Viscoelastic damping in high rise structures

A feasibility study on the development of a prototype tool for engineering firms which can be used to determine the required amount of viscoelastic damping in a high rise structure to reduce accelerations from wind-induced vibrations to a comfortable level.

Appendices

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A feasibility study on the development of a prototype tool to determine the required amount of viscoelastic damping in a high rise structure to reduce accelerations from wind-induced vibrations

APPENDICES
Contents

E Definition equivalent stiffness and damping 1
  E.1 Series and parallel systems ........................................ 1
  E.2 Model of the bracing .............................................. 4

E Method to determine matrices 6
  E.1 Stiffness of the structure ...................................... 6
  E.2 Rotational stiffness of the foundation .................... 7
E Definition equivalent stiffness and damping

E.1 Series and parallel systems

In correspondence with Figure E.1, the equivalent stiffness of parallel connected springs is found by:

\[ F = F_{k_1} + F_{k_2} = k_1 u + k_2 u = (k_1 + k_2)u \]  

(E.1)

The equivalent spring stiffness of springs in series is determined by:[35]

\[ u = u_1 + u_2 = \frac{F_1}{k_1} + \frac{F_2}{k_2} = F \left\{ \frac{1}{k_1} + \frac{1}{k_2} \right\} \]

(E.2)

![Figure E.1: Springs in series and parallel and equivalent models](image)

An identical approach can be employed to find the equivalent damping of a system, see Figure E.2. Correspondingly, the equivalent damping of parallel connected dampers is found by:

\[ F = F_{c_1} + F_{c_2} = c_1 u + c_2 u = (c_1 + c_2)u \]

(E.3)
Additionally, the equivalent spring stiffness of dampers is expressed by:

\[
\frac{du}{dt} = \frac{du_1}{dt} + \frac{du_2}{dt} = \frac{F_1}{c_1} + \frac{F_2}{c_2} = F \left[ \frac{1}{c_1} + \frac{1}{c_2} \right]
\]

(E.4)

In case the system is expressed by a combination of parallel and series elements, the following procedure may be followed in accordance with Figure E.3. It must hold that:

\[
F(t) = k_3u_3(t) + c_3\dot{u}_3(t) = k_2u_2(t) + c_2\dot{u}_2(t) = k_1u_1(t) + c_1\dot{u}_1(t)
\]

\[
= \left( k_3 + c_3 \frac{d}{dt} \right) u_3(t) = \left( k_2 + c_2 \frac{d}{dt} \right) u_2(t) = \left( k_1 + c_1 \frac{d}{dt} \right) u_1(t)
\]

(E.5)

In this case, an equivalent stiffness cannot be found due to the time dependency of the damping. Therefore, an operator is now introduced instead:

\[
O = \frac{1}{k_3 + c_3 \frac{d}{dt}} + \frac{1}{k_2 + c_2 \frac{d}{dt}} + \frac{1}{k_1 + c_1 \frac{d}{dt}}
\]

(E.6)
In the frequency domain the above equations are expressed by:

\[
F(\omega) = (k_3 + i\omega c_3) \tilde{u}_3(\omega) = (k_2 + i\omega c_2) \tilde{u}_2(\omega) = (k_1 + i\omega c_1) \tilde{u}_1(\omega)
\]  \hspace{1cm} (E.7)

And:

\[
O = \frac{1}{\frac{1}{k_3} + \frac{1}{k_2} + \frac{1}{k_1}}
\]  \hspace{1cm} (E.8)

A similar procedure can be used in case one element is not present in comparison with Figure E.3. For example, in Figure E.4 the spring in element 2 is not present and thus:

\[
F = \left( k_3 + c_3 \frac{d}{dt} \right) u_3(t) = c_2 \frac{d}{dt} u_2(t) = \left( k_1 + c_1 \frac{d}{dt} \right) u_1(t)
\]  \hspace{1cm} (E.9)

And correspondingly:

\[
O = \frac{1}{\frac{1}{k_3+c_3} \frac{d}{dt} + \frac{1}{c_2} \frac{d}{dt} + \frac{1}{k_1+c_1} \frac{d}{dt}}
\]  \hspace{1cm} (E.10)

In the frequency domain the above equations are expressed by:

\[
F(\omega) = (k_3 + i\omega c_3) \tilde{u}_3(\omega) = i\omega c_2 \tilde{u}_2(\omega) = (k_1 + i\omega c_1) \tilde{u}_1(\omega)
\]  \hspace{1cm} (E.11)

And:

\[
k_{eq} = \frac{1}{\frac{1}{k_3} + \frac{1}{k_2} + \frac{1}{k_1}}
\]  \hspace{1cm} (E.12)
E.2 Model of the bracing

