Graduation Plan

Master of Science Architecture, Urbanism & Building Sciences

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Personal information	
Name	Kaan Akbaba
Student number	4655923

Studio		
Name / Theme	Computational Design	
Main mentor	Pirouz Nourian	Design informatics
Second mentor	Fred Veer	Structural design
Guest supervisor	Anjali Mehrotra	Applied mechanics (3MD - CEG)
Argumentation of choice of the studio	The choice of the studio comes naturally, as I enjoy making (or studying) models that help us get a better idea of how the world works and I believe that computers are a powerful tool to aid in this goal. As for the mentors and supervisor: • Pirouz Nourian: the choice of having Pirouz as my main mentor is based on his expertise on computer modelling and algorithms, and the implementations thereof. Moreover, Pirouz is knowledgeable about the mathematical theories and models that are necessary in developing such algorithms. • Fred Veer: a considerable part of this project is about the development of tessellations and the lattices their dual graphs create. As a material scientist, I believe Fred can aid in the development and structural evaluation of such lattices. • Anjali Mehrotra: the implementation of a discrete element model is a central topic in this project. As a researcher in the field of structural dynamics, I believe Anjali will aid in the formulation, verification and implementation of the discrete element model.	

<table>
<tr><th colspan="2">Graduation project</th></tr>
<tr><td>Title of the graduation project</td><td>Topology optimization of isotropic homogeneous discrete element structures</td></tr>
<tr><th colspan="2">Goal</th></tr>
<tr><td>Location:</td><td>Topic is independent of location, but experiments can be conducted using existing structures (see: Process - Experiments)</td></tr>
<tr><td>The posed problem</td><td>The main problem this project aims to solve can be stated as follows:
Find the optimal structural topology of a structure comprised of isotropic homogeneous discrete elements, maximizing its structural efficiency.
This problem comes with a set of foreseeable sub-problems, such as:
Find practical or relevant shapes and topologies for the discrete elements.
Find a way to model the structural behaviour of an assembly of discrete elements.
Find a valid and quantifiable definition for “structural efficiency”.
Find a way to maximize this “structural efficiency”.</td></tr>
<tr><td>research questions and</td><td>The main research question this project aims to answer is as follows:
How can a topology optimization problem be formulated and solved for isotropic homogeneous discrete element using an arbitrary tetrahedral or cubic tessellation?
Sub-questions can be directly derived from the main question. These questions include:
What repeating tessellation pattern can be used to acquire indefinitely many unique geometrical shapes?
How can the discrete elements be abstracted to simplify the modelling procedure?
How can isotropic and homogeneous discrete elements be accurately modelled to obtain values for the optimization?
Which numerical value must be obtained to begin optimizing the structure and how can this value be calculated?
What are relevant constraints for the optimization process?
What method or algorithm can be used to either maximize or minimize the objective function?</td></tr>
<tr><td>design assignment in which these result.</td><td>A cubic or tetrahedral tessellation must be developed. With these tessellations a set of rules must be formulated to find their connectivity. This results in a dual graph (or</td></tr>
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	a spring model) which can be used to perform a discrete element analysis. After such an analysis, a formulation for the work done by the structural system and its gradient functions should be derived with which the topology can be optimized using either existing implementations, or a self-implemented optimization algorithm (which is based on existing algorithms)

Process

Method description

3D-tessellations

- Base geometries will be used to assemble larger elements. In other words: a tessellation will be developed for larger elements
- Rules will be introduced to define the connectivity of these base geometries:
  - The centroid of a geometry represents a node
  - Nodes will be connected by an edge if the geometries they represent share a face
- A graph will be constructed in 3D-space using nodes and edges

