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CONSIDERATIONS OF ENGINE-OUT FLIGHT AND AIRCRAFT SIZE EFFECTS IN TRANSPORT DESIGN

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SUMMARY

Some major design decisions in subsonic transport aircraft design are dominated by consideration of engine failure during flight. Basic performance requirements and analysis for critical sectors of the flight are presented and evaluated to obtain a design case for the minimum thrust required to lift an aircraft from a given airfield. Observations are also made on the Minimum Control Speed and the design of the vertical tailplane to cope with the case of engine failure.

In the development of all-new aircraft, as opposed to derivative or stretched aircraft, there is a very remote chance of success when the economic improvements relative to existing airliners are marginal. Using different scaling rules, it will be shown that the increase in size of the aircraft has a significant and unfavorable effect on its weight breakdown. Improvements in the state-of-the-art and design refinements are therefore compulsory to make a large aircraft competitive with the smaller type it is intended to replace.

PART A

ENGINE FAILURE CONSIDERATIONS IN CIVIL TRANSPORT AIRCRAFT DESIGN

A-1 Introduction

Although the modern aeronautical turbine engine is an extremely reliable piece of machinery, the possibility of engine failure or a forced throttling down cannot be discarded. Engine failure results in a considerable decrease in thrust and hence performance deterioration, causes disturbing rolling and yawing momemts and a drag penalty, dependent on the airplane layout.

The probability of a single (isolated) engine failure is remote, e.g. once in 10^4 flights, but the application of multiple engines per aircraft enhances the probability of a failure. Theory shows that simultaneous failure of two engines on a twin-engine aircraft is extremely unlikely, provided the engines operate completely independently, so that failure of one engine does not trigger the second to fail. This very fact makes the twin-engine aircraft acceptable from the public safety point of view, but the attentive observer will probably point at the recent crash of a DC-9 in New Hope, Ga., preceded by a severe hail storm, knocking out both of its engines. Indeed, the elements and the inborn tendency of human beings to make mistakes continuously make it necessary to increase the inherent safety level of transport aircraft, even though it is already at a remarkably high level as compared to other means of transport.

In order to ensure a safe abortion of the take-off or continuation of the flight after such an event, airworthiness rules have been established that are of paramount importance to the aircraft design characteristics, not as an afterthought but from the very beginning of its inception.

The subject of the first part of this lecture will be to discuss what the implications on the design will be of engine failure during or immediately after take-off. During this critical segment of the flight there must always be a sufficient amount of excess thrust to safely climb away once the airspeed has been exceeded below which the aircraft can be safely brought to a standstill.

Engine failure enroute is considerably less critical but nevertheless, on a twin-engine aircraft the marginal performance available after such an event makes it necessary to land as soon as practical. Operational rules, therefore, do not allow public transportation in twin-engine aircraft on extended overwater flights.

Directional control of the aircraft during the yawing motion following engine failure is of particular concern to the pilot when engine thrust is high and airspeed and rudder effectiveness are low. For aircraft with wing mounted engines this consideration usually sizes the vertical tailplane and the rudder.

A-2

Airworthiness rules - FAR 25 - specify the minimum permissible climb performance, the most demanding of them with one engine inoperative, and the climb gradient available after a rejected landing attempt.

The take-off is divided into a number of nominally distinct segments:

(a) The first segment, defining an engine-out climb potential immediately after liftoff, without taking advantage from the generally favorable ground effect.
(b) The second segment, extending from the point where the undercarriage is retracted up to 400 ft above the airfield.
(c) The third or final take-off segment, extending from 400 ft to at least 1500 ft altitude, during which the aircraft is accelerated, flaps are retracted and power reduced from take-off to operational climb rating.

Minimum permissible climb performance is also required during the phases of flight prior to landing:

(a) The one engine-out approach climb potential, which in effect limits the approach flap deflection. Since the approach and landing speeds are closely related the landing distance is frequently affected by this requirement.

(b) The landing climb potential, which is intended to ensure adequate performance during a wave-off and subsequent climbout for a go-around.

The table shows that all performances are defined as a minimum gradient at a specified speed. In terms of flight mechanics, the climb gradient is simply the difference between the thrust-to-weight and the drag-to-lift ratios. These have to be established for the appropriate values of airplane weight, flap settings, engine ratings and installation losses as well as extra drag associated with the asymmetric flight condition. Although all of these requirements may have an impact, the second segment climb gradient is often found to be of particular significance in the preliminary design.

A-3

The desirable flexibility of an aircraft to variations in ambient conditions is illustrated clearly in so-called W(weight) A(altitude) T(temperature) diagrams. For a particular aircraft these curves define an upper limit to the operational take-off weight, set by the climb gradient requirements, as a function of the airfield elevation and ambient temperature. The present figure depicts a simplified example of a hypothetical aircraft design equipped with flat-rated engines, delivering an almost constant thrust up to 15 degrees Centigrade above the standard. The "Design Point" reflects one of the aircraft's specification items, stipulating that on a hot and high airfield of given size no penalty is allowed in the useful load relative the capacity load. For the present design this item has in fact, sized the engines.

A-4

The engine thrust to be installed to comply with a climb gradient is readily derived in terms of a thrust/weight ratio at the take-off safety speed V_2. Eventually this can be converted into a nominal thrust loading, using the thrust decay with altitude, temperature and speed. The lift/drag ratio being of primary interest here, it is worth considering this aerodynamic quality index into more detail.

A-5

Relative to the idealized, extrapolated en-route drag, the lift/drag ratio with flaps deflected is reduced primarily due to extra profile drag, trim drag and extra lift-induced drag associated with the lift increment on the inboard wing where the part-span flaps are deflected. Relative to the ideal polar L/D is lower by an appreciable amount, say 30%. In this area there is scope of continuous aerodynamic development, both in theoretical aerodynamics and wind-tunnel testing of new flap configurations. Multiple-slotted trailing-edge devices, variable-camber leading-edge slats, flap endplates and full-span flaps may, for example, be investigated during the configuration development phase, when initial predictions have to be substantiated.

The broken line in the figure refers to the take-off safety speed for various flap angles and is of particular significance for the engine sizing.

A-6

In summary, this figure shows the most pertinent factors and elements contributing to the required thrust/weight ratio. The flap deflection is an important variable in the designer's hands, which can be used to advantage to reduce the engine thrust required. But as both the maximum lift and the thrust loading are affected by flap deflection it is obvious that the take-off field length will be the limiting factor.

A-7

The take-off of a civil transport is undoubtedly the most complicated item of performance. In order to ensure the highest level of safety, the designer has to prove that the airfield size is adequate under the most adverse conditions of engine failure, runway precipitation and asymmetric thrust.

A safety margin of 15% to the all-engines take-off distance allows for variation in piloting technique and operational deteriorations in thrust and aerodynamic performance and for weight increases during service.

No such margin is thought to be necessary for the case of engine failure as the failure is assumed to occur at the most critical speed, this being equivalent to an added margin of safety.

A-8

The most critical moment of failure is that at which the distance required to safely continue the take-off or to bring the airplane to rest are just equal. In this case we refer to the field length required as being balanced (Balanced Field Length, BFL). Failure of an engine has less effect on multi-engine aircraft than on a twin-engine type and consequently the all-engines take-off distance, factored by 1.15, is often critical for a 4-engine aircraft. The figure also demonstrates that any improvement in the braking deceleration is effective - as far as the aborted take-off is concerned - up to the point where V_1 equals the rotation speed. Any further increase in breaking effectiveness will unbalance the field, as is sometimes the case with short-field, twin-engine aircraft.

A-9

For a given aircraft, the determination of the field length required as a function of the take-off weight is complicated by the fact that variations in the flap deflection are required to comply with the engine-out climb performance. The associated variations in C_L-max are manifest in the figure as discrete steps, corresponding to different flap angles.

Ideally, of course, the envelope of this curve represents the minimum field length, which could be obtained provided the flap setting were continuously variable. Although mechanically this is not (yet) practical, the project designer may conviently vary the flap angle to his advantage, the discrete flap angles being decided downstream the design or flight testing process.

For a given flap angle, the airplane weight can be increased beyond the value where the second segment climb requirement can be satisfied with $V_2 = 1.2\ V_S$, by increasing the rotation, lift-off and take-off safety speeds, as generally this improves the climb performace. This technique, referred to as "overspeed", is also useful in case the aircraft cannot be rotated over its full angle due to a tail clearance rotation. However, overspeed progressively penalizes the take-off distance and the condition where V_2 is equal to the minimum drag speed sets the limit, usually at an impractically long field length required.

Take-off performance degrades with increased airport elevation and ambient temperature. Design specifications usually state the airport performance with MTOW on a hot (tropical) day, designated in the figure as "Design Point". Incidentally, note that the steps in both curves occur for the same field length. Also, the flap angle at the Design Point is only 5^o.

<u>A-10, A-11, A-12</u>

A simplified derivation is given for take-off and climb performance. These equations are not accurate enough to be used for design purposes, but they do indicate design trends. The combination of both performances yields the minimum engine thrust required to safely lift-off the aircraft from a given airfield, satisfying both take-off and climb requirements simultaneously. A number of interesting observations can be made from this relatively crude approach:

(a) The span loading W/b^2 appears to be the primary design parameter sizing the engines, and not primarily the wing loading W/S, as is often stated.

(b) A high take-off lift coefficient is not necessarily the primary aerodynamic quality factor for the high-lift system to be aimed at, but rather the highest possible lift/drag ratio. Trailing-edge flaps with a fair amount of backwards motion (Fowler motion) are generally favorable for take-off, whereas leading edge slats are useful for obtaining high lift and good pitching moments without **spoiling** the L/D ratio.

It should be said here that real life is complicated by factors such as the wing weight, which obviously increases when the span loading decreases.

After all, the designer is not necessarily interested in a low thrust/weight ratio, but rather in the minimum engine size for a given payload, to be carried over a specified range. Also, the engines are not always sized by field performance requirements. The present considerations are of particular significance for short-haul aircraft. Engine size for long-range aircraft is frequently based on optimum cruise conditions.

A-13

The controllability of the aircraft engine failure must be investigated and tested for all configurations in a specified range of speeds. Four-engine aircraft must also be controllable after failure of two engines in the en-route configuration.

Since engine failure causes a disturbing yawing moment - and on propeller driven aircraft also an appreciable rolling moment - deflection of both the rudder and the ailerons will be required. Therefore, engine failure considerations will frequently determine the vertical tailplane size and rudder capacity of aircraft with wing-mounted engines.

A-14

The yawing oscillation after failure of an outboard engine will be most violent at low speeds and high engine thrust. The sideslip angle shows a peak immediately after the thrust reduction, which may amount to 20 or 25 degrees, and fin stalling is an obvious danger here as it would bring the airplane completely out of control. The overshoot can be reduced by installing a yaw damper, which superimposes appropriate rudder deflections on those made by the pilot.

Other design principles reducing the probability of a fin stall are:
- the use of a low aspect ratio vertical tail,
- a sweptback fin leading edge,

both measures reducing the tailplane's lift-curve slope and increasing the critical angle of attack. Some aircraft feature a dorsal fin, which does not materially contribute to the directional stability at small sideslip angles, but they are very effective in postponing the stall on the root region of the fin by forming a vortex which cleans up the flow.

A-15

Since for a given rudder deflection the side force on the vertical tail is proportional to the dynamic pressure $1/2\ \rho\ V^2$, the rudder angle required to maintain equilibrium with on engine out increases rapidly when the airspeed is reduced. The minimum control speed V_{MC} is the lowest speed at which the airplane can be kept under control. It is obvious that the airplane may not be lifted off the ground at any lower speed and V_{MC} therefore is a link between take-off performance and vertical tailplane design.

A-16

An impression of the design factors governing the size of the tailplane is readily gained from an equilibrium consideration, assuming the sideslip angle to be zero. It is demonstrated that aircraft with high T/W ratios, high lift coefficients and engines located far outboard need large vertical tailplanes. Introduction of an approximate relationship derived in one of the previous figures shows that the vertical tailplane size is related almost directly to the take-off distance. Alternatively, for a given empennage design, the take-off distance is proportional to the weight for those combinations of thrust and weight where V_{MC} determines the lift-off speed.

