Design of a statically balanced mechanism using magnets and springs

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November 1, 2015
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Introduction

This thesis is part of the graduation process of the author to achieve the title of Master of Science at the Delft University of Technology. It presents the efforts done during a graduation-internship performed at InteSpring B.V. in Delft.

The work done is part of the Exobuddy project, a collaboration between TNO and InteSpring. The goal of the project is to design and build an exoskeleton which provides load carrying augmentation, aimed for military use.

The current work explains the process of designing, building and testing of a statically balanced mechanism, including magnets and springs. The prototype made serves as a proof-of-concept, which shows that a working statically balanced system can be designed which delivers a high force in a relatively small design space. The force generated could be used to actuate a friction brake, useable for the Exobuddy exoskeleton.

The work done can be found in concentrated form in the paper, which the thesis commences with, and serves as the foundation of the thesis. The chapters following the paper can be regarded as a further elaboration of the work done. These chapters are less concentrated and provide additional information about certain processes, and serve as appendices to the paper. To prevent repetition of information, references have been made to the paper. The paper itself however is intended as a read-alone document, so no references have been made to the rest of the thesis.
Design of a statically balanced mechanism using magnets and springs

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Abstract—This paper presents a statically balanced system with novel design, which incorporates compression springs as a positive stiffness and magnets as a negative stiffness.

The mechanism varies the force in the positive stiffness elements from zero to maximum. In this way, the balancing system can be used as a force amplifier. The goal of the current design is to be used in a friction brake.

A specific magnet configuration is used to achieve a relatively linear force-distance relationship. The configuration consist of three block-magnets, from which one magnet translates and two magnets are placed in a stationary steel U-shape. This force-distance relationship is balanced by a piecewise linear characteristic achieved by using multiple compression springs with different action-initiation points.

The proof-of-concept build consists of a lever mechanism with a lever arm of 100 mm, one set of magnets and three compression springs. By varying the position of the lever from 0° to 5.04°, the force in the springs is varied between 0 N and 872 N.

Experimental evaluation of the prototype reveals a minimum of 97% force reduction in the prototype compared to the unbalanced prototype.

I. INTRODUCTION

Modern day exoskeletons used for load carrying augmentation often suffer from a high power consumption. This leads to a limited battery life and usage time. The main components responsible for a high power consumption are the actuators, which provide the torques needed at the joints. An increasing focus on exploiting semi-passive elements can be witnessed in the field of robotics [1], which is aimed to limit this problem. The semi-passive elements provide a counter-torque, inhibiting the movement of joints, requiring a lower power consumption compared to conventional actuators.

Often used devices providing a counter-torque are magneto-rheological brakes [2], electro-rheological brakes [3], hydraulic systems [4] or DC-motor brakes [5]. However these solutions are heavy, provide a low maximum torque, are hard to control or require more power than is desired. Furthermore these solutions provide a counter-torque when no control input is present, inhibiting the user when used in a exoskeleton.

A viable solution is the use of a friction brake. Friction brakes make use of a normal force acting on a friction surface, generating a friction force. Commercially available devices use electromagnetic actuation to generate the force needed. While highly controllable, these devices are either too heavy or generate too little force to be used in a wearable exoskeleton. A possible solution to this is the use of a piezo-electric elements as an actuator. These elements are able to provide an actuation force in excess of 1 kN, but tend to suffer from deformation of the construction due to their micro-meter range stroke, leading to a lower performance than expected. Furthermore, piezoelectric elements are generally too expensive, and require a actuation voltage of several hundreds of volts, which is undesirable from a safety standpoint.

An alternative to reach a high actuation force with a lower power consumption is the use of static balance [6]. Static balance decouples the actuation force moving a balanced object and the force in the stiffness elements balancing the object. By using the force generated in the positive stiffness elements as a normal force on a friction surface, such a device can be used to actuate a friction brake. Two examples found using this principle are the devices by Plooij and van der Hoeven [7].

Any statically balanced mechanism incorporating stiffness elements can be used for force generation. However static balanced mechanisms found in literature are often either too large or achieve a relatively low force.

To reach a compact design, the use of a stiffness element with a intrinsic negative stiffness is proposed. This is likely to result into a simpler coupling between positive and negative stiffness elements, reducing mechanism size and complexity.

Often used stiffness elements with an intrinsic negative stiffness are leaf springs and disc springs (also known as Belleville springs). These elements undergo buckling, which results in negative stiffness when the design parameters are chosen correctly. However these elements can show hysteretic effects [8], compromising the performance of the balancing system. In the current paper the use of magnets is proposed. Magnets are able to provide a relatively high maximum force while achieving negative stiffness. Furthermore, preliminary tests performed on the attraction force between magnets show (near-)zero hysteresis, minimising imbalance and thus actuation force.

The goal of the current paper is to design a statically balanced mechanism incorporating magnets, which serves as a proof-of-concept to be used in future statically balanced brakes.

The outline of this paper is as follows: First the design requirements will be given in section II. A short initial research is given in section III. What follows is the theoretical design of the magnet configuration (negative stiffness element) in section IV. The guiding mechanism in which the magnets
are placed in, will be discussed in section V. What follows is the prototype build of the magnet configuration inside the guiding mechanism, found in section VI. Based on the results of the negative stiffness, the positive stiffness is designed in section VII, leading to the statically balanced prototype in section VIII. Section IX describes both the testing procedure of the prototype and the results. The paper ends with a discussion and conclusion, found in sections X and XI respectively.

II. DESIGN REQUIREMENTS

The design space in which the statically balanced system has to fit is cylindrical, with a diameter of 140 mm and a height of 50 mm.

By moving the balanced object the force generated in the positive stiffness element should be able to vary between 0 N and 750 N. The actuation force should be less than 2 % of the force generated.

It is desired that the force-distance characteristic of the positive stiffness element is linear. A linear relationship leads to a performance of the device which is less sensitive to the actuation accuracy, friction material wear and imperfect balance compared to an exponential relationship.

Since it is a proof-of-concept the weight of the design will not be considered.

III. INITIAL RESEARCH

Initial research has been performed to be able to design a suitable magnet configuration, leading to the current section. The research includes the state of the art of comparable devices, methods to change magnet behaviour and the investigation of magnets using FEM (Finite Element Analysis) software.

A. Internally Balanced Magnet

Only one example is found of a device which balances the force of a magnet from zero to maximum, which is the 'internally balanced magnet' device by Hirose and Suzuki [9], [10]. The device, intended for wall-climbing robots, balances the attraction force between a permanent magnet and a steel wall. The attraction force be can varied with small effort, yielding an energy efficient alternative to conventional energy consuming electromagnets.

The approach by Hirose and Suzuki was to choose a permanent magnet as a negative stiffness. The force-displacement characteristic of the magnet was measured, on which the design of a complementary positive stiffness element was based. Coupling of the permanent magnet and positive stiffness element resulted into a zero-stiffness mechanism.

A nonlinear force-displacement characteristic for the positive stiffness was required. To achieve this Hirose and Suzuki proposed three possible solutions: 1) a multi-stage spring unit consisting of compression springs, 2) a multi-stage spring unit consisting of leaf springs and 3) a spring unit based on a rubber o-ring moving between two special shaped shells.

Three main downsides arise from using a standard set of magnets as a negative stiffness: 1) Due to the exponential force-displacement curve it is hard to balance with a positive stiffness. This leads to either a large (in case of piece-wise linear approximation mechanisms), inadjustable (in case of cam mechanisms) or poor performing positive stiffness system. Inadjustable systems are often undesirable due to possible errors in simulation, measurement and fabrication during the design process, leading to a greater balancing error in the final product. 2) In case of force generation, a small deviation in input will lead to an large deviation in force output at small distances, and the other way around. This will increase the demand on the actuator in terms of precision and speed. 3) In case of force generation, it may be desired to generate a (near) zero force. Due to the exponential nature of the force-displacement characteristic this will lead to a large travel of the balanced object, increasing the size of the system.

Dispite the downsides Hirose and Suzuki achieved a sticking force of 1100 N with a maximum actuation force of 100 N using leaf springs, a reduction in actuation force of 91 %.

B. Changing magnet behaviour

To change magnet behaviour some design choices can be made. A common example is the use of iron in combination with magnets. Iron can be used to redirect field lines, shaping the magnetic field. An example of an application is the iron yoke (Fig. 1a). The yoke redirects the field lines, yielding an uniform field strength between two magnets. While Hirose and Suzuki use a magnetic yoke, no fundamental change was achieved in the magnet behaviour.

Another example is to place the magnets in an array. A well known example is the Halbach array (Fig. 1b), which concentrates the magnetic flux to one side of the array, while on the other side the flux is weakened. This increases the maximum force delivered by the magnet, while minimising flux in other directions, making it more suitable for practical applications.

A curious example in a commercial product using a specific magnet configuration is the MagSpring® by LinMot® [11] (Fig. 1c). The Magspring consists of a cylindrical stator, which surrounds a slider. The stator is able to provide constant force over a certain range of motion, without the need of an external power source. To achieve this permanent magnets are used, which are located in either the stator, slider, or both, dependent on the force rating.

From the solutions found it is reasoned that altering the behaviour of the magnet is a viable option. Doing this will have the potential of simplifying the positive stiffness solution, decreasing the device size and complexity.
C. 2D simulations of attracting magnets

A series of initial two-dimensional magnet simulations have been performed using FEM software to achieve a better understanding of magnet behaviour. The software package used for this is COMSOL Multiphysics® 5.0 [12]. These simulations investigated two block magnets, for which the distance between them was varied. The simulation showed the field strength, field lines (field direction) and attraction force between the magnets.

