Experimental Characterization and Analytical Modelling of Rotor Tonal Noise

MSc Thesis Report

by

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Preface

"When defeat comes, accept it as a signal that your plans are not sound, rebuild those plans, and set sail once more toward your coveted goal" - Napoleon Hill

"Taking pride in one's duty is the key to ethical professionalism" - Dr. A.P.J. Abdul Kalam

This project has been partially performed at ENSTA ParisTech, France for 6 months and at Delft University of Technology for 3 months. First and foremost, I would like to express my deepest gratitude to my thesis supervisors, Dr. Benjamin Cotté (ENSTA ParisTech) and Dr. Daniele Ragni (Delft University of Technology), for allowing me to pursue this project. Dr. Benjamin has been a great mentor to me, he has been really supportive both on personal as well as professional fronts. During the project, there were regular update meetings with Dr. Daniele, to discuss the progress of the project. Dr. Daniele has constantly helped during this project with his valuable feedback and expertise. Furthermore, I would like to thank my daily supervisor at ENSTA ParisTech, Tommy Rigall (Ph.D. candidate). He has been extremely kind, helpful and patient throughout this project. He has contributed to this project in many ways with his enthusiastic interests in my work and helped me to critically solve the problems in the project. It was a pleasure working with you all, I learned a lot during this project.

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Regards,
Chakshu Deora
Rotors have played a huge role in the history of the aerospace industry. It is being widely used in small scale aircraft, helicopters, ships, wind turbines, to name a few. The study of noise generated by the rotors is an important subject of research. This project aims to characterize the open rotor tonal noise using the experimental and analytical model with a primary focus on Blade Tower Interaction (BTI) noise.

The objective of the project is to implement an analytical acoustics model with input from an aerodynamics solver for calculating the noise radiation from a rotating blade. Fan noise experiments are conducted in the anechoic chamber at IMSIA, ENSTA ParisTech, France, which are used to validate the model and also characterize BTI noise. The acoustic model is based on Ffowcs-Williams and Hawkings (FW-H) acoustic analogy for moving sources. The analysis is performed in the frequency domain. The implemented model can be used to obtain noise radiations from propeller, wind-turbines or any rotating source, given the appropriate aerodynamic inputs. The implementation of the model is validated with the benchmark results in literature.

The steady loading noise predicted by the analytical model matches very well with the experiments at first blade passage frequency (BPF) harmonic. The experiments offer a deep insight into the mechanism of noise radiation focusing mainly on tonal components associated with blade tower interaction. Different configurations with variation of pitch angle, blade tower distance are employed to study the impact on BTI noise. The effect of BTI noise is more pronounced at smaller blade tower distance and is almost negligible in front of the rotor when the blade tower distance is equal to the tower diameter. Further, pressure taps are used to study the pressure variations on the tower as the blade passes. An empirical model is suggested based on the pressure tap results to mimic the pressure variations on the tower.

**Keywords:** Rotor tonal noise, Blade tower interaction noise (BTI), Fan noise experiment, Analytical modeling, Ffowcs Williams and Hawkings
List of Abbreviations

**BEM**  Blade Element Momentum  
**BPF**  Blade Passage Frequency  
**BTI**  Blade Tower Interaction  
**FFT**  Fast Fourier Transform  
**FW-H**  Ffowcs Williams and Hawkings  
**PSD**  Power Spectral Density  
**RAS1**  Rotor Axis Set 1  
**RPS1**  Rotor Plane Set 1  
**RPS2**  Rotor Plane Set 2  
**SNR**  Signal to Noise Ratio
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Introduction

1.1. Background

A sound is a form of energy, defined as the vibrations that propagate in the form of longitudinal pressure waves through a medium such as air and water. These pressure waves are received by the human ear or animal’s ear and are interpreted by the brain. We, humans, use sound extensively in our day to day activities, such as communication, social interaction, music, education, and many others. It is hard to imagine a life without sound. However, there are numerous sources of unwanted sound or noise, which are annoying not only to humans but to animals as well. One of the noise sources is generated by the fluid flows. It is an important topic of research, which in itself is a very broad topic. Few examples can be aircraft noise, helicopter noise, wind turbine noise, and ship noise. The study of these noises can be categorized under the subject of aeroacoustics. As the name suggests, “aeroacoustics is the study of noise generation due to aerodynamic phenomena” [18]. It is a relatively new field of study in the history of aerospace engineering, with most of the research starting in 1950’s.

There is an enormous increase in energy required to supply for the recent technological advancements and increasing population. Over the last three decades, researchers are shifting the focus on the renewable sources of energy to meet the energy demands, of which, wind energy being one of the largest contributors. A wind turbine may produce a swishing/whooshing noise while in operations [36]. People living in close proximity to wind farms often complain about this modulated noise. Though there is no clear study on the health effects due to the noise, one of the common health impacts is sleep disturbance, which further might result in psychological distress [40, 45]. With the increase in air travel across the globe, people living close to the airport also feel annoyed by the aircraft noise during take-off and landing. In the case of aircraft, propeller aircraft generates more noise compared to the turbofan aircraft. Figure 1.1 presents the close response survey of annoyance due to different sources like a wind turbine, aircraft, road traffic, and railways. It can be observed that even at low sound pressure level (dBA), people tend to find wind turbine more annoying. Note that the annoyance is a subjective quantity, and can be different for each individual.

It is interesting to note that energy dissipated into sound is very small as compared to the overall energy requirement of an aircraft or wind turbine. So, the major motivation for conducting research in aeroacoustics is to reduce the noise in order to minimize the annoyance, but not to increase the efficiency.

For eliminating the noise, firstly, it is important to identify the sources of noise. Sir James Lighthill was the first one to identify or quantify turbulence as a source of the noise from jet aircraft. In 1952, he published the inhomogeneous wave equation by re-arranging the Navier-Stokes equation [29]. It relates the aerodynamic behavior of the flow with the acoustics and is famously known as Lighthill’s acoustic analogy. It serves as a starting point for most of the acoustic analysis. A physical interpretation of Lighthill’s analogy is given as the conversion of kinetic energy to acoustic energy and is represented by the small fluctuations in the flow of momentum across the surface [29].

Since 1952, there has been a great development in the field of aeroacoustics. Lighthill’s theory is used to derive the noise radiation of complex phenomena. The problem of jet noise has been mitigated with the introduction of turbofan engines in the 1970s [28]. With the quieter turbofan engines, more emphasis was put on other sources of noise such as landing gear noise, propeller noise, air-frame component noise, trailing edge noise, to name a few. This project revolves around the characterization of rotor tonal noises, which
1. Introduction

Figure 1.1: Annoyance level for different sources [37]

The focus of the project is to study the tonal noise of an open rotor. Along with steady sources of noise, BTI acts as an unsteady source of loading with a period of blade passing frequency (BPF) and hence acts a source of tonal noise. BTI noise mainly comprises of two effects: blade passage effect and velocity deficit effect, explained in Chapter 2. As the name suggests, BTI noise is most commonly studied for wind turbines. Though, the understanding of BTI can also help to extend the knowledge to understand propeller pylon interaction noise and helicopter rotor interaction with its tail. For the wind turbines, BTI is a source of low frequency noise or infra-sound. The importance and effect of low frequency noise is still a debatable topic [2, 43]. Moreover, experimental study on BTI noise is very limited in the literature or very recent [50], so it would be interesting to obtain further details on BTI noise with help of this project. Additionally, pressure taps are mounted on the tower to study the pressure variations as the blade passes, inspired by the results of Yauwenas [48].

1.2. Outline of the Report

This report is submitted in the partial fulfillment of masters thesis (AE5110) for obtaining the MSc Aerodynamics degree at Delft University of Technology. Chapter 2 presents the literature study related to the project and later, research objectives are defined. Chapter 3 focuses on experimental setup and procedures used in the project. Formulation of the analytical model is discussed in Chapter 4. Chapter 5 discusses the results obtained from the experiments and analytical model. Finally, Chapter 6 concludes the report with concluding remarks and future recommendations. Appendix A presents the far-field and near-field formulations for the analytical model. Appendix B explores the application of AeroDyn tool.
Rotors are found in a great deal of application in aerodynamics and hydrodynamics, such as, propellers (aircraft, marine, airships), fans, wind turbines, and helicopter rotor. A brief introduction to the aerodynamic sources of rotor noise is presented in this chapter.

For low Mach number applications such as wind turbine blades, axial fans and marine propeller, the noise can be divided into tonal (periodic) and broadband components. **Tonal noise** can mainly be identified with discrete frequencies occurring at blade passage frequency (number of blade times the rotational frequency) and its harmonics, whereas **broadband noise** is characterized by a continuous spectra over a wide range of frequencies, as shown in Figure 2.1. Tonal components can be easily identified by the discrete peaks presented in Figure 2.1. As the definition of broadband components suggests, it does not possess energy at any particular frequency. Energy is distributed over a wide range of frequencies.

![Figure 2.1: Tonal and Broadband components. Taken from Ref. [18]](image)

A broad classification of rotor noise based on tonal and broadband components is presented by Marte et al. [34], shown in Figure 2.2. For this thesis project, thrust and torque noise, thickness noise, trailing edge noise and blade tower interaction (wake and field interaction) are the main subject of interest. Further, only these sources are discussed in detail.

1. **Thrust and Torque Noise**: Thrust and torque (in Figure 2.2) noise can be considered as a part of loading noise. As the propeller rotates, it produces pressure fluctuations along its blades or airfoil section. This pressure distribution can be divided into thrust (axial - normal to the plane of rotation) and torque component (in the plane of rotation). For a rotating source like a propeller, loading noise can occur due to unsteady as well as steady loading. A propeller produces steady loading in propeller's reference frame when the inflow is uniform. For a stationary observer, the force component is fluctuating with the propeller's rotation. This fluctuating force appears as an oscillating pressure for a stationary observer and acts as a source of the noise.
2. Literature Study

Sources of unsteady loading can be distorted inflow, pitching propeller axis, helicopter advancing and retreating blades, fluctuations in the boundary layer. All these sources change the angle of attack of the blade as it rotates resulting in unsteady loading on the blade. So, the time variation of the loading produces noise. Unsteady loading noise is effective source of sound as compared to steady loading [18].

2. **Thickness noise** represents the noise due to volume displacement of the air owing to the rotation of the blade. It can also be understood as a periodic injection and ejection of mass in a fixed reference frame (similar to a monopole). It is dependent on the normal surface velocity and the profile of the blade (thickness). Thickness noise is generally an inefficient source of noise at low Mach number [34]. At high Mach number, it becomes an important source of noise and cannot be neglected. Thickness noise can be reduced by decreasing the thickness of the blade. Halving the thickness reduces the noise by 6 dB [18]. Acoustic radiation of thickness noise is maximum in its plane of rotation, as shown in Figure 2.3.

3. **Trailing Edge Noise (TEN)**: TEN is caused by the interaction of unsteady flow (turbulent boundary layer) with the sharp trailing edge. Turbulent flows scattered by a trailing edge is a very efficient source of noise as compared to the turbulence on the blade itself. It is widely studied that turbulent boundary layer TEN is a broadband source of the noise because turbulent flow contains eddies of different sizes. It also shows the randomness of pressure fluctuations in the turbulent boundary layer. TEN approxi-
mately scales with the fifth power of flow velocity \( (U^5) \) [18]. The scaling factor has also been validated experimentally by Zajamsek et al. [49] for NACA 0012 rotating airfoils for the majority of the Strouhal number range. TEN has a cardioid directivity pattern, as shown in Figure 2.4. Maximum noise is in the upstream direction of the airfoil and minimal amount of sound is radiated downstream.

![Figure 2.4: Directivity pattern for trailing edge noise [18]](image)

4. **Blade Tower Interaction (BTI)**

   ![Figure 2.5: Blade tower interaction noise in upwind configuration, Ref. [48]](image)

   This source of noise is commonly discussed for wind turbine applications, where the tonal noise is generated when the blade passes in front of the tower. However, this concept can be extended to propellers as well, where propeller mounting can act as an obstruction and be a source of the noise. It is an unsteady aerodynamic source of noise caused due to interaction of blade and tower. As the interaction of the blades with tower is periodic, a periodic source of noise is produced with periodicity depending on blade passage frequency (BPF). The mechanism of BTI noise generation can be attributed to the unsteady loading on both tower and blades. Two sources of aerodynamic unsteady loading are; a) reduced velocity field upstream of the tower b) blade passage effect.

   When the blade passes through **velocity deficit** region in front of the tower, the blade observes a sudden change in loading due to change in angle of attack and hence acts as a source of noise (see Figure 2.5). A very popular example of BTI is the downwind wind turbine design, in which, the rotor is in the wake of the tower. In the downwind setting, tower produces a massive velocity deficit and blade passing through this deficit acts as a very effective source of the noise. To overcome this problem, the upwind wind turbine design was introduced. In this design, there is also a velocity deficit due to the tower but it is much less compared to the downwind design. **Blade passage effect** can be understood as the interaction of aerodynamic pressure on the blade with the tower. This effect is more common for application in propellers and helicopter interaction noise.
In the case of wind turbines, BTI is a source of low-frequency noise, see Ref. [31]. In the literature, the unsteady force fluctuations on the blade is often considered as the source of BTI, while neglecting the forces on the tower. However, Yauwenas et al. [48] presented a numerical study of BTI for a three-bladed rotor mounted on a vertical cylindrical tower in an upwind configuration. It is found that pressure fluctuation on the tower is more prominent as compared to that on the blade itself as can be seen in Figure 2.6. Stronger pressure variation on the tower indicates its importance in the generation of BTI. So, it is suggested to study the effect of tower fluctuations in the prediction of BTI.

![Figure 2.6: Acoustic pressure waveform comparing the tower and blade. [48]](image)

2.1. Models for noise prediction
The initial process of predicting the noise from a stationary source is in itself a challenging task. In recent times, a lot of progress has been made in the field of aeroacoustics, in terms of predicting noise radiation even from a rotating source or a complex geometry. Different models available for tonal and broadband components are discussed in the following subsections.

2.1.1. Theories for Noise Prediction of Rotating Sources
The first ever theory proposed for calculating the noise radiation from a rotating propeller is given by Gutin [20] in 1948. This model gives noise radiation by a rotating force or a dipole, of constant strength. Gutin's work has been extended by Garrick and Watkins [17] for a propeller in forward flight. The model proposed by Gutin does not take into account the effect of the quadrupole term (noise due to turbulence), as turbulence is identified as a source of broadband noise by Lighthill [29] in 1952. In 1965, Lowson [30] defined the formulation for noise radiation from a source in an arbitrary motion using point singularities such as monopoles and dipoles. Ffowcs Williams and Hawking [16] developed a general expression for a source in an arbitrary motion including the aerodynamic bounding surfaces (monopole and dipole distribution) and turbulence flow (quadrupole distribution).

Two most common methods used in industry or academia for predicting the noise from rotating sources are Kirchoff's formulation and Ffowcs-Williams and Hawking's analogy. Kirchoff's formulation for moving sources is based on the inhomogeneous wave equation, in which sources are located on Kirchoff's surface [15]. Kirchoff's surface should be defined in the linear flow region, otherwise, the formulation is not valid. On the other hand, FW-H is an extension of Lighthill's acoustic analogy for moving surfaces. It is an exact rearrangement of the Navier-Stokes equation into the inhomogeneous wave equation with one volume term and two surface source terms [16]. A rigorous analytical comparison of the two formulations is presented in [5].

