On the excavation process of a moving vertical jet in cohesive soil
to Wietske
On the excavation process of a moving vertical jet in cohesive soil

Proefschrift

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aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus Prof. ir. K.C.A.M. Luyben,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen
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Summary

In this study the excavation process of a moving vertical jet in cohesive soil has been investigated. Based on experimental research, different soil failure mechanisms are defined and modeled. The main result of this study is a calculation model to predict the jet cavity width ($W_c$) and depth ($Z_c$) as a function of the main jet and soil parameters.

Cohesive soil contains clay particles. These particles are responsible for the main characteristics of cohesive soils: a very low water permeability, a relatively high skeleton compressibility and plasticity. The moving jet generates an undrained response in cohesive soil, due to a high loading rate of the jet in combination with the low water permeability and relative high skeleton compressibility of the soil. This makes it possible to use the total stress analysis and characterize the soil with an undrained shear strength ($s_u$) only.

The behavior of a moving jet is mainly determined by entrainment of soil at the front side of the jet and the entrainment of ambient water at the rear side of the jet. The jet is momentum driven, as a result entrainment of ambient water and soil slows down the jet.

The entrainment of ambient water is calculated with the use of an empirical entrainment coefficient. To determine this entrainment coefficient, for both the cavitating and non-cavitating case, water submerged (free) jet tests were conducted in a pressure vessel. The dynamic pressure (in the present study called the stagnation pressure, $p_{stag}$) in the center of the jet was measured, at several distances to the nozzle. A broad range of parameters was tested: nozzle diameter ($D_n = 3, 5$ and $7$ mm), jet pressure ($p_j = 1.5-20$ MN/m$^2$), distance to the nozzle ($s = 6D_n - 72D_n$) and ambient pressure ($p_{a0} = 0.1-0.57$ MN/m$^2$).

To identify and document the different failure mechanisms of the soil, 72 exploratory jet tests on artificially prepared cohesive soil samples were conducted. A broad range of parameters was tested: peak undrained shear strength ($s_{up} = 20-70$ kN/m$^2$), jet pressure ($p_j = 0.41-15.6$ MN/m$^2$), nozzle diameter ($D_n = 3-32.5$ mm), traverse velocity ($v_t = 0.12-2.0$ m/s) and distance between nozzle and original soil surface ($SOD = 20-30$ mm).

From the jet tests on soil it was possible to conclude that for a large range of test settings ($5.4 < p_j/s_{up} < 200, 0.15 < v_t < 2$) the cavity depth normalized with the nozzle diameter ($Z_c/D_n$) is linearly proportional to the normalized jet pressure ($= \text{jet ratio: } p_j/s_{up}$). For
these tests the influence of the traverse velocity of the nozzle on the normalized cavity depth is limited. For higher jet ratios the influence of the traverse velocity of the nozzle cannot be neglected.

The stand off distance (SOD) can be discounted by using the jet diameter ($D_{j,ss}$) and stagnation pressure ($p_{stag,ss}$) just above the original soil surface (for small stand off distances $D_{j,ss} \approx D_n$ and $p_{stag,ss} \approx p_j$).

The measured minimum stagnation ratio at the original soil surface ($p_{stag,ss}/su_p$) for soil failure was 5.4. Theoretically, this minimum ratio should be 6.2.

Important in the analysis of the excavation process are the conditions at which the jet will penetrate the soil nearly vertically. These conditions are:

1. The cavity depth ($Z_c$) must be larger than about 2.5 times the jet diameter at the soil surface ($D_{j,ss}$). To accomplish this the stagnation ratio at the soil surface must be at least 12 ($p_{stag,ss}/su_p > 12$).

2. The traverse velocity of the nozzle ($v_t$) must be lower than the maximum feasible horizontal propagation velocity of the soil front ($u_{f,h}$).

If the cavity depth is smaller than $2.5D_{j,ss}$ and/or traverse velocity of the nozzle is higher than $u_{f,h}$ the jet will deflect backwards, opposite the traverse direction of the nozzle, directly after penetrating the soil.

Four different types of jets/failure modes can be distinguished, based on the cavity dimensions and cavity wall structure, see also Table 1:

- **Penetrating jet**: This type of jet penetrates the soil nearly vertically. This results in a soil wall structure with straight small vertical nerves (non-deflection zone). Only at the bottom of the cavity the jet will deflect backwards, opposite the traverse direction of the nozzle (deflection zone). The jet cavities of the penetrating jet are deep and rectangular, with a constant cavity width of about 1 to 1.4 times the jet diameter at the original soil surface ($W_c = 1 \sim 1.4D_{j,ss}$).

- **Deflecting jet**: This type of jet deflects backwards directly after penetrating the soil (non-deflection zone is missing).

- **Dispersing jet flow**: If the cavity depth is smaller than 0.8 times the jet diameter at the original soil surface ($Z_c < 0.8D_{j,ss}$), the jet (radial dispersing jet flow) is able to push the soil up to the original soil surface. This results in relatively wide cavities.

- **Hydro-fracturing**: At low traverse velocity of the nozzle the jet can penetrate the planes of weaknesses, if present in the soil. This results in very irregular cavity shapes, which can be much wider and deeper than expected.

It is assumed that the soil in the non-deflection zone of a penetrating jet fails in very small discrete soil elements (order of magnitude 2 mm). In the deflection zone the soil front is supported by the deflecting jet. This prevents the forming of small discrete soil elements. It is assumed that the soil fails in small strips with typically a length equal to the length of the curvature of the jet.

To substantiate these assumptions, in a second test program the soil failure process was recorded with a high speed camera (1000-3000 frames per second). Unfortunately, the
Table 1: Theoretical conditions under which the different failure modes occur.

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>( \frac{p_{stag,ss}}{su_p} [-] )</th>
<th>( v_t [\text{m/s}] )</th>
<th>( \frac{Z_c}{D_{j,ss}} [-] )</th>
<th>( \frac{W_c}{D_{j,ss}} [-] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penetrating jet</td>
<td>&gt; 12 and ( \leq u_{f,h} )</td>
<td>&gt; 2.5</td>
<td>1.1-1.4</td>
<td></td>
</tr>
<tr>
<td>Deflecting jet</td>
<td>7.3-12 or ( &gt; u_{f,v} )</td>
<td>0.8-2.5</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>Dispersing jet flow</td>
<td>5.4-7.3 or ( \gg u_{f,v} )</td>
<td>&lt; 0.8</td>
<td>2.45-1.7</td>
<td></td>
</tr>
<tr>
<td>Hydro-fracturing</td>
<td>-</td>
<td>&lt; 0.15</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The shearing process of discrete soil elements was not visible on the high speed recordings. The recordings only showed a thin horizontal front, below which deforming material occurs and above which the material has been removed already. It is possible that the frame rate used, was too low. The shearing of strips in the deflection zone was visible in the recordings.

The different failure modes were modeled to predict the cavity dimensions. These models were verified with the test data. In conclusion it is possible to predict cavity dimensions accurately. In these models hydro-fracturing was not included.

The cavity depth can be calculated \( Z_c \), assuming that the stagnation ratio at the cavity bottom must be 8.2 times the peak undrained shear strength: \( p_{stag}(SOD + Z_c) = 8.2su_p \). If the jet is assumed to have a uniform velocity profile and the increase in jet flow density is neglected, the uniform stagnation pressure as function of the distance to the nozzle, and thus the cavity depth, can be solved analytically. This analytical solution is only applicable for non-cavitating jets with a low traverse velocity. The development of the uniform stagnation pressure can also be calculated numerically, taking into account cavitation and the increase in jet flow density due to the entrainment of soil.

Also a more advanced calculation model has been developed, taking into account the failure of small discrete soil elements in the non-deflection zone and the shearing of strips in the deflection zone. Because the duration of the failure process of the discrete soil elements is considered in this model, the jet trajectory can be predicted as a function of the stagnation ratio at the original soil surface \( \frac{p_{stag,ss}}{su_p} \) and the traverse velocity of the nozzle \( v_t \).
Samenvatting

In deze studie wordt het losmaakproces van een transiterende verticale jet in cohesieve grond onderzocht. Gebaseerd op experimenteel onderzoek worden verschillende bezwijkmechanismen van de grond beschreven en gemodelleerd. Deze studie heeft geresulteerd in een model waarmee de jetkuilbreedte ($W_c$) en -diepte ($Z_c$) berekend kunnen worden als functie van de belangrijkste jet- en grondparameters.

In cohesieve grond zitten kleideeltjes. Deze deeltjes geven cohesieve grond de kenmerkende eigenschappen: een zeer slechte waterdoorlatendheid, een hoge samendrukbaarheid van het korrelskelet en plasticiteit. De eerste twee eigenschappen in combinatie met de hoge belastingsnelheid tijdens het jetproces zorgen voor een ongedraineerd gedrag. Hierdoor is het mogelijk een ongedraineerde totaal-spanningsanalyse te maken en de grond slechts te karakteriseren met een ongedraineerde schuifsterkte ($su$).

Het gedrag van een transiterende jet wordt voornamelijk bepaald door de opname van omgevingswater aan de achterzijde en van grond aan de voorzijde van de jet. Hierdoor nemen het debiet en de dichtheid van de jet toe. Uitgaande van behoud van impuls, zorgt dit ervoor dat de snelheid van het jetmengsel afneemt. De hoeveelheid omgevingswater die wordt opgenomen door de jet, wordt berekend met een empirische entrainment coëfficiënt. Om deze coëfficiënt vast te kunnen stellen voor zowel een caviterende als een niet-caviterende jet, zijn in een druktank jetproeven uitgevoerd. Tijdens deze testen is op verschillende afstanden vanaf de nozzle de dynamische druk in het hart van de jet gemeten. Hierbij is een groot aantal parameters gevarieerd: nozzlediameter ($D_n = 3, 5$ and $7$ mm), jetdruk ($p_j = 1.5$-$20$ MN/m$^2$), afstand tot de nozzle ($s = 6D_n - 72D_n$) en omgevingsdruk ($p_{a0} = 0.1$-$0.57$ MN/m$^2$).

Om de verschillende bezwijkmechanismen van de grond te identificeren zijn 72 oriënterende jettetsen op kunstmatig aangemaakte cohesieve grondsamples uitgevoerd. Een grote variatie in parameters is getest: ongedraineerde piek-schuifsterkte ($su_p = 20$-$70$ kN/m$^2$), jetdruk ($p_j = 0.41$-$15.6$ MN/m$^2$), nozzlediameter ($D_n = 3$-$32.5$ mm), voortgangssnelheid van de nozzle ($v_t = 0.12$-$2.0$ m/s) en afstand tussen de nozzle en het originele grondoppervlak ($SOD = 20$-$30$ mm).

Op basis van deze testen is vastgesteld dat de jetkuildiepte, genormeerd aan de nozzlediameter, recht evenredig toeneemt met de verhouding tussen de jetdruk en de ongedraineerde piek-schuifsterkte ($Z_c/D_n \propto p_j/su_p$). Deze afhankelijkheid is vastgesteld.
binnen de volgende range aan parameters: $5.4 < \frac{p_j}{su_p} < 200$, $0.15 < v_t < 2$, $SOD < 2D_n$. De invloed van de voortgangssnelheid van de nozzle op de jetkuil diepte is in dit geldigheidsgebied beperkt. Voor grotere waardes van $\frac{p_j}{su_p}$ speelt de voortgangssnelheid van de nozzle wel een rol.

De invloed van de stand off distance kan verdisconteerd worden door te rekenen met de stuwdruk ($p_{stag,ss}$) en de diameter van de jet ($D_{j,ss}$) ter hoogte van het originele grondoppervlak (voor een kleine $SOD$ geldt: $p_{stag,ss} \approx p_j$ en $D_{j,ss} \approx D_n$).

Uit de metingen volgt dat voor penetratie $p_{stag,ss}/su_p$ minimaal 5.4 moet zijn (theoretisch is dit 6.2).

Belangrijk bij de analyse van het jetproces is het omslagpunt waarbij de jet de grond niet meer verticaal penetreert. Voor het verticaal penetreren moet aan twee voorwaarden worden voldaan:

1. De kuil diepte ($Z_c$) moet groter zijn dan ongeveer 2.5 keer de diameter van de jet ter hoogte van het grondoppervlak ($D_{j,ss}$). Om dit te realiseren moet de stuwdruk van de jet aan het oppervlak groter zijn dan 12 keer de ongedraineerde piek-schuifsterkte ($p_{stag,ss}/su_p > 12$).

2. De voortgangssnelheid van de nozzle ($v_t$) moet kleiner zijn dan de maximaal haalbare horizontale voortgangssnelheid van het front dat door de jet kan worden losgespoten ($u_{f,h}$).

Gebaseerd op de afmetingen en de wandstructuur van de jetkuilen, zijn vier verschillende bezwijkregimes/mechanismen gedefinieerd, zie ook Tabel 1:

- Verticaal penetrerende jet: Dit type jet penetreert de grond verticaal. Dit resulteert in een wandstructuur met verticale nerven (‘non-deflection zone’). Onderin de jetkuil buigt de jet pas om naar achteren (‘deflection zone’). De jetkuilen zijn diep en hebben een constante breedte ($W_c = 1 \sim 1.4D_{j,ss}$).

- Afbuigende jet: Dit type jet buigt direct na het penetreren van de grond naar achteren om (‘non-deflection zone’ ontbreekt).

- Divergerende jetstroom: Bij zeer ondiepe jetkuilen ($Z_c < 0.8D_{j,ss}$) is de jet in staat om de grond zijdelings naar het oppervlak weg te drukken. Dit resulteert in extra brede kuilen met een onregelmatige kuilvorm en wandstructuur.

- Scheurvorming: Bij zeer lage voortgangsnelheden van de nozzle is de jet in staat om eventueel aanwezige voorkeursvlakken aan te spreken. Dit resulteert in zeer onregelmatige kuilvormen die aanzienlijk dieper en breder kunnen zijn dan op basis van $p_{stag,ss}/su_p$ verwacht kan worden.

Aangenomen wordt dat de grond in de ‘non-deflection zone’ van de penetrerende jet bezwijkt in zeer kleine blokjes (orde grootte 2 mm) die afzonderlijk afschuiven. In de ‘deflection zone’ wordt de grond door het ombuigen van de jet ondersteund, hierdoor wordt het bezwijken van de grond in zeer kleine brokjes tegengegaan. Aangenomen wordt dat de grond in deze zone afschuift in stroken, met een lengte gelijk aan de kromming van de jet.

Met ‘high speed’ filmopnamen (1000-3000 beelden per seconde) is geprobeerd dit te onderbouwen. Het afzonderlijk afschuiven van kleine kleibrokjes in de ‘non-deflection
zone’ was niet zichtbaar. Op de beelden is alleen een dun horizontaal front zichtbaar waaronder continu grond wordt losgemaakt en wordt opgenomen in de jetstraal. Het door de jet wegdrukken van stroken grond in de ‘deflection zone’ was wel zichtbaar.

De verschillende bezwijkmechanismen zijn gedefinieerd om de afmetingen van een jetkuil te kunnen berekenen. Deze modellen zijn geverifieerd aan de hand van de metingen. Geconcludeerd kan worden dat de jetkuilafmetingen goed voorspeld kunnen worden. De jetkuilafmetingen kunnen berekend worden door aan te nemen dat de stuwdruk onderin de jetkuil minimaal 8.2 keer de ongedraineerde piek-schuifsterkte moet zijn: $p_{stag}(Z_c+SOD) = 8.2s_{up}$.

De stuwdruk als functie van de afstand tot de nozzle, en dus de jetkuilafmetingen $(Z_c)$, kan analytisch worden opgelost, uitgaande van een uniform snelheidsprofiel en een constante mengseldichtheid in de jet. Deze analytische oplossing is alleen geldig voor niet-cavitierende jets met een zeer lage voortgangssnelheid. Door het verloop van de stuwdruk numeriek te berekenen, kan ook rekening gehouden worden met cavitatie en de toename van de mengseldichtheid door de opname van grond.

Er is ook een geavanceerd rekenmodel ontwikkeld, waarbij het afzonderlijk afschuiven van kleine gronddeeltjes in de ‘non-deflection zone’ en het afschuiven van stroken in de ‘deflection zone’ wordt meegenomen in de modellering. Met dit model kan ook de loop van de jet in de grond voorspeld worden als functie van de voortgangssnelheid van de nozzle $(v_t)$ en de verhouding tussen de stuwdruk van de jet aan het oppervlak en de ongedraineerde piek-schuifsterkte $(p_{stag,ss}/s_{up})$. 

Tabel 1: Randvoorwaarden waarbinnen de verschillende bezwijktypen plaatsvinden.

<table>
<thead>
<tr>
<th>Bezwijktype</th>
<th>$p_{stag,ss}/s_{up}$ [-]</th>
<th>$v_t$ [m/s]</th>
<th>$Z_c/D_{j,ss}$ [-]</th>
<th>$W_c/D_{j,ss}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penetrerende jet</td>
<td>$&gt;$ 12 en $\leq u_{f,h}$</td>
<td>$&gt;$ 2.5</td>
<td>1-1.4</td>
<td></td>
</tr>
<tr>
<td>Afbuigende jet</td>
<td>7.3-12 of $&gt; u_{f,v}$</td>
<td>0.8-2.5</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>Divergerende jet</td>
<td>5.4-7.3 of $\geq u_{f,v}$</td>
<td>$&lt; 0.8$</td>
<td>2.45-1.7</td>
<td></td>
</tr>
<tr>
<td>Scheurvorming</td>
<td>-</td>
<td>$&lt; 0.15$</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
## Nomenclature

### Roman letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{1,2,..} )</td>
<td>Auxiliary variable</td>
<td>-</td>
</tr>
<tr>
<td>( a_\omega )</td>
<td>Angular acceleration</td>
<td>rad/s(^2)</td>
</tr>
<tr>
<td>( A )</td>
<td>Surface</td>
<td>m(^2)</td>
</tr>
<tr>
<td>( A_c )</td>
<td>Activity</td>
<td>-</td>
</tr>
<tr>
<td>( A_w )</td>
<td>Pore water pressure coefficient</td>
<td>-</td>
</tr>
<tr>
<td>( b_{1,2,..} )</td>
<td>Auxiliary variable</td>
<td>-</td>
</tr>
<tr>
<td>( b_j )</td>
<td>Radial distance, where the jet velocity is ( 1/e \cdot u_j(s,0) )</td>
<td>m</td>
</tr>
<tr>
<td>( B_w )</td>
<td>Pore water pressure coefficient</td>
<td>-</td>
</tr>
<tr>
<td>( c_{1,2,..} )</td>
<td>Auxiliary variable</td>
<td>-</td>
</tr>
<tr>
<td>( c )</td>
<td>Concentration</td>
<td>-</td>
</tr>
<tr>
<td>( c' )</td>
<td>Cohesion</td>
<td>N/m(^2)</td>
</tr>
<tr>
<td>( c_c )</td>
<td>Clay content, fraction of total solid mass</td>
<td>-</td>
</tr>
<tr>
<td>( c_f )</td>
<td>Friction coefficient; ( 2\tau_b/\left(\rho_m u_{bl,max}^2\right) )</td>
<td>-</td>
</tr>
<tr>
<td>( c_g )</td>
<td>Gas concentration in fluid</td>
<td>mol/m(^3)</td>
</tr>
<tr>
<td>( c_i )</td>
<td>Isotropic consolidation coefficient</td>
<td>m(^2)/s</td>
</tr>
<tr>
<td>( c_v )</td>
<td>Vertical consolidation coefficient</td>
<td>m(^2)/s</td>
</tr>
<tr>
<td>( C )</td>
<td>Compressibility</td>
<td>m(^2)/N</td>
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<tr>
<td>( C_c )</td>
<td>Compression index</td>
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<tr>
<td>( C_s )</td>
<td>Swell index</td>
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</tr>
<tr>
<td>( d_{bc} )</td>
<td>Empirical depth factor in equation for the bearing capacity</td>
<td>-</td>
</tr>
<tr>
<td>( D )</td>
<td>Diameter</td>
<td>m</td>
</tr>
<tr>
<td>( D_{50} )</td>
<td>Median particle diameter</td>
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<tr>
<td>( D_{90} )</td>
<td>Particle diameter at which 90% of particles are coarser</td>
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</tr>
<tr>
<td>( e )</td>
<td>Void ratio</td>
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<tr>
<td>( E )</td>
<td>Erosion rate</td>
<td>g/m(^2)/s</td>
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<tr>
<td>( F )</td>
<td>Force</td>
<td>N</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravitational constant</td>
<td>m/s(^2)</td>
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<tr>
<td>( h )</td>
<td>Drainage length</td>
<td>m</td>
</tr>
<tr>
<td>( h_e )</td>
<td>Erosion depth</td>
<td>m</td>
</tr>
<tr>
<td>( h_s )</td>
<td>Swelling depth</td>
<td>m</td>
</tr>
</tbody>
</table>
I  Momentum flux  N
Ip  Polar moment of inertia  kg \cdot m^2
k  Experimental constant  -
k_w  Water permeability  m/s
k_s  Equivalent roughness height  m
K_0  Coefficient of lateral effective pressure at rest  -
L  Length  m
LI  Liquidity index  %
LL  Liquid limit  %
m  Mass  kg
m_{nc}  Empirical parameter (exponent)  -
m_{sr}  Empirical rate parameter (exponent)  -
m_{tv}  Vertical compressibility  m^2/N
M  Moment force  Nm
M_c  Slope of the CS-line on the p'-q plane  -
M_e  Empirical Erosion rate parameter  g/m^2/s
n_{sr}  Empirical rate parameter  -
n_0  In-situ porosity  -
N_{bc}  Bearing capacity factor  -
p  Pressure  N/m^2
P  Power  W
PI  Plasticity index  %
PL  Plastic limit  %
P_w  Wetted perimeter  m
q  Deviatoric stress  N/m^2
q_{bc}  Bearing capacity  N/m^2
q_z  Specific discharge  m/s
Q  Flow rate  m^3/s
r  Radial distance to centerline of the jet  m
R  Radius  m
Re  Reynolds number  -
s  Jet distance, measured along the centerline of the jet  m
s_{bc}  Empirical shape factor in equation for the bearing capacity  -
su  Undrained shear strength  N/m^2
S  Degree of saturation  -
S_s  Sensitivity  -
S_t  Surface tension  N/m
SOD  Stand off distance  m
t  Time  s
T  Temperature  °C
T  Dimensionless time-scale  -
u  Velocity  m/s
U  Degree of consolidation  -
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>$v$</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$v_\omega$</td>
<td>Angular velocity</td>
<td>rad/s</td>
</tr>
<tr>
<td>$v_t$</td>
<td>Horizontal traverse velocity of the nozzle</td>
<td>m/s</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$w$</td>
<td>Water content in terms of mass</td>
<td>-</td>
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<tr>
<td>$W_c$</td>
<td>Jet cavity width</td>
<td>m</td>
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<tr>
<td>$Z_c$</td>
<td>Jet cavity depth</td>
<td>m</td>
</tr>
<tr>
<td>$z$</td>
<td>Distance in z-direction</td>
<td>m</td>
</tr>
<tr>
<td>$z_d$</td>
<td>Distance to drainage surface</td>
<td>m</td>
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</tbody>
</table>

**Greek letters**

<table>
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<td>Unified Soil Classification System</td>
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<td>WID</td>
<td>Water Injection Dredging</td>
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Chapter 1
Introduction

1.1 Background

The main activity of dredging companies is to remove and transport soil from the seabed. This is done for different purposes:

- The construction and maintenance of ports
- Land reclamation
- Coastal protection
- The burying and deepening of infrastructure for communication and the energy markets (oil, gas and wind)

The soil can be excavated hydraulically by the use of jets or mechanically by the means of cutting.

1.1.1 Hydraulic excavation of soil

Water jets are widely used, to excavate soil. A jet having a high flow velocity, penetrates the soil and brings the sediment into suspension. Figure 1.1 shows some pictures of a moving vertical jet penetrating cohesive soil. Because of the horizontal movement of the nozzle, the jet deflects backwards in the cavity.

Almost all dragheads of Trailing Suction Hopper Dredgers (TSHD) are equipped with a water jet system, see Figures 1.2 and 1.3. A number of vertically orientated jets are positioned at the front side of a draghead. Soil is continuously loosened and suspended, by trailing the draghead over the seabed. The loosened soil is sucked up and pumped into the hopper.
1. Introduction

The Dutch dredging companies use relatively low jet pressures (jet flow velocities) in combination with a large flow rate, order of magnitude 1 MN/m² at 0.03 m³/s/jet, whereas other dredging companies also use much higher jet pressures, up to 36 MN/m². The trailing speed (traverse velocity) is typically 0.5 to 1.5 m/s.

The suspended soil has not to be removed in all cases by suction devices. When a gentle slope and/or a strong current is present, the suspended material will flow away under the influence of gravity and density gradients. This process is called
1.1. Background

Water Injection Dredging (WID). Only a jet pipe is needed in these cases, see Figure 1.4. WID is usually used in soft cohesive soils or fine graded sands, like silts. The jet pressures are therefore lower than used on a draghead, typically 0.1 to 0.2 MN/m².

Trenching machinery is also often equipped with a jetting system. A trenching machine creates a slot in the soil (trench), in which a pipe or cable can be buried. Soil will be loosened with one or two almost vertically positioned jet swords, see Figure 1.5. The suspended soil is sucked up behind the jet swords and side-casted or will settle again. Main difference between the jetting process of a trencher and a draghead is the traverse velocity. The traverse velocity of a trencher is much lower, typically 0.1 m/s, whereas the jet pressures and flow rate are comparable.

![Figure 1.4: Jet beam of a water injection dredger.](image1)

Figure 1.4: Jet beam of a water injection dredger.

![Figure 1.5: Jet trenching machine.](image2)

Figure 1.5: Jet trenching machine.

Another application of a jet system is Mass Flow Excavation (MFE). In MFE the soil is loosened and blown away with one or two big jets with relatively low jet pressures. The influence length of the jet is large because of the large flow rate, making it possible to position the jet system at a large distance from the soil surface, named the stand off distance (SOD). There are stand-alone systems where the nozzle is constructed directly behind the pump (see Figure 1.6) but it is also possible to use the suction tube of a TSHD as jet pipe and replace the draghead with a nozzle, see Figure 1.7.

**Mechanical excavation of soil**  Another frequently used method to loosen the soil is to cut it mechanically by chisels. Chisels can for example be constructed at the back side of a draghead, on a bucket of a backhoe dredger or on a cutter. In cohesive soil mechanical cutting is more efficient than jetting, contrasting to non-cohesive soil, where the jetting process is by far the most efficient process to loosen the soil. In cohesive soil the specific energy for jetting (Joules per suspended soil volume [J/m³]) is an order of magnitude higher than for cutting.
1. Introduction

However, mechanical excavation of soil is not always allowed and/or possible:

- A jetting system can be prescribed in the specifications of a project to prevent damage on vulnerable infrastructure (cables and pipelines) on the seabed.
- The water depth can be too large to transfer sufficient horizontal towing (cutting) force to the draghead or trenching tool.
- At very large water depths often a remotely operated underwater vehicle (ROV) is used. The horizontal towing (cutting) force that can be provided with tracks and/or thrusters of a ROV is also limited.

1.1.2 Process description

Soil  Soil is composed of solid particles. At the seabed the pores in between these particles are almost fully saturated with water. The solids consist of a mixture of clay, silt and sand. The presence of clay particles is mainly responsible for the cohesive behaviour of the soil. Therefore cohesive soils are often called clays. Clay particles are very small flat platy shaped particles (< 2µm), formed by chemical weathering. Clay particles have the ability to attract and bind water, in contrary with ‘non-clay’ granular particles. Characteristics of cohesive soils are a very low water permeability and a high compressibility of the particle skeleton. This results usually in an undrained behaviour (pore water flow through the soil is negligible), making it possible to characterize the soil with the undrained shear strength \( su \). Non-cohesive soils mainly consists of non-clay particles, like sand. The water permeability of these granular soils is relatively high, generally resulting in a drained behaviour.
1.1. Background

**Main soil failure mechanisms caused by a moving jet** The jet can exert different forces on the soil. The main forces are:

- A normal load in the main direction of the jet flow (stagnation pressure), due to the mass flow of the jet.
- A shear force parallel to the flow direction (shear stress), due to the high flow velocity and the viscosity of the water.

In non-cohesive soils the shear stresses exerted by the jet \( \tau_b \) detach individual grains from the seabed. To remove a grain the created void space behind this grain must be filled up with water, see Figure 1.8 (a). The velocity of this erosion process depends mainly on the permeability of the soil and the flow velocity of the jet; the lower the water permeability, the slower the erosion process (Rhee, 2010). This process is a drained process and is often called *surface erosion*.

![Figure 1.8: Failure mechanisms of (a) non-cohesive soil and (b) cohesive soil, caused by a moving vertical jet.](image)

Surface erosion, caused by a moving turbulent jet, rarely occurs in cohesive soils, because the water permeability is too low in relation to the time scale of the jetting process. Depending on the loading conditions, different shear surfaces are formed in the soil. When the (jet) load exceeds the soil resistance in these surfaces, the soil will fail. In the case of a moving vertical jet, these surfaces are mainly formed by the stagnation pressure \( p_{stag} \) of the jet, see Figure 1.8 (b).

**Cavitating jets** The stagnation pressure in the vertical moving jet as a function of the distance to the nozzle (jet distance) is one of the more important parameters
1. Introduction

for the excavation process of cohesive soil. The stagnation pressure is a function of the jet flow velocity and the jet flow density (which is constant for a free water jet). The jet velocity decreases with jet distance, because at the interface between the jet and ambient fluid transfer of mass and momentum takes place. Ambient fluid is accelerated and entrained in the jet. As a result the jet slows down.

Under conditions of cavitation, a cone of bubbles forms around the jet (compare Figures 1.1 and 1.9). The momentum exchange between the jet and ambient fluid is hindered by this cone of bubbles. As a result the decrease in jet velocity with jet distance is smaller.

Figure 1.9: Moving cavitating vertical jet in cohesive soil.

1.1.3 Literature review

The behaviour of cohesive soil (Leroueil et al., 1990), (Mitchell and Soga, 2005) and the behaviour of submerged jets (Rajaratnam, 1976) was studied extensively by different researchers. Existing literature on those two topics will be reviewed in detail in the Chapters 2 and 3, respectively. Little is known about the interaction between both. The fundamental research is limited to the erosion of cohesive soil induced by a turbulent flow parallel to the soil surface (Winterwerp and van Kesteren, 2004) and the erosion induced by a stationary low pressure jet (Mazurek et al., 2001) and (Mazurek et al., 2006). A description of the relevant failure mechanisms of cohesive soil exposed to a moving vertical jet is still missing. Only simple experimental correlations between the most important jet parameters and cohesive soil parameters have been published (Machin et al., 2001) and (Machin and Allan,
1.2 Scope and outline of this thesis

The published data of moving jet tests in cohesive soil is limited to one small dataset (Rockwell, 1981). The erosion induced by a turbulent flow parallel to the soil surface is discussed in Section 2.6. The available literature on the jetting process in cohesive soil is mainly analyzed in Section 5.2.3.

A lot of research has been done on the (abrasive) jet cutting of hard materials, like steel and concrete (Summers, 1995). Already in 1978, Hashish and duPlessis proposed a generalized non-dimensional cutting equation for all kind of materials, see Section 7.2.2. The jet pressures used for jet cutting are usually significantly higher than used in dredging practice. As a result the determining failure processes differ and the generalized cutting equation cannot be used.

At larger jet pressures the submerged jet starts to cavitate. This will reduce the entrainment of ambient water. Despite the amount of research on cavitating jets, the literature does not provide an equation to quantify the entrainment and the stagnation pressure decay in the case of a cavitating jet (Soyama and Lichtarowicz, 1996). In Chapter 4 the existing literature about cavitating jets will be summarized.

1.2 Scope and outline of this thesis

Questions about the excavation process of a moving jet in cohesive soil are asked more frequently, to optimize and develop (new) jet devices and to predict the productions of existing jet equipment. A good understanding of the behaviour of the moving submerged water jet, the cohesive soil failure mechanisms and their interactions is essential to answer these questions.

The main objectives of this PhD-study are:

- To describe the failure mechanisms of cohesive soil exposed to a moving vertical submerged water jet.
- To develop a physical and mathematical model to predict the jet cavity dimensions (depth and width) as a function of the main jet parameters and soil properties.

The development of the stagnation pressure, and thus the jet velocity, with distance must be known to model the excavation process of a moving vertical jet in cohesive soil. The jet velocity decreases with distance. This is mainly caused by the entrainment of ambient fluid at the interface between jet and ambient water. The amount of entrainment decreases by cavitation. A quantitative description of the entrainment as function of the jet pressure and ambient pressure is lacking.
1. Introduction

Therefore one of the sub objectives is:

- To quantify the entrainment for a cavitating jet as a function of the jet pressure, ambient pressure, nozzle diameter and the distance to the nozzle.

1.2.1 Scope

This thesis is limited to fully saturated homogeneous cohesive soil exposed to a moving submerged vertical circular jet. The soil strength investigated is soft to stiff \((su = 20-200 \text{ kN/m}^2)\). The scope of the jet parameters is listed in Table 1.1.

<table>
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<td>Jet pressure ((p_j))</td>
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<td>Nozzle diameter ((D_n))</td>
<td>3.0 - 40 mm</td>
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<tr>
<td>Jet momentum flux ((M_0))</td>
<td>150 - 500 kN</td>
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<td>Jet power ((P_0))</td>
<td>3.5 - 65 kW</td>
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<td>Stand off distance ((SOD))</td>
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<tr>
<td>Reynolds number ((Re))</td>
<td>(&gt; 4 \cdot 10^5)</td>
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</table>

The jet momentum flux \((M_0)\) is defined as: \(\rho_w Q_0 u_0\), where \(\rho_w\) is the density of the jet water in [kg/m³], \(Q_0\) is the flow rate in [m³/s] and \(u_0\) the jet flow velocity in [m/s], both at nozzle exit. The Reynolds number \((Re)\) is defined as: \(u_0 D_n / \nu\), where \(\nu\) is the kinematic viscosity of the jet water in [m²/s].

1.2.2 Outline

In Chapter 2 the characteristics of cohesive soils are elaborated. The conclusion of this chapter is that the cohesive soil can be modeled as a plastic material with an undrained shear strength, which increases with the deformation velocity of the soil.

In Chapter 3 the behaviour of non-cavitating submerged turbulent jets is investigated. This research is mainly focused on the entrainment, the major cause of the decrease in jet velocity with jet distance.

At relative high jet pressures the jet starts cavitating. This phenomena decreases the entrainment. In Chapter 4 an empirical relation for the entrainment coefficient as a function of the jet pressure and ambient fluid pressure is derived and verified by experiments.
Jet tests on artificially prepared cohesive soil samples are described and analyzed in Chapter 5. Two test programs were conducted. The objective of the first test program was to visualize and to explore the different failure mechanisms. A wide range of jet parameters was tested and different failure patterns were observed, see Figure 1.10. Working hypotheses were formulated based on these test results concerning the occurrence of the different failure modes. In a second test program the failure of the soil was recorded with a high speed camera (1000-3000 frames per second) to substantiate these hypotheses. The high speed recordings are also analyzed in this chapter.

Figure 1.10: Top view of the results of different jet tests. Test settings: (a) $D_n = 3$ mm, $p_j/su = 223$, $v_t = 2$ m/s, (b) $D_n = 20$ mm, $p_j/su = 41$, $v_t = 0.25$ m/s, (c) $D_n = 20$ mm, $p_j/su = 32$, $v_t = 1$ m/s, (d) $D_n = 32.5$ mm, $p_j/su = 10$, $v_t = 1$ m/s, (e) $D_n = 32.5$ mm, $p_j/su = 12$, $v_t = 1.5$ m/s.

In Chapter 6 and 7 the jetting process is modeled. Four types of jets / failure modes are defined.

In Chapter 6 the modeling of the deflecting jet type will be discussed. The jet flow of this type of jet will deflect backwards, directly after penetrating the original soil bed. It is assumed that the soil fails in large curved strips / lumps. With the proposed model the cavity depth can be calculated as a function of the assumed cavity width ($W_c$) and the stagnation ratio at the original soil surface ($p_{stag, ss}/su_p$, in which $p_{stag, ss}$ is the stagnation pressure at the original surface and $su_p$ is the peak undrained shear strength). The threshold value of the stagnation ratio at the original soil surface for soil failure will be derived. Based on this minimum stagnation ratio, also a minimum jet ratio as a function of the distance to the
soil surface ($SOD$) can be determined, where the jet ratio is defined as the ratio between the jet pressure and the undrained shear strength of the soil ($p_j / su_p$). The jet flow will (partly) disperse in radial direction on the top of the soil strip. When the stagnation pressure of the radial dispersing flow is large enough, it can widen the cavity. Two failure mechanisms are defined and modeled to calculate the final cavity width.

