Nanomechanical Magnetization Reversal

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The dynamics of the ferromagnetic order parameter in thin magnetic films is strongly affected by the magnetomechanical coupling at certain resonance frequencies. By solving the equation of motion of the coupled mechanical and magnetic degrees of freedom we show that the magnetic field induced magnetization switching can be strongly accelerated by the lattice and illustrate the possibility of magnetization reversal by mechanical actuation.

The dynamics of the order parameter of small magnetic clusters and films is a basic problem of condensed matter physics with considerable potential for technological applications [1]. The magnetic field induced reversal in such systems is perhaps the most active field of research. In clusters as small as 1000 atoms the magnetization carries solutions are robust beyond their formal regime of applicability.

We consider a small dielectric cantilever with a single-domain ferromagnetic layer deposited on its far end (see Fig. 1) in the presence of an external field \( \mathbf{H}_0 \).

The dynamics of the magnetization \( \mathbf{M} \) is well described by the Landau-Lifshitz-Gilbert equation [15]:

\[
\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{M} \times \left( \frac{d\mathbf{M}}{dt} \right)_{\text{cant}},
\]

where \( \gamma \) denotes the gyromagnetic ratio. The phenomenological Gilbert constant is typically \( \alpha \leq 0.01 \), and the derivative \( \left( \frac{d\mathbf{M}}{dt} \right)_{\text{cant}} = \frac{d\mathbf{M}}{dt} + \frac{d}{dt} \left( -\mathbf{M}_x \mathbf{x} + \mathbf{M}_z \mathbf{z} \right) \) is taken in the reference system of the cantilever (see Fig. 1). For small \( \varphi = \varphi(L) \), where \( \varphi(y) \) is the torsion angle at position \( y \) of the cantilever, the effective field is \( \mathbf{H}_{\text{eff}} = (\nu \mathbf{M}_x \varphi - \nu \mathbf{M}_x \mathbf{z} + \mathbf{H}_0 \mathbf{z} + \nu \mathbf{M}_x \mathbf{z} + \mathbf{H}_0 \mathbf{z} \), where \( \nu \) describes the demagnetizing dipolar field (\( \nu \approx 4\pi \) for our geometry). The coupling originates here from the demagnetizing field and since the crystal anisotropy field can be small (e.g., in permalloy) it is disregarded initially.

Without coupling and Gilbert damping the dynamics of the magnetic subsystem can be solved analytically [16], leading to the trajectories and oscillation periods depicted in Fig. 2. The period of the motion can be expressed by elliptic integrals \( K \) as

FIG. 1. A nano-magnetomechanical cantilever supporting magnetovibrational modes. On a dielectric substrate (such as Si) a single-domain ferromagnetic film is deposited at the free end.
FIG. 2. The periods $T_1$ ($E_{mg} < 1$) and $T_2$ ($E_{mg} > 1$) of the magnetization dynamics in Eqs. (2) in units of $2\pi/\sqrt{[H_0 + \nu M]/H_0}$ as a function of energy in units of $H_0 M$. The inset shows a plot of typical trajectories at different energies on the unit sphere ($\nu M = 10 H_0$).

\begin{align*}
T_1 &= 4\sqrt{2.K(p_-/p_+)} / (\gamma \sqrt{p^-}) ;
E_{mg} < MH_0 , \\
T_2 &= 2\sqrt{2.K(1 - p_+/p_-)} / (\gamma \sqrt{p^-}) ;
E_{mg} > MH_0 , \\
p_\pm &= H_0^2 - \nu E_{mg} \pm H_0 \sqrt{H_0^2 - 2\nu E_{mg} + \nu^2 M^2} ,
\end{align*}

(2)

where $E_{mg} = - H_0 M + \nu M^2/2$. The strong variation of the periodicity has important consequence for the coupling to the lattice as explained below.

