Viscoelastic damping in high rise structures

A feasibility study on the development of a prototype tool for engineering firms which can be used to determine the required amount of viscoelastic damping in a high rise structure to reduce accelerations from wind-induced vibrations to a comfortable level

Literature review

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Glossary of terms

**Aeroelastic**: Structure where the aerodynamic, elastic and inertial forces interact

**Amorphous**: solid which is not fluid below its solidification temperature and which does not contain a crystalline or orderly structure

**Drag**: The drag of an object is the resistance to motion caused by air

**Gusts**: Fluctuations of wind velocity around the mean

**High rise structures** buildings which are affected by lateral forces due to wind or earthquake actions to an extent that they play an important role in the structural design because of the building’s height and usually contain elevators and a lot of vertical transportation of persons takes place in vertical direction

**Passive damper**: dampers which do not require additional energy and dissipate energy from a structural system. The response of the damper is quite slow but costs are relatively low and it will never behave as a resonance system

**Reynolds number**: The Reynolds number is defined as the ratio of fluid inertia forces (the resistance to change or motion) to viscous forces and it is a dimensionless number. The Reynolds number is used to determine pressure coefficients and to characterize different flows, like laminar or turbulent flows. The formula to determine the Reynolds number is: [11] page 72

\[ Re = \frac{\mu V L}{\mu} \]  

**Reynolds stress**: The Reynolds stress is defined as the net rate of transfer of momentum across a surface in a fluid resulting from turbulence in the fluid

**Slip trigger displacement**: the value of the displacement of a system at which it leaves its stick state

**Steady state motion**: The stable motion of a system in its equilibrium

**Vortices**: Circular, spinning motions of wind in a turbulent wind flow by a periodic release of Van Karman vortices. This release causes an alternating load perpendicular to the wind direction.

**Wind**: The perceptible flow of a current of air from high pressure areas to low pressure areas which arise from various causes
Chapter 1

Introduction

This literature review is part of the thesis on viscoelastic damping in high rise structures. The thesis consists of three different parts, namely the literature study, the main report and the appendices. This report is a literature review on high rise structures, dynamic wind load, structural dynamics and damping. The literature review has been the basis for the modeling phase of the thesis.
Chapter 2

High rise structures

In this first chapter a brief description of high rise buildings will be presented. First, will be described why high rise structures are built and will be built in the future. Then a historical overview of high rise buildings will be presented. Thirdly, an overview of the structural systems used in high rise structures will be displayed. Next, some issues regarding high rise structures in the Netherlands will be discussed.\[35\]

2.1 Why high rise structures?

As the population in the Netherlands increases, the available space in the entire country decreases. Therefore, a more efficient way of using this space will be used now and in the future. A solution to the effective use of space is to build in an upward direction, which only requires a limited surface area at ground level, but provides a much larger floor area in the building due to the many available floor levels. In addition to that, the use of high rise buildings also is interesting from an economical point of view. The situation of having a small surface at ground level and many floor levels to achieve a large surface area is much cheaper than having this same floor area spread out over a larger area at ground level when making use of buildings with only a few floor levels.

From an aesthetic point of view, architects and structural engineers want to design a taller and more appealing building than the ones designed by others. Of course the development of high rise buildings brings negative aspects along as well. For example, the required fire safety mentioned in regulations is much harder to achieve due to the height of the building. Past and current developments have led to better conditions in terms of fire safety so it is expected this will become less of an issue over time.\[1,35\]

High rise buildings in the Netherlands are by far not as tall as structures elsewhere in the world. This is caused by several factors; from which the main factor is caused by building codes. Each building in the Netherlands has to meet the requirements mentioned in the codes. For example, a maximum depth of 7.2 meters is allowed for offices in day light regulations. This causes a smaller surface area per floor level in the Netherlands because the same value for slenderness is used here as elsewhere in the world. Having a smaller surface area and the same slenderness, this immediately leads to lower buildings.\[1,9\]
2.2 Structural systems

The structural systems used in high rise buildings can roughly be divided into two groups, namely interior and exterior structural systems. Interior structures are integrated into the building while exterior structures are constructed at the exterior of a building. This is only a rough division: interior structures can have exterior structural elements and the other way around. Each of these two groups contains subgroups of structures, which is displayed in Figure 2.1. This figure also displays the type of material which is used per type of structure, either steel, concrete or a combination of steel and concrete.

![Figure 2.1: Group division of high rise structures](image)

All structural systems are limited to a certain height, see Figure 2.2. A combination of these
systems is also possible.

For this thesis only the interior structures are considered because these are more often used in the Netherlands. Most of these systems are discussed below.

**Core structures**

Core structures are the most applied type of structures in high rise building. Other structural systems are often a coupled system of a core structure and some other extensional structure. A core structure consists of a group of shear walls applied over the height of the building and its deformation is caused by bending, see Figure 2.3. Due to its large bending stiffness, the core provides horizontal stiffness to the building. The attachment of floors and beams to the core also contributes to the stiffness of the overall stiffness.

**Frame structures**

The group of frame structures is divided into three other groups, namely rigid frame structures, braced frame structures and rigid braced frame structures. The latter is a combination of the other two structures so only the first two will be discussed here.
CHAPTER 2. HIGH RISE STRUCTURES

Figure 2.4: Model for a rigid frame structure

The rigid frame structure consists of unbraced frames which are connected by moment resisting connections, see Figure 2.4. Horizontal loads are transferred both by bending and shear. Hence, the deformation of the structure is a combination of bending mode and shear mode. In addition to that, both beams and columns consist of a finite bending stiffness. Therefore, the members will deform as is also displayed in Figure 2.4. This figure also shows the shape of deformation of the entire structure. Braced frame structures on the other hand provide higher horizontal stiffness: in this case the moment resisting connections are replaced by hinged connections to which diagonals are added for the sake of stability, see Figure 2.5. The vertical members of the frame are loaded by axial forces to deal with bending and the diagonals are activated as a response to shear forces, which is also displayed in Figure 2.5. Thus, the deformation is again a combination of shear and bending deformations.

Outrigger systems

An outrigger system consists of a core structure which is connected to façade columns by outriggers, see Figure 2.6. Usually, outriggers consist of a story height truss structure. These outriggers are used to activate the façade columns to which parts of the horizontal loads are transferred. When the system is loaded, one façade is loaded by tension and the other by compression. The maximum bending moment at the base of the core is reduced when this system is used compared to the use of a simple core structure. However, the stability of the building is only supplied by the core structure itself: the façade columns do not contribute to the stability of the structure.
2.2. STRUCTURAL SYSTEMS

Coupled systems
With the structures above, the possibility arises to combine these structures to coupled structures. A few of these structures will be discussed now.

Shear wall + hinged frame
The beams and columns are connected by hinges. The substructure is connected to the shear wall and the shear wall transfers horizontal loads to the hinged frame substructure.

Coupled walls
A coupled wall structure consists of two or more horizontally connected shear walls. The connections are usually floor beams with moment resisting connections. The bending stiffness of the coupling beams determines the composite action of the system. Coupled shear walls are often used to create three dimensional core structures to accommodate functions such as elevators, stair wells and service shafts. The coupled system possesses a larger bending stiffness than a single shear wall. Consequently, the possibility arises to increase the total height of the structure. [10:1:29]
2.3 Design aspects

High rise structures differ from other structures due to the height and slenderness. This requires special design aspects for the structural designer in case a high rise building is considered. These aspects are listed below.

1. **Strength:** The building must be able to resist the most critical combination of loads acting on the building to prevent failure of a single member or the entire structure. Important design aspects to take into account are:
   - External forces cause a moment at the base of the building and thus stresses since vertical and horizontal forces are transferred to the foundation. The external force considered in this thesis is wind load of which the value is expressed by a function of height. The maximum tensile and compressive stresses at the ends at the bottom of the structure must be checked by the structural designer.
   - The failure of a single structural member can initiate progressive collapse of the member or the entire structure. The effect of the collapse of the member should be estimated in the design phase by the structural designer.
   - High temperatures cause changes in a material’s behavior and consequently the structural behavior of the building. Hence, collapse can occur in case of fire which could be prevented by the application of fire resistant coatings, sprinklers or composite materials.
   - Time dependent factors such as creep, shrinkage and fatigue cause local stresses and possibly collapses of members.

2. **Stiffness:** The building must be able to resist the most critical combination of loads acting on the building to reduce deformations to an acceptable level. The critical value for deformation is not just determined by failure of the member but mainly to an occupant’s perception of deformations. Important design aspects to take into account are:
   - The deformation of a high rise structure depends on shear action an bending. However, for slender structures shear action is negligible and its deformation shape is therefore assumed to be caused by bending only.

3. **Stability:** The building must be able to resist the most critical combination of loads acting on the building to prevent failure of a single member or the entire structure. Important design aspects to take into account are:
   - To prevent the building form tipping over, the moment capacity of the building should be larger than the moment caused by wind load. In case of too little capacity, more dead weight could be added to the building, for instance by the application of more dense materials. Another solution is the application of tensile piles in the foundation of the building.
   - For certain building shapes and for light weight buildings the torsion stability is critical. The application of additional braces or mass to the structure could stabilize the structure. In addition to that, the structural designer could find a solution to
reduce the asymmetry in mass or stiffness to reduce the torsional motion of the structure.

4. **Human comfort**: Criteria with respect to the comfort of occupants in the building other than mentioned above:

- In case of a fire occupants must be able to leave the building within a certain time period. Therefore, precautions must be taken to guarantee a safe escape for occupants at each floor level.

- Due to its height, high rise structures could undergo large deflections and consequently accelerations at the top of the building. Humans are sensitive to motion and thus to accelerations. This will be elaborated on further in Chapter 4, Section 4.3.2.

### 2.4 Remarks

- Core structures are the most applied type of structure for tall buildings. Therefore, it is likely to use this type of structure as a starting point for the prototype tool. Its model can be adjusted to other types of interior structures.

- Time dependent factors (creep and shrinkage) are negligible in this research because these are only local effects.
Chapter 3

Dynamic wind load

This chapter provides the reader with information regarding aspects related to wind. First it will be explained what factors the wind profile is constructed from and secondly a mathematical description of wind load will be presented for different types of wind load.

3.1 General

Different causes for wind currents will now be summed up and explained:

1. Coriolis force:

the wind flow is influenced by the rotation of the earth and it will deviate at different positions on earth. The deviation is caused by the Coriolis effect, which is the deflection of moving objects (the air) compared to the rotating reference object (the earth). This effect causes a deviation of the air perpendicular to the displacement so a shear force is introduced. The deviation at the northern hemisphere is to the right and on the southern hemisphere to the left, see Figure 3.1.

![Figure 3.1: Deviations in wind flow](image-url)
A mathematical expression for this deviation is:

\[ F_c = 2mvω\sin(φ) = fmv \] (3.1)

This is called the Coriolis force. However, it is not an actual force since it is not able to transfer energy. At gradient height, the Coriolis force and gradient force are in equilibrium. This is called geostrophic wind.

2. **Boundary layer**: in general it holds for gases that when an object moves through the gas, molecules of that gas stick to the surface of the object. This new layer is called the boundary layer of the object and it can entirely change an object’s effective shape.

Boundary layers are either laminar or turbulent depending on the value of the Reynolds number. Boundary layers are laminar for lower Reynolds numbers and turbulent for higher Reynolds numbers. In the latter case the velocity is unsteady with swirling flows. Wind is considered to be laminar, turbulences occur only locally.

