Calibration Report Annular Flume

At the Faculty of Coastal Engineering, Zhejiang University

Report of measurements and observations

Steven te Slaa*

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* Section of Environmental Fluid Mechanics, Delft University of Technology, P.O. Box 5048, 2600 GA Delft, The Netherlands. Tel. + 31 15 27 83348; Fax: +31 15 27 85124 e-mail: S.teSlaa@tudelft.nl
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East China Normal University, State Key Laboratory of Estuarine and Coastal Research, Shanghai, China;
Steven te Slaa
Calibration Report Annular Flume
Faculty of Coastal Engineering
Zhejiang University

Steven te Slaa¹²
¹Delft University of Technology
²East China Normal University, State Key Laboratory of Estuarine and Coastal Research
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1 Introduction

1.1 Application of annular flumes

Estuarine systems worldwide contain large quantities of fine (cohesive) sediment. Fate and transport of these sediments is to a large extent governed by erosion and deposition processes. Knowledge on the behaviour of these sediments is required for modeling purposes in order to predict impacts on morphology and environment as a result of e.g. engineering purposes. Physical laboratory experiments are strong instruments to obtain this knowledge under confined conditions. Ongoing research regarding the behavior of sediment from the Yangtze River Estuary is carried out in the framework of the Sino-Dutch cooperation project “Effects of Human Activities on Eco-Morphological processes”. Within this project the State Key Laboratory of Estuarine and Coastal Research (SKLEC) in Shanghai and Delft University of Technology (DUT) share a collaboration partnership. Up to now, sedimentation experiments with Yangtze Estuary sediment are carried out at SKLEC in order to understand its settling and consolidation behavior (Te Slaa et al., 2012). Erosion behaviour is another key process of which its understanding is desired. SKLEC however, doesn’t (yet) have the facilities to perform erosion experiments. Therefore, collaboration was agreed with the Ocean and Hydraulic Engineering Faculty of the Zhejiang University in Hangzhou. This faculty has the availability of an annular flume, which is a powerful tool in determining erosion rates of cohesive sediment. The flume however, has not been in operation for several years and data and knowledge about the flume characteristics is lacking. Therefore a (re-) calibration of this flume is done of which this document describes the procedure and subsequent results. Calibration is carried out with state of the art knowledge on annular flumes, obtain at DUT (Booij, 1994).

1.2 Annular Flume characteristics

An annular flume is a typical shaped ring flume with two rotating elements, the top lid and the flume, which can rotate independently. A uniform tangential flow is generated by rotating the top lid. In order to minimize secondary currents, the flume rotates in opposite
direction. As a result, the flow exerts a bed shear stress. For erosion experiments, quantification of the exerted bed shear stress as function of the rotation velocities is essential, since erosion rates dependent on it. The largest advantage of the use of annular flumes is the absence of boundary problems since in theory this flume is infinitely long. The main assumption in the use of annular flumes is that the walls of the flume and the top lid are considered as hydrodynamically smooth. A disadvantage of annular flumes is the generation of secondary flows. When rotating the individual components, different tangential flow directions along the top lid and the bottom of the flume result in two secondary flow cells. The vertical component of the secondary flow affects the turbulence structure with the result that secondary currents exceed settling velocities ($u_{sec} >> w_s$).

Also, the bed shear stress $\tau_{bed}$ is not homogeneously distributed over the width of the flume. Therefore, this secondary flow should be considered in case of sedimentation or erosion studies. Booij (1994) studied the characteristics of these secondary flows in annular flumes, resulting in a relation for the optimum ratio between the rotation speed of the top lid and the flume at which secondary flows are minimized.
1.3 Calibration procedure

The hydrodynamic conditions inside the flume can be altered with three externally adjustable variables: 1) the rotation speed of the top lid 2) the rotation speed of the flume and 3) the water depth. Calibration of the annular flume holds that these variables will be related to the hydrodynamic conditions and has the goal to gain control over the exerted bed shear stress. The calibration will consist out four steps (Table 1):