The equation of motion of the cantilever is [17]

\begin{equation}
C \frac{\partial^4 \varphi}{\partial y^4} = \rho I \frac{\partial^2 \varphi}{\partial t^2} + 2\beta \rho I \frac{\partial \varphi}{\partial t},
\end{equation}

(3)

where $C$ is an elastic constant defined by the shape and material of the cantilever ($C = \frac{1}{2}\mu da^3$ for a plate with thickness $a$ much smaller than width $d$ and $\mu$ is the Lamé constant), $I = \int (x^2 + y^2) dx dy = ad^3/12$ is the moment of inertia of the cross-section about its center of mass, $\rho$ the mass density, and $\beta$ a phenomenological damping constant related to the quality factor $Q$ at the resonance frequency $\omega_r$ as $Q = \omega_r / (2\beta)$ (at 1 GHz, $Q \sim 500$) [11]. Note that $\omega_r$ can be also a higher harmonic resonance frequency in what follows. The clamping boundary condition is $\varphi|_{y=0} = 0$. The conservation law for the mechanical angular momentum $\mathbf{V}^\text{el}(y)$ for a thin slice at point $y \in [0,L]$ (without magnetic overlayer) $d\mathbf{V}^\text{el}(y)/dt = \mathbf{T}(y)$, where the torque $\mathbf{T}(y)$ flowing into the slice, is modified by the coupling to the magnet in a region $y \in [L, L + \Delta L]$ ($\Delta L$ is the length of the cantilever covered by the magnetic layer) as

\begin{equation}
\frac{d}{dt}\left[\mathbf{V}^\text{el}(L) + \left(-\frac{1}{\gamma}\right)\mathbf{M}(L)\mathbf{V}\right] = \mathbf{T}(L) + \mathbf{T}_\text{field}.
\end{equation}

(4)

where $\mathbf{T}_\text{field} = \nu M \times \mathbf{H}_0$, $\nu$ is the volume of the magnet, and $\mathbf{T}|_y = -C \mathbf{r}(y)$ is the torque flowing through the cantilever at point $y$ ($\tau = \partial \varphi(y)/\partial y$). When $\Delta L \ll L$, internal strains in the magnetic section may be disregarded. The magnetovibrational coupling can then be treated as a boundary condition to the mechanical problem [14], which is expressed as the torque $C \mathbf{r}|_{y-L}$ exerted by the magnetization on the edge of the cantilever:

\begin{equation}
C \mathbf{r}|_{y-L} = \frac{V}{\gamma}(\frac{d\mathbf{M}}{dt} + \gamma \mathbf{M} \times \mathbf{H}_0) |_y.
\end{equation}

(5)

The effect of the coupling between Eqs. (1) and (3) may thus be summarized as

\begin{equation}
\frac{\partial \varphi}{\partial y} |_{y-L} = \sqrt{\nu M/H_0}\sqrt{\frac{\pi \varphi_0^2}{(cM)}} \left(\frac{d\mathbf{M}}{dt} + \gamma \mathbf{M} \times \mathbf{H}_0\right).
\end{equation}

(6)

where $c^2 = C/(\rho l)$ and $\varphi_0^2 = MV\sqrt{H_0/\nu M}/(\gamma 2\rho l 2\omega_c)$. In the absence of damping, the above system of equations can be obtained as well from the free energy:

\begin{equation}
F = V\left[-\nu M_0 + \frac{\nu}{2}M_z + \phi(L)M_z^2\right] + C \int_0^L \tau^2 dy.
\end{equation}

(7)

We first address the coupling strength [18] of a system described by Eq. (7). Consider the two subsystems oscillating at common frequency $\omega$. The total mechanical energy is then $E_m = \rho l 2\omega_0^2 \varphi_0^2$, where $\varphi_0$ is the maximal angle of the torsional motion [that will turn out to be identical to the parameter introduced below Eq. (6)]. By equipartition this energy should be of the order of the magnetic energy $E_{mg} = MVH_0$. The maximal angle would correspond to the mechanical motion induced by full transfer of the magnetic energy to the lattice. By equalizing those energies we find an estimate for the maximal angle of torsion $\varphi_0 = \sqrt{MVH_0/(\rho l 2\omega_0^2)}$, which at resonance is identical to the parameter introduced above. The coupling between the subsystems can be measured by the distribution of an applied external torque (e.g., applied by a magnetic field) over the two subsystems. The total angular momentum flow into the magnetic subsystem by the effective magnetic field is $(MV/\gamma)\omega$, whereas that corresponding to the mechanical subsystem at the same frequency is $(\rho l 2\omega_0^2)\omega$. Their ratio is $\varphi_0 \nu M/H_0$. The maximum angle $\varphi_0$ derived above is therefore also a measure of the coupling between the magnetic and mechanical subsystems. This estimate is consistent with the splitting of polariton modes at resonance of $G = \varphi_0^2 \omega_0^2 M/\nu M/H_0$ [14]. An estimate for a cantilever with $\rho = 2330$ kg/m$^3$ (Si) and $d = 100$ nm ($\omega \sim 1$ GHz) leads to $\varphi_0^2 \nu M/H_0 \sim (\gamma 2\rho d^2 H_0) \sim 10^{-3}$. Decreasing $d$, $\rho$, $H_0$, or $1/M$ is beneficial for the coupling.