In this thesis two main objects can be considered which influence the wind flow. These objects are the earth and the building to be studied. The boundary layer of the earth is called the atmospheric boundary layer. As sub-objects a building’s surroundings are considered; this will be discussed later in this chapter. For now only the two main boundary layers will be discussed further.[25]

(a) **Atmospheric boundary layer (ABL)**: This is defined as the region of wind which is affected by the earth’s surface. The wind flow is expected to be turbulent at any wind speed due to the large Reynolds numbers in the ABL. However, the turbulence could be stabilized by thermal effects, for instance by warm air flowing over a cold surface. In that case, the vertical turbulence components are completely suppressed. On the other hand, in case of strong winds the thermal effects are suppressed and the ABL is considered to be adiabatic and therefore stable.

The ABL consists of two sub-layers: the inter-facial layer and the Ekman layer. The inter-facial layer is the very bottom of the ABL where the Reynolds stress changes from its maximum to zero at ground level. This is also displayed in Figure 3.2.

![Figure 3.2: Atmospheric boundary layer](image)

The height of the zero-plane displacement equals the average height of the buildings
in a certain area. The net flow at this point equals zero due to wind flows that skip over the tops of buildings, which leaves sheltered areas between them.

The Ekman layer is the remainder of the ABL. Figure 3.2 shows a reduction of the Reynolds stress by the increase of height until it reaches zero at gradient height. Other forces in the Ekman layer are the Coriolis force and the pressure force. Together these forces form the static pressure field: at gradient height the gradient pressure force and the Coriolis force form the geostrophic balance (Figure 3.3.a) and at any other height the force balance is as displayed in 3.3.b. Then, the Reynolds stress does not equal zero so a shear force is introduced. This shear force acts in the opposite direction of the smooth gradient wind. The radius $\alpha$ changes along the height, until it reaches zero at gradient height where the shear force becomes zero. (author?)[7, Ch.7]

(b) **Building’s boundary layer (BBL):** the boundary layer of a building is used to find the drag on a building. The drag of a building is explained further in the next section.

3. **Structure:** Buildings are assumed to be bluff bodies. Bluff bodies have a broad flattened body, such as rectangular shaped buildings. The effective shape of the building determines the wind flow around it. These flows cause pressure areas where local currents occur such as turbulence. The region behind the building is called the wake; the surrounding air currents cause recirculating flow in that region. Figure 3.4 shows a bluff building and its wake region.

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Figure 3.3: Force balance in ABL

Figure 3.4: Bluff body and wake region
3.2 Characteristics

Wind loads have a random and unpredictable character which makes regular load cases such as harmonic ones, not useful in a model. Consequently, regular load cases and models do not apply to wind load. The wind spectrum is referred to as a random process, which comes down to the following:

**Definition 1** A random process is an unpredictable function from which the outcome is never the same by any set of initial data.

With respect to the wind spectrum, the following is observed:\textsuperscript{39,40}

**Observation 1** A wind spectrum depends on many factors and even when the same factors are used to generate a spectrum, the wind spectrum will never be the same. Figure 3.5 displays a scheme of factors that influence the along-wind load spectrum, which is obtained from (author?)\textsuperscript{530, page 6}.

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figure3.5.png}
  \caption{Scheme of factors that influence along-wind load spectrum}
\end{figure}

Since the wind spectrum is never the same, it is modeled as a stochastic process with a mean wind load and peaks. These are defined by making use of the Gumbel distribution; this is based on extreme value theory. The mathematical definition of wind load will be discussed later. The wind spectrum over time is constructed from different types of wind loads, namely gusts and vortices. The contribution of each type of wind load to the total wind load varies over time.

Structures under dynamic wind load are to be divided into two groups, namely conservative structures and non-conservative structures. The former does not exchange energy with the outside world while the latter does exchange energy. As for the latter case, the load on the structure depends on the structural motion of the building. This is also called an aeroelastic structure; in these type of structures the aerodynamic, elastic and inertial forces interact.

Regular along wind and torsional motions are typical movements for conservative structures. Non-conservative structures are more complex so a few examples are to be discussed now:

1. **Galloping**: An alternating motion perpendicular to the wind direction for long linear elements with a non-circular cross-section. The load is maintaining itself by motion. The model for galloping is displayed in Figure 3.6.a;

2. **Flutter**: When bending and torsion have about the same natural frequency, an amplified motion could occur for a strip shaped structure. This motion is called flutter; the model
for flutter is displayed in Figure 3.6.b. \cite{author??}, page 141-150 \cite{author??}, par. 10.6; page 87.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure36.png}
\caption{Galloping and flutter. a. Galloping and b. Flutter}
\end{figure}

\section{Important factors according to NEN-EN 1991-1-4}

To get a better understanding of the mathematical theory on wind loads, it is important to gain insight into the factors that influence the wind flow around a building in a certain area. A brief description is already presented in Section 3.1 and this section will further discuss the factors by NEN-EN 1991-1-4, accordingly.

- **Location**  The wind speed profile depends on the location the building is built at. The zero-plane displacement in the open field is at a lower altitude than in built-up areas, and therefore the wind speed will be higher at lower altitudes as well. This is displayed in Figure 3.7. The shape of the wind profile is explained by the decreasing angles of the shear layers along the height, as is displayed in Figure 3.3. At gradient height the velocity profile becomes vertical since $\alpha = 0$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure37.png}
\caption{Wind speed profile for different terrains}
\end{figure}

- **Wind area**: The Netherlands is divided into three different wind areas according to their basic wind speeds, see Table 3.1.
3.3. IMPORTANT FACTORS ACCORDING TO NEN-EN 1991-1-4

Figure 3.8: Wind area division in the Netherlands

Table 3.1: Basic wind speed per wind area in the Netherlands

<table>
<thead>
<tr>
<th>Wind area</th>
<th>Basic wind speed $v_{b,0}[m/s]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29.5</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>24.5</td>
</tr>
</tbody>
</table>

Figure 3.8 shows the area division on a map of the Netherlands. Correspondingly, the basic wind speeds are higher at sea level than at the inland. This is explained by the change of pressure between an area above the water and an area above the land. Usually, this pressure change is larger compared to a change between two pressure areas above the land.

- **Roughness:** The zero-plane displacement depends on the height of obstacles (such as buildings and trees) in a building’s surroundings. The deviation of heights of these obstacles determines the roughness of the terrain. This is also known as the texture of a surface area. The terrain is considered rough in case of large deviations, see Figure 3.9 (left), and smooth in case of small deviations, see Figure 3.9 (right).

Figure 3.9: Roughness rough surface area (left); smooth surface area (right)
In NEN-EN 1991-1-4 three different categories are considered, see Table 3.2 although Davenport has distinguished eight different classes. The roughness’s influence on the wind speed decreases as the altitude above the earth’s surface increases.\[27\]

<table>
<thead>
<tr>
<th>Terrain category</th>
<th>$z_0 [m]$</th>
<th>$z_{\text{min}} [m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0  Sea or coastal area</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>I  Area with few buildings</td>
<td>0.2</td>
<td>4</td>
</tr>
<tr>
<td>II Built-up area</td>
<td>0.5</td>
<td>7</td>
</tr>
</tbody>
</table>

The zero-plane displacement is moved upwards from ground level by the displacement height $d$. The velocity equals zero at height $z = z_0 + d$ in dense forests and cities. In general $d = 0.5h_{\text{surroundings}}$ to $0.7h_{\text{surroundings}}$. However, this is only applicable above the minimum height of $z$, also denoted as $z_{\text{min}} = 20z_0 + d$. The displacement height is not used in case of only a few obstacles in a building’s surroundings. In mathematical form this is denoted as:

$$v = 0 \quad \text{at} \quad z = \begin{cases} z_0 + d & \text{for } z > z_{\text{min}} \\ z_{\text{min}} & \text{for } z < z_{\text{min}} \end{cases}$$ \hspace{1cm} (3.2)

**Peak factor:** The peak factor is often approximated by $g \approx 3.5$. This value is obtained by the use of the dominant frequency in wind gusts, which is $0.13\text{Hz}$ for $\Delta t = 10\text{min}$ in the formula below.

$$g = \sqrt{2\log N}$$ \hspace{1cm} (3.3)

In Section 3.1 is explained that a wind flow changes by the approach of a bluff body. In NEN-EN 1991-1-4 these changes are adopted in coefficients that are used to define the wind load.

**Stagnation pressure:** Wind load causes pressure on a building and the wind flows towards both sides of the building as demonstrated in Figure 3.10. These flows are dependent on a building’s shape so the flow pattern changes for non-rectangular cross sections. However, the principle remains the same and thus the rectangular cross section will be used to explain the stagnation pressure principle.

![Figure 3.10: Principle of stagnation pressure](image)

In the top view of the building in Figure 3.10 different streamlines can be distinguished. One of these streamlines divides the flow in half and is called the stagnation streamline.
Above this line the flow goes to one side of the building and below this line it goes to the opposite side of the building. According to Bernoulli’s law the total pressure in a fluid remains the same at every point. Hence, the following expression is applicable on the stagnation line:

\[ p_e + \frac{\rho v_e^2}{2} = p_0 + \frac{\rho v_0^2}{2} \]  

(3.4)

In this expression \( p_e \) and \( v_e \) are the pressure and velocity at a certain distance from the building at the stagnation line. The air flows towards the building along the stagnation line. Since the flow cannot pass through the building, the air comes to a standstill at the point where the stagnation line meets the building. In Figure 3.10 this is displayed as the stagnation point. At the stagnation point applies \( v_0 = 0 \) and \( p_0 \). Therefore, the above expression can be simplified to:

\[ p_e + \frac{\rho v_e^2}{2} = p_0 \]  

(3.5)

Where \( p_e \) is the stagnation pressure and equals the sum of the static pressure and dynamic pressure. The stagnation pressure coefficient becomes:

\[ C_p = \frac{2(p_0 - p_e)}{\rho v_e^2} \]  

(3.6)

\( C_p \) is largest at the stagnation point where \( C_p = 1 \). The coefficient equals one at the stagnation point but at any other point it is smaller than one. In the wake of a building the vortices cause a negative pressure coefficient varying between -0.5 and -1.5. At the sides of the building \( C_p \) varies between -0.5 and -1.0. Figure ?? shows the values of \( C_p \) for every facade of the building. The pressure coefficient applies to solid structure and thus to buildings.

### 3.4 Wind load

Wind loads are dependent on many factors as is discussed in the previous sections. In addition to that, its fluctuations over time are considered random which makes the overall definition rather complex. The total wind load on a building as a whole is equal to the summation of loads on parts of the surface area of the building. In mathematical form this leads to the following equations:

\[ F_w = \sum C_{p,i} A_i q_{w,i} \]  

(3.7)

\[ q_w = \frac{1}{2} \rho \bar{v}^2 = \frac{1}{2} \rho (\bar{v} + \tilde{v})^2 = \frac{1}{2} \rho \bar{v}^2 + \frac{1}{2} \rho \tilde{v} \bar{v} \]  

(3.8)

Where \( q_w \) is a function for the wind speed. The wind spectrum over time is modeled as a stochastic process. Therefore, the wind speed in Equation 3.8 is split up into a mean \( \bar{v} \) and fluctuating part \( \tilde{v} \), see also Figure 3.11. In the equation the factor \( \tilde{v}^2 \) is being neglected because it is relatively small.
Chapter 3. Dynamic Wind Load

Due to the random character of the wind, the load and response function of the wind load must be obtained by means of a spectral analysis. In this part of the thesis the mathematical definition of these functions is explained step by step in accordance with NEN-EN 1991-1-4.

