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A hybrid scheme for real-time prediction of bus trajectories

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SUMMARY

The uncertainty associated with public transport services can be partially counteracted by developing real-time models to predict downstream service conditions. In this study, a hybrid approach for predicting bus trajectories by integrating multiple predictors is proposed. The prediction model combines schedule, instantaneous and historical data. The contribution of each predictor as well as values of respective parameters is estimated by minimizing the prediction error using a linear regression heuristic. The hybrid method was applied to five bus routes in Stockholm, Sweden, and Brisbane, Australia. The results indicate that the hybrid method consistently outperforms the timetable and delay conservation prediction method for different route layouts, passenger demands and operation practices. Model validation confirms model transferability and real-time applicability. Generating more accurate predictions can help service users adjust their travel plans and service providers to deploy proactive management and control strategies to mitigate the negative effects of service disturbances.

KEY WORDS: travel time prediction; hybrid model; real-time information; vehicle trajectory; bus reliability; linear regression heuristic

1. INTRODUCTION

Reliability is one of the most important determinants of public transport level of service as well as service efficiency. Service attributes such as reliability of operations and prior timely information on unplanned changes are among the most important determinants of travellers' satisfaction with public transport service [1]. Public transport services, in particular urban bus services, are subject to inherent sources of uncertainty. In addition to physical, operational and technological measures to improve service reliability, service providers can improve service predictability by developing more accurate and reliable predictions concerning downstream service conditions. Generation and provision of bus travel time/departure time predictions relies on the acquisition and transmission of instantaneous data and is one of the primary online applications of advanced public transport systems. In this study, a method that integrates multiple data sources for generating projections of downstream vehicle trajectories is proposed. The predictions facilitate the dissemination of real-time travel information as well as support real-time control and fleet management decisions.

Bus travel times are determined by various inter-dependent stochastic factors such as traffic congestion, intersection delay, passenger demand, driver's behaviour and weather conditions. A disturbance in any of these factors can potentially propagate and consequently lead to deterioration in schedule adherence. The provision of real-time bus arrival/departure predictions can help service users to adjust their travel plan, make more informed decisions and thus reduce the adverse effect of service irregularity [2]. It hence can yield increase of the service ridership [3]. Moreover, accurate

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predictions of downstream conditions facilitate the deployment of proactive management and control strategies designed to mitigate the negative effects of service disruptions.

Several methods have been introduced for bus downstream trajectories prediction in the last two decades, all aiming at providing fast and more accurate travel time prediction in various prediction circumstances (e.g. prediction horizon, route characteristics, traffic condition and data availability). However, it is still contentious to choose the 'best model' being capable of delivering fast and accurate prediction over a wide range of transportation networks. Moreover, existence of such model is doubtful and unverified in the literature [4]. Hence, development of a fusion framework, in which the advantage of different models and data sources considering the targeted prediction circumstance are combined, is highly desirable. The combination process has been generally performed in two different ways: serial and parallel. In the first way, two methods are employed with a different functionality. The former is implemented to pre-process, simplify or group the input data (e.g. clustering to diminish the number of data features). The latter method then obtains predictions using the first method output [5, 6]. In a parallel way, two (or more) models are parallely implemented to sum up the advantages of each individual model [7, 8]. In this study, the second approach is considered, because the relevant data are wildly available in the appropriate format and can therefore be integrated simultaneously to improve the performance of the prediction models.

This study proposes a hybrid prediction model that combines the advantages of three independent prediction methods. The hybrid model integrates prediction methods, which are based on *schedule*, *instantaneous* and *historical*. In the training phase, a weight for each prediction method is determined by a heuristic algorithm on the basis of its prediction error. Prediction is performed on a rolling horizon basis with each prediction projecting the remaining bus trajectory (i.e. departure time predictions for all the downstream stops). Previous work tested the feasibility of estimating parts of the prediction model using a genetic algorithm [9] and compared the performance of the model from passengers' and operators' perspective when estimated for each route separately as opposed to joint estimation [10]. This study builds up on the rolling horizon prediction approach while devising an efficient model calibration technique and performing an extended validation. The main contribution of this study is developing and implementing a methodology for integrating and estimating the contribution of different prediction models and data sources while satisfying practical requirements related to the generation of real-time information. In addition, the prediction model is calibrated and validated on the basis of data of five routes from two public transport systems, confirming method transferability. This study builds up on our previous works [9, 10] by devising a hybrid prediction model with significant advances in prediction methodology, parameter calibration and an extended validation.

The remaining of this paper is organized as follows: we first review previous studies in the context of bus travel time prediction (Section 2). Then, the proposed hybrid prediction method is described in detail (Section 3). Five bus routes in two case study areas, Brisbane and Stockholm, are described (Section 4), followed by an explanation of implementation details including data processing and specifications for the optimization process (Section 5). Then, the proposed method is applied to the case study routes, and the results are benchmarked against the currently deployed prediction methods in Brisbane and Stockholm (Section 6). Finally, we conclude with an overall assessment of the proposed approach, discuss its advantages and shortcomings and outline directions for further research (Section 7).

2. LITERATURE REVIEW

Previous research in traffic predictions has developed a large range of methodologies that are often categorized into *data-driven* and *model-driven* methods.

Data-driven methods are generally empirical models that statistically model the relationships between the variables. Such models can be classified into *parametric* and *non-parametric* models. Parametric methods are based on a structural pre-defined function with a number of independent variables, whereas the structure and parameters of the model are mined from data in the case of non-parametric methods. In the context of bus travel time predictions, the two most commonly used parametric models are *linear regression models* and *time-series models*. Linear regression models formulate bus travel time as a linear function of independent variables [2, 11, 12] such as distance,

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number of stops, dwell times, boarding information and weather descriptor parameters [13]. Time-series models assume that travel time patterns are recurring, and therefore, past patterns can be used for predicting travel times in the future. Kalman filter techniques are commonly used for estimating these models [14–16]. The main drawback of parametric models is that their simple structure may not depict the complex inter-actions that are inherent to urban bus operations and underlie its uncertainty. Non-parametric data-driven methods consist of various machine learning techniques that have been deployed to predict bus travel times. In particular, artificial neural networks and support vector machines have gained popularity among data scientists in recent years [12, 17–19], thanks to their capability to capture complex non-linear relationships [20]. However, the training and estimation of these methods require large amounts of data and the application of designated techniques to reveal the underlying relation between inputs and outputs [21].

Model-based methods are generally mathematical models that provide functional relationship between different variables. These models can be classified into analytical [23, 24] and numerical models [25], where the former provides a closed-form solutions whereas the latter generally consider a time stepping function to obtain model behaviour over time. While the advantage of model-based methods is that they provide direct insight into the impact of independent variables on travel time predictions, their performance depends on model capability to represent the related mechanisms.

In addition to data-driven and model-driven methods, two simple prediction rules were proposed in the literature that requires no training or estimation. The first rule is based on the assumption that travel times remain constant during the same period of time on different days [26]. The second rule assumes that travel time remains stable within a sufficiently short time interval, and thus, short-time predictions are equal to the latest travel time that has been recorded for the same road segment [27]. While the first rule neglects day-to-day variations, the latter neglects within-day variations. However, empirical analysis suggests that both kinds of variations are not negligible [13]. Timetables are often constructed by implicitly assuming the former for a given day category (e.g. summer weekdays, and holidays). Predictors that rely solely on historical or instantaneous data are simple, fast and easily applicable. Mori *et al*. [4] concluded that instantaneous predictors yield better results for short prediction horizons (one-step-ahead, which is generally 5 minutes) while historical predictors were found superior for longer horizons.

The complexity of bus travel time variations and the advantages of individual prediction methods call for the development of a hybrid prediction scheme that allows the integration of several methods and data sources.

Hybrid models have recently emerged as a promising prediction approach in the context of freeway traffic where it obtained high-accuracy predictions [8, 28–30]. However, on the basis of a review of the literature and to the best of our knowledge, no hybrid prediction model has been developed and applied hitherto for real-time predicting bus travel times. Although hybrid models perform well, simple models are still popular in the most recent advanced travel information system [4]. Furthermore, simple prediction models remain popular among real-time information system providers in practice [31]. The inherent complexity of hybrid models presented in the literature can hinder their deployment. Methods for generating real-time information need to be fast, to be reasonably simple, handle noisy data from various sources, to be scalable (to generate predictions for the entire network) and to be robust when unexpected disruptions occur. The following section describes the modelling approach adopted in this study to address the aforementioned requirements.