In Chapter 7 the modeling of the most common jet type, the penetrating jet, will be discussed. The cross section of the cavity created by a penetrating jet is rectangular, narrow and deep. This type of jet penetrates the soil almost vertically and only after a certain penetration length the jet flow deflects. First of all a simple analytical model is derived to predict the cavity width and depth. This model is only applicable for very low traverse velocities, because the entrainment of solids is neglected. Secondly a simple 1D-approach of the jetting process is described. In this approach the entrainment of solids and thus the development of the jet concentration is included. Also a more advanced model is derived. In this model the jetting process is divided into two zones, the non-deflection zone and the deflection zone. In the non-deflection zone the soil is assumed to fail in small discrete soil elements. The duration of the shearing process of these small discrete soil elements is taken into account. In the deflection zone the soil is assumed to fail in large curved strips. This failure process is also considered.

Finally, the main results of the modeling are summarized in Chapter 8. In this chapter also the conclusions and recommendations for further research are given.
Chapter 2
Cohesive soil

2.1 Introduction

Cohesive soils are usually composed of a mixture of clay, silt and sand, in which the clay particles are mainly responsible for the cohesive behaviour. The pores in between the particles are fully or partially filled with water. In the present study only fully saturated soils will be considered.

The main characteristics of cohesive soils are:

- very low water permeability
- high skeleton compressibility
- plasticity
- swell properties

Loading generally generates an undrained response, because of the low water permeability and high skeleton compressibility. An isotropic total stress increment will result in an almost identical increment in pore water pressure and will not influence the (inter-particle) effective stress, under undrained conditions (pore water flow through the soil is negligible). This makes it possible to characterize the soil strength with the undrained shear strength ($s_u$), which is independent of rapid changes in external pressure conditions.

The shear strength ($\tau$) of a soil is mostly given by an equation of the form (Mohr-Coulomb law):

$$\tau = c' + \sigma' \cdot \tan \varphi$$  \hspace{1cm} (2.1)
2. Cohesive soil

where \( c' \) is the cohesion, \( \sigma_{\perp}' \) is the effective compressive normal stress on the shear plane and \( \varphi \) is the internal friction angle. The effective normal compressive stress is defined as the total normal compressive stress (\( \sigma_{\perp} \)) minus the pore water pressure (\( p_w \)): \( \sigma_{\perp}' = \sigma_{\perp} - p_w \) (Terzaghi, 1943).

Despite the terminology a significant cohesion, defined as shear strength in the absence of effective normal compressive stress, does not exist in cohesive soils, except for cemented cohesive soils (Mitchell and Soga, 2005). However, also in the absence of an external load, a significant apparent cohesion can exist due to a water under pressure (suction) which results in a (internal) normal compressive effective stress. This apparent cohesion results in a certain coherence.

The mobilized internal friction angle (\( \varphi \)) takes into account the sliding resistance between the particles, particle rearrangements, dilation and eventually crushing (Rowe, 1962). Therefore the internal friction angle depends on a lot of parameters. The main parameters are the clay content (\( c_c \), defined as the mass of the clay particles divided by the total mass of the soil particles), the clay mineral, the chemical properties of the pore water, the initial fabric (structure) and the over-consolidation ratio.

The internal friction angles of normally consolidated cohesive soils range from 20\(^o\) to 25\(^o\). Usually, for over-consolidated cohesive soils the peak values of the internal friction angle are higher. Values up to 50\(^o\) have been reported (Kulhawy and Mayne, 1990). Whereas for large strains the internal friction angle can decrease to values between the 15\(^o\) and 5\(^o\) (Mitchell and Soga, 2005).

In general, the higher the clay content (\( c_c \)), the lower the water permeability (\( k_w \)), the higher the compressibility (\( C \)), the lower the internal friction angle (\( \varphi \)) and the higher the plasticity (\( PI \)). In the following sections the above statements will be discussed more in detail.

2.2 Standard classification

2.2.1 Particle size

The solid particles in the soil vary in size. The particle size distributions, according to the Unified Soil Classification System (USCS) and the Dutch classification system (NEN) are listed in Table 2.1. Soils are classified as coarse grained, granular, and cohesion-less if the amount of gravel and sand exceeds 50\% by weight, or fine grained and cohesive if the amount of fines (silt and clay-size material) exceeds 50\% by weight. However the terms cohesion-less and cohesive must be used with care, as even a few percent of clay minerals in a course-grained soil can impart plastic characteristics (see Section 2.2.4).
2.2. Standard classification

Table 2.1: Particle size distribution, particle diameter ($D_p$) in mm.

<table>
<thead>
<tr>
<th></th>
<th>USCS</th>
<th>NEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>$D_p &gt; 4.75$</td>
<td>$D_p &gt; 2$</td>
</tr>
<tr>
<td>Sand</td>
<td>$D_p &gt; 0.075$</td>
<td>$D_p &gt; 0.063$</td>
</tr>
<tr>
<td>Silt</td>
<td>$D_p &gt; 0.005$</td>
<td>$D_p &gt; 0.002$</td>
</tr>
<tr>
<td>Clay</td>
<td>$D_p &gt; 0.002$</td>
<td>$D_p$</td>
</tr>
</tbody>
</table>

2.2.2 Clay minerals

Clay can refer both to a size and to a class of minerals. As a size term, it refers to all constituents of a soil smaller than a particular size of 2 \( \mu \text{m} \). Therefore not all clay particles are finer than 2 \( \mu \text{m} \). However, the amount of clay minerals in a soil is often closely approximated by the material finer than 2 \( \mu \text{m} \).

Characteristics of clay minerals are: (1) a small particle size, (2) a flat platy shape, (3) a net negative electrical charge and (4) a very large specific surface which interacts with pore water. Therefore clay particles have the ability to attract and bind pore water, whereas non-clay particles cannot.

Clay-minerals are built up of combinations of two sheets, the silica-sheet composed of silica tetrahedra and the octahedral-sheet composed of aluminum or magnesium octahedra. Different clay mineral groups are characterized by the stacking of these sheets and the manner in which two- or three sheet layers are held together. Most common clay minerals are Kaolinite, Illite and Montmorillonite (see Figure 2.1). In Table 2.2 the main properties of these clay minerals are listed. Below also a brief description of the composition of the common clay minerals is given, according to Mitchell and Soga (2005).

**Kaolinite** The basal layer of the kaolinite-mineral is composed of one silica tetrahedral sheet and one aluminum octahedral sheet (1:1 clay mineral). Multiple basal layers are attached by strong hydrogen bonds, forming crystalline flake-like particles with a thickness of about 100 nm and plate dimensions of 2 \( \mu \text{m} \). The large crystalline particles have a negative charge on their flat face and a positive charge on their edges. Because the attached basal-layers cannot be separated easily by water molecules, Koalinite is known as a non-swelling clay mineral with relative large particles, resulting in a low specific surface area ($A_s$).

**Montmorillonite** Montmorillonite consists of an aluminum octahedral sheet sandwiched between two tetrahedral silica sheets (2:1 clay mineral). In the oc-
Table 2.2: Main properties most common clays, data from (Mitchell and Soga, 2005).

<table>
<thead>
<tr>
<th></th>
<th>Kaolinite</th>
<th>Montmorillonite</th>
<th>Illite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle dimensions</td>
<td>[µm]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1-4 x</td>
<td>&gt;0.001 x</td>
<td>0.003-0.1 x</td>
</tr>
<tr>
<td></td>
<td>0.05-2</td>
<td>&lt;10</td>
<td>&lt;10</td>
</tr>
<tr>
<td>Specific surface ($A_s$)</td>
<td>[m²/g]</td>
<td>15 ± 5</td>
<td>770 ± 70</td>
</tr>
<tr>
<td>Specific density ($\rho_s$)</td>
<td>[kg/m³]</td>
<td>2670 ± 10</td>
<td>2530 ± 170</td>
</tr>
<tr>
<td>Liquid limit (LL)</td>
<td>[%]</td>
<td>30-110</td>
<td>100-900</td>
</tr>
<tr>
<td>Plastic limit (PL)</td>
<td>[%]</td>
<td>25-40</td>
<td>50-100</td>
</tr>
<tr>
<td>Activity ($A_c$)</td>
<td>[-]</td>
<td>0.5</td>
<td>1-7</td>
</tr>
</tbody>
</table>

Tahedral sheets of Montmorillonite every sixth aluminum ion is substituted by a magnesium ion. Charge deficiencies in the basal layers are balanced by exchangeable cations on the surfaces of the particles. The bonds between the basal layers are weak and can easily be separated by the adsorption of water, resulting in a highly plastic, swelling clay. The particles remain thin (> 1 nm) due to the weak bonds, resulting in a large surface area. The larger the specific surface the larger the ability to attract and bind water. Montmorillonite has by far the largest specific surface and as a result the highest plasticity and swelling properties.

**Illite** The composition of the basal layer of Illite is similar to that of Montmorillonite. In Illite about one-fourth of the silicon positions are filled by aluminum. The resulting charge deficiency is balanced by potassium ($K^+$-ions) between the basal-layers. The number of basal layers is limited by the presence of potassium. Only at the outside of the crystalline particles, negative charges are maintained on the flat side. The interlayer (ionic) bonding is so strong that the basal spacing remains fixed. No polar water molecules are able to enter between the basal layers. The crystalline particle thickness is about 3 nm and plate dimensions from 0.1 µm up to several micrometers.

Non-clay particles are predominantly rock fragments or grains of the common rock-forming minerals, formed by physical weathering, unlike clay-minerals which are formed by chemical weathering. By far Quartz is the most abundant non-clay mineral (80-90 %). The other non-clay mineral frequently present in small percentages is Feldspar. Both are built up of three-dimensional silica tetrahedra ($SiO_2$).
2.2. Standard classification

Aluminum octahedral sheet

Strong hydrogen bonds

Exchangeable cations

Potassium

Silica tetrahedral sheet

Kaolinite

Montmorillonite

Illite

Figure 2.1: Composition of the basal layers in most common clay minerals (Mitchell and Soga, 2005).

2.2.3 Plasticity

The Atterberg limits, the plastic limit \((PL)\) and the liquid limit \((LL)\), are extensively used for identification and classification of cohesive soils. The plastic limit is the water content forming the boundary between the solid and plastic states (lowest water content at which the soil can be deformed without volume change or cracking) and the liquid limit is the water content forming the boundary between the plastic and liquid states. Where the water content \((w)\) is defined as the ratio of the mass of water to the mass of solids in a sample.

Both limits can be determined with two simple standardized soil tests, described in the ASTM D4318-10. These tests can be considered as undrained shear tests on remoulded samples, reflecting the mechanical behaviour as function of the water content. The plastic limit corresponds approximately to a water content at which the soil has an undrained shear strength in the range of 100 to 300 kN/m\(^2\), with an average value of 170 kN/m\(^2\), while the liquid limit corresponds to a water content at which the soil has an undrained shear strength of about 1.7 to 2.0 kN/m\(^2\) (Wroth and Wood, 1978).

A high liquid limit means that the soil has a great capacity to bind pore water. For example a pure Montmorillonite clay with a liquid limit of 300% can bind a volume of water that is about 8 times the volume of the clay particles.\(^1\) Whereas for Kaolinite with a \(LL\) of 70% this ratio is only 1.8. The capacity to bind pore water is a function of the specific surface \((A_s)\). Therefore the liquid limit is also a function of the specific surface (Farrar and Coleman, 1967):

\[
LL = 19 + 0.56A_s
\]

\(^1\) \(w = 300\% \rightarrow V_w/V_p = \rho_s/\rho_w \cdot w/100 = 2650/1000 \cdot 3 \approx 8\). Note that the corresponding density at liquid limit is only 1185 kg/m\(^3\).
The difference in water content between the liquid and plastic limit is called the plasticity index:

\[ PI = LL - PL \]  

(2.3)

The higher the plasticity index, the higher the compressibility and the volume change when the soil is (un)loaded. A plot of the plasticity index as function of the liquid limit that is divided into different zones, is termed the plasticity chart (see Figure 2.2). This chart forms an essential part of the USCS.

Figure 2.2: Plasticity chart, with the classification of the soil samples used for the jet tests (asterisks), see Section 5.1.

The so-called A-line distinguishes between inorganic clays and soils rich in organic matter and silt, whereas the so-called U-line envelops soils found in the natural environment.

Both type and amount of clay-particles influence the plasticity. To separate them the activity \( (A_c) \) can be used:

\[ A_c = \frac{PI}{\% < 2\mu m} \]  

(2.4)

where \( \% < 2\mu m \) is the clay content. The activity of Kaolinite is low, around 0.4, for Illite it is around 0.9 and for Montmorillonite it can go up to 7 (Mitchell and Soga,
2.2. Standard classification

The activity of a soil is related to the ability of the clay particles to isolate the non-clay particles, preventing them to form a non-clay-skeleton (see Section 2.2.4).

The high value of the activity of Montmorillonite can be explained by the large capacity to bind pore water. The larger the capacity to bind water, the larger the volume of the clay-water mixture at the same clay content and the lower the clay content at which the non-clay particles are isolated and cohesive behaviour is possible.

For expressing and comparing the consistencies of different cohesive soils often the liquidity index ($LI$) is used, defined as:

$$LI = \frac{w - PL}{PI}$$  \hspace{1cm} (2.5)

When $LI > 1$ the soil can be regarded as a fluid and when $LI < 0$ the soil can be regarded more or less as a non-plastic granular solid.

### 2.2.4 Skeleton

The compressibility of a granular-skeleton is usually lower than that of a clay-skeleton (see Table 2.3). The relative particle movement is also more constrained in a granular-skeleton than in a clay-skeleton (Winterwerp and van Kesteren, 2004). This is caused by the shape and stiffness of the granular particles (sand or silt). Therefore an important discriminator for soil behaviour is whether the granular particles are able to form a skeleton or not.

When the actual void ratio, defined as the volume of the void-space between the specific particles (pores) divided by the volume of the specific particles, of the granular-skeleton exceeds the maximum void ratio of the granular-skeleton ($e_{gr,\text{max}}$), the granular particles will be isolated in a clay-water mixture. As a result the compressibility of the skeleton will increase and the mobility of the granular particles will increase as well.

Assuming that (1) the soil is fully saturated, (2) all of the pore water is associated with the clay phase and (3) the specific density ($\rho_s$) of the granular and clay particles are equal, for any water content ($w$) the minimum clay content ($c_c$) of a clay-skeleton dominated soil can be calculated:

$$c_c > \frac{e_{gr,\text{max}}}{1 + e_{gr,\text{max}}} - \frac{w}{1 + e_{gr,\text{max}}} \frac{\rho_s}{\rho_w}$$  \hspace{1cm} (2.6)

This relation indicates that for a water content of 0.25 and a maximum void ratio for the granular particles ($e_{gr,\text{max}}$) of 0.9, only 13% of the total solid mass needs
2. Cohesive soil

to be clay to prevent direct inter-particle contact between the granular particles.\(^2\)

This corresponds well with the critical clay content of 15% between cohesive and cohesion-less soil, determined by Skempton and Bishop (1985).

The standard classification by grain size distribution and Atterberg limits is based on the remoulded state, and does not account for the actual in-situ fabric. Therefore Winterwerp and van Kesteren (2004) propose a classification based on the pore size distribution that reflects such in-situ conditions in combination with the above-mentioned geotechnical standards.

2.3 Drained versus undrained behaviour

Effective stress

The effective stress concept of Terzaghi (1943) states that the total stress \((\sigma)\) acting on an element of soil is counteracted by two stress components: the effective stress \((\sigma')\) and the pore water pressure \((p_w)\). In the general form:

\[
\sigma = \sigma' + \eta p_w
\]  

(2.7)

where \(\eta\) is the fraction of the pore water pressure that is influenced by the total stress. The effective stress determines the stress-strain, volume change and strength behaviour of the soil skeleton.

According to Terzaghi (1943) \(\eta\) is 1. This value is valid when the solid particle compressibility \((C_p)\) and the pore water compressibility \((C_w)\) are negligible, compared to the soil skeleton compressibility \((C_{sk})\). For almost all saturated soils this is the case, see Table 2.3.

Volume strain

The volume strain increment of the skeleton \((\Delta \varepsilon_{sk})\), positive for an increase of volume), due to a change in isotropic total stress \((\sigma_p)\) and pore water pressure \((\Delta p_w)\), is:

\[
\Delta \varepsilon_{sk} = -\Delta(\sigma_p - p_w)C_{sk} - \Delta p_w C_p = -\Delta\sigma'_p C_{sk} - \Delta p_w C_p
\]  

(2.8)

where the isotropic stress is defined as the mean stress in the principal directions: \(\sigma_p = (\sigma_1 + \sigma_2 + \sigma_3)/3\) (in geo-technical literature the isotropic stress is commonly denoted as \(p\)). The volume strain of the skeleton must be equal to the volume strain of the pore water \((\Delta \varepsilon_w)\) plus the volume strain of the solid particles \((\Delta \varepsilon_p)\).

\(^2\)In terms of porosity: \(n_{situ} = 0.4\) and \(n_{gr,max} = 0.47\). Where the porosity is defined as the ratio of the volume of the pores (\(V_{void}\)) to the volume of the soil sample (\(V\)): \(n = V_{void}/V = e/(1 + e)\).
2.3. Drained versus undrained behaviour

Table 2.3: Compressibility of the soil particles (\(C_p\)) and skeleton (\(C_{sk}\)) for different soils, after (Skempton, 1960) and (Winterwerp and van Kesteren, 2004).

<table>
<thead>
<tr>
<th></th>
<th>(C_{sk}) ([10^{-9}\text{m}^2/\text{N}])</th>
<th>(C_p) ([10^{-9}\text{m}^2/\text{N}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand (dense-loose)</td>
<td>18-92</td>
<td>0.028</td>
</tr>
<tr>
<td>Clay (very stiff-very soft; mud)</td>
<td>10-1000</td>
<td>0.020</td>
</tr>
<tr>
<td>London clay</td>
<td>75</td>
<td>0.020</td>
</tr>
<tr>
<td>Organic (without gas - with gas)</td>
<td>-</td>
<td>0.5-100</td>
</tr>
<tr>
<td>Pure water</td>
<td>-</td>
<td>0.49</td>
</tr>
</tbody>
</table>

The volume strain of the pore water can be caused by either compression of the pore water due to a change in pore water pressure, or by a net outflow of pore water:

\[
\Delta \varepsilon_{sk} = \Delta \varepsilon_p + \Delta \varepsilon_w = -(1 - n)C_p \Delta p_w - nC_w \Delta p_w - \nabla q_w \Delta t \tag{2.9}
\]

where \(n\) is the porosity and \(\nabla q_w\) is the divergence of the specific discharge in the main directions, which depends on the gradient of the head of excess pore pressure, through Darcy’s law:

\[
\nabla q_w = \frac{\partial q_{w,x}}{\partial x} + \frac{\partial q_{w,y}}{\partial y} + \frac{\partial q_{w,z}}{\partial z} = \frac{k_w}{\rho_w g} \left( \frac{\partial^2 p_w}{\partial x^2} + \frac{\partial^2 p_w}{\partial y^2} + \frac{\partial^2 p_w}{\partial z^2} \right) \tag{2.10}
\]

where \(k_w\) is the water permeability and \(g\) is the gravitational constant.

**Drained behaviour**

In a drained deformation process the time scale is such that the generated water pressures dissipate by pore water flow. Equating the Equations 2.8 and 2.9 and forming time derivatives gives the following equation for the pore water pressure dissipation:

\[
\frac{\partial p_w}{\partial t} = \frac{C_{sk}}{C_{sk} + n(C_w - C_p)} \cdot \frac{\partial \sigma_p}{\partial t} + c_i \left( \frac{\partial^2 p_w}{\partial x^2} + \frac{\partial^2 p_w}{\partial y^2} + \frac{\partial^2 p_w}{\partial z^2} \right) \tag{2.11}
\]

where \(c_i\) is the isotropic consolidation coefficient:

\[
c_i = \frac{k_w}{\rho_w g} \cdot \frac{1}{C_{sk} + n(C_w - C_p)} \tag{2.12}
\]
2. Cohesive soil

The process of dissipation of generated water over pressures by a net pore water outflow is called consolidation. The reversed process of dissipating water under pressures by a net pore water inflow is called swelling. For swelling the unload-reload compressibility of the skeleton has to be taken in the Equations 2.11 and 2.12. The unload-reload compressibility of the skeleton is about 5 times lower than the normal one (Kulhawy and Mayne, 1990).

In the one-dimensional consolidation case (vertical deformation and flow only) with a constant vertical load, Equation 2.11 reduces to (Terzaghi, 1943):

$$\frac{\partial p_w}{\partial t} = c_v \frac{\partial^2 p_w}{\partial z^2}$$  \hspace{1cm} (2.13)

$$c_v = \frac{k_w}{\rho_w g m_v + n(C_w - C_p)} \approx \frac{k_w}{\rho_w g m_v}$$  \hspace{1cm} (2.14)

where $c_v$ is the vertical consolidation coefficient and $m_v$ is the vertical compressibility, obtained from a confined compression test (oedometer test). The typical time scale for consolidation ($T_c$) of a soil layer with a height $h_c$ and only drained at one side, follows from the solution of Equation 2.13:

$$T_c \propto \frac{h_c^2}{c_v}$$  \hspace{1cm} (2.15)

In Section 2.6.2 is concluded that the timescale of the jetting process in cohesive soil is much shorter than that of the consolidation or swelling processes. Therefore, the failure process of cohesive soil can be considered as fully undrained during the jetting process.

**Undrained behaviour**

The pore water flow through the sediment is negligible in a fully undrained deformation process ($\nabla q_w \Delta t \approx 0$). An undrained process is also considered as a deformation process with a volume strain equal to zero (see Equation 2.9), while the compressibilities of the solids and pore water usually can be neglected.

### 2.4 Strength in triaxial undrained compression

The undrained shear strength ($s_u$) of a soil sample can be determined with the Mohr-Coulomb failure criterium. This strength is not a overall constant, but depends on the effective stress path followed to failure. The effective stress path is determined by (1) the initial stress state, (2) the followed total stress path and (3) the excess pore water generation. This will be discussed below.
2.4. Strength in triaxial undrained compression

2.4.1 Mohr-Coulomb failure criterium

For a Mohr-Coulomb model the strength in triaxial compression can be expressed in terms of maximum (\(\sigma'_1\)) and minimum (\(\sigma'_3\)) effective principal stresses and involving the parameters of cohesion (\(c'\)) and internal friction angle (\(\varphi\)) by:

\[
\frac{\sigma'_1 - \sigma'_3}{2} = c' \cos \varphi + \frac{\sigma'_1 + \sigma'_3}{2} \sin \varphi
\]  
\[(2.16)\]

This relation can be expressed in terms of the corresponding isotropic effective stress (\(\sigma'_p\)) and the deviatoric stress (\(q\)), which are related to the maximum and minimum effective principal stresses, by:

\[
\begin{align*}
\sigma'_p &= \frac{\sigma'_1 + 2\sigma'_3}{3} = \frac{\sigma'_v + 2\sigma'_h}{3} \\
q &= \sigma'_1 - \sigma'_3 = \sigma_1 - \sigma_3 = \sigma_v - \sigma_h
\end{align*}
\]  
\[(2.17)\]
\[(2.18)\]

In triaxial compression the maximum (\(\sigma'_1\)) and minimum (\(\sigma'_3\)) effective principal stresses can also be denoted as the axial and radial or vertical (\(\sigma'_v\)) and horizontal (\(\sigma'_h\)) effective stresses, respectively. Substituting the Equations 2.17 and 2.18 in 2.16 gives for the strength in triaxial compression:

\[
q = M(\sigma'_p + \frac{c'}{\tan \varphi})
\]  
\[(2.19)\]

\[
M = \frac{6 \sin \varphi}{3 - \sin \varphi}
\]

2.4.2 Effective stress path in undrained triaxial compression

The effective stress path starts at the initial effective stress state:

\[
\begin{align*}
\sigma'_{p0} &= \sigma_{p0} - p_{w0} = \frac{\sigma'_{v0} + 2\sigma'_{h0}}{3} - p_{w0} \\
q_0 &= \sigma'_{v0} - \sigma'_{h0}
\end{align*}
\]  
\[(2.20)\]
\[(2.21)\]

in which \(p_{w0}\) is the initial pore water pressure. The quantities at failure can be expressed as follows:

\[
\begin{align*}
\sigma_p &= \sigma'_{p0} + p_{w0} + \Delta \sigma_p = \sigma'_{p0} + p_{w0} + \zeta \Delta q \\
q &= q_0 + \Delta q \\
p_w &= p_{w0} + \Delta p_w \\
\sigma'_p &= \sigma_p - p_w = \sigma_{p0} + \zeta \Delta q + \Delta p_w
\end{align*}
\]  
\[(2.22)\]
\[(2.23)\]
\[(2.24)\]
\[(2.25)\]
in which parameter $\zeta$ is defined as the isotropic total stress increment $(\Delta \sigma_p)$ divided by the deviatoric stress increment $(\Delta q)$. For standard triaxial compression, for which $\Delta \sigma_3' = \Delta \sigma_h' = 0$, parameter $\zeta$ is $1/3$.

**Excess pore pressure generation**

The excess pore water pressure ($\Delta p_w$) generation can be expressed in terms of the increments of (maximum or) vertical ($\sigma_v$) and (minimum or) horizontal ($\sigma_h$) stresses and involving parameters $A_w B_w$, by (Skempton, 1954):

$$\Delta p_w = B_w [\Delta \sigma_h + A_w (\Delta \sigma_v - \Delta \sigma_h)] \tag{2.26}$$

The same relation in terms of increments of the isotropic total stress and deviatoric stress and involving parameters $\alpha_h$ and $\beta_h$ can be written as (Henkel, 1960):

$$\Delta p_w = \beta_h (\Delta \sigma_p + \alpha_h \Delta q) \tag{2.27}$$

Comparison of the Equations 2.26 and 2.27 leads to the following relations between both types of parameters, namely:

$$\alpha_h = A_w - 1/3; \beta_h = B_w \tag{2.28}$$

From Equation 2.11 it follows that in the undrained case the parameter $\beta_h$, defined as the ratio between of the pore water pressure increment to the isotropic total stress increment, is:

$$\beta_h = \frac{\Delta p_w}{\Delta \sigma_p} = \frac{1}{1 + n(C_w - C_p)/C_{sk}} \tag{2.29}$$

When the solid particle compressibility ($C_p$) and the pore water compressibility ($C_w$) are negligible, compared to the soil skeleton compressibility ($C_{sk}$), the parameter $\beta_h$ is about 1; an isotropic total stress increment results in an identical increment in pore water pressure and do not affect the effective stresses.

**Compressibility gas-water mixture** When gas is present in pore water the compressibility increases tremendously. Assuming that the solubility of gases in water is negligible (time scale of jet load is relatively short in comparison with that of gas diffusion), the compressibility of a gas-water mixture can be approximated by Fredlund (1976):

$$C_w = SC_{w0} + \frac{1 - S}{p_g} \tag{2.30}$$
in which $S$ is the degree of saturation ($V_w/V_p$), $C_{w0}$ is the compressibility of pure water and $p_g$ is the partial pressure of (free) gases inside the pore water. In equilibrium, the partial pressure interior a spherical gas bubble with a radius $R_{bu}$ is related to to the exterior liquid pressure ($p_w$) by, see also Section 4.2.1:

$$ p_g + p_{va} = p_w + \frac{2S_t}{R_{bu}} $$

(2.31)

where $p_{va}$ is the vapour pressure ($\approx 2.3$ kN/m$^2$) and $S_t$ is the surface tension of water ($\approx 0.075$ N/m). For bubble radii larger than about 20 $\mu$m the pressure interior ($p_g$) is about equal to the water pressure ($p_w$). In this case Equation 2.30 simplifies to Verruijt (2006):

$$ C_w = SC_{w0} + \frac{1 - S}{p_w} $$

(2.32)

Assuming a degree of saturation ($S$) of 95%, a pore water pressure ($p_w$) of 300 kN/m$^2$, a solid particle compressibility ($C_p$) of $0.02 \cdot 10^{-9}$ m$^2$/N and a soil skeleton compressibility ($C_{sk}$) of $500 \cdot 10^{-9}$ m$^2$/N, the pore water compressibility ($C_w$) is $1.7 \cdot 10^{-7}$ m$^2$/N and the corresponding value of $\beta_h$ becomes 0.87.

The parameter $\alpha_h$ depends largely on the initial state. The initial state can be denoted in the over consolidation ratio ($OCR$), defined as the ratio of the maximum isotropic effective pre-consolidation pressure ($\sigma_{p0,max} = \sigma_{pc,max}$) to the present isotropic effective consolidation pressure ($\sigma_{p0} = \sigma_{pc}$). For normally consolidated soils ($OCR = 1$), the pore water pressure will increase due to compaction, while for heavily overconsolidated soils ($OCR \geq 4$) the pore water pressure will decrease due to the tendency to dilate. This will be discussed in the Section 2.5. In Table 2.4 the ranges of $\alpha_h$ are listed for different initial states.

Table 2.4: Values of $\alpha_h$ for cohesive soils, converted from (Skempton, 1954).

<table>
<thead>
<tr>
<th>Initial State</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normally consolidated ($OCR = 1$)</td>
<td>0.15</td>
<td>0.65</td>
</tr>
<tr>
<td>Lightly overconsolidated ($OCR &lt; 4$)</td>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>Heavily overconsolidated ($OCR \geq 4$)</td>
<td>-0.8</td>
<td>0</td>
</tr>
</tbody>
</table>

**Quantities at failure** Substituting the Equations 2.23, 2.25 and 2.27 in 2.19 gives for the deviatoric stress increment to failure:

$$ \Delta q = \frac{6c' \cos \varphi + 6\sigma_{p0}' \sin \varphi - q_0(3 - \sin \varphi)}{3 - [1 + 6(1 - \beta_h)\zeta - 6\beta_h\alpha_h] \sin \varphi} $$

(2.33)


2. Cohesive soil

Substituting 2.33 in 2.23 gives for the deviatoric stress at failure:

\[
q = \frac{6c' \cos \varphi + 6\sigma_{p0}' \sin \varphi - 6(1 - \beta_h)\zeta - \beta_h\alpha_h\sigma_0}{3 - 1 + 6(1 - \beta_h)\zeta - 6\beta_h\alpha_h} \sin \varphi
\]  

(2.34)

For the isotropic effective stress \(\sigma_p'\) and the pore water pressure \(p_w\) at failure the following relations can be derived:

\[
\sigma_p' = \frac{2\sigma_{p0}' \sin \varphi + [1 - \beta_h)\zeta - \beta_h\alpha_h][6c' \cos \varphi - \sigma_0(3 - \sin \varphi)]}{3 - 1 + 6(1 - \beta_h)\zeta - 6\beta_h\alpha_h} \sin \varphi
\]  

(2.35)

\[
p_w = \sigma_{p0}' + \frac{\beta_h(\zeta + \alpha_h)[6c' \cos \varphi + 6\sigma_{p0}' \sin \varphi - \sigma_0(3 - \sin \varphi)]}{3 - 1 + 6(1 - \beta_h)\zeta - 6\beta_h\alpha_h} \sin \varphi
\]  

(2.36)

**Effective stress path, starting from** \(K_0\) **effective stress state**

The stress path in undrained triaxial compression, starting from a \(K_0\) effective stress state for a normally consolidated cohesive soils, is depicted in Figure 2.3 (a). The \(K_0\) initial effective stress state is expressed by the vertical \(\sigma_{v0}'\) and horizontal effective \(\sigma_{h0}'\) stresses, namely:

\[
\sigma_{h0}' = \sigma_{v0}'K_0
\]  

(2.37)

The corresponding isotropic effective and deviatoric stresses are:

\[
\sigma_{p0}' = \sigma_{v0}'(1 + 2K_0) ; q_0 = \sigma_{v0}'(1 - K_0)
\]  

(2.38)

in which \(K_0\) is the so-called coefficient of earth pressure at rest. For normally consolidated soils \(K_0\) can be estimated with the equation of Jaky (1944):

\[
K_0 \approx 1 - \sin \varphi
\]  

(2.39)

The \(K_0\) for overconsolidated cohesive soils is larger, because the horizontal stresses do not decrease in the same proportion as the vertical stresses during vertical unloading.

Substituting the initial stresses 2.38 in Equation 2.34 and neglecting the cohesion,\(^3\) gives for the corresponding undrained shear strength \((su)\) in terms of deviatoric stress:

\[
su = \frac{q}{2} = \frac{\sigma_{v0}'[2(1 + 2K_0) - 6(1 - K_0)(1 - \beta_h)\zeta - \beta_h\alpha_h]}{6 - [2 + 12(1 - \beta_h)\zeta - 12\beta_h\alpha_h]} \sin \varphi
\]  

(2.40)

\(^3\)For non-cemented cohesive soils, the cohesion \((c')\) is practically zero and can be neglected.
2.4. Strength in triaxial undrained compression

Effective stress path, starting from an isotropic effective stress state

In Figure 2.3 (c) the stress path in undrained triaxial compression, starting from an isotropic effective stress state for a normally consolidated cohesive soil (position B) is depicted. For undrained unloading from a $K_0$ effective stress state (position A) it may be assumed on the basis of elastic behaviour that the isotropic effective stress ($\sigma_{p0}'$) remain constant, while the initial deviatoric stress ($q_0$) reduces to zero.

$$\sigma_{p0}' = \frac{\sigma_{v0}'(1 + 2K_0)}{3}; \quad q_0 = 0 \quad (2.41)$$

For the calculation of the corresponding failure state, starting from isotropic effective stress state $\sigma_{p0}'$ in Figure 2.3 (c), these initial stresses could be substituted in Equation 2.34:

$$su = \frac{q}{2} = \frac{2\sigma_{v0}'(1 + 2K_0) \sin \varphi}{6 - [2 + 12(1 - \beta_h)\zeta - 12\beta_h\alpha_h] \sin \varphi} \quad (2.42)$$

The stress path followed is such as shown in Figure 2.3 (c): B → B’. In that case it would be assumed that during the complete effective stress path irreversible deformation would occur, while this may not be realistic for the part of the effective stress path passing the elastic domain, developed during previous unloading to the
2. Cohesive soil

isotropic stress state. It is more realistic to assume that the effective stress path followed is such as shown in Figure 2.3 (c): B → A → A’. In this case Equation 2.40 can be used to calculate the undrained shear strength ($su$).

Evaluation

In the Equations 2.40 and 2.42 the variable $\varphi$ is still unknown, this variable depends largely on the initial stress state and strain level. For normally consolidated cohesive soils the internal friction angle is about equal to the critical state friction angle and ranges globally from $20^\circ$ to $25^\circ$ (Mitchell and Soga, 2005). For heavily overconsolidated soils the maximum or peak internal friction angle ($\varphi_p$) is often higher than the critical state friction angle ($\varphi_{cs}$). Values up to $50^\circ$ have been reported (Kulhawy and Mayne, 1990). This will be discussed in the next section.

In Table 2.5 for different situations the calculated undrained shear strengths ($su$) normalized with the vertical effective consolidation pressure ($\sigma'_{v0}$) are listed.

Table 2.5: Calculated undrained shear strength ($su$) normalized with the vertical effective consolidation pressure ($\sigma'_{v0}$).

<table>
<thead>
<tr>
<th>Over Cons. Ratio (OCR)</th>
<th>[-]</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>$\geq$ 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree of saturation ($S$)</td>
<td>[-]</td>
<td>0.95</td>
<td>1</td>
<td>1</td>
<td>1-0.95</td>
</tr>
<tr>
<td>Parameter $\beta_h$</td>
<td>[-]</td>
<td>0.87</td>
<td>1</td>
<td>1</td>
<td>1-0.87</td>
</tr>
<tr>
<td>Parameter $\alpha_h$</td>
<td>[-]</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>Friction angle ($\varphi$)</td>
<td>[$^\circ$]</td>
<td>20-25</td>
<td>20-25</td>
<td>20-25</td>
<td>35</td>
</tr>
<tr>
<td>Equation number</td>
<td></td>
<td>2.40</td>
<td>2.40</td>
<td>2.42</td>
<td>2.40</td>
</tr>
<tr>
<td>$su/\sigma'_{v0}$</td>
<td>[-]</td>
<td>0.27-0.31</td>
<td>0.26-0.31</td>
<td>0.22-0.24</td>
<td>0.81-0.76</td>
</tr>
</tbody>
</table>

The difference in normalized undrained shear strength ($su/\sigma'_{v0}$) between a normally consolidated and a heavily overconsolidated sample is significant, due to the difference in excess pore pressure generation. For a normally consolidated cohesive soil sample the excess pore pressure generation will be positive due to compaction, whereas for a heavily overconsolidated sample this will negative due to the tendency to dilate.

The influence of the degree of saturation ($S$) on the normalized undrained shear strength ($su/\sigma'_{v0}$) is limited. For an unsaturated normally consolidated soil sample ($S = 95\%$) the normalized undrained shear strength is only a little higher than for a fully saturated sample, whereas for a not fully saturated heavily overconsolidated soil sample the normalized shear strength will be a little lower than for a fully saturated sample (see Table 2.5).
It can be concluded that the undrained shear strength will be underestimated, assuming that the deformations developed during unloading to the isotropic stress state are irreversible (Equation 2.42). For the normally consolidated situation given in Table 2.5, the difference in normalized shear strength is about 15%, between an elasto-plastic behaviour starting at the $K_0$-state as indicated by point A in Figure 2.3 (c) (Equation 2.40) and an elastic behaviour during unloading from the $K_0$-state following path A-B in Figure 2.3 (c) and elasto-plastic behaviour during reloading path B-B’ to failure (Equation 2.42).