Brentner et al. [5] have mathematically shown that FW-H and Kirchoff's formulation are equivalent and predict the results equally well if the integration surface is considered permeable and lies in the linear flow regime. The disadvantage of Kirchoff's formulation is its invalidity when the integration surface does not lie in the linear flow region. One of the main reasons for FW-H's success over Kirchoff's formulation is its ability to represent the source terms physically, whereas Kirchoff's formulation offers less insight into the physics
of sound generation. For designing purposes, FW-H is a natural choice, as source components can be calculated individually as well. However, FW-H also poses a computational disadvantage for high Mach number flows due to the presence of the quadrupole term (volume integral). At low Mach number, quadrupole noise can be neglected [32]. A numerical approximate method for predicting the quadrupole term for high-speed applications is presented in [7].

FW-H analogy is widely popular for the acoustic analysis. In the past, different formulations are proposed in the time and frequency domains. Brentner et al. [6] discuss three numerical algorithm for obtaining a solution in the time domain. First of which is retarded time, which is the most used of all the algorithms. However, there is a limitation in case of high Mach number flows (M>1), multiple solutions exist for M>1 which can be overcome with the suitable root solvers but the bigger problem is the singularity of Doppler term at M = 1. Collapsing sphere algorithm described in [6] removes the Doppler singularity but a singularity arises when radiation vector and surface normal vector are aligned or parallel. Collapsing sphere algorithm is computationally more expensive than the retarded time algorithm. The third algorithm is the emission surface or advanced time approach [10], in which acoustics signal from all the sources is collected at the same observer time. This technique is based on the retarded time algorithm. Of the three techniques, the emission surface algorithm is found to be the most robust and efficient computationally [10, 47].

Time domain analysis can be advantageous when detailed time-domain CFD results are available. In case of unavailability of detailed time-domain CFD results, frequency domain formulations are usually better suited. Frequency domain analysis has been extensively studied in the literature [22, 33, 38]. The analysis in the frequency domain has been successful for both subsonic as well as supersonic applications. Also, frequency spectra are more intuitive for analyzing the acoustic characteristics of the flow. For tonal components of the noise, frequency domain analysis is very efficient computationally. For this thesis project, modeling will be done using FW-H formulations in the frequency domain.

2.1.2. Acoustic Model for Tonal Components

Hanson and Parzych [23] proposed a derivation for the propeller tonal noise in the frequency domain using the Goldstein's formulation for noise calculation. Goldstein [19] presented a generalized formulation for calculating noise radiations in the presence of solid boundaries. Figure 2.7 presents the summary of the acoustic analogies in the literature and Goldstein has shown that different theories can be obtained from the generalized equation derived in [19] with appropriate use of Green's function.

![Figure 2.7: Structure of acoustic analogy theory [19]](image)

Hanson and Parzych [23] gives the formulation for the exact acoustic model by keeping the sources on the surface of the body. An approximate method has also been presented by placing the sources at the mean chord of the airfoil throughout the blade. This approximate method simplifies the formulation but acoustic
predictions are off by around 1.5dB. Numerical integration over the tangential co-ordinate is to be performed for the near field. However, simplified analytical expressions are also presented for the far-field approximations. This method has been successfully implemented in the NASA prediction code WOBBLE. Results are compared with the experimental data for prop-fan and propellers. Propeller measurements shows an excellent match with the predicted radiations. However, prop-fan measurement does not correspond very well with the predicted results. As there are no assumptions made in the acoustics formulation, the problem persists in obtaining the exact aerodynamics inputs.

The terminology used for three sources in [23] is thickness noise (surface term), loading noise (surface term) and quadrupole noise (volume term), which is only true when the surface is considered as impermeable [6]. Harmonic formulation derived in [23] is best suited for predicting the tonal components. As quadrupole term is volume integral, it requires relatively high computational power. However, at low tip speeds, the contribution from the quadrupole term is very small and can be neglected.

Blade tower interaction noise has been a topic of interest as it produces low-frequency noise (LFN), which is under investigation for potential health impacts. Viterna [46] developed a NASA LeRC sound prediction code, particularly for wind turbines noise prediction. This acoustic model is based on the Goldstein’s formulation [19]. Aerodynamic blade force is required as an input for this acoustical model. Madsen [31] presented a model for prediction of low frequency noise (blade tower interaction) based on Viterna [46] model. Madsen [31] has studied the blade tower interaction noise mechanism for both upwind and downwind configuration of wind turbines. Aerodynamic input is obtained from the HAWC2 code [27]. HAWC2 is the general aeroelastic code for wind turbines, developed at Risoe DTU that models the structural dynamics, aerodynamics and the control system. Yauwenas et al. [48] presented a numerical study on the BTI for the upwind configuration using unsteady Reynolds Averaged Navier Stokes (URANS) equations, to understand the effect of blade tower distance on noise generation. Also, Yauwenas [48] has highlighted in detail the mechanism of sound generation due to BTI. The model proposed by Hanson and Parzych [23] can be used to calculate the BTI noise using appropriate aerodynamic inputs.

2.1.3. Acoustic Model for Broadband Components
Turbulent boundary layer trailing edge noise is the primary contributor to the broadband noise, though there are other sources of broadband noise as well. The general formulation for calculation of broadband noise is based on the FW-H equation [16]. Homicz and George [24] extended the FW-H equations for the broadband noise due to turbulence interaction of the flow with propellers in 1974. At that time, their formulation was considered computationally very expensive or in other words, the computational capacity at that time was limited.

Amiet [1] introduced an approximate method for calculation of noise from rotating sources. In this method, the rotational motion of the blade is approximated by the average over the angular position of the noise from
the translating blade. Basically, the rotational motion of the blade is approximated by a succession of translation motion (see Figure 2.8). Amiet identified the application of this approximation for high frequencies and low rotor speeds, where the effect on rotation on the noise can be neglected. The correction of rotation is taken into account by a Doppler shifted frequency [1].

Amiet’s model is widely used for predicting broadband noise (both leading edge and trailing edge noise) as it reduces the computational power. Blandeau and Joseph [3] validated Amiet’s model with the exact FW-H equations and obtained the range of frequency for its application. It has been found that Amiet’s model predicts the noise radiation very well in the mid-frequency range and diverges for very low and high frequencies.

Amiet and Schlinker [39] used the Amiet’s approximated method [1] for analyzing the trailing edge noise from a propeller. Sinayoko et al. [42] has highlighted the discrepancies with the original theory and presented a new model for trailing edge noise. Calculations of the trailing edge noise are based on FWH acoustic analogy with the input of pressure distribution given by Amiet’s theory [39]. Also, they have presented the validity of the new model and suggested the correct value of Doppler weighting exponent (a).

2.1.4. Benchmark Results
Further, for validating the implementation of the model and experiments, following benchmark cases from the literature can be used.

- Elementary sources: In the literature, reference analytical solutions have been proposed for rotating monopoles and dipoles in the frequency domain [33, 44]. An elementary solution such as monopole and dipole can correspond to the thickness and loading terms of FW-H analogy in the compact limit.
- Steady thickness and loading noise: Simplest reference solution used for validation purpose can be the one presented by Yang et al. [47].
- Turbulent boundary layer trailing edge noise: Refs. [3, 42] presents the analytical solution for validating the trailing edge noise implementation.
- Blade-tower interaction noise: Yauwenas et al. [48] and Zajamsek et al. [50] present a numerical and experimental study of blade tower interaction and can be used as benchmark results.

2.2. Experimental Studies
Experiments play a great role in validating the prediction of the models. As mentioned in Chapter 1, the aim of the project is to characterize the open rotor tonal noise using the analytical model and fan noise experiments. This thesis project involves the fan noise experiments to validate the approximation of the models. Another motivation for conducting fan noise experiments is the lack of experimental acoustical data for the rotating propeller/airfoils compared to the stationary airfoils. In this project, fan noise experiments are conducted at the anechoic chamber of IMSIA at ENSTA ParisTech, France.

Oerlemans et al. [36] conducted an experimental study on a two bladed scaled wind turbine model to validate the new airfoil designs for reduction of trailing edge noise. A beamforming technique with 136 microphones was employed to capture the acoustical phenomenons. Cho et al. [11] also conducted an experimental investigation of noise sources on a scaled model of the NREL wind turbine model. Cho et al. studied that the source of noise moves closer to the tip with an increase of frequency. Fan noise experiments to be conducted for this project are inspired by the rotor model used by Zajamsek et al. [49, 50] (see Figure 2.9).

Zajamsek et al. [49] conducted an experimental study on NACA 0012 airfoils (see Figure 2.9) to validate the BPM model [8] to predict the trailing edge noise from rotating propellers. Zajamsek et al. observed a M^5 scaling for the trailing edge noise from rotating propeller, as stated in [18] as well. Another similar experimental study has been conducted by Boorsma and Schepers [4] at DNW wind tunnels to study the wind turbine trailing edge noise.

An important aspect of this thesis project is to characterize the effect of BTI experimentally. There are number of experimental studies on BTI for downwind configuration [26, 41], but very limited experimental studies on the upwind configuration are available in the literature. A very recent experimental study on BTI is conducted by Zajamsek et al. [50] for an upwind configuration. As explained earlier, the contribution of pressure fluctuation on the tower contributes more towards the BTI noise as compared to the pressure variation on the blade itself [49, 50] (see Figure 2.6). To further understand the effect of pressure variation on the tower when the blade is passing, pressure taps are to be mounted on the tower. The location of the pressure taps on the tower is inspired by the numerical results by Zajamsek [49]. The region of interest on the tower is shown
2. Literature Study

Figure 2.9: Experimental Setup used by Zajamsek [49]. The indicated sections are - 1. blades, 2. slip ring, 3. torque sensor, 4. motor and 5. support tower.

It can be seen that major pressure variation occurs from 0.5 to 1.1 time the blade radius and will be an area of interest for mounting the pressure taps.

Figure 2.10: Pressure variation on the tower (fan case) [50]
2.3. Research Objectives

As mentioned in Chapter 1, the objective of the thesis is to characterize the tonal noise of an open rotor with main focus on BTI noise for various kinematics and geometric configuration with the help of analytical acoustic prediction model and fan-noise experiments. To achieve the desired goal, the project can be divided into four primary tasks, which are listed below with the following sub-tasks.

1. Implement the analytical model (Ffowcs Williams and Hawkings Analogy) for rotating sources in the frequency domain.
   - Characterize the tonal noise components based on suitable models.
   - Choose appropriate aerodynamic solver as an input for FW-H acoustic analogy.
2. Validate the model predictions with the benchmark results from the literature.
   - Validate the assumptions made in models (e.g. far-field approximations).
   - Convergence study of the model.
3. Perform fan noise experiments using various configurations such as rotational speed, the angle of attack and blade tower distance.
   - Test the experimental setup.
   - Compare the experimental measurements with the acoustical model predictions.
   - Study the effect of blade-tower distance, pitch angle on noise radiation.
4. Pressure variation study on the tower.
   - Chose appropriate location of pressure taps.
   - Semi-analytical method using Curle's Analogy.
3

Experimental Setup and Procedures

3.1. Experimental Facility
In the framework of this project, a new fan noise experiment has been built in the anechoic chamber of IMSIA, ENSTA ParisTech, France. The dimensions of the anechoic chamber are $3.5 \times 3.0 \times 2.6$ m and it provides a near reflection-free environment up to a minimum frequency of approximately 100 Hz.

![Experimental Setup](image)

The design of the experiment is inspired by the fan noise experiment conducted by Zajamsek et al. [50] to study the blade tower interaction noise and is designed by Tommy Rigall (PhD candidate at ENSTA ParisTech). The experiment setup was mounted from scratch. The initial design posed some vibration problems and
rotor axis alignment issues. These problems were rectified using suitable modifications to the experimental setup to improve the structural rigidity of the test setup. The final experimental setup used for this project is presented in Figure 3.1. The whole setup is mounted on a turning table which can be rotated 360°. The motor is fixed on a table which rests on 4 legs with additional diagonal supports to eliminate the torsional vibration modes. A 3 kW motor (Figure 3.1(b)) is employed to rotate the blades. The speed and direction of motor rotation are controlled by the speed variator. In Figure 3.1(a), the motor is covered by the absorbing material (Abs. Mat.) to reduce noise by the motor. The maximum achievable speed of the motor is 1440 revolutions per minute (RPM). Range of RPM’s considered for experiments is from 600 to 1200 RPM. Below 600 RPM, the Signal to Noise Ratio (SNR) was too low.

![Figure 3.2: NACA 0012 blade, with tripping](image)

<table>
<thead>
<tr>
<th>RPM</th>
<th>Re_{tip} (x 1e+5)</th>
<th>M_{tip}</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>1.05</td>
<td>0.08</td>
</tr>
<tr>
<td>750</td>
<td>1.32</td>
<td>0.10</td>
</tr>
<tr>
<td>900</td>
<td>1.58</td>
<td>0.12</td>
</tr>
<tr>
<td>1050</td>
<td>1.85</td>
<td>0.14</td>
</tr>
<tr>
<td>1200</td>
<td>2.11</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 3.1: Variation of tip Reynolds Number and Mach Number, \( r_{tip} = 40 \) cm

![Figure 3.3: Pitch Angle rigs](image)

NACA 0012 blades of chord 0.07 m and span 0.4 m made of aluminium are used as rotor blades for this experiment, as seen in Figure 3.2. Blades can be mounted using a rig of radius 0.035 m manufactured for pitch
angles of 3° and 8° (see Figure 3.3). Variation of tip Mach Number and chord-based tip Reynolds number with RPM variation is given in Table 3.1. Even for maximum RPM, Reynolds number is relatively low, and which implies that the boundary layer will remain laminar on large extent of the chord surface. To avoid laminar instabilities, tripping is used at 10% of the chord on both sides of the blades. This tripping corresponds to the white strip on the blade shown in Figure 3.2. Axis of the rotor is supported by two bearings and is connected to the torque sensor (component 4 in Figure 3.1(a)). Torque sensor measures the torque and rotational angle of the axis. The torque signal is transmitted from the rotating shaft and is measured using the principle of strain gauge [13]. For the angle measurement, the sensor sends 360 pulses per revolution, with the option of 2 channels to increase the resolution. Next important component of the setup is the tower (component 2) mounted on a translating cart (component 6). The diameter (D) of the tower is 11 cm. Translating cart can be used to adjust the blade-tower distance. Blade tower distance (d) is measured from the leading edge of the blade to the tower, as shown in Figure 3.4. Pitch angle of the blade is shown by \( \theta \) in Figure 3.4.

![Figure 3.4: Schematic for blade tower distance, top-view](image)

![Figure 3.5: Pressure Taps on the tower](image)
Additionally, pressure taps are mounted on the tower to capture the pressure variations due to passage of blade. A total of 14 pressure taps are used to have the spatial distribution of pressure on the tower and are distributed as shown in Figure 3.5. The position of the pressure taps is selected based on the numerical simulation of blade tower interaction by Zajamsek [50], can be seen in Figure 2.10. Maximum variation of pressure along the curvature of the tower (‘s’ direction) is observed at 80% of blade radius (Figure 2.10). The spatial position of the pressure taps along the curvature are chosen based on these results.

3.2. Microphones

For acoustic measurements, 3 Brüel & Kjær free-field 1/2" microphones are used. All the three microphones are condenser microphones externally polarized with suitable polarizing voltage. These microphones are selected because of the wide frequency range of 10 Hz - 20 kHz and its suitability for measurement in the anechoic chamber [9]. Two sets of microphone locations are used for this project and are presented below.

- **Set 1**: Locations of the microphones are specified in Figure 3.6. One of the microphones (Mic 1) is located in the rotor plane (RPS1) at a distance of 1.2 m from the rotor center. The distance for the microphones is restricted due to the size of the anechoic chamber. Further, in the report, Mic 1 (RPS1), Mic 2 (RAS1), Mic 3 (Bottom S1) notation will be used to refer to the microphone's location of set 1. The Cartesian coordinates for the position of mics, with the center of rotation as origin, are given in Table 3.2.