3. METHODOLOGY

The proposed hybrid model is a linear combination of three prediction methods.

- *Scheduled:* The static time-dependent schedule is used as a source of information (*Timetable*) for the former.
- *Instantaneous* based: This method can be considered an extension of simple methods where instead of choosing the last observed value for prediction, the last few observed values are taken into account for prediction. This number of observations should be selected so that the prediction error is minimized.

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- *Historical* based: This part mines historical data on the basis of certain selection criteria designed to maximize prediction accuracy.

Automated vehicle location (AVL) data are used for the aforementioned two latter cases.

In the following section, the model and its components are described. In the next step, linear regression-based heuristic is introduced as an estimator of model parameters (Section 3.2). The proposed model is evaluated by measuring model accuracy (to be defined later in Section 3.3.1) and compared with two benchmarks (Section 3.3.2). Finally, model validity is examined in the last part of model evaluation procedure (Section 3.3.3).

3.1. Model description

Assume a bus trip k assigned to a route consisting of an ordered set of stops, $S=\{s_1, \dots, s_{|S|}\}$. As bus trip k departs from stop $s_p \in S$, the prediction for the remaining bus departure times regarding all downstream stops $s_i \in \{s_{p+1}, \dots, s_{|S|}\}$ is updated (Figure 1).

Let Π^c and Π^o be the matrices of scheduled and observed departure times, respectively (c stands for constant because scheduled times are assumed to be taken from the respective timetable; o stands for observed). Π^c_{k,s_i} and Π^o_{k,s_i} are specific cell entries corresponding to the scheduled and observed departure times of trip $k \in K$ at stop $s_i \in S$, respectively. K is the set of bus trips scheduled for the route under consideration. Hence, each row of Π^c, denoted by Π^c_k, refers to a single trip $k \in K$ and the same for Π^o. Also, each column of Π^c, denoted by $\Pi^c_{s_i}$, refers to all the departure times corresponding to stop s_i and the same applies to Π^o.

A rolling horizon procedure is proposed for generating bus trajectory predictions on the basis of the hybrid scheme. In this procedure, predicted departure time from stop s_i depends on predicted departure time from the previous stop s_{i-1}, and the departure time is an incrementally accumulated value, constituting a Markov process. Hence, the predicted departure time from each stop depends solely on the summation of the predicted departure time from the previous stop and the marginal addition of the travel time predicted between the last pair of stops (with the exception of stop s_{p+1}, where the departure time from the previous stop, Π^o_{k,s_p}, has already been observed):

$$\Pi^f_{k,s_p,s_i} = \Pi^f_{k,s_p,s_{i-1}} + \hat{t}_{k,s_{i-1}} \tag{1}$$

where Π^f_{k,s_p,s_i} denotes the predicted departure time (f stands for forecast) for trip k from stop s_i generated when the bus departs from stop s_p $\left(\Pi^o_{k,s_p}\right)$ and $\hat{t}_{k,s_{i-1}}$ denotes predicted travel time for trip k connecting two successive stops, s_{i-1} to stop s_i.

Bus operators commonly apply a schedule-based control strategy where bus drivers are instructed to hold at a designated set of time point stops (TPS), $S' \subseteq S$, in order to maintain the timetable in case they run ahead of schedule [22]. In case that this holding strategy is implemented, the control strategy logic can be explicitly incorporated into the prediction method by introducing the following condition:

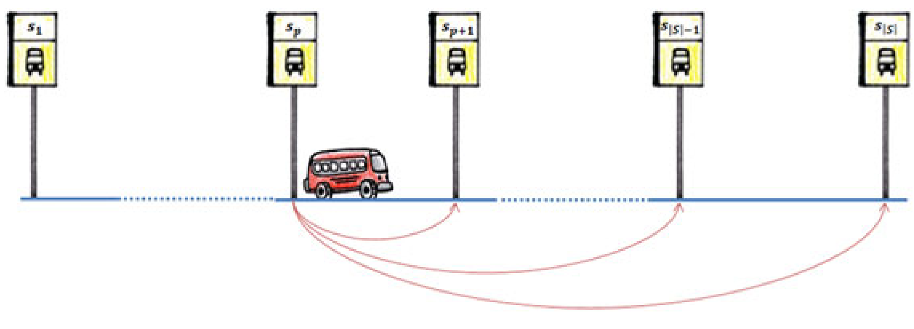

Figure 1. Rolling predictions for all downstream stops.

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$$\Pi^{f}_{k,s_p,s_i} = \begin{cases} \max\left(\Pi^{f}_{k,s_p,s_{i-1}} + \hat{t}_{k,s_{i-1}}, \Pi^{c}_{k,s_i}\right) & \text{if} \quad s_i \in S' \\ \Pi^{f}_{k,s_p,s_{i-1}} + \hat{t}_{k,s_{i-1}} & \text{otherwise} \end{cases} \tag{2}$$

It is thus assumed that drivers fully comply with the control scheme and hold the bus at time point stops in case of an early arrival, and therefore, overtaking does not occur under this control scheme.

As mentioned earlier in this section, scheduled, instantaneous and historical travel time information is fused to make a prediction for travel time. For trip k on a specific road segment connecting two successive stops, s_{i-1} to stop s_i a hybrid prediction is formulated as

$$\hat{t}_{k,s_i} = \beta_c \cdot t^{c}_{k,s_i} + \beta_r \cdot t^{r}_{k,s_i} + \beta_h \cdot t^{h}_{k,s_i} \tag{3}$$

where $t^{c}_{k,s_i}, t^{r}_{k,s_i}$ *and* t^{h}_{k,s_i} correspond to scheduled, instantaneous and historical travel time predictions, respectively (r stands for real time) for trip k on road segment connecting stops, s_{i-1} and s_i. β_c, β_r and β_h denote the weights assigned to the corresponding predictors. As described earlier, the scheduled travel time is obtained from a timetable, and the instantaneous and historical travel times are calculated on the basis of AVL data or can be potentially inferred from automatic passenger count and automatic fare collection data (Figure 2). While t^h is obtained by mining AVL data archive, t^r refers to recent vehicle position probes. In this study, travel times refer to the time between bus departure times and hence include running times between stops and dwell times at the downstream.

Scheduled travel time is retrieved from the timetable and is formulated as follows:

$$t^{c}_{k,s_i} = \Pi^{c}_{k,s_i} - \Pi^{c}_{k,s_{i-1}} \tag{4}$$

where t^{c}_{k,s_i} is the scheduled travel time between two consecutive stops s_{i-1} and s_i. The timetable is the outcome of the negotiations between the regulator (i.e. public transport authority) and the service provider (i.e. operator). The timetable is then used as a reference for regulating the service by control centre dispatchers and drivers through the deployment of control strategies and speed adjustments. The timetable is therefore also a determinant of bus travel times and not merely an outcome of traffic conditions. Therefore, the scheduled travel time can be informative in forecasting downstream vehicle trajectories.

Instantaneous travel time refers to the most recently observed travel time. On the one hand, using a single observation can potentially introduce bias due to interruption in data collection system or by an unexpected circumstance (e.g. vehicle failure). On the other hand, this numerical range should not be too large in order to still remain representative of the instantaneous information. A numerical range can be defined for each route segment. The optimal value, η, is estimated in this study using a heuristic algorithm. Thus, the predicted travel time between two consecutive stops (s_{i-1}, s_i) depends on the η most recent downstream bus travel times, which passed the same road segment.

The set of trips that are used for estimating downstream travel times is defined separately for each route segment. This is illustrated in Figure 3 where bus trip k passed stop s_p, and we are interested in calculating instantaneous prediction for all downstream stops (marked by stars). Here, for simplicity of illustration, the value of η is two. The statuses of six downstream buses are illustrated, where the first two (shown in black) have already arrived at the last stop and the rest (shown in green) are still on their

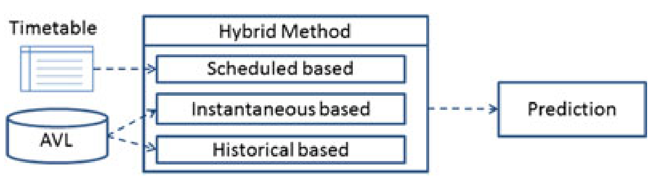

Figure 2. Schematic block diagram of proposed hybrid prediction method. AVL, automated vehicle location.

