REPORT No. 12

THE COLLEGE OF AERONAUTICS

CRANFIELD

Addendum to Report No. 12

THE AERODYNAMIC DERIVATIVES WITH RESPECT TO SIDESLIP FOR A DELTA WING WITH SMALL DIHEDRAL AT SUPERSONIC SPEEDS

by

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The Aerodynamic Derivatives with respect to Sideslip for a Delta Wing with Small Dihedral at Supersonic Speeds

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SUMMARY

The axis of a delta wing under sideslip is not aligned with that of the apex Mach cone. In calculating the forces by first order methods this fact may be ignored when the wing is at zero incidence, but when it is at incidence first order effects are introduced due to the distortion of the existing flow. In the original paper the latter were ignored; the corresponding forces are derived in the present addendum.

Neglecting leading edge suction the terms in the three sideslip derivatives dependent on incidence are discontinuous in changing from the quasi-subsonic to the definitely supersonic condition, generally involving a marked decrease in the numerical value and for the rolling derivative always a change of sign.

The leading edge suction due to incidence drops rapidly to zero as an edge approaches the apex Mach cone with the result that the suction contributions to the derivatives become indefinitely large in the limiting case though the actual forces are small.

MEP

1. Results

To the non-dimensional derivatives quoted in the original College of Aeronautics Report No. 12 must be added the following quantities:-

(a) For the leading edges inside the apex Mach cone

(i) $$\ell_v = \frac{\pi a}{3E'}$$

(ii) $$n_v = -\frac{4a\delta}{3E'}\cot\gamma - \frac{\pi a^2 M^2 \sec^2\gamma}{3E'^2\sqrt{1-\lambda^2}}$$

(iii) $$y_v = -\frac{2a\delta}{E'} - \frac{\pi a^2 M^2 \sec\gamma \tan\gamma}{2E'^2\sqrt{1-\lambda^2}}$$

(The terms in a^2 are due to leading edge suction)

(b) For the leading edges outside the apex Mach cone

(i) $$\ell_v = -\frac{2a\cot\gamma}{3\beta^3}$$

(ii) $$n_v = \frac{8a\delta\cot\gamma}{3\pi(\lambda^2-1)^{3/2}}\cdot\left\{\sec^2\gamma\sec^{-1}\lambda - \frac{M^2}{\beta^2}\sqrt{\lambda^2-1}\right\}$$

(iii) $$y_v = \frac{4a\delta}{\pi(\lambda^2-1)^{3/2}}\cdot\left\{\sec^2\gamma\sec^{-1}\lambda - \frac{M^2}{\beta^2}\sqrt{\lambda^2-1}\right\}$$

In Fig. 5 the contributions to ℓ_v, n_v and y_v from the pressure distribution are plotted against Mach numbers for different aspect ratios. In Fig. 6 the contributions to n_v and y_v from the leading edge suction are similarly plotted.

2. Pressure Distribution due to Incidence and Sideslip For the Leading Edges lying within the Apex Mach Cone

Consider the linear transformation:-

$$\left.\begin{aligned} x' &= x\,\mathrm{ch}\theta - \beta y\,\mathrm{sh}\theta \\ y' &= -\frac{x}{\beta}\,\mathrm{sh}\theta + y\,\mathrm{ch}\theta \\ z' &= z \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots(1)$$

/ The cone ...

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The cone $x^2 - \beta^2 y^2 - \beta^2 z^2 = 0$ and the operator $\left(-\beta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$ are unaltered by the transformation while a delta wing of semi-vertex angle γ' in the primed space with its axis coincident with the x'-axis corresponds to a delta wing of semi-vertex angle γ in the unprimed space with its axis yawed through an angle ψ with respect to the x-axis, where:-

$$\frac{\tan\gamma'}{1+\beta^2\tan^2\gamma'} = \frac{\tan\gamma}{1+\beta^2\tan^2\gamma - M^2\sec^2\gamma\,\sin^2\psi} \qquad \text{......(2)}$$

and

$$\theta = \frac{1}{4}\log\frac{\sin(\mu+\gamma+\psi)\sin(\mu-\gamma+\psi)}{\sin(\mu+\gamma-\psi)\sin(\mu-\gamma-\psi)}$$

where $\cot\mu = \beta$

It will be noted that the transformation is real if $\mu > \gamma + \psi$.

Let $\phi \equiv \phi(x',y',z')$ be the induced velocity potential for the delta wing of angle γ' unyawed but at incidence α. Under the transformation ϕ becomes a function of (x, y, z) which is clearly the induced potential for a wing of angle γ at incidence α yawed through an angle ψ with the free stream along the x-axis.