Magnetization reversal by a magnetic field in the coupling regime can be realized even without any damping by transferring magnetic energy into the mechanical system. Since we find that $\varphi_0 \ll 1$ for realistic parameters, the subsystems undergo many precessions/oscillations before the switching is completed. The switching is then associated with a slow time scale corresponding to the global motion governed by the coupling on the (reintroduced) weak damping relative to a fast time scale characterized by the
Larmor frequency. The equation of motion for the slow dynamics (the envelope functions) can be derived by averaging over the rapid oscillations. To this end we substitute Eq. (6), linearized in the small parameters $\alpha$, $\beta$, and $\varphi_0$, into the equations for the mechanical and the magnetic energies:

$$\frac{d}{dt} E_{me} = -2\beta E_{me} + C_1 r_{x-L} \frac{d\varphi}{dt} |_{x-L},$$
$$\frac{d}{dt} E_{mg} = -H_0 M_z + \nu M_x M_y.$$ (8)

We focus in the following on the regime $H_0 \ll \nu M$, which usually holds for thin films and not too strong fields, in which the magnetization motion is elliptical with a long axis in the plane and small $M_z$, even for larger precession cones. Disregarding terms containing higher powers of $M_z$ and averaging over one period as indicated by $\langle \rangle$

$$\langle \frac{dE_{me}}{dt} \rangle + 2\beta \langle E_{me} \rangle = -V \nu \langle M_z M_x \varphi \rangle.$$ (9)
$$\langle \frac{dE_{mg}}{dt} \rangle + \langle \alpha \nu^2 M_M^2 \rangle = \nu \langle M_x M_y \varphi \rangle.$$ (9)

By adiabatic shaping of time-dependent magnetic fields we can keep the two subsystems at resonance at all times. The slow dynamics $\varphi(L,t) \sim A(t) e^{i(\omega + \pi/2)t}$ and $M_x \sim W(t) e^{i\omega t}$ in time domain is then governed by the equation

$$A + \beta A = -\frac{\varphi_0^2 \gamma \nu}{H_0 M} \sqrt{\nu M / H_0} (\nu H_0 + \frac{\nu W^2}{4}) W,$$
$$W + \alpha \sqrt{\nu M / H_0} \omega W / 2 = \frac{\omega}{H_0} (\nu H_0 + \frac{\nu W^2}{4}) A.$$ (10)

$\varphi(L,t) \sim \widetilde{A}(t) e^{i\omega t}$, and $M_x \sim \widetilde{W}(t) e^{i(\omega - \pi/2)t}$ corresponding to $\pi/2$-shifted harmonics are also solutions, and the initial conditions determine the linear combination of two envelope functions, i.e., the beating pattern of two hybridized polariton modes [14]. When initially all energy is stored in 1 degree of freedom, $M_x$ is $\pi/2$ shifted from $\varphi(L,t)$ and $\widetilde{A}(t) = 0$. Equations (10) describe a (damped) harmonic oscillator with frequency $\omega \varphi_0 \sqrt{\nu M / H_0} = G \ll \omega$ when $\nu W^2 / 4 \ll \nu H_0$. Such oscillatory behavior persists for general angles (except for motion with very large angle cones close to the antiparallel configuration). This is illustrated by Fig. 3 which shows a numerical simulation of Eq. (6) for an undamped system excited at $t = 0$ by a magnetic field $H_0$ at an angle $2\pi/3$ with the initial magnetization. The number of periods necessary to transfer all energy from one subsystem to the other is therefore given by $\sim 1/(4\varphi_0 \sqrt{\nu M / H_0})$. Equation (10) also shows that for damping constants $\alpha > \varphi_0 / \pi$ or $\beta / \omega > \varphi_0 \sqrt{\nu M / H_0} / \pi$ the beating is suppressed.