Found was that when the distance between the magnets increases, two interesting things happen: 1) The field lines 'peel' off: a lower number of field lines travel from one magnet to the other. 2) The field lines between the magnets spread out: The field lines which travel from one magnet to the other tend to spread out. Both peeling and spreading of the field lines lead to a reduction in field strength between the magnets. Both field strength and attraction force decrease exponential for increasing distance. It was concluded that the field strength seems to be roughly related to attraction force between the magnets.

It is hypothesised that peeling of the field lines would be mandatory for a reduction in attraction force. However, spreading of the field lines, not. So a possible solution for a more linear force-distance relationship between magnets would be a design in which the spreading does not occur.

IV. Magnet Configuration Design

The current section describes the theoretical design of the magnetic configuration used. The theoretical design forms the basis of the prototype build in section VI.

A. Desired properties

A compact negative stiffness element is desired. This negative stiffness element generates a force which decreases for increasing displacement.

The most obvious solution is to use a magnet as-is. However, magnets have an undesired exponential force-distance as explained in section III. It is wished that this force-displacement is linear, in which the force is maximum at zero distance and decreases to zero at relatively short distance.

B. Design Method

To synthesise a new type of configuration the standard case investigated in section III-C will be used as a basis. This basis consists of a vertical stack of two magnets: one stationary and one translating vertically (‘Normal’ configuration in Fig. 2a), which will serve as a baseline to compare the results to. Both magnets have an upwards magnetisation, attracting each other.

It is hypothesised that minimising the spreading of the field lines between the magnets will lead to a more desired behaviour. To achieve this the stationary magnet will be converted into a configuration which shows a similar path of field lines, but less spreading. This path of field lines points upwards and sideways. To realise this, extra block magnets will be added next to the original magnet with an identical size. The individual magnetisation of the magnets will be either up, down, left or right. For simplification purposes, the magnets are placed in a square lattice.

When a plausible configuration is synthesised, it is attempted to improve the design by adding an iron structure. This structure can either shield the magnetic field or replace magnets. This would result in a final two-dimensional design.

If the final two-dimensional design is known, a three-dimensional design will be simulated. This three-dimensional design will be an extruded version of the two-dimensional design, in which magnets will be used that are available at Supermagnete [13]. The dimensions of this three-dimensional design will be the basis of the design of the constructed prototype in section VI.

C. Results

The results of the initial synthesis is the 'Initial' configuration shown in Fig. 2b. It is found that the design could be simplified by introducing iron, replacing seven magnets by one iron U-shape. This led to the 'Final' configuration in Fig. 2c. No significant change in maximum force or behaviour was found by the replacement of the magnets by the steel structure.

The force-displacement curves found using 3D FEM simulations is shown in Fig. 3. The design is based of the 'Q-40-10-10-N' magnet from Supermagnete (Table I). The force-displacement curve of the 'Final' configuration shows a roughly linear force-distance behaviour with a negative stiffness. Also an increase in maximum holding force is achieved compared to the 'Normal' configuration. Zero force is reached within 9 mm, while for the 'Normal' configuration this distance is over 60 mm.

D. Discussion

No linear force-displacement curve was achieved using the synthesised configuration. However, an improvement is reached in terms of maximum holding force and linearity of the curve.

The transition between the 'Initial' configuration and the 'Final' configuration can be understood by realising that the outer and lower magnets simply redirect the field lines. The steel replaces this function. Addition of iron to the top of the structure did not improve the performance in terms of linearity of the force displacement curve or maximum force.
Table 1: Magnet used from Supermagnete [13] to construct the magnet configuration shown in Fig. 2c.

While the results are satisfactory, it may be argued if the best possible solution is found. Weaknesses of the method used is that shape, magnetisation direction and positioning are limited. Commercially available magnets however are often block-shaped and possess a magnetisation perpendicular to one of the surfaces, which makes the current method practical. To investigate the performance of rectangular magnets instead of square magnets a parametric sweep was simulated. In the parametric sweep the dimensions of the magnets were varied using the same configuration. This did not result in an improved performance in terms of maximum force or linearity of the force-displacement curve compared to the solutions presented. An improved performance may be reached by positioning the magnets at an angle or varying the exact position with respect to each other. However this increases the complexity of the synthesis process.

V. GUIDING MECHANISM

To restrict the positive and negative stiffness elements along a trajectory a guiding mechanism is needed. For this, a lever mechanism (Fig. 4) was chosen for several reasons: 1) It provides a relatively straight path since the maximum rotational angle is small, 2) Small number of parts, ease of manufacturing and reliability, 3) Ball bearings can be used which allow an high amount of force with little play, 4) By decreasing the lever arm of the positive stiffness with respect to the negative stiffness the force amplification is increased.

The lever arm was chosen to be 100 mm, combining a relatively straight trajectory for small angles and sufficient design space for the positive stiffness system while still achieving a compact design.

VI. MAGNET CONFIGURATION PROTOTYPE

After the synthesis of the magnet configuration and the guiding mechanism, these can be build and tested, which is described in the current section. The results of the tests form the basis of the design of the positive stiffness discussed in section VII.

A. Construction

The steel parts of the magnet configuration are milled from a solid block of structural steel. The structure designs are split through their symmetry axis, thus two pieces of plain carbon steel are manufactures to accommodate one ‘Q-40-10-10-N’ magnets each. The magnets were glued into place using widely available metal-metal glue. Two lips were added to the base of the steel structure to accommodate screws, as seen in Fig.5, to mount the magnets to the lever mechanism.

For the upper magnet an aluminium block was designed in which the ‘Q-40-10-10-N’ magnet could be placed snugly, in order to maximise the holding force of the glue to prevent failure. Again screws could be used to mount the magnet to the guiding mechanism.

B. Testing

The performance of the magnets was measured in a test bench, in which the lower half of the lever was attached rigidly, while the top lever was mounted using a steel eye and a steel cable (Fig. 6). Regular force-displacements curves were plotted using the test bench. The force-displacement curves were then corrected for other stiffnesses, like the cable, which were measured separately. The force-displacement curves were then
Fig. 6: Testing the magnet configurations in the guiding mechanism. The test consists of pulling the lever apart with a steel cable. A large bolt was added to be able to separate the lever arms without assistance of the test bench.

transformed to torque-rotation curves that could be compared to torque-rotation data taken from 3D COMSOL simulations. These COMSOL simulations are similar to the ones performed in section IV, but include the rotational guiding instead of a linear guiding.

C. Results

The results of the test and the COMSOL simulation can be seen in Fig. 7. The curves achieved from the physical prototype shown a high resemblance with simulated data. However the physical force is found to be around 16% lower.

D. Discussion

In the model a steel U-shape was used while for the physical prototype two L-shapes were used to construct one U-shape. Due to fabrication inaccuracies an air gap between L-shapes was present which increases the distance between the magnets and decreases the field strength. This is expected to be the main cause of the force-difference between the simulation and prototype. Other causes may be a weaker and/or inhomogeneous magnetisation of the magnets and saturation of the steel material (which was not modelled).

Another expected influence on the behaviour was the addition of the mounting lips on the steel structures. Additional FEM simulations were performed in which these lips were added to the design, however no significant change was witnessed.

VII. THE POSITIVE STIFFNESS

With the negative stiffness determined, a complementary positive stiffness has to be found. Preferably, the positive stiffness element does not increase the size of the unbalanced prototype. Furthermore, it is wished that the stiffness is tunable. This way the device can be adjusted after assembly, minimising balancing errors caused by errors in design, calculations and manufacturing.
The prototype build (Fig. 11) consists of two lever arms, hinged by two bearing blocks and an axis. Three stainless 1.4310 steel compression springs [15] are accommodated at the centre of the prototype, and seated in spring holders, which are fastened in the adjustment slots. By fastening the holders at different locations in the slot the stiffness introduced by the springs is altered. By introducing a spacer ring between the levers and the spring holders the initiation points are altered. The magnet configuration is located at the end of the levers. An endstop is added to prevent the prototype from opening beyond its working range.

IX. Evaluation

The design of the negative stiffness, guiding mechanism, and positive stiffness have led to the design of the static balance mechanism. The current section evaluates the design.

A. Method

To be able to test the prototype with a steel cable the prototype was underbalanced, in which the magnets dominate the springs. The results of such a test will prove the ability of the prototype to achieve zero stiffness.

The performance of the prototype is defined by the balancing quality \( b_{\text{quality}} \). The balancing quality is defined by the deviation of the torque measured around the underbalance torque \( T_{\text{underbalance}} \):

\[
b_{\text{quality}} = 1 - \frac{\max(\left| \frac{T_{\text{measured}} - T_{\text{underbalance}}}{T_{\text{magnets}}} \right|)}{\max(T_{\text{magnets}})}
\]

In which \( T_{\text{measured}} \) is the torque measured after balancing, \( T_{\text{underbalance}} \) is the torque around which is balanced and \( T_{\text{magnets}} \) is the maximum torque exerted by the magnets.

To properly adjust the springs in the prototype will be opened and closed several times in the test bench with data recording. When there is no range present in which a constant force is present, the spring stiffnesses will be adjusted by altering their position in the slots. If there are multiple constant force ranges present, the action initiation points are adjusted by adding or removing spacer rings to achieve one constant force range, achieving a (close to) zero stiffness characteristic.
B. Results

The prototype is balanced around 5.0 N m with a maximum deviation of 1.77 N m. This leads to a balancing quality of 0.96 (Fig. 12).

C. Discussion

The balancing quality reached is lower than expected based on the simulations, which predicted a balancing quality of 0.99.

The lower balancing quality is mainly caused by the behaviour of the largest spring after 3.15°. This behaviour is present in the phase between full-contact and point-contact between the spring and the lever (Fig. 13). In this phase there is no uniform deflection of the spring, leading to gradually decreasing stiffness contribution of the spring to the lever. This gradually increases the balancing error, since a constant stiffness was expected.

The contact behaviour explained is also present at the two remaining springs, however these contribute to a gradually change in stiffness, possibly leading to an improved balancing quality between 0° and 3.15°.

