HARD LIMITING AND
SEQUENTIAL DETECTING
LORAN-C SENSOR
Aan Elvira en Robbert
en aan allen die op enigerlei
wijze meegewerkt hebben aan het
tot stand brengen van dit proefschrift

HARD LIMITING AND
SEQUENTIAL DETECTING
LORAN-C SENSOR

Proefschrift

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1 ABSTRACT

The primary objective of the study reported in this thesis is the investigation of the possibilities for large scale integration of a Loran-C navigation receiver.

The basic concept of a Loran-C receiver consists of three parts. The antenna with the RF front-end, the digital timer, and finally the digital signal-processor with the associated software. The number of transistors required to realize such a receiver is almost within reach of the nowadays production technics of VLSI circuits. However, the combination of linear and digital circuits, and the large dynamic range of the received signals are serious obstacles to come to a single chip design.

This thesis gives the results of the design-process of the most critical parts of a Loran-C sensor. After a short survey of the Loran-C system and some typical Loran-C receiver configurations, it is explained why the hard-limiter receiver is the optimal choice in respect of low power consumption and low-cost.

The first analysis concerns the antenna. Magnetic- and electric antennas are introduced. The selective- as well as the non-selective configurations are discussed in respect with antenna length and dynamic range.

The received Loran signals and the less wanted interfering signals (including noise) must be band-pass filtered, to discriminate these two types of signal as much as possible, while preserving sufficiently the original envelope waveform.
Then the design objectives of hard-limiting amplifiers are discussed in relation to power consumption and signal amplitude-dependent propagation-delay variation. A low-power hard-limiter integrated circuit is shown.

The digital timing circuits and the micro computer of the Loran-C receiver are left to the reader, as adequate general information is available from other sources.

However, the sequential signal processing and the zero-crossing tracking loop are discussed in detail, as they form a very essential part of the receiver. Much attention is also paid to the initialization procedure of the receiver. It is shown that, even for large initial clock errors, acquisition of noisy signals is feasible.

Finally, a short treatment is given of range-range and pseudo range-range navigation. It is shown how to keep geodetic calculations simple and accurate by approximating the Gauss ellipsoid-sphere transformations by polynomials.

### 2 INTRODUCTION

There would appear no end to developing new systems for electronic navigation. Indeed, limiting our view to long-range navigation, we can distinguish five rather well-known systems: Omega, Decca, Loran-C, GPS (Global Positioning System) and Transit. The first four of these systems are based on range or range-difference measurements, while the Transit navigation system uses the Doppler shift phenomenon. It is also the only non-continuous navigation system available, as the observer may derive his position about once in every hour and a half. The above-mentioned systems can be characterized with regard to range and the 95% accuracy (table 2-1).

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>RANGE</th>
<th>ACCURACY (95%)</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omega</td>
<td>World</td>
<td>16 km</td>
<td>10-14 kHz</td>
</tr>
<tr>
<td>Decca</td>
<td>200 km</td>
<td>300 m (?)</td>
<td>70-130 kHz</td>
</tr>
<tr>
<td>Loran-C</td>
<td>2000 km</td>
<td>300 m</td>
<td>90-110 kHz</td>
</tr>
<tr>
<td>GPS/p-code</td>
<td>World</td>
<td>15 m (3 dim.)</td>
<td>1.5 GHz</td>
</tr>
<tr>
<td>Transit</td>
<td>World</td>
<td>600 m</td>
<td>150/400 MHz</td>
</tr>
</tbody>
</table>

Table 2-1. Approximate specifications of five long-range navigation systems. The GPS has three-dimensional capabilities. The Transit data are based on an assumed speed error of 1 knot. Data (excl. Decca) are from [7-1].

The Decca system basically uses for each transmitting station a unique frequency. In a four transmitter chain, all four different frequencies are derived from a common subharmonic frequency f of around 14.2 kHz. The various Decca transmitters emit the fifth, the sixth, the eighth and the
ninth harmonic of $f$. In the receiver the $6f$ signal is multiplied by three, the $9f$ signal by two, and then these two resulting $18f$ signals are phase compared. In a similar way the $6f$ signal is multiplied by four and the $8f$ by three, yielding two $24f$ signals. Finally, the fifth harmonic of the $6f$ signal is phase compared with the sixth harmonic of $5f$. These two phase-difference comparisons serve as a measure for the position of the observer. Thus, the Decca system applies frequency diversity to identify the various stations. 

Decca stations basically transmit continuous-wave signals with an inherently very narrow bandwidth. This is advantageous with respect to moderately required transmitter power for obtaining an acceptable post-detection signal-to-noise ratio at the receiver site. There is also an important disadvantage: the radio signals will arrive at the antenna of the receiver via different paths. The first and the shortest path (the ground wave), passes just over the surface of the earth. A second path (the sky wave) traverses the ionosphere. At the receiver site the two signals will be added vectorially. As the path via the ionosphere is very unstable in length, the propagation time will be just as unstable. The measured "delay" will fluctuate unpredictically.

Electronic navigation means, in fact, measuring the propagation time of radio signals traveling from the transmitter to the receiver. Therefore, as soon as the sky wave signal level can not be neglected in comparison with the ground wave signal level, navigation accuracy will deteriorate. In general, the ground wave signal level decreases faster than the sky wave level as the range increases. This phenomenon reduces the accurate operating range to about 200 - 400 km, depending on the propagation conditions. [1-2, 1-12]

Omega uses very low frequencies from 10 to 14 kHz. In this frequency range the ionosphere and the earth's surface act as a duct for the propagating radio waves. This duct varies in height with variations in the solar-radiation intensity, which in turn causes propagation-velocity variations. Although there is some regularity from day to day in the propagation anomalies, it is almost impossible to obtain accurate position fixes. The 2-sigma accuracy as given in Table 2-1 is somewhat pessimistic. The 1-sigma accuracy is generally in the order of 2-4 nautical miles. However, the sometimes occurring "sudden ionospheric disturbances" are behind the poor 2-sigma figure, as the probability function of the error is far from Gaussian. Each one of the eight Omega stations transmits consecutively four different frequencies of 10.2, 11.05, 11.333 and 13.6 kHz as well as a frequency unique for each transmitter. Each frequency is emitted as a burst lasting about one second. The transmitting sequence of all transmitters is the same, and the pattern is repeated every 10 seconds. A single frequency is sufficient for navigation, however, the additional frequencies help to reduce propagation anomalies by means of redundancy. Station identification in the Omega system is based on frequency and time diversity. The eight stations are spread all over the globe: Norway, Liberia, Australia, North Dakota, Hawaii, La Reunion, Argentina and Japan. [1-11]

Loran-C uses almost the same frequency band as Decca. But there is a significant difference between these two systems. Loran-C stations transmit sharply rising high-energy bursts with a central frequency of 100 kHz. The length of a single burst is limited to 270 µs. At the receiver site the ground wave signals will always arrive first, and the sky wave will come in after some extra delay. The sharply rising edge of the envelope of the signal
provides the unique opportunity to distinguish the ground wave from the sky wave signals by using just the very first part of the burst. The ground-wave propagation velocity is stable in time, so that navigational accuracy is guaranteed even in the presence of sky waves.

However, sharp leading edges consume much spectral bandwidth, which in turn requires a large receiver bandwidth. Clearly, the large bandwidth together with the long range necessitate enormous transmitter powers to guarantee an acceptable signal-to-noise ratio in the receiver. And, indeed, some Loran-C stations emit peak power in excess of 1.5 MW!

Each Loran-C station emits consecutively eight bursts, accurately spaced in time at 1 millisecond intervals. These eight bursts are repeated at the Group Repetition Interval (GRI), which is the identifier for a specific chain. Each burst can be transmitted with or without a 180 degree phase shift. This phase polarity modulation is used for phase coding to identify the various stations in a particular chain. The Loran-C navigation system is in wide use in the northern hemisphere. [1-8]

To demonstrate the advantages of the sky wave-ground wave discrimination capability of the Loran-C system over the Decca and Omega systems, we compared the phase stability of five signals as received at the Department of Electrical Engineering of the Delft University of Technology in Delft in The Netherlands. The first two signals originate from the Omega stations in Norway and Liberia. The results of a 24 hour tracking period are given in Fig. 2-1. The phase jumps at sunrise and sunset are really impressive, but frustrating. Fig. 2-2 gives the phase stability of two continuous wave signals from the Droitwich transmitter in London at 200 kHz and the HBG transmitter at 60 kHz in Neuchatel in Switzerland. The very different behavior of these two signals
might be attributed to the different distances from the two stations and the different frequencies. The same figure depicts the stability of the Loran-C signal as received from the Sylt transmitter in Germany. The phase stability of the Loran-C signal is excellent. The stability of the frequency standards in the various transmitters and the receiver is such that there will not be a noticeable instability in the phase measurements.

NAVSTAR GPS (Global Positioning System) is the newest branch on the tree of long range navigation systems. According to the plans, a series of 18 satellites will be deployed in six orbital planes, which will be equally spaced 60 degrees apart in longitude and inclined to the equator at 55 degrees. Each plane will contain 3 satellites, spaced at 120 degrees in that particular plane. Each satellite will transmit continuously a direct sequence spread spectrum signal, containing orbit parameters and the transmission time of the signal. The receiver will then despread and decode the signal, and calculate the propagation times of the signals from at least four satellites. Due to the stable and accurately known propagation velocity of the 1.5 GHz signals, the navigational accuracy will be superb. In the precision mode (the military mode) the fixes show a two-sigma accuracy of 12 m [7-1]. American national-security requirements will unfortunately cause a deterioration in the civil (coarse) mode accuracy to an estimated two-sigma value of 100 meters. GPS is still in the test phase. The system is expected to be fully operational in 1987 [7-2, 7-3, 7-4].

Transit was the first widely accepted satellite navigation system. The nearly polar-orbiting satellites transmit 150 and 400 MHz signals, which contain the orbit parameters. The receiver measures the Doppler shift of the satellite signal for a period of about 10 to 20 minutes. The resulting Doppler curve uniquely indicates the position of the receiver in relation to the satellite. For high accuracy the satellite should not show an elevation too low (short time of radio contact and "flat" Doppler curve) or too high an elevation. The latter will introduce large errors in a direction perpendicular to the flight path of the satellite. Another serious constraint is that during the measurement of the Doppler shift, the true speed and course of the observer must be known. Otherwise it is not possible to accurately recover the correct Doppler curve. The Transit system is very popular for determining bias errors in other navigation systems, such as Omega and INS (Inertional Navigation System).

In general, the navigation receiver does not "know" the time so accurately that it may measure the propagation time of signals from a single transmitter. Therefore, it needs two transmitters to establish a difference in propagation time. All positions showing the same Difference in Time of Arrival (DTA) will form a hyperbolic Line Of Position (LOP) on an ellipsoid (the earth's surface). A third transmitter makes it possible to derive a second LOP. The navigator will then be at the crossing of the two LOP's. However, if exact times (from transmission and reception) were known, two transmitters would be sufficient for a fix. The application of atomic clocks makes such a form of navigation feasible for a limited period of time. This method of operation, called RHO-RHO navigation, is actually dead reckoning. In the case of satellite navigation we operate in a three-dimensional space, which implies that at least four transmitters are required for a three-dimensional position fix. It means also that the altitude of the navigator is determined, which might be advantageous to airplanes.
The traditional users of navigation equipment are airplanes and ships. However, two new user fields are arising: land-based vehicles and small boats for fishing and leisure. The latter is understandable: due to the decreasing prices of large-scale integrated circuits, electronic navigation has provided a really affordable improvement in security in dangerous shipping areas.

The application of navigation in cars is intended as a navigation facility for the driver or as an aid to a fleet controller. There is also a wide-spread interest in aiding a driver to find his way in an unknown area or a city. In big cities, it would be an impressive help to ambulance, police, fire department and taxicab strategies if all vehicle positions were known continuously. The simplest way to accomplish this task is to have drivers frequently reporting their positions, a method extensively applied nowadays. Much easier, from the viewpoint of the driver and the dispatcher, is the application of a navigation system in the car. The computer of the central dispatcher would then regularly call up the positions of all cars via the mobilophone network. The selective "navigator"-calling unit will prevent the driver from being disturbed. Furthermore, it could also provide information about current jobs.

Especially in cities, the navigation accuracy obtained might be insufficient for accurately determining the street the car is on. Land-based radio navigation systems show very peculiar accuracy limits. These limits originate from the somewhat troublesome radio path and the limited information bandwidth with respect to the highly dynamic maneuverability of the vehicles. In downtown areas, the reception of navigation transmitters is seriously hampered by multi-path, or even no-path, propagation. So, vehicles might frequently be located in tunnels and under power lines, where no position data can be obtained. In such cases a car can only navigate in the dead-reckoning mode.

Still, upcoming mass storage devices, e.g. the compact disk, will make it possible to store street and road maps for wide areas. A combination of odometer, heading sensors, electronic maps and radio navigation will then yield the accurate position of the driver. It is a small step towards routing advice to the driver or towards radio relaying a car position to the fleet controller station [4-12].

Finally, another promising application of electronic navigation is as an aid to airplanes during the approach phase to airstrips not provided with an Instrument Landing System (ILS). The pilot of the plane may be supported by the following relative navigational data: distance to go to the beginning of the runway and the amount of cross track (XTK) from the center line of the runway. Thousands of runways, mainly in the USA, could be instrumented in this way at a relatively low cost, as compared to that required for traditional ILS [4-7].

Until now, we have not indicated any preference for one of the five above-discussed navigational systems in terms of finding a generally applicable and low-cost navigation unit. Although the title of this thesis already gives a hint, we came to our final choice in a rather negative way: rejecting the four others for one reason or another. Omega is less ideal for reasons of limited accuracy. Decca is too limited in range, mainly from sky wave pollution of the ground wave. A second objection is that the Decca Company tries to keep the receiver design under its own control, though without success.
A problem not yet mentioned is the lane ambiguity in the Omega and Decca systems. Phase differences will be repeated every full cycle of the reference frequency. Rather ingenious techniques have been developed to find the correct lane number. This lane ambiguity is much less of a problem in a Loran-C receiver. The burst character of the signals is a great advantage in preventing lane or cycle uncertainties. The highly accurate three-dimensional GPS is a very promising system. However, three serious objections arise: first, the rather high cost of even a simple set; second, the high power consumption of the UHF part of the receiver; and third, the complex mathematical routines required because of the nonstationarity of the satellites.

And finally, the Transit system will probably be phased out some years after the GPS system is fully operational as the primary DOD (Department Of Defence) navigation system.

In conclusion, Loran-C is in our opinion the best choice for low-cost navigation. It provides acceptable accuracy for large parts of the northern hemisphere. The low frequency is very convenient for implementation in VLSI. This low frequency is also of prime importance in reducing the power consumption drastically. The antenna can be made in a small size. And last, but not least, the world-wide competition in the design of Loran-C receivers challenges designers to come up with new ideas for such receiver/navigators. This is particularly true for the European continent, where the interference from so many radio stations threatens the proper functioning of a Loran-C set. Ironically enough, much of the interference comes from the competing Decca transmitters. As will be shown later, such interference might be rejected by notch filters.

However, it should be said that the way we receive, detect and consecutively process the Loran-C signals is also applicable to a Decca or Omega receiver [5-10].

Finally, it should be observed that our preference for the Loran-C system is not without any doubts. But this is more the rule than the exception for any complicated technology.
3 INPUT AND OUTPUT CONSTRAINTS OF A LORAN-C SENSOR

In the foregoing chapter we wound up selecting the Loran-C system as the most suitable for low-cost accurate electronic navigation. Before designing a receiver, we must thoroughly acquaint ourselves with what sort of signals the receiver may encounter and what the receiver is expected to do with these signals. In other words, we need to know what type of information should be supplied to the operator, regardless of whether the operator is a human being, a computer or an auto-pilot system.

The United States Coast Guard has been assigned as the manager of the Loran-C system network. However, the Loran-C system is no longer an exclusively American facility. Two chains operate in the Soviet Union, and the French have two transmitters under construction. In spite of this multi-national character, the cooperation between the various countries has thus far proved satisfactory.

The same can be said for the broad group of users and manufacturers of Loran-C equipment. Representatives of these groups are members of the Special Committee No. 70 of the Radio Technical Commission For Marine Services. Almost all members of the committee have the American nationality.

However, a more international Committee could give an extra impulse to expediting the wide introduction of Loran-C in Europe and Asia.

This Special Committee No. 70 published the significant report "Minimum Performance Standards (MPS) Marine Loran-C Equipment", [5-5]. The MPS applies only to equipment which is allowed on board ships with a dead weight in excess of 1600 gross tons. This does not mean that all commercially available receivers meet the MPS specifications. Many low-cost units are meant for pleasure boats etc. However,
when safety is of prime importance, the MPS specifications really do make sense.

The new field of Automatic Vehicle Monitoring (AVM) requires rather different receiver specifications. The main reason is that a vehicle is much more dynamic in its maneuvering than a crude-oil carrier. Dean has done some very interesting automotive tests in the city of New York. He found accuracies of better than 100 meters in 50% of the fixes [4-6]. However, Dean applied corrections for propagation anomalies. Parness proposes a set of specifications especially for AVM applications [4-4]. Although these proposals are not yet official, they should be considered as meaningful.

Applying Loran-C as a navigational aid (in the differential mode) for landing airplanes is so tentative that, until now, nobody has come up with aeronautical specifications. But with the MPS and typical maneuvering parameters of small airplanes in mind, adequate specifications should be attainable. However, in this thesis we will not pay attention to the application of Loran-C in small airplanes.

3.1 INPUT CONSTRAINTS

In an idealized situation the receiver has to process nothing but the Loran-C signal as emitted from the various stations of a particular chain. As could be expected this is far from reality. The Loran-C signal itself is corrupted by its own reflected signals from the ionosphere. And due to a non-linear relation between phase and frequency along the transmission path, the envelope of the burst is distorted. In addition, the receiver faces propagation velocity anomalies. This phenomenon is difficult to predict in areas with strongly varying conductivities of the earth's surface. Such variations will especially occur at transitions from land to sea and in mountainous areas.

To make the situation even worse, the signals are embedded in a blend of Gaussian noise from the receiver electronics, atmospheric noise, and interferences from many other stations in the LF band.

A navigational receiver will strive by means of signal processing to minimalize the effects of the signal disturbers. However, to design high-performance signal processors, one must first know the relations between the wanted Loran-C signal and the unwanted interferences and noises. Therefore, we will start by specifying each of the above-mentioned signals separately, and also in relation to the Loran-C signal. All signal levels will be expressed in field strengths, as this frees us from specifying the effective length and the directivity of the receiving antenna.

3.1.1 LORAN-C SIGNAL

The Loran-C signal is basically a burst of 100 kHz sine waves. The signal \( L(t) \) as emitted can be expressed by:

\[
L(t) = A (t/t_p)^2 \exp\left(2(1-t/t_p)\right) \sin(\omega t) \quad \text{V/m} \tag{2-1}
\]

where 
- \( t = \text{time for } t \geq 0 \)
- \( t_p = 65 \mu s \)
- \( A = \text{peak amplitude of the burst} \)
Fig. 3-1 graphically represents the basic burst. The envelope of the burst is such that 99% of the radiated energy will be in the frequency band of 90 to 110 kHz.

With the assumption of an ideal transmission medium, this signal will arrive attenuated, but undistorted at the receiver site with a phase delay which depends on the distance from the transmitter. Unfortunately, this is far from what happens in reality. The received signal differs from the emitted signal because of multipath transmission, envelope distortion, and secondary phase effects. We will mention each of these effects separately.

The emitted signal can reach the receiver's antenna via different paths. The shortest path extends just over the earth's surface, and is therefore called the ground wave. A second path is formed via reflections of the signal in the ionosphere. These reflected signals are called sky waves. The delay of the sky wave in respect to the ground wave depends on the altitude of the ionosphere and the distance from the transmitter. In the case of a short distance, the delay is roughly the time the signal takes to travel from the ground to the ionosphere and back again. When the receiver is further away from the transmitter, the delay is not just a measure of the difference in path lengths, but it also depends on the propagation velocities which are different for the ground-wave and the sky-wave signals. The vertical gradient of the refraction (n) of the atmosphere reduces the free-space propagation velocity (V_c) of radio waves close to the earth's surface somewhat more than in the ionosphere. This phenomenon reveals a tendency to diminish the time difference between the two signals at greater distances from the transmitter.

The finite conductivity of the earth's surface introduces an extra delay, called the Secondary Factor (SF). The SF is specified for an all sea-water path. The signal will be additionally delayed when propagating over ground with various conductivities and profiles, such as mountains and coasts. This additional delay is denoted by the Additional Secondary Factor (ASF). The effective propagation velocity is

\[ \omega = 2\pi \cdot 10^5 \text{ rad/s} \]
time $t$ over the distance $L$ amounts to:

$$t = L \cdot (\frac{n}{V_c} + SF + ASF)$$  \hspace{1cm} (2-2)$$

where $t$ = effective propagation time over distance $L$

$V_c = 2.99792458 \times 10^8$ m/s (propagation velocity in free space)

$n = 1.000338$ (refraction of the standard atmosphere at the surface of the earth)

$SF = $ Secondary Factor, about $-0.3 \mu s + 2.1 \mu s/Mm$

for $\gamma = 5$ mhos/m & $L > 400$ km

$ASF = $ Additional Secondary Factor, varying from

$0.1 \mu s + 0.6 \mu s/Mm \ (\gamma = 0.05$ mhos/m) to $2.5 \mu s + 3.0 \mu s/Mm \ (\gamma = 0.001$ mhos/m) for $L > 400$ km.

Clearly, the correct knowledge of $n$, $SF$ and $ASF$ is of prime importance as it sets the attainable accuracy of radio navigation. The trouble, however, is that the $ASF$ strongly depends on the conductivity of the ground, as can be seen from Fig. 3-2. Therefore, in practical situations the transmission path will show rather varying values for the conductivities. This means that for precision navigation, the navigator must analyze the trajectories and determine the effective $ASF$'s along the propagation path.

Fig. 3-3 shows the sky-wave delay with respect to the ground wave as a function of the height of the ionosphere and distance from the transmitter. The relevant data have been taken from [1-2]. During the daytime, the apparent ionospheric height amounts to approximately 70 km, but in the nighttime the height increases to about 90 km. At 2000 km from a transmitter, the sky-wave delay might be as low as 30 $\mu s$ during the daytime.
The complex signal such as the one received is represented by \( L(t) \):

\[
L(t) = A \cdot (t'/t_p)^2 \exp(2(1-t'/t_p)) \sin(w(t+\psi)) + B \cdot ((t''-t_s)/t_p)^2 \exp(2(1-(t''-t_s)/t_p)) \sin(\omega(t-t_s)+\psi_s) \text{ V/m} \tag{2-3}
\]

where

- \( t = \text{time for } t \geq 0 \text{ and } (t-t_s) > 0 \)
- \( t_p = 65 \mu s \)
- \( \omega_p = 2\pi \times 10^5 \text{ rad/s} \)
- \( t' = t-ECD \text{ ground wave for } t' > 0 \)
- \( t'' = t-ECD \text{ sky wave for } t'' > 0 \)
- \( \psi = \text{phase of the ground wave} \)
- \( \psi_s = \text{phase of the sky wave} \)

Fig. 3-5 shows a complex Loran-C burst where the sky wave amplitude is 12 dB more than that of the ground wave, while the latter leads the sky wave by 32.5 \( \mu s \) (a typical MPS value).
delay of 45 μs, the sky-wave signal might reach a level of 26 dB over the ground-wave signal level.

3.1.2 NOISE

Noise is characterized by the spectral energy-density level and by the distribution curve of the amplitudes. It is the latter that gives rise to numerous discussions as soon as VLF and LF communications are involved. Noise generated in resistors or semiconductors has a mainly Gaussian distribution, and is therefore easy to specify. But at the output of the receiver antenna, we will see atmospheric noise which deviates strongly from the Gaussian distribution. Atmospheric noise at these low frequencies is mainly generated by lightning discharges. These discharges are transmitted over very long distances, as the propagation loss at 100 kHz is rather low. Lightning discharges from large distances, in excess of 10,000 km, will be received randomly spaced in time and at a rather high rate, while the amplitudes will not vary over extreme values. Adding all these lightning strokes vectorially results in a more or less Gaussian distribution. But as soon as the discharges take place at distances closer to the receiving antenna, the amplitudes are far from equal in energy level and the repetition rate will be much lower. This will cause a deviation from the Gaussian distribution for high noise levels. The number of lightning discharges per unit of time depends on the latitude and the season of the year. And as the lightning phenomenon is not even regular in character, the reader will understand that there are almost as many atmospheric noise models as there are people dealing with LF or VLF communications.

The CCIR has carried out intensive measurements on noise levels and noise distributions all over the world and at various times of the day and the year. The results are published in [2-3]. Although this report has been very useful to system designers, there are some doubts about the exactness of the amplitude distribution due to the lack of sophistication of the available equipment in 1964.

More recently, Feldman [2-14] of the United States Coast Guard repeated the noise-level measurements under three different types of weather conditions: "quiet", "tropical" and "frontal". These three types of weather are intended to be representative for real-life situations. The probability density (pd) of the noise’s amplitudes for the three types of weather shows a Gaussian character for the low amplitudes, while the pd’s for the higher amplitudes depend strongly on the type of weather. As could be expected, the "quiet" type tends more towards Gaussianity than the "frontal" type of atmospheric noise.

Van der Wal and van Willigen [3-3] made a mathematical approximation of the pd’s provided by Feldman. It resulted in a Gaussian part and one or more polynomial parts. The equations are listed below.

Quiet type:

\[ pd(y) = \begin{cases} 
0.787 \cdot 10^1 \cdot \exp(-0.5(y/0.05)^2) & \text{for } 0 \leq y < 0.1 \\
0.106 \cdot 10^{-4} \cdot y^{-5} & \text{for } 0.1 \leq y < 0.2 \\
0.595 \cdot 10^{-3} \cdot y^{-2.5} & \text{for } 0.2 \leq y < 13 \\
0 & \text{for } y \geq 13 
\end{cases} \]

rms value = 0.101
Tropical type:

\[
pd(y) = 0.599 \times 10^{-1} \times \exp(-0.5(y/0.057)^2) \quad \text{for} \quad 0 \leq y < 0.1
\]
\[
= 0.669 \times 10^{-2} \times y^{-2.3} \quad \text{for} \quad 0.1 \leq y < 20
\]
\[
= 0 \quad \text{for} \quad y \geq 20
\]

rms value = 0.392

Frontal type:

\[
pd(y) = 0.428 \times 10^{-1} \times \exp(-0.5(y/0.065)^2) \quad \text{for} \quad 0 \leq y < 0.05
\]
\[
= 0.414 \times 10^{-1} \times y^{-1.45} \quad \text{for} \quad 0.05 \leq y < 1
\]
\[
= 0.414 \times 10^{-1} \times y^{-1.80} \quad \text{for} \quad 1 \leq y < 2.6
\]
\[
= 0.578 \times 10^{-1} \times y^{-2.15} \quad \text{for} \quad 2.6 \leq y < 20
\]
\[
= 0.147 \times 10^{-2} \times y^{-2} \quad \text{for} \quad y \geq 20
\]

rms value = 1.77

Fig. 3-6 graphically represents these equations. For comparison the pd for Gaussian noise with a rms value of 0.05 is also depicted. Note that the pd’s of the Gaussian part of the quiet-type noise model and the purely Gaussian noise model correspond very well if the purely Gaussian noise has a 6 dB lower rms value than the quiet type noise.

For practical design- and test purposes a representative, yet easy to implement noise model is very desirable. The Minimum Performance Standards [5-5] prescribe, besides a conventional Simulated Random Noise (SRN) model, a type of Simulated Atmospheric Noise (SAN) to evaluate the performance of marine Loran-C receiving equipment. The SAN model comes very close to the amplitude probability distribution of Feldman's "quiet" type.

SAN is composed of two components. The first component, contributing 15.85% of the noise energy, consists of Gaussian background noise. The second component consists of randomly distributed bursts of a sine wave. The frequency of the sine wave is 100 kHz, which is the operating frequency of Loran-C. The occurrence of a burst is Poisson-distributed, nominally 50 per second, with a duration of three full cycles or 30 microseconds. The top of the sine-wave burst is fixed at 30.5 dB above the SAN rms level, i.e. 38.5 dB above the rms level of the Gaussian background noise.

The MPS states that the noise’s field-strength level is within the range of 12 to 75 dB/μV/m. These values apply for Simulated Random Noise and Simulated Atmospheric Noise. Combining the specifications for signal and noise must always result in a signal-to-noise ratio of greater than 0 dB.
3.1.3 INTERFERENCE

The rather steep leading edge of the Loran-C bursts makes a broad pre-detection bandwidth of the receiver necessary. A typical value amounts to 25 kHz, as will be shown later. In this way we get a reasonable signal-to-noise ratio, while preserving the steep slope of the leading edge of the envelope. Bear in mind that this leading edge contains the information for the correct selection of the cycle to be tracked! This wide bandwidth makes the receiver unfortunately also very susceptible to interference signals whose frequencies lie near or even within the Loran-C spectrum. The way in which the signal processor is disturbed depends on the type of interference. In those cases where the interferences has no harmonic relationship with the navigational signal whatsoever, the processor might kill the disturbance effectively by signal averaging. However, if there is some kind of synchronism, the wanted signal might not be curable. In addition to the CW signals causing synchronous interference, such situations might also be encountered when signals from two different Loran-C chains are received simultaneously. This phenomenon is called cross-rate interference.

In coastal areas, Decca signals often show a rather high field strength. And as these signals are close in frequency to the Loran-C band, the signal processor will find the disturbance at its input. Fortunately, these signals are very narrow banded. They can almost be considered as continuous waves. This creates the opportunity to apply narrow-band notch filters which reject the interference, while not seriously distorting the Loran-C signal. The problem of how to determine the frequency of the interferences, to tune the notch filters, remains.

Usually we will encounter more than a single interference signal. Although we may apply quite a number of notch filters, we still have to decide which interferences should be killed. Generally speaking, this means the stronger ones, as they do the most harm. However, when the observer moves around, the rank ordering of the disturbances in field strength will vary frequently. This implies that effective notch filtering should be realized by an automated process. To illustrate real-life conditions, Fig. 3-7 depicts the spectrum of 50 - 150 kHz as received at the Delft University of Technology. It shows clearly that fighting all the interferences resembles Don Quixote's battles against windmills.

The Minimum Performance Standards specify various sorts of interference, listing three types of interference signals accordingly to their frequencies.
In-band interference, in the frequency band of 90-110 kHz.

Near-band interference, in the 70-90 kHz and the 110-130 kHz band.

Out-of-band interference, for frequencies below 70 kHz or above 130 kHz.