<table>
<thead>
<tr>
<th>Mic Number</th>
<th>Cartesian co-ordinate (x, y, z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mic 1 (RPS1)</td>
<td>(0 m, 1.2 m, 0 m)</td>
</tr>
<tr>
<td>Mic 2 (RAS1)</td>
<td>(1.1 m, 0 m, 0 m)</td>
</tr>
<tr>
<td>Mic 3 (Bottom S1)</td>
<td>(1.1 m, 0 m, -1.1 m)</td>
</tr>
</tbody>
</table>

Table 3.2: Cartesian co-ordinates for Set 1 Microphones

Absorbing material is used to avoid reflections from the setup itself. The rotating table is also covered with absorbing material. Figure 3.7 presents the spectra (PSD) of motor background noise without blades, with and without the absorbing material covering around the motor. It can be observed that
absorbing material helps to reduce the noise. The electrical noise is characterized by the high frequency tonal peaks clearly visible above 5kHz (Figure 3.7). Background noise at low frequency is primarily due to the mechanical noise and is not efficiently reduced by the absorbing material.

- **Set 2**: For this set, the location of Mic 1’ is at same location as in set 1. Mic 2’ and Mic 3’ are in the same x-y plane (see Figure 3.8), located below the center of rotation. Mic 3’ lies in the plane aligned with the rotor axis and perpendicular to the x-y plane. Further, in the report, Mic 1’ (RPS2), Mic 2’ (Bottom Side S2) and Mic 3’ (Bottom S2) notation will be used to refer to the microphone’s location of set 2. The Cartesian coordinates for set 2 microphones, with respect to the center of the rotor, are given in Table 3.3.

<table>
<thead>
<tr>
<th>Mic Number</th>
<th>Cartesian co-ordinate (x, y, z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mic 1’ (RPS2)</td>
<td>(0 m, 1.2 m, 0 m)</td>
</tr>
<tr>
<td>Mic 2’ (Bottom Side S2)</td>
<td>(0.25 m, 0.8 m, -1.1 m)</td>
</tr>
<tr>
<td>Mic 3’ (Bottom S2)</td>
<td>(0.44 m, 0 m, -1.1 m)</td>
</tr>
</tbody>
</table>

Table 3.3: Cartesian co-ordinates for Set 2 Microphones

Figure 3.7: Motor Background Noise by Mic 1 (RPS1) at 1200 RPM
3. Experimental Setup and Procedures

3.3. Data Acquisition
An 8-channel National Instrument Daq - USB 5261, is used for acquisition. The maximum acquisition frequency for this model with multiple channels in use is $1E + 6$ Hz. For this experiment, 6 channels are employed to perform the measurements. 3 channels are used for 3 microphones, 3 channels for torque sensor (1 for torque and 2 for angle measurement). Acquisition frequency for the microphones is 48 kHz. This acquisition frequency allows visualizing the frequency content of the signal up to a frequency of 24 kHz.

Zoc-22b Scani-Valve pressure scanner is employed for acquiring pressure variations on the tower. It has maximum of 32 channels. Acquisition frequency for the pressure taps depends on the number of channels in use. Acquisition frequency can be increased by decreasing the number of channels. There was a problem in acquiring the data at high frequency due to some issue with the LabView program. Due to lack of time, the problem in the LabView program was not addressed. And the maximum achievable frequency was 500 Hz while using 8 channels.

3.4. Signal Processing
Signal processing is a very important tool for acoustical analysis. The most important application of signal processing in this project is analyzing the time signal by transforming into the frequency domain. Fast Fourier Transform (FFT) is used to convert the original signal (in time or space domain) into a depiction in the frequency domain. To analyze the signal obtained from the experiment, a hanning window is used on every signal to avoid the signal truncation.

To analyze the signals, power spectral density (PSD) and spectrum level are used. A Power Spectral Density (PSD) is a measure of the signal’s power content as a function of frequency. PSD’s are more commonly used to analyze broadband stationary signals. If the unit of the measured signal is Volts (V), the units of PSD is $V^2/Hz$.

Another important tool is the spectrum level. Its units are defined as $V^2$, if the signal is in Volts (V). As PSD is defined per unit frequency, spectrum level is defined as the root mean square of the signal, which has passed through a frequency filter of bandwidth $\Delta f$ and centered at frequency $f$. This kind of analyses is important for analyzing the tonal noise as compared to PSD. If the bandwidth of the analysis is increased, the spectrum level for broadband noise will also be increased because more energy is passing through the filter, whereas the tone level will remain the same. In contrast, if the power spectrum is used, and bandwidth is increased, the broadband level will remain the same because energy per unit Hz remains the same. However, the tone level in PSD is reduced when bandwidth is increased. Generally, in rotor applications, spectrum level is used more frequently and it is important to specify the bandwidth when reporting the results. PSD can be obtained from the spectrum level by dividing it with frequency resolution. Welch’s average method is used for obtaining the PSD and spectrum level.

A sinusoidal signal mixed with random noise is used as an example to illustrate the difference between PSD and spectrum level. The sinusoidal signal consists of two frequencies 50 Hz and 120 Hz. With the help of this example (see Figure 3.9), it can be easily seen that for spectrum level analyses, tone level remains same.
3.5. Uncertainty Analysis

with the variation of frequency resolution and broadband level changes. For PSD, the tone level changes and broadband level remains the same with the variation of frequency resolution [18].

Parameters used for signal processing to analyze the signals from the experiments are given in Table 3.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Window</td>
<td>Hanning Window</td>
</tr>
<tr>
<td>Frequency Resolution ($\Delta f_r$)</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Sampling Frequency ($f_s$)</td>
<td>48kHz</td>
</tr>
<tr>
<td>% Overlap</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 3.4: Parameters for signal processing

3.5. Uncertainty Analysis

For any experiment, it is crucial to perform an uncertainty analysis in order to have confidence in the results. In this section, uncertainty analysis is performed for both sets of microphone locations defined in Section 3.2. This analysis is performed in a controlled environment. The temperature sensor is used to observe the variations of the temperature before, during and after the experiment. As this project is focused on characterizing the tonal noise, only tonal peaks will be considered for uncertainty quantification. The result and analysis for both sets are presented.

3.5.1. Set 1

Location of set 1 microphones are given in Figure 3.6. For this analysis, two cases are considered, with tower and without the tower. Without tower, results are used to study the steady loading noise and validate the model. A set of 10 consecutive measurements with a time gap of 30 s between each measurement are considered. Parameters used for the experiment (both with and without tower) to analyze the uncertainty characteristics are given in Table 3.5. Duration of each measurement is 30 sec, which allows measuring of 600 revolutions for 1200 RPM.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation Speed</td>
<td>1200 RPM</td>
</tr>
<tr>
<td>Pitch of blades</td>
<td>3°</td>
</tr>
<tr>
<td>Acquisition Frequency</td>
<td>48 kHz</td>
</tr>
<tr>
<td>Duration of measurement</td>
<td>30 s</td>
</tr>
<tr>
<td>Blade tower distance (d)</td>
<td>D/2</td>
</tr>
</tbody>
</table>

Table 3.5: Experiment’s parameters - uncertainty analysis of Set 1

- **Without Tower**: Before analyzing the microphone results, first, the temperature variation during the experiment is presented in Table 3.6. It can be observed that temperature variation is not significant and can be neglected in terms of altering the characteristics of the microphone with time. There is not much change in speed of the sound owing to small variations in the temperature.

For post-processing the microphones data, parameters defined in Table 3.4 are used for signal processing. Power spectra is obtained using the `pwelch` function in MATLAB. Then the peaks at blade passing frequency (BPF) harmonics are extracted for first 15 harmonics and are compared for 10 measurements. First 15 harmonics are selected because the blade tower interaction noise is mainly dominant up to 10-15 harmonics of BPF and steady loading noise is most dominant for the first few harmonics. Figure 3.10 presents the results for uncertainty analysis for the three microphones. Black curves represent the 10 measurements and the red curve shows the average of 10 measurements. It can be seen from Figure 3.10(a), that Mic 1 produces acceptable results with a difference of few decibels between measurements. It matches very well at lower harmonics. The maximum variation is around 4 dB for Mic 1. For Mic 2 and Mic 3, the results are not at all acceptable. For 1st harmonic the maximum variation is around 6 dB and 7 dB for Mic 2 and Mic 3 respectively.

---

1For with tower case, where D is the diameter of tower i.e. 11cm.
### Table 3.6: Temperature and Speed of sound variation during the experiment - without tower

<table>
<thead>
<tr>
<th>Measurement (#)</th>
<th>Temperature (°C)</th>
<th>Speed of sound (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before start</td>
<td>23.6</td>
<td>345.4</td>
</tr>
<tr>
<td>1</td>
<td>23.4</td>
<td>345.2</td>
</tr>
<tr>
<td>2</td>
<td>23.1</td>
<td>345.1</td>
</tr>
<tr>
<td>3</td>
<td>22.8</td>
<td>344.9</td>
</tr>
<tr>
<td>4</td>
<td>23.1</td>
<td>345.1</td>
</tr>
<tr>
<td>5</td>
<td>23.2</td>
<td>345.1</td>
</tr>
<tr>
<td>6</td>
<td>22.9</td>
<td>345.0</td>
</tr>
<tr>
<td>7</td>
<td>23.4</td>
<td>345.2</td>
</tr>
<tr>
<td>8</td>
<td>22.8</td>
<td>344.9</td>
</tr>
<tr>
<td>9</td>
<td>23.1</td>
<td>345.1</td>
</tr>
<tr>
<td>10</td>
<td>23.4</td>
<td>345.2</td>
</tr>
</tbody>
</table>

Furthermore, time signals were analyzed to understand the variation observed for Mic 2 and Mic 3. One of the 10 measurement is used to plot the pressure signal in Figure 3.11. Low pass filter is used with a cut off frequency of 600 Hz and is plotted along with the original signal. The rationale behind choosing the limit to 600 Hz is to visualize only the first 10 harmonics of blade passage frequency i.e 60 Hz. Figure 3.11(a) presents the time signal for Mic 1. Mic 1 is in the rotor plane and the blade passage time period can be clearly observed. The low pass filter removes the high-frequency noise, probably due to the trailing edge noise. Signal for Mic 1 is stationary and so its variance with different measurements is very small. For Mic 2 and Mic 3, the signals are non-stationary, and even with low pass filter, it is difficult to see the effect of steady loading at blade passage frequency. SNR for Mic 2 and Mic 3 is too
low to exploit the data. That is why a new set of microphone locations will be used later.

Figure 3.11: Time Signal - Uncertainty analysis without tower, for Set 1 Microphones

- **With Tower:** For this case, the parameters used for the experiment are given in Table 3.5. Temperature variation for this series of experiments is the same as discussed earlier.

Figure 3.12 presents the uncertainty analysis for 3 microphones with tower. In this case, along with steady loading noise, blade tower interaction noise will be a source of tonal noise. It is expected to have better results for the tower configuration, as a stronger peak is produced at BPF due to blade tower interaction and hence the signal to noise ratio would be better. Also, there is an increase in the SPL with tower for higher harmonics as compared to Figure 3.10 (without tower). The results also correspond to the presence of blade tower interaction. Mic 1 without tower was already very good but is even better with tower (see Figure 3.12(a)). There is a huge improvement in Mic 2 and Mic 3 as compared to the previous case with a maximum deviation of 2 dB and 2.5 dB respectively.

**Conclusion:** From this analysis, it can be observed that Mic 1 produces precise results both with and without tower whereas Mic 2 and Mic 3 signals vary a lot without tower. For the configuration without tower, the main source of tonal noise is steady loading noise along with some interaction noise between blade's flow and set-up support. The SNR of steady loading noise is very low for Mic 2 and Mic 3, producing non-stationary signals. With tower, the blade tower interaction noise is an additional source of noise which produces a good SNR as compared to the case without tower and hence the results for Mic 2 and Mic 3 are far more precise. So, it is not recommended to use Mic 2 and Mic 3 for analyzing the tonal noise sources without a tower.

### 3.5.2. Set 2

Location of set 2 microphones is presented in Figure 3.8. For set 2, only the case without a tower is considered as it was observed from the uncertainty analysis of set 1 microphones that the microphones produces accept-
able results with tower. Once again, the temperature variation was not significant in this case, so it is being omitted here in the report. As no particular pattern was observed in the variation of peaks for a sequence of experiments for set 1 microphones, only 5 measurements are considered for uncertainty analysis for set 2. Parameters used for conducting the uncertainty analysis are given in Table 3.7.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPM</td>
<td>1200</td>
</tr>
<tr>
<td>Pitch of blades</td>
<td>3°</td>
</tr>
<tr>
<td>Acquisition Frequency</td>
<td>48 kHz</td>
</tr>
<tr>
<td>Duration of measurement</td>
<td>30 s</td>
</tr>
</tbody>
</table>

Table 3.7: Experiment's parameters - uncertainty analysis of Set 2

Figure 3.13 presents the uncertainty analysis for set 2 measurements. The position of Mic1’ is unchanged and Figure 3.13(a) is quite similar to Figure 3.10(a). Position of Mic 2’ is changed completely as compared to set 1. In set 1, Mic 2 was in rotor axis (i.e. in induced flow), but now it is at approximately 45° from rotor axis (i.e. relatively out of flow). The result for Mic 2’ is presented in Figure 3.13(b). It shows a good result for first harmonic and even for higher harmonics. The maximum variation is around 3-4 dB. Mic 3’ is brought closer to the rotor in order to have a strong signal to noise ratio. Bringing the Mic 3’ closer to the rotor also allows it to be out of the induced flow. Figure 3.13(c) presents the result for Mic 3’. It can be seen that the results are very good for first harmonics with a variation of 0.5 dB, and the maximum variation is around 2-3 dB.

Further, the time signal for all the microphones is presented in Figure 3.14. Two signals are presented in these plots, the blue one is the original signal and the orange is the filtered signal. Low pass filter is used to
3.5. Uncertainty Analysis

Measurements

Mean

(a) Mic 1' - RPS2

Mean

Measurements

(b) Mic 2' - Bottom Side S2

Mean

Measurements

(c) Mic 3' - Bottom S2

Figure 3.13: SPL - Uncertainty analysis without tower, for Set 2 Microphones

pass the signal below the frequencies of 600 Hz. It can be seen in Figure 3.14 that the time signal is stationary as compared to set 1 location, which is also reflected in the uncertainty quantification as well.

**Conclusion:** For the configuration with tower, both sets of locations work equally well because of stronger peaks due to blade tower interaction noise. However, while increasing the blade tower distance, it should be kept in mind that the effect of BTI reduces with increasing blade tower distance. Without tower, the results for set 1 location is not too good (especially for Mic 2 and Mic 3), because the signal to noise ratio of steady loading noise is not strong enough. For set 2 locations, there is a lot of improvement as compared to set 1. This analysis is performed for 3° pitch angle. A similar study was conducted for 8° pitch for set 2 locations, and as expected, good results were obtained for all 3 microphones. In conclusion, while studying the steady loading noise without a tower, set 2 location will be used.
Figure 3.14: Time Signal - Uncertainty analysis without tower, for Set 2 Microphones
Theoretical Acoustic Model

4.1. Hanson Model

A detailed frequency domain derivation for calculating tonal components of noise radiation from a rotating source is presented by Hanson and Parzych [23]. This formulation is based on the Ffowcs-Williams & Hawking (FWH) acoustic analogy [16]. The theory described in Ref. [23] is presented for the propeller noise application, but in general, it can be used for any rotational source, with appropriate aerodynamic inputs.

Sir James Lighthill introduced an acoustic analogy which is suitable for stationary sources. For real applications of interests in aeroacoustics, sources are generally moving. Few examples can be a rotating propeller, helicopter rotor, and a wind turbine. Ffowcs-Williams and Hawking [16] established the formulation for moving sources using the concept of generalized derivatives. Generalized derivatives allow differentiating discontinuous functions. This concept can be extended to define a surface bounding a region with a discontinuity at the surface, such that the continuity and momentum equations can be defined in the whole region. Then, by manipulating the equations in a very similar way as done to obtain Lighthill’s equation, Ffowcs-Williams and Hawking’s equation for moving surfaces can be obtained. Integral solutions to FW-H equation can be obtained by using a suitable Green’s function [18].