First, the wind spectrum of wind velocities must be defined mathematically. This is used as input in order to find the dynamic wind load on a structure. Then, the dynamic response can be formulated. The steps needed to be taken are displayed in Figure 3.12.

\[ x(t) = x_0 + \sum_{j=1}^{N} \hat{x}_j \sin(\omega_j t + \theta_j) = x_0 + \sum_{s=1}^{N} x_j(t) \]  

3.4.1 Mathematical theory on stochastic processes

Although regular harmonic loads are not applicable to a structure loaded by wind, the wind load can be modeled as a summation of sine function. Therefore, the stationary function in a stochastic process as is displayed in Figure 3.13, is expressed by:
In this expression, the phase angle $\varphi_j$ is generated randomly.

The average of the process is:

$$\bar{x} = \frac{1}{T} \int_0^T x(t) dt = x_0 \quad (3.10)$$

In statistics, this is also known as the mean ($\mu$) of the process. The variance of the spectrum is expressed by:

$$\sigma^2_x = \frac{1}{T} \int_0^T [x(t) - \bar{x}]^2 dt = \sum_{k=1}^N \frac{1}{2} x_j^2 \quad (3.11)$$

Both the mean and standard deviation are constant in time. Hence, the process is a stationary process with respect to these quantities. The variance spectrum of the process becomes:

$$S_{xx}(\omega_j) = \frac{1}{2} x_j^2 \frac{\Delta \omega}{\Delta \omega} \quad \text{with} \quad \Delta \omega = \frac{1}{T}; \quad \omega_j = \frac{s}{T} \quad (3.12)$$

So the variance of the spectrum equals:

$$\sigma^2_x = \int_0^\infty S_{xx}(\omega) d\omega \quad (3.13)$$

The variance of the spectrum shows the distribution of the total variance over the frequencies.

The Fourier transform of the spectrum for $N \to \infty$ is expressed by:

$$S_x = \frac{1}{\pi} \int_{-T/2}^{T/2} x(t) e^{-i\omega t} dt = \frac{1}{\pi} \int_{-T/2}^{T/2} x(t) [\cos(\omega t) - i \sin(\omega t)] dt \quad (3.14)$$

with $T \to \infty$

If:

$$A(\omega) = \frac{1}{\pi} \int_{-T/2}^{T/2} x(t) \sin(\omega t) dt \quad \text{with} \quad T \to \infty \quad (3.15a)$$

$$B(\omega) = \frac{1}{\pi} \int_{-T/2}^{T/2} x(t) \cos(\omega t) dt \quad \text{with} \quad T \to \infty \quad (3.15b)$$
Then, Equation 3.14 reduces to:

\[ S_x = B(\omega) - iA(\omega) \]  
\[ S_x^* B(\omega) + iA(\omega) \]  

(3.16a)

(3.16b)

Where \( S_x \) and \( S_x^* \) are each other’s conjugates. It holds that:

\[ S_x(\omega)S_x^*(\omega) = (B - iA)(B + iA) = B^2 + A^2 \]  

(3.17)

And thus the spectrum can be rewritten as:

\[ S_{xx} = \frac{1}{2} \{ A^2(\omega) + B^2(\omega) \} \Delta\omega \]  

(3.18)

With \( \Delta\omega = \frac{2\pi}{T} \) the expression for the spectrum is expressed by:

\[ S_{xx} = \frac{\pi}{T} S_x(\omega)S_x^*(\omega) \]  

(3.19)

For more information on stochastic processes see\(^{[40]}\). The input on the system is considered as a sine function and consequently, the output of the system is a sine function with a different amplitude and a phase shift, see Equations 3.20a and 3.20b.

\[ x = \hat{x} \sin (\omega t + \theta) \quad \text{input} \]  
\[ y = |H|\hat{x} \sin (\omega t + \theta + \psi) \quad \text{output} \]  

(3.20a)

(3.20b)

This leads to the following set of equations:

\[ S_{xx}(\omega) = \lim_{\Delta\omega} \frac{1}{2} \frac{\hat{x}^2(\omega)}{\Delta\omega} \]  

(3.21a)

\[ S_{yy}(\omega) = \lim_{\Delta\omega} \frac{1}{2} \frac{\hat{x}^2(\omega)|H(\omega)|^2}{\Delta\omega} = |H(\omega)|^2 S_{xx}(\omega) \]  

(3.21b)

### 3.4.2 Wind load in along wind direction

The theory of the previous section will now be adopted to a dynamical system loaded by wind. First, the mathematical theory is applied to a dynamic wind load. Next, formulas from NEN-EN 1991-1-4 are adopted for a description of the wind spectrum and the response of a building.

**Application of theory**

**Gust spectral density function**

The gust spectral density function is described by:

\[ S_{vv} = \sigma_v^2 F_D \]  

(3.22)

In this equation \( \sigma_v \) is the standard deviation and \( f \) is the frequency of the wind speed. \( F_D \) is the autospectrum of the wind load, which is used for calculations in different directions. The autospectrum used in NEN-EN 1991-1-4 is Solari’s spectrum:

\[ F_D = \frac{6.8x}{(1 + 10.2x)^{5/3}} \quad \text{where} \quad x = \frac{fL}{\bar{v}(10)} \]  

(3.23)

Where \( L \) is the gust length and \( \bar{v}(10) \) is the mean wind speed at a height of 10 meters.
Intermezzo 1  The wind load is never at its peak value at two different points at the façade at the same time. Therefore, a relation must be obtained to describe this phenomenon. This relation is called coherence in the frequency domain and correlation in the time domain. The coherence takes the size of the gusts and different phases at these two points at a certain time into account. The gust spectral density function of multiple points is described by:

\[ |S_{v_i v_j}|^2 = S_{v_i v_i}(f)S_{v_j v_j}(f)\text{coh}_{v_i v_j}(f) \] (3.24)

Now from Intermezzo 1 it follows that the new spectrum becomes:

\[ S_{\bar{v}_1 \bar{v}_2} = \text{coh}_{\bar{v}_1 \bar{v}_2} \sqrt{S_{\bar{v}_1 \bar{v}_1}S_{\bar{v}_2 \bar{v}_2}} = \text{coh}_{\bar{v}_1 \bar{v}_2} \sigma_{\bar{v}_1} \sigma_{\bar{v}_2} \frac{F_D}{f} \] (3.25)

Where the coherence is expressed by:

\[ \text{coh}_{\bar{v}_1 \bar{v}_2} = \exp \left( -\frac{f}{\bar{v}_{10}} \sqrt{C_z^2(z_1 - z_2)^2 + C_y^2(y_1 - y_2)^2} \right) \] (3.26)

**Aerodynamic admittance**

The aerodynamic admittance to the load spectrum is expressed by:

\[ \chi^2 = \frac{1}{A^2} \int_A \int_A \left( \frac{C_1 \bar{v}_1 \sigma_{\bar{v}_1}}{C_h \bar{v}_h \sigma_{\bar{v}_h}} \right) \left( \frac{C_2 \bar{v}_2 \sigma_{\bar{v}_2}}{C_h \bar{v}_h \sigma_{\bar{v}_h}} \right) \text{coh}_{\bar{v}_1 \bar{v}_2} dA_1 dA_2 \] (3.27)

**Load spectrum**

The next step is to find the load spectrum by implementation of the gust spectral density function and the aerodynamic admittance:

\[ S_{f f}(z,n) = \chi^2 \{C_p \rho \bar{v}(z)A \}^2 S_{v v} \] (3.28)

\[ C_p \rho \bar{v}(z)A \] is squared because the same coordinates on the façade are chosen for point 1 and 2. Now the variance of the load spectrum is expressed by:

\[ \sigma_f^2 = \int_0^{\infty} S_{f f} df \] (3.29)

**Mechanical admittance**

The mechanical admittance to the dynamic system is represented by:

\[ H(\omega) = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\zeta \omega_0}{\omega_0}\right)^2}} \] (3.30)

This value will be derived later in Section 4.2.1.

**Response spectrum**

The response spectrum consists of a quasi-static part and a dynamic part. The total response of a system is obtained by:

\[ S_{uu} = \chi^2 |H(\omega) - H(0)|^2 S_{f f} \] (3.31)
CHAPTER 3. DYNAMIC WIND LOAD

$H(0)$ is the contribution of the quasi-static part where all terms with respect to time are neglected. If the quasi-static part is neglected and the real spectrum is approximated by a white-noise spectrum (see Definition 2) $S_0 = S_{ff}(\omega_e)$, see figure 3.14, the dynamical contribution to the response spectrum becomes:

$$S_{u_D u_D} = \chi^2 |H(\omega)|^2 S_{ff}(\omega_e)$$  \hspace{1cm} (3.32)

**Definition 2** A white-noise spectrum consists of equal contributions of the loads per frequency. In other words: the load distribution is constant over time and the definition of the spectrum becomes:

$$S_{ff}(\omega) = S_0 = constant$$  \hspace{1cm} (3.33)

The approximation of a load spectrum by a white-noise spectrum is displayed in Figure 3.14.

![Figure 3.14: Approximation by a white noise spectrum of the load spectrum](image)

Again, the variance of the dynamic part of the response spectrum is obtained from the surface area of the spectrum:

$$\sigma_{u_D}^2 = \chi^2 \int_{0}^{\infty} |H(\omega)|^2 S_{ff}(\omega_e) = \chi^2 \left\{ \frac{\pi \omega_e S_{ff}(\omega_e)}{4\zeta k^2} \right\}$$  \hspace{1cm} (3.34)

The total variance now becomes:

$$\sigma_u^2 = \chi^2 \left\{ \sigma_{u Q S}^2 + \sigma_{u D}^2 \right\} = \chi^2 \left\{ \frac{\sigma_f^2}{k^2} + \frac{\pi \omega_e S_{ff}(\omega_e)}{4\zeta k^2} \right\}$$  \hspace{1cm} (3.35)

**Application according to NEN-EN 1991-1-4**

According to NEN-EN 1991-1-4 the wind load on a structure must be determined by:

$$q_w = c_sc_d c_f q_p(z_e)$$  \hspace{1cm} (3.36)

$c_s c_d$ = Structural factor that takes the shape of the building and dynamic wind effects into account

$c_f$ = Force coefficient for the structure or structural element

$q_p(z_e)$ = Extreme wind pressure at reference height $z_e$

These factors are obtained from other factors that are based on the theory explained in the previous section.
The mean wind load is found by:

\[ \bar{v} = v_{b,0} k_r \ln \left( \frac{z}{z_0} \right) \]  

(3.37)

\[ k_r = 0.19 \left( \frac{z_0}{0.05} \right)^{0.07} \]  

(3.38)

The value for \( v_{b,0} \) depends on the wind area and is displayed in Table 3.1 and the roughness length is displayed in Table 3.2.

**Gust spectral density function**

In the spectral density function for wind speed \( S_L(z, n) \), the wind speed is divided over the frequencies for one point in a building:

\[ S_L(z, n) = \frac{6.8 f_L(z, n)}{[1 + 10.2 f_L(z, n)]^{5/3}} \]  

(3.39)

The dimensionless frequency \( f_L(z, n) \) in Equation 3.39 is expressed by:

\[ f_L(z, n) = \frac{n L(z)}{\bar{v}(z)} \]  

(3.40)

\[ L(z) = 300 \left( \frac{z}{200} \right)^{\alpha} \quad \text{where} \quad \alpha = 0.67 + 0.05 \ln (z_0) \]  

(3.41)

In Equations 3.40 and 3.41 the turbulence scale \( L(z) \) describes the average size of a wind gust and \( f_L(z, n) \) is determined by the eigenfrequency of the building. Now the wind spectrum \( S_{vv}(z, n) \) is obtained by:

\[ S_{vv}(z, n) = \frac{\sigma_v^2}{n} S_L(z, n) \]  

(3.42)

The factor \( \sigma_v^2 \) is called the variance of the spectrum, where \( \sigma_v \) is the standard deviation of the spectrum.

