"Urban road network size: some Dutch findings".

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Abstract: "The size of urban road networks - some Dutch findings".

This article deals with a descriptive-statistical analysis of urban road networks. In six Dutch towns of varying size and character road network size and structure have been determined. Size is expressed by the numbers of network elements, i.e. the numbers of intersections, road sections and cells. Network structure is measured by the distribution of intersections by degree. A comparison with the few other investigations into this subject is made. For predictive purposes network size is related to city population and total road length. In one town the networks of two districts are compared.
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1. Introduction

There is a growing need in transport planning practice for more knowledge of urban transport networks. This knowledge pertains to the size, shape, structure of these networks and also to relationships between these network characteristics and other urban phenomena such as population size, population distribution etc.

First of all, such knowledge is of importance when performing a transport system analysis in which the impacts of alternative plans are estimated. In such an analysis the transport network is a key element as it is mainly on the basis of this network that the traffic flows and service levels, being the variables from which most impacts can be derived, are determined. Through the models that have to be built of this network, it has a considerable effect on both the accuracy of the transport system analysis outputs and the amount of computer time required for this analysis. More insight into the structure, size, density, etc. of such networks could be of great value in constructing efficient, which means accurate and cheap, models of transport networks.

More knowledge of the make-up of transport networks also indirectly adds to the improvement of the transportation planning process. It enables the development of efficient computer programs, especially assignment programs.

The design of data banks is a third potential field where information about urban transport networks will find an application. The number of urban data banks in which all data relevant to spatial planning are contained in a systematic and easily accessible way, is rapidly growing. It appears very efficient to organize such data banks on the basis of the complete actual transport network. The intersections, sections and cells (building blocks) of this network are the locational references of all kinds of urban data, as an alternative for squares or coordinates. The data (e.g. population, car ownership, land
use, accidents) are then associated with one of the aforementioned network elements. Operations having a spatial dimension (e.g. determining the number of cars, trips per km² in district A, estimating the average distance to the town centre, or to a hospital, etc.) can be carried out using the coordinates that have been determined for the intersections. The usefulness of such network-oriented data bases will be clear from the fact that in the U.S.A. every town over 50,000 inhabitants has stored the 1970 census data in such a way [1]. Similar data banks are created for purposes of maintenance and administration of transport facilities.

The availability of a reliable estimate of the numbers of network elements and their relationships is essential for an apriori judgement of the feasibility of such a data system and for its design. For instance, when a traffic accident data bank, based on intersections and road sections, is to be developed such estimates can answer questions like: what will be its size, how much will it cost to use it, how should we program the necessary software? [see e.g. 19]

However, the topology of urban transport networks has not only practical relevance. It is also gaining increased interest in recent times as a subject of scientific investigations aiming at the development of theory explaining these physical-artifactual phenomena on the basis of socio-geographic facts. Questions fitting in this theoretical framework are: can the increase in the size and the change in structure of a network be explained by e.g. population growth? What is the relationship between size and density of a network. Research into these matters has clear theoretical relevance and constitutes an important area of concern of transport geography. Graph theory is one of the basic means to bring rigor into this field.

The amount of previous work done in determining the characteristics of actual urban transport networks is very limited. The major part of this work has been carried out in the U.K., resulting in some articles in this journal [2 to 9].
In the following section we will point out why we have performed an investigation into urban transport networks. Our method of analysis is outlined in section 3, followed by some empirical findings on the size of urban road networks. In section 5 attention is turned to the structure of these networks. Section 6 deals with the relationship between network size and city size, after which our results are compared with corresponding findings of previous investigations (section 7). Finally, attention will be given to inner-city variations in road network structure.
2. Background of this study

The main reason to do some work in this field is to be found in the research project "Spatial abstraction in transportation planning". In short (a more detailed exposition can be found elsewhere [10]), the first objective of this project is to establish the relationships, if there are any, between the level of spatial detail applied in the transportation systems analysis on the one hand and the accuracy of the estimates of the plan's consequences (e.g. travel times, noise levels) and the costs of the analysis on the other hand. The level of spatial detail is primarily determined by the zone size and the degree to which the transport network is abstracted. These effects on the precision and on the costs of the transport system analysis will be investigated in the first instance by an experimental approach. This implies the performance of a great number of analyses (including traffic assignments) with varying levels of spatial detail for a real town. One of these analyses will be carried out for the "real" situation, that is to say: zones are identical with building blocks, being the smallest areas not divided by streets in a town, whereas almost the complete actual road network will enter into this step.

It will be clear that the computer costs involved with this experimental program can easily go beyond the budgets available. The enormous costs and risks make a thorough feasibility study indispensable. Such a study should enable us to derive the maximum possible city size we can cope with, that is, given the available computer hardware and software, and other resources. It should also give us estimates of the experimental costs to be expected. Crucial is the analysis step having the highest level of spatial detail. Details on this feasibility study can be found in [31 to 14].