Now at the aerofoil:-

$$\phi = \frac{V\alpha\sqrt{x'^2\tan^2\gamma' - y'^2}}{E'(\beta\tan\gamma')} \qquad \text{............(3)}$$

To find the potential for the yawed wing referred to body axes we substitute from (1) and (2) into (3) and make the further orthogonal transformation:-

$$\left.\begin{aligned} x &= \bar{x}\cos\psi - \bar{y}\sin\psi \\ y &= \bar{x}\sin\psi + \bar{y}\cos\psi \end{aligned}\right\}$$

and obtain:-

$$\phi = \frac{V\alpha}{E'(\beta\tan\gamma')}\sqrt{\frac{\bar{x}^2\tan^2\gamma - \bar{y}^2}{1-\sec^2\gamma\sin\psi(\sin\psi+\beta\cos\psi\,\mathrm{th}\,\theta)}}\,.$$

/ For small ...

For small angles of yaw it will be seen from (2) that $\gamma' = \gamma + O(\psi^2)$ and that, provided the leading edges both remain inside the apex Mach cone, for ψ sufficiently small, $\theta = O(\psi)$.

Hence the potential at the aerofoil referred to axes fixed in it remains unaltered by a small yaw.

For a small sideslip $\bar{v}$ the excess pressure distribution referred to axes fixed in the wing is

$$-\rho\left\{V\frac{\partial\phi}{\partial\bar{x}} - \bar{v}\frac{\partial\phi}{\partial\bar{y}}\right\}$$

so that there are additional forces due to a sideslip corresponding to the pressure distribution;-

$$-\frac{\rho\bar{v}V\alpha}{E'}\frac{\bar{y}}{\sqrt{\bar{x}^2\tan^2\gamma-\bar{y}^2}} \qquad \ldots\ldots\ldots\ldots(4)$$

3. Suction Forces at the Leading Edges When Lying within the Apex Mach Cone

At page 10 of the original paper it was quoted that if the total induced velocity at the leading edge was of the form ($C/\sqrt{\xi}$ + bounded terms), where ξ is the distance in from the leading edge that the suction force was $\pi\rho C^2\cos\gamma\sqrt{1-\lambda^2}$ per unit length.

Now $\cos\gamma\sqrt{1-\lambda^2} = \sqrt{1-M^2\sin^2\gamma}$ is the compressibility or Glauert factor for the flow normal to the leading edge of which the Mach number is $M\sin\gamma$. For a yaw ψ this should be $M\sin(\gamma+\psi)$ at the leading edge $x\tan\gamma - y = o$ and $M\sin(\gamma-\psi)$ at the other one.

Now the value of C was found to be

$$\left\{\frac{V\alpha}{E'} + \frac{2\bar{v}\delta}{\pi}\right\}\sqrt{\tfrac{1}{2}x\tan\gamma\sec\gamma}$$

so that the resulting suction is

$$\frac{\pi}{2}\rho x\tan\gamma\sec\gamma\left\{\frac{V\alpha}{E'} + \frac{2\bar{v}\delta}{\pi}\right\}^2\sqrt{1-M^2\sin^2\gamma \mp \frac{2M^2\bar{v}}{V}\sin\gamma\cos\gamma}$$

where ψ has been replaced by $\bar{v}/V$, yielding an additional term at the edge $x\tan\gamma - y = o$

$$- \frac{\pi \rho \bar{v} V\alpha^2 M^2 x \tan^2\gamma}{2 E'^2 \sqrt{1 - \lambda^2}} \qquad \ldots\ldots\ldots\ldots (5)$$

with opposite sign at the other edge.

The two results (4) and (5) combine to give the additions to the sideslip derivatives quoted in Section 1.

4. The Pressure Distribution due to Incidence and Sideslip for the Leading Edges Outside the Apex Mach Cone.

Though the method of Section 2 can be employed to deduce the potential from that corresponding to an unyawed wing, the pressure distribution in the definitely supersonic case is more readily derived directly. The potential in this case is no longer independent of the yaw.

Consider the wing to be yawed through an angle $\psi = \bar{v}/V$ and the free stream direction to be that of the x-axis. If the yaw is such that both leading edges lie outside the apex Mach cone, the potential may be regarded as the result of a distribution of sources of strength $V\alpha/\pi$: the difference, ϕ_1, between the potentials in the yawed and unyawed cases can be regarded as due to a narrow wedge of sources of strength $+ V\alpha/\pi$ between the lines $y - x \tan\gamma = 0$ and $y - x \tan(\gamma + \psi) = 0$ of strength $- V\alpha/\pi$ between the lines $y + x \tan\gamma = 0$ and $y + x \tan(\gamma - \psi) = 0$. For a small yaw this distribution on neglecting terms of order ψ^2 is equivalent to a line of strength $V\alpha\psi\ell/\pi$ along the leading edge $y > 0$, and a line of strength $- V\alpha\psi\ell/\pi$ along the leading edge $y < 0$ where ℓ is the distance from the apex.