<table>
<thead>
<tr>
<th></th>
<th>Goal</th>
<th>Related Calibration Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Determine operation characteristics of driving mechanisms</td>
<td>Obtain individual relations between the power supply and rotation velocities of flume and top lid from measurements</td>
</tr>
<tr>
<td>2</td>
<td>Minimize secondary currents and turbulence structure</td>
<td>Theoretical study and visualization of flow field inside the rotation flume</td>
</tr>
<tr>
<td>3</td>
<td>Determine exerted bed shear stress ($\tau_{bed}$) as function of operation characteristics</td>
<td>Observation of “initiation of motion” for various sand fractions for stepwise increasing $\tau_{bed}$</td>
</tr>
<tr>
<td>4</td>
<td>Validate relationship between theoretical derived $\tau_{bed}$ and measured $\tau_{bed}$</td>
<td>Correct theoretical relations between rotation velocities and ($\tau_{bed}$) with observations</td>
</tr>
</tbody>
</table>

Table 1 Calibration steps
2 Flume dimensions and optimum operation characteristics

2.1 Dimensions annular flume Zhejiang University

Since technical drawings of the flume are not available (anymore) dimension are taken from measurements (Figure 1). Therefore, at 8 locations around the flume distances between inner and outer wall of the flume, and the central axis cylinder are measured (Table 2). At 4 locations this axis cylinder diameter was measured as well. Average inner radius, outer radius and diameter are calculated from the measurements in Table 3. The average width of the flume (b) is 13.71 cm.

![Figure 1 Flume dimensions](image)

<table>
<thead>
<tr>
<th>Location</th>
<th>Outer radius [cm]</th>
<th>Inner radius [cm]</th>
<th>Axis Cylinder diameter [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>41.70</td>
<td>28.00</td>
<td>3.79</td>
</tr>
<tr>
<td>2</td>
<td>41.90</td>
<td>28.05</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>41.90</td>
<td>28.80</td>
<td>3.79</td>
</tr>
<tr>
<td>4</td>
<td>41.85</td>
<td>28.00</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>41.85</td>
<td>28.00</td>
<td>3.79</td>
</tr>
<tr>
<td>6</td>
<td>41.90</td>
<td>27.92</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>41.65</td>
<td>28.00</td>
<td>3.79</td>
</tr>
<tr>
<td>8</td>
<td>41.65</td>
<td>27.98</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>41.80</td>
<td>28.09</td>
<td>3.79</td>
</tr>
</tbody>
</table>

Table 2 Measured flume dimension
### Table 3 Average flume dimensions

<table>
<thead>
<tr>
<th></th>
<th>Radius [cm]</th>
<th>Diameter [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner</td>
<td>29.99</td>
<td>59.98</td>
</tr>
<tr>
<td>Outer</td>
<td>43.70</td>
<td>87.39</td>
</tr>
</tbody>
</table>

#### 2.2 Operation characteristics for optimum flow conditions

The rotation velocities of the top lid and the flume are controlled individually. This is done by turning a handle on the control panel for respectively the top lid and the flume. The relationship between the handle positions and rotation velocities of the top lid and the flume is the first step in calibration of the flume. The number of rotations with corresponding duration is measured for varying handle positions for both the top lid and the flume (Table 4). Results reveal a linear relationship between handle position and rotation velocity for both the top lid and the flume (Figure 2).