If only the range between 0° and 3.15° is considered a balancing quality of 0.99 is reached, which is in line with the simulations.

Compared to the works of Hirose and Suzuki a higher balance quality is reached with a similar design of the positive stiffness. This leads to the conclusion that the improved shape of the magnet force-distance characteristic leads to a lower balancing error.

A weakness of the current design is that the effect of the friction material and the friction brake construction are not considered. If the springs are coupled to the friction block these will compress. This would result in a decrease in balancing quality, leading to a higher actuation force than discussed. However, the construction stiffness may be corrected for, when designing the positive stiffness system of the statically balanced device.

A possibility to solve the contact behaviour problem discussed in section IX could be the use of rotating spring seatings. These seatings would have a rotation axis parallel to the rotation axis of the lever, ensuring full contact between the springs and seating when needed. An alternative is to use seatings with a spherical head, at which a spring can seat itself at the head surface.

The use of a translational guiding mechanism is a viable alternative, since this would solve the spring-lever contact behaviour. Furthermore, it may result in a more compact design. However, the use of a lever-arm to increase the force in the positive stiffness elements would not be possible.

XI. Conclusion

In this work a statically balanced mechanism is presented. The goal of the mechanism was to be used as a force-amplifier in a friction brake, serving as a proof-of-concept.

Magnets were used as an alternative to conventional leaf springs and disc springs as a negative stiffness. The magnets were placed in a novel configuration, consisting of two stationary magnets in a steel U-shape and one translating magnet. The configuration used showed an improved behaviour in terms of linearity of the force-displacement curve, maximum force achieved, and travel distance compared to the behaviour of a conventional attracting magnet pair.

The magnets were balanced by a piecewise linear approximation using three compression springs. A reduction of 96% in actuation torque of the lever was achieved. This leads to a reduction of 98% in actuation force, compared to using no balancing system.
REFERENCES


Appendix A

Problem description

The current chapter describes motivation behind the project and the considerations leading to the design requirements. The information presented is identical to the information presented in section I and II of the paper, but in a more extended form and linked to the project it is part of.

A.1 The Exobuddy project

*This information is not included in the public version of the thesis. For further information about the Exobuddy project contact InteSpring B.V. (info@intespring.nl).*

A.2 Possible solutions

A short literature research has been performed on existing solutions, leading to Table A.1.

Three main groups were identified: one based on fluid flow, one based on friction and the last based on magnetism. Nine points of interest were defined, which are in random order:

- The velocity dependency of the device. For some devices the counter-torque delivered is dependent on the rotational velocity. This increases the complexity of the control system used in the exoskeleton, as an extra angular measurement has to be performed at all times.

- Damping directions. Some devices are able to deliver a counter-torque in both directions of rotation. For the Exobuddy exoskeleton only one damping direction is sufficient. However it may be useful for future designs.

- Power consumption. This is one of the most important factors, since a low power consumption increases the battery-life of the exoskeleton.

- Size. Desirably the device is as small as possible. It will be located at the hips which could inhibit arm-movement of the user, which is naturally undesired, especially in the line of combat.
• Weight. While the exoskeleton is designed to carry heavy loads, it may occur that the it is turned off when certain movements are performed which are not augmented. An increased weight will lead to a lower agility of the user.

• Price. Price is of lower importance when considering military devices.

• Torque lower limit. Some devices deliver a torque when no input signal is given. This is undesired when the exoskeleton is turned off, since it inhibits the user.

• Torque upper limit. The maximum torque delivered is important, since it directly determines the amount of load carrying augmentation provided by the exoskeleton. The maximum torque may be easily increased by introducing a gearbox to the design. However, this would increase the size, weight, complexity and backlash of the design. So the devices found are assessed on their performance without gearbox.

• Controllability. Ideally, the control input (position, voltage, etc.) is proportional to the torque delivered. This decreases the demand on the controller output.

<table>
<thead>
<tr>
<th>Principle</th>
<th>Type</th>
<th>Velocity independent</th>
<th>Damping directions</th>
<th>Power consumption</th>
<th>Size (Nm/mm)</th>
<th>Weight</th>
<th>Price</th>
<th>Torque lower limit</th>
<th>Torque upper limit</th>
<th>Controllability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid flow</td>
<td>Magneto-rheological</td>
<td>No</td>
<td>2</td>
<td>++</td>
<td>++</td>
<td>+</td>
<td>0</td>
<td>–</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>Electro-rheological</td>
<td>No</td>
<td>2</td>
<td>0</td>
<td>–</td>
<td>0</td>
<td>–</td>
<td>–</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>Pump &amp; solenoid</td>
<td>No</td>
<td>2</td>
<td>+</td>
<td>–</td>
<td>0</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Friction</td>
<td>Electromagnetic</td>
<td>Yes</td>
<td>2</td>
<td>0</td>
<td>–</td>
<td>0</td>
<td>+</td>
<td>++</td>
<td>0</td>
<td>++</td>
</tr>
<tr>
<td></td>
<td>Piezoelectric</td>
<td>Yes</td>
<td>2</td>
<td>++</td>
<td>0</td>
<td>–</td>
<td>+</td>
<td>–</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>Statically balanced</td>
<td>Yes</td>
<td>2</td>
<td>++</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Magnetism</td>
<td>Eddy current</td>
<td>No</td>
<td>2</td>
<td>–</td>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>Hysteresis</td>
<td>Yes</td>
<td>2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>DC motor</td>
<td>No</td>
<td>2</td>
<td>++</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>+</td>
<td>–</td>
<td>0</td>
</tr>
</tbody>
</table>

Table A.1: Table used to compare different types of dampers and brakes.

A.3 Choosing a solution

Based on the assessment factors described and their scores in Table A.1, the solutions could be compared for their suitability for the exoskeleton.

A.3.1 Solution principle

The fluid flow based dampers excel in their upper limit torque. The torque lower limit, however, is found to be undesirable. In all cases there is a fluid flow present when the input axis is rotated.
A.3. CHOOSING A SOLUTION

Even when there is no control input present this will lead to the dissipation of energy, generating a counter-torque. For the most desirable type of the three, the magneto-rheological brake, the minimum torque is about 10% of the maximum torque generated for the designs found. This is deemed unacceptable.

The friction brakes dampers score best on their lower limit torque, which is (close to) zero for all designs. The controllability is high, since the torque delivered is directly related to the force generated at the friction surfaces. The power consumption is potentially low and the weight is reasonable. The torque upper limit scores reasonable.

The magnetism group scores best on their controllability. The the rest of the performance however is weak, especially the weight and size of the devices. The DC motor type scores very good on the power consumption, potentially generating energy.

Based on the table the most viable solution is found to be a friction brake. Friction brakes have the ability to produce zero counter torque, which is not achievable by an hydraulic system alone. The counter-torque is independent on the velocity, which makes it a predictable solution while walking at several velocities and decreases the complexity of a control system. Furthermore the weight is lower and the torque achieved higher compared to the magnetic solutions.

A.3.2 Solution type

For the friction brake an actuation mechanism has to be chosen. Three main types are shown in the table.

The electromagnetic solution [2] scores weak on power consumption. Of the three this type is most commonly available commercially, however no suitable type has been found which suits the demands of the exoskeleton. Most likely this type will not result in a successful design.

Both piezoelectrical [3] and statically balanced friction brakes [6, 7] are found to be viable solutions. Piezoelectric elements are able to provide a high force at a low power consumption. However, these often suffer from a low construction stiffness, leading to a lower force output than expected. Furthermore, these devices require voltages in the range of several hundreds of volts, which is less desirable from a safety standpoint. Statically balanced brakes are only found in the form of friction lockers, and only three prototypes exist from one research group. The maximum force achieved by these, however, does not exceed 1.08 N m, while 100 N m is required.

It was concluded that both piezoelectrical and statically balanced brakes are potential solutions. However the statically balanced brake is most likely to be improved on its design, given...
that the designs available were not designed for the amount of torque needed for the exoskeleton. Furthermore, InteSpring has experience with building and designing balancing systems, so this type of design is most likely to succeed given the expertise available. Furthermore, the statically balanced brake would be able to lock (provide near-maximum force) at no energy consumption, which is useful if the user stands still.

A.4 Friction brake design

Several types of friction systems exist, of which some possess a self energising effect. When a self energising effect is present, the friction force generated is used to increase the actuation force present, increasing the friction force even more.

An example of a self-energising brake is shown in Figure A.2 [8]. A drum is shown with radius $R$ rotating around it’s axis in the direction of $\omega$. Actuation force $F_u$ acts on the lever at distance $p$ from it’s rotation point, which leads to normal force $N$ on the friction pad. This normal force generates a friction force $W$ on the drum. $W$ induces a moment around the point of rotation of the lever in the same direction as actuation force $F_u$, which increases $N$, leading to a higher $W$. The torque on the drum can be written as:

$$M_w = F_u \frac{p R \mu}{b - a \mu}$$

Figure A.2: Example of a friction brake using a self-energising effect.

Where $\mu$ is the friction coefficient between the friction block and the drum. To achieve a high force the denominator has to be small, leading to a small difference between $b$ and $a\mu$. However the friction coefficient is expected to vary, given that the exoskeleton has to be usable in demanding conditions. So if $\mu$ varies the torque delivered will vary by a larger extend. This effect is shown in Figure A.3, where a comparison is made for a drum brake with self energising effect and a traditional brake, using realistic parameters. The exponential curve reaches an asymptote when $b = a\mu$, which causes the friction brake to lock without actuation force, which is very undesirable. Self energising variations of internal drum brakes, external drum brakes, band brakes, capstan brakes and wedge brakes have been analysed, however all these types show this effect [9, 10, 11]. Furthermore, the control of the torque may be more challenging.
One example is the wedge brake, were a actuation torque may be needed to reduce the torque generated [12].

