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THE INFLUENCE OF WATER DEPTH ON THE HEAVING AND PITCHING MOTIONS OF A SHIP MOVING IN LONGITUDINAL REGULAR HEAD WAVES

by

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CONTENTS

ABSTRACT

The heaving and pitching motions of a Series 60 model of $C_B=0.7$ moving in longitudinal regular head waves of shallow water are calculated by Watanabe's strip method[1], [2], [3]. The results are represented in Tables and Figures and the shallow water effect is discussed.

NOMENCLATURE

a, b, c, d, e, g :	coefficients of heave equation
A, B, C, D, E, G	coefficients of pitch equation
A_w	waterplane area
B(x)	beam of a section
F_a	exciting force amplitude
g	gravity constant
G_o	center of gravity (C.G.)
h	water depth
$\bar{h}$	wave amplitude
H	half-beam draft ratio
I_w	moment of waterplane area
I_{yy}	moment of inertia of the ship about y-axis
L	length between perpendiculars
m''	sectional added mass of unit thickness for heave
M_a	exciting moment amplitude
N	sectional heave damping coefficient
S_w	sectional area under calm water surface
t	time
T	draft
$\bar{T}$	mean draft
v	ship velocity
∇	displacement volyme
w	suffix designating wave
x, y, z	body coordinates
X, Y, Z	space coordinates

β	section fullness coefficient
$\varepsilon_{\zeta W}$	phase difference between heave and wave
$\varepsilon_{\psi W}$	" " " pitch and wave
$\varepsilon_{\psi\zeta}$	" " " pitch and heave
ε_{FW}	" " " exciting force and wave
ε_{MW}	" " " " moment " "
ζ	heave at time t
ζ_a	heave amplitude
ζ_W	wave elevation at time t
ζ_{AB}	amplitude of absolut bow motion
ζ_{AS}	" " " stern motion
ζ_{RB}	" " relativ bow motion
λ	wave length
ν	ω^2/g
ν_o	shallow water number
ρ	water density
ψ	pitch at time t
ψ_a	pitch amplitude
ω	circular frequence
ω_e	circular frequency of encounter

THE INFLUENCE OF WATER DEPTH ON THE HEAVING AND PITCHING MOTIONS OF A SHIP MOVING IN LONGITUDINAL REGULAR HEAD WAVES

INTRODUCTION

By applying Watanabe's strip theory some important hydrodynamic forces and moments acting on a Series 60 model of $C_B = 0.7$ moving in a longitudinal head wave system of shallow water are calculated. The calculated results are represented in non-dimensional forms and shown in Tables A and Figs. A.

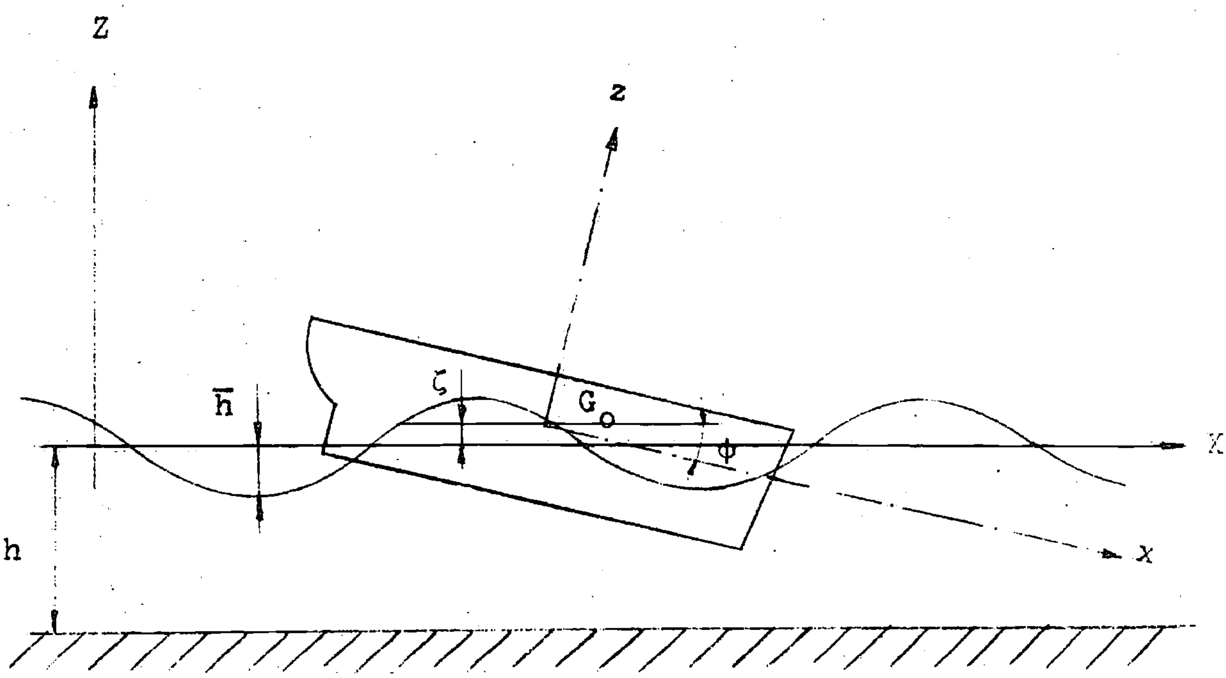
Subsequently, heaving and pitching motions of the ship are calculated and the results are presented in Tables B and Figures B.

The influence of shallow water depth on the hydrodynamic forces and motions as shown in the figures are discussed.

It is revealed by the calculations that the heaving and pitching motions are remarkably damped by the shallow water effect.

DEFINITION OF SHIP MOTIONS AND WAVES

The coordinate systems here utilized are space- and body-coordinate system O-XYZ and G_o-xyz respectively. X-axis lies on the undisturbed water surface and Z-axis points vertically upward. x-axis is longitudinal passing through the center of gravity G_o of the ship, while y- and z-axis point port and upward, respectively. The coordinate system G_o-xyz coincides with the system O-XYZ at the initial rest condition. We follow the convention of right-handed coordinate system.



Assuming only heaving and pitching motions of a ship at the speed V in a longitudinally oncoming wave system, we describe the surface wave as follows

$$\zeta_W = \bar{h} \cos (\nu_o x + \omega_e t) \qquad \text{.........(1)}$$

, where $\bar{\eta}$ wave amplitude

ν_o shallow water wave number, i.e. $\left(\frac{\omega^2}{g} = \nu_o \tanh \nu_o h\right)$

ω_e circular frequency of encounter i.e. $(\omega + \nu_o V)$

The heaving and pitching motions of the ship corresponding to the wave defined above are then expressed by

$$\left.\begin{aligned} \zeta &= \zeta_a \cos\,(\omega_e t + \varepsilon_{\zeta w}) \\ \psi &= \psi_a \cos\,(\omega_e t + \varepsilon_{\psi w}) \end{aligned}\right\} \quad \ldots\ldots\ldots(2)$$

respectively, where ζ_a, ψ_a are heave and pitch amplitudes and $\varepsilon_{\zeta w}$, $\varepsilon_{\psi w}$ phase differences between heave and wave and pitch and wave, respectively.

THE COUPLED EQUATIONS AND COEFFICIENTS

The coupled equations of heave and pitch of a ship moving in longitudinal regular waves [1], [2] are written in the form

$$\left.\begin{aligned} a\ddot{\zeta} + b\dot{\zeta} + c\zeta - d\ddot{\psi} - e\dot{\psi} - g\psi &= F_a \cos\,(\omega_e t + \varepsilon_{FW}) \\ A\ddot{\psi} + B\dot{\psi} + C\psi - D\ddot{\zeta} - E\dot{\zeta} - G\zeta &= M_a \cos\,(\omega_e t + \varepsilon_{FW}) \end{aligned}\right\} \quad \ldots\ldots(3)$$

The coefficients on the left-hand sides of the above equations are

$$a = \rho\nabla + \int_L m'' \, dx$$

$$b = \int_L N \, dx$$

$$c = 2\rho g \int_L y_w dx$$

$$d = \int_L m'' x \, dx + \rho \int_L S_w x \, dx$$

$$e = \int_L N x \, dx - V \int_L m'' \, dx$$

$$g = 2\rho g \int_L y_w x \, dx - V \int_L N \, dx$$

$$A = I_{yy} + \int_L m'' x^2 \, dx$$

$$B = \int_L N x^2 \, dx$$

$$C = 2\rho g \int_L y_w x^2 \, dx - VE$$

$$D = \int_L m'' x \, dx + \rho \int_L S_w x \, dx$$

$$E = \int_L N x \, dx + V \int_L m'' \, dx$$

$$G = 2\rho g \int_L y_w x \, dx$$

, where ρ water density

g	gravity constant
∇	displacement volume
S_W	sectional area under calm water level
y_W	half-breadth of a section on the calm water-line
I_{yy}	longitudinal moment of inertia of the ship's mass about G_o-y-axis
m''	sectional added mass of unit thickness for heavee
N	sectional heave damping coefficient of unit thickness

The exciting forces and moments on the right-hand sides of the equations (3) are represented in the form

$$F_a \begin{Bmatrix} \cos \varepsilon_{FW} \\ \sin \varepsilon_{FW} \end{Bmatrix} = 2\rho g \bar{h} \int_L y_W \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \cos \nu_o x \\ \sin \nu_o x \end{Bmatrix} dx$$

$$- \omega \bar{h} \, (\omega + \nu_o V) \int_L m'' \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \cos \nu_o x \\ \sin \nu_o x \end{Bmatrix} dx$$

$$\mp \omega \bar{h} \int_L N \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \sin \nu_o x \\ \cos \nu_o x \end{Bmatrix} dx$$

$$M_a \begin{Bmatrix} \cos \varepsilon_{MW} \\ \sin \varepsilon_{MW} \end{Bmatrix} = \bar{h} \int_L (\omega^2 m'' - 2\rho g y_W) \, x \, \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \cos \nu_o x \\ \sin \nu_o x \end{Bmatrix} dx$$

$$\pm \omega \bar{h} \int_L \left(N - V \frac{dm''}{dx} \right) x \, \frac{\cosh \nu_o (h - \bar{T})}{\cosh \nu_o h} \begin{Bmatrix} \sin \nu_o x \\ \cos \nu_o x \end{Bmatrix} dx$$

where h water depth

$\bar{T}$ mean draft of a section

$\varepsilon_{FW}, \varepsilon_{MW}$ phase differences between exciting force and wave and exciting moment and wave, respectively.

Sectional values of added mass and damping coefficient m" and N for heave are obtained from [3]. In the case of deep water these values are obtained from [8]. If $h \to \infty$ then ν_o and $\frac{\cosh \nu_o(h - \bar{T})}{\cosh \nu_o h}$ are replaced by $\nu = \frac{\omega^2}{g}$ and $e^{-\nu\bar{T}}$, respectively.