Obviously, the most effective method of reducing the vertical tailplane size is to bring the engines as far inboard as possible.

A-17

In addition to the yawing moment equilibrium, it is necessary to maintain equilibrium of the lateral forces. The side force on the vertical tailplane can be counteracted by banking and/or slipping the aircraft away from the dead engine. A limitation is imposed on the angle of bank for safety reasons and the figure shows that there is a definite limit to the yawing that can be accomodated by aerodynamic means only. Unconventional features, such as

- engines interconnected by a torsion shaft,
- cyclic propeller pitch, etc.

may be unavoidable on some STOL aircraft.

A-18

A summary of design considerations for the vertical tailplane and rudder of aircraft with wing-mounted engines serves to illustrate the interactive nature of the many design variables facing the designer in the preliminary stages of the design. It is emphasized that other considerations may dominate the vertical tailplane design:

- controllability during crosswind landings
- lateral/directional stability requirements
- rudder pedal control forces.

PART B

THE SQUARE-CUBE LAW AND SIMILARITY RULES FOR CONVENTIONAL AIRCRAFT

B-1 Introduction

Once in about ten to fifteen years discussion within aeronautical industries and organizations concentrates on the feasibility of very large transport aircraft and the balance of advantages and disadvantages of such aircraft.

Do we actually need bigger aircraft?

It is generally conceded that the economic case for the very large aircraft and, more generally, for aircraft with increased capacity relative to a previous generation - depends on having the traffic to justify its production in large enough quantities.

The world wide increase in passenger-miles produced by the scheduled and charter airlines has been at an average rate of almost 15% since the introduction of the high-subsonic jets. Although the vastly increased operation costs since 1973, caused by a four-fold increase in the fuel price, has resulted in increased fare prices and reduced growth, there is no indication that the air transport market is saturated as yet.

Traditionally, the growth in transport demand has been negotiated by the airliners partly by increasing the frequency of the flights, partly by the introduction of bigger aircraft. It has recently become apparent that many airports have become saturated, that the airspace around centers of transportation has limited capacity and that environmental problems due to the large numbers of departures and landings have become almost insurmountable. In spite of the reduced growth of air transport, we will have to face that new aircraft in all categories will be bigger than ever.

Although we will concentrate on the technical design aspects associated with the growth of aircraft sizes, we have to face that larger problems have to be solved in the area of operating such aircraft:

- Handling of passenger flows at the terminal buildings.
- Maintenance and overhaul of these colossusses, requiring new buildings and other capital investments.
- The loading of airports, ramps and, in particular, bridges.
- The problem of safe flying and the air traffic control problem. aggravited by the large atmospheric vorticity caused by the trailing vortices of huge aircraft.

B-2

An appropriate way of illustrating the effects of aircraft growth is to extrapolate large aircraft from a set of smaller aircraft, that are already in operation. The basis for this extrapolation is formed by scaling and similarity rules. Simplifying assumptions can be made, such that dimensions, weights, thrust, etc. for the large aircraft can be approximated by applying rationally derived scale factors to the smaller aircraft.

In spite of the danger of oversimplification, these scaling rules are realistic in that they rely on available factual technical data of existing, certified aircraft.

B-3

Different scaling laws can be devised. The one most frequently used is the "square-cube law", stating that for constant aircraft shape and density the areas vary proportional to the square, and weights vary proportional to the cube of the linear dimensions (Case A).
An alternative approach assumes a matching of dimensional and weight growth such that the performance - in its most general meaning - remains constant (Case B).

B-4

Simple approximate relationships between some major flight performances and design parameters such as wing loading and thrust loading, can be used to derive the scaling factors. The assumption will also be made that certain non-dimensional aerodynamic coefficients and propulsion performance characteristics are held constant. In other words, the scaling is effected for a constant technology.

B-5

Scaling laws A and B defined previously are applied to derive the effects of scaling on flight performance.

In case A we note that both the wing loading and the thrust loading increase as a consequence of the square-cube relationship. An aircraft weight increase, for example, of 100% relative to the baseline aircraft results in a 26% increase in the landing run and 50% increase of the take-off run required. Climb performance, not shown here, also detoriorates in a generally unacceptable fashion.

The column on the r.h.s. shows a constant wing loading and thrust loading and obviously constant performance levels. The fuel weight will have to increase proportional to the all-up weight in order to maintain range performance uncompromised.

B-6

The previous figure indicates that, in order to satisfy scaling law A, it is necessary to reduce the cruise altitude. This is generally undesirable both from an operational and economic standpoint (S.F.C. deteriorates below the tropopause). This figure shows other scale factors that could be applied to cope with the increase in wing loading with size. Of these, only the first obeys the scaling law strictly, but an increase in speed, C_L or a combination of these measures is more likely to result in acceptable characteristics. These measures require improved aerodynamic technology, unless the t/c ratio is decreased, which may result in a structural design and weight problem.

B-7

In conclusion, the scaling for constant performance according to method B is definitely preferred, as method A results in a significant sacrifice in flight performance with increasing size. However it will be shown later that following rule B the designer will have a hard time in preserving an adequate useful load.

B-8

At this point it is interesting to look at the variation in some performance characteristics of present-day high-subsonic jet transports and business aircraft, of widely different sizes. Although these aircraft by no means are designed by a scaling process and serve widely different purposes, the data show that a statistical trend places most of the aircraft in between the two scaling laws. It is also indicative that the smaller sized aircraft follow scaling law A more closely, while aircraft with a take-off weight above 100,000 lb. are closer to the constant performance trend

B-9

Size effects pertaining to the wing structure can be derived readily for about half the wing structure which is designed to withstand the in-flight bending induced by the wing lift. The derivation shows this weight contribution, concentrated mainly in the upper and lower skin-stiffener structures, to be proportional to:

- the load-factored All-Up Weight,
- the structural wing span ($b_s = b/\cos \Lambda$)
- the cantilever ratio, i.e. structural wing span divided by root thickness (b_s/t_r)

and inversely proportional to the stress-to-density ratio. Is is obvious that this structure weight as a fraction of the AUW tends to increase with the size of the wing. A very embarrassing tred becomes obvious: when structures get heavier, they get heavier, provided the stress/density ratio is not allowed to increase with size.

Following scaling law A, however, requires the average stress level to increase proportional to the airplane dimensions.

B-10

Scale factors are now available, showing that the designer either has to face an increasing stress level with size (case A) or an increasing structural weight fraction for the bending material if the wing loading and stress/density ratio are constant (case B), or combine both nuisances.

Note that this table assumes a constant load factor n. This is not necessarily correct for aircraft of widely different sizes as the gust load is more critical than the maneuver load for aircraft with low wing loadings, generally resulting in higher load factors for small aircraft.

B-11

The achievable stress level in the wing is not altogether constant throughout the structure. In the lower (tension) side it is frequently set by the 1-g loading condition (i.e. level flight) in view of fatigue properties limiting the structural lifetime. Accordingly, the ultimate stress in Al-alloys generally does not exceed 50,000-60,000 lb/sq.in.

Buckling theory may be used to show that in the compression-loaded structure the achievable stress level increases with the loading intensity, characterized by the end load per unit width of the compression panel, divided by the rib distance. However, for large transport aircraft the loading intensity in the wing root is so high that the **maximum** compressive stress is approached.

Composite materials show great potential, particularly in the highly loaded regions of the structure.

B-12

Scaling factors for the fuselage structure can also be derived from basic sizing considerations. In large regions of a pressure cabin the skin thickness is determined by the design pressure differential. For a given hoop stress level this causes the skin thickness to increase proportional to the cabin diameter.

Other parts of the fuselage structure are designed to withstand the bending load induced by the payload upon touchdown or by in-flight maneuvering forces induced by the tailplane.

It may be noted here that the loads on the wing and fuselage are in equilibrium and in view of the approximately equal moment arms the maximum bending moment in the wing is of the same magnitude as that in the fuselage. However, the fuselage diameter being much larger than the wing root thickness, the loading intensity in the fuselage shell and the achievable compressive stress are much lower as compared to the wing root.

B-13

Scaling factors for the fuselage indicate that, in order to scale up all fuselage dimensions with the same factor (case A), the stresses have to be increased. This is not desirable for those regions where the pressure differential determines the skin-thickness and consequently, this condition is dominating in large fuselages, shifting the emphasis from scaling rule A to rule B.

As regards the fuselage geometry, it is noted that, following scaling rule B, the fuselage volume is scaled up by λ^3 and the payload by λ^2. This would reduce the volumetric efficiency of the fuselage, and make the fuselage unnecessarily bulky, heavy and draggy. It is also contrary to statistical evidence, which shows that, except for the small general aviation aircraft, the max. payload-to-cabin volume ratio is approximately invariable with size. (about 5 lb/cu. ft., typically).
Different scaling rules should therefore be devised for the fuselage and the wing.

B-14

Scaling considerations similar to those for the fuselage and wing can be derived for other weight elements, such as engines, landing gears and tailplanes.

Other weight contributions, such as furnishings and airconditioning system weights, are directly related to the payload, while some equipment and system weight elements are hardly subjected to the weight growth at all. This figure shows the dramatic reduction of the fuel fraction available when the empty weight elements follow scaling method B. When the size grows by a factor of 2 and the weight by a factor of 4 relative to the reference aircraft, there is no trip fuel available. In other words, by adhering strictly to the principle of maintaining performance, the designer has made himself the trap of producing a concept of zero productivity.

B-15

Fortunately, the actual design situation is not so bad as the previous figure suggests and the technical prospects of aircraft of increased size are much brighter than the economic feasibility.

The table compares weight fractions of two airplane designs (1 and 2) with the same flight performance, differing by a factor 2 in All-Up Weight and payload. Both designs have gone through a detailed sizing process and are optimized under similar ground rules. The comparison with scaling methods A and B reveals that the wing scale factors closely follow method B. The fuselage weight is overpredicted by both scaling rules A and B, although method A is fairly close. The primary reason is that the fuselage of the big airplane (2) can be designed with a better ratio of wetted area to payload. Actual fuel weight is between methods A and B, but again much closer to method A.

B-16

In conclusion, although with regard to performance considerations the balance was in favor of scaling method B, basic structural sizing considerations make it clear that some kind of a compromise is necessary. If no significant technological advancements can be made, deterioration in both flight performance and structural quality has to be faced. This generally being unacceptable, the designer has to find other ways.

B-17

Defeating the problems inherent to the design of large transports is generally a combination of many relatively modest technology advancements in virtually all areas of the design. Some of the improvements shown in the figure are general in nature (e.g. the favorable scale effect on aerodynamic performance), others pertain to present technology advancements considered feasible in the near future.
The net effect of the unfavorable scale effects and the improvements possible should result in at least 10 to 15% gain in economic performance, in order to make a new design competitive with existing aircraft.