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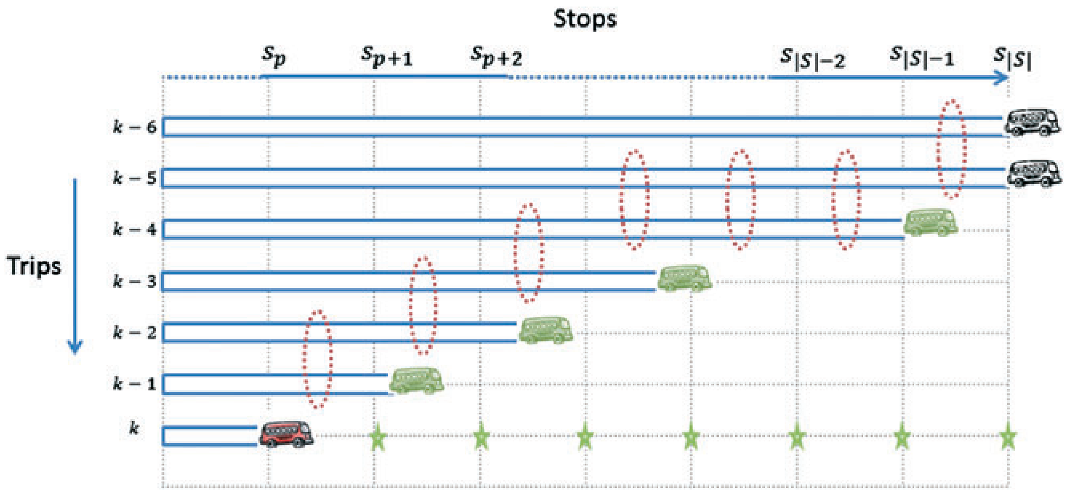

Figure 3. Instantaneous prediction, η = 2.

way. The trips considered when generating travel time predictions between each pair of consecutive stops are indicated by red dashed ellipse.

For a given road segment, the median value of the observed travel times pertaining to the last η buses is computed. The median rather than the mean value is taken to reduce the impact of extreme values. The instantaneous part of the prediction method between two consecutive stops s_{i-1} and s_i is formulated as follows, where η is subject to calibration and *Md* denotes median.

$$t^r_{k,s_i}(\eta) = Md\left[\Pi^o_{k-n,s_i} - \Pi^o_{k-n,s_{i-1}}\right]^{\eta}_{n=1} \tag{5}$$

Historical travel time refers to previous instances that exercise similarity to the circumstances of the target trip. For historical travel time to be indicative of future conditions, some discriminative traits need to be determined. Then, a similarity function is defined on the basis of these discriminative traits to select the most relevant travel time observations. The discriminative traits in this study are time and location (road segment). Regarding time, it could be discriminative to check if historical data for the same road segment exist in the same day of the week and time of day (denoted *reference time* in the rest of paper) from the previous week. While last week might be the most informative historical source of information, data over a longer period of time might allow for more robust predictions. The determination of the optimal number of past weeks that should be considered in the prediction is left for future research. In this context, the similarity function constitutes a time window pointing to records archived from 7 days ago. The time window is determined by $t^* \pm \delta$, where t^* is the reference time and δ is a tolerance parameter. The calibration of the latter is explained in detail in Section 3.2.

Thus, the set of trips included in the historical travel time component of the prediction scheme for the road segment connecting stops s_{i-1} and s_i is

$$\hat{K}(s_i, t^*, \delta) = \left\{\forall k^{-w} \in K^{-w} : t^* - \delta \leq \Pi^o_{k^{-w},s_i} \leq t^* + \delta\right\} \tag{6}$$

where k^{-w} is an observation from the set of bus trips (K^{-w}) observed in the same day of the last week (i.e. 7 days ago) for the same route.

For the most relevant historical data for each road segment to be referred to, the time window is dynamically shifted forward on the basis of scheduled travel times. In other words, for any stop s_i, the reference time equals to the observed time (Π^o_{k,s_p}) for the currently passed stop s_p plus a summation over all scheduled travel times for road segments connecting s_p and s_i. For example, if bus A just departed from stop 1 at 15:10 h, then scheduled departure times from stops 1, 2, 3 and 4 are 15:05,

DOI: 10.1002/atr

15:15, 15:30 and 15:40 h accordingly. The reference time will be 15:20 (15:10 + [15:15 − 15:05]), 15:35 and 15:45 h for stops 2 to 4.

$$t^* = \Pi^o_{k,s_p} + \sum_{j=p}^{i-1}\left(\Pi^c_{k,s_{j+1}} - \Pi^c_{k,s_j}\right) \tag{7}$$

Finally, the median of all involved historical travel times is computed for the road segments connecting s_{i-1} to s_i. Mean has been widely used to summarize the data because of its ease of computation; however, recently the use of median in travel time estimation is advocated owing to its robustness to the extreme values. For the current modelling, we have used median. The historical part of the hybrid prediction method is formulated as

$$t^h_{k,s_i}(\delta) = Md\left[\Pi^o_{k^{-w},s_i} - \Pi^o_{k^{-w},s_{i-1}}\right]_{k^{-w}\in\hat{K}(s_i,t^*,\delta)} \tag{8}$$

By integrating the aforementioned elements, Equation (3) can be re-written as

$$\hat{t}_{k,s_i} = \beta_c \cdot t^c_{k,s_i} + \beta_r \cdot t^r_{k,s_i}(\eta) + \beta_h \cdot t^h_{k,s_i}(\delta) \tag{9}$$

By integrating the aforementioned elements, the general form of the proposed prediction method (Equation (2)) is obtained:

$$\Pi^f_{k,s_p,s_i} = \begin{cases} \max\left(\Pi^f_{k,s_p,s_{i-1}} + \beta_c \cdot t^c_{k,s_i} + \beta_r \cdot t^r_{k,s_i}(\eta) + \beta_h \cdot t^h_{k,s_i}(\delta), \quad \Pi^c_{k,s_i}\right) & \text{if} \quad s_i \in S' \\ \Pi^f_{k,s_p,s_{i-1}} + \beta_c \cdot t^c_{k,s_i} + \beta_r \cdot t^r_{k,s_i}(\eta) + \beta_h \cdot t^h_{k,s_i}(\delta) & \text{otherwise} \end{cases} \tag{10}$$

3.2. Model calibration

The aforementioned hybrid prediction model involves the specification of five parameters:

- Three prediction weights (β_c, β_r and β_h) pertaining to scheduled, instantaneous and historical predictors.
- Number of buses used in the instantaneous predictor (η).
- Tolerance of the time window used in the historical predictor (δ).

The search space is defined by the 5-D space including four continuous float (β_c, β_r, $\beta_h$and δ) and one discrete integer variable (η). In accelerating the convergence of the optimization process, the search space is constrained by considering the dependency among the relative prediction weights by normalizing their contribution ($\beta_c+\beta_r+\beta_h=1$, i.e. each pair strictly determines the third weight). Therefore, the optimization problem dimensionality is reduced to 4-D with the two prediction weights constrained into the feasible range of [0, 1].

Suitable ranges have to be defined also for η and δ on the basis of data availability and case study circumstances according to the following considerations:

I. The more frequent the service is, the larger the number of recent bus trips that should be included in the instantaneous predictor because they can convey information on recent travel conditions. Conversely, a large number of η can potentially induce bias in case of infrequent or irregular services. Hence, the upper limit for this variable should be kept reasonably low.
II. The more frequent the service and vehicle probes are, the shorter the tolerance parameter that can be used. In contrast, infrequent services or sparse data transition will require longer tolerance to ensure that a sufficient number of observation is available for estimating the historical predictor.

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Thus, the upper limit should be sufficiently large to accommodate the lower end of the range of data availability and service headway.

Given the discontinuity (including maximum function) and complexity (involve median expressions and integer η) associated with the optimization problem, a heuristic algorithm is needed for estimating the model parameters. A linear regression heuristic algorithm is proposed for estimating departure time prediction model parameters.

