while the additional costs for the slope angle increases nonlinear.

In the case of Pulau Tekong, the model for the cost optimal dike is clearly looking for a dike section with a small footprint and a relative steep slope. In section 3.4, it is determined that the cost of the design consists out of four parts: the footprint, the dike area, the berm area and the stones for the revetment. In the figures the contribution of these parts is given while the slope angle is changing. The other variables are kept constant.
5. Sensitivity analysis

Figure 5.4: Increase in cost due to a gentler slope angle. All the other variables are kept the same.

Figure 5.5: Change in contribution of the four elements to the total costs because of a change in slope angle.

It is concluded from the above figures that, although the footprint becomes larger due to a gentler slope, the main reason of the increase in costs are the stones for the revetment. The slope length increases because of a gentler slope and thus the contribution of the stones increases.

For an increasing dike height the costs are mainly dominated by an increasing dike area, while the costs due to increasing berm dimensions are mainly dominated by an increasing berm area. The cost of the armour layer mainly dominates the increasing costs due to an increasing $d_{50}$.

In Appendix I the same graphs are presented for the other variables. When other values for the different components of the total costs are taken, it is possible that a completely different cross section is cost optimal.
5.2. Influence of each variable on the cost optimal design

5.2.2. Influence of each variable on the failure probability of the design

Before selecting the cost optimal design based on the investment costs, the model selects all the designs that fulfil the probability requirement. In this section the influence of the variables on the failure probability is investigated. The same strategy as in the previous section is used, the cost optimal design is used and only one of the variables changes while the other are kept constant. The variables and steps in that are used are given in Table 5.2. These values are the same values that are used to find the cost optimal design of the sea dike at Pulau Tekong. The values at step 2 represent the cost optimal design.

<table>
<thead>
<tr>
<th>Hdike (m)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer slope (tanα)</td>
<td>-</td>
<td>0.40</td>
<td>0.36</td>
<td>0.32</td>
<td>0.28</td>
<td>0.24</td>
<td>0.20</td>
<td>-</td>
</tr>
<tr>
<td>Hberm (m)</td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dn50 (m)</td>
<td>0.46</td>
<td>0.48</td>
<td>0.50</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.2: Minimum, maximum and step size for each variable for the probability investigation.

In the figure above the cost optimal design is given at step 2. It is already visible that a change in slope angle means the largest decrease or increase in failure probability. Not only a change in slope angle has the largest influence on the cost, it also has the largest influence on the total failure probability. The total failure probability is determined based on the failure probability of overtopping, armour layer stability and macro-stability. In the figures the contribution of these parts is given while the slope angle is changing. The other variables are kept constant.
5. Sensitivity analysis

Figure 5.7: Influence of the outer slope angle on the failure probability of the cost optimal design.

Figure 5.8: Change in contribution of the three failure mechanisms to the total failure probability because of a change in slope angle.

From the above figures it is concluded that a gentler slope means a decrease of the total failure probability because of a decrease of the failure probability due to overtopping and armour layer stability. Although the failure probability of macro-stability remains constant for a gentler outer slope, the contribution to the total failure probability becomes significantly larger.

In the following two figures the influence of changing berm dimensions on the total failure probability is presented.
5.2. Influence of each variable on the cost optimal design

The contribution of macro-stability to the total failure probability reduces rapidly and when a berm height of 3 metres is chosen the contribution is already practically zero. Based on the above two figures it is concluded that changing berm dimensions have a large influence on the total failure probability. It might not be worth it to increase the other variables so much that smaller berm dimensions are possible, or the other way around. This might be the reason that the top 5 cost optimal designs all have the same berm dimensions. The effect may be less when a smaller step size is taken for the berm dimension. Another and may be better solution is to also consider the inner slope angle as a variable. The inner slope angle might have a large influence on the macro-stability, while the effect of the variable dike height is only limited and the effect of the outer slope angle and stone size of the armour layer is zero.

The same analysis is done for a changing dike height and a changing stone size of the armour layer. The results are given in Appendix J.
5.3. Effect of the sand key on the cost optimal design

The first change is the removal of the sand key. This sand key was already installed before the first dike alternatives were developed. In the Netherlands often different subsoil is found on which a sea dike is built. A significant disadvantage of this sub layer is that it increases the instability of the dike section. In Singapore a large sand key is placed in order to guarantee the stability of the dike section. It can be expected that, with the removal of the sand key, a larger part of the failure probability is reserved for the failure mechanism macro-stability. Another possible result is that the cost optimal design will include a larger berm to cope with the larger instability of the dike section.

5.3.1. Change of input and effect on the deterministic geometry

In Figure 5.11 the new cross section is given. In this cross section the sand key of the reference geometry is removed and is replaced by marine clay.

![Figure 5.11: The sand key of the reference geometry is removed and is replaced by marine clay.](image)

The values used in the calculation are the same as given in section 4.4.2. A different sub layer does not have any effect on the deterministic calculation for overtopping and the armour layer. The values found from these calculations remain the same. The removal of the sand key has an effect on the macro-stability and therefore the new cross section is tested in D-Geo Stability (Figure 5.12).

![Figure 5.12: Critical slip circle for the reference geometry without the sand key as given in D-Geo Stability.](image)
5.3. Effect of the sand key on the cost optimal design

The critical slip circle is determined with the use of D-Geo Stability. This is another slip circle than found for the design including the sand key. The calculated safety factor decreased from 2.13 to 1.80. This is still above the required partial factor of safety of 1.25 that is recommended. Hence, the new cross section fulfills the requirements.

5.3.2. Cost optimal design

With the use of Matlab and D-Geo Stability again 1470 different cross-sections have been tested and this time only 786 different cross-sections have a lower failure probability than 1:10,000. A total of 146 geometries have lower investment costs than the reference geometry. The characteristic values of the top five optimal designs are given in Table 5.3. Also the reference geometry is given and the cost optimal design with the sand key.

<table>
<thead>
<tr>
<th></th>
<th>Reference design</th>
<th>Optimal design with sand key</th>
<th>Optimal designs without a sand key</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1st</td>
</tr>
<tr>
<td>( P_{f,\text{model}} )</td>
<td>1.11E-05</td>
<td>9.79E-05</td>
<td>8.77E-05</td>
</tr>
<tr>
<td>Cost ((\text{€}/\text{m}))</td>
<td>€ 23,612</td>
<td>€ 20,547</td>
<td>€ 21,144</td>
</tr>
<tr>
<td>Total cost ((\text{€}*10^6))</td>
<td>€ 236.1</td>
<td>€ 205.5</td>
<td>€ 211.4</td>
</tr>
<tr>
<td>( H_{dike} \text{ (m+CD)} )</td>
<td>6.1</td>
<td>5.8</td>
<td>5.8</td>
</tr>
<tr>
<td>Outer slope (\text{ (tan}a))</td>
<td>0.33</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>( D_{\text{n50}} \text{ (m)} )</td>
<td>0.51</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>( H_{\text{berm}} \text{ (m+CD)} )</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( W_{\text{berm}} \text{ (m)} )</td>
<td>8</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 5.3: Top five optimal designs

In Figure 5.13 the contribution of the various failure mechanisms to the total failure probability is given.

![Figure 5.13: Contribution of the failure mechanisms to the failure probability.](image-url)
5.3.3. Conclusions and recommendations

- The sand key in the model has influence on the macro-stability of the dike section; the berm dimensions increase.
- Although the dike section has a lower stability without the sand key, the contribution of the failure mechanism macro-stability decreases because of a larger berm (maximum of 9%).
- The cost optimal design without the sand key is only six million euros more than the cost optimal design with the sand key. The sand key in the model improves the macro-stability of the dike section, however a larger berm compensates for the lower stability of the dike section without the sand key. The construction of the sand key may have been a large investment and it is shown that a larger berm can take over the stability at most probably lower costs. The costs for the construction of the sand key are not taken into account in this research, because the sand key was already installed before starting with the design of the sea dike.
- Less seepage can be expected with a sub layer that consists entirely of clay. The second key driver for the dike alternatives is the seepage of saline water under and through the dike. Keeping this second key driver in mind, the dike section without a sand key is a good alternative. Although the dike section is slightly more expensive.

5.4. Effect of correlation between wave height and water level

In section 4.2 it has been noted that almost no correlation was observed between the water level and the wave height that acts as the hydraulic boundary conditions for the sea dike for the polder at Pulau Tekong. The crest height of the dike section is raised with an additional 0.4 metres to cope with, among others, the uncertainty in the amount of correlation. Until now for all the calculations a correlation of zero has been assumed. The effect of correlation is investigated in this section. First a small correlation is observed (\(\rho=0.3\)) before more correlation is added (\(\rho=0.6\)). Finally the dike section is moved to a more typically Dutch situation, with a relative high correlation (\(\rho=0.9\)). In the Netherlands the wave height is correlated to the water level because during storms they are both generated by local wind. In Singapore it was observed that there is little correlation between local surge and the local wind speed.