ENGINE FAILURE CONSIDERATIONS IN CIVIL TRANSPORT AIRCRAFT DESIGN

- TAKEOFF AND CLIMB PERFORMANCE
- MATCHING AIRCRAFT TO AIRFIELD CONDITIONS

- ENGINE-OUT CONTROL
- VERTICAL TAILPLANE DESIGN

FAR-25 ENGINE-OUT CLIMB REQUIREMENTS* FOR JET TRANSPORTS

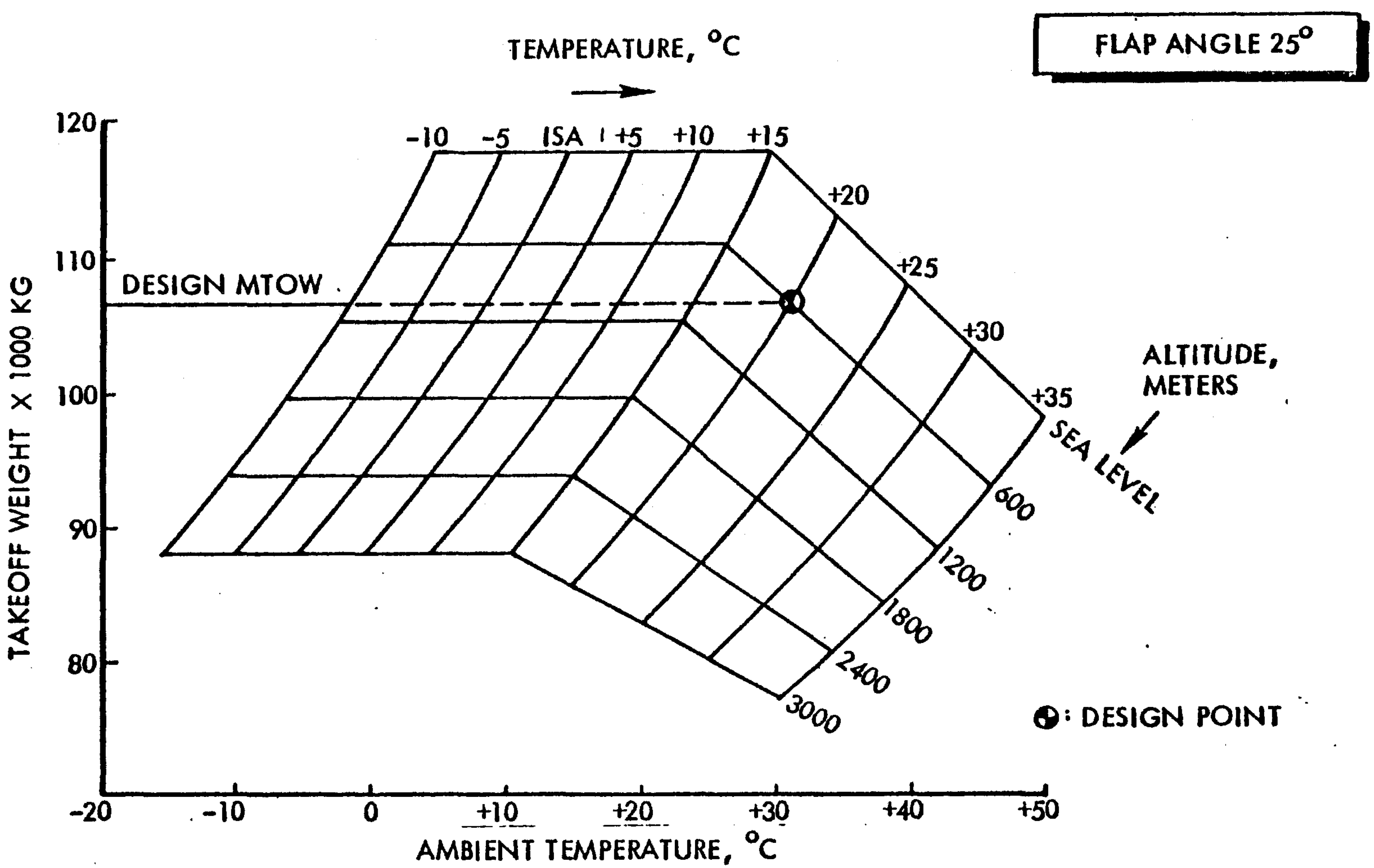
PHASE OF FLIGHT		AIRPLANE CONFIGURATION				FLIGHT SPEED	MINIMUM CLIMB GRADIENT, %		
		FLAPS	U.C.	ENGINES			N=2	N=3	N=4
LIFTOFF	1ST. SEGMENT	TAKE OFF	↓	ONE ENGINE OUT	TAKEOFF	LIFT-OFF	0	.3	.5
TAKEOFF FLIGHT PATH	2ND. SEGMENT	TAKE OFF	↑	ONE ENGINE OUT	TAKEOFF	V_2	2.4	2.7	3.0
TAKEOFF FLIGHT PATH	FINAL TAKEOFF	EN ROUTE	↑	ONE ENGINE OUT	MAX. CONT.	$\geq 1.25V_s$	1.2	1.5	1.7
APPROACH CLIMB		APPR.	↑	ONE ENGINE OUT	TAKEOFF	$\leq 1.5V_s$	2.1	2.4	2.7
LANDING CLIMB		LAND.	↓	ALL ENGINES TAKEOFF		$\leq 1.3V_s$	3.2	3.2	3.2

V_s = STALLING SPEED

V_2 = TAKEOFF SAFETY SPEED ($\geq 1.2\ V_s$)

* SUMMARY ONLY

WAT CURVES FOR A HYPOTHETICAL TRIJET



ENGINE SIZE BASED ON 2ND SEGMENT CLIMB

CLIMB GRADIENT:

$$\gamma_2 = \frac{N-1}{N} \frac{T_{V_2}}{W} - \left(\frac{C_D}{C_L}\right)_{V_2}$$

N = NO. OF ENGINES

V_2 = TAKEOFF SAFETY SPEED (TYPICALLY 1.2 TIMES STALLING SPEED)

W = AIRCRAFT WEIGHT

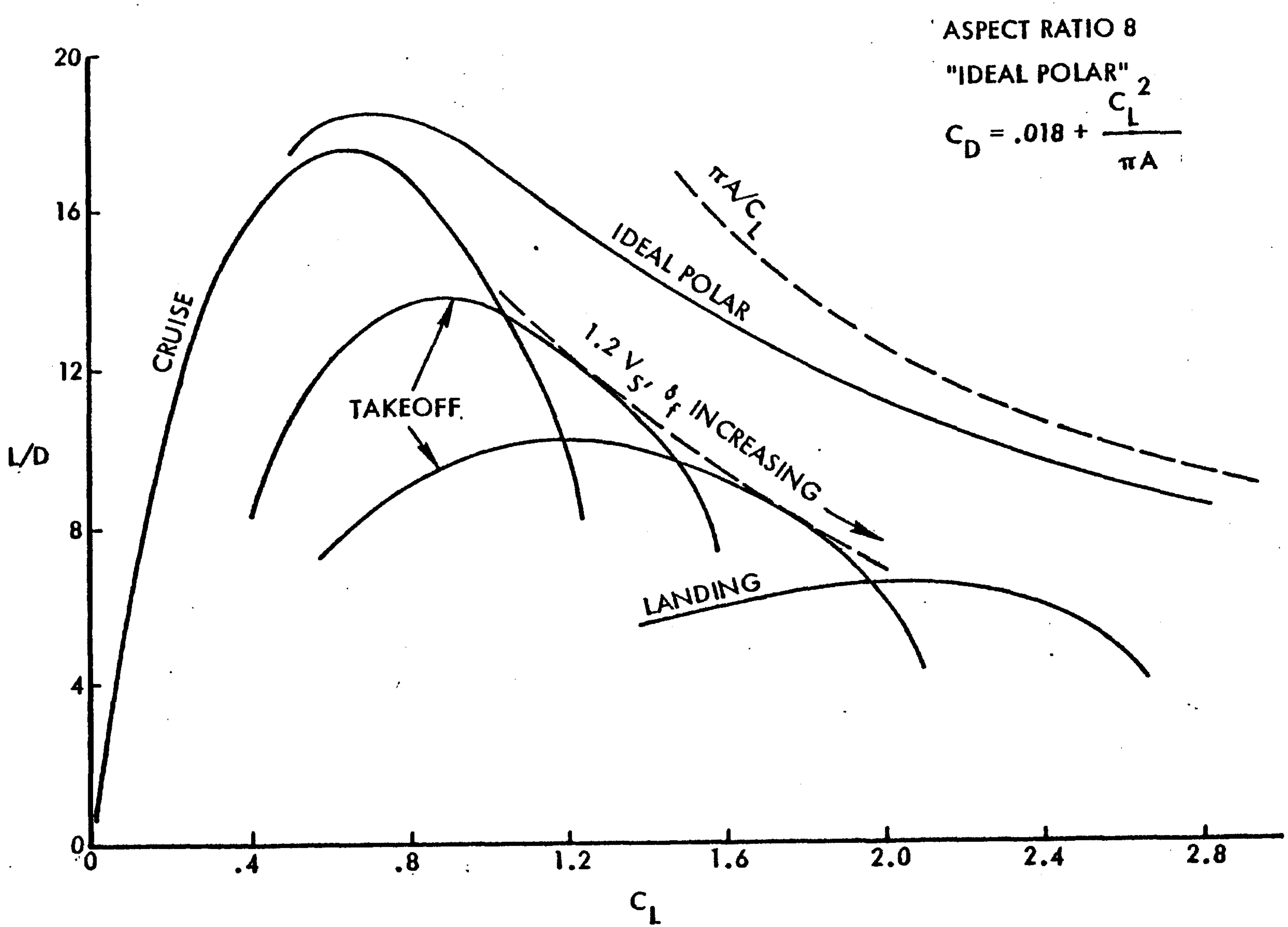
T_{V_2} = ALL-ENGINES TAKEOFF THRUST AT V_2

$\left(\frac{C_D}{C_L}\right)_{V_2}$ = DRAG/LIFT RATIO AT V_2

THRUST REQUIRED:

$$\frac{T_{V_2}}{W} = \frac{N}{N-1} \left\{ \gamma_2 + \left(\frac{C_D}{C_L}\right)_{V_2} \right\}$$

LOW-SPEED DRAG POLARS



PRIMARY PARAMETERS AND DESIGN VARIABLES, TAKEOFF CLIMB

- AMBIENT CONDITIONS
- NUMBER OF ENGINES AND ENGINE CHARACTERISTICS
- FOR GIVEN FLAP ANGLE AND $C_{L_{MAX}}$: ASPECT RATIO, AFFECTING INDUCED DRAG
- DRAG DUE TO FLAP DEFLECTION:
  - DEFLECTION ANGLE
  - AERODYNAMIC QUALITY OF WING/FLAP DESIGN
- DRAG DUE TO THRUST ASYMMETRY