![Figure A.3: The dependency of the countertorque delivered to the friction coefficient.](image)

One alternative is to use a cone brake [9], which has a inherent mechanical advantage. However even this type shows a similar high dependency on the friction coefficient.

Another possibility to decrease the actuation force needed is the incorporation of a simple lever-arm or other mechanical advantage to increase performance the force delivered. This decreases the force needed in a simple manner. However, this leads to a higher input/output displacement ratio, which may pose a problem in case of friction pad wear or construction deformation. Since an mechanical advantage is reasonably simple to incorporate, it is chosen to neglect this option for now, since the rest of the brake design is not yet know.

It is expected that a minimum amplification of 4 will be achieved by either a self-energising system or a mechanical advantage. This is a conservative number, so the balancing system designed may be scaled down in the future.

### A.5 Design specifications

The friction brake design will likely be a drum brake or disc brake. Since the primary design space available has a diameter of 150 mm it is assumed that a maximum effective radius usable is 70 mm. A friction coefficient of 0.5 is assumed, leading to a normal force on the friction blocks of 2857 N. Assumed is a force amplification of 4, leading to a required actuation force if 714 N. This way, the balancing system has to reach a force of 714 N in the positive stiffness elements. It is assumed that a small servo motor will be used to control the system, which needs a reduction of 98 % in the actuation force to operate.

The thickness of the primary design space is 80 mm, for which it is judges that at least 50 mm will be available for the actuation system. Leaving a design space for the balancing system with a radius of 140 mm and a thickness of 50 mm.

Given the time constraints of the graduation process, it is chosen to limit the project to the design of the balancing system alone. The balancing system designed serves as a proof-of-concept, which can be optimised for future design.
Appendix B

Theory and State of the Art

The current section provides some information about the state of the art left out of section III of the paper.

B.1 Internally balanced magnet by Hirose

See section III-A of the paper. Illustrations of the workings of the three positive stiffness systems proposed are shown in Figure B.1.

![Diagram of internally balanced magnet by Hirose](image)

**Figure B.1:** Three positive stiffness elements proposed by Hirose and Suzuki.

The maximum adhering force of the magnets achieved was 1100 N. The maximum actuation force of the leaf spring solution was about 100 N [14]. The maximum actuation force of the coil springs was 130 N and the o-ring solution had a maximum actuation force of 170 N. This leads to an reduction in actuation force of 91%, 88% and 84%. Hirose claims in his original paper that an actuation force of less than 10 N was achieved [13]. With a sticking force of 1400 N this leads to a reduction of 99%. However this seems to be in a limited range of motion, given that there will be an asymptote present at zero distance, and the relative poor performance of the very similar coil-spring based device.

B.2 MagSpring by LinMot

See section III-B of the paper. The construction and shape of the force-deflection curve is shown in Figure B.2.
APPENDIX B. THEORY AND STATE OF THE ART

Figure B.2: The MagSpring by LinMot, a device which is able to provide a constant force of a certain range of motion.

B.3 Halbach Array

A common used special arrangement of magnets is the Halbach array. The array was invented by John C. Mallinson and Klaus Halbach independently in the 1970’s and 1980’s. The configuration of the magnets inside the array enhance the field strength on one side of the array, while cancelling out the field strength on the other side of the array. A visualisation of the principle can be seen in Figure B.3. On the top left an alternating up and down configuration can be seen with corresponding field lines. On the top right an alternating left-right configuration can be seen with corresponding field lines. In both cases the top field lines follow the same path, while on the bottom the field lines follow opposite path. When combining (superimposing) the two configurations the top field lines would be enhanced, while the bottom lines would cancel out each other. Ideally this would double the field strength on the top of the array while no field strength is presented at the bottom. The length of the array can be increased by repeating the alternating field orientations.

The array is often used in cases in which the magnetic strength of a design is important for it’s performance. Examples range from fridge magnets to electromotors and particle beam accelerators.

Figure B.3: The diagram of Mallison visualising magnetic field cancellation

A simulations has been done in COMSOL Multiphysics® 5.0 to investigate the Halbach array. A plot is shown of the field strength of similar magnet configurations in Figure B.4. What can be seen is that facing all magnets in the same direction results in a relatively low field strength, except for the left and right sides. Alternating the field directions increases the strength in both up and down directions. The Halbach array finally increases the field strength
B.3. HALBACH ARRAY

Magnetisation order:

↑ ↑ ↑ ↑ ↑

Magnetisation order:

↑ ↓ ↑ ↓ ↑

Magnetisation order:

↓ → ↑ ← ↓

Figure B.4: Field strength of three magnet-array configurations: up, alternating and Halbach.

even more on the top side, both in magnitude and range, while on the bottom weaker short distance field is present.

A small prototype has been made of the Halbach array. The prototype consisted of three cubicle block magnets glued on an aluminium plate to keep them in place, where the aluminium plate was oriented 'in plane' in the same plane as the diagrams shown. It was found that the assembly of the prototype was challenging, since interaction forces between the magnets occur. These interaction forces cause the magnets to repel/rotate/attract each other. To be able to glue them the magnets were pinched between two pieces of ferromagnetic metal, fixing them in place, allowing the glue to harden.

Experimentally it was found that indeed attraction force at one side of the array was considerable larger as the other side.
Appendix C

Using magnets as a negative stiffness

The current section describes all the work done on the negative stiffness system, incorporating magnets.

C.1 Testing force between magnets

As mentioned in the introduction of the paper, initial tests were done on the attraction force between magnets. The goal was to investigate the behaviour of the magnets with different shapes, as well to achieve some practical experience using them.

C.1.1 Magnet types

A set of neodymium magnets was ordered from Supermagnete [16], including different shapes, being blocks, rings and discs. All the magnets possess a maximum holding force (maximum attraction force between the magnet and a ferromagnetic material) of around 80 N. This holding force was chosen since magnets of different shapes were available possessing this force, for easier comparison. Furthermore the force is considered to be substantial, while still being relatively safe. Bigger magnets would require an increase in safety precautions, which is considered to be unnecessary in this stage. The spherical magnet available were not chosen since these would be impractical to test or use in a design.

The data from the datasheet of the magnets acquired can be seen in Table C.1. The magnets would be tested on their attraction force in the test bench available at Intespring [15].

C.1.2 Mounting

Mounting the magnets into the test bench is found to be a challenge, given that the mounting equipment of the test-bench is fabricated from ferromagnetic material. The attraction to parts other than the test samples was expected to interfere with the measurement.

To prevent interference by ferromagnetic material, mounting parts were fabricated from aluminium. These mounting parts consist of a set of small sheets of aluminium, measuring
APPENDIX C. USING MAGNETS AS A NEGATIVE STIFFNESS

<table>
<thead>
<tr>
<th>Magnet shape</th>
<th>Type number</th>
<th>Measurements/mm</th>
<th>Holding force/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring</td>
<td>R-27-16-05-N</td>
<td>$\varnothing 26.75/16 \times 5$</td>
<td>83</td>
</tr>
<tr>
<td>Block</td>
<td>Q-15-15-08-N</td>
<td>$15 \times 15 \times 8$</td>
<td>76</td>
</tr>
<tr>
<td>Block</td>
<td>Q-40-15-05-N</td>
<td>$40 \times 15 \times 5$</td>
<td>80</td>
</tr>
<tr>
<td>Disc</td>
<td>S-25-05-N</td>
<td>$\varnothing 25 \times 5$</td>
<td>83</td>
</tr>
</tbody>
</table>

Table C.1: Magnets acquired from supermagnet for testing. All magnets are Neodymium magnets with a magnetisation of 1.29 T to 1.32 T.

44 mm × 47 mm × 2 mm. These aluminium sheets were glued to two magnets of each type, resulting in eight aluminium sheets with magnets attached to them (four sets of two). The magnets were glued to the aluminium using 'KOMBI Metaal-Metaal' glue by Bison® (Figure C.1a), assuming no interference on the magnetic field. The aluminium plates can be mounted on aluminium blocks 40 mm × 48 mm × 10 mm (Figure C.1b) using small steel bolts, which are assumed to have a negligible effect on the measurement. The aluminium blocks can easily be attached to the test bench.

The aluminium block which would move during the test (in z-direction) was bolted directly on a mounting block of the test bench. The bottom block was mounted using a vice grip. Using this vice both x- and y-direction could be adjusted to ensure proper alignment between the magnets. This was done by mounting the top block to the test bench and clamping the bottom block to the top one by using the magnets. This ensures proper alignment between the magnets. Once the magnets were clamped against each other, the bottom block could be attached to the test bench by using the vice. The vice was adjusted until tightening the vice would not result in a visible shift of the bottom block. This way of mounting still introduces a chance of the blocks being not completely aligned due to rotational angle around the z-axis. However the magnets were aligned while gluing them. During the mounting of the blocks no significant rotational shift was witnessed. So it is assumed that this has a negligible effect. To prevent damage to the magnets during testing due to contact between them the bottom magnets were covered with a single layer of masking tape to avoid peak pressures.

C.1.3 Testing

The initial distance between the magnets was roughly 1 mm. The attraction force between the magnets was defined as positive. The test definition is shown in Table C.2. The magnets pairs were tested (e.g. ring-ring, block-block and disc-disc) and each type was tested on their attraction force against steel (Figures C.1d and C.1e). This was done by removing the bottom magnet and clamping a piece of construction steel into the vice. The piece of steel faced the magnet with it’s largest surface area and measures 170 mm × 40 mm × 8 mm (Figure C.1c). During the magnet-steel test the steel was covered with a single layer of masking tape. It was found that during some tests the magnets were not perfectly parallel to each other. Expected is that the effect is small. Each test (both magnet-magnet tests and magnet-steel tests) was repeated three times. During the repeated tests the test-pieces were not re-clamped.
C.1. TESTING FORCE BETWEEN MAGNETS

(a) Magnet samples used, compromising different shapes of magnets glued on aluminium sheets. Of each type two were made.