Our special interest in the relationship between city size and road network size, expressed by the numbers of intersections, street sections and regions, immediately follows from the purpose of the feasibility study.
3. Approach

3.1. Terminology and symbols

Object of our study is the road network of cities, comprising all public roads and streets usable for car traffic. In such networks the following elements are distinguished:
- intersections
- street sections
- regions (or cells)

An important property of an intersection is its degree, that is the number of street sections interconnected by it. The notion of degree, denoted by $\delta$, is illustrated by the following figures:

- $\delta = 1$
- $\delta = 2$
- $\delta = 3$
- $\delta = 4$

It should be noted that an end point of a cul-de-sac is treated as an intersection of degree 1. Characteristics of the street sections like e.g. length, one-way working, capacity, speed, are not of any relevance in our analysis; our interest is in the topological structure of the network only. This means that the way the intersections are connected by street sections is of prime concern.
We express the size of a network by its numbers of intersections, street sections and regions. The following symbols will be used:

\[ I_\delta = \text{number of intersections of degree } \delta; \]
\[ I = \text{total number of intersections}; \]
\[ S = \text{number of street sections}; \]
\[ R = \text{number of regions}. \]

3.2. Method of analysis

The aim of the analysis is to estimate the size and structure of the network in cities of different size and character, and to relate network size to city size expressed by variables like population, area, total road length, etc. Furthermore, we want to explore differences in network structure between various parts of cities.

Values of the explanatory variables can easily be obtained from official state and municipal statistics. However, figures on the numbers of intersections, street sections, and regions in real world networks of entire cities are very rare. For the purpose we had in mind we had, therefore, to determine these figures ourselves. To this end the following procedure is used:

- the total city is subdivided into squares of 500 m x 500 m using the grid system indicated on a city map;
- in each square the number of intersections by degree is counted;
- by simple adding the total number of intersections by degree is obtained;
- from these total figures total numbers of street sections and regions are derived by using simple graph-theoretic formulae.

The intersections are counted by square, because of the following reasons:

- it is an efficient technique;
- its results permit more detailed analysis, e.g. concerning spatial distributions, spatial autocorrelation;
- the flexibility with respect to studying various subareas;
the possibility of estimating total city figures by using a sample of the squares. (In our case no sampling is used: all intersections in every square are counted). Below it will become clear that the counting of the intersections by degree is an essential feature of our approach. It obviates counting street sections and regions, the numbers of which can be computed as follows,

a. **Intersections**

The total number of intersections \( I \) is obtained by simple addition:

\[
I = \sum_{\delta} I_{\delta} \tag{1}
\]

However, two refinements of earlier definitions are to be presented. The first one refers to the treatment of the municipal boundary, which encloses the urban road network. Crossings of roads with this boundary are treated as intersections with degree 1.

The second refinement has to do with fly-overs. They do not constitute real intersections as no street sections are interconnected. On the other hand a fly-over leads to a partition of space into four cells, just as in the case of a 4-degree intersection.

Because of this it is decided to treat the very few fly-overs as 4-degree intersections. Consequently, the numbers of real intersections and street sections will be somewhat overestimated.
Finally, we mention the absence of 2-degree intersections because of our interest in structural aspects of road networks only.

b. Street sections
The number of street sections \( S \) can be calculated from the number of intersections by degree \( I_6 \), using the formula:

\[
S = \frac{1}{2} \sum_6 I_6
\]  

(2)

This is immediately clear from the fact that the degree of an intersection represents the number of links joining in that intersection, and from the fact that every link is incident with two nodes and therefore appears twice in the summation of degrees over nodes.

c. Regions
The number of regions in a network (also called cells or faces) can be computed using the well-known Euler formula from graph theory:

\[
R + I - S = 2
\]  

(3)

The formula holds only for so-called planar networks, that is, networks which can be projected onto the plane in such a way that the links have no common points except at the intersections.

By representing fly-overs by intersections of degree-4 our non-planar actual road networks are converted into planar networks.

Since an area enclosed by a number of street sections and the municipal boundary is also regarded as a region, Euler's formula has to be enlarged by the number of boundary sections.
A further remark concerning the application of Euler's formula refers to the so-called outside region, the area beyond the network area. In the formula this area is treated as one distinct region. Because we are only interested in the number of regions within the municipal boundary the following expression results:

\[ R = S + SB - I + 1 \]  \hspace{1cm} (4)

where \( SB \) = number of boundary sections, which is equal to the number of boundary crossings.

We have determined our figures using ordinary city maps with scales ranging from 1:10,000 to 1:12,500. Since we are interested only in the topological and not the metrical characteristics of the networks, we consider them sufficiently accurate.

Contrary to most previous investigations, our approach does not apply a sampling procedure, like Tanner [9] and Liivamagi and Vaughan [7] did. The determination of the degree of each intersection enabling an easy computation of the number of street sections as a substitute for direct counting of these elements, is an idea proposed by Howe [6].

3.3. Selected cities

This study is part of the feasibility study described in section 2. That purpose required that Dutch cities with population ranging from 80,000 to 300,000 were investigated.
A kind of systematic sample has been drawn in such a way that the sample cities cover this range more or less evenly.