At points inside the apex Mach cone the change in potential due to the sideslip is:-

$$\phi_1 = \frac{1}{\pi} \bar{v} \alpha \sec^2\gamma \int_{x_0 > 0} \frac{x_0 \, d x_0}{\sqrt{(x-x_0)^2 - \beta^2(y-x_0\tan\gamma)^2}}$$

$$+ \frac{1}{\pi} \bar{v} \alpha \sec^2\gamma \int_{x_0 > 0} \frac{x_0 \, d x_0}{\sqrt{(x-x_0)^2 - \beta^2(y+x_0\tan\gamma)^2}}$$

where the limits of integration of either integral are such that the integrands are real.

/ Putting ...

Putting $x_o(\lambda^2-1) = -(x-\beta\lambda y) - (\lambda x-\beta y)\sin\theta_1$ in the first integral and

$$x_o(\lambda^2-1) = -(x+\beta\lambda y) + (\lambda x+\beta y)\sin\theta_2$$

in the second integral we obtain:-

$$\phi_1 = \frac{\bar{v}a\sec^2\gamma}{\pi(\lambda^2-1)^{3/2}}\left\{\int_{-\pi/2}^{\theta_{01}}\left[x - \beta\lambda y + (\lambda x-\beta y)\sin\theta_1\right]d\theta_1\right.$$

$$\left. - \int_{\theta_{02}}^{\pi/2}\left[x + \beta\lambda y - (\lambda x+\beta y)\sin\theta_2\right]d\theta_2\right\}$$

where θ_{01} and θ_{02} are the values of θ_1 and θ_2 at the apex.

Hence the change in pressure $p_1 = -\rho V\dfrac{\partial\phi_1}{\partial x}$ due to sideslip is:-

$$p_1 = \frac{\rho\,\bar{v}\,Va\sec^2\gamma}{\pi(\lambda^2-1)^{3/2}}\left\{\int_{-\pi/2}^{\theta_{01}}(1+\lambda\sin\theta)d\theta - \int_{\theta_{02}}^{\pi/2}(1-\lambda\sin\theta)\,d\theta\right\}$$

$$= \frac{\rho\,\bar{v}\,Va\sec^2\gamma}{\pi(\lambda^2-1)^{3/2}}\left\{\tan^{-1}\left[\frac{2y\tan\gamma\sqrt{(\lambda^2-1)(x^2-\beta^2y^2)}}{x^2\tan^2\gamma-(2\lambda^2-1)y^2}\right]\right.$$

$$\left. - \frac{2y\tan\gamma\sqrt{(\lambda^2-1)(x^2-\beta^2y^2)}}{x^2\tan^2\gamma - y^2}\right\}$$

where the inverse tangent takes the value zero at $y = 0$ and $\pm\pi$ at $x = \pm\beta y$ respectively.

At points outside the Mach cone the pressure is constant and since it is continuous it must take the value given by the above expression at the Mach cone, that is:-

$$p_1 = \rho\,\bar{v}\,Va\sec^2\gamma\,/\,(\lambda^2-1)^{3/2}, \quad \beta y > x$$

with opposite sign for $\beta y < -x$.

If p_o is the excess pressure resulting from incidence in the unyawed condition then the total excess pressure is $(p_o + p_1)$. The distribution p_o being symmetrical about the zx-plane is not symmetrical with regard to a yawed wing

/ and therefore ...

and therefore produces contributions to the lateral derivatives.

If p_a, p_e and p_t are the values of p_o at the axis, the leading edge $(y>0)$ and the trailing edge respectively then the couples and sideforce acting on the wing are to first order:-

$$L = 4 \left\{ \int_o^c \int_o^{x \tan\gamma} (\psi p_o x - p_1 y)\, dx\, dy + \psi \int_o^{c \tan\gamma} p_t y^2 dy - \frac{1}{3} \psi p_e c^3 \sec^2\gamma \tan\gamma \right\}$$

$$N = 4\delta \left\{ \int_o^c \int_o^{x \tan\gamma} (\psi\, p_o y + p_1 x)\, dx\, dy - \psi\, c \int_o^{c \tan\gamma} p_t\, y\, dy - \frac{1}{3} \psi\, p_a c^3 + \frac{1}{3} \psi\, p_e c^3 \sec^2\gamma \right\}$$

$$Y = 4\delta \left\{ \int_o^c \int_o^{x \tan\gamma} p_1\, dx\, dy - \psi \int_o^{c \tan\gamma} p_t\, y\, dy - \frac{1}{2} \psi\, p_a c^2 + \frac{1}{2} \psi\, p_e c^2 \sec^2\gamma \right\}$$