The MPS gives a maximum signal level for CW interference not exceeding 120 dB/μV/m. The relative value of this interference to the Loran-C signal depends on the frequency and the number of interferences. The reader is further referred to the MPS. [5-5]

3.1.4 VECTOR REPRESENTATION OF DISTURBERS

All the disturbances as described above can be represented as a vector added to the Loran-C vector. See Fig. 3-8. Let's assume that the signal vector is normalized. In the case of Gaussian noise the disturbing vector on top of the signal vector will have a uniformly distributed phase, while the amplitude has a Rayleigh distributed probability density. Atmospheric noise will differ from the Gaussian noise in the probability-density function of the amplitudes. The pdf's will be extended more towards higher amplitude levels.

Sky waves and synchronous interference have a constant or at least a near-constant phase relative to the signal vector. The length of the disturbing vector represents the amplitude of the interference. Non-synchronous interference will exhibit a varying phase relationship with the Loran-C signal.

Its influence on the tracking accuracy can easily be seen from the phase alteration stemming from the disturbing signal; nonetheless, it is noise or interference. The speed of the relative phase alteration in relation to the slew rate of the tracking algorithm indicates the vulnerability of the signal to the disturbance.

Software tracking filters can very effectively narrow the post-detection bandwidth. However, no tracking algorithm can manage to kill any kind of synchronous interference. And only narrow-banded synchronous interferences might be attenuated with notch filters, leaving us with a great desire not to encounter broad-banded synchronous signal polluters.

3.2 OUTPUT CONSTRAINTS

Opting for a universally applicable Loran-C sensor forces the designer of such a sensor to supply the output data free of any navigational pre-processing. This implies that navigational data be given as Times Of Arrival (TOA) and not as Difference Times of Arrival (DTA). However, TOA's can only be given relative to an internal reference time. Absolute time is generally not available. Hatch [6-1] showed
extensively that hyperbolic navigation is often far from optimum with respect to accuracy. Chapter 13 explains how to find one's position from three TOA's with an unknown time error in the receiver.

Besides the basic position information, the sensor should also give a status report about the quality of the received signals such as signal-to-noise ratios, ECD's, sky waves, and alarms. However, the reader is first invited to proceed to the next chapter, which will explain how the Loran-C system and a Loran-C receiver are organized.

4 GENERAL LORAN-C SENSOR PRINCIPLES

In the previous chapter the description of the Loran-C system has been limited to a specification of the envelopes of the bursts as transmitted, and of how these signals thereafter arrive at the receiver's antenna. These are usually very noisy and quite often have a distorted envelope. Thus far nothing has been said about the physical lay-out and timing of the system. However, at this stage it is interesting to know how to distinguish the master signal from the signals of the secondary transmitters, and what the terms phase coding and secondary blinking mean.

Such matters must be settled before one can even consider designing a Loran-C receiver and the associated signal processor. Therefore, we will start with a functional description of the Loran-C system as far as is needed for our purposes of designing a receiver. We will subsequently give an explanation of three different types of receiver concepts: the linear, the clipped-linear and the hard-limiter type.

4.1 THE LORAN-C SYSTEM

A typical Loran-C navigation systems consists of a set of transmitters, with each of them emitting high-energy bursts in the 90 – 110 kHz frequency band. The number of stations per chain varies normally from three to five. Should three stations be involved, one master and two secondaries, their geographical lay-out is preferably such that the base lines between the master and the two secondaries are nearly orthogonal [1-12]. In the case of four transmitters, we opt for a starlike configuration, where the master station is the center. See Fig. 4-1. Chains which have
an excess of four stations, quite often display a geographical configuration which anticipates specific difficult navigational areas. Such difficulties might be due to a lack of suitable sites for erecting transmitter towers, or to the areas to be served being very extensive.

Each transmitter sends a sequence of eight bursts, with an interval of exactly 1 ms between the beginnings of the bursts. The master station emits an additional ninth burst at an interval time of 2 ms after the eighth burst. The secondary stations will transmit their eight bursts with a specific emission delay after the start of the master sequence. The emission delay is different for each secondary station, and depends highly on the geographical positions of the transmitters. Fig. 4-2 depicts a chain consisting of a master and two secondary stations. We specify the various timings as:

$$t_m = \text{time of beginning of transmission master sequence}$$

$$t_{s1}, t_{s2} = \text{time of beginning of transmission secondary 1 or 2 sequence}$$

$$b_1, b_2 = \text{propagation time from master to secondary 1 or 2}$$

$$c_1, c_2 = \text{coding delay of secondary 1 or 2}$$

Further:

$$e_1, e_2 = \text{emission delay of secondary 1 or 2}$$

$$t_0, t_1, t_2 = \text{propagation time from master, secondary 1 and 2 to receiver}$$

The coding delays are essential as they prevent simultaneous reception of more than a single transmitter: Loran-C timing is based on time diversity.
The master emits nine bursts instead of eight as the secondaries do. In the older generations of Loran-C receivers this ninth burst was used for master/secondary recognition. The ninth burst is sometimes used for chain messages to users, in which case the burst is switched on and off in a specific code, which can be deciphered by the user.

The station identification is achieved by phase coding. The phase of the carrier of a burst can be altered by 180 degrees. We call the phase '+' when there is no phase inversion, and '-' when there is phase inversion. The phase-code pattern of the master and the secondaries is repetitive for every two GRI's, called the A and the B group. Table 4-1 shows the phase coding pattern for the group A and the group B sequences.

<table>
<thead>
<tr>
<th>Station</th>
<th>Group A</th>
<th>Group B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7 8 9</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>Master</td>
<td>+ - - + - + +</td>
<td>- - - + + +</td>
</tr>
<tr>
<td>Secondary</td>
<td>+ + + + - - +</td>
<td>+ - - + - -</td>
</tr>
</tbody>
</table>

Table 4-1. Phase coding of the master and secondary bursts for groups A and B.

As all slaves have the same phase coding, additional information derived from the coding delay of the various secondaries is needed to solve the who's-who problem in secondary-station identification. The coding delay is such that in no part of the service area might the sequence of the secondary stations be altered. The phase coding of the transmitted signals is also needed for determining whether they belong to the A or the B group.

The coding pattern is selected in such a way that the disturbing effects of very long sky-wave delays are partially cancelled. Delays up to 7 ms are sometimes noticeable. This may mean, for example, that the sky wave of the first burst is interfering with the ground wave of the seventh burst. Based on the idea that delay characteristics will not vary from burst to burst, the selected coding will suppress sky waves for delays of 1 to 7 ms [1-9]. Fig. 4-3 represents the resulting vector from the ground-wave signal and the sky-wave signal from a preceding burst. The phase relation between the two signals is randomly chosen. A '+' phase ground wave added to a '+' phase sky wave causes an apparent phase delay, called r (retard). The same delay will be experienced for a '-' phase ground wave and a '-' phase sky wave. The other two possible combinations cause a phase advance, denoted by a. We may now check the results of the number of delays and advances over all 16 bursts of the A and B groups. It should be noted that the delays and advances are in general not
equal. Therefore, the rejection efficiency for sky-wave influences depends mainly on the phase angle between the ground-wave and the sky-wave vector. For a phase angle equal to 90 degrees, such cancelation might be ideal. But if the phase angle approaches 0 or 180 degrees, the cancelation efficiency diminishes severely.

Master sequence

<table>
<thead>
<tr>
<th>Delay</th>
<th>Group A</th>
<th>Group B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ms</td>
<td>++-----+-----+</td>
<td>++-----+-----+</td>
</tr>
<tr>
<td>2 ms</td>
<td>x r a r a a a a a x</td>
<td>x r a r r r r r x</td>
</tr>
<tr>
<td>3 ms</td>
<td>x x a a a a r r r</td>
<td>x x a a a a a a a</td>
</tr>
<tr>
<td>4 ms</td>
<td>x x x r a r r a</td>
<td>x x x r a r r a</td>
</tr>
<tr>
<td>5 ms</td>
<td>x x x x x r a r</td>
<td>x x x x x r a a</td>
</tr>
<tr>
<td>6 ms</td>
<td>x x x x x x r a</td>
<td>x x x x x x r a</td>
</tr>
<tr>
<td>7 ms</td>
<td>x x x x x x x a</td>
<td>x x x x x x x r</td>
</tr>
</tbody>
</table>

Table 4-2. Effects of a long-delayed sky-wave interference on the ground wave. x = not disturbed, a = phase advance, r = phase retardation, e.g., at a delayed sky wave of 3 ms, six ground-wave vectors will be advanced in time, while six vectors will be delayed over a series of 18 bursts from groups A and B.

When it is assumed that there is a 90 degree phase shift between the ground wave and the sky wave, and only the first 8 bursts of the A and B sequences are considered, Table 4-2 shows that a perfect cancelation occurs for sky-wave to ground-wave delays ranging from 1 to 7 ms. It must be mentioned that the net result in sky-wave pollution reduction depends on the phase relation between the ground and the sky wave. This can be seen from Fig. 4-3. The full cancelation of the phase shifts will be lost if the ninth burst is included. However, this is of less importance, as this burst is often "forgotten" in modern receivers. The ninth burst is a heritage from the time when the master station was visually identified from the CRT screen. An equivalent suppression of sky-wave interference is found for the secondary phase-coding pattern.

The carrier frequency of the Loran-C signal is 100 kHz with a very high precision. The stations will try to maintain the offset within $1 \times 10^{-11}$. This frequency might also be present as a spurious signal from the receiver itself, or from other electronic systems on board the ship or aircraft. The phase coding of the Loran-C system is not perfectly balanced with respect to suppressing 100 kHz spurious signals. However, it is possible, by excluding some received bursts from the final signal processing, to make the receiver less sensitive to such interference. The phase coding has 10 '+' phases and 6 '-' phases per 2 GRI's. Thus, eliminating 4 of the '+' phase samples will balance the code. This will also slightly reduce the information bandwidth. Another drawback is the loss of long-delayed sky-wave rejection capabilities. We have yet to see a suitably balanced phase coding where the sky-wave interference rejection is also ideal.

The timing and the envelope of the burst are permanently controlled by monitor receiving stations. If a secondary station does not meet the specifications, it will notify the
The information concerning the Loran-C system as given above should be adequate for starting the design of a navigation receiver. The essence of electronic navigation is the determination of the propagation time from a known transmitter to the navigation receiver. The most accurate way to accomplish this task is to track one of the zero crossings of the received signal. The first thing the receiver must do, after being switched on, is to determine where the various signal bursts are situated in the group repetition interval. Next, the leading edge of the ground-wave part of the received bursts must be found, a process known as the initializing phase. And finally, a specific zero crossing must be found. This is called the cycle-identification process. Thereafter the selected zero crossing must be accurately tracked.

As the receiver generally encounters rather low SNR signals, thorough signal processing is absolutely necessary to obtain adequate navigation accuracy. As the Loran-C system has been operational since 1956, the reader will not be surprised to read that many receiver designs are already in existence. The concepts of the designs mainly vary in the way the receiver converts the LF antenna signals to a signal which can be processed by analog or digital integrators. Three types of receiver concepts are of interest:

a - the full linear receiver,
b - the hard-limiter receiver, and
c - the clipped-linear receiver.

Most known receiver designs fall into one of the above-mentioned categories.

4.2 THE LINEAR RECEIVER

The full linear receiver is the most cumbersome to design, the most difficult to implement and also the ultimate with respect to signal recovery. The two main difficulties facing the designer, and this is valid for all receiver concepts, are the extreme ranges of signal amplitudes and the amplitude distribution of the atmospheric noise.

Fig. 4-4 depicts the functional block diagram of the receiver. The signals as received by the antenna are band-pass filtered to eliminate as much as possible unwanted frequencies. Thereafter the Loran-C signal and the atmospheric noise passes through a variable-gain amplifier in order to bring the received signals into the working range of the analog signal sampler.

![Fig. 4-4 Functional block diagram of linear receiver.](image)

As stated earlier, the heart of electronic navigation - the determination of the propagation time from the known position of a transmitter to the navigation receiver - can be most
accurately accomplished by tracking one of the zero crossings of the received signal. Clearly, the selected zero crossing must be in the front part of the burst, where the sky-wave content can be disregarded. The receiver's signal processor estimates the time of the zero crossing. Then, at that estimated position of time, a sample of the signal is taken. When the amplitude of the received signal is known, the amplitude of the signal sample is a measure for the error in the estimated position of the zero crossing. The sample is fed to the signal integrator, which makes a new estimate of the position of the zero crossing.

The received signals, as said before, are very noisy, which means that many samples have to be taken and integrated before a conclusion can be drawn about the estimation error of the signal's zero crossing. This integration process can easily be disturbed by the extravagant noise peaks occurring in the LF band. These peaks might exceed 60 dB relative to the top of the received Loran-C sine wave. If we further realize that our interest is mainly concentrated around the zero crossing of the signal, at a level of about 30 dB below the top of the sine wave, we will be required to integrate signals which show a dynamic range greater than 90 dB. To make things even worse, integration times in excess of 10 minutes may be necessary.

The above-mentioned requirements are for electronic circuits difficult to achieve. The sampling gate must operate over extreme amplitude levels. Moreover, highly accurate linear integration over hundreds of seconds is almost impossible. Fast analog-to-digital converters with a very large linear range ease the implementation of such integrators. Integrating over long periods of time is a simple task for digital signal processors. However, to reduce the enormous amplitude range, a system of noise censoring is sometimes applied. The receiver measures the average signal amplitude. When a signal which exceeds the average value by a predetermined value is received, the sample is discarded, as being useless for the integration process. However, it cannot easily be said which samples are useful. Nonetheless, the dynamic range of the received signals becomes more manageable. Still, the differences in signal levels from the various stations of a particular chain might show differences exceeding 100 dB.

The variable-gain amplifier stage is controlled by the signal integrator. As each received Loran-C station has its own field strength, during a single GRI the gain has to be varied three or four times.

It is not sufficient to take a single sample of the received burst. More information concerning the burst is required, e.g., is the ground wave or the sky wave sampled, or, which zero crossing is actually tracked? Therefore, we take more than a single sample per signal burst. Fig. 4-5 gives an impression of a possible set of samples. The first two samples test whether a signal is present in front of the estimated burst, which could indicate that we are tracking a
sky wave. The two samples are taken in quadrature to prevent
the sample from being taken exactly at a zero crossing. With
this configuration the signal at one of the sampling
positions will never be below 3 dB if related to the top of
the sine wave. The third sample is taken at the top of the
sine wave preceding the zero crossing (sample 4), which is
used for tracking, as is the fifth sample, taken at the top
of the sine wave trailing the zero crossing. The ratio of
the amplitudes of samples three and five is a measure for the
zero crossing which is tracked. As the differences in
amplitude from samples three and five do not differ
significantly, these samples must be accurately estimated to
avoid arriving at an incorrect zero-crossing number. This
also explains why the leading edge of the transmitted burst
is so carefully monitored.

In spite of all the technical problems of the full linear
receiver, it is still a sound concept in receiver design.
That does not particularly include the noise-handling
characteristics. It is the supreme rejection capability for
interfering continuous waves. The linearity ensures that no
modulation of the Loran-C signal will be introduced by
interference signals. Synchronous detection and low-pass
filtering can then be applied to reject the interference from
the navigational signals. The amount of rejection will only
be determined by the integration time. Where ultimate
signal-processing quality prevails, the linear approach makes
sense.

4.3 THE HARD-LIMITER RECEIVER

The complexity of the full linear receiver is sharply
contrasted by the simplicity of the hard-limiter type of
receiver. Fig. 4-6 gives the functional plan. The band-pass
filtered Loran-C signals are fed to a hard-limiting amplifier
with a very high gain. The hard limiter is basically a
single-bit analog-to-digital converter, or merely a polarity
detector. If we were now to take samples from the received
signals, we should obtain nothing more than information about
the polarity of the sampled signal. The samples for the
zero-crossing tracker only indicate whether the estimated
time of the zero crossing is too late or too early. There is
absolutely no information derived about the magnitude of the
estimation error.

![Fig. 4-6 Functional block diagram of a hard-limiter receiver.](image)

Although one would appear to be seriously hampered by this
rather primitive phase-error detector, it will be shown later
that this simple piece of hardware is very powerful in
combination with efficient detection algorithms. However,
this highly non-linear device behaves far from optimally with
respect to its susceptibility to interfering continuous-wave
signals, as it just mixes all received signals. And no
tracking algorithm may be applied to repair the damage once
done. The only solution is to prevent any interference signal
from reaching the limiter stages, which implies introducing
notch filters in front of the limiter stages. Such filters
block a narrow band of the spectrum. We recall Fig. 3-7 to
demonstrate the numerous interference signals inside and
outside the Loran-C band. Note that the spectrum shown is already band-pass filtered. So, if the Loran-C signal is to be cleaned up, there is work enough for many notch filters: a single notch filter rejects just a single narrow-banded part of the spectrum. The rejection bandwidth should indeed be narrow in order to avoid distortion of the envelope of the Loran-C burst. The remaining question is how to tune all these filters accurately to the interference signals. Chapter 8 describes a versatile notch filter.

However, the hard limiter is second to none when it comes to efficiency of rejecting atmospheric noise. We remind the reader that atmospheric noise is characterized by extremely high amplitudes which might exceed the average value of the noise by more than 60 dB. Such high peaks disturb analog integrating processes completely. But in a hard-limiting receiver such extreme disturbances do not cause any more harm than a low-amplitude noise peak. The occurrence of high peaks is rather seldom, and the polarity is on the average as often positive as negative. So, if a Loran-C sample coincides with a noise peak, only that particular sample is lost if the noise's value exceeds the signal's value and is opposite in polarity.

To find the correct zero crossing for tracking, we must derive this information from the incoming signal before it is hard-limited. Fig. 4-7 depicts one of the possibilities for identifying the wanted zero crossing. The original signal is envelope delayed by 5 μs, and then subtracted from the slightly attenuated non-delayed signal. To determine the 100 kHz cycle, we sample the hard-limited derived signal 2.5 μs before and 2.5 μs after the zero crossing which is tracked. If the correct zero crossing is tracked, both samples will be positive. If a zero crossing is too early, the third sample will be positive and the fifth sample negative. If the tracking is one or more cycles too late, opposite polarities are detected. The first and the second samples provide ground-wave detection to avoid sky-wave tracking. See Fig. 4-8a and Fig. 4-8b.
4.4 THE CLIPPED-LINEAR RECEIVER

As could be predicted, one should preferably opt for a combination of a linear receiver and a hard-limiting receiver, which would combine the positive features of both, while avoiding either specific limitation. The linear receiver behaves excellently with respect to interference rejection, but poorly under atmospheric noise conditions, while the hard-limiter receiver shows the complementary qualities.

Such a concept can be realized up to some extent by limiting the linear range of the receiver. If signals exceed the upper and lower bounds of the linear range, the maximum, and the minimum value, respectively, will be given to the signal processor. Usually the limits of the linear range are chosen such that the signal and the continuous-wave interferences are handled linearly. But the high amplitudes of the atmospheric noise are limited. This type of receiver needs, just as the linear receiver does a variable-gain-controlled amplifier for normalizing the signal amplitudes. For each station the top of the sine wave is to be estimated for the signal-processing algorithms. Otherwise no information can be derived about the amount of the zero-tracking error. As we recall the large dynamic range of the incoming Loran-C signals, the required linear range of the sampling devices is still impressive.

Although the atmospheric-noise-handling capabilities of the hard-limiter receiver are superior to the clipped-linear receiver, the latter is really a technically sound concept. It is probably the best concept known to us. However, it should also be realized that it is probably the most complex, a factor of dominant influence on the costs. In the following chapter we will see that are many more items of interest, which should be given thought finally setting on the most appropriate receiver concept.

Finally, the transfer characteristic of each of the three concepts is depicted in Fig. 4-9.

![Transfer Characteristics](image-url)
5 MOTIVATING FACTORS IN THE SELECTION OF THE HARD-LIMITER RECEIVER TYPE

In the previous chapter we introduced the various basic types of Loran-C receivers. However, we have indicated only the signal-handling properties of the three classes of receivers. Nothing has been mentioned about other important items, such as:

1 - suitability of the RF part of the receiver for integration.
2 - complexity of the pre-processor circuits which has a large impact on the size of the chip.
3 - complexity of the mathematical routines needed for signal processing, which determines the processor speed requirements and the amount of software needed.

The linear receiver, as depicted in Fig. 4-4, differs in the RF parts completely from the hard-limiter receiver (Fig. 4-6). The linear receiver uses a variable-gain amplifier and an analog signal sampler, followed by an analog signal processor. The same configuration is also the basis of the clipped-linear type. The signal processor nowadays most frequently consists of an analog-to-digital converter and a digital signal processor.

The hard-limiter set-up applies a limiting amplifier as polarity deriver, followed by a digital sampler such as a D-type flip-flop. The signal processor is entirely digital.

The antenna signals are thus amplified by the gain-controlled amplifier or the hard limiter. Both should potentially have enough gain to bring the level of even the weakest-received signals up to a level which suits the ADC or
the D-type flip-flop. Generally speaking, it can be said that the hard limiter must have a higher gain than the linear amplifier, although this statement needs some amplification.

Let us consider a linear receiver. A logical assumption is that the minimal input signal to be processed amounts to 1 μV. The imaginary 10-bit analog-to-digital converter applied has an input-voltage range of 10 V. 10 bits is equivalent to 1024 steps, where a single step equals about 10 mV. If the smallest input signal were to consume the complete range of the ADC, then the required gain would be 140 dB. However, this is an unrealistic assumption. One of the strengths of linear receivers is their capability to reject unwanted signals by synchronous detection and subsequent low-pass filtering. This means that there should be adequate head room for signals riding upon the wanted Loran-C burst. The amount of sensible head room is dictated by the number of bits of the ADC, and the resolution of the timer which controls the sampling gate. Let the resolution of the timer be 100 ns, which is a realistic value. If the zero crossing to be tracked is halfway between two possible consecutive sample-point positions, then the samples are taken not closer than 50 ns from the zero crossing. For a 100 kHz sine wave this means not closer than 1.8 degrees from the ZC. At that position the signal level equals \( \sin(1.8) \), or 0.031 times the peak value of the sine. In our example this is 16 nV. Now let's consider this smallest voltage to be sampled equal to a least-significant step of the ADC. In the case of a 10-bit ADC, we may then linearly handle signals up to 1024•16 nV, or about 16 μV. This value exceeds the peak-to-peak value of the sine 15 times. The conclusion can now be drawn that interference signals up to 15 μV can be rejected by means of low-pass filtering after the sample procedure. So, the maximum linear gain of the amplifier amounts to 116 dB.

For the hard-limiter case the situation is completely different, as we face now a digital sampling circuit which shows an input-voltage zone, where the logical output of the sampler is not necessarily equal to the logical input at the sample moment. This undefined zone is, say, 1 V, a typical value for CMOS logic. If we once again intend to determine the polarity of the input sine wave at a voltage level of 16 nV, we must have adequate limiter gains to make the polarity decision independent of the dead zone of the sampler. So, the gain should exceed 0.5 V/16 nV, or 150 dB. In chapter 9 we will see that the determination of the required limiter gain is actually much more complicated than the simple analysis given would indicate.

For both types of signal amplifiers it can be said that the degree of difficulty in implementation is roughly equal. The same holds for the size of the chip. We thereby conclude that there is no significant reason to prefer either type of receiver as far as the amplifier part is concerned.

As far as linear receivers are concerned we note that we will deal with only the version equipped with a sampling ADC and a digital signal processor. From an implementer's point of view, the entirely analog integration is far from attractive. The accuracy of the sampling gate must exceed the accuracy of the ADC, which for a 10-bit ADC means an absolute accuracy of at least 0.1%. The next point to be raised is the settling time. The voltage value of the signal must be fetched in such a short time that the input signal does not alter more than, say, 1/2 LSB. Let us take a 10-bit ADC, which will be needed in full for the input sine wave, regardless of
whether this is the signal or the interference. Then a half LSB amounts to 1/2000 of the peak-to-peak value of the sine. For a sine wave of 100 kHz, this voltage change will occur in $2500/90 \cdot \arcsin(0.001) = 1.6$ ns. Such settling times are technically unrealistic when the required precision is taken into account. The conclusion is that we have to decrease our demands on ADC specifications. However, the quasi-linear approach is not as hopeless as we indicated above. The almost ever-present noise on the signal has a dithering and a linearizing effect on the quantizing of the signal by the ADC, assuming that many samples are taken and averaged. This reduces the stress on the ADC with respect to the number of bits and the settling time. In some receiver designs, noise is deliberately added to the signal, to force the quantizing ADC into a more linear mode.

The signal processor of a linear receiver processes multi-bit words from the ADC. As many samples have to be averaged before an acceptable accuracy is achieved, multi-byte operations are to be performed. The linear signal processing is complicated. It must not only establish an average value of the input signal, but it must also determine the gain-control parameters for the linear-amplifier circuits. The complexity of the input signal puts a heavy burden on the cleverness of the software (and its designer) of such a processor.

The sampling-gate settling time and precision problems are less severe in the hard-limiter approach. The gain of the limiter must be large enough to make the sample taking insensitive to dead zones in the input signal of the digital sampler. However, we must take care of the required set-up and hold times of the sampler. The input signal is not expected to change polarity during the set-up and hold times, as this may give random outputs of the sampler. To eliminate the zero-crossing tracking error due to this timing problem, we must take care to keep the sum of the set-up and hold times less than half the resolution of the timing of the sample taking. So, for a resolution of 100 ns, this value should not exceed 50 ns. This is a feasible value for digital-signal polarity samplers.

The signal processing of the single-bit information from the digital-signal polarity sampler is rather uncomplicated. The integration process is limited to the incrementing and decrementing functions of certain registers of the processor. Chapter 10 describes in detail how such signal processing is realized, and will show the simplicity of the process.

Although the hard-limiter approach is very attractive with respect to hardware and software simplicity and to signal processing in the presence of atmospheric noise, the interference rejection capabilities are poor. Consequently, a hard-limiter receiver needs additional filters to eliminate such interferences before they may bring about any damage to the signal.

The last aspect, power consumption, has not been dealt with in the foregoing discussion. Nevertheless, this aspect is crucial for miniature equipment. The absence of an ADC and the simpler signal processor in the hard-limiter concept, favors the latter in this respect.

To ease the receiver selection procedure, we have constructed a table, containing the most important aspects:
In surveying all the aspects discussed for both concepts, we clearly perceived the hard-limiter type to be the most suitable for the receiver applications we have in mind. For that purpose small size, low power consumption and reasonable performance are the main parameters. However, we do so with the full knowledge that the explosive growth in the integration technology may one day make our "state of the art" amusing.

6 PHYSICALLY SMALL LOW-POWER ACTIVE ANTENNA

The antenna of a Loran-C receiver can be configured in many different physical ways. We are familiar with the magnetic types, such as the single loop, the crossed-loop and the rotating-field antenna, and the electric antenna, which is rather common for navigational purposes. How one should go about selecting the optimum type might at first appear confusing. However, in our case there are some very clear specifications that the antenna must meet: small physical size, high sensitivity and a polarization of the antenna which must coincide with the polarization of the received electromagnetic field. We begin the selection process by differentiating between the magnetic and the electric antenna.

6.1 MAGNETIC ANTENNA

Loran-C uses vertically polarized electromagnetic signals, so that the polarization of the magnetic component of the received signal is horizontal and perpendicular to the direction of the transmitter. This requires a vertical loop antenna. The voltage induced in the loop is determined by the field strength, the number of turns and the area of the loop, and finally by the angle of incidence of the magnetic field relative to the plane of the loop, or:

\[ U_o = H \cdot C \cdot \cos(i) \cdot \sin(\omega t + a) \]  

(6-1)

where

- \( U_o \) = output voltage of the loop
- \( H \) = field strength of the magnetic component
- \( C \) = constant
- \( i \) = angle of incidence
- \( \omega \) = radian frequency of the received signal
6-2

\[ a = \text{phase of the received signal} \]

Fig. 6-1 shows the relative amplitude and phase of the output voltage \( U_0 \) as a function of the angle of incidence \( i \). The phase jumps at \( i = \pi/2 \) and \( i = 3\pi/2 \) are a serious problem, as a \( \pi \) phase jump corresponds to a 5 \( \mu \)s time jump, or the equivalent of a 1500 meter long jump. In cases where the the Loran-C sensor is part of an integrated navigation system, we may anticipate this jump, as the angle of incidence is known from the computer. The amplitude dependence from \( i \) is more of an inconvenience than a serious drawback. A useful solution for the latter problem is the application of two vertical loops, perpendicular to each other (Fig. 6-2). The computer of the integrated navigation system then determines which of the two loops will perform best, and thereafter switches the selected loop to the receiver. In this case the possible phase jumps of 180 degrees are known, and are accordingly taken into account. The amplitude response of this crossed-loop configuration will be no more than 3 dB below the maximum value for all angles of incidence.

However, the usefulness of this crossed-loop configuration depends on the availability of information about the attitude of the loops relative to the direction towards the various Loran-C transmitter stations. In other words, a compass device and navigation computing facilities are essential, although the amplitudes and the phases of the two signals also basically contain the heading information.

Fig. 6-2 Dual-loop magnetic antenna configuration.

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The rotating-field magnetic antenna does not need these facilities, as it also uses two perpendicular loops. The output signal of one of the loops is phase shifted over \( \pi/2 \) radians and then added to the output of the other loop. Fig. 6-3 gives the lay-out of the antenna. We find the following relations:
6-4

\[ U_1 = H \cdot C \cdot \sin(i) \cdot \sin(\omega t + a) \]  
\[ U_2 = H \cdot C \cdot \sin(i) \cdot \sin(\omega t + a + \pi/2) \]  
\[ U_3 = H \cdot C \cdot \cos(i) \cdot \sin(\omega t + a) \]  
\[ U_2 + U_3 = H \cdot C \cdot \sin(\omega t + a + i) \]

Fig. 6-4 represents the relative amplitude and the relative phase of the received signals as a function of the angle of incidence. Note that the phase difference between signals from two different transmitters also contains a part equivalent to the difference in the two headings towards the transmitters. We conclude that the phase difference between two signals does not alter, while the absolute phase of each signal does alter with a rotating antenna. The absolute-phase variations of the output signal per unit of time for antennas on board a maneuvering ship or car might exceed by far the tracking slew rates of the receiver. Rotation-rate-aiding sensing devices will then be necessary for accurate tracking.

6.2 ELECTRIC ANTENNA

As the polarization of the received Loran-C signals is vertically oriented, a vertical electrical receiving antenna is not direction-sensitive. This frees the system of heading-estimation and heading-dynamics problems.

We clearly opt for the shortest possible antenna. The question now arises of how to determine this minimal length. We will start with a qualitative approximation.

Antennas which are short in relation to the wavelength may be represented by a voltage source \( U_a \) in series with a capacitance \( C_a \) (Fig. 6-5), where

\[ U_a = F \cdot l_{\text{eff}} \]  
\[ C_a = C \cdot l \]  

and \( F \) = field strength of the electric component  
\( l \) = physical length of the antenna  
\( l_{\text{eff}} \) = effective length of the antenna (appr. 1/2)  
\( C \) = capacitance of the antenna per unit of length

In sum, the magnetic antenna does not lend itself well to use in our Loran-C receiver design. However, applications in aircraft may be more successful, as the necessary heading sensors and more extensive computer facilities are then available.