**Aerodynamic admittance**

The aerodynamic admittance to the load spectrum is expressed by:

\[ \chi^2 = \frac{1}{\eta} - \frac{1}{2 \eta^2} [1 - e^{-2\eta}] \quad \text{where} \quad \eta = \frac{4.6 h}{L(z_e)} f_L(z, n) \]  

(3.43)

**Load spectrum**

The load spectrum may be obtained from:

\[ S_{ff} = \chi^2 \{ C_p \rho \bar{v}(z) A \}^2 S_{vv} \]  

(3.44)

**Response spectrum**

The response spectrum is found by multiplication of the load spectrum with the mechanical admittance function, which is the same as discussed in the previous section. A more detailed description regarding calculations with NEN-EN 1991-1-4 is in Appendix ??.
3.4.3 Torsional wind load

In all codes torsional wind loads are neglected in regular cases and are only considered in case of aeroelastic structures. This arises from the assumption that wind loads on a building are uniformly distributed across its faces, and the resultant shear force acts in the direction of the wind. However, this is rarely the case since torsional motion is possibly caused by the following causes:

1. Wind is a stochastic process which results in an unequal pressure distribution along the façade. In its turn, this unequal distribution causes a torsional motion of the building.
2. Wind acting at a random angle to the building
3. Asymmetry in mass and stiffness of the building
4. Non-symmetric cross-sections.\textsuperscript{[22]}

Torsional motion is rarely taken into account but it is important that it will be taken into account in the future. The main reason for this is the bi-lateral motion, which persons in the building are more sensitive to with respect to accelerations. This was also discussed in section 4.3.2. High rise structures become more complex and asymmetry in the structure is not a rare phenomenon.

In literature, models for torsional motion are mainly constructed from measurements in wind tunnels and of pressure areas at existing buildings. Nevertheless, the ideal situation for the structural engineer would be to have insight into the structure’s behavior in the preliminary design phase. In case of unwanted torsional motion of the structure, the design can still be adjusted rather easily in this phase of the building process. In addition to that, measurements in wind tunnels are expensive so an accurate theoretical model is also desirable with respect to the reduction of costs.

It is expected such model will be suitable for structures with symmetry in cross-section, mass and stiffness. Non-symmetric structures should still be tested in wind-tunnels or the model should be adjusted to implement these properties.

Table 3.3 shows a comparison of literature. Together with the response spectrum for lateral load, this comparison is used to derive an expression for torsional wind load on buildings.

3.4.4 Summation of loads

If the eigenfrequencies of the torsional and translational motion are about the same, the deformation shape will be a summation of both motions. In that case also the velocity and acceleration profile will contain a torsional and translational component. However, unlike the summation of displacements of both motions the summation of accelerations is slightly different. \textsc{Reference Literature} states that the equivalent acceleration in the along wind direction is found by:

$$a_{\text{equivalent}} = \sqrt{a_{\text{along}}^2 + a_{\text{torsional}}^2}$$  \hspace{1cm} (3.45)

In this formula the maximum accelerations of both motions are taken into account but the actual acceleration is reduced by making use of the square root of the equation.
### Table 3.3: Comparison of literature

<table>
<thead>
<tr>
<th>Literature</th>
<th>Basis</th>
<th>Domain</th>
<th>Assumptions</th>
<th>Mean torque coefficient</th>
<th>Spectral density function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liang</td>
<td>Empirical</td>
<td>Frequency</td>
<td>sdfsf</td>
<td>sfdsfs</td>
<td>sfdfsdf</td>
</tr>
</tbody>
</table>
| Elsevier   | Empirical | Frequency | sdfsf       | sfdsdf                   | \[
\frac{fs_{ST}(f)}{\sigma^2_{ST}} + B_3 f_{ST}(f)\frac{fs_{SV}(f)}{\sigma^2_{SV}} + B_4 f_{ST}(f)\frac{fs_{VP}(f)}{\sigma^2_{VP}}
\] |
Chapter 4

Structural dynamics

In this chapter the structural dynamics of a high rise structure is considered. First, the directions of motion will be discussed after which simple models of structures are presented. This model will be extended to a more complex model for other structures. Then, important dynamic properties with respect to a high rise building will be discussed.

4.1 General

In a dynamical system with one mass, spring and damper three different directions of motion can be distinguished. The elements in the dynamic system are:

1. **Mass**: Represents the mass of the system in [kg]

2. **Spring**: Represents the spring stiffness of the system in [N/m]. The spring takes the bending stiffness of the structure into account.

3. **Damper**: Dissipates energy from the system and transfers this energy to heat. The damping device for the linear model is a dashpot system; this system resists motion by means of viscous friction. Dampers will be further discussed in Chapter ??.

The three considered directions are displayed in Figure 4.1:

1. **Alongwind direction**: main concern for slab like structures

2. **Cross-wind direction**: main concern for buildings with a rectangular or circular floor plan

3. **Torsional displacement**: main concern for buildings with asymmetry in shape, mass or stiffness[30] page 2

The dynamic response of a building is influenced by many factors. These factors are shape, mass, stiffness and damping from which damping is the most effective one.[30] page 3

4.2 Systems

The simplest dynamic model is the one degree of freedom model with one mass, a spring and a damper. Once the the corresponding equations are clear for this type of model, the
4.2. SYSTEMS

4.2.1 Mass spring damper systems

External loads on a system cause displacement, velocities and accelerations. Once a system is loaded, the motion of the structure is disturbed for a certain time period. This is the transient phase of the motion. In case of a stable system, the transient phase will last for a while but eventually it will return to its undisturbed and stable motion pattern. This is called the steady-state motion of the system. In case of a stable structure, it is important to check the steady-state motion of the system. The way to do this will be discussed in this section. First, only single mass spring damper systems (1-MSD systems) are considered.

Alongwind direction

The 1-MSD system for the alongwind direction is displayed in Figure 4.2. This system is a 1-MSD system since the mass will only move vertically in the $x_1$-direction.
The application of Newton’s laws and the equilibrium of forces to the system in Figure 4.2, leads to the equation of motion for the 1-MSD system:

\[ m\ddot{x} + c\dot{x} + kx = F_x(t) \]  

(4.1)

In this second order differential equation, \( x \) is the displacement, \( \dot{x} \) is the velocity and \( \ddot{x} \) is the acceleration in the along-wind direction. \( F_x(t) \) is an external load acting on the structure. The load can be described as a sine or cosine function:

\[ F_x(t) = F_0 \cos(\omega t + \theta) \]  

(4.2)

Where \( \theta \) is the phase shift which equals \( \pi/2 \) in case of a sinusoidal load.

Since the external load is co-sinusoidal, it is likely for the motion of the system to be co-sinusoidal as well:

\[
\begin{align*}
X_1 &= \text{Re} \{X_0 e^{i\omega t}\} \quad (4.3a) \\
\dot{X}_1 &= \text{Re} \{i\omega X_0 e^{i\omega t}\} \quad (4.3b) \\
\ddot{X}_1 &= \text{Re} \{-\omega^2 X_0 e^{i\omega t}\} \quad (4.3c)
\end{align*}
\]

In case of a sinusoidal load the solution must be sought for by taking the imaginary part of the exponent. Substitution of Equation 4.3a into Equation 4.1 leaves:

\[ \text{Re} \{(-\omega^2 m + i\omega c + k) X_0\} = F_0 \quad (4.4) \]

From which \( U_0 \) can be obtained:

\[ X_0 = \frac{F_0}{k - \omega^2 m + i\omega c} = \frac{F_0}{a + ib} \quad \text{where} \quad a = k - m\omega^2, \quad b = \omega c \]  

(4.5)

**Theorem 1** Rules trigonometry

\[ e^{i\omega t} = \cos(\omega t) + i \sin(\omega t) \]  

(4.6)

With the application of Theorem 1, the steady-state solution of the equation of motion becomes:

\[
\begin{align*}
X_1(t) &= \text{Re} \{X_0 e^{i\omega t}\} \\
 &= \text{Re} \left\{ \frac{F_0}{a - ib} e^{i\omega t} \right\} \\
 &= \text{Re} \left\{ \frac{F_0 (a - ib)}{a^2 + b^2} (\cos(\omega t) + i \sin(\omega t)) \right\} \\
 &= \frac{1}{a^2 + b^2} \text{Re} \left\{ [aF_0 - ibF_0] [\cos(\omega t) + i \sin(\omega t)] \right\} \\
 &= \frac{1}{a^2 + b^2} [F_0a \cos(\omega t) + F_0b \sin(\omega t)] \\
 &= \frac{F_0}{(k - m\omega^2)^2 + (\omega c)^2} \left[ (k - m\omega^2) \cos(\omega t) + \omega c \sin(\omega t) \right]
\end{align*}
\]

(4.7)
The amplitude of the displacement is obtained from:

\[
\hat{X}_1(t) = \frac{F_0}{F_0} \sqrt{\left(\frac{k - m\omega^2}{(k - m\omega^2)^2 + (\omega c)^2}\right)^2 + \left(\frac{c\omega}{(k - m\omega^2)^2 + (\omega c)^2}\right)^2}
\]

\[
= \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (\omega c)^2}}
\]

\[
= \frac{F_0}{k\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\omega_\zeta}{\omega_0}\right)^2}}
\]

(4.8)

Where \( \omega_0^2 = k/m, c = 2k\zeta\omega_0 \) and \( F_0/k \) is the static amplitude. The frequency response function of the 1-MSD system now becomes:

\[
H(\omega) = \frac{X_{dyn}}{X_{stat}} = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\omega_\zeta}{\omega_0}\right)^2}}
\]

(4.9)

The shape of the frequency response function is displayed in figure 4.3. The peak in the graph of this figure is at the point where \( \omega = \omega_0 \). Consequently, this is the governing situation for the system.

![Mechanical admittance](image)

*Figure 4.3: Frequency response function of the 1-MSD system*

The same coefficients are used to determine the phase shift \( \theta(\omega) \), which is expressed by:

\[
\theta(\omega) = \frac{-1}{\frac{F_0(k - m\omega^2)}{(k - m\omega^2)^2 + (\omega c)^2}} = \arctan\left(\frac{-c\omega}{k - m\omega^2}\right)
\]

(4.10)

To determine the eigenfrequencies of the system, the load and damping is now set to zero. The form of the solution is still sought for in the same form as in Equation 4.3a. The following equation is now obtained:

\[
X_0 (-\omega^2 m + k)
\]

(4.11)
Since $X_0 \neq 0$, it must hold that:

$$k - m\omega^2 = 0 \quad (4.12)$$

With $m \neq 0$ and $k \neq 0$, the solution to this equation leaves the natural frequency $\omega$ of the 1-MSD system:

$$\omega = \sqrt{\frac{k}{m}} \quad (4.13)$$

**Torsional direction**

Now the external load is replaced by an external moment as is displayed in figure 4.4, and thus the equation of motion with respect to torsion becomes:

$$J\ddot{\phi} + c_r \dot{\phi} + k_r \phi = M_0 \cos (\omega t + \theta) \quad (4.14)$$

![Figure 4.4: 1-MSDS for torsion](image)

Again, the co-sinusoidal character of the moment is used by assuming the following motion:

$$\Phi = Re \{\Phi_0 e^{i\omega t}\} \quad (4.15)$$

The same procedure is used as in the previous situations to find the steady-state solution to this equation:

$$\Phi = \frac{M_0}{(k_r - J\omega^2)^2 + (\omega c)^2} \left[ (k_r - J\omega^2) \cos(\omega t) + \omega c \phi \sin(\omega t) \right] \quad (4.16)$$

Making use of $\omega_0^2 = k_r/J$ and $c_r = 2k_r\zeta\omega_0$ and $M_0/k_r$ as the static amplitude, the frequency response function with respect to torsion becomes:

$$H(\omega) = \frac{X_{dyn}}{X_{stat}} = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{2\zeta}{\omega_0}\right)^2}} \quad (4.17)$$
Multiple degrees of freedom system

Until now the 1MSD system with one degree of freedom is considered. The loads applied to the system did not interact and were not applied simultaneously. Now what happens if the load in x-direction and the external moment are applied at the same time? This will be demonstrated by an example. The considered system is the same as before but is now assumed to have three degrees of freedom instead of one. Its assumed directions for displacements are displayed in figure 4.5.