(b) Aluminium mounting block used.

(c) Piece of steel used as a ferromagnetic material.

(d) A close-up of a magnet-magnet test (initial position).

(e) A close-up of a magnet-steel test (initial position).

Figure C.1: Pictures of the test samples and materials used.
APPENDIX C. USING MAGNETS AS A NEGATIVE STIFFNESS

<table>
<thead>
<tr>
<th>Stage</th>
<th>Record</th>
<th>Action</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No</td>
<td>Go to −5 N with 1 mm/min</td>
<td>This results into a compression force between the magnets, thus the magnets touching.</td>
</tr>
<tr>
<td>2</td>
<td>Yes</td>
<td>Decrease deflection by 60 mm with 60 mm/min</td>
<td>The distance between the magnets is increased. Thus the force goes to positive immediately and decreases.</td>
</tr>
<tr>
<td>3</td>
<td>Yes</td>
<td>Increase deflection by 59.5 mm with 60 mm/min</td>
<td>The distance between the magnets is decreased. Thus the force increases again. Chosen was to not decrease the distance by 60 mm, since this option is distance controlled and could result in damaging the magnets by compression.</td>
</tr>
<tr>
<td>4</td>
<td>No</td>
<td>Return to start position with 5 mm/min</td>
<td>This increases the distance between the magnets until the initial distance before stage 1 is reached. Thus increasing roughly 0.5 mm.</td>
</tr>
</tbody>
</table>

Table C.2: Test protocol used for magnet-magnet tests and magnet-steel tests.

Results

The results of the tests can be seen in Figure C.2. In all cases both the attraction force to the same type of magnet and a piece of steel is shown. Also the hysteresis is plotted, which increased by a factor 10 to be visible in the graph. In all cases the magnet-magnet force is higher than the magnet-steel force except at zero distance, where the force is roughly the same. An exponential behaviour is shown, as expected. The measured hysteresis is found to be very low, with a maximum of 1.25%. Due to the peak in force at near-zero distance and the limited resolution of the test bench, as well as the possible existence of some hysteresis in the test bench itself, it is expected that the hysteresis is even lower in reality.

To compare the effect of the shape of the magnet, the graphs of the magnet-magnet force graphs are plotted over each other in one graph. All graphs lie very close to each other, except the graph of the ring magnet, for which a higher drop in force is measured at the first few millimetres.

C.1.4 Insights

It seems that the shape of the magnet has a influence on the shape of the force-displacement curve. The ring magnet seems to have a relatively straight force-displacement curve for the first few millimetres. One idea is to fit a compression spring, which has a linear force-displacement curve, over the first few millimetres of the curve of the ring magnet. This should result in a relatively good performing balanced system. However, the force still does not reach zero in a reasonable distance, so this solution is not suitable for the problem to be solved.

From a practical standpoint, it is clear that using magnets is challenging. Mounting the magnets was found to be problematic. It seems that gluing is the most practical solution to fixate the magnets in an assembly. To keep the magnets in place during gluing the aluminium plates were placed between the magnets and a steel plate. This fixates the magnets which allows the glue to dry, which could be usable for assembly of further prototypes.
C.1. TESTING FORCE BETWEEN MAGNETS

Measured force of attraction of a 26.75/16 mm x 5 mm ringmagnet

Measured force of attraction of a 15 x 15 x 8 mm blockmagnet

Measured force of attraction of a 40 x 15 x 5 mm blockmagnet

Measured force of attraction of a 25 x 5 mm discmagnet

Figure C.2: Plots produced from the first series of magnet tests, including magnets of different shapes with a similar maximum force of attraction.
C.2 Investigating a standard case

To get a better understanding of the behaviour of magnets and how to improve it some 2D simulations are performed using COMSOL, as explained in the section III-C of the paper. The goal was to achieve a visual representation of the magnetic field generated by magnets in what their relation is to the force generated.

C.2.1 Results

In Figure C.3 the results of two 2D simulations are shown. Block magnets are shown with an upward magnetisation. The magnets increase the magnetic field strength, which is indicated by the colours. Blue indicates the lowest field strength and red the highest. Field lines are plotted, which indicate the direction of the magnetisation. The lines plotted originate from the centre of the single magnet, or the centre of the bottom magnet in case of a pair. A limited set of field lines is plotted, while in reality an infinite amount can be defined. The field strength is related to the density of field lines: when the density is high at a specific point the field strength is also high.

In case of the single magnet (Figure C.3a) the field lines travel closely to each other with an upward direction inside the magnet. When reaching the top of the magnet the field lines travel outward and downward, going to the bottom of the magnet.

In Figure C.3b a pair of block magnets is simulated, both with an upwards magnetisation and zero distance between them. Both magnets combine seem to act as the single magnet: the field lines travel from the bottom of the bottom magnet to the top of the top magnet internally. When reaching the top the field lines spread outward and downward to the bottom of the bottom magnet. An higher field density is shown at the centre of the magnets compared to the previous case. This is mainly caused by the fact that the field lines travel straight for a longer distance, causing a higher field strength over a larger area, which is not noticeable in the single magnet case.

In Figure C.3c two blocks on top of each other are simulated, both with an upwards magnetisation and a distance of 5 mm between them. Some interesting things can be seen. Not all the field lines travel from the bottom magnet to the top, some 'take the short route' by going directly to the bottom magnet. Furthermore the field lines between the magnets spread out, causing a lower field strength between the magnets compared to the magnets themselves.

Figure C.3d shows a distance of 10 mm. The effect witnessed in the 5 mm case is now more pronounced: More field lines take shorter route, and the field lines between the magnets are more spread out, leading to an even lower field strength. These effects increase with increasing distance, what can be seen in Figure C.3e and Figure C.3f. Eventually, when the distance between the magnets is large enough, all the field lines of the bottom magnet do not reach the field lines of the top magnet. At this point there is no interaction between the magnets.
C.2. INVESTIGATING A STANDARD CASE

(a) Single block magnet

(b) Two block magnets, 0 mm distance

(c) Two block magnets, 5 mm distance

(d) Two block magnets, 0 mm distance

(e) Two block magnets, 10 mm distance

(f) Two block magnets, 20 mm distance

(g) Two block magnets, 0 mm

Figure C.3: Plots of 2D COMSOL simulations of block magnets with upward magnetisation. In case of two magnets the field lines originate from the bottom magnet.
C.2.2 Insights

Based on these findings some assumptions are made about magnets and their attraction force:

- The force of attraction between the magnets seems to be related to the field strength between the magnets.
- The field strength is related to the density of the field lines between the magnets.
- For higher distances between the magnet the field lines seem to 'peel' off. This means that the field lines going to the top magnet initially take the 'short route' to the bottom of the bottom magnet when distance increases.
- For higher distances the field lines tend to spread out, lowering the field strength even more.

It seems that the combination between peeling en spreading of the field lines leads to the exponential decrease in force.

C.3 Magnet configuration synthesis

In Figure C.4d a block magnet is shown, where a box is drawn above the magnet. This is the location were the largest changes in the magnetic field occurs as shown previously. The field lines travel upwards and sideways when no other magnet is present. It is hypothesised that guiding the field lines to take this path will decrease the spreading of the field lines. This would result in an higher field strength directly above the magnet when the distance between magnets increases compared to the standard case in Figure E.1a. Peeling of the field lines would still occur, however it is hypothesised that this is necessary for an decrease in force.

It is concluded that 'guiding the field lines' could be easily done by adding two magnets with an outward magnetisation. This would result in a configuration with a minimum of three magnets: one magnet upwards and two sideways. To guide the field lines back to the bottom magnet extra magnets were added. The reasons for this are two-fold: the field lines are less likely to bend back to the translating magnet after going trough the side magnets and the magnetic field is somewhat contained for practical reasons. This resulted in two main configurations, shown in Figure C.4b and C.4c. Configuration 1 shows an higher resemblance to the Halbach array, but in a U-shape. Configuration 2 is a variation which is lower but wider.

Comparing the field lines of these configurations (Figures C.4e and C.4f) to the original case (Figure C.4d), it is found that indeed the field lines take roughly the same route. In all cases the field lines travel upward and outward.

The force-displacement curves of the two configurations (Figure C.4g) show a much more desired behaviour. In both cases there is a range in which a relatively linear force-displacement curve is shown with a negative stiffness. In case of configuration 1 there is first a force build-up, after the force drops again. In configuration 2 the force drops immediately. In both cases the force decreases when peeling of the field lines occurs.

C.4 Simplifying design

See section IV of the paper.
C.5 Testing a simple magnet bridge prototype

C.5.1 About the prototype

As a proof of concept a simple version of the design synthesised has been constructed and tested. The main goal was to see if the configuration would work in reality based on a quick and simple prototype. Furthermore potential problems could be identified before a final prototype was constructed.

The prototype consists of six cube magnets with a edge length of 7 mm. A steel U-shape was constructed by bending a piece of 1 mm thick steel. The prototypes (Figure C.5) were glued together and mounted to the aluminium blocks used for the magnet testing in section C.1.

Both 2D and 3D simulations were done in COMSOL Multiphysics® as well as the physical
testing, shown in Figure C.5b. It is not clear which what simulation represents the physical test the most, however it is clear that the achieved force is lower. Furthermore the test results do not show the full curve. It is found that an excess of glue causes the prototype to touch before zero distance was reached between the translating magnets and the configuration, so no maximum force was reached.

The results show that the the quick prototype shows a curve which is less linear compared to the earlier simulations. However it is expected that this is caused by the relative short length of the prototype. Due to the short length, field lines travelling out of plane can have a more influential effect. In the 2D simulation this effect is neglected, which shows the most linear curve of the three.