Fig. 6-4 Absolute amplitude and relative phase response of a rotating-field magnetic antenna as a function of the angle of incidence.

Fig. 6-5 Equivalent circuit of the short electric antenna.
The voltage source of the antenna $U_a$ actually represents two signals: the received Loran-C signal $U_{as}$ and the received noise $U_{an}$. See Fig. 6-6a.

The antenna is connected to an electronic amplifier, a combination known as an "active antenna". The active antenna may be coupled to the receiver directly or via a coaxial cable. We will now represent all noise sources related to the antenna amplifier and the connected receiver by an equivalent input voltage noise source $U_{en}$ and an equivalent input current noise source $I_{en}$. Thus, the connected amplifiers are assumed to be noise-free. By transforming $U_{en}$ and $I_{en}$ into a single equivalent input-noise voltage $U_n$ which is allowable as $U_{en}$ and $I_{en}$ are independent, we are able to compare the received signal, the received noise and the equivalent input noise of the active antenna (Fig. 6-6b).

![Equivalent amplifier voltage- and current-noise sources (a) transformed into a single equivalent voltage-noise source at the input of the amplifier (b).](image)

Let's assume that $U_{en}$ and $I_{en}$ do not depend on the antenna length. From (6-6) we see that the received noise is linearly dependent on the antenna length. We may now shorten the antenna to a length at which the received noise $U_{an}$ just exceeds the internally generated noise $U_n$. This means that we will not significantly deteriorate the signal-to-noise ratio of the received signal. The spectrum of $U_n$ is a function of $U_{en}, I_{en}$ and the antenna impedance:

$$S(U_n) = S(U_{en}) + S(I_{en}) \cdot \frac{1}{(wC_a)^2}$$

(6-8)

We also learn from (6-7) that the magnitude of $C_a$ is linearly related to the length of the antenna. This results in an inverse dependence of $C_a$ on $U_{en}$, as far as the noise contributed by $I_{en}$ is concerned. As $C_a$ is relatively small, it is of prime importance to reduce $I_{en}$ as much as possible.

Our interests extend to two types of antennas: the selective and the non-selective antenna. The words "selective" and "non-selective" indicate the presence or absence of selectivity in the antenna system preceding any electronic amplification. Hence, a non-selective active antenna with a band-pass filter behind it, is still called a non-selective antenna, although the final frequency response is band-limited.

The advantages of a selective antenna are evident in the presence of strong out-of-band interference signals. On board ships and aircraft, where many communication systems may simultaneously be active, the interference level may be of such a magnitude that passive band-pass filtering is the only remaining solution to facilitate proper linear operation of the active antenna. However, this selectivity is not easily effectuated without a substantial loss in the dynamic range of the antenna system.

Both antenna types may either be an integral part of a portable receiver or a separate unit positioned at a remote distance (e.g. mounted on a mast) from the receiver.
6.2.1 SELECTIVE ELECTRIC ANTENNA

Fig. 6-7 depicts a selective antenna with a series-tuned band-pass filter in front of the electronic amplifier. The filter is terminated by the real impedance $R$, which determines the bandwidth of the tuned circuit. This impedance $R$ is the sum of the internal input impedance of the amplifier $R_1$ and the loss resistance $R_s$ of the inductor $L$. As we are interested in the length of the antenna, we will express the values for $C$, $L$ and $R$ as functions of $l$. The center frequency, $f_0$, and the bandwidth $B$ of the filter are set by the specifications of the Loran-C signal: $f_0 = 100$ kHz and $B = 30$ kHz at the $-3$ dB positions of the frequency-response curve. The equivalent input-noise sources from the amplifier are again given as $U_{en}$ and $I_{en}$, respectively. The current noise source of the resistor $R_1$ is represented by $I_{rsn}$, while $U_{rsn}$ denotes the noise of $R_s$. Initially it is assumed that resistor $R_1$ produces normal thermal noise. Later it will be seen that "electronic cooling" can be effectively applied to reduce the generation of noise. We now find the following relations for $U_{as}$, $U_{an}$, $C_a$, $L$ and $R$:

$$U_{as} = F_s \cdot 1/2 \quad \text{(approximation for thin-wire antenna)}$$  \hspace{1cm} (6-9)

$$U_{an} = F_n \cdot 1/2 \quad \text{(approximation for thin-wire antenna)}$$  \hspace{1cm} (6-10)

$$C_a = C \cdot l \quad \text{(end effects neglected)}$$  \hspace{1cm} (6-11)

The electrical length of the antenna, $l$ (end effects neglected), is given by:

$$l = 1/(\omega_0^2 \cdot C \cdot l)$$  \hspace{1cm} (6-12)

$$R = R_1 + R_s = 1/(\omega_0^2 \cdot C \cdot l)$$  \hspace{1cm} (6-13)

where $U_{as}$ = antenna voltage of the received Loran-C signal

$F_s$ = Loran-C signal field strength

$U_{an}$ = antenna voltage of the received noise

$F_n$ = noise field strength

$Q$ = quality factor of the tuned circuit

In order to judge their effects, we transform the various noise sources into an equivalent input-voltage noise source $U_n$ (Fig. 6-8). We may then compare $U_n$ with $U_{an}$.

Multiplying the sum of the spectra of $I_{rn}$ and $I_{en}$ by the square of the modulus of the impedance $Z$ and adding the result to the spectra of the noise sources $U_{en}$ and $U_{rsn}$ yields the spectrum of $U_n$. This addition is permitted, as the spectra of the four noise sources are not correlated. The complex impedance of $Z$ is given by:

$$Z = R_s + j \omega L + 1/(\omega C_a) = R_s + j(\omega L - 1/(\omega C_a))$$  \hspace{1cm} (6-14)

As $Z$ is frequency-dependent, we must perform the transformation by means of an integration over the frequency range of interest, from $f_1$ to $f_2$. The MIPS specifies a Gaussian-noise bandwidth of 47.2 kHz. For comparative
purposes, we will adopt the same bandwidth, so that
\[ f_1 = 76.4 \text{ kHz and } f_2 = 123.6 \text{ kHz} \]

\[ u_n^2 = [S(U_{en}) + S(R_{sn})] \cdot (f_2 - f_1) \]

\[ + [S(I_{en}) + S(I_{rn})] \cdot \left| \frac{Z}{f_2} \right| \left\{ \left| \frac{Z}{f_1} \right|^2 \right. \text{ df} \]

Combining (6-14) and (6-15) yields:

\[ u_n^2 = [S(U_{en}) + S(U_{rsn})] \cdot (f_2 - f_1) \]

\[ + [S(I_{en}) + S(I_{rn})] \cdot \left[ 4 \pi^2 f_2^3 / (3 \omega_0^2 C_a^2) \right] \left\{ \left| \frac{Z}{f_2} \right|^2 \right. \text{ df} \]

\[ - 1 / (4 \pi^2 C_a^2 f_1^2) \left\{ \left| \frac{Z}{f_1} \right|^2 \right. \text{ df} \]

\[ - 2 f / (\omega_0^2 C_a^2) \left\{ \left| \frac{Z}{f_1} \right|^2 \right. \text{ df} \]

(6-16)

Substituting (6-11), (6-12) and (6-13) in (6-16) now yields

\( u_n \) as a function of \( U_{en}, I_{en} \) and \( L \).

Fig. 6-9 gives \( u_n \) and \( L \) as a function of \( L \), with \( U_{en}, I_{en} \) and \( R_s \) as parameters. The chosen values for \( U_{en} \) and \( I_{en} \) are typical of a JFET input stage.

Fig. 6-9 also depicts the received noise-voltage level \( u_n \). The noise field strength \( F_n \) is derived from the MPS in the following way. The lowest signal field strength is given as 25 dBuV/m. The MPS also specifies a minimum signal-to-noise ratio of 0 dB for received signals. In other words, the minimum noise field strength to be taken into account is 25 dBuV/m. Of the two possible noise models, Gaussian noise and atmospheric noise, the latter is the most advantageous for hard limiter receivers. However, to get the most out of these specification, we must design the active antenna in such a way that the internally generated noise does not exceed the Gaussian part of the atmospheric noise model. This Gaussian part of the noise is, according to the MPS, 3 dB below the RMS value of the atmospheric noise. So, the minimum noise field strength to be reckoned with actually goes down to 17 dBuV/m, or 7.1 \( \mu \)V/m. It should be borne in mind that these values are related to a noise bandwidth of 47.2 kHz.

To enable comparison, \( u_{an} \) is depicted for the Simulated Gaussian-Noise model in the same picture. The drawing clearly indicates that, for this type of noise, a much shorter antenna will perform adequately, 0.50 m as compared to 0.93 m.
6.2.1.1 DESIGN CONSIDERATIONS FOR SELECTIVE ELECTRIC ANTENNA

The electronic realization of the selective antenna as indicated above is a careful compromise between many conflicting specifications, i.e. short physical length of the antenna, low noise contribution from the electronic circuit, high dynamic range, bandwidth, etc.

The way the bandwidth of the tuned circuit is controlled in the foregoing subsection is far from elegant, as the noise from $R_1$ and $R_S$ can not be disregarded. Generally speaking, $R_1$ contributes more noise to the system than $R_S$. Therefore, the application of an "electronically cooled" resistor for $R_1$ may reduce the total noise source $U_n$ significantly. However, before discussing such techniques, we must pay attention to a more physical problem: the small values for $C_a$, and hence the large values for $L$!

The capacitance of the antenna rod, $C_a$, is tuned by the inductor $L$. Although the Q of the circuit is rather low, about 3, we must continue to be careful not to detune the circuit by varying the capacitance of the antenna rod, by introducing hand effects. The detuning will affect the cycle-selection procedure, as will be explained in chapter 7. This detuning sensitivity may be reduced by terminating the antenna in an additional external capacitor $C_b$ (Fig. 6-10).

Unfortunately, this method will adversely affect the sensitivity of the antenna system. However, this increase of capacitance also has some advantages, as a smaller self-inductance, with an inherent smaller noise resistance $R_e$, is then required. Fig. 6-9 also gives the necessary inductance for various antenna lengths. These values for the inductance are rather unpracticable for the frequency band of interest, e.g. 0.33 H for an antenna length equal to 0.93 meters! Fig. 6-10 gives the basic circuit of an active antenna, including an additional parallel capacitance $C_b$, with all relevant noise sources. As before, all these noise sources should be transformed into a single equivalent input voltage-noise source $U_n$, where we may compare this value with the received noise $U_{an}$ (Fig. 6-11). As can be seen, the capacitances $C_a$ and $C_b$ are combined into a single capacitance $C_t$, where

$$C_t = C_a + C_b$$  \hspace{1cm} (6-17)

The received signals, $U_{as}$ and $U_{an}$, are attenuated by $C_b$. Accordingly we introduce $U_{as}'$ and $U_{an}'$:

$$U_{as}' = \frac{U_{as}}{C_b}$$

$$U_{an}' = \frac{U_{an}}{C_b}$$

Fig. 6-10 Basic circuit of selective electric antenna with additional shunt capacitor at the amplifier input.

Fig. 6-11 All noise and signal sources from the circuit in Fig. 6-10 transformed into equivalent input sources.
For these corrected values of the capacitance and the received signals, we may now use the same formulas as given in (6-13), (6-14) and (6-15). The results for $C_b = 50$ pF are depicted in Fig. 6-12. All other parameters are as given in Fig. 6-9.

An examination of Fig. 6-9 and Fig. 6-12 reveals the not totally unexpected penalty for adding $C_b$, an increase in required antenna length for the Gaussian- and the atmospheric-noise models. The revenues are the more feasible values for the inductor, which in turn means a lower parasitic capacitance, a lower loss resistance, and therefore, a lower noise contribution. It implies as well a smaller susceptibility to detuning because of hand effects. However, we are still opting for the shortest antenna possible. Therefore, we must know the influences of the various parameters on the value of $U_n$. Figures 6-13a, 6-13b and 6-13c show the values of $U_{an}$ for the isolated parameters of $S(U_{en})$, $S(I_{en})$ and $Q_l$ respectively.

When a JFET is applied to the input stage, the amplifier may exhibit rather low values for the equivalent input current and voltage noises. The most potentially harmful noise source comes from the input terminating resistor $R_I$. Applying multi-loop feedback techniques introduces the possibility of reducing the noise contribution from this terminating resistor in the filter while maintaining the

![Graph showing the equivalent circuit and antenna (SRN and Gaussian part of SAN) input-noise sources, required inductance as a function of the antenna length for a pre-loaded selective electric antenna.](image)

![Graph showing the equivalent input-noise voltage of pre-loaded selective electric antenna as a function of the antenna length with $S(U_{en})$ as parameter.](image)
Fig. 6-13c Equivalent input-noise voltage of pre-loaded selective electric antenna as a function of the antenna length with the Q of the inductor as parameter.

required input impedance [5-11]. This technique, called "electronic cooling", might significantly enhance the noise performance of the active antenna. Fig. 6-14 shows the result of electronic cooling of the terminating resistor $R_1$. The applied values for $S(U_{en}), S(I_{en})$ and $Q_1$ are typical for practical circuits, while the cooling factor $X$ is the parameter. If the input resistance is $R_1$, then the spectrum of the noise current from $R_1$ equals $4kT/(X\cdot R_1)$.

6.2.1.2 ELECTRONIC ASPECTS OF AMPLIFIER CONFIGURATION

In the foregoing subsection nothing has been said about the equivalent noise sources originating from the electronic amplifier circuit. Nor has a realization method for electronic cooling of the terminating resistance of the band-pass filter been given.

According to Nordholt [5-11], an accurately determined input resistance of an electronic amplifier can be realized by a multi-loop feedback. One loop sets the gain of the amplifier, while the other determines the input impedance of the terminating resistor $R_1$. The applied values for $S(U_{en}), S(I_{en})$ and $Q_1$ are typical for practical circuits, while the cooling factor $X$ is the parameter. If the input resistance is $R_1$, then the spectrum of the noise current from $R_1$ equals $4kT/(X\cdot R_1)$.

Fig. 6-15 Basic circuit model of electronic amplifier providing electronic cooling of the input resistance via multiple feedback.

![Basic circuit model of electronic amplifier](image)

Fig. 6-16 gives the complete functional circuit for noise calculations. The spectrum of the equivalent input voltage-noise source $U_{en}$ in Fig. 6-17 can be found from:

$$U_{o}/U_1 = -Z_2/Z_1 \quad (Z_3>Z_2) \quad (6-20)$$

$$Z_1 = Z_3/(1+Z_2/Z_1) \quad (6-21)$$

The spectrum of the equivalent input voltage-noise source $U_{en}$ in Fig. 6-17 can be found from:

$$S(U_{en}) = 4kT\cdot[(c/g_m+R_1+R_s)+(1/R_3+1/R_s)|Z_s|^2] \quad (6-22)$$
where $R_1$ = resistive part of $Z_1$
$R_2$ = resistive part of $Z_2$
$R_S$ = loss resistance of the inductor
$R_g$ = effective resistance from gate to ground
$C$ = FET device dependent constant ($0.5 < c < 2$)
$g_m$ = transconductance of the FET
$Z_S$ = complex impedance of the tuned circuit

For a low noise contribution from the amplifier, we must strive for a low $R_1$ value and a high $R_3$ value. However, the effective input impedance must agree with the required bandwidth of the tuned circuit. $R_g$ usually exceeds $R_3$ by far, and is therefore non-dominant.

Nevertheless, we must realize that this noise reduction is inherently concatenated to the closed loop gain of the amplifier. This implies that the output stage of the amplifier must still be able to handle the maximum signal levels that the antenna will pick up. This might be a severe limitation for low power supply voltages. However, there is an escape hatch: the application of a step-up transformer at the output of the amplifier [5-11].

The final selection of the circuit parameters turns out to be a compromise between amplifier noise, dynamic range and power consumption.

- Short antennas require very low noise amplification, which will be obtained by selecting high values for $g_m$, $R_3$ and $R_2/R_1$, and by taking a low value for $R_1$.
- A high dynamic range requires higher values for $R_1/R_2$, as the noise current from $R_1$ is approximately linear with the square root of $R_1/R_2$, while the maximum signal-handling capability is linear with $R_1/R_2$.
- Low power unfortunately generally accompanies low $g_m$'s, which inherently generate more noise. On the other hand, the amplifier needs to exhibit only a modest linearity, as the signals at the input and at the output of the amplifier are band-pass filtered, which will reduce the intermodulation product terms. Consequently, low frequency operation (low $F_t$) and moderate open loop gain are applicable, which is to the good of lower power consumption.

Fig. 6-17 depicts a realized circuit. For the given
parameters we find with (6-22) a spectrum value for $U_\text{an}$ equal to $4 \times 10^{-18} \text{ V}^2/\text{Hz}$. This value comes close to the received atmospheric-noise value $U_\text{an}$. The maximum Loran-C signal field strength the amplifier can handle amounts to about 0.5 V/m.

6.2.2 NON-SELECTIVE ELECTRIC ANTENNA

The non-selective electric antenna lacks the main technical drawbacks of the selective antenna: the large inductance and the required bandwidth-determining input resistor $R_4$, which limits the dynamic range. But in accordance with Murphy's law, we inherit another serious problem in return: an increase in both the spurious response and in the power consumption.

A non-selective antenna can be obtained in at least two different ways. The first method applies an amplifier with a very high input impedance. If this impedance is large in comparison with the impedance of the antenna itself, the response will be frequency-independent. The second method of obtaining a broadband response is to connect the antenna to a charge amplifier. This transimpedance amplifier has a capacitive feedback from the output to the inverting input of the amplifier. Where the capacitance of the antenna is $C_a$, and the feedback capacitance $C_f$, the gain of the active antenna is given by:

$$\frac{U_o}{U_a} = -\frac{C_a}{C_f}$$

(6-23)

This broadband response introduces a serious load on the performance of the electronic amplifier. The complete spectrum as received by the antenna must pass the amplifier before it can be fed back via the capacitance $C_f$ to the input. This means in effect that we are faced with the spectrum from power-line frequencies up to frequencies in the UHF band, radiated by TV transmitters. The penalty of such wide band amplifiers is the inherently high power consumption.

Lindenmeier [5-12] introduced the high input-impedance approach. Although the noise performance of this type of active antenna is reasonable, active antennas should be protected against static discharges. This can be realized by shunting the input of the antenna amplifier by high-speed diodes or surge absorbers. However, the non-linearity of such diodes easily introduces intermodulation distortion. We will therefore focus exclusively on the charge amplifier configuration which is an almost ideal circuit for static protection.

Nordholt and van Willigen [5-9] have investigated the performance of a wide-band active antenna, based on a charge-amplifier circuit. The noise and the dynamic range of this antenna is adequate for our purpose, but the power consumption of 3 Watts is absolutely unacceptable. At the expense of some degradation in the intermodulation performance, it proved to be possible to diminish the power consumption to 85 mW. We will start with a short discussion of the noise properties of this active-antenna type.

6.2.2.1 NOISE AND ELECTRONIC ASPECTS OF A NON-SELECTIVE ACTIVE ANTENNA

With reference to Fig. 6-6a, we see that for the spectrum of the equivalent input-noise source the following can be
\[ S(U_{en}) = S(U_{en}) + S(I_{en})/(\omega C_a)^2 \]  

(6-24)

The small values for \( C_a \) with short antennas make a very low \( a \) mandatory. The obvious input device is once again a field-effect transistor. Fig. 6-18 gives that part of the circuit most relevant to the noise behavior. Under the assumption of a reasonable gain of the FET stage, which allows us to disregard the noise contribution of the following stages, we may state that the spectrum \( S(U_{en}) \) originates from the drain current \( I_d \). Via transformation techniques for noise sources, as described in [5-11], we find:

\[ S(U_{en}) = 4kT c/g_m \frac{((C_{iss} + C_d + C_f)/C_a)^2}{g_m} \]  

(6-25)

where \( c = \) FET device-dependent constant \((0.5 < c < 2)\)

- \( g_m = \) transconductance
- \( C_{iss} = \) total input capacitance of the FET including the diode and wiring capacitances
- \( C_d = \) antenna capacitance
- \( C_f = \) feedback capacitance

The noise spectrum \( S(I_{en}) \) stems mainly from the bias resistance \( R_{bias} \) and the gate leakage resistance \( R_{leakage} \) of the FET, so:

\[ S(I_{en}) = 4kT (1/R_{bias} + 1/R_{leakage}) \]  

(6-26)

The noise sources \( U_{en} \) and \( I_{en} \) are considered white and uncorrelated at the frequencies of interest. As we opt for low noise, we must strive for high values for \( g_m, R_{bias} \) and \( R_{leakage} \) and for low values for \( C_{iss} \) and \( C_f \). However, the value for \( C_f \) is not entirely open to selection. For a given antenna length, and thus for a given \( C_a \), we must ensure that the output stage of the amplifier is not overloaded for the largest signal field strengths. The then fixed \( C_f \) makes the determination of the spectrum of the equivalent input-noise field strength feasible. Fig. 6-19 illustrate, for a prototype of a non-selective active antenna, the equivalent input-noise contribution of the amplifier together with the received signal and noise. The maximum allowable Loran-C signal field strength and the maximum output-voltage level are the parameters.

Fig. 6-18 Basic model of non-selective electric antenna.

Fig. 6-19 Equivalent circuit and antenna (SRN and Gaussian part of SAN) input-noise sources as a function the antenna length for a non-selective electric antenna.
The low power constraints forces the designer to carefully select the amplifier's active devices. As previously stated, the optimal choice for the input device is a FET. However, this FET must combine a high transconductance at a low drain-source voltage and a low drain current, as well as a low $C_{iss}$. The device selected (BF510) has the following parameters:

$$g_m = 3.3 \text{ mA/V} \quad \text{(at } I_d = 1 \text{ mA and } V_{ds} = 1 \text{ V)}$$
$$C_{iss} = 5 \text{ pF}$$

The basic diagram of the amplifier circuit is given in Fig. 6-20. To provide an excellent linearity from VLF up to UHF, a high loop gain over a very wide frequency band is mandatory. Therefore, we select transistors which possess a very high $F_t$ at low collector currents and low collector-emitter voltages: BFT92R (PNP) and BFR92 (NPN). For both types, $F_t$ exceeds 2 GHz at 1 mA collector current.

![Fig. 6-20 Basic circuit of a non-selective electric antenna.](image)

FET $T_1$ and the PNP transistor $T_2$ form a cascode stage. Behind this stage the main current gain of the amplifier is realized with the transistors $T_3$ and $T_5$, while the main voltage gain comes from transistor $T_4$. This gain is linear with the collector load impedance $Z_c$ of $T_4$. To obtain a high dynamic resistance, while not wasting supply voltage, bootstrapping is provided from the emitter-follower output stage. Resistor $R_6$ matches the characteristic coaxial-cable impedance $R_7$. The dynamic load impedance $Z_c$ at the collector of $T_4$ equals the parallel impedance of $Z_a$, $Z_b$ and $Z_c$, where:

$$Z_a = R_4 \cdot (1 + 1/X) \quad \text{with } X = r_e \cdot (1 + R_5 + R_7)/R_5 \cdot (R_6 + R_7) \quad (6-27)$$
$$Z_b = a \cdot e \cdot (1 + R_6 + R_7) \quad (6-28)$$
$$Z_c = 1/(pC_2) \quad (6-29)$$

As we opt for a high dynamic range at high frequencies, we must minimize parasitic capacitances at the collector of $T_4$. Otherwise, due to signal-current splitting, it will be difficult to attain a full output-voltage swing at low collector currents of $T_4$. As there are so many high-intensity radio signals at relatively low frequencies, it is crucial to achieve high loop gain at these low frequencies. This high loop gain, thanks to the bootstrapping, will keep the inter-modulation distortion at an acceptable level. However, unconditional stability requires the introduction of a pole-zero pair in the transfer function of the gain to compensate for the two poles in the transfer functions of the transistors $T_2$ and $T_4$. This will reduce the gain for higher frequencies without introducing additional phase shift.

This pole-zero pair is constructed around $T_3$. The transfer function from the base current to the emitter current of $T_2$ can be approximated by:

$$1 \frac{e_2}{b_2} = (R_2 + R_3 + 1/(pC_1))/(R_3 + 1/(p0^{-E}C_1)) \quad (6-30)$$
The zero is located at:

\[ f = \frac{1}{2\pi \cdot (R_2 + R_3) \cdot C_1} \]  

(6-31)

and the pole at:

\[ f = \frac{1}{2\pi \cdot R_3 \cdot \alpha \cdot C_1} \]  

(6-32)

The resistor \( R_1 \) introduces a phantom zero in the feedback path. Although the effect is minimal, it does serve as an elegant way to minimize the slight peak in the frequency response at high frequencies. However, \( R_1 \) should be kept small, as the noise from this resistor is in series with the input signal.

At last Fig. 6-21 presents a diagram of the full circuit, from which it can be seen that a more complex feedback loop is applied. The loop is split up into a DC and an AC path.

The AC feedback is reduced by about 10 dB around the Loran-C band of 100 kHz. This gives us an increased headroom for non-Loran-C signals, thereby reducing the risk of overloading. The emitter follower \( T_5 \) is connected to a signal-modulated current source \( T_6 \). This class-AB output-stage solution is attractive with respect to a saving in power consumption. At the input of the amplifier are two high-speed, low-capacitance diodes connected in anti-parallel. As long as the amplifier is not overdriven, the virtual ground prevents distortion of the signal by the diodes. As soon as overloading comments, the virtual ground will be lost, and the diodes will then absorb the excess energy. This must be paid for in the form of a slight increase in noise, due to the capacitance of the diodes. Because of the high \( F_e \)'s of the applied transistors, no phantom zero is needed at the input of the circuit. The amplifier is powered through the coaxial cable.

The calculated and measured results of a non-selective active antenna are:

- Supply voltage: 5 V
- Supply current: 17 mA
- Physical length of the antenna: 0.144 m
- Effective length of the antenna rod: 0.115 m
- Antenna capacitance \( C \): 5.3 pF
- Effective length of the active antenna: 0.20 m
- \( S(U) \) - calculated: \( 2.4 \times 10^{-16} \text{ V}^2/\text{Hz} \)
- \( S(U) \) - measured: \( 4.8 \times 10^{-16} \text{ V}^2/\text{Hz} \)
- Frequency: 100 kHz
- Bandwidth: 40 kHz
- Output voltage (in 50 Ohms): 0.707 V rms
- Maximum field strength (100 kHz): 3.5 V rms/m
- Maximum field strength (non 100 kHz): 11 V rms/m

The intermodulation measurements are carried out at an output-signal level of -10 dBm for each signal and with an IM-product frequency of 2 Mhz.
Test frequencies | Second-order | Third-order |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8 &amp; 10 MHz</td>
<td>+60 dBm</td>
<td>n.a.</td>
</tr>
<tr>
<td>28 &amp; 30 MHz</td>
<td>+40 dBm</td>
<td>n.a.</td>
</tr>
<tr>
<td>68 &amp; 70 MHz</td>
<td>+20 dBm</td>
<td>+30 dBm</td>
</tr>
<tr>
<td>8 &amp; 14 MHz</td>
<td>n.a.</td>
<td>+25 dBm</td>
</tr>
<tr>
<td>16 &amp; 30 MHz</td>
<td>n.a.</td>
<td>+20 dBm</td>
</tr>
<tr>
<td>36 &amp; 70 MHz</td>
<td>n.a.</td>
<td></td>
</tr>
</tbody>
</table>

The 3 dB discrepancy between the calculated and the measured equivalent input-noise sources may be caused by the inaccuracy of the noise-power measurement set-up, a faulty estimation of the parameter c of the FET, and a non-negligible noise-power supplement from the second amplifier stage.

The very short length of this antenna is responsible for a noise contribution of the amplifier which exceeds the received noise of the atmospheric-noise model. However, the antenna is designed for application in mobile equipment, where small physical properties are more important than the highest signal-to-noise ratios. Another argument is that in automotive applications the antenna will be prone to a higher noise floor.

In marine applications the active antenna will generally be mounted on the mast. As the mast is a non-separable part of the antenna configuration, the effective length will increase drastically. The extent of such lengthening depends on the length and the slenderness of the supporting structure, as is described by Lindenmeier [5-12]. Therefore, the given of the active-antenna example may perform well, even under very low atmospheric-noise conditions. The optimum selection of the length of the physical antenna part is a trade-off between the sensitivity of the system and the risk of overloading the output stage of the amplifier.
Fig. 6-22 shows the realistic test of an active antenna. The input of the amplifier is exposed to a series of high-voltage sparks from an automobile ignition system. After one million sparks, there was no noticeable deterioration in the specifications. Finally, Fig. 6-23 contains a photograph of the thin-film substrate carrying the amplifier components.

It would seem that the non-selective, charge-amplifier type of active antenna is the best choice for general applications. It provides a satisfactory dynamic range and is power-efficient. However, it must be realized that the risk of overloading the antenna, especially in strong VHF and UHF fields, may be a serious danger with respect to intermodulation distortion. Under such conditions, the selective antenna may be more suitable.

7 BAND PASS FILTER SELECTION

The next step after having designed the antenna is to select the band-pass filter which is to be placed between the antenna and the receiver (Fig. 4-6). It is trivial but crucial that the band-pass filter should let the Loran-C signal pass, while blocking all threats such as atmospheric noise and interfering signals. Our first thought could be to diminish the bandwidth of the band-pass filter so that it lets just the Loran-C signal pass. This minimized bandwidth would then assure a maximal protection against interference and noise. Unfortunately, this simplified approach does not at all lead to an optimally designed input filter.

In chapter 3 we already noted that the received signal is composed of the ground-wave Loran-C signal and its contaminants, such as atmospheric noise, sky waves and continuous-wave interferences (CWI). The mutual relations between these four parts of the complex signal will significantly alter while passing through the band-pass filter. Before going into quantitative analysis, we will give some qualitative indications of the filtering effects.

The response of a band-pass filter to the modulation of a carrier can be reasonably characterized by the bandwidth and the step-response delay time. The rise time of the filter is here of less relevance, as it is already partly included in the step-response time. The power bandwidth is a measure for the reduction of the received noise. The bandwidth and the steepness of the filter slopes determine the amount of rejection of interfering signals outside the Loran-C band. The calculations on noise reduction and interference rejection are simple. With respect to noise and interference minimization, one must strive for high-order,
narrow-bandwidth filters. Butterworth and Chebyshev types are then natural candidates.

The way the filter deteriorates the ratio of the ground-wave and sky-wave signals, and the signal-to-noise ratio at the sample-point position is much more complicated. To prevent pollution of the phase of the ground wave by the sky wave, the sampling of the ground-wave signal should take place before the sky wave arrives. If we assume a relative sky-wave delay of 32.5 us (specified by the MPS), the phase-tracking samples are to be taken at the latest at the 30.0 us zero crossing. We assume further a step-response time of the band-pass filter in excess of the sum of the rise time of the filter and 32.5 us. This implies that the amplitude of the ground-wave signal at the sample-point position of 32.5 us will still be very close to zero! In the light of the signal-to-noise ratio, this is a very unfortunate result. Our first thought might be to sample somewhat later, so that the envelope may grow to an acceptable level. However, as the ratio of the ground-wave and the sky-wave signal levels do not change when both signals pass the filter, intolerable sky-wave pollution may be introduced. So, a short step-response delay time of the filter is essential. This favors the choice of low-order, large-bandwidth filters of the Gauss or the Bessel type.