DIMENSIONLESS REPRESENTATION

In representing the calculated results, the following dimensionless forms are used:

$\frac{h}{T}$ depth parameter

$\frac{\lambda}{L}$ wave length to ship length ratio

$\frac{V}{\sqrt{g L}} = F_n$ Froude Number

$\omega_e\sqrt{\frac{L}{g}}$ frequency of encounter

$\frac{a}{\rho \nabla}$ virtual mass coefficient

$\frac{b\sqrt{L g}}{\rho g \nabla}$ heave damping coefficient

$\frac{A}{\rho \nabla L^2}$ virtual inertia coefficient

$\dfrac{B\sqrt{L\,g}}{\rho\, g\, \nabla\, L^2}$ pitch damping coefficient

$\dfrac{F_a}{\rho\, g\, A_W \bar{h}}$ exciting force coefficient

$\dfrac{M_a}{\rho\, g\, I_W \nu_o \bar{h}}$ exciting moment coefficient

$\dfrac{\zeta_a}{\bar{h}}$ heave amplitude ratio

$\dfrac{\psi_a}{\nu_o \bar{h}}$ pitch amplitude ratio

By assuming that C.G. lies at midship the absolute bow and stern motions and relative bow motion are expressed in non-dimensional forms as follows

$$\left\{\begin{matrix} \dfrac{\zeta_{AB}}{\bar{h}} \\ \dfrac{\zeta_{AS}}{\bar{h}} \end{matrix}\right\} = \left\{\left(\frac{\zeta_a}{\bar{h}} \cos \varepsilon_{\zeta W} \mp \frac{\psi_a L}{2\bar{h}} \cos \varepsilon_{\psi W}\right)^2 + \left(\frac{\zeta_a}{\bar{h}} \sin \varepsilon_{\zeta W} \mp \frac{\psi_a L}{2\bar{h}} \sin \varepsilon_{\psi W}\right)^2\right\}^{\frac{1}{2}}$$

$$\zeta_{RB} = \left\{\left[\frac{\zeta_a}{\bar{h}} \cos \varepsilon_{\zeta W} - \frac{\psi_a L}{2\bar{h}} \cos \varepsilon_{\psi W} - \cos\left(\frac{\pi L}{\lambda}\right)\right]^2 + \left[\frac{\zeta_a}{\bar{h}} \sin \varepsilon_{\zeta W} - \frac{\psi_a L}{2\bar{h}} \sin \varepsilon_{\psi W} - \sin\left(\frac{\pi L}{\lambda}\right)\right]^2\right\}^{\frac{1}{2}}$$

where λ wave length

L length between perpendiculars

A_W waterplane area

I_W moment of waterplane area about y-axis.

CALCULATION AND DISCUSSION

For the numerical calculations we adopt a Series 60 model of C_B=0.7 having following particulars.

Length between perpendiculars	3.000	m
Displacement volume	0.1537	m^3
Draft	0.171	m
Beam	0.428	m
Radius of gyration	0.750	m

Station	B(x)	H(x)	β (x)
1	0.0830	0.2425	0.8386
3	0.2803	0.8186	0.8716
5	0.4001	1.1685	0.9301
7	0.4280	1.2498	0.9761
9	0.4280	1.2498	0.9860
11	0.4280	1.2498	0.9850
13	0.4280	1.2498	0.9633
15	0.4113	1.2010	0.8660
17	0.3372	0.9848	0.6794
19	0.1575	0.4599	0.3751

,where B(x) beam of a section

H(x) half-beam draft ratio of a section

β(x) fullness coefficient of a section

The calculations are carried out for the following speeds, waves and depths.

F_n = 0.0, 0.1 and 0.2.

$\frac{\lambda}{L}$ = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.3, 1.5, 1.7, and 2.0.

$\frac{h}{T}$ = ∞, 10.0, 4.0, 2.5, 2.0 and 1.5.

In the calculation the following assumptions are made:

1. Although trim and parallel sinkage exist they are not considered.
2. The center of gravity lies at L/2.

The Virtual Mass, Virtual Inertia, Heave and Pitch Damping Coefficients are represented as functions of frequency for different depth parameter h/T at F_n=0.0 (Table A-1 & A-2). They are also illustrated in Figures A-1 to A-4. It is seen from Fig. A-1 that at high frequency range the added mass for the depth h = 1.5 T is approximately twice as large as that for h/T = ∞, 10.0, 4.0, and 2.5. This suggests that the natural heaving period of a ship in shallow water will be longer than that in deep water, provided that damping is comp comparatively small. From Fig. A-3 we observe that heave damping coefficients increase noticeably as the depth decreases and at the depth of h/T = 1.5 they are nearly twice as bi big as those for h/T = ∞ and 10.0.

The Exciting Forces and Moments acting on the restrained ship moving at the velocity of F_n = 0.2 are illustrated in Fig. A-5 and A-6 as functions of wave length to ship length ratio. It is found that these forces and moments generally increase as the depth decreases.

<u>The Heave and Pitch Amplitudes</u> together with the phase differences with respect to the waves are given in Tables B and illustrated in Figs B. In general the motions are remarkably damped as the depth decreases. This tendency is more significant as the Froude Number increases.

Although our theory cannot solve the motion problem in very shallow water, it will be quite useful to consider underkeel clearences. Provided that this theoretical calculation is proved to be reasonable in an experimental study, this method will be a routine technique for further research on ship's behaviour in resricted waters.

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TABLE A-1

	$\omega_e \sqrt{\frac{L}{g}}$	$\frac{a}{\rho \nabla}$	$\frac{b \sqrt{L g}}{\rho g \nabla}$	$\frac{A}{\rho \nabla L^2}$	$\frac{B \sqrt{L g}}{\rho g \nabla L^2}$
h/T= ∞	3.963	1.824	1.120	0.095	0.077
	3.545	1.792	1.405	0.094	0.089
	3.236	1.781	1.626	0.095	0.098
	2.996	1.783	1.794	0.096	0.104
	2.802	1.794	1.920	0.097	0.109
	2.642	1.811	2.016	0.098	0.112
	2.390	1.855	2.146	0.101	0.115
	2.198	1.905	2.212	0.104	0.116
	2.047	1.957	2.249	0.107	0.117
	1.922	2.008	2.265	0.109	0.116
h/T=10.0	3.963	1.824	1.124	0.095	0.077
	3.545	1.791	1.406	0.094	0.089
	3.236	1.779	1.622	0.095	0.098
	2.996	1.782	1.782	0.096	0.103
	2.802	1.794	1.899	0.097	0.107
	2.641	1.812	1.982	0.098	0.110
	2.386	1.857	2.078	0.101	0.111
	2.190	1.905	2.111	0.104	0.111
	2.029	1.949	2.109	0.106	0.109
	1.894	1.986	2.088	0.109	0.106
h/T=4.0	3.960	1.819	1.169	0.094	0.078
	3.533	1.775	1.463	0.093	0.090
	3.209	1.749	1.694	0.093	0.098
	2.946	1.734	1.874	0.093	0.104
	2.726	1.724	2.014	0.093	0.108
	2.535	1.717	2.121	0.093	0.111
	2.219	1.707	2.271	0.093	0.115
	1.968	1.703	2.360	0.094	0.117
	1.763	1.707	2.413	0.094	0.118
	1.594	1.706	2.456	0.094	0.119
	1.390	1.647	2.538	0.091	0.121

TABLE A-2

	$\omega_e \sqrt{\frac{L}{g}}$	$\frac{a}{\rho \Delta}$	$\frac{b \sqrt{L g}}{\rho g \Delta}$	$\frac{A}{\rho \Delta L^2}$	$\frac{B \sqrt{L g}}{\rho g \Delta L^2}$
h/T=2.5	3.919	1.833	1.489	0.094	0.090
	3.448	1.788	1.909	0.093	0.105
	3.076	1.754	2.228	0.091	0.117
	2.772	1.727	2.463	0.091	0.125
	2.518	1.707	2.628	0.090	0.131
	2.303	1.694	2.743	0.090	0.135
	1.959	1.673	2.915	0.089	0.141
	1.699	1.593	3.112	0.085	0.148
	1.497	1.460	3.283	0.079	0.155
	1.336	1.407	3.324	0.077	0.156
	1.149	1.586	3.224	0.086	0.152
h/T=2.0	3.854	1.924	1.913	0.096	0.106
	3.348	1.891	2.393	0.095	0.124
	2.951	1.856	2.720	0.093	0.136
	2.631	1.826	2.931	0.092	0.144
	2.368	1.804	3.076	0.092	0.149
	2.149	1.781	3.212	0.091	0.154
	1.808	1.666	3.536	0.085	0.167
	1.556	1.538	3.719	0.080	0.175
	1.364	1.591	3.652	0.083	0.171
	1.213	1.816	3.510	0.094	0.165
	1.039	2.309	3.341	0.118	0.158
h/T=1.5	3.702	2.597	3.015	0.111	0.144
	3.153	2.561	3.407	0.109	0.160
	2.734	2.459	3.606	0.107	0.169
	2.407	2.370	3.795	0.105	0.178
	2.145	2.278	4.050	0.101	0.190
	1.932	2.174	4.274	0.097	0.200
	1.679	2.089	4.316	0.094	0.201
	1.756	2.100	4.342	0.094	0.202
	1.840	2.130	4.330	0.095	0.202
	1.608	2.099	4.265	0.095	0.199
	1.375	2.315	3.982	0.107	0.187

TABLE B-1

$h/T = \infty$		$\frac{\lambda}{L}$	$\omega_e\sqrt{\frac{L}{g}}$	$\frac{\zeta_a}{\bar{h}}$	$\frac{\phi_a}{\nu_o \bar{h}}$	$\varepsilon_{\zeta w}$	$\varepsilon_{\phi w}$	$\varepsilon_{\phi\zeta}$	$\frac{\zeta_{AB}}{\bar{h}}$
	Fn=0.0	0.50	3.545	0.268	0.110	81	3	-78	0.691
		0.60	3.236	0.384	0.021	115	-4	-119	0.449
		0.70	2.996	0.272	0.141	123	-122	115	0.787
		0.80	2.802	0.143	0.289	105	-116	139	1.247
		0.90	2.642	0.121	0.411	42	-111	-153	1.545
		1.00	2.507	0.213	0.510	12	-108	-120	1.717
		1.10	2.390	0.312	0.588	4	-105	-109	1.805
		1.30	2.198	0.470	0.702	-1	-101	-100	1.847
		1.50	2.047	0.591	0.778	-2	-99	-97	1.806
		1.70	1.922	0.675	0.830	-2	-98	-96	1.739
		2.00	1.772	0.761	0.882	-2	-96	-94	1.632
	Fn=0.1	0.50	4.802	0.051	0.044	-5	-64	-59	0.258
		0.60	4.283	0.161	0.046	29	-14	-43	0.162
		0.70	3.894	0.141	0.129	58	122	64	0.533
		0.80	3.588	0.190	0.388	-64	170	-126	1.647
		0.90	3.340	0.554	0.620	-29	-162	-133	2.578
		1.00	3.135	0.655	0.769	-6	-142	-136	2.928
		1.10	2.961	0.643	0.841	2	-129	-131	2.867
		1.30	2.682	0.667	0.903	1	-116	-117	2.554
		1.50	2.466	0.726	0.933	-1	-109	-106	2.288
		1.70	2.292	0.780	0.950	-2	-105	-103	2.081
		2.00	2.087	0.840	0.964	-2	-101	-99	1.847
	Fn=0.2	0.50	6.058	0.014	0.020	-34	-80	-46	0.118
		0.60	5.330	0.064	0.026	9	-48	-57	0.113
		0.70	4.791	0.091	0.039	30	63	33	0.110
		0.80	4.373	0.058	0.168	101	107	6	0.604
		0.90	4.038	0.266	0.426	-153	134	-73	1.438
		1.00	3.763	0.761	0.745	-109	164	-87	2.419
		1.10	3.532	1.217	0.918	-72	-171	-99	3.054
		1.30	3.165	1.365	1.069	-23	-145	-122	3.493
		1.50	2.884	1.137	1.122	-7	-127	-120	3.091
		1.70	2.662	1.028	1.104	-3	-117	-114	2.631
		2.00	2.401	0.985	1.066	-3	-109	-106	2.168