Two main assumptions have been made to convert the proposed prediction model into a linear model: (i) the control strategy constraint (i.e. holding at time point stop if bus is ahead of time) is relaxed and (ii) the sets of η and δ values are exogenously fixed. The former has been applied by realizing the implemented schedule-based control strategy introduced in Equation (2). It leads that Equation (2) simplified and transformed into $\Pi^{f}_{k,s_p,s_i} = \Pi^{f}_{k,s_p,s_{i-1}} + \hat{t}_{k,s_{i-1}}$. These assumptions transform the proposed model into

$$\Pi^{f}_{k,s_p,s_i} = \Pi^{f}_{k,s_{i-1}} + \beta_c \cdot t^{c}_{k,s_i} + \beta_r \cdot t^{r}_{k,s_i}(\eta) + \beta_h \cdot t^{h}_{k,s_i}(\delta) \tag{11}$$

Given the linear form of Equation (11), β_c, β_r and β_h are optimized using a linear regression method. The optimum weights are calculated for a different combination of η and δ. The combination, resulting in the lowest prediction error, represents the final solution for all five parameters. The pseudo-code of the heuristic algorithm is detailed as follows, where RMSE denotes root mean square error between predicted and observed values and is explicitly defined in the next section.

The Linear Regression Heuristic Algorithm

Given Timetable and AVL data
Set *the suitable range for* $\boldsymbol{\eta}$ *and* $\boldsymbol{\delta}$ ($\eta \epsilon \{1, \ldots, H\}, \delta \epsilon [0, \Delta]$)
Discretize *the continuous range of* $\boldsymbol{\delta}$ *into* ***M*** *discrete counterparts*
for $a = 1, \ldots, H$
  for $m = 1, \ldots, M$
    Set $\boldsymbol{\eta = a}$ *and* $\boldsymbol{\delta = m\Delta/M}$
    Calculate $\boldsymbol{t^{c}_{k,s_p,s_m}}, \boldsymbol{t^{r}_{k,s_p,s_m}}$ ***and*** $\boldsymbol{t^{h}_{k,s_p,s_m}}$ $\forall s_p, s_m \in S, p \prec m$ *and* $k \in K$
    Given ($\boldsymbol{\Pi^{o}_{k,s_m} - \Pi^{o}_{k,s_p}}$)$\forall s_p, s_m \in S, p \prec m$ *and* $k \in K$
    Deploy Linear Regression to estimate weights in Eq 11
    Calculate $RMSE^{\boldsymbol{a,m}}$ *obtained by* $\left(\hat{\beta}^{a,m}_{c}, \hat{\beta}^{a,m}_{r}, \hat{\beta}^{a,m}_{h}, a, m\Delta/M\right)$
    end for
  end for
Find $\arg\min_{a,m} RMSE^{a,m}$
Return the $\boldsymbol{\eta}$ ***and*** $\boldsymbol{\delta}$ *values and estimated weights as estimates for the hybrid model*

3.3. Model evaluation

First, the prediction method performance is evaluated on the basis of its capability to predict actual departure times. Second, prediction errors are benchmarked against the errors yield by static information (timetable) and a commonly deployed prediction method. The combination of this two is instrumental in assessing prediction discrepancy and the added value of the hybrid scheme in generating short-term predictions. Finally, the model is validated by testing the performance of the prediction scheme with the estimated parameter values when applied on a different dataset.

3.3.1. Prediction accuracy

Root mean square error is used in this study as the primary measure of performance in evaluating the discrepancy between predicted and actual departure times. Deployment of RMSE penalizes variance in prediction error in which larger residuals obtain higher weight than smaller ones. The RMSE is computed at the stop-to-stop level over all trajectory predictions that were made at stop s_p for stop s_i:

DOI: 10.1002/atr

$$RMSE^{s_p,s_i} = \sqrt{\frac{\sum_{k \in K} \left[\min\left(0, \Pi^f_{k,s_p,s_i} - \Pi^o_{k,s_i}\right) + \alpha \cdot \max\left(0, \Pi^f_{k,s_p,s_i} - \Pi^o_{k,s_i}\right) \right]^2}{|K|}} \tag{12}$$

As late arrival is more adverse than earliness from operational management approach, a higher weight is assigned to overestimations of the departure time in order to reflect the operator perspective in the evaluation process. In this study, the penalty associated with overestimated predictions is doubled in the RMSE formula; hence, $\alpha = 2$. Conversely, higher weight could be assigned to underestimations to tailor predictions to passengers' perspectives (e.g. $\alpha = 0.5$).

This measure can then be aggregated at the origin and destination levels as well as for all predictions made along the route. The aggregate RMSE value could be summarized over all stops and their respective upstream predictions as

$$RMSE = \frac{1}{|S| - 1} \sum_{p=1}^{|S|-1} \left(\frac{1}{|S| - p} \sum_{m=p+1}^{|S|} RMSE_{s_p,s_i} \right) \tag{13}$$

This global measure of performance enables us to directly compare the performance of alternative prediction methods in terms of root mean square prediction error. Consequently, this performance indicator is used as the objective function in the optimization process when estimating model parameters.

3.3.2. Added value of hybrid method

The relative improvement of the proposed model over the static timetable information and a conventional prediction method is used to assess its potential added value. On the basis of the timetable, the operator expects trip k to arrive at stop s_i at time Π^c_{k,s_i}, regardless of the time and place that the prediction is made. This can be formulated as

$$\Pi^f_{k,s_p,s_i} = \Pi^c_{k,s_i}; \qquad \forall p \prec i \tag{14}$$

Alternatively, operator expectations may rely on a delay conservation prediction method, which is commonly used for generating real-time passenger information [31]. According to the delay conservation method, the predicted departure time of trip k from stop s_i generated at bus departure time from stop $s_p \left(\Pi^o_{k,s_p}\right)$ is

$$\Pi^f_{k,s_p,i} = \begin{cases} \Pi^c_{k,s_i} & if \left(\Pi^o_{k,s_p} < \Pi^c_{k,s_p} \wedge \exists j, p < j \leq i : s_j \in \hat{S}\right) \\ \Pi^o_{k,s_p} + \left(\Pi^c_{k,s_i} - \Pi^c_{k,s_p}\right) & \text{otherwise} \end{cases} \tag{15}$$

The scheduled departure time is used for prediction if the vehicle runs ahead of schedule and there is an intermediate time point stop. In all other cases, the fundamental assumption underlying this prediction method is that the latest schedule deviation will be maintained.

3.3.3. Model validation

A sound validation of the developed hybrid prediction method is essential prior to any further applications. In the case of a prediction method, the result of the validation process should reflect a degree of consistency for predicted values. The prediction model is validated by predicting bus

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trajectories for a period other than the one used for model calibration, while model parameters are specified on the basis of the values estimated in the calibration phase.

The validation results are assessed by comparing the forecasted vehicle trajectories with the corresponding actual departure times available from the respective AVL dataset. The average value of the mean absolute error (MAE), maximum relative error (MRE) and RMSE are calculated for the validation experiment. The RMSE is computed as described in Section 3.3.1, and MAE and MRE are calculated for all predictions, as follows:

$$MAE = \frac{1}{|K|*|S|*(|S|-1)} \sum_{k \in K} \sum_{p=1}^{|S|-1} \sum_{i=p+1}^{|S|} \left| \Pi^{f}_{k,s_p,s_i} - \Pi^{o}_{k,s_i} \right| \tag{16}$$

$$MRE = \max \frac{\left| \Pi^{f}_{k,s_p,s_i} - \Pi^{o}_{k,s_i} \right|}{\Pi^{o}_{k,s_i} - \Pi^{o}_{k,s_p}} \qquad \forall s_p, s_i \in S, p \prec i \text{ and } k \in K \tag{17}$$

In addition, the degradation in prediction accuracy due to the usage of parameters that were not specifically tailored for the analysis period is assessed by comparing the aforementioned measures with those obtained when optimizing their values for the validation period.

4. CASE STUDIES

A total of five bus routes (three in Stockholm, Sweden, and two in Brisbane, Australia) were chosen as case studies for the proposed hybrid prediction scheme. The two distinctive case study areas allow testing the performance of the hybrid scheme under different operational and urban environments. Moreover, the five routes vary in their length, demand patterns and service headway. This diversity is important for investigating model transferability. The prediction scheme was tested for three time-of-day periods: morning peak (07:00–09:00 h), off peak (09:00–15:00 h) and afternoon peak (15:00–19:00 h), in order to assess model performance under different levels of passenger demand and traffic congestion.

4.1. Stockholm, Sweden

The first case study area is located in Stockholm, Sweden. Four trunk bus routes (labelled 1 to 4) operate in Stockholm inner city. These routes are the busiest and the most frequent bus routes in Stockholm and enhance the backbone of the rapid rail services by offering frequent connections between main points of interest and transport hubs along the main urban arterials. They account for 58% of the total passenger-kilometre by bus in the inner city.