5.4.1. Change of input

The only change in input for this investigation is in the hydraulic boundary conditions. The correlation is raised in small steps and the effect is observed. In section 2.3.3 the construction of the joint probability density curve of water level and wave height has already been treated. Below the contour lines of the joint probability density curves are given for the various correlations that are investigated here.
5.4. Effect of correlation between wave height and water level

Figure 5.14: Contour of the joint probability curve for various correlations.

In the figures it is visible that a higher correlation implies that higher waves occur more often during high water and lower waves occur more often during low water. In a situation with low correlation higher waves occur during both high and low water.

It is also visible in the figures above that for this specific case the difference between the lowest and highest waves is only several decimetres. The difference between the lowest and highest water levels is also several decimetres. This is because at the island Pulau Tekong a low decimation height is observed for the water level and wave height. In the Netherlands typical hydraulic boundary conditions may consist of surge levels of 5 meters and waves heights of 4-5 meters. A significantly higher decimation height is found in the Netherlands. Because of this significantly higher decimation height for the hydraulic boundary conditions, the influence of correlation becomes also significantly higher.

In the next section the effect of correlation on the cost optimal design is discussed.

5.4.2. Cost optimal design

In Table 5.4 the characteristic values for the reference design and for the cost optimal designs for various correlations are given.
Table 5.4: The cost optimal design for various correlations.

From the table above the following can be observed:

- The only variable that needs to change to meet the safety requirement is the crest height; all the other variables remain the same. The increase in crest level is linear.
- An increase in correlation also increases the costs because of a higher crest level. This increase is almost linear as explained in section 5.2.
- For every correlation the model searches for the dike section with a small footprint. Hence, steep slopes and small berms.
- The influence of correlation on the costs is relative low. This is expected to be more for the Dutch situation, where higher boundary conditions are found.

Based on these observations it apparently does not make sense, in terms of cost efficiency, to increase the stability of the dike to keep the same crest height.

It can be expected that the distribution of the failure probability over the various failure mechanisms changes for a higher correlation. In Figure 5.15 this distribution is presented.

![Figure 5.15: Distribution of the failure probability over the failure mechanisms for various correlations.](image)

From this figure it can be concluded that there are no significant changes in the distribution due to correlation. It has been noted earlier in the report that the dike area (without a berm) is, from
5.4. Effect of correlation between wave height and water level

A geotechnical perspective, very stable. An inner slope of 1:3 already provides a large part of the inner slope stability. When a steeper inner slope is considered, another conclusion may be possible.

Another way to look at the influence of correlation is to keep the dimensions of the dike section the same and calculate the failure probability. The dike section that is used is the cost optimal dike section found in section 4.5.1. The results are presented in Table 5.5 and Figure 5.16.

<table>
<thead>
<tr>
<th>Correlation</th>
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<td>6.37E-04</td>
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</tr>
<tr>
<td>H_{dike} (m+CD)</td>
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<td>5.8</td>
<td>5.8</td>
<td>5.8</td>
</tr>
<tr>
<td>Outer slope (tanα)</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>D_{50} (m)</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>H_{berm} (m+CD)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>W_{berm} (m)</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5.5: Influence of correlation for one dike section.

From the above it can be concluded that an increase in correlation results in higher failure probabilities for one dike geometry.

5.4.3. Conclusions

- An increase in correlation has a negative influence on the failure probability given a certain dike geometry.
- For the cost optimal dike section of the sea dike at Pulau Tekong an increase in dike height compensates the higher failure probability.
- It apparently does not make sense, in terms of cost efficiency, to increase the stability of the dike to keep the same crest height.
- In the distribution of the failure probabilities over the various mechanisms there are no significant changes. Therefore the choice of Royal HaskoningDHV to take an extra allowance
for the crest level into account to cope with uncertainties as, among others, correlation seems to be a good choice.

5.5. Effect of a lower failure probability

The new design standards for the Netherlands may lead to lower design failure probabilities. Also in Singapore, while keeping the third key driver (phasing of the project) in mind, a lower failure probability may be required when more urbanisation will take place in the polder thirty years after the construction.

5.5.1. Change of input

In section 4.1.2 two dike alternatives are given that are so-called over-dimensioned. These alternatives are developed for the second user phase of the project. In this phase urbanisation will take place and a higher safety level is recommended. Instead of a maximum return period of 10,000 years a lower return period 50,000 years is used. All the other input parameters are kept the same.

5.5.2. Cost optimal design

In Table 5.6 the results are given for a failure probability of 1/50,000 per year. The top five cost optimal designs are given. To be able to compare also the reference design is given and the cost optimal design for a failure probability of 1/10,000 per year.

<table>
<thead>
<tr>
<th>Reference design</th>
<th>Optimal design 1/10,000</th>
<th>Optimal designs 1/50,000</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Pₖ, model</td>
<td>Cost (€/m)</td>
</tr>
<tr>
<td></td>
<td>1.11E-05</td>
<td>9.79E-05</td>
</tr>
<tr>
<td></td>
<td>€ 23,612</td>
<td>€ 20,547</td>
</tr>
<tr>
<td></td>
<td>Total cost (€*10⁶)</td>
<td>€ 236.1</td>
</tr>
<tr>
<td>H dike (m+CD)</td>
<td>6.1</td>
<td>5.8</td>
</tr>
<tr>
<td>Outer slope (tana)</td>
<td>0.33</td>
<td>0.36</td>
</tr>
<tr>
<td>D₅₀ (m)</td>
<td>0.51</td>
<td>0.50</td>
</tr>
<tr>
<td>H berm (m+CD)</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>W berm (m)</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5.6: The top five cost optimal designs for a probability of failure of 1/50,000.

Remarkable is that there is one design (4th) with the same berm dimensions as the cost optimal design for a return period of 10,000 years. A lower failure probability means that higher hydraulic boundary conditions will be used for the design. A higher design water level increases the risk on instability of the dike section. This can be compensated by a higher crest or a larger berm. The 4th optimal design has a higher crest and the same berm dimensions. It can be expected that the contribution of macro-stability becomes larger. In Figure 5.17 the change in contribution is given.
5.5. Effect of a lower failure probability

As expected the contribution of macro-stability increased (from 13% to 69%). It is not recommended to assign such a significant part of the failure space to the failure mechanism macro-stability of the inner slope. In general the failure of a dike section because of overtopping and outer slope instability can be recognised before the actual failure occurs. High water levels and waves can be predicted and erosion of the armoured layer and crest can be observed. The failure due to macro-instability can occur quite suddenly and with no warning in advance.

The distribution of the failure probability can be a reason not to choose the cost optimal design. Another possible reason is to keep the opportunity to reinforce the dike section when a higher safety level is suddenly required, for example when more and more people are living behind the dike.

In Singapore urbanisation will start thirty years after the development of the polder at Pulau Tekong. In this second phase of the polder a higher safety level is required and the sea dike needs reinforcement. The possibility to change the slope angle in Singapore is very limited, because the dike section is located on the border between Singapore and Malaysia. A change in stone dimensions is also a limited option; the entire revetment needs to be replaced. It is more efficient to raise the crest level or increase the berm. In Table 5.7 two different optimal designs are presented for a failure probability of 1/10,000 per year and the cost optimal design for a failure probability of 1/50,000 per year.
5. Sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>Reference design</th>
<th>Optimal design 1/10,000</th>
<th>2nd optimal design 1/10,000</th>
<th>Optimal design 1/50,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{T_{\text{model}}} )</td>
<td>1.11E-05</td>
<td>9.79E-05</td>
<td>8.82E-05</td>
<td>1.85E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 23,612</td>
<td>€ 20,547</td>
<td>€ 20,698</td>
<td>€ 21,928</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 236.1</td>
<td>€ 205.5</td>
<td>€ 207.0</td>
<td>€ 219.3</td>
</tr>
<tr>
<td>Hdike (m+CD)</td>
<td>6.1</td>
<td>5.8</td>
<td>5.8</td>
<td>6</td>
</tr>
<tr>
<td>Outer slope (tan( \alpha ))</td>
<td>0.33</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>Dn50 (m)</td>
<td>0.51</td>
<td>0.50</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>Hberm (m+CD)</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Wberm (m)</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 5.7: Optimal designs for various safety levels.

From the table above it follows that to reinforce the 2nd best design for the 1/10,000 per year failure to the cost optimal design for the 1/50,000 per year the crest height has to be raised with 0.20 metres and the berm should be enlarged with several metres. This is visualised in Figure 5.18.