<table>
<thead>
<tr>
<th>Handle Flume</th>
<th>Handle Top Lid</th>
<th>Flume</th>
<th>Top lid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>time [s]</td>
<td>laps [#]</td>
<td>wb [rpm]</td>
</tr>
<tr>
<td>4.00</td>
<td>4.71</td>
<td>191.7</td>
<td>25</td>
</tr>
<tr>
<td>4.25</td>
<td>5.13</td>
<td>180.7</td>
<td>25</td>
</tr>
<tr>
<td>4.50</td>
<td>5.38</td>
<td>170.4</td>
<td>25</td>
</tr>
<tr>
<td>4.75</td>
<td>5.63</td>
<td>161.4</td>
<td>25</td>
</tr>
<tr>
<td>5.00</td>
<td>6.06</td>
<td>153.5</td>
<td>25</td>
</tr>
<tr>
<td>5.25</td>
<td>6.31</td>
<td>175.2</td>
<td>30</td>
</tr>
<tr>
<td>5.50</td>
<td>6.56</td>
<td>167.0</td>
<td>30</td>
</tr>
<tr>
<td>5.75</td>
<td>6.81</td>
<td>160.0</td>
<td>30</td>
</tr>
<tr>
<td>6.00</td>
<td>7.23</td>
<td>153.7</td>
<td>30</td>
</tr>
<tr>
<td>6.25</td>
<td>7.48</td>
<td>171.8</td>
<td>35</td>
</tr>
<tr>
<td>6.50</td>
<td>7.73</td>
<td>165.1</td>
<td>35</td>
</tr>
<tr>
<td>6.75</td>
<td>8.15</td>
<td>159.0</td>
<td>35</td>
</tr>
<tr>
<td>7.00</td>
<td>8.41</td>
<td>153.3</td>
<td>35</td>
</tr>
<tr>
<td>7.25</td>
<td>8.66</td>
<td>168.8</td>
<td>40</td>
</tr>
<tr>
<td>7.50</td>
<td>9.08</td>
<td>162.9</td>
<td>40</td>
</tr>
<tr>
<td>7.75</td>
<td>9.33</td>
<td>158.1</td>
<td>40</td>
</tr>
<tr>
<td>8.00</td>
<td>9.58</td>
<td>172.1</td>
<td>45</td>
</tr>
<tr>
<td>8.25</td>
<td>10.00</td>
<td>166.5</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 4 Computed rotation velocities derived from observed number of rotations and corresponding duration
Analysis of the results lead to the following linear relations between rotation velocity and handle positions \((H_t\) and \(H_f\)) of the flume and top lid, respectively:

\[
\omega_f = 1.96 \times H_f - 0.17 \\
\omega_t = 3.58 \times H_t - 0.99
\]

(1.1)

Where \(\omega_t\) [rpm] is the rotational speed of the top lid, \(\omega_f\) [rpm] the rotational speed of the flume. Now, the optimum rotation velocity ratio between top lid and flume can be defined for which Booij (1994) defined a relationship between individual rotation velocities of top lid and flume for which secondary currents are minimized, with water depth \((h)\) and the width \((b)\) of flume as variables.

\[
\frac{\omega_t}{\omega_f} + 1 = -1.17 \frac{h}{b}
\]

(1.2)

Substitution of the design parameter for the present flume \((b = 0.137 \text{ [m]}\) and \(h = 0.140 \text{ [m]}\)) lead to the following optimum relation:

\[
\frac{\omega_t}{\omega_f} = -2.1947
\]

(1.3)

Substitution of equation (1.1) in (1.3) leads to the relation for optimum handle positions for a water depth of 14 cm:

\[
H_f = -0.828 \times H_t + 0.236
\]

(1.4)
2.3 Secondary Currents

Now that optimum operation characteristics are known, a qualitative interpretation of the secondary currents and turbulence structure can be composed. Therefore, several tests are carried out in which the flow field is visualized while the rotation velocities of the flume were set at the optimum conditions according to equation (1.2) with a water depth of 20 cm. Dye is injected into the flume (Figure 3) to visualize the turbulence structure. Visual observations showed that the colored fluid maintained at its vertical position in the flow field for at least half a rotation at various flow velocities. In addition, during the erosion experiment, the movement of two organic particles (leaves and/or roots) was observed in the flume. These pieces (< 0.5 cm) maintained their vertical position in the flow field for several rotation after which they only slowly altered their vertical position in the flow field. From the observations it is concluded that the influence of the turbulence structure and secondary currents in the flume are indeed minimized and are therefore neglected.