TABLE B- 2

		$\frac{\lambda}{L}$	$\omega_e\sqrt{\frac{L}{g}}$	$\frac{\zeta_a}{\bar{h}}$	$\frac{\psi_a}{\nu_o\ \bar{h}}$	$\varepsilon_{\zeta w}$	$\varepsilon_{\psi w}$	$\varepsilon_{\psi\zeta}$	$\frac{\zeta_{AB}}{\bar{h}}$
h/T= 10.0	Fn=0.0	0.50	3.545	0.268	0.110	81	3	-78	0.691
		0.60	3.236	0.384	0.021	115	-4	-119	0.448
		0.70	2.996	0.272	0.142	123	-122	115	0.789
		0.80	2.802	0.143	0.290	105	-116	139	1.251
		0.90	2.641	0.122	0.413	42	-111	-153	1.550
		1.00	2.505	0.214	0.512	12	-107	-119	1.721
		1.10	2.386	0.314	0.590	4	-105	-109	1.810
		1.30	2.190	0.476	0.703	0	-101	-101	1.849
		1.50	2.029	0.592	0.778	-1	-99	- 98	1.806
		1.70	1.894	0.675	0.829	-1	-97	-96	1.736
		2.00	1.724	0.760	0.878	-1	-95	-94	1.625
	Fn=0.1	0.50	4.802	0.051	0.044	-5	-64	-59	0.259
		0.60	4.283	0.161	0.046	29	-14	-43	0.163
		0.70	3.893	0.141	0.129	58	122	64	0.534
		0.80	3.588	0.191	0.389	-64	170	-126	1.650
		0.90	3.339	0.556	0.622	-28	-162	-134	2.586
		1.00	3.133	0.655	0.772	-5	-142	-137	2.937
		1.10	2.958	0.643	0.843	2	-129	-131	2.874
		1.30	2.673	0.665	0.905	1	-116	-117	2.556
		1.50	2.448	0.724	0.934	-1	-109	-108	2.287
		1.70	2.264	0.777	0.950	-1	-104	-103	2.075
		2.00	2.038	0.835	0.961	-1	-100	-99	1.836
	Fn=0.2	0.50	6.058	0.014	0.020	-34	-80	-46	0.119
		0.60	5.330	0.064	0.026	9	-48	-57	0.114
		0.70	4.791	0.091	0.039	30	63	33	0.109
		0.80	4.373	0.058	0.168	101	107	6	0.603
		0.90	4.038	0.266	0.427	-152	134	-74	1.440
		1.00	3.761	0.764	0.746	-109	164	-87	2.426
		1.10	3.529	1.222	0.919	-71	-170	-99	3.066
		1.30	3.156	1.360	1.074	-22	-144	-122	3.509
		1.50	2.867	1.124	1.125	-6	-127	-121	3.088
		1.70	2.633	1.014	1.104	-2	-116	-114	2.619
		2.00	2.352	0.971	1.064	-2	-101	-99	2.151

TABLE B- 3

		$\frac{\lambda}{L}$	$\omega_e\sqrt{\frac{L}{g}}$	$\frac{\zeta_a}{\bar{h}}$	$\frac{\phi_a}{\nu_o\ \bar{h}}$	$\varepsilon_{\zeta w}$	$\varepsilon_{\phi w}$	$\varepsilon_{\phi\zeta}$	$\frac{\zeta_{AB}}{\bar{h}}$
h/T= 4.0	Fn=0.0	0.50	3.533	0.273	0.111	85	4	-81	0.708
		0.60	3.209	0.369	0.022	119	-8	-127	0.450
		0.70	2.946	0.253	0.142	126	-117	117	0.784
		0.80	2.726	0.134	0.284	102	-111	147	1.224
		0.90	2.535	0.125	0.400	40	-102	-147	1.503
		1.00	2.368	0.218	0.493	13	-104	-117	1.661
		1.10	2.219	0.315	0.568	6	-101	-107	1.741
		1.30	1.968	0.471	0.677	2	-98	-100	1.779
		1.50	1.763	0.584	0.750	0	-96	-96	1.742
		1.70	1.594	0.666	0.802	0	-95	-95	1.681
		2.00	1.390	0.750	0.853	0	-94	-94	1.582
	Fn=0.1	0.50	4.790	0.052	0.045	-3	-64	-61	0.263
		0.60	4.256	0.168	0.046	30	-12	-42	0.162
		0.70	3.844	0.135	0.144	59	130	71	0.618
		0.80	3.511	0.265	0.421	-41	175	-144	1.860
		0.90	3.233	0.545	0.642	-5	-150	-145	2.702
		1.00	2.996	0.541	0.743	10	-150	-140	2.770
		1.10	2.791	0.531	0.786	9	-119	-128	2.608
		1.30	2.451	0.597	0.833	4	-108	-112	2.304
		1.50	2.182	0.674	0.864	1	-103	-104	2.083
		1.70	1.963	0.736	0.886	0	-100	-100	1.916
		2.00	1.704	0.803	0.910	0	-98	-98	1.730
	Fn=0.2	0.50	6.047	0.014	0.021	-33	-80	-47	0.121
		0.60	5.303	0.066	0.026	9	-48	-57	0.117
		0.70	4.742	0.094	0.042	32	67	35	0.124
		0.80	4.296	0.058	0.195	122	112	-10	0.708
		0.90	3.931	0.380	0.510	-135	144	-81	1.762
		1.00	3.624	0.980	0.809	-84	179	-97	2.836
		1.10	3.362	1.303	0.936	-42	-159	-117	3.453
		1.30	2.934	1.065	1.074	-3	-130	-127	3.349
		1.50	2.601	0.901	1.045	1	-115	-116	2.713
		1.70	2.333	0.875	1.014	1	-108	-109	2.309
		2.00	2.018	0.888	0.990	0	-102	-102	1.951

TABLE B- 4

$h/T = 2.5$		$\frac{\lambda}{L}$	$\omega_e\sqrt{\frac{L}{g}}$	$\frac{\zeta_a}{\bar{h}}$	$\frac{\psi_a}{v_o\ \bar{h}}$	$\varepsilon_{\zeta w}$	$\varepsilon_{\psi w}$	$\varepsilon_{\psi\zeta}$	$\frac{\zeta_{AB}}{\bar{h}}$
	Fn=0.0	0.50	3.448	0.250	0.110	100	10	-90	0.736
		0.60	3.076	0.314	0.029	125	-15	-140	0.440
		0.70	2.772	0.223	0.136	126	-107	127	0.764
		0.80	2.518	0.124	0.269	98	-105	157	1.172
		0.90	2.303	0.133	0.382	39	-103	-142	1.441
		1.00	2.118	0.223	0.475	15	-100	-115	1.600
		1.10	1.959	0.316	0.549	8	-99	-107	1.686
		1.30	1.699	0.470	0.660	3	-97	-100	1.736
		1.50	1.497	0.582	0.736	1	-95	-96	1.710
		1.70	1.336	0.664	0.790	1	-94	-95	1.658
		2.00	1.149	0.750	0.846	0	-94	-94	1.570
	Fn=0.1	0.50	4.704	0.060	0.050	4	-61	-65	0.291
		0.60	4.124	0.195	0.043	39	-3	-42	0.152
		0.70	3.670	0.140	0.172	44	161	115	0.845
		0.80	3.303	0.323	0.418	7	-155	-162	1.950
		0.90	3.001	0.402	0.563	19	-131	-150	2.324
		1.00	2.746	0.417	0.640	16	-119	-135	2.326
		1.10	2.530	0.458	0.693	10	-112	-122	2.260
		1.30	2.182	0.564	0.766	4	-105	-109	2.108
		1.50	1.916	0.654	0.814	2	-101	-103	1.964
		1.70	1.705	0.721	0.846	1	-99	-100	1.835
		2.00	1.463	0.792	0.880	0	-97	-97	1.682
	Fn=0.2	0.50	5.961	0.015	0.022	-27	-79	-52	0.131
		0.60	5.171	0.076	0.027	12	-45	-57	0.120
		0.70	4.567	0.099	0.057	37	85	48	0.202
		0.80	4.089	0.089	0.274	-159	131	-69	1.049
		0.90	3.699	0.583	0.595	-87	171	-112	2.265
		1.00	3.375	0.961	0.777	-41	-160	-119	3.024
		1.10	3.101	0.986	0.886	-13	-142	-129	3.235
		1.30	2.666	0.833	0.936	2	-120	-122	2.791
		1.50	2.335	0.809	0.937	2	-111	-113	2.392
		1.70	2.075	0.827	0.938	1	-106	-107	2.126
		2.00	1.777	0.862	0.938	0	-102	-102	1.857