Goldstein [19] presented a generalized formulation for calculating noise radiations in the presence of solid boundaries. Hanson and Parzych [23] used Goldstein’s formulation for calculating the tonal noise source from moving surfaces in a moving medium. The acoustic density fluctuations presented in Hanson and Parzych [23] is given as:

\[
\frac{c_o^2 \rho'(x,t)}{V_{N}} = \left( \int_{-T}^{T} \int_{A(t)} \rho_o V_N \frac{DG}{Dt} dA(y) \, dt \right) + \left( \int_{-T}^{T} \int_{A(t)} f_i \frac{\partial G}{\partial y_i} dA(y) \, dt \right) + \int_{-T}^{T} \int_{V(t)} T_{ij} \frac{\partial^2 G}{\partial y_i \partial y_j} dy \, dt ,
\]

where \( c_o \) is the speed of sound in ambient fluid, \( \rho' \) is the density variation, \( \rho_o \) is the ambient density, \( V_N \) is the source normal velocity, \( G \) is the free field moving medium Green function, \( f_i \) is the force on the blade surface, and \( T_{ij} \) is Lighthill’s stress tensor. Thickness and loading noise are represented by surface integrals over \( A(t) \) and quadrupole noise is calculated by the volume integral \( V(t) \).

**Thickness noise** represents the noise due to volume displacement of the air owing to the rotation of the blade. It can also be understood as a periodic injection and ejection of mass in a fixed reference frame at each element of air (similar to monopole). It is dependent on the normal surface velocity \( (V_n) \) and the profile of the blade (thickness). For low Mach number regime, thickness noise is generally neglected [18]. **Loading noise** can be produced due to steady as well as unsteady pressure fluctuations in case of rotating source like propeller or wind turbines. When the inflow is not distorted, there is a steady loading on the blade in propeller’s reference frame. But a stationary observer notices the component of force rotating and changing direction...
with the propeller’s rotation, which acts as a source of noise. Sources of unsteady loading can be distorted/turbulent inflow, pitching propeller axis, advancing and retreating blades, fluctuating boundary layer. All these sources except the fluctuating boundary layer change the angle of attack of the blade as it rotates resulting in generating unsteady loading on the blade. The time variation of the loading produces noise. At low Mach number, unsteady loading noise is generally dominating over steady loading [18]. **Quadrupole noise** mainly accounts for non-linear effects like wake interaction which can be neglected at low Mach number.

Equation (4.1) can also be written in a concise way as follows,

\[ c_o^2 \rho' = p'(x, t) = p_T(x, t) + p_L(x, t) + p_Q(x, t) \]

where \( p_T, p_L \) and \( p_Q \) corresponds to the thickness noise, loading noise and quadrupole noise respectively. Thickness noise is generally an inefficient source of noise at low Mach number [34]. At high Mach number, it becomes an important source of noise and cannot be neglected. This project focuses on low tip Mach number regime for the rotating source, where thickness noise can be neglected. In the framework of this project, only loading noise is implemented considering the low Mach number regime.

### 4.1.1. Coordinate System and Green’s Function

Before deriving the Equation (4.1) further, let us first define the coordinate system and Green’s function. A right-hand system is used for co-ordinates, with the x-axis pointing in the flight direction. The y-axis could lie in any direction, it is considered to be in the horizontal direction with z-axis pointing downwards (see Figure 4.1).

![Figure 4.1: Coordinate system. Taken from Ref.[23]](image)

Observer coordinates are denoted by \( (x, y, z) \) and source coordinates are denoted by \( (x_o, y_o, z_o) \). Propeller aligned coordinates are defined as \( x_o', y_o', z_o' \) and can be obtained according to the yaw, pitch and roll angles.

Free space, moving medium, time-dependent Green’s function is suitable for this system and is given as,

\[ G = \frac{\delta(t - \tau - \frac{\sigma}{c_o})}{4\pi S} \]

where \( \tau \) is the source time (time at which source emits radiation), \( t \) is the observer time (time at which observer receives the radiation), and \( S \) is the amplitude radius, defined as:

\[ S = \sqrt{(x - x_o)^2 + \beta^2((y - y_o)^2 + (z - z_o)^2)} \]

with \( \beta^2 = 1 - M^2 \) and \( \sigma \) is the phase radius:

\[ \sigma = \frac{M(x - x_o) + S}{\beta^2} \]
4.1.2. Loading Noise

Loading noise formulation will be derived for near-field and far-field approximations. Note that, the first derivation for a single blade is presented, the effect of multiple blades can be taken into account by superposition. From Equation (4.1) and (4.2), loading noise is given as:

\[ P_L(x, t) = \int_{-T}^{T} \int_{A(t)} f_i \frac{\partial G}{\partial y_i} dA(y) d\tau. \] (4.6)

The notation \( A(\tau) \) in Equation (4.6) means that the integration surface depends on time due to blade motion. It is preferred to perform the source integration in blade fixed cylindrical coordinates \((r_o, \phi_o, x'_o)\). Cylindrical coordinates simplifies the source coordinates system as rotation of the blade involves only the \( \phi_o \) as variable coordinate whereas \( r_o \) and \( x'_o \) are fixed. So, the area element is located at \((r_o, \phi_o, x'_o) = (r_s, \phi_s + \Omega \tau, x_i)\). Equation (4.6) can thus be written as:

\[ P_L(x, t) = \int_{-T}^{T} \int_{A} f_i \frac{\partial G}{\partial y_i} |_{\phi_o=\phi_s+\Omega \tau} dA(y) d\tau. \] (4.7)

Green’s function derivative and force \((f_i)\) is to be evaluated in terms of \( r_o, \phi_o \) and \( x'_o \). It can be seen in Equation (4.7) that the inner integral on \( A \) is independent of source time. As the integration limits do not depend on the integration variables, the order of integration can be interchanged and Equation (4.7) can be written as:

\[ P_L(x, t) = \int_{A} \int_{-T}^{T} f_i \frac{\partial G}{\partial y_i} |_{\phi_o=\phi_s+\Omega \tau} d\tau dA(y). \] (4.8)

The first hypothesis of this model is to consider only periodic signal such that \( P_L(x, t) \) can be defined as a periodic function at a frequency of \( \omega/2\pi \), and its Fourier series representation can be written as:

\[ P_L(x, t) = \sum_{n=-\infty}^{\infty} P_{Ln}(x)e^{-i\Omega t}, \] (4.9)

where the Fourier coefficients \( P_{Ln} \) can be evaluated according to:

\[ P_{Ln}(x) = \frac{\Omega}{2\pi} \int_{0}^{2\pi/\Omega} p_L(x, t)e^{i\Omega t} dt. \] (4.10)

The \( \tau \) limits in Equation (4.8) can be considered as one revolution of the rotor, i.e. \( 0 < \tau < 2\pi \), as it will produce noise equals to one period of the noise signal. The limits can vary between \( t_1 \) to \( t_1 + 2\pi/\Omega \), where it is not necessary to know the exact value of \( t_1 \). It can also be represented by:

\[ \tilde{p}_L(x, t) = p_L(x, t) \quad t_1 < t < t_1 + 2\pi/\Omega \]

\[ = 0 \quad \text{elsewhere}. \] (4.11)

With the modified range of \( \tau \) integral, the Fourier coefficients can be calculated from an adapted form of Equation (4.10):

\[ P_{Ln}(x) = \frac{\Omega}{2\pi} \int_{-\infty}^{\infty} \tilde{p}_L(x, t)e^{i\Omega t} dt. \] (4.12)

Now, using Equation (4.8), (4.11) and (4.3) in Equation (4.12), the following result is obtained:

\[ P_{Ln}(x) = \frac{\Omega}{2\pi} \int_{A} \int_{-\infty}^{2\pi/\Omega} \left[ f_i(\Omega \tau) \frac{\partial}{\partial y_i} \left| \frac{1}{4\pi S} \int_{-\infty}^{\infty} \delta(t - \tau - \sigma \Omega c_o) e^{i\Omega \sigma t} d\sigma \right| \right] dA(y) d\tau. \] (4.13)

Performing the \( t \)-integration and defining the harmonic Green’s function by

\[ G_n = \frac{e^{i\Omega \sigma t_c o}}{4\pi S}, \] (4.14)
Equation (4.13) becomes:

\[
P_{ln}(x) = \frac{\Omega}{2\pi} \int_{0}^{\frac{2\pi}{\Omega}} \int_{A} f_i(\Omega \tau) \frac{\partial G_m}{\partial y_j} \phi_{o_{n}+\phi_{s}} e^{im\Omega \tau} \, d\tau \, dA. \tag{4.15}
\]

Now, the variable of integration \(\tau\) can be changed using \(\phi_o = \phi_s + \Omega \tau\) to get,

\[
P_{ln}(x) = \frac{1}{2\pi} \int_{A} e^{-in\phi_s} \int_{0}^{2\pi} \left[ f_i(\phi_o - \phi_s) \frac{\partial G_m}{\partial y_j} \right] e^{im\phi_o} \phi_o d\phi_o dA. \tag{4.16}
\]

So far, the contribution of only one blade is considered, the effect of multiple blades is considered using the principle of superposition. If each of the blades is assumed to be identical, then loading experienced by each blade would be same. Thus, an identical noise signal is produced with time lags as per the blade position.

For \(B\) equally spaced blades, only the blade passing harmonics \((n = mB)\) contribute. So, the equation for the loading noise due to multiple blades become:

\[
P_{lnB}(x) = \frac{B}{2\pi} \int_{A} e^{-im\phi_s} \int_{0}^{2\pi} \left[ f_i(\phi_o - \phi_s) \frac{\partial G_m}{\partial y_j} \right] e^{im\phi_o} \phi_o d\phi_o dA. \tag{4.17}
\]

The pressure \(P_{lnB}(x)\) is written simply as \(P_{ln}\) in Equation (4.17). Similarly, \(G_{mB}\) is written as \(G_m\). Green's function is modified as:

\[
G_m = \frac{e^{ik_m\sigma}}{4\pi S}, \tag{4.18}
\]

where \(k_m = mBM_t\) and \(M_t = \Omega r_i/c_o\) (tip rotational Mach number).

For programming purposes, a non-dimensional form of Equation (4.17) is preferred. The length is non-dimensionalized by propeller tip radius \(r_i\), area by \(r_i^2\), time by \(\Omega\), pressure by ambient pressure \(p_o = \rho_o c_o^2/\gamma\), with \(\gamma\) is the ratio of specific heats, and force \((f_i)\) by \(p_o r_i^2\). Equation (4.17) will remain the same after non-dimensionalizing, as normalizing parameters are hidden in either the Green's function or the force \((f_i)\).

### 4.1.3. Generalized form of Loading Noise in Discrete form

Equation (4.17) represents the exact form of Hanson model for calculation of tonal noise. To implement this equation in a program, it is required to discretize the blade into elements.

Radial and chordwise discretization is represented by \(\mu\) and \(\nu\) respectively. The area integral \(\int_{A} dA\) is transformed into a summation along radial and chord direction as \(\sum_{\mu,\nu}\). The forcing term in Equation (4.17) i.e. \(f_i dA\) is discretized as follows:

\[
F_{i\mu,\nu} = (f_i dA)_{\mu,\nu}. \tag{4.19}
\]

The index \(i\) for the forcing components is along the 3 cylindrical components and is interpreted as:

\[
F_{1\mu,\nu} = F_{1\mu,\nu}, \quad F_{2\mu,\nu} = F_{2\mu,\nu}, \quad F_{3\mu,\nu} = F_{3\mu,\nu}. \tag{4.20}
\]

With the discretized elements, the equation for loading noise (Equation 4.17) can be written as:

\[
P_{ln}(x) = \frac{B}{2\pi} \sum_{\mu,\nu} e^{-im\phi_s} \int_{0}^{2\pi} \left[ F_{i\mu,\nu}(\phi_o - \phi_s) \frac{\partial G_m}{\partial y_j} \right] e^{im\phi_o} \phi_o d\phi_o. \tag{4.21}
\]

Equation (4.21) represent the general form of loading noise in discretized form. The value of force can be obtained from an aerodynamic solver and can be used to obtain the noise radiation. Discretization on the blade will depend on the aerodynamic input. For instance, if the aerodynamic input is distributed radially along the blade, then the discretization will be considered only along the radial direction with the assumption that each radial location represents the equivalent load along the chord.

Further, the time dependence of the forcing term \((F_{i\mu,\nu})\) is given by the tangential angle \((\phi_o)\). This forcing term can be expressed as a Fourier series in \(\phi_o\) and is given as,

\[
F_{i\mu,\nu} = \sum_{k=-\infty}^{\infty} F_{ik\mu,\nu} e^{-ik(\phi_o - \phi_{ref})}. \tag{4.22}
\]
where, $\phi_0$ is the initial position of the blade, $\phi_{\text{ref}}$ is the reference position that can be chosen arbitrarily, and $F_{i k \mu \nu}$ are the Fourier coefficient of the force. Using Equation (4.22) in Equation (4.21):

$$P_{Lm}(x) = \frac{B}{2 \pi} \sum_{k=0}^{\pi} \left[ F_{i k \mu \nu} \frac{\partial G_m}{\partial y} e^{i(mB-k)\phi_{\text{ref}}} \right] \int_0^{2\pi} \left[ F_{i k \mu \nu} \frac{\partial G_m}{\partial y} e^{i(mB-k)\phi_0} \right] d\phi_0.$$  \hspace{1cm} (4.23)

The indices $i$ in Equation (4.23) correspond to the Einstein summation convention. Using Equation (4.20), (4.23) can be expanded as:

$$P_{Lm}(x) = \frac{B}{2 \pi} \sum_{k=0}^{\pi} \left[ F_{i k \mu \nu} \frac{\partial G_m}{\partial y} e^{i(mB-k)\phi_{\text{ref}}} \right] \int_0^{2\pi} \left[ F_{i k \mu \nu} \frac{\partial G_m}{\partial y} e^{i(mB-k)\phi_0} \right] d\phi_0.$$  \hspace{1cm} (4.24)

Equation (4.24) is the generalized form of loading noise in discrete form. The derivation of this form in terms of far-field approximation and near field calculation is given in Appendix A.

### 4.1.4. Numerical Implementation

Both near-field and far-field calculations are implemented. The implementation of far-field is straightforward, using the analytical expression given by Equation (A.9). For implementing the near-field code, a numerical integration is to be performed. A `quad` function from the `scipy` module of Python is used. It integrates the function between infinite limits using the Fortran QUADPACK library. This function is used to calculate the $I_{\text{near}}(k, \mu, \nu)$ from Equation (A.14) within 0 to $2\pi$ limits.

#### 4.1.4.1. Convergence of Integration

In the experiments, the microphones are located in the near-field. So, it is important to check the convergence of integration for near-field calculations in order to compare it with the experimental measurements. The default value of absolute tolerance for `quad` is 1.49e-08. It works for most of the cases but shows some errors when the value of integrand is of the order of absolute tolerance. In that case, relative tolerance is required to be used. The value of integrand reduces if the observer distance is increased or if the frequency is reduced. Increasing the observer distance is not an issue in this case, as far-field approximations can be used for that case. But in order to compare the model with the experiment, the lowest rotational speed of rotor is 600 RPM, which corresponds to a frequency of 10 Hz and tolerances needs to be adjusted for low frequency accordingly.