![Figure 3 Injection of coloring agent](image)
3 Bed shear stress calibration

3.1 *Theoretical bed shear stress formulation*

The aim of using an annular flume is to determine erosion rates of cohesive sediment as function of the bed shear stress. In order to do so, it is necessary to know the magnitude of the exerted bed shear stress for given rotation velocities of the flume components. A thorough analysis of flow fields in annular flumes has been done by Booij (1994) who theoretically derived flow characteristics. Booij estimates that the average tangential flow velocity ($u_{av}$) in the central region of the flume with the following relation:

$$u_{av} \approx \omega_t R \frac{1}{1+\beta} + \omega_f R \frac{\sqrt{\beta}}{1+\sqrt{\beta}}$$  \hspace{1cm} (1.5)

Where $\omega_t$ and $\omega_f$ are the rotation velocities of the top lid and flume respectively and $R$ the radius of the flume. $\beta$ is the ratio between surface area of the flume and the top lid given by:

$$\beta = \frac{b+2h}{b}$$  \hspace{1cm} (1.6)

Next, he compared these theoretically derived flow velocities with laser-doppler velocimeter measurement of primary and secondary flow velocities in an actual flume in the fluid mechanics laboratory of Delft University of Technology. From the average tangential flow velocity an estimation of the friction velocity can be made, given that $c_i$ is nearly constant ($c_i \approx 25$ in Booij):

$$\frac{u_{av} - u_w}{u_*} \geq c_i$$  \hspace{1cm} (1.7)

Now, the friction velocity can be used to estimate the bed shear stress following:

$$u_* = \sqrt{\frac{\tau_b}{\rho}}$$  \hspace{1cm} (1.8)
Where $\rho$ is the fluid density. After comparison of estimations and measurements Booij (1994) states that the proposed method to estimate the bed shear stress (friction velocity) appears to give sufficiently reliable results to be useful for erosion predictions, especially around the optimum condition for the secondary flow. Following this method, predictions for the flume in Zhejiang University annular flume are made for a water depth of 14.0 cm. By combining equations (1.5), (1.6) and (1.7) in (1.8) a theoretical relation between the applied bed shear stress and the rotation velocity of the flume is found:

$$\tau_{\text{bed(Booij)}} = \frac{1}{10} \left( \frac{1.64 \omega_f + 3.89}{c_t} \right)^2$$

(1.9)

The theoretical relation found above still needs an experimental validation for the present flume. This will be described in the next section.

### 3.2 Validation of theoretical bed shear stress

**Principle**

Since the theoretical derived bed shear stresses (Booij, 1994) are estimates which are derived for a much larger flume ($R = 1.7$ m), validation is needed for the flume at Zhejiang University. This is done by performing erosion experiments with coarse granular material (sand) with different grain size distributions. The critical shear stress for the initiation of motion of these sands are well known in relation to its grain size (Shields, 1936). Via the principle of “initiation of motion”, using the well known Shields curve, observed movement of a sediment particle can be related to the exerted bed shear stress. In this study the criterion for initiation of motion is defined as the formation of ripples.

For the present validation of the bed shear stress sand beds with varying $d_{50}$ are placed in the flume. Next, for each separate bed, the rotation velocity of the flume is increased stepwise until “initiation of motion” of is observed. A comparison of the observed critical shear stress for the “initiation of motion” for the corresponding $d_{50}$ with the bed shear stress (Booij) for the given rotation velocities, will validate the results obtained by the method of Booij for the Zhejiang University annular flume.
Sand fractions
Sediment samples with the narrowest possible grain size distribution are produced by sieving. After sieving, fines and small plates are washed out. For each size class enough sediment is produced to form at least a 0.5 cm thick bed in the flume. Sand samples are taken for grain size analysis using laser diffraction for samples 2 - 5. Sample 1 appeared to be to course for laser diffraction analysis and therefore the cumulative grain size distribution was obtained by sieving. The cumulative volume fractions are presented in Figure 4. The $d_{50}$’s of sand fraction 1 to 5 are respectively 1321, 938, 646, 447 and 290 $\mu$m.

![Figure 4 Grain size distribution of calibration sands](image)

Critical shear stress Shields – Van Rijn
A practical relation to determine the critical bed shear stress as function of particle size ($d$) has been proposed by Van Rijn (1984) with the Shields – Van Rijn formulation. This method avoids iteration since the particle Reynolds number was replaced by the dimensionless particle diameter. Following this formulation, the critical bed shear stress for the initiation of motion for a given particle diameter is determined. The critical shear stresses for the initiation of motion for all sand fractions are calculated by making use of the Shield – Van Rijn diagram (Figure 5). Therefore, first the dimensionless particle
diameter \((D_*)\) on the \(x\)-axis are calculated \((1.10)\). Next, the corresponding value for the Shields parameter \(\Theta_{cr}\) is determined from the Shield – Van Rijn diagram. Figure 5 shows a small range of \(\Theta_{cr}\) for the onset of initiation of motion (lower shaded area in Figure 5). This range is taken into account in assessing the critical bed shear stress. Lower and upper limits of this range are used. Next, from the critical mobility number (\(y\)-axis) the critical bed shear stress for initiation of motion \(\tau_{b,cr}\) can be derived for each sand fraction via equation\((1.11)\). Results are shown in Table 5.