FAR 25 TAKEOFF REQUIREMENTS

CIVIL TAKEOFF FIELD LENGTH IS THE LARGEST OF

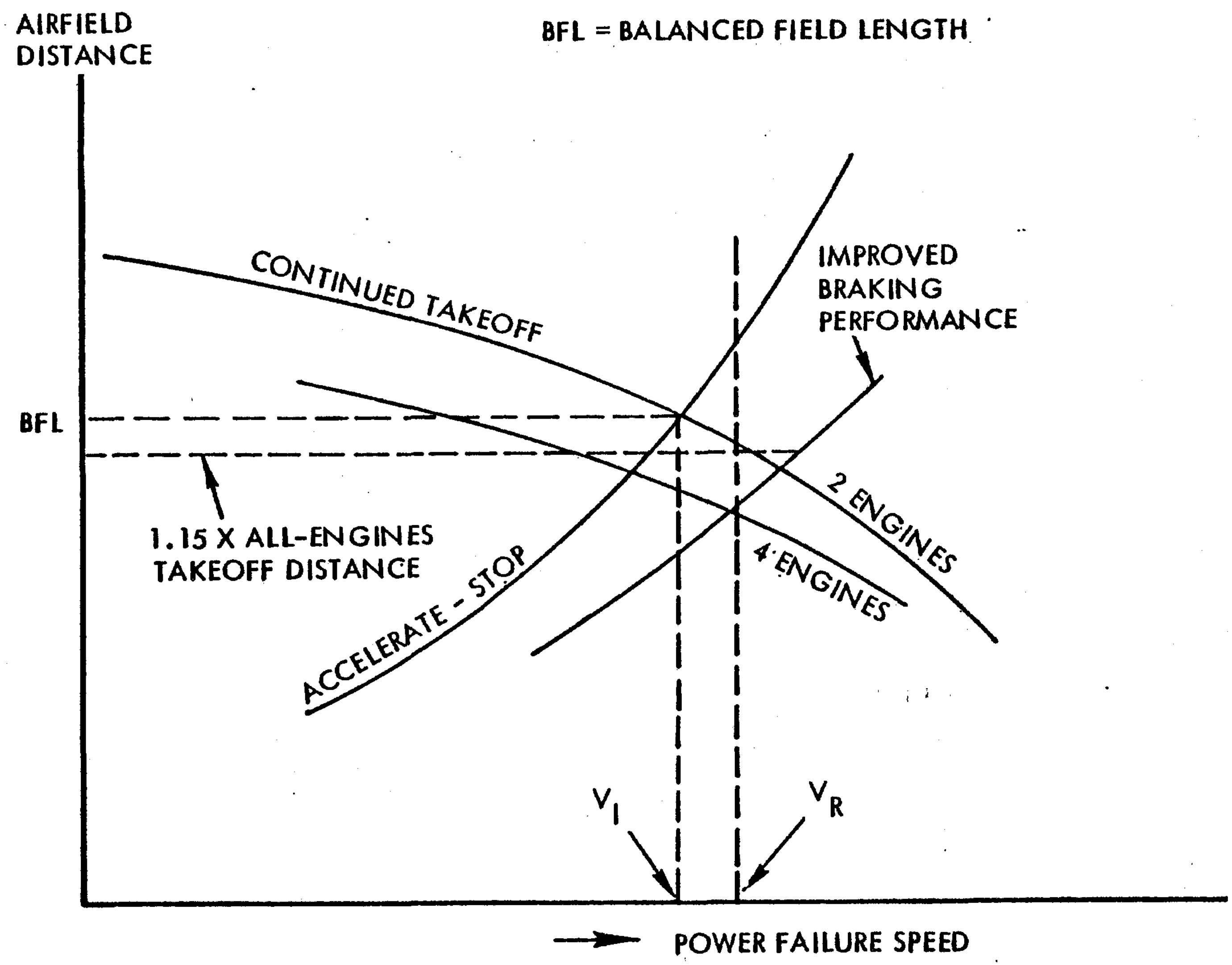
- 1.15 X ALL-ENGINES TAKEOFF DISTANCE TO 35 FT.
- DISTANCE TRAVELLED TO ATTAIN 35 FT. FOR A TAKEOFF CONTINUED AFTER ENGINE FAILURE
- DISTANCE REQUIRED FOR ACCELERATION TO ENGINE FAILURE POINT AND ABORTED TAKEOFF (ACCELERATE-STOP DISTANCE)

NOTE:

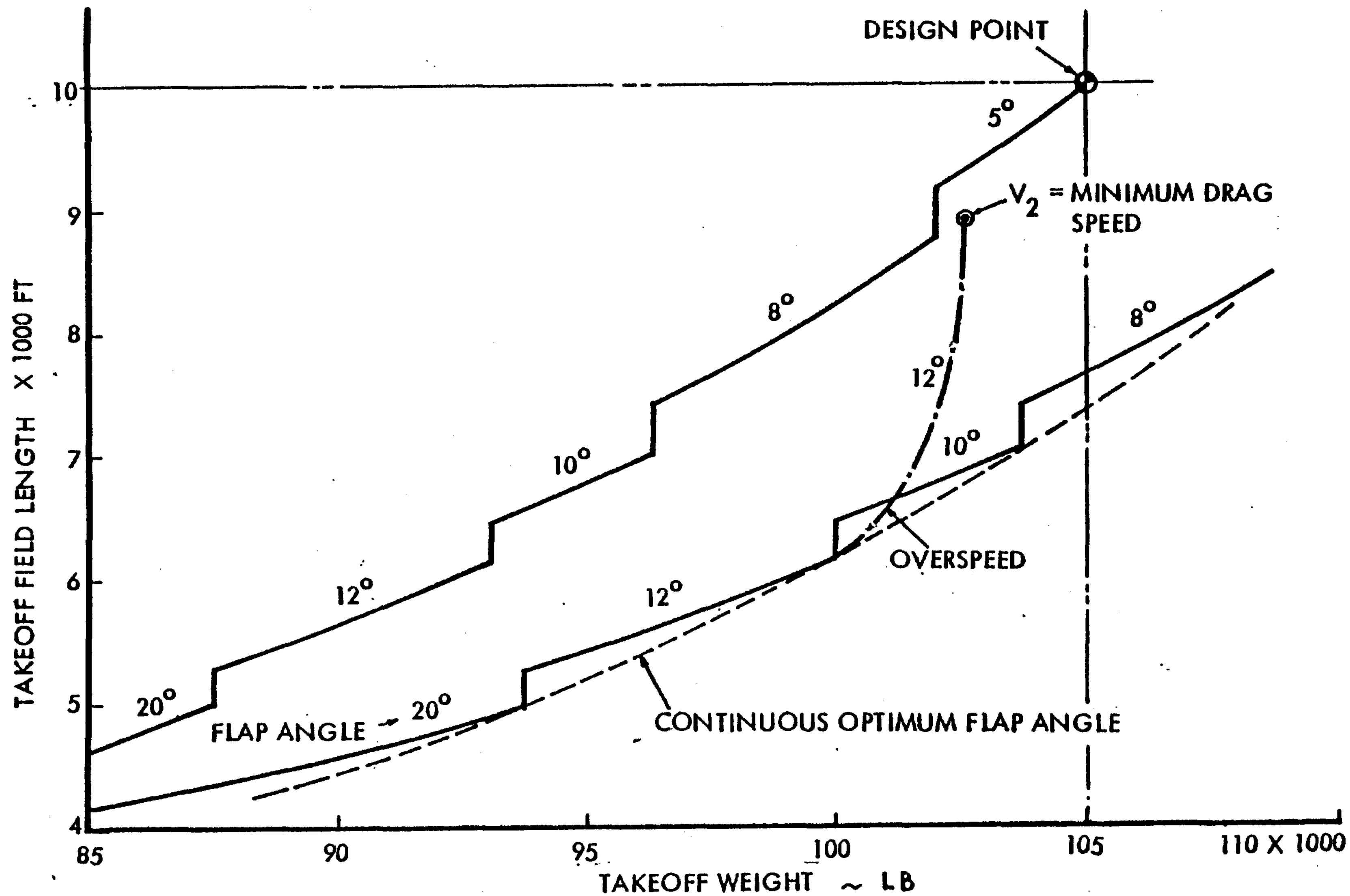
- TAKEOFF SAFETY SPEED AT 35 FT: 1.2 V_S OR MORE
- FLAP SETTING LIMITED BY CLIMB REQUIREMENT

A- 8

BALANCED FIELD LENGTH CONCEPT



TAKEOFF DISTANCE VS. WEIGHT



APPROXIMATE TAKEOFF DISTANCE

SIMPLIFIED APPROACH:

- TAKEOFF DISTANCE = TAKEOFF RUN (ALL ENGINES) + AIR DISTANCE (ONE ENGINE OUT)
- MINIMUM THRUST WHEN 2ND SEGMENT CLIMB GRADIENT IS JUST SATISFIED
- AIR DISTANCE INVERSELY PROPORTIONAL TO CLIMB GRADIENT AND THEREFORE APPROXIMATELY CONSTANT, E.G. 2,000 FT. TYPICALLY

$$S_{TO} = S_{RUN} + S_{AIR}$$

$$S_{RUN} \approx \frac{\text{KINETIC ENERGY AT LIFTOFF}}{\text{ACCELERATING FORCE}} \qquad \text{(ALL ENGINES OPERATING)}$$

$$S_{RUN} \doteq 1/2 \; \frac{W}{g} \; \frac{V_2^{\,2}}{T_{V_2}}$$

MINIMUM ENGINE SIZE FOR GIVEN FIELD LENGTH

- THRUST REQUIRED FOR GIVEN TAKEOFF

$$\frac{T_{V_2}}{W} \approx \frac{V_2^{\,2}}{2g\,(S_{TO} - S_{AIR})} = \frac{W/S}{\rho g\,(C_L)_{V_2}\,(S_{TO} - S_{AIR})}$$

FOR GIVEN $(C_L)_{V_2} = C_{L_{MAX}}/1.44$, THE THRUST/WEIGHT RATIO IS PROPORTIONAL TO THE WING LOADING.

- THRUST REQUIRED FOR 2ND SEGMENT CLIMB (ONE ENGINE OUT):

$$\frac{T_{V_2}}{W} = \frac{N}{N-1}\left\{\gamma_2 + \frac{(C_L)_{V_2}}{\pi A E}\right\}, \text{ WITH } E \equiv \frac{(C_L)_{V_2}}{\pi A\,(C_D/C_L)_{V_2}} = \frac{\text{IDEAL DRAG}}{\text{ACTUAL DRAG}}$$

- ELIMINATION OF $(C_L)_{V_2}$, APPROXIMATED:

$$\frac{T_{V_2}}{W} = \text{FACTOR}\sqrt{\frac{N}{N-1}\,\frac{W/b^2}{\pi E \rho g\,(S_{TO} - S_{AIR})}}$$

CONCLUSIONS FROM FIELD CONSIDERATIONS

CONSTANT FLAP ANGLE:

- FOR GIVEN TAKEOFF DISTANCE T/W IS PROPORTIONAL TO WING LOADING; MINOR ASPECT RATIO INFLUENCE

- FOR SPECIFIED CLIMB GRADIENT T/W IS PRIMARILY AFFECTED BY ASPECT RATIO; MINOR WING LOADING INFLUENCE

MINIMUM ENGINE SIZE:

- SPAN LOADING W/b^2 IS THE PRIMARY PARAMETER AND SHOULD BE LOW

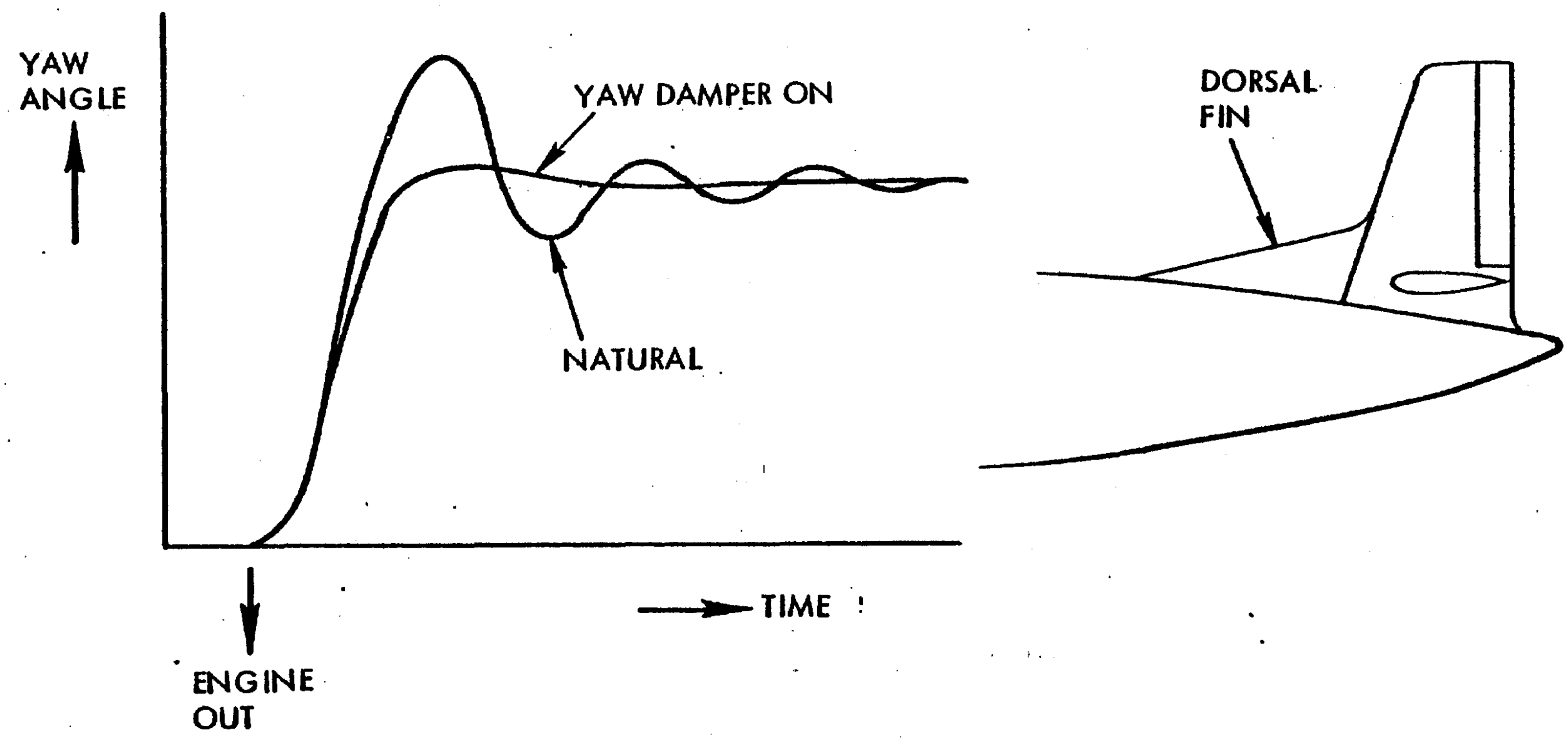
- A LOW DRAG PENALTY DUE TO FLAP DEFLECTION (FACTOR "E" HIGH) IS MORE ESSENTIAL THAN HIGH MAXIMUM LIFT