To illustrate how the envelope of the Loran-C burst is affected, Fig. 7-1 gives the input and the output signal of a 5th-order Butterworth band-pass filter with a bandwidth of 20 kHz. The step-response delay time amounts to 55 us. Note that the step-response delay is more visible than the rise time of the filter. The next plot in Fig. 7-2 shows us the

![Fig. 7-1 Input and output Loran-C signal of a 6th-order transitional (Gaussian down to -12 dB) band-pass filter with 25 kHz bandwidth.](image)

![Fig. 7-2 Output signals of a 25 kHz bandwidth, 6th-order transitional (Gaussian down to -12 dB) band-pass filter of a ground-wave signal and its 32.5 us delayed sky-wave signal with a +12 dB relative amplitude.](image)
two output signals of a 25-kHz wide, 6th-order transitional (Gaussian to -12 dB) filter of a ground-wave signal and its 32.5 μs delayed sky-wave signal which has a +12 dB relative amplitude. The step-response time delay of this filter equals 47 μs. At 32.5 μs after the start of the Loran-C burst no output signals can be seen on the plot, in spite of the presence of both signals. Fig. 7-3 depicts the envelopes of the input and the output signals, where the amplitude scale is made logarithmic. The applied filters are of the same type as the one from Fig. 7-1. However, a second bandwidth of 60 kHz is also given. The step-response time delays are 55 μs and 18.3 μs. If a certain minimal difference between the outputs from the ground wave and the sky wave have to be maintained, the sample-point position must move further to the leading edge of the burst. The consequence, as discussed above, is a serious reduction in the ground-wave-signal amplitude level.

It would appear as if we have come into a deadlock position: high-order, narrow-bandwidth opposite low-order, large-bandwidth filters. In the following sections we will see that for a specific amount of sky-wave pollution and for an also specific maximum tracking error, low-order, large-bandwidth filters provide superior SNR at the output of the filter and poor interference suppression. So, a compromise must be found which gives acceptable noise and interference reductions while simultaneously keeping the tracking error due to sky waves at a low value. Such a compromise could be found with the help of filter tables [5-14] & [5-15]. However, there is a serious complication. The curves of the step responses are not accurately given at the beginning of the leading edges. It is at just this position where it is crucial to analyze the amount of sky-wave signal content in the ground-wave signal. However, with the help of transient-analysis computer programs, accurate determination of the performance of various filter types is feasible. We will now discuss the various indicated effects in more detail, so that we may finally come to a selection procedure for the band-pass filter.

7.1 ATMOSPHERIC NOISE

The phenomena of noise will produce randomly varying Loran-C signal measurements. The tracking algorithm will integrate the collected samples to acquire an improved
post-detection signal-to-noise ratio with respect to the pre-detection signal-to-noise ratio. Atmospheric noise has a non-Gaussian probability distribution: it shows a greater abundance of high peaks, which necessarily slows down the signal-integration process. Therefore, analog receivers are forced to apply noise-censoring techniques to eliminate the samples disturbed by these high-amplitude noise peaks. Hard-limiter receivers have "built-in" noise-censoring properties in that a disturbance of a sample by a high noise peak has no more weight than a disturbance by a lower one [3-1 & 3-3].

In the Minimum Performance Standards [5-5], the choice is between Simulated Random (Gaussian) Noise (SRN) and Simulated Atmospheric Noise (SAN), which is composed of two components. The first component, contributing 15.85% (~8 dB) of the noise energy, consists of Gaussian background noise. The second component is a series of randomly distributed bursts of a 100 kHz sine wave. The occurrence of a burst is Poisson-distributed, nominally 50 per second, with a duration of 30 μs (3 full cycles). The top amplitude of the sine wave is fixed to 30.5 dB above the SAN RMS-level, i.e. 38.5 dB above the RMS-value of the Gaussian background noise. The probability that a sample will be taken in a burst amounts to 30 μs * 50/s = 1.5*10^-3.

When the SAN signal is fed through a band-pass filter, the envelope of the sine-wave burst will get stretched out. By defining the pulse length at the filter output as the time the pulse envelope exceeds the Gaussian-background-noise RMS-level, we can calculate the pulse length for different filter types and bandwidths. Using Laplace transformations on the band-pass filter's low-pass equivalent yields a pulse length of 353 μs for a 6th-order Butterworth filter of 15 kHz bandwidth, and 128 μs for a transitional (Gaussian to -12 dB) filter of the same bandwidth. The probability that a sample will be taken in such an extended burst is now 1.8*10^-2. This means that in a hard-limiter receiver less than one sample per 100 samples might show a wrong polarity. Thus, this error is rather harmless. Later it will be seen that only filter bandwidths of 15 kHz to 60 kHz are of interest, so that no significant errors will be introduced when we use only the Gaussian background noise, which is ~8 dB of the SAN RMS-level.

The Minimum Performance Standards (see also section 3.1.2) define a lower limit on the signal-to-noise ratio (for tracking) of 0 dB, regardless of whether the noise type is Gaussian or atmospheric. Applying SAN then yields a minimum SNR of +8 dB, a most welcome gain, as it facilitates the selection procedure of the optimum band-pass filter.

7.2 TRACKING ERROR DUE TO SKY WAVES

The phase relation between the ground-wave and the sky-wave signals is to be considered stable for the integration times of the signal averagers. This implies that tracking samples must be taken before any intolerable sky-wave content in the Loran signal arises, as it cannot be discarded by signal integration. For all further calculations we will use one of the most stringent MPS conditions, namely a ground-wave-to-sky wave ratio of ~12 dB for a relative sky wave delay of 32.5 μs.

Fig. 3-7 gives a typical representation of the European interference situation in the LF band. A number of CW interferers are visible next to the Loran-C spectrum. Notch filters can very effectively suppress such a CWI signal.
However, a single notch filter can only kill a single interferer. The number of interferers will adequately account for a preference towards a rather narrow band-pass filter which could possibly reject a number of interfering signals simultaneously. The penalty for such a narrow band-pass filter is the inevitable reduction in the signal-to-noise ratio of the ground-wave signal for a specific tracking error due to sky waves. However, this SNR reduction is minimal for filters with a minimum envelope delay. Gaussian filters will perform very well. Unfortunately such a filter will perform poorly with respect to near-band interference suppression. Butterworth filter types have the opposite qualities. The Chebyshev filters show even better CWI characteristics. However, their step-response times are so long that excessive bandwidths are necessary to establish adequate GSR at the output of the filter for acceptable SNR's. A quick examination of the tables and graphs of [7-15] then shows that the resulting near-band interference rejection is poor.

With the aid of Laplace transformations on the equivalent low-pass filters, the tracking error due to the MPS-specified sky-wave contamination can be visualized. The tracking algorithm will estimate the position in time of a specific zero crossing of the received navigation signal. Therefore, we will find just a few trackable, i.e. clean, zero crossings. Computer analyses give in Fig. 7-4a-c the difference between the SNR at the output and at the input of the filter as a function of the amount of tracking error. The bandwidth of the filters is the parameter and varies between 15 kHz and 50 kHz. The results are depicted for the following filter types: 6th-order Butterworth, Transitional Gaussian to -6 dB and Transitional Gaussian to -12 dB.

In agreement with what was stated in the first section, it can be concluded that all three filter types share a tendency to having higher bandwidths accompanied by a superior SNR at the output of the filter in conjunction with a specified maximum tracking error. Not surprisingly, that this conclusion conflicts with the interference rejection performance.

Finally, it should be remarked that only short sky-wave contamination effects have been taken into account here. Long sky waves, which extend into the next burst or series of bursts, are far less sensitive to the step-response delay times of the band-pass filter. Long sky-wave effects are described in section four.
7.3 TRACKING ERROR DUE TO CONTINUOUS-WAVE INTERFERENCE

Continuous wave interference can be characterized by its absolute frequency or by its relational frequency with respect to the Loran-C signal. The MPS handbook specifies near-band (70 kHz - 90 kHz and 110 kHz - 130 kHz) interference and out-band interference (<50 kHz or >200 kHz). The interference can be classified into three types, i.e. synchronous, near-synchronous and non-synchronous. The interference is said to be synchronous if its frequency equals \( \frac{N}{2 \text{ GRI}} \) for \( N=1,2,3,... \), with a sub-division into odd- and even-synchronous for \( N \) is even and odd, respectively. If the interfering frequency exhibits a difference with the closest \( \frac{N}{2 \text{ GRI}} \) frequency exceeding the tracking bandwidth of the receiver, the interference is said to be non-synchronous. If the frequency difference is less than the tracking bandwidth, it is called a near-synchronous interference. A zero frequency difference will result from synchronous interference.

Fig. 7-5 depicts part of a positive- and a negative-going sine wave around the zero crossing of the signal together
with a horizontal line, representing the interference level. It is assumed that the interference is synchronous, and that the interference level is equal at all zero-crossing positions of the Loran signal. The positive- and the negative-going slopes of the sine waves are related to the phase coding of the Loran-C signals.

The samples of the signal will finally be taken at the position where the tracking algorithm estimates that the average value of the samples equals zero. However, if only positive-going Loran signals are taken, the average zero position will be found at the time position where the sum of the Loran signal and the interference is zero. In our example it will result in too early an estimation of the zero crossing of the Loran signal. The opposite will happen for the negative-going slope of the Loran signal.

The signal processor will make its estimation based on the average of a large series of samples, where the number of samples to be averaged strongly depends on the SNR of the received signal. If we bear in mind that in a sequence of two group-repetition intervals 10 positive- and 6 negative-going slopes will be found, the average value of the time-position error will then be 4/16 times the error of a non-phase-coded Loran signal. This error reduction of 75% is only valid for linear averaging. Hard-limiter receivers only determine the average value of the polarity samples. The probability of a correct observation of the signal polarity in a single sample is denoted by \( p_{\text{obs}} \). For a plus infinite SNR, \( p_{\text{obs}} \) will approach 1, while for a minus infinite SNR, \( p_{\text{obs}} \) will asymptotically go to 0.5. As will be seen later in chapter 10, the tracking algorithm will settle for a sample position where the average value of \( p_{\text{obs}} \) equals 0.5. Due to the unbalanced phase coding of the signal, the hard-limiter receiver will establish tracking errors equal to the error of a non-phase-coded signal under high SNR conditions. For lower SNR's, \( p_{\text{obs}} \) will come closer to 0.5, and this in turn has a strong linearizing effect (see chapter 10) on the signal averager. Fig. 7-6 gives the tracking error as a function of the signal-to-synchronous interference ratio, \( \text{SI}_{\text{syn}} \), where the SNR is the parameter. It is clearly demonstrated that the tracking error decreases sharply in the case of lower SNR's. The ultimate gain in the allowable signal-to-interference ratio amounts to 12 dB for the same tracking-error level. It can be said that for realistic tracking errors (e.g. <100 ns) and for real-life SNR's (<<0 dB near the zero crossing) the hard-limiter-receiver performance equals that of a linear receiver.

The tracking bandwidth of Loran receivers is generally very small. We should therefore momentarily ponder the risk
of meeting a synchronous or near-synchronous interferer. Assuming a tracking bandwidth of 0.1 Hz and a frequency range of 40 kHz where interferers might be found, the probability of running across such an interferer amounts to $5 \times 10^{-6}$. Whether or not this risk is worrisome depends on the application of the navigation set.

A far more serious problem is non-synchronous interference. Especially in Europe, we face a myriad of uninvited stations, as can be seen in Fig. 3-7. As will be discussed in chapter 9, the effect of non-synchronous interference can be expressed as an apparent increase in the Gaussian noise level. This increase depends on the ratio of the RMS levels of the interference and the Gaussian noise, $I_{\text{nsyn}} / \text{NR}$. Fig. 7-7 depicts $p_{\text{obs}}$ as a function of the SNR with $I_{\text{nsyn}} / \text{NR}$ as parameter. To facilitate the determination of the detection performance of a hard limiter, Fig. 7-8 plots the constant-observation reliabilities as a function of the signal-to-non-synchronous interference ratio ($S_{\text{nsyn}}$) and the signal-to-noise ratio (SNR).

![Diagram](Image)

Fig. 7-7 $p_{\text{obs}}$ as a function of the SNR with $I_{\text{nsyn}} / \text{NR}$ as parameter.

The amount of interference the hard limiter has to undergo depends on the frequency of the interferer and the performance of the band-pass filter. As the interference and the noise are both influenced by the filter, Fig. 7-9a-c gives the differences between the interference-to-noise ratio at the output $\text{INR}_{\text{out}}$ and the interference-to-noise ratio at the input of the filter $\text{INR}_{\text{in}}$, as a function of the frequency. The bandwidth of the filter is the parameter. The type of filters are the same as given in Fig. 7-4a-c. As far as interference reduction is concerned, the smaller bandwidths will perform the best. However, this conflicts with the preference for wide bandwidths which is suitable for operation under sky-wave conditions.

We have faced various, almost confusing, aspects of band-pass filters with respect of signal contaminators. In the next section will be demonstrated how to come to a selection of a filter which will perform adequately for our purpose.

### 7.4 FILTER SELECTION

The previous sections dealt with two types of tracking errors:

- errors with an average value equal to zero, due to noise and non-synchronous interference.
- errors with an average value not equal to zero, due to sky waves and (near-) synchronous interference.

For a certain receiver performance, the signal requirements at the output of the band-pass filter may be formulated. In our example the required output conditions
will be:

A - $\text{SNR}_{\text{out}} > 0$ dB (to satisfy, for example, dynamic tracking specifications)

B - Tracking error due to sky-wave contamination $\leq 50$ ns

C - Tracking error due to synchronous interference $\leq 50$ ns

The band-pass filter effect will be studied for the underlying four different reception conditions, assuming that in all four cases $\text{SNR} > 0$ dB and that Simulated Atmospheric Noise (SAN) is applied:

1 - Skywave: 32.5 us delay; $\text{GSR} = -12$ dB

2 - One (near-) synchronous CWI at 110 kHz; $\text{SIR} = 0$ dB

3 - One non-synchronous CWI at 110 kHz; $\text{SIR} = -20$ dB

4 - One non-synchronous CWI at 200 kHz; $\text{SIR} = -60$ dB

These input conditions have been drawn from the MPS, although it prescribes two instead of one interferer simultaneously. This has been done to simplify the selection example. The selected 110 kHz is the worst-case situation of near-band interference as defined in the MPS.

Let's assume that a single filter has to be found that meets the output requirements for all four input conditions separately. In this example we will study a 6th-order transitional (Gaussian to -12 dB) filter type.

7.4.1 SKY-WAVE CONTAMINATION

The maximal allowed tracking error ($< 50$ ns) and the minimal required $\text{SNR}_{\text{out}} (> 0$ dB) set bounds to the bandwidth which can be used. Fig. 7-4c provides us with the possible bandwidth and tracking-error combinations, and it also lists the change in the $\text{SNR}$, $(\text{SNR}_{\text{out}} - \text{SNR}_{\text{in}})$, as is tabulated below:

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>$\text{SNR}<em>{\text{out}} - \text{SNR}</em>{\text{in}}$ (dB)</th>
<th>$\text{SNR}<em>{\text{out}}$ (dB) for $\text{SNR}</em>{\text{in}} = +8.0$ dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-9.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>20</td>
<td>-5.5</td>
<td>+3.5</td>
</tr>
<tr>
<td>25</td>
<td>-4.0</td>
<td>+4.0</td>
</tr>
<tr>
<td>30</td>
<td>-1.5</td>
<td>+6.5</td>
</tr>
<tr>
<td>35</td>
<td>+0.2</td>
<td>+8.2</td>
</tr>
<tr>
<td>40</td>
<td>-1.1</td>
<td>+6.9</td>
</tr>
<tr>
<td>50</td>
<td>+0.0</td>
<td>+8.0</td>
</tr>
</tbody>
</table>
Because of the atmospheric noise model, $\text{SNR}_{\text{in}}$ is greater than 8 dB, i.e. the Gaussian background noise. $\text{SNR}_{\text{out}} > 0$ dB is required, so that $\text{SNR}_{\text{out}} - \text{SNR}_{\text{in}}$ has to be greater than -8 dB. All bandwidths from the above table may be used, except the 15 kHz bandwidth.

### 7.4.2 CONTINUOUS WAVE INTERFERENCE (CWI)

To study the effect of CWI, we are interested in the resulting $\text{SIR}_{\text{out}}$ at 110 kHz (the worst case for near-band interference) and 200 kHz. Fig. 7.9c provides a plot of $\text{INR}_{\text{out}} - \text{INR}_{\text{in}}$ as a function of frequency and with the bandwidth as parameter. As $\text{SIR}_{\text{in}}$ is specified, to find $\text{SIR}_{\text{out}}$, we need to determine $\text{SIR}_{\text{out}} - \text{SIR}_{\text{in}}$. The latter can be found from the combination of Fig. 7-4c and Fig. 7-9c because:

$$\text{SIR}_{\text{out}} - \text{SIR}_{\text{in}} = (\text{SNR}_{\text{out}} - \text{SNR}_{\text{in}}) - (\text{INR}_{\text{out}} - \text{INR}_{\text{in}})$$  \hspace{1cm} (7-1)

The results are tabulated below:

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>$\text{SNR}<em>{\text{out}} - \text{SNR}</em>{\text{in}}$ (dB)</th>
<th>$\text{INR}<em>{\text{out}} - \text{INR}</em>{\text{in}}$ (dB)</th>
<th>$\text{SIR}<em>{\text{out}} - \text{SIR}</em>{\text{in}}$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-9.0</td>
<td>+3.6</td>
<td>-12.6</td>
</tr>
<tr>
<td>20</td>
<td>-5.5</td>
<td>+3.6</td>
<td>-9.1</td>
</tr>
<tr>
<td>25</td>
<td>-4.0</td>
<td>+2.7</td>
<td>-6.7</td>
</tr>
<tr>
<td>30</td>
<td>-1.5</td>
<td>+1.7</td>
<td>-3.2</td>
</tr>
<tr>
<td>35</td>
<td>+0.2</td>
<td>+0.8</td>
<td>-0.6</td>
</tr>
<tr>
<td>40</td>
<td>-1.1</td>
<td>+0.0</td>
<td>+1.1</td>
</tr>
<tr>
<td>50</td>
<td>+0.0</td>
<td>-1.5</td>
<td>+1.5</td>
</tr>
</tbody>
</table>

Fig. 7-9a $\text{INR}_{\text{out}} - \text{INR}_{\text{in}}$ as a function of frequency for a 6th-order Butterworth filter with the bandwidth as parameter.

Fig. 7-9b $\text{INR}_{\text{out}} - \text{INR}_{\text{in}}$ as a function of frequency for 6th-order transitional (Gaussian down to -6 dB) filter with the bandwidth as parameter.

Fig. 7-9c $\text{INR}_{\text{out}} - \text{INR}_{\text{in}}$ as a function of frequency for a 6th-order transitional (Gaussian down to -12 dB) filter with the bandwidth as parameter.
This table provides the data needed to analyze the filter effects on various forms of interference.

7.4.3 (NEAR-) SYNCHRONOUS CWI AT 110 kHz

With a maximum allowed tracking error of 50 ns, a minimum required SIR\(_{\text{out}}\) of 30 dB can be found from Fig. 7-6. The table in section 7.4.2 shows that no bandwidth gives the desired SIR\(_{\text{out}}\) for the given SIR\(_{\text{in}}\) = 0 dB. Thus, additional notch filters are necessary. The required notch depth is easy to find from the same table via:

Required notch depth = 30 dB - (SIR\(_{\text{out}}\) - SIR\(_{\text{in}}\)) \[(7-2)\]

In table form we give the results below:

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>SIR(<em>{\text{out}}) - SIR(</em>{\text{in}}) (dB)</th>
<th>Required notch depth (dB) for SIR(_{\text{out}}) &gt; 30 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-12.6</td>
<td>&gt; 42.6</td>
</tr>
<tr>
<td>20</td>
<td>-9.1</td>
<td>&gt; 39.1</td>
</tr>
<tr>
<td>25</td>
<td>-6.7</td>
<td>&gt; 36.7</td>
</tr>
<tr>
<td>30</td>
<td>-3.2</td>
<td>&gt; 33.2</td>
</tr>
<tr>
<td>35</td>
<td>-0.6</td>
<td>&gt; 30.6</td>
</tr>
<tr>
<td>40</td>
<td>-1.1</td>
<td>&gt; 31.6</td>
</tr>
<tr>
<td>50</td>
<td>+1.5</td>
<td>&gt; 28.5</td>
</tr>
</tbody>
</table>

The notch depth may be relaxed as we move further away from the 110 kHz, which can be seen by studying Fig. 7-9. The band-pass filter becomes more effective. It should be recalled that the probability of meeting a (near-) synchronous interferer is very low. Therefore, these notch-depth requirements should be observed with some reserve.

7.4.4 NON-SYNCHRONOUS CWI AT 110 kHz

The effect of non-synchronous interference can be expressed as an increase in the Gaussian noise level (see Fig. 7-7). The input conditions are:

* \(\text{SNR}_{\text{in}} > +8\) dB
* \(\text{SIR}_{\text{in}} > -20\) dB

To meet the output requirement \(\text{S}(\text{N}+\text{I})\text{R} > 0\) dB (where \(\text{N}+\text{I}\) represents the Gaussian noise plus the non-synchronous interference effect), the minimum SIR\(_{\text{out}}\) may be found from Fig. 7-8. With the help of the table in section 7.4.2, we may then determine the notch performances required to meet the necessary SIR\(_{\text{out}}\) values. The results are given in the table below:

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>Required SNR(_{\text{out}}) (dB)</th>
<th>Required SIR(_{\text{out}}) (dB)</th>
<th>Required notch depth (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>&gt; -1.0</td>
<td>impossible</td>
<td>impossible</td>
</tr>
<tr>
<td>20</td>
<td>&gt; +3.5</td>
<td>&gt; 5.0</td>
<td>&gt; 34.1</td>
</tr>
<tr>
<td>25</td>
<td>&gt; +4.0</td>
<td>&gt; 5.0</td>
<td>&gt; 31.7</td>
</tr>
<tr>
<td>30</td>
<td>&gt; +6.5</td>
<td>&gt; 5.0</td>
<td>&gt; 28.2</td>
</tr>
<tr>
<td>35</td>
<td>&gt; +8.2</td>
<td>&gt; 5.0</td>
<td>&gt; 25.6</td>
</tr>
<tr>
<td>40</td>
<td>&gt; +6.9</td>
<td>&gt; 5.0</td>
<td>&gt; 26.1</td>
</tr>
<tr>
<td>50</td>
<td>&gt; +8.0</td>
<td>&gt; 5.0</td>
<td>&gt; 23.5</td>
</tr>
</tbody>
</table>

The values for SNR\(_{\text{out}}\) are found by adding 8 dB (advantage of the SAN) to the values in the (SNR\(_{\text{out}}\) - SNR\(_{\text{in}}\)) column in section 7.4.1. From the table it can be seen that the notch-depth requirements are relaxed by 5 dB as compared to the synchronous interference condition in section 7.4.3.
7.4.5 NON-SYNCHRONOUS CWI AT 200 KHZ

The required SIR$_{\text{out}}$ is the same as found for 110 kHz in the previous sub-section. Now the band-pass filter itself provides more suppression than at 110 kHz. For the given SIR$_{\text{in}}$, equal to -60 dB, the resulting SIR$_{\text{out}}$ can be found from the table in section 7.4.2 and checked with the minimum required SIR$_{\text{out}}$ values (> 5 dB) as found in the foregoing sub-section.

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>SIR$_{\text{out}}$ (dB)</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>57.0</td>
<td>+</td>
</tr>
<tr>
<td>20</td>
<td>31.5</td>
<td>+</td>
</tr>
<tr>
<td>25</td>
<td>22.0</td>
<td>+</td>
</tr>
<tr>
<td>30</td>
<td>15.5</td>
<td>+</td>
</tr>
<tr>
<td>35</td>
<td>10.2</td>
<td>+</td>
</tr>
<tr>
<td>40</td>
<td>2.9</td>
<td>-</td>
</tr>
<tr>
<td>50</td>
<td>-7.0</td>
<td>-</td>
</tr>
</tbody>
</table>

Without notch filters, the 40 kHz and 50 kHz bandwidths are too wide for a hard-limiter receiver for this out-band interference.

7.4.6 FINAL FILTER CHOICE

Input condition #1 (sky waves) excludes the 15 kHz bandwidth, while input condition #4 (200 kHz CWI) excludes the 40 kHz and the 50 kHz bandwidth. The results for the remaining filters are listed below:

When designing strictly on the given input conditions, a bandwidth of 35 kHz would be the best choice, considering the SNR$_{\text{out}}$ and the required notch depth. However, in real-life situations there are far more continuous-wave interferers than stated here. Thus it might make sense to choose the narrowest filter possible to reduce the required number of notch filters. From Fig. 7-9 the operating range of the notch filters, i.e. the region where the bandwidth does not provide the required SIR$_{\text{out}}$ - SIR$_{\text{in}}$ can be found. To read this value from the figures, we must use the conversion:

$$\text{INR}_{\text{out}} - \text{INR}_{\text{in}} < (\text{SNR}_{\text{out}} - \text{SNR}_{\text{in}}) - (\text{SIR}_{\text{out}} - \text{SIR}_{\text{in}})$$

where SNR$_{\text{out}}$ - SNR$_{\text{in}}$ is found from the table in section 7.4.2. SIR$_{\text{out}}$ - SIR$_{\text{in}}$ is greater than 30 dB, which follows from SIR$_{\text{in}}$ = 0 dB and from Fig. 7-6, or in table form:

<table>
<thead>
<tr>
<th>Bandwidth (kHz)</th>
<th>SIR$<em>{\text{out}}$ - SIR$</em>{\text{in}}$ (dB)</th>
<th>INR$<em>{\text{out}}$ - INR$</em>{\text{in}}$ (dB)</th>
<th>Notch tuning range (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-5.5</td>
<td>&lt; -35.5</td>
<td>74 - 134</td>
</tr>
<tr>
<td>25</td>
<td>-4.0</td>
<td>&lt; -34.0</td>
<td>71 - 141</td>
</tr>
<tr>
<td>30</td>
<td>-1.5</td>
<td>&lt; -31.1</td>
<td>68 - 146</td>
</tr>
<tr>
<td>35</td>
<td>+0.2</td>
<td>&lt; -29.8</td>
<td>66 - 151</td>
</tr>
</tbody>
</table>
Now there is a trade-off between the available SNR at the band-pass filter output and the frequency range to be protected by notch filters. In spite of the attraction of using a larger bandwidth to obtain a high SNR\textsubscript{out}, the associated wider frequency range increases the probability that more interferers will have to be notched. So, it will result in a greater number of notch filters and in a wider tuning range.

In this chapter it has been demonstrated how to select the most suitable bandwidth for a particular type and order of filter. However, an investigation into many types of filters showed that the optimum filter performance is not very distinct. Generally speaking, it seemed that the Gauss-type filters gave the best overall results. The question remains of what the best overall result is. In the European areas, with so many interfering stations, the selection will undoubtedly move towards the higher-order, narrow-bandwidth types, thereby sacrificing some performance with respect to sky-wave contamination resistance. In the United States, the final choice for the band-pass filter will in all likelihood be somewhat different.

8 SOME ASPECTS OF NOTCH FILTERS

As we have seen in the foregoing chapter, not all interfering signals can be rejected by the band-pass filter. Especially in the event of continuous-, or nearly continuous-wave interference, additional notch filters might improve the apparent signal quality considerably. However, just as with band-pass filters, notch filters more or less distort the envelope of the received Loran-C signal bursts. As the part of the frequency spectrum where notching is most needed varies with the position of the receiver, notch filters are generally manually or automatically tunable. That will introduce an unpredictable envelope distortion which may in turn cause malfunctioning of the cycle-selection circuits, as these circuits are calibrated for a very specific envelope of the burst. The amount of distortion depends on the number of applied notch filters, and on the bandwidth and the frequency of the notch. The distortion can be kept at harmless levels if we opt for a narrow bandwidth, while keeping the notch frequency out of the 90 - 110 kHz range, which contains 99% of the energy of the burst.

Notch filters may be realized in various techniques: passive or active, analog or digital, manual or automatical tuning, etc. However, every design should face the following constraints:

1 - Bandwidth at the mouth of the filter
2 - Notch depth
3 - Tuning
4 - Maximum signal-handling capability
5 - Noise properties
6 - Circuit complexity
7 - Power consumption
Digital filters would be very attractive from the viewpoint of VLSI. However, the relatively high notch frequencies require high clock rates, which is an adequate reason for power drain. And, unfortunately, such filters also have a rather limited dynamic signal-level range. However, digital filters can easily be tuned by a computer, and their frequency and phase responses are accurately predictable.

Analog filters, in either the passive or the active composition, are generally based on either LC or RC circuits. Because of the absence of inductors in the RC-type filters, they are somewhat attractive with respect to integration. However, the intrinsic low-Q properties of RC twin-T type filters limit their usefulness for our purpose. Therefore, such filters will not be discussed in this thesis.

What remains are the LC-type notch filters. Although basically of a passive nature, electronic circuits will be applied to facilitate impedance matching and eventually to improve the Q of the resonant circuit. In this chapter, special attention will be given to the dynamic range of such filters. This is of prime importance to maintain high tracking accuracy on weak Loran-C signals while simultaneously receiving strong Loran-C signals and interference.