The calculated values of the normalized undrained shear strength ($su/\sigma'_v$) correspond with those found in literature. Jamiolkowski et al. (1985) found for the ratio $su/\sigma'_v$ of normally consolidated cohesive soils a value of about 0.24, while Ladd (1991) found a value of 0.33. Wroth and Houlsby (1985) found that the normalized undrained shear strength for normally consolidated cohesive soils depends also on the plasticity, in the following way:

$$\frac{su}{\sigma'_v} = 0.129 + 0.00435 \cdot PI$$

For the normalized undrained shear strength of overconsolidated cohesive soils Jamiolkowski et al. (1985) found the following experimental relation:

$$\frac{su_p}{\sigma'_v} = \left(\frac{su}{\sigma'_v}\right)_{NC} OCR^{m_j}$$

where $(su/\sigma'_v)_{NC}$ is the strength ratio for normal consolidated cohesive soil and $m_j$ is a material constant with a value between 0.7 and 0.9.

### 2.5 Stress-strain behaviour

#### 2.5.1 Critical-state

When cohesive soil is deformed uniformly to large strains it reaches a stationary state, known as the critical state. Under drained conditions it holds that:

$$\frac{de}{d\varepsilon^d} = \frac{d\sigma'_p}{d\varepsilon^d} = \frac{dq}{d\varepsilon^d} = 0$$

And under undrained conditions:

$$\frac{dp_w}{d\varepsilon^d} = \frac{d\sigma'_p}{d\varepsilon^d} = \frac{dq}{d\varepsilon^d} = 0$$
in which $\varepsilon^d$ is the deviatoric strain. The basic concept of the critical state is that under sustained uniform shearing at failure, there exists a unique combination of void ratio ($e$), effective mean stress ($\sigma'_p$) and deviatoric stress ($q$). Projected onto the $\sigma'_p$-$q$ plane this gives a straight line, the critical state line (CS-Line), see Figure 2.5 (c). This line can be defined with Equation 2.19, by substituting the friction angle ($\phi$) with the critical state friction angle ($\phi_{cs}$). This angle ($\phi_{cs}$) ranges globally from $20^\circ$ to $25^\circ$ (Mitchell and Soga, 2005). Kenny (1959) established the following dependency between $\phi_{cs}$ and the plasticity index ($PI$):

$$\sin \varphi_{cs} = 0.8 - 0.094 \ln (PI) \quad (2.47)$$

On the $e - \log \sigma'_p$ plane the critical state line for cohesive soil tends to be linear and parallel to the normal compression line (NC-line), see Figure 2.4 (b):

$$\Delta e = -C_c \cdot \log \left( \frac{\sigma'_p + \Delta \sigma'_p}{\sigma'_p} \right) \quad (2.48)$$

where $C_c$ is the compression index of the soil.\(^4\) The position of the critical state line can globally be linked to the Atterberg limits. In terms of liquidity index, see Figure 2.4 (a):

$$LI_{cs} = \frac{w_{cs} - PL}{PI} = -\log \left( \frac{\sigma'_{p,PL}/\sigma'_p}{\sigma'_{p,LL}/\sigma'_{p,LL}} \right) \quad (2.49)$$

where $w_{cs}$ is the water content at critical state, $\sigma'_p$ is the effective mean stress at critical state, $\sigma'_{p,LL}$ and $\sigma'_{p,PL}$ are the mean effective stresses at liquid limit and plastic limit, respectively.

The stress-strain behaviour of cohesive soil depends largely on the initial state of the soil related to the CS-line on the $e - \log (\sigma'_p)$ plane and the drainage conditions, see Figure 2.4 (b).

When the state of the soil is above the CS-line it will contract under drained deviatoric deformation conditions, or will generate excess pore water pressures under undrained conditions. This is the case for normally and lightly overconsolidated cohesive soils with an over consolidation ratio ($OCR$) up to about 4 (Mitchell and Soga, 2005). When the state of the soil is below the CS-line ($OCR \geq 4$) it will dilate under drained deviatoric deformation conditions or will generate negative pore water pressures, thus pore water suction, under undrained conditions.

\(^4\)Note that the unload-reload compression or swelling line is less steep than the normal compression line, because the soil behaves stiffer, resulting in a lower unload-reload compression index or swelling index ($C_s$), see Figure 2.4 (b).
2.5. Stress-strain behaviour

2.5.2 Normally and slightly overconsolidated cohesive soil

First consider a normally consolidated soil sample, indicated by point A in Figure 2.5 (a). Note that point A is located on the loose, contractive side of the CS-line. When this sample, that is consolidated under isotropic pressure \( \sigma'_{p,cA} \), is subjected to an undrained triaxial compression test at constant mean total pressure, the effective stress path followed is such as shown in Figure 2.5 (c) \((A \rightarrow C)\). In case of uniform contractive deformation (compaction) the pore water stresses increase such, that the effective stress path curves to lower values of the mean effective stresses. The deviatoric stress increases till the stress path reaches the CS-line. The sample is said to be at the critical (failure) state.

In the case the soil has a flocculated open structure (metastable fabric), it is possible that the soil exhibits a peak shear stress. When such a soil is deformed, the fabric is disrupted and breaks down and the mobilized friction angle decreases, while excess pore water pressure increases towards failure. Also the process of aging, under a constant effective stress, can increase the rigidity of the structure with phenomena such as cementation and thixotropy and will result in a peak shear stress, which is a little higher than the critical state shear strength (Leroueil et al., 1990). See also Section 2.5.4.

For most cohesive soils the platy clay particles align along a localized failure plane after large shear displacements in the order of 30 to 200 mm (Skempton and Bishop,
Figure 2.5: Undrained soil behaviour for normally and heavily overconsolidated cohesive soils for triaxial compression at constant mean total pressure and uniform deformation: (a) void ratio \( (e) \) versus mean effective stress \( (\sigma'_p) \), (b) excess pore water pressure \( (\Delta p_w) \) versus deviatoric strain \( (\varepsilon^d) \), (c) deviatoric stress \( (q) \) versus mean effective stress \( (\sigma'_p) \), (d) deviatoric stress \( (q) \) versus deviatoric strain \( (\varepsilon^d) \).

This is called the *residual state*. As a result the internal friction angle and undrained shear strength decreases further, compared to the critical state, see Figures 2.5 (c) and (d). The internal friction angles at residual state \( (\varphi_r) \) for most common clay minerals ranges approximately from 15° for Kaolinite, to a value between 5° and 10° for Montmorillonite (Mitchell and Soga, 2005).
2.5.3 Heavily overconsolidated cohesive soil

Figure 2.5 (c) shows also the stress path for a heavily overconsolidated cohesive soil \((OCR > 4)\), subjected to an undrained triaxial compression test at constant mean total pressure \((B \rightarrow C)\). Heavily overconsolidated cohesive soils exhibit a stiff response initially until the stress state reaches the yield envelope. After reaching the yield envelope the state of the soil progressively moves toward the critical state, exhibiting softening behaviour due to a decrease in friction angle \((\varphi_p \rightarrow \varphi_{cs})\). At a certain strain level the soil reaches the critical state at which under sustained uniform shearing the pore water pressure and effective stresses remain constant.

The passage through the peak is generally associated with the formation of shear planes. Simultaneously, negative pore water stresses are generated due to the tendency to dilate. The height of the negative water pressures depends on the compressibility of the skeleton. The lower the compressibility, the higher the pore water under pressures. Because the compressibility of a granular (non-clay)-skeleton is significantly lower than a clay-dominated skeleton, the generated water under pressures in a granular-skeleton due to dilation are much higher than in a clay-dominated skeleton. The absolute magnitude of the water under pressures is limited. When the under pressures become less than the vapor pressure of the pore water, cavitation occurs (Winterwerp and van Kesteren, 2004). At locations where cavitation occurs, the soil dilates, like for drained behaviour. Because the pores in cohesive soils are commonly of colloidal size \((< 0.1 \text{ µm})\), progressive cavitation is suppressed and probably will not occur (Kesteren et al., 1992).

2.5.4 Remoulded state

When an undisturbed sample of cohesive soil is remoulded the undrained shear strength decreases. The ratio between the peak undisturbed strength \((s_{up})\) to the remoulded strength \((s_{urem})\) is called the sensitivity \((S_s)\). All cohesive soils exhibit some amount of sensitivity, varying from 1 to 8 for slightly sensitive to very sensitive, respectively. The main causes for the development of sensitivity are (Mitchell and Soga, 2005):

- **Metastable fabric** During consolidation the flake-like particles can flocculate and form an open unordered initial fabric (metastable state). The particles are arranged in edge-to-edge and edge-to-face associations. This fabric can carry effective stresses at a void ratio higher than would be possible if the particles were arranged in a parallel array (fully remoulded state). When such a soil is deformed, the fabric is disrupted and breaks down. As a result the internal friction angle decreases. The tendency for a volume decrease, also decreases the effective stresses. Both, the decrease in internal friction angle and effective stresses, cause a drop in strength.
2. Cohesive soil

- **Cementation** On disturbance the cemented bonds are destroyed leading to a loss of strength.

- **Leaching and changes in monovalent/divalent cation ratios** Reduction in salinity by leaching (decrease in cation concentration) increases the diffuse double layer thickness and as a result the inter-particle repulsion. The increase in inter-particle repulsion is responsible for deflocculation and dispersion of the soil on mechanical remoulding. Reduction in salinity by leaching is the main cause for the development of quick clay’s ($S_s > 8$).

At the remoulded state the undrained shear strength is correlated well with the liquidity index. Wroth and Wood (1978) found the following relation:

$$su_{rem} = 170e^{-4.6LI}$$

(2.50)

where $su_{rem}$ is the undrained shear strength at the remoulded state in kPa. Leroueil et al. (1983) proposed the following relation for values of the liquidity index between 0.4 and 3.0:

$$su_{rem} = \frac{1}{(LI - 0.21)^2}$$

(2.51)

### 2.5.5 Strain-rate dependent behaviour

At high strain-rates ($\dot{\varepsilon}$) the viscous resistance of the soil structure increases the undrained shear strength ($su$):

$$\frac{su}{su_{ref}} = 1 + n_{sr} \log \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}} \right)$$

(2.52)

where $su_{ref}$ is the reference undrained shear strength determined at a strain rate $\dot{\varepsilon}_{ref}$ and $n_{sr}$ is an empirical constant. Kulhawy and Mayne (1990) found for $n_{sr}$ a value of 0.1, based on 209 shear tests on 26 different cohesive soils. According to Dayal and Allen (1975) and Winterwerp and van Kesteren (2004) $n_{sr}$ depends on the undrained shear strength. The higher the $su$, the lower the strain rate dependency. Dayal and Allen (1975) found values for $n_{sr}$ between 0.03 ($su_{ref} = 80$ kN/m$^2$) and 0.25 ($su_{ref} = 9.5$ kN/m$^2$), based on cone penetration tests. According to Soga et al. (1996) the strain rate dependency of the undrained shear strength can be expressed as follows:

$$\frac{su}{su_{ref}} = \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}} \right)^{m_{sr}}$$

(2.53)

where $m_{sr}$ is a rate parameter with a value between 0.018 and 0.087. The corresponding values for $n_{sr}$ are 0.04 and 0.25, respectively ($1 < \dot{\varepsilon}/\dot{\varepsilon}_{ref} < 10.000$).
2.5. Stress-strain behaviour

These values correspond well with the values found by (Dayal and Allen, 1975). Figure 2.6 shows the strain rate dependency of the undrained shear strength. The data is obtained by constant rate of strain undrained triaxial tests. Globally the data can be approximated with the Equations 2.52 and 2.53. The values for $n_{sr}$ and $m_{sr}$ are 0.12 and 0.06, respectively.

![Figure 2.6: Strain rate dependency of the undrained shear strength.](image)

During a triaxial test, one or more shear bands are formed in the soil sample. The amount and thickness of these bands are usually unknown. Because of this, the actual strain-rates in the shear bands are also unknown.

Karlsson (1963) investigated the effect of the angular velocity ($v_\omega$) of a vane on the undrained shear strength, see Figure 2.7. Unfortunately Karlsson only used one vane diameter. The vane diameter was 15 mm, with a height of double the diameter. The results can be approximated with an relation similar to Equation 2.53:

$$\frac{su}{su_{ref}} = \left(\frac{v_\omega}{v_{\omega,ref}}\right)^{m_{sr,v}}$$

(2.54)

where $m_{sr,v}$ is an empirical rate parameter, with a value of about 0.1. Before localization of the deformations the thickness of the deformation band is more or less linearly proportional to the vane diameter. In this case Equation 2.54 is independent of the vane diameter. After localization of the deformations and the formation of shear bands, the thickness of the deformation zone becomes independent of the vane diameter. The localization of the deformations usually starts after reaching the peak shear strength. For granular materials like sands, the thickness of the shear bands is about $10D_{50}$ (Oda and Iwashita, 1999). For the shear band thickness in cohesive soils this relation is probably also valid. This should be further investigated. In this case the strain rate dependency of the
2. Cohesive soil

undrained shear strength can be predicted with the relative velocity over the shear plane \((v_\omega R_v)\):

\[
\frac{s_u}{s_{u,ref}} = \left(\frac{v_\omega R_v}{v_{\omega,ref} R_{v,ref}}\right)^{m_{sr,v}}
\] (2.55)

where \(R_v\) is the radius of the vane.

When jetting cohesive soil, it is assumed that the soil fails by the shearing of small discrete soil elements, see Figure 1.8. When the jet load on a soil element exceeds the resistance in the curved shear surfaces, the element starts to rotate. This failure mechanism is broadly similar with that during a vane test. In both cases the soil fails along a curved shear surface. It is assumed that the shear resistance in the curved shear surface after localization can be predicted with Equation 2.55, substituting the vane radius \((R_v)\) by the radius of the shear surface of the discrete soil element \((R_{de})\).

![Figure 2.7: Effect of angular velocity \((v_\omega)\) on the undrained shear strength, for lab-vane tests with a vane diameter of 15 mm (Karlsson, 1963).](image)

2.6 Soil failure mechanisms

In this section the occurrence of the following three possible failure mechanisms will be discussed:

- Fracturing (tensile cracking)
- Surface erosion
- Bulk erosion
2.6. Soil failure mechanisms

2.6.1 Fracturing

Micro-cracks are present in cohesive soils. When the micro-cracks coalescence, macro-cracks are formed and failure occurs, known as fracturing. Fracturing is caused by tensile stresses. These tensile stresses will be induced by the moving vertical jet at different locations in the soil, see Figure 2.8. This failure process was observed during the jet tests with very low traverse velocities, see Section 5.2. Van Kesteren has investigated this failure process extensively (Winterwerp and van Kesteren, 2004). His findings are briefly described below.

![Figure 2.8: Tensile stresses induced by the moving vertical jet, (a) behind a shearing soil element (b) behind a shearing soil strip (c) in the extension of a small crack present in the soil surface by an increase in static pressure.](image)

Micro-cracks are present in cohesive soils as discontinuities in the skeleton, for instance at the interfaces of large particles. Under tensile load around these cracks stress concentrations occur. Due to these stress concentrations the soil between the cracks can fail at stresses much smaller than the undrained shear strength (Au et al., 2003), (Winterwerp and van Kesteren, 2004), (Mitchell and Soga, 2005).

The forming of stress concentrations takes time. When a saturated soil sample is loaded with a tensile stress ($\Delta \sigma_t$), simultaneously an identical water pressure drop in the micro-cracks is generated, see Figure 2.9 (a) and (b). As a result initially no stress concentrations around the edges of the cracks occur. When the influence of dilation and compaction on the pore water pressure is neglected ($A_w = 1/3$), the pore water pressure surrounding the micro-cracks drops with $1/3\Delta \sigma_t$, see Figure 2.9 (c). The water pressure in the cracks is now lower, namely $\Delta p_w = \Delta \sigma_t$, than in the surrounding pores. Due to this water pressure difference pore water flows towards the cracks, the water pressure inside the cracks increases and subsequently, stress concentrations around the cracks will appear, see Figure 2.10 (a).

The time-scale of this drainage (consolidation) process is governed by the size of the cracks, the drainage length and the consolidation coefficient ($c_v$). The development of the water pressure inside a penny shaped crack ($p_{w,mc}$) as a function of time...
Figure 2.9: (a) Definition sketch of fracturing. (b) Water pressure in mini-crack immediately after tensile loading \((t = t_0)\). (c) Stresses in soil skeleton at \(t_0\).

can be predicted with the radial consolidation theory of Barron (1948):

\[
\frac{\Delta p_{w,mc}}{\Delta \sigma_t} = \frac{1}{3} + \frac{2}{3} e^{-\frac{\Delta \sigma_t}{T_{mc}}} \tag{2.56}
\]

in which the dimensionless time scale \(T_{mc}\) and the shape factor \(b_{mc}\) are defined as:

\[
T_{mc} = \frac{c_v t}{(2h_{rad})^2} \tag{2.57}
\]

\[
b_{mc} = \frac{n_{mc}^2}{n_{mc}^2 - 1} \ln (n_{mc}) - \frac{3n_{mc}^2 - 1}{4n_{mc}^2} \tag{2.58}
\]

\[
n_{mc} = \frac{h_{rad}}{R_{mc}} \tag{2.59}
\]

where \(h_{rad}\) is the drainage length and \(R_{mc}\) is the radius of a penny-shaped crack, see Figure 2.9 (a). Van Kesteren derived a similar solution for the case of a constant tensile loading rate \((\dot{\sigma}_t)\), see Winterwerp and van Kesteren (2004). When the water pressure evolution in the cracks is known, the critical stress concentrations can be calculated with the standard fracture mechanics. For the calculation
of the stress concentrations the critical fracture energy has to be known. Because
the radius of the cracks and the critical fracture energy are no standard soil pa-
rameters this theory is not yet useful in practice.

When 60% of the pore water pressure increment in the cracks has dissipated
\( \Delta p_{w,mc} / \Delta \sigma_t < 0.4 \), the stress concentrations around the cracks are almost
fully developed, see Figure 2.10 (b) and (c). It is assumed that stress con-
centrations can be neglected until 10% of the pore water pressure increment
has dissipated \( \Delta p_{w,mc} / \Delta \sigma_t > 0.9 \). This critical time after loading \((t_{10})\) can
be calculated with Equation 2.56. In Table 2.6 for different radial drainage
lengths the calculated values of \( t_{10} \) are listed, assuming a crack radius of 0.1 mm
(Winterwerp and van Kesteren, 2004) and a consolidation coefficient of \( 10^{-9} \text{ m}^2/\text{s} \).

\[
t = t_{60} \geq \frac{V}{\sqrt{V t_{60}^{2/3}}} \cdot \frac{\Delta \sigma}{\Delta \sigma_t}^{2/3}
\]

\[
\Delta p_w / \Delta \sigma_t
\]

\[
0.4 \Delta \sigma_t / \Delta \sigma
\]

\( t_{10} \quad t_{60} \quad t \)

\( (a) \quad (b) \quad (c) \)

Figure 2.10: (a) Effective soil stress and water pressure increment in the horizontal
plane through the mini-crack at \( t_{60} \). (b) Normalized water pressure development
in the mini-crack as a function of time. (c) Normalized effective soil stress concen-
tration at the edge of the mini-crack as a function of time.

The loading time of the jet is assumed to be in the order of \( D_n/v_t \), where \( D_n \)
is the nozzle diameter and \( v_t \) is the traverse velocity of the nozzle. Assuming
a commonly used nozzle diameter of 0.03 m, the corresponding critical traverse
velocity of the nozzle for which the stress concentrations can be neglected, can be
calculated. For lower traverse velocities of the nozzle fracturing can occur. The
results are also listed in Table 2.6.

At large tensile stresses it is also possible that in the cracks cavitation occurs and
enclosed cavities are formed which weaken the soil. Due to the colloidal size of
the pores progressive cavitation is suppressed (Kesteren et al., 1992). Therefore
in cohesive soil this phenomena will only occur at very high tensile stresses.
Table 2.6: Calculated critical loading time for which the stress concentrations are assumed to be negligible \( t_{10} \), and the corresponding critical traverse velocity of the nozzle \( v_t \), assuming a crack radius \( R_{mc} \) of 0.1 mm, a consolidation coefficient \( c_v \) of \( 10^{-9} \) m\(^2\)/s and a nozzle diameter of 0.03 m.

<table>
<thead>
<tr>
<th>( h_{rad} ) [mm]</th>
<th>( n_{mc} ) [-]</th>
<th>( t_{10} ) [s]</th>
<th>( v_t ) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>1.5</td>
<td>0.17</td>
<td>0.18</td>
</tr>
<tr>
<td>0.2</td>
<td>2</td>
<td>0.77</td>
<td>0.04</td>
</tr>
<tr>
<td>0.4</td>
<td>4</td>
<td>9.68</td>
<td>0.003</td>
</tr>
</tbody>
</table>

2.6.2 Water-submerged surface erosion

During water-submerged surface erosion continuously layers of sediment are removed at relatively small values of bed shear stresses \( \tau_b \ll su \). During this process the strength of the soil bed slowly (i.e. in a drained manner) decreases by swelling (reversed consolidation), until an erodible soil surface is obtained \( \tau_c < \tau_b \), where \( \tau_c \) is the critical shear stress for erosion at the soil surface). This process is briefly described below, from a soil mechanical perspective.

**Sedimentation** During water-submerged sedimentation of cohesive soils, clusters of cohesive sediment (flocs), having been transported by water flow, settle on the interface between the body of flowing water and the water-saturated soil layer underneath. Consequently each cluster of cohesive soil, together composing the soil layer with increasing thickness, is gradually experiencing increasing vertical effective stress \( \sigma_v' \) and related 1-dimensional vertical compressive strain \( \varepsilon_v \) and densification (virgin loading), while the horizontal effective stress \( \sigma_h' \) is gradually increasing proportionally with the vertical effective pressure: \( \sigma_h' = K_0 \sigma_v' \) (Jaky, 1944).

This sedimentation process is often sufficiently slow to avoid significant excess pore water pressure by allowing time for the excess pore water to be gradually drained to the soil surface. In Figure 2.11 (a) the depths below the submerged soil surface of three saturated soil clusters (I, II and III) are indicated at reference time \( t_0 \) by \( I_0 \), \( II_0 \) and \( III_0 \). The corresponding effective stress states are depicted in Figure 2.11 (a) located along the so-called \( K_0 \)-line.

**Surface erosion** During the inverse process of water-submerged surface erosion, starting from reference time \( t_0 \), the subsequent velocity profile of the water flow just above the soil surface, including effects of turbulence and surface roughness, exerts
2.6. Soil failure mechanisms

Figure 2.11: Illustration of processes of sedimentation and submerged surface erosion (a) levels of soil surface and soil clusters I, II and III, (b) effective stress states of soil clusters, (c) vertical strain of soil clusters.

A shear stress on this soil surface ($\tau_b$), which is able with increasing magnitude to initially unlock some individual clusters of cohesive sediment. Consequently due to this submerged erosion the soil clusters of this cohesive soil layer are experiencing decreasing vertical effective stress ($\sigma_v'$) and related 1-dimensional vertical swelling $\varepsilon_v$ if the process remains slow enough to avoid swelling-induced excess pore water suction by allowing time for pore water flow into the swelling soil volume.

In that case the horizontal effective stress ($\sigma_h'$) gradually decreases with the vertical effective stress ($\sigma_v'$). However, due to the initially mainly elastic soil behaviour during vertical 1-dimensional unloading, the coefficient of incremental decrease of lateral pressure for mainly elastic unloading ($K_e = \Delta \sigma_h' / \Delta \sigma_v'$) is initially much smaller than the coefficient of lateral effective pressure at rest ($K_0$) during initial loading.

When during continued slow surface erosion the saturated soil volume under consideration is approached by the soil surface, the decreasing horizontal effective
stress ($\sigma_h'$) becomes larger than the local decreasing vertical effective stress ($\sigma_v'$), while the soil behaviour changes from mainly elastic unloading towards elasto-visco-plastic passive compressive. For passive compression the horizontal effective stress can be expressed by $\sigma_h' = K_p \sigma_v'$, in which the coefficient of lateral passive pressure is expressed by:

$$K_p = \frac{1 + \sin(\varphi_{cs})}{1 - \sin(\varphi_{cs})}$$

Figure 2.11 (a) illustrates the levels of the submerged soil surfaces at two consecutive times, namely $t_1$ and $t_2$ during continuing submerged erosion, starting at reference time $t_0$. At time $t_1$ the submerged soil surface is approaching the level of soil cluster II while at time $t_2$ the level of deeper soil cluster III is approached. In Figure 2.11 (b) the effective stress states at time $t_1$ of soil clusters II and III are indicated as II$_1$ and III$_1$, while for time $t_2$ the effective stress state of soil cluster III is indicated by III$_2$. In Figure 2.11 (b) also the lines of hydrostatic pressure and passive pressure are shown. In Figure 2.11 (c) the corresponding vertical effective stress-strain relations are illustrated for initial virgin loading due to sedimentation and subsequent unloading due to submerged erosion. See also Figure 2.12 (a).

The top layer can fail under deviatoric deformation in a drained (B $\to$ C) or undrained (B $\to$ D) manner, see Figure 2.12. The critical bed shear stress ($q_{u,m}/2$) under undrained conditions will be higher than under drained conditions ($q_{d,m}/2$) because of the negative pore water pressures. Therefore, for relatively small values of bed shear stresses ($\tau_b \ll su$), the top layer will fail in a drained manner.

**Critical shear stress for surface erosion** Although a lot of research has been conducted on the surface erosion of cohesive soils, no general erosion model is presently available to quantify the erosion resistance and the erosion velocity ($v_e$), defined perpendicular to the soil surface. The difficulty is the large number of parameters which affect the erosion resistance of cohesive soils. Besides the standard soil properties (such as the undrained shear strength, plasticity, clay content and particle size distribution) also the effective stress state near the submerged horizontal soil surface, the chemical and electrochemical properties of the soil (representing the internal structure and inter-particle forces) and the water quality and temperature are of interest, see e.g. (Mostafa et al., 2008) and (Partheniades, 2009).

The surface erosion process is often expressed with the following equation (Partheniades, 1990) and (Whitehouse et al., 2000):

$$E = M_e(\tau_b - \tau_c), \text{ for } \tau_b > \tau_c$$

where $E$ is the erosion rate in $\text{kg/m}^2/\text{s}$ and $M_e$ is an empirical erosion rate parameter. $M_e$ may increase from $10^{-10}$ to $10^{-4}$ $\text{kg/N/s}$, respectively for (strongly)
2.6. Soil failure mechanisms

consolidated cohesive soils to loosely packed muds (Jacobs, 2011).

Different empirical correlations have been established for the critical shear stress. Smerdon and Beasley (1959) found a correlation between $\tau_c$ [N/m$^2$] and $PI$ [%], based on series of laboratory erosion tests on soft clays ($0.1 < su < 10$ kN/m$^2$):

$$\tau_c = 0.163P I^{0.84}$$  \hspace{1cm} (2.62)

A number of researchers established a correlation between $\tau_c$ [N/m$^2$] and the density of the soil ($\rho_s$ [kg/m$^3$]). The most common is (Whitehouse et al., 2000):

$$\tau_c = 0.015(\rho_s - 1000)^{0.73}$$  \hspace{1cm} (2.63)

Other less common correlations were found between $\tau_c$ and the water content ($w$) (Otsubo and Muraoka, 1988), and between $\tau_c$ and a combination of the liquidity index ($LI$) and the mean particle size ($D_{50}$) (Mostafa et al., 2008). Typical values of $\tau_c$ given in literature range from 0.1 till 10 N/m$^2$ (Jacobs, 2011). These values are three to four orders of magnitude smaller than the standard shear strengths.

Figure 2.12: Soil behaviour of the swelling top layer: (a) void ratio versus mean effective stress, (b) deviatoric stress versus mean effective stress, (c) deviatoric stress versus deviatoric strain.
Surface erosion velocity Winterwerp and van Kesteren (2004) proposed a method to predict the (surface) erosion velocity of an eroding bed. This method is further elaborated by Jacobs (2011). This method is briefly described below.

From the one-dimensional consolidation case of Terzaghi (1943) (Eq. 2.13) it follows that the rate of swelling is proportional to the ratio between the vertical swelling coefficient \( c_{v,sw} \) and the depth below the soil surface \( h_{sw} \):

\[
v_{sw} = \frac{dh_{sw}}{dt} \propto \frac{c_{v,sw}}{h_{sw}}
\]

in which \( v_{sw} \) is the propagation velocity of a swelling front at a depth \( h_{sw} \), defined as the depth at which the water pressure increment starts to dissipate, see Figure 2.13 (a). In the top (swelling) layer the shear strength is assumed to increase linearly from the shear strength at zero effective stress \( \tau_{dr} = c' \) at the soil surface to the original undrained shear strength \( su \) at the swelling front \( h_{sw} \), see Figure 2.13 (c). In this case a (turbulent) flow, inducing a bed shear stress \( \tau_b \), is able to erode a layer with a thickness of:

\[
h_e = h_{sw} \frac{\tau_b - \tau_{dr}}{su - \tau_{dr}} \approx h_{sw} \frac{\tau_b - \tau_{dr}}{su}
\]

Figure 2.13: Surface erosion in a turbulent flow: (a) definition of different parameters, (b) total and effective stress development in top soil layer, (c) strength gradient in the swelling layer.

According to Winterwerp and van Kesteren (2004) and Jacobs (2011) the erosion layer \( h_e \) has a minimum thickness. According to Winterwerp and van Kesteren
the minimum thickness of $h_e$ scales with the median diameter of the primary particles in the soil ($D_{50}$), corrected for interstitial water:

$$h_e \approx \frac{10D_{50}}{1 - n_0} \quad (2.66)$$

where $n_0$ is the situ-porosity of the top soil layer. Jacobs (2011) assumes that the thickness of the erosion layer is typically equal to the size of the flocs (particulate clusters of cohesive sediment, formed during the deposition process). For a consolidated bed the floc size and thus the minimum erosion layer can be approximated with the following equation:

$$h_e \approx D_{50}(1 - n_0)^{-2.85} \quad (2.67)$$

The swelling depth decreases with an increase in bed shear stress ($\tau_b$), assuming that for erosion the induced bed shear stresses are equal to the strength of the bed at $h_e$. The erosion velocity for a continuously eroding bed, defined perpendicular to the soil surface, must be equal to the swelling rate: $v_e = v_{sw}$, see Figure 2.13 (a). Substituting the Equations 2.65 and 2.66 into Equation 2.64, results in the following equation for the erosion velocity:

$$v_{sw} = v_e \propto c_{v,sw} \frac{c_v \phi_s}{h_{sw}} \frac{\tau_b - \tau_{dr}}{10D_{50} \rho_{dry}} \quad (2.68)$$

It follows that the erosion velocity ($v_e$) increases with the bed shear stress ($\tau_b$). In the critical case $\tau_b = \tau_{dr}$, the depth of the swelling front is infinite and as a result the erosion velocity will be almost zero, leading to drained soil behaviour. Therefore, the critical bed shear stress $\tau_c$ will be theoretically a little higher than $\tau_{dr}$.

From Equation 2.68 the following description of the erosion rate parameter ($M_e$, see Equation 2.61) can be derived, assuming $\tau_c = \tau_{dr}$:

$$M_e = \frac{c_{v,sw} \phi_s \rho_{dry}}{10D_{50} \rho_{dry}} \quad (2.69)$$

where $\rho_{dry}$ is the dry density of the soil. Theoretically, the maximum surface erosion velocity ($v_{e,max}$) occurs for $\tau_b = su$, for which $h_{sw} = h_e$:

$$v_{e,max} \propto \frac{c_{v,sw} \phi_s}{10D_{50}} \quad (2.70)$$

Assuming the following set of parameters: $c_{v,sw} = 10^{-8}$ m$^2$/s, $D_{50} = 5$ $\mu$m and $n_0 = 0.42$ ($\rho_b = 1950$ kg/m$^3$), the maximum erosion velocity is about 0.1 mm/s. For $\tau_b > su$, the erosion process transfers into an undrained failure process.
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**Surface erosion in the case of a moving vertical jet** In the case of a moving vertical jet it is possible that the traverse velocity of the jet \( (v_t) \) is much larger than the theoretically surface erosion velocity \( (v_e) \). In this case the soil will not fail by the induced bed shear stresses. The soil has to fail in an undrained manner by the stagnation pressure of the jet \( (p_{stag}) \) exerted at the original soil surface, see Figure 2.14. Winterwerp and van Kesteren (2004) state that for a fully undrained process the Péclet number \( (Pe = v_t h_{sw}/c_{v,sw}) \) must be greater than 10. When the traverse velocity of the jet is about the erosion velocity \( (v_t \approx v_e = v_{sw}) \) and the bed shear stresses are smaller than the undrained shear strength of the soil, the process is assumed to be fully drained. Winterwerp and van Kesteren (2004) state that for a drained process the Péclet number must be \( < 1 \).

![Figure 2.14: Definition sketch of surface erosion in the case of a moving jet.](image)

The traverse velocity of a TSHD is typical somewhere between 0.5 and 1.5 m/s. For jet trenchers the traverse velocities are usually much lower, somewhere between 0.03 and 0.15 m/s. These traverse velocities are orders higher than the maximum erosion velocity \( (v_{e,max}) \) which is about 0.1 mm/s (Equation 2.70). It can be concluded that the failure process of cohesive soil during the jetting process is fully undrained. As a result surface erosion rarely occurs.

On the other hand, in non-cohesive soils surface erosion is the main failure mechanism during the jetting process. Because the water permeability of non-cohesive soils is significantly higher than that of cohesive soils, the erosion process is commonly drained.

### 2.6.3 Bulk (or mass) erosion

For fast erosion of cohesive soils, the available time is insufficient for enabling water to be gradually sucked into the swelling soil volume. The soil volume re-
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mains restraint by the pore water, delaying the occurrence of volume increase and corresponding decrease of mean effective pressure (undrained conditions). Consequently, with the mean effective stress being maintained by the pore water suction also the effective strength remains unchanged, thereby improving the resistance against surface erosion.

In the undrained case shear planes/surfaces are formed in the soil. The shear stresses along these shear surfaces are equal to the undrained shear strength ($s u$), see Figure 2.15. When a (jet) load exceeds the resistance in these shear surfaces, the soil will fail, assuming that tensile cracking (see Section 2.6.1 and cavitation (see also Section 2.5.3)\(^5\) can be excluded. For a vertical load on a flat horizontal soil surface the shear resistance is called the bearing capacity ($q_{bc}$), which can be calculated with the Bearing Capacity Theory, see Appendix A. The bearing capacity for a circular load at the soil surface is:

$$q_{bc} = N_{bc} su \approx 6.2 su$$  \hspace{1cm} (2.71)

where $N_{bc}$ is the bearing capacity factor. When the soil surface is not completely flat the bearing capacity factor will decrease. Theoretically $N_{bc}$ can decrease to a value of 2.

The normal stresses induced by a turbulent water flow, scale with the stagnation pressure of that flow ($p_{stag}$), defined as:

$$p_{stag} = \frac{\rho_m u^2}{2}$$  \hspace{1cm} (2.72)

where $\rho_m$ is the density of the jet flow and $u$ is the mean velocity in the jet. Therefore the criterion for undrained soil failure can be written as:

$$p_{stag} > 2 \sim 6.2 su$$  \hspace{1cm} (2.73)

During bulk erosion, discontinuously lumps are ruptured from the soil surface, induced by a turbulent flow parallel to the soil surface. Winterwerp and van Kesteren (2004) suggest for the threshold of bulk erosion a bearing capacity factor ($N_{bc}$) between 2 and 5. These values correspond well with the theoretical ones.

A moving vertical jet exerts continuously a pressure on the soil surface, equal to the stagnation pressure, see Figure 2.14. As a result more continuously lumps of soil will be ruptured (hereafter referred as the shearing of small discrete soil elements). This failure process will be elaborated more in detail in the next chapters.

\(^5\)For heavily overconsolidated soils, simultaneously with the forming of shear planes, negative pore water pressures are generated due to the tendency to dilate. These negative pore water pressures can cause cavitation, when the under pressures are large enough. Then the soil will dilate, like for drained behaviour.
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![Figure 2.15: Definition sketch of bulk erosion induced by a turbulent flow.](image)

**Non-cohesive soil** In non-cohesive soils also an undrained erosion process can occur. The water permeability of a non-cohesive soil can decrease significantly by the presence of fines.\(^6\) As a result, also in fine grained sands, like silts, in combination with a high traverse velocity a similar failure process can occur as in cohesive soil. In this case the apparent cohesion of a non-cohesive soil is equal to the generated water under pressure in the shear plane times the tangent of the internal friction angle.

In fine grained sands it is also possible that, as a result of high exerted shear stresses in combination with a low water permeability, the absolute water pressure behind the grains drops to the vapour pressure. The water will cavitate and the grains can be removed more easily.