Note that the tolerance depends on the different cases and different locations. One example is used to highlight the effect of tolerance and then a suitable criteria given considering all the cases. A rectangular NACA 0012 blade is considered with the same dimensions as the experimental blades (given in Section 3.1). The blade is discretized only along the spanwise direction. The loading is obtained by Xfoil, which is explained in detail in Section 4.1.5. Note that it is steady loading, so it expected to only have peaks for the first few harmonics of BPF, with rapid decrease for higher harmonics. The parameters for the case under study are given in Table 4.1. The observer is in the rotor plane with spherical co-ordinates $(r, \theta, \phi)$ of $(1 \, \text{m}, \pi/2, 0)$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPM or Rotation Frequency $(F_x)$</td>
<td>600 or 10 Hz</td>
</tr>
<tr>
<td>BPF</td>
<td>30 Hz</td>
</tr>
<tr>
<td>Observer Location $(r, \theta, \phi)$</td>
<td>$(1 , \text{m}, \pi/2, 0)$</td>
</tr>
<tr>
<td>Spanwise discretization $(\mu)$</td>
<td>30</td>
</tr>
<tr>
<td>Number of Blades</td>
<td>3</td>
</tr>
<tr>
<td>Blade Pitch</td>
<td>$3^\circ$</td>
</tr>
</tbody>
</table>

Table 4.1: Near-field convergence parameters for model simulation

Figure 4.2 presents the results for different values of the relative error $(\epsilon_r)$. The near-field calculations specified in Appendix A is used to calculate the steady loading noise. As expected, there is a sharp decrease in the SPL for higher harmonics. It can be seen that for higher harmonics, unexpected pattern is observed with $\epsilon_r = 1$ and $1 \epsilon - 1$. With further decrease in $\epsilon_r$, the pattern remains the same. As already mentioned, it is not suitable to select the value of relative tolerance $\epsilon_r$ just based on this case. Other extreme cases were selected to find a global value of $\epsilon$ suitable for all the cases. CPU time of the simulation was also observed for

---

$^1$Origin at center of rotation
each value of tolerance. There is no significant increase in the value of CPU time with $\epsilon_r$. Considering various cases and lack of dependence on CPU time, it is suggested to used the relative tolerance value of 1e-4. This value is found to be suitable for all the cases under consideration.

![Figure 4.2: Convergence of near-field integration for different values of the relative error ($\epsilon_r$)](image)

**4.1.4.2. Validation with Reference**

For comparison, a test case proposed by Mao et al. [33] is used. They have used a unit strength dipole placed at the tip of each of the three blades of radius ($r_{tip}$) 0.5 m. Rotational frequency is 100 Hz with a tip Mach number of 0.92, whereas the fluctuating frequency of the dipole source is 300 Hz. The location of the stationary observer is given by spherical co-ordinates ($r, \theta, \phi$) = (1 m, $\pi$/4, 0). For the coordinate system used by our model, the observer is situated in the X-Y plane, while blades are rotating in the Z-Y plane, as depicted by Figure 4.3. The observer location is clearly in the near field, as the blade passage frequency is 300 Hz, corresponding to a wavelength ($\lambda$) of 1.14 m. For an observer to be in far-field, observer distance ($r_{obs}$) should be much greater than the wavelength ($\lambda$), $r_{obs} \gg \lambda$.

![Figure 4.3: Observer location for the test case of Mao et al. [33]](image)

Mao [33] et al. have presented three cases for three different dipole directions i.e. axial, radial and circumferential. The reference results by Mao corresponds to an exact analytical model in the frequency domain.
The comparison for the three cases is presented in Figure 4.4. It can be observed from the results that near field predictions for all the three cases match very well with Mao et al. [33], whereas far-field predictions are off as expected.

![Graphs showing comparison of near-field and far-field predictions with Mao et al. [33] predictions.](image)

Further, to verify the far-field model implementation, the observer distance is increased. All the parameters are the same as defined for comparison in Figure 4.4, except the observer location. The observer location is increased from the position defined for the previous comparison i.e. \((r, \theta, \phi) = (1 \text{ m}, \pi/4, 0)\) till far-field approximation matches the near field calculations. The comparison is made for the radial dipole direction and plotted in Figure 4.5. With the increase of observer distance, far-field approximation approaches the near-field calculations. It is important to note that the convergence of far-field approximation to the near-field depends on the source of noise. From this analysis it can be concluded that both far-field and near field calculations are implemented correctly. It can be used to obtain the noise calculations for any rotating body with loading information.

### 4.1.5. Aerodynamic Input

An acoustic model for loading noise is presented in the preceding sections. Appropriate aerodynamic input is required to calculate the noise due to the source. The objective of the project is to compare the noise predicted by the model with the experimental measurements. For the experiments, a fan configuration is used as the experiments are performed in the anechoic chamber without wind tunnel. The rotor is rotated by the motor.

It was desired to obtain the loading on the blade including the effect of interaction with the tower. AeroDyn tool was considered for this purpose. AeroDyn is a time-domain wind turbine aerodynamics module that
has been coupled into the FAST version 8 multi-physics engineering tool to enable aero-elastic simulation of horizontal axis turbines [35]. This tool considers the effect of tower by using potential flow around a cylinder, so velocity deficit due to the tower is considered and is taken into account while calculating the loads on the blade. For a wind turbine, this is a very good tool as it takes into account various corrections such as tip loss, hub loss, Glauert’s correction for heavily loaded rotor and skewed wake correction. To simulate the fan configuration similar to the experiment, AeroDyn was not found to be suitable. As the tool is originally designed for wind turbines, it requires incoming velocity as an input. But for the experiments, there is no incoming velocity (as the blades are forced to rotate), although there will be small induced flow due to the rotation of the blades. While calculating the effect of tower, the change in loading on the blades is only considered due to velocity deficit effect but not the blade passage effect. Some preliminary results of AeroDyn for NREL 5MW reference wind turbine are presented in Appendix B. HAWC2 is a similar tool for wind turbine, which was also not suitable for simulating the fan configuration.

Further, to obtain the steady loading on the blade, Xfoil [14] is used. The induced flow due to the rotation is assumed to be sufficiently small such that the pitch angle is equal to the angle of attack. Blade is discretized along the spanwise direction. The discretization of the blade is shown in Figure 4.6. The parameter N-crit that govern the laminar -turbulent transition is adjusted to mimic the effect of tripping on the blades [14]. N-crit is set to 1 both on top and bottom side of the blade. The lift coefficient ($C_l$) and drag coefficient ($C_d$) are obtained from the Xfoil for each element. The force for each element is assumed to be acting at its center. As the induced flow is ignored, the angle of attack of the blade is same as the pitch angle and the velocity is equal to the rotational velocity $\omega r$ of the blade, where $\omega$ is the rotational velocity and $r$ is distance of center of the element from the root. Torque and thrust for each element of the blade is obtained by Equations (4.25).
and (4.26) respectively. It is a very simple model to simulate the steady loading of a fan.

\[ F_q = C_l \sin \phi + C_d \cos \phi \]  
\[ (4.25) \]

\[ F_t = -C_l \cos \phi + C_d \sin \phi \]  
\[ (4.26) \]

In Equation (4.24), \( F_r \), \( F_\phi \) and \( F_x' \) are three components of force in radial, circumferential and axial directions respectively. Calculation from Xfoil does not provide the radial loading. Circumferential and axial forces are nothing but the torque (\( F_\phi \)) and thrust (\( F_t \)) loading respectively, obtained from Xfoil (Equation 4.25 and 4.26). Note that \( F_{\phi k, \mu} \) and \( F_{x' k, \mu} \) are the Fourier components of the force to be summed over \( k \) harmonics. Also, Equation (4.24) is summed over \( \mu \) (spanwise) and \( \nu \) (chordwise). As explained in Figure 4.6, the blade is discretized only along spanwise direction, so summation along \( \nu \) can be ignored.

This method is used to obtain the steady loading on the blade. In order to proceed, a convergence study is performed to select a suitable value of \( \mu \). A test case of NACA 0012 blades rotating at 1200 RPM at a pitch of 3° is performed. The reference value of \( \mu \) for comparison is 90. The observer is located in the rotor

![Figure 4.6: Discretization of blade along the span for Xfoil Calculations](image)

![Figure 4.7: Convergence of \( \mu \) - 1200 RPM, Pitch = 3°, Obs = Mic1 RPS1](image)
plane where steady loading is dominant (Mic 1 RPS1 of set-1). Figure 4.7 presents the results. First subfigure gives the SPL curve at each harmonic for $\mu = 90$, and the second curve presents the difference between SPL calculations for different values of $\mu$. It can be seen that for $\mu = 10$, there is substantial difference with respect to the reference for higher harmonics. However, for this example, the value of SPL at higher harmonics is around -100 to -200 dB. The error is improved with the increase in value of $\mu$. It can be observed that for $\mu = 20$, the error has already reduced a lot and is further reduced for higher values of $\mu$. Note that, similar analysis was performed for other locations of observer. Similar results were obtained from other locations as well. Considering the analysis for other locations and the current location (Figure 4.7), the value of $\mu$ to be used for results is 40. For $\mu = 40$, the error is very small, even for higher harmonics.

**Tip Loss Factor:** Tip loss factor is used to correct the assumption of infinite number of blades [21]. There is a loss in thrust force due to the tip vortices. This loss of thrust force can be taken into account by using the Prandtl’s tip loss factor. Correction factor ($F_{tip}$) is given as,

$$F_{tip} = \frac{2}{\pi} \cos^{-1}(e^{-f}),$$

and:

$$f = \frac{B(R - r)}{2r \sin \phi},$$

where $B$ is the number of blades, $R$ is tip radius of the blade, $r$ is the local radius, and $\phi$ is the flow angle. The plot for correction factor is given in Figure 4.8. Correction factor ($F_{tip}$) is multiplied by the calculated force to have the corrected value.

![Figure 4.8: Prandtl’s Tip Loss Factor](image)

**4.2. Simplified BTI Model for Tower**

As mentioned in Section 2.2, the pressure fluctuations on the tower are found to be the primary contributors to the BTI noise, in accordance to [48]. So, it is interesting to study these fluctuations on the tower as the blade passes. Some numerical simulations have already been carried out by Yauwenas et al. [48] to study this phenomenon. In this project, the pressure taps are mounted on the tower, as explained in Section 3.1. Pressure variations on the tower are obtained and inverted Ricker or Mexican Hat wavelet is found to be a close match to the pressure fluctuations, as will be seen in Section 5.3. Ricker wavelet is the second-order derivative of Gaussian probability density function and is shown in Figure 4.9(a). The expression for Ricker wavelet is given in Equation (4.27).

$$f(t) = (1 - 2\pi^2 f_0^2 t^2)e^{-\pi^2 f_0^2 t^2},$$  (4.27)
where, $f_M = \frac{\sqrt{2}}{T_d}$ and $T_r = T_d/\sqrt{3}$. $T_r$ and $T_d$ are the width parameters shown in Figure 4.9(a). Width of the wavelet can be adjusted by changing the values of $T_r$ & $T_d$. Figure 4.9(b) shows the results by Yauwenas et al. [48], the Ricker wavelet could be a good match for representing the pressure fluctuations on the tower. Experimental results for the pressure taps will be discussed in Chapter 5. Ricker wavelet is used to fit the experimental pressure tap data to obtain the best value of width parameter $T_r$. It is a semi-empirical way of modeling and requires a lot of experimental data to achieve a rational model which takes into account the influence of RPM, pitch angle, blade tower distance $(d)$ on BTI noise.

![Inverted Mexican Hat Wavelet](image1)

![Thrust Coefficient on tower as the blade passes, 900 RPM, pitch = $0^\circ$, (48)](image2)

**Figure 4.9**: Mexican Hat Wavelet and Shape of pressure fluctuations on tower from Ref. [48]

Force distribution ($F_i$) on the tower can be obtained using the pressure taps. Further, the relationship between the far field acoustic pressure, $p$, at a location $x$ due to the total unsteady force, $F_i$, is defined according to Curle’s analogy in the compact approximation [18]:

$$p(x,t) = -\frac{1}{4\pi c_o} \frac{x_i}{r^2} \left[ \frac{\partial F_i}{\partial t} \right],$$

where subscript $i$ represents the three components of the aerodynamic force and $c_o$ is the speed of sound. Note that Equation (4.28) is only valid for a compact source i.e. the size of the source is significantly smaller than the acoustic wavelength. In the experiment, the diameter of the tower is 0.11 m and the acoustic wavelength for the maximum rotation speed of 1200 RPM (BPF = 60 Hz) will reach 1 m for approximately 6$^{th}$ harmonic of BPF. Even for the acoustic wavelength of 1m, the tower will act as a compact source. It will be shown in Chapter 5, that the effect of BTI is observed till 8$^{th}$ - 10$^{th}$ harmonic. So, it is a reasonable assumption to consider the tower as a compact source for BTI noise.
Results and Discussions

In this chapter, the results from the experiment and models are presented. Section 5.1 presents the fan noise experiment results both with and without tower for different pitch angles. Section 5.2 compares the analytical model for steady loading noise with the experimental results. Lastly, preliminary results with the pressure taps are shown in Section 5.3.

5.1. Experimental Results

In this section, experimental results for different configurations are presented and discussed. Section 5.1.1 presents the effect of frequency resolution on the experimental signals. Experimental results without tower are discussed in Section 5.1.2. The effect of the tower presence on the broadband and tonal noise is presented in Section 5.1.3. Section 5.1.4 discusses the effect of blade tower distance on the tonal noise. Finally, Section 5.1.5 presents the effect of pitch angle on BTI noise.

5.1.1. Frequency Resolution

The effect of frequency resolution for PSD and spectrum level has been discussed in Section 3.4, using a sinusoidal signal. In this section, the same analysis is presented using experimental measurements. The measurement under consideration is the one with the tower, at a blade tower distance (d) of D, 1200 RPM for Mic 3 Bottom S1. A sampling frequency of 48kHz with a 50% overlap is used in the Welch’s average method to obtain the PSD and spectrum level. Figure 5.1 presents the PSD and spectrum level for 3 frequency resolutions.

![Figure 5.1: Effect of Frequency Resolution, Mic 3 - Bottom S1, With Tower, d = D, 1200 RPM](image)

As observed in Section 3.4, with the increase in frequency resolution, there is a decrease in the tone levels of PSD, whereas the broadband level remains the same, as the energy per unit Hz is the same. Whereas for
the spectrum level, the tone level remains the same with an increase in frequency resolution, and broadband level increases as more energy is passing through the frequency bandwidth (see higher harmonics in Figure 5.10(b)). As the primary focus of the project is on the tonal noise, spectrum level will be used to analyze the tonal noise. Further, in this report, a frequency resolution of 1 Hz is used for signal processing to minimize the effect of broadband noise on tonal peak amplitude.

5.1.2. Without Tower

The first set of experiments is conducted without the tower. As concluded from the uncertainty analysis (Section 3.5), set 2 locations are used for analyzing the configuration without tower. Location of set 2 microphones is described in Figure 3.8. Figure 5.2 presents the PSD plots on the whole frequency range, without tower, for pitch angles of 3° and 8°, for 1200 RPM. The results are presented for the 3 microphones. Background noise without the blades is plotted with a dashed black line. In general, the level of background noise is lower than the aerodynamic noise with blades. At a low frequency of 25 Hz, a peak is observed due to background noise for all the three microphones. The source of this noise could be the mechanical noise in the experiment. At high frequency, the motor's electric noise is visible and is dominant around 1kHz frequency. Its levels are comparable to the noise levels in the experiment with blades.

Broadband noise is present above 1kHz and could be attributed to the trailing edge noise. A strange pattern is observed in the frequency range of 4kHz-7kHz (green box in Figure 5.11(a)) for all the three microphones. There is a frequency shift for this phenomenon for 3 microphones due to the presence of Doppler's effect (different locations of the microphones). The source of this noise is not identified and is not studied further as primary focus of the project is on tonal noise. The level of broadband noise is minimum for Mic 1’ (in the rotor plane), and maximum for Mic 3’ (in front of the rotor). This pattern also corresponds to the
directivity pattern of the trailing edge. Trailing edge noise is expected to have a cardioid directivity (Figure 2.4), directing maximum noise in the upstream direction of the airfoil.