From Ref. 2 $$p_o = - \frac{2\rho V^2 \alpha \tan\gamma}{\pi\sqrt{\lambda^2 - 1}} \tan^{-1} \left[x \sqrt{\frac{\lambda^2 - 1}{x^2 - \beta^2 y^2}} \right]$$

The above expressions, together with the values of p_o, p_1 given, lead to the results quoted in Section 1.

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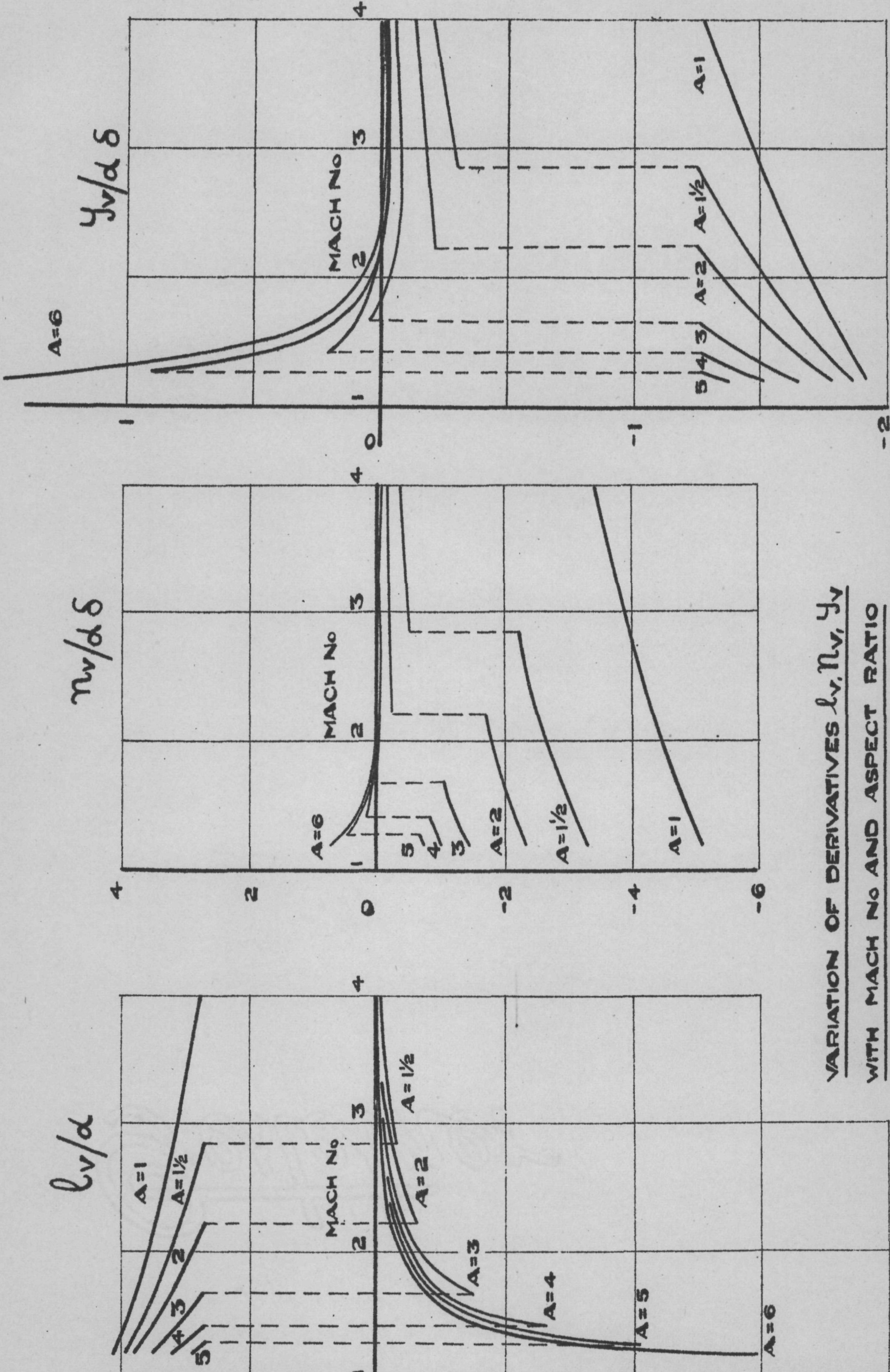