### 2.7 Conclusions

The main conclusions of this chapter are:

1. The jetting process generates an undrained response in cohesive soil, because of the low water permeability and relative high skeleton compressibility. This makes it possible to use the undrained total stress analysis and characterize the soil with the undrained shear strength \((su)\) only.

2. For heavily overconsolidated soils \((OCR > 4)\), and to a lesser extent for normally consolidated cohesive soils with a flocculated open structure, the peak shear strength is higher than the shear strength at the critical state. After reaching the peak shear strength, under sustained shearing the soil exhibits softening behaviour due to dilation and the formation of shear bands, leading to a decrease in internal friction angle.

\(^6\)The permeability of saturated sands can be predicted with the well known Hazen-equation:
\[ k_w = 100D_{10}^2 \] (Hazen, 1892), where \(k_w\) is the permeability in [cm/s] and \(D_{10}\) is the grain size in [cm] for which 10% is smaller.
3. Under sustained uniform shearing and undrained conditions the soil reaches a (stationary) critical state at which the pore water pressure and effective stresses remain constant. For normally and slightly overconsolidated soils \((OCR < 4)\) the generated pore water pressures at the critical state are positive due to compaction. For heavily overconsolidated soils \((OCR > 4)\) the generated pore water pressures are negative due to dilatation.

4. The undrained peak shear strength of cohesive soils also depends on the strain-rate. The higher the strain-rate, the higher the viscous resistance and the higher the undrained shear strength.

5. For fracturing micro-cracks, present in cohesive soils, have to coalesce. Under tensile loading around these micro-cracks stress concentrations occur. Due to these stress concentrations the soil between the micro-cracks can fail at stresses much lower than the undrained shear strength. The forming of these stress concentrations takes time. When the loading time of the jet is short enough the stress concentration can be neglected and no fracturing occurs. The critical loading time depends on the consolidation coefficient \((c_v)\), the drainage length \((h_{rad})\) and the characteristic radius of the micro-cracks \((R_{mc})\). The loading time of the jet is only large enough to cause fracturing at very low traverse velocities \((v_t)\).

6. Two mechanisms of erosion can be distinguished: surface erosion and bulk erosion.

   Surface erosion is a drained erosion process, at which continuously layers of sediment are removed at relatively small values of bed shear stresses \((\tau_b \ll su)\). During this process the strength of the soil bed slowly (i.e. in a drained manner) decreases by swelling, until an erodible soil surface is obtained \((\tau_b > \tau_c)\), where \(\tau_c\) is the critical shear stress for erosion at the soil surface). Because the traverse velocity of the jet is much larger than the propagation velocity of the swelling front, this mechanism will not occur during the jetting process.

   Bulk erosion is an undrained process, at which discontinuously lumps are ruptured from the soil surface by shearing. For bulk erosion the turbulent normal pressures have to overcome the resistance in the shear surfaces \((\sigma_n \approx 2 \cdot su)\). A jet exerts continuously a normal pressure on the soil surface. As a result more continuously soil lumps will be ruptured, but the failure mechanism is in essence the same.
2. Cohesive soil
Chapter 3
Velocity development in turbulent jets

3.1 Introduction

The moving vertical jet exerts a normal pressure on the cohesive soil, equal to the stagnation pressure \( p_{stag} \). This pressure will form shear surfaces in the soil. The soil will fail along these surfaces, when the stagnation pressure exceeds the resistance in the shear surfaces. In contrast to a non-cohesive soil where the bed shear stresses, exerted by jet, are mainly responsible for the erosion of individual soil particles.

The stagnation pressure is a function of the axial jet velocity. Assuming the principle of Bernoulli the stagnation pressure can be calculated with the following equation:

\[
p_{stag} = \frac{\rho_m u_u^2}{2}
\]  

where \( \rho_m \) is the mixture density of the jet and \( u_u \) is a characteristic jet velocity. In the present study \( u_u \) is assumed to be the uniform flow velocity, defined as:

\[
u_u = \frac{I}{\rho_m Q}
\]  

where \( I \) is the momentum flux and \( Q \) is the jet flow rate. The uniform flow velocity is a theoretical velocity, to simplify the problem. In reality always a velocity profile exists. The uniform flow velocity decreases with distance to the nozzle \((s)\), because of the entrainment of ambient fluid and soil in the jet.

The nozzle of the vertical jet is positioned a certain distance above the original soil surface, called the stand off distance \((SOD)\). In the region between nozzle
and soil surface the jet is fully surrounded by water (free jet). After penetrating the original soil surface, the jet is partly enclosed by soil and can be considered as a confined jet, see Figure 3.1 (a). In the special case where the jet flow disperses in radial direction, the jet can be considered as a radial wall jet, see Figure 3.1 (b). The behavior of these different types of turbulent jets, in particular the jet flow velocity development with distance, will be discussed in this chapter.

![Figure 3.1: Sketch of confined jet (a) and radial wall jet (b).](image)

### 3.2 Free jet

Figure 3.2 shows a schematic depiction of the free jet. At the interface between jet and ambient water a mixing (shear) layer is formed as a result of the velocity difference and the water viscosity. In the mixing layer transfer of mass and momentum takes place. Ambient water is accelerated and entrained in the jet. The development of this mixing layer starts immediately after leaving the nozzle.

The jet velocity is usually assumed to be uniform at the nozzle exit ($u_0$). Because of the development of the mixing layer, the region of unhindered velocity becomes smaller with distance. This region is called the potential core. When the turbulence of the mixing layer has penetrated to the axis, the mixing layer is fully developed and the potential core has disappeared. The region where the potential core exists, is called the flow development region. The region where the mixing layer is fully developed is called the region of fully developed flow.

#### 3.2.1 Region of fully developed flow

In the region of fully developed flow, the velocity profile of a turbulent circular free jet is Gaussian, see Figure 3.2. This results in the following equation for the
velocity development in this region:

\[ u_j(s, r) = \sqrt{\frac{k_1}{2} u_0} \frac{D_n}{s} e^{k_2 \frac{r^2}{s^2}} \]  

(3.3)

where \( u_j(s, r) \) is the jet velocity at a (axial) distance \( s \) and a radial distance \( r \), \( u_0 \) is the initial jet velocity at nozzle exit, \( D_n \) is the nozzle diameter and \( k_1 \) and \( k_2 \) are empirical constants. For these constants different values can be found in literature. Fischer et al. (1979) found for \( k_1 \) an average value of 77 ± 2 and for \( k_2 \) an average value of 87.3 ± 5, based on 13 experimental investigations. Fundamentally, the turbulent jet is momentum driven. The momentum flux \( (I) \) on any cross section of the jet flow is conserved (Rajaratnam, 1976):

\[ I(s) = \rho_w 2\pi \int_0^{\infty} u_j(s, r)^2 r \cdot dr = I_0 \]  

(3.4)

where \( I_0 \) and \( Q_0 \) are the momentum flux and flow rate at nozzle exit, respectively:

\[ I_0 = \rho_w Q_0 u_0 \]  

(3.5)

\[ Q_0 = \mu_n \frac{1}{4} \pi D_n^2 u_0 \]  

(3.6)

in which \( \mu_n \) is the nozzle discharge coefficient. This coefficient depends on the nozzle inflow and geometry, and varies globally between 0.6, for a bad geometry, and 1 (White, 2009). In this chapter \( \mu_n \) is assumed to be 1 and will be omitted. By substituting the equations 3.3, 3.5 and 3.6 into Equation 3.4 and integrating,
3. Velocity development in turbulent jets

the momentum flux can be calculated. It follows that, assuming $\mu_n = 1$, $k_1$ and $k_2$ must be equal for preservation of momentum. Therefore, in the present study for $k_1$ and $k_2$ a value of 77 is used, analogue to Albertson et al. (1950): $k_1 = k_2 = k = 77$.

The region of fully developed flow starts when the jet velocity in the centerline equals the initial jet velocity: $u_j(s = s_{dr}, 0) = u_0$. It follows that the region of fully developed flow starts at a distance of:

$$s_{dr} = \sqrt{k_1/2} \cdot D_n \approx 6.2D_n$$ (3.7)

The flow rate in the region of fully developed flow as a function of distance is obtained by integrating the velocity profile (Equation 3.3):

$$Q(s) = Q_0 \sqrt{\frac{8}{k} \cdot \frac{s}{D_n}} \approx 0.32Q_0 \frac{s}{D_n}, \text{ for } s \geq s_{dr}$$ (3.8)

The flow rate is found to linearly increase with distance. Differentiating Equation 3.8 results in the equation for the entrainment per unit length:

$$\frac{dQ}{ds} = \alpha_{mom} \pi D_n u_0 \approx 0.081\pi D_n u_0, \text{ for } s \geq s_{dr}$$ (3.9)

where $\alpha_{mom}$ is the entrainment coefficient, defined as:

$$\alpha_{mom} = \frac{1}{\sqrt{2k}} \approx 0.081, \text{ for } s \geq s_{dr}$$ (3.10)

It follows that the entrainment coefficient is constant in the region of fully developed flow.

Similar to the circular free jet, the entrainment coefficients for other types of jets can be derived based on empirical equations for the jet velocity development, see Appendix B. It follows that for a plane or rectangular free jet and a smooth wall jet the entrainment coefficient in the region of fully developed flow is almost the same as for the circular free jet. For these jets $\alpha_{mom} = 0.085$. Given the variation in the empirical constants, the difference in $\alpha_{mom}$ can be neglected.

The values of 0.081 and 0.085 are only valid in the non-cavitating case. When the jet cavitates the entrainment coefficient decreases. The values of $\alpha_{mom}$ for the cavitating jet will be derived in Chapter 4.

3.2.2 Flow development region

In the flow development region the mixing layer is not yet fully developed. Therefore the entrainment coefficient is not constant and increases with distance.
Albertson et al. (1950) derived an equation for the flow rate in the flow development region of a free circular jet ($Q_{dr}$). He assumed a half Gaussian velocity distribution in the shear zone, see Figure 3.2:

$$Q_{dr}(s) = Q_0 \left[1 + f_1 \frac{s}{D_n} + f_2 \left(\frac{s}{D_n}\right)^2\right]$$

(3.11)

$$f_1 = 2\sqrt{\frac{\pi}{k}} - \sqrt{\frac{8}{k}} \approx 0.081$$

$$f_2 = \frac{6 - \sqrt{8\pi}}{k} \approx 0.013$$

In Figure 3.3 the normalized flow rate according to Albertson is plotted as a function of normalized distance. Differentiating Equation 3.11 results in the equation for the entrainment in the flow development region:

$$\frac{dQ_{dr}}{ds} = \alpha_{mom,dr} \pi D_n u_0$$

(3.12)

$$\alpha_{mom,dr} = \left(\frac{1}{4} f_1 + \frac{1}{2} f_2 \frac{s}{D_n}\right)$$

At the transition to the fully developed flow region ($s = s_{dr} = \sqrt{k/2D_n}$) the entrainment coefficient is discontinuous; $\alpha_{mom}$ increases abruptly from 0.06 to 0.081, see Figure 3.4. This discontinuity is counter intuitive.

Lee et al. (2011) proposed an empirical relation for the velocity profile in the flow development region. However this relation was not physically substantiated.

Because an unambiguous description of the flow rate in the flow development region is missing, in the present study this is described by a quartic polynomial transfer function, see Figure 3.3 (1D-approach).

$$Q(s) = a_1 s^4 + b_1 s^3 + c_1 s^2 + d_1 s + e_1, \text{ for } s < s_{dr}$$

(3.13)

$$a_1 = \frac{Q_0 - \frac{2}{3} \alpha_{mom} \pi D_n u_0 s_{dr}}{\frac{1}{3} s_{dr}^4}$$

$$b_1 = \frac{\alpha_{mom} \pi D_n u_0 - 8 a_1 s_{dr}^3}{3 s_{dr}^2}$$

$$c_1 = -6 a_1 s_{dr}^2 - 3 b_1 s_{dr}, \quad d_1 = 0, \quad e_1 = Q_0$$
3. Velocity development in turbulent jets

Figure 3.3: Normalized flow rate of the circular free jet as a function of the normalized distance.

Figure 3.4: Entrainment coefficient in the flow development region of the circular free jet.

This transfer function is based on the following conditions:

\[
\begin{align*}
    s = 0 : & \quad Q_{dr} = Q_0, \quad \frac{dQ_{dr}}{ds} = 0 \\
    s = s_{dr} : & \quad Q_{dr} = Q(s_{dr}) = 2Q_0, \quad \frac{dQ_{dr}}{ds} = \frac{dQ}{ds} = \alpha_{mom}\pi D_n u_0 \\
    & \quad \frac{d^2 Q_{dr}}{ds^2} = \frac{d^2 Q}{ds^2} = 0
\end{align*}
\]

The entrainment coefficient can be derived by dividing the derivative of the flow rate by \( \pi D_n u_0 \), see Figure 3.4. This definition of the entrainment coefficient is used in the 1D-approach of the jet, see Section 3.2.4.

A simple estimate of the flow rate in the flow development region can be made, assuming that the entrainment coefficient in this region is half the entrainment coefficient in the region of fully developed flow: \( \alpha_{mom,dr} = 1/2\alpha_{mom} \). This definition is used in the analytical approach of the jet, see Section 3.2.4. In this approach the entrainment per unit length immediately after leaving the nozzle is overestimated until halfway the flow development region. Beyond this point the entrainment per unit length is underestimated. The flow rate development, based on a constant entrainment coefficient, is also plotted in Figure 3.3.
3.2. Free jet

3.2.3 Modeling of the mixing layer

It is assumed that the entrainment of a turbulent jet can be calculated with the following equation:

\[
\frac{dQ}{ds} = \alpha_{mom} P_w(s) \Delta u(s)
\]  

(3.14)

where \(P_w\) is the perimeter of the interface between jet and ambient water and \(\Delta u\) is the axial jet velocity difference at the interface between jet and ambient fluid. The difficulty is to define \(P_w\) and \(\Delta u\), because of the Gaussian velocity profile. In the present study \(\Delta u\) is assumed to be the fictitious uniform flow velocity \((u_u, \text{see Equation 3.2})\). Based on this uniform velocity also a fictitious interface (perimeter) can be defined. These assumptions make it also possible to calculate the entrainment for the confined jet.

For a submerged circular free water jet, the flow density is constant and equal to the water density; \(\rho_m(s) = \rho_w\). Substituting the equations 3.5 and 3.8 into Equation 3.2 results in the following equation for the uniform jet velocity development in a fully developed circular free jet \((s > s_{dr})\):

\[
u_u(s) = \frac{1}{2} \sqrt{k u_0} \frac{D_n}{s} = \frac{1}{2} u_j(s, 0)
\]  

(3.15)

Based on this uniform velocity, the fictitious jet diameter is:

\[
D_j(s) = \sqrt{\frac{4 Q(s)}{\pi u_u(s)}} = \sqrt{\frac{8}{k} s}
\]  

(3.16)

With this equation also the perimeter of the jet interface can be calculated. This results in the following equation for the entrainment per unit length of a submerged circular free jet:

\[
\frac{dQ}{ds} = \alpha_{mom} \pi D_j(s) u_u(s) = \alpha_{mom} \pi D_n u_0
\]  

(3.17)

This equation is the same as the derivative of the flow rate (Equation 3.9). It is concluded that the entrainment can be estimated by multiplying the entrainment coefficient with the uniform flow velocity and the surface of the fictitious interface between jet and ambient water.

3.2.4 Modeling free jet

The circular free jet can be described with two first order differential equations, with the uniform flow velocity \((u_u)\) and the fictitious jet radius \((r_u = D_j/2)\) as
3. Velocity development in turbulent jets

variables. These differential equations can be derived from the continuity equation for water and the conservation equation for the momentum flux:

\[
\frac{dQ}{ds} = \alpha_{mom} 2\pi r_u u_u \tag{3.18}
\]

\[
\frac{dI}{ds} = \frac{d(\rho_w Q u_u)}{ds} = 0 \tag{3.19}
\]

Substituting the one into the other and rewriting these equations, results in:

\[
\frac{du_u}{ds} = -\frac{dQ}{ds} \frac{1}{\pi r_u^2} = -2\alpha_{mom} \frac{u_u}{r_u} \tag{3.20}
\]

\[
\frac{dr_u}{ds} = \frac{du_u}{ds} \frac{r_u}{u_u} = \frac{dQ}{ds} \frac{1}{\pi r_u u_u} = 2\alpha_{mom} \tag{3.21}
\]

This system can be solved analytically when the entrainment coefficient is assumed to be constant. The boundary conditions are: \(u_u(0) = u_0\) and \(r_u(0) = 1/2D_n\). Assuming that the entrainment coefficient in the flow development region is half the entrainment coefficient in the region of fully developed flow, the following equations for the uniform jet velocity and fictitious jet radius can be derived (analytical approach):

\[
\text{for } s < s_{dr}: \quad u_u(s) = u_0 \frac{D_n \sqrt{k/2}}{s + D_n \sqrt{k/2}} \tag{3.22}
\]

\[
r_u(s) = \frac{1}{\sqrt{2k}}s + \frac{1}{2}D_n \tag{3.23}
\]

\[
\text{for } s \geq s_{dr}: \quad u_u(s) = \frac{1}{2} \sqrt{\frac{k}{2}} u_0 \frac{D_n}{s} \tag{3.24}
\]

\[
r_u(s) = \sqrt{\frac{2}{k}}s \tag{3.25}
\]

Assuming a gradually increasing entrainment coefficient in the region of fully developed flow, according to Figure 3.4 (transfer function), the system of differential equations must be solved numerically by discretizing the jet in this region. In the 1D-approach the jet is divided into infinitesimal thin slices in axial direction, see Appendix C. For each slice a mass and momentum balance is solved, separately.

The normalized uniform jet velocity development, according to the analytical and 1D-approach, are plotted in Figure 3.5. In the analytical approach the entrainment coefficient is initially larger than in the 1D-approach, see Figure 3.4. Therefore, the uniform jet velocity is initially lower in the analytical approach. In the
3.3 Confined jet

region of fully developed flow the velocity development is the same for both approaches. As a reference also the maximum velocity in the center of the jet is plotted ($u_{max} = u(s,0)$). Note that at the end of the flow development region, the uniform flow velocity is already decreased with 50%, whereas in the center of the jet the maximum flow velocity is still the initial jet velocity.

In Figure 3.6 the normalized fictitious jet radii, according to the analytical approach (equations 3.23 and 3.25) and the 1D-approach, are plotted as a function of the normalized jet distance. The fictitious jet radius is initially larger in the analytical approach, because the entrainment coefficient is larger than in the 1D-approach. In the region of fully developed flow the fictitious jet radius is the same for both approaches and can be calculated with Equation 3.16 ($r_u = D_j/2$).

The radius of the circular free jet is often characterized with parameter $b_j$, which is defined as the radial distance at which $u(s,b_j) = 1/e \cdot u(s,0)$, see e.g. (Fischer et al., 1979)$^1$. The radius $b_j$ can be calculated using Equation 3.3. As a reference the normalized values of $b_j$ are also plotted in Figure 3.6.

Figure 3.5: Uniform velocity development of a circular free jet, based on a constant (analytical approach) and a gradually increasing (1D-approach) entrainment coefficient in the flow development region.

Figure 3.6: Fictitious normalized jet radius, based on an uniform velocity profile, as a function of the normalized distance.

3.3 Confined jet

The moving penetrating jet is partly enclosed by soil, see Figure 3.7 (a). In this case the development of the mixing layer and thus the entrainment of ambient

$^1$The parameter $b_j$ is also defined as the radial distance at which $u_j(s,b_j) = 1/2 \cdot u_j(s,0)$, see e.g. Rajaratnam (1976).
water is limited to the backside of the jet. It is assumed that the behavior of the mixing layer of the moving penetrating jet is largely comparable with that of the free jets. The length of the flow development region and the development of the entrainment coefficient in this region are assumed to be the same.

At the front side of the jet, soil is entrained. This increases both the jet flow rate and the jet flow density. The entrainment of soil per unit length depends mainly on the traverse velocity of the jet ($v_t$) and the cavity width ($W_c$). At the interface between jet and soil a boundary layer is formed, see Figure 3.7 (b) and (c). In this boundary layer the no-slip condition gives rise to a large velocity gradient and associated shear stresses parallel to the wall. The boundary layer thickness increases, while the main jet velocity decreases with distance. As a result the velocity gradient and associated wall shear stresses decrease with distance. The wall shear stresses decreases the jet momentum flux and as a result slows down the jet.

In cohesive soil the cavity width ($W_c$) is more or less constant (unlike the cavity width in sand which increases with distance by erosion). Therefore the cross section of the jet can be schematized by a rounded rectangle with a variable
3.3. Confined jet

length \((L_j)\) and a constant width \((W_c)\), see Figure 3.8. The length of the jet cross section \((L_j)\) is called the jet thickness. The cavity width can be approximated by the fictitious jet diameter at the original soil surface: \(W_c \approx 2r_u(SOD)\), see also Section 7.2.1.

![Figure 3.8: Definition sketch moving penetrating jet (cross section perpendicular to the jet direction).](image)

3.3.1 Calculation of bed shear stresses

**Boundary layer development in confined jet**  At the interface between jet and soil a boundary layer is formed. The development of the boundary layer and the exerted bed shear stresses are discussed in Appendix D. It is concluded that the boundary layer development in the jet is comparable to the development of the boundary layer in an uniform flow on a rough plate. The influence of the decelerating flow and the entrainment of soil is not included. These influences are assumed to be negligible.

The bed shear stresses can be calculated with the following equation:

\[
\tau_b = c_f \frac{1}{2} \rho_m u_{bl,max}^2
\]  \hspace{1cm} (3.26)

where \(c_f\) is the friction coefficient and \(u_{bl,max}\) is the jet velocity at the edge of the boundary layer.

The flow in the boundary layer starts laminar and becomes turbulent after a certain distance. For highly turbulent flows it is assumed that the flow becomes turbulent at a local Reynolds number of about \(5 \cdot 10^5\) (White, 2009). In the laminar zone the friction coefficient is assumed to be constant and equal to the

\footnote{The local Reynolds number is defined as: \(Re_s = s_{bl}u_0/\nu\), where \(s_{bl}\) is the distance to the initiation of the boundary layer and \(\nu\) is the kinematic viscosity of the fluid. Therefore the turbulent zone starts when \(s_{bl}\) becomes \(5 \cdot 10^5\nu/u_0\).}
3. Velocity development in turbulent jets

(maximum) value at the beginning of the turbulent zone. In the turbulent zone the friction coefficient is (Schlichting, 1979):

$$c_f = \left(2.87 + 1.58 \log \frac{s_{bl}}{k_s}\right)^{-2.5}$$  (3.27)

where $k_s$ is the equivalent roughness height and $s_{bl}$ is the distance to the initiation of the boundary layer. The equivalent roughness height is assumed to be in the order of $5D_{50} - 1D_{90} = 25 - 110 \mu m$, see Appendix D.4.1. The minimum friction coefficient occurs when the boundary layer is fully developed. The boundary layer is assumed to be fully developed when the boundary layer height ($\delta$) is about half the cavity width: $\delta = 1/2W_c$. The boundary layer height can be estimated with the following equation, Appendix D.2:

$$\delta = \frac{k_s}{30} e^{\kappa/\sqrt{1/2c_f}}$$  (3.28)

where $\kappa$ is the Kármán constant ($\approx 0.41$).

To give an indication of the maximum values of the friction coefficients, these are calculated for the ultimate settings of the exploratory jet tests, described in Section 5.1. The tested initial jet velocities ($u_0$) range from 28.6 to 176.6 m/s ($p_j = 0.41$-15.6 MN/m$^2$). Assuming a roughness height ($k_s$) of 110$\mu$m, the maximum friction coefficients range from 0.01 to 0.017.

The bed shear stresses are a function of the jet velocity at the edge of the boundary layer ($u_{bl,max}$). In Appendix D.4.1 a calculation method is proposed to calculate this velocity as a function of the uniform flow velocity ($u_u$). It follows that the ratio $u_{bl,max}/u_u$ ranges from 1 to about 1.15. This means that the maximum bed shear stresses are only 1.3 to 2.2% of the stagnation pressures.$^3$

**Decay of momentum flux**  The bed shear stresses result in a decay of the jet momentum flux $I$:

$$\frac{dI}{ds} = -\tau_b \left(\frac{1}{2} \pi W_c + 2L_{jb}\right) = -\lambda \frac{1}{2} \rho_m u_u^2 \left(\frac{1}{2} \pi W_c + 2L_{jb}\right)$$  (3.29)

where $L_{jb}$ is the length of the rectangle in the middle of the jet cross section (see Figure 3.8) and $\lambda$ is a sort of friction factor, defined as: $c_f(u_{bl,max}/u_u)^2$.

$^3\tau_b = c_f \frac{1}{2} \rho_m u_u^2 \left(\frac{u_{bl,max}}{u_u}\right)^2 = c_f \left(\frac{u_{bl,max}}{u_u}\right)^2 p_{stag}.$
3.3.2 Entrainment of water and soil

The entrainment of ambient water and soil per unit length for the moving vertical penetrating jet can be calculated with the following equation:\(^4\)

\[
\frac{dQ}{ds} = W_c v_t + \frac{1}{2} \pi W_c u_u \alpha_{mom} \tag{3.30}
\]

The continuity equations for water \((Q_w)\) and solids \((Q_s)\) becomes:

\[
\frac{dQ_w}{ds} = \frac{dQ(1 - c_j)}{ds} = W_c v_t n_0 + \frac{1}{2} \pi W_c u_u \alpha_{mom}(1 - c_a) \tag{3.31}
\]

\[
\frac{dQ_s}{ds} = \frac{dQ c_j}{ds} = W_c v_t (1 - n_0) + \frac{1}{2} \pi W_c u_u \alpha_{mom} c_a \tag{3.32}
\]

where \(n_0\) is the in-situ porosity of the soil, \(c_j\) is the soil concentration in the jet and \(c_a\) is the soil concentration of the ambient fluid, which is neglected \((c_a = 0)\).

The soil concentration reads \((\rho_m - \rho_w)/(\rho_s - \rho_w)\). With \(\rho_m\) the jet flow density, \(\rho_w\) the water density and \(\rho_s\) the density of the solids.

From the continuity equations for water and soil a differential equation for the soil concentration in the jet \((c_j)\) can be derived. Elaborating the continuity equation for soil \((\text{Equation 3.32})\) and substituting the continuity equation for the jet flow rate \((\text{Equation 3.30})\), results in:

\[
\frac{dc_j}{ds} = \frac{1}{1/4\pi W_c + L_{jb}} \left[ \frac{v_t}{u_u} (1 - n_0 - c_j) + \alpha_{mom} \frac{1}{2} \pi (c_a - c_j) \right] \tag{3.33}
\]

where \(L_{jb}\) is the length of the rectangle in the middle of the jet cross section. This differential equation can be rewritten in a differential equation for the jet flow density:

\[
\frac{d\rho_m}{ds} = \frac{dc_j}{ds} (\rho_s - \rho_w) \tag{3.34}
\]

The differential equations for the flow rate and the density are needed to derive the uniform jet velocity from the conservation equation of the momentum flux, Equation 3.29 \((dI/ds = d\rho_m Q u_u / ds)\).

\(^4\)It is also conceivable that the ambient water flows more parallel to the cavity wall towards the jet. In this case the last term changes in \(\alpha_{mom}, W_c u_u\).
3.3.3 Modeling confined jet

To model the confined jet two calculation models are derived. A simple analytical model in which the entrainment of soil and the influence of the bed shear stresses are not included (analytical approach) and a more advanced model in which these processes are included (1D-approach). Another difference is the definition of the entrainment coefficient in the flow development region. In the analytical approach the entrainment coefficient is constant and in the 1D-approach this coefficient is gradually increasing with distance, see Section 3.2.2. These two models will be discussed below.

1D-approach

From the differential equations for the jet momentum flux, the jet flow rate and the jet flow density, the following differential equation for the jet velocity can be derived:

\[
\frac{du_u}{ds} = -\frac{u_u}{1/4\pi W_c + L_{jb}} \cdot \left[ \lambda \left( \frac{1}{4} \pi + \frac{L_{jb}}{W_c} \right) + \frac{v_t}{u_u} \left( 1 + \frac{\Delta_d}{1 + \Delta_d c_j} (1 - n_0 - c_j) \right) \right] - \frac{1}{2} \pi \alpha_{mom} \left( \frac{\Delta_d c_j}{1 + \Delta_d c_j} - 1 \right) \]

(3.35)

where \( \Delta_d \) is the relative density difference defined as: \((\rho_s - \rho_w)/\rho_w\). For the length of the rectangle in the middle of the jet cross section \(L_{jb}\) also a differential equation can be derived:

\[
\frac{dL_{jb}}{ds} = -\lambda \left( \frac{1}{4} \pi + \frac{L_{jb}}{W_c} \right) + \frac{v_t}{u_u} \left[ 2 + \frac{\Delta_d}{1 + \Delta_d c_j} (1 - n_0 - c_j) \right] + \pi \alpha_{mom} \left( 1 - \frac{1}{2} \frac{\Delta_d c_j}{1 + \Delta_d c_j} \right) \]

(3.36)

The system of differential equations can be solved numerically, according to the 1D-approach, see Appendix C. For different scenarios the calculated normalized uniform jet velocities of a moving vertical penetrating jet are plotted in the Figures 3.9 and 3.10 as a function of the normalized distance. In the calculations the nozzle diameter \(D_n\) is 0.02 m, the stand off distance \(SOD\) is \(2D_n\), the jet pressure \(p_j\) is 0.8 MN/m\(^2\) and the in-situ porosity of the soil \(n_0\) is 0.52.

In Table 3.1 the calculated normalized uniform jet velocities at normalized distances \(s/D_n\) of 10, 20 and 30 are listed. The effect of the bed shear stresses on
3.3. Confined jet

the uniform jet velocity is very limited, see Figure 3.9. In this particular case the difference between the calculated uniform jet velocity with and without bed-shear stresses is smaller than 6%.

Table 3.1: Calculated normalized uniform velocities ($u_u$) for a moving vertical penetrating jet.

<table>
<thead>
<tr>
<th></th>
<th>$s/D_n = 10$</th>
<th>$s/D_n = 20$</th>
<th>$s/D_n = 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s = 0 , \mu m, v_t = 0 , m/s$</td>
<td>0.53</td>
<td>0.37</td>
<td>0.30</td>
</tr>
<tr>
<td>$k_s = 110 , \mu m, v_t = 0 , m/s$</td>
<td>0.52 (2%)</td>
<td>0.35 (3%)</td>
<td>0.28 (4%)</td>
</tr>
<tr>
<td>$k_s = 110 , \mu m, v_t = 0.1 , m/s$</td>
<td>0.51 (2%)</td>
<td>0.34 (3%)</td>
<td>0.27 (4%)</td>
</tr>
<tr>
<td>$k_s = 110 , \mu m, v_t = 0.5 , m/s$</td>
<td>0.47 (10%)</td>
<td>0.30 (14%)</td>
<td>0.23 (18%)</td>
</tr>
<tr>
<td>$k_s = 110 , \mu m, v_t = 1.5 , m/s$</td>
<td>0.39 (25%)</td>
<td>0.23 (34%)</td>
<td>0.17 (39%)</td>
</tr>
</tbody>
</table>

The influence of the entrainment of solids on the development of the uniform jet velocity is also limited ($\leq 5\%$) for traverse velocities lower than about 0.1 m/s, see Figure 3.9. For higher traverse velocities the influence of the entrainment of solids in the jet cannot be neglected. At a jet distance of $30D_n$ the normalized uniform jet velocity decreases about 37\% by an increase of the traverse velocity from 0.1 to 1.5 m/s, see Figure 3.10.
3. Velocity development in turbulent jets

Analytical approach

In previous paragraph it was concluded that for low traverse velocities of the nozzle \((v_t < 0.1 \text{m/s})\), the influence of the bed shear stresses and the entrainment of solids are limited. This makes it possible to simplify the system of differential equations. When the influence of the bed shear stresses and the entrainment of solids is neglected, the system becomes \((s > \text{SOD})\):

\[
\frac{d\rho_m}{ds} = 0, \quad \frac{dI}{ds} = 0
\]

\[
\frac{dQ}{ds} = \frac{1}{2} \pi W_c u_u \alpha_{mom}
\]

\[
\frac{d u_u}{ds} = - \frac{dQ \rho_w u_u}{ds \rho_w Q} = - \frac{\alpha_{mom} u_u}{1/2 W_c + 2L_{jb}/\pi}
\]

\[
\frac{dL_{jb}}{ds} = - \frac{d u_u \pi W_c/2 + 2L_{jb}}{u_u} = \alpha_{mom} \pi
\]

This system can be solved analytically, when the entrainment coefficient in the flow development region is assumed to be constant \((\alpha_{mom,dr} = 1/2\alpha_{mom})\). The initial boundary conditions for the theoretical case \(\text{SOD} = 0\), are: \(u_u(0) = u_0\), \(W_c(0) = D_n\) and \(L_{jb}(0) = 0\). If the stand off distance is non-zero, the initial boundary conditions for the uniform flow velocity and the cavity width follow from the system of equations for the free jet (Equations 3.22 and 3.23). The other boundary condition is: \(L_{jb}(\text{SOD}) = 0\).

Assuming that the \(\text{SOD}\) is smaller than the flow development region the uniform jet velocity in the soil cavity can be calculated with the following equations:

\[
\text{for } s_{dr} > s > \text{SOD}: \quad u(s) = u_0 \sqrt{\frac{k}{2 \sqrt{(N_1 + 1)}}} \frac{1}{\sqrt{s/D_n + \sqrt{k/2}}}
\]

\[
\text{for } s \geq s_{dr} \geq \text{SOD}: \quad u(s) = u_0 \sqrt{\frac{k}{2 \sqrt{(2N_1 + 2)}}} \frac{1}{\sqrt{D_n/s}}
\]

where \(N_1\) is defined as: \(\text{SOD}/s_{dr}\). In this case the cavity width becomes:

\[
W_c = D_n (1 + N_1)
\]

The entrainment of ambient water per unit length decreases with distance, because the cavity width is constant, in contrast with that of a free jet which is independent of distance. As a result the uniform jet velocity in the cavity decreases with
3.4. Radial wall jet

the inverse of the square-root of $s$, whereas the uniform jet velocity of the free jet decreases with the inverse of $s$ (compare the Equations 3.40 and 3.24).

In Figure 3.11 the normalized uniform jet velocity of the confined jet, calculated according to the analytical approach and the 1D-approach, are plotted as a function of the normalized distance. As previously mentioned, in the analytical approach the entrainment coefficient is initially higher than in the 1D-approach. Therefore, in the analytical approach the uniform jet velocity is initially lower. As a reference, in Figure 3.11 also the normalized uniform jet velocity development of the free jet is plotted. The jet velocity decay with distance of the confined jet is clearly lower than for the free jet. The uniform jet velocity for the confined jet at a distance of $12D_n$ is almost twice as high as for the free jet. It can be concluded that the jet velocity decay will decrease significantly, by limiting the entrainment of ambient water.

![Graph showing normalized calculated uniform velocity for confined jet and free jet](image)

Figure 3.11: Normalized calculated uniform velocity for the confined jet ($SOD = 0$ m) and the free jet, according to the analytical approach and the 1D-approach, as a function of the normalized distance.

3.4 Radial wall jet

A part of the jet flow will disperse/deflect in radial direction, when the penetration depth of the vertical jet in the soil is relatively small, resulting in relatively shallow wide cavities, see Figure 3.12 (a). This occurs at a relatively high traverse velocity and/or a low jet ratio ($p_j/su$). In Section 6.3 this mechanism will be discussed more in detail.

To predict the cavity width the flow velocity in radial direction must be known. The radial flow is comparable with the flow in a radial wall jet, impinging per-
3. Velocity development in turbulent jets

The radial flow can be described with the following set of differential equations, when the entrainment of solids and the influence of the bed sheaf stresses are assumed to be negligible:

\[
\frac{dQ_r}{dr} = 2\pi r u_{u,r} \alpha_{mom, rw} \tag{3.42}
\]

\[
\frac{d u_{u,r}}{dr} = -\frac{dQ_r}{dr} \frac{\rho_w u_{u,r}}{\rho_w Q_r} = -\frac{\alpha_{mom, rw} u_{u,r}}{h_r} \tag{3.43}
\]

\[
\frac{d h_r}{dr} = -\frac{d u_{u,r}}{dr} \frac{2h_r}{r} - \frac{h_r}{r} = 2\alpha_{mom, rw} - \frac{h_r}{r} \tag{3.44}
\]

Where \(h_r\) is the fictitious height of the radial jet and \(\alpha_{mom, rw}\) is the entrainment coefficient of the radial wall jet.

Poreh et al. (1967) published velocity profiles of a turbulent radial wall jet, produced by an impinging circular air jet. From these profiles the flow rate distribution in radial direction and the entrainment coefficient can be derived. The entrainment coefficient is found to be about 0.128, see Appendix B.3. This is
about 1.5 times the found values for the circular and rectangular jet \( (\alpha_{\text{mom},rw} \approx f_1 \alpha_{\text{mom}} = 1.5 \alpha_{\text{mom}}) \).