The following six cities have been selected:

- Delft: 87,000 population
- Leiden: 100,000 population
- Breda: 122,000 population
- Tilburg: 154,000 population
- Haarlem: 172,000 population
- Utrecht: 275,000 population

Relevant data on these cities are summarized in Table 1 and Figure 1 gives an impression of the six road networks to be investigated.

Information on the representativeness of the cities selected can be found elsewhere [13].
<table>
<thead>
<tr>
<th>Name of City</th>
<th>Population 1)</th>
<th>Area</th>
<th>road network length 4)</th>
<th>% houses built after 1965 2)</th>
<th>Density of population in built-up area 3)</th>
<th>road length in built-up area 5)</th>
<th>road length per km² in built-up area 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>munici-pality</td>
<td>munici-pality 2)</td>
<td>built-up area 3)</td>
<td>munici-pality</td>
<td>built-up area 5)</td>
<td>(1) : (3)</td>
<td>(5) : (1)</td>
</tr>
<tr>
<td>Delft</td>
<td>87,000</td>
<td>2628</td>
<td>1250</td>
<td>184</td>
<td>149</td>
<td>25.5</td>
<td>69.6</td>
</tr>
<tr>
<td>Leiden</td>
<td>100,000</td>
<td>2305</td>
<td>1300</td>
<td>202</td>
<td>182</td>
<td>11.5</td>
<td>76.9</td>
</tr>
<tr>
<td>Breda</td>
<td>122,000</td>
<td>6055</td>
<td>2150</td>
<td>384</td>
<td>300</td>
<td>22.5</td>
<td>65.7</td>
</tr>
<tr>
<td>Gilburg</td>
<td>154,000</td>
<td>8018</td>
<td>2650</td>
<td>418</td>
<td>363</td>
<td>23.5</td>
<td>58.1</td>
</tr>
<tr>
<td>Haarlem</td>
<td>172,000</td>
<td>3212</td>
<td>2270</td>
<td>365</td>
<td>355</td>
<td>13.0</td>
<td>75.8</td>
</tr>
<tr>
<td>Utrecht</td>
<td>275,000</td>
<td>5273</td>
<td>3390</td>
<td>522</td>
<td>461</td>
<td>15.5</td>
<td>81.1</td>
</tr>
</tbody>
</table>

3) We have determined these figures on the basis of city maps. On the maps we have drawn a ring enclosing the area which is indicated as built-up.
5) This "area" is defined in quite another way (by the Central Bureau of Statistics) than the area of note 3), but we assume comparability of the related figures.
6) We assume that the total municipal population is concentrated in the built-up area.
4. Urban road network size

The results of the counts and computations of the numbers of network elements - intersections, road sections and regions - are shown in Table 2. The class of 1-degree intersections is divided into two categories, boundary crossings (I_{1b}) and end points of cul-de-sacs (I_{1c}).

Table 2. Number of network elements

<table>
<thead>
<tr>
<th>city: population:</th>
<th>DELFT (87,000)</th>
<th>LEIDEN (100,000)</th>
<th>BREDA (122,000)</th>
<th>TILBURG (154,000)</th>
<th>HAARLEM (172,000)</th>
<th>UTRECHT (275,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>boundary crossings</td>
<td>I_{1b}</td>
<td>I_{1c}</td>
<td>I_{1b}</td>
<td>I_{1c}</td>
<td>I_{1b}</td>
<td>I_{1c}</td>
</tr>
<tr>
<td>No.</td>
<td>21</td>
<td>180</td>
<td>25</td>
<td>158</td>
<td>49</td>
<td>168</td>
</tr>
<tr>
<td>end points</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cul-de-sacs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>54</td>
<td>289</td>
<td>54</td>
<td>289</td>
<td>54</td>
<td>289</td>
</tr>
<tr>
<td>INTERSECTIONS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with δ = 1</td>
<td>I_1</td>
<td>I_1</td>
<td>I_1</td>
<td>I_1</td>
<td>I_1</td>
<td>I_1</td>
</tr>
<tr>
<td>No.</td>
<td>201</td>
<td>183</td>
<td>217</td>
<td>343</td>
<td>206</td>
<td>350</td>
</tr>
<tr>
<td>δ = 2</td>
<td>I_2</td>
<td>I_2</td>
<td>I_2</td>
<td>I_2</td>
<td>I_2</td>
<td>I_2</td>
</tr>
<tr>
<td>No.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>δ = 3</td>
<td>I_3</td>
<td>I_3</td>
<td>I_3</td>
<td>I_3</td>
<td>I_3</td>
<td>I_3</td>
</tr>
<tr>
<td>No.</td>
<td>837</td>
<td>1140</td>
<td>1418</td>
<td>1955</td>
<td>1940</td>
<td>2756</td>
</tr>
<tr>
<td>δ = 4</td>
<td>I_4</td>
<td>I_4</td>
<td>I_4</td>
<td>I_4</td>
<td>I_4</td>
<td>I_4</td>
</tr>
<tr>
<td>No.</td>
<td>166</td>
<td>313</td>
<td>344</td>
<td>419</td>
<td>420</td>
<td>876</td>
</tr>
<tr>
<td>δ = 5</td>
<td>I_5</td>
<td>I_5</td>
<td>I_5</td>
<td>I_5</td>
<td>I_5</td>
<td>I_5</td>
</tr>
<tr>
<td>No.</td>
<td>6</td>
<td>5</td>
<td>23</td>
<td>18</td>
<td>20</td>
<td>32</td>
</tr>
<tr>
<td>δ = 6</td>
<td>I_6</td>
<td>I_6</td>
<td>I_6</td>
<td>I_6</td>
<td>I_6</td>
<td>I_6</td>
</tr>
<tr>
<td>No.</td>
<td>3</td>
<td>-</td>
<td>2</td>
<td>-</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>TOTAL NUMBER OF INTERSECTIONS</td>
<td>I</td>
<td>1213</td>
<td>1641</td>
<td>2004</td>
<td>2735</td>
<td>2590</td>
</tr>
<tr>
<td>STREET SECTIONS</td>
<td>S</td>
<td>1712</td>
<td>2440</td>
<td>2987</td>
<td>3987</td>
<td>3915</td>
</tr>
<tr>
<td>REGIONS</td>
<td>R</td>
<td>521</td>
<td>825</td>
<td>1033</td>
<td>1307</td>
<td>1366</td>
</tr>
</tbody>
</table>

