Slope Stability Analysis Based on Random Finite Element Method and Probabilistic Approach

Final Presentation

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Introduction

\[ p_f = f(H, \beta, V_c, \theta) \]
Structure

Problem Description

History of Slope Stability Analysis

Failure Probability Determination

- Probabilistic Approach
- RFEM

Regression Analysis

- Direct Method
- Indirect Method

Conclusions and Recommendations
History of Slope Stability Analysis

Conventional Methods

• Limit Equilibrium Analysis
  a) Swedish Method;
  b) Bishop’s Method;
  c) Janbu’s Simplified Method;

• Limit Analysis
  a) Lower Bound Approach;
  b) Upper Bound Approach.

• Design Chart and Tables
  a) Taylor’s Chart;
  b) Baker-Tanake Chart;
  c) Cheng’s Table.
History of Slope Stability Analysis

Numerical Method

- Simple Finite Element Method (FEM)

- Modified Finite Element Method
  a) Rigid Element Method;
  b) Random Finite Element Method (RFEM).
Failure Probability Analysis

Probabilistic Approach

- Theoretical Foundation
- Probabilistic Description of Shear Strength ($c_u$)
- Single Random Variable Approach
- Correction due to Variance Reduction
- Results and Interpretations
Theoretical Foundation

Probabilistic Approach

\[ f(x; \mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2} \]

\[ F(x; \mu, \sigma^2) = \Phi \left( \frac{x - \mu}{\sigma} \right) \]

\[ \sigma_X^2 = \ln \left( 1 + \frac{\sigma_Y^2}{\mu_Y^2} \right) \]
\[ \mu_X = \ln(\mu_Y) - \frac{\sigma_X^2}{2} \]

\[ f_X(x; \mu, \sigma) = \frac{1}{x\sigma \sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}, \quad x > 0 \]

\[ P[Y < b] = \Phi \left( \frac{\ln(b) - \mu_X}{\sigma_X} \right) \]
Probabilistic Description of $c_u$

Probabilistic Approach

A typical log-normal distribution with a mean of 20Kpa and a standard deviation of 10KPa ($v_c=0.5$)

\[
\begin{align*}
\mu_{c_u} &= \exp(\mu_{\ln c_u} + \frac{1}{2} \sigma_{\ln c_u}^2) \\
\sigma_{c_u} &= \mu_{c_u} \sqrt{\exp(\sigma_{\ln c_u}^2) - 1} \\
\sigma_{\ln c_u}^2 &= \ln(1 + \frac{\sigma_{c_u}^2}{\mu_{c_u}^2}) \\
\mu_{\ln c_u} &= \ln(\mu_{c_u}) - \frac{\sigma_{\ln c_u}^2}{2}
\end{align*}
\]
Single Random Variable Approach

Probabilistic Approach

\[ F_s = \frac{c_u}{\gamma_s H N_s} \]

\[ P[c_u < 10.88] = \Phi\left(\frac{\ln(10.88) - 2.9708}{0.2231}\right) \approx 0.11 \]

\[ C = \frac{c_u}{\gamma_s H} \]

\[ \mu_{inc} = \mu_{incu} - \ln(\gamma_s H) \]

\[ \sigma_{inc}^2 = \sigma_{incu}^2 \]

\[ P[C < N_s] = \Phi\left(\frac{\ln(N_s) - \mu_{inc}}{\sigma_{inc}}\right) \]
Correction due to Variance Reduction

Probabilistic Approach

\[ \gamma = \left( \frac{\sigma_{\ln \sigma_{\ln C}}}{\sigma_{\ln C}} \right)^2 \]

\[ \mu_{\ln C} = \exp\left( \mu_{\ln C} + \frac{1}{2} \sigma_{\ln C}^2 \right) \]

\[ \sigma_{\ln C} = \sqrt{\exp\left( \sigma_{\ln C}^2 \right) - 1} \]

\[ \sigma_{\ln \sigma_{\ln C}}^2 = \gamma \sigma_{\ln C}^2 \]

\[ \mu_{\ln \sigma_{\ln C}} = \mu_{\ln C} \]

Variance reduction function over a square element of side length with a Markov correlation function
[Griffiths and Fenton, 2004]
Results and Interpretations