C.5.2 Insights

It is clear that for the final prototype all excess glue has to be removed for proper functioning. An attempt was made to remove the glue, however the prototype broke before re-testing. The last lesson of this prototype is that building the configuration out of cube magnets is challenging. The magnets tend to propel each other. Using three bigger magnets instead of six smaller ones would be easier to assemble. However, the magnets stick to the steel structure, which makes further clamping relatively easy.
C.6 Optimisation of design

As explained in section IV-D of the paper a attempt was made to improve the design made. To do this the geometry of the magnet configuration in the FEM simulations were defined as parameters. The parameters defined were:

1. width of translating magnet
2. height of translating magnet
3. width of outer magnets
4. height of outer magnets
5. iron thickness
6. air gap between base magnets
7. air gap between base and translating magnet

These parameters were each altered individually by performing a ‘parametric sweep’. This means that the force-displacement curve was calculated dependent on a given parameter.

It was found that both translating magnet width and height had no significant positive effect on the behaviour, as well as the iron thickness and the air gaps. The width of the outer magnets had a relatively low impact on performance, where decreasing the width would lead to a decrease in force. The height of the outer magnets however had a influence on the shape of the curve. Increasing the height of the outer magnets by would lead to an increasingly linear plot (Figure C.6). However a dip would occur at the last third of the plot, that may or may not pose a problem.

The optimisation led to an alternative design, with slightly higher outer magnets, seen in Figure C.8b. During manufacturing the steel structure was split in two. This leads to an alternative version that can be made, combining the 'high' design and original 'low' design, seen in Figure C.8c. This ‘hybrid’ configuration is also constructed, using the 'Q-19-13-06-N’ magnets by Supermagnete [16].
Figure C.6: Parametric sweep performed in COMSOL, altering the height of the outer magnets. Initial block magnets modelled had a side length of 12 mm.

Figure C.7: Prototypes constructed to assemble the 'low', 'high' and 'hybrid' solutions deducted.

Figure C.8: The initial configuration ('low') and two variations deducted ('high' and 'hybrid').
C.7 Testing the magnet configuration

To mount the prototype on the test-bench (Figure C.9) a mounting block was used which fits the test bench. This block can be bolted with four bolts onto the bottom plate of the prototype, centred underneath the bottom magnet-bridge assembly. The block is secured to the test-bench using a pin and a nut. The prototype was oriented with the axis of rotation on the left.

To secure the top lever to the loadcell a steel cable was used with an eye on each end. One eye was attached to a ring nut on the top lever of the prototype, centred above the top magnet. The other eye of the steel cable was attached to the load cell using a mounting block, on which a pin was attached.

The use of a cable was the most easy way to connect the top lever of the prototype. This way only a force greater than zero can be measured, however this is the only force of interest at this stage.

C.7.1 Test protocol

A simple test protocol was written to test the two magnet configurations. Initially, the load cell was set to zero before mounting the steel cable. To avoid drifting of the loadcell reading the testbench was put on 30 min before the tests would commence, to allow the load cell to warm up. The steps were as follows:

1. Increase force by 5 N while increasing deflection with 10 mm min$^{-1}$, no data recording.
2. Increase deflection by 15 mm with 10 mm min\(^{-1}\), with data recording

3. Return to initial position with 10 mm min\(^{-1}\), no data recording

The initial position of the loadcell was such that there was some slack in the steel cable. The first step of the test protocol is to ensure that this slack was removed before commencing the test. However in the initial position the cable was able to provide a slight resistance to compression, due to the stiffness of the cable. Thus still some slack was possible before the data recording commenced. However this was not a problem, since the test was based on deflection. The deflection was chosen such that the whole force-deflection curve was recorded.

### C.7.2 Correction data

To be able to correct for prototype stiffness a separate test was done in which the prototype was locked using a clamp. This way the deflection of the cable and other parts of the prototype could be determined given a force. These deflections will be subtracted from the deflection measured during testing of the magnet configurations, since we are only interested in the force-displacement curve of the magnet configuration itself.

Another test was done with no magnets present to measure the friction in the prototype. Some friction was measured, mainly caused by misalignment of the bearings due to fabrication imperfections. However the friction measured was less 0.25\%, so it was neglected.

### C.7.3 Data analysis

Data analysis was done using MATLAB. The test-bench was able to export the raw data into a text format which could easily be imported and used in MATLAB. A Matlab script was written to analyse the data. The script is subdivided into the following steps:

1. Import RAW force-displacement data.
2. Correct for cable and prototype stiffness and shift data.
3. Transform linear force data to rotational torque data.
4. Import COMSOL data and compare to test data.

These steps will be further elaborated in the current section.

#### Import Raw force-displacement data

To have a first look at the raw data the data was simply imported and plotted, shown in Fig. C.10. Only the 'lower' and 'high configuration are shown.

What can be seen are the force-displacement data for the two different test samples, the locked prototype and the empty prototype. The first three plots consist of five curves since these were repeated five times. The last plot consists of one curve, since this test was only done once. For the repeated tests the curves have a slight offset to each other. This is caused by the initial tension force before the data recording commences. This causes the initial displacement of each curve to be different.
Figure C.10: RAW test data of the unbalanced prototype. Shown are the force-displacement curves from the test bench for the 'low' and 'high' configurations at the top. At the bottom left the force-displacement curve is shown for the locked prototype. At the bottom right the friction measured is shown.

For both the magnet configurations a roughly straight line can be seen, as expected. For the first two plots an exponential curve can be seen in the first part of the plot, corresponding to deformation in both the cable and prototype. This deformation seems to be exactly the same curve as the locked prototype plot, as expected. The first two plots conclude with an rounded edge followed by a horizontal line. The horizontal line corresponds to the upper lever jumping upwards, which could not be recorded in the current setup. The rounded edge can be explained using the locked prototype curve. At low forces a relatively large slack can be seen for a small variation of force. Thus during the test at the end of the curve a sudden increase in slack occurs, causing the prototype to virtually stand still while a decrease in force occurs. However this is not properly registered because only the force and displacement of the loadcell itself was measured. Correcting the data of the first two plots with the third will give a much clearer inside in what happens. In the fourth plot the force-displacement curve can be seen of the empty prototype, which represents the friction in the lever and gravity. The friction is noticeable when moving the prototype by hand, which is confirmed by the test data, reaching almost 1 N. However the influence of gravity and friction is insignificant compared to the large forces present at the first
two plots. So it was chosen to neglect these effects in the rest of the analysis.

**Correct for cable and prototype stiffness and shift data**

From the raw data it could be seen that a significant slack effect was present in the cable and prototype. The largest slack is expected to be present in the rings of the steel cable, which will deform when a force is applied. This effect is illustrated in Fig. C.11. This effect have a distorting effect on the displacement measured. However it has no effect on the force measured, since this is independent on the deflection.

![Illustration of the cable slack effect.](image)

**Figure C.11:** Illustration of the cable slack effect. The loop of the cable deforms given normal force in the cable.

To correct the data the five stiffness measurements were given an mean offset. Since the stiffness data contained some noise an polynomial fit was taken through the mean of the five measurements. Finally for all the magnet measurements at each datapoint the displacement was corrected for a given force, based on the polynomial fit.

**Transform linear force data to rotational torque data**

Based on the geometry of the prototype and test setup the force-displacement data measured is transformed to torque-rotation data of the lever. By a simple experiment it was verified that the loadcell only measures the vertical component of the force applied, so the data was also corrected for that.

**Import COMSOL data and compare to test data**

The results of the measurements are shown in Figure C.12 as well as the simulated data. The conclusions for the 'high' and 'hybrid' configurations are the same as the conclusions for the 'low' configuration discussed in the paper.

As expected from the simulations a dip is shown in the force-displacement curves for the 'hybrid' configurations. Where 'high-in' resembles a hybrid configuration where the high side is placed at the inside of the lever, and 'high-out' resembled a hybrid configuration where the high side is placed at the outside of the lever.
Figure C.12: Comparison between the test data and the data from the 3D simulations.
Appendix D

Guiding Mechanism

To force the positive and negative stiffness elements along a trajectory a guiding mechanism has be designed. Three main options were considered, which were a) Linear guiding using sliders on guiding rods, b) the sarrus mechanism, and c) a lever mechanism, seen in Figure D.1. Due to several considerations the lever mechanism was used for the final prototype, as explained in the paper. The considerations to use this solutions can be found in this chapter.

D.1 Guiding rods

The linear guiding concept consists of a stationary base with guiding rods on it. A slider would move up and down along the rods using linear bearings. Based on the results of the first magnet balancer prototype (section E.1.2) it was found that this solution would perform poorly due to high friction if a moment would be introduced on the moving platform. This is likely if the attracting magnets are placed off-centre. One strategy to minimise this problem is to place the magnets in the centre of the construction. However this results in an off-centre placement of the positive stiffness elements. This will result in a unwanted moment on the slider, if the positive element does not consist of two identical pairs at each side of the magnet. This would increase the size of the positive stiffness elements thus the whole construction.

An alternative is to place the negative and positive stiffness elements on top of each other, however still the same problem is likely to arise if the positive stiffness is anything other than a

![Figure D.1: Different types of guidance mechanisms considered.](image)

(a) Slider  (b) Sarrus  (c) Lever
single element acting on a point in the middle of the platform with a force parallel to the slider. Furthermore this would increase the height of the solution to much, given a limited design space.

D.2 Sarrus mechanism

Due to the poor performance of the slider mechanism of the magnet balancer build previously, a better solution was sought to guide the magnets through a path in space. During a conversation with prof.dr.ir. J.L. Herder it was found that one possible solution would be a sarrus mechanism. The sarrus mechanism is a linkage is a straight-line mechanism, which converts the circular motion of a set of joints to a linear motion of two parallel sheets.