TABLE B- 5

h/T	Fn	$\frac{\lambda}{L}$	$\omega_e\sqrt{\frac{L}{g}}$	$\frac{\zeta_a}{\bar{h}}$	$\frac{\phi_a}{\nu_0 \bar{h}}$	$\varepsilon_{\zeta w}$	$\varepsilon_{\phi w}$	$\varepsilon_{\phi\zeta}$	$\frac{\zeta_{AB}}{\bar{h}}$
h/T= 2.0	Fn=0.0	0.50	3.348	0.228	0.109	105	13	-92	0.734
		0.60	2.951	0.301	0.037	124	-17	-141	0.466
		0.70	2.631	0.224	0.132	124	-101	135	0.769
		0.80	2.368	0.131	0.263	98	-102	160	1.156
		0.90	2.149	0.138	0.375	42	-101	-143	1.419
		1.00	1.965	0.225	0.467	17	-99	-116	1.578
		1.10	1.808	0.317	0.542	9	-98	-107	1.666
		1.30	1.556	0.471	0.654	4	-96	-100	1.723
		1.50	1.364	0.582	0.732	2	-95	-97	1.703
		1.70	1.213	0.664	0.788	1	-94	-95	1.653
		2.00	1.039	0.750	0.844	1	-93	-94	1.567
	Fn=0.1	0.50	4.605	0.068	0.054	13	-57	-70	0.325
		0.60	3.998	0.208	0.038	45	-47	-92	0.173
		0.70	3.529	0.184	0.165	42	180	132	0.884
		0.80	3.154	0.313	0.381	23	-145	-168	1.805
		0.90	2.847	0.367	0.517	25	-126	-151	2.134
		1.00	2.593	0.396	0.601	19	-116	-135	2.187
		1.10	2.379	0.445	0.661	12	-111	-123	2.161
		1.30	2.040	0.556	0.741	5	-104	-109	2.045
		1.50	1.783	0.648	0.791	2	-101	-103	1.915
		1.70	1.583	0.717	0.829	1	-99	-100	1.805
		2.00	1.353	0.789	0.873	1	-97	-98	1.673
	Fn=0.2	0.50	5.862	0.017	0.006	-20	-76	-56	0.141
		0.60	5.046	0.084	0.027	16	-43	-59	o.122
		0.70	4.426	0.098	0.065	39	101	62	0.260
		0.80	3.939	0.117	0.296	-111	147	-102	1.194
		0.90	3.546	0.559	0.563	-63	-176	-113	2.237
		1.00	3.222	0.825	0.717	-29	-152	-123	2.790
		1.10	2.951	0.859	0.811	-9	-136	-127	2.919
		1.30	2.523	0.793	0.879	2	-119	-121	2.619
		1.50	2.202	0.792	0.897	2	-110	-112	2.300
		1.70	1.952	0.816	0.903	1	-106	-107	2.062
		2.00	1.667	0.854	0.912	0	-102	-102	1.819

TABLE B- 6

		$\frac{\lambda}{L}$	$\omega_e\sqrt{\frac{L}{g}}$	$\frac{\zeta_a}{\bar{h}}$	$\frac{\phi_a}{\nu_o \bar{h}}$	$\varepsilon_{\zeta w}$	$\varepsilon_{\phi w}$	$\varepsilon_{\psi\zeta}$	$\frac{\zeta_{AB}}{\bar{h}}$
h/T= 1.5	Fn=0.0	0.50	3.153	0.221	0.131	87	8	-79	0.810
		0.60	2.734	0.368	0.067	108	-8	-116	0.587
		0.70	2.407	0.301	0.124	117	-89	154	0.839
		0.80	2.145	0.185	0.251	101	-97	162	1.161
		0.90	1.932	0.153	0.363	55	-98	-153	1.405
		1.00	1.765	0.222	0.457	24	-98	-122	1.561
		1.10	1.608	0.310	0.535	12	-96	-108	1.651
		1.30	1.375	0.463	0.650	5	-95	-100	1.712
		1.50	1.200	0.577	0.729	2	-95	-97	1.694
		1.70	1.063	0.660	0.785	1	-94	-95	1.645
		2.00	0.908	0.747	0.841	1	-93	-94	1.562
	Fn=0.1	0.50	4.410	0.074	0.065	26	-55	-81	0.403
		0.60	3.782	0.219	0.064	42	-33	-75	0.348
		0.70	3.305	0.254	0.097	41	-156	163	0.674
		0.80	2.931	0.343	0.313	27	-143	-170	1.567
		0.90	2.630	0.412	0.469	28	-127	-155	2.017
		1.00	2.385	0.423	0.565	24	-117	-141	2.119
		1.10	2.180	0.453	0.627	16	-111	-127	2.096
		1.30	1.858	0.552	0.712	7	-105	-112	1.992
		1.50	1.619	0.642	0.777	4	-101	-105	1.897
		1.70	1.433	0.711	0.825	2	-99	-101	1.806
		2.00	1.222	0.784	0.872	1	-97	-98	1.676
	Fn=0.2	0.50	5.666	0.021	0.027	-4	-74	-70	0.163
		0.60	4.829	0.099	0.032	20	-54	-74	0.170
		0.70	4.202	0.128	0.035	27	128	101	0.220
		0.80	3.716	0.102	0.213	-29	159	-172	0.939
		0.90	3.329	0.379	0.445	-50	-171	-121	1.780
		1.00	3.031	0.660	0.597	-31	-150	-119	2.272
		1.10	2.751	0.801	0.694	-14	-137	-123	2.508
		1.30	2.342	0.813	0.805	1	-120	-121	2.473
		1.50	2.037	0.799	0.840	2	-112	-114	2.210
		1.70	1.802	0.817	0.862	1	-107	-108	2.010
		2.00	1.536	0.854	0.895	1	-103	-104	1.812

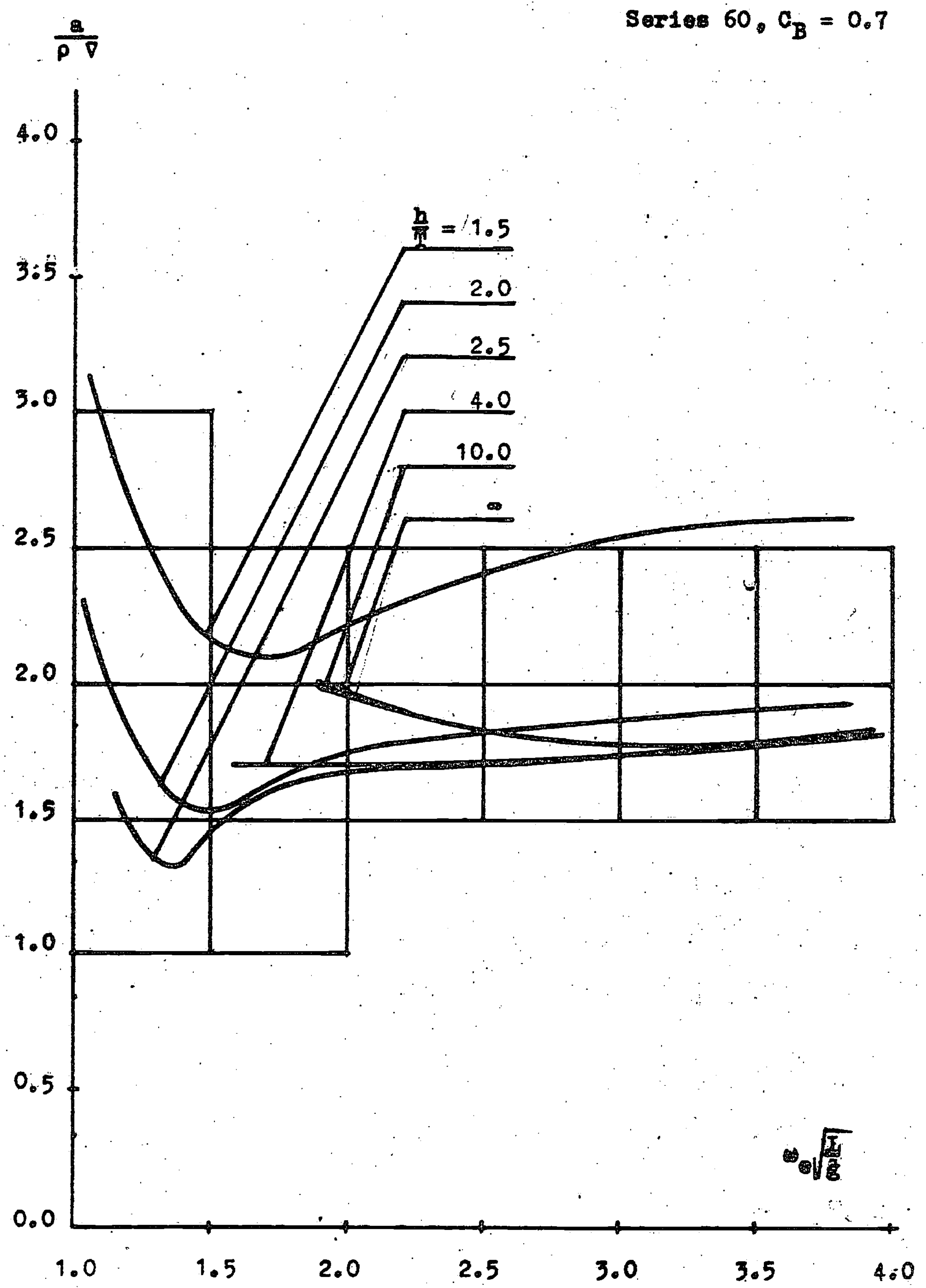
TABLE B- 6

		$\frac{\lambda}{L}$	$\omega_e\sqrt{\frac{L}{g}}$	$\frac{\zeta_a}{\bar{h}}$	$\frac{\phi_a}{\nu_o\ \bar{h}}$	$\varepsilon_{\zeta w}$	$\varepsilon_{\phi w}$	$\varepsilon_{\psi\zeta}$	$\frac{\zeta_{\Lambda B}}{\bar{h}}$
h/T= 1.5	Fn=0.0	0.50	3.153	0.221	0.131	87	8	-79	0.810
		0.60	2.734	0.368	0.067	108	-8	-116	0.587
		0.70	2.407	0.301	0.124	117	-89	154	0.839
		0.80	2.145	0.185	0.251	101	-97	162	1.161
		0.90	1.932	0.153	0.363	55	-98	-153	1.405
		1.00	1.765	0.222	0.457	24	-98	-122	1.561
		1.10	1.608	0.310	0.535	12	-96	-108	1.651
		1.30	1.375	0.463	0.650	5	-95	-100	1.712
		1.50	1.200	0.577	0.729	2	-95	-97	1.694
		1.70	1.063	0.660	0.785	1	-94	-95	1.645
		2.00	0.908	0.747	0.841	1	-93	-94	1.562
	Fn=0.1	0.50	4.410	0.074	0.065	26	-55	-81	0.405
		0.60	3.782	0.219	0.064	42	-33	-75	0.348
		0.70	3.305	0.254	0.097	41	-156	163	0.674
		0.80	2.931	0.343	0.313	27	-143	-170	1.567
		0.90	2.630	0.412	0.469	28	-127	-155	2.017
		1.00	2.385	0.423	0.565	24	-117	-141	2.119
		1.10	2.180	0.453	0.627	16	-111	-127	2.096
		1.30	1.858	0.552	0.712	7	-105	-112	1.992
		1.50	1.619	0.642	0.777	4	-101	-105	1.897
		1.70	1.433	0.711	0.825	2	-99	-101	1.806
		2.00	1.222	0.784	0.872	1	-97	-98	1.676
	Fn=0.2	0.50	5.666	0.021	0.027	-4	-74	-70	0.163
		0.60	4.829	0.099	0.032	20	-54	-74	0.170
		0.70	4.202	0.128	0.035	27	128	101	0.220
		0.80	3.716	0.102	0.213	-29	159	-172	0.939
		0.90	3.329	0.379	0.445	-50	-171	-121	1.780
		1.00	3.031	0.660	0.597	-31	-150	-119	2.272
		1.10	2.751	0.801	0.694	-14	-137	-123	2.508
		1.30	2.342	0.813	0.805	1	-120	-121	2.473
		1.50	2.037	0.799	0.840	2	-112	-114	2.210
		1.70	1.802	0.817	0.862	1	-107	-108	2.010
		2.00	1.536	0.854	0.895	1	-103	-104	1.812

