Statically Balanced Compliant Mechanisms by Tuned Post-Buckling Behaviour

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Outline

- Compliant Mechanism
- Statically Balancing
- Buckling
- Case study
- Conclusions
Compliant Mechanism

- Single piece structure
- Motion by deformation

- Advantages
  - No friction
  - No hysteresis
  - No backlash
  - Less pieces
- Disadvantages
  - Energy storage
  - Limited elastic motions

- Application
  - precision mechanisms

Statically Balancing

- For every position in equilibrium
- No external force
- Actuation force only for acceleration

Conditions for Statically Balancing

- Constant speed
- Constant kinetic energy
- No actuation force

- Constant potential energy
  - Strain energy, gravity energy, work by other conservative forces
  - Interchange due to motion

- Equilibrium of all forces
  - Continuous equilibrium

- No stiffness
  - Start at equilibrium state
  - No change of force equilibrium
Buckling

- Non-linear behaviour of a structure
- Force as function of deformation
- Geometric non-linearities
  - Shaping
  - Reduced stiffness
- Spring
  - Linear: Shortening
  - Non-linear: Buckling by bending

Buckling of Simple Rod Model

- External force \( \lambda \)
- Equilibrium for position \( \xi \)

  - Pre-buckling
    - axial direction
    - Equilibrium state: \( \xi = 0 \)
  - Post-buckling
    - Rotational direction
    - Deviation \( \delta \xi \) around \( \xi = 0 \)

- Total stiffness
  - Physical stiffness
  - Geometric stiffness

Zero Stiffness: Equilibrium Path

\[ c_{\lambda} \delta \xi = (\xi - A \cos \xi) \delta \xi = 0 \]

- Zero total stiffness
- No change of equilibrium due to \( \delta \xi \)
- Geometrical stiffness increases for increasing \( \lambda \)
- Equilibrium path
  - External force for each position
Stability

- Equilibrium path
  - External load for each equilibrium state
- Simple rod
  - Mass exerts constant external load
  - Torsional spring restoring effort

Stable  Neutrally stable  Unstable

Buckling Applied for Statically Balancing

- Statically balancing:
  - Equilibrium state at a zero stiffness trajectory
- Buckling:
  - Initial pre-buckling equilibrium state
  - External loads that create zero stiffness
- Compliant mechanism
  - Varying external buckling load according to its equilibrium path
  - Statically balanced
- Couple compliant mechanism
  - Total behaviour becomes neutrally stable

Interface

- Force equilibrium at interface
  - Starts with external buckling load $u(ξ)$
  - Adds up equilibrium behaviour of compliant elements
- Requirement
  - Interface displacement
  - Displacement of coupling point $u(ξ)$
Statically Balanced Simple Rod Model

Energy Curves Simple Rod Model

Imperfections

- Assumption: perfectly aligned forces
- Imperfections due to
  - Manufacturing
  - Material irregularities
  - Temperature
- Consequences
  - Perfect equilibrium path is never reached
  - Hard to anticipate since imperfections unpredictable
Conclusions and Recommendations

- A buckling analysis results in an equilibrium path which predicts how to statically balance a mechanism.
- Elements can be combined in the coupling point based on their equilibrium paths, which should be in equilibrium.
- Accurate pre-stressing is required, but the required pre-stressing is difficult to predict due to imperfections.
- Tuning possibilities of the pre-stress elements can help to anticipate on reproducible imperfections.

Questions?

Statically Balanced Compliant Mechanisms by Tuned Post-Buckling Behaviour
Statically Balanced Compliant Mechanisms by Tuned Post-Buckling Behaviour
Conditions for Static Balancing

- Mechanism
  - Isolated system
  - Conservative forces
  - Statically balancing
  - Constant speed
  - No actuation force

- Mechanical energy
  \[ E_{mech} = P + K = \text{constant} \]
  - Law of conservation of energy
  - Constant speed
  \[ K = \frac{1}{2} m v^2 = \text{constant} \]
  - Potential energy: Total strain energy + external potential energy
  \[ P = U(u) + E(u) = \text{constant} \]

Buckling of Simple Rod

- Potential energy
  \[ \phi = \frac{1}{2} c \xi^2 + A(L + L \cos \xi) = \text{constant} \]

- Moments
  \[ \frac{\partial^2 \phi}{\partial \xi^2} = \Sigma M = c \xi - AL \sin \xi = 0 \]

- Stiffness
  \[ \frac{\partial^2 \phi}{\partial \xi^2} = c = - AL \cos \xi = 0 \]

Unique Load per Position

- Unstable
  - Can be statically balanced
  - Requirement: unique load per position

- Snap-buckling
  - Jump to lower branch segment
  - No unique load per position
Displacement field for buckling

- 1 DoF mechanism
- $\xi$ measure for deviation from $\xi = 0$
- More complex displacement field for compliant mechanism
- $x_1$ and $x_2$
- Mode shapes --- $FXME$
- 1st mode shape $u_1$ is initial displacement
- 2nd independent mode shape $u_2$
- $w(\xi) = u_0 + u_1 \xi + u_2 \xi^2 + ...$

Simply Supported Euler Beam