Hydraulic fracture characterization with dispersion measurements of seismic waves

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Summary
During hydraulic fracturing experiments in our laboratory the opening of hydraulic fractures is monitored with ultrasonic transducers. Transmitted and reflected signals show a distinct and strong dispersive behavior. An experiment with a controlled fracture width shows that although the width is very small, the fracture itself can explain the measured dispersion of compressional waves.

The strong impedance contrast magnifies the one-way time delay. The ratio of dimensionless fracture width and impedance ratio determines the responses and jump in particle displacement. This is in agreement with the linear slip approximation when the fracture compliance is taken proportional to the fracture width. The product of dimensionless fracture width and impedance ratio acts as the dimensionless fracture density and is very small for hydraulic fractures. The jump in delections at the fracture interface can thus be neglected.

The dispersion measurement approach can be used to monitor the width of hydraulic fractures and could therefore lead to an increased phenomenological understanding of hydraulic fracture growth.

Introduction
Hydraulic fracturing is a technique to stimulate the production of oil and gas reservoirs. Seismic monitoring of hydraulic fracture growth is becoming increasingly popular [WILLS ET AL. 1992; MHOODS AND WINTERSTEIN 1994; PAIGE ET AL. 1995], since it is one of the few tools which can provide information of the fracture away from the wellbore. Although it is long known that fluids can have a dramatic impact on the propagation of seismic waves, less is known about the actual physical interaction of the seismic wave with the fracture and how the seismic data can be used for fracture characterisation.

During scaled hydraulic fracturing experiments in our laboratory, we monitor the growth of the fractures with ultrasonic waves, which are actively induced by piezoelectric sources, [Savic, 1995]. The principle of seismic monitoring, or time-lapse seismics, is that we can distinguish between the seismic wavefield for the embedding (incident wavefield) and the wavefield related with the interaction with the fracture (scattered wavefield). Advantage must be taken of the fact that we can relate the scattered wavefield with the known incident field. In that manner the interaction of the seismic waves with the hydraulic fracture can be studied in detail, since the complexity of the seismic inversion problem can be reduced to a much more simple (fracture) parameter estimation.

The laboratory experiments clearly show that for transmitted waves, the effect of the fracture is a dispersion in terms of an attenuation and a time delay of the transmitted wave, both for compressional waves, (see Figure 1) and shear waves, (see Figure 2). Although the fracture width is extremely small, around one percent of the incident wavelength, the measured time shift and attenuation relative to the incident wavefield is substantial; 0.4 μs time delay for compressional waves and 4.0 μs time delay for shear waves. These delays are much larger than expected on a simple one-way travel time argument (see Table 1).

Most theoretical approaches for describing the interaction of the wavefield with the fracture postulate the use of the classical boundary conditions, such as the 'void' and the 'fluid filled' boundary conditions. These boundary conditions do not result in a dispersion of the transmitted waves, [cf. DE HOOG, 1958]. In the literature it is common to use the modified boundary conditions which allow for a displacement discontinuity (1) proportional to the traction on the fracture interface and the fracture compliance tensor (2) [cf. Schoenberg, 1980; Pyran-Nolte ET AL., 1990]:

\[ \Delta u = Z_0 t \]

Equivalently, Eq. (1) can be formulated with a fracture stiffness tensor, the inverse of the fracture compliance tensor. However, no clear physical interpretation exists for the fracture compliance tensor that arises in this theory. Moreover, no experimental verification of the displacement discontinuity model exists in combination with a meaningful interpretation of the fracture compliance tensor.

In this paper it is shown that an extremely thin hydraulic fracture can explain the large time-delays seen in laboratory experiments. More specifically, the ratio of the acoustic impedances of the fluid and the embedding rock acts as a blow up factor for the one-way time delay. This quantity determines the dispersive behavior of the transmitted waves.