Prediction methods are applied and evaluated for both directions of three of the trunk bus routes: 1, 2 and 4. Route 3 was excluded from the analysis because of incomplete data. Table I presents a summary of route characteristics, and Figure 4 presents the case study routes and their corresponding stops. Time

Table I. Summary of route characteristics in Stockholm.

Route	Distance (km)	No. stops	Planned headway [min, max]; avg (minutes)	Stop distance avg; std; [min, max] (meters)	No. data records
1_Eastbound	11.5	33	[5, 7]; 6	335; 133; [166, 750]	181 104
1_Westbound	11	32	[5, 7]; 6	340; 118; [169, 754]	159 040
2_Northbound	7.9	22	[5, 7]; 6	380; 158; [163, 803]	125 352
2_Southbound	8	24	[5, 7]; 6	342; 145; [149, 718]	116 402
4_Northbound	12.1	31	[4, 6]; 5	410; 243; [213, 1264]	182 100
4_Southbound	12.1	30	[4, 6]; 5	418; 239; [223, 1239]	186 124

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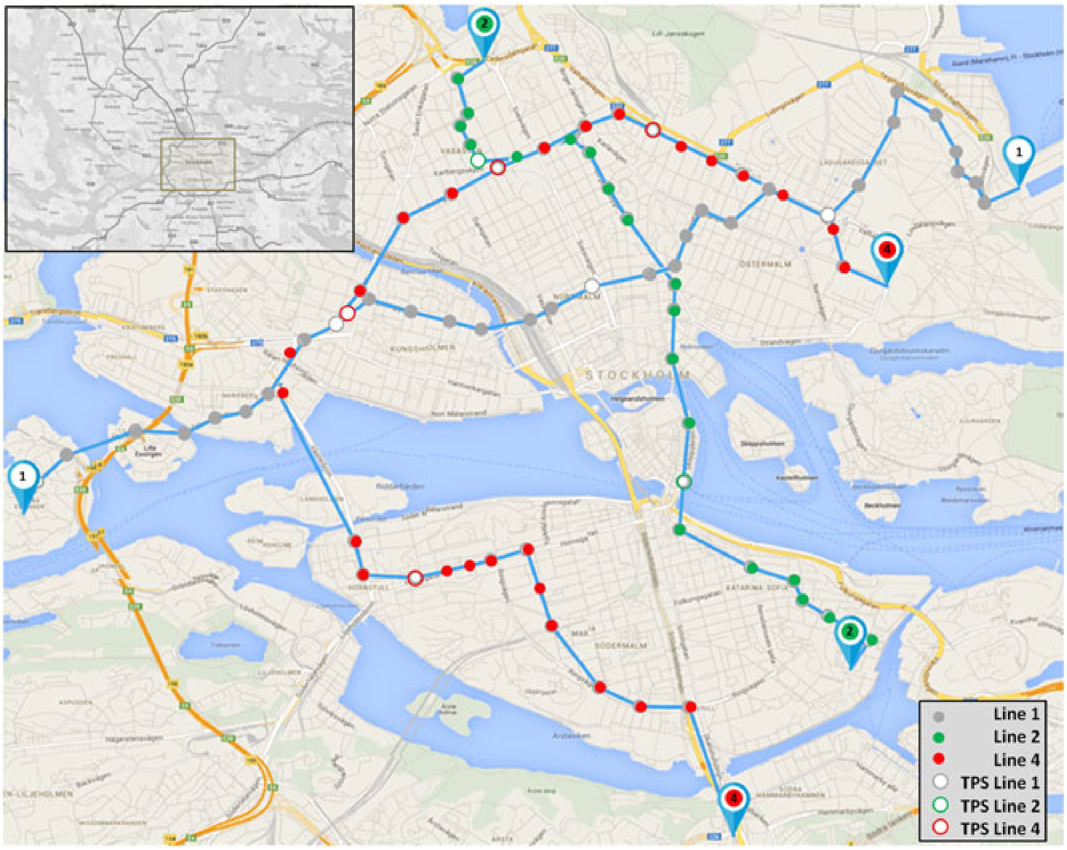

Figure 4. Case study network—Stockholm. TPS, time point stop.

point stops are also visible. These stops are of particular importance from operator's perspective when evaluating the prediction accuracy because they are used as driver relief points and potential locations for fleet management strategies. Route 1 connects the main eastern harbour to a residential area in the western part of the city through the commercial centre. Route 2 serves as a south–north connection through Stockholm's old city. Its route provides a cross-radial service on the main urban arterials. Route 4 is the longest route with a daily ridership of 60 000 passengers, the highest in Sweden, which is about 22% of all travel by bus in Stockholm inner city, while the share for routes 1 and 2 are 13% and 10%, respectively [32].

The AVL data were available for this study from all buses running on routes 1, 2 and 4. AVL devices report the time and location of bus arrival and departures from each stop along the route. We consider two datasets: (i) AVL data from 1 Dec 2011 to 31 Jan 2012 for all four routes and (ii) AVL data from 1 Aug 2011 to 30 Oct 2011 for route 1. A schedule-based control strategy was implemented in Stockholm during the analysis period. Only records generated on working days are used in the analysis. The datasets contain a total of 1 546 721 data records of which 596 599 are of the dataset used for validation, each record representing a stop visit. The second dataset is used solely for model validation. The validation of the hybrid model using AVL data is explained in detail in Section 6.3.

4.2. Brisbane, Australia

Brisbane (the capital city of Queensland) is the third largest city in Australia. Translink, a division of the Department of Transport and Main Roads, is the regulator for the entire South East Queensland public transport [33]. The network has an integrated ticketing options allowing for a seamless travel between Translink buses, train, ferries and trams. For this study, two routes-route 60 (known as Blue CityGlider) and route 555 from Brisbane-are selected (refer to Figure 5; the top left map illustrates the

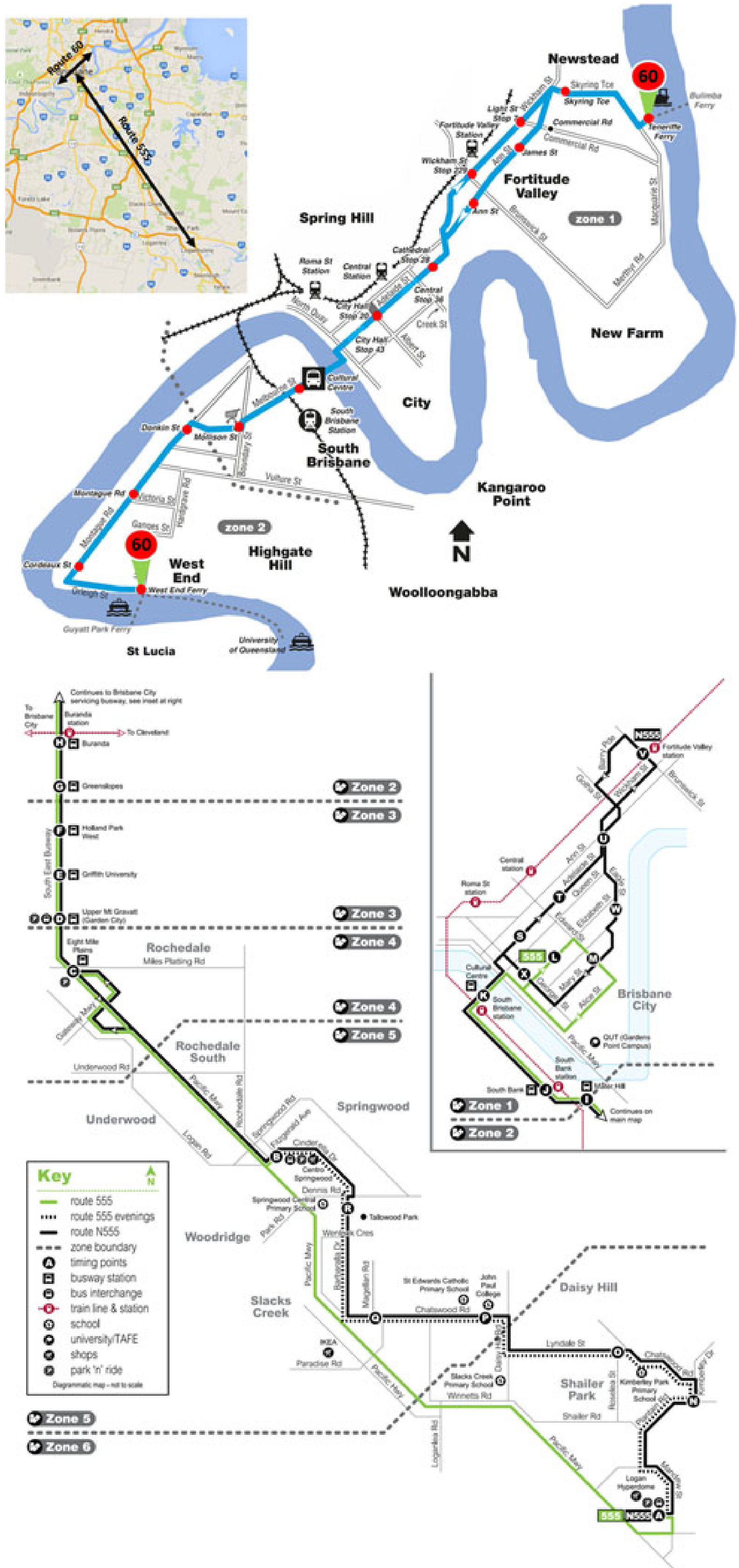

Figure 5. Route 60 and route 555—Brisbane.