8.1 FREQUENCY RESPONSE OF A BASIC NOTCH FILTER

LC-type notch filters can be designed with a single resonator or with multiple resonators. For reasons of miniaturization we will discuss only the single-resonator type. The LC resonator can be used either in a parallel- or a series-tuned mode, as is, in its basic configuration, depicted in Fig. 8-1. The two main parameters of a notch-filter response curve are the mouth bandwidth and the notch depth. The mouth bandwidth, \( B_m \), is defined as the bandwidth in Hz where the attenuation amounts to 3 dB with respect to the far-from-resonance response. The notch depth, \( ND = \frac{U_o}{U_{\text{min}}} \) at \( f_0 \), is also related to the far-from-resonance response. For the parallel type we find the following approximations:

\[
\begin{align}
  f_0 &= \frac{1}{2\pi \sqrt{LC}} \text{ Hz} \\
  B_m &= \frac{1}{2\pi R C} \text{ Hz} \\
  ND &= \frac{R_o}{R_p + R_o}
\end{align}
\]
And for the series type:

\[
\begin{align*}
\frac{\mathbf{f}_0}{\mathbf{B}_m} &= \frac{1}{(2\pi\sqrt{\mathbf{L}\mathbf{C}})} \text{ Hz} \\
\mathbf{B}_m &= \frac{(\mathbf{R}_p + \mathbf{R}_s)}{(2\pi\mathbf{L})} \text{ Hz} \\
\mathbf{ND} &= \frac{\mathbf{R}_s}{\mathbf{R}_g}
\end{align*}
\]  

\[\mathbf{R}_p \text{ and } \mathbf{R}_s, \text{ respectively, represent the combined losses of the inductor } \mathbf{L} \text{ and the capacitor } \mathbf{C}. \text{ The } \mathbf{Q} \text{ of the parallel resonator is then given by } \frac{\omega\mathbf{L}}{\mathbf{R}_p} \text{ or } \frac{1}{\omega\mathbf{C} \mathbf{R}_p}, \text{ and for the series resonator the } \mathbf{Q} \text{ is set by } \frac{\omega\mathbf{L}}{\mathbf{R}_s} \text{ or } \frac{1}{\omega\mathbf{C} \mathbf{R}_s}. \text{ We may now give, for } \mathbf{Q} \gg 1, \text{ the relation between } \mathbf{Q}, \mathbf{f}_0, \mathbf{B}_m \text{ and } \mathbf{ND}:
\]

\[
\mathbf{B}_m \cdot \mathbf{ND} = \frac{\mathbf{f}_0}{\mathbf{Q}} \text{ Hz}
\]

The notch filter can be tuned by varying either the \( \mathbf{L} \) or the \( \mathbf{C} \) or both. Low power drain limits the designer to the application of variable capacitance diodes. The relatively wide tuning range and the low operating frequency necessitates the use of high capacitance diodes. Such diodes have a rather low \( \mathbf{Q} \), in the order of 100. This forces the designer into a rather frustrating compromise when choosing the best performing combination of bandwidth and notch depth. If the notch depth should be, say, -20 dB (1/10) at 100 kHz, \( \mathbf{B}_m \) will be not less than 10 kHz. This clearly illustrates that the above simple basic circuit is not able to fulfill the required task. Either the losses of the tuned circuit are to be reduced significantly, or some form of bridging technique is to be utilized in order to acquire a better notch depth. However, we will first discuss some examples of basic notch-filter circuits before the so called electronic \( \mathbf{Q} \)-multipliers are introduced to decrease the \( \mathbf{B}_m \cdot \mathbf{ND} \) product.

\[\text{8.2 NOTCH-FILTER CIRCUITS}\]

Before going into the electronic circuitry, we briefly review the very distinct difference in maximum signal-handling capability between the series and the parallel-tuned filters. In the series types the signal to be rejected will be attenuated by shunting the signal source with low impedances, while in the case of parallel resonators, this rejection is accomplished by isolating the load from the source by high impedances. The signal levels over the varactor diode will, in the case of parallel resonance, not exceed the source voltages \( \mathbf{U} \). With series-tuned resonators, the varactor voltage will exceed the source voltage \( \mathbf{U} \) by about \( \frac{\omega_0 \cdot \mathbf{L}}{(\mathbf{R}_p + \mathbf{R}_s)} \) times. As the varactor is not allowed to handle ac voltages comparable in level to that of the dc control voltage, the series-type notch filter is less suited to handle high level input signals. However, this does not directly imply that the dynamic range of the series-type filter is worse than that of parallel-type filters. The dynamic range is defined as the difference between the noise level and the maximum signal-handling level of the filter. So, we should first examine the noise properties of both types of filters before any judgement can be made. Three basic circuits will be introduced, and for each circuit the noise and the signal-handling properties will be given.

The frequency spectrum at the 100 kHz band is rather crowded; therefore, we will generally apply more than one notch filter. To facilitate the cascading of a large number of filters it is practical to apply unity gain (far from the notch frequency) for each filter. As we have gainless filters, it seems, at least for these types of circuits, more practical to specify the equivalent noise voltages at the
output of the filter circuits, rather than at the input, as is customary.

Fig. 8-2a depicts a series-tuned notch filter driven from the emitter follower $T_1$, while transistor $T_2$ acts as an isolator of the output. Far from resonance of the tuned circuit, the complex impedance of this resonator is large compared to $R_1$. By neglecting the noise contribution from the dc current sources, we can write the spectrum of the noise-voltage source, $U_n$, at the output of the filter as:

$$S(U_n)_{out} = 4kT \cdot \frac{(R_g + R_1 + (r_{e1} + r_{e2})/2)}{2} \quad (8-8)$$

At the resonant frequency $\omega_0$, the noise power will be diminished because of the shunting of the source by $R_s$, which is usually very low. The noise spectrum of $U_n$ is given in Fig. 8-2b. The noise dip does not greatly affect the total noise power, as this dip is under practical conditions rather narrow.

The voltage over the varactor is called $U_c$. At resonance we may now determine the ratio of the source voltage $U_g$ and $U_c$:

$$|U_c/U_g| = 1/(\omega_0 C \cdot (r_{e1} + R_1 + R_2)) \quad (8-9)$$

which can be approximated by:

$$|U_c/U_g| = f_0/B_m \quad (8-10)$$

Example: $f_0=100$ kHz, $B_m=2$ kHz, then $|U_c/U_g|=50$

We assume that $U_c$ is the maximum allowed ac voltage over the varactor diode C. The maximum tolerable source voltage $U_g$ and the noise voltage $U_n$ can now be determined. The ratio of these two voltages is called the dynamic range DR and equals:

$$DR = U_c \cdot B_m/(4f_0 \cdot 4kT \cdot (R_g + R_1 + (r_{e1} + r_{e2})/2)) \quad (8-11)$$

The values for $U_c$ and $B_m$ are generally specified, and because of the limited freedom in selecting the capacitance $C$, the value for $R_1$ is also set. From (8-11) it can be observed that the only way to improve the DR is by minimizing $R_g$, $r_{e1}$ and $r_{e2}$. In the ultimate case, we find for DR:

$$DR = U_c \cdot B_m/(4f_0 \cdot 4kT \cdot R_1) \quad (8-12)$$
An example of a parallel-tuned notch filter can be seen in Fig. 8-3a. The resonant circuit is fully floating, what might be a disadvantage for the varactor control circuit. However, this configuration is attractive as far as the dynamic range is concerned. The voltage over the varactor $U_c$, at the resonance frequency, will not exceed the source voltage $U_g$. The ratio of $U_c$ and $U_g$ amounts:

$$|U_c/U_g| < R_p/(R_p + R_1)$$

(8-13)

If we continue to neglect the noise from the dc current source for $T_1$, the noise behavior far from resonance will mainly depend on the source resistance $R_g$ and the emitter resistances of the transistors $T_1$ and $T_2$:

$$S(U_n)_{\text{out}} = 4kT \cdot (R_g + (r_{e1} + r_{e2})/2)$$

(8-14)

At resonance, the noise spectrum $S(U_n)_{\text{out}}$ will show a peak equivalent to:

$$S(U_n)_{\text{out}} = 4kT \cdot (R_g + (r_{e1} + r_{e2})/2)$$

(8-15)

which can be approximated by:

$$S(U_n)_{\text{out}} = 4kT \cdot R_p$$

(8-16)

The frequency response of $S(U_n)_{\text{out}}$ is depicted in Fig. 8-3b. The noise contribution from the peak in the spectrum depends on the noise power bandwidth $B_n$ of the tuned circuit. The additional noise power at the output $S(U_n)_{\text{out}} \cdot B_n$ can be approximated by:

$$S(U_n)_{\text{out}} \cdot B_n = 4kT \cdot L \cdot (\pi f_0)^2$$

(8-17)

The additional noise contribution can be minimized by striving for low values for $L$. When this extra noise is neglected, the dynamic range for the parallel-type notch filter can be given by:

$$DR = U_c/4kT \cdot (R_g + (r_{e1} + r_{e2})/2)$$

(8-18)

The last circuit, given in Fig. 8-4, is based on parallel resonance and feedback. At resonance of the tuned circuit the feedback of the CE stage will increase, thereby reducing the gain. To provide unity gain of the filter, the values for $R_1$...
and \( R_2 \) should be equal. The notch depth \( ND \) is then given by:

\[
ND = \frac{R_1}{(R_2 + R_p)} \quad (8-19)
\]

This filter will also not suffer from voltage swing-up in the resonant circuit. The highest value for \( U_C \) equals \( U_g \). The spectrum of the output noise source, far from resonance, approximates:

\[
S(U_n)_{\text{out}} = 4kT \cdot \left[ \frac{1}{2} (r_{e1} + r_{e2}) + \frac{1}{2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \right] \quad (8-20)
\]

As \( r_{e1} \ll R_2 \) and \( r_{e2} \ll R_1 \), (8-20) will be approximately:

\[
S(U_n)_{\text{out}} = 4kT \cdot \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad (8-21)
\]

For \( R_1, R_2 \) and \( R \ll R_p \), (8-22) can be reduced to:

\[
S(U_n)_{\text{out}} = 8kT \cdot R_1 \quad (8-23)
\]

Equation (8-23) closely matches (8-21), and therefore an almost flat noise spectrum will be found. The dynamic range can now be determined:

\[
DR = \frac{U_C}{\sqrt{4kT \cdot (R_1 + R_2 + R_p)}} \quad (8-24)
\]

To give the reader an idea of the performance of these three circuits, some typical circuit data will be given. For all circuits, we will assume that \( R = 50 \) Ohms, that \( B_m = 2 \) kHz and that the collector currents of \( T_1 \) and \( T_2 \) are less than 0.5 mA. Next we assume a receiver noise bandwidth of 45 kHz and a maximal signal voltage over the varactor of 1 V.

For the circuit of Fig. 8-2 we give: \( L = 10 \) mH, \( C = 270 \) pF and \( R_1 = 100 \) Ohms. The DR then amounts to 101 dB. The circuit of Fig. 8-3 yields under the same conditions a DR of 131 dB. Finally, for the notch filter as depicted in Fig. 8-4 we used the following parameters: \( L = 1 \) mH, \( C = 2.7 \) nF and \( R = 29.5 \) kOhms, which yields a DR of 110 dB.

Until now, we have given no indication of our preference for either circuit. Although the circuit from Fig. 8-2a shows rather good noise properties, it suffers from the relatively poor, large signal-handling capability, which severely reduces the dynamic range. The circuit given in Fig. 8-3a could score very high if there were no floating resonator, as this makes it difficult to keep the parasitic capacitances of the resonator at a low level. It also hampers the application of electronic tuning. The last circuit, as shown in Fig. 8-4, seems a reasonable compromise with respect to power.
consumption, dynamic range and suitability for Q-multipliers.

The reader may wonder why the basic Q of the resonator in the circuits of Fig. 8-3a and 8-4 is apparently spoiled by adding external resistors. The main reason for this is to keep the bandwidth of the filter accurately under control. And this in turn is necessary for predicting the amount of envelope distortion of the Loran-C burst, although at the cost of some extra noise, and a reduction in the mouth bandwidth of the notch filter.

In the next section an example will be given of a notch filter whose performance is adequate for our Loran-C receiver. It combines a narrow mouth bandwidth and an adequate notch depth by applying an electronic Q-multiplier circuit.

Before moving on to the Q-multiplier, it should be mentioned that there are other, elegant ways to improve the notch depth drastically: the application of balanced types of filters such as the bridged-T filter. As these filters are extensively discussed in the literature [5-17], no further attention will be given to this method. Still, the reader is cautioned against concluding that the bridged-T filter is inferior to the method given below of compensating the positive loss resistance by an appropriate negative resistance.

8.3 Q-MULTIPLYING NOTCH FILTER

To prevent noticeable envelope distortion of the Loran-C signal, the mouth bandwidth $B_m$ of the filter should preferably not exceed 2 kHz. Further, as could be seen from section 7.4.6, a notch depth $ND$ of about 35 dB will be sufficient. With (8-7) we find a required Q of about 2000 for a 70 kHz notch frequency, and about 3600 for a 130 kHz notch frequency. Such high Q values for LC resonators cannot be obtained without electronic aids such as Q-multipliers.

![Fig. 8-5 Basic Q-multiplying circuit.](image)

The principle of Q-multiplication is addition of a negative resistance in series with the positive series loss resistance or in parallel with the positive parallel resistance of the resonator. Fig. 8-5 shows a simplified circuit of a Q-multiplier circuit configuration. The Q of the resonator will be infinite if $R_s$ equals the negative input impedance $Z$ of the amplifier. For infinite gain $A$ of the active amplifier, this condition is met if:

$$R_s = -Z_1 Z_2 / Z_3$$

(8-25)

When $R_s$ is less than the negative impedance of the amplifier, the circuit will oscillate. So, the condition for attaining a high Q while maintaining unconditional stability will strongly depend on the tolerances of the impedances $Z_1$, $Z_2$ and $Z_3$, and on the amplifier gain $A$. However, when the Q-multiplying capability is added to the notch filter circuit
of Fig. 8-4, the stability criterion is met much more easily. $R_s$ in Fig. 8-6 expresses the total loss resistance of the resonance circuit. The values for $Z_1$, $Z_2$ and $Z_3$ are now selected for maximum notch depth. This implies that the equivalent parallel loss resistance $R_p$ will be infinite. In contrast to the circuit in Fig. 8-5, the resonator is also loaded with the parallel impedance $R_2$. In the case of an infinite value for $R_p$, a lossless resonator, there will be no voltage drop over $R_2$, and is therefore apparently fully isolated from the resonator. However, as soon as $R_p$ tends to become negative, $R_2$ forms a very effective damping, thereby preventing any instability of the $Q$-multiplier. In spite of the newly acquired stability, errors in the compensation of the loss resistance $R_S$ will still definitely cause a reduction in the notch depth.

Although very high values for notch depths can be attained, they can only be achieved for rather limited frequency spans, by virtue of the frequency dependence of the loss resistances of the varactor, and to some extent also of the inductor. Therefore, the final notch filter as given in Fig. 8-7 has a tuning range of either 70 - 90 kHz, or of 110 - 130 kHz. In these limited frequency ranges, a notch depth in excess of 35 dB is obtained.

Finally, it should be remarked that we have limited the discussion on notch filters to the noise and the large signal-handling problems, as we believe these are so often overlooked in many designs.
9 HARD-LIMITER SENSOR

Until now all received signals have been handled in a linear way. Although there might be a slight difference in selecting the band-pass filter and the associated notch filters, the design of a purely linear concept will not significantly differ from that of our hard-limiter receiver. However, from here on, the linear and the hard-limiter paths diverge completely. In addition, we will not have any other information about our blend of signals than its polarity. Therefore, before all amplitude information is killed in the hard-limiter stages, we must derive, from the envelope of the incoming burst, information about which zero crossing of the signal is actually tracked. (See chapter 4, and more specifically figures 4-8a and 4-8b.) Having transformed the envelope information into polarity information, we are from now on free to kill the envelope. The polarities of the two cycle samples now contain all the basic information needed for a correct cycle selection procedure.

An ideal hard limiter will correctly determine the polarity of the signal samples, irrespective of their value. The extreme amplitude range of the received signals necessitates large gains of the hard limiter. As the effective gain changes for varying input signals, care must be taken to keep delay variations in the amplifier stages within acceptable limits. Otherwise, signal-level variations will be transformed into apparent time shifts of the zero crossing at the output of the limiting amplifier. In the next sections we will analyze the effects on zero-crossing accuracy, or tracking error, caused by a non-ideal hard limiter and a non-ideal digital polarity sampler. As our aim is to design a low-power integrated limiter, we should recognize the influences of noise, dc-offset, signal level...
variations and power consumption on the tracking accuracy.

The basic hard-limiter tracking system is depicted in Fig. 9-1. The D-type flip-flop DFF takes samples from the limited input signals at the estimated position of the zero crossing. The following non-ideal properties of the limiter and the DFF set bounds to the ultimate performance of the tracker:

* $-A$ = finite gain of the limiter
* $L_{\text{off}}$ = dc-offset at the input of the limiter
* $D_{\text{off}}$ = dc-offset at the input of the DFF

We will subsequently analyze what influence these limitations of real-world electronics have on the tracking accuracy.

### 9.1 TRACKING ERROR DUE TO LIMITER WITH FINITE GAIN AND TO OFFSET IN DIGITAL SAMPLER

We are interested in the polarity-detection accuracy of the combination of hard limiter and D-type flip-flop. Fig. 9-2 represents input and output signals of an ideal hard limiter. Samples are taken at the sample-point position, denoted by SPP. The transfer function of the limiter is
depicted in Fig. 9-3. The navigation signal \( U_s \) is considered to be polluted by Gaussian noise with the rms value \( \sigma \). The probability density of \( U_1 \) in the SPP is shown in Fig. 9-4. \( U_{spp} \) is the amplitude value of \( U_s \) at the SPP. If the momentary value of \( U_1 \) exceeds \( B/A \), the limiter outputs \( -B \), and the DFF will generate a logical "one". A logical "zero" will be the result for \( U_1 < -B/A \). For these two input-signal conditions, the hard limiter produces solid "ones" and "zeroes", and will then also correctly be sampled by the D-type flip-flop, whatever the value of \( D_{off} \) might be. It is assumed, of course, that the logical swings of the hard limiter and the flip-flop match.

However, for \( -B/A < U_1 < B/A \), the limiter acts as a linear amplifier with the gain \( -A \), in which case there are no definite "ones" or "zeroes" at the input of the DFF. So, the dc-offset at the input of the flip-flop might substantially influence the polarity-detection reliability, which is undesirable.

When \( \sigma \gg B/A \) the mean inverted output level of the DFF, \( U_Q \), can be written as:

\[
U_Q = \frac{B}{\sqrt{2\pi}} \int_{-B/A}^{B/A} \exp\left(-\frac{(U_1 - U_{spp})^2}{2\sigma^2}\right) dU_1
\]

\[
U_Q = \frac{B}{\sqrt{2\pi}} \int_{-B/A}^{B/A} \exp\left(-\frac{(U_1 - U_{spp})^2}{2\sigma^2}\right) dU_1
\]

\[
U_Q = \frac{B}{\sqrt{2\pi}} \int_{-\infty}^{-B/A} \exp\left(-\frac{(U_1 - U_{spp})^2}{2\sigma^2}\right) dU_1
\]

\[
U_Q = \frac{B}{\sqrt{2\pi}} \int_{-B/A}^{B/A} \exp\left(-\frac{(U_1 - U_{spp})^2}{2\sigma^2}\right) dU_1
\]

\[
U_Q = \frac{B}{\sqrt{2\pi}} \int_{-B/A}^{B/A} \exp\left(-\frac{(U_1 - U_{spp})^2}{2\sigma^2}\right) dU_1
\]

\[
U_Q = \frac{B}{\sqrt{2\pi}} \int_{-\infty}^{-B/A} \exp\left(-\frac{(U_1 - U_{spp})^2}{2\sigma^2}\right) dU_1
\]

For \( U_{spp} \ll |B/A| \), the first-order approximations for the integral expressions are reasonably accurate, yielding:

\[
1/(2\pi) \int_{-B/A}^{B/A} \exp\left(-\frac{(U_1 - U_{spp})^2}{2\sigma^2}\right) dU_1 \approx 0.5 + 1/(2\pi) \cdot (U_{spp} - B/A)/\sigma
\]

or:

\[
U_Q \approx (2/\pi) \cdot (U_{spp} - D_{off}/A) \cdot B/\sigma
\]

When the polarity decision algorithm of the signal processor is based on the average value of the polarity samples, the phase tracker will act properly as long as \( U_{spp} \gg D_{off}/A \). Assuming that the timing resolution \( \Delta t \) is infinitely small, we may conclude from (9-3) that the amount of \( D_{off} \) never sets a minimum for the final tracking error SPP-ZC, as \( D_{off} \) can be
largely compensated by making the limiter gain high. Fig. 9-5 shows the minimum values of $U_{\text{sp}}$ for the correct polarity decisions as a function of limiter gain $-A$ and DFF offset voltage $D_{\text{off}}$.

We use expression (9-3) to calculate as well the error made in the ZC detection for a given value of $D_{\text{off}}$ and limiter gain $-A$. The system will now track at a distance of $\Delta t/T$ from ZC, where $U_{\text{sp}}$ equals $D_{\text{off}}/A$. The resulting tracking error is given by:

$$\Delta t/T = 1/(2\pi) \cdot \arcsin\left(\frac{D_{\text{off}}}{U_{\text{sp}} A}\right)$$

(9-4)

To summarize, we note that $D_{\text{off}}$ and the maximum allowed tracking error $\Delta t/T$ determine the minimum-required hard-limiter gain $-A$. The noise level is irrelevant at this point. However, $L_{\text{off}}$, the dc-offset at the input of the limiter, also affects the tracking accuracy. This error can be calculated, as will be shown in the next section.

9.2 TRACKING ERROR DUE TO DC-OFFSET AT LIMITER INPUT

Let us consider the circuit in Fig. 9-6 in order to analyze the $L_{\text{off}}$ effect. The offset of the hard limiter is represented by the voltage source $L_{\text{off}}$. The network $R_{1}, C_{1}$ and $R_{2}$ forms a dc-feedback loop for the bias input current. The input resistance and the gain of the limiter are assumed to be infinite. The time constant $R_{2}C_{1}$ is very large compared to the signal frequency $^{1}$.

The presence of $L_{\text{off}}$ results in a mean input voltage $U_{\text{i}}$ which differs from zero. As a consequence, the duty cycle of the output signal $U_{O}$ does not equal exactly 50%. As the average value of $U_{\text{i}}$ is fed back to the input of the hard limiter, the effect of the input offset voltage is reduced, but not completely canceled. Consequently, the ZC of the navigation signal will be offset by $\Delta t$ at the input of the dc-offset free limiter (Fig. 9-7).

The tracking error $\Delta t/T$ can be given by:

$$\Delta t/T = 1/(2\pi) \cdot \arcsin\left(\frac{\bar{U}_{\text{i}}}{U_{\text{sp}} A}\right)$$

(9-5)

Further, because of the feedback loop and the assumption that the voltage drop over the feedback resistors due to the dc input current can be neglected, we can write:

![Fig. 9-6 Hard limiter with dc feedback circuit.](image)

![Fig. 9-7 Tracking error $\Delta t$ due to $L_{\text{off}}$.](image)
Studying Fig. 9-7, we see:

\[ U_o = -4B \Delta t/T \]  

(9-7)

From (9-5) and (9-7) we derive:

\[ U_l = \hat{U}_s \sin(\pi U_o / 2B) \]  

(9-8)

or for small values of \( \pi U_o / 2B \):

\[ U_o = -\pi \hat{U}_s U_o / 2B \]  

(9-9)

With (9-6) and (9-9) we find:

\[ U_l = L_{off} / (1 + 2B/\pi \hat{U}_s) \]  

(9-10)

and for \( \hat{U}_s \gg U_l \):

\[ U_l = \frac{L_{off}}{2B} \]  

(9-11)

Formula (9-11) expresses the apparent shifting of the ZC due to the limiter offset for a given signal amplitude \( U_s \), while Fig. 9-8 demonstrates the tracking error \( \Delta t/T \) as a function of signal amplitude \( U_s \) with \( L_{off} \) as parameter. These graphs are valid for \( D_{off} = 0 \), \( A = -\infty \), and \( B = 0.5 \) V. We see that the tracking error is rather low, even for very small input signals. However, in real life, the input signal will almost continuously change as we consecutively receive signals from stations at different distances from the receiver. The weaker signals may be endangered if the stronger signals have a relatively large duty cycle. Joosse and van Willigen gave in [5-13] detailed tracking-error calculations for multiple-input signal conditions.

9.3 TRACKING ERROR DUE TO LIMITER WITH OFFSET IN THE PRESENCE OF NOISE

We will now take a closer look at the effects of limiter offset on the polarity determination of a signal \( U_s \) corrupted by Gaussian noise with an rms value \( \sigma \). We extend the circuit of Fig. 9-6 with a noise source and a sampling DFF without dc-offset (Fig. 9-9). Fig. 9-10 depicts the momentary
Fig. 9-10 Momentary amplitude probability of $U_1$ as a function of $U_{\text{smom}}$.

The momentary amplitude probability of $U_1$, where $U_{\text{smom}}$ represents the momentary value of the navigation signal $U_s$. In more mathematical form:

$$p(U_1)_{\text{mom}} = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{(U_1 - U_{\text{smom}} - L_{\text{off}} - U_s)^2}{2\sigma^2}\right)$$  \hspace{1cm} (9-12)

The mean value of $U_1$ is fed back to the input of the limiter. Because we want to know how much $L_{\text{off}}$ is canceled, we must determine the average exceedance probability $p(U_1 > 0)$. As $U_{\text{smom}}$ represents only the momentary value of the input signal $U_s$, we should find the average value of the exceedance probability $p(U_1 > 0)$ over a whole number of periods of $U_s$, thus:

$$p(U_1 > 0) = \frac{1}{nT} \int_{0}^{nT} \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{(U_1 - U_{\text{smom}} - L_{\text{off}} - U_s)^2}{2\sigma^2}\right) \cdot dU_1 \cdot dT$$  \hspace{1cm} (9-13)

And if $|U_s + U_1| < \sigma$, we find for $U_1$ [5-13]:

$$V_0 = L_{\text{off}} / (1 + \sqrt{2\pi} \cdot B / \sigma)$$  \hspace{1cm} (9-14)

This result becomes very interesting when we compare (9-14) and (9-10). We see that $U_s$ is now replaced by $\sigma / \sqrt{2\pi}$. The polarity detection of the Loran-C signal is somewhat easier to understand than the determination of the average value of $U_1$. The exceedance-probability curve is not "riding" on top of a moving signal vector, but is positioned at a steady position SPP of the signal sine wave: the sampling rate is synchronous with the signal $U_s$. Fig. 9-11 shows $p(U_1)_{\text{spp}}$ as a function of $U_1$. The probability-density curve of the noise is positioned symmetrically around $U_1 + U_{\text{spp}}$, where $U_{\text{spp}}$ is the signal level at the sample-point position SPP.

Again, it is the tracking algorithm which makes us curious about the average value of $U_1$. This value can be expressed by:

$$V_0 = B / \sqrt{2\pi} \int_{0}^{\infty} \exp\left(-\frac{(U_1 - U_{\text{spp}})^2}{2\sigma^2}\right) \cdot dU_1$$

$$-B / \sqrt{2\pi} \int_{-\infty}^{0} \exp\left(-\frac{(U_1 - U_{\text{spp}})^2}{2\sigma^2}\right) \cdot dU_1$$  \hspace{1cm} (9-15)

Fig. 9-11 Amplitude probability of $U_1$ at SPP as a function of $U_1$. 

\[\text{Fig. 9-10 Momentary amplitude probability of } U_1 \text{ as a function of } U_{\text{smom}}.\]
If once again $\sigma \gg |U_0 + L_{\text{off}} + U_s|$ and $B \gg \sigma$, we may simplify (9-15) to:

$$U_Q = L_{\text{off}} + \sqrt{(2/\pi) \cdot B \cdot U_{\text{spp}} / \sigma}$$  \hspace{1cm} (9-16)

Clearly, as long as the polarity of $U_{\text{spp}}$ equals the polarity of $U_Q$, the signal processor will operate properly. This sets the minimum value for $U_{\text{spp}}$ for proper handling. Or with (9-16) we may write:

$$U_{\text{spp}} > \sqrt{(\pi/2) \cdot \sigma \cdot L_{\text{off}} / B}$$  \hspace{1cm} (9-17)

This lower limit for $U_{\text{spp}}$ also sets the tracking error $\Delta t/T$:

$$\Delta t/T = -1/2 \sqrt{(2\pi) \cdot \sigma / U_s \cdot L_{\text{off}} / B}$$  \hspace{1cm} (9-18)

Finally, full cycle-slip or signal killing will occur if $U_s < U_1$ or:

$$U_s < \sqrt{(\pi/2) \cdot \sigma \cdot L_{\text{off}} / B}$$  \hspace{1cm} (9-19)

Fig. 9-12 depicts the resulting tracking error $\Delta t/T$ for $B=0.5$ V as a function of $U_s/\sigma$ with $L_{\text{off}}$ as parameter.

As already stated in section 9.2, the tracking error will worsen as well if more than one single Loran-C signal is received with substantial differences in signal levels. The interested reader is referred to [5-13].

Finally, it must be said that the negative influence of $L_{\text{off}}$ can be largely reduced by applying polarity-chopping techniques [3-3]. The phase coding of the Loran-C signals therefore already partially reduces the tracking error.

### 9.4 Hardware Realization

In the foregoing sections we investigated the properties of the hard limiter as a signal polarity sensor under very poor signal-to-noise ratios and signal-to-offset conditions. We have seen that a high tracking performance may be expected when the limiter gain is high, the dc-offset is small and the logic swing at the limiter output is large.

In order to be able to estimate the required gain, we must know the expected signal levels and the offset of the DFF. The maximum input-signal level which the proposed hard-limiter circuit can process, without noticeable saturation effects in the bipolar silicon transistors of the input stage, amounts to about 300 mV. The Loran-C signal levels might show a dynamic range of 120 dB. So, when the strongest signal equals 300 mV, set by the active antenna and the band-pass filter, the weakest signal will then amount to 300 nV$\text{rms}$. Further, CMOS (low power!) DFF's show typical offsets of 1 V in the trigger level. Assuming that the
weakest signals must be correctly detected at an SPP of 25 ns from the ZC, we can derive from (9-4) a required gain of 160 dB. This high gain is needed for correct tracking under low SNR conditions.

In anticipating the practical realizations of hard limiters, we face an additional problem: the transition from linear to non-linear operation of the various amplifier stages. This transition causes variations in the time constants of the amplifier circuit, depending on the input-signal amplitude. This delay variation, called $\tau_{\text{diff}}$, depends on transistor and interconnection capacitances, emitter currents, collector voltages and the signal-level variation.

The final electronic realization of the hard limiter is intended to have less than a 100 ns delay variation (a realistic value from the perspective of the Minimum Performance Standards) for an input-signal variation of 120 dB. This includes effects due to the offsets in the hard limiter and the DFF. Because of low power requirements, the total power consumption of the limiter circuit with a gain of 160 dB at 100 kHz should not exceed 10 mW.

The hard-limiter design is based on the buffered current switch as shown in Fig. 9-13. This circuit is favored because of the following considerations:

* The delay variation can be kept low by carefully selecting the bias currents $I_0$ and $I_1$.
* A supply voltage of 2 V is sufficient, which helps to minimize power consumption. In spite of this low supply voltage, signals up to $1 \, \text{V}_{\text{pp}}$ can be handled without transistors driving into collector saturation.

By selecting a supply voltage of 2 V and $R \cdot I_0$ equal to 0.6 V, many stages can be cascaded without unfavorable dc-level shifts.

The selection of $R \cdot I_0 = 0.6 \, \text{V}$ implies a maximum voltage swing of $0.6 \, \text{V}_{\text{pp}}$. The linear gain $G$ of a single amplifier stage is then expressed by:

$$G = R/r_e = I_o \cdot R/0.0054 = 11 \, (\approx 21 \, \text{dB}) \quad (9-20)$$

As a total linear gain of 160 dB is required, we should cascade eight equivalent stages.