**Initial boundary conditions** The difficulty is to define the initial boundary conditions to solve the system of differential equations. To establish the boundary conditions a zone is defined in which the jet flow is fully deflected, see Figure 3.12. This zone is called the zone of impingement. The height of this zone is \( 1/2D_j(s_s) \), where \( D_j(s_s) \) is the fictitious jet diameter at the top of the zone of impingement. \( D_j(s_s) \) can be calculated with the Equations 3.23 and 3.25, assuming that \( s = s_s \).

Assuming a constant stagnation pressure on the base of \( 1/2\rho_wu_0(s_s)^2 \), a base diameter of \( \sqrt{2}D_j(s_s) \) is needed to compensate the vertical momentum flux of the jet. Therefore, the diameter of the zone of impingement is assumed to be \( \sqrt{2}D_j(s_s) \). This results in the following equations for the height \( (h_{zi}) \) and radius \( (r_{zi}) \) of the zone of impingement and the jet velocity at the top of this zone \( (u_{zi}) \):

\[
\begin{align*}
\text{for } SOD < D_n(1 + \sqrt{k/2}) : \quad u_{zi} &= \frac{1}{N_2 + 1} u_0 \\
h_{zi} &= \frac{1}{2} D_n (N_2 + 1) \\
r_{zi} &= \frac{1}{2} \sqrt{2}D_n (N_2 + 1)
\end{align*}
\]

\[
N_2 = \sqrt{\frac{k}{2} + \frac{1}{2}} - \frac{SOD}{D_n} - \frac{1}{2}
\]

\[
\text{for } SOD \geq D_n(1 + \sqrt{k/2}) : \quad u_{zi} &= \frac{1}{2N_3} u_0 \\
h_{zi} &= D_n N_3 \\
r_{zi} &= \sqrt{2}D_n N_3 \\
N_3 &= \frac{SOD}{D_n \left(1 + \sqrt{\frac{k}{2}}\right)}
\]

where \( N_2 \) and \( N_3 \) are defined as \( s_s/s_{dr} \).

If the stand off distance is much larger than the region of flow development \( (SOD \gg D_n(1 + \sqrt{k/2})) \), it follows that the height of the impinging zone is about \( 0.14SOD \). This height corresponds well with the empirical determined influence height of a plane surface placed perpendicularly in a jet flow. Beltaos and Rajaratnam (1974) found that the jet flow of an impinging jet is affected by the boundary,
3. Velocity development in turbulent jets

from a distance of 0.86 times the stand off distance \( (SOD) \).\(^5\) Beyond this point the measured jet velocities deviate from the velocities of a free jet and decrease significantly.

The flow rate leaving the zone of impingement in radial direction \( (Q_{r0}) \) is assumed to be equal to the flow rate that enters it at the top side:

\[
Q_{r0} = Q(s_s) = u_u(s_s) \frac{1}{4}\pi D_j(s_s)^2
\]  \( (3.54) \)

The uniform jet velocity at a distance \( s_s \) can be calculated with the Equations 3.45 and 3.49, while \( u_u(s_s) = u_{zi} \). The radial uniform jet velocity can be derived from the horizontal (radial) momentum flux:

\[
u_{u,r} = \frac{I_r}{\rho u Q_r} = \frac{f_2 I}{\rho u Q_r}
\]  \( (3.55) \)

where \( I_r \) and \( Q_r \) are the momentum flux and flow rate in radial direction, respectively and \( f_2 \) is an empirical constant. From measurements it follows that the vertical momentum flux is not fully converted in radial direction, see Appendix B.3. About 30% of the momentum flux is lost in the zone of impingement \( (f_2 \approx 0.7) \).

For the initial radial jet velocity \( (u_{u,r0}) \) and jet flow height \( (h_{r0}) \) at the edge of the zone of impingement \( (r = r_{zi}) \), the following equations can be derived:

\[
\begin{align*}
\text{for } SOD < D_n(1 + \sqrt{k2}) & : \quad u_{u,r0} = \frac{f_2}{N_2 + 1} u_0 \\
& \quad h_{r0} = \frac{Q_{r0}}{u_{u,r0} 2\pi r_{r0}} = \frac{1}{\sqrt{32}} \frac{N_2 + 1}{f_2} D_n
\end{align*}
\]  \( (3.56, 3.57) \)

\[
\begin{align*}
\text{for } SOD \geq D_n(1 + \sqrt{k2}) & : \quad u_{u,r0} = \frac{1}{2} \frac{f_2}{N_3} u_0 \\
& \quad h_{r0} = \frac{1}{\sqrt{8}} \frac{N_3}{f_2} D_n
\end{align*}
\]  \( (3.58, 3.59) \)

\(^5\)Substituting this distance in Equation 3.3 \( (s = 0.86 \cdot SOD, r = 0) \) and converting the velocity into pressure, results in the following relation for the pressure in the stagnation point of an impinging jet:

\[
\frac{p_{stag}}{p_j} = \frac{k_1}{2} \frac{1}{0.86^2} \left( \frac{D_n}{SOD} \right)^2 = k_3 \left( \frac{D_n}{SOD} \right)^2
\]  \( (3.53) \)

Assuming \( k_1 = 77 \), \( k_3 \) becomes 52. Other researchers found similar values for \( k_3 \) between 48 and 60, see Rajaratnam and Mazurek (2005).
3.5. Conclusions

With the initial conditions for the radial jet flow height and radial jet flow velocity, the system of differential equations (Equations 3.42 to 3.44) can be solved analytically.

This results in the following equations for the radial jet velocity and radial jet flow height as a function of the radial distance:

\[
\begin{align*}
SOD < D_n(1 + \sqrt{k/2}), r > r_{zi}: \\
&u_{u,r}(r) = u_0 \left[ \frac{f_2 D_n^2}{8f_1 \alpha_{mom} r^2 + D_n^2(N_2 + 1)^2 \left( \frac{1}{f_2} - 4f_1 \alpha_{mom} \right)} \right] \\
h_r(r) = f_1 \alpha_{mom} r + \frac{D_n^2(N_2 + 1)^2 \left( \frac{1}{f_2} - 4f_1 \alpha_{mom} \right)}{8r} \\
\end{align*}
\]

\[
\begin{align*}
SOD > D_n(1 + \sqrt{k/2}), r > r_{zi}: \\
&u_{u,r}(r) = u_0 \left[ \frac{f_2 D_n^2}{8f_1 \alpha_{mom} r^2 + 4(N_3 D_n)^2 \left( \frac{1}{f_2} - 4f_1 \alpha_{mom} \right)} \right] \\
h_r(r) = f_1 \alpha_{mom} r + \frac{(N_3 D_n)^2 \left( \frac{1}{f_2} - 4f_1 \alpha_{mom} \right)}{2r} \\
\end{align*}
\]

In Figure 3.13 the normalized calculated uniform radial flow velocity is plotted as function of the normalized radial distance, for three stand off distances. The influence of the stand off distance decreases with radial distance. For stand off distances larger than about 12\(D_n\), the influence of the stand off distance can be neglected. This corresponds with the measurements of Poreh et al. (1967).

In Figure 3.14 the normalized calculated radial jet flow height is plotted as a function of the normalized radial distance, for three stand off distances. The radial jet flow height is needed for the calculation of the cavity width of a dispersing jet. This will be discussed in Section 6.3.

3.5 Conclusions

In this chapter equations have been derived for the jet velocity of a free jet, a confined jet (jet partly enclosed by soil) and a radial wall jet (jet flow dispersing
3. Velocity development in turbulent jets

![Graph](image1.png)  ![Graph](image2.png)

Figure 3.13: Normalized uniform radial flow velocity as a function of the normalized radial distance, for different standoff distances.

Figure 3.14: Normalized radial jet flow height as a function of the normalized radial distance, for different standoff distances.

on the soil surface). These equations will be used, in combination with the fundamentals of cohesive soil behavior (Chapter 2), for the modeling of the excavation process of the jet in cohesive soil (Chapters 6 and 7).

With respect to the modeling of the jet velocity development, the following conclusions can be drawn:

- The jet flow velocity decreases with distance. This is mainly caused by the entrainment of water and soil.

- The entrainment of water can be estimated by multiplying the empirical entrainment coefficient with the uniform jet velocity and the surface of the interface between jet and ambient fluid.

The empirical entrainment coefficient in the region of fully developed flow is constant for the confined and free jet ($\alpha_{mom} = 0.081$). The entrainment coefficient in the flow development region is assumed to increase gradually from 0 to 0.081.

The entrainment coefficient for the radial flow of an impinging jet is about 0.128, approximately 1.5 times higher than for the confined and free jet.

- The entrainment of soil depends mainly on the traverse velocity of the jet. When the stagnation ratio ($p_{stag}/su$) is sufficiently high the entrainment of soil can be calculated by multiplying the frontal jet surface with the traverse velocity of the jet. The influence of the entrainment of soil on the jet velocity development is limited and can be neglected for jet traverse velocities lower
than about 0.1 m/s. For higher jet traverse velocities the influence of soil entrainment cannot be neglected.

- At the interface between jet and soil a boundary layer is formed. The development of this boundary layer can be approximated by the development of the boundary layer on a rough plate in an uniform flow. The exerted shear stresses on the soil are only a few percent of the stagnation pressure. The effect on the momentum flux is also very limited, and can be neglected for a quick estimate of the jet velocity.

- For low jet traverse velocities (influence of bed shear stresses and the entrainment of soil are neglected) there is an analytical solution for the uniform jet velocity in the region of fully developed flow of the confined and free jet. Assuming that the entrainment coefficient in the flow development region is also constant, there is also an analytical solution for the uniform jet velocity in this region.

- The uniform jet velocity of the radial flow of an impinging jet can also be calculated analytically. For the radial flow the entrainment of soil and the influence of the bed shear stresses are assumed to be negligible.
3. Velocity development in turbulent jets
Chapter 4
Cavitating jet

4.1 Introduction

The moving vertical jet exerts a normal pressure on the soil equal to the stagnation pressure ($p_{stag}$). This stagnation pressure causes the soil to fail. The stagnation pressure is a function of the axial jet flow velocity, which decreases with distance. The axial velocity development of a non-cavitating jet is well known, see Chapter 3. Despite the amount of research on cavitating jets, the literature does not provide a description of the velocity development of a cavitating jet.

Under conditions of cavitation, a cone of bubbles forms around the jet (see Figure 4.1), which decreases the momentum exchange between the jet and the ambient water and the associated entrainment. As a result the decay of the jet velocity and thus the stagnation pressure with jet distance decreases (Yahiro and Yoshida, 1974), (Soyama and Lichtarowicz, 1996). This will increase the penetration depth of the jet in cohesive soil.

Measurements on cavitating jets are usually restricted to their erosion rates and cutting capacities. For many materials the cutting depths and/or productions are published as functions of various jet parameters. Usually the erosion rate is based on free submerged cavitating jets impinging a surface perpendicular to the jet direction. In these cases the erosion rate is largely determined by the implosions of cavitation bubbles. The amount of cavitation bubbles at the interface between the moving vertical jet and soil is assumed to be limited. Therefore, the direct contribution of bubble implosions on the excavation capacity is probably negligible.

In this chapter a literature review on the forming of cavitation bubbles and the behavior of cavitating jets is given. In order to determine a description for the entrainment of a cavitating jet, jet tests were carried out at various ambient fluid pressures ($p_{ao}$) in fresh and saline water. The experimental setup and results of these tests will be discussed in this chapter.
4.2 Literature review

4.2.1 Cavitation

The velocity difference between a jet and the ambient water creates a mixing layer in which the transfer of mass and momentum takes place. Ambient water is entrained in the jet. To replace the entrained water, ambient water must flow towards the jet and decreases the static pressure around it. The higher the jet velocity, the more water entrained and the greater the pressure drop. Turbulent vortices locally further reduce the static pressure.

Vapour is formed inside the water when the static pressure drops below the vapour pressure \( p_{va} \). This can occur only at free surfaces.\(^1\) In natural water, free surfaces are normally present as micron-sized bubbles of contaminant gas called nuclei; these can be present in crevices within the solid boundary or within suspended particles, or be freely suspended within the water. Typical nucleus radii are between 5 and 10 \( \mu m \) (Brennen, 1995).

The amount of nuclei depends on the ambient fluid pressure and temperature and can be changed by filtering and degassing the fluid. In water the measured numbers of nuclei with a radius between the 5 and 10 \( \mu m \) vary for instance from 25 to 200 \( cm^{-3} \), see (Ooi, 1985) and (Brennen, 1995).

---

\(^1\)For a pure liquid, free of nuclei, the pressure difference between the vapour pressure and the critical pressure (called the tensile strength of a liquid) increases significantly. Berthelot (1850) was able to achieve 5 MN/m\(^2\) of tension in pure water at normal temperatures.
4.2. Literature review

**Critical local pressure**  Because of the surface tension of water ($S_t$), the exact critical ambient pressure ($p_{a,c}$) at which a micro-sized bubble is unstable, and will grow explosively, is often a little lower than the vapour pressure. The critical ambient pressure of a spherical micro bubble with an equilibrium radius $R_{bu0}$ will be derived below, see also (Brennen, 1995).

The pressure interior a bubble is related to the exterior liquid pressure ($p_{a0}$) by:

$$p_g0 + p_{va} = p_{a0} + \frac{2S_t}{R_{bu0}} \tag{4.1}$$

The pressure interior the bubble is the sum of the partial pressures of the gases inside ($p_g0$) and the vapor pressure ($p_{va}$). Vaporization of liquid at the liquid bubble interface occurs very fast, relative to the time scale of the bubble dynamics. Therefore, the vapor pressure is assumed to be constant at every instant. On the other hand gas diffusion is a relatively slow process. This leads to the assumption that the mass of contaminant gases inside remains constant and the partial pressures varies with bubble volume. A small change in ambient pressure ($\Delta p_a = p_{a0} - p_a$) will lead to a new equilibrium:

$$p_g0 \left( \frac{R_{bu0}}{R_{bu}} \right)^3 + p_{va} = p_a + \frac{2S_t}{R_{bu}} \tag{4.2}$$

Below a certain ambient pressure there is no new static equilibrium and a bubble will grow explosively. Solving for the minimum of $p_a$ using Equation 4.2 provides the values of the critical ambient pressure $p_{a,c}$ and the corresponding critical radius $R_{bu,c}$:

$$p_{a,c} = p_{va} - \frac{4S_t}{3R_{bu,c}} \tag{4.3}$$

$$R_{bu,c} = \sqrt[3]{\frac{3R_{bu0}^3 p_{g0}}{2S_t}} \tag{4.4}$$

It follows that the critical pressure of a micro-bubble with an initial radius smaller than about 9 $\mu$m at a exterior liquid pressure ($p_{a0}$) of 0.1 MN/m$^2$ ($S_t = 0.075$ N/m, $p_{va} = 2.3$ kN/m$^2$), is lower than the absolute vacuum, and will not cavitate. On the other side the critical ambient pressure for micro bubbles present in crevices within the solid boundary or within suspended particles can also be a little higher than the vapour pressure, see also (Brennen, 1995).

For this study, the critical ambient pressure is assumed to be equal to the vapour pressure. Hence, the required pressure drop for cavitation ($\Delta p_a$) is about $p_{a0} - p_{va}$, where $p_{a0}$ is the initial ambient fluid pressure.
4. Cavitating jet

Response time It takes time to grow from the initial radius to the critical radius. Therefore, the required pressure drop must persist for a period that is longer than the response time of the bubble, namely the time the bubble takes to grow to its critical diameter (Ooi, 1985). The Rayleigh-Plesset equation (Rayleigh, 1917) can be used to estimate the response time of a bubble:

\[ \frac{p_{va} - p_a}{\rho_m} = R_{bu} \frac{d^2 R_{bu}}{dt^2} + \frac{3}{2} \left( \frac{dR_{bu}}{dt} \right)^2 + \frac{2S_t}{\rho_m R_{bu}} + \frac{4\nu}{R_{bu}} \frac{dR_{bu}}{dt} \]  

(4.5)

where \( t \) is time, \( \rho_m \) is the liquid density and \( \nu \) is the kinematic viscosity of the fluid. Integrating Equation 4.5, without the viscous term, and forming time derivatives, yields:

\[ \left( \frac{dR_{bu}}{dt} \right)^2 = \frac{2}{3} \frac{p_{va} - p_a}{\rho_m} \left[ 1 - \left( \frac{R_{bu0}}{R_{bu}} \right)^3 \right] + 2 \frac{p_{a0}}{\rho_m} \left( \frac{R_{bu0}}{R_{bu}} \right)^3 \ln \frac{R_{bu}}{R_{bu0}} \]

\[ -2 \frac{S_t}{\rho_m R_{bu}} \left[ 1 - \left( \frac{R_{bu0}}{R_{bu}} \right)^2 \right] \]  

(4.6)

Note that this equation is confined to a spherical bubble in an infinite inviscid incompressible fluid, without thermal effects. With Equation 4.6 the response time of a bubble with an initial radius \( R_{bu0} \) to grow to its critical radius \( R_{bu,c} \) (Equation 4.4), when it is exposed to a liquid pressure drop of \( \Delta p_a = p_{a0} - p_{a,c} \), can be evaluated numerically. Table 4.1 presents this response time (\( t_{bu} \)) for various initial bubble radii.

Table 4.1: Response time for a bubble to grow from \( R_{bu0} \) to \( R_{bu,c} \), when it is exposed to a liquid pressure drop of \( p_{a0} - p_{a,c} \) (\( p_{a0} = 0.1 \) MN/m\(^2\), \( p_{va} = 2.34 \) kN/m\(^2\) and \( S_t = 0.073 \) N/m).

<table>
<thead>
<tr>
<th>( R_{bu0} ) [( \mu )m]</th>
<th>( R_{bu,c} ) [( \mu )m]</th>
<th>( p_{a,c} ) [kN/m(^2)]</th>
<th>( t_{bu} ) [( \mu )s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>35</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>20</td>
<td>132</td>
<td>1.6</td>
<td>63</td>
</tr>
<tr>
<td>50</td>
<td>509</td>
<td>2.1</td>
<td>398</td>
</tr>
<tr>
<td>100</td>
<td>1429</td>
<td>2.3</td>
<td>1680</td>
</tr>
</tbody>
</table>

The time a micro bubble is exposed to a pressure drop depends on the timescale of the turbulent pressure fluctuations and the residence time of the bubble in the region of sufficiently low pressure.
4.2. Literature review

**Salinity** In dredging practice, jetting mostly takes place in seawater that has a salinity \(c_{sal}\) of about 35%/o.\(^2\) However, most research on cavitation is done in fresh water. Cavitation-determining parameters – namely vapour pressure \(p_{va}\) and surface tension \(S_t\) – differ a little with salinity (see Table 4.2).

The nuclei distribution is also different. A typical nuclei distribution measured in the Pacific Ocean shows the number of nuclei with radii < 20µm is much higher than the number for fresh water (O’Hern et al., 1988). For cavitation inception these differences are important, but for more developed cavitation they are negligible.

Another probably more important difference between fresh and saline water is the susceptibility of the bubbles to coalesce. In fresh water, bubbles tend to coalesce and form large bubbles. In saline water, free surfaces are slightly negatively charged. The resulting electrical repulsive forces prevent bubbles for coalescing (Weissenborn and Pugh, 1995). Hence, in saline water the cavitation bubbles remain relatively small.

Due to their buoyancy, the bubbles will rise and escape the cone of cavitation bubbles. Because the buoyancy of the bubbles increases with volume, they rise and escape the cone earlier in fresh water than in saline water. Hence, the effective length of the cavitation cone in saline water is possibly greater than in fresh water, (Summers and Sebastian, 1980).

Table 4.2: Vapor pressure \(p_{va}\) and surface tension \(S_t\) of fresh \(c_{sal} = 0\%_o\) and saline water \(c_{sal} = 35\%_o\).

<table>
<thead>
<tr>
<th>(T [\degree C])</th>
<th>(p_{va} [MN/m^2])</th>
<th>(S_t \cdot 10^{-2}[N/m])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{sal} = 0%_o)</td>
<td>(c_{sal} = 42%_o)</td>
<td>(c_{sal} = 0%_o)</td>
</tr>
<tr>
<td>10</td>
<td>1.23</td>
<td>1.21</td>
</tr>
<tr>
<td>20</td>
<td>2.34</td>
<td>2.30</td>
</tr>
</tbody>
</table>

### 4.2.2 Incipient cavitation

Besides the research on the cutting capacity of a cavitating jet, research has also concentrated on incipient cavitation. Cavitation inception is defined as the moment that at least five bubbles expand and then implode (Ooi, 1985). The con-

\(^2\) A salinity of about 35%/o means that every kilogram of seawater has 35 grams of dissolved salts, predominantly Sodium Chloride ions: \(Na^+, Cl^-\).
conditions for cavitation inception are typically indicated by the cavitation inception index:

\[
\sigma_i = \frac{p_{a0} - p_{va}}{1/2\rho_m u_0^2} \approx \frac{p_{a0}}{p_{cav,i}}
\]  

(4.7)

where \(\rho_m\) is the liquid density, \(u_0\) is the jet velocity at nozzle exit and \(p_{cav,i}\) is the jet pressure at cavitation inception. The published cavitation inception numbers for submerged jets differ significantly. A collection of measured indices for untreated water shows values between 0.12 (Lienhard and Stephenson, 1966) and 1.62 (Ran and Katz, 1994). This means that the static pressure fluctuations in the jet can reach a value of 1.62 times the dynamic head of the jet at nozzle exit \((1/2\rho_w u_0^2 = p_j)\).

To explain cavitation inception different researchers have measured the turbulent pressure fluctuations in a cavitating jet. The measured maximum pressure drops are in the order of 0.97 (Ran and Katz, 1994)\(^3\) to 1.3 (Ooi and Acosta, 1984)\(^4\) times the dynamic head. See also Rood (1991), Franklin (1994) and Arndt (2002). Two trends are found: \(\sigma_i\) increases with nozzle diameter and with air content. Other trends are conflicting (Gopalan et al., 1999). An insufficient understanding of the underlying flow makes it difficult to interpret all differences.

The location where the first cavitation events occur, depends on the nozzle geometry, nozzle diameter and jet pressure. For most horn shaped nozzle’s cavitation starts inside the nozzle, see (Yanaida et al., 1985) and (Saito and Sato, 2006), while for small nozzle’s \((D_n = 3.17-4.76 \, \text{mm})\) the published locations increases up to 6 times the nozzle diameter (Ooi, 1985).

It has been determined that the first cavitation bubbles do not influence the jet flow. Because of this, the exact value of \(\sigma_i\) is of little relevance to the jetting of cohesive sediments. What is more relevant is the condition: cavitation influences the maximum stagnation pressure firstly, which is expressed in the cavitation number of cone development:

\[
\sigma_d = \frac{p_{a0} - p_{va}}{p_{cav}} \approx \frac{p_{a0}}{p_{cav}}
\]  

(4.8)

Where \(p_{cav}\) is the jet pressure for cavitation cone development, defined as the lowest jet pressure (pressure difference) at which the influence of cavitation on the stagnation pressure is measurable. The value of \(\sigma_d\) is necessarily smaller than \(\sigma_i\). However, no \(\sigma_d\) values are given in the literature.

\(^3D_n = 25.4 \, \text{mm}, u_0 = 17.5 \, \text{m/s}, p_{a0} = 0.15 \, \text{MN/m}^2.\)

\(^4D_n = 3.17 \, \text{mm}, u_0 = 12.2 \, \text{m/s}, p_{a0} = 0.11 \, \text{MN/m}^2.\)
4.2.3 Developed cavitation

When the pressure drop required for cavitation is about constant and extends over a long distance, a cone of cavitation bubbles forms around the jet. The jet core remains free of cavities (Ran and Katz, 1994). This cone reduces the exchange of momentum between the jet and the ambient water. As a result, less ambient water is entrained and the decrease in jet velocity and stagnation pressure with distance is reduced.

With an increase in jet pressure, the pressure drop required for cavitation extends over a longer distance. Hence, the length of the cone increases with jet pressure.

The structure of a cavitating jet and the behaviour of unsteady cavitation bubbles are still unclear. In addition, modeling numerically a cavitating jet is very challenging. In the cavitation region the flow is compressible, while in the non-cavitating regions the flow is incompressible. Although a number of attempts have been made to model a cavitating jet, see e.g. Xing and Frankel (2002), Peng and Fujikawa (2006) and Alehossein and Qin (2007), an integral model is not yet available.

4.2.4 Bubble implosions

Because of pressure peaks, cavitation bubbles will collapse / implode. Near a (solid) boundary these implosions are very destructive and therefore subject of many investigations (see e.g. Figure 4.5). Two characteristic effects are believed to be mainly responsible for the destructive action: the emission of shock waves upon the collapse of the bubble and the generation of a high-speed liquid jet directed towards the boundary. As the thin liquid jet reaches very high velocities, its impingement onto a boundary causes high water hammer pressures of short duration. Vogel and Lauterborn (1988) and Phillipp and Lauterborn (1998) report maximum jet velocities of about 100 m/s and water hammer pressures up to 450 MN/m². These velocities and hammer pressures are depending on the ambient pressure, initial bubble diameter and the distance between the boundary and bubble center. These values are consistent with measured pressure peaks in jets of 120 MN/m² (\(D_n = 1\) mm, \(u_0 = 245\) m/s), see (Sato et al., 1988).

The largest erosive force is caused by a collapse of a bubbles in direct contact with the boundary, where pressures up to several GN/m² act on the material surface, Vogel and Lauterborn (1988) and Phillipp and Lauterborn (1998).

4.2.5 Other datasets

Three datasets of measurements on the stagnation pressure of cavitating jets are found.
Yahiro and Yoshida (1974) studied the influence of an air film around a submerged jet on the stagnation pressure decay. In the context of their study, they also made a lot of measurements on cavitating jets without an air film. The influence of cavitation on the stagnation pressure decay measured by Yahiro and Yoshida is much more significant than the influence measured in the context of present study. We have not found a convincing explanation for this difference. Other researchers on the influence of an air film around a jet also found a discrepancy between their datasets and that of Yahiro and Yoshida, see (Berg et al., 2006), (Vinke, 2009).

Shen and Sun (1988) studied the stagnation pressure decay of a submerged non-free jet. Although the jet pressures were such that the jets should have been cavitating, the researchers do not mention the influence of cavitation. The trends they found are the same as those presented in this thesis.

Soyama and Lichtarowicz (1996) investigated the structure of a cavitating jet, by measuring the stagnation pressure in the center and at different radial distances. Also numerous high speed recordings of the cavitating jets were taken. The main interest of this work was to correlate the stagnation pressure measurements with previously measurements on the maximum erosion rate of cavitating jets. Stagnation pressures are published at different jet distances and ambient pressures (0.16-0.4 MN/m$^2$) for a cylindrical nozzle with a diameter of 2.5 mm and a jet pressure of 8 MN/m$^2$. They provide empirical equations for the length of the visible cavitation cone length, optimum stand off distance for erosion. The constants in the latter equations are not quantified.

4.2.6 Conclusion

Despite the amount of research there are still a lot of uncertainties. The structure of a cavitating jet and the behavior of unsteady (clusters of) cavitation bubbles are still unclear and an integral mathematical model is missing.

Some uncertainties are of limited importance. Because the first cavitation bubbles will not influence the jet flow the exact value and position of cavitation inception are of little interest. More relevant are the conditions at which the cone of bubbles has developed such that entrainment decreases. This will result in higher stagnation pressures which are important for the jetting of cohesive soil. In order to determine these conditions and to determine an empirical formula for the entrainment coefficient, cavitating free jet tests were carried out in fresh and saline water at different ambient fluid pressure levels.

4.3 Experimental set-up cavitating free jet tests

Cavitating jet tests were carried out in a 1.6 m long, 1.2 m diameter cylindrical pressure vessel (see Figure 4.3). A height-adjustable vertical jet pipe was installed
on the top of this vessel. The vertical and horizontal position of the jet could be adjusted above the measuring panel within a tenth of a millimeter.

The jet flow was provided by an electrically operated, frequency-regulated piston pump (38 kW). The maximum jet flow was 235 l/min. The flow could be reduced by means of a pressure relief valve in the bypass at the discharge side of the pump. The pressure peaks of the three pistons were damped by an accumulator.

The ambient pressure in the vessel was adjusted with a regulating valve in the drain. The pressure fluctuations in the vessel were damped by a large accumulator (50 l) behind the vessel.

**Measuring panel** The measuring panel, which was located in the middle of the vessel, was designed such that it disturbed the jet only minimally. This panel consisted of a structure in which eight very thin ($D_{out}$ 1.58 mm) hollow tubes were clamped vertically along the two axes (see Figure 4.4). These tubes were the measuring points for the stagnation pressure and were connected to separate (calibrated) pressure transducers. The range of the used pressure transducers was matched to the expected stagnation pressures.

The first design of the panel was too fragile to withstand the jet pressure of 200 bar for a long time. After some tests a more robust panel was designed. Comparative tests show no difference between the two panels. For the exact dimensions of the two panels see Appendix E.
4. Cavitating jet

Nozzles Most tests were carried out with short nozzles of the brand Woma. Three nozzle diameters \( (D_n) \) were tested: 3, 5 and 7 mm, see Appendix E, Figure E.7. These nozzles were relatively short, which results in low discharge coefficients, defined as the jet flow divided by the nozzle surface multiplied by the jet velocity; \( \mu_n = Q_0/(A_n u_0) \). The calculated discharge coefficients of these nozzles were 0.89, 0.88 and 0.89, respectively. To investigate the influence of nozzle design on the cavitation behavior also a long conical nozzle was tested. The calculated discharge coefficient of this nozzle was significantly higher 0.99.

In almost the same experimental set-up the influence of an air film around a jet was investigated (Berg et al., 2006). The results of reference tests with this nozzle, without air, are included in the analysis. The nozzle was designed according to Yahiro and Yoshida (1974), and is therefore called the Yahiro-nozzle. Its nozzle diameter was 3 mm, and it had a discharge coefficient of 0.82.

Test settings Table 4.3 presents the test settings. The jet pressure \( (p_j) \) is the dynamic head of the jet at nozzle exit. The initial ambient pressure \( (p_{a0}) \) is the measured absolute pressure in the pressure vessel.

To ensure that the water properties were constant, continually aerated and filtered (5 \( \mu \)m) tap water was used. Some tests were carried out with saline water. A solution of NaCl was added to tap water to produce saline water. The measured salinity was about 42\% in terms of mass (which is a little higher than that of normal seawater, i.e. 35\%).

Repeatability and accuracy measurements To verify the repeatability of the measurements some test settings were repeated at the end of most test series.
4.3. Experimental set-up cavitating free jet tests

Table 4.3: Test settings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet pressure</td>
<td>1.5-20 [MN/m²]</td>
</tr>
<tr>
<td>Nozzle diameter</td>
<td>3, 5 and 7 [mm]</td>
</tr>
<tr>
<td>Stand off distance</td>
<td>6-72Dₙ [mm]</td>
</tr>
<tr>
<td>Ambient pressure</td>
<td>0.10-0.57 [MN/m²]</td>
</tr>
</tbody>
</table>

Table 4.4: Water properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air content</td>
<td>10-12 [ppm]</td>
</tr>
<tr>
<td>Water temperature</td>
<td>12-15 [°C]</td>
</tr>
<tr>
<td>Nuclei diameter</td>
<td>&lt; 5 [µm]</td>
</tr>
<tr>
<td>Salinity</td>
<td>0 &amp; 42 [% e]</td>
</tr>
</tbody>
</table>

There was also some overlap in test settings in different test series. The maximum difference in measured stagnation pressures is of the order of 10%, as can be noticed in some of the figures.

It was found that at high jet pressures (>15 MN/m²) the jet pipe shifted slightly. In these cases the center line stagnation pressure was obtained, in back-analysis, by shifting the nozzle position fictitious such that all eight measured stagnation pressures lay on a symmetric profile (close to Gaussian).

The tests into the influence of an air film around the jet were carried out some months later in the same pressure vessel (Berg et al., 2006). All sensors were re-connected, the measuring panel was adapted, an air-inlet was provided and the Yahiro-nozzle was mounted. The jet pipe was fixated better than during previously test series. Also the alignment of the nozzle was checked regularly by shifting the nozzle a little in the horizontal plane to find the highest stagnation pressure.

The results of reference tests without air film showed the same trends. Because of a different nozzle design the absolute values of the measured stagnation pressures were slightly different, see Section 4.4.3, Figure 4.9.

The measured stagnation pressures of present study globally correspond with the data of Shen and Sun (1988) (see Figure 4.7) and Soyama and Lichtarowicz (1996) (see Figures, 4.12 and 4.18).
4. Cavitating jet

4.4 Results free cavitating jet tests

In the present study the jet pressure \( (p_j) \) is defined as the dynamic head of the jet at nozzle exit:

\[
p_j = \frac{1}{2} \rho w u_0^2
\]  \hspace{1cm} (4.9)

The stagnation pressure \( (p_{stag}) \) is defined with respect to the initially ambient fluid pressure \( (p_{a0}) \):

\[
p_{stag}(s) = p_{me}(s) - p_{a0}
\]  \hspace{1cm} (4.10)

where \( p_{me}(s) \) is the measured absolute pressure in the center of the jet at a distance \( s \) from the nozzle. The stagnation pressure is about equal to the dynamic pressure, because the measuring point was disturbing the jet flow minimally.

4.4.1 Jet pressure

The stagnation pressures normalized with jet pressure for the three nozzle diameters are plotted in Figure 4.6 as function of the jet pressure. These pressures were measured at a jet distance of \( 12D_n \) and an initially ambient fluid pressure \( (p_{a0}) \) of 0.13 MN/m\(^2\).

For a non-cavitating jet, the normalized stagnation pressure in the center of the jet can be derived from Equation 3.3:

\[
\frac{p_{stag,nc}}{p_j} = \frac{k}{2} \left( \frac{D_n}{s} \right)^2 \approx 38.5 \left( \frac{D_n}{s} \right)^2
\]  \hspace{1cm} (4.11)

It follows that the normalized stagnation pressure for a non-cavitating jet only depends on the normalized jet distance \( (D_n/s) \). At a jet distance of \( 12D_n \), the calculated normalized stagnation pressure is about 0.27 (striped line in Figure 4.6).

The normalized stagnation pressures start to increase with respect to the reference value of the non-cavitating jet, at a jet pressure of about 2.5 MN/m\(^2\). At this pressure, the cone of bubbles reduces the entrainment. This is an important pressure and is defined as the jet pressure for cavitation cone development \( (p_{cav}) \). The corresponding cavitation number for cone development \( (\sigma_d) \) is 0.052.

At higher jet pressures, more and more cavitation bubbles are formed. As a result, the effectiveness of the cone of bubbles increases and the direct interaction surface between the jet and ambient fluid shrinks. This interaction area is responsible for the exchange of momentum between the jet and the ambient water, and thus
4.4. Results free cavitating jet tests

for cavitation. A reduction of this area results in a decrease in the momentum exchange and in the static pressure drop.

At a certain jet pressure, the effectiveness of the cavitation cone, expressed in the normalized stagnation pressure, reaches a maximum value. A further increase in jet pressure results only in a negligible increase in the normalized stagnation pressure. The number of newly formed bubbles per interaction area is optimized/maximized.

Shen and Sun (1988) measured the stagnation pressure with a pitot tube in a jet with a nozzle diameter of 2.85 mm. They found the same trend, see Figure 4.7. The increase in the normalized stagnation pressure for the 2.85 mm nozzle is stronger than for the 3 mm nozzle. The exact explanation for this difference cannot be given. Probably the difference can be partly explained by the difference in nozzle design, see Paragraph 4.4.3.

### 4.4.2 Nozzle diameter

As shown in Figure 4.6, the differences between the three nozzle diameters are limited. Table 4.5 presents $p_{cav}$ and corresponding $\sigma_d$ for the nozzles. The jet pressure for cavitation cone development decreases with an increase in nozzle diameter. The same trend was found for the cavitation inception number. Cavitation inception numbers for similar nozzle diameters measured by Ooi (1985) are listed in Table 4.6. Necessarily the value of $\sigma_d < \sigma_i$. Also the calculated critical jet pressures for cavitation inception ($p_{cav,i}$) at an ambient pressure of 0.13 MN/m$^2$...
are listed. These pressures are significantly lower than the pressures for cavitation cone development.

Table 4.5: Jet pressure for cavitation cone development and the corresponding cavitation number for the three nozzle diameters ($p_{a0} = 0.13$ MN/m$^2$).

<table>
<thead>
<tr>
<th>$D_n$ [mm]</th>
<th>$p_{cav}$ [MN/m$^2$]</th>
<th>$\sigma_d$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>0.065</td>
</tr>
<tr>
<td>5</td>
<td>2.4</td>
<td>0.054</td>
</tr>
<tr>
<td>3</td>
<td>2.6</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Table 4.6: Cavitation inception numbers measured by Ooi (1985) and calculated critical jet pressures for cavitation inception at an ambient pressure of 0.13 MN/m$^2$ (air content 10.5 ppm).