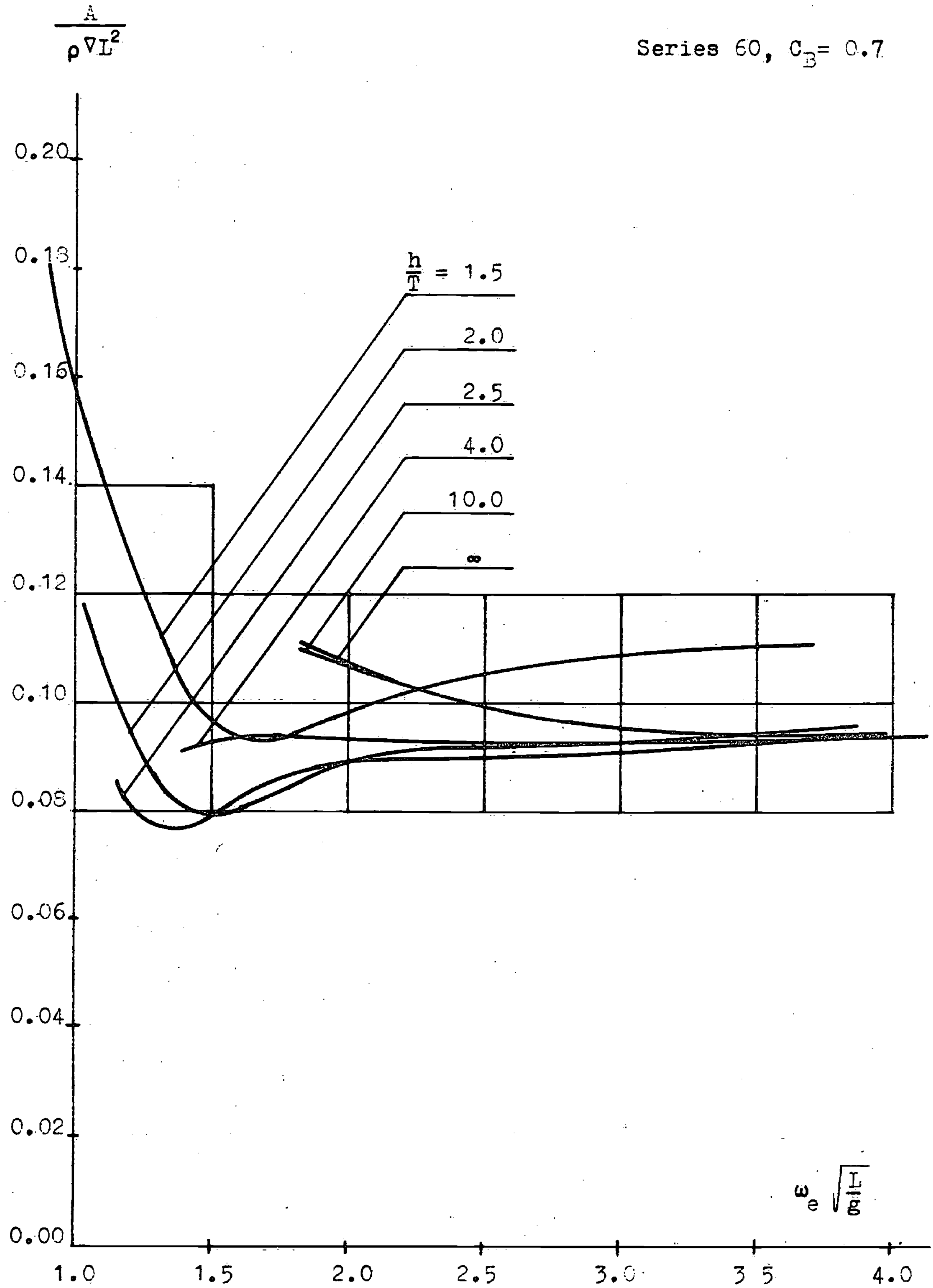
TABLE B- 6

		$\frac{\lambda}{L}$	$\omega_e\sqrt{\frac{L}{g}}$	$\frac{\zeta_a}{\bar{h}}$	$\frac{\phi_a}{\nu_o\ \bar{h}}$	$\varepsilon_{\zeta w}$	$\varepsilon_{\phi w}$	$\varepsilon_{\phi\zeta}$	$\frac{\zeta_{AB}}{\bar{h}}$
h/T= 1.5	Fn=0.0	0.50	3.153	0.221	0.131	87	8	-79	0.810
		0.60	2.734	0.368	0.067	108	-8	-116	0.587
		0.70	2.407	0.301	0.124	117	-89	154	0.839
		0.80	2.145	0.185	0.251	101	-97	162	1.161
		0.90	1.932	0.153	0.363	55	-98	-153	1.405
		1.00	1.765	0.222	0.457	24	-98	-122	1.561
		1.10	1.608	0.310	0.535	12	-96	-108	1.651
		1.30	1.375	0.463	0.650	5	-95	-100	1.712
		1.50	1.200	0.577	0.729	2	-95	-97	1.694
		1.70	1.063	0.660	0.785	1	-94	-95	1.645
		2.00	0.908	0.747	0.841	1	-93	-94	1.562
	Fn=0.1	0.50	4.410	0.074	0.065	26	-55	-81	0.403
		0.60	3.782	0.219	0.064	42	-33	-75	0.348
		0.70	3.305	0.254	0.097	41	-156	163	0.674
		0.80	2.931	0.343	0.313	27	-143	-170	1.567
		0.90	2.630	0.412	0.469	28	-127	-155	2.017
		1.00	2.385	0.423	0.565	24	-117	-141	2.119
		1.10	2.180	0.453	0.627	16	-111	-127	2.096
		1.30	1.858	0.552	0.712	7	-105	-112	1.992
		1.50	1.619	0.642	0.777	4	-101	-105	1.897
		1.70	1.433	0.711	0.825	2	-99	-101	1.806
		2.00	1.222	0.784	0.872	1	-97	-98	1.676
	Fn=0.2	0.50	5.666	0.021	0.027	-4	-74	-70	0.163
		0.60	4.829	0.099	0.032	20	-54	-74	0.170
		0.70	4.202	0.128	0.035	27	128	101	0.220
		0.80	3.716	0.102	0.213	-29	159	-172	0.939
		0.90	3.329	0.379	0.445	-50	-171	-121	1.780
		1.00	3.031	0.660	0.597	-31	-150	-119	2.272
		1.10	2.751	0.801	0.694	-14	-137	-123	2.508
		1.30	2.342	0.813	0.805	1	-120	-121	2.473
		1.50	2.037	0.799	0.840	2	-112	-114	2.210
		1.70	1.802	0.817	0.862	1	-107	-108	2.010
		2.00	1.536	0.854	0.895	1	-103	-104	1.812

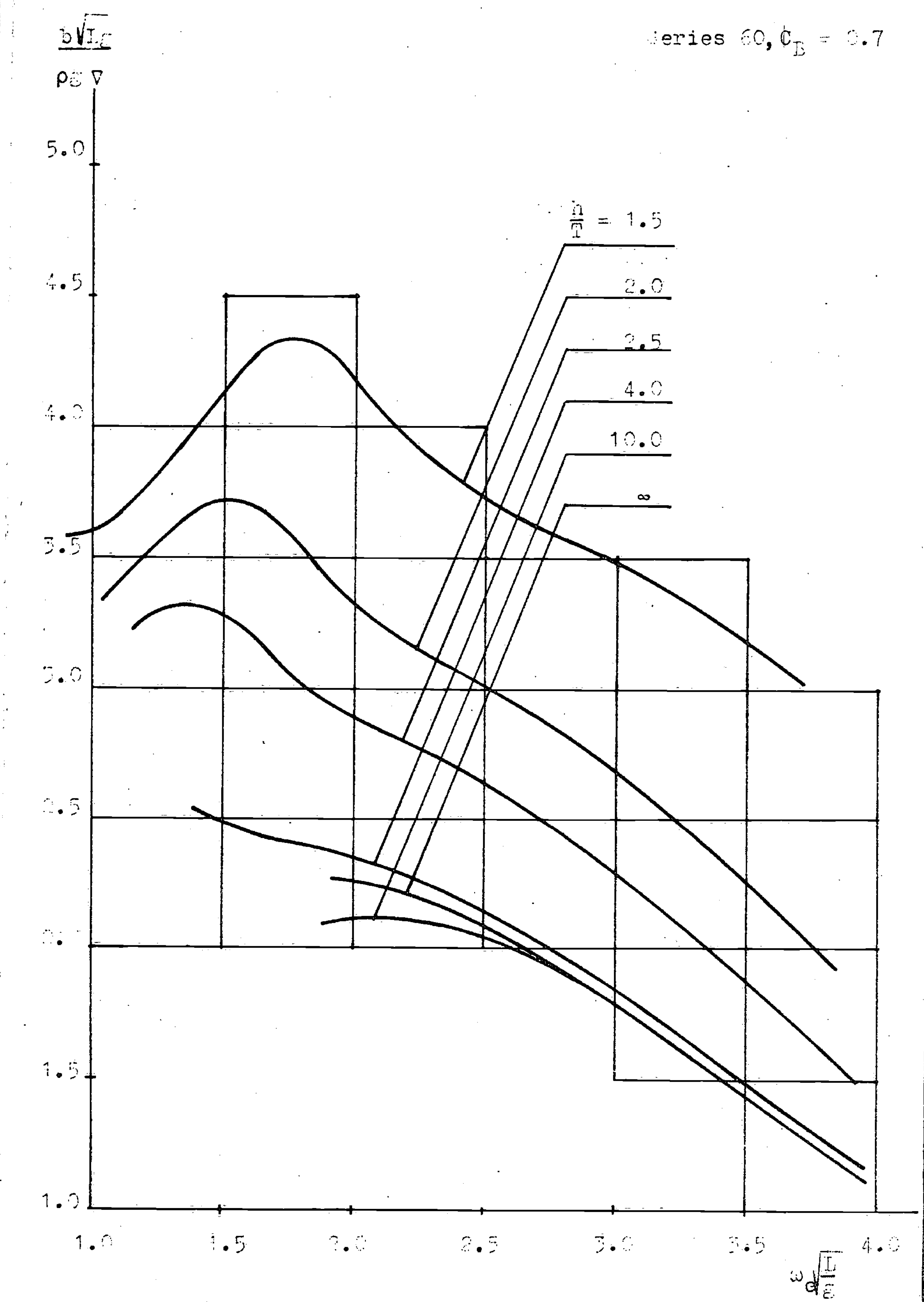
CHALMERS TEKNISKA HÖGSKOLA	Virtual Mass Coefficient at $F_n = 0.0$	Fig. A-1
		CTH - SH
		Report 44