NOTE: $\frac{W}{b^2} = \frac{W}{S}\,\frac{S}{b^2} = \frac{W/S}{A}$

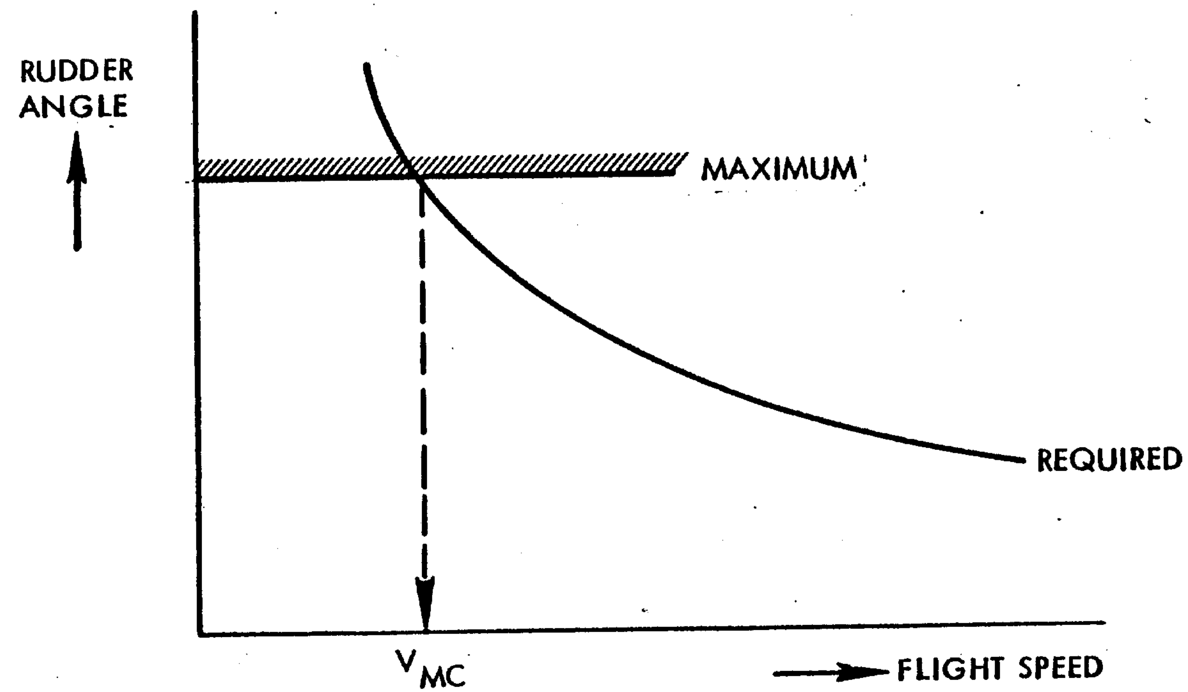
ENGINE-OUT CONTROL

- TRANSIENT MOTION AFTER ENGINE FAILURE
- FIN STALL
- MINIMUM CONTROL SPEED
- FIN SIZE AND RUDDER CAPACITY

YAWING MOTION AND DAMPING



A-15 MINIMUM CONTROL SPEED (V_{MC})



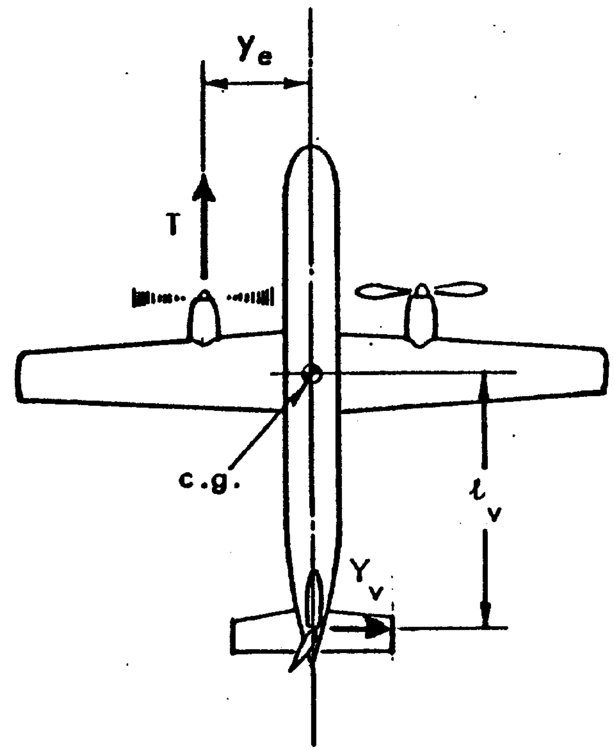
NOTE: $V_2 \geq 1.1\ V_{MC}$

$V_R \geq 1.05\ V_{MC}$

TAKEOFF PERFORMANCE WILL BE PENALIZED WHEN V_{MC} IS TOO HIGH

EQUILIBRIUM CONDITION - ZERO SIDESLIP

THE YAWING MOMENT DUE TO ASYMMETRIC THRUST IS BALANCED BY THE VERTICAL TAILPLANE:



$$Y_v = \frac{T\,y_e}{\ell_v} \qquad \text{- REQUIRED}$$

$$Y_v = \frac{d\,C_Y}{d\,\delta_r}\,\delta_r\,\frac{1}{2}\,\rho\,V^2\,S_v \qquad \text{- AVAILABLE}$$

MAXIMUM RUDDER ANGLE;

$$\delta_{MAX} = \frac{T\,y_e/\ell_v}{\frac{d\,C_{Y_V}}{d\,\delta_r}\;\frac{1}{2}\,\rho\,V_{MC}^{\;2}\,S_v}$$

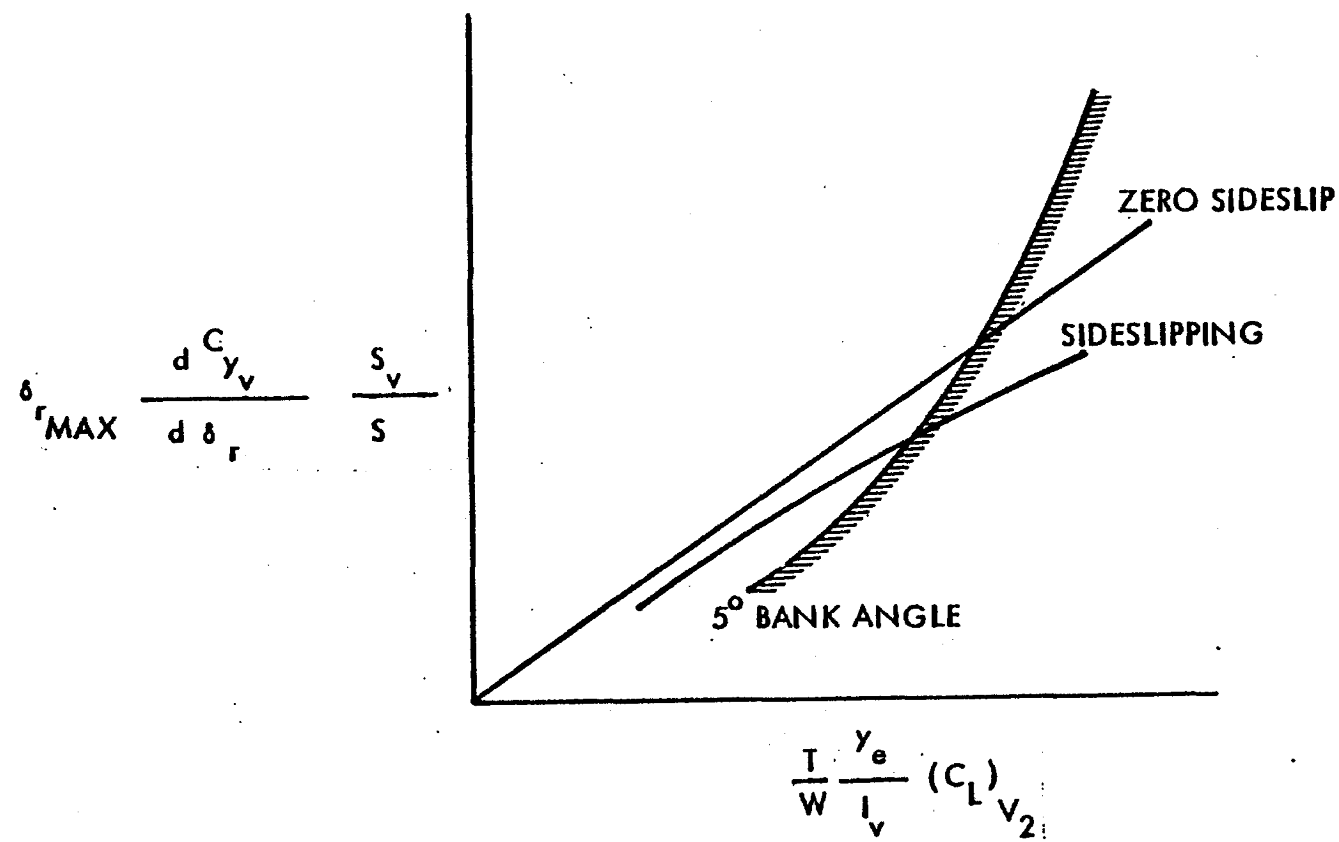
OR:

$$\delta_{r_{MAX}}\,\frac{d\,C_{Y_V}}{d\,\delta_r}\,\frac{S_v}{S} = \frac{T}{W}\,\frac{W}{\frac{1}{2}\rho\,V_2^2\,S}\,\frac{y_e}{\ell_v}\left(\frac{V_2}{V_{MC}}\right)^2$$

$$= \frac{T}{W}\,(C_L)_{V_2}\,\frac{y_e}{\ell_v}\left(\frac{V_2}{V_{MC}}\right)^2$$

ALTERNATIVELY: $\delta_{r_{MAX}}\,\frac{d\,C_{Y_V}}{d\,\delta_r}\,S_v \doteqdot \frac{W/N}{\rho\,g\,S_{RUN}}\,\frac{y_e}{\ell_v}\left(\frac{V_2}{V_{MC}}\right)^2$

A-17 RUDDER CAPACITY FOR ENGINE-OUT CONTROL



VERTICAL TAILPLANE DESIGN CONSIDERATIONS

AVOID FIN STALLING:

- ADEQUATE VERTICAL TAIL AREA AND TAIL ARM
- LOW ASPECT RATIO
- FIN LEADING EDGE SWEEPBACK
- DORSAL FIN
- YAW DAMPER

REDUCE FIN SIZE :

- POWERED CONTROLS: RUDDER DEFLECTION AS LARGE AS PRACTICABLE (UP TO 35 DEGREES)
- USE OF A SEGMENTED RUDDER (VARICAM)
- INCREASE TAIL ARM BY FIN SWEEPBACK
- LOCATE HORIZONTAL TAILPLANE EITHER LOW OR HIGH TO ENHANCE THE END-PLATE EFFECT
- LOCATE THE ENGINES AS FAR INBOARD AS POSSIBLE, UNLESS OTHER CRITERIA BECOME DOMINANT

THE SQUARE-CUBE LAW

AND

OTHER SCALING RULES FOR TRANSPORT AIRCRAFT

OR:

"WHEN AIRPLANES GET HEAVIER, THEY GET HEAVIER"

OUTLINE OF THE SUBJECT

- EFFECTS OF AIRCRAFT SIZE VARIATION ON
    - (AERODYNAMIC) PERFORMANCE
    - STRUCTURAL DESIGN
    - WEIGHT FRACTIONS
- USING ASSUMPTIONS SIMPLIFYING THE ANALYSIS

NOTE: "SIZE" DEFINED BY LINEAR DIMENSION, E.G. WING SPAN, FUSELAGE DIAMETER, ETC.