![Figure 5.18: Extra space needed for reinforcement.](image)

Only on the inner side of the dike reinforcement is necessary and this is relative simple to construct; the hydraulic boundary conditions do not act on the inner side of the dike and construction can be realised from land. It is possible to keep the reinforcement in mind in the first design. The footprint becomes 3.16 metres larger and this area can be reserved. Therefore the second optimal design for a failure probability of 1/10,000 per year (Figure 5.18) is advised for the sea dike at Pulau Tekong. This design is easily adaptive when in the near future a lower failure probability is considered.

In Figure 5.19 the change in contribution of the various failure mechanisms to the total failure probability is given.
5.5. Effect of a lower failure probability

As can be expected the contribution of the armour layer stability becomes larger; both variables where it depends on (slope angle and $d_{50}$) do not change while a lower failure probability is chosen.

5.5.3. Conclusions

- Although the cost optimal design for a return period of 1:50,000 years has the lowest amount of investment costs, it is not necessarily the best option. The distribution of the failure mechanisms over the failure space or the opportunity to reinforce the dike are reasons not to choose the cost optimal design.
- With only a small investment the design for a failure probability of 1:10,000 can be reinforced to a design for a failure probability of 1:50,000. A relative small area (3.16 metre) should be reserved for the second phase of the project.
- A lower failure probability does not directly lead to a higher crest level; it can be compensated by increasing the stability of the dike section. In a deterministic calculation it will directly lead to a higher crest level. The same can be said about the $d_{50}$. 

![Figure 5.19: Change in contribution of the failure mechanisms to the total failure probability.](image)
6. Conclusions & recommendations

6.1. Introduction

The aim of the research is given by the following objective:

"Find a cost optimal design of a sea dike by using probabilistic methods and flexibility in the distribution of the total failure probability over the various failure mechanisms"

This research describes a specific approach for the calculation of the cost optimal design, followed by a case study and a sensitivity analysis. For the purpose of this study a probabilistic model has been set up to calculate the cost optimal design for a sea dike while taking into account:

- the failure mechanisms overtopping, armour layer stability and macro-stability;
- a number of input geometries that differ in crest height, outer slope angle, berm dimensions and \( d_{50} \);
- various correlations between water level and wave height;
- the prices for the various elements of the dike section.

With the model it is possible to quantify the design in terms of the contribution of the selected failure mechanisms to the total failure probability and select the cost optimal design. To be able to find the cost optimal design, interaction between the selected failure mechanisms within the failure probability of the sea dike is necessary.

The model is set up in such a way that it is relatively easy to:

- change the input values for the variables, hydraulic boundary conditions, parameters and costs;
- add another failure mechanism;
- take other variables into account.

In this chapter the main conclusions are presented followed by a recommendation for the design of the sea dike at Pulau Tekong. Based on the case study and the sensitivity analysis the research questions, which were presented in chapter 1, are answered. In the final section several recommendations for further research are given.

6.2. Main conclusions and recommendation

Based on this research it is concluded that:

- When using the current Dutch guidelines, the dike is designed on hydraulic boundary conditions corresponding to an exceedance probability of 1/10,000 per year. The failure probability of the sea dike may differ from this value. With the model it is possible to calculate this failure probability and to determine the contribution of each failure mechanism.

- When using a probabilistic approach, instead of a deterministic approach, a cost saving design may be found that still fulfils the requirement of the current Dutch guidelines. Although this design costs less, it does not directly mean that the design is less robust. It is also possible to find a design with a lower failure probability and with less investment
6.3. Research questions

costs. For the sea dike at Pulau Tekong, using a probabilistic approach and no flexibility in the contribution of the various failure mechanisms, it does not only result in a design with less investment costs (3%), but also a stronger design is found.

- Allowing more flexibility in the distribution of the total failure probability over the various failure mechanisms, may result in an even more cost saving design. This is achieved by changing the key dimensions of the dike section in such a way that trade-off of failure space between the various failure mechanisms occurs. For the sea dike at Pulau Tekong, adding flexibility in the distribution of the total failure probability, results in a reduction of the investment costs of 31 million euro (13%) compared to the deterministic design.

- It is possible to fulfil the demands of specific user requirements by changing the contribution of the failure mechanisms. This is achieved by the trade-off of failure space between the selected failure mechanisms. An example was given where a smaller stone size of the armour layer is compensated by a larger berm. The failure mechanism armour layer stability takes over the contribution of the failure mechanism macro-stability.

For the sea dike at Pulau Tekong the recommended design and key dimensions for a total failure probability of 1/10,000 per year is given in Figure 6.1. This design is easily adaptive when in the future the dike should have to comply with a lower total failure probability. In the figure also the extra space needed for dike design that satisfies to a lower failure probability (1/50,000 per year) is given.

![Figure 6.1: The design for a probability requirement of 1/10,000 per year is easily adaptive to a design that fulfils a probability requirement of 1/50,000 per year.](image)

For both designs still the majority of the available failure space is taken up by overtopping.

6.3. Research questions

What is the difference with a design based on the current guidelines?

Using a probabilistic method and flexibility in the distribution of the total failure probability over the various failure mechanisms cost savings can most possibly be achieved. In Figure 6.2 the design based on the current Dutch guidelines and the cost optimal design calculated with the model are given.

![Figure 6.2: Deterministic design and cost optimal probabilistic design given in one figure.](image)
6. Conclusions & recommendations

Given the fact that the resistance against failing of the dike section is determined by the height and strength of the dike section, it is concluded from this figure that the cost optimal dike section is less robust than the reference design.

This conclusion is confirmed with the developed model. A probabilistic evaluation of the reference model shows that the total failure probability of the cost optimal design is almost ten times higher than the total failure probability of the reference design.

What variables characterise the cost optimal design and what is the effect of the exchange of variables on the distribution of the total failure probability over the various mechanisms for the cost optimal design?

In the case study it is observed that both overtopping and armour layer stability take up most of the available total failure space. There is interaction between the variables and it is possible to exchange failure space between the failure mechanisms. High interaction is observed between armour layer stability and overtopping and lower interaction between these two and macro-stability. Based on the interaction found in this research, from a geotechnical point of view a sea dike itself appears to be already very stable and especially on stability a sea dike can be cost optimised.

What element that characterises the geometry of a dike section has the largest influence on the cost optimal design?

For the sea dike at Pulau Tekong, it is clearly observed that the model is looking for a dike section with a small footprint and a relative steep slope. The slope angle has the largest influence on the cost of the design. A gentler slope means an increase in berm area and footprint, but the main reason of the increase in costs are the stones for the revetment (Figure 6.3). The slope length increases because of a gentler slope and thus the costs of the revetment increases.

What is the effect of different soil parameters on the cost optimal design?

The sand key in the model improves the macro-stability of the dike section, however a larger berm compensates for the lower stability of the dike section without the sand key. The construction of the sand key may have been a large investment and it is shown that a larger berm can take over the stability at most probably lower costs.
6.5. Recommendations

*What is the effect of correlation between wave height and water level on the dike section?*

An increase in correlation has a negative influence on the total failure probability given a certain dike geometry. For the cost optimal dike section of the sea dike at Pulau Tekong an increase in dike height compensates for this higher failure probability. The influence of correlation on the costs is relative low for the sea dike at Pulau Tekong. This is expected to be more for the Dutch situation, where a larger decimation height, higher water levels and higher wave heights are typically found.

*What is the effect when a lower failure probability is chosen?*

A lower failure probability means that higher hydraulic boundary conditions will be used for the design. Although a higher design water level is used, a lower failure probability does not directly lead to a higher crest level; it can be compensated by increasing the stability of the dike section. In a deterministic calculation it will directly lead to a higher crest level. The same can be said about the $d_{50}$.

6.5. Recommendations

To further increase the performance of the model several recommendations are listed below.

*Use a set of boundary conditions with larger decimation heights.*

Only one set of boundary conditions has been used in this research. Another set of boundary conditions, with higher decimation heights of the water level and wave height, can lead to different designs and more interaction between the different variables can be expected. For that case it is recommended to use a set of boundary conditions that are typical for the Dutch coast.

*Include other distributions for the water level and wave height.*

In this research the Gumbel distribution is assumed for both water level and wave height. Although it is an often used distribution for yearly maxima of wave height and high sea water levels, other extreme value distributions may fit the data better (e.g. the Rayleigh distribution).

*Include surging waves.*

All waves are assumed to be plunging. Although the majority of waves fall within the plunging region, surging waves can have an effect on the geometry of the most cost optimal dike section.