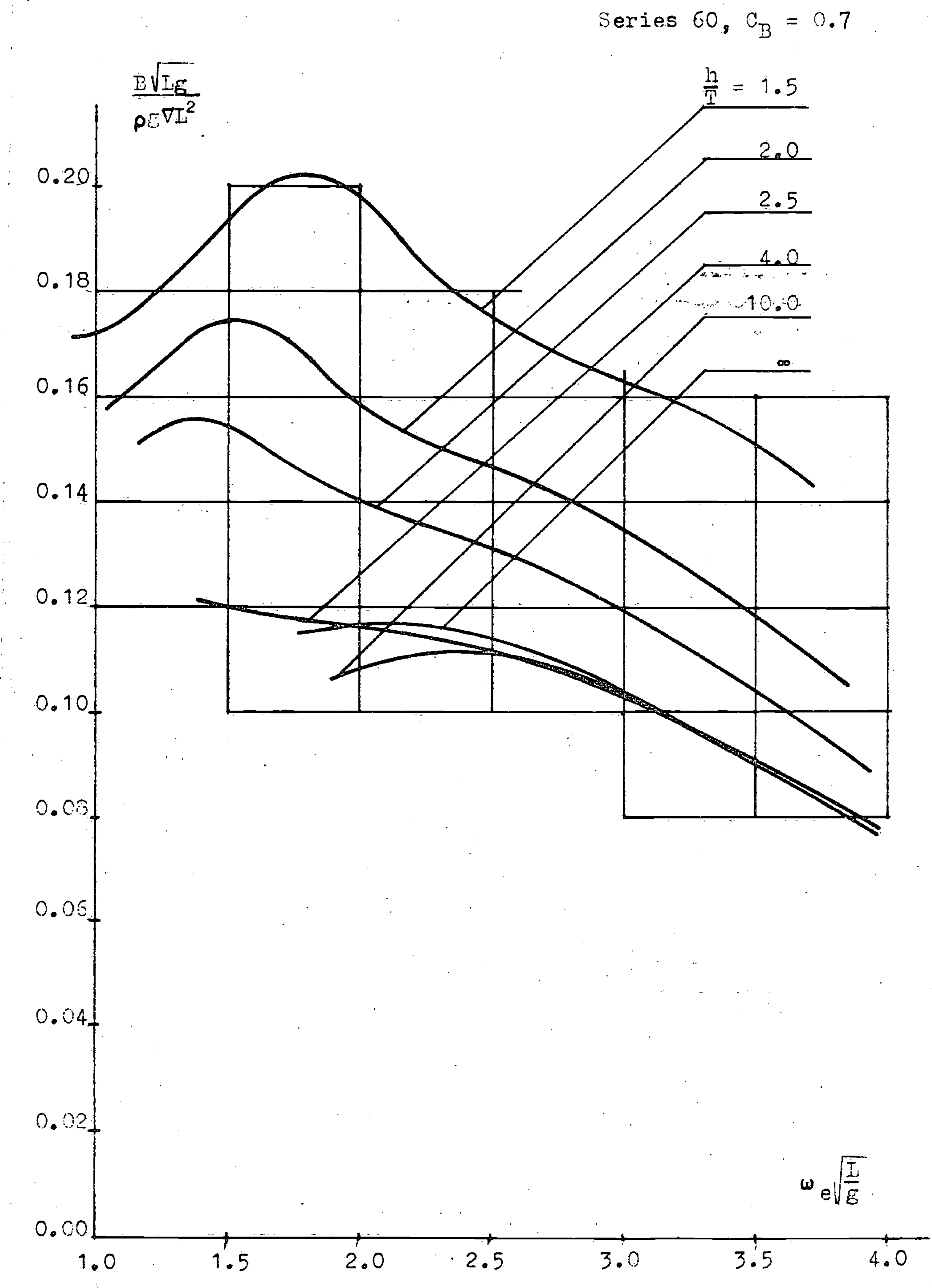
CHALMERS TEKNISKA HÖGSKOLA	Virtual Inertia Coefficient at F_n=0.0	Fig. A - 2 CTH - SH Report 44



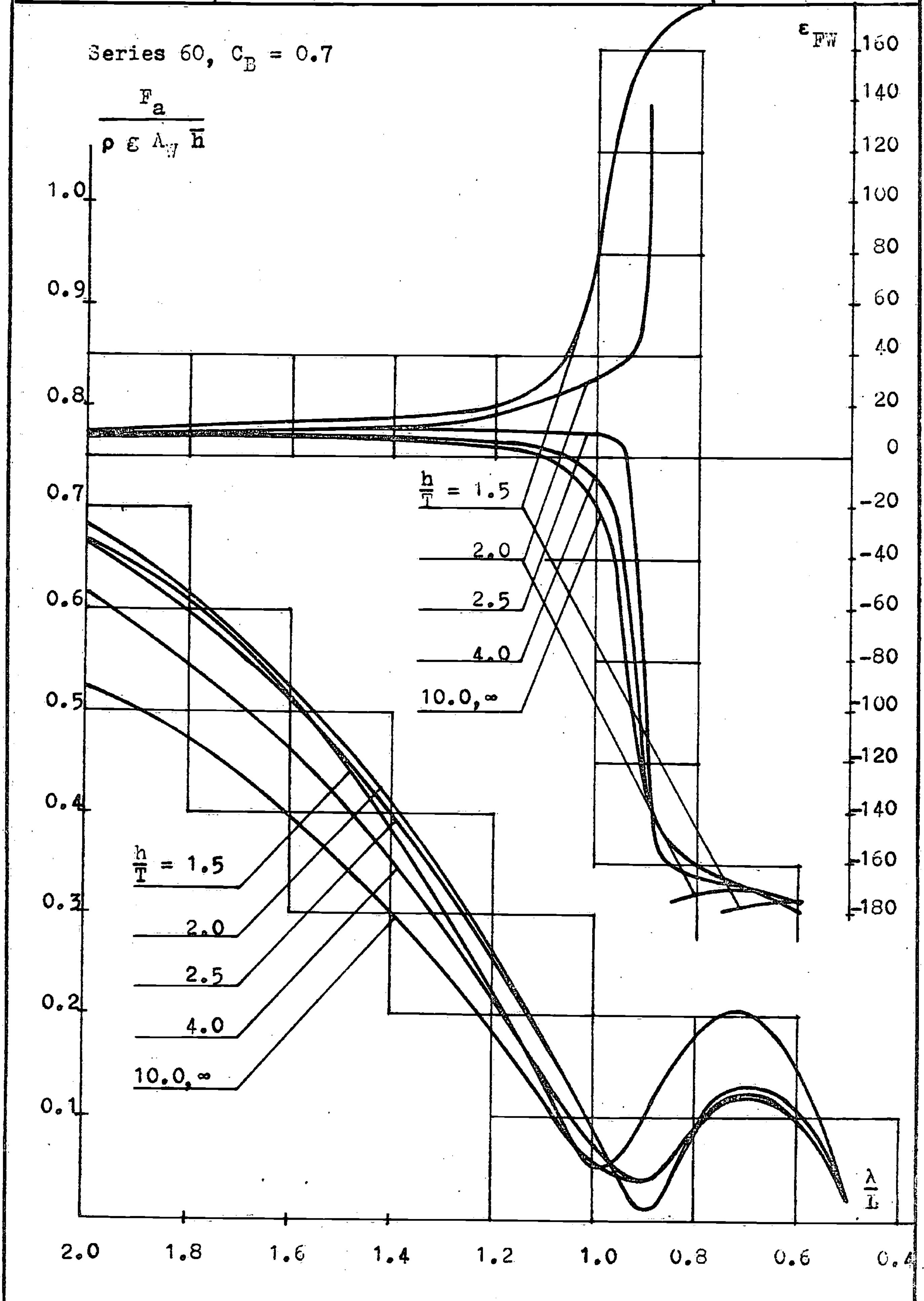
CHALMERS TEKNISKA HÖGSKOLA	Heave Damping Coefficient at $F_n = 0.0$	Fig. [illegible] - 3 CTH - [illegible] Report [illegible]4



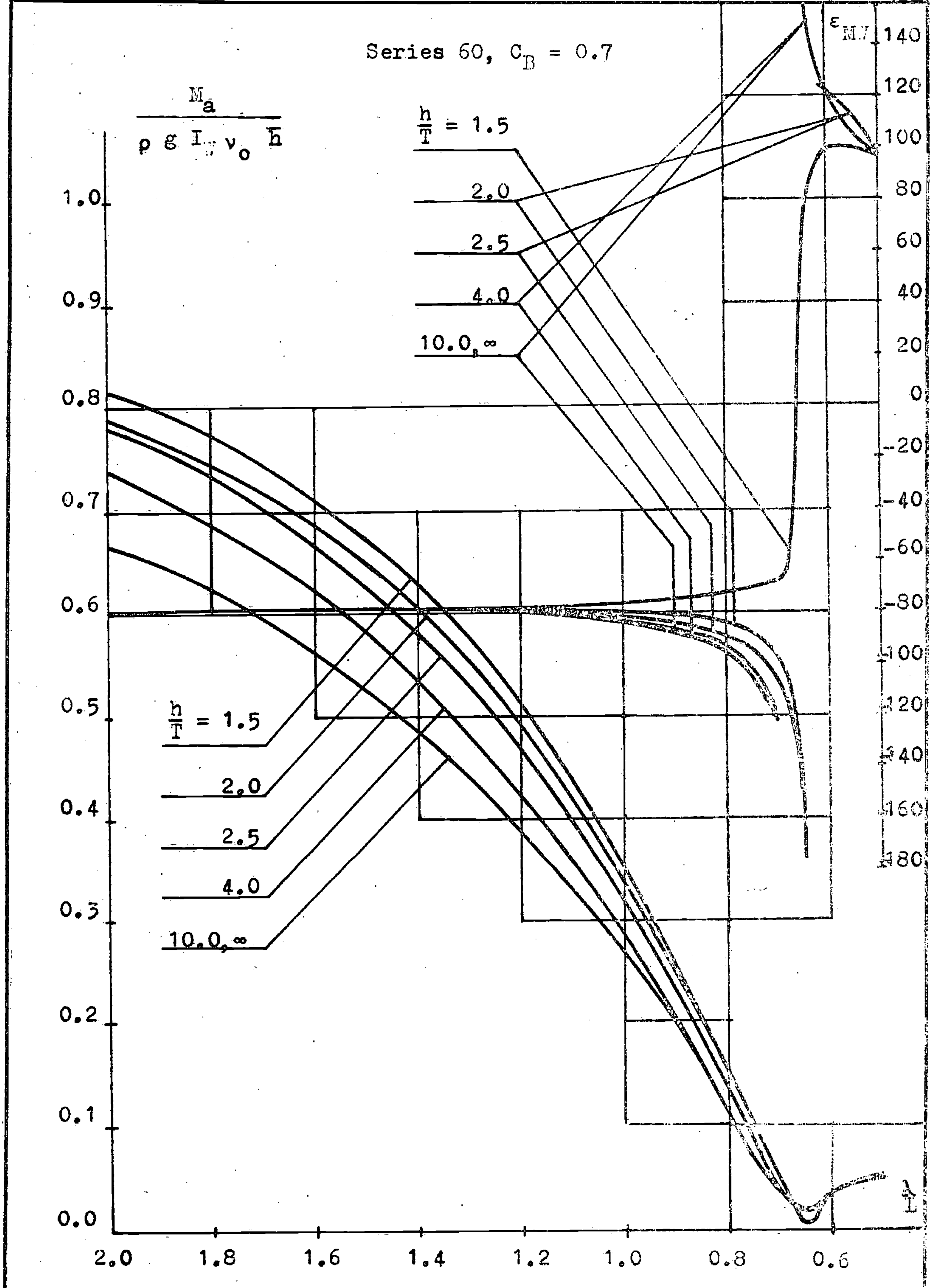
CHALMERS TEKNISKA HÖGSKOLA	Pitch Damping Coefficient at $F_n = 0.0$	Fig. A - 4 CTH - SH Report 44