A general equation of motion is described by:

\[ M \ddot{x}_i + C \dot{x}_i + K x_i = F_i \cos(\omega t + \theta) \quad i = 1, 2, 3 \]  

(4.18)

Where \( M, C \) and \( K \) are the mass-, damping- and stiffness matrices respectively. For each direction of motion a balance of forces can be obtained:

\[ m \ddot{x}_1 = -kx_1 - cx_1 - \frac{a}{2}kx_3 + \frac{a}{2}cx_3 + F(t) \]  

(4.19a)

\[ m \ddot{x}_2 = 0 \]  

(4.19b)

\[ J \ddot{x}_3 = -\frac{a}{2}kx_1 + \frac{a}{2}cx_1 - \frac{a^2}{4}kx_3 - \frac{a^2}{4}cx_3 + M(t) \]  

(4.19c)

Corresponding to Equation 4.18, this system can be written in matrix notation:

\[
\begin{bmatrix}
  m & 0 \\
  0 & J
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_3
\end{bmatrix}
+ \begin{bmatrix}
  1 & \frac{-a}{2} \\
  \frac{-a}{2} & \frac{-a^2}{4}
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_3
\end{bmatrix}
+ \begin{bmatrix}
  1 & \frac{a}{2} \\
  \frac{a}{2} & \frac{a^2}{4}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_3
\end{bmatrix} = \begin{bmatrix}
  F(t) \\
  M(t)
\end{bmatrix}
\]

(4.20)

From this system it is clear that the assumed three degrees of freedom system is actually a two degrees of freedom system. This is the case because no spring and dashpot elements are attached in the \( x_2 \)-direction and no external force is applied either. Hence, \( m \ddot{x}_2 = 0 \).

If such element or load were applied to the system, the system would be a three degrees of freedom system. In addition to that, the angular motion \( x_3 \) would be coupled both to the horizontal and vertical direction instead of just to the vertical direction \( x_1 \).
The steady-state solution of this system is expressed by:

\[ x = \text{Re} \left\{ (X_1 + X_2 + X_3) e^{i\omega t} \right\} \]
\[ = \text{Re} \left\{ \sum_{i=1}^{i=3} X_i \right\} e^{i\omega t} \]
\[ i = 1, 2, 3 \]  \hspace{1cm} (4.21)

The natural frequencies of the system are found by solving the following set of equations:

\[
\begin{bmatrix}
  m - \omega^2 & 0 & 0 \\
  0 & J - \omega^2 & 0 \\
  0 & 0 & k
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_3 \\
  x_3
\end{bmatrix}
+ \begin{bmatrix}
  1 & a & 0.5 \\
  a & a^2 & 1
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0
\end{bmatrix} \]
\hspace{1cm} (4.22)

4.2.2 Continuous systems

In the previous section a MSD systems were considered. However, structures could often be modeled as a continuous system as well. In a high rise structure the dominating axis is the height of the building. Its corresponding model is a clamped beam loaded by wind, see Figure 4.6.

Corresponding to the model, the differential equation becomes:

\[ \rho A \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} = F(t) \]  \hspace{1cm} (4.23)

Equation 4.23 is adopted according to the Euler-Bernoulli beam theory. Figure 4.7 shows an element $dx$ of an Euler-Bernoulli beam from which the Differential Equation 4.23 is derived. It takes into account the shear deformation as well as the bending deformation of the beam. This theory is based on the basic hypothesis that the plane cross-sections remain perpendicular to the neutral axis of the beam during bending. The Timoshenko beam theory relaxes the assumption of plane and normal cross sections and does take into account a certain shear angle during bending. As for this thesis this shear angle is irrelevant and therefore the Euler-Bernoulli beam theory will be elaborated on further. The solution to Equation 4.23 can be sought for in the following form:

\[ w(x, t) = \sum_{n=1}^{\infty} \psi_n(t) W(x) \]  \hspace{1cm} (4.24)
In the above equation $W(x)$ corresponds with the normal modes of the system. In accordance with the clamped beam in Figure 4.6, the boundary conditions of the model become:

\[ w(0, t) = 0 \]  
\[ \frac{\partial u}{\partial x}\bigg|_{z=0} = 0 \]  
\[ EI \frac{\partial^3 u}{\partial x^2} \bigg|_{z=h} = 0 \]  
\[ EI \frac{\partial^4 u}{\partial x^4} \bigg|_{z=h} = 0 \]

The steady state solution of the problem can again be sought for in the form of Equation 4.3a. What remains is a differential equation which is dependent on the coordinate only and therefore it is solvable. However, in case of additional masses, springs or dampers the differential equation becomes more complex. Two methods can be employed to find the steady-state solution of the new system:

1. The equilibrium of forces of the structures has an additional component which could be implemented into the differential equation by means of a delta function. The delta function is the mathematical representation of a pulse. If a MSD-system is connected to the beam at a certain coordinate $z_a$, the balance of forces of this system could be multiplied by the delta function at the point $z_a$. Hence, the practical representation of the delta function is to take into account a situation at some coordinate that is different from the rest of the beam. In this case, the balance of forces is changed at the coordinate $z_a$.

2. At $z_a$ new boundary conditions (interface conditions) can be introduced to maintain the balance of forces at this point. The system is now split into two fields and thus the displacement of the system is described by these two fields. Again, the solution can be sought for in the form of Equation 4.3a. What remains is the solution dependent on the $z$-coordinate only to describe the displacement of the beam along the $z$-axis.\[24;33\]

### 4.2.3 Comparison of methods

The MSD systems are easy to adopt when only a few MSDs are considered. It becomes more complex for NMSD systems especially when MSDs are attached to the system in different directions. The Delta function is an easy way to describe the motion of a system when a MSD is attached to the clamped beam. However, if the number of MSDs increases, the model becomes more complex to solve analytically. Nevertheless, computationally the
solution can be derived quite easily. Also, the beam its motion is described by a single equation instead of multiple equations, which is the case for the other methods.

The method of finding a solution by the application of boundary conditions is simple in case of a few fields since it makes use of basic similarities at interfaces. In case of a larger number of MSDs, the number of boundary conditions increases by \( n \times 4 \) (in case of a beam). All boundary conditions should be derived analytically but the solution to the problem could be found computationally.

The latter two methods take into account the exact position of the MSDs while the former does not. Additionally, the clamped beam could be extended to a larger structure more easily, for instance for an outrigger structure.

The first method is better applicable in case of combined motions, such as lateral and torsion. The matrix vector system clearly shows how the motions are coupled.

### 4.3 Structural properties of high-rise structures and its dynamic response

In this part the requirements with regard to high rise structures will be discussed.

#### 4.3.1 Slenderness

The height of a structure (\( h \)) in itself is not governing for wind-induced dynamic responses. Increasing the height of a building with the same floor plan (where the depth is proportionally growing with increasing height) leads to more mass in a building which has a positive effect on the accelerations. the slenderness or aspect ratio of a building is defined as \( h/d \) and is kept constant for this situation. This is displayed in Figure 4.8(middle).

Now, when the depth (\( d \)) is kept constant and the height of the building increases, the aspect ratio increases, see Figure 4.8(right). The higher the aspect ratio, the higher the accelerations become as a consequence of reduced resistance of the structure to the loads. It also leads to higher stresses at the base of the building. Oosterhout made a graph for
the acceleration as a function of the aspect ratio, see Figure 4.9. Here the depth was kept to 20 meters and the height was varied between 8 and 240 meters. This graph clearly shows a large increase of the acceleration below $h_r = h/d < 5$. The function becomes approximately linear for values of $h_r > 5$.

![Figure 4.9: Acceleration as a function of the aspect ratio](image)

In Figure 4.10 a building with a constant and increasing aspect ratio are considered. From this graph it is clear that a constant value of the aspect ratio meets the requirements from the ISO criterium. A variable aspect ratio on the other hand does not meet the requirements and requires adjustment in the structural design of the building.\cite{30} page 46-51

![Figure 4.10: Acceleration as a function of the aspect ratio ISO](image)

As mentioned in Chapter 2, structures in the Netherlands are not as tall as structures elsewhere in the world. However, the slenderness of Dutch buildings and other buildings is the same. Hence, the wind-induced dynamic response is still a problem for buildings in the Netherlands even for lower buildings. Therefore, the definition of a high rise building in chapter 1 from Smith et al., could be extended to a better applicable definition, see Definition 3.
Definition 3  High rise structures are buildings which are affected by lateral forces due to wind actions with an aspect ratio of $h/d\geq 5$ because for these buildings the dynamic response is largest which plays an important role in the structural design of the building.\cite{35} page 11\cite{35} page 16

4.3.2 Accelerations

Due to the height of the structure, accelerations in high rise buildings could increase to levels which are experienced as uncomfortable to people in the building. It depends on a persons sensitivity to acceleration if acceleration is observed and therefore each person has a different perception threshold. It depends on several factors if acceleration is observed:

1. **Frequency / period of the building:** perception threshold increases when the frequency decreases. This does not apply to very large frequencies, for which the effect is adverse.

2. **Age:** children are more sensitive to motion than adults

3. **Body posture:** the sensitivity to motion is proportional to the distance of a persons head to the floor. A person lying or sitting down is therefore less sensitive to motion than a standing person

4. **Expectancy of motion:** prior knowledge of motion leads to a reduction of 50% of a perception threshold

5. **Body movement:** a moving person perceives motion at a higher acceleration level than a person that is not moving

6. **Visual cues:** visual signs confirm a persons perception to motion and it tends to exaggerate the magnitude of occurring motion

7. **Acoustic cues:** persons find the same acceleration level less acceptable for higher values of an acoustic level caused by wind

8. **Type of motion:** persons can experience axial and bi-axial motion. The former is motion in one direction and the latter in two directions, which is the case for the torsional movement. In that case, a person experiences balance shifts in two orthogonal direction. Torsional motion is mostly perceived by visual signs and therefore are experienced at a lower perception threshold than lateral motion.