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relative geographical locations of the two routes; the top route map is for route 60 and the bottom route map is for route 555).

- Route 60 connects Brisbane central business district and its nearby north-east and south-west suburbs. It is one of the highest-frequency bus services in Brisbane. It runs every 5 minutes between 07:00–09:00 h and 16:00–18:00 h on weekdays, and every 10 to 15 minutes during other operation hours.
- Route 555 connects Logan city (Loganholme station) with Brisbane central business district (Elizabeth Street stop) with a frequency of four buses per hour serving 12 stops inbound and 13 stops outbound. The route spans over both busway and motorway. The green line in Figure 5 for route 555 indicates the corresponding path to the considered time period (07:00–19:00 h) in the case study.

Table II presents a summary of these two route characteristics.

5. IMPLEMENTATION

The proposed linear regression heuristic was applied for estimating the values of the hybrid model parameters. The aggregate RMSE value calculated using Equation (13) is used as the objective function for evaluating each individual solution, which consists of a combination of η and δ and obtained weights (β_c, β_r and β_h) from linear regression. Considering the aforementioned criteria in Section 3.2, suitable ranges for the instantaneous and historical predictors' parameters were defined ($\eta \epsilon \{1, \ldots, 4\}, \delta \epsilon [0, 1]$). Then, the range for δ is discretized into 10 distinct counterparts ($M = 10$). Forty different combinations of η and δ are hence considered in the estimation procedure.

There are a few issues that require special consideration when processing AVL data. In both case study areas, each route is also operated by several partial service patterns-traversing only a subset of the route's stops. The respective AVL records require special treatment in order to ensure the correct inclusion of partial trips in the prediction scheme. In addition, some trips have incomplete information for some stops. This happens occasionally in case the bus drives through the stop without stopping. In such cases, missing records are generated by linear interpolating on the basis of the distances between successive stops. Finally, bus drivers are in rare cases instructed by control centre dispatchers to terminate their trip and disembark passengers, a fleet management strategy known as short turning. In these unusual cases, no records are added and the trip is handled as a partial trip. The estimation of model parameters, generation of predictions and model evaluation were all programmed in MATLAB.

6. RESULTS

The proposed hybrid method was applied to five bus routes in the case study areas of Stockholm and Brisbane, enabling the investigation of its performance under different circumstances. Model parameters were estimated separately for each bus route direction on the basis of the estimation method described in Section 3.2. We first report the attained values for the model parameters-weights of prediction elements ($\beta_c, \beta_r, \beta_h$), number of considered buses in instantaneous part (η) and the tolerance parameter in historical part (δ)-and discuss the influence of route characteristic and time-of-day periods (Section 6.1). The added value of the hybrid method is then assessed by comparing its performance with the reference cases (Section 6.2). Finally, the validity of the method is examined using a separate dataset (Section 6.3).

Table II. Summary of route characteristics in Brisbane.

Route	Distance (approx.) (km)	No. stops	Planned headway [min, max]; avg (minutes)	Stop distance (approx.) avg; std; [min, max] (m)	No. data records
60_Inbound	8	13	[5, 15]; 10	700; 220; [450, 1230]	31 512
60_Outbound	8	13	[5, 15]; 10	700; 200 [370, 1110]	32 409
555_Inbound	30	12	[15, 15]; 15	2800; 2400 [800, 8800]	57 984
555_Outbound	35	13	[15, 15]; 15	3000; 3500 [480, 13 500]	67 405

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6.1. Hybrid method calibration

Model calibration was performed separately for each route direction and time-of-day period (morning, off peak, evening). The estimated values of the hybrid method parameters are listed in Table III. The objective function value-the average over the RMSE values of all the predictions in the study period (Equation (13))-is reported for the final solution obtained from the heuristic algorithm. The aggregate RMSE reports the overall error for the prediction method, quantifying prediction accuracy. The lower the RMSE, the more trustworthy it is for estimating passenger waiting time and as a real-time management tool for operators. In the case of route 4 in Stockholm and routes 60 and 555 in Brisbane, a lower prediction error is attained in the off-peak periods when compared with that in the morning peak and evening peak. These routes run along radial corridors that are characterized by peak commuting congestion, where off-peak periods are more predictable owing to lower levels and lesser fluctuations in passenger demand and traffic flow. In contrast, routes 1 and 2 in Stockholm serve the main streets of the commercial core, where the differences among time-of-day periods are less pronounced.

As evident in Table III, the parameter values vary over routes and time periods. The historical predictor parameter, β_h, ranges from a low weight of 26% for the inbound direction of route 60 at the off peak to 82% for the northbound direction of route 4 for the off-peak period. The relative contribution of instantaneous data varies from 15% for the northbound direction of route 4 to 72% for the inbound direction of route 60, both under the off-peak period. For all routes and time periods, the scheduled travel time is assigned with the lowest share compared with historical and instantaneous data. This indicates that in most cases, instantaneous and even archived AVL records are more indicative of the prevailing traffic conditions than the untrustworthy schedule. This is especially remarkable in the Stockholm case, where drivers are instructed to hold until the scheduled time at selected time point stops.

Table III. Estimated parameters of the hybrid prediction method.

Route	Time period	β_c	β_r	β_h	δ (hours)	η	Objective function value
1_Eastbound	Morning peak	0.06	0.42	0.52	0.5	2	105.8
	Off peak	0.05	0.50	0.45	1	4	105.8
	Evening peak	0.05	0.39	0.56	0.7	4	112.1
1_Westbound	Morning peak	0.04	0.31	0.65	0.4	2	91.8
	Off peak	0.05	0.34	0.61	0.7	4	94.8
	Evening peak	0.04	0.42	0.54	0.6	4	105.1
2_Northbound	Morning peak	0.06	0.31	0.63	0.8	2	95.8
	Off peak	0.06	0.29	0.65	0.8	4	98.8
	Evening peak	0.06	0.26	0.67	0.7	4	95.7
2_Southbound	Morning peak	0.03	0.20	0.77	0.4	2	85.5
	Off peak	0.04	0.21	0.75	1	4	80.2
	Evening peak	0.03	0.30	0.67	0.7	4	92.2
4_Northbound	Morning peak	0.05	0.31	0.64	0.6	2	105.0
	Off peak	0.04	0.15	0.82	1	2	89.3
	Evening peak	0.05	0.23	0.72	0.9	4	103.3
4_Southbound	Morning peak	0.05	0.26	0.69	0.7	2	106.8
	Off peak	0.05	0.55	0.40	0.9	4	97.3
	Evening peak	0.04	0.60	0.35	0.9	4	135.2
60_Inbound	Morning peak	0.09	0.37	0.54	0.7	4	131.4
	Off peak	0.02	0.72	0.26	0.5	4	108.2
	Evening peak	0.04	0.69	0.27	0.4	4	151.5
60_Outbound	Morning peak	0.06	0.46	0.49	0.3	2	111.4
	Off peak	0.00	0.69	0.31	0.2	4	88.2
	Evening peak	0.07	0.53	0.40	0.7	4	140.6
555_Inbound	Morning peak	0.04	0.60	0.36	1	1	180.6
	Off peak	0.00	0.49	0.51	1	2	88.6
	Evening peak	0.02	0.38	0.60	1	1	106.2
555_Outbound	Morning peak	0.06	0.39	0.55	0.9	2	311.3
	Off peak	0.03	0.36	0.61	1	4	84.1
	Evening peak	0.03	0.36	0.61	1	4	84.1

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Although all the bus routes in our case study in Stockholm operate in the inner city and are all high-frequency trunk routes, parameter values vary. This suggests that the results of the prediction method are dependent on route's characteristics such as day-to-day and within-day regularity of traffic conditions and passenger demand. Therefore, the model should be calibrated for routes with different characteristics.