<table>
<thead>
<tr>
<th>$D_n$ [mm]</th>
<th>$\sigma_i$ [-]</th>
<th>$p_{cav,i}$ [MN/m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.35</td>
<td>0.24</td>
<td>0.54</td>
</tr>
<tr>
<td>4.76</td>
<td>0.11</td>
<td>1.2</td>
</tr>
<tr>
<td>3.17</td>
<td>0.08</td>
<td>1.6</td>
</tr>
</tbody>
</table>

4.4.3 Nozzle design

The Figures 4.8 and 4.9 show the influence of nozzle design on the normalized stagnation pressure. The normalized stagnation pressures of the conical nozzle are structurally higher. The comparison between the Woma-nozzle with the Yahiro-nozzle show a similar difference.

These differences can partly be explained by the differences in discharge coefficients; the higher the discharge coefficient the higher the normalized stagnation pressures. To discount the effect of nozzle design, it is better to normalize the jet distance ($s$) with the initial jet diameter ($D_{j0} = \sqrt{\rho_n D_n}$) instead of the nozzle diameter ($D_n$). This was already suggested by Soyama and Lichtarowicz (1996).

4.4.4 Ambient pressure

The pressure drop required for cavitation ($\Delta p_n$) increases linearly with the ambient pressure. Thus, the jet pressures for cavitation cone development also increase with ambient pressure. Figure 4.10 shows the measured stagnation pressures normalized with jet pressure for the 5 mm nozzle at various ambient pressures as function of the jet pressure. These pressures were measured at a normalized jet distance of $12D_n$.

At all jet pressures, the normalized stagnation pressure decreases with ambient pressure. It is remarkable that the normalized stagnation pressures at certain conditions are lower than can be expected for a non-cavitating jet. This decrease
4.4. Results free cavitating jet tests

Figure 4.8: Influence of nozzle design on the normalized stagnation pressure at a jet distance of $12D_n$ ($p_{a0} = 0.13$ MN/m$^2$).

Figure 4.9: Influence of nozzle design on the normalized stagnation pressure at a jet distance of $18D_n$ ($p_{a0} = 0.13$ MN/m$^2$).

Figure 4.10: Influence of ambient fluid pressure on the normalized stagnation pressure for the 5mm-nozzle at a jet distance of $12D_n$.

is probably caused by the distortionary impact of imploding bubbles on the jet flow.

Figure 4.11 shows the jet pressures of cavitation cone development ($p_{cav}$) as function of the ambient pressure. The pressure for cone development increases approximately linearly with ambient fluid pressure. It follows that the average value of $\sigma_d$ is about 0.045.
4. Cavitating jet

Figure 4.11: Jet pressures for cavitation cone development as function of the ambient fluid pressure for the 5mm-nozzle at a jet distance of $12D_n$.

4.4.5 Jet distance

Figure 4.12 shows the measured stagnation pressures normalized with jet pressure as function of jet distance for the 3mm-nozzle. These pressures are measured at an ambient pressure of $0.13 \text{ MN/m}^2$. As a reference, the normalized jet pressures for a non-cavitating jet are also plotted (striped line).

Figure 4.12: Influence of jet distance on the normalized stagnation pressure for the 3mm-nozzle at an ambient fluid pressure of $0.13 \text{ MN/m}^2$.

The stagnation pressures published by Soyama and Lichtarowicz (1996) are also plotted in Figure 4.12.\footnote{Because the stagnation pressures are read from a figure the accuracy is not high.} The data of Soyama and Lichtarowicz show the same trend. The ambient fluid pressure was however a little higher ($0.16 \text{ MN/m}^2$),
4.4. Results free cavitating jet tests

therefore the data is not fully comparable. The normalized stagnation pressures lie a little higher than one would expect on basis of present data.

To investigate the effectiveness of the cavitation cone, the measured stagnation pressures given in Figure 4.13 are normalized with the calculated stagnation pressures for a non-cavitating jet ($p_{stag,nc}$, see Equation 4.11). For a non-cavitating jet, the stagnation pressure in the center equals the jet pressure up to a jet distance of $\sqrt{k/2D_n} \approx 6.2D_n$. Thus, up to a jet distance of $6.2D_n$ no effect of cavitation can be measured ($p_{stag}/p_{stag,nc} = 1$). For all jet pressures the effectiveness of the cavitation cone is maximal at a jet distance of about $18D_n$. For a jet pressure of 19.5 MN/m², the measured stagnation pressure is almost 4 times higher than that calculated for a non-cavitating jet. At a jet distance of $18D_n$ and further, the effectiveness of the cavitation cone decreases almost linearly with jet distance (see dotted line). This decrease is also due to the limited length of the cone. At a certain jet distance, the jet velocity has decreased such that no new bubbles are formed.

![Figure 4.13: Effectiveness of cavitation cone ($D_n = 3$ mm, $p_{a0} = 0.13$ MN/m²).](image)

4.4.6 Salinity

To investigate the influence of salinity on cavitation, some jet tests were carried out in water with a salinity ($c_{sal}$) of 42‰ (in terms of mass). Figure 4.14 shows the measured stagnation pressures normalized with jet pressure for fresh and saline

---

Soyama and Lichtarowicz (1996) measured the stagnation pressure on a plate orientated perpendicular to the main flow direction. In such a flow, the jet is disturbed downstream from about 0.86 times the stand off distance (Rajaratnam, 1976). This means that the real stagnation point lie about 0.14 times the stand off distance before the plate. In the present study, the jet flow was disturbed minimally at the measuring points. To compensate for this, the jet distance ($s$) given by Soyama and Lichtarowicz (1996) was corrected with a factor 0.86.
water. The nozzle diameter was 5 mm, the jet distance $12D_n$ and the ambient pressure 0.13 MN/m$^2$. The differences between the pressures in fresh and saline water are negligible. For more developed cavitation, the influences of the differences in $p_{v0}$ and $S_t$ for fresh and saline water are apparently negligible.

![Figure 4.14: Results for fresh water ($c_{sal} = 0\permil$) and saline water ($c_{sal} = 42\permil$) water: $D_n = 5$ mm, $s/D_n = 12\cdash$.](image)

During some tests, the cavitation cone was captured with a high resolution video camera. These images allowed the length of the cavitation cone to be measured. Table 4.7 presents the measured lengths of the cavitation cone under similar test conditions in fresh and saline water. The differences are negligible.

Table 4.7: Measured length of the cavitation cone ($L_{cone}$) in fresh and saline water ($D_n = 5$ mm).

<table>
<thead>
<tr>
<th>$p_j$ [MN/m$^2$]</th>
<th>$p_{a0}$ [MN/m$^2$]</th>
<th>$L_{cone}$ [mm] ($c_{sal} = 0\permil$)</th>
<th>$L_{cone}$ [mm] ($c_{sal} = 42\permil$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.13</td>
<td>98</td>
<td>110</td>
</tr>
<tr>
<td>2.5</td>
<td>0.5</td>
<td>38</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>62</td>
<td>60</td>
</tr>
</tbody>
</table>
4.5 Analyses free cavitating jet tests

4.5.1 Development of stagnation pressure

Under conditions of cavitation, a cone of bubbles forms around the jet, which decreases the momentum exchange between the jet and the ambient fluid. The effectiveness of the cavitation cone depends on the quantity and radii of the cavitation bubbles. The thickness of the cavitation cone layer is not constant over the full length of the cone, see Figure 4.15.

![Figure 4.15: Definition sketch of the various developing regions of the cavitation cone.](image)

The cone needs to develop over some length; this is called the cone development region. Both the number of bubbles per unit length and their radii increase in this region. After this region is a second region, in which the cone is fully developed and the effectiveness is roughly constant. The length of this region depends on the jet pressure, ambient pressure and nozzle diameter. At a certain distance, the jet velocity decreases such that the conditions for cavitation are no longer satisfied and no new bubbles are formed. The cavitation cone loses its effectiveness and disappears, after which the entrainment per unit length equals the entrainment of the non-cavitating jet.

When the cavitation cone is fully developed, the entrainment per unit length is roughly constant and can be expressed by the following equation (similar with a non-cavitating jet):

\[
\frac{dQ}{ds} = \frac{1}{\sqrt{2k_{\text{cav}}}} \pi D_n u_0 = \alpha_{\text{mom,cav}} \pi D_n u_0
\]

(4.12)
where $k_{cav}$ is an empirical constant and $\alpha_{mom,cav}$ is the entrainment coefficient. The larger $k_{cav}$, the more the entrainment is reduced. For a non-cavitating jet, $k \approx 77$ (Fischer et al., 1979).

It follows from the measurements that the normalized stagnation pressure increases with the square root of the jet pressure for jet pressures above $p_{cav}$, see also Section 4.4.1 and Figure 4.6. Therefore, it is assumed that $k_{cav}$ increases with $\sqrt{\frac{p_{j}}{p_{cav}}}$ for jet pressures above $p_{cav}$. With $p_{cav} = \frac{1}{\sigma_d}p_{a0}$, see Section 4.4.4, the empirical constant for a developed cavitation cone $k_{cav}$ can be written as:

$$k_{cav} = k\sqrt{\sigma_d}\sqrt{\frac{p_{j}}{p_{a0}}}$$  \hspace{1cm} (4.13)

This results in the following relation for the entrainment coefficient:

$$\alpha_{mom,cav} = \sqrt{\frac{1}{2k}}\sqrt{\frac{p_{a0}}{\sigma_dp_{j}}}$$  \hspace{1cm} (4.14)

Substituting $k = k_{cav}$ into Equation 4.11 results in the following relation for the normalized stagnation pressure ($p_{j} > p_{cav}$):

$$\frac{p_{stag}}{p_{j}} = \frac{k}{2}\sqrt{\sigma_d}\sqrt{\frac{p_{j}}{p_{a0}}} \left(\frac{D_n}{s}\right)^2$$  \hspace{1cm} (4.15)

### 4.5.2 Measured vs calculated stagnation pressures

The measured and the calculated normalized stagnation pressures (Equation 4.15) are plotted in Figure 4.16 as function of the jet pressure. The jet distance is $12D_n$ and the nozzle diameter is 5 mm. The values for $k$ and $\sigma_d$ are 77 and 0.045, respectively. The measured and the calculated normalized stagnation pressures are in good agreement with each other.

Figure 4.17 shows all measured stagnation pressures for the 5 mm nozzle at a distance of $12D_n$ according to Equation 4.15. The correlation is quite good.

The measured stagnation pressures for the Yahiro-nozzle and the obtained stagnation pressures from Soyama and Lichtarowicz (1996) are plotted in Figure 4.18 in the same way as in Figure 4.17. As a reference, the estimated values (according to Equation 4.15) are also plotted (striped line). Again the value of $\sigma_d$ is 0.045, but for $k$, a value of 95 is used. This value results in a better correlation than the previously used value of 77. A value of 95 is not unrealistic. Depending on the outlet conditions of the jet, values up to 115 are mentioned, see Fondse et al. (1983).
4.5. Analyses free cavitating jet tests

Figure 4.16: Measured (Eq. 4.15; \( \sigma_d = 0.045, k = 77 \)) and calculated normalized stagnation pressures as function of the jet pressure for different ambient pressures (\( D_n = 5 \text{ mm}, s/D_n = 12 [-] \)).

Figure 4.17: Correlation according to Equation 4.15 (\( D_n = 5 \text{ mm}, s/D_n = 12 [-] \)).

All data points obtained by Soyama and Lichtarowicz (1996) lie a little above the reference line. Probably this is due to the differences in nozzle geometry. By increasing the value of \( k \) a little, a better correlation will be found.

The values of the Yahiro-nozzle measured at long jet distances are beneath the reference line (data points located near the origin), which means that the stagnation pressures calculated by equation Equation 4.15 are slightly overestimated. This can be explained by a decrease in the effectiveness of the cavitation cone at long distances. Equation 4.15 assumes a constant effectiveness over the whole jet distance.
4. Cavitation jet

Figure 4.18: Correlation according to Equation 4.15 for the Yahiro nozzle ($D_n = 3$ mm, $p_{a0} = 0.13$ MN/m$^2$) and the data obtained from Soyama and Lichtarowicz (1996) ($D_n = 2.5$ mm, $p_{a0} = 0.16-0.4$ MN/m$^2$).

4.5.3 Effective length cavitation cone

Equation 4.15 is valid only in the fully developed cone region ($s \geq s_{cd}$). In the cone development region the stagnation pressure in the center of the jet equals the jet pressure ($s < s_{cd}$: $p_{stag} = p_j$). The length of the cone development region can be calculated by substituting $p_j$ for $p_{stag}$ in Equation 4.15. This results in the following relation for the normalized length of the cone development region:

$$s_{cd} = \sqrt{\frac{k}{2}} \sqrt[4]{\frac{\sigma_d p_j}{p_{a0}}} = \frac{1}{2\alpha_{mom,cav}}$$

(4.16)

It is difficult to estimate the length of the fully developed cone region. The jet conditions for which no new bubbles are formed are unknown. It was initially assumed that the jet pressure for cavitation cone development ($p_{cav}$) is also the minimum pressure required for the formation of new bubbles. In this case, the fully developed cone region ends when the maximum stagnation is decreased to the value of $p_{cav}$ ($= p_{a0}/\sigma_d$). The jet distance that satisfies this condition can be calculated with Equation 4.15. It follows that:

$$s_{fde} = \sqrt{\frac{k}{2}} \sigma_d^{1.5} \left(\frac{p_j}{p_{a0}}\right)^{1.5} \approx b_{fde} \sigma_{cav}^{-0.75}$$

(4.17)

where $b_{fde}$ is an empirical constant, depending on the nozzle geometry and $\sigma_{cav}$ is a sort of cavitation index, defined as $p_{a0}/p_j$. For the used nozzle the value of $b_{fde}$ ranges from about 0.6 to 0.67.

Assuming that the influence of the cone on the momentum exchange in the disappearing cone region is negligible, Equation 4.15 is valid from $s_{cd}$ to $s_{fde}$. For
4.5. Analyses free cavitating jet tests

jet distances \( s > s_{f_{dc}} \) the value of the entrainment coefficient is the same as for a non-cavitating jet. At these distances, the influence of cavitation can be taken into account by introducing a fictitious displacement \( \Delta s \) in Equation 4.11, see Figure 4.19:

\[
\frac{p_{stag}}{p_j} = \frac{k}{2} \left( \frac{D_n}{s - \Delta s} \right)^2, \text{ for } s > s_{f_{dc}}
\]  

(4.18)

![Figure 4.19: Definition sketch of the shift in jet distance in the equation for the stagnation pressure of a non-cavitating jet to discount for the influence of cavitation.](image)

The fictitious displacement \( \Delta s \) can be calculated by substituting \( s_{f_{dc}} \) (Equation 4.17) for \( s \) and \( p_{cav} \) for \( p_{stag} \) in Equation 4.18. This results in the following relation for \( \Delta s \):

\[
\Delta s = \sqrt{\frac{k}{2} \sigma_d \frac{p_j}{p_{a0}} \left( \sqrt{\frac{\sigma_d p_j}{p_{a0}}} - 1 \right)} D_n
\]  

(4.19)

The calculated normalized stagnation pressures for the Yahiro-nozzle, according to the calculation method as described above, are plotted as a function of the normalized jet distances in Figures 4.20 and 4.21. As a reference, the measured values are also plotted. To fit the data also into the non-cavitating regime \( (p_j = 2.5 \text{ MN/m}^2) \), a k-value of 95 is used (see the section above). All measured values (except that at a normalized jet distance of 9) are in good agreement with the calculated ones.

For a jet pressure of 20 MN/m\(^2\), the fully developed cone region ends at a normalized jet distance of 33. Beyond the end of the fully developed cone region, the
entainment per unit length increases to the value of a non-cavitating jet, resulting in a stronger decrease in stagnation pressure with jet distance, see Figure 4.21.

![Figure 4.20: Measured and calculated (Equation 4.15 and 4.18) normalized stagnation pressures for the Yahiro-nozzle at an ambient pressure of 0.13 MN/m² (Dₙ = 3 mm, k = 95, σ₃ = 0.045).](image)

Figure 4.20: Measured and calculated (Equation 4.15 and 4.18) normalized stagnation pressures for the Yahiro nozzle (close-up of Figure 4.20).

![Figure 4.21: Measured and calculated (Equation 4.15 and 4.18) normalized stagnation pressures for the Yahiro nozzle (close-up of Figure 4.20).](image)

Figure 4.21: Measured and calculated (Equation 4.15 and 4.18) normalized stagnation pressures for the Yahiro nozzle (close-up of Figure 4.20).

### 4.5.4 Visible length cavitation cone

In Table 4.8, some observations of the visible cone length (s_nₐₚ) are compared with the calculated distance of the end of the fully developed cone region (s_fdₑₚ, for definitions see Figure 4.15). It appears that s_fdₑₚ is significantly shorter than s_nₐₚ. The visible cone is apparently not effective over the full length. This is consistent with the observation that even before any influence of cavitation on the stagnation pressure is measured (p_j < p_cav = 2.4MN/m²), already a cone is clearly visible.
Table 4.8: Comparison between the observed cavitation cone length ($s_{nc}$) and the calculated end of the fully developed cone region ($s_{f,dc}$) for the 5mm-nozzle at an ambient fluid pressure of 0.13 MN/m$^2$.

<table>
<thead>
<tr>
<th>$p_j$ [MN/m$^2$]</th>
<th>$s_{f,dc}$ [mm]</th>
<th>$s_{nc}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>2.6</td>
<td>33</td>
<td>98</td>
</tr>
<tr>
<td>5.0</td>
<td>54</td>
<td>133</td>
</tr>
<tr>
<td>7.0</td>
<td>70</td>
<td>155</td>
</tr>
</tbody>
</table>

It is found that the visible cone length corresponds with the condition $p_{stag} = 0.15 p_{cav}$, see Figure 4.22:

$$\frac{s_{nc}}{D_n} = \sqrt{\frac{1}{0.15^2}} k \sigma_d^{1.5} \left( \frac{p_j}{p_{a0}} \right)^{1.5} \approx b_{nc} \sigma_{cav}^{-0.75}$$ \hspace{1cm} (4.20)

Where $b_{nc}$ is an empirical constant. For the used nozzle the value of $b_{nc}$ ranges from about 1.57 to 1.74. Soyama and Lichtarowicz (1996) found an identical type of relation for the visible cone length:

$$\frac{s_{nc}}{D_n} = b_{nc} \sigma_{cav}^{-m_{nc}}$$ \hspace{1cm} (4.21)

Table 4.9 shows the values found by Soyama and Lichtarowicz (1996) for the empirical constants $b_{nc}$ and $m_{nc}$, depending on the nozzle geometry. These values produce a similar result as the values found in the present study, see also Figure 4.22.

Table 4.9: Empirical constants $b_{nc}$ and $m_{nc}$ found by Soyama and Lichtarowicz (1996).

<table>
<thead>
<tr>
<th>Nozzle type</th>
<th>$b_{nc}$ [-]</th>
<th>$m_{nc}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical</td>
<td>1.73</td>
<td>0.83</td>
</tr>
<tr>
<td>Conical</td>
<td>3.04</td>
<td>0.67</td>
</tr>
</tbody>
</table>
4. Cavitating jet

Figure 4.22: Measured and calculated normalized visible cone length for the 5mm-nozzle at an ambient fluid pressure of 0.13 and 0.5 MN/m².

4.5.5 Modeling the entrainment coefficient

To model the moving cavitating jet penetrating cohesive soil, the entrainment coefficient must be known. In Table 4.10 a summery is listed, how to calculate the entrainment coefficient of the cavitating jet in the different regions.

Table 4.10: Summary calculation of the entrainment coefficient of the cavitating jet in the 1D-approach.

<table>
<thead>
<tr>
<th>Region</th>
<th>Distance [m]</th>
<th>Entrainment coefficient [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone Development</td>
<td></td>
<td>Transfer function&lt;sup&gt;1)&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>$s &lt; s_{cd}$</td>
<td>$p_{stag}(s_{cd}) = p_j$ (Eq. 4.16)</td>
</tr>
<tr>
<td>Fully developed cone</td>
<td>$s_{cd} \leq s &lt; s_{fdc}$</td>
<td>$\alpha_{mom,cav} = \sqrt{\frac{1}{2k}} \sqrt{\frac{P_{ad}}{\sigma_d p_j}}$</td>
</tr>
<tr>
<td></td>
<td>$p_{stag}(s_{fdc}) = p_{cav}$ (Eq. 4.17)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s \geq s_{nc} \approx s_{fdc} + 2D_n$&lt;sup&gt;2)&lt;/sup&gt;</td>
<td>$\alpha_{mom} = \frac{1}{\sqrt{2k}}$</td>
</tr>
</tbody>
</table>

<sup>1)</sup> The transfer function can be derived similar as for the non-cavitating jet, see Section 3.2.2.  <sup>2)</sup> The length of the disappearing region is assumed to be $2D_n$. So in the model the entrainment coefficient changes gradually from $\alpha_{mom,cav}$ to $\alpha_{mom}$ over a length of $2D_n$.

Figure 4.23 shows an example of the development of the entrainment coefficient for a 5mm-nozzle at an ambient pressure of 0.13 MN/m². At a jet pressure of 20 MN/m² the entrainment is over a length of about 30 times the nozzle diameter nearly 40% lower than in the non-cavitating case.
4.6 Conclusions

The maximum jet pressure of the exploratory low pressure tests in cohesive soil was 0.82 MN/m², for a nozzle diameter of 32.5 mm. The maximum jet pressure of the second test program was 1.06 MN/m², for a fictitious nozzle diameter of 20 mm. These jet pressures are a little lower or equal to the expected jet pressures for cavitation cone development. Because of this it is assumed that only for the high pressure jet tests, the influence of cavitation is of importance. For the low pressure jet tests in cohesive soil the influence of cavitation is neglected.

Another uncertainty is the influence of the enclosed area, in the case of a moving jet in soil. Because the jet cavities at high jet pressures are relatively narrow and deep the entrained ambient water is more hindered than for the cavitating free jet. Because of this, it is expected that the enclosed jet will cavitate earlier. This influence is not yet taken into account.
4. Cavitating jet

**Cavitation in dredging practice** The jet pressure for cavitation cone development, increases linearly with ambient pressure, see Figure 4.11. In dredging practice the ambient pressures are usually higher than about 2.5 MN/m$^2$ (water depths are larger than about 15 m). At these ambient pressures the needed jet pressures for cavitation are even higher than normally applied ($p_j < 2$ MN/m$^2$). It can be concluded that in the current dredging practice cavitation of jets does not occur.
Chapter 5
Jet tests in cohesive soil

5.1 Experimental setup

5.1.1 Introduction
Two test programs with a moving vertical submerged jet acting on artificially prepared cohesive soil surfaces were conducted.

The first program was conducted in the summer of 2004. The main objective of this exploratory jet program was to visualize and document the different failure mechanisms of the soil. A broad range of jet parameters was tested. In total 72 jet tests were conducted in this program. Based on these test results some hypotheses were formulated concerning the occurrence of the different failure modes.

Because of the relatively high traverse velocity of the jet, it was not possible to record the failure of the soil by a video camera with a standard frame rate. For this reason and to substantiate the hypotheses, in 2007 a second jet program was conducted, where the failure of the soil was recorded with a high speed camera (1000-3000 frames per second). For both test programs more or less the same experimental setup was used. This setup is discussed in Section 5.1.2.

All tests were conducted on artificially prepared cohesive soil samples with the same particle composition. Only the undrained shear strength ($su$) was varied, because this parameter is assumed to be the most important one. The composition and preparation of the soil samples is discussed in Section 5.1.3.

5.1.2 Setup
The jet tests were conducted in the Dredge Flume of Deltares (formerly Delft Hydraulics), see Figure 5.1. One side of this flume consists of a glass wall. Soil samples (width x height x length: 0.2x0.5x1.5 m) were placed and pre-stressed against this glass wall, by means of a rigid plate on the opposite side which can
5. Jet tests in cohesive soil

Figure 5.1: Dredge Flume of Deltares. On the background the carriage with the piston pump.

Figure 5.2: Position of the soil samples in the Dredge Flume.

be moved in the direction of the glass wall, see Figures 5.2 and 5.3. On the window side of the soil samples a grid of small squares was printed to visualize the deformations in the soil during the jetting process. The dislodged soil lumps (clusters of soil particles) were collected in a chamber in the front of the soil samples.

Figure 5.3: Setup jet tests (a) top view (b) cross section, with the jet pipe positioned for jetting along the glass wall.

The jet installation was placed on a moving carriage. The velocity of this carriage could be accurately adjusted. The jet installation consisted of a variable speed jet pump and a vertical jet pipe with at the bottom a nozzle. The nozzle was
5.1. Experimental setup

positioned about 25 mm above the soil samples. Two jet pumps were used. A centrifugal pump was used for the low pressure tests ($p_j < 1 \text{ MN/m}^2$). For the high pressure jets tests a piston pump was used. Fluctuations of the discharge pressure of the piston pump were smoothed with an accumulator.

The water level was kept constant about 0.5 m above the surface of the soil samples during the tests. The time of submergence of soil samples was as short as possible (within a hour), to prevent softening.

The horizontal traverse velocity of the nozzle, the jet flow and the jet pressure were logged.

**Exploratory jet program**

One sample was used for every test. Most tests were conducted in the middle of a soil sample. To observe/record the failure of the soil in the vertical plane, also tests were conducted jetting directly along the glass wall.

Two different nozzle diameters (20 and 32.5 mm) were used for the low pressure tests ($p_j = 0.41-0.82 \text{ MN/m}^2$), see Appendix E, Figure E.8. The measured discharge coefficient of these nozzle’s ($\mu_n$) was about 1. Three nozzle diameters (3, 5 and 7 mm) were used for the high pressure tests ($p_j = 2.6-15.6 \text{ MN/m}^2$). The same nozzle’s were used for the cavitating jet tests, as described in previous chapter, see Figure E.7. The calculated discharge coefficients of these nozzle’s were: 0.89, 0.88 and 0.89, respectively.

The failure of the soil in the vertical plane was recorded with two stationary video cameras positioned behind the glass wall in the flume parallel to the Dredge Flume. The deformations on the soil surface were recorded with a stationary underwater video camera.

After every test the following parameters were determined: the characteristic dimensions of the jet cavity, the jet production by measuring the difference in weight of the soil sample before and after the test, the characteristic dimensions of the dislodged soil fragments and the undrained shear strength of the soil sample.

The main parameters of the exploratory test program are listed in Table 5.3. The stand off distance ($SOD$) was held constant for all tests.

**Second jet program**

Main objective of this test program was to visualize the failure mechanisms in the cohesive soil more in detail, so that the formulated working hypotheses could be verified, see Section 5.3. Therefore only jet tests directly along the glass wall were conducted.
5. Jet tests in cohesive soil

Table 5.1: Test settings of the exploratory test program.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low jet pressure ($p_j$)</td>
<td>0.4, 0.6, 0.8 [MN/m²]</td>
</tr>
<tr>
<td>High jet pressure ($p_j$)</td>
<td>3.5, 5.4, 7.0, 10.4, 11.2 and 17 [MN/m²]</td>
</tr>
<tr>
<td>Nozzle diameter ($D_n$)</td>
<td>3, 5, 7, 20 and 32.5 [mm]</td>
</tr>
<tr>
<td>Stand Off Distance ($SOD$)</td>
<td>20-30 [mm]</td>
</tr>
<tr>
<td>Traverse velocity nozzle ($v_t$)</td>
<td>0.12, 0.25, 0.5, 1.0, 1.5 and 2.0 [m/s]</td>
</tr>
<tr>
<td>Strength of the soil ($s_u$)</td>
<td>20, 45 and 70 [kN/m²]</td>
</tr>
</tbody>
</table>

The glass wall was assumed to be a vertical symmetry plane through the centerline of the jet. Therefore for these tests nozzles with a section of half a circle were used. The used diameters were 28 and 45 mm, see Appendix E, Figure E.9. The measured discharge coefficients were 0.81 and 0.94, respectively. It was determined that, despite the shape of the nozzles, the jet section was circular. Therefore the glass wall could not be considered as a clear symmetry plane. To characterize the jets a fictitious nozzle diameter was defined: $D_{n,f} = \sqrt{4A_n/\pi}$ ($D_{n,f,28} \approx 20$ mm and $D_{n,f,28} \approx 32$ mm).

The failure of soil in the plane of symmetry of the jet was recorded with a high speed video camera. The camera was zoomed on a small area. In Table 5.2 the main settings of the camera are listed.

Table 5.2: Main settings high speed camera.

<table>
<thead>
<tr>
<th>Frame rate [Hz]</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution [pixels]</td>
<td>1024x1024</td>
<td>1024x512</td>
<td>512x512</td>
</tr>
<tr>
<td>Image dimension [mm]</td>
<td>200x200</td>
<td>100x50</td>
<td>50x50</td>
</tr>
<tr>
<td>Recording time [sec]</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Because of the circular shape of the jets, between the front side of the jet and the glass wall a thin slice of soil remained. Unfortunately, this slice impeded the sight on the jet front at some tests. As a result it was not always clear if the visible deformations are from the actual jet front or from a thin soil layer between the jet and the glass wall. To improve the sight on the jet front the shape of the 28mm-nozzle was changed a little, see Appendix E, Figure E.9 (a).
Another problem occurred with the stronger soil samples. It was not possible to
attach the soil sample over the full surface to the glass wall. Because of this the
jet could penetrate between the soil sample and the glass wall. Resulting in a
different failure pattern of the soil.

The main parameters of the second test program are listed in Table 5.3.

Table 5.3: Test settings of the second test program.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet pressure ($p_j$)</td>
<td>0.6, 8.8, 1.06</td>
<td>[MN/m²]</td>
</tr>
<tr>
<td>Nozzle diameter ($D_n$)</td>
<td>28 and 45</td>
<td>[mm]</td>
</tr>
<tr>
<td>Stand Off Distance ($SOD$)</td>
<td>≈ 20</td>
<td>[mm]</td>
</tr>
<tr>
<td>Traverse velocity nozzle ($v_t$)</td>
<td>0.15, 0.5, 1.0, 1.4 and 1.8</td>
<td>[m/s]</td>
</tr>
<tr>
<td>Strength of the soil ($s_u$)</td>
<td>29-60</td>
<td>[kN/m²]</td>
</tr>
</tbody>
</table>

5.1.3 Used soil samples

Composition and preparation

The jet tests were conducted on artificially prepared cohesive soil samples. The particle composition of all samples was the same, only the water content (undrained shear strength) and preparation process was varied.

The soil samples were composed of Koalin WBH (55%), Colclay D90 (10%), Assersand (35%) and tap water. Koalin WBH is a grinded Kaolinite, Colclay D90 is a grinded Sodium-bentonite (Montmorillonite) and Assersand is sand (quartz) from a quarry near Assen (The Netherlands) with a mean particle size of 95 µm. The particle size distribution of the different materials are listed in Table 5.4. The mean specific density of the particles ($\rho_s$) was 2607 ±2 kg/m³ (Greeuw, 2004), determined by Deltares, according to NEN5111 (Pycnometer method). The specific densities of the different components, listed in Table 5.4 are indicative, see Table 2.2.

Two different processes were used to prepare the soil samples.

**DH-samples** Most of the soil samples were prepared at Deltares in Delft (formerly Delft Hydraulics). These samples were prepared by mixing the soil particles with a measured quantity of water for some minutes, and subsequently extruding the soil by means of a screw-conveyor to unsaturated clay-samples. The dimensions of the soil samples were: width 0.2 m, height 0.5 m and length 1.5 m.
5. Jet tests in cohesive soil

Table 5.4: Particle size distribution and specific density ($\rho_s$) of different soil components.

<table>
<thead>
<tr>
<th>Percentage of weight larger than [%]</th>
<th>$\rho_s$ [kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2$\mu$m 63$\mu$m 90$\mu$m 125$\mu$m 200$\mu$m</td>
</tr>
<tr>
<td>Asserzand (35%)</td>
<td>-        85      55      15      2       2650</td>
</tr>
<tr>
<td>Colclay D90 (10%)</td>
<td>-        12      5       2       0       2530 ±170</td>
</tr>
<tr>
<td>Koalin WBH (55%)</td>
<td>-        25      15      10      0       2670 ±10</td>
</tr>
<tr>
<td>Soil samples (100%)</td>
<td>64       31      20      13      2       2607 ±2</td>
</tr>
</tbody>
</table>

amount of the added water determines the undrained shear strength of the soil samples. The samples were engineered to have an undrained shear strength ($su$) of 20, 45 and 70 kN/m$^2$, referred as DH$_{20}$, DH$_{45}$ and DH$_{70}$, respectively.

The main advantages of this preparation process, compared with a natural consolidation process, are: (1) in a relative short time a large volume/amount of soil samples can be prepared (2) at low costs. However, the disadvantages of this preparation process are: (1) occurrence of planes of weakness, (2) incompletely saturated and (3) no natural consolidation process.

The DH-samples had a curved and layered structure, which was caused by the rotating screw which presses thin layers of soil to the sample, see Figure 5.4 and 5.5. Failure of the soil would preferably occur along these curved planes of weakness.

The DH-samples were not completely saturated. The degree of saturation ($S$) was about 95%, see Table 5.5. The soil bulk density ($\rho_b$), water content ($w$) and
5.1. Experimental setup

Table 5.5: Soil properties DH- and GD-samples.

<table>
<thead>
<tr>
<th></th>
<th>DH$_{20}$</th>
<th>DH$_{45}$</th>
<th>DH$_{70}$</th>
<th>GD$_{40}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil bulk density ($\rho_b$) [kg/m$^3$]</td>
<td>1837</td>
<td>1944</td>
<td>1982</td>
<td>1890</td>
</tr>
<tr>
<td>Water content ($w$) [%]</td>
<td>30.0</td>
<td>23.7</td>
<td>21.8</td>
<td>31.5</td>
</tr>
<tr>
<td>Degree of saturation ($S$) [%]</td>
<td>93</td>
<td>94</td>
<td>95</td>
<td>100</td>
</tr>
<tr>
<td>Porosity ($n$) [-]</td>
<td>0.46</td>
<td>0.40</td>
<td>0.37</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Porosity ($n$) are also listed in this table. These parameters were determined by measuring the wet ($m_w$) and dry ($m_d$) mass of 13 samples with a known volume ($V_{sample}$):

$$\rho_b = \frac{m_w}{V_{sample}} \quad (5.1)$$
$$w = \frac{m_w - m_d}{m_d} \quad (5.2)$$
$$S = \frac{V_w}{V_p} \cdot 100\% = \frac{\frac{m_w - m_d}{\rho_w}}{V_{sample} - \frac{m_d}{\rho_s}} \cdot 100\% \quad (5.3)$$
$$n = \frac{\rho_b - \rho_s}{S\rho_w - \rho_s} \quad (5.4)$$

where $\rho_s$ is the specific density of the particles/solids and $\rho_w$ is the water density. The porosity ($n$) can also be checked with the following relation:

$$n = \frac{\rho_b w}{S\rho_w(1 + w)} \quad (5.5)$$

The water content ($w$) and bulk density ($\rho_b$) of the DH-samples were also determined by Deltares (Greeuw, 2004).

Another disadvantage of the DH-samples, more difficult to quantify, is that the preparation process differs from the natural consolidation process. During a natural consolidation process pore water is slowly driven out of the pores between the particles due to a top load (or self weight) until the excess pore water pressure is dissipated. This process is much more gradually.

Most soil samples used in the exploratory jet program were recycled and again used in the second test program. After the exploratory jet program the samples were chopped up and again extruded to new samples.
5. Jet tests in cohesive soil

**GD-samples** To investigate the influence of the preparation process also some samples were prepared by a natural consolidation process. This consolidation process was carried out by Deltares in Delft (formerly GeoDelft).

In a consolidation cell with a diameter of 1.25 m a mud, with the same particle composition as the DH-samples, was consolidated under a vertical consolidation pressure ($\sigma_{v0}$) of 330 kN/m$^2$. The mud was mixed under vacuum. The initial height of the mud sample was 690 mm with a water content of 82%. The consolidation process is analyzed in Appendix G. The cell was drained on both sides. After 5616 hours (234 days) the consolidation process was stopped. The final settlement was about 325 mm. The measured water content of the consolidated soil sample was 31.5% The corresponding undrained shear strength ($su$) was about 40 kN/m$^2$. During the consolidation process the measured pore water pressure ($p_w$) reduced from 330 to 9.6 kN/m$^2$. The measured pore water pressure and settlement time-development of the consolidation process are listed in Appendix G, Figures G.3 and G.4.

The degree of saturation ($S$) of the GD-samples was about 100%, based on two measurements, see Table 5.5.

**Storage** After preparation the soil samples were packet air- and watertight and stored in air at a temperature between 15 and 20 °C and a relative humidity of 80 to 90%, for shorter than a month. It was determined that the undrained shear strength of the samples remained more or less constant, during the storage.

**Geotechnical parameters**

The determined particle size distribution in percentages of weight (classification according to NEN, see Table 2.1) is: 36% clay (< 2 µm), 33% silt and 31% sand (> 63 µm), see also Table 5.4. The clay fraction globally consists of 85% Kaolinite and 15% Montmorillonite. The determined $D_{50}$ is 5 µm.