For the broadband noise, there is a slight increase in the $8^\circ$ pitch as compared to the $3^\circ$. It is in agreement with the theory as the trailing edge noise is caused by the turbulent fluctuations leaving the sharp edge. And the turbulent boundary layer fluctuations will be stronger for $8^\circ$ pitch, owing to the stronger adverse pressure gradient, which leads to a higher level of noise as compared to $3^\circ$ pitch.

Figure 5.3 presents the spectrum levels plotted with respect to BPF harmonics for the same configuration. It can be seen that SNR is very good for the tonal peaks at BPF harmonics as compared to the background noise. The rotation speed of the blades is 1200 RPM, which corresponds to tip Mach Number of 0.16. It is expected that the effect of thickness noise will be small at this Mach number, such that the loading noise is the primary source of tonal noise. For this experiment, at low frequencies, there are no sources of unsteady loading on the blade if the interactions with the experimental setup are neglected. Trailing edge noise will be a source of unsteady loading but it will be present at high frequencies and will have broadband characteristics. As the observers (microphones) are situated away from the rotor axis, steady loading on the blade will act as a source of noise due to the Doppler's effect. Loading noise is observed with a strong peak at the BPF of 60 Hz for all the 3 microphones and then a sharp decrease for the further harmonics of BPF.

![Figure 5.3: Spectrum Level, Without Tower, 1200 RPM, Different Pitch Angles, BPF = 60 Hz](image)

For Mic 1’, the SPL level of 1st harmonic for $3^\circ$ and $8^\circ$ is almost same, and the noise level the for next harmonics is higher for $8^\circ$ pitch as compared to $3^\circ$. A similar trend is observed for Mic 2’ and Mic 3’. First harmonic noise level for Mic 3’ is slightly higher than the Mic 2’ and Mic 1’. Also, the decrease for next harmonics for Mic 3’ is not as sharp as for Mic 1’ and Mic 2’. Mic 3’ is situated in front of the rotor, below the rotor axis (Figure 3.8). From these results, it can be observed that the effect of loading noise is more prominent at Mic 3’ location as compared to the other two microphones. Loading noise is directly proportional to the loading
5. Results and Discussions

on the surface. In this case, loading on the blade for 8° pitch will be higher as compared to the 3° pitch. The increase in loading noise with the increase of pitch angle agrees well with the theory.

5.1.3. Effect of Tower Presence

In this section, the effect of tower presence is studied on the whole spectra. For this analysis, a case with blade tower distance d\textsuperscript{1} of D/2 is compared to the case without tower. Set 1 locations are used to study the effect of tower (given in Figure 3.6), since the uncertainty was low with tower as seen in Section 3.5. The uncertainty without the tower for set 1 was very high, especially for Mic 2 and Mic 3. So, uncertainty analysis should be kept in mind while discussing these results. Parameters for the experiment under study are given in Table 5.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational Speed</td>
<td>1200 RPM</td>
</tr>
<tr>
<td>BPF</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Blade Tower Distance (d)</td>
<td>D/2</td>
</tr>
<tr>
<td>Blade Pitch</td>
<td>3°</td>
</tr>
<tr>
<td>Microphone Set</td>
<td>Set - 1</td>
</tr>
</tbody>
</table>

Table 5.1: Parameter for experiment - Effect of tower

Figure 5.4: PSD, 1200 RPM, Pitch = 3°, BPF = 60 Hz, Effect of Tower

Figure 5.4 presents the PSD plots to visualize the impact of the tower. Broadband levels, in this case, are almost the same with and without tower for all the microphones. Again, the broadband level for Mic

\textsuperscript{1}d is defined in Section 3.1, D = Diameter of the tower = 11 cm
5.1. Experimental Results

1 is minimum, which corresponds to the directivity pattern of trailing edge noise. Further, the spectrum level is plotted to analyze the peaks at BPF harmonics in Figure 5.5. It can be seen in Figure 5.5(a), that the first harmonic is almost the same with and without a tower. As will be seen in Section 5.2, steady loading is dominant at first harmonic and hence the SPL is almost the same. For the next harmonics, the effect of the tower can be seen distinctively at BPF harmonics. The effect of the tower is prominent around 3-5 BPF harmonics and is visible up to 11\textsuperscript{th} BPF harmonic for Mic 1.

Now, Mic 2 and Mic 3 should be compared carefully, the uncertainties for both the microphones are very high, especially for 1st harmonics (Figure 3.10). For higher harmonics, uncertainties are less than the difference between the noise level with and without a tower. So, it is suggested to consider the BPF harmonics > 1 for comparing Mic 2 and Mic 3 results. The effect of the tower is visible up to 8\textsuperscript{th} BPF harmonic for both Mic 2 and Mic 3 in contrast to Mic 1. Also, the difference between with and without tower noise levels for Mic 1 is greater than that for Mic 2 and Mic 3. It shows that the effect of the tower is prominent in the rotor plane as compared to the front of the rotor. In this experimental setup, the primary source for the BTI noise is the blade passage effect as there is no incoming flow on the rotor. Although, a little induced flow will be generated by the propeller, which might contribute to BTI noise through reduced velocity field mechanism.

![Graphs showing spectrum level for different microphones with and without a tower.](image)

**Figure 5.5: Spectrum Level, 1200 RPM, Pitch = 3°, BPF = 60 Hz, Effect of Tower**

In Figure 5.5, apart from the peaks at BPF harmonics, there are peaks present at BPF/3 harmonics as well. These peaks occur at the rotational frequency of the blade i.e. 20 Hz and its harmonics. The amplitude of these peaks is very low as compared to the peaks at the BPF harmonics. The reason for these peaks could be the abnormality in one of the blade or misalignment in rotor axis. **It is interesting to note that the amplitude of these peaks is increased with the presence of tower.** The noise at BPF/3 could also be attributed to the mechanical noise from the experiment.
5.1.4. Effect of Blade Tower Distance

In this section, the effect of blade tower distance is studied. Set 1 location is used for this comparison with rotation speed of 1200 RPM and 3° pitch angle. Three blade tower distances with $d = D$, $D/2$ and $2D/7$ are tested. As seen in Section 5.1.3, the presence of the tower does not have much effect on broadband noise, only spectrum levels focusing on tonal peaks are plotted here.

Figure 5.6 presents the spectrum level for three microphones. Results without tower are also plotted along with the three blade tower distances. The dominant peaks are observed at BPF of 60 Hz and its higher harmonics. As expected, the blade tower interaction noise increases with the decrease of blade tower distance ($d$). For the first harmonic, the effect of BTI is not as visible as observed at higher harmonics. Even for minimum blade tower distance of $2D/7$, the first harmonic noise level for Mic 1 without a tower is comparable (see Figure 5.6(a)). Thus, the loading noise seems to be dominant at the first harmonic even with the tower. At higher harmonics, the effect of BTI can be visualized clearly, with maximum noise level for minimum blade tower distance. For Mic 1, BTI noise is present up to $14^{th}$ harmonic.

As the uncertainties for Mic 2 and Mic 3 are higher (explained in Section 3.5), no comment has been made on the first harmonic. For the higher harmonics, the effect of three blade tower distances is visible for Mic 2 and Mic 3. Another interesting observation from Figure 5.6 is that for Mic 2 and Mic 3, the noise level for $d = D$ is almost equal to the noise level in the configuration without tower, whereas for Mic 1 it is not the case. The noise level for $d = D$ is slightly higher than without a tower for Mic 1. These results also indicate that the blade tower interaction noise is more dominant in the rotor plane (Mic 1).
5.1.5. Effect of Pitch Angle with tower
For this analysis, blade tower distance (d) is fixed to D/2. Two pitch angles of 3° and 8° are tested at 1200 RPM. Set 2 microphone locations are used. Figure 5.7 presents the PSD to visualize the whole spectra. It can be observed that broadband noise is slightly increased for 8° pitch as compared to the 3° pitch. It is similar to the case without tower, where broadband noise due to the trailing edge noise increases with the pitch angle.

Figure 5.7: PSD, Comparison for different pitch angles, d = D/2, 1200 RPM

Further, to understand the effect of pitch angle on the tonal noise, spectrum level for the 3 microphones are plotted in Figure 5.8. It can be observed that the first harmonic level is almost the same for both pitch angles for all microphones. For the next harmonics, the noise level for 3° pitch is higher than the 8° pitch. The difference in noise level for both the pitch angles is maximum for the 2nd and the 3rd harmonics. It is not really intuitive that the 3° pitch produces more noise with the tower as compared to 8° pitch at lower frequencies. In the literature, there has been no explicit study to observe the effect of pitch angle on BTI noise. However, articles by Yauwenas et al. [48] and Zajamsek et al. [50] can be used as a reference. Yauwenas et al. did a numerical simulation to study the BTI noise focusing on blade passage effect with 0° pitch and Zajamsek et al. performed the numerical simulation and experimental study on BTI at 5° pitch on the same geometry used by Yauwenas et al. The observer is situated at the rotor axis at a distance of 1.4 m in both the references. Total acoustic pressure is calculated using the Curle's analogy and is presented in Figure 5.9 for one rotation. Although it is not very clear from the Figure 5.9, it can be observed that the acoustic pressure drop for 0° pitch is more pronounced compared to the 5° pitch and hence leading to more noise for 0° pitch. It also points out to the fact that low pitch angle produces more tonal noise with the tower. The mechanism governing this phenomenon is not clear yet, but the trend has been observed in the literature. Further, a detailed study of pressure distribution over the tower could help to understand the phenomenon better.

One of the possible reasons for this could be understood using the pressure variations on the tower. Yauwenas et al. [48] have presented the numerical simulation on the tower while the blade is passing. The
5. Results and Discussions

Figure 5.8: Spectrum Level, Comparison for different pitch angles, \( d = D/2 \), 1200 RPM

Figure 5.9: Effect of pitch angle on BTI noise in literature, Taken from Refs. [48] & [50]

Thrust variation on the tower as the blade passes is presented in Figure 4.9(b). Stronger peaks are observed for lower blade tower distance (\( d \)). The shape of pressure variation on the tower is very interesting, as the blade approaches the tower, there is an increase in the tower pressure, followed by a sharp decrease as the blade is completely overlapping with the tower. When the blade is in front of the tower, the flow is accelerated due to reduced area between tower and blade and hence pressure decreases on the tower. As the blade is going away...
from the tower, its trailing edge pressure interacts with the tower and leads to an increase in pressure. Note 
that the increase in pressure is stronger when the blade is approaching the tower rather than leaving, due to 
strong pressure fluctuations on the leading edge as compared to the trailing edge [48].

When the pitch angle is increased, the area between the tower and the blade increases. This would lead 
to less acceleration of the flow when the blade is in front of the tower and, less drop in pressure on the tower, 
which might lead to lower noise levels as well. This could be a possible reason for less noise in the present 
experiments when switching from $3^\circ$ to $8^\circ$, as shown in Figure 5.8.
5.2. Steady Loading - Model and Experiment (Without Tower)

In this section, the steady loading noise predicted by the model is compared with the experimental measurements. The model for calculations of the loading noise is explained in detail in Chapter 4. In order to compare the steady loading noise with the experiments, the configuration without a tower is used in the experiments. The NACA 0012 blades are tripped at 10% of the chord on both suction as well as pressure sides. Two configurations for pitch angles are compared i.e. 3° and 8°. The loading on the blades is obtained from Xfoil, as explained in Section 4.1.5. For experimental measurements, set-2 microphone locations are chosen because of the lower uncertainty without tower as compared to set-1 (see Section 3.5).

Signal processing parameters used for treating the experimental measurements are given in Table 3.4. A frequency resolution of 1 Hz is used to obtain the spectra. Acquisition frequency for the microphones is 48 kHz. Spectrum level (dB) for each microphone is obtained from the measurements. As the model predicts only the tonal noise at blade passing frequency (BPF), the peaks at BPF are extracted from the spectrum level and are compared with the model.

![Comparison of steady loading noise - model and experiment, pitch = 8°, 1200 RPM.](image)

First, the results for 8° pitch at 1200 RPM are presented. Figure 5.10 presents the comparison of model with the experimental measurements for 3 microphone locations. A solid black line presents the experimental measurements with uncertainty bars, dashed black line represents the background (BG) noise from the experiment, the red curve is for model predictions and the blue curve represents the model with the inclusion of tip loss factor. Background noise is well below the measurements (except for harmonic 8) such that signal to noise ratio is very good. It can be observed that model prediction for the first harmonic is very good for all the microphones. However, the model over-predicts the noise without the inclusion of the tip loss factor. Globally, the model predicts that the steady loading noise is predominantly occurring at first harmonic and decreases very fast for higher harmonics. This pattern can also be observed in the experimental results,
not as strongly but the decrease is less sharp compared to model predictions.

This discrepancy between experiment and model could be attributed to various reasons. In the experiments, the 3 NACA 0012 blades are supposed to be identical, but there might be a small manufacturing abnormality in the blades or a misalignment while mounting the blades. If one of the blades is not identical to the other two, that blade will have different loading and will be observed by the observer as a change at one third of blade passage frequency (BPF/3). This change in loading could be a reason for the peaks observed in the experiment at higher harmonics. For the model, all the blades are identical and hence this effect is not captured. Another possible reason could be due to the interaction of the blade loading with the component of the experiments (like table, supports, table legs). However, it is not expected to have a dominant effect, but could be a potential reason for peaks at higher harmonics, as the amplitude of the peaks is not too high. The misalignment of the rotor axis could also be a source of these peaks. All these effects are very small and are normally present in every experimental setup. These effects are not modeled in the loading noise model, which explains the mismatch between experimental and model predictions at higher harmonics.

Thickness noise is expected to be low for the current Mach number regime. However, a recent study by Doolan et al. [12] shows that the component of steady thickness noise can be observed at higher harmonics even at low Mach number. Its amplitude will not be as high as that of steady loading noise. Steady thickness noise could also be a potential reason for the peaks at higher harmonics in Figure 5.10. An analytical model for steady thickness noise could be a useful tool to study the effect of thickness noise.

A similar analysis is performed for different rotational speeds, ranging from 600 to 1200 RPM. Only the first harmonic is considered for comparison of loading noise. Results for different rotation speeds are presented in Figure 5.11. The results are consistent with the variation of rotation speed. Also, the same pattern of the sharp decrease in loading noise with higher harmonics is observed for different rotation speeds both in experiments as well as model.
The results for $3^\circ$ pitch are now presented. Set-2 microphones are used for the comparison. Results for 3 microphones at 1200 RPM are presented in Figure 5.12. For $3^\circ$ pitch, the model predictions are not well aligned with the experimental results especially for Mic 1 and Mic 2' (Figures 5.12(b) and 5.12(c) respectively). For Mic 3', the model prediction for the first harmonic is relatively better than Mic 1 and Mic2'. For Mic1 and Mic2', there is approximately 10 dB difference for the first harmonic between experimental measurements and model prediction. Whereas, the first harmonic model prediction for Mic 3 is within 1 dB difference from the experimental measurement.

The reason behind the mismatch between experiment and model prediction could be the presence of thickness noise. For $8^\circ$ pitch, the loading noise seems to be the dominant source of noise as the experiment and model prediction match very well. With the decrease of the pitch, it is expected that loading on the blade will be decreased as compared to the $8^\circ$ pitch and hence the loading noise. However, there would not be much change in the thickness noise, as it depends on the thickness of the blade (airfoil profile) and its velocity. As the velocity (RPM) and thickness of the blade are same as compared to the $8^\circ$ case, it could be possible that loading noise has decreased while the level of thickness noise remains the same. This might explain the discrepancies in the results for $3^\circ$ pitch. It could also be said that the thickness noise is very small for lower Mach Number. In that case, the above explanation fails. But with the decrease of pitch angle, the loading noise is decreased and the thickness noise remains the same (however small it is). So, the decrease in the loading noise might lead the thickness noise to become the dominant source of noise. It is just an assumption which needs to be validated further with an appropriate thickness noise formulation for rotating source.