We will demonstrate the procedure described in section 3.2. by performing the computations for Breda. The numbers of intersections (I_{1b}, I_{1c}, I_1, ..., I_6) have been counted. The number of street sections can be derived using formula (2):

\[
S = \frac{1}{2} \sum \delta \cdot I_\delta = \frac{3}{2}(1.217 + 2.0 + 3.1418 + 4.344 + 5.23 + 6.2) = \frac{3}{2}(217 + 4254 + 1376 + 115 + 12) = \frac{3}{2}(7094) = 2987
\]

The number of regions is obtained from expression (4):

\[
R = S + SB - I + 1 = 2987 + 49 - 2004 + 1 = 1033.
\]

In the next sections these raw data will be further analysed.
5. Urban road network structure

5.1. General

An important characteristic of networks is the way nodes and links are connected. This property is often denoted by the term "structure". It is emphasized again that in our investigation we are dealing with network structure only in topological sense. This means that only the fact that a pair of intersections is connected by one street section is relevant, and not the length of this section.

Generally, a great number of different networks can be constructed, given a set of nodes. A rather extreme example may show this:

![Diagram of network structures](image)

It will be clear from this, that beyond the network size parameters there is a need for measures describing network properties such as structure, in order to improve the possibilities of specification of networks and discrimination between networks.

An excellent survey on methods for analysing network structure has been produced recently by Leusmann [15]. Our limited data, consisting of numbers and frequencies of network elements, only permit the application of very simple and global measures for quantifying network structure. In this article we shall not pay much attention to the quality of various kinds of structure measures, see [16].

5.2. Average degree

In section 3.1. we have defined the degree of an intersection as the number of interconnected street sections. Therefore the average degree of all intersections gives an idea of how strongly the intersections are interconnected. For
example, an average degree of 2 means that on the average every intersection is joined with two other intersections.

The average degree of all intersections is shown in Table 3, bottom line, columns 1a, 2a, ..., 6a, for each of the six cities. It has been obtained from:

$$\bar{\delta} = \frac{\sum \delta \cdot I_\delta}{I}$$  \hspace{1cm} (5)

Table 3: Distribution of intersections by degree and average degree

<table>
<thead>
<tr>
<th>city: population:</th>
<th>DELFT (87,000)</th>
<th>LEIDEN (100,000)</th>
<th>BREDA (122,000)</th>
<th>TILBURG (154,000)</th>
<th>HAARLEM (172,000)</th>
<th>UTRECHT (275,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>type of intersection</td>
<td>all %</td>
<td>real %</td>
<td>all %</td>
<td>real %</td>
<td>all %</td>
<td>real %</td>
</tr>
<tr>
<td>column</td>
<td>(1a)</td>
<td>(1b)</td>
<td>(2a)</td>
<td>(2b)</td>
<td>(3a)</td>
<td>(3b)</td>
</tr>
<tr>
<td>boundary crossings</td>
<td>2 -</td>
<td>1 -</td>
<td>2 -</td>
<td>2 -</td>
<td>2 -</td>
<td>1 -</td>
</tr>
<tr>
<td>end points cul-de-sacs</td>
<td>15 -</td>
<td>10 -</td>
<td>9 -</td>
<td>10 -</td>
<td>6 -</td>
<td>8 -</td>
</tr>
<tr>
<td>$\delta = 1$</td>
<td>17 -</td>
<td>11 -</td>
<td>11 -</td>
<td>12 -</td>
<td>8 -</td>
<td>9 -</td>
</tr>
<tr>
<td>$\delta = 2$</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
</tr>
<tr>
<td>$\delta = 3$</td>
<td>69 83</td>
<td>70 79</td>
<td>71 79</td>
<td>72 82</td>
<td>81 75</td>
<td>69 75</td>
</tr>
<tr>
<td>$\delta = 4$</td>
<td>14 16</td>
<td>19 21</td>
<td>17 19</td>
<td>15 17</td>
<td>16 18</td>
<td>22 24</td>
</tr>
<tr>
<td>$\delta = 5$</td>
<td>- 1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\delta = 6$</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
</tr>
<tr>
<td>average degree $\bar{\delta}$</td>
<td>2.81</td>
<td>3.18</td>
<td>2.98</td>
<td>3.22</td>
<td>2.98</td>
<td>3.22</td>
</tr>
</tbody>
</table>