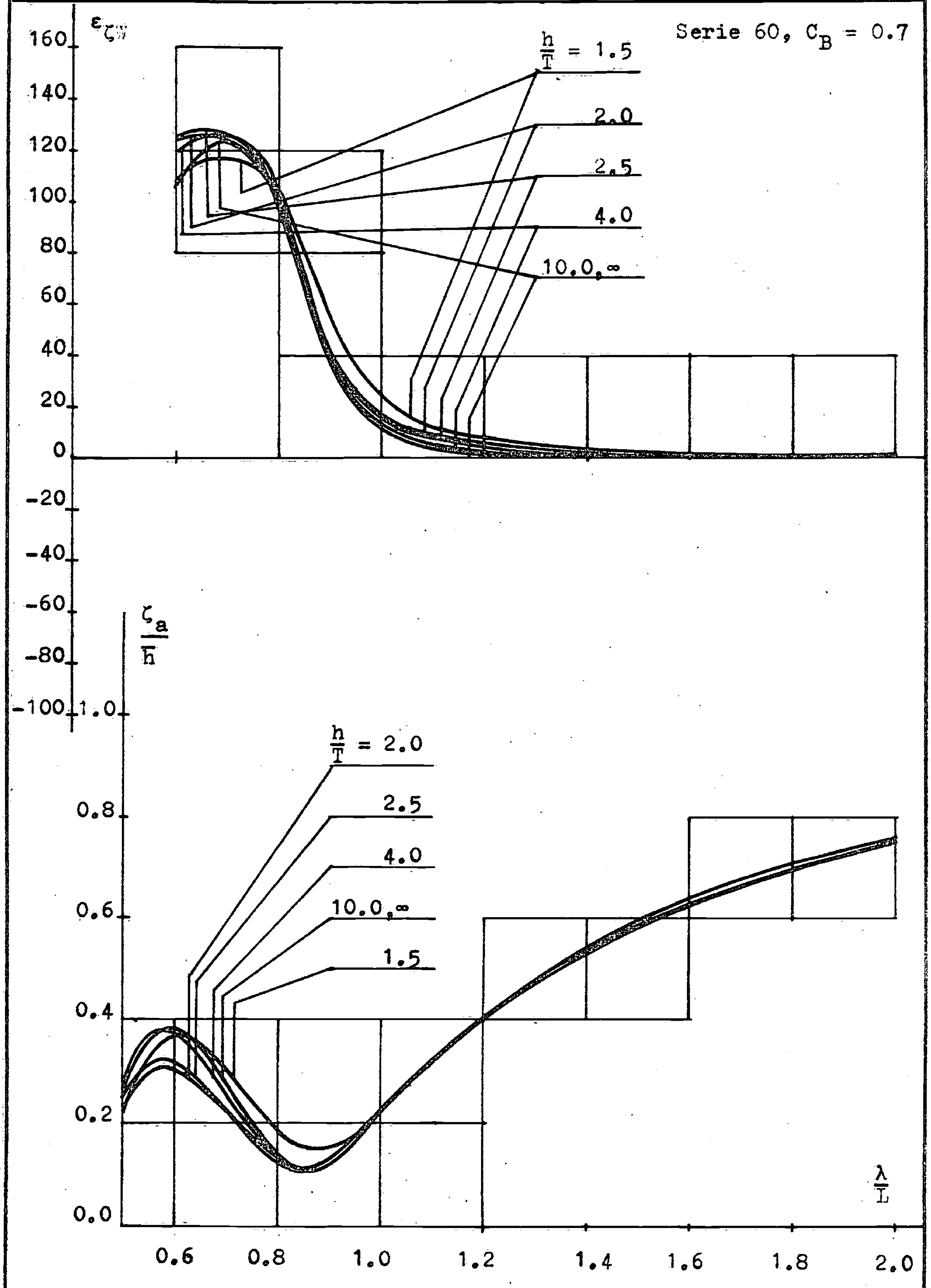
CHALMERS TEKNISKA HÖGSKOLA	Exciting Force Coefficient on the Restrained Ship Moving at $F_n = 0.20$	Fig. A - 5 CTH - SH Report 44



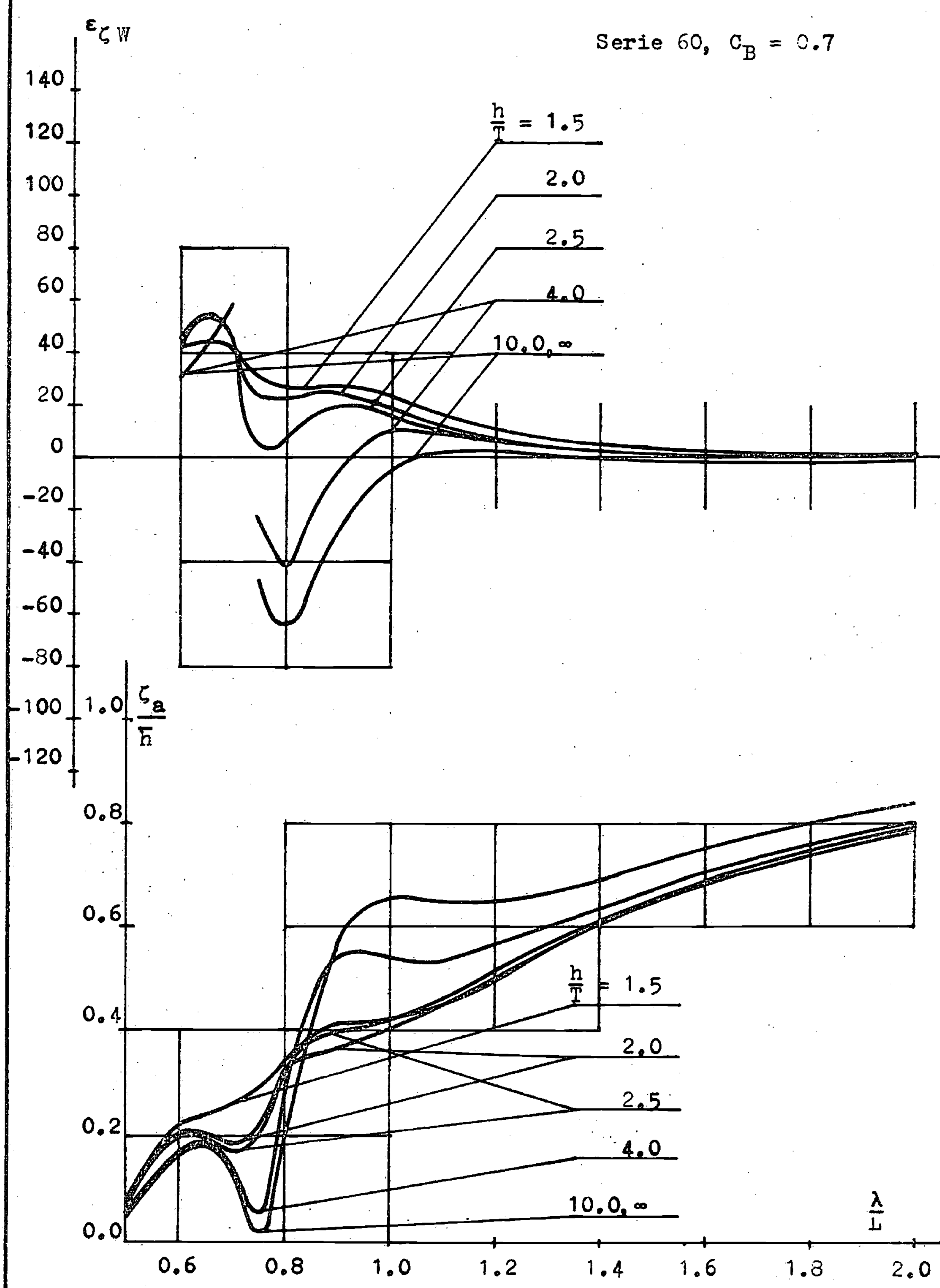
CHALMERS TEKNISKA HÖGSKOLA	Exciting Moment Coefficient on the Restrained Ship Moving at $F_n = 0.20$	Fig. A - 6 CTH - SH Report 44