Probabilistic Approach

Theoretical solutions for slopes with $\beta = 50^0$:

- $V_c = 0.3$
- $V_c = 0.5$

- Taylor's solution, deterministic, $C_u = \text{mean}$
- Theoretical $\theta/H = 0$, deterministic, $C_u = \text{median}$
- Theoretical $\theta/H = \infty$, deterministic, $C_u = \text{mean}$
Failure Probability Analysis
Random Finite Element Method (RFEM)

- Parametric Study
- Mesh Study
  a) Horizontal Boundary
  b) Number of Elements
- Simulation Process
- Results and Interpretations
Parametric Study

Random Finite Element Method (RFEM)

- Gradient (β) : 10°, 20°, 30°, 40°, 50° and 60°;
- Height (H) : 2, 4, 6 and 8m;
- Coefficient of variation (v_c) : 0.1, 0.2, 0.3, 0.4 and 0.5;
- Correlation length (θ) : 0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.2, 1.5, 2.0, 5.0, 10.0 and 50.0

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>Ns</th>
<th>Fs</th>
<th>H (m)</th>
</tr>
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<tr>
<td>10</td>
<td>0.081</td>
<td>0.9</td>
<td>14</td>
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<td>3</td>
<td>4</td>
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<tr>
<td>20</td>
<td>0.109</td>
<td>0.9</td>
<td>10</td>
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<td>3</td>
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<tr>
<td>30</td>
<td>0.136</td>
<td>0.9</td>
<td>8</td>
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<td>3</td>
<td>2</td>
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<tr>
<td>40</td>
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<td>0.9</td>
<td>7</td>
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<td>2</td>
</tr>
<tr>
<td>50</td>
<td>0.174</td>
<td>0.9</td>
<td>6</td>
</tr>
<tr>
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<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>60</td>
<td>0.191</td>
<td>0.9</td>
<td>6</td>
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<tr>
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<td></td>
<td>3</td>
<td>2</td>
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### Parametric Study

Random Finite Element Method (RFEM)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^{(0)}$</td>
<td>$10^0, 20^0, 30^0, 40^0, 50^0, 60^0$</td>
<td>6</td>
</tr>
<tr>
<td>$H(m)$</td>
<td>$2.0, 4.0, 6.0, 8.0$</td>
<td>4</td>
</tr>
<tr>
<td>$v_c$</td>
<td>$0.1, 0.2, 0.3, 0.4, 0.5$</td>
<td>5</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>$0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.2, 1.5, 2.0, 5.0, 10.0, 50.0$</td>
<td>12</td>
</tr>
<tr>
<td>$c_u/Y_c$</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$E$</td>
<td>$1000*\mu_u$</td>
<td>1</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$0.3$</td>
<td>1</td>
</tr>
</tbody>
</table>

$6 \times 4 \times 5 \times 12 = 1440$
Mesh Study
Random Finite Element Method (RFEM)
Mesh Study

Random Finite Element Method (RFEM)

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>Height (m)</th>
<th>0.5H</th>
<th>R-error</th>
<th>H</th>
<th>R-error</th>
<th>2H</th>
<th>R-error</th>
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<tbody>
<tr>
<td>10</td>
<td>2</td>
<td>0</td>
<td>None</td>
<td>0</td>
<td>None</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>0</td>
<td>None</td>
<td>0</td>
<td>None</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>40</td>
<td>4</td>
<td>0.2805</td>
<td>0.0450</td>
<td>0.2260</td>
<td>-0.0095</td>
<td>0.2355</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>8</td>
<td>0.8170</td>
<td>0.0400</td>
<td>0.7720</td>
<td>-0.0050</td>
<td>0.7770</td>
<td>0</td>
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<tr>
<td>60</td>
<td>2</td>
<td>0.0580</td>
<td>0.0165</td>
<td>0.0385</td>
<td>-0.0030</td>
<td>0.0415</td>
<td>0</td>
</tr>
<tr>
<td>60</td>
<td>8</td>
<td>0.9335</td>
<td>0.0435</td>
<td>0.8930</td>
<td>0.0030</td>
<td>0.8900</td>
<td>0</td>
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</tbody>
</table>
Mesh Study
Random Finite Element Method (RFEM)

\[ v_c = 0.5, \theta = 1 \]
\[ \beta = 45^\circ \]