The average degree is about 3 and does not show great differences. In average an intersection has about three joining street sections.

It must be remembered that in this analysis the end points of cul-de-sacs are treated as intersections. However, for the purpose of many transportation-oriented analyses (e.g. traffic safety studies), it is much more relevant to consider only the real intersections, that are points where several streets intersect. For this reason the average degree of all real intersections has been determined and displayed in Table 3, last line, columns 1b, 2b, ..., 6b. These values don't show large variation either (3.18 - 3.26). Our observations in six towns indicate that the typical real
intersection from a transportation point of view has in average three incident street sections.

It might be useful to point out the relationship between the average degree of all intersections of a network and the so-called $\beta$-index, that has been developed in graph theory. This index is the simplest measure for the strength of the connectedness of the nodes. It can be applied to differentiate between simple topological structures (low $\beta$-values) and complex structures (high $\beta$-values)\(^1\) The $\beta$-index is defined as:

$$\beta = \frac{I}{S}$$

(6)

By simply substituting equation (2) in equation (5) it can be shown that $2\beta = \bar{\lambda}$. This means that the average degree gives no additional information about network structure over the $\beta$-index, and vice versa.

\(^1\) The significance and meaning of this index will not be discussed here, see \([16]\).
5.3. Distribution by degree

Although the average degree of the intersections reveals something about network structure, more detailed information is necessary. The average degree is a very rough measure for network structure. This can be seen from the fact that there are many, topologically highly different networks having the same average degree and the same size (See section 5.1.).

More insight is provided by the distribution of the intersections by degree. The distributions of all intersections for the six cities are presented in Table 3, columns 1a, 2a, ..., 6a; Figure 2 displays these results.

The high similarity of the six distributions is striking. They reveal an overwhelming majority of degree-3 intersections of about 70%. The remaining intersections are almost wholly end points of cul-de-sacs (+ 10%) and degree-4 intersections (15-20%). Regarding differences between the cities, the relatively high share of end points of cul-de-sacs in Delft is noticed.

In this section too, it seems appropriate to perform the same analysis on the real intersections only, that is excluding the degree-1 intersections. The frequency distributions of real intersections by degree are shown in Table 3, columns 1b, 2b, ..., 6b; see also Figure 3.

Recalling earlier results, the remarkable similarity of the distributions was to be expected. It can be seen that by far the most (+ 80%) real intersections of the urban road networks are T-junctions. This being, contrary to a priori expectations, could perhaps be explained by the fact that the most frequently used routes, which contribute most to our mental picture of the networks, have relatively many four-arm intersections.
5.4. Circuits around regions

In this section we will examine the number of street sections surrounding a region in average. This figure is especially relevant when linking up zone centroids in an almost unabstracted spatial model of a road network. In such a network model every region (building block) is a zone and all streets are incorporated into it.

In the above Figure region 88 is surrounded by seven street sections in case (a), and by four sections in case (b). The average length of the circuits around regions can be defined as

$$\overline{S}_R = \frac{2S - \delta_1}{R}$$  \hspace{1cm} (7)^1

This formula stems from the fact that all street sections, except dead-end streets, are shared by two contiguous regions. Our observations are summarized in Table 4, showing an average value of $\overline{S}_R$ of just below 6.

---

1) Boundary sections are not accounted for by $\overline{S}_R$
Table 4. Average number of street sections surrounding a region.

<table>
<thead>
<tr>
<th>city</th>
<th>$\overline{S}_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delft</td>
<td>6.38</td>
</tr>
<tr>
<td>Leiden</td>
<td>5.62</td>
</tr>
<tr>
<td>Breda</td>
<td>5.67</td>
</tr>
<tr>
<td>Tilburg</td>
<td>5.84</td>
</tr>
<tr>
<td>Haarlem</td>
<td>5.59</td>
</tr>
<tr>
<td>Utrecht</td>
<td>5.54</td>
</tr>
</tbody>
</table>
6. Network size related to city size

The ordering of the findings presented in Table 2 suggests that there may be quite strong relationships between network size and city population. This can be visualized in a scatter diagram depicting our six observations (see Fig. 4). Since there exist no sufficient theoretical grounds for a particular model specification we hypothesize, given the available statistical evidence, a linear model between network size and city population. We are not aware of any previous work on this subject, that might have suggested any specification.