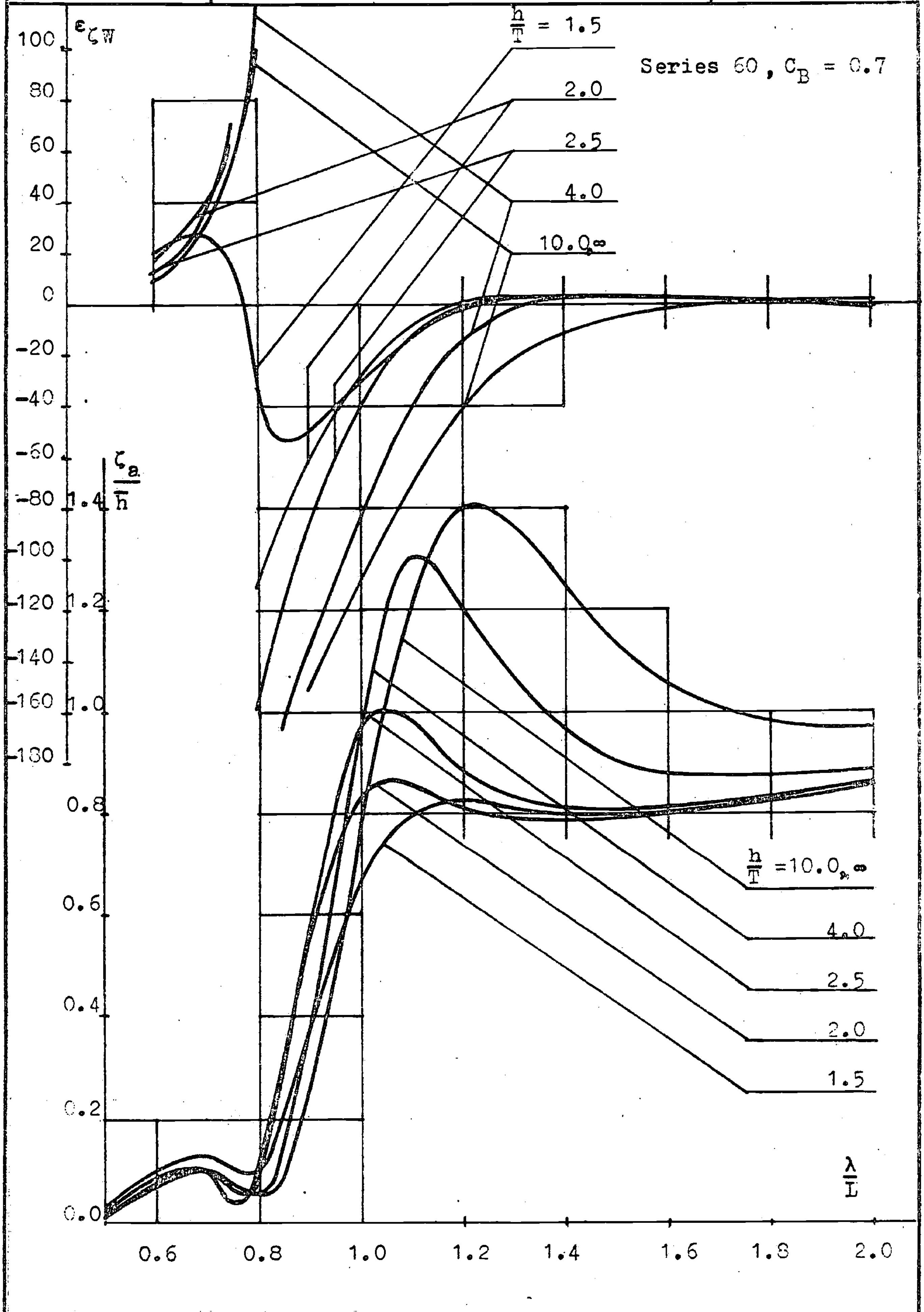
CHALMERS TEKNISKA HÖGSKOLA	Heave Amplitude Ratio at $F_n = 0.0$	Fig. B - 1 CTH - SH Report 44



CHALMERS TEKNISKA HÖGSKOLA	Heave Amplitude Ratio at $F_n = 0.10$	Fig. B - 2 CTH - SH Report 44

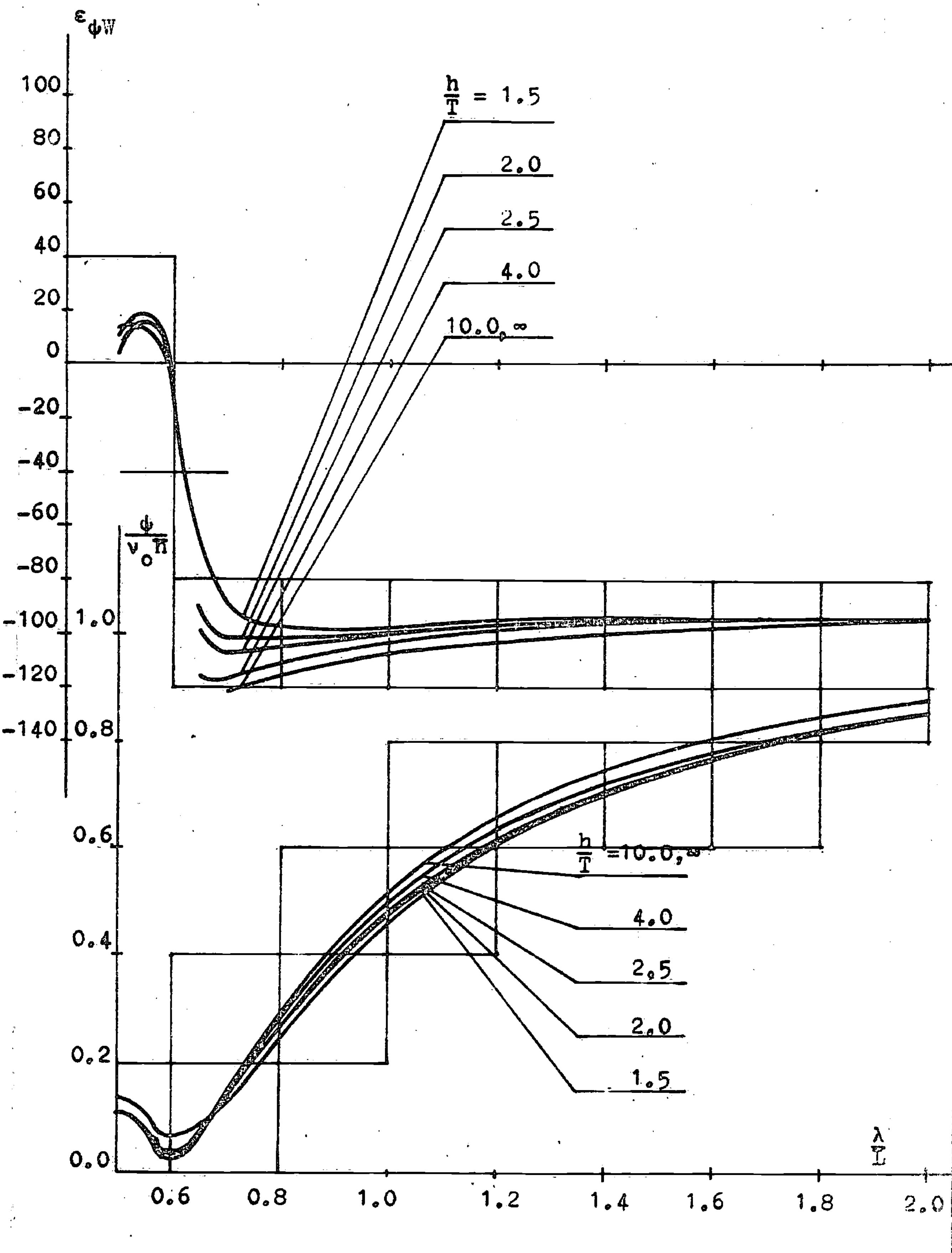


CHALMERS TEKNISKA HÖGSKOLA	Heave Amplitude Ratio at F_n = 0.20	Fig. B - 3 CTH - SH Report 44

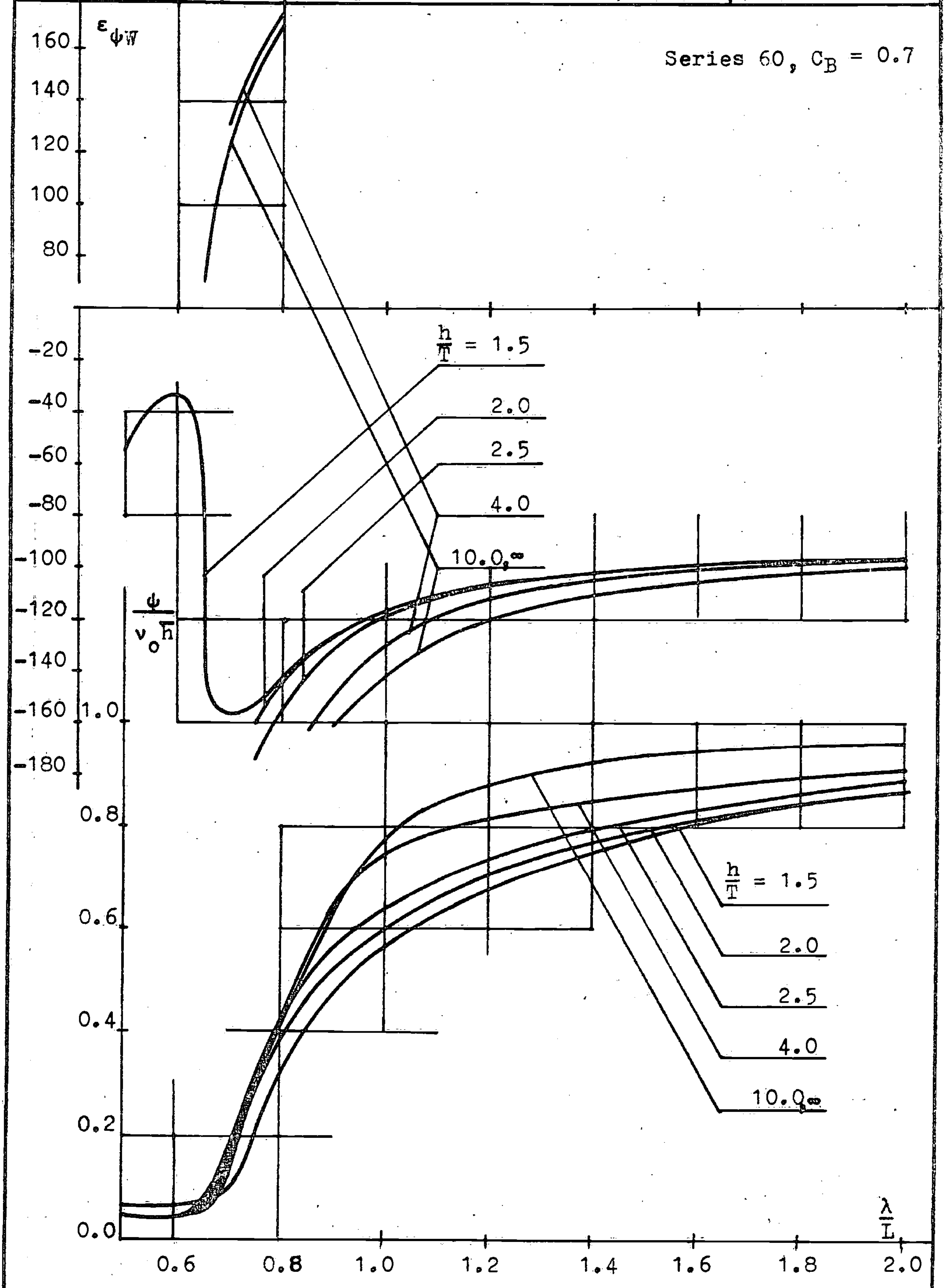


CHALMERS TEKNISKA HÖGSKOLA	Pitch Amplitude Ratio at $F_n = 0.0$	Fig. B - 4
		CTH - SH
		Report 44

Series 60, $C_B = 0.7$



CHALMERS TEKNISKA HÖGSKOLA	Pitch Amplitude Ratio at $F_n = 0.1$	Fig. B - 5 CTH - SH Report 44



CHALMERS TEKNISKA HÖGSKOLA	Pitch Amplitude Ratio at $F_n = 0.20$	Fig. B-6 CTH - SH Report 44