\[
D_* = \left[ \frac{(s-1)g}{\nu^2} \right]^{1/3} d_{50} \quad \quad (1.10)
\]

\[
\Theta_{cr} = \frac{\tau_{b,cr}}{(\rho_i - \rho)gd_{50}} \quad \quad (1.11)
\]

Figure 5 Initiation of motion and suspension for a current over a plane bed (Van Rijn, 1984)
Validation experiment procedure

For each sand fraction a layer of at least 0.5 cm is placed on the bottom of the annular flume after which the bed is leveled. Next, at 8 locations around the flume the height of the sediment bed is measured, and the flume is filled with water up to a level of 14.0 cm above the average height of the bed. The bed level variation around the flume showed deviation of around 2.0 mm, which produces an inaccuracy < 1% in the theoretical bed shear stress. The starting conditions were chosen below the “calculated critical shear stress according to Shield – Van Rijn”. Next, then the flume is put in operation at optimum rotation velocities, and the bed shear stresses are increased in steps of 0.03 – 0.04 Pa. Every step is maintained for 2 minutes. During each step the processes at the sediment-water interface are observed. After two minutes the rotation speed of the top lid and bottom were increased to the next step. This is done until severe scour occurred.

Results

While stepwise increasing the bed shear stress, various transport mechanisms are observed. The three most obvious observations are:

1. Rolling and jumping of smallest particles over the bed: since the particle size gradation of the bed is non homogeneous, the smallest particles will start to move first. These particles are removed from the whole width of the flume and are transported to the outer bend where they keep rolling and jumping.

2. Ripples start to occur in the bed: these ripples start to occur in the outer bend and develop over approximately half of the width of the flume.

<table>
<thead>
<tr>
<th>D50</th>
<th>d*</th>
<th>Θ_ε_min</th>
<th>Θ_ε_max</th>
<th>Θ_ε_mean</th>
<th>τ_ε_min</th>
<th>τ_ε_max</th>
<th>τ_ε_mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1321</td>
<td>52.22</td>
<td>0.037</td>
<td>0.046</td>
<td>0.042</td>
<td>0.79</td>
<td>0.98</td>
<td>0.89</td>
</tr>
<tr>
<td>938</td>
<td>37.08</td>
<td>0.032</td>
<td>0.040</td>
<td>0.036</td>
<td>0.49</td>
<td>0.61</td>
<td>0.55</td>
</tr>
<tr>
<td>646</td>
<td>25.54</td>
<td>0.028</td>
<td>0.033</td>
<td>0.031</td>
<td>0.29</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>447</td>
<td>17.67</td>
<td>0.027</td>
<td>0.031</td>
<td>0.029</td>
<td>0.20</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>290</td>
<td>11.46</td>
<td>0.029</td>
<td>0.034</td>
<td>0.032</td>
<td>0.14</td>
<td>0.16</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 5 Range of critical shear stresses for initiation of motion according to Shields – Van Rijn for the applied sand fractions. The most right column represents the handle position for the flume on the moment of observed initiation of motion.
3. Severe erosion occurs in the flume: from the inside deep scour holes develop and all sediment is transported towards the outer bends. At the outer bend all sediment accumulates and large bed forms develop.

These three observed mechanisms are tabulated as function of the applied theoretical bed shear stress (Table 6). Black numbers indicate no observed transport mechanisms. The **green**, **blue** and **red** numbers represent above mentioned observations 1 – 3, respectively. The underlined number depicts the event of the initial formation of ripples, corresponding with the theoretical bed shear stress. Now, the theoretical estimation can be validated by comparing the observed (Shields – Van Rijn) and estimated (Booij) bed shear stress.