We also related the three network variables to another city size statistic, namely total road length. This figure is easily available from official statistics. These data are also depicted in Figure 4. Apart from the statistical evidence, the modular make up of urban road networks may serve as an additional theoretical argument in favor of a linear relationship between network size and total road length.

Going into more detail, it has been found that the intercepts of all six lines do not differ significantly from zero, so we fitted regression lines, forced through the origin. The following relationships result:

\[
\begin{align*}
I &= 15.3 \cdot P \\
S &= 23.3 \cdot P \\
R &= 8.0 \cdot P \\
I &= 7.7 \cdot L \\
S &= 11.7 \cdot L \\
R &= 4.0 \cdot I.
\end{align*}
\]

where:  \( P = \) city population (in thousands)

\( L = \) road network length [km]

It is necessary to emphasize that no attempt was made to introduce statistical rigor into the analysis: the small sample size as well as the lack of a true random sample are but two reasons for this. The relationships are only to be used to get rough estimates of the numbers of network elements.
A second remark concerns the strong interdependencies of the relationships within each triple, the reasons for which can be found in section 3.2.

We refer to [13] for a more detailed statistical evaluation of the relationships and a comparison of city population and road length as descriptive variables.
7. Our findings compared with other investigations

In order to check the validity of our study we compare our results with corresponding findings of other investigations mainly from Britain. However, as other researchers already mentioned: "The amount of previous work done in assessing the characteristics of actual urban road networks has been surprisingly limited, ...." [?].

The comparability of most of these few studies with our work is limited due to different definitions, different kinds of networks, etc. The publications reviewed are references [2-9]. The following comparisons are confined to empirical studies of urban areas only.

a. The first item to be compared is the distribution of intersections by degree.

No source has been found presenting figures on first degree intersections (or cul-de-sacs).

Concerning real intersections (third or higher degree) our findings can be compared with two British studies.

Table 5: Comparison of distributions of real intersections by degree.

<table>
<thead>
<tr>
<th></th>
<th>= 3</th>
<th>= 4</th>
<th>= 5,6</th>
<th>total</th>
<th>average degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>Tanner et al. [9, p.24]</td>
<td>76</td>
<td>23</td>
<td>1</td>
<td>100</td>
<td>3.25</td>
</tr>
<tr>
<td>Our study</td>
<td>79</td>
<td>20</td>
<td>1</td>
<td>100</td>
<td>3.22</td>
</tr>
<tr>
<td>Liivamagi [?, p.337]</td>
<td>unknown</td>
<td>unknown</td>
<td>unknown</td>
<td>---</td>
<td>3.3</td>
</tr>
</tbody>
</table>

The figures of Tanner et al. have been gathered by using an ingenious sampling procedure. It was a modified form of a random selection of points representative for the whole road system of Great Britain [9, pp. 2-11].

The figures given in Table 5 refer to urban areas only.
Liivamagi & Vaughan also used a sampling technique. A number of sample areas in London of 1 square kilometre each were selected systematically from a National Grid Base Map. For each of the sample squares the total number of intersections, by type of road, was counted. The value of the average degree in Table 5 refers to the "major road" network of London [7, p. 338].

Our figures were determined by a complete count in six Dutch towns. The percentages refer to the sum of the numbers of the six towns [13, Tables 3 and 5].

Table 5 reveals a close agreement of our findings with the results of the two British studies.

b. A major aim of our study is to relate numbers of intersections, street sections, and regions of road networks to total city population.

No other data have been found which would facilitate comparison with our relationships. Statistics on the so called Geographic Base (DIME) Files of the U.S. Bureau of the Census [17, 18] enables development of similar relationships for U.S. urban areas. However, the network definition used for the DIME files precludes comparison with our data because, for instance, railroads, rivers, political boundaries are all part of the network.

c. Another useful item to compare is the so-called intersection frequency, i.e., the number of intersections per unit of network length. Liivamagi & Vaughan [7, p. 337] have calculated figures on the intersection frequency for London as a function of the distance from the city centre. Near the city centre there are on average approximately 12 intersections per kilometre of road (all classes) decreasing to about 3 in the outskirts.

From the aforementioned study by Tanner et al., a figure of 5.77 can be derived as an average for all urban areas in Great Britain [9, p. 22]. However in our opinion the comparability of this figure is questionable.
The figures of both Tanner and Liivamagi apply to intersections of degree three and higher only. From Table 6 in which our observations are contrasted with those obtained in Great Britain, it can be seen that the Dutch observations are in agreement with both British studies.