![Figure 1: Attenuation and dispersion of transmitted compressional waves relative to the incident wavefield.](image1.png)

![Figure 2: Attenuation and dispersion of transmitted shear waves relative to the incident wavefield.](image2.png)
Hydraulic fracture characterization

Theory

We define the temporal Fourier forward and inverse transform pair for causal functions as:

\[ \hat{t}(\omega) = \int_{\mathbb{R}} \exp(-j\omega t) \chi(t)u(t)dt, \]

\[ \int_{\mathbb{R}} \exp(j\omega t) \hat{t}(\omega) d\omega = \chi(t)u(t), \]

where the characteristic function \( \chi(t) \) is defined as:

\[ \chi(t) = \{ 0, \frac{1}{2}, 1 \} \quad t \in \{ \mathbb{R}^-, 0, \mathbb{R}^+ \}. \]

The transmission and reflection response of a compressional wave for perpendicular incidence is found after a recursion procedure, [cf. Brekhovskikh, 1960):

\[ \hat{T}(\alpha, \zeta) = \frac{(1 - R^2)}{2} \exp(-j\alpha) \left[ \frac{1}{2} \frac{R}{1 - R^2} \exp(-2j\alpha) \right], \]

\[ \hat{R}(\alpha, \zeta) = \frac{R}{1 - R^2} \exp(-2j\alpha). \]

The symbol \( \alpha \) denotes the dimensionless width of the layer, or the one-way phase delay:

\[ \alpha = \frac{\omega h}{c_{p}}, \]

with circular frequency \( \omega \) and (P-wave) fluid velocity \( c_{p} \).

The limiting case of a fracture of vanishing width is found by taking the limit \( \alpha \downarrow 0 \):

\[ \lim_{\alpha \downarrow 0} \hat{T}(\alpha, \zeta) = 1, \quad \lim_{\alpha \downarrow 0} \hat{R}(\alpha, \zeta) = 0. \]

For finite impedance ratio a case corresponding to perfect bonding is found. The impedance ratio between the liquid and rock is very small. Taking the limiting case \( \zeta \downarrow 0 \) for finite width,

\[ \lim_{\zeta \downarrow 0} \hat{T}(\alpha, \zeta) = 0, \quad \lim_{\zeta \downarrow 0} \hat{R}(\alpha, \zeta) = 1, \]

a vanishing transmission response is found, corresponding with the 'void' boundary condition. In case of a hydraulic fracture both the width and the impedance ratio are small (cf. Table 1 on page 6). Note that the limiting process in case that both quantities are small is ambiguous:

\[ \lim_{\alpha \downarrow 0} \lim_{\zeta \downarrow 0} \hat{T}(\alpha, \zeta) \neq \lim_{\zeta \downarrow 0} \lim_{\alpha \downarrow 0} \hat{T}(\alpha, \zeta). \]

A similar statement is true for the reflection response. Therefore it is more appropriate to make a change of variables. Define the dimensionless fracture compliance \( \zeta \) and the dimensionless fracture density \( \eta \) as:

\[ \zeta = \frac{\alpha}{\zeta_r}, \]

\[ \eta = \frac{\alpha}{\zeta_r}. \]

The inverse transform is then found as:

\[ \alpha = \text{sgn}(\zeta) \sqrt{\frac{\eta}{\zeta_r}}, \]

\[ \zeta_r = \left( \frac{\eta}{\zeta_r} \right)^{1/2}. \]

Since the fracture density is small, the transmission and reflection response can be approximated:

\[ \hat{T}(\zeta, \eta) = \frac{2}{2 + i\zeta} + O(\eta) = \frac{2}{2 + i\zeta} + O(\eta), \]

\[ \hat{R}(\zeta, \eta) = \frac{i\zeta}{2 + i\zeta} + O(\eta) = \frac{i\zeta}{2 + i\zeta} + O(\eta). \]

In Eq. (12) the characteristic time \( t_c \) has been defined as:

\[ t_c = \frac{h}{c_{p}}, \]

(13)

Note that Eq. (12) shows that in lowest order of the fracture density, the frequency response of a thin layer comprises a single pole. The transmission and reflection response in the time domain are found after inversion with the Fourier inversion integral:

\[ T(t) = \frac{2}{t_c} \chi(t) \exp(-2t/t_c), \]

\[ R(t) = \frac{t_c}{2} \partial_{t}[T(t)] = \left[ \delta(t) - \frac{2}{t_c} \chi(t) \exp(-2t/t_c) \right]. \]

The total transmission and reflection response is found approximately by convolving the incident wavefield with the responses in the time domain. Apparently, in first order the characteristic time determines the kind of relaxation behavior seen in the experimental data. The impedance ratio acts as a blow up factor; the one-way time delay is multiplied with the inverse of the impedance ratio to determine the characteristic relaxation behavior and therefore the visibility and dispersion of the transmitted and reflected waves. The approximations made in Eq. (12) are only valid for a bandlimited response, since the dimensionless fracture density must be small:

\[ \alpha \zeta_r \ll 1, \quad \omega \ll \frac{c_{p}}{h \zeta_r}. \]