TWO APPROACHES TO SCALING AND SIMILARITY

A CONSTANT SHAPE AND DENSITY:

DIMENSIONS OF STRUCTURES, PAYLOAD, ETC. ARE SCALED UP PROPORTIONALLY TO A SCALE FACTOR RAISED TO THE APPROPRIATE POWER

B CONSTANT PERFORMANCE:

DIMENSIONS AND WEIGHTS ARE SCALED SUCH THAT CONSTANT FLIGHT PERFORMANCE AND STRESS LEVELS ARE ACHIEVED

ASSUMPTIONS AND DERIVATIONS

- TAKEOFF & APPROACH SPEED $= \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}}$
- TAKEOFF RUN $\doteqdot \frac{W}{S} \times \frac{W}{T}$
- LANDING RUN $\doteqdot \frac{W}{S}$
- CONSTANT ROLLING & BRAKING FRICTION COEFFICIENTS, LIFT & DRAG COEFFICIENTS
- LEVEL FLIGHT SPEED $= \sqrt{2 \frac{T/\rho}{C_D S}}$ (FROM $T = D$)
- CRUISE ALTITUDE : $\rho = \frac{2 W/S}{C_L V^2}$ (FROM $L = W$)
- SPECIFIC RANGE $= \frac{V}{C_T T} = \frac{V L/D}{C_T W}$ (C_T MEANS SFC)

BASIC AIRCRAFT/PERFORMANCE SCALING

	ITEM	SCALE FACTOR (A)	SCALE FACTOR (B)
AIRFRAME	LINEAR DIMENSION	λ	λ
	(WING) AREA $\sim$ S	λ^2	λ^2
	(A.U.) WEIGHT $\sim$ W	λ^3	λ^2
	WING LOADING $\sim$ W/S	$\lambda \doteqdot W^{1/3}$	1
ENGINES	ENGINE DIAMETER	λ	λ
	INTAKE AREA	λ^2	λ^2
	THRUST $\sim$ T	λ^2	λ^2
	THRUST LOADING $\sim$ W/T	$\lambda \doteqdot W^{1/3}$	1
PERFORMANCE	TAKEOFF SPEED	$\lambda^{1/2} \doteqdot W^{1/6}$	1
	TAKEOFF RUN	$\lambda^2 \doteqdot W^{2/3}$	1
	APPROACH SPEED	$\lambda^{1/2} \doteqdot W^{1/6}$	1
	LANDING RUN	$\lambda \doteqdot W^{1/3}$	1
	CRUISE SPEED	1	1
	CRUISE DENSITY	$\lambda \doteqdot W^{1/3}$	1
	SPECIFIC RANGE	$\lambda^{-3} \doteqdot W^{-1}$	$\lambda^{-2} \doteqdot W^{-1}$

B-4 (CONT'D) **ASSUMPTIONS**

- PROPULSION SYSTEM PROPERTIES INDEPENDENT OF THRUST LEVEL:
  - SPECIFIC TAKEOFF THRUST: THRUST PER UNIT AIR MASS FLOW/SECOND
  - SPECIFIC FUEL CONSUMPTION: FUEL FLOW (LB/H) PER LB THRUST
- $\frac{\text{THRUST}}{\text{AIR DENSITY}}$ INDEPENDENT OF ALTITUDE AND SPEED

CRUISE CONDITIONS FOR SCALING METHOD (A)

VERTICAL EQUILIBRIUM: $W = L = C_L \cdot \frac{1}{2} \gamma p M^2 S$

FACTOR	CONSTANT	SCALE FACTOR
p	C_L, M	$W/S \doteqdot \lambda \doteqdot W^{1/3}$
C_L	p, M	$W/S \doteqdot \lambda \doteqdot W^{1/3}$
M	C_L, p	$\sqrt{W/S} \doteqdot \lambda^{1/2} \doteqdot W^{1/6}$

NOTE:
- DECREASING ALTITUDE IS GENERALLY UNFAVORABLE
- INCREASING M IS LIMITED BY COMPRESSIBILITY EFFECTS

B-7 OBSERVATIONS ON THE SQUARE-CUBE SCALING

- FLIGHT PERFORMANCE DETERIORATES WITH INCREASING SIZE

- TAKEOFF RUNS INCREASE MORE RAPIDLY WITH SIZE THAN LANDING RUNS

  COROLLARY: FOR SMALL AIRCRAFT THE LANDING IS MORE CRITICAL THAN THE TAKEOFF

- SMALL AIRCRAFT HAVE GOOD FIELD PERFORMANCE BY VIRTUE OF THE SIZE EFFECT

- SCALING REQUIRES CHANGES IN ALTITUDE, SPEED AND/OR C_L

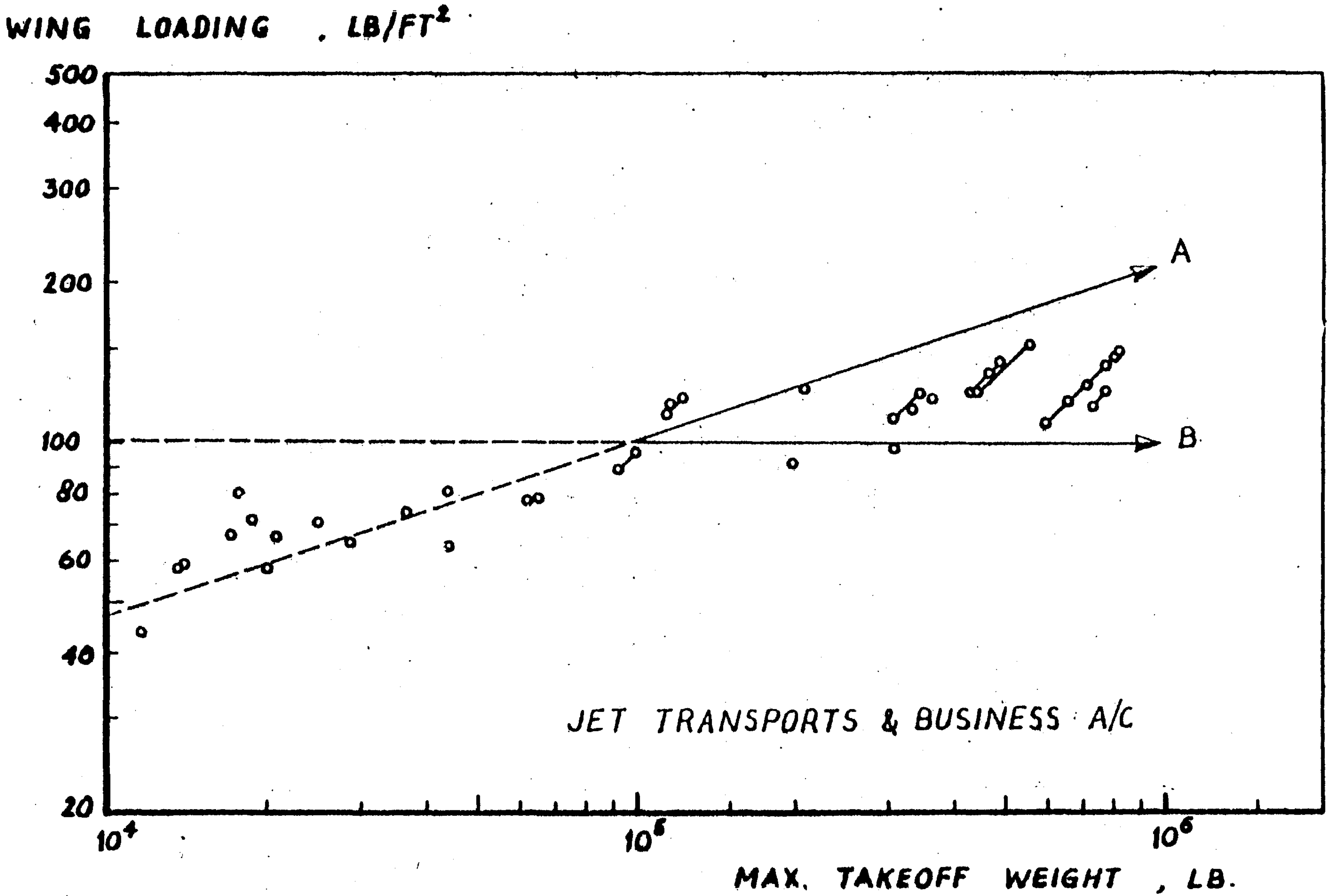
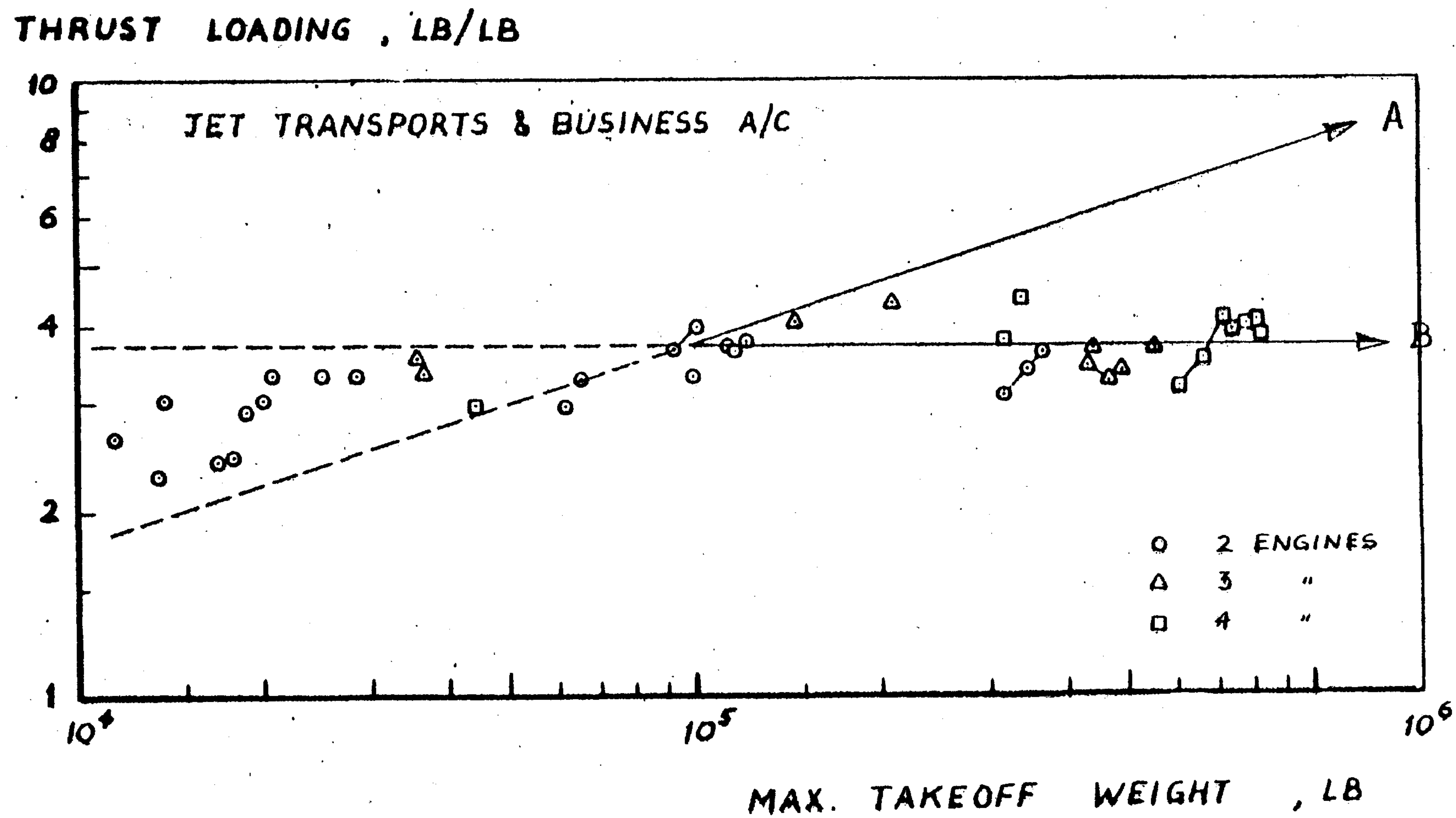
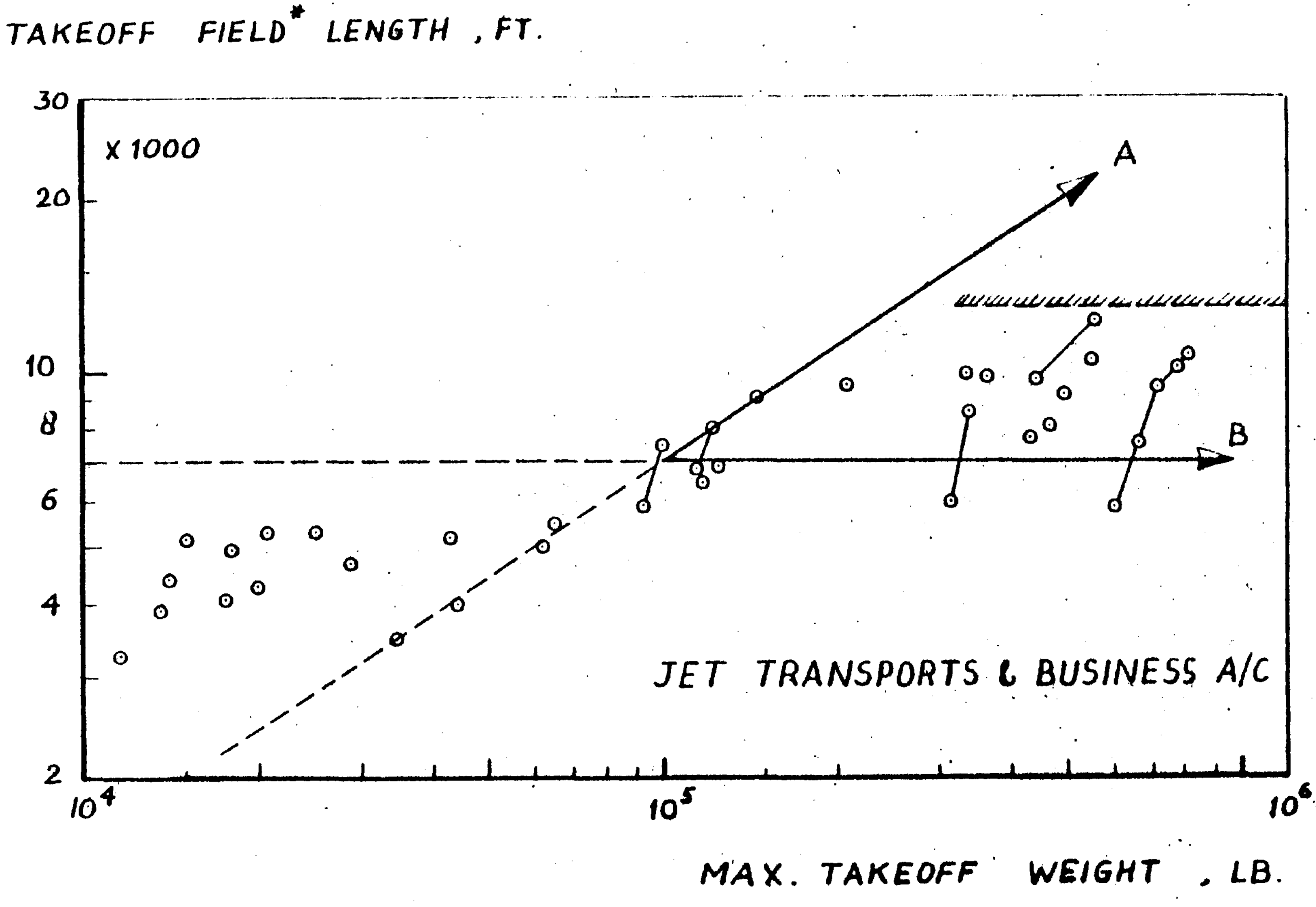