Table 6: Comparison of intersection frequencies

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>L</th>
<th>I/L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No.</td>
<td>km</td>
<td>No./km</td>
</tr>
<tr>
<td>DELFT</td>
<td>1012</td>
<td>149</td>
<td>6.8</td>
</tr>
<tr>
<td>LEIDEN</td>
<td>1458</td>
<td>182</td>
<td>8.0</td>
</tr>
<tr>
<td>BREDAS</td>
<td>1787</td>
<td>300</td>
<td>6.0</td>
</tr>
<tr>
<td>TILBURG</td>
<td>2392</td>
<td>363</td>
<td>6.6</td>
</tr>
<tr>
<td>HAARLEM</td>
<td>2384</td>
<td>356</td>
<td>6.7</td>
</tr>
<tr>
<td>Utrecht</td>
<td>3664.</td>
<td>461</td>
<td>7.9</td>
</tr>
<tr>
<td>Dutch cities total</td>
<td>12,697</td>
<td>1811</td>
<td>7.0</td>
</tr>
<tr>
<td>Great Britain [9]</td>
<td></td>
<td></td>
<td>5.8</td>
</tr>
</tbody>
</table>

I = intersections of degree three or more
L = road network length in built-up area
I/L = intersection frequency

(d. A further network characteristic is the intersection density, that is, the number of intersections per unit area.

Liivamagi & Vaughan [7, p. 336] have estimated intersection densities for London as a function of the distance from the city centre. Near the centre there are on average approximately 120 intersections per square kilometre decreasing to about 20 in the outskirts.

Clayton [3, p. 371] reports two comparable observations: in a small area in the centre of London he found 61 intersections per sq. km. In a "19th century inner suburb" of Sydney a figure of 68 was determined. A study by Borchert is focused on the usefulness of the intersection density as a demarcator of the urban area. He observed
that near the city boundary the intersection density was about 20 and increased to a maximum value of 66 [2].
Table 7 summarizes all findings.

Table 7: Comparison of intersection densities

<table>
<thead>
<tr>
<th></th>
<th>I No.</th>
<th>A km²</th>
<th>I/A No./km²</th>
</tr>
</thead>
<tbody>
<tr>
<td>DELFT</td>
<td>1012</td>
<td>12.50</td>
<td>81</td>
</tr>
<tr>
<td>LEIDEN</td>
<td>1458</td>
<td>13.00</td>
<td>112</td>
</tr>
<tr>
<td>BREDA</td>
<td>1787</td>
<td>21.50</td>
<td>83</td>
</tr>
<tr>
<td>TILBURG</td>
<td>2392</td>
<td>26.50</td>
<td>90</td>
</tr>
<tr>
<td>HAARLEM</td>
<td>2384</td>
<td>22.70</td>
<td>105</td>
</tr>
<tr>
<td>UTRECHT</td>
<td>3664</td>
<td>33.90</td>
<td>108</td>
</tr>
<tr>
<td>Dutch cities total</td>
<td>12,697</td>
<td>130.10</td>
<td>98</td>
</tr>
<tr>
<td>London [7]</td>
<td></td>
<td></td>
<td>20-120</td>
</tr>
<tr>
<td>Small area in London Centre [3]</td>
<td></td>
<td></td>
<td>61</td>
</tr>
<tr>
<td>Small area in Sydney [3]</td>
<td></td>
<td></td>
<td>68</td>
</tr>
<tr>
<td>Minneapolis [2]</td>
<td></td>
<td></td>
<td>20-66</td>
</tr>
</tbody>
</table>

It can be seen that our observations are within the range reported by Liivamagi & Vaughan for London. The intersection density of the Dutch cities appears to be relatively high. This difference cannot be due to differences in the network structure (see Table 5). Perhaps it stems from differences in population density and/or delineation criteria applied.

e. Urban road networks may also be compared with respect to road length per capita and road length per unit area. The necessary figures can easily be taken from the official statistical sources, as for instance the Municipal Yearbooks and The Statistics of Roads. However, it was not within the scope of this study to perform such a comparative analysis.
8. Comparison between two city districts

8.1. Approach

In the preceding sections entire cities have been examined. In one city, Tilburg, we have tried to find out what differences and similarities exist between parts of the network of a city with respect to numbers of elements and structural characteristics. To this end we have isolated two relatively big residential districts of Tilburg, called Noord and Oud-Noord respectively.

District "Noord" is a modern residential area, built-up in the last ten years, having about 20,000 inhabitants and an area of 5 km$^2$. District "Oud-Noord" is a much older, mainly residential area, having about 37,000 inhabitants on an area of 4 km$^2$. It has some factory premises. Both districts are fairly well-bounded and are separated from each other by a canal. The networks of both districts are depicted in Fig. 5. As the count for Tilburg as a whole was performed by squares (see section 3.2.), the data for both districts can be easily obtained.

8.2. Results

Table 8 summarizes the observations for both districts as well as for the entire city of Tilburg. The differences in network lay-out and network density that are apparent in Figure 5 are confirmed by the data in Table 8.