The perception threshold also depends on the function of the building. For example, restrictions of residential buildings should be more severe than those of a commercial building. A person spends more time at home and thus the chances of exceeding a persons perception threshold increases. In addition to that, persons are less tolerant to their home environment than to their working environment.\cite{35} page 26-31

Perceptions of accelerations are very subjective because the sensitivity to it varies from person to person. Nevertheless, it is certain the frequency plays a dominant role in the perception of accelerations regardless of a persons sensitivity. In NEN-EN 1990+A1+A1/C2-2011/NB:2011 the allowable value for acceleration is displayed as a function of frequency, see Figure 4.11. Actually two graphs are displayed here, because of the distinction between residential (function 2) and other buildings (function 1).\cite{37} page 192-193\cite{28}
4.3. STRUCTURAL PROPERTIES OF HIGH-RISE STRUCTURES AND ITS DYNAMIC RESPONSE

4.3.3 Modal mass

The modal mass or equivalent mass of a building is defined as the part of the building which contributes to the dynamic behavior of the building per mode shape. It is usually displayed as a percentage of the total mass of the building. This percentage depends on the mode shape of the building which in its turn depends on the eigenfrequencies of the building. Mathematically, the modal mass of building can be derived through:

\[
m^* = \int_0^h m(z)\phi^2(z)dz
\]  

(4.26)

In Equation 4.26, \( m(z) \) is the mass per unit height and \( \phi(z) \) is the eigenvector matrix of the building and defines a function for the first mode shape. This equation is solved for the first mode shape in [Jeary, 1981, page 47-49] for two important types of structures, namely the cantilever beam and shear beam. The mode shapes for the fundamental frequency are determined, which is used to find the modal mass, see Table 4.1.

<table>
<thead>
<tr>
<th>Type of beam</th>
<th>( \phi(z) )</th>
<th>( m^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever (bending) beam</td>
<td>( 1-\cos(\pi z/(2H)) )</td>
<td>0.227 m(_{tot})</td>
</tr>
<tr>
<td>Shear beam</td>
<td>( \sin(\pi z/(2H)) )</td>
<td>0.5 m(_{tot})</td>
</tr>
</tbody>
</table>

A more general definition of modal mass is presented in [Irvine, 2012, page 2]:

\[
m^*_i = \phi_i^T M \phi_i
\]  

(4.27)

This formula enables a calculation of the modal mass for the \( i \)-th frequency. Similar formulas hold for modal stiffness and modal damping of a building respectively [Jeary, 1981, page 35]:

\[
k^*_i = \phi_i^T K \phi_i
\]  

(4.28)

\[
c^*_i = \phi_i^T C \phi_i
\]  

(4.29)
These equations do not have to be solved for each type of structure. Oosterhout also estimated the error of neglecting the second mode shape of a structure. This error is negligible and therefore the assumption made to only take into account the first mode shape, is acceptable. [Oosterhout, 1996, page 37]

Accordingly, the modal mass is between 0.25 and 0.5 times the total mass, depending on the ratio of bending and shear. This is approximately the same as was found before in [Jeary, 1981]. The ratio between bending and shear is defined as: $\alpha = EI/GA$. When $\alpha = \infty$, the effective mass is 0.25 times the total mass and when $\alpha = 0$, the effective mass is 0.5 times the total mass.

Another way to find the modal mass of a building is by first finding the modal stiffness of a building through the application of a forget-me-not. For example, the modal stiffness for a clamped beam with an applied load to the top of the building is found by:

$$ k^* = \frac{3EI}{h^3} \quad (4.30) $$

Then, by the application of Equation 4.13 the equivalent mass of the building becomes:

$$ \omega_0 = \sqrt{\frac{k^*}{m^*}} \rightarrow m^* = \frac{k^*}{\omega_0^2} \quad (4.31) $$

4.4 Conclusion

- Buildings with an aspect ratio of $h_r \geq 5$ are sensitive to vibrations
- Damping is the most effective way to reduce accelerations
- Torsional motion is the most disturbing type of motion for occupants in a building
4.4. CONCLUSION

41em
Chapter 5

Damping

This chapter provides information on damping in buildings. First, different types of damping will be discussed after which several auxiliary damping systems are selected which fit best to the purpose of this thesis. These damping systems will be described into more detail and will be compared afterwards. In addition to that, the phenomenon at the end of this chapter a conclusion will be presented regarding damping and the prototype tool.

5.1 General

In general it holds that systems undergoing vibrations will reach a steady vibration mode after a certain time frame. This is caused by the damping of such system, which in general is defined as:

**Definition 4** Damping is best defined as the dissipation of mechanical vibration energy from the system [Spijkers et al, 2005, page 163]

Damping can be expressed by the damping ratio, $\zeta$:

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}}$$

$c$ = damping

$c_{cr}$ = critical damping

With regards to the damping ratio, four different situations can be considered for damped systems:

1. Undamped: $\zeta \rightarrow 0$
2. Underdamped: $\zeta \leq 1$
3. Overdamped: $\zeta \geq 1$
4. Critically damped: $\zeta = 1$

The higher the amount of damping, the quicker the system reaches its steady-state motion. This is demonstrated in Figure 5.1 for different values of the damping ratio $\zeta$. If the damping of the system increases, the peak at $\omega = \omega_0$ in the frequency response function reduces, as was presented in Figure 4.3. This is an important observation with respect to the reduction of the accelerations in a high rise structure.
5.2 Types of damping

In high rise structures many types of damping can be distinguished, for instance aerodynamic damping; soil damping; structural damping; artificial damping; auxiliary damping. According to NEN-EN 1991-1-4, the value for damping in a high rise structure is the summation of only three types of damping sources:

\[ \delta_{\text{tot}} = \delta_s + \delta_{ae} + \delta_d \]  

(5.2)

Where \( \delta \) is the logarithmic decrement of the underdamped system. The logarithmic decrement is related to the damping ratio by the following relation:

\[ \delta = \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}} \]  

(5.3)

From these type of damping (respectively: structural, aerodynamic and auxiliary damping), the auxiliary damping source has the largest influence on the total damping of the system. In the next paragraphs these types of damping are explained into more detail.

5.2.1 Structural damping

The value for structural damping depends on the the applied structural material to the building. For each type of material a specific value for the damping coefficient can be distinguished, see Table 5.1.
Table 5.1: Damping factors for materials according Spijkers, et al., 2005, page 180 and NEN-EN 1991-1-4

<table>
<thead>
<tr>
<th>Material</th>
<th>Damping factor $\delta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.05</td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>0.1</td>
</tr>
<tr>
<td>Structures of both concrete and steel</td>
<td>0.08</td>
</tr>
<tr>
<td>Masonry</td>
<td>0.05</td>
</tr>
<tr>
<td>Wood</td>
<td>0.05</td>
</tr>
</tbody>
</table>

As mentioned in Chapter 2, high rise structures are usually constructed from steel, concrete or both. Additions of other materials to the building such as masonry claddings have a positive effect on the damping factor since these materials have a higher damping factor. However, as the height of a building increases, the influence of these additional materials decreases. Hence, in this thesis the influence of additional materials will be neglected; only the material applied to the load bearing structure will be taken into account.

### 5.2.2 Aerodynamic damping

The effect of aerodynamic damping is mainly determined by the shape of a building. In addition to that, modifications to corners could add a positive effect to the aerodynamic damping. This holds for both the alongwind and acrosswind responses, which are reduced by, for example, rounded corners.

Other effective adjustments to buildings are tapering effects and the change of cross-sectional shape along the height of the building. A combination of the two especially has a positive effect on the alongwind responses.

Additional openings could positively influence the aerodynamic damping as well. However, this effect is adverse for incorrectly applied openings and the effect is highest for openings positioned near the top of the building. In case the openings are positioned elsewhere in the building, the effect decreases enormously.

The Shanghai World Financial Center is a 460m tall building in which most modifications described above come together, see Figure 5.2. The building is tapered and the shape of the cross-section changes along the height. A 51m diameter opening is applied near the top.
5.2. TYPES OF DAMPING

Appendix F.5 of NEN-EN 1991-1-4+A1+C2 shows the following equation to calculate the aerodynamic damping:\cite{16,27}

\[ \delta_{ae} = \frac{c_f \rho \beta v_m (z_s)}{2n_1 m_e} \]  

(5.4)

5.2.3 Auxiliary damping

The group of auxiliary damping systems consists of three sub-groups, namely: active systems, semi-active systems and passive systems. Each sub-group is divided into other sub-groups, see Figure 5.3. Each system is briefly described below.

Damping systems

Active damping systems are systems that require an external energy source in order to work as a damping system. These type of systems are mainly used in seismic areas. An advantage of the system is the adjustability of the natural frequency of the system. The system can be coordinated and adjusted to whatever situation. Disadvantageous is when the natural frequency of the damping systems is about the same as the natural frequency of the building: the system now does not work as a damping system but as a resonance system, which results in higher displacements, velocities and accelerations.

Passive damping systems do not require additional energy and it dissipates energy from the structural system. The response of this system is slower than the response of an active system and it cannot be tuned to the expected behavior of the building. Advantageous are the relatively low costs of the system and the possibility to integrate the system into the load bearing structure. In addition to that, a passive damping system will never behave as a resonance system so it will never intensify the motions of the building. Passive damping systems are mainly used to suppress wind-induced and light earthquake responses.\cite{16}

Semi-active systems respond quicker to a buildings motion than passive systems and the natural frequency is adjustable just like the active system. It does not need much power: the damper moves their weight into the opposite direction of the buildings motion. By
Figure 5.3: Groups of damping systems
5.3. DAMPING SYSTEMS

Doing this, it counteracts the swaying of the building. Disadvantageous are the limited possibilities to integrate the system into the structure.\cite{34}

5.3 Damping systems

Passive damping systems are the best suitable systems to suppress wind-induced vibrations in the Netherlands. The active and semi-active systems could jeopardize a stable motion because incorrect tuning might lead to adverse effects. In addition to that, the systems are too expensive for small vibrations and it is hard to implement these systems into the structural system of a building.

Abroad commonly used passive systems are: viscoelastic dampers (VEDs), viscous dampers (VDs), and friction dampers (FDs). It is expected these type of dampers are going to be used regularly in high rise buildings in the Netherlands. In the following paragraphs the working principle of each damper system is further analyzed but first the phenomenon hysteresis is discussed due to its relevance for the energy dissipation in all damper types.

The general shape of current damping predictor models is shown in Figure 5.4 and in mathematical form as follows:

\[
\zeta_s(X) = \zeta_b + \zeta_c(X) \leq \zeta_{s,max} \tag{5.5}
\]

\[
\zeta_c(X) = \beta X^a \leq \zeta_{c,max} \quad \text{for} \quad X \in X_l, X_m > \tag{5.6}
\]

![Figure 5.4: General shape of current damping predictor models](image)

In the above equation, \(\zeta_b\) is the base line damping ratio at the low-amplitude plateau. If the amplitudes of vibration are above a certain value \(X_l\), the damping ratio is determined as a nonlinear function of amplitudes of vibration response within a certain range. Once the amplitude \(X_m\) is reached, the damping remains constant at the high amplitude plateau at its maximum \(\zeta_{c,max}\) for \(X \geq X_m\).

The above damping model can be adopted directly for steady state motions under sinusoidal vibrations. However, in case of random vibrations, the amplitudes of the responses will be varied. By an increase of amplitude the natural frequencies tend to decrease, which is an illustration of the amplitude dependency of the natural frequencies. However, the contribution of this factor to the frequency changes contributes in the third decimal only which suggests that frequency nonlinearity is relatively small for tall buildings and thus for this thesis a linear situation is assumed.\cite{2,19}
5.3.1 Hysteresis

Until now only linear systems are considered. Nevertheless, many damping materials do not behave as such and therefore the model becomes non-linear. In that case, the damping system is usually proportionally to both the velocity as the displacement. As a result the force-displacement diagram is formed by a loop. The shape of the loop changes in case the damping system does not depend on the displacement but simply on the velocity (thus, this is the linear model).