*Use the armour layer as a variable in the calculation of the fragility curve of overtopping.*

In the calculation of overtopping, the parameter $y_f$ (factor for slope roughness) changes depending on the type of revetment. In this research it is taken as a constant in the calculations for overtopping. In this model only a stone revetment is used and the difference in roughness, because of larger armour stones, is difficult to estimate. Therefore the $d_{50}$ is taken as a variable for the armour layer stability, but the parameter $y_f$ remains constant for the failure mechanism overtopping.

*Include different kinds of armour layer.*

In this model only a stone revetment is considered. For higher waves the elements at the most external layer of a revetment are often concrete elements (e.g. Xbloc and Accropode).

*Use the inner slope angle as a variable.*

It is assumed that the influence of a berm is larger than the influence of the *inner* slope angle and that it has a larger effect on the geometry of the dike section. Based on the results it is concluded that an inner slope of 1:3 already provides a large part of the inner slope stability. More interaction between the variables is expected when also the inner slope angle is used as a
variable. In Appendix G the effect of an inner slope angle of 1:2 on the cost optimal design is presented.

*Use the width and the height of the berm as two separate variables.*

In this research it is assumed that the width and height of the berm are in proportion. Instead of two variables (width and height) only one variable is used for the berm. It is expected that more interaction between the variables occur, when the width and height of the berm are used as separate variables. Another advantage is that the influence of a wider and lower berm or a smaller and higher berm on the cost optimal design can be investigated.

*Include other failure mechanisms.*

The failure or breaching of a dike can occur due to twelve typical mechanisms. It is assumed that the three selected failure mechanisms (overtopping, armour layer stability and macro-stability of the inner slope) do influence the geometry of the dike the most. However a dike section should fulfil the requirements of all the failure mechanisms. Other failure mechanism can have influence on the dike geometry as well, especially piping and stability of the outer slope.

*Increase the accuracy of the model.*

The accuracy of the model depends on the computation time. The computation time is mainly influenced by a multiplication of the following elements:

- the number of Monte Carlo simulations ($N_s$);
- the number of combinations between the (deterministic) water level and the (deterministic) wave height used for the calculation of the various fragility curves;
- the number of failure mechanisms;
- the number of cross sections.

The relation between these elements in the model is given in Figure 6.4.

![Figure 6.4: Relation between the elements that determine the computation time.](image)

The number used for the different elements are chosen such that the model has a reasonable computation time without losing too much accuracy. In Appendix F. the effect of a higher
6.5. Recommendations

number of Monte Carlo simulations on the accuracy and computation time is given. It is also possible to apply importance sampling.

*Investigate the effect of the costs on the cost optimal design.*
The costs are based on the investment costs of the dike section and are derived from figures determined by Royal HaskoningDHV. In the Netherlands other figures may be used. The value for land expropriation is relative high in Singapore. For every metre extra width of the dike section, the investment costs are raised with 216 euro. In the Netherlands often a value of 100 euro or less is used in calculations.

Other costs than the investment costs can be included in the model. Often a larger berm is designed for multifunctional purposes. The berm can be used as a maintenance road, for recreation or even for urbanisation. The extra costs for a larger berm and consequently a larger footprint are then compensated with the benefits retrieved from these multifunctional purposes.
6. Conclusions & recommendations
6.5. Recommendations
References


Appendices
A. Background failure probabilities and new Dutch design guidelines

Because of its location, the Netherlands is always threatened by floods and already since the middle ages, the building of flood defences increased considerably. In the beginning the flood defences appear to be designed by a trial-and-error process. Gradually the people discovered that local protection was not useful if water could still enter the area from other sides and the flood defences were extended to enclosed dike rings. Over the centuries new methods were applied and measurement programs of water levels were implemented. New design crest levels were based on the highest water level measured and extra height was added based on experience.

The 1953 flood disaster in the southwest of the Netherlands causing the highest water level observed to date resulted in major investments to improve the water defences. A new committee was installed and on the advice of this Delta Committee the first return periods for the Netherlands were defined.

Nowadays the dikes in the Netherlands are still designed to safely withstand predefined loads based on a return period. This means that during design conditions the dike is sufficiently strong and remains accessible. Not all dikes in the Netherlands are designed on the same return periods. This depends on the type of threat and the economic value of the protected area. The return periods currently vary from 1/1,250 per year for certain river dikes and 1/10,000 per year for dikes on the sea side (TAW, 2000).

Summary new design strategy 2014

In contradiction with the current design strategy of a dike body in the new design guidelines of 2014 there will be looked at the inundation norm for an entire dike trajectory. From this norm the failure probability of a dike section can be deducted and finally also the hydraulic loads and safety factors for the dike section can be obtained. The final step is to design the dike using the current guidelines and the newly obtained hydraulic loads and safety factors. The result of the new guidelines is that the design of the dike will be slightly heavier and the goal is that it will not be rejected directly after the building phase of the dike.

In the report ‘Achtergrondrapport Ontwerpinstrumentarium 2014’ of Rijkswaterstaat the focus is on the failure mechanisms overflow and wave overtopping, uplift and piping en macro stability of the inner slope (Rijkswaterstaat, 2013). These failure mechanisms can be seen as the dominant mechanisms for the dimensions of a dike section.

Some of the requirements of the new design guidelines is that the design process is semi probabilistic and the design must be dependent on the norm height and also on the length of the trajectory (the length effect).

This can be illustrated with the following flow diagram:
For the contribution of the separate failure mechanisms on cross-sectional level to the total failure probability budget the figures are given in Table A.1. In the current Dutch guidelines 90 percent of the available failure space within the assigned probability of failure is taken up by overflow and overtopping. For the other failure mechanisms the probability of failure should be less than 10 percent of the assigned failure probability.

![Table A.1: Contribution of separate failure mechanisms on cross-sectional level (Rijkswaterstaat, 2013).](image)

The advantage of working with a failure probability budget is that it may lead to a conservative design. This slightly conservative design ensures that the design of the dike will pass all the requirements for the failure mechanisms of the dike on the trajectory level. The disadvantage is that it can lead to somewhat uneconomical designs.

**Example for designing the crest height**

According to the current guidelines the inner slope will fail when a certain critical overtopping...
discharge is reached. Which critical discharge should be used is depended on the design and the quality of the inner slope. The limit state function for overtopping is:

$$Z = q_{adm} - q_{over}$$ \hspace{1cm} (7.1)

In this equation $q_{over}$ is the amount of overtopping and $q_{adm}$ is the admissible overtopping rate.

A probabilistic approach is the basis for the semi-probabilistic approach that will be used. This means for the failure probability of a certain cross-section is the following:

$$P_{f,i} = P(Z_i < 0) = P(q_{over,i} > q_{adm,i})$$ \hspace{1cm} (7.2)

And for the complete trajectory:

$$P_f = P(Z_1 < 0 \cup Z_2 < 0 \cup \ldots Z_n < 0)$$ \hspace{1cm} (7.3)

From this it follows that the complete dike trajectory fails when at least one of the cross-sections inside the trajectory fails (length effect).

When using a deterministic approach for failure due to overtopping the start is to transform the maximum inundation failure probability ($P_{norm}$) into a failure probability for the failure mechanism wave overtopping and overflow ($P_{ov}$). The factor $\omega$ (0.24 for overtopping and overflow, see Table A.1) is used for the conversion:

$$P_{ov} = P_{norm} \cdot \omega$$ \hspace{1cm} (7.4)

The determination of the crest height is normally done for separate cross-sections and not for the dike trajectory in total. The result is that the failure probability for a certain cross-section is smaller than the failure probability given above. This length effect ($N$) is given in de formula as:

$$P_{ov,i} = \frac{P_{ov}}{N}$$ \hspace{1cm} (7.5)

The two main contributors for the length effect $N$ are:

- Variations in the orientation of the trajectory; there is a variation in loads that act on the defence on different locations. Wind, for example, will not come simultaneously from the north and the south.
- The presence of various loads systems; the loads on a defence can come from different sources at the same time. For example a river outflow and wave setup.

In the Netherlands several studies are done to find a value for $N$ (values reaching from 1-3) for every dike trajectory. The result is visible in Figure A.2.
Figure A.2: N-value for every dike trajectory (Rijkswaterstaat, 2013).
B. General classification of dikes

In general three types of dikes can be distinguished:

- Sea dikes
- River dikes
- Lake dikes

On the coast the threat is usually short and violent; the duration of a storm is in the range from hours to a day and the violence of the waves plays a dominant role in combination with high water (Figure B.1). The rise of water is very fast and difficult to predict and also the tide plays an important role.

Along the rivers the threat is less violent, but lasts much longer, which may lead to dike weakening or collapse due to the flushing out of soil. The design water level for river dikes depends on the discharge of the river and overflow is a serious threat. In contradiction to a sea dike waves play a less dominant role and there is a low correlation with the high water level Figure B.2. Although it is still difficult but in general the level can be predicted several days before the maximum level occurs. The top of the flood-wave in the river has a much longer duration and may last for several days (Tonneijck & Weijers, 2009).