FIG. B-8 : VARIATION OF AIRCRAFT CHARACTERISTICS WITH SIZE

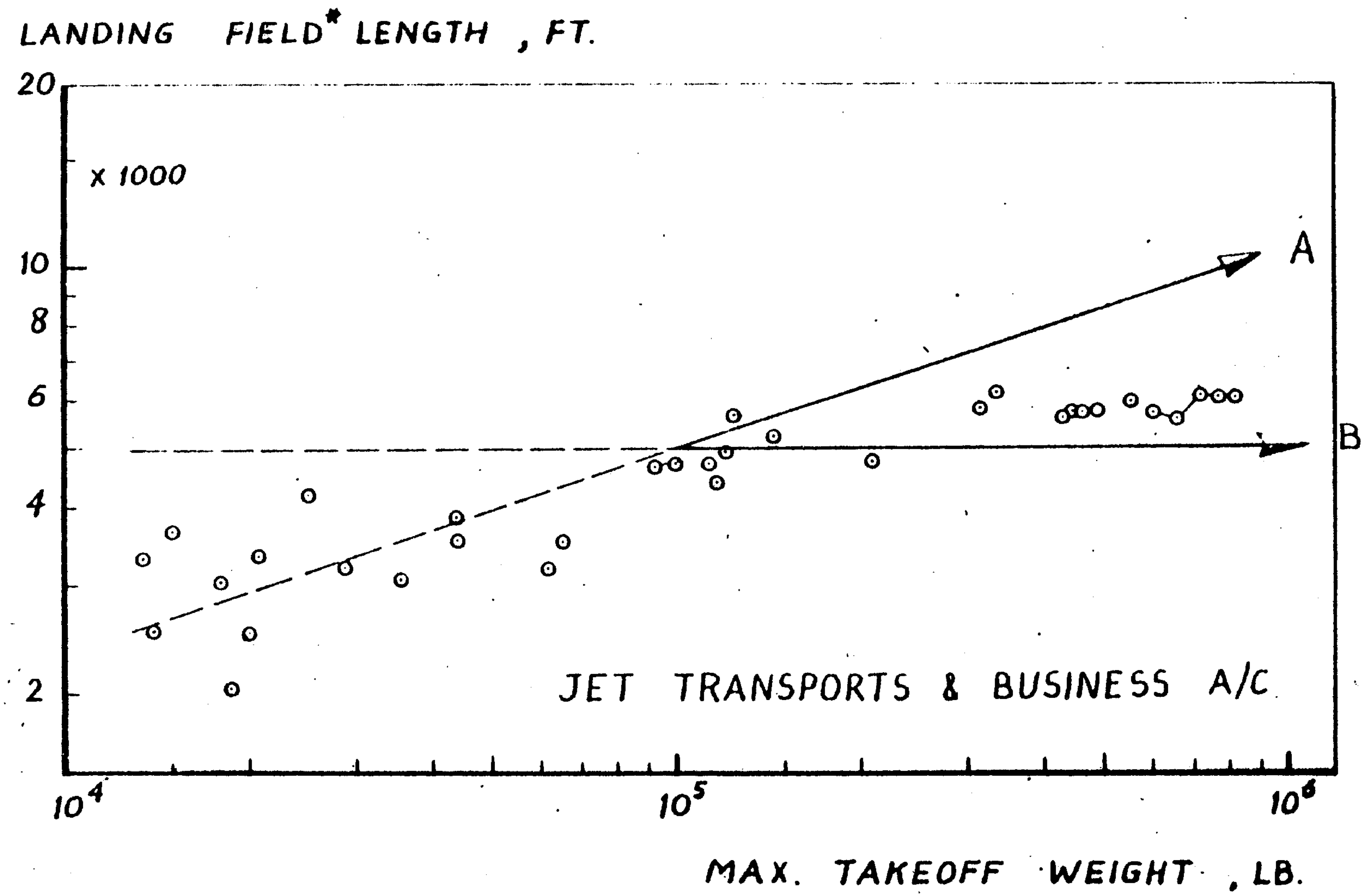


B-8 (CONTINUED)



*FAA BALANCED FIELD LENGTH
SEA LEVEL , ISA STANDARD

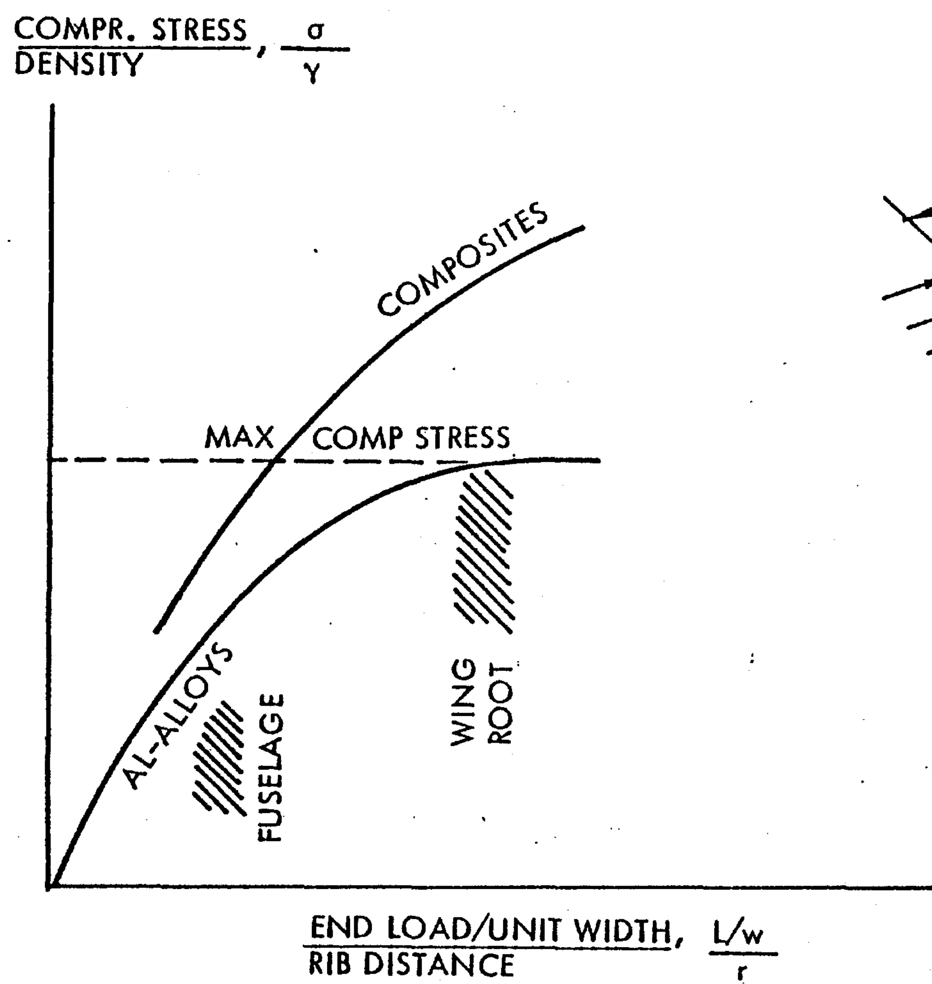
B-8 (CONTINUED)