- (a) $0.1 \times 0.1 \text{ m}$
- (b) $0.125 \times 0.125 \text{ m}$
- (c) $0.2 \times 0.2 \text{ m}$
- (d) $0.25 \times 0.25 \text{ m}$
- (e) $0.5 \times 0.5 \text{ m}$
- (f) $1 \times 1 \text{ m}$
- (g) $2 \times 2 \text{ m}$
## Mesh Study

Random Finite Element Method (RFEM)

<table>
<thead>
<tr>
<th>Element size (m)</th>
<th>Number of elements</th>
<th>( p_f )</th>
<th>R-error</th>
<th>Time (s)</th>
</tr>
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<tbody>
<tr>
<td>0.1</td>
<td>3220</td>
<td>0.188</td>
<td>0</td>
<td>16273</td>
</tr>
<tr>
<td>0.125</td>
<td>2064</td>
<td>0.184</td>
<td>-0.004</td>
<td>8586</td>
</tr>
<tr>
<td>0.2</td>
<td>810</td>
<td>0.176</td>
<td>-0.012</td>
<td>2272</td>
</tr>
<tr>
<td>0.25</td>
<td>520</td>
<td>0.161</td>
<td>-0.027</td>
<td>1255</td>
</tr>
<tr>
<td>0.5</td>
<td>132</td>
<td>0.136</td>
<td>-0.052</td>
<td>219</td>
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<tr>
<td>1</td>
<td>34</td>
<td>0.100</td>
<td>-0.088</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>0.065</td>
<td>-0.123</td>
<td>8</td>
</tr>
</tbody>
</table>
Mesh Study
Random Finite Element Method (RFEM)
# Mesh Study

Random Finite Element Method (RFEM)

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>Angle (°)</th>
<th>Number of elements</th>
<th>Element size (m)</th>
<th>$P_r$</th>
<th>Time (hour)</th>
<th>R-error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(50)</td>
<td>10</td>
<td>11400</td>
<td>0.04</td>
<td>0.000</td>
<td>2.1</td>
<td>-</td>
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<tr>
<td>2(25)</td>
<td>10</td>
<td>2950</td>
<td>0.08</td>
<td>0.000</td>
<td>0.2</td>
<td>0.000</td>
</tr>
<tr>
<td>4(50)</td>
<td>10</td>
<td>11400</td>
<td>0.08</td>
<td>0.000</td>
<td>2.4</td>
<td>-</td>
</tr>
<tr>
<td>4(25)</td>
<td>10</td>
<td>2950</td>
<td>0.16</td>
<td>0.000</td>
<td>0.3</td>
<td>0.000</td>
</tr>
<tr>
<td>6(50)</td>
<td>10</td>
<td>11400</td>
<td>0.12</td>
<td>0.000</td>
<td>3.0</td>
<td>-</td>
</tr>
<tr>
<td>6(25)</td>
<td>10</td>
<td>2950</td>
<td>0.24</td>
<td>0.000</td>
<td>0.3</td>
<td>0.000</td>
</tr>
<tr>
<td>8(40)</td>
<td>10</td>
<td>11400</td>
<td>0.16</td>
<td>0.008</td>
<td>5.2</td>
<td>-</td>
</tr>
<tr>
<td>8(25)</td>
<td>10</td>
<td>2950</td>
<td>0.32</td>
<td>0.010</td>
<td>0.7</td>
<td>0.002</td>
</tr>
<tr>
<td>2(50)</td>
<td>60</td>
<td>5025</td>
<td>0.04</td>
<td>0.001</td>
<td>1.2</td>
<td>-</td>
</tr>
<tr>
<td>2(25)</td>
<td>60</td>
<td>1325</td>
<td>0.08</td>
<td>0.001</td>
<td>0.1</td>
<td>0.000</td>
</tr>
<tr>
<td>4(50)</td>
<td>60</td>
<td>5025</td>
<td>0.08</td>
<td>0.211</td>
<td>9.2</td>
<td>-</td>
</tr>
<tr>
<td>4(25)</td>
<td>60</td>
<td>1325</td>
<td>0.16</td>
<td>0.230</td>
<td>1.4</td>
<td>0.019</td>
</tr>
<tr>
<td>6(50)</td>
<td>60</td>
<td>5025</td>
<td>0.12</td>
<td>0.737</td>
<td>20.0</td>
<td>-</td>
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<tr>
<td>6(25)</td>
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<td>1325</td>
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<td>8(40)</td>
<td>60</td>
<td>5025</td>
<td>0.16</td>
<td>0.943</td>
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<td>-</td>
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<tr>
<td>8(25)</td>
<td>60</td>
<td>1325</td>
<td>0.32</td>
<td>0.956</td>
<td>4.7</td>
<td>0.013</td>
</tr>
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</table>
Simulation Process