In (shallow) lakes there is a constant water level with on top a surge effect due to wind. In a lake with a diameter of 50 km there may occur water level differences between one side of the lake and the other side of more than one metre (Verhagen, 1998). The increase in water level due to wind in an enclosed basin needs to be compensated by a decrease of the water level on the
other side of the basin (e.g. the Black Sea). In case the basin is connected to a large ocean, the supply of water from the ocean ensures that no decrease of water level occurs (e.g. the North Sea). Both situations are visualised in Figure B.3.

![Figure B.3: Longitudinal section of a rectangular basin with wind setup. Upper figure: enclosed basin. Lower figure: basin open at the upwind side (Voortman, 2003).](image)

The various types of load on the dikes result in various geometries. A typical river dike has another shape than a typical sea dike. As mentioned in the previous section a sea dike has to resist the short and violent attack of waves and therefore the outer slope has elements to reduce the wave attack (e.g. a revetment and an outer berm). A river dike has to resist high water levels with long durations and therefore has elements to reduce the flow through the dike and guaranty stability (e.g. an inner berm). Schematic profiles of both typical dike cross sections are given in Figure B.4 and Figure B.5.

![Figure B.4: Schematic profile of sea dike (Verhagen, 1998).](image)
Although there is a large difference between the loads on the different classifications of dikes, there is a general profile that can be used as a starting point to show the different elements of a dike. Only the dimensions of the elements differ because of a different load. The general profile is shown in Figure B.6 and shows possible combinations of elements in a dike.
C. Design methods

Once the limit state functions of the failure mechanisms are defined there are two fundamentally different ways to judge the performance of a dike section:

- Deterministic analysis;
- Probabilistic analysis.

A deterministic analysis assumes that its outcome is certain if the input to the calculation is fixed. No matter how many times the calculation is done, the result remains exactly the same. The result of a deterministic analysis of a design is binary; the structure fails under the prescribed load and strength combination or it does not. There is a unique input leading to a unique output. No estimate of the likelihood of failure is given in any way by this type of methods. Probabilistic design on the other hand explicitly takes the uncertainties in load, strength and physical models into account (Voortman, 2003).

A probabilistic approach is often called a reliability approach. The reliability of a structure depends on the margin between the resistance to failure and the loads. The way this margin is calculated can differ per case. Generally the probabilistic analysis can be divided into three levels (Vrijling & Van Gelder, 2002):

- Level III: The level III methods calculate the probability of failure, by considering the probability density functions of all strength and load variables. The reliability of an element is linked directly to the probability of failure. An often used method is the Monte Carlo simulation.
- Level II: This level comprises a number of methods for determining the probability of failure and thus the reliability. It entails linearizing the reliability function in a carefully selected point. These methods approximate the probability distribution of each variable by a standard normal distribution. An often used method is FORM (First Order Reliability Method).
- Level I: At this level no failure probabilities are calculated. The level I calculation is a design method according to the standards, which consider an element sufficiently reliable if a certain margin is present between the representative values of the strength and the loads. This margin is created by taking so-called partial safety factors into account in the design.

Most civil engineering projects follow a level I approach, better known as a semi-probabilistic approach. In a semi-probabilistic approach, the variations in, and uncertainties associated with many of the variables are quantified and the approach is still based on discrete values. Because of this the approach often leads to over-dimensioned and economically inefficient structures.

A level III method can provide a more economical efficient design. The Monte Carlo method is an example of a level III method. During a Monte Carlo simulation a large number (N) of sample values of the random variables according to their distribution functions and their statistical properties are drawn. For every draw (i) a combination of sample values will be used as input in the so called limit state function (Z). When \( Z \leq 0 \) it is assumed that failure will of the structure will occur and when \( Z > 0 \) no failure will occur. In section 2.2.1 the limit state function is threatened in more detail. When \( N_F \) indicates the number of times the limit state function is negative, the
estimation of the probability of failure can be expressed by \( P_f = N_f / N \). The total procedure of the Monte Carlo simulation is shown in Figure C.1.

It has to be emphasized that a large number of samples is required in order to estimate the required probabilities with the desired accuracy. An often used method to estimate the number of simulations based on the failure probability is (Vrijling & Van Gelder, 2002):

\[
N > 400 \left( \frac{1}{P_f} - 1 \right)
\]  

For small probabilities of failure, the required number of simulations becomes very large.
D. D-Geo Stability calculation and derivation fragility curve macro-stability

D-Geo Stability is a software tool developed to analyse slope stability in a two-dimensional geometry. The tool comes as a standard module that can be extended with other modules to fit more advanced applications. The standard module uses the bishop method as explained in section 3.2.3 to calculate the safety factor (Fs) along a given slip plane. An infinite number of slip planes can occur in a given geometry. Therefore, a search algorithm needs to find which slip plane is representative.

The geometry is considered safe against macro-stability when the lowest calculated safety factor (Fs) is higher than a predefined partial factor of safety (FoS). This partial factor of safety usually is higher than one to deal with uncertainties in the model and data. In

A geometry can exist out of multiple soil layers and the calculated safety factor depends on the following soil parameters of each layer:

- Total unit weight above phreatic level [kN/m²];
- Total unit weight below phreatic level [kN/m²];
- Cohesion c [kN/m²];
- Friction angle \( \phi \) [°].

The phreatic level (or groundwater level) is used to mark the border between dry and wet soil. Each soil layer is connected to a Piezometric Level line (PL-line). The PL-lines represent the pore pressures in the soil. An input file can contain more PL-lines as different soil layers can have different piezometric levels (Figure D.1). One of the PL-lines is selected and acts as a phreatic level.

The phreatic line on the outer side of the dike section has the same level as the water level.

**Probabilistic calculation within D-Geo Stability**

The Bishop probabilistic random field model is an extension of the standard module and performs a probabilistic slope stability analysis, in order to determine the probability that the safety factor is less than the required factor of safety. The computation model is based on
Bishop’s method of slices for equilibrium analysis and random field modelling of spatial variability in soil strength and pore pressures. In other words, D-Geo Stability uses stochastic values for its calculations.

In D-Geo Stability it is possible to give various graphical representations of the calculated results. In Figure D.2 the critical slip circle is shown. This is the slip circle with the lowest safety factor and therefore the highest probability of failure. Key information like the safety factor and the probability of failure are printed in the status panel at the bottom.

![Critical slip circle and key information for a probabilistic calculation with D-Geo Stability.](image)

It is important to mention that in a probabilistic calculation the factor of safety (FoS) is not of any importance. A separate limit state stability factor is used. This factor is given the value 1 and the probability of failure is therefore calculated around the absolute limit state of the structure.

**Construction of the fragility curve of macro-stability with D-Geo Stability**

In the previous section the calculation of the probability of failure is shown for a phreatic line that coincides with the water level on the outer side of the dike section. In section 2.2.1 it is explained that to be able to construct a fragility curve the water level should be assumed deterministic and taken in small steps. For the failure mechanism macro-stability of the inner slope this means that for every water level step a separate model run should be performed which results in a very long computation time to construct the fragility curve for one geometry. To reduce the computation time the water level is divided into six steps with equal distance between them (Figure D.3).
D. D-Geo Stability calculation and derivation fragility curve macro-stability

For each water level a failure probability is calculated with D-Geo Stability and for the intermediate water levels the failure probability is obtained by means of interpolation. The result is a fragility curve for the failure mechanism macro-stability of the inner slope (Figure D.4).

Usually a fragility curve reaches the value 1 for the failure probability when the loads are large enough. In above figure it is visible that the fragility curve stops at a certain level and remains horizontally. D-Geo Stability is not able to calculate a failure probability for water levels higher than the crest level and the maximum failure probability is found just below the crest level. In reality the sea dike section will always completely fail due to overtopping before the water level reaches the crest level and therefore the upper horizontal part of the fragility curve is not used in the calculations of the total failure probability of the dike section.
E. Polder variants

Four variants for the polder layout are proposed in the first preliminary phase of the project based on information given by HDB. The key drivers for the Pulau Tekong polder variants are:

- Land use requirements;
- Drinking water requirement;
- Soil balance;
- Water management requirements.

For the design of the dike the key driver soil balance is important. The original plan for the area was to do reclamation with traditional landfill. Parts of the area are already filled up till above polder level and parts are still below. The key driver soil balance focus on the reuse of the soil in order to minimise extra import of soil.