The Atterberg limits are listed in Table 5.6. The plasticity index of the GD-sample is higher than that of the DH-sample. As a result the liquidity index ($LI$) of the GD-sample at the same porosity, assuming a fully saturated sample, will be smaller whereas the undrained shear strength ($su$) will be higher than that of the DH-sample.\(^1\) The higher plasticity of the GD-sample means that the ability the bind water of this sample is higher than that of the DH-sample. Probably, the double layers between the clay particles are developed much better in the GD-sample, due to the preparation process.

\(^1\)The $LI$ of the GD-sample is 0.24 (w = 31.5%). Assuming that the DH$_{20}$-sample would be fully saturated the water content ($w$) should also be 31.5% for this sample. In this case the $LI$ of the DH$_{20}$-sample would be 0.29. Substituting these values of $LI$ in Equation 2.50 gives a difference in (remoulded) undrained shear strength of about 20%.  

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5.1. Experimental setup

Table 5.6: Atterberg limits determined by Deltares (NEN 5104).

<table>
<thead>
<tr>
<th></th>
<th>PL [%]</th>
<th>LL [%]</th>
<th>PI [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GD-sample</td>
<td>17</td>
<td>77</td>
<td>60</td>
</tr>
<tr>
<td>DH-sample</td>
<td>17</td>
<td>67</td>
<td>50</td>
</tr>
</tbody>
</table>

According to ASTM D2487, the soil can be classified as: inorganic clay with high plasticity.

The Montmorillonite appears to be very active. The activity of the clay \( (A) \) in the samples is about 1.5. Assuming an activity of 0.4 for the Kaolinite the activity of the Montmorillonite must be about 7.5.\(^2\) This is very high, in literature values up to 7 are mentioned (Mitchell and Soga, 2005).

To determine the water permeability \( (k_w) \), consolidation coefficient \( (c_v) \) and a rough estimate of the mean consolidation pressure \( (\sigma_{v0}) \) of the different DH-samples one-dimensional compression tests (oedometer tests) were conducted, according to NEN 5118, see Appendix F.1. Because the water permeability of the unsaturated soil samples was very low, the standard duration of the load-steps were too short to determine the consolidation coefficients accurately. Additionally two Constant Rate of Strain oedometer tests (CRS-tests) were conducted, on a sample with a height of 20 mm and a strain rate of 0.5 and 0.1%/hour, see Appendix F.2. The main results are listed in Table 5.7. For the GD-sample the different parameters were derived from the consolidation process in a large consolidation cell at a vertical pressure of 330 kN/m\(^2\), see Appendix G.

**Undrained shear strength** The mean undrained shear strength of all samples was determined before and after the jet tests. At 3 predefined positions the strength was determined with a pocket-penetrometer (ppm) and a hand-torvane. The used undrained shear strength was defined as the average value of the shear strengths determined with the ppm before the tests. The standard deviation of the measured shear strength with the ppm was 7.3%. The shear strengths measured with hand-torvane were about 2% higher than the ppm-values.

To correlate the shear strengths determined with the ppm, the undrained shear strengths of the three strength classes were also determined by means of unconsolidated undrained triaxial tests (UU-triaxial tests, according to NEN5117, see Appendix F.4) and lab-vane tests, see Appendix F.3. The determined peak shear strengths \( (s_{up}) \) are listed in Table 5.8. The undrained shear strengths of these

\[^2\]15% \cdot 7.5 + 85% \cdot 0.4 \approx 1.5 [-].
5. Jet tests in cohesive soil

Table 5.7: Geotechnical parameters DH- and GD-samples. The values without reference are from the Geotechnical soil report of Deltares (Greeuw, 2004).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DH20</th>
<th>DH45</th>
<th>DH70</th>
<th>GD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cons. coeff. ((c_v) \cdot 10^{-9}) [m²/s]</td>
<td>0.5-1</td>
<td>2-5</td>
<td>&lt; 1</td>
<td>7</td>
</tr>
<tr>
<td>Permeability ((k_w) \cdot 10^{-10}) [m/s]</td>
<td>0.9</td>
<td>-</td>
<td>-</td>
<td>1.1</td>
</tr>
<tr>
<td>Compression index ((C_c)) [-]</td>
<td>0.35</td>
<td>0.22</td>
<td>0.15</td>
<td>0.54</td>
</tr>
<tr>
<td>Vert. cons. press. ((\sigma_{v0})) [kN/m²]</td>
<td>50-65</td>
<td>80</td>
<td>140</td>
<td>-</td>
</tr>
</tbody>
</table>

more controlled tests, in particular for the DH45-sample, are significantly lower than the ppm-values. The undrained shear strengths of the GD-samples were unfortunately only determined by means of pocket-penetrometer and hand-torvane.

Table 5.8: Peak undrained shear strengths of DH- and GD-samples in [kN/m²]. For the UU-triaxial tests also the axial strain \((\varepsilon_b)\) at the peak are listed.

<table>
<thead>
<tr>
<th>Method</th>
<th>DH20</th>
<th>DH45</th>
<th>DH70</th>
<th>GD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pocket-penetrometer (s_{u_p})</td>
<td>22</td>
<td>45</td>
<td>82</td>
<td>41</td>
</tr>
<tr>
<td>Hand-torvane (s_{u_p})</td>
<td>27</td>
<td>48</td>
<td>85</td>
<td>39</td>
</tr>
<tr>
<td>Lab-vane (see Tbl. F.2) (s_{u_p})</td>
<td>18</td>
<td>35</td>
<td>86</td>
<td>-</td>
</tr>
<tr>
<td>UU-triaxial (see Fig. F.16) (s_{u_p})</td>
<td>18.3</td>
<td>32.6</td>
<td>71.2</td>
<td>-</td>
</tr>
<tr>
<td>(\varepsilon_b) [%]</td>
<td>(6.4)</td>
<td>(9.6)</td>
<td>(7.2)</td>
<td>-</td>
</tr>
</tbody>
</table>

Because of the differences in determined undrained shear strength, according to different methods, a bandwidth of ±20% is assumed. This bandwidth is based on the difference between the maximum \((s_{u_{max}})\) and minimum value \((s_{u_{min}})\) of the determined undrained shear strengths on a sample, divided by two times the mean value of the four determination methods \((s_{u_{mean}})\), see Table 5.8, and taking in consideration the variation in shear strength over a sample.

In Appendix F.4 the effective stress paths of the UU-triaxial tests are analyzed. The DH20-sample behaves like a normally consolidated soil, however the softening
5.1. Experimental setup

after reaching the peak deviates significantly. During pre-peak deformation the soil sample shows a strong contraction, accompanied by a pore water pressure increase. The softening is possibly caused by the degree of saturation (see Table 5.5 for the measured degree of saturation).

The other two samples behave more like lightly over-consolidated and heavily over-consolidated soils, respectively. After a limited contraction, accompanied by a water pressure increase, the pore water stresses decreases due to the tendency to dilate, see also Figure 2.5. This corresponds with the determined parameters of Henkel ($\alpha_h$) and the estimated mobilized peak friction angles ($\varphi_p$), see Table 5.9. The values of parameter $\alpha_h$ decreases with increasing strength. This indicates that the over-consolidation ratio increases with the strength from normally consolidated ($OCR = 1$) for the DH$_{20}$-sample to heavily over consolidated ($OCR \geq 4$) for the DH$_{70}$-sample, see Table 2.4. For normally consolidated cohesive soils a practical value of $\varphi_p$ is about 20°. For heavily over-consolidated soils values up to 50° are possible, see Section 2.1.

Table 5.9: Determined parameters of Henkel ($\alpha_h$, $\beta_h$) and estimated mobilized peak friction angles ($\varphi_p$), see Appendix F.4, Table F.5.

<table>
<thead>
<tr>
<th>Parameter of Henkel (1960)</th>
<th>DH$_{20}$</th>
<th>DH$_{45}$</th>
<th>DH$_{70}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_h$ [-]</td>
<td>0.62</td>
<td>-0.01</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\beta_h$ [-]</td>
<td>0.83</td>
<td>0.87</td>
<td>0.81</td>
</tr>
<tr>
<td>Mobilized peak friction angle $\varphi_p$ [°]</td>
<td>16-20</td>
<td>19-25</td>
<td>25-33</td>
</tr>
</tbody>
</table>

Based on the UU-triaxial tests also an estimation of the stress state at the critical state was made see Appendix F.4, Figure F.17. The mobilized friction angle at the critical state ($\varphi_{cs}$) is assumed to range from 12° to 15°. The estimated values of the undrained shear strength at the critical state ($su_{cs}$) and the corresponding sensitivity $S_s$, defined as $su_p/su_{cs}$ are listed in Table 5.10. It is assumed that sensitivity of the GD-sample is equal to that of the DH$_{20}$-sample, because both behave normally consolidated.

**Swelling**  The time of submergence of the soil samples during the tests was as short as possible (within one hour), to prevent softening. The undrained soil strength ($su_p$) at the contact surfaces between soil and ambient water decreased

---

3For unsaturated soils menisci increase the inter-particle attraction by surface tension. When the soil is sheared, these menisci break down partly, the mobilized resistance reduces, causing the soil to undergo softening (Mitchell and Soga, 2005).
5. Jet tests in cohesive soil

Table 5.10: Estimated friction angles and undrained shear strengths of DH- and GD-samples at Critical State (CS).

<table>
<thead>
<tr>
<th></th>
<th>DH&lt;sub&gt;20&lt;/sub&gt;</th>
<th>DH&lt;sub&gt;45&lt;/sub&gt;</th>
<th>DH&lt;sub&gt;70&lt;/sub&gt;</th>
<th>GD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction angle at CS</td>
<td>( \varphi_{cs} ) [(^{\circ})]</td>
<td>12-15</td>
<td>12-15</td>
<td>12-15</td>
</tr>
<tr>
<td>Undr. shear strength at CS</td>
<td>( s_{u,cs} ) [kN/m(^2)]</td>
<td>15</td>
<td>23</td>
<td>44</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>( S_s ) [-]</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
</tr>
</tbody>
</table>

about 20\% during the tests. However the thickness of the swelling layer was limited: about 10 mm.<sup>4</sup>

**Evaluation of differences in soil properties due to preparation process**

The main differences of both preparation methods of the cohesive soil samples are:

- **Degree of saturation** \( (S) \). The DH-samples were not fully saturated, whereas the GD-samples were. As a result in the undrained triaxial compression tests to failure the DH-samples will not behave fully saturated \( (\beta_h < 1) \), whereas the GD-samples will.

- **Loading time**, allowing for creep and related aging. The loading time of the GD-samples was much longer than that of the DH-samples (months for the GD-samples versus minutes for the DH-samples). By aging the undrained shear strength can increase significantly, due to reorganization of the (clay-)particles and the water cation-structure (Leroueil et al., 1990), which simultaneously also affect the creep rate of deformation. The peak undrained shear strength of the GD-sample was significantly higher than that of the DH<sub>20</sub>-sample with roughly the same porosity \( (s_{u,p} = 40 \text{ kN/m}^2 \) for the GD-sample versus 22 kN/m\(^2\) for the DH<sub>20</sub>-sample), see Tables 5.5 and 5.8. This difference could be due to aging.

Furthermore the degree of saturation differed and was higher for the GD-sample. However, for normally consolidated soil the peak undrained shear strength is expected to decrease with increasing degree of saturation and therefore cannot explain the difference in peak strength.

- **Pre-loading conditions**. The pre-loading condition of the DH-samples is unknown, whereas the pre-loading of the GD-samples is well known. For the

<sup>4</sup>The predicted thickness of the swelling layer was smaller than 20 mm, assuming a negative consolidation coefficient of \( 5c_v \approx 5 \cdot 2 \cdot 10^{-10} \) (m\(^2\)/s). The swelling layer was defined as the layer in which the water under pressures were dissipated with at least 2\%. 

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DH-samples the pre-loading condition is estimated from the oedometer tests ($\sigma_{v0}$), see Appendix F.4, Table F.5. This makes it difficult to determine the initial effective stresses in the DH-samples.

- Homogeneity. The variability of the DH-samples is expected to be larger than that of the GD-samples. Consequently the band-width of the undrained shear strength for the DH-samples will also be larger than for the GD-samples.

The jet tests however show no significant difference between the DH- and GD-samples, see the next Section Figure 5.15, despite the above mentioned differences. The only observed difference is that at low traverse velocities in the DH-samples hydro-fracturing occurred, whereas in the GD-samples not.

5.2 Test results

5.2.1 Observations

Based on the cavity dimensions and wall structure, four different types of jets / failure modes can be distinguished:

- Penetrating jet (Figures 5.6 to 5.9), characterized by: (1) narrow deep cavities, (2) a soil wall structure with small straight nearly vertical nerves, (3) excavated soil that dissolves completely in the jet and ambient water.

- Deflecting jet (Figure 5.10), characterized by: (1) shallow cavities, (2) a soil wall texture with nerves deflecting in the direction opposite the traverse direction of the nozzle, (3) limited dislodged soil lumps that can be found after the tests.

- Dispersing jet flow (Figures 5.12 and 5.11), characterized by: (1) wide shallow cavities, (2) an irregular soil wall structure, (3) dislodged soil lumps that can be found after the tests.

- Hydro-fracturing (Figure 5.13), characterized by: Irregular cavity dimensions. The soil fails along preferred weak surfaces. Because of this the cavity dimensions can increase significantly, compared to the penetrating and deflecting jet.

Most tests were of the penetrating jet type, with the same characteristic cavity shape and cavity wall structure. The cross section of these cavities is rectangular, with a constant cavity width of about 1 to 2 times the nozzle diameter, see Figure 5.6. The soil surface is mostly pushed up, see Figure 5.6 (a) and (b). In some cases the soil at the edge of the cavity is pushed up in a regular pattern, see Figure 5.7.
5. Jet tests in cohesive soil

Figure 5.6: Typical jet cavity traversal cross-sections. Test settings: (a) $p_j/su_p = 33$, $D_n = 32.5$ mm, $v_t = 1$ m/s, (b) $p_j/su_p = 25$, $D_n = 20$ mm, $v_t = 1.5$ m/s, (c) $p_j/su_p = 35$, $D_n = 7$ mm, $v_t = 0.25$ m/s.

Figure 5.7: Regular pattern of soil that is pushed up at the cavity edge (a) overview, (b) detail (top view). Test conditions: $p_j/su_p = 21$, $D_n = 32.5$ mm, $v_t = 0.25$ m/s.
5.2. Test results

The wall structure on both sides of the cavity can be divided into three zones, see Figure 5.8:

1. Non-deflection zone
2. Deflection zone
3. Transport zone

![Diagram showing the division of the cavity into different zones](image)

Figure 5.8: Dividing cavity depth in different zones with a characteristic wall texture pattern. Test conditions: \( p_j/su_p = 29.4 \), \( D_{nf} = 20 \text{ mm} \), \( v_t = 0.15 \text{ m/s} \).

The non-deflection zone is situated directly below the soil surface. In this zone the wall structure consists of small nearly vertically orientated nerves. The distance between these nerves is for all tests about 2 to 5 mm, see Figure 5.9. The height of this zone depends mainly on the jet ratio \( (p_j/su_p) \) and to a lesser extent to the traverse velocity of the nozzle \( (v_t) \).

At a certain distance the nerves start to deflect backwards (deflection zone). The distance between the nerves in this zone is an order larger than in non-deflection zone. When the non-deflection zone is missing the jet is called a deflecting jet, see also Figure 5.10. The cavity depth of these tests is about \( 2.5D_n \).

In the transport zone near the cavity bottom, the wall texture has disappeared.

Tests with a cavity depth smaller than about \( 2.5D_n \) have an irregular cavity shape and wall structure, see Figures 5.11 and 5.12. In the present thesis these tests are called dispersing jet tests. For these tests the cavity width increases with a decrease in cavity depth. The soil surface also becomes irregular. After ending these tests, soil lumps (clusters of soil particles) can be retrieved, see Figure 5.12.
5. Jet tests in cohesive soil

Figure 5.9: Vertically oriented nerves in non-deflection zone. Test conditions: (a) \( \frac{p_j}{su_p} = 25.8, D_{n,f} = 20 \text{ mm}, v_t = 0.15 \text{ m/s} \), (b) \( \frac{p_j}{su_p} = 8.9, D_n = 5 \text{ mm}, v_t = 1 \text{ m/s} \).

Figure 5.10: Typical strip thickness of a deflecting jet (a) overview, (b) detail. Test conditions: \( \frac{p_j}{su_p} = 15.8, D_{n,f} = 20 \text{ mm}, v_t = 1 \text{ m/s} \).

(d) (in contrast with the penetrating jet tests where no lumps can be retrieved from the soil surface and in the area around).

For some tests with a very low traverse velocity of the nozzle a different phenomena occurs. The soil fails along the planes of weaknesses, resulting in relatively large cavity depths, see Figure 5.13. It seems that the jet flow has penetrated along these planes of weaknesses, and subsequently has pushed the soil apart. Apparently, a low traverse velocity is needed to let the micro cracks in the weak surfaces grow and coalesce, see Section 2.6.1.
5.2. Test results

Figure 5.11: Typical shallow cavity shape (a) top view, (b) transverse cross section. Test conditions: $p_j/su_p = 9$, $D_n = 32.5$ mm, $v_t = 1$ m/s.

Figure 5.12: Typical shallow cavity shape (a) top view, (b) transverse cross section, (c) longitudinal cross section, (d) collected lumps of dislodged soil. Test conditions: $p_j/su_p = 9$, $D_n = 20$ mm, $v_t = 1$ m/s.

5.2.2 Dimensional analysis

Several researchers have used dimensional analysis to determine the appropriate parameters to describe the excavation process of a (moving) vertical submerged circular jet in cohesive soil, see e.g. Mazurek et al. (2001). The main jet parameters are: the initial jet velocity ($u_0$), the nozzle diameter ($D_n$), the stand off distance ($SOD$), the traverse velocity of the nozzle ($v_t$), the jet flow density (initially the density of water $\rho_w$) and the kinematic viscosity ($\nu$). The soil is only characterized by the critical shear stress ($\tau_c$), defined as the bed shear stress at
5. Jet tests in cohesive soil

Figure 5.13: Hydro-fracturing (a) top view, with two planes of weaknesses along which the jet had penetrated the soil surface, (b) longitudinal cross section. Test conditions: \( D_n = 32.5 \text{ mm}, p_j = 0.82 \text{ MN/m}^2, v_t = 0.125 \text{ m/s}, su_p = 70 \text{ kN/m}^2. \)

which surface erosion is firstly observed, see Equations 2.62 and 2.63.

For a stationary (not-moving) jet they found that the maximum depth at equilibrium normalized by the stand off distance is a function of the Reynolds number, defined as \( u_0 D_n/\nu \), and the non-dimensional parameter \((D_n/SOD)^2 \rho_w u_0^2/\tau_c\). They concluded that for Reynolds numbers greater than about \( 1 \cdot 10^4 \), the influence of this number can be neglected (turbulent flow). The lowest Reynolds number of the jet tests in the present study was about \( 5 \cdot 10^5 \). From the analysis these moving jet tests indeed no effect of the Reynolds number was found.

The non-dimensional parameter reduces to \( \rho_w u_0^2/\tau_c \), because the SOD was not varied structurally in the present study and was more or less constant. Assuming that for high jet pressures the critical shear stress is highly correlated to the undrained shear strength \( (\tau_c \propto su_p) \), this parameter can also be written as \( p_j/su_p \), while \( p_j \propto \rho_w u_0^2 \). This parameter \((p_j/su_p)\) is called the jet ratio. From the data analysis it was found that the jet ratio was the decisive parameter to correlate the different cavity dimensions, see Figures 5.14 to 5.20. There was no convincing correlation found between the cavity dimensions and any non-dimensional parameter with the traverse velocity included.

The dependency between the cavity dimensions and the jet ratio \((p_j/su_p)\) can also be derived by coupling the needed stagnation pressure for cohesive soil failure to the bearing capacity of the soil (the minimum load under which the soil fails). The bearing capacity of cohesive soil is a function of the undrained shear strength: \( q_{bc} = N_{bc} su_p \), where \( N_{bc} \) is a semi empirical constant, depending on the occurring slip surfaces, see Appendix A.
5.2. Test results

The stagnation pressure ($p_{stag}$) at a certain jet distance can be derived from the equations for the velocity development in a confined jet, see Section 3.3. It follows that the stagnation pressure in the cavity linearly increases with the jet pressure ($p_j$) and nozzle diameter ($D_n$) and decreases inversely with the cavity depth ($Z_c$), namely:

$$p_{stag} \propto p_j \frac{D_n}{Z_c}$$

(5.6)

Combining the needed stagnation pressure ($q_{bc}$) with the stagnation pressure development, results in the following non-dimensional dependency:

$$\frac{Z_c}{D_n} \propto \frac{p_j}{su_p}$$

(5.7)

5.2.3 Measured cavity depth

In Figure 5.14 the measured cavity depths ($Z_c$) of the low-pressure tests (LP), normalized by the nozzle diameter ($D_n$), are plotted as function of the jet ratio ($p_j/su_p$). In addition also a trend line with a slope of 0.2 and an uncertainty bandwidth are plotted. The assumed bandwidth for the undrained shear strength is ±20%. Next to this the inaccuracy of the measured cavity depth is in the order of ±5 mm. The inaccuracy of the other parameters ($p_j, v_t, D_n$) is an order lower and can be neglected.

Excluding three tests (left upper corner), there is a clear correlation between both non-dimensional parameters; the non-dimensional cavity depth increases linearly with the jet ratio. Given the inaccuracies and the differences in traverse velocity of the nozzle ($v_t = 0.15 - 2$ m/s), the correlation between the normalized measured cavity depths and the trend line is quite good: $R^2 = 0.82$.

The three tests with a divergent cavity depth, showed hydro-fracturing. As a result the cavity depth is significantly larger. The traverse velocity of the nozzle of these tests was about 0.15 m/s. Not all tests with a traverse velocity of 0.15 m/s, showed hydro-fracturing. In Subsection 2.6.1 it was found that the critical traverse velocity for hydro-fracturing depends on the presence of planes of weakness in the soil and the nozzle diameter (loading time). Machin and Allan (2011) determined that the critical traverse velocity for hydraulic fracture to occur is somewhere between 0.1 and 0.5 m/s.

In Figure 5.15 the normalized cavity depths are plotted as a function of the traverse velocity of the nozzle ($v_t$) for three jet ratios. The normalized cavity depth decreases about 20%, by increasing the traverse velocity from 0.5 to 1.5 m/s. It can be concluded that for the low pressure tests the influence of the traverse velocity on the normalized cavity depth is limited.
5. Jet tests in cohesive soil

Figure 5.14: Normalized measured cavity depth ($Z_c/D_n$) as a function of the jet ratio ($p_j/su_p$), for all low-pressure tests.

Figure 5.15: Normalized measured cavity depth as a function of the traverse velocity ($v_t$), for two jet ratios ($p_j/su_p$) in DH- and GD-samples.

From the Figures 5.14 and 5.15 it can also be concluded that the differences in the normalized cavity depth ($Z_c/D_n$) between the DH- and GD-samples are not significant and are assumed to be negligible.

In Figure 5.16 the normalized measured cavity depths of the exploratory high-pressure tests are plotted as function of the jet ratio. The normalized cavity depths increase to a value of 100 at a jet ratio of about 800. The correlation between the data of the high-pressure tests and the derived trend-line for the moving non-cavitating jet is quite good till a jet ratio of about 400. This is remarkable, because at high jet pressures the influence of cavitation is expected to be non-negligible;
the higher the jet pressure, the less ambient water is entrained at the backside of the jet and as a result the larger is the penetration depth.

\[
\frac{Z_c}{D_n} \propto p_j \frac{s_u}{p_a \infty} \sqrt[4]{\frac{p_j}{p_a \infty}}
\]  

(5.8)

The influence of cavitation is probably limited because of the relatively small entrainment surfaces at the rear side of the jet (very small jet cavities). From the equations 3.40 and 4.14 the following non-dimensional dependency can be derived:

The influence of the nozzle traverse velocity \((v_t)\) on the cavity depth \((Z_c)\) at high jet ratios \((p_j/s_u)\) is also expected to be non-negligible. The influence of the nozzle traverse velocity is clearly visible in Figure 5.16 for the tests with the 3mm-nozzle at a jet ratio of about 800. By increasing the nozzle traverse velocity from 0.25 m/s to 1.5 m/s the normalized cavity depth decreases from 95 to 65.

**Other data sets** Only one public data set of moving jet tests in cohesive soil was found. Rockwell (1981) published cavity depths of 23 jet tests with a moving jet on cohesive soil \((D_n = 25.4-63.5 \text{ mm}, p_j = 1.1-15.9 \text{ MN/m}^2, v_t = 0.25-1.32 \text{ m/s}, s_u = 3.5 \sim 13.8 \text{ kN/m}^2)\).

In Figure 5.17 the measured cavity depths are compared with the data of Rockwell (1981). Because the undrained shear strength for all tests was indicated as 3.5 to 13.8 kN/m², the uncertainty bandwidth is very large. It seems that the impact of the jet ratio on the normalized cavity depth according to the data of Rockwell is smaller than expected.
5. Jet tests in cohesive soil

![Graph showing measured cavity depths compared to data of Rockwell (1981).](image)

Figure 5.17: Comparison of the measured cavity depths with the data of Rockwell (1981).

In the past the Dutch dredging companies Boskalis and Van Oord conducted a large number of moving jet tests on both cohesive and non-cohesive soils. This data set corresponds well with the data measured in the context of this thesis (Nobel and Vlasblom, 2005).

Also other researchers conducted several moving jet tests, but did not publish the data. For example Machin and Allan (2011) only describes the test results globally.

5.2.4 Measured cavity width

In Figure 5.18 the measured cavity widths ($W_c$) normalized by the nozzle diameter ($D_n$) are plotted as function of the jet ratio ($p_j/s_u$). Only the results of the low pressure tests in the middle of a soil sample are plotted (the results of the tests directly behind the glass wall are omitted).

For the penetrating jet, the cavity width remains constant after penetrating and is only 1 to 1.5 times the nozzle diameter. These values are only valid for small stand off distances ($SOD \approx 1D_n$). For higher stand off distances the cavity width will increase. The penetrating jet tests are clearly distinguishable; the jet ratio ($p_j/s_u$) must be higher than about 12.

When the jet ratio is lower than about 12, the jet deflects backwards directly after penetrating the soil bed, for the tested range of traverse velocities of the nozzle.\footnote{For higher traverse velocities, the jet ratio at the transition between the penetrating jet and the deflecting jet will increase, see also Section 8.1.1.}

A clear non-deflection zone is missing (deflecting jet). The lower the jet ratio, the
5.2. Test results

Figure 5.18: Normalized measured cavity width ($W_c/D_n$) as function of the jet ratio ($p_j/su_p$), for the low-pressure tests (for legend see Figure 5.19).

Figure 5.19: Normalized measured cavity width as function of the normalized cavity depth ($Z_c/D_n$), for the low-pressure tests.

larger the jet surface that is acting on the soil bed, see Figure 5.24 (c). As a result the impinging zone on the soil bed will increase. This will widen the cavity width.

The transition between the deflecting jet and the penetrating jet is also visible when the normalized cavity width ($W_c/D_n$) is plotted as a function of the normalized cavity depth ($Z_c/D_n$), see Figure 5.19. The transition takes place at a normalized cavity depth of about $2.5 \sim 3$. This corresponds with the minimum height of the cavity front, needed to bend the momentum flux of the jet completely backwards, see later Section 5.3.

When the normalized cavity depth ($Z_c/D_n$) is smaller than about $2.5 \sim 3$ the jet will not penetrate fully. A part of the jet flow will disperse in the transverse direction (dispersing jet flow). At the bottom of shallow cavities it is also possible that the horizontal jet pressure/flow is able to push away the soil in radial direction, while the jet is not able to penetrate any deeper ($p_{stag}/su_p < N_{bc}$), see Section 6.2.1 and Figure 6.1 (c) and (d).

The transition between the deflecting jet and the dispersing jet cannot be derived from Figure 5.19. For the deflecting and dispersing jet the cavity width ($W_c$) increases for low jet ratios to about $3D_n$. Machin and Allan (2011) also conducted numerous jet tests on cohesive soils with a moving jet at small stand off distances. They found similar cavity widths, between 1 and 3 times the nozzle diameter.

In Figure 5.20 the results of the high-pressure tests are plotted. These results are less reliable. Because of the small cavity widths the measurement accuracy was limited. For some tests the cavity width was zero. After passing of the jet the cavity was closed again (denoted in the figure as $W_c/D_n = 0.1$). Until a jet ratio
of 200 the normalized cavity width is constant between 1 and 1.5. For jet ratios larger than 200 the cavity width increases.

**Large scale tests**  Jet tests with a TSHD of Boskalis near the coast of Mumbai \((v_t = 0.5 \text{ m/s}, p_j/su_p \approx 30, SOD = 3 \text{ m}, D_n = 0.5 \text{ m}, \text{see Figure 1.7})\) showed trenches with a depth of about 1 m and a cavity width of 3.5 to 5 m. These dimensions were measured with a single beam.

For these tests the jet diameter at the original soil surface \((D_{j,ss})\) was about \(2D_n = 1 \text{ m}\). Consequently, the trench width \((W_c)\) normalized by the jet diameter at the original soil surface \((D_{j,ss})\) was \(3.5 \sim 5\) for these large scale tests. This is significantly larger than the normalized measured cavity widths of the laboratory jet tests: \(W_c/D_n < 3\), see Figure 5.18.\(^6\) The difference between the normalized cavity widths of the laboratory- and large scale tests will be discussed in Section 6.3.

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\(^6\)To compare the laboratory- and large scale tests the stagnation ratio \((p_{stag,ss}/su_p)\) at the original soil surface is of importance. For the laboratory jet tests the stagnation ratio at the original soil surface is about equal to the jet ratio \((p_j/su_p)\), because of the small stand off distances \((SOD)\). For the large scale tests the stagnation ratio at the original soil surface was about 7.5.

---

**5.2.5 Curvature of cavity front**

Another characteristic of the jet cavities is the radius of the curvature \((R_{cu})\) in the deflecting region, see Figure 5.8. It is assumed that the radius of the curvature is

![Figure 5.20: Normalized measured cavity width \((W_c/D_n)\) as a function of the jet ratio \((p_j/su_p)\), for the high-pressure tests.](image)

![Figure 5.21: Measured radius of the curvature \((R_{cu})\) as a function of the calculated jet thickness just before the curvature \((L_{j,cu})\).](image)

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5.3 Analyses

Determined by the radius needed to bend the momentum flux of the jet backwards. In this case the required radius is a function of the dimensions of the jet cross section, just before the curvature. The determining dimension is the length of this section \( L_{j, cu} \), see Figure 5.22 (note that for a deflecting jet this length is about equal to the jet diameter at the original soil surface: \( L_{j, cu} = D_{j, ss} \)).

\[
D_n \quad D_{j, ss} \quad L_{j, cu} \quad R_{cu} = R_{st} \approx 2L_{j, cu} \\
\Delta P_{st} \quad \Delta P_{sl} \\
\]

Figure 5.22: Definition sketch of jet cross sections.

In Figure 5.21 the measured radii \( R_{cu} \) are plotted as a function of the calculated jet thickness \( L_j \). The average radius is about 2.5 times the thickness of the jet. Globally the ratio between the radius of the curvature and the jet thickness ranges from 2 to 4. Not for all tests the radii were measurable. For some tests the pattern of curves was not well visible (shallow cavities) or the cavities were too narrow (tests with 5mm and 3mm nozzles).

5.3 Analyses

Based on the results of the exploratory jet tests, some hypothesis were formulated about the main failure mechanisms in the non-deflection zone and the deflection zone of a the penetrating jet and the main failure mechanism of a deflecting jet. Below the assumed failure mechanisms are only discussed briefly and will be verified on the basis of the high speed recordings of the second test program. In next chapter the failure mechanisms will be discussed more in detail.
5.3.1 Failure mechanisms

The soil fails along localized shear planes. The jet load has to exceed the undrained shear strength in these shear planes.

**Loading conditions** The different loading conditions of the jet on the soil sample are, see Figure 5.23:

- Normal pressure parallel to the jet flow, equal to the stagnation pressure ($p_{stag}$) of the jet (a).
- Normal pressure perpendicular to the main jet flow direction (b).
- Normal pressure perpendicular to the jet flow, because of the deflection of the jet (c).
- Shear stresses as exerted by the jet flow (d).
- Temporary normal pressure difference perpendicular to the jet flow, by turbulent pressure fluctuations (e).

![Figure 5.23: Loading conditions of a moving vertical jet penetrating a cohesive soil bed.](image)

The first three loading conditions are the most important ones. The shear stresses exerted by the jet are an order magnitude lower than the normal pressures, and in most cases also smaller than the undrained shear strength of the cohesive soil, see Section 3.3.1. Therefore the shear stresses are initially ignored. Initially the turbulent pressures fluctuations are also neglected, see Section 7.4.5.

**Non-deflection zone of the penetrating jet** In the non-deflection zone mainly the first loading condition occurs. It is assumed that the stagnation pressure of the jet dislodges small discrete soil elements by shearing, see Figure 5.24 (a). The shearing of a discrete soil element takes time. The smaller the jet ratio, the longer the duration of this process. The shearing time of the elements in combination
with the traverse velocity of the nozzle can be such that more soil elements are simultaneously dislodged at different heights, see zone I in Figure 5.24 (b). Because of this the jet can already achieve a small inclination angle in the non-deflection zone. The higher the traverse velocity of the nozzle, the larger the inclination angle of the jet in the non-deflection zone.

Figure 5.24: Influence of the traverse velocity ($v_t$) on the cavity depth ($Z_c$). (a) Very low traverse velocity; the influence of the traverse velocity on the cavity depth can be neglected. (b) The traverse velocity is such that more soil elements are dislodged simultaneously at different heights. (c) The traverse velocity is such that the non-deflection zone is missing.

On the top of a shearing soil element a part of the jet flow deflects backwards. As a result the static pressure directly above a shearing soil element also increases in horizontal direction. This horizontal pressure will also start elements to shear /rotate, see Figure 5.25. However, the shear surface of a soil element that must rotate upwards (in the opposite direction of the jet flow) is much larger than that of a soil element that rotates downwards (in the jet flow direction). Because of this, it is more likely that most elements are dislodged in vertical direction, see Figure 5.25 (b). Only just below the original soil surface, the shear surface of a soil element that rotates upwards is smaller or equal to that of an element that rotates downwards, see Figure 5.25 (a). This results for most of the tests in a small elevation of the soil surface directly sidewards at the edges of the cavity, see Figures 5.6 and 5.7.

Note that fluctuations in the jet and imperfections in the soil will result in a less structured failure pattern then suggested above.

**Deflection zone of the penetrating jet** The jet will deflect backwards, due to the traverse velocity. To bend the vertical jet momentum ($I$) completely in
horizontal direction, the soil has to provide a vertical force equal to this. The pressure exerted by the jet in the curvature is maximal equal to the dynamic head of the jet, just before the curvature \((p_{stag,cu})\). Therefore the soil pressure has to increase over a horizontal soil surface \((A_{s,h})\) that is minimal equal to: \(I/p_{stag,cu}\). The vertical jet momentum \((I)\) can be written as: \(\rho mQu_u = 2A_{j,cu}p_{stag,cu}\), where \(A_{j,cu}\) is the jet cross section just before the curvature, by combining the Equations 3.1 and 3.2. It follows that the soil surface, projected in the horizontal plane, over which the pressure will increase \((A_{s,h})\) is minimal equal to two times the jet cross section \((A_{j,cu})\).

In Figure 5.22 the different cross sections are shown. It follows that the radius of the curvature of the jet \((R_{cu})\) is minimal equal to the length of the jet cross section just before the curvature \((L_{j,cu})\). In this case it is assumed that the soil pressure increases over the full curvature with a value equal to the stagnation pressure just before the curvature \((p_{stag,cu})\). In practice the soil pressure is not constant over the full curvature, because the radius of the curvature and the jet velocity are not constant. Therefore the radius will be equal or larger than \(2L_{j,cu}\).

The exerted pressure in the curvature supports the cavity front and prevents the shearing of small discrete soil elements. The soil will fail in strips with a length equal to the curvature of the jet main flow, see zone II in Figure 5.24 (a). The
5.3. Analyses

shear surface of a strip is much larger than that of a small discrete soil element in the non-deflection zone. Because of this the vertical jet load needed for soil failure is also larger than for a soil element. As a result the strip thickness is much larger than that of a soil element.

Deflecting jet  The failure mechanism of the deflecting jet is comparable with that in the deflection zone of the penetrating jet. The difference is that the jet flow of the deflecting jet can disperse freely in all directions, while the jet flow of the penetrating jet is hindered, because it is partly enclosed. As a result for the deflecting jet, the width of the soil strips will increase with the strip thickness, while for the penetrating jet the width of the soil strips is more or less independent of the strip thickness and is equal to the cavity width in the non-deflection zone.

The strip thickness and width depend on the jet ratio. The smaller the jet ratio, the larger the strip thickness and width. In the ultimate case the total jet section is needed for soil failure. In this case the strip thickness and width are about \(\sqrt{2}\) times the jet diameter at the original soil surface \(D_{j,ss}\).