Random Finite Element Method (RFEM)

\[ \hat{p}_f = \frac{N_f}{N_{\text{tot}}} \]
Results and Interpretations

Random Finite Element Method (RFEM)

Failure Probability (%)

- $v_c = 0.1$
- $v_c = 0.2$
- $v_c = 0.3$
- $v_c = 0.4$
- $v_c = 0.5$
Results and Interpretations
Random Finite Element Method (RFEM)
Results and Interpretations

Random Finite Element Method (RFEM)
Results and Interpretations
Random Finite Element Method (RFEM)
Regression Analysis

\[ p_f = f(H, \beta, V_c, \theta) \]
Regression Analysis

Direct Method

\[ p_r = A \cdot \Theta^B + C \]

\[ p_r = -0.295 \cdot \Theta^{-0.209} + 0.5379 \]

\[ p_r = 0.3627 \cdot \Theta^{-0.1525} + 0.522 \]
Regression Analysis

Direct Method

Relative differences with respect to \( \theta \)
Regression Analysis

Direct Method

\[ p_f = 0.7915 \times e^{\left(\frac{\beta - 58.7919}{23.8532}\right)^2} \]
Regression Analysis

Direct Method

\[ p_f = A + e^{-\left(\frac{\beta - B}{C}\right)^2} \]

\[ p_f = 1 - \frac{1}{e^{A \beta^B}} \]

\[ p_f = 1 - \frac{1}{e^{0.0000003 \beta^{0.00}}} \]
Regression Analysis

Indirect Method

\[ p_f = P[F_s < 1] \]

\[ \mu_{F_s} = \mu_{inc} = \frac{\mu_{cu}}{\gamma_s H N_s} \]

\[ p_f = \Phi\left( \frac{\ln(F_{sc}) - \mu_{lnF_s}}{\sigma_{lnF_s}} \right) = \Phi\left( \frac{-\mu_{lnF_s}}{\sigma_{lnF_s}} \right) \]

\[ p_f = \Phi\left( \frac{\ln(1) - \mu_{lnF_s}}{\sigma_{lnF_s}} \right) = \Phi\left( \frac{-\ln(\mu_{F_s}) + \frac{\sigma_{lnF_s}^2}{2}}{\sigma_{lnF_s}} \right) \]
Regression Analysis

Indirect Method
Regression Analysis

Indirect Method
Regression Analysis

Indirect Method

\[ \sigma_{F_s} = A \cdot \theta^B + C \]

\[ \sigma_{F_s} = -0.7106 \theta^{-0.1696} + 1.1661 \]

\[ \sigma_{F_s} = A \cdot v_c + C \]

\[ \sigma_{F_s} = 1.08 \cdot v_c + 0.0057 \]
Regression Analysis

Indirect Method

Relative differences with respect to $\theta$

Relative differences with respect to $v_c$
Conclusions & Recommendations

Conclusions

• Advantages of RFEM over traditional ways;
• Superiority of failure probability over the factor of safety;
• Strengths of both probabilistic approach and RFEM;
• Defection of the regression analysis.
Conclusions & Recommendations

Recommendations

- Limitation of Computing Ability $\iff$ More simulations;
- Distribution of Fs for every single slope is needed;
- More appropriate function types are need for regression analysis.
Questions?
Thank you!

Thanksgiving wishes for you and your family!