![Comparison of steady loading noise - model and experiment, pitch = $3^\circ$, 1200 RPM.](image-url)
5.2. Directivity of loading noise

In this subsection, the directivity of steady loading noise is presented. The implemented model is used to calculate the directivity pattern for steady loading noise. The directivity pattern is visualized on two planes. Before discussing the results, it is useful to go back to the definition of the coordinate system. The flow direction is in the -x direction, such that rotor axis point in x. The y-axis is pointing in the horizontal direction and the z-axis points downwards (see Figure 4.1). Two planes under consideration are the XY plane passing through the rotor axis and the XZ plane through the rotor axis. Observers are placed on a circle of radius 1.2 m from the center of rotation. Directivity is calculated for two pitch angles, i.e. $3^\circ$ and $8^\circ$. RPM of the rotor is 1200. The result presented includes the effect of tip loss factor.

Figure 5.13 and 5.14 presents the directivity pattern in the XY and XZ plane respectively. Note that only the first harmonic of steady loading noise is used to produce these results. It can be seen from Figure 5.13 that steady loading has dipole-like directivity in XY plane with minimum on the rotor axis. As expected, the noise level for $3^\circ$ pitch is lesser than the $8^\circ$ pitch and the directivity pattern is same for both pitch angles. Also, similar results are obtained for the XZ plane (see Figure 5.14). A dipole-like directivity is obtained in XZ plane as well.

![Figure 5.13: Directivity Analysis - Top View (XY Plane)](image1)

![Figure 5.14: Directivity Analysis - Side View (XZ Plane)](image2)
5.3. BTI Model for Tower

Pressure taps are mounted on the tower, as explained in Section 3.1. This exercise is an attempt to establish a model for pressure fluctuations on the tower as the blade passes in front of the tower. Due to lack of time and some issues with the pressure scanner, the measurements are limited and hence only the preliminary results are discussed in this section.

The case under consideration is 300 RPM. It is a relatively low velocity as compared to the real applications, but the BTI noise is present even for this velocity. This low velocity is selected because of some problems in acquiring pressure data at high frequencies. The low rotational speed is more suitable to have a sufficient number of samples per cycle.

Eight pressure taps are used with an acquisition frequency of 500 Hz and are mounted in a straight line on the tower covering from 40 to 110 percent of the blade’s span. In this analysis, the amplitude of force on the tower is not considered, as the spatial distribution of the pressure on the tower in ‘s’ direction (Figure 3.5) is not measured and hence it is difficult to consider the area of the tower for calculating the actual force. Force is obtained by linear summation of pressure acquired by 8 pressure taps multiplied by the area. For the area, a rectangle of 2 cm × 3 cm, consisting of the pressure tap, is considered. So, a qualitative study is performed to model the shape of pressure variation on the tower with a Mexican hat wavelet (Ricker wavelet).

The modeling of the tower pressure is needed to compute the far-field noise using Curle’s analogy, described in Section 4.2.

Figure 5.15(a) presents the axial force acting on the tower with a variation of blade tower distance (d). One period of rotation is shown in the Figure. Three distinctive peaks can be observed owing to the three-bladed rotor. The two distance under consideration are 2D/7 and D/2, where D is the diameter of the tower (D = 11cm). For lower blade tower distance (2D/7), the variation in the axial force is more pronounced as compared to the D/2 case, whereas the width of the peaks does not vary much with the variation of blade tower distance. Experiments show the same pattern of pressure variation on the tower as observed by Zajamsek et al. [50]. They presented a simulated force acting on the tower (Figure 5.15(b)) using an unsteady Reynolds Average Navier-Stokes (URANS) computations. Note that the parameters used by the ref. [50] and in the present experiments are different. The purpose of this study is to perform a qualitative analysis. Comparing Figures 5.15(a) and 5.15(b), it can be observed that the same pattern is obtained from the experiments as the numerical simulation by Zajamsek et al. [50].

Now, an inverted Mexican hat wavelet (explained in Section 4.2) is used to fit the experimental data obtained by the pressure taps. Figure 5.16 presents the fitting of an inverted Mexican hat filter with the experimental data for two blade tower distances. The values of width parameter $T_r$ (described in Section 4.2) for 2D/7 and D/2 are 0.0074 s and 0.0068 s respectively. It is not an exact match for the pattern observed in the experiments, but it is very close. The Mexican hat filter fits better with the smaller blade tower distance (d = 2D/7) as compared to d = D/2. The increase in pressure when the blade is approaching the tower is greater than when the blade is leaving, as seen in Figure 5.15. This behavior is not accurately reproduced using the Mexican hat filter, as the amplitude of its side peaks is the same.
Far-field acoustic pressure is calculated from the modeled force on the tower using Curle's analogy, as discussed in Section 4.2. Acoustic pressure calculated by the model is qualitatively compared with the numerical simulation by Zajamsek et al. [50] in Figure 5.17. The observer location specified in Ref. [50] is used to calculate the acoustic pressure due to the tower. The observer is situated at the rotor axis, 1.4 m away from the center of the rotor. Blade tower distance (d) is equal to 2D/7, both in the model and the reference. Again, only a qualitative comparison is performed. Figure 5.17(a) presents only the contribution of the tower, whereas the numerical simulation by Zajamsek et al. (Figure 5.17(b)) presents total acoustic pressure and also the separate components of blade and tower. It can be observed that the shape obtained by the modeled tower wall pressure is the same as the tower contribution in Figure 5.17(b). Also, Figure 5.17(b) shows that the tower is the major contributor to the BTI noise as compared to the blades, that is why only tower is considered in this work.

Further, the acoustic pressure calculated for the pressure variation on the tower using Curle's analogy is compared with the present experimental data. For this comparison, Mic 3’ location of set 2 is selected. Mic 3’ is located in front of the rotor, right below the rotor axis (see Figure 3.8(a)). The experiment is conducted at a rotational speed of 300 RPM, 3° pitch and d = 2D/7. The pressure variations on the tower are modeled using the Mexican hat wavelet and Curle's analogy is used to obtain the acoustic pressure. It is worth mentioning again that only qualitative analysis is performed here. As previously found the BTI noise is mostly dominant.
up to the 10th harmonic, so a band-pass filter of frequency range 30 - 150 Hz is used to filter the microphone signal. The lower limit of the band-pass filter is selected such that first BPF harmonic (i.e. 15 Hz) is omitted as steady loading noise is dominant at the first BPF harmonic. The higher limit corresponds to the 10th BPF harmonic.

Figure 5.18(a) presents the normalized acoustic pressure using the modeled tower pressure and the experiment. The acoustic pressure is normalized with the maximum absolute value of the respective signals. Note that, the Curle's model takes into account only the contribution of the tower, whereas the experimental data, although filtered, corresponds to the total contribution. It can be seen that Curle’s model predicts the drop in normalized acoustic pressure accurately and the pattern is the same for both the experiment and model. Further, the increase in pressure predicted by the model does not match very well with the experimental data. It might be possible that some other sources of noise like the contribution of the blade, other mechanical noise sources are showing up in the experiment.

Figure 5.18(b) presents the normalized PSD for the pressure presented in Figure 5.18(a). The pressure is normalized by the pressure corresponding to the 5th BPF harmonic of the respective signals. For the experiment, the noise level increases with the BPF harmonics and is maximum for 3rd harmonic whereas, for the modeled tower contribution, the maximum is occurring at 5th BPF harmonic. The shape of the pattern observed by the model is consistent with the simulated tower contribution by Zajamsek [50], as shown in Figure 5.19. Globally, it can be said that the pattern obtained by the model is similar to the experiment. First, there is an increase in the noise level up to a certain number of BPF harmonic and then there is a decrease with further increase in frequency.

![Figure 5.18](image1.png)

**Figure 5.18:** Comparison of Acoustic Pressure and PSD for the model and experiment, d = 2D/7, 300 RPM, pitch = 30°, BPF = 15 Hz

![Figure 5.19](image2.png)

**Figure 5.19:** d = 2D/7, 900 RPM, pitch = 50°, BPF = 45 Hz, Taken from Ref. [50]
Finally, to have a global analytical model for the pressure fluctuation on the tower, more experimental data or numerical simulations are required. In this project, it was not possible to acquire more data and to post-process it due to the time constraints. From this analysis, it appears that the width of peaks in the force fluctuations on the tower is not dependent on the blade tower distance. However, the amplitude of the peaks does depend on the blade tower distance. It would be interesting to study the effect on the width with the variation of different rotational speeds in the future.
6

Conclusions and Recommendations

The project aims to study the tonal noise sources of an open rotor with a primary focus on steady loading and blade tower interaction (BTI) noise using experimental and analytical models. An analytical model based on the Ffowcs-Williams and Hawkings (FW-H) analogy is implemented for loading noise. Aerodynamic steady loading on the blades is obtained using Xfoil. The experiments are conducted in the anechoic chamber of ENSTA ParisTech, France. The experimental setup was built from scratch with suitable modifications to the initial design to make the setup more reliable. NACA 0012 blades with tripping are rotated by a 3kW motor. Further, a tower is installed on a translating cart which allows changing its distance from the rotor plane. The important conclusions and findings from this study are listed in Section 6.1. Recommendation for the future work are presented in Section 6.2.

6.1. Conclusions

1. The model for loading noise is successfully implemented for both far-field and near-field. Far-field approximation allows to simplify the expression, and is completely analytical. For the near-field, numerical integration needs to be performed. The implementation of the model is verified by using a benchmark result of the rotating dipoles from the literature.

2. Xfoil is used to obtain the steady aerodynamic loading input for the loading noise model. AeroDyn tool by NREL was considered to take into account the effect of the tower on aerodynamic loading. This tool is designed for wind turbines, but in the experiments, the rotor is in fan configuration rather than being in wind turbine. In the experiment, there is no incoming velocity and AeroDyn fails to converge at zero incoming velocity. Also, while calculating the effect of the tower, it only takes into account the effect of reduced velocity but not the blade passage effect. Blade passage effect is expected to be a dominant contributor to the BTI noise in the current experimental setup. So, following this reasoning, the AeroDyn tool was not used in the analysis. The preliminary results obtained with AeroDyn are presented in Appendix B.

3. Steady loading noise obtained by the analytical model is compared with the experimental results without tower for two pitch angles of 3° and 8°. In the experiment, steady loading noise and trailing edge noise are identified as the primary sources of noise. Trailing edge noise has a broadband characteristic and steady loading noise is observed at BPF harmonics. Signal to noise ratio (SNR) for both the tonal and broadband noise is very good as compared to the background noise. Broadband noise and steady loading noise for 8° is slightly higher as compared to 3°, owing to strong turbulent fluctuations and higher loading for the 8° pitch respectively. The analytical model shows that the steady loading noise is dominant only at the first blade passage frequency (BPF). A similar trend is also observed in the experiment, although the decrease of higher harmonic levels in the experiments is not as sharp as observed by the model because of the presence of other sources of noise in the experiment. For the 8° pitch angle, the predicted noise level at first BPF harmonic matches pretty well with experiments for various rotational speeds as well. For 3° pitch, the results for experiment and model are off by around 10 dBs, with the model under predicting the noise. The presence of thickness noise could be a possible reason for this mismatch, as thickness noise remains the same for both 3° and 8° pitch and loading...
noise decreases for 3° pitch as compared to 8°. It is just an assumption that needs to be validated with the model for thickness noise in the future. In the experiment, steady loading noise is found to be dominant in front of the rotor, with higher amplitude for BPF harmonics greater than 1 as compared to the other two microphone locations.

4. The effect of tower presence is visible in the tonal components. As expected, BTI noise is present at BPF harmonics. There is not much change in the broadband noise with the presence of the tower. Also, the effect of BTI noise is not visible at the first BPF harmonic, as steady loading noise is a dominant source of noise. For higher BPF harmonics, the BTI noise can be identified distinctively. The influence of the tower is visible up to 11th harmonic in the rotor plane and up to 8th harmonic in front of the rotor.

5. Three blade tower distances \( d = (D, D/2, 2D/7) \) are compared with the configuration without tower. BTI noise increases with a decrease in the blade tower distance. In front of the rotor, the noise levels for \( d = D \) case are almost equal to the case without tower. The same is not true in the rotor plane, one can see the difference between \( d = D \) configuration and the case without tower. The effect of BTI noise is visible up to 14th BPF harmonic in the rotor plane and up to 11th BPF harmonic in front of the rotor. This analysis points out that the effect of BTI is maximum in the rotor plane.

6. Another interesting case study is the variation of pitch angle with the tower, two pitch angles of 3° and 8° are tested. The broadband noise shows the same trend with the pitch angle as observed for the configuration without tower. Surprisingly, the noise level for 3° pitch is higher as compared to the 8° pitch. A possible reason for this behavior could be the relative increase of blade tower distance with the increase of pitch angle, which leads to lower noise levels for a higher pitch angle. There is not an explicit study in the literature comparing the effect of pitch angle with the tower, to the author’s knowledge.

7. Pressure taps are mounted on the tower to study the pressure variations on the tower as the blade passes. An empirical approach is used to model the pressure fluctuations on the tower. Due to lack of time, only one rotational velocity of 300 RPM was considered for two blade tower distances. Because of limited data, only a qualitative study is conducted. An inverted Ricker wavelet is found to be a close match to the pressure fluctuations on the tower, with appropriate width and amplitude. Further, Curle’s analogy is used to calculate the acoustic pressure. The modeled pressure fluctuations on the tower display a good trend as compared to the literature.

### 6.2. Future Recommendations

The current work can be extended further to better understand the open rotor noise mechanism. The following are the recommendations for future work.

1. In this project, the analytical model for loading noise is implemented. In the future, the analytical model for thickness noise would complete the model for calculations of noise radiation, especially for high Mach number flows. Thickness noise was not considered in this project because the rotor was operating in low Mach number flow but some discrepancies between the model and the experiment could be due to the lack of thickness noise model.

2. The model for loading noise can be used for any rotating source given that the aerodynamic loading is known for the source. It can easily be used for wind turbines with appropriate aerodynamic input being obtained from any open-source tool like AeroDyn, HAWC2 or blade element momentum (BEM) theory.

3. In the experiment, the whole setup was mounted on the turning table to check the directivity. Considering the complexity of the setup, it would be more suitable to use more microphones or beam-forming array to visualize the directivity patterns better.

4. The work done on the BTI model for tower fluctuations is an initial step. This work can be extended by using more pressure taps to have a fine spatial resolution of the pressure variations on the tower. It will allow to calculate the exact pressure acting on the tower and hence allows to better model the fluctuations.

\( ^1d = \text{blade tower distance (see Figure 3.4), D = diameter of the tower} = 11\text{cm} \)
5. In this project, a limited set of data was obtained for the pressure taps. To obtain a global model for pressure fluctuations on the tower, it is required to have the data for different rotational speeds and different blade tower distances. These measurements will allow obtaining a trend for the width parameter \((T_r)\) and amplitude for the Ricker wavelet. Furthermore, a model for the tower pressure based on a more physical reasoning could be obtained.

6. The current experiments performed with pressure taps and microphones are not synchronized. The pressure taps were directly connected to the computer using a USB rather than the connection via NI-DAQ. It is suggested to use the pressure taps which can be connected to the same NI-DAQ as that of microphones, to synchronize the data from both pressure taps and microphones.
A.1. Far-Field Approximations

In order to calculate the loading noise term (Equation 4.24), Green’s function derivative are needed. In this subsection, far-field approximations are presented for the Green function’s derivative. Green’s function defined in Equation (4.3) is used to further derivation.