<table>
<thead>
<tr>
<th>Sand</th>
<th>Applied theoretical bed shear stress (τ_{bed}) [Pa]</th>
<th>H_f initiation of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.591 0.638 0.687 0.738 0.791 0.845 0.901 0.956 1.020</td>
<td>6.5</td>
</tr>
<tr>
<td>#2</td>
<td>0.345 0.382 0.420 0.460 0.502 0.545 0.591 0.638 0.687</td>
<td>5</td>
</tr>
<tr>
<td>#3</td>
<td>0.191 0.218 0.247 0.278 0.310 0.345 0.382 0.420 0.460</td>
<td>3.75</td>
</tr>
<tr>
<td>#4</td>
<td>0.142 0.165 0.191 0.218 0.247 0.278 0.311 0.345 0.382</td>
<td>3.25</td>
</tr>
<tr>
<td>#5</td>
<td>0.082 0.100 0.112 0.142 0.165 0.191 0.218 0.247 0.278</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 6 Observations of transport mechanisms in the flume during verification tests for each sand fraction. Black numbers indicate that no transport is observed. Green numbers indicate the observation of rolling and jumping of the smallest particles. Blue numbers indicate the formation of ripples. Underlined blue numbers indicate the observation of first initiation of motion. Red numbers indicate severe erosion in the flume. The most right column shows the handle position of the flume at the first formation of ripples.

### 3.3 Validation results

The theoretical derived bed shear stresses by Booij, and the observed critical shear stresses according to Shields – Van Rijn are presented as a function of the rotation velocity of the flume (Figure 6). Corresponding top lid velocities obey equation (1.3). The range of critical shear stresses as a result of the range of Φ_{cr} is presented with vertical errorbars. In his estimation of the bed shear stress (equation 1.9), Booij suggested a c_t value of 25 for a flume with a radius of 1.7 m. This value appears to be unsatisfactory with the observed data (black dashed line in Figure 6). The exponential trend however, seems to match the trend of observed bed shear stresses. Therefore equation (1.9), with c_t as unknown coefficient, is fitted to the measured data in order to obtain a correct value for c_t. The best fit (r=0.989) corresponds with a c_t value of 8.425 (red dashed line in Figure 6). Since this formulation shows a deviation near the origin (a nonzero bed shear stress is exerted while ω_f = 0) a small correction is applied. The final formulation for the
bed shear stress as function of the rotation velocity of the flume is given by (blue solid line in Figure 6):

\[
\tau_{bw} = 1/10 \left( \frac{1.637 \omega_f + 3.872}{c_t} \right)^2 - 0.0183; \quad c_t = 8.425
\tag{1.12}
\]

Figure 6 Calibration results of the “theoretical bed shear stresses estimated by Booij” and the observed “critical shear stress according to Shields – Van Rijn”

### 3.4 Discussion

It can be observed that the modeled bed shear stress (1.12) deviates slightly from the measured critical bed shear stresses. The measurement of the grain size by laser diffraction might deviate little from the actual particle size. However, this deviation is small compared to the error that is introduced by the manual reading of \( \Theta_{cr} \) from Figure 5. A second inaccuracy is introduced by the stepwise increase of the bed shear stress. The actual critical shear stress for the formation of ripples occurs in-between these successive steps. Nonetheless we can conclude that the measured and modeled bed shear stresses are in good agreement.
4 Conclusions

The combination of a theoretical derived formulation and experimental measurements for the bed shear stress in the annular flume are in good agreement. The relation between the rotation velocity of the flume and the exerted bed shear stress of the Zhejiang University Annular Flume is found for a waterdepth of 14.0 cm for which a $c_t$ value of 8.425 is found:

$$\tau_{\text{bed}} = \frac{1}{10} \left( \frac{1.637 \omega f}{c_t} + 3.872 \right)^2 - 0.0183$$

The reason for the difference between the $c_t$-value obtained for the Zhejiang flume and the Delft flume is unclear, however, the variation in curvature of the flumes is expected to be responsible for this difference. Visual observation showed that turbulence structure and secondary currents in the flume are of secondary order and can therefore be neglected in erosion studies.
References