A prototype of this mechanism has been made, using 44 mm × 47 mm × 1 mm aluminium plates. Regular transparent packing tape is used as hinges between the parts, applying it on both the inside and outside. The prototype and its workings can be seen in Figure D.2.

The prototype has virtually no stiffness for movement of the parallel plates, while the stiffness in other directions is very high. The parallel plates are able to rotate very slightly with respect to each other, mainly caused by play in the tape hinges. Often the mechanism is constructed with square plates, however, the rectangular plates were already available so these were used. Still the mechanism works. Furthermore now there is only one singular position of the hinges instead of two, since in case one of the hinges is fully extended the other is not.

The performance of the mechanism is very good as a demonstrator. However the hinges are fairly large. One possibility to counter this is to make them smaller, according to the needed range of motion. Another possibility is to let the hinges fall inwards, making them relative smaller to the top and bottom plate. The joints are now made with simple tape, which introduces some play, which is expected to let the mechanism work so well. When a more rigid mechanism is manufactured with higher precision hinges, the alignment of them will be more important, which will introduce a higher chance of poor performance. A possibility to counter this is to use only one hinge instead of the sarrus mechanism. This will decrease the number of parts greatly. However the path will not be straight anymore, but this may not be a problem.

The lever arm of the hinge has to be chosen such that the path of the magnets is usable while still being compact.

D.3 Lever mechanism

As a simple and reliable alternative to the slider and sarrus mechanisms it was chosen to use a lever mechanism. When choosing the lever length sufficiently large the trajectory at the end
of the lever would still be relatively linear, since the maximum rotational angle would be small
due to the short working distance of the magnets. As a hinge a small rod on ball bearings can
be used, which can handle a high amount of force with little play. Finally, by altering the lever
arm of the positive stiffness, the force generation of the mechanism can be increased.
Appendix E

Choosing a positive stiffness

To balance the negative stiffness provided by the magnets, a positive stiffness element had to be chosen. Solutions considered were:

- A repulsive magnet pair.
- Tension springs
- Compression springs
- A cam mechanism

The current chapter shows the design considerations leading to the chosen positive stiffness system.

E.1 Repulsive magnet pairs

The orientation of the magnetisation of magnets determines if magnets attract or repel each other. If the magnetisation is in the same direction the magnets attract, if the directions are reversed the magnets repel each other (Figure E.1). Furthermore, it was found using FEM simulations that the attractive and repelling force of the pairs are identical but reversed, which makes coupling between the two the theoretical best solution to achieve a balanced system.

(a) An attracting magnet pair. (b) A repulsing magnet pair.

Figure E.1: Magnet pairs attracting and repulsing each other
APPENDIX E. CHOOSING A POSITIVE STIFFNESS

Figure E.2: *Attraction force and repulsing force between two ring magnets (26.75/16 mm × 5 mm)*

E.1.1 Testing the repulsive force between magnets

Very similar to the tests assessing the attracting force between magnets (Appendix C.1), a test was done testing the repulsive force between magnets (Figure E.2). What can be seen is that the curves lie close to each other, where the biggest deviation is seen between 2 mm and 4 mm. The differences might be explained by misalignment between the magnets or fabrication errors in the prototypes. Another difference might be some fundamental effect that is not shown in the FEM simulations, which causes the forces not to have the same magnitude at all times.

E.1.2 Magnet balancer prototype

To get a feeling of potential problems when building a static balancer using solely magnets, a quick and easily manufacturable prototype has been designed and made (Figure E.3).

This prototype would include similar sets of attracting and repulsing magnet pairs. The force-distance curves of the attracting and repulsing magnets are expected to be the same, so combining these pairs should result in a prototype which is (roughly) statically balanced. To achieve this these pairs must have the same distance to each other at all times to ensure proper balancing. The basic idea was to have 2 sets of attracting and 2 sets of repulsing magnets ring magnets. Ring magnets were chosen since these would be able to slide along round rods with slightly smaller diameter compared to the inner diameter of the ring magnets. These 4 pairs would be placed on square base, with one pair in each corner. The bottom magnets of all pairs would be attached to the base. A top plate with similar dimensions to the base plate would function as a coupling to all magnets. This top plate would have a hole at each corner to be able to slide with the magnets along the sliders.

Construction

The construction is done by cutting square aluminium plates, drilling holes in each corner for both top and bottom plates. The sliders were manufactured in a lathe. The sliders were bolted onto the bottom plate. The magnets were glued to their corresponding top and bottom plate.
E.1. REPULSIVE MAGNET PAIRS

(Using the same glue as used in the bench test.) This is done when the magnets were positioned on the sliders, to assure proper alignment. To prevent the top magnets and top plate to be glued to the sliders, some grease was applied to the sliders before applying glue. Once the glue was dry the top plate with the magnets was able to be removed. Some excess glue was removed from the top magnets for the top assembly to slide along the sliders.

Performance

The construction performed reasonably well on the force required to move the top assembly. The structure however was not perfectly balanced. The major problem of the prototype however was "schranken". The attracting magnets pulled on the plate, inducing a rotation to an arbitrary direction. This causes one pair of attracting magnets to be slightly closer to each other than the other. This causes the attracting pair to deliver an even higher force, pulling the plate skewed and blocking it.

Insights

The most obvious insight from this prototype is to not use sliders if not necessary, so another solution has to be sought or the current has to be improved considerably. If sliders are used proper measures have to be taken to minimise the chance of "schranken". Another lesson is that while in theory the magnets may counter-balance each other, due to imperfections in the construction there always will be an imbalance. To minimise this problem the balancing system should be able to be adjustable to correct for this.

E.1.3 Evaluation of the repelling magnet concept

Coupling between a attracting pair and repelling pair is theoretical the best solution, since the set would be perfectly balanced. However it was found that the introduction of iron diminishes this effect, so the magnet configuration synthesised will not be suitable. The alternative is to use the unsimplified solution without iron, increase the size and weight of the solution.
If suitable magnet configurations are found which are reversible, there are two factors which could lead to imbalance between the magnet pairs. The first factor is the alignment of the magnets within one magnet pair. When one set is aligned and the other not, the forces will not compensate each other perfectly. The second factor is the manufacturing errors within the magnets. When the magnets are not manufactured perfectly the same, forces will not be perfectly reversed.

To prevent imbalance in the final prototype, it was reasoned that an adjustment mechanism should be installed. Adjustment of the magnet sets is not intuitive and was deemed impossible. The alternative is the add an adjustable spring system, which increases the complexity of the total design. Given this complexity, the limitations of the guidance systems which can be used, and the need of alternative magnet configurations, led to the conclusion that the use of magnets for the positive and magnet stiffness system was not realistic for the given system.

### E.2 Tunable cantilever beam

The tunable cantilever beam consists of a cantilever mounted rigidly to a ground, as seen in Fig. E.4a. When applying a force to the end-point the cantilever will deflect, generally modelled as a linear relationship for relative small angles. This linear relationship is defined as:

\[
\delta = F \cdot k, \quad k = \frac{3EI}{L^3}
\]

Where \( F \) is the applied force and \( k \) is the stiffness of the beam, which depends on beam length \( L \), modulus of elasticity \( E \) and moment of inertia \( I \) (around the axis perpendicular to the plane). If a nodge is placed, the length of the beam is virtually altered, leading to a change in stiffness.

What can be seen is that there is a high dependency on the length of the lever. If the lever length is decreased by 20\%, the stiffness is roughly doubled. So if the stiffness is to be doubled a notch has to be placed at roughly 20\% (in reality this distance will be slightly different, due to the difference in fixation between the nodge and the full constrained model). The displacement at this distance however, is very small. This results in an high dependency of the force-deflection curve on the notch placement. Furthermore, the contact between the cantilever and the nodge will be a line contact, probably resulting in high stresses.

A quick COMSOL simulation is done involving a cantilever touching a nodge, to investigate the viability of this design. Found was that that indeed there was a high accuracy required for the notch placement to achieve a certain stiffness, in the order of 0.01 mm. The desired curve has a relative small change in stiffness, which can be reasonably followed using three notches, however these have to be placed within millimetres of each other. Finally the stresses on the nodge and cantilever are found to be high, resulting in a construction which is either too large or too weak. Given these points it was reasoned that this design would be too challenging.

### E.3 Tension spring concept

The tension spring concept relies on tension springs from which one attachment point is attached to the top part of the lever, while another attachment point is chosen at a specific point above the lever (Figure E.5). This way, the linear force-displacement characteristic of the tension spring will result in an non-linear moment-rotation curve.
Five design parameters were defined: length $b$ between the rotation point of the lever and attachment point 1 of the spring, the $x$ and $y$ coordinate of spring attachment point 2, and the stiffness $k$ and zero-length $l_0$ of the spring. Expected is that when choosing these parameters correctly, the torque-rotation curve caused by the spring could match the torque-rotation curve of the magnets closely.

To achieve the most optimal design an optimisation has been written using MATLAB® 2015. The input of the optimisation was a catalogue of springs from a spring manufacturer [20], which consisted of 967 springs. Each spring could be used once or multiple times in parallel, for a maximum of $n$ times, resulting in a total of $n \cdot 967$ spring configurations. For each spring configuration the optimisation searches the most optimal values for the given design parameters. Given the limited design space, $n = 5$ was chosen.

The most optimum value is determined by the maximum absolute balancing error $E_b$ in the torque-rotation curve:

$$E_b = \max(||T_p - T_s||) \quad (E.2)$$

In which $T_p$ is the torque needed around the rotation point for perfect balance of the prototype, and $T_s$ is the theoretical torque curve provided by the springs.

The results of the optimisation can be seen in Table E.1. For an increase in the number of springs there is an increase in accuracy. This is primarily caused by the maximum allowable force generated by a single tension spring. Each time the maximum force was exceeded, the spring was omitted. This leads to a limited set of possible springs for a lower number of springs. In some cases, no decrease in balancing error was found for a larger number of springs, leading to an empty field in the table.