Discrete element analysis

- Edges will have either of the following properties:
  - Rigid element to connect nodes rigidly (to create assemblies of elements)
  - Spring-like interfaces that model the interaction between two elements in 3D-space
- The spring constants of each interface will be determined,
- The global stiffness and mass matrices will be assembled and the adaptive global damping value will be calculated
- restrained degrees of freedom will be crossed out of the matrices
- applied force vector is constructed
- the system $\boldsymbol{M}\ddot{\boldsymbol{u}}+C\dot{\boldsymbol{u}}+\boldsymbol{K}\boldsymbol{u}=\boldsymbol{f}$ is solved for $\ddot{\boldsymbol{u}}$ (i.e., the accelerations are calculated by using the above-mentioned matrices, force vector, and boundary conditions for the displacements and velocities for all elements)

Topological optimization

- Derive the work done on the system (compliance) by using the expressions for the energy of a spring and the kinetic energy:

  $$c=\frac{1}{2}k u^2+\frac{1}{2}m\dot{u}^2$$

  Which results in:

  $$c(\chi)=\frac{1}{2}\boldsymbol{u}^{(t_{limit})T}\boldsymbol{K}^{(t_{limit}-\Delta t)}\boldsymbol{u}^{(t_{limit})}+\frac{1}{2}\dot{\boldsymbol{u}}^{(t_{limit})T}\boldsymbol{M}^{(t_{limit}-\Delta t)}\dot{\boldsymbol{u}}^{(t_{limit})}$$
- The work function is minimized using with the following constraints:
  - $\frac{V(\chi)}{V_0}=f$

    this means that the ratio between the volume of the optimized solution and the starting volume should be equal to *f*; this is to prevent 'overoptimization', which can lead to impractical structures
  - $0<\chi_{min}\leq\chi_i\leq 1$

    design variables that indicate the density of an element. Values closer to 0 mean that elements do not exist/do not contribute much to the system matrices, while values closer to 1 mean that elements are present and contribute (fully) to the system matrices
- The following existing algorithms can be used:
  - Optimality criteria method (OCM)
  - Method of moving asymptotes (MMA)
  - Sequential linear programming method (SLP)
  - A self-implemented algorithm based on existing ones

Experiments

- Some structures will be optimized using both the cubic and tetrahedral tessellations:
  - A tunnel/corridor
  - A multi-storey corridor (may be offset)
  - A rectangular room
  - Multiple rectangular rooms on top of each other (may be offset)
  - Case study of Villa Rotonda, as this building combines the use of corridors and a central (square) room. The structure will be approximated to fit the developed tessellation and scaled down to cut down on computational costs
- The results of cubic and tetrahedral tessellations will be compared

Verification

- Results are verified using DEA software 3DEC
- In the future, verifications could also be done using compas_rbe, which is currently under development

Literature and general practical preference

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Svanberg, K. (1987). The method of moving asymptotes—a new method for structural optimization. *International journal for numerical methods in engineering*, *24*(2), 359-373.

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Reflection

1. *What is the relation between your graduation (project) topic, the studio topic, your master track (A,U,BT,LA,MBE), and your master programme (MSc AUBS)?*

   The relation between this project and the studio topic (design informatics) is the required computational effort to calculate and keep track of the data involving the tessellation, discrete element modelling, and topology optimization. Another important point is the abstraction of reality in a numerical model, including the assumptions made and the logic to achieve this abstraction. The result of all of this is (hopefully) a well-founded design.

   Using computational tools and reasoning in order to find a certain design is inherently related to Building Technology and MSc AUBS, as it can be seen as a form of design by research.

2 *What is the relevance of your graduation work in the larger social, professional and scientific framework?*

The relevance of this research on the building industry can be seen in the optimization of masonry designs. This project may result in a useful tool to create efficient structures that save on labour and resources. An important note is that the shape of these masonry elements can be chosen arbitrarily, as long as they can be approximated by voxels or tetrahedra.

Regarding the scientific relevance, this project may serve as a stepping stone for some in the field of topology optimization. Especially in the field of discrete element topology optimization this project may be useful work, as this field is not nearly as developed as finite element based topology optimization.

An interesting tangent of this research may be the field of molecular sciences. The discrete element model somewhat resembles the structure and behaviour of molecules. It is of course for the scientific community to decide how relevant this research is in the field of molecular sciences.