For a far-field observer, the source distance can be neglected as compared to observer distance such that amplitude radius (Equation 4.4) becomes

\[ S_o = \sqrt{x^2 + \beta^2(y^2 + z^2)} . \]  
(A.1)

For the phase radius (σ), its first order terms in source distance ÷ observer distance are retained, resulting in

\[ \sigma = -\frac{1}{\beta^2}(Mx + S_o) - \frac{1}{\beta^2}(x + MS_o) \frac{x}{S_o} - \frac{y}{S_o} y_o - \frac{z}{S_o} z_o . \]  
(A.2)

Considering certain manipulations presented in Hanson report [23], the expression of the phase radius for far-field simplifies to

\[ \sigma = r - Sc \cdot x' - Ss \cdot y, \]  
(A.3)

where

\[ Sc = \frac{\cos \theta'}{1 - M \cos \theta}, \quad Ss = \frac{\sin \theta'}{1 - M \cos \theta}, \]

\[ \theta' \] and \[ \theta \] are the spherical coordinate angles to the observer based on retarded coordinates aligned with propeller axis and flight direction respectively. The upper/lower (±) implies the right/left hand rotation.

Using the value of \[ S_o \] and \[ \sigma \] from Equation (A.1) and (A.3) respectively in Green’s function:

\[ G_m = \frac{e^{ikm(r-Sc \cdot x')}}{4\pi S_o} e^{-ikmSc \cdot x' \cdot x} \cdot (SsF_{k \mu, \nu}I_\phi + SsF_{k \mu, \nu}I_\phi + ScF_{k \mu, \nu}I_\phi), \]  
(A.4)

The Green function’s derivative to be used in loading noise equation (4.23) is obtained by simply calculating the derivatives of Equation (A.4).

\[ \frac{\partial G_m}{\partial y_1} = \frac{\partial G_m}{\partial r_o} = -ikmSc \cdot x' \cdot x \cdot (SsF_{k \mu, \nu}I_\phi + SsF_{k \mu, \nu}I_\phi + ScF_{k \mu, \nu}I_\phi), \]  
(A.5)

Inserting the Green function’s derivatives (A.5) in Equation (4.23) gives:

\[ P_{Lm} = -\frac{iBk_m e^{ik_m r}}{4\pi S_o} \sum_{\mu, \nu} \sum_k e^{-i[mB-k\cdot e^{k\cdot e^{k\cdot f}}} \times (S_kF_{\mu, \nu}I + S_kF_{\mu, \nu}I + S_kF_{\mu, \nu}I), \]  
(A.6)
where the tangential integrals are given as

\[
I_r = \frac{1}{2\pi} \int_0^{2\pi} \cos(\pm \phi' - \phi_o) e^{i(mB-k)\phi o} e^{-ik_m S_r r_o \cos(\pm \phi' - \phi_o)} d\phi_o,
\]

\[
I_\phi = \frac{1}{2\pi} \int_0^{2\pi} \sin(\pm \phi' - \phi_o) e^{i(mB-k)\phi o} e^{-ik_m S_r r_o \cos(\pm \phi' - \phi_o)} d\phi_o,
\]

\[
I_x' = \frac{1}{2\pi} \int_0^{2\pi} e^{i(mB-k)\phi o} e^{-ik_m S_r r_o \cos(\pm \phi' - \phi_o)} d\phi_o.
\]

Now, using the definition of the Bessel's function, the tangential integrals (A.7) can be expressed as:

\[
I_r = i e^{i(mB-k)(\pm \phi' - \pi/2)} J_m(k_m S_r r_o),
\]

\[
I_\phi = \frac{mB-k}{k_m S_r r_o} e^{i(mB-k)(\pm \phi' - \pi/2)} J_m(k_m S_r r_o),
\]

\[
I_x' = e^{i(mB-k)(\pm \phi' - \pi/2)} J_m(k_m S_r r_o).
\]

where \( J_n(x) \) is the Bessel's function of order \( n \) and argument \( x \) and \( J'_n(x) \) is the derivative of Bessel's function with respect to the argument. Using Equation (A.8) in (A.6) gives the analytical expression for loading noise:

\[
P_{Lm} = -\frac{ik_m e^{i k_m r_o}}{4\pi S_o} \sum_{\mu, \nu} \sum_k e^{i(mB-k)(\pm \phi' - \pi/2 - \phi_{\mu, \nu}) + k_m r_o} e^{-ik_m S_r x_{\mu, \nu}} \times
\]

\[
\left[ i S_x F_{k, \mu, \nu} J_m(k_m S_r r_o) + \frac{mB-k}{k_m r_o} F_{\phi, \mu, \nu} J_m(k_m S_r r_o) + S_z F_{k, \mu, \nu} J_m(k_m S_r r_o) \right].
\]

Equation (A.9) is the analytical expression for loading noise when the observer is in the far-field. Tangential integrals for the far-field can be analytically expressed in terms of Bessel's function. For a near field observer, a numerical integration is performed, discussed in the next subsection A.2.

### A.2. Near-Field Calculations

As mentioned earlier, Equation (4.23) is the exact expression without any approximation. For an observer in the near field, the source distance cannot be neglected as compared to the observer, as done for the far-field observer.

The Green's function is defined in Equation (4.18), with value of amplitude radius (\( S \)) and phase radius (\( \sigma \)), defined in Equation (4.4) and (4.5) respectively. These equations are directly used to calculate the Green function's derivative in Equation (4.24).

To simplify the source observer distance, the following notations are used:

\[
X = x - x_o,
\]

\[
Y = y - y_o,
\]

\[
Z = z - z_o,
\]

where \( x \) is the observer distance and \( x_o \) is the source distance which is rotating and depends on cylindrical coordinates \( r_o, \phi_o \) and \( x_o \). Amplitude radius (\( S \)) and phase radius (\( \sigma \)) are redefined using Equation (A.10):

\[
S = \sqrt{X^2 + \beta^2(Y^2 + Z^2)},
\]

\[
\sigma = \frac{1}{\beta^2} (MX + S).
\]
A.2. Near-Field Calculations

Green function derivative can be defined as,

\[ G_r = \frac{\partial G_m}{\partial r_o} = (i k_m a_r - S_r / S) G_m , \]
\[ G_\phi = \frac{1}{r_o} \frac{\partial G_m}{\partial \phi_o} = (i k_m a_\phi - S_\phi / S) G_m , \]
\[ G_{x'} = \frac{\partial G_m}{\partial x'_o} = (i k_m a_{x'} - S_{x'} / S) G_m , \]  
(A.12)

where subscript \((r, \phi, x')\) for \(\sigma\) and \(S\) represents the derivative with respect to \(r, \phi, x'\) respectively, as depicted by \(G_r, G_\phi\) and \(G_{x'}\). Using Green's function derivative (A.12) in Equation 4.24, gives the expression:

\[ P_{Lm}(x) = \frac{B}{2\pi} \sum_{\mu, \nu} \sum_k e^{-i[(mB - k)\phi_s - k\phi_{ref}]} \int_0^{2\pi} \left[ F_{r k\mu,\nu} G_r + F_{\phi k\mu,\nu} G_\phi + F_{x' k\mu,\nu} G_{x'} \right] e^{i(mB - k)\phi_o} d\phi_o . \]  
(A.13)

\[ P_{Lm}(x) = \frac{B}{2\pi} \sum_{\mu, \nu} \sum_k e^{-i[(mB - k)\phi_s - k\phi_{ref}]} \times I_{near}(k,\mu,\nu) . \]  
(A.14)

So, to obtain the acoustic radiation for a near field observer, numerical integration needs to be performed over the cylindrical co-ordinate \(\phi_o\) on the Equation (A.13).
A.3. List of Symbols

- \( B \) = number of blades
- \( k_m = mBM_t \), wave number in harmonic Green’s function
- \( r = \sqrt{x^2 + y^2 + z^2} \), radiation distance from center of propeller
- \( x_r \) = retarded x co-ordinate
- \( r_{\mu} \) = radius to \( \mu^{th} \) row of source element on blade
- \( m \) = harmonic of blade passing frequency
- \( k \) = index for loading harmonics
- \( \phi' \) = tangential co-ordinate to observer in retarded system in propeller system
- \( \phi_{\mu, \nu} \) = cylindrical coordinate to source element
- \( \phi_{ref} \) = reference angle at starting, \( t = 0 \)
- \( S_o = \sqrt{x^2 + \beta^2(y^2 + z^2)} \), \( (x, y \text{ and } z \text{ are observer co-ordinates}) \), amplitude radius for far field
- \( S_c = \frac{\cos \theta'}{1 - M \cos \theta} \)
- \( S_s = \frac{\sin \theta'}{1 - M \cos \theta} \)

- \( \theta, \theta' \) = angle to observer from flight direction, shaft axis
- \( x_{\mu, \nu} = x \text{ position of source element} \)
- \( F_{rk\mu, \nu}, F_{\phi k\mu, \nu}, F_{xk\mu, \nu} \) = loading components in cylindrical co-ordinates direction
- \( f_n(x) \) = Bessel function of order \( n \) and argument \( x \)
- \( f'_n(x) \) = derivative of Bessel function of order \( n \) and argument \( x \)
- \( \mu \) = index for counting source elements in radial direction (while discretization)
- \( \nu \) = index for counting source elements in chordwise direction (while discretization)
- \( \beta^2 = 1 - M^2 \)
- \( M \) = flight mach number

Retarded x co-ordinate \( (x_r) \) is calculated as follows,

\[
x = x_r - Mr
\]

\( x_r \) is found by squaring both sides in above equation, the resulting value of \( x_r \) is given as,

\[
x_r = \frac{1}{\beta^2}(x + MS_o)
\]
AeroDyn is a time-domain wind turbine aerodynamics module that has been coupled into the FAST version 8 multi-physics engineering tool to enable aero-elastic simulation of horizontal axis turbines. AeroDyn can also be used standalone without being coupled to FAST. AeroDyn consists of four submodels:

- rotor wake/induction
- blade airfoil aerodynamics
- tower influence on the fluid local to the blade nodes
- tower drag

Blade Element Momentum (BEM) theory is used for wake modelling. Various corrections such as tip-loss, hub-loss, Glauert’s correction for heavily loaded rotor and skewed wake correction can be also be taken into account while modeling the wake. A detailed theory implemented in AeroDyn can be found in its theory manual [35]. The influence of the tower is taken into account by calculating the velocities around the cylinder using the potential flow theory [35].

For the current project, it is desired to obtain the aerodynamic calculation for the fan model with the influence of the tower. AeroDyn is originally designed to calculate loads of a wind turbine configuration (tower model can be implemented). A series of test cases were simulated using AeroDyn to study the possibility of its application for the required geometry (fan-model).

The instructions for handing the AeroDyn and various parameters are given in the user manual [25]. NREL 5MW wind turbine is used as a test case which is one of the reference models given in AeroDyn. A list of parameters used for the initial reference design is given in Table B.1. Most of the parameters are self-explanatory by their names. Figure B.1 shows the wind turbine geometry parameters. Note that for all the cases with the influence of tower, d/D is approximately 1, where d is the distance between tower & rotor plane and D is the diameter of the tower.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
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<tbody>
<tr>
<td>Number of Blades</td>
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</tr>
<tr>
<td>Hub Radius (m)</td>
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<tr>
<td>Hub Height (m)</td>
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</tr>
<tr>
<td>Overhang (m)</td>
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</tr>
<tr>
<td>Shift Tilt (deg)</td>
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</tr>
<tr>
<td>Precone (deg)</td>
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<tr>
<td>Pitch Angle (deg)</td>
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<tr>
<td>Yaw Angle (deg)</td>
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</tr>
<tr>
<td>Wind Speed (m/s)</td>
<td>7</td>
</tr>
<tr>
<td>Rotational Speed (rpm)</td>
<td>12.1</td>
</tr>
</tbody>
</table>

Table B.1: List of parameters for 5MW NREL Wind Turbine
Figure B.1: Wind Turbine Geometry [25]
B.1. Comparison of Steady and Unsteady Models

It is possible to choose a steady or unsteady blade aerodynamic model in AeroDyn. Unsteady models are based on Leishman-Beddoes model [25].

Figure B.2 shows the comparison of the steady and unsteady model without the influence of the tower for one rotation of the rotor. Two parameters: angle of attack (at $r/R = 0.8$) and rotor thrust coefficient are chosen. It can be seen in Figure B.2(a), angle of attack for steady and unsteady model matches exactly. It is not expected to have a change in the angle of attack over one rotation at one location of the blade if the influence of the tower is not considered. However, reduction, in this case, is caused due to the yaw angle which is $20^\circ$ for this simulation.

Thrust coefficient is different for both steady and unsteady models. Thrust coefficient predicted by the unsteady model is lower than the steady model.

Figure B.2: Steady and unsteady comparison without tower influence

Figure B.3 shows the similar comparison to that of Figure B.2, but with the influence of tower. Both steady and unsteady models can predict the peaks in the thrust coefficient, however, peaks predicted by the unsteady model are sharp and its value is lower than the steady model.

Figure B.3: Steady and unsteady comparison without tower influence

Figure B.4 shows the effect of tower with unsteady model. The effect of the tower can be visualized by the reduction of the angle of attack when the blade passes the tower. A similar effect can be noticed in the thrust coefficient with three peaks (3 blades) corresponding to the effect of the tower.
B.2. Effect of parameters

B.2.1. Effect of geometry parameters

For all the comparisons in this section, only the unsteady model is considered. Figure B.5 shows the effect of different parameters like yaw angle, shift tilt and precone (values of the parameter for the earlier configuration is given in Table B.1). To see the effect on the angle, first, the case without tower influence is considered. It can be seen that the yaw angle affects the angle of attack and thrust coefficient as compared to other parameters. The angle of attack is almost constant for the case when the yaw angle is zero (0°).

B.2.1.1. Effect of multiple parameters

In the previous section, the effect of each parameter is studied independently. In this section, the effect of multiple parameters will be studied simultaneously. Figure B.7 shows the effect of multiple parameters with the tower. All the parameters under consideration are set to zero and as expected it can be seen the angle of attack is constant when the blade is not passing the tower.

B.2.2. Effect of Velocity

This section includes the effect of the tower for the results presented. And the geometry used for this simulation is the reference geometry without any modifications.
Figure B.6: Effect of different geometric parameters with tower

Figure B.7: Effect of multiple geometric parameters with tower

Figure B.8 shows the effect of different incoming velocities on the angle of attack, thrust coefficient, and thrust force. As expected, the angle of attack reduces with a decrease in incoming velocity and is even negative for very low velocities (due to twist in the blade). To understand the effect on thrust, it is better to visualize through the thrust force Figure B.8(c). Thrust force reduces with a decrease in velocity. Also, it is interesting to note that the thrust force reduces when the blade passes in front of the tower, whereas the thrust coefficient increases. It suggests that local velocity is being used in AeroDyn for normalizing the thrust force.

Also, for $U_{inf} = 0 \text{m/s}$ case, there is some value of thrust force (which is due to twist, precone and shift tilt). As there is no variation in the thrust value for blade passing the tower, it can be said that AeroDyn only takes into account the velocity deficit effect due to tower but not the blade passage effect (which is due to the interaction between tower and blade pressure's).

B.3. Concluding Remarks

For this project, experiments are to be conducted in an anechoic chamber without any incoming velocity and blades are to be rotated using a motor. So, it is required to obtain the aerodynamic forces with tower influence due to the blade passage effect rather than due to the velocity deficit. AeroDyn is a good tool with the inclusion of various corrections and tower influence for simulating wind turbine loads, but it is not suitable for this project.
Figure B.8: Effect of velocity with tower influence
Bibliography