The non-linear behavior of the damping material is implemented into the equation of motion as follow:

\[ m\ddot{u} + F_{\text{damper}}(u, \dot{u}) + ku = F(t) \]  

This new equation makes the calculations more complex since it cannot be solved as a linear differential equation. Therefore, other methods as described before must be considered, like:

1. Time-domain analysis, which requires an estimation for the acceleration at \( t = t_i \). In order to do this, the boundary conditions for \( u \) and \( \dot{u} \) at \( t = 0 \) must be determined. Then, \( \ddot{u}_0 \) can be determined. Linear interpolation is used to find the accelerations at some \( t \) at the interval \( t_0 < t < t_i \).

2. Method of equivalent damping, which is described further in the next section.

However, only the second method will be considered in this thesis.

The observed loop is observed by quickly loading and unloading a material, and is called a hysteresis loop. This loop is different for all kind of materials but a general form is displayed in Figure 5.5. If such loop is observed, it means the material is stretched more easily during unloading than during loading. This loop employs to harmonically loaded materials and the area enclosed by the loop resembles the energy being dissipated per cycle from the material. More energy is required during loading than during unloading and therefore it holds:[37]

\[ E_{\text{dissipated}} = E_{\text{loading}} - E_{\text{unloading}} \]  

In case of a harmonic load \( u = \hat{u} \sin (\omega t) \), the spring and damping force are described by:

\[ F_s = k\hat{u} \sin (\omega t) \]
\[ F_d = c\hat{u}\omega \sin (\omega t) \]
Then, the force displacement diagram is described as in Figure 5.6. Therefore, if the spring stiffness of a damping device is negligible, the hysteresis loop follows the shape of the left figure. In case the damper does contain spring stiffness, the loop is tilted according to the middle figure.

![Figure 5.6: Hysteresis loop](image)

As mentioned before only a few damping systems will be considered. The hysteresis loops of these systems are displayed in Figure 5.7.

![Figure 5.7: Hysteresis loops of different type of dampers](image)

From this figure it is clear that friction dampers dissipate the largest amount of energy for any given force and displacement compared to viscous and viscoelastic dampers. Calculations on energy dissipation of viscous dampers are the easiest and are therefore the starting point of most calculations on dampers. As for other damping systems it is possible to find an equivalent value for damping by equalizing the energy dissipation of that system to the energy dissipation of a viscous damper. This will be further described in the next paragraph.\[31\]p.5

### 5.3.2 Viscous dampers

According to [Kareem, et al., 1999] viscous damping devices dissipate energy from the structural system as follows:

**Definition 5** This form of damper dissipates energy by applying a resisting force over a finite displacement through the action of a piston forced through a fluid-filled chamber for a completely viscous, linear behavior.

Examples of practical applications of viscous dampers are in Figure 5.8.
Working principle

Viscous materials dissipate energy from systems by fluid friction. According to\cite{17}, four different ways lead to the increase of the viscous damping force. This clarifies what parameters influence the behavior of a viscous damper:

\textbf{There are four ways to gain increased viscous damping force:}

1. \textit{Raise the viscosity of viscous material}
2. \textit{Enlarge the area containing the viscous material}
3. \textit{Increase the velocity applied to the viscous material}
4. \textit{Amplify the damping force of the viscous material}

The first parameter depends on the type of material being used in the piston and determines the storage capacity of the material. Increasing viscosity leads to intensification of temperature dependence of the viscous damping force. Increasing the velocity leads to a reduction of the damping coefficient, which is intensified for higher values of viscosity.\cite{17}

The motion of the piston in the viscous damper moves around an equilibrium point and will always return to this point.\cite{4}

Viscosity is a measure of a non-crystalline material’s resistance to deformation. According to Figure 5.9, the viscosity is derived from the ratio of an applied shear stress $\tau$ and the change in velocity $dv$ with distance $dy$:

\[ \eta = \frac{\tau}{dv/dy} = \frac{F/A}{dv/dy} \]  

(5.10)

Consequently, if $\eta$ increases, the resistance to deformation of the viscous material also increases.\cite{11}p.415-416
The viscosity of the material is used to find the stress of the material:

\[ \sigma(t) = \eta \frac{de(t)}{dt} \]  

(5.11)

Or, equivalently for shear stress:

\[ \tau(t) = G\gamma(t) + \eta \frac{d\gamma(t)}{dt} \]  

(5.12)

**Advantages and disadvantages**

The advantages and disadvantages of viscous dampers are in Table 5.2.

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculations simplify due to the full energy dissipation by the damper</td>
<td>Multiple dampers are required for a proper energy dissipation</td>
</tr>
<tr>
<td></td>
<td>Possibly difficult to integrate into the structural system</td>
</tr>
</tbody>
</table>

**Mathematical model**

Viscous damper systems are linear which makes calculations rather easy. In addition to that, it is time independent and the principle of superposition is applicable. Other types of damper systems with a non-linear behavior can often be modeled as a viscous damper by equalizing the energy dissipated per cycle of the non-linear system to the energy dissipation of a viscous damper system. This is advantageous because the non-linear calculations become linear. For specific cases, this will be done in the next paragraphs. First, it is important to get insight into the mathematical model of a linear viscous damper because this is the starting point of all other mathematical damping models.

Constant viscous damping is proportional to the velocity by the relation:

\[ F_d = cv = c\dot{x} \]  

(5.13)

For a harmonic motion \( x = \dot{x}\sin(\omega t) \) the hysteresis loop is displayed in Figure 5.7. The hatched area equals the amount of dissipated energy per cycle:

\[ E_{diss} = \int Fdx = \int F\dot{x}dt = c\pi\omega\dot{x}^2 \]  

(5.14)

It holds for some materials that the area enclosed by the hysteresis loop grows quadratically with the square of the amplitude, which is called frequency dependent viscous damping. In this case and in many other case the calculations become non-linear. In general, if a non-linear model is considered, the equivalent viscous damping of the non-linear system becomes:

\[ E_{diss,viscous} = E_{diss,other} \rightarrow c_{eq}\pi\omega_{eq}\dot{x}^2 = E_{diss,other} \rightarrow c_{eq} = \frac{E_{diss,other}}{\pi\omega_{eq}\dot{x}^2} \]  

(5.15)
As for the case of frequency dependent viscous damping, the hysteresis loops become as in Figure 5.10 and therefore:

\[ c_{eq} = \frac{\alpha \dot{x}^2}{\pi \omega_{eq} \dot{x}^2} = \frac{\alpha}{\pi \omega_{eq}} \]  

(5.16)

![Figure 5.10: Frequency dependent hysteresis loop where \( \alpha \) is the magnification factor](image)

Then, the equation of motion derived in Chapter 4 for the 1-MSD system changes into:

\[ m\ddot{x} + \frac{\alpha}{\omega_{eq}} \dot{x} + ku = F(t) \]  

(5.17)

Using once again the equations derived in Chapter 4, the following equation for this situation is found:

\[ H(\omega) = \frac{k\dot{x}}{F} = \frac{1}{\sqrt{(1 - \frac{\omega^2}{\omega_0^2})^2 + \left(\frac{\alpha}{\pi k}\right)^2}} \]  

(5.18)

So the maximum amplitude at \( \omega = \omega_0 \), becomes:

\[ H = H_{max} = \frac{\pi k}{\alpha} \]  

(5.19)

### 5.3.3 Viscoelastic dampers

Amorphous polymers behave like glass at low temperatures, while at intermediate temperatures above the glass transition temperature (\( T_g \) in Figure 5.11) the polymer behaves as a rubbery solid. Once the melting temperature \( T_m \) is exceeded, the material behaves as a viscous liquid. For relatively small deformations the mechanical behavior at low temperatures may be elastic and at the highest temperatures viscous behavior occurs. The rubbery solid behavior in the intermediate region is a combination of these two extreme characteristics and is named viscoelasticity. The (energy) storage capacity of the material is expressed by the storage modulus of the material, which changes by any shift in temperature. In a building, temperature control is required in the surrounding area of the VED for instance by the integration of the VEM with steel members because of the high conductivity of steel.\(^{[6]}\) The ratio between the viscous and elastic part differs for each material and defines the exact behavior of the material. The stress-strain behavior of a fully elastic and viscous material are displayed in Figure ??.

Viscoelastic (VE) dampers dissipate energy from the load bearing structure partly by storage and partly by fluid friction. However, only 2-4% of the energy is dissipated by
5.3. DAMPING SYSTEMS

Figure 5.11: Temperature dependence of VEM (Figure based on Callister, 2003)

Working principle

The shape of the hysteresis loop can be explained by the dependency on both velocity and displacement of the material. The viscous part of the material is related to the velocity and the elastic part to the displacement.

The deformation of the VE material is mainly caused by shear forces; this is also the most efficient way to dissipate energy from the structure but for safety reasons, $\tau \leq 1\%$

Additionally, the costs of VE dampers are relatively low. For instance, in Colombia Center Building VE dampers were only 1.5% of the total building costs. This included an extensive testing program.\[30\] p.100-102

Calculations on VE dampers are often costly due to non-proportional damping which must be approached often by numerical methods. However, some methods have been developed to overcome this problem. One of these methods will be discussed in Section 5.3.4.\[3\]
Advantages and disadvantages

A list of advantages and disadvantages is in Table 5.3.

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>VE dampers have been applied many times before to suppress wind-induced responses</td>
<td>The capacity for dissipated energy decreases when temperature increases</td>
</tr>
<tr>
<td>VE dampers could easily be integrated into the structural system of a building</td>
<td>Computations can be costly due to non-proportional damping</td>
</tr>
<tr>
<td>Costs are relatively low compared to other damper types</td>
<td>-</td>
</tr>
<tr>
<td>High damping at lower frequencies</td>
<td>-</td>
</tr>
</tbody>
</table>

5.3.4 Mathematical model

The behavior of VE materials is dependent on an elastic and viscous part, accordingly:

$$\sigma(t) = \sigma_{\text{elastic}} + \sigma_{\text{viscous}} = E\varepsilon(t) + \eta \frac{d\varepsilon(t)}{dt}$$  \hspace{1cm} (5.20)

At the application of an external load, elastic materials instantaneously deform. Upon the release of the load, the material recovers and assumes its original dimensions, see Figure 5.13 (left). As for viscous materials, the deformation is not instantaneous and is dependent on time, see Figure 5.13 (middle). The deformation is not reversible. Viscoelastic materials undergo an instantaneous strain, which is followed by a time-dependent strain, see Figure 5.13 (right).

\[\text{Elastic} \hspace{1cm} \text{Viscous} \hspace{1cm} \text{Viscoelastic}\]

Hence, the behavior of viscoelastic materials is dependent on time as well as temperature. The time-dependent Young’s modulus is expressed by:

$$E_r(t) = \frac{\sigma(t)}{\varepsilon_0}$$  \hspace{1cm} (5.21)

$$\begin{align*}
\sigma(t) & = \text{Time-dependent stress} \\
\varepsilon_0 & = \text{Constant strain level}
\end{align*}$$
On the other hand, when the stress is kept constant, the time-dependent creep is derived by:\[6\] p.484-489

\[ E_r(t) = \frac{\sigma_0}{\varepsilon(t)} \]
\[ \sigma(t) = \text{Constant stress level} \]
\[ \varepsilon_0 = \text{Time-dependent strain level} \] (5.22)

In case of a full elastic material, the stress and strain are in phase, see Figure 5.14 (left). The phase shift for viscous materials is \( \Psi = \pi/4 \). The conclusion with regard to the phase shift for VE materials becomes:

\[ 0 < \Psi < \pi/4 \] (5.23)

This phase shift is used to find the longitudinal storage capacity of the VE material, which is expressed by:

\[ \eta_E = \tan \Psi = \frac{E''(\omega)}{E'(\omega)} = \frac{|E^*| \sin \Psi}{|E^*| \cos \Psi} \]

Where:
\[ -E^* = \sqrt{|E'(\omega)|^2 + |E''(\omega)|^2} = \frac{\sigma_a}{\varepsilon_a} \] (5.24)

And, equivalently for shear:\[38\]

\[ \eta_G = \tan \Psi = \frac{G''(\omega)}{G'(\omega)} = \frac{|G^*| \sin \Psi}{|G^*| \cos \Psi} \] (5.25)

From literature follows the loss factor of the material should be measured by experiments. Correspondingly, the volume is determined by:\[32\]

\[ V = \frac{1}{\pi G'' \gamma_{max}^2} \]
\[ G'' = |G^*| \sin (\Psi) = \text{Loss modulus} \]
\[ \gamma_{max} = \text{Maximum shear strain} \]
\[ G^* = \text{Dynamic modulus} \]
\[ \Psi = \text{Phase shift between } \tau(t) \text{ and } \gamma(t) \] (5.26)
Implementation of the VED into the equation of motion could be done by a representative model of the VED as is displayed in Figure 5.15. The spring represents the spring stiffness of the VED and the dashpot system represents the viscous properties of the VED.