Note the following two frequency domain relations, using Eq. (12):

\[ 1 - \hat{R} - \hat{T} = O(\eta), \]

\[ 1 + \hat{R} - \hat{T} = i\zeta \hat{T} + O(\eta). \]

Equations (16) state that the jump in the traction across the fracture interface is proportional to the dimensionless fracture densi-
by \( \eta \), and that the jump in particle velocity is proportional to the dimensionless fracture compliance \( \zeta \). Supposing the fracture density can be neglected, a consequence of the approximations made in Eq.(12) is the assumption that the fractures are continuous across the interface. Calculating the jump in the displacements, the linear slip discontinuity model is found where the (normal) fracture compliance can be interpreted proportional to the fracture width:

\[
Z'' = \frac{h}{\rho' c_s^2} = h \kappa, \tag{17}
\]

or with Eq.(1), and introducing the (longitudinal) strain \( \varepsilon_{xx} \)

\[
\varepsilon_{xx} = \frac{\Delta u}{h} = \kappa f_j \tag{18}
\]

with \( \kappa \) the compressibility of the fluid. Apparently, on such a small scale, the constructive interference of all multiples expresses itself in a steep amplitude gradient, which is created inside the hydraulic fracture. Eq.(18) states the quasi-static constitutive equation of a fluid. Hence, for a thin low impedance layer, under the condition of small dimensionless layer density, the quasi-static constitutive relation can be used. The steep amplitude gradient might as well be interpreted in terms of the linear slip discontinuity model [Schoenberg, 1980], in which the fracture compliance is taken proportional to fracture width.

**Experimental results**

An experimental set-up has been built in which an adjustable slit can be made with a fluid layer of controlled width, simulating a hydraulic fracture. The accuracy of the width measurements is on the order of a few microns, or a percent of the total width of the hydraulic fracture. The dispersion of the transmitted waves is measured as a function of the width of the fluid layer, similar as in the hydraulic fracturing experiment. Indeed the characteristic relaxation behavior described in Eq.(14) is seen in the data (Figure 3). For increasing width of the fracture the transmitted wave is more slowly attenuated and delayed in time.

Let’s assume that the dispersive seismic signal as a function of the fracture width \( S(t, h) \) can be predicted by convolving the seismic signal without fracture, the base signal \( \delta_B(t) \), with the transmission response in Eq.(5). In the frequency domain this is expressed as:

\[
\tilde{S}(w, h) = \tilde{T}(w, h) \tilde{\delta}_B(w). \tag{19}
\]

However, in this experiment the case of a closed fracture still produced a small reflected signal. This means that for dry-fractures or the ‘zero width’ case, the plane layer approach is inappropriate; the roughness of the fracture interface probably induces a partial contact mechanism. Hence, in this experiment the base signal is a ‘missing link’.

Alternatively, for the whole range of values for the fracture width the base signal can be estimated by deconvolving the measured signal with the theoretical transmission response:

\[
\tilde{\delta}_B(w) = \frac{\tilde{S}(w, h)}{\tilde{T}(w, h)}. \tag{20}
\]

The theoretical transmission response is a known quantity since the width and the fluid properties have been measured. We postulate that if the approximations made are valid, then all transmitted signals should estimate the same base signal. This procedure has been applied in Figure 4. Note that no fitting procedure has been applied, since all unknown parameters have been measured and have a clear physical interpretation. The estimated base signals agree remarkably well. The ‘zero-width’ case shows as expected a slight deviation, since the present analysis does not incorporate a partial contact mechanism.

**Conclusions**

Concluding from Figure 4 we state that, even for a very small width, standard linear acoustic theory can predict the measured dispersion of transmitted compressional waves. Since the linear slip model is presented as a valid approximation of the thin low impedance layer, the results in Figure 4 can be regarded as a verification of the linear slip model. At this moment this verification is limited to hydraulic fractures and compressional waves. By carefully designing a seismic monitoring experiment of hydraulic fractures, the width of hydraulic fractures can be monitored. This application of seismic monitoring could lead to an increased phenomenological understanding of hydraulic fracture growth. Future research will focus on the accuracy that can be attained with this method and the extension to shear waves in the model.

**Acknowledgements**

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**References**


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Figure 3: Measured transmitted compressional waves as a function of artificial hydraulic fracture width.

Figure 4: Estimated base signals by correcting for theoretical transmission response.

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