In Brisbane, instantaneous travel time and historical travel time generally take the highest weights for route 60 and 555, respectively. This indicates that the accuracy of model based on *timetable* in Brisbane is generally low. This finding should encourage further study to evaluate the timetable in Brisbane. Our observation on the timetable for route 555 indicates that the trip travel time, as per the timetable, for route 555 during peak period considers only 5 minutes of the delay compared with that during off peak (e.g. trip staring at 07:15 h from Loganholme station should arrive at 08:00 h at Elizabeth Street (travel time 45 minutes) and 10:15 h from Loganholme station should arrive at 10:55 h at Elizabeth Street (travel time 40 minutes)). Past research on the day-to-day variability of bus travel time during peak and off-peak period in Brisbane is reported over 40% and around 20%, respectively [34]. High day-to-day variability generally makes timetable less effective.

The value for the historical travel time window, δ, fluctuates between ±12 minutes (0.2 hour) to 1 hour. The minimum value has been obtained for the inbound direction of route 60 at the off peak, while $\delta = 1$ is reported for several instances. In addition, the most common value for η is 4. Services with lower frequency may need wider time window (larger tolerance) to capture sufficient number of observations from historical data. This is demonstrated by the larger values of δ for route 555, which has the lowest frequency among our case study routes.

6.2. Added values of the proposed method

Bus departure time predictions were generated for the case study routes for all downstream vehicle trajectories by applying the hybrid method. In addition, the prediction accuracy when relying on the timetable and the delay conservation method is evaluated by calculating the aggregate RMSE value for each prediction method. Table IV presents the results for each of the five bus routes in both directions, and the percentage difference obtained by hybrid method compared with the timetable and the delay conservation method is presented in brackets on the last column. The results reported for the hybrid prediction method are based on the estimated parameter values reported in Table III.

The hybrid method yields the highest prediction accuracy and consequently more reliable service. In this context, reliability refers to deviation of actual departure time from predicted value. Reliability improvement hence enables passengers to reduce their waiting time. The added value of the hybrid method varies considerably among bus routes and time periods. The deployment of the hybrid method reduced the prediction error from [215, 1608] and [136, 710] seconds for timetable and delay conservation predictions, respectively, down to [80, 311] seconds when calculated over bus trajectories of different routes. Overall, the accuracy of predictions obtained from the hybrid method is improved by 72% and 48% when compared with timetable and delay conservation prediction method. The largest relative improvement in prediction accuracy is observed in the case of the inbound direction of route 555 in Brisbane during the morning-peak period, with a decrease of 89% and 75% in the RMSE compared with the timetable and delay conservation predictions. Bus departure time predictions, also when generated by the hybrid scheme, are in general more reliable for the Stockholm case study routes. This is presumably attributed to the more balanced demand patterns and lesser traffic congestion as well as the more rigorous control scheme.

The performance of the aforementioned methods is further investigated by analyzing how their performances evolve spatially. Figure 6 shows the RMSE values for the timetable, delay conservation and hybrid methods along each route (note that the scale varies for different routes). It can be observed that the hybrid method yields persistently more accurate predictions than the timetable and delay conservation methods with only few exceptions. However, the magnitude of the improvement varies considerably. Also, regardless of the prediction method considered, some of the routes are consistently more predictable than others as discussed earlier. In Stockholm, the prediction error of the delay

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Table IV. Root mean square error (in seconds) of prediction methods.

Route	Period	Timetable	Delay conservation	Hybrid
1_Eastbound	Morning peak	374.5	201.1	105.8 [−72%; −47%]
	Off peak	442.5	207.7	105.8 [−76%; −49%]
	Evening peak	446.6	216.8	112.1 [−75%; −48%]
1_Westbound	Morning peak	374.3	165.0	91.8 [−75%; −44%]
	Off peak	408.4	176.9	94.8 [−77%; −46%]
	Evening peak	406.2	191.7	105.1 [−74%; −45%]
2_Northbound	Morning peak	263.7	145.4	95.8 [−64%; −34%]
	Off peak	295.9	160.9	98.8 [−67%; −39%]
	Evening peak	326.0	155.4	95.7 [−71%; −38%]
2_Southbound	Morning peak	304.0	177.7	85.5 [−72%; −52%]
	Off peak	264.2	151.6	80.2 [−70%; −47%]
	Evening peak	343.0	178.4	92.2 [−73%; −48%]
4_Northbound	Morning peak	331.3	162.6	105 [−68%; −35%]
	Off peak	366.0	136.1	89.3 [−76%; −34%]
	Evening peak	628.1	162.9	103.3 [−84%; −37%]
4_Southbound	Morning peak	474.2	205.4	106.8 [−77%; −48%]
	Off peak	366.3	165.1	97.3 [−73%; −41%]
	Evening peak	668.1	282.6	135.2 [−80%; −52%]
60_Inbound	Morning peak	234.1	228.8	131.4 [−44%; −43%]
	Off peak	215.1	196.1	108.2 [−50%; −45%]
	Evening peak	508.6	292.3	151.5 [−70%; −48%]
60_Outbound	Morning peak	237.2	261.7	111.4 [−53%; −57%]
	Off peak	223.2	198.5	88.2 [−60%; −56%]
	Evening peak	458.8	309.6	140.6 [−69%; −55%]
555_Inbound	Morning peak	1608.2	709.9	180.6 [−89%; −75%]
	Off peak	392.0	187.9	88.6 [−77%; −53%]
	Evening peak	532.8	207.5	106.2 [−80%; −49%]
555_Outbound	Morning peak	1588.8	668.7	311.3 [−80%; −53%]
	Off peak	545.7	180.0	84.1 [−85%; −53%]
	Evening peak	828.3	322.2	84.1 [−90%; −74%]

conservation method tends to increase further downstream because the delay preservation assumption is more likely to be invalidated the longer the prediction horizon is. In Brisbane, two distinctive patterns are observed for the relation between the prediction errors of the timetable and delay conservation method. For route 60, the prediction errors of both schemes follow the same pattern, suggesting that the error is induced by the scheduled travel times. In contrast, the prediction error of the delay conservation method levels with the hybrid scheme rather than with the timetable. This pattern arguably pertains to unreliable dispatching times while the travel times follow closely the scheduled ones.

6.3. Validation

The results presented in Section 6.2 demonstrate that the proposed hybrid method outperforms the reference methods. The transferability of the model is also confirmed for different route layouts, passenger demands and operation practices. The average computational time for generating a prediction is 5×10^{-4} seconds and is thus well suited for real-time applications. However, the calibration of the hybrid model parameters is computationally expensive (e.g. approximately 1 hour for route 1, eastbound direction, morning peak). Therefore, it is necessary to test the following hypothesis to validate model capability for real-time application: 'Estimated values for the proposed hybrid model parameters can be successfully applied for generating future predictions.'