On the contrary in the Netherlands we are used to polders and we design an optimal dike to limit the costs as much as possible, while in Singapore they are used to the traditional landfill and a polder means a large saving on the import cost of soil. It is not necessary to optimise the dike and keep the cost as low as possible, because a somewhat larger dike still means a large saving on the import of soil. This is one of the two main reasons that it could be attractive to design a dike that is over-dimensionalized. The other reason is that Singapore is used to the traditional landfill, which has in fact a very large safety level compared to a polder. The step to a polder with a dike design that is optimised on the basis of costs and thus has a “minimal” design concerned the safety level that is chosen, is a large step for Singapore. A safer, over-dimensionalized, design could be a good option for a country that is not used to live below sea level.

Also the key driver water management is important for the design of a dike. Since the water level in the polder will be artificially lower than the surrounding areas, the polder will receive water from seepage through and under the dike. A requirement from a water management perspective is to limit seepage into the polder.

Based on the four key drivers four preliminary designs are made of which each one will focus on one of the key drivers. To give an impression the variant based on soil balance is presented below.
As can be seen in the section view above the polder will be designed with a polder level of CD-1 and CD-2 m. In the other variants the maximum assumed polder level is CD-3m. The polder has a surface area of plus minus 1.000 ha and the length of the ring dike will be approximately 10.000 m.
The effect of a higher number of Monte Carlo simulations

The accuracy of the model depends on the computation time. The computation time is mainly influenced by a multiplication of the following elements:

- the number of Monte Carlo simulations ($N_S$);
- the number of combinations between the (deterministic) water level and the (deterministic) wave height used for the calculation of the various fragility curves;
- the number of failure mechanisms;
- the number of cross sections.

The relation between these elements in the model is given in Figure F.1.

![Diagram showing the relation between the elements that determine the computation time.](image)

**Figure F.1**: Relation between the elements that determine the computation time.

The number used for the different elements are chosen such that the model has a reasonable computation time without losing too much accuracy.

In this research the number of Monte Carlo simulation used is $N_S=10,000$. The step size ($\Delta$) for the water level and wave height is 0.1, which results in 4,900 combinations. The effect on the computation time and the accuracy of the model is investigated for two times as many Monte Carlo simulations ($N_S=20,000$). Also the effect of a smaller step for the water level and wave height is investigated. A step size of 0.05 is chosen, which results in 19,600 combinations.

The computation time of the model consists of three parts:

1. Creating the input files for D-Geo Stability with Matlab;
2. Perform the macro-stability calculations with D-Geo Stability;
3. Obtain the failure probability and other output values with Matlab.

The effect on the computation time is given in
From this table it follows that for two times as many Monte Carlo simulations ($N_S=20,000$), the computation time also nearly doubles. When taking a step size ($\Delta=0.05$) that is half of the original step size, the computation time is eleven times longer. The number of combinations for $\Delta=0.05$ is only four times larger than the number of combinations for $\Delta=0.1$.

The effect on the accuracy of both a larger number of Monte Carlo simulations and a smaller step size for the water level and wave height is given in Table F.2, Table F.3 and Table F.4.
### F. The effect of a higher number of Monte Carlo simulations

<table>
<thead>
<tr>
<th>Reference design</th>
<th>Cost optimal design $N_s=10,000$ step size $WL &amp; H_s=0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_f$</td>
<td>1.00E-04, 9.38E-05, 8.81E-05, 9.53E-05, 8.76E-05, 5.94E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 23,612, € 20,547, € 20,698, € 20,709, € 20,848, € 20,872</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 236.1, € 205.5, € 207.0, € 207.1, € 208.5, € 208.7</td>
</tr>
<tr>
<td>$H_dike$ (m+CD)</td>
<td>6.1, 5.8, 5.8, 5.6, 5.8, 5.6</td>
</tr>
<tr>
<td>Outer slope (tanα)</td>
<td>0.33, 0.36, 0.36, 0.32, 0.36, 0.32</td>
</tr>
<tr>
<td>$D_{n50}$ (m)</td>
<td>0.51, 0.50, 0.52, 0.46, 0.54, 0.48</td>
</tr>
<tr>
<td>$H_{berm}$ (m+CD)</td>
<td>3, 1, 1, 1, 1, 1</td>
</tr>
<tr>
<td>$W_{berm}$ (m)</td>
<td>8, 4, 4, 4, 4, 4</td>
</tr>
</tbody>
</table>

Table F.4: Top 5 cost optimal designs for $N=10,000$ and $Δ=0.05$.

It can be concluded that the top 5 cost optimal designs do not change when a larger number of Monte Carlo simulations is chosen or the step size between water level and wave height is decreased. It does have an influence on the failure probability of the dike sections and the distribution of the total failure space over the various failure mechanisms is given in Table F.5.

<table>
<thead>
<tr>
<th>$N_s=10,000 &amp; Δ=0.1$</th>
<th>$N_s=10,000 &amp; Δ=0.1$</th>
<th>$N_s=10,000 &amp; Δ=0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>53 % 34 % 13 %</td>
<td>53 % 34 % 13 %</td>
</tr>
<tr>
<td>2nd</td>
<td>71 % 11 % 18 %</td>
<td>71 % 11 % 18 %</td>
</tr>
<tr>
<td>3rd</td>
<td>29 % 50 % 21 %</td>
<td>29 % 50 % 21 %</td>
</tr>
<tr>
<td>4th</td>
<td>72 % 10 % 18 %</td>
<td>72 % 10 % 18 %</td>
</tr>
<tr>
<td>5th</td>
<td>34 % 42 % 24 %</td>
<td>34 % 42 % 24 %</td>
</tr>
</tbody>
</table>

Table F.5: Distribution of the various failure mechanisms over the total failure probability for different numbers of Monte Carlo simulations and different step sizes between water level and wave height.

Although there is a difference in the distribution of the failure space when more accuracy is used, it is not significant for the conclusions of this report. The interaction between the variables remains the same.

When more accuracy is necessary, it is recommended to increase the number of steps between water level and wave height. To reduce the computation time, a more powerful computer may be used.
G. The effect of a steep inner slope

For the research it was assumed that the influence of a berm is larger than the influence of the inner slope angle and that it has a larger effect on the geometry of the dike section. Based on the results it is concluded that an inner slope of 1:3 already provides a large part of the inner slope stability. More interaction between the variables is expected when also the inner slope angle is used as a variable. In this appendix the effect of an inner slope angle of 1:2 on the cost optimal design is presented.

Both the crest height and \( d_{50} \) of the reference design remain the same. The cross-section is checked in D-Geo Stability (Figure G.1). The critical slip circle has a safety factor of 1.90 and fulfils the requirements.

![Reference design, with an inner slope of 1:2, is checked in D-Geo Stability.](image)

In Table G.1 the characteristic values for the reference geometry and the top 5 cost optimal designs with an inner slope of 1:3 are given. The effect of an inner slope of 1:2 on these characteristic values of the designs is given in Table G.2.

<table>
<thead>
<tr>
<th>Reference design</th>
<th>Cost optimal design with inner slope 1:3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_f )</td>
<td>1.00E-04</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 23,612</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 236.1</td>
</tr>
<tr>
<td>H_{dike} (m+CD)</td>
<td>6.1</td>
</tr>
<tr>
<td>Outer slope (tana)</td>
<td>0.33</td>
</tr>
<tr>
<td>D_{n50} (m)</td>
<td>0.51</td>
</tr>
<tr>
<td>H_{berm} (m+CD)</td>
<td>3</td>
</tr>
<tr>
<td>W_{berm} (m)</td>
<td>8</td>
</tr>
</tbody>
</table>

Table G.1: Characteristic values for the reference geometry and the top 5 cost optimal designs with an inner slope of 1:3.
### Table G.2: Characteristic values for the reference geometry and the top 5 cost optimal designs with an inner slope of 1:2.

<table>
<thead>
<tr>
<th></th>
<th>Reference design</th>
<th>Cost optimal design with inner slope 1:2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st</td>
</tr>
<tr>
<td>$P_f$</td>
<td>1.00E-04</td>
<td>8,36E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 21,385</td>
<td>€ 19,702</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 213.9</td>
<td>€ 197.0</td>
</tr>
<tr>
<td>$H_{dike}$ (m+CD)</td>
<td>6.1</td>
<td>5.8</td>
</tr>
<tr>
<td>Outer slope (tanα)</td>
<td>0.33</td>
<td>0.36</td>
</tr>
<tr>
<td>$D_{n50}$ (m)</td>
<td>0.51</td>
<td>0.5</td>
</tr>
<tr>
<td>$H_{berm}$ (m+CD)</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$W_{berm}$ (m)</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

From the above table it follows that because of a steeper inner slope the berm dimensions increase. The top 5 cost optimal designs all have the same berm dimensions as the reference design. All the other variables stay the same as for the top 5 cost optimal designs with an inner slope of 1:3.