The most striking difference in network structure pertains to the number and share of degree-1 intersections. This share of the modern residential area is no less than seven times the older residential district's share (28% and 4% respectively). This is a direct consequence from the fact that relatively many cul-de-sacs have been constructed in the modern area (2.9% against 2.5% of all...
Table 8. Comparison of road network structure in two urban districts

<table>
<thead>
<tr>
<th></th>
<th>Modern residential area &quot;Noord&quot;</th>
<th>Older residential area &quot;Oud-Noord&quot;</th>
<th>Tilburg</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>population</strong></td>
<td>20,000</td>
<td>37,000</td>
<td>154,000</td>
</tr>
<tr>
<td><strong>area [km^2]</strong></td>
<td>5</td>
<td>4</td>
<td>26.5^1)</td>
</tr>
<tr>
<td></td>
<td>all</td>
<td>real</td>
<td>all</td>
</tr>
<tr>
<td>intersections by degree:</td>
<td>No.</td>
<td>%</td>
<td>No.</td>
</tr>
<tr>
<td>δ = 1</td>
<td>142</td>
<td>28</td>
<td>-</td>
</tr>
<tr>
<td>δ = 2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>δ = 3</td>
<td>321</td>
<td>63</td>
<td>321</td>
</tr>
<tr>
<td>δ = 4</td>
<td>49</td>
<td>9</td>
<td>49</td>
</tr>
<tr>
<td>δ = 5</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td><strong>network size:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>intersections</td>
<td>513</td>
<td>371</td>
<td>423</td>
</tr>
<tr>
<td>street sections</td>
<td>653</td>
<td>662</td>
<td>237</td>
</tr>
<tr>
<td>regions</td>
<td>157</td>
<td></td>
<td>237</td>
</tr>
<tr>
<td><strong>average degree</strong></td>
<td>2.55</td>
<td>3.03</td>
<td>3.13</td>
</tr>
</tbody>
</table>

1) Built-up area. The number of intersections outside this area is negligible.

2) Boundary crossings are not included.

street sections). Figure 6 shows the distributions of the intersections by degree. After the foregoing remarks it is not surprising that the average degree of all intersections is considerably lower in the modern residential area than in the older district.

The differences regarding network structure become even much more pronounced when compared to the small differences that have been found between the six cities. A comparison between Table 3 and Table 6 reveals that all figures of the six cities fall within the range defined by the values
of the two districts. The same remark applies to the network size figures per capita. Whereas these ratios are almost the same for all six entire cities (see Fig. 4), fairly different values are apparent here within one town. As an example, the number of intersections per 1000 inhabitants in the six cities ranges from 13.7 (Delft) to 17.4 (Tilburg), whereas the district values are 11.4 (old) and 25.6 (modern).

It will be a question of further research to find out to what extent this observed big difference between the within-town and between-town variability of network characteristics has more general validity. Some factors support such a supposition:

- the changing principles of network design in various time periods: e.g. districts designed in the motor car era, let's say in the last 20 years, look quite different from those designed in earlier periods;
- specific land uses and city functions require specific network forms. The network lay-out in an industrial area with big parcels is quite different from that in a high-density residential area.
- given a certain amount of variability between city districts, the use of aggregate measures, like the ones we applied, always leads to a loss of variance, when comparing entire cities.
9. Concluding remarks

The preliminary and tentative nature of our findings cannot be overemphasized. This caution stems in the first place from the kind of sample applied. The sample has a very limited size, only six cities, and is not really random.

Lacking a theoretical basis, the relationships established between network size and city size are purely descriptive.

The main empirical findings can be summarized as follows. All urban road networks are highly similar with regard to their distributions of intersections by degree. In that respect the predominant share of the degree-3 intersections (T-junctions) of about 75 percent should be noted. A strong linear relationship between road network size, i.e. numbers of intersections, street sections and regions respectively, and city size expressed as population and total road length, has been established. A result, being at first sight somewhat in contradiction with the aforementioned finding, is the fact that different districts of one city exhibit large differences in network size and network structure. A comparison with the limited empirical evidence gathered elsewhere, did not invalidate our findings.

Finally we mention two suggestions for further research that can be performed with our data: the numbers of intersections by degree for each 500 x 500 m² square. First, the spatial distributions of various network characteristics, like e.g. average intersection degree or intersection density, could be investigated in each of the towns. The analysis could be performed using isolines or 'trend surface analysis'. Second, the data available enables the development and evaluation of sampling procedures for estimating network characteristics such as total number of network elements and the spatial distributions of these elements. Inevitably, the problem of spatial autocorrelation, meaning that
observations in contiguous or nearby zones are not independent, will then have to be dealt with.

Anyhow, it will be clear that a lot of research is needed before the findings presented in this article are theoretically well-founded.
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Figure 1. Road networks of six Dutch cities.
Fig. 2: Distributions of intersections by degree.
Figure 3. Distributions of real intersections by degree.
Figure 4. Relationship between city size and network size.

1. Relationship between city population and number of intersections:
   \[ I = 15.3 \cdot P \]

2. Relationship between city population and number of street sections:
   \[ S = 23.3 \cdot P \]

3. Relationship between city population and number of regions:
   \[ R = 8.0 \cdot P \]

4. Relationship between road network length and number of intersections:
   \[ I = 7.7 \cdot L \]

5. Relationship between road network length and number of street sections:
   \[ S = 11.7 \cdot L \]

6. Relationship between road network length and number of regions (building blocks):
   \[ R = 4.0 \cdot L \]
Figure 3. Road network in district "Noord" and district "Oud-Noord".
Figure 6. Distributions of intersections by degree of both districts and Tilburg.