The parameters estimated using the first dataset for route 1 were applied to the second dataset for the same route to test its validity for future predictions (both datasets are described in Section 4.1.). Furthermore, model parameters were also estimated for the second dataset to assess the extent of performance deterioration caused by the usage of a non-tailored hybrid method. Table V presents the performance indicators described in Section 3.3.3 for both cases. Evidently, prediction accuracy

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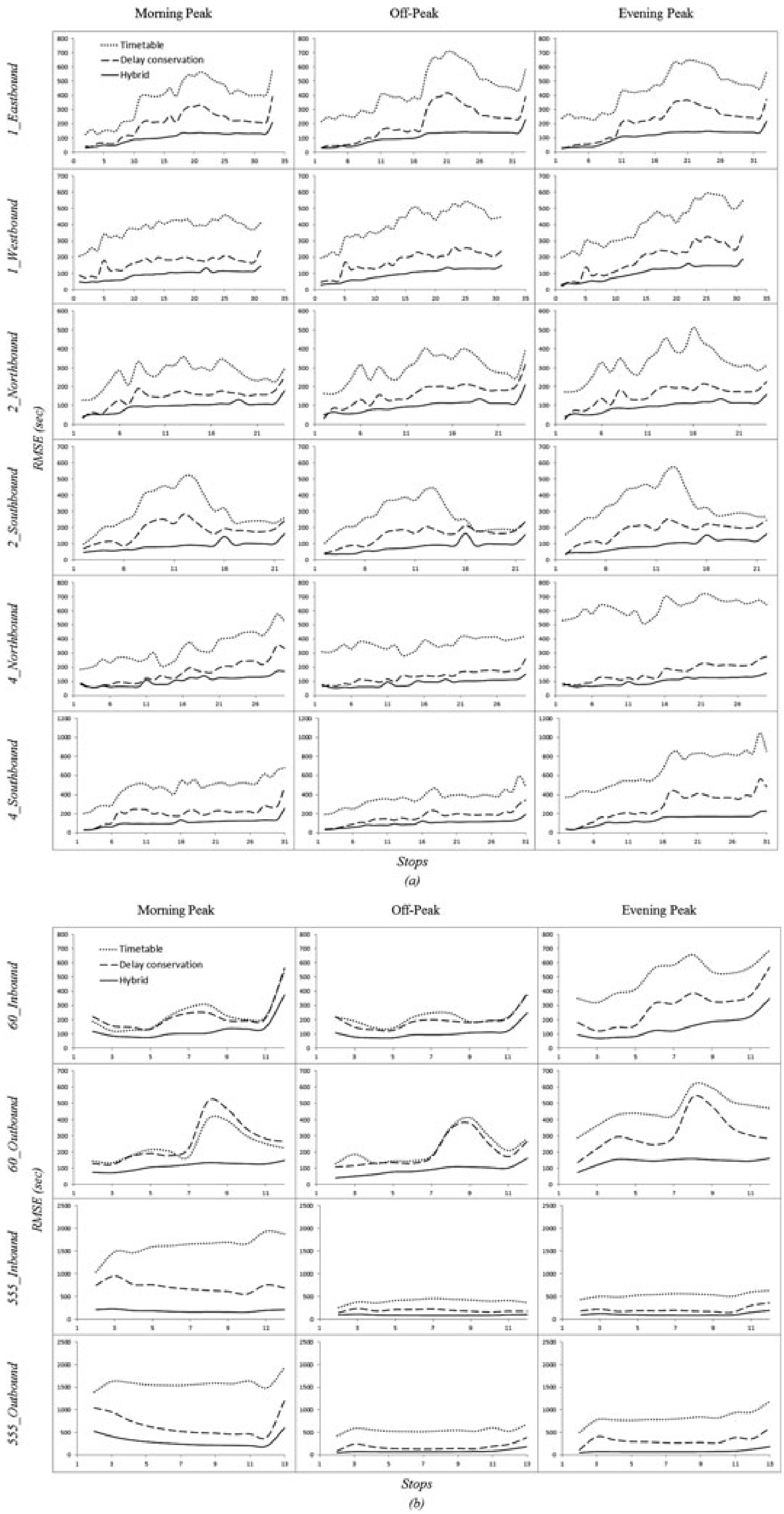

Figure 6. RMSE (y-axis) of the timetable, delay conservation and hybrid prediction schemes along each bus route (x-axis) during each of the time-of-day periods in (a) Stockholm and (b) Brisbane. RMSE, root mean square error.

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Table V. Validation results for route 1 in Stockholm.

Route		RMSE (seconds)			MRE (%)			MAE (seconds)		
		7–9	9–15	15–17	7–9	9–15	15–17	7–9	9–15	15–17
1_Eastbound	Calibration	113.0	105.0	137.8	8.0	6.8	12.0	80.5	76.4	84.4
	Validation	114.8	105.2	140.9	7.9	6.7	12.2	81.1	76.3	83.7
1_Westbound	Calibration	91.3	93.6	109.5	4.0	4.0	3.6	64.3	66.7	76.0
	Validation	92.7	93.8	109.9	4.0	4.0	3.6	64.7	66.8	75.6

RMSE, root mean square error; MRE, maximum relative error; MAE, mean absolute error.

is almost unaffected—a maximum of 3-second difference in RMSE—by the application of model parameters that have been estimated for a different period. The mean absolute error even decreased slightly for evening-peak period. However, no significant change was observed in maximum error with the MRE remaining almost unchanged in both applications. The validation results indicate that the hybrid method is robust with respect to the estimated parameter values, confirming the aforementioned hypothesis. Hence, the computational effort associated with model calibration can be conducted once at the route level and then be applied in future predictions, enabling real-time applications of the hybrid prediction method.

7. CONCLUSION

Various hybrid models have been developed in the last two decades in the context of freeway travel time prediction. These hybrid models combine different predictors and proved to outperform individual models [8, 29, 30]. Hitherto, no such model has been proposed and evaluated in the public transport domain. A hybrid model was constructed in this paper on the basis of a linear combination of schedule, instantaneous and historical predictors. Model specification requires estimating the contribution of each predictor as well as parameters associated with the extent of instantaneous and historical data to be considered in the prediction.

The proposed hybrid method was applied to five bus routes in the case study areas of Stockholm and Brisbane. Prediction accuracy of the hybrid method is then assessed by comparing its performance with that of alternative prediction methods that are used in practice. Overall, the hybrid method results with an improvement of 72% and 48% in prediction accuracy when compared with timetable and delay conservation prediction method, respectively. The accuracy level attained by the proposed prediction scheme is 132.5, 93.5 and 112.6 seconds in morning-peak, off-peak and evening-peak periods, respectively. More reliable predictions can potentially result with operational efficiency gains, and improvement in users' satisfaction and their loyalty to the system. Although the added value for the hybrid method varies among bus routes and time periods, the results confirm the transferability of the model for different route layouts, passenger demands and operation practices. Model validation suggests that model parameters can be estimated once and then used in subsequent applications with high accuracy, enabling real-time applications. While model parameters are estimated separately for different route directions and time-of-day periods, the parameters are used uniformly throughout the route. The prediction model could be further enriched by updating individual predictors' weights dynamically, for example, by incorporating time-series analysis methods. This learning mechanism will allow the prediction scheme to evolve so that the contribution of each predictor on the basis of its recent performance is dynamically altered. The performance of such improvements can be benchmarked against state-of-the-art machine learning algorithms for bus travel time predictions.

The hybrid method proposed in this study can embrace a wide range of predictors and be used in various applications. The method could be extended to predict total journey time to support travel journey planners by inferring expected headways and thus deduce the transfer times. Moreover, the prediction method could be embedded in a decision support system to facilitate the implementation of proactive fleet management and control measures.

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8. SYMBOL DEFINITION

k	Bus trip
s_i	Bus stop i
Π^c	Matrix of scheduled departure times
Π^o	Matrix of observed departure times
Π^f	Matrix of predicted departure times
Π^c_{k,s_i}	Scheduled departure time of trip k at stop s_i
Π^o_{k,s_i}	Observed departure time of trip k at stop s_i
Π^f_{k,s_p,s_i}	Predicted departure time of trip k from stop s_i generated when the bus departs from stop s_p
S'	Set of time point stops
$\hat{t}_{k,s_{i-1}}$	Predicted travel time for trip k connecting two successive stops, s_{i-1} to stop s_i
t^c_{k,s_i}	Scheduled travel time prediction for trip k on road segment connecting stops, s_{i-1} and s_i
t^r_{k,s_i}	Instantaneous travel time prediction for trip k on road segment connecting stops, s_{i-1} and s_i
t^h_{k,s_i}	Historical travel time predictions for trip k on road segment connecting stops, s_{i-1} and s_i
β_c	Weight assigned to a scheduled travel time prediction
β_r	Weight assigned to an instantaneous travel time prediction
β_h	Weight assigned to a historical travel time prediction
η	Number of downstream bus travel times which is considered in instantaneous prediction
t^*	Reference point of a time window in a historical travel time prediction
δ	Tolerance parameter of a time window in a historical travel time prediction
Md	Median

9. ABBREVIATION

APTS	Advanced Public Transport Systems
ANN	Artificial Neural Networks
SVM	Support Vector Machines
AVL	Automated Vehicle Location
APC	Automatic Passenger Count
AFC	Automatic Fare Collection
TPS	Time Point Stop
RMSE	Root Mean Square Error
MAE	Mean Absolute Error
MRE	Maximum Relative Error

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