In Figure G.2 and Figure G.3 the contribution of the various failure mechanisms to the total failure probabilities is given for respectively an inner slope of 1:3 and 1:2.

![Figure G.2: Contribution of each failure mechanism to the total failure probability for an inner slope of 1:3.](image)
The effect of a steep inner slope

From the above figures it follows that there is no significant change in the distribution of the total failure probability over the various failure mechanisms. Because of a steeper inner slope the interaction between the failure mechanisms did not increase and a larger berm compensates the extra instability of the sea dike due to the steeper inner slope.

To achieve more interaction between macro-stability and the other failure mechanisms, it is recommended to take the inner slope angle as a variable. Also the step size of the berm dimensions can be decreased.

The output, as given by the model, for the top 5 optimal designs with an inner slope of 1:2 is given in Table G.3.
<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hdike</strong></td>
<td>5.8</td>
<td>5.8</td>
<td>5.6</td>
<td>5.8</td>
<td>5.6</td>
</tr>
<tr>
<td><strong>Slope</strong></td>
<td>0.36</td>
<td>0.36</td>
<td>0.32</td>
<td>0.36</td>
<td>0.32</td>
</tr>
<tr>
<td><strong>Dn50</strong></td>
<td>0.5</td>
<td>0.52</td>
<td>0.46</td>
<td>0.54</td>
<td>0.48</td>
</tr>
<tr>
<td><strong>Hberm</strong></td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>Wberm</strong></td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td><strong>P_1</strong></td>
<td>8.36E-05</td>
<td>7.39E-05</td>
<td>6.28E-05</td>
<td>7.36E-05</td>
<td>5.27E-05</td>
</tr>
<tr>
<td><strong>P_1 (overt)</strong></td>
<td>7.01E-05</td>
<td>7.02E-05</td>
<td>2.62E-05</td>
<td>7.00E-05</td>
<td>2.62E-05</td>
</tr>
<tr>
<td><strong>P_1 (armour)</strong></td>
<td>4.51E-05</td>
<td>1.10E-05</td>
<td>4.52E-05</td>
<td>9.19E-06</td>
<td>3.26E-05</td>
</tr>
<tr>
<td><strong>P_1 (macro)</strong></td>
<td>3.66E-06</td>
<td>3.66E-06</td>
<td>7.13E-06</td>
<td>3.66E-06</td>
<td>7.13E-06</td>
</tr>
<tr>
<td><strong>Overt (%)</strong></td>
<td>59%</td>
<td>83%</td>
<td>33%</td>
<td>85%</td>
<td>40%</td>
</tr>
<tr>
<td><strong>Armour (%)</strong></td>
<td>38%</td>
<td>13%</td>
<td>58%</td>
<td>11%</td>
<td>49%</td>
</tr>
<tr>
<td><strong>Macro (%)</strong></td>
<td>3%</td>
<td>4%</td>
<td>9%</td>
<td>4%</td>
<td>11%</td>
</tr>
<tr>
<td><strong>Cost</strong></td>
<td>€ 19,702</td>
<td>€ 19,853</td>
<td>€ 19,945</td>
<td>€ 20,003</td>
<td>€ 20,107</td>
</tr>
<tr>
<td><strong>Footprint</strong></td>
<td>€ 10,690</td>
<td>€ 10,690</td>
<td>€ 10,978</td>
<td>€ 10,690</td>
<td>€ 10,978</td>
</tr>
<tr>
<td><strong>Dike area</strong></td>
<td>€ 4,721</td>
<td>€ 4,721</td>
<td>€ 4,703</td>
<td>€ 4,721</td>
<td>€ 4,703</td>
</tr>
<tr>
<td><strong>Berm area</strong></td>
<td>€ 528</td>
<td>€ 528</td>
<td>€ 528</td>
<td>€ 528</td>
<td>€ 528</td>
</tr>
<tr>
<td><strong>Stones</strong></td>
<td>€ 3,764</td>
<td>€ 3,915</td>
<td>€ 3,736</td>
<td>€ 4,065</td>
<td>€ 3,898</td>
</tr>
<tr>
<td><strong>Footprint (%)</strong></td>
<td>56%</td>
<td>55%</td>
<td>56%</td>
<td>55%</td>
<td>56%</td>
</tr>
<tr>
<td><strong>Dike area (%)</strong></td>
<td>25%</td>
<td>24%</td>
<td>24%</td>
<td>24%</td>
<td>24%</td>
</tr>
<tr>
<td><strong>Berm area (%)</strong></td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td><strong>Stones (%)</strong></td>
<td>17%</td>
<td>18%</td>
<td>17%</td>
<td>18%</td>
<td>18%</td>
</tr>
</tbody>
</table>

*Table 6.3: Output as given by the model for the top 5 optimal designs with an inner slope of 1:2.*
**H. The effect of sea level rise**

The planning horizon for the sea dike taken into account is 100 years. An indication for this period is a sea level rise of 0.85, although this is much higher than historical sea level rise, which is around 0.20m per century (Royal HaskoningDHV & Surbana, 2014b).

Including the sea level rise, the new crest height for the reference design of the sea dike at Pulau Tekong is 6.95+CD. The $d_{50}$ remains the same and the cross-section is checked in D-Geo Stability (Figure H.1). The critical slip circle has a safety factor of 2.04 and fulfils the requirements.

*Figure H.1: Reference design, including sea level rise, is checked in D-Geo Stability.*

In Table H.1 the characteristic values for the reference geometry and the top 5 cost optimal designs without sea level rise are given. The effect of sea level rise on these characteristic values of the designs is given in Table H.2.

<table>
<thead>
<tr>
<th></th>
<th>Reference design</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_f$</td>
<td>1.00E-04</td>
<td>9.79E-05</td>
<td>8.82E-05</td>
<td>7.42E-05</td>
<td>8.80E-05</td>
<td>8.80E-05</td>
</tr>
<tr>
<td>Cost (€/m)</td>
<td>€ 23,612</td>
<td>€ 20,547</td>
<td>€ 20,698</td>
<td>€ 20,709</td>
<td>€ 20,848</td>
<td>€ 20,872</td>
</tr>
<tr>
<td>Total cost (€*10^6)</td>
<td>€ 236.1</td>
<td>€ 205.5</td>
<td>€ 207.0</td>
<td>€ 207.1</td>
<td>€ 208.5</td>
<td>€ 208.7</td>
</tr>
<tr>
<td>$H_{dike}$ (m)</td>
<td>6.1</td>
<td>5.8</td>
<td>5.8</td>
<td>5.6</td>
<td>5.8</td>
<td>5.6</td>
</tr>
<tr>
<td>Outer slope (tana)</td>
<td>0.33</td>
<td>0.36</td>
<td>0.36</td>
<td>0.32</td>
<td>0.36</td>
<td>0.32</td>
</tr>
<tr>
<td>$D_{n50}$ (m)</td>
<td>0.51</td>
<td>0.50</td>
<td>0.52</td>
<td>0.46</td>
<td>0.54</td>
<td>0.48</td>
</tr>
<tr>
<td>$H_{berm}$ (m+CD)</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$W_{berm}$ (m)</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

*Table H.1: Characteristic values for the reference geometry and the top 5 cost optimal designs without sea level rise.*
From the above table it follows that because of sea level rise:

- The crest height increases. Some of the cost optimal designs have a significant lower crest height than the reference design.
- The cost optimal designs with a crest height of 7+CD or higher, all have a relative steep slope.
- The berm dimensions increase for the top 4 cost optimal designs. The 5th cost optimal design has the same berm dimensions as the cost optimal design without sea level rise. This is compensated by a higher crest and a larger $d_{50}$.

Based on these observations it is expected that for the 1st and 3rd design, overtopping will have a significant contribution to the total failure space (relative low crest). Because of the relative small berm, it is expected that macro-stability will have a significant contribution for the 5th design. In Figure H.2 and Figure H.3 the contribution of the various failure mechanisms to the total failure probabilities is given.

![Figure H.2: Contribution of each failure mechanism to the total failure probability without sea level rise.](image-url)
The effect of sea level rise

Figure H.3: Contribution of each failure mechanism to the total failure probability including sea level rise.

The following is observed in Figure H.3:

- The expectation, that for the 1st and 3rd design overtopping will have a significant contribution to the total failure space, is not correct.
- The contribution of a failure mechanism not only depends on the dimensions of the variables that are relevant for that failure mechanism, but it also depends on the dimensions of the variables that are relevant for the other failure mechanisms. The 4th design is a good example. The design has a relative high crest and a low contribution of overtopping is expected. Because the berm dimensions and $d_{50}$ are also relatively large, the result is that overtopping still has the largest contribution to the total failure space.
- It was expected that for the 5th design the failure mechanism macro-stability has a large contribution. This expectation is confirmed by Figure H.3.
- Significant interaction is observed between all failure mechanisms.