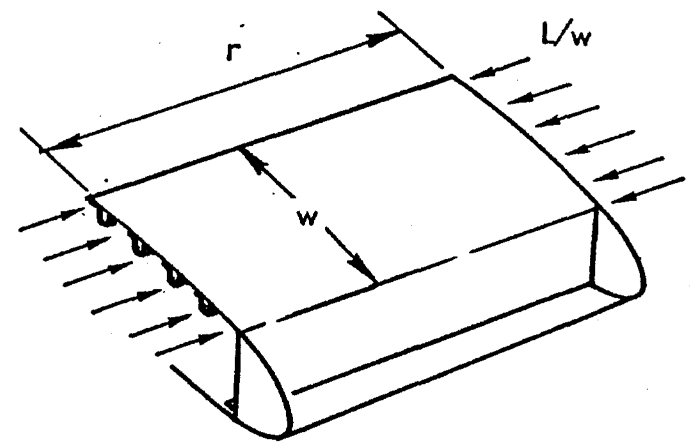


* FAA , FACTORED

B-8 (CONCLUDED)

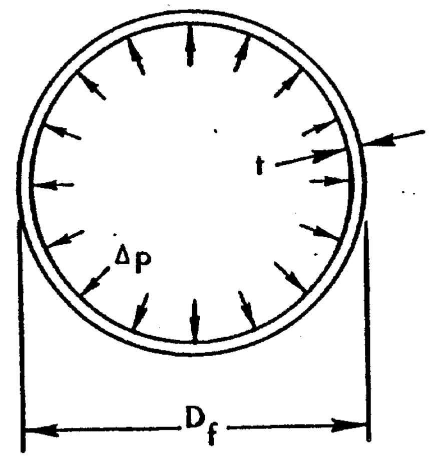
STRESSES IN COMPRESSION STRUCTURES





ACHIEVABLE STRESSES IN COMPRESSION - LOADED STRUCTURES
CAN BE INCREASED WHEN LOADING INTENSITIES INCREASE

SCALING THE FUSELAGE STRUCTURE



(a) LOADING DUE TO PRESSURE DIFFERENTIAL, PER UNIT LENGTH:

$$\frac{\text{SKIN TENSION}}{\text{UNIT LENGTH}} = \frac{\Delta p\, D_f}{2} = \sigma t$$

$$t = \frac{\Delta p\, D_f}{2\,\sigma}$$

$$\text{SKIN WEIGHT} = \gamma \times t \times S_{wet} \doteq \frac{\Delta p}{\sigma/\gamma}\, D_f\, S_{wet}$$

(b) BENDING MOMENT, $M_b \doteq$ LOAD FACTOR X PAYLOAD X CABIN LENGTH

$$\left.\begin{array}{l} M_b \doteq n\, W_p\, \ell_f \\ \text{ALSO: } M_b \doteq \bar{t}\, D_f^2\, \bar{\sigma} \end{array}\right\} \rightarrow \bar{t} = \frac{n\, W_p\, \ell_f}{\bar{\sigma}\, D_f^2}$$

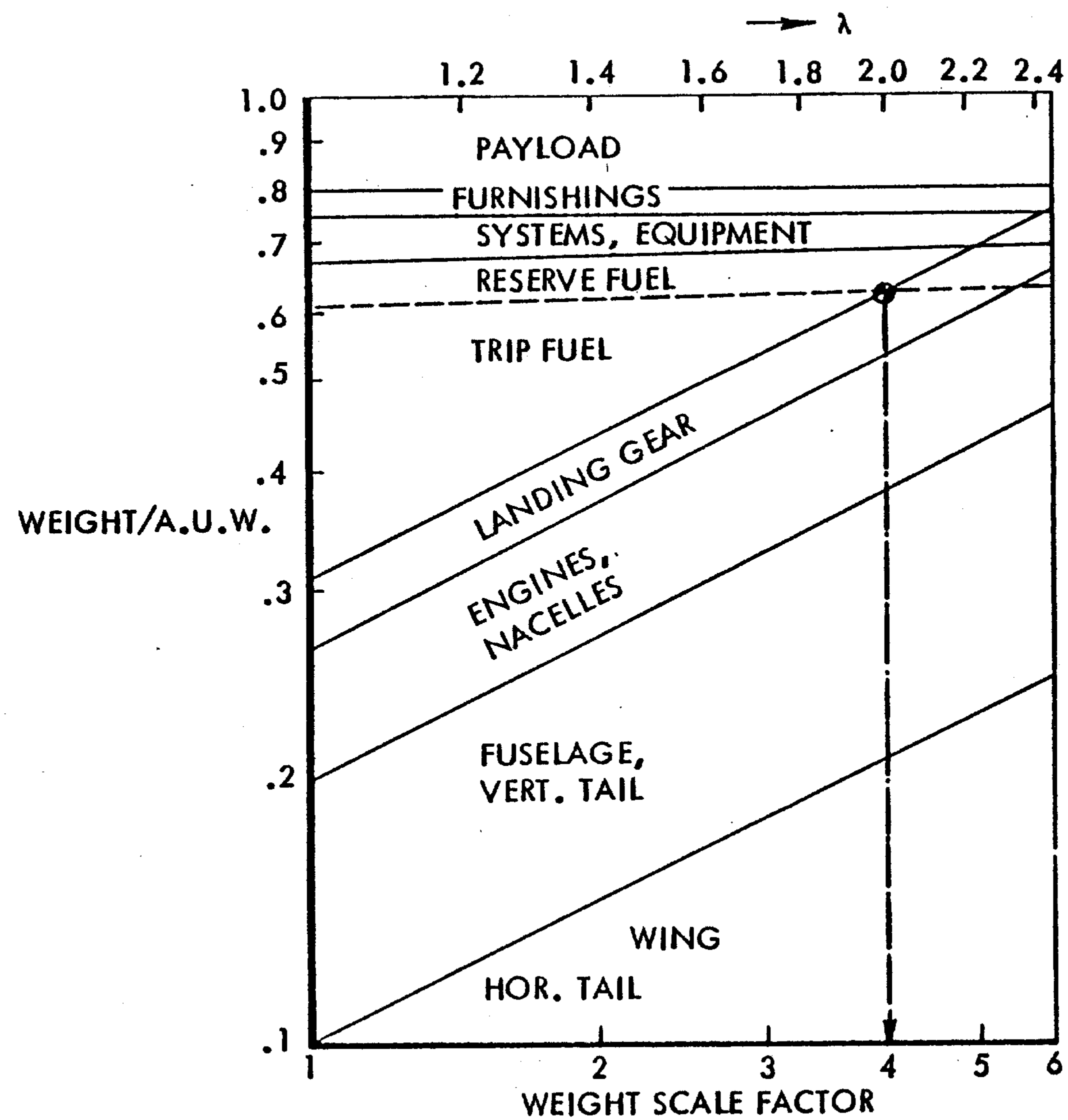
$$\bar{\sigma} = \frac{n\, W_p\, \ell_f}{\bar{t}\, D_f^2}$$

$$\text{SKIN/STRINGER WEIGHT: } \gamma \times \bar{t} \times S_{wet} \doteq \frac{n\, W_p}{\bar{\sigma}/\gamma} \quad \frac{\ell_f\, S_{wet}}{D_f^2}$$

FUSELAGE SCALING FACTORS

ITEM		SCALING FACTOR	
		A	B
DIAMETER	D_f	λ	λ
SKIN THICKNESS	t	λ	λ
PAYLOAD	W_p	$\lambda^3 \doteq W$	$\lambda^2 \doteq W$
BENDING MOMENT	M_b	$\lambda^4 \doteq W^{4/3}$	$\lambda^3 \doteq W^{3/2}$
EQ. SHELL THICKNESS	$\bar{t}$	$\lambda \doteq W^{1/3}$	$\lambda \doteq W^{1/2}$
(COMPRESSIVE) STRESS	$\bar{\sigma}$	$\lambda \doteq W^{1/3}$	1
STRUCTURAL WEIGHT	W_s	$\lambda^3 \doteq W$	$\lambda^3 \doteq W^{3/2}$
RELATIVE WEIGHT	W_s/W	1	$\lambda \doteq W^{1/2}$
WETTED AREA, DRAG		λ^2	λ^2
DRAG/ALL-UP WEIGHT		$\lambda^{-1} \doteq W^{-1/3}$	1

SCALING OF AIRPLANE WEIGHT FRACTIONS



ACTUAL COMPONENT WEIGHT SCALE FACTORS

ITEM	AIRPLANE 1	AIRPLANE 2	RATIO	SCALING RULE A	SCALING RULE B
ALL-UP WEIGHT	320,000	640,000	2.00	2.00	2.00
PAYLOAD	80,000	160,000	2.00	2.00	2.00
EMPTY WEIGHT	131,200	277,125	2.11	2.00	2.59
• WING	35,000	94,000	2.69	2.00	2.82
• FUSELAGE	29,300	54,000	1.84	2.00	2.82
ETC.					
FUEL WEIGHT	108,800	202,875	1.86	2.00	1.33

ALL WEIGHTS IN LB.

CONCLUSIONS FROM STRUCTURAL SCALING

- MAINTAINING CONSTANT PERFORMANCE AND STRESS LEVELS (METHOD B) RESULTS IN EMPTY WEIGHT FRACTIONS INCREASING WITH SIZE
- FUEL WEIGHT FRACTIONS, REQUIRED FOR CONSTANT RANGE PERFORMANCE, CANNOT BE ACHIEVED.
- SCALING UP FOR CONSTANT SHAPE (METHOD A) ENTAILS INCREASED STRESS LEVELS AND PERFORMANCE DETERIORATION
- WING VOLUME INCREASES MORE RAPIDLY WITH SIZE THAN FUEL VOLUME (METHOD B). SMALL, LONG-RANGE AIRCRAFT GENERALLY HAVE A FUEL VOLUME PROBLEM.

DEFEATING SCALING PROBLEMS

- LARGE STRUCTURES CAN BE DESIGNED MORE EFFICIENTLY:
  - MORE SOPHISTICATED STRESS ANALYSIS
  - LESS NON-OPTIMUM WEIGHT (JOINTS)
  - MORE REFINED FABRICATION METHODS CAN BE AFFORDED
- LARGE AIRCRAFT HAVE LOWER SKIN FRICTION DRAG AND HIGHER LIFT COEFFICIENTS (HIGHER REYNOLDS NUMBERS), PROVIDED THE SURFACE SMOOTHNESS IS GOOD ENOUGH.
- LARGE ENGINES ACHIEVE BETTER COMPONENT AND OVERALL EFFICIENCIES AND BETTER PERFORMANCE.
- POTENTIAL IMPROVEMENT AND ADVANCED AERODYNAMIC WING DESIGN:

CONSTANT	IMPROVEMENT
GEOMETRY, WEIGHT	SPEED ↑
SPEED, ASPECT RATIO	WEIGHT ↓
SPEED, WEIGHT	ASPECT RATIO ↑

DEFEATING SCALING PROBLEMS (CONCLUDED)

- COMPOSITE MATERIALS
  - HIGHER STRESS/DENSITY
  - HIGHER STIFFNESS/WEIGHT
  - IMPROVED "FATIGUE" LIFE
  - REDUCED CORROSION PROBLEMS
- MORE EFFICIENT ENGINE CYCLES AND TECHNOLOGY
  - REDUCED SFC AND ENGINE WEIGHT
  - IMPROVED TAKEOFF PERFORMANCE
  - REDUCED NOISE LEVELS
- IMPROVED (LIGHTER) SYSTEMS, EQUIPMENT, FURNITURE

  NOTE: AVIONICS, FLIGHT DECK FURNITURE AND CREW WEIGHTS DO NOT INCREASE WITH AIRPLANE SIZE
- UNCONVENTIONAL AIRPLANE CONFIGURATIONS, E.G. SPAN-LOADER
- CCV - TECHNOLOGY
  - RELAXED STATIC STABILITY
  - MANEUVER AND GUST LOAD ALLEVIATION
  - FLUTTER SUPPRESSION