In correspondence with Figure E.5, the equivalent stiffness of the bracing in the frame is calculated by:

\[ F = k u \]

\[ \varepsilon = \frac{N}{EA} = \frac{\Delta l}{l} \rightarrow N = \Delta l \frac{EA}{l} \]

\[ \Delta l = u \cos \alpha \]

\[ = u \frac{b}{\sqrt{b^2 + h^2}} \]

\[ F = N \cos \alpha \]

\[ = \frac{N}{\sqrt{b^2 + h^2}} \]

\[ = \frac{EA}{l} \frac{b}{\sqrt{b^2 + h^2}} \]

\[ = \frac{EA}{l} \frac{b}{\sqrt{b^2 + h^2}} \frac{b}{\sqrt{b^2 + h^2}} \]

\[ = \frac{EA}{\sqrt{b^2 + h^2}} \] \[ \frac{b}{\sqrt{b^2 + h^2}} \frac{b}{\sqrt{b^2 + h^2}} \]

\[ = \left[ \frac{EA}{\sqrt{b^2 + h^2}} \frac{b}{\sqrt{b^2 + h^2}} \frac{b}{\sqrt{b^2 + h^2}} \right] u \]

\[ = \left[ \frac{b^2}{\{b^2 + h^2\}^{3/2}} \right] u \]

stiffness bracing

\[ (E.13) \]
Similarly, the equivalent damping coefficient can be determined for the horizontal direction: The equivalent horizontal damping from the bracing is determined by:

$$
\Delta \dot{u} = \dot{u} \cos \alpha \\
= \dot{u} \frac{b}{\sqrt{b^2 + h^2}} \\
F = c \dot{u} \\
= N \cos \alpha \\
= c_{d,\text{bracing}} \Delta \dot{u} \cos \alpha \\
= \left[ c_{d,\text{bracing}} \frac{b}{\sqrt{b^2 + h^2}} \sqrt{b^2 + h^2} \right] \dot{u} \\
= \left[ c_{d,\text{bracing}} \frac{h^2}{b^2 + h^2} \right] \dot{u} \quad \text{eq. damping bracing}
$$

(E.14)
E  Method to determine matrices

E.1  Stiffness of the structure

The stiffness is determined in accordance with Figure E.1. The stiffness of the springs is expressed by:\cite{6,7}

\[ M = Ke \]  \hspace{1cm} (E.1)

The rotations are described by:

\[ \phi_{ij} = \frac{1}{h}(w_j - w_i) \]  \hspace{1cm} (E.2a)
\[ \phi_{jk} = \frac{1}{h}(w_k - w_j) \]  \hspace{1cm} (E.2b)

Correspondingly:

\[ e = \phi_{ij} - \phi_{jk} = \frac{1}{h}(-w_i + 2w_j - w_k) \]  \hspace{1cm} (E.3)

And thus:

\[ M = Ke = \frac{K}{h}(-w_i + 2w_j - w_k) \]  \hspace{1cm} (E.4)

\[ \text{Model of a single element} \]

\[ \text{Model of a multiple elements} \]

\textit{Figure E.1: Model to determine the stiffness of the core structure}
E.2. ROTATIONAL STIFFNESS OF THE FOUNDATION

Then, the forces are:

\[ F_i = -\frac{M}{h} = \frac{K}{h^2}(-w_i + 2w_j - w_k) \quad (E.5a) \]
\[ F_j = 2\frac{M}{h} = \frac{K}{h^2}(-2w_i + 4w_j - 2w_k) \quad (E.5b) \]
\[ F_k = -\frac{M}{h} = \frac{K}{h^2}(-w_i + 2w_j - w_k) \quad (E.5c) \]

And in matrix notation:

\[
\begin{bmatrix}
  F_i \\
  F_j \\
  F_k
\end{bmatrix} = \frac{K}{h^2}
\begin{bmatrix}
  1 & -2 & 1 \\
  -2 & 4 & -2 \\
  1 & -2 & 1
\end{bmatrix}
\begin{bmatrix}
  w_i \\
  w_j \\
  w_k
\end{bmatrix} \quad (E.6)
\]

E.2 Rotational stiffness of the foundation

In correspondence to Figure E.9 the rotational stiffness of a foundation is to be determined by:

\[ K_r = \frac{M}{\theta} \quad (E.7) \]

Correspondingly, the following applies:

\[ \theta = \frac{u}{h_{storey}} \]
\[ M = K_r \theta = K_r \frac{u}{h_{storey}} \]
\[ F = \frac{M}{h_{storey}} = \left( \frac{K_r}{h^2_{storey}} \right) u \quad (E.8) \]

The corresponding \( n \times n \) stiffness matrix becomes:

\[
K_{\text{foundation}} = \begin{bmatrix}
  k_{11} & \cdots & \cdots & \cdots \\
  \cdots & \ddots & \cdots & \cdots \\
  \cdots & \cdots & k_{nn}
\end{bmatrix} = \begin{bmatrix}
  K_r & \cdots & 0 \\
  \vdots & \ddots & \vdots \\
  0 & \cdots & 0
\end{bmatrix} \quad (E.9)
\]
The normal force in the piles is to be calculated by:

\[ M = F h_{\text{storey}} = N_{\text{pile}} 2a \quad \text{(E.10a)} \]

\[ N_{\text{pile}} = \frac{F h_{\text{storey}}}{2a} \quad \text{(E.10b)} \]

The rotation \( \theta \) of the foundation is to be found by:

\[ \varepsilon_{\text{pile}} = \frac{N_{\text{pile}}}{(E A)_{\text{pile}}} = L_{\text{pile}} \Delta L_{\text{pile}} \]

\[ \Delta L_{\text{pile}} = \frac{N_{\text{pile}}}{L_{\text{pile}} (E A)_{\text{pile}}} \quad \text{(E.11)} \]

\[ \theta = \tan^{-1} \left\{ \frac{\Delta L_{\text{pile}}}{a} \right\} \]

Hence, the rotational stiffness \( K_r \) becomes:

\[ K_r = \frac{F h_{\text{storey}}}{\tan^{-1} \left\{ \frac{\Delta L_{\text{pile}}}{a} \right\}} \quad \text{(E.12)} \]
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