The output for the top 5 optimal designs including sea level rise, as given by the model, is given in Table H.3.
Optimal design with SLR

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hdike</td>
<td>6.4</td>
<td>7</td>
<td>6.4</td>
<td>7</td>
<td>7.2</td>
</tr>
<tr>
<td>Slope</td>
<td>0.32</td>
<td>0.4</td>
<td>0.32</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Dn50</td>
<td>0.46</td>
<td>0.52</td>
<td>0.48</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>Hberm</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Wberm</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>P_f (overt)</td>
<td>9.74E-05</td>
<td>8.18E-05</td>
<td>8.90E-05</td>
<td>7.48E-05</td>
<td>8.17E-05</td>
</tr>
<tr>
<td>P_f (armour)</td>
<td>4.04E-05</td>
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<td>4.05E-05</td>
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<td>1.57E-05</td>
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<tr>
<td>P_f (macro)</td>
<td>4.52E-05</td>
<td>4.51E-05</td>
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<td>2.02E-05</td>
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<tr>
<td>Overt (%)</td>
<td>34 %</td>
<td>51 %</td>
<td>38 %</td>
<td>65 %</td>
<td>17 %</td>
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<tr>
<td>Armour (%)</td>
<td>38 %</td>
<td>38 %</td>
<td>31 %</td>
<td>22 %</td>
<td>22 %</td>
</tr>
<tr>
<td>Macro (%)</td>
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<td>11 %</td>
<td>31 %</td>
<td>13 %</td>
<td>61 %</td>
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<tr>
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<td>€ 24,141</td>
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<td>€ 13,030</td>
<td>€ 12,744</td>
<td>€ 12,550</td>
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</tr>
<tr>
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<td>€ 297</td>
<td>€ 132</td>
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<td>Dike area (%)</td>
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</tbody>
</table>

*Table H.3: Output as given by the model for the top 5 optimal designs including SLR.*
I. Cost investigation

In Figure I.1 the influence of each variable on the cost optimal design is presented in a graph. In this graph the increase or decrease of the total costs are given when each variable changes per step and all the other variables are kept constant. The variables and steps in these graphs are given in Table I.1. Also the minimum and maximum are given that are used in the costs investigation. These values are the same values that are used to find the cost optimal design of the sea dike at Pulau Tekong. The values at step 2 represent the cost optimal design.

<table>
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<tbody>
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<td>6.2</td>
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<td>6.6</td>
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</tr>
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<td>0.40</td>
<td>0.36</td>
<td>0.32</td>
<td>0.28</td>
<td>0.24</td>
<td>0.20</td>
<td>-</td>
</tr>
<tr>
<td>Hberm (m)</td>
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<td>3</td>
<td>4</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dn50 (m)</td>
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<td>0.50</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>-</td>
</tr>
</tbody>
</table>

Table I.1: Minimum, maximum and step size for each variable for the costs investigation.

In the figure above the cost optimal design is given at step 2. It is already visible that a change in slope angle means the largest decrease or increase in costs. In Figure I.2 the additional costs are further specified. The additional costs are the increase of the total costs when one variable changes and all the other variables are kept constant. In the left figure the increase compared to the cost optimal design is given, while in the right figure the increase compared to the previous step is given.
Several observations that follow from Figure I.3 and Figure I.4 are:

- It can be clearly observed that a smaller slope angle means the largest increase in costs.
- In the left figure it is visible that additional costs for a change in the dike height and the stone size are constant (straight line). The additional costs for a change in berm dimensions and slope angle increase for every step. This observation is confirmed in the right figure.
- A constant increase is observed for the berm dimensions, while the additional costs for the slope angle increases nonlinear.

In the case of Pulau Tekong the model for the cost optimal dike is clearly looking for a dike section with a small footprint and a relative steep slope. The cost of the design consists out of four parts: the footprint, the dike area, the berm area and the stones for the revetment. In
It is concluded from the above figures that, although the footprint becomes larger due to a gentler slope, the main reason of the increase in costs are the stones for the revetment. The slope length increases because of a gentler slope and thus the contribution of the stones increases.

Below the same figures are presented for the variable dike height.

Figure I.5: Increase in cost due to a higher dike. All the other variables are kept the same.
It is visible that because of an increasing dike height the cost mainly are increased by an increasing dike area. Although the footprint also increases, the contribution to the total costs becomes less.

Also for the $d_{50}$ the figures are presented here:

Figure I.7: Increase in cost due to a larger stone size of the armour layer. All the other variables are kept the same.
As could have been expected the increasing costs due to an increasing stone size of the armour layer is mainly caused by the increasing costs for the stones of the armour layer.

And finally the graphs for the variable berm height are presented:

Figure I.8: Change in contribution of the four elements to the total costs because of a change in stone size of the armour layer.

Figure I.9: Increase in cost due to a larger berm. All the other variables are kept the same.
The largest contribution to the total costs for increasing berm dimensions is the increasing berm area.
J. Probability investigation

Before selecting the cost optimal design based on the investment costs, the model selects all the designs that fulfill the probability requirement. In this section the influence of the variables on the failure probability is investigated. The same strategy as in the previous section is used, the cost optimal design is used and only one of the variables changes while the other are kept constant. The variables and steps in that are used are given in Table J.1. These values are the same values that are used to find the cost optimal design of the sea dike at Pulau Tekong. The values at step 2 represent the cost optimal design.

<table>
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<th></th>
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<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
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<td>H_dike (m)</td>
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<td>6.2</td>
<td>6.4</td>
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</tr>
<tr>
<td>Outer slope (tanα)</td>
<td>-</td>
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<td>0.36</td>
<td>0.32</td>
<td>0.28</td>
<td>0.24</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>D_n50 (m)</td>
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<td>0.48</td>
<td>0.50</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>-</td>
</tr>
</tbody>
</table>

Table J.1: Minimum, maximum and step size for each variable for the probability investigation.

In the figure above the cost optimal design is given at step 2. It is already visible that a change in slope angle means the largest decrease or increase in failure probability. Not only a change in slope angle has the largest influence on the cost, it also has the largest influence on the total failure probability. The total failure probability is determined based on the failure probability of overtopping, armour layer stability and macro-stability. In the figures the contribution of these parts is given while the slope angle is changing. The other variables are kept constant.
From the above figures it is concluded that a gentler slope means a decrease of the total failure probability because of a decrease of the failure probability due to overtopping and armour layer stability. Although the failure probability of macro-stability remains constant for a gentler outer slope, the contribution to the total failure probability becomes significantly larger.

In the next two figures the influence of the dike height on the total failure probability of the cost optimal design is presented.
In the above figures it is visible that for an increasing dike height the contribution of overtopping reduces to zero. Most of the failure space that becomes available is taken by the failure mechanism armour layer stability.

The influence of the dike height on the total failure probability of the cost optimal design is presented in the two figures below.
In the above figures it is visible that for an increasing stone size the contribution of armour layer stability reduces to zero. Most of the failure space that becomes available is taken by the failure mechanism overtopping. It is concluded that the contribution to the failure space of the failure mechanisms overtopping and armour layer stability is interchangeable.

Also for increasing berm dimensions the figures are presented here.
The contribution of macro-stability to the total failure probability reduces rapidly and when a berm height of 3 metres is chosen the contribution is already practically zero. Based on the above two figures it is concluded that changing berm dimensions have a large influence on the total failure probability. It might not be worth it to increase the other variables so much that smaller berm dimensions are possible, or the other way around. This might be the reason that the top 5 cost optimal designs all have the same berm dimensions. The effect may be less when a smaller step size is taken for the berm dimension. Another and may be better solution is to also consider the inner slope angle as a variable. The inner slope angle might have a large influence on the macro-stability, while the effect of the variable dike height is only limited and the effect of the outer slope angle and stone size of the armour layer is zero.
### The top 50 cost optimal dike designs

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<th>Tamp Rate</th>
<th>Width (m)</th>
<th>Ht (foot)</th>
<th>Ht (m)</th>
<th>Ht (m) (m)</th>
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<th>Cost ($k)</th>
<th>Cost (%)</th>
<th>Cost ($k)</th>
<th>Cost (%)</th>
<th>Cost ($k)</th>
<th>Cost (%)</th>
<th>Cost ($k)</th>
<th>Cost (%)</th>
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<td>€11,294</td>
<td>€5,357</td>
<td>€132</td>
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<td>26%</td>
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<th>Pt (kN/m)</th>
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<th>Arma [%]</th>
<th>Mono [%]</th>
<th>Cost</th>
<th>Footprint</th>
<th>Dilemma</th>
<th>Bermant</th>
<th>Stanes</th>
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