The representation of a viscoelastic damper by a spring-dashpot system

\[ k_{\text{ved}} \quad C_{\text{ved}} \]

Figure 5.15: Representation of a viscoelastic damper by a spring-dashpot system

The method of equivalent damping should now be extended to a method of equivalent damping and spring stiffness:

\[ m \ddot{x} + c_{eq} \dot{x} + k_{eq} x = F(t) \] (5.27)

However, VED’s are most effective in case it is loaded by shear. The above equation does not take into account the shear force and should therefore be adjusted in case of shear action.

The specific damping capacity of the structure is \( \psi_{st} \) is represented by the ratio of dissipated energy and stored energy per cycle:

\[ \eta = \frac{E_{\text{diss}}(\omega)}{E_{\text{stored}}(\omega)} = \frac{\pi \gamma_0^2 G''(\omega)}{\frac{3}{4} G'(\omega)} = 4\pi \tan \Psi \] (5.28)

By the introduction of the frequency equation, the above equation can also be expressed as:

\[ \eta = -2 \frac{E_{\epsilon} \frac{\partial f}{\partial \epsilon} + G_{\gamma} \frac{\partial f}{\partial \gamma}}{\omega \frac{\partial f}{\partial \omega}} \] (5.29)

Where:

- \( \eta_{\epsilon} = 2\pi \tan \Psi_{\epsilon} \) = storage capacity longitudinal
- \( \eta_{\gamma} = 2\pi \tan \Psi_{\gamma} \) = storage capacity shear

The storage capacity can be converted to the damping ratio:\[13]\n
\[ \zeta = \frac{\psi}{4\pi} = -\frac{E_{\epsilon} \frac{\partial f}{\partial \epsilon} + G_{\gamma} \frac{\partial f}{\partial \gamma}}{2\pi \omega \frac{\partial f}{\partial \omega}} \] (5.30)

Calculations on VEDs are often costly due to the non-linear behavior of the material. Numerical methods are often adopted for calculations on VEM, for example in case of the above equation. However, the behavior can also be described by linear relations, which is the case for small values of the damping ratio. In that case, the loss modulus is related directly to the damping ratio and damping coefficient of the VED by:\[15;18]\n
\[ \zeta = \frac{\eta}{2} ; \quad c_{\text{VEM}} = \eta \frac{K'}{\omega} \] (5.31)
5.3. DAMPING SYSTEMS

Where $K'$ is the storage modulus of the VEM. The behavior of the VEM becomes non-linear for higher values of $\zeta$. In that case, the loss modulus is to be obtained from:

$$\eta = \frac{1 - \left\{1 - 0.5(2\zeta)^2\right\}^2}{s^2 - 1} ; \quad 2\zeta = \frac{\Delta\omega}{\omega_n}$$

(5.32)

Where $s$ takes into account the reduction of the amplitude by $1/s$ from $\omega_n$ at $\omega_1$ and $\omega_2$. This is the basis of the half-power bandwidth method, which is a method to determine the damping ratio of a structure or material from the frequency response function, see Figure ??.

This method will be employed in Chapter ?? to determine the damping in the VEM in the physical test set-up.

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n}$$

(5.33)

5.3.5 Friction dampers

Friction dampers dissipate energy from the load bearing structure through dry friction. It consists of frictional interfaces and a clamping system. For examples of practical applications of friction dampers, see Figure 5.17.

Friction damping or Coulomb damping is divided into static and kinetic friction from which only the latter will be considered in this thesis. Kinetic friction is defined as follows:

**Definition 6** Kinetic friction is the friction that occurs between objects when these objects slide against each other and are in relative motion.
The friction force between the surfaces equals the product of the normal force and the kinetic friction coefficient, or in symbols:

\[ F_F = \mu_D F_N \quad (5.34) \]

**Working principle**

Friction dampers show almost perfect rectangular behavior, which is interpreted as constant energy dissipation from the system because of carefully controlled sliding friction. This value remains constant over time and is independent on factors such as surface area, displacement, velocity and temperature. The objects vibrate around an equilibrium point, which could change over time due to the non-linear character of the damper. Due to this provision of non-linearity, the friction damper enables plastic behavior while the structure itself remains elastic.

Friction dampers are usually applied in steel-framed buildings where two main groups of friction dampers can be distinguished, namely rigid frame friction dampers and braced frame friction dampers. The former provides real plastic hinges which can easily be replaced after deformation while the latter are integrated into diagonal bracings that slips at a predetermined stress value.

Two motion states can be distinguished for the damping system, being slip and stick state. This is caused by the so-called stick-slip phenomenon and occurs between two sliding objects. Between these sliding systems, shocks occur from time to time due to friction. As for the loads the following holds: when \( F_{\text{shear}} \leq F_{\text{friction,max}} \), the damper is in stick state and \( F_{\text{friction}} = F_{\text{loading}} \). When \( F_{\text{friction,max}} \) is reached, the system is in slip state and \( F_{\text{friction}} = F_{\text{slip}} \). This is also displayed in Figure 5.18 c and d, where a friction damper is added to the partition wall. In case of small displacements, the system is in stick state (Figure 5.18c); in case of large displacements, the system is in slip state (Figure 5.18d).

\[ \text{Figure 5.18: Model to illustrate slip-stick motion (Obtained from [2;23])} \]

In the above example of a partition wall, the friction damper contains a certain stiffness \( k_0 \). Therefore, the equation of motion changes to:

\[ m_b \ddot{x} + c_b \dot{x} + [k_b + k_0 H(d_c - X)]x = 0 \quad \text{for} \quad x \geq d_c \quad (5.35) \]

In the above equation \( d_c \) is the slip trigger displacement of the system. Now if only the Coulomb friction is considered (Figure 5.19a), the damping starts at a very high value at zero amplitude and decreases by an increase of amplitude according to:

\[ \zeta_c = \frac{1}{2} \frac{Q}{X \pi m_b \omega_b'^2} \quad (5.36) \]
In case only the nonlinear contribution is taken into account and thus the stick-slip phenomenon, dependent on the hysteresis loop, the damping increases from zero at amplitude zero to a higher value at higher amplitudes. However, the shape of the displacement-amplitude graph is still incorrect, see Figure 5.19b. If the linear and non-linear contribution are added up, the frequency dependent damping becomes as is displayed in Figure 5.19c. [2]

The stick-slip vibration can be reduced by improving the surface’s roughness. Surfaces with small surface roughness have a small maximum static coefficient and dynamic friction coefficient. Additionally, the maximum sliding force is reduced.

Figure 5.20 shows three different forces acting on the slider during the slip period. The spring force acts in the direction of the slider at a height $h_2$, which is at a different height than the center of gravity at $h_1$. The equilibrium of forces is given by:[23]

$$M = kx(h_1 - h_2) - Fh_1 = m\ddot{y} - kxh_2 \quad \rightarrow \quad J\varphi = m\dot{y}h_1 - kxh_2 \quad (5.37)$$

Advantages and disadvantages

Advantages and disadvantages of friction dampers are in Table 5.4.
### Table 5.4: Advantages and disadvantages of friction dampers

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials are not affected by degrada-</td>
<td>Mainly applicable to steel, not to other</td>
</tr>
<tr>
<td>tion due to aging</td>
<td>materials</td>
</tr>
<tr>
<td>Insensitive to changes in ambient</td>
<td>Unknown</td>
</tr>
<tr>
<td>temperature</td>
<td></td>
</tr>
<tr>
<td>No fluid leaking problems</td>
<td>Unknown</td>
</tr>
</tbody>
</table>

#### 5.3.6 Mathematical model

Figure 5.21 displays a 1-MSD system with a friction damper. The equivalent viscous damper is visualized by the dashed damper named $c_{eq}$. Unlike viscous damping, friction damping is not proportional to velocity. Consequently, the equation of motion changes into:

$$m\ddot{x} + F_c \frac{\dot{x}}{|\dot{x}|} + kx = F(t)$$  \hspace{1cm} (5.38)

![Dynamic model of a 1-MSD system with a friction damper](image)

It holds that $F(t) > F_c$ and when $F(t)$ is large enough, the damper follows the rectangular shaped hysteresis loop as displayed in Figure 5.7. The independence of velocity and spring stiffness explains the shape of the loop and the energy dissipation of the friction damper is:

$$E_{diss} = 4F_c\dot{x}$$  \hspace{1cm} (5.39)

From Equation 5.14 it followed that the energy dissipation for a viscous damper equals $c\pi \omega \dot{x}^2$. Now, the equivalent damping of the friction damper can be determined:

$$4F_c\dot{x} = c_{eq}\pi \omega \dot{x}^2 \rightarrow c_{eq} = \frac{4F_c}{\pi \omega \dot{x}}$$  \hspace{1cm} (5.40)

Using the equations derived in Chapter 4, the amplification factor for the friction damper is derived:

$$A = \frac{k\dot{x}}{F} = \frac{\sqrt{1 - \left(\frac{4\alpha}{\pi V}\right)^2}}{\left|\frac{1 - \omega^2}{\omega_0^2}\right|} \quad \text{where} \quad \alpha = \frac{F_c}{F}, \quad V = \frac{k\dot{x}}{F}$$  \hspace{1cm} (5.41)
5.3.7 Comparison of types of dampers

A comparison is in Table 5.5.

Table 5.5: Comparison of types of dampers

<table>
<thead>
<tr>
<th></th>
<th>Viscous</th>
<th>Viscoelastic</th>
<th>Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsensitive to ambient temperature changes</td>
<td>-</td>
<td>-</td>
<td>++</td>
</tr>
<tr>
<td>Easy to implement into mathematical model</td>
<td>++</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Inexpensive (practical)</td>
<td>+/-</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>Inexpensive (calculations)</td>
<td>+</td>
<td>-</td>
<td>+/-</td>
</tr>
<tr>
<td>Large energy dissipation (see hysteresis loop)</td>
<td>+/-</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Requires no maintenance</td>
<td>+/-</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>No extra effort is needed to integrate damper into structure</td>
<td>-</td>
<td>++</td>
<td>-</td>
</tr>
</tbody>
</table>

5.4 Conclusion

A list of conclusions regarding the prototype tool:

- **Structural damping:** Only the load bearing structure is considered for structural damping and therefore this is a conservative value. Though, for a first approximation of the required auxiliary damping this will suffice. Besides that, the effect of structural damping is much smaller than the effect of auxiliary damping.

- **Aerodynamic damping:** The aerodynamic damping of a structure must be determined by Equation 5.4.

- **Positions of dampers:** The most favorable position of each type of damper at a position where the relative velocity is highest
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