VARIATION OF DERIVATIVES l_v, n_v, y_v
WITH MACH No AND ASPECT RATIO
NEGLECTING LEADING EDGE SUCTION

FIG 5

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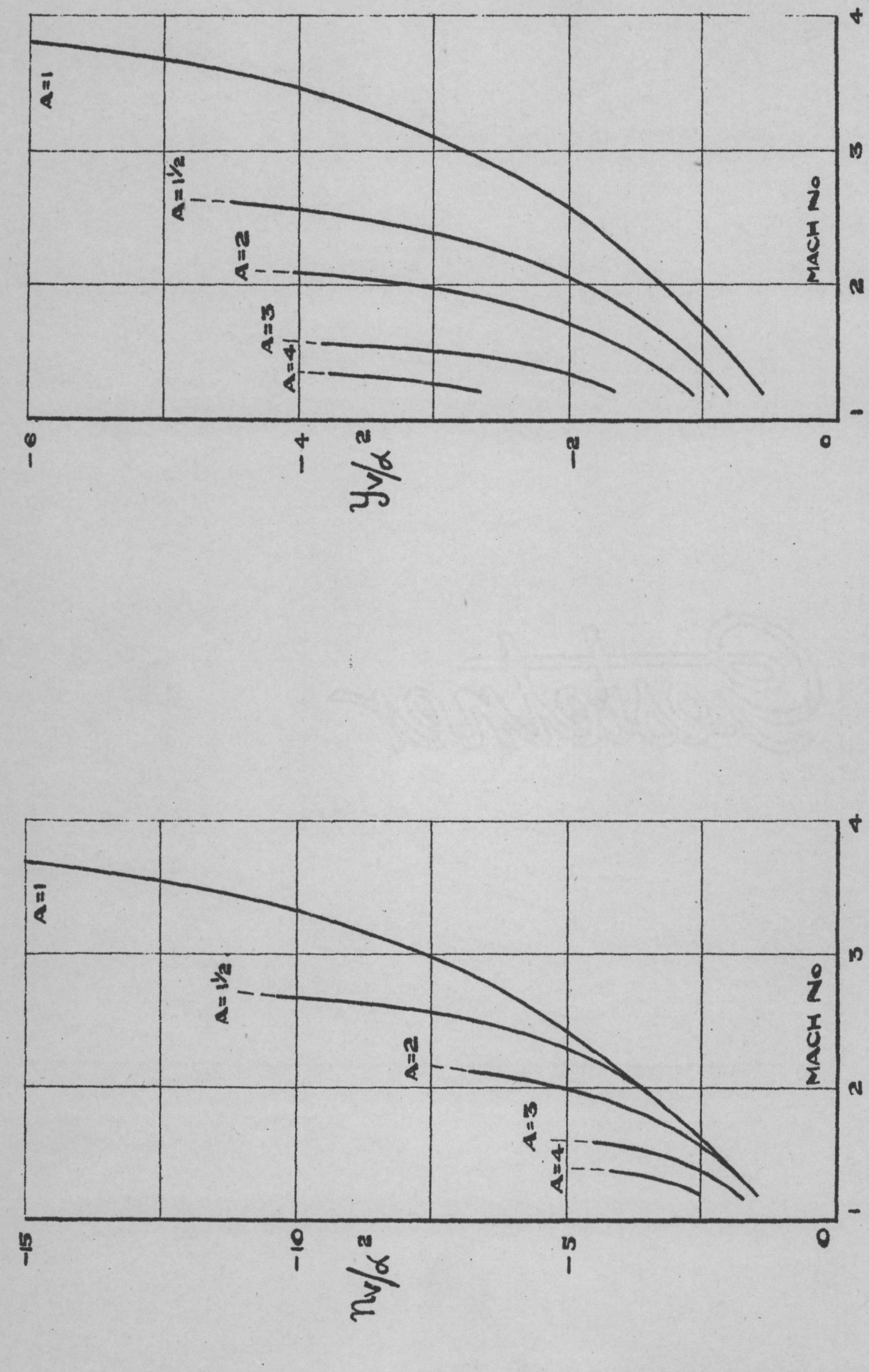

CONTRIBUTION OF LEADING EDGE SUCTION
TO THE DERIVATIVES n_v AND y_v
FOR VARYING MACH No AND ASPECT RATIO

FIG 6