Figure 3 illustrates that the mechanical system absorbs energy from the magnetic subsystem and gives it back repeatedly in terms of violent oscillations that are modulated by an envelope function on the time scale derived above. An efficient coupling requires that the frequencies of the subsystems are close to each other at each configuration, which was achieved in the simulation by the adiabatic modulation of the magnetic field $H_0$ according to Fig. 2. However, the reversal process is robust; an estimate from Eqs. (9) for the necessary proximity of the resonant frequencies of the mechanical and the magnetic subsystems is $\Delta \omega \sim (\nu M / H_0)^{1/2} \varphi_0 \omega$. In that case the above estimates still hold.

Let us now consider the magnetization reversal by an antiparallel magnetic field. Since we assume zero temperature, the dynamics in Fig. 4 is initiated by assuming a small mechanical twist at $t = 0$. We wish to illustrate here that making use of the magnetoelastic coupling can accelerate the reversal importantly. We can suppress the backflow of mechanical energy, for example, by a sufficiently damped mechanical subsystem, as evident in Fig. 4 by comparing the two curves for $\beta = 0$ and $\beta = 0.04$ with vanishing Gilbert damping, $\alpha = 0$. Alternatively, we may detune the external magnetic field out of the resonance precisely.

![FIG. 3. Time-dependent response of the magnetomechanical system to an external magnetic field switched on at $t = 0$, in the absence of dissipation. Plotted are the $x$ and $z$ components of the magnetization ($\nu M = 10H_0$).](image)

![FIG. 4. Time-dependent response of the magnetomechanical system to an external magnetic field switched on at $t = 0$ ($\nu M = 10H_0$, $\alpha = 0$). The importance of mechanical damping can be seen by comparing the dotted line ($\beta = 0$) with the solid line ($\beta = 0.04$). The dashed line illustrates the dynamics when the external magnetic field is reduced to half of its initial value at $t = 3.5$ ($\beta = 0$).](image)
after the first reversal, effectively rectifying the energy flow from the magnetic into the mechanical subsystem. We observe that even without any intrinsic damping ($\alpha = \beta = 0$) the unwanted “ringing” can be strongly suppressed (dashed line in Fig. 4).

Finally we propose a nonresonant mechanical reversal scheme analogous to “precessional” switching [4]. The effective field $H_{\text{eff}}$ [see Eq. (6)] has a component perpendicular to the plane of the film $H_x \sim M\varphi$. Under a sudden mechanical twist this component acts like a transverse magnetic pulse about which the precessing develops. Alternatively, we can suddenly release a twist preinstalled on the cantilever. This could be achieved by an STM tip bonded to an edge of the cantilever at $(L, d)$ and pulling it up slowly to the breaking point. The mechanical response should be fast, i.e., react on a time scale $(\gamma\varphi r M)^{-1}$, but there are no resonance restrictions now. We integrate the equation of motion numerically for a strongly damped cantilever $\beta/\omega \approx 1$ initially twisted by $\varphi = 0.2$ and suddenly released at $t = 0$. We reintroduce an easy axis anisotropy described by $DM_{\text{z}}$. Adopt $D = 0.05$, $\alpha = 0.01$ and no external fields. Figure 5 displays the desired reversal. The rather severe overshoot, in the case of the precessional switching technique, can be minimized by carefully engineering the mechanical actuation to be closer to the optimum “ballistic” path between $M_z = \pm 1$.

The experimental realization will be a challenge since the cantilever has to work at high frequencies $\sim 1$ GHz. The magnetic film should be small enough to form a single domain. In the resonant reversal a significant coupling strength of $G/\omega^2 \sim 10^{-3}$ requires that one tenth of the cantilever volume is a ferromagnet. Mechanical reversal by breaking a STM tip-cantilever bond might be easier to realize since sharp control of the resonance condition is not required.

Summarizing, we investigated the nonlinear dynamics of coupled magnetic and mechanical fields for a cantilever with a ferromagnetic tip. Employing the new dissipation channels we propose three strategies for fast magnetization reversal and suppressed ringing. We can (i) make use of the additional mechanical damping and (ii) shape the external magnetic field pulses, thus quickly channeling-off magnetic energy when damping is weak. Finally, (iii) we propose a precessional reversal scheme based on the mechanically generated out-of-plane demagnetizing field without applied magnetic fields.

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18. In principle a ferromagnetic cantilever would achieve maximal coupling when the volume of the magnet equals the volume of the cantilever.