We are now interested in $\tau_{\text{diff}}$, which is the difference between the large signal delay $\tau_{\text{IS}}$ and the small signal delay $\tau_{\text{SS}}$. Analyzing the small-signal linear model of the circuit yields for $\tau_{\text{SS}}$ [5-13]:

$$\tau_{\text{SS}} = R(C_{\text{cs}} + 2 C_{b'c}) \cdot 0.027/I_1 \cdot (C_{b'e} + 13 C_{b'c}) \quad (9-21)$$

The first term in [9-21] represents the delay in the

![Fig. 9-13 Basic circuit of limiter stage with dual dc feedback loop over n stages.](image-url)
Under the restriction that $2 \times I_1 < I_0$ we may write the following expression for $\tau_{1s}$:

$$I_1 \times \tau_{1s}/(C_{b'c} + C_{cs}) = R \times I_0 \times \exp(\tau_{1s}/(R(C_{cs} + 2C_{b'c}))$$  \hspace{1cm} (9-22)

From [9-21] and [9-22] $\tau_{\text{diff}}$ can now be calculated via numerical methods and the results are depicted in Fig. 9-14. For verification purposes, we also derived $\tau_{\text{diff}}$ by using the network-analysis program SPICE. The resulting values, shown in Fig. 9-15 are in reasonable agreement with the graphs from Fig. 9-14. The values for the parasitic capacitances of the transistors used are:

* $C_{cs} = 1.7 \text{ pF}$
* $C_{b'c} = 0.4 \text{ pF}$
* $C_{b'c} = 0.5 \text{ pF}$.

Let us assume that $\tau_{\text{diff}}$ should not exceed 50 ns for a 120 dB input-signal variation. Fortunately, this does not mean that six limiter stages are moving from a linear to a limiting operation, as the input noise will cause continuous limiting of the final stages. Although the source of the noise is immaterial, the noise level will never go below the equivalent noise input voltage from the limiter. This noise-floor voltage is given by:

$$\bar{u}_n = \sqrt{(4kT \times (2r_b + r_c + R_s) \times \Delta f)}$$  \hspace{1cm} (9-23)

where $r_b =$ base resistance of input transistors and $R_s =$ source resistance.

The process used to manufacture the hard-limiter chip, leads to $r_b = 300 \text{ Ohm}$ and $R_s = 1 \text{ kOhm}$. So we may encounter an input-noise voltage of 5 $\mu V$ for values of $I_0$ which are not too low and a noise bandwidth of 1 MHz. As the maximum input-signal level amounts to 300 $mV_{\text{rms}}$, the linear-to-limiting transition point shifts over 80 dB through the amplifier stages. This implies that the maximum allowed delay...
variation of 50 ns may be spread over four stages, which means that $\tau_{\text{diff}}$ should not exceed 12.5 ns! From figures 9-14 or 9-15 we may now select values for $I_0$ and $I_1$ that give the required $\tau_{\text{diff}}$ for the lowest power consumption: 160 $\mu$A for $I_0$ and 20 $\mu$A for $I_1$. 

Fig. 9-17 Photograph of chip containing the circuit of Fig. 9-16.

Fig. 9-18 Circuit diagram of hard limiter with digital sampler and shift register.
To avoid intolerable parasitic on-chip capacitive feedback, two cascaded limiter chips are applied with an 80 dB linear gain per chip. The complete circuit diagram is depicted in Fig. 9-16, which also shows an additional level shifter to facilitate matching to the levels of the digital circuits. Finally the picture in Fig. 9-17 gives an impression of the chip, manufactured in a standard bipolar process in the IC-Workshop of the Delft University of Technology. The total power consumption amounts to 2 mA at 2 V and 0.16 mA at 5 V for the complete limiter.

The two cascaded chips are mounted together with the digital sampler and an 8-bit shift register on a thin-film ceramic substrate (Fig. 9-18 & Fig. 9-19). The big blocks visible on the thin film are the decoupling capacitors in the dc-feedback loops. The circuit is mounted in a shallow metal box to avoid spurious capacitive feedback, which will inevitably introduce a signal-level-dependent phase delay.

![Fig. 9-19 Photograph of thin film containing the circuit of Fig. 9-18.]

10 SIGNAL PROCESSING BY MEANS OF SEQUENTIAL DETECTION

In the foregoing chapters we have dealt with the hardware part of the Loran-C receiver. The received signal is amplified, band-pass filtered and consecutively sampled at precisely predetermined positions in time. We now come to the software part of the receiver set. This software must accurately estimate, from a large series of signal samples, the position of the zero crossings at 25 μs after the beginning of the various Loran-C bursts. All the collected samples have one thing in common: they are usually extremely contaminated by noise and interference. In those cases in which the contamination is not systematic, the quality of the recovered Loran signals may be improved through integration. This, of course, occurs at the expense of the information bandwidth. The needed amount of signal integration depends on three items: the pre-detection quality of the received Loran-C signals, the minimum required quality of the recovered signal and the reponse time of the signal averager. As these three items are mutually related, only two of the three variables are open to choice.

The input signal in non-stationary receivers varies almost continuously; consequently, it is attractive to let the software filter also continuously adapt to these varying input-signal conditions.

To keep the power consumption of the signal processor at a low level, the signal-averaging software should be simple. Having as few as possible computer instructions per unit of time aids this low-power design goal, as this tends to reduce the clock frequency of the computer. The remaining question is whether algorithms which meet these demands can be developed.

For our purpose we can make fruitful use of a
signal-processing technique developed by Wald [3-2], generally known as "sequential detection". A sequential-detection signal detector collects just as many signal samples as are needed to give an estimate of either the polarity or the presence or absence of a signal. This estimate is performed with a predetermined accuracy for a predefined minimum signal-to-noise ratio of the input signal. The number of samples to collect, needed to achieve such an estimate, depends on the SNR of the input signal. Or, in other words, the response time of the estimator varies with the SNR of the signals to be processed. This adaptation to the quality of the input signals is a very welcome attribute to navigation receivers. It may prevent receivers from going bananas during temporary signal reductions, or even signal losses. On the other hand, high-quality signals make a fast-reacting "navigator".

Van der Wal and van Willigen [3-1] suggested how to apply this sequential-detection technique to a first-order tracking loop of a Loran-C receiver. Although this applied form of sequential detection performs well for stationary purposes, it must be extended in order to anticipate the physical velocity of a receiver. However, before discussing such second-order tracking loops, a brief explanation of sequential detection will be given.

10.1 SIGNAL-POLARITY-DETECTION PROPERTIES

The tracking of a zero crossing (ZC) of the received Loran-C signals is in the case of hard-limiter receivers based on the determination of the polarity of the signal at the sample-point position (SPP). Let's assume a negative-going slope of the signal at 25 μs after the start of the burst. Finding a positive signal at the SPP will then lead us to conclude that our estimated position of the ZC is too early. The digital timing circuits of the receiver then start to take samples of the burst some time later, e.g. one resolution step of the timer. The minimum resolution time step is generally equal to the inverse value of the clock frequency of the timer. If a positive signal is found at this new SPP again, the SPP must be delayed once more. This process is continued until a negative signal is detected. The ZC is now ahead of the SPP, so that the SPP must be advanced in time. This first-order ZC-tracker will find in the case of a noise-free sine wave the zero crossing with an accuracy of better than 50% of the minimum time step of the digital timer.

However, owing to the presence of noise, it may happen that a step is made in the wrong direction of time. The detector moves the SPP away from the ZC instead of moving towards it. Fig. 10-1 represents the probability density of the Gaussian noise, with its mean value offset by the Loran-signal vector S. The probability that a correct polarity observation will be performed is given by the area under the probability density curve on the right side of the vertical axis, and is known as the exceedance probability.
\( P_{\text{obs}} \). It is, of course, assumed that a perfect hard limiter without offset is applied. The area under the curve on the left side of the vertical axis represents the probability that a faulty polarity will be detected is denoted by \( q_{\text{obs}} \). The relation between \( P_{\text{obs}} \) and \( q_{\text{obs}} \) is given by:

\[
P_{\text{obs}} + q_{\text{obs}} = 1
\] (10-1)

We define the observation likelihood ratio as:

\[
L_{\text{obs}} = \frac{P_{\text{obs}}}{q_{\text{obs}}}
\] (10-2)

The combination of the hard limiter and the low occurrence of high noise peaks justifies our exclusive use of the Gaussian part of the atmospheric-noise model. However, the noise level is then effective decreased by 8 dB with respect to the atmospheric-noise level. Therefore, it must be emphasized that for all of the following SNR calculations, the Gaussian distribution is exclusively applied! The exceedance probability \( P_{\text{obs}} \) is then given by:

\[
P_{\text{obs}} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{t^2}{2}\right) dt
\] (10-3)

To demonstrate the order of magnitude of \( P_{\text{obs}} \) for realistic SNR's of received Lorans-C signals, which seldom exceeds 0 dB, Fig. 10-2 shows \( P_{\text{obs}} \) as a function of the SPP in relation to the ZC of the sine wave with the SNR as parameter. It clearly shows that when approaching the ZC, the \( P_{\text{obs}} \) almost goes down to 0.5, even for relatively strong signals. This means that the zero-crossing tracker will perform very noisily, especially around the zero crossing of the signal. The tracking behavior can also be given as a Markov chain. Fig. 10-3 shows the transfer probabilities for correct and incorrect time jumps of the SPP at various discrete positions in time. Moving further away from the ZC also means that \( L_{\text{obs}} \) increases, which implies a higher probability that a correct time step will be effectuated. The distribution function can be calculated from the transfer probabilities, as given in the Markov chain in Fig. 10-3. For a 0 dB SNR, the sigma then equals 0.27 μs. This is equivalent to about 100 meters.

So far, the tracking error has been determined by a single observation of the signal at the SPP. And as a consequence, the decision to move the SPP to a more correct position is also based on a single observation. This means that the probability of a correct decision equals the probability of a
To attain a decision reliability exceeding the observation reliability, a batch of samples should be integrated. Integration may simply be performed by means of an up-down counter, abbreviated as UDC. A positive observed signal increments the counter, while a negative observation decrements the UDC. The question now arises of how many samples are to be taken before a decision can be made about the polarity of the sampled signal. And what is then the likelihood ratio of the decision? Two very distinct ways to reach a polarity decision must first be introduced:

1. Take $n$ samples, and determine, after having integrated these $n$ samples in a UDC, the position of the counter. If the final position exceeds the starting position of the UDC, it is decided that the signal is positive; otherwise the signal is negative.

2. Apply two thresholds at equal distances from the starting position of the UDC. Then take as many samples as are needed to reach either the positive or the negative threshold. If the positive threshold is passed, the signal is considered to be positive, and in the case of negative threshold passing, the signal is considered negative. When no threshold is passed, no decision is made about the polarity of the sampled signal. More samples are needed to afford a polarity estimate. This process is the so-called sequential detection.

The fixed-sample-number method (1) does not anticipate the SNR of the incoming sampled signals. It always processes a batch containing a fixed number of samples. If the SPP is close to the ZC, the SNR will be very low. Therefore, rather many samples may be required before a polarity decision with an adequate likelihood ratio can be made. However, in the case of a non-determined difference between the clock frequencies of the transmitter and the receiver, the SPP and the ZC will drift relative to each other. This clock error may originate from physical clock errors or from the Doppler velocity of the receiver. The large amount of samples needed to perform a "noise free" ZC tracking now brings with it the highly undesirable risk of cycle slip under clock-error conditions. A second drawback of the fixed-sample-number signal averager is its poor step response. When the SPP is far from the ZC, e.g. near the top of the sine wave, the SNR at that particular SPP is relatively high. Thus, the number of samples taken far exceeds the required amount. At a rather
unimportant time position in the SPP, a superfluous likelihood ratio of the polarity decision $L_{\text{dec}}$ is obtained, which unnecessarily slows down the lock-in procedure of the ZC tracker. The last, but certainly not the least point to be mentioned is that in the case of temporary signal loss, the tracking algorithm will still take decisions about the polarity of a non-existent signal, causing a real random walking SPP! Such situations easily occur while driving through tunnels or while encountering high amounts of radio interference or precipitation static.

The second method, where a variable number of samples is taken to estimate the polarity of the signal samples, does not have the disadvantages of the first method. The higher the SNR of the collected samples, the faster the decision about the polarity is taken. It means that a high likelihood ratio of the decision can be combined with a good step response of the tracking loop, thereby significantly reducing the risk of cycle slip.

To understand the principles of sequential detection, it is necessary to explain some integration mechanisms of noisy signal samples and the properties of the consecutive decision making [3-1].

To facilitate the explanation of sequential detection, Fig. 10-4 shows the position of the up-down counter while integrating the received signal samples. If no noise is present, the position of the UDC will be identical to the number of samples collected, assuming that the counter was initially set to zero. If only noise is received, the average UDC position will be zero. However, by virtue of the very-low frequency spectral contents of the noise, the counter will sooner or later exceed one of the thresholds. Integrating samples with a specific SNR makes the average growth of the UDC position predictable. The direction of the slope depends on the polarity of the signal, while the slant is related to the SNR. The average counter increase or decrease per sample amounts to $p_{\text{obs}} - q_{\text{obs}}$. So, the expected number of samples, $E_{SN}$, needed to reach the relevant threshold $T$ or $-T$ equals:

$$E_{SN} = \frac{T}{(p_{\text{obs}} - q_{\text{obs}})}$$

When the threshold $T$ is passed, a decision is made about the polarity of the received signal samples. For convenience we may, just as in the case of observation reliability, introduce the likelihood ratio of the correct decision $p_{\text{dec}}$ and the incorrect decision $q_{\text{dec}}$, or:

$$L_{\text{dec}} = \frac{p_{\text{dec}}}{q_{\text{dec}}}$$

As could be expected, $L_{\text{dec}}$ is larger than $L_{\text{obs}}$. The relation
between \( L_{\text{dec}} \), \( L_{\text{obs}} \) and the threshold \( T \) is given by [3-1]:

\[
L_{\text{dec}} = (L_{\text{obs}})^T
\]

(10-6)

With these three simple formulas, we may now derive the decision performance, expressed in terms of speed and reliability, of the sequential polarity detector.

Before discussing the properties of signal averaging any further, we will demonstrate the basic functioning of a zero-crossing tracking loop by citing a Pascal program representing the tracking algorithm:

```
PROGRAM FirstOrderTrackingLoop(Input,Output);

{ SignalSample = binary output from hard limiter;
a logical '1' stands for a positive signal,
while a logical '0' means a negative signal.
Sum = position of up-down counter
SPP = Sample Point Position
dSPP = resolution of digital timer circuit }

VAR Sum,Threshold,SPP,dSPP: Integer;
   SignalSample : Boolean;
BEGIN
Sum:= 0;
WHILE signal samples in storage DO
BEGIN
   IF SignalSample = 1 THEN
      BEGIN
         Sum:= Sum + 1;
         IF Sum = +Threshold THEN
            BEGIN
               Sum:= 0;
               SPP:= SPP + dSPP;
            END;
      END
   ELSE
      BEGIN
         Sum:= Sum - 1;
         IF Sum = -Threshold THEN
            BEGIN
               Sum:= 0;
               SPP:= SPP - dSPP;
            END;
      END;
   END;
END.
```

Although the tracking loop has now been described, the performance of this algorithm has not yet been determined. This can be best expressed when the probability distribution of all the relevant sample-point positions is resolved. The standard deviation is then from this probability distribution derived, which is a meaningful indication of the zero-crossing tracking performance.

To find this deviation, we must for all relevant SPP's determine the probability that at that particular SPP either a correct or an incorrect decision will be made, or that no decision will be made at all. This is illustrated in Fig. 10-5, which closely resembles Fig. 10-3. However, there is now the added probability of staying at a particular SPP. This needs further explanation.

In the foregoing sections it was already mentioned that after some time, depending on the SNR of the samples and the level of the thresholds, one of the thresholds will be passed. That might be either the correct or the incorrect threshold. It would be interesting to know how many samples on the average
The relation between $p_{\text{dec}}$ and $q_{\text{dec}}$ is given by:

$$p_{\text{dec}} + q_{\text{dec}} = 1 \quad (10-8)$$

From (10-6) and (10-8) the following can be derived:

$$p_{\text{dec}} = \frac{L_{\text{dec}}}{(L_{\text{dec}} + 1)} \quad (10-9)$$

Now all probabilities, as denoted in Fig. 10-5, are available for the derivation of the standard deviation of the SPP. However, the results obtained depend also on the resolution of the timer, i.e., the time difference between two successive SPP's. This can be seen from the figures given in Table 10-1. It is assumed that the ZC is situated equally far from two successive SPP's. The standard deviations are derived for timer resolutions varying from 200 to 25 ns. The chosen values for the SNR are typical for European conditions.

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>+12</td>
<td>210</td>
<td>152</td>
<td>116</td>
<td>102</td>
<td>100</td>
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<td>391</td>
<td>276</td>
<td>198</td>
<td>146</td>
<td>114</td>
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<td>-12</td>
<td>785</td>
<td>550</td>
<td>385</td>
<td>273</td>
<td>197</td>
<td>146</td>
<td>114</td>
<td>101</td>
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<tr>
<td>-24</td>
<td>1192</td>
<td>1023</td>
<td>781</td>
<td>548</td>
<td>384</td>
<td>273</td>
<td>197</td>
<td>145</td>
</tr>
</tbody>
</table>

Table 10-1a Standard deviation in ns of the probability distribution of the SPP as a function of SNR and threshold. Timing resolution = 200 ns. ZC is at 100 ns from SPP.
Table 10-1b Standard deviation in ns of the probability distribution of the SPP as a function of SNR and threshold. Timing resolution = 100 ns. ZC is at 50 ns from SPP.

<table>
<thead>
<tr>
<th>Thresholds</th>
<th>SNR (dB)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
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<tr>
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<td>271</td>
<td>191</td>
<td>136</td>
<td>99</td>
<td>73</td>
<td>57</td>
<td>51</td>
<td>50</td>
</tr>
<tr>
<td>- 12</td>
<td></td>
<td>548</td>
<td>382</td>
<td>269</td>
<td>190</td>
<td>136</td>
<td>98</td>
<td>73</td>
<td>57</td>
</tr>
<tr>
<td>- 24</td>
<td></td>
<td>1042</td>
<td>784</td>
<td>546</td>
<td>380</td>
<td>268</td>
<td>190</td>
<td>136</td>
<td>98</td>
</tr>
</tbody>
</table>

Table 10-1c Standard deviation in ns of the probability distribution of the SPP as a function of SNR and threshold. Timing resolution = 50 ns. ZC is at 25 ns from SPP.

<table>
<thead>
<tr>
<th>Thresholds</th>
<th>SNR (dB)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 12</td>
<td></td>
<td>97</td>
<td>69</td>
<td>50</td>
<td>37</td>
<td>29</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>190</td>
<td>134</td>
<td>95</td>
<td>68</td>
<td>49</td>
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<td>25</td>
</tr>
<tr>
<td>- 12</td>
<td></td>
<td>381</td>
<td>267</td>
<td>189</td>
<td>133</td>
<td>95</td>
<td>68</td>
<td>49</td>
<td>36</td>
</tr>
<tr>
<td>- 24</td>
<td></td>
<td>784</td>
<td>545</td>
<td>380</td>
<td>267</td>
<td>188</td>
<td>133</td>
<td>95</td>
<td>68</td>
</tr>
</tbody>
</table>

Table 10-1d Standard deviation in ns of the probability distribution of the SPP as a function of SNR and threshold. Timing resolution = 25 ns. ZC is at 12.5 ns from SPP.

<table>
<thead>
<tr>
<th>Thresholds</th>
<th>SNR (dB)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 12</td>
<td></td>
<td>68</td>
<td>48</td>
<td>34</td>
<td>25</td>
<td>18</td>
<td>14</td>
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<tr>
<td>0</td>
<td></td>
<td>133</td>
<td>94</td>
<td>67</td>
<td>48</td>
<td>34</td>
<td>26</td>
<td>18</td>
<td>14</td>
</tr>
<tr>
<td>- 12</td>
<td></td>
<td>267</td>
<td>188</td>
<td>133</td>
<td>94</td>
<td>67</td>
<td>47</td>
<td>34</td>
<td>25</td>
</tr>
<tr>
<td>- 24</td>
<td></td>
<td>545</td>
<td>379</td>
<td>266</td>
<td>188</td>
<td>133</td>
<td>94</td>
<td>66</td>
<td>47</td>
</tr>
</tbody>
</table>

Instead of assuming that the zero crossing of the received signal is situated halfway between two successive sample-point positions, we may also consider a situation where the ZC coincides with an SPP. The resulting standard deviations are given in Table 10-2.

Table 10-2a Standard deviation in ns of the probability distribution of the SPP as a function of SNR and threshold. Timing resolution = 200 ns. ZC coincides with an SPP.

<table>
<thead>
<tr>
<th>Thresholds</th>
<th>SNR (dB)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 12</td>
<td></td>
<td>165</td>
<td>115</td>
<td>85</td>
<td>64</td>
<td>47</td>
<td>34</td>
<td>24</td>
<td>17</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>333</td>
<td>225</td>
<td>155</td>
<td>110</td>
<td>82</td>
<td>62</td>
<td>45</td>
<td>33</td>
</tr>
<tr>
<td>- 12</td>
<td></td>
<td>711</td>
<td>485</td>
<td>327</td>
<td>222</td>
<td>154</td>
<td>110</td>
<td>82</td>
<td>61</td>
</tr>
<tr>
<td>- 24</td>
<td></td>
<td>1092</td>
<td>934</td>
<td>707</td>
<td>483</td>
<td>325</td>
<td>221</td>
<td>153</td>
<td>110</td>
</tr>
</tbody>
</table>
In studying all the results from Table 10-1 and Table 10-2, we found some interesting relations between the various design parameters of the tracking loop. However, it should be mentioned that these relations are only valid if the standard deviations do not tend to fixed values because of a limited timer resolution \( d_{SPP} \). From the above-given numerical tests, the approximate formula for the standard deviation can be found:

\[
SD = \frac{C}{d_{SPP}/(T - SNR)}
\]

(10-10)

where, in the worst case, \( C \) can be approximated by \( 8.6 \times 10^{-4} \) s. The form of the above expression was more or less expected. However, it is much easier to conclude that afterwards than to make such a prophecy beforehand without numerical control.

After having seen so many numerical data, one might wonder how a sequential tracking loop actually behaves. This tracking behavior can be demonstrated by computer simulation. White Gaussian noise is simulated by software by adding 12
uniformly distributed random numbers in the range from zero to one. The value 6 is subtracted from the sum, and the thus-acquired number represents a sample from a white Gaussian noise source with a variance equal to one. The signal is calculated from the relative position of the SPP and the amplitude, the SNR controlling parameter, of the signal. Then, the signal sample and the noise sample are added, and the sum is tested for either positive or negative polarity. This simple simulation technique is applied in all further demonstrations.

Fig. 10-6 shows the step response of a first-order loop for various SNR's. It can clearly be seen that when approaching the zero crossing of the signal, or when reducing the SNR, the integration time increases. For comparative purposes, Fig. 10-7 gives the step response of a fixed-sample-number averaging tracking loop, where the slew rate at 2.5 ms from the ZC equals the slew rate from the sequential-detection tracker for an SNR equal to -10 dB. In this way both tracking algorithms are equally subject to cycle slip. The SNR of the input signals for both loops varies from -infinite to +10 dB. The varying response time is striking and the quietness of the sequential detector is superior to the noisy walking of the fixed-sample-number detector.

This simple first-order tracking loop will lead or lag if the receiver is moving relative to the transmitter. For constant speeds and headings, the tracking error will stabilize at a value where the time it takes the UDC to reach the correct threshold exactly equals the time the receiver needs to travel the equivalent distance of a single time step dSPP of the SPP. At this equilibrium position of the SPP, the expected sample number ESN amounts to \( \frac{T}{p_{\text{obs}} - q_{\text{obs}}} \). This number of samples is to be collected in \( d_{\text{SPP}} \cdot V \) seconds. If

Fig. 10-6  First-order sequential-detecting tracking-loop step response for SNR = 10 dB (a), 0 dB (b), -10 dB (c). Plot (d) given dead reckoning at signal loss. Threshold = 40, dSPP = 100 ns, SN = 6000, simulated white Gaussian noise.
we bear in mind that 8 samples are taken per group repetition interval GRI, the sampling will finally take place at the position where the observation reliability $p_{\text{obs}}$ equals:

$$p_{\text{obs}} = 0.5 + \frac{V \cdot T \cdot \text{GRI}}{(16 \cdot \text{dSPP} \cdot V_c)}$$  \hspace{1cm} (10-11)$$

where $V$ = Doppler velocity of the receiver in m/s
$T$ = threshold
GRI = group repetition interval in s
dSPP = timing resolution in s
$V_c$ = propagation velocity ($3 \cdot 10^8$ m/s)

For a known SNR of the input signal the tracking error $T_e$ can now be determined:

$$T_e = 5 \cdot 10^{-6}/m \cdot (2 \cdot \sinh(SNR_{spp} / SNR_{ssp}) \cdot V_c \cdot m$$  \hspace{1cm} (10-12)$$

where SNR_{spp} = SNR at sample-point position
SNR_{ssp} = SNR at standard sample-point position

Example: $V = 28$ m/s (100 km/hr)  
$T = 16$  
GRI = 0.1 s  
dSPP = 50 ns

Then $p_{\text{obs}}$ yields 0.6867. The equivalent SNR equals 0.6571 or -3.6 dB. So, the sampling-point position SPP will settle at the position where the SNR amounts to -3.6 dB. For an SNR of 0 dB at the SSP, the tracking error $T_e$ will then amount to 484 m.

In the next section a second-order sequential-detection tracking algorithm which anticipates velocities of the receiver, will be introduced.
10.2 SEQUENTIAL-DETECTION SECOND-ORDER ZERO-CROSSING TRACKER

The simple tracking loop of the foregoing sections does not adequately perform under dynamic navigation conditions. First-order trackers on moving receivers will always lag or lead. However, sequential detection can also be applied to second-order loops. Unfortunately, estimating their tracking performance is much more complicated than for first-order loops. Therefore, the performance analysis is established by computer simulations.

Second-order tracking is based on the estimate of the position and the velocity of the ZC. However, just as with the first-order tracker, the only information available is the possible passing of one of the thresholds. Therefore, a positive threshold passing can hardly reveal more than that our estimate of the ZC position is obviously too early, and that the estimate of the velocity of the ZC is evidently too high, and vice versa for negative threshold passing.

In the foregoing section we have seen that a faulty estimate of the position of the ZC subsequently caused sample-point position SPP to change. But how must we anticipate a continuously moving zero crossing? This can be effectuated by moving the SPP at the same speed as the ZC. If the SPP moves at exactly the same velocity as the ZC, the UDC will not vary its average position.

In a digitally controlled timing system the SPP can only be moved in discrete time steps. To minimize the inevitable timing quantizing error, the moving of the SPP must happen as regularly as possible, while keeping the mean value of the difference of the velocities of the ZC and the SPP at the lowest attainable value. According to Oberman [5-16], the SPP time steps should occur in an optimally spaced rate-multiplier pattern. In that case the least amount of excess noise will be introduced. The way an optimally spaced rate multiplier can be configured is formulated by the following expression:

\[ V_{\text{spp}} = \frac{A}{B} \cdot \frac{d\text{SPP}}{GRI} \cdot V_c \]  (10-13)

where \( V_{\text{spp}} \) = velocity of SPP in m/s  
\( A/B \) = rate factor ( \( 0 \leq A/B \leq 1 \) )  
\( d\text{SPP} \) = resolution of timer circuit  
\( GRI \) = group repetition interval  
\( V_c \) = propagation velocity (\( 3 \cdot 10^8 \) m/s)

The rate-multiplier function is established by repetitively adding A to a summing register SR. If the value of the register exceeds the value B, a time step \( d\text{SPP} \) is added to the SPP, and B is subtracted from the summing register SR. Negative speeds are established in a comparable way. The speed of the SPP is varied by incrementing or decrementing A (see Pascal program further on). The resolution \( dV_{\text{spp}} \) of \( V_{\text{spp}} \) in expression (10-13) amounts to:

\[ dV_{\text{spp}} = \frac{1}{B} \cdot \frac{d\text{SPP}}{GRI} \cdot V_c \text{ m/s.} \]  (10-14)

The Loran-C transmitters emit per GRI a series of eight bursts at 1 ms intervals. So, we receive from each transmitter \( 8/GRI \) bursts per second. However, during the 7 ms time span of the 8 bursts, the SPP will be relatively constant. If, for example, the SPP should move over \( d\text{SPP} \) in the 7 ms time interval, the equivalent speed of the SPP would equal \( d\text{SPP}/(7 \cdot 10^{-3}) \cdot V_c \). Assuming \( d\text{SPP} = 100 \) ns, \( V_{\text{spp}} \) will then amount to 4.3 km/s. This example may explain why it will
be sufficient to limit the SPP corrections to once per GRI (10-13).

The second-order sequential-detection tracking-loop algorithm is still rather simple, as may be seen from the Pascal implementation below:

PROGRAM SecondOrderTrackingLoop(Input,Output);

{ SignalSample = binary output from hard limiter; a logical '1' stands for a positive signal, while a logical '0' means a negative signal
Sum = position of up-down counter
SPP = sample point position
dSPP = resolution of digital timer circuit
A = factor controlling the speed of SPP
B = factor controlling the speed resolution of the velocity estimator
SR = summing register
N = loop counter }

VAR A,B,dSPP,N,SPP,SR,Sum,Threshold: Integer;
SignalSample : Boolean;
BEGIN
A:= 0;
Sum:= 0;
SR:= 0;
WHILE receiving signal samples DO
BEGIN
SR:= SR + A;
IF SR >= B THEN
BEGIN
SR:= SR - B;
END;
IF SignalSample = 1 THEN
BEGIN
Sum:= Sum + 1;
IF Sum = +Threshold THEN
BEGIN
Sum:= 0;
SPP:= SPP + dSPP; {SPP updating}
A:= A - 1; {V_spp updating}
END;
ELSE
BEGIN
Sum:= Sum - 1;
IF Sum = -Threshold THEN
BEGIN
Sum:= 0;
SPP:= SPP - dSPP; {SPP updating}
A:= A + 1; {V_spp updating}
END;
END;
END;
END.

Although the above-given algorithm is a clear
demonstration of the simplicity of the second-order tracker, no indications have yet been given of how to select appropriate values for the parameters of the loop. The parameters of interest are:

- \( T \) - threshold of the up-down-counter
- \( dSPP \) - resolution of timer circuit
- \( dV_{spp} \) - resolution of the speed \( V_{spp} \)

To get a better insight into the behavior of the sequential-detection second-order loop, we will degenerate the loop by alternatively making the parameters \( dSPP \) and \( dV_{spp} \) equal to zero. Let \( dV_{spp} \) be zero. Assume further that \( A_{spp} \) also equals zero, which implies that no velocity anticipation exists. The second-order loop is then completely degenerated to a first-order tracking loop as described in section 10-1. The tracking may lead or lag, but the loop stability is unquestionable.