The most optimal solution found was using three springs in combination with the ‘hybrid-out’ configuration, leading to a balancing error of $0.3 \text{ N m} \ (0.78 \%)$. A torque-rotation curve of this solution can be found in Fig. E.6, showing the measured data of the magnet, the theoretical balancing force, and the theoretical absolute balancing error.
E.4 Compression spring concept

See section VII of the paper. Additional simulations have been performed for the other magnet configurations deducted. The results can be seen in Table E.2.

E.5 Cam-mechanism

Several possible cam mechanisms exist incorporating spring components. Examples are the O-rubber ring spring by Hirose (as explained in the state of the art section), the wrap cam by Ostler and Zwick [17], the guiding cams by Wisse, Barents, Dorsser and Herder [18] and the rotational cams by van der Hoeven [7] and Zhou [19].

These types of mechanisms have the advantage that they are able to follow a precise pre-determined curve. The disadvantage of these mechanisms is that the cam shape is based is pre-calculated and is not adjustable after manufacturing. So if any measurement and/or calculation errors exist these will have a direct impact on the performance. Furthermore if any change in spring behaviour occurs due to creep or other effects, the spring has to be replaced instead of a re-adjustment of the mechanism.

E.6 Chosen positive stiffness concept

For the purpose of balancing, both the extension spring concept and compression spring concept were deemed realistic. For the given application, force generation, the compression-spring concept is deemed most suitable due to several reasons:

- The springs in which the force are generated act directly on the base, making it suitable to be used directly. In the case of generating force on a friction block, the friction block could be placed directly under the springs.
Table E.1: Maximum balancing error for the extension spring concept for a given amount of springs for each magnet configuration. In the 'Hybrid-in' configuration the high side of the 'Hybrid' configuration is pointed inward, while at the 'Hybrid-out' configuration it is pointed outward. A '-' sign indicates no improvement compared to one spring less.

- Decreasing the distance between the springs and the rotation point of the lever increases the achieved maximum force in the springs. This also holds for the extension spring concept. However, the maximum allowable force in compression springs is relatively high compared to extension springs. This leads to a relative low spring mass in the compression spring concept.

- Lower complexity in construction. In the extension spring concept, the design of the adjustable mounting points of the top spring would be challenging.

- More intuitive balancing system adjustment. While the extension-spring concept achieves a theoretical lower balancing error, a manual adjustment to correct imbalance will be hard without recalculation. In the compression spring concept a balancing error can directly be linked to either a change in stiffness or initiation point. A change of stiffness can easily be achieved by changing the distance to the point of rotation of the lever, while a change of initiation point can be achieved by placing a spacer between the lever and the springs.

- The design has a higher inherent safety. Since the springs are enclosed they are not likely to jump away during failure.

### E.7 Design of the positive stiffness

The design and construction of the positive stiffness system has been described in the paper in section VII and VIII. Some extra photo’s can be seen in Appendix F.
### APPENDIX E. CHOOSING A POSITIVE STIFFNESS

<table>
<thead>
<tr>
<th></th>
<th>Number of springs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td><strong>Normal</strong></td>
<td>Balancing error (Nm)</td>
<td>3.80</td>
<td>1.82</td>
<td>0.61</td>
<td>0.46</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>Balancing error (%)</td>
<td>23.53</td>
<td>11.27</td>
<td>3.78</td>
<td>2.85</td>
<td>2.41</td>
</tr>
<tr>
<td><strong>Low</strong></td>
<td>Balancing error (Nm)</td>
<td>1.64</td>
<td>0.90</td>
<td>0.52</td>
<td>0.22</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Balancing error (%)</td>
<td>4.10</td>
<td>2.27</td>
<td>1.30</td>
<td>0.54</td>
<td>-</td>
</tr>
<tr>
<td><strong>High</strong></td>
<td>Balancing error (Nm)</td>
<td>1.33</td>
<td>1.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Balancing error (%)</td>
<td>3.72</td>
<td>3.47</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
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<td><strong>Hybrid-in</strong></td>
<td>Balancing error (Nm)</td>
<td>1.32</td>
<td>0.60</td>
<td>0.38</td>
<td>0.21</td>
<td>0.19</td>
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<td>1.60</td>
<td>1.02</td>
<td>0.56</td>
<td>0.50</td>
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<tr>
<td><strong>Hybrid-out</strong></td>
<td>Balancing error (Nm)</td>
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<td>0.50</td>
<td>-</td>
<td>0.29</td>
<td>0.23</td>
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<td>1.30</td>
<td>-</td>
<td>0.76</td>
<td>0.60</td>
</tr>
</tbody>
</table>

**Table E.2:** Maximum balancing error for the compression spring concept for a given amount of springs for each magnet configuration. In the 'Hybrid-in' configuration the high side of the 'Hybrid' configuration is pointed inward, while at the 'Hybrid-out' configuration it is pointed outward. A '-' sign indicates no improvement compared to one spring less.
Appendix F

Prototype construction

F.1 Design and fabrication

The design of the prototype is largely described in sections VI and VIII of the paper.

The primary material for the prototype was aluminium. This was used for the lower lever, top lever, bearing blocks, axis and upper magnet block. Structural steel was used for the U-shape of the magnet configuration, consisting of two L-shapes. The bearings used were 'HK1010' needle bearings by ZEN, since these were already available and would be sufficient for the prototype.

All the parts were designed in such a way that the parts could be fabricated manually in either a mill or lathe. The parts were constructed by the author. The prototype is assembled by using small screws. The screws used to mount the magnetic parts were made of stainless steel to prevent influence on the magnetic field and performance, the rest were made of regular steel.

Some pictures of the prototype can be seen in Figure F.1: Before the design of the positive stiffness system was known the lever arms of the prototype were left intentionally too long, seen in F.1a. Spring seatings were manufactured using a lathe, seen in F.1b. Before assembly, the spring stiffness of each spring was evaluated in the test bench, seen in Figure F.1c and F.1d. The tests showed a maximum deviation in stiffness of less than 5%, for which the slot lengths were accounted for. The steel structures to place the magnets in are shown in Figure F.1e. The magnets were glued into place on the steel structures, and mounted in the prototype, seen in Figure F.1f. Figures F.1g and F.1h show that one of the magnets broke off during testing. This magnet was re-glued and tests were reperformed.

F.2 Evaluation

The bearing blocks in the prototype are not perfectly aligned, caused extra friction in the bearings when assembled (about 0.1 N m). Chosen was to make no alterations, since this could lead to play in the mechanism, compromising the performance.

When assembled the bottom steel structures do not touch, causing a gap (Figure F.1f). When the screws are not tightened enough the steel structures are able to shift, causing the the magnets to not be parallel. Both problems are expected to have a influence on the performance on the device.
Figure F.1: Pictures of the prototype fabrication, parts and construction.
Appendix G

Prototype evaluation

G.1 Performance of the prototype

The performance of the prototype is largely discussed in the paper in section IX and X, except for the performance of the balancer with the remaining three magnet configurations, and hysteresis curves.

G.1.1 Performance of the remaining configurations

The performance of the balancer with the remaining configurations is shown in Figure G.1. The 'low' configuration and 'hybrid-out' configuration perform the same in terms of balancing error. The 'high' configuration performs worse, while the 'hybrid-in' configuration performs best in terms of balancing error. The maximum torque reached is 40 N·m, which would result in the highest force generated in the springs. The 'high' configuration scores with 30 N·m lowest while the hybrid configurations reach a maximum torque between in between those: 37.5 N·m. The contact behaviour discussed in the paper is present in all cases. The range before this behaviour is the same for the 'high' and 'hybrid-out' configurations (0° to 4°), while the range is smaller for the 'low' configuration (0° to 3°). The range for the 'hybrid-in' configuration is not clear, since the deviation increases from 2°. This is likely caused by the stiffness of the largest spring, since the adjustment was at its limit.

G.1.2 Hysteresis effects in the prototype

A hysteresis test has been performed; opening and closing the prototype with increasing displacements steps of the loadcell. An unaltered curve of the raw data from the test bench for the 'high' configuration is shown in Figure G.2. For the first two millimetres the cable stretches, while for the rest of the curve the lever opens and closes. The curves for increasing and decreasing displacements do not follow the same path. A difference of up to 12 N can be seen for the curve after 2 mm. For the range from 0 mm to 2 mm also an hysteresis effect is shown, with a difference up to 14 N in force. Similar plots were achieved for all configurations. Since based on these data no clear conclusions can be drawn on the amount of hysteresis present in the balancer, it was chosen to neglect this in the paper.
APPENDIX G. PROTOTYPE EVALUATION

Figure G.1: Resulting torque-rotation plots using the four different magnet configurations.

G.2 Possible improvements of the prototype

For the purpose of being a proof-of-concept the current prototype serves well. Some points for possible improvement have already been discussed in the paper in section X and appendix F. However, some further pointers can be made for future improvements of the device overall.

As an alternative to the lever with ball bearings, a compliant 'living' hinge could be used. This reduces the number of parts needed. Furthermore a compliant hinge could provide a rotational stiffness, (partly) balancing the negative stiffness of the magnets. For the current prototype it was reasoned that such a hinge could also flex along an axis perpendicular to the axis it is intended for. This effect could be considerable due to the negative stiffness of the magnets along this perpendicular axis. The stiffness would also decrease the force in the positive stiffness elements, which is not desirable from a force-generation standpoint. So some
design challenges are present for such a hinge.

The balancer made is in no way optimised in terms of stiffness or weight. The constructive parts could be made thinner to achieve a lower weight. However, this may result in some non-neglectable deformation of the lever parts. This deformation may be accounted for in the design of stiffness elements.

Figure G.2: Hysteresis curve of the 'high' configuration, raw data from the testbench.
Bibliography