The next experiment involves making \( dSPP \) zero. Now we have a control loop where only the speed \( V_{spp} \) is effective and controlled by threshold exceedances. Then, if the SPP is lagging the ZC, the loop will increment the velocity estimate \( V_{spp} \) as long as the lagging occurs. However, the estimated speed must now exceed the true velocity of the ZC in order to make it feasible to diminish the lagging of the ZC. At the time the SPP passes the ZC, the speed estimate is exaggerated and will result in a leading SPP. In this way we have introduced a "bang-bang" type of control loop. The loop properties now resemble the double analog integrator oscillator.

Designing second-order sequential-detection tracking loops means here selecting optimal values for the resolution of the digital timer \( dSPP \), the threshold \( T \) and the velocity estimator \( dV_{spp} \) as a function of SNR. Properly chosen parameters will yield a critically damped step response on velocity jumps of the Loran-C receiver. It is preferable that the once-selected values for the control parameters meet the performance requirements under strongly varying signal conditions.

In the first part of this section it was already mentioned that the second-order character of the loop introduces a number of difficulties in the performance analysis. The grounds of this problem are the continuously changing SNR conditions for every sample to be collected. In spite of this somewhat disappointing fact, it is still possible to indicate how to design the loop so that it adequately anticipates the expected signal dynamics. And in addition, some simulation plots can be given to show the typical behavior of the second-order loop under varying signal conditions.

The size of the velocity step of the tracking loop is especially important in case the radio signals are temporarily interrupted. This may happen during precipitation conditions or upon strong radio interferences from onboard transmitters. The tracking is then temporarily forced into the dead-reckoning mode, i.e. no thresholds will be reached. Understandably, a good estimate of the velocity of the ZC will then be of prime importance to provide accurate dead reckoning. However, the accuracy of the estimate is limited by the velocity resolution \( dV_{spp} \). It is naturally assumed that the loop was in a stable condition before the radio interference began.

The velocity estimate of the ZC can only be corrected if one of the thresholds is exceeded. This then directly
explains why low SNR and small \( dV \) 's make a slowly anticipating velocity estimator. The SNR is generally not open to choice, and therefore, the value for the threshold is more or less dictated by the maximally allowed standard deviation of the tracking results. So, the size of the selected \( dV \) will be a compromise between the tracking performance on a maneuvering vessel and the dead-reckoning capability during signal loss.

It is interesting to consider how the loop reacts to a continuous acceleration and to a sudden step in the velocity of the carrier. In both cases errors will inevitably develop in the tracking of the zero crossing of the signal. Continuous accelerations can, at least in principle, be made ineffective by introducing third-order loops. In spite of the low software overhead, the results from many simulations did not indicate that significant gains in accuracy are obtainable. Therefore, third-order loops will not be discussed further in this thesis.

Let's assume a positive step in the acceleration from zero to a specified value. The increasing negative error in the \( V_{zc} \) estimate will force the SPP from the ZC position, thereby increasing the SNR at the SPP. The resulting negative threshold passings will increase the \( V_{spp} \) by a number of units of \( dV_{spp} \). The time it takes to increase \( V_{spp} \) by \( dV \) is a direct measure for the acceleration of the SPP. If the accelerations of the SPP and of the ZC are equal, the velocity estimate error will not further increase. This equilibrium will be reached if:

\[
dV_{spp} \times (p_{obs} - q_{obs}) \times 8 \times GRI/T = A_{zc}
\]

or:

\[
p_{obs} = 0.5 + A_{zc} \times T \times GRI/(16/dV_{spp})
\]

where \( A_{zc} = \) acceleration of the ZC.

From (10-16) it can be deduced that low tracking errors require low thresholds and a large \( dV_{spp} \). Both requirements contradict wishes concerning low-noise recovered position data of the zero crossing. The mean tracking error \( T_e \) can be derived from (10-16) and amounts to:

\[
T_e = 10^{-5}/2\pi \times \text{asin}(SNR_{spp}/SNR_{ssp}) \ s
\]

where \( SNR_{spp} = \) SNR at the sampling-point position
\( SNR_{ssp} = \) SNR at the standard sampling point (MPS!)

The velocity error can be found from:

\[
V_e = A_{zc} \times dSPP \times V_c/dV_{spp} \ m/s
\]

It is noteworthy that the velocity error is independent of the SNR of the received signal. Of course, the latter is only valid for signal conditions where the tracking error will stabilize.

Example: \( SNR_{spp} = -15 \) dB (\( = 0.178 \))
\( A_{zc} = 25.7 \times 10^{-3} \) m/s² (standard MPS acceleration)
\( T = 32 \)
\( dSPP = 100 \) ns
\( dV_{spp} = 0.5859 \) m/s
\( GRI = 100 \) ms

Then we find:

\[
p_{obs} = 0.5088
\]
This example is graphically depicted in Fig. 10-8c. The initial velocity of the receiver carrier is zero. The vessel accelerates during 400 seconds by 3 knots per minute, which is a standard test condition in the Minimum Performance Standards [5-5]. The various plots in Fig. 10-8 also show the tracking-error dependence on the SNR of the received signal, as well as the absence of a relation between the velocity error and the SNR. The tracking performance tests as given in Fig. 10-8a,b,c,e are based on exceedance probabilities instead of Gaussian noise. The results then indicate the mean tracking errors, so that the plots are not blurred with noise. The plots in Fig. 10-8d,f are based on real synthetic white Gaussian noise samples. The plot of Fig. 10-8e is less smooth than the one in Fig. 10-8f. The parameters of both simulations are identical and the currently simulated SNR of -33 dB is very low. The transient response shows a substantial overshoot. However, the loop remains stable.

Fig. 10-9a-d depicts the responses of the second order tracking loop upon velocity steps of the input signal. It can be noted that the amount and the duration of the tracking error depends on the SNR, while the resulting velocity error is almost non-related to the SNR. The following simulations, shown in Fig. 10-10 a-d, concern the tracking error and the velocity error resulting from an initial tracking error, but without any initial velocity error.

A very important property of the second-order loop is the noise performance. Because of the computational burden, we will give only an approximation of the noise properties of

---

**Fig. 10-8 Second-order sequential-detecting tracking-loop response to acceleration of 25.7 m/s² (MPS) from 60th to 460th second. Threshold = 32, dSPP = 100 ns, dV = 0.6 m/s, GRI = 100 ms. Vert. scale = 100 ns/div (Tₚ in upper track) and 1 knot/div (Vₛ in lower track). Hor. scale = 30 s/div. SNR = -3 dB (a), -9 dB (b), -15 dB (c), -33 dB (d), -33 dB (e), -33 dB (f). Simulations of (d) and (f) are based on Gaussian noise, the others on exceedance probabilities.**
Fig. 10-9 Second-order sequential-detecting tracking-loop response to velocity step of 20 knots. Loop parameters are as in Fig. 10-8. SNR = +3 dB (a), -3 dB (b), -9 dB (c), -15 dB (d). Vert. scale as in Fig. 10-8, hor. scale = 15 s/div. Simulations are based on exceedance probabilities.

The tracking accuracy can be expressed as the square root from the sum of the squared errors $E_1$, $E_2$, $E_3$ and $E_4$:

$$T_e = \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2}$$  \hspace{1cm} (10-19)

where $E_1$ = standard deviation of the tracking loop as derived in section 10-1 for first-order loops without any velocity errors.

---

Fig. 10-10 Second-order sequential-detecting tracking-loop response to initial track error of -1.5 ms. Loop parameters are as in Fig. 10-8, SNR = +3 dB (a), -3 dB (b), -9 dB (c), -15 dB (d). Scales as in Fig. 10-9. Simulations are based on exceedance probabilities.

$E_2$ = quantizing noise from the optimally spaced rate multiplier

$E_3$ = quantizing noise from tracking error build-up due to long-lasting velocity-estimate errors for low SNR at the SPP

$E_4$ = mean value of the tracking error due to acceleration.
The term $E_1$ may be determined from the formulas as given in section 10-1. The error $E_2$ represents the rms error due to the finite steps of SSP to anticipate the velocity of the receiver. The error amounts to:

$$E_2 = \frac{d\text{SPP}}{2/3}$$  \hspace{1cm} (10-20)

The error $E_3$ is not simple to derive. Its value depends on the relation between $d\text{SPP}$ and $dV_{\text{spp}}$. In well-designed second-order loops, one should strive for the situation in which the maximum tracking-error build-up, due to the velocity-error estimate, equals the step size of the sample-point position. Then the value of $E_3$ can be given as:

$$E_3 = \frac{d\text{SPP}}{\sqrt{3}}$$  \hspace{1cm} (10-21)

The last error $E_4$ is already given in expression 10-17. It is now possible to estimate the rms value of the tracking error for the example given before:

- $E_1 = 113.9$ ns (from (10-10))
- $E_2 = 28.9$ ns
- $E_3 = 57.7$ ns
- $E_4 = 99.1$ ns

Then we find for $T_e$:

$$T_e = 164.2 \text{ ns or } 49.3 \text{ m.}$$

When the acceleration ceases, $E_4$ will become zero, thereby yielding for $T_e$:

$$T_e = 130.9 \text{ ns or } 39.3 \text{ m.}$$

Simulation is a very efficient method of acquiring an adequate set of loop parameters. However, it requires knowledge of the positional dynamics of the receiver and the expected radio-signal qualities. We will give two very distinct simulation demonstrations. The first, Fig. 10-11, is meant for a large ship, traveling at a near-constant speed and with a stable heading. What happens if, due to precipitation static, the SNR of the received Loran-C signals is temporarily diminished from 0 dB to -30 dB is simulated. During the precipitation period the receiver is almost completely in the dead-reckoning mode of operation.

The second simulation, depicted in Fig. 10-12, represents a typical city condition. A car is waiting for a traffic

![Fig. 10-11 Simulation of sudden SNR reduction, due to precipitation statics, from 0 dB to -30 dB during the 250th second until the 1000th second of the simulation time. Ship's heading is towards the Loran-C transmitter and ship's speed amounts to 19.4 knots and is constant during the simulation run. Vert. scale = 30 m/div (T in upper track) and 2 m/s (V in lower track). Hor. scale is 100 s/div, GRI = 80 ms. At the start $T_e = -285$ m, and $V_e = 19.4$ knots. dSPP = 100 ns, $dV_{\text{SPP}} = 0.25$ m/s.](image)
light at the entrance of a 285 m long tunnel. At the moment the car starts driving, the radio signal is completely lost. While driving through the tunnel, the car accelerates up to 72 km/hr. When leaving the tunnel, the Loran signals are coming in again. At that particular moment, the zero-crossing estimate is in error for 285 meters or 0.95 μs (the worst case) and the velocity-estimate error will in the worst case amount to 72 km/hr, or 20 m/s. The last two simulations show the strong points of sequential detection. The variable response time of the loop provides excellent dead-reckoning properties, while preserving the good cycle holding and fast lock-in capabilities.

In spite of the tracking-simulation availability, the loop-parameter optimizing process is complex. The final selection of the parameters depends highly on the expected maneuverability of the receiver carrier and on the encountered SNR of the received signals. The set of loop parameters for car applications in cities will strongly differ from those used on crude-oil carriers.

With respect to the dynamic tracking capability, a Loran-C receiver that meets the Minimum Performance Standards [5-5] has, in our opinion, not yet been evaluated enough to anticipate real-life navigation conditions. However, the fast-growing market for small aircraft and car applications will probably favor the appearance of standards for these uses of electronic navigation.
11 TRACKING INITIALIZING FOR LOW SNR AND LARGE CLOCK ERRORS

In the foregoing part of this thesis, the Loran signals are observed as they propagate from the transmitter towards the active receiving antenna. After the signals are band-pass filtered and hard limited, polarity samples are taken as raw data for the digital signal processor. The processor then derives via the sequentially detecting zero-crossing tracking algorithm the control signals for the digital timing circuits. Until now, it was assumed that the signal samples were correctly taken around a specific zero crossing of a particular burst coming from a selected transmitter. However, after the receiver is switched on, only information concerning the group-repetition interval and the phase coding of the bursts of the various transmitters belonging to the selected Loran-C chain is available. So, first, some form of signal search has to be performed before any zero-crossing tracking makes sense.

This rough determination of the position of the received bursts in the selected GRI is generally known as the search procedure. The basic search process starts by taking samples of the received signal at discrete positions in time in the GRI. Samples from each time position in that GRI are then integrated to improve the signal-to-noise ratio. After having collected a predetermined number of samples, a search routine tests the results of the integrators by correlating them with the expected results. The expectation is based on the knowledge of the 1 ms time differences between the eight consecutive bursts of a single transmitter and the phase-code patterns of the master and the slave transmissions. By shifting the receiver-time reference with respect to the transmitter time, the processor searches for peaks in the correlation. It is assumed that these peaks coincide with the
course positions of the master and one or more slaves.

The searching procedure faces two primary problems. The first is the generally poor SNR of the received signals. This low SNR inevitably requires considerable signal averaging to make a reliable correlation test feasible. Unfortunately, designing low-cost receivers involves the application of relatively low-quality clocks. This brings us to the second problem: a drifting time difference between the clocks of the transmitter and the receiver makes signal averaging questionable, if not impossible. However, phase locking the receiver frequency standard to one of the Loran-C signals may cure that problem sufficiently. To make things complicated, the two processes are mutually exclusive, i.e. the local frequency standard synchronization will not work without signal averaging and vice versa. However, in the next sections it will be demonstrated that the situation is not as hopeless as it looks now.

11-1 FRAME SYNCHRONIZATION

To start with, we assume a perfect receiver clock, i.e. the receiver and the transmitter clocks have a constant, albeit unknown time difference. It is then possible to determine where signals can be detected in the GRI and from which transmitter these signals are coming. This can be accomplished by taking samples of all received signals at regular time intervals in the GRI. If we recall equation (2-1) which gives a Loran-C burst, and assume a band-pass filter with a bandwidth of about 30 kHz, the envelope of the Loran signal will exceed the level at the standard sampling point (25 µs) for approximately 125 µs. So, if the GRI is sampled every 125 µs, the envelopes of the signals, if present of course, will not lie below the 25 µs level. However, samples may just accidentally be taken at, or near, a zero crossing of the signal, in which case the SNR would be very low. This accidental signal loss can be circumvented by taking two samples in quadrature, i.e. at a 2.5 µs time distance. In this way, at least one of the samples will be taken at a position in the sine wave where the signal reduction with respect to the peak does not exceed 3 dB. As a consequence, the number of sampling positions N per GRI now equals:

\[ N = \frac{2 \cdot \text{GRI}}{125 \cdot 10^{-6}} \]  

Due to the low SNR, it is not possible to deduce the presence or absence of a signal from a single sample. A number of samples from a particular position in the GRI frame must be integrated to obtain a more concise decision about the occurrence of a possible signal. However, the phase coding of the Loran-C signals ensures that the signals repeat every two GRI's instead of every one. That would mean that only a single sample per two GRI's of the received signal at the selected position in the GRI can be collected. This disadvantage can be avoided by collecting the sample from a single particular time position simultaneously in two different integrators. The second integrator then collects samples such that every second sample is phase inverted (see also Table 4-1). The total number M of required integrators then amounts to:

\[ M = \frac{4 \cdot \text{GRI}}{125 \cdot 10^{-6}} \]  

Van der Wal & van Willigen [3-1] designed high-performance signal detectors based on sequential detection, in which the final decision is rather precise, although the sequential
procedure may lead to unacceptably long waiting times. Unfortunately, the imperfect synchronous, i.e. drifting, local clock forces one to limit the sample collection process to a certain maximum amount of time. And, therefore, a decision about the presence or absence of the signal must consequently be based on a limited number of signal samples.

As before, the signal averager applies a counter as integrator. If a positive sample is found, the initially zero'ed counter advances; otherwise the counter position is not altered. After having collected SN samples, the probability distribution of all possible counter positions equals the well-known binomial distribution. The mean value and the standard deviation of this probability function highly depend on the SNR and the polarity of the sampled signal. Fig. 11-1 gives an example of the probability function of 40 samples for white Gaussian noise and for positive signals with SNR's of -6 and 0 dB.

After having collected SN samples, the counter position CP is tested against two threshold values, $T_1$ and $T_2$, to deduce either the presence of a positive or negative signal or no signal at all. After SN samples, the counter position indicates:

- If $CP \leq T_1$, then a signal with negative polarity is present.
- If $T_1 < CP < T_2$, then no signal is present.
- If $CP \geq T_2$, then a signal with positive polarity is present.

The reliability of such a decision-making procedure can be determined from the probability function of the counter position after SN samples. Let's assume in Fig. 11-1 that the positive threshold $T_2$ equals 25 and the negative threshold $T_1$ amounts to 15. Then the area under the noise curve on the right side of $T_2$ is a measure for taking noise for positive signal, and is indicated by $p_0$. The $+$ sign stands for 'positive signal detected', and the 0 means that no signal is actually transmitted at the time of sample taking. The area under the -6 dB curve on the left side of $T_2$ represents the probability of not detecting the -6 dB positive signal, and is denoted by the sum of $p_{-1}^+$ and $p_{-2}^-$ respectively. These error probabilities are more easily determined via distribution or exceedance probability curves as depicted in Fig. 11-2. The exceedance probabilities are shown on a logarithmic scale and are given for 40 and 400 samples. It can be seen that for 40 noise samples, the threshold at 25 will be exceeded in roughly 4% of all tests. The -6 dB (SNR)
positive signal will on the average be detected for 75% of all tests. The probability that this signal will be denoted as a negative signal (p_{-,-}) amounts to about \( 5 \cdot 10^{-5} \), assuming \( T_1 \) positioned at 15. Fig. 11-2 shows clearly the improvement in decision reliability achieved by increasing the number of samples for integration by a factor of ten. Setting the threshold levels at 160 and 240 brings the detection errors for noise and signals (SNR >= -6 dB), respectively, down to the \( 10^{-4} \) level.

The selection of the threshold levels must be based on minimalization of the failure rate of the search procedure. The search procedure is assumed to be erroneous if the correct start position of the eight bursts from a particular transmitter is not found, or if the start position of a series of non-existing bursts is detected. The Minimum Performance Standards [5-5] state that not more than one failure in ten search procedures is allowed. The total search error \( P_e \) is built up from various types of errors \( P_{el...} \). The values of \( P_{el...} \) depend greatly on the number of samples per position in time and the level of the thresholds \( T_1 \) and \( T_2 \). The main errors contributing to the total error budget will be explained in the next sections.

The correlation process effectively ensures that the contents from eight - one millisecond spaced - integrators are compared with the expected contents to see whether the receiver time frame is correctly aligned with the received master or slave time frame. This can be accomplished by adding the contents of the eight integrators according to the phase coding of the sought for signal. If the summing result then does not exceed one of the thresholds, the expected bursts were probably not at that particular time position. Fig. 11-3 shows the inverse distribution of 240 noise samples and the normal distributions of 240 correctly phase-decoded samples from positive signals with SNR's ranging from -12 dB to +6 dB. The decoded signal is assumed positive. To improve the readability of small exceedance probabilities, the distribution values are again logarithmically scaled. In the next section, concerning receiver-frequency standard synchronization, we will explain why 240 samples are collected.

Assume that the thresholds, \( T_1 \) and \( T_2 \), are positioned at 90 and 150, respectively. It can be derived from the graph that \( p_{+1,0} \) then amounts to about \( 2 \cdot 10^{-5} \). As we test for threshold passings at \( T_1 \) as well at \( T_2 \), the error is to be multiplied by 2. Assume further that the GRI equals 100 ms, which yields 3200 for \( M \) in equation (11-2). Although not strictly correct, we assume further that noise is collected in 3000 time positions. This implies that the error \( p_{+1,0}' \) as determined from the graph in Fig. 11-3, must be multiplied by 3000. The final error, denoted by \( P_{el} \), then amounts to 0.12. This high value for \( P_{el} \) already exceeds the maximum allowable value of 0.10 for \( P_e \). The threshold levels must therefore be positioned further away from the center position of CP=120.
However, we now face the second error $P_{e2}$, which gives the probability of missing a correct signal. Once again it can be seen from Fig. 11-3 that for $T_{2} = 150$, the loss probability for positive signals with an SNR equal to -6 dB amounts to 0.02. If three transmitters are to be found, $P_{e2}$ then turns out to be 0.06. Lower SNR values will certainly increase $P_{e2}$ significantly, while a signal with a 0 dB SNR will be found with a high degree of reliability. Conclusion: shifting $T_{2}$ to a higher value increases $P_{e2}$, while it decreases $P_{e1}$. Therefore, minimizing $P_{e}$ by choosing the optimum value for $T_{1}$ and $T_{2}$ implies a specification of the lowest signal-to-noise ratio for signals to be found.

According to the "law of never-fading obstructions", the passing of one of the thresholds does not guarantee that a found signal is really the expected signal. The phase codes of the Loran-C signals are designed for optimal reduction of sky-wave influences, but they are, unfortunately, not free from spurious cross correlations [3-8 & 3-9]. This is shown in Table 11-1, where the absolute correlation terms are given from the comparison tests of the detected polarities of the actually received signals with each of the four possible series of polarities from the master or the slave transmissions for either the A interval leading the B interval of the GRI or vice versa. The so-called search frames are thereby shifted in 1 ms time steps throughout the GRI.

<table>
<thead>
<tr>
<th>correlation</th>
<th>frame offset RX versus TX in milliseconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>test</td>
<td>-7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7</td>
</tr>
<tr>
<td>$M[A&amp;B]&lt;-M[A&amp;B]$</td>
<td>0 0 0 0 0 0 0 16 0 2 0 2 0 2 0 0</td>
</tr>
<tr>
<td>$M[A&amp;B]&lt;-M[B&amp;A]$</td>
<td>0 4 0 0 0 4 0 0 0 2 0 2 0 6 0</td>
</tr>
<tr>
<td>$M[A&amp;B]&lt;-S[A&amp;B]$</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>$M[A&amp;B]&lt;-S[B&amp;A]$</td>
<td>0 4 0 8 0 4 0 0 0 4 0 8 0 4 0</td>
</tr>
<tr>
<td>$S[A&amp;B]&lt;-S[A&amp;B]$</td>
<td>0 0 0 0 0 0 0 16 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>$S[A&amp;B]&lt;-S[B&amp;A]$</td>
<td>0 4 0 0 0 4 0 0 0 4 0 0 0 4 0</td>
</tr>
</tbody>
</table>

Table 11-1 Absolute correlation terms from comparing the polarities of the search frame with the detected polarities of the received frames. $M[A/B]<-S[B/A]$ indicates that the frame (16 time positions) of the A and B intervals of the master is compared with the frame of the B and A intervals of the slave.

The table demonstrates that for certain time positions high false-detection probabilities can be expected, e.g. in case the master A/B search frame is +4 ms or -4 ms offset with respect to the actually received slave B/A frame. Frank [3-8] proposed summing the contents from the first four and the last four integrators of the A and the B interval separately. If the product of the two integration results exceeds a positive threshold, the search frame should be correctly aligned with one of the the transmission frames. If this threshold is not reached, the conclusion is that either noise or wrongly aligned signals have been sampled. Frank did not at that time take signals with low SNR's into account, or at least did not publish it, as Tatebayashi, Nakamura and Yamada [3-10] recently did.

The false correlation problem needs further investigation.
It is interesting to check how Frank's solution behaves under real-world conditions. The application of hard limiting requires a slight deviation from the method of Frank. Our receiver signal detector is not able to find the signal strength in an absolute way. The best that can be done is to derive the signal-to-noise ratio of the received signals in a rather inaccurate way. The now proposed solution is illustrated by the following Pascal program:

```
PROGRAM SignalSearch(Input,Output);

{ This program correlates the 8 bursts belonging to a specific position in the GRI with one of the selected phase codes. }

VAR T1,T2,T1',T2': Integer;
    SignalFound: Boolean;

PROCEDURE CORRELATE_X;
VAR X: Integer;
BEGIN
X := Correlation of bursts 1 through 4 from the A and B intervals with selected phase code;
END;

PROCEDURE CORRELATE_Y;
VAR Y: Integer;
BEGIN
Y := Correlation of bursts 5 through 8 from the A and B intervals with selected phase code;
END;

BEGIN
CORRELATE_X;
CORRELATE_Y;
IF ((X+Y) <= T1) OR ((X+Y) >= T2)) THEN
IF ((X <= T1') AND (Y <= T1')) OR
((X >= T2') AND (Y >= T2')) THEN SignalFound := 1;
ELSE SignalFound := 0;
END.
```

Table 11-1 shows the various possible spurious correlations. It is, however, more meaningful to analyze the exceedance probabilities from all these responses. First, Table 11-2 will give a more detailed picture of how these spurious correlations are established; to this end, each possible false correlation term is specified separately as S(k,l,m) which means that the exceedance probability is derived from the convolution of the probability functions of k samples of noise, l samples with positive correlation and m samples with negative correlation. It is further illustrated how each correlation term is built up from the two correlation terms from the first four and the last four bursts of a series of eight bursts. Note that for clarity the terms are arranged so that the final result always represents a positive correlation. So, only the positive threshold, T_2 or T_2', is used for the determination of the error rates.

In Fig. 11-4a-c the exceedance probabilities for noise are depicted, as well as all possible correlations of 240 samples, either true or false, for positive signals with an SNR of -6, 0 and +6 dB. As the correlation test is often performed with merely noise samples, the threshold must be chosen such that the false-alarm rate is very low. Let us take 89 for T_1 and 151 for T_2, yielding P_{el}=10^{-5}. We can assume further that the pure noise case repeats 2000 times throughout the GRI. Then we find for P_{el}: 2\cdot1000\cdot10^{-5}=0.02. In Fig. 11-3, P_{0,+1} amounts to about 0.02 for positive
11-12

signals with an SNR equal to -6 dB, and less than $10^{-6}$ for a signal-to-noise ratio of 0 dB. But what happens if spurious correlation is encountered? In the graphs from Fig. 11-4a it can be seen that $p_{+1,sp}$ (sp denotes spurious signal) will exceed the 0.01 level only for the $S(10,6,0)$ and the $S(8,8,0)$ terms. For signals with an SNR of 0 dB and better, two more terms will serve as candidates for incorrect threshold passings: $S(4,8,4)$ and $S(12,4,0)$. The conclusion is that the correct term $S(0,16,0)$ and the four false terms need further investigation.

<table>
<thead>
<tr>
<th>case</th>
<th>sum of 8 bursts</th>
<th>sum of group 1</th>
<th>sum of group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$S(0,16,0)$</td>
<td>$X(0,8,0)$</td>
<td>$Y(0,8,0)$</td>
</tr>
<tr>
<td>2</td>
<td>$S(2,8,6)$</td>
<td>$X(0,6,2)$</td>
<td>$Y(2,4,2)$</td>
</tr>
<tr>
<td>3</td>
<td>$S(6,6,4)$</td>
<td>$X(0,4,4)$</td>
<td>$Y(6,2,0)$</td>
</tr>
<tr>
<td>4</td>
<td>$S(10,4,2)$</td>
<td>$X(2,4,2)$</td>
<td>$Y(8,0,0)$</td>
</tr>
<tr>
<td>5</td>
<td>$S(4,8,4)$</td>
<td>$X(4,2,2)$</td>
<td>$Y(0,6,2)$</td>
</tr>
<tr>
<td>6</td>
<td>$S(12,4,0)$</td>
<td>$X(4,4,0)$</td>
<td>$Y(8,0,0)$</td>
</tr>
<tr>
<td>7</td>
<td>$S(10,0,0)$</td>
<td>$X(2,6,0)$</td>
<td>$Y(8,0,0)$</td>
</tr>
<tr>
<td>8</td>
<td>$S(8,8,0)$</td>
<td>$X(8,0,0)$</td>
<td>$Y(0,8,0)$</td>
</tr>
</tbody>
</table>

Table 11-2 Specification of all possible true and false correlation terms for the sum of 8 bursts and for the subgroups of both 4 consecutive bursts each. $S(10,4,2)$ indicates that the correlation test is performed on the set of 10 noise samples, 4 samples with positive correlation and 2 samples with negative correlation. The terms are arranged so that the final correlation is always positive. The selection of the X and Y terms for the first four bursts or the last four bursts is random.

Fig. 11-4a Exceedance probability functions of the counter positions after 240 samples (S terms) for noise and for all cases of correlation. SNR = -6 dB.

Fig. 11-4b The same as Fig. 11-4a, except that SNR = 0 dB.

Fig. 11-4c The same as Fig. 11-4a, except that SNR = +6 dB.
Therefore, the correlation results from the first four and the last four bursts are compared. First we have to test whether both correlations yield threshold passings, thereby indicating that no noise is involved in either group, and then check whether the polarities of the found signals are equal. If they are, the correlation is from a valid signal, and otherwise the correlation is due to misalignment of the transmission and the receiver frames.

The exceedance probabilities for the X and Y terms are depicted in Fig. 11-5a-c. The SNR again ranges from -6 to +6 dB. The thresholds to be determined now must ensure an appropriate reliability with respect to the noise and the polarity tests for the following four spurious conditions (see also Table 11-2):  

<table>
<thead>
<tr>
<th>Case</th>
<th>X-term</th>
<th>Y-term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X(8,0,0)</td>
<td>Y(8,0,0)</td>
</tr>
<tr>
<td>5</td>
<td>X(4,2,2)</td>
<td>Y(0,6,2)</td>
</tr>
<tr>
<td>6</td>
<td>X(4,4,0)</td>
<td>Y(8,0,0)</td>
</tr>
<tr>
<td>7</td>
<td>X(2,6,0)</td>
<td>Y(8,0,0)</td>
</tr>
<tr>
<td>8</td>
<td>X(8,0,0)</td>
<td>Y(0,8,0)</td>
</tr>
</tbody>
</table>

Table 11-3 Specifications of the subgroup which needs to be investigated further concerning either true or false correlation.

From Fig. 11-5a-c we see that positioning T_1' at 45 and T_2' at 75 will result in a test failure rate not exceeding 0.01 for all five conditions and for SNR's in excess of -6 dB. However, we have not yet mentioned the occurrence rate of the various spurious conditions. As this phenomena is dependent on the number of transmitters configured in the selected
chain, as well as on the geographical position of the receiver in relation to the various transmitter stations, only the basic ideas of false correlation detection are given. The reader may now easily determine the optimum thresholds for his or her specific purpose. The next section discusses a receiver clock synchronization method, which is absolutely essential for the correlation tests just described.

11.2 CLOCK SYNCHRONIZATION

Integration of signal samples requires all samples of a particular position in time in the GRI to be taken at the same position. Obviously, a drifting clock will not obey this condition. If the sampling-point position drifting is limited to a quarter period of the sampled sine wave, the signal loss will not exceed $3\,\text{dB}$. The maximum allowable sample-collecting time $T_c$ can be found from:

$$T_c = \frac{2.5 \cdot 10^{-6}}{(df/f)}$$

where $df/f$ is the relative clock error. Low-cost crystal oscillators exhibit typical $df/f$ values of $10^{-5}$. This limits $T_c$ to 0.25 seconds, yielding not more than 24 samples for integration. This small amount of samples is far too low to accomplish an accurate search procedure. Therefore, the search process must be preceded by synchronization of the local clock with one of the received Loran-C signals.

Under typical operational conditions, the receiver will generally encounter signals with a reasonable quality ($\text{SNR} > 0\,\text{dB}$) from at least one of the transmitters from the selected chain. The collected samples are tested for correlation with one of the four possible transmission frames. The signal at the position with the highest correlation, regardless of whether this correlation is either correct or false, is then used for zero-crossing tracking in the way described in chapter 10. The large clock error of $10^{-5}$ actually means an initial velocity error of 3000 m/s. Although second-order sequential-detection zero-crossing trackers are hardly limited with respect to the velocities they can anticipate, it takes some time before a particular velocity is accurately estimated. It corrects its velocity estimation step by step $- dV$ until the estimated velocity and the physical velocity become approximately equal. Clearly, in order to prevent cycle slip, the slew rate of the loop under completely misaligned velocity estimation must be adequate to counteract the large initial velocity estimation error.

The slew rate of a second-order tracking loop for a given velocity estimation error equals the slew rate of a first-order loop for the same given velocity. Therefore, the first-order model will be used for the following calculation. Let's assume that the sample-point position SPP can be altered once per GRI. The slew rate $SR$ can then be determined from:

$$SR = 8 \cdot (2 \cdot p_{obs} - 1) \cdot dSPP \cdot V_c / (T \cdot GRI)$$

where $SR$ = slew rate in m/s
$p_{obs}$ = observation reliability
$dSPP$ = timer resolution in s
$V_c$ = propagation velocity in m/s
$T$ = absolute value of the thresholds
$GRI$ = group repetition interval.

Let us take the following loop parameters, which are typical for a standard tracking loop for Loran applications:
\[ P_{\text{obs}} = 0.8413 \text{ (signal with SNR = 0 dB)} \]
\[ d_{\text{SPP}} = 100 \text{ ns} \]
\[ V = 3 \times 10^8 \text{ m/s} \]
\[ T = 16 \]
\[ \text{GRI} = 100 \text{ ms}. \]

We then find a slew rate equal to 102 m/s. Although this figure is a good demonstration of the maintainability of the correct cycle, even after velocity jumps of 102 m/s (367 km/hr), it is far from the required 3000 m/s.

A further study of expression (11-4) shows that the slew rate can be improved by increasing \( d_{\text{SPP}} \) and by decreasing \( T \). Note, moreover, that the slew rate should be adequate even for signals with an SNR going down to -9 dB. This low SNR is, due to high interference levels, often encountered in Europe. The threshold level \( T \) must maintain a sufficiently high value, as otherwise the likelihood ratio of the polarity decision will be too low, thereby reducing the effective slew rate. A good compromise is reached for the next parameter values:

\[ P_{\text{obs}} = 0.6386 \text{ (signal with SNR = -9 dB)} \]
\[ d_{\text{SPP}} = 3.2 \text{ us} \]
\[ T = 8 \]

The slew rate and the likelihood ratio of the polarity decision then amounts to:

\[ \text{SR} = 2661 \text{ m/s (equivalent to an 8.9 } 10^{-6} \text{ clock error)} \]
\[ L_{\text{dec}} = (P_{\text{obs}}/q_{\text{obs}})^T = 95 \]

The second-order tracking loop now starts to correct the velocity estimation until the SPP "jumps" alternately around the zero crossing. It does not make sense to apply high-resolution velocity steps in the control loop. The velocity step size must be balanced well with the timing step size, so as to reach a fast velocity estimation while not introducing intolerable tracking errors by velocity steps which are too large. When the equilibrium in the tracking process is reached, the tracking parameters may be altered, as the velocity error is then reduced to the resolution of \( dV_{\text{spp}} \). The tracking accuracy will be improved by reducing the \( d_{\text{SPP}} \) and the \( dV_{\text{spp}} \) values and by increasing the threshold level \( T \).

After some time the alternate passing of the positive and the negative thresholds will indicate that the loop is stabilized again, and the process of varying the loop parameters will continue until the final values for \( d_{\text{SPP}}, dV_{\text{spp}} \) and \( T \) are

![Fig. 11-6 Simulation of lock-in procedure of receiver clock. Initial clock error = 10^{-5} and SNR = -9 dB.](image-url)
Simulations have indicated that good results can be obtained if, at all parameter updatings, the velocity steps are divided by four, the timer steps are divided by two and the thresholds are increased by two, until the final values are reached. Fig. 11-6 demonstrates the simulation of the lock-in procedure. It takes about 20 seconds to reduce the initial clock error of $10^{-5}$ to $10^{-7}$, which is very adequate for frame synchronization as described in the foregoing section.

11.3 CONCLUSION

As we have seen in this chapter, the start-up procedure of a Loran-C receiver is rather complicated. To recapitulate:

1 - Collect a small batch of 24 samples from all relevant positions in the GRI and perform a correlation test on the collected samples with all possible phase codes. The signal with the highest correlation will be used in step two.

2 - Synchronize the receiver clock with the signal found from step one. It is assumed that the clock error is now at least less than $10^{-6}$.

3 - Repete the correlation test of step one, but now on a larger batch of 240 samples. This batch size agrees with expression (11-3) if the clock error is less than $8.3 \times 10^{-7}$. Check all resulting transmission positions in the GRI for false correlation.

4 - Find the leading edge of the ground wave from all signals found in step three in the selected chain.

5 - Establish stable and accurate zero-crossing tracking.

6 - Select the 25 μs zero crossing for tracking.

7 - The Loran-C receiver is then ready for supplying navigational data!

The next and last chapter of this thesis will outline how the actual position is derived from the raw tracking data.
12 LATITUDE-LONGITUDE COORDINATE CONVERSION

12.1 INTRODUCTION

Up to this point we have expressed all retrieved navigational data in terms of Times Of Arrivals (TOA). It was, and it sometimes still is, common practice to find the position by determining two Differences Times of Arrivals (DTA) from three TOA's. The DTA of a pair of transmitters, let us say the master and the first slave, yields a hyperbolic Line Of Position, or LOP on the map. A second pair of transmitters, e.g. the master and the second slave, gives a second LOP. The two LOP's will coincide at the position of the observer. It should be noted that generally two crossings of the two LOP's exist. However, in the areas of interest, it is not difficult to identify the correct crossing.

The high resolution of Loran-C receivers makes the application of special Loran-C charts unpractical. If the full capability of the resolution were to be applied for operational use, one would have to have hundreds of charts available for all the areas covered by high-quality Loran-C signals. Therefore, number-crunching facilities in a Loran-C receiver are a very welcome attribute to the navigator. They may show the position in latitude and longitude directly, thus freeing the operator from guessing around on a map crowded with hyperboles such as can be seen in Fig. 12-1. Moreover, the latitude-longitude notation of position is more generally accepted than the notation in hyperbolic LOP's. However, the somewhat unpredictable propagation velocity of radio signals, especially in coastal waters, may also introduce an uncertainty in the calculated latitude-longitude position. As there are no standard correction tables for propagation anomalies generally available, coordinate
conversion may pose a substantial risk if exact positions in
dangerous areas are imperative. LOP data are free from the
above-mentioned errors, as the Loran-C charts are calibrated
for propagation anomalies.

As Hatch [6-1] pointed out, the hyperbolic way of
processing the raw receiver data is suboptimal. We could
reach an optimal solution if all possible hyperbolic LOP's
from the transmitters involved in a specific chain are used
for position determination. Hatch suggests opting for
range-range navigation, with as many transmitters as there
are available. Basically, two transmitters will suffice if the
navigator "knows" the times of transmission and reception.
This is not a realistic assumption, as it would require an
on-board atomic clock without offset to the transmitter
clock. However, for short periods of time, this technique is
feasible.

A third transmitter makes time determining attainable. It
makes the time and the position fixing in two degrees of
freedom solvable. The gain in navigation accuracy in this so-
called "pseudo range-range" system might be significant, as
the receiver clock stability causes a substantial correlation
in time for consecutive fixings. This procedure is based on
the determination of the averages of the bias and the rate of
change of the bias in the clock from many previous time or
position fixes. The knowledge of the average bias and rate of
change gives us a set of overdetermined equations. It is
precisely this feature which causes the improvement in
attainable accuracy.

The procedure of deriving the position in latitude and
longitude from DTA or TOA data is generally known as "lat-lon
conversion". The calculation is somewhat complicated for two
reasons. First, the earth's surface is not a sphere, but an ellipsoid and second, the propagation velocity of radio signals varies slightly with the physical properties of the atmosphere and the earth's surface.

12.2 PROPAGATION ASPECTS AND COORDINATE CONVERSION

The permittivity of the atmosphere reduces the free-space propagation velocity \( V_c \) of radio waves with a ratio equal to the index of refraction \( n \) of the atmosphere. However, further delay is introduced if the signals propagate over an all-sea-water path. This delay is called the secondary factor (SF). And finally, the signal is additionally delayed by the propagating over ground of various conductivities and profiles, such as mountains and coasts. This is denoted as the additional secondary factor (ASF). The distance \( L \) from the transmitter to the receiver can then be calculated from the measured signal-propagation delay \( t_p \) by:

\[
L = \frac{t_p}{n/V_c + SF + ASF}
\] (12-1)

The following data, partly based on WGS 72 data, can be applied for position calculations:

- Equatorial radius of ellipsoid \( R_e = 6,378,135.0 \) m
- Polar radius of ellipsoid \( R_p = 6,356,750.5 \) m
- Speed of light in free space \( V_c = 2.99792458 \times 10^8 \) m/s
- Refraction index of the standard atmosphere \( n = 1.000338 \)
- Secondary Factor of all-sea-water path \( SF = -0.3 \) \( \mu \)s + 2.1 \( \mu \)s/Mm (approx.)
- Additional Secondary Factor of land path; conductivity equals 0.05 S/m (\( L > 400 \) km) \( ASF = 2.5 \) \( \mu \)s + 3.0 \( \mu \)s/Mm (approx.)

Mathematical routines for coordinate conversion can be realized in two forms. The most straightforward method is the closed-form formula, where the position is derived in a single-pass calculation. The second method starts with an estimated position. If the measured and the calculated values do not agree within certain limits, the estimated position is updated accordingly with the estimated error in the calculated DTA or TOA. This procedure is repeated until the error is so small that the position is determined with the required accuracy. Due to the linear approximation of the errors and the recursive structure of the algorithm, only a small amount of software is needed. Therefore, we unreservedly prefer the iteration method.

![Diagram](https://via.placeholder.com/150)

**Fig. 12-2** Lat-lon representation of range computation from estimated position to transmitter and of range measurement from true position to transmitter.
In Fig. 12-2 the estimated position \((N_0, E_0)\) and the true position \((N, E)\) is shown in a latitude-longitude representation. The heading towards transmitter \(TX_n\) is \(A_n\), and as we simplify the system, we assume the same heading from the true position. We may now calculate the distance from the estimated position to \(TX_n\) and check for a discrepancy between the measured distance and the true position. However, the measurement of the distance from the true position to the transmitter \(TX_n\) can only be done if one knows the range bias of the receiver. If this range bias cannot be determined, the only solution for position fixing is the hyperbolic technique. To avoid any dimensional error, we will exclusively use range data, thus no time data.

So, we are interested in the northern, eastern and range-bias corrections, that is \(dN\), \(dE\) and \(dR\), respectively. The old estimated position and the range bias will then be updated with these correction data, which then gives a better estimate of the true position. Using the simplified equation to express the difference between the measured and the computed ranges as a function of \(dN\), \(dE\) and \(dR\) yields:

\[
C_n - M_n = dN \cdot \cos(A_n) + dE \cdot \sin(A_n) + dR
\]

and

\[
R_n = M_n - R
\]

where \(N_0, E_0\) = estimated position

\(N, E\) = true position

\(C_n\) = computed range from \(N_0, E_0\) to \(TX_n\)

\(M_n\) = measured range with bias \(R\) from \(N, E\) to \(TX_n\)

\(R_n\) = true range from \(N, E\) to \(TX_n\)

\(dN\) = \(N - N_0\), or northern adjustment

The equation given above opens the way to various concepts of position determination. We may operate in the hyperbolic mode or in the range-range mode. Or, we may use more than the minimum required number of transmitters, and we may even navigate with the help of more than one chain. Although most variants have their particular charms, we will only deal with the most interesting. However, to avoid wasting paper, only the full solution for the hyperbolic, dual-chain configuration will be given. For the two remaining interesting modes of operation, the basics will be outlined.

12.3 HYPERBOLIC POSITION FIXING ON THE SPHERE

Applying the general error formula to four transmitters, where \(TX_1, 2\) belong to chain 1, and \(TX_3, 4\) to chain 2, gives the following matrix:

\[
\begin{bmatrix}
C_1 - M_1 & \cos(A_1) & \sin(A_1) & 1 & 0 & dN \\
C_2 - M_2 & \cos(A_2) & \sin(A_2) & 1 & 0 & dE \\
C_3 - M_3 & \cos(A_3) & \sin(A_3) & 0 & 1 & dR_1 \\
C_4 - M_4 & \cos(A_4) & \sin(A_4) & 0 & 1 & dR_2
\end{bmatrix}
\]

where \(dR_1\) = range-bias adjustment in \(R_1\)

\(dR_2\) = range-bias adjustment in \(R_2\)

Although there is a dictated relation between the individual chain timings, it is computationally more convenient to introduce two range biases \(R_1\) and \(R_2\).
Reducing the matrix to a two-LOP hyperbolic system and dropping \( dR_1 \) and \( dR_2 \) gives:

\[
\begin{vmatrix}
D_{12} & D_{34}
\end{vmatrix} = \begin{vmatrix}
\cos(A_1) - \cos(A_2) & \sin(A_1) - \sin(A_2)
\cos(A_3) - \cos(A_4) & \sin(A_3) - \sin(A_4)
\end{vmatrix} \star \begin{vmatrix}
dN
\end{vmatrix}
\] (12-5)

where 

\[
D_{12} = (C_1 - M_1) - (C_2 - M_2)
D_{34} = (C_3 - M_3) - (C_4 - M_4)
\]

We must now find \( dN \) and \( dE \). The above matrix function can be written as:

\[
E = A \times X
\] (12-6)

or

\[
X = (A^{-1}) \times E
\] (12-7)

This gives us:

\[
A = \begin{bmatrix}
B_1 & B_2 \\
B_3 & B_4
\end{bmatrix}
\] (12-8)

or,

\[
A^{-1} = \begin{bmatrix}
B_4 & -B_2 \\
-B_3 & B_1
\end{bmatrix}
\] (12-9)

where

\[
B_1 = \cos(A_1) - \cos(A_2)
B_2 = \sin(A_1) - \sin(A_2)
B_3 = \cos(A_3) - \cos(A_4)
B_4 = \sin(A_3) - \sin(A_4)
\]

For the northern and eastern corrections, \( dN \) and \( dE \) respectively, we write:

\[
dN = \frac{D_{12} \times B_4 - D_{34} \times B_2}{B_1 \times B_4 - B_2 \times B_3}
\] (12-10)

\[
dE = \frac{D_{34} \times B_1 - D_{12} \times B_3}{B_1 \times B_4 - B_2 \times B_3}
\] (12-11)

We call \( S \) the arc distance in radians from \( N_0, E_0 \) to \( N_n, E_n \), the position of \( TX_n \). So,

\[
S = \arccos\left(\sin(N_0) \times \sin(N_n) + \cos(N_0) \times \cos(N_n) \times \cos(E_0 - E_n)\right)
\] (12-12)

If we express this distance in meters, we find:

\[
L = R \times S \times \pi / 180
\] (12-13)

where \( R \) = radius of the sphere

The heading from \( N_0, E_0 \) to \( N_n, E_n \) equals \( A_n \). We find \( A_n \) from:

\[
A = \arccos\left(\frac{\sin(N_n) - \sin(N_0) \times \cos(S)}{\sin(S) \times \cos(N_0)}\right)
\] (12-14)

If \( \sin(E_n - E_0) < 0 \) then \( A_n = A \), else \( A_n = 360 - A \).

Now we can calculate \( B_1, B_4 \).

The position-fixing procedure is summarized below:

1 - Estimate the current position \( N_0, E_0 \).
2 - Calculate the distances \( C_1 \ldots 4 \) and the bearings \( A_1 \ldots 4 \) from \( N_0, E_0 \) to the four transmitters \( T_X_1 \ldots 4 \).

3 - Determine the terms \( B_1 \ldots 4 \).

4 - Read the DTA's from the receiver and compute the equivalent range differences \( M_2-M_1 \) and \( M_4-M_3 \).

5 - Determine \( D_{21} \) and \( D_{43} \).

6 - Calculate the northern and eastern corrections, \( dN \) and \( dE \) respectively, and update the estimated position \( N_0, E_0 \).

7 - If \( dN \) or \( dE \) exceeds the error limit, return to step 2.

8 - Output the last updated estimated position \( N^E \).

In the above-given procedure, we applied four transmitters from two independent chains. Generally, three transmitters from a single chain may yield a fix. However, at the outer limits of a particular chain, it is sometimes more fruitful to use two LOP's from different chains. This may significantly improve the accuracy. This procedure is known as "dual rate", "cross rate" or "dual chain" navigation.

12.4 PSEUDO RANGE-RANGE NAVIGATION ON THE SPHERE

As was stated in the introduction, range-range navigation is definitely superior to hyperbolic navigation. The only reason for this is the overdetermination in a set of equations, due to the inherent stability of the time bias or the equivalent range bias. To avoid dimensional errors, we will again exclusively use range data, thus excluding time data.

The procedure for reaching this gain in accuracy is the following:

1 - Determine \( dN \), \( dE \) and \( dR \).

Step 1 uses a set of equations, which are given below in matrix form:

\[
\begin{pmatrix}
C_1-M_1 \\
C_2-M_2 \\
C_3-M_3
\end{pmatrix} =
\begin{pmatrix}
\cos(A_1) & \sin(A_1) & 1 \\
\cos(A_2) & \sin(A_2) & 1 \\
\cos(A_3) & \sin(A_3) & 1
\end{pmatrix}
\begin{pmatrix}
dN \\
dE \\
dR
\end{pmatrix}
\] (12-15)

Step 4 requires the updating of the average value of the range bias \( R \). Computationally speaking, the easiest way of updating is applying the floating average, or:

\[
\text{AVER}(R)_n = \text{AVER}(R)_0 + \frac{(R - \text{AVER}(R)_0)}{NS}
\] (12-16)

where \( \text{AVER}(R)_0 \) = old average value of \( R \)
\( \text{AVER}(R)_n \) = new average value of \( R \)
\( NS \) = number of samples over which the averaging takes place
\( R \) = range bias.

Step 5 is somewhat more of a computational burden. The determination of the range bias \( R \) ensures that we now face an overdetermined set of equations. And in an overdetermined set of equations all ranges need not go through the very same position. Therefore, we will apply the least-squares error (LSE) method to find our position. The set of overdetermined equations can be written as:
The LSE solution for \( X \) is now:

\[
T^{-1}X = (A^T A)^{-1}A^T e
\]

The way in which the mathematical job can be done depends on the type of number cruncher. Nevertheless, a description of the routines for matrix transposition and inversion lies beyond the scope of this thesis.

A very promising method is the dual-chain concept in combination with range-range navigation. The same set of equations as in the hyperbolic case is to be used. The procedure looks like the following:

1. Determine \( dN, dE, dR_1 \), and \( dR_2 \).
2. Update the position \( N, E \), and the range biases \( R_1 \) and \( R_2 \).
3. If necessary, repeat steps 1 and 2.
4. Determine the average values of \( R_1 \) and \( R_2 \).
5. Compute \( dN \) and \( dE \) in an LSE method with \( \text{AVER}(R_1) \) and \( \text{AVER}(R_2) \).

The last-given method makes full use of the potentials of Loran-C. The additionally required computational power is relatively small. But it will increase the effective operational area enormously.

It should be noted that range-range navigation is more sensitive to unknown \( \text{ASF}'s \) than hyperbolic navigation. In the latter there is a tendency toward the effects of two \( \text{ASF}'s \) being cancelled if the two paths show more or less the same amount of \( \text{ASF} \).

12.5 ELLIPSOID -SPHERE TRANSFORMATIONS

All calculations up to now have been done on a sphere. However, the latitude and longitude positions on a sphere do not coincide with those on an ellipsoid. There are a number of well-known transformation methods for determining great circle distances on an ellipsoid. Some of them are accurate up to a high degree, and others offer easier calculability. Unfortunately, we do not know a transformation method that is accurate over a large surface area and which is also easy to implement in a simple, yet efficient algorithm.

However, our investigation revealed that a good compromise is feasible. The positions of all relevant transmitters on the sphere are derived from the ellipsoidal positions by using the conformal transformation by Gauss. This has to be done only once, so the complexity of the computations is not important. With the data from the Loran-C receiver we may now derive our position on the sphere. The only thing yet to be done is to transform this position to coordinates on the ellipsoid. For a limited service area, e.g., 4,000,000 km², this transformation can be accomplished with a polynomial approximation. Tests have demonstrated that a third-order polynomial approximation yields errors not exceeding 20 meters for an area of 2000 km by 2000 km. This algorithm is very easy to implement.

The general ellipsoid-to-sphere transformation polynomials are:

\[
N_s = A_0 + A_1 E + A_2 e^2 + A_3 e^3
\]

\[
E_s = E' + k(E - E')
\]
where \( N_s, E_s \) = sphere position
\( N_e, E_e \) = ellipsoid position
\( N', E' \) = transformation reference position
\( k \) = constant
\( A_0, A_1, A_2 \) = constants

The third order polynomial can be rewritten as:

\[
N_s = A_0 + N_e \cdot (A_1 + N_e \cdot (A_2 + N_e \cdot A_3))
\]

(12-21)

This implies just three multiplications and three additions.

For the inverse transformation we have:

\[
N_e = B_0 + N_s \cdot (B_1 + N_s \cdot (B_2 + N_s \cdot B_3))
\]

(12-22)

\[
E_e = (E_s + (k-1) \cdot E')/k
\]

(12-23)

where \( B_0, B_1, B_2 \) = constants

As the applications of Loran-C navigation are so diverse, the reader of this thesis should consider the above-given lat-lon conversion methods as just an indication of the possibilities. As a low-cost realization of navigation equipment was one of our priorities, we had to be very thrifty with the amount of required software. Nonetheless, the solutions given here do not undermine the final accuracy of the position fixing.

13 REFERENCES

13-1 VLF AND LF NAVIGATION


13-2 PROPAGATION


13-3 SIGNAL DETECTION AND PROCESSING


13-4 VLF AND LF NAVIGATION APPLICATIONS


[5-8] Specifications of the Transmitted Loran-C Signal, DOT, USCG, COMDTINST M16562.4.


13-6 GEODETICS


13-7 - MISCELLANEOUS


SAMENVATTING

Het belangrijkste doel van de studie zoals weergegeven in dit proefschrift is het onderzoeken van de mogelijkheden om een Loran-C navigatie-ontvanger in chip-vorm te implementeren.

Het basisconcept van een Loran-C ontvanger bestaat uit drie delen. De antenne met de HF-ingangscircuits, de digitale klok en tenslotte de digitale signaal-verwerker met de bijbehorende programmatuur. Het aantal benodigde transistoren om zo'n ontvanger te implementeren is vrijwel in het bereik van de hedendaagse productietechnieken van zeer grote geïntegreerde circuits. Echter, de combinatie van lineaire en digitale schakelingen, en het grote dynamische bereik van de ontvangen signaal vormen ernstige obstakels om de totale ontvanger-schakeling op een enkele chip onder te brengen.

Het proefschrift beschrijft de resultaten van het ontwerpproces van de meest kritische delen van een Loran-C ontvanger. Na een kort overzicht van het Loran-C systeem en enkele karakteristieke Loran-C ontvanger-concepten wordt vervolgens uiteengezet dat in het kader van laag energieverbruik en lage produktiekosten de begrenzende ontvanger de optimale keuze is.


De ontvangen Loran signalen en de minder gewenste interfererende signalen, met inbegrip van ruis, moeten m.b.v. een filter in bandbreedte begrensd worden om zoveel mogelijk
deze twee typen van signalen te kunnen scheiden zonder daarbij de omhullende van het oorspronkelijke signaal te veel aan te tasten.

Vervolgens worden de ontwerp kriteria van begrenzende versterkers besproken in relatie tot energieverbruik en signaalafhankelijke looptijd variaties. Tenslotte wordt een klein vermogen begrenzende versterker in geïntegreerde vorm getoond.

De digitale klok-schakelingen en de micro-computer van de Loran-C ontvanger worden niet verder behandeld daar voldoende algemene informatie beschikbaar is uit andere bronnen.

Echter, de sequente signaal bewerking en de regellussen, benodigd om de nuldoorgang van het ontvangen signaal te bepalen, worden in detail behandeld daar deze een essentieel deel van de ontvanger vormen. Veel aandacht is tevens gegeven aan de inschakelprocedure van de ontvanger. Aangetoond wordt dat zelfs bij grote initiële fouten van de klok, de acquisitie van verruiste Loran-C signalen mogelijk is.

Tot slot wordt een korte uiteenzetting gegeven van rho-rho en speudo rho-rho navigatie. Getoond wordt hoe geodetische berekeningen eenvoudig en nauwkeurig gehouden kunnen worden door het toepassen van polynoom benaderingen van de Gauss ellipsoïde-bol transformaties.

About the author

Durk van Willigen was born in Gouda on April 27, 1934.

He obtained the Ingenieurs-degree from the Delft University of Technology, Delft, The Netherlands, in January 1963. In the same year he started his career at that as a member of the scientific staff of the Radio Laboratory, later called the Laboratory for Pulse and Digital Electronics, first under the direction of the late Prof.ir. J.W. Alexander and then under the direction of Prof.ir. T. Poorter. With Prof. Alexander he was first involved in the research and design of frequency synthesizers for application in HF communication receivers. This resulted in the early seventies in the development of ground stations for meteorologic satellites. Special attention was paid to scan-conversion systems. In 1977 the author initiated, together with Peter W. van der Wal, the work in the field of designing Loran-C receivers, suitable for large-scale integration. Prof. Poorter, the original promotor, has always been an inspiring and stimulating mentor. His death, which left the author with a gap in his personal relations, was a great loss for the Laboratory for Pulse and Digital Electronics over which he presided.

Durk van Willigen is author and co-author of several articles concerning scan-conversion systems for meteorological satellite pictures, HF communication receivers, and signal-processing and electronics for Loran-C receivers. In 1975 he decided to start a small consulting company for the development of electronic systems, called Reelektronika. As a consequence, he had to reduce his work at the Delft University of Technology.
His continuous research work, along with that of many students, has ultimately resulted in this thesis.

Within the Institute of Electrical and Electronics Engineers he is a member of, among others, the Aerospace and Electronic Systems and the Solid-State Circuits Societies. He is also a member of the Institute of Navigation, the Wild Goose Association, the Nederlands Elektronica- en Radiogenootschap and the Koninklijk Instituut van Ingenieurs.

His extra-professional interests include music and sailing.

STELLINGEN

1 - Physiek kleine en toch ruisarme antennes voor Loran-C navigatie-ontvangers moeten helaas van het actieve type zijn.


3 - Het toepassen van sequente detectie is een krachtig hulpmiddel bij het tegengaan van "cycle slip" zonder daarbij concessies te hoeven doen aan de operationele detectiebandbreedte.

4 - Het is merkwaardig dat bij elektronische navigatie veelal verkwistend wordt omgegaan met de totale hoeveelheid beschikbare informatie.


Zie ook hoofdstuk 12 van dit proefschrift.

5 - Ten aanzien van integreerbaarheid, energieverbruik en kostprijs zijn ontvangers voor VLF en LP navigatie principieel superieur aan die voor satelliet-navigatie.
6 - Aanvaarding van het Global Positioning System als algemeen mondiaal navigatiesysteem maakt Europa navigatorisch volkomen afhankelijk van een niet-europees monopolistisch militair systeem.

7 - Het operationeel worden van GPS zal waarschijnlijk gepaard gaan met een verminderde belangstelling van de Verenigde Staten van Amerika voor het Loran-C navigatiesysteem in Europa. Frankrijk is met de bouw van twee nieuwe Loran-C zenders aan de atlantische kust in een vergevorderd stadium. "Dual raten" van deze zenders met reeds bestaande europese Loran-C zenders kan het dekkingsgebied sterk vergroten. Beide bovenstaande uitspraken rechtvaardigen de wens om alle europese Loran-C zenders onder gezamenlijk europees beheer te plaatsen en de totale kosten over de lid-staten te verdelen in evenredigheid met het belang van ieder land.


8 - Elektronica ontwerpers dienen meer dan tot nu toe zelf nieuwe toepassingsgebieden te exploreren, en niet steeds te wachten tot eventuele klanten naar hen toekomen.

9 - Met het aktief propageren van "gate arrays" bij het midden- en kleinbedrijf vangt men later misschien de "full custom VLSI" vissen.

10 - Het valt te verwachten dat in de naaste toekomst een schaarste zal ontstaan aan elektronica ingenieurs die nog in staat zijn schakelingen op transistornivo te ontwerpen. De elektrotechnische industriën doen er daarom goed aan nu reeds te proberen om deze laatste exemplaren van de soort voor hun bedrijf te winnen.

11 - Hoewel het grootschalig toepassen van getijdencentrales ambtenaren nog verder van de dertiende maand brengt, kunnen zij wel langer genieten van hun vrije dag.

12 - De democraat gekozen bestuursorganen zijn helaas veelal niet in staat de samenleving uitvoerig in te lichten over de voor- en nadelen van het invoeren van nieuwe technologieën. Technici dienen daarom op uiterst loyale wijze, zelfs ongevraagd, informatie te verstrekken over de consequenties van hun werk. Misschien kunnen we dan het aantal dioxine- en vuilstortings-verrassingen beperken.

Stellingen behorende bij het proefschrift "Hard Limiting and Sequential Detecting Loran-C Sensor"  
Delft, 8 januari 1985  
Durk van Willigen
Errata

page 3-6, line 4: \[ V_c = 2.99792458 \times 10^8 \text{ m/s} \]

page 3-6, line 8:

\[ SF = \text{Secondary Factor, about } -0.3 \times 10^{-6}/L \text{ s} + 2.1 \times 10^{-12} \text{ s/m} \]

\[ \text{for } \gamma = 5 \text{ mhos/m } \& \text{ } L > 400 \text{ km} \]

page 3-6, line 10:

\[ ASF = \text{Additional Secondary Factor, varying from } \]

\[ 0.1 \times 10^{-6}/L \text{ s} + 0.6 \times 10^{-12} \text{ s/m } \text{(} \gamma = 0.05 \text{ mhos/m) to } \]

\[ 2.5 \times 10^{-6}/L \text{ s} + 3.0 \times 10^{-12} \text{ s/m } \text{(} \gamma = 0.001 \text{ mhos/m) for } \]

\[ L > 400 \text{ km} \]

page 12-4, line 26:

\[ SF = -0.3 \times 10^{-6}/L \text{ s} + 2.1 \times 10^{-12} \text{ s/m } \text{(approx.)} \]

page 12-5, line 3:

\[ ASF = 2.5 \times 10^{-6}/L \text{ s} + 3.0 \times 10^{-12} \text{ s/m } \text{(approx.)} \]