5.3.2  Hypotheses

Based on the analyses above, the following working hypotheses are formulated:

1. The soil fails along localized shear planes. The jet load has to exceed the undrained peak shear strength in these shear planes.

2. For the penetrating jet two failure zones can be distinguished: the non-deflection zone and the deflection zone, like Figure 5.24.
   - In the non-deflection zone the stagnation pressure of the jet dislodges very small discrete soil elements, by shearing.
   - In the deflection zone the soil front is loaded/supported by the deflecting jet, like Figure 5.22. This prevents the forming of small discrete soil elements. As a result the soil fails in small strips with a characteristic length equal to the length of the curvature of the jet.

3. The failure mechanism of the deflecting jet is comparable with that in the deflection zone of the penetrating jet.

5.3.3  Experimental evidence supporting hypotheses

With high speed camera it was tried to record the failure process in the non-deflection and deflection zone more in detail. The aim was to verify the defined hypothesis about the dislodgement of small discrete soil elements in the non-deflection zone and the shearing of curved strips in the deflection zone, see also (Nobel et al., 2010).
5. Jet tests in cohesive soil

Non-deflection zone  It proved to be impossible to record the assumed shearing process of the discrete soil elements in the non-deflection zone. The recordings only showed that the soil was removed in small vertical strips/columns. The thickness of these columns were consistent with the observed distances between the vertical oriented nerves on the cavity walls. At the top of these strips soil was removed continuously.

Figure 5.26 shows the development of the jet front just below the original soil surface for a test with the following settings: \( p_j / s u_p = 37.9, \) \( D_{n,f} = 20 \text{ mm} \) (\( D_n = 28 \text{ mm} \)), \( v_t = 0.5 \text{ m/s} \). The vertical uniform jet velocity \( (u_u) \) was about 45 m/s

and the vertical velocity of a front \( (u_f) \) below which deforming material occurs and above which the material has been removed already (upside of thin deforming layer) is about 12 m/s (47 [mm] / 4 \( \cdot 10^{-3} \text{ [s]} \)). Unfortunately, the velocities of the deforming soil could not be determined.

The high speed recordings show large deformations in the soil in the front of the jet. Figure 5.27 shows also the deformation of the soil just below the original soil surface for a test with a low nozzle traverse velocity \( (v_t = 0.1 \text{ m/s}) \). The other test settings were: \( p_j / s u_p = 25.8, \) \( D_{n,f} = 20 \text{ mm} \). The circled soil element deforms significantly in the first four pictures, before it is dislodged in picture 5. These deformations are caused by the exerted shear stresses.

At low traverse velocities of the nozzle the influence of the exerted shear stresses become more important and probably cannot be neglected. The effect of these
5.3. Analyses

deformations on the failure process of the soil elements is difficult to estimate.

Figure 5.27: Deformations in jet front. Test settings: $p_j/su_p = 26$, $D_{n,f} = 20$ mm, $v_t = 0.1$ m/s. The image dimension is 50x50 mm. The time interval between the frames is $13 \cdot 10^{-3}$ s.

Unfortunately, the velocities of the soil deformations in the deforming layer could not be determined visually; they were too high. With special PIV (Particle Image Velocimetry) software it was also not possible. Because of this it was not possible to determine the actual failure mechanisms. In Section 7.4.3 it is concluded that the used frame rate of maximal 3000 frames per second is, in hindsight largely insufficient to visualize the failure mechanisms of the soil.

With PIV software the displacements in the soil for the deforming layer were calculated, to get more insight in the deformations in the soil. Figure 5.28 shows the calculated displacements in the soil ($\Delta t = 0.5 \cdot 10^{-3}$ s).

At two levels the jet deforms the soil layer. Till 15 mm inside the soil front deformations are visible. With this software it should be possible to calculate the actual strain levels in the soil layer. However the high speed recordings are deficient to calculate the strain levels (and thus the actual failure mechanisms) in the deforming layer.

**Deflection zone** Figure 5.29 shows the soil failure in the deflection zone. The test settings were: $p_j/su_p = 20$, $D_{n,f} = 20$ mm, $v_t = 0.5$ m/s.
5. Jet tests in cohesive soil

Figure 5.28: With PIV-software calculated soil deformations (b) for the soil bed as visible in picture (a). Test settings: $\frac{p_j}{su_p} = 37.8$, $D_{n,f} = 20$ mm, $v_t = 0.5$ m/s. The image dimension is 50x100 mm. The time interval is $0.5 \cdot 10^{-3}$ s.

Figure 5.29: Failure of a soil strip in the deflection zone. Test settings: $\frac{p_j}{su_p} = 20$, $D_{n,f} = 20$ mm, $v_t = 0.5$ m/s. The image dimension is 50x100 mm. The time interval between the frames is $5 \cdot 10^{-3}$ s.

It seems that a strip with a thickness of about 15 mm is pushed downwards by the jet. The velocity of the strip was about 6 m/s. The uniform jet velocity at
the top of the soil strip was significantly higher, about 40 m/s. Unfortunately, it is not clear from these recordings what type of failure mechanisms occur.

5.4 Conclusions

The main conclusions of this chapter are:

1. Four different types of jets / failure modes can be distinguished:
   - Penetrating jet, characterized by: (1) narrow deep rectangular cavities with a constant cavity width of about 1 to 1.5 times the jet diameter at the soil bed, (2) a soil wall structure with small straight nearly vertical nerves, (3) dislodged soil that is fluidized completely.
   - Deflecting jet, characterized by: (1) shallow cavities; smaller than 2.5 times the jet diameter at the soil bed, (2) a soil wall texture with nerves deflecting in the direction opposite the traverse direction of the nozzle, (3) limited number of dislodged soil lumps that can be found after the tests.
   - Dispersing jet flow, characterized by: (1) wide, up to 5 times the jet diameter at the original soil surface, shallow cavities, (2) an irregular soil wall structure, (3) dislodged soil lumps that can be found after the tests.
   - Hydro-fracturing, characterized by: Irregular cavity dimensions. The soil fails along preferred weak surfaces. This can increase the cavity dimensions significantly, compared to the penetrating and deflecting jet.

2. A linearly dependency was found between the normalized cavity depth \( Z_c/D_n \) and the jet ratio \( p_j/su_p \), up to a jet ratio of 400 (excluding the tests at a very low traverse velocity, where hydro-fracturing occurred: \( v_t \leq 0.15 \text{ m/s} \)). For higher jet ratios the nozzle traverse velocity \( v_t \) and cavitation will also influence the cavity depth.

3. The measured cavity width \( W_c \) for the laboratory tests with a jet ratio between 12 and 200 varied between 1 and 1.5 times the nozzle diameter. For jet ratios smaller than 12, cavity widths up to 2.8 times the nozzle diameter were found. For the high jet ratios \( p_j/su_p > 200 \) the traverse velocity also affects the cavity width. For traverse velocities smaller than about 1.5 m/s the cavity seems to increase with jet ratio. For the high traverse \( v_t \geq 1.5 \text{ m/s} \) velocities the cavity width decreased to zero, only a cut was visible.
4. The following working hypothesis about the soil failure mechanisms were formulated:

- The soil fails along localized shear planes. The jet load has to exceed the undrained peak shear strength in these shear planes.

- For the penetrating jet two failure zones can be distinguished: the non-deflection zone and the deflection zone. In the non-deflection zone the stagnation pressure of the jet dislodges very small discrete soil elements, by shearing. In the deflection zone the soil front is loaded/supported by the deflecting jet. This prevents the forming of small discrete soil elements. As a result the soil fails in small strips with a characteristic length equal to the length of the curvature of the jet.

- The failure mechanism of the deflecting jet is comparable with that in the deflection zone of the penetrating jet.

5. The working hypotheses could not be confirmed with the high speed recordings, because the velocities of the deforming soil could not be determined visually. Also with special PIV (Particle Image Velocimetry) software it was not possible to determine the velocities in the deforming layer, they were too high. Consequently, it was not possible to determine the actual failure mechanisms.
Chapter 6
Modeling deflecting jet and dispersing jet flow

6.1 Introduction

In Section 5.2 four types of jets/failure modes are defined:

- Penetrating jet, see Figures 6.1 (a) and (e).
- Deflecting jet, see Figures 6.1 (b) and (f).
- Dispersing jet flow, see Figures 6.1 (c) and (d).
- Hydro-fracturing

First, the modeling of the deflecting jet and the dispersing jet flow will be discussed. The modeling of the penetrating jet type will be elaborated in next chapter. Hydro-fracturing was no part of this study and is recommended for further research.

The jet flow of the deflecting jet type, deflects backwards directly after penetrating the soil. In this case the whole jet front is supported by the deflecting jet flow. This prevents the shearing of small discrete soil elements. As a result the soil fails in large strips, see Figure 6.2 (a). In the previous chapter it has been established, that this is the case for cavity depths smaller than about 2.5 times the jet diameter at the original soil surface \((Z_c \leq 2.5D_{j,ss})\). In next section the modeling of the deflecting jet will be discussed.

A large part of the jet flow has to impinge on the original soil surface, before the soil strips fails/starts to shear. There exists a zone of impingement on the soil surface, see Figure 3.12. In this zone the jet load is distributed over a large surface area. As a result the cavity width at the soil surface is larger than the jet diameter at that location \((W_c > D_{j,ss})\). For the penetrating jet type the cavity
Figure 6.1: Defined soil failure mechanisms: (a) and (e) Penetrating jet, (b) and (f) Deflecting jet, (c) Dispersing jet flow, straight penetration, (d) Dispersing jet flow, gradual penetration (a-d: cross sections, perpendicular to the traverse direction of the nozzle, e-f: longitudinal cross sections).

width is much smaller and is about equal to the jet diameter, compare the Figures 6.1 (b) and (f) for the deflecting jet with those of the penetrating jet, Figures 6.1 (a) and (e).

The impinging jet flow will (partly) disperse in radial direction on the top of the soil strip. When the stagnation pressure of the radial dispersing flow is large enough, it will widen the cavity. For small scale jets it is assumed that the jet penetrates directly to the final depth before dispersing, see Figure 6.1 (c). For large scale jets it is assumed that the radial flow is continuously dispersing during the shearing process of the soil strip. This results in very wide shallow cavities, see Figure 6.1 (d). This will be discussed in Section 6.3. All calculations will be compared with the measurements of the jet tests, which are discussed in the previous chapter. The chapter is closed with conclusions.

6.2 Deflecting jet

For the deflecting jet, two failure mechanisms (shear surfaces) are defined. The main difference between these mechanisms is the assumed cavity width. Based on these mechanisms the cavity depth as a function of stagnation ratio at the original
soil surface can be calculated. In this Section these two failure mechanisms will be described and the calculated cavity depths will be compared with the measured ones. Also the minimum stagnation ratio \( (p_{stag}/su_p) \) for soil failure at the original soil surface is calculated. Based on this minimum stagnation ratio, the associated minimum jet ratio \( (p_j/su_p) \) as a function of the stand off distance will be derived.

### 6.2.1 Failure mechanisms

To investigate the behavior of the deflecting jet and to be able to predict the jet cavity dimensions as a function of the stagnation ratio at the original soil surface, two failure mechanisms (shear surfaces) are defined, see Figure 6.3. The main assumptions for both mechanisms are:

1. The total jet impinges on the original soil surface.

2. The soil fails along a shear surface that is curved in two directions. The formed soil strips rotate backwards along a horizontal axis.

3. The shear stresses along the shear plane are initially equal to the peak undrained shear strength \( (su_p) \).

The difference between both mechanisms is the assumed cavity width. In the first mechanism the cavity width \( (W_c) \) is equal to the diameter of the jet load \( (L_{st}) \), whereas the cavity width in the second mechanism is two times the jet load diameter \( (2L_{st}) \), see Figure 6.3. The jet load diameter is assumed to be equal to the zone of impingement: \( L_{st} = \sqrt{2D_{j,ss}} \), see Section 3.4. It follows that

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**Figure 6.2:** Deflecting jet: (a) Failure mechanism (longitudinal cross section), (b) Definition deflecting jet (cross section, perpendicular to the traverse direction of the nozzle).
6. Modeling deflecting jet and dispersing jet flow

the cavity width ($W_c$) of mechanism I and II are about $1.4D_{j,ss}$ and $2.8D_{j,ss}$, respectively. These values correspond with the measured cavity widths of the low pressure jet tests. The measured cavity widths for the low pressure deflecting jet lay somewhere between $1.17D_n$ and $2.6D_n$, see Figure 5.18.\(^1\)

**Equilibrium of moments**

The required stagnation ratio for soil failure follows from the equilibrium of moments, with respect to the axis of rotation ($AR$), see Figure 6.3:

\[
M_j(p_{stag,ss}) = -M_{r,f}(su_p) - M_{r,s}(su_p)
\]  

where $M_j$ is the driving jet moment, which is a function of the stagnation pressure at the original soil surface ($p_{stag,ss}$), $M_{r,f}$ is the resisting soil moment of the shear surface at the front side of the strip and $M_{r,s}$ is the resisting soil moment of the flat shear surfaces at both sides of the strip. The resisting soil moments are a function of the peak undrained shear strength ($su_p$).

In the special case that the cavity depth ($Z_c$) is equal to the diameter of the jet load ($Z_c = L_{st} = \sqrt{2}D_{j,ss}$), the axis of rotation lies in the horizontal plane of the

\(^1\)For these tests the diameter at the soil surface is about equal to the nozzle diameter ($D_{j,ss} \approx D_n$), because of the small stand off distances.
6.2. Deflecting jet

soil surface and passes through the hindmost point of the jet load, see Figures 6.3 and 6.4 (a). If the cavity depth is smaller than the diameter of the jet load ($Z_c < L_{st}$), the axis of rotation is assumed to move upwards in the vertical plane, see Figure 6.4 (b). Whereas for cavities larger than the thickness of the jet load ($Z_c > L_{st}$) the axis of rotation is assumed to move backwards in the horizontal plane, see Figure 6.4 (c).

![Diagram](Image)

Figure 6.4: Position of the axis of rotation ($AR$) as function of the cavity depth ($Z_c$) for mechanism I and II (is similar for both mechanism): (a) Cavity depth is equal to the jet load diameter ($L_{st}$), (b) Cavity depth is smaller than the jet load diameter, (c) Cavity depth is larger than the jet load diameter. (longitudinal cross section through the center line)

**Driving jet moment**  The driving jet moment ($M_j$) with respect to the axis of rotation ($AR$) is equal for both mechanisms (I and II, see Figure 6.3) and can be calculated with the following equation:

$$M_j = F_{jl}L_{jl} = p_{stag,ss}A_{jl}L_{jl}$$  \hspace{1cm} (6.2)

In which $F_{jl}$ is the vertical jet load and $L_{jl}$ is the moment arm of jet load with respect to the axis of rotation, which depends on the cavity depth ($Z_c$):

- for $Z_c < L_{st}$ : $L_{jl} = \frac{1}{2}L_{st}$  \hspace{1cm} (6.3)
- for $Z_c \geq L_{st}$ : $L_{jl} = Z_c - \frac{1}{2}L_{st}$  \hspace{1cm} (6.4)
Modeling deflecting jet and dispersing jet flow

Substituting these relations for the moment arm in Equation 6.2, gives for driving jet moment ($M_j$):

$$
\text{for } Z_c < L_{st} : \quad M_j = p_{stag,ss} \frac{1}{8} \pi L_{st}^3
$$

$$
\text{for } Z_c \geq L_{st} : \quad M_j = p_{stag,ss} \frac{1}{4} \pi L_{st}^2 \left( Z_c - \frac{1}{2} L_{st} \right)
$$

Resisting soil moment from the curved shear surface at the front

The resisting soil moment from the curved shear surface at the front side of the soil strip ($M_{r,f}$), with respect to the horizontal axis of rotation ($AR$), can be calculated by dividing it in infinitesimal small vertical strips and calculating the resisting soil moment for each strip separately. The resisting soil moment of an infinitesimal small strip with a width of $R_h d\alpha_r$ and a length of $\gamma R_{st}$ is, see Figure 6.5:

$$
M_{r,\alpha} = dM_{r,f}(\alpha_r) = suR_{st}(\alpha_r)^2 \gamma(\alpha_r) R_h \cdot d\alpha_r
$$

in which $\alpha_r$ and $\gamma$ are auxiliary angles as defined in Figure 6.5, $R_{st}$ is the radius of the curvature in the vertical plane, $R_h$ is the radius of the curvature in the horizontal plane at the soil surface. Note that $R_{st}$ and $\gamma$ are functions of $\alpha_r$. Collecting the resisting soil moments of the infinitesimal small vertical strips results in the following integral:

$$
M_{r,f} = -2 \int_{\alpha_r=0}^{\alpha_r=\frac{1}{2}\pi} dM_{r,f}(\alpha_r) = -2 suR_{st} \int_{\alpha_r=0}^{\alpha_r=\frac{1}{2}\pi} R_{st}(\alpha_r)^2 \gamma(\alpha_r) \cdot d\alpha_r
$$

The different definitions of the parameters used in Equation 6.8 are listed in Table 6.1. Two situations can be distinguished: the cavity depth is smaller than the diameter of the jet load ($Z_c < L_{st}$) and the cavity depth is equal or larger than the diameter of the jet load ($Z_c \geq L_{st}$). The definitions for mechanism I ($W_c = 1/2L_{st}$) and II ($W_c = L_{st}$) are almost the same. The only difference is the relation for $R_h$. For mechanism I and II $R_h$ is $L_{st}$ and $1/2L_{st}$, respectively, compare Figures 6.6 (c) and (e).

By substituting these parameters in Equation 6.8, the resisting soil moment from the curved shear surface at the front side of the soil strip can be calculated. In the present study this is done numerically.

Resisting soil moment from the flat shear surfaces on both sides

The resisting soil moment from the flat shear surfaces on both sides ($M_{r,s}$) is more difficult to calculate and must be calculated numerically. In Figure 6.7 five situations are shown schematically for which the resisting soil moment from both sides can
6.2. Deflecting jet

![Diagram](image)

Figure 6.5: Definition of a strip with a width of \( R_h d\alpha_r \) and a length of \( \gamma R_{st} \), (a) longitudinal cross section and (b) top view.

be calculated analytically (3 for mechanism II and 2 for mechanism I). For these situations the shear surfaces have a well defined standard shape:

- No shear surface on both sides: \( H_{st} \left( \frac{1}{2} \pi \right) = 0 \) (only occurs at mechanism II), see Figure 6.7 (a):

  \[
  M_{r,s} = 0 \quad (6.9)
  \]

- Shape of the shear surfaces on both sides is a semicircle: \( Z_c \leq L_{st} \) and \( R_{st} \left( \frac{1}{2} \pi \right) = \frac{1}{2} L_{st} \), see Figures 6.7 (b) and (d):

  \[
  M_{r,s} = -2suA_{r,s}L_{r,s} = -su_p \pi \left( \frac{1}{2} L_{st} \right)^2 \frac{1}{3} L_{st} = -su_p \frac{1}{12} \pi L_{st}^3 \quad (6.10)
  \]

- Shape of the shear surfaces on both sides is one-third of a circle: \( Z_c \geq L_{st} \) and \( R_{st} \left( \frac{1}{2} \pi \right) = L_{st} \), see Figures 6.7 (c) and (e):

  \[
  M_{r,s} = -2suA_{r,s}L_{r,s} \approx -2su_p \frac{1}{3} \pi L_{st}^2 \frac{2}{3} L_{st} = -su_p \frac{4}{9} \pi L_{st}^3 \quad (6.11)
  \]

6.2.2 Comparison calculated and measured jet cavity dimensions

From the equilibrium of moments (Equation 6.1) the cavity depth \( (Z_c) \) as a function of the stagnation ratio at the soil surface \( (p_{stag,ss}/su_p) \) can be calculated. This is done numerically. In Figure 6.8 the calculated normalized cavity depth
6. Modeling deflecting jet and dispersing jet flow

Table 6.1: Definition of the parameters used in Equation 6.8.

<table>
<thead>
<tr>
<th></th>
<th>$Z_c &lt; L_{st}$</th>
<th>$Z_c \geq L_{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Figure 6.6 (a)</td>
<td>Figure 6.6 (b)</td>
</tr>
<tr>
<td>$R_h$ (mechanism I)</td>
<td>$\frac{1}{2} L_{st}$</td>
<td>$\frac{1}{2} L_{st}$</td>
</tr>
<tr>
<td>$R_h$ (mechanism II)</td>
<td>$L_{st}$</td>
<td>$L_{st}$</td>
</tr>
<tr>
<td>$R_{st,0} = R_{st}(\alpha_r = 0)$</td>
<td>$\frac{L_{st}^2 + Z_c^2}{2Z_c}$</td>
<td>$Z_c - R_h (1 - \cos \alpha_r)$</td>
</tr>
<tr>
<td>$R_{st} (\alpha_r)$</td>
<td>$\sqrt{H_{st}(\alpha_r)^2 + (R_{st,0} - Z_c)^2}$</td>
<td>$Z_c - R_h (1 - \cos \alpha_r)$</td>
</tr>
<tr>
<td>$H_{st} (\alpha_r)$</td>
<td>$L_{st} - R_h (1 - \cos \alpha_r)$</td>
<td>$Z_c - R_h (1 - \cos \alpha_r)$</td>
</tr>
<tr>
<td>$\gamma (\alpha_r) = \gamma_1 (\alpha_r) + \gamma_2 (\alpha_r)$</td>
<td>$\cos (\gamma_1 (\alpha_r)) = \frac{R_{st,0} - Z_c}{R_{st}(\alpha_r)}$, $\frac{1}{2} \pi$</td>
<td>$\sin (\gamma_2 (\alpha_r)) = \frac{1}{2} \pi$, $\frac{1}{2} L_{st}$</td>
</tr>
<tr>
<td>$\gamma_1 (\alpha_r)$</td>
<td>$\frac{1}{2} L_{st}$</td>
<td>$\frac{1}{2} L_{st}$</td>
</tr>
<tr>
<td>$\gamma_2 (\alpha_r)$</td>
<td>$\frac{1}{2} L_{st}$</td>
<td>$\frac{1}{2} L_{st}$</td>
</tr>
</tbody>
</table>

$(Z_c/D_n)$ is plotted as a function of the required jet ratio ($p_{j}/su_{p}$) for both mechanisms (I: $W_c = L_{st} \approx 1.4D_n$, II: $W_c = 2L_{st} \approx 2.8D_n$). As a reference also the measured cavity depths of the low pressure tests are plotted. This figure is only valid for small stand off distances: $D_{j,ss} \approx D_n$ and $p_{stag,ss}/su_{p} \approx p_{j}/su_{p}$. In Figure 6.9 the corresponding normalized cavity widths ($W_c/D_n$) are plotted.

The calculated minimum stagnation ratio for soil failure is 6.2, assuming a cavity width of 1.4$D_n$. This value corresponds with the bearing capacity factor of a circular load on cohesive soil ($N_{bc} = 6.2$), see Appendix A. The minimum stagnation ratio is expected to be a little smaller than the bearing capacity factor, because at the backside of the soil strip the soil is already removed. The minimum stagnation ratio of the jet tests was also smaller, about 5.5. Probably there is an other position of the axis of rotation for which a lower value of the stagnation ratio is obtained.

According to the calculations the minimum cavity depth is about $1D_n$. For smaller cavity depths a higher stagnation ratio is required. For these higher stagnation ratios there is also an equilibrium at a larger cavity depth, which is apparently more preferable. The minimum cavity depth corresponds with the diameter of the shear surfaces in cohesive soil beneath a circular load. This diameter is about 0.7 times the load diameter ($L_{st}$) (Leroueil et al., 1990): $0.7L_{st} \approx 0.7\sqrt{2}D_n = 1D_n$.

For very shallow cavities ($Z_c \approx 1D_n$), it is expected that the final cavity width is
6.2. Deflecting jet

the result of the radial dispersing jet flow, see next Section. Therefore the expected cavity width of the deflecting jet will be somewhere between $1.4D_n$ and $2.8D_n$. A cavity width of about $1.7D_n$ fits best with the measured ones.

In Section 5.2 it has been established, that for cavities deeper than about 2.5 times the jet diameter at the original soil surface ($Z_c > 2.5D_{j,ss}$) another failure mechanism occurs. The jet front is no longer fully supported by the deflecting jet flow, a non-deflection zone exists. As a result the soil will fail in small discrete soil elements. This type of jet is called a penetrating jet and will be discussed in the next chapter. The transition between the deflecting jet type and the penetrating jet type, takes place at a stagnation ratio of about 12, see Figures 6.8 and 6.9.
6. Modeling deflecting jet and dispersing jet flow

Figure 6.7: Shear surface on both sides of the jet (a) Mechanism II: no shear surface on both sides, (b) Mechanism II: semicircle (c) Mechanism II: one-third of circle (d) Mechanism I: semicircle, (e) Mechanism I: one-third of circle. (top view and longitudinal cross section; striped lines longitudinal cross section through the center line)

6.2.3 Minimum jet ratio

For cohesive soil failure the stagnation ratio at the original soil surface must be at least equal to the bearing capacity factor of a circular load on cohesive soil ($N_{bc} = 6.2$, see Appendix A).
The stagnation pressure at the original soil surface as a function of the SOD can be derived from the Equations 3.45 and 3.49:

for $SOD < D_n(1 + \sqrt{k}/2)$:

$$p_{stag} = \frac{p_j}{(N_2 + 1)^2}$$  \hspace{1cm} (6.12)

$$N_2 = \frac{SOD}{D_n} - \frac{1}{2} = \frac{N_1\sqrt{\frac{k}{2}} - \frac{1}{2}}{\sqrt{\frac{k}{2}} + \frac{1}{2}}$$  \hspace{1cm} (6.13)

for $SOD \geq D_n(1 + \sqrt{k}/2)$:

$$p_{stag} = \frac{p_j}{(2N_3)^2}$$  \hspace{1cm} (6.14)

$$N_3 = \frac{SOD}{D_n\left(1 + \frac{k}{2}\right)} = \frac{N_1}{\sqrt{\frac{k}{2}} + 1}$$  \hspace{1cm} (6.15)
Combining the calculated stagnation pressure with the minimum stagnation ratio for soil failure, the minimum jet ratio \( \frac{p_j}{s_{up}} \) as a function of the stand off distance becomes:

for \( SOD < D_n(1 + \sqrt{k/2}) \):

\[
\frac{p_j}{s_{up}} > (N_2 + 1)^2 N_{bc} \quad (6.16)
\]

for \( SOD \geq D_n(1 + \sqrt{k/2}) \):

\[
\frac{p_j}{s_{up}} \geq 4(N_3)^2 N_{bc} \quad (6.17)
\]

Assuming a stand off distance of \( 1D_n \), the minimum jet ratio becomes 7.2 (\( D_j \approx D_n \) and \( N_{bc} = 6.2 \)). This jet ratio can also be calculated according to the 1D-approach, see Appendix C. For a stand off distance of \( 1D_n \), the minimum jet ratio according to the 1D-approach becomes 6.2. This is significantly lower than in the analytical approach, also shown in Figure 6.10. The difference is due to the lower entrainment in the 1D-approach, see also Section 3.2.4. It must be concluded that for small stand off distances, the analytical approach is less accurate.

Figure 6.10: The minimum jet ratio for cohesive soil failure as a function of the normalized stand off distance, calculated according to the analytical approach and the 1D-approach.
6.3 Dispersing jet flow

The impinging jet flow will (partly) disperse in radial direction on the top of the soil strip. When the stagnation pressure of the radial dispersing flow is large enough, it can widen the cavity. Two failure mechanisms caused by the dispersing jet flow are defined:

- In the first mechanism the jet penetrates directly to the final depth before dispersing (straight penetration). The jet flow disperses only at the bottom of the cavity. This mechanism was observed at the laboratory tests with a small jet ratio \( \left( \frac{p_j}{s u_p} \right) \), see Section 5.2 Figure 5.11.

- In the second mechanism the jet flow is continuously dispersing in radial direction during the shearing process of the soil strip. This radial jet flow is assumed to widen the cavity gradually in multiple layers, by pushing away small discrete soil elements. This results in very wide shallow cavities. This mechanism probably occurred at jet tests with a TSHD, see Section 5.2.4.

First, the equations to calculate the radial stagnation pressure and to calculate the required stagnation ratio for radial soil failure will be derived. These equations are necessary to model the two failure mechanisms. In Subsection 6.3.2 the first mechanism will be modeled. With this model the cavity depth \( (Z_c) \) and width \( (W_c) \) can be calculated. The calculated cavity widths will be compared with the measured cavity widths of the laboratory jet tests. In Subsection 6.3.3 the second failure mechanism will be modeled and will be compared with the results of jet tests with a TSHD.

6.3.1 Radial jet flow

In Section 3.4 the modeling of the radial wall jet is discussed. The equations for the radial jet flow velocity and radial jet flow height as a function of the radial distance are given. These equations will be used to model the dispersing jet flow.

**Radial stagnation pressure**  Assuming that the amount of dislodged soil does not influence the radial jet flow density significantly, the stagnation pressure at a certain radial distance \( (r) \) can be derived from the Equations 3.60 and 3.62:
6. Modeling deflecting jet and dispersing jet flow

for $SOD + Z_c < D_n(1 + \sqrt{k/2})$, $r \geq r_{zi}$:

$$p_{stag,r}(r) = p_j \frac{f_2 D_n^2}{8f_1 \alpha_{mom} r^2 + D_n^2 (N_2 + 1)^2 \left( \frac{1}{f_2} - 4f_1 \alpha_{mom} \right)}$$

$$N_2 = \frac{SOD + Z_c - \frac{1}{2}}{\sqrt{\frac{k}{2} + \frac{1}{2}}}$$

for $SOD + Z_c \geq D_n(1 + \sqrt{k/2})$, $r \geq r_{zi}$:

$$p_{stag,r}(r) = p_j \frac{f_2 D_n^2}{8f_1 \alpha_{mom} r^2 + 4(N_3 D_n)^2 \left( \frac{1}{f_2} - 4f_1 \alpha_{mom} \right)}$$

$$N_3 = \frac{SOD + Z_c}{D_n \left( 1 + \sqrt{\frac{k}{2}} \right)}$$

where $r_{zi}$ is the radial distance at the edge of the zone of impingement (see Section 3.4), $f_1$ is an empirical constant to discount the higher value of the entrainment coefficient in radial direction ($\alpha_{mom,r} = f_1 \alpha_{mom}$, see Appendix B) and $f_2$ is an empirical constant to account for the loss of momentum flux in the zone of impingement. Physically this constant must be 1. However, for an impinging jet with a stand off distance between $8D_n$ and $24D_n$, $f_2$ is found to be about 0.7, see Appendix B.3.

The cavity depth ($Z_c$) can be calculated, according to the proposed calculation model for the deflecting jet, assuming a cavity width of $1.7D_{j,ss}$, see previous section.

The cavity depth can also be predicted, by equating the axial stagnation pressure with the bearing capacity of the soil ($q_{bc}$, see Appendix A). The stagnation pressure as a function of distance can be derived from the Equations 3.45 and 3.49, see also Section 6.2.3. The bearing capacity of the soil is: $q_{bc} = N_{bc} su_p$, where $N_{bc}$ is the bearing capacity factor. This factor increases linearly from 6.2 at the original soil surface to about 8.2 at a cavity depth of 1.6 times the diameter of the jet load. This factor is constant, for cavity depths larger than about 1.6 times the jet load diameter. It follows, that the cavity depth can be calculated with the following equations:

for $SOD + Z_c < D_n(1 + \sqrt{k/2})$:

$$\frac{Z_c}{D_n} = \left( \frac{1}{2} + \sqrt{\frac{k}{2}} \right) \sqrt{\frac{1}{N_{bc}} \sqrt{\frac{p_j}{su_p}} - \frac{SOD}{D_n} - \sqrt{\frac{k}{2}}}$$

(6.20)
for \( SOD + Z_c \geq D_n (1 + \sqrt{k/2}) \):

\[
\frac{Z_c}{D_n} = \left( \frac{1}{2} + \sqrt{\frac{k}{8}} \right) \sqrt{\frac{1}{N_{bc}} \frac{p_j}{s_{up}} - \frac{SOD}{D_n}}
\]  

(6.21)

**Radial stagnation ratio for soil failure** The required radial stagnation pressure for cohesive soil failure follows from the equilibrium of moments and depends largely on the radial jet flow height \( h_r \) and the height of the radial soil front \( (Z_{rl}) \), see Figure 6.11. When the radial jet flow height is larger than about half the radial soil front height \( (h_r \geq 1/2 Z_{rl}) \), the jet can pressurize the radial soil front over the full height. In this case the required stagnation ratio for soil failure can be approximated by the following equation, see Figure 6.11 (a):

\[
\begin{align*}
M_j &= -M_r \\
\pi r Z_{rl}^2 p_{stag,r} &= -(\pi^2 r Z_{rl} + 2\pi Z_{rl}^2) Z_{rl} s_{up} \\
p_{stag,r} / s_{up} &= \pi + 2 \frac{Z_{rl}}{r}
\end{align*}
\]

(6.22)

It follows that, when the layer height is negligible compared to the radial distance \( (Z_{rl} \ll r) \), the required stagnation ratio becomes \( \pi \).

![Figure 6.11: Definition sketch of radial soil failure mechanism.](image)

When the radial jet height is smaller than half the cavity height \( (h_r < 1/2 Z_{rl}) \), the radial soil front is not pressurized over the full height. This will increase the required stagnation ratio, see Figure 6.11 (b):

\[
p_{stag,r} / s_{up} = \frac{\pi r + 2Z_{rl}}{4\pi r \frac{h_r}{Z_{rl}} \left( 1 - \frac{h_r}{Z_{rl}} \right)}
\]

(6.23)
6. Modeling deflecting jet and dispersing jet flow

6.3.2 Straight penetration

For small scale tests with a low stagnation ratio at the original soil bed, the (deflecting) jet is assumed to penetrate directly to the final depth before dispersing (straight penetration). The jet flow disperses only at the bottom of the cavity, see Figures 6.12 and 6.13.

Figure 6.12: Top view of a jet cavity of a dispersing jet straight penetrating to the maximum depth before dispersing.

Figure 6.13: Definition sketch of a jet straight penetrating to the maximum depth before dispersing.

The dispersing jet flow can only widen the cavity, when the required stagnation ratio for radial soil failure is smaller than the required stagnation ratio in vertical direction. The required stagnation ratio in vertical direction is assumed to be equal to the bearing capacity factor (N_{bc}), see Appendix A. This factor linearly increases with the cavity depth. The required stagnation ratio in radial direction at the edge of the zone of impingement (r = r_{zi}) can be calculated with Equation 6.22. It follows that the required stagnation ratio in radial direction also increases with the cavity depth.
6.3. Dispersing jet flow

The critical cavity depth \((Z_{rl} = Z_c)\), at which the radial jet flow is no longer able to widen the cavity, can be calculated by substituting the equations for the initial radial jet flow height (Eq. 3.57 and 3.59) and the equations for the radius of the zone of impingement (Eq. 3.47 and 3.51) into Equation 6.22. The critical cavity depth can also be calculated according to the 1D-approach, see Appendix C. In Figure 6.14 the calculated critical cavity depth, normalized with the nozzle diameter, is plotted as a function of the normalized stand off distance. For a stand off distance of \(1D_n\), the critical depth is about \(0.8D_n\). In Figure 6.15 the corresponding jet ratios are plotted. When the actual jet ratio is larger than the critical jet ratio, the jet penetrates too deep in the soil and cannot push away the soil in radial direction any longer.

![Figure 6.14: Equilibrium depth at which the required stagnation ratios for soil failure in axial and radial direction are equal, normalized with the nozzle diameter, as a function of the normalized stand off distance.](image1)

![Figure 6.15: Corresponding jet ratios for the equilibrium depth, as a function of the normalized stand off distance.](image2)

At small jet distances, in the analytical approach more ambient water is entrained in the jet than in the 1D-approach. As a result in the analytical approach, the radial jet flow height at a certain distance is larger. Therefore the critical depth according to the analytical approach is found at a larger cavity depth. The required jet ratio to penetrate to this critical depth is also larger in the analytical approach, see Figure 6.15.

For cavity depths smaller than the critical depth, the cavity width can be calculated by substituting the equation for the radial stagnation pressure (Eq. 6.18 or Eq. 6.19) and the equation for the radial jet flow height (Eq. 3.57 or Eq. 3.59) into the equation for the required stagnation ratio (if \(h_r \geq 1/2Z_c\): Eq. 6.22 else Eq.
6. Modeling deflecting jet and dispersing jet flow

In Figure 6.16 the calculated normalized cavity width for the dispersing jet flow is plotted as a function of the normalized cavity depth for a stand off distance of $1D_n$. In the 1D-approach, the radial jet height at a cavity depth of $0.4D_n$ is half the cavity depth. At this depth the radial jet flow is still able to pressurize the soil front completely. If the cavity depth becomes larger than $0.4D_n$, the radial jet flow is not able to pressurize the soil front over the full height any longer. As a result the required stagnation pressure in radial direction increases quickly with cavity depth. Resulting in a strong decline in cavity width.

![Figure 6.16: Normalized calculated cavity width for a jet straight penetrating to the maximum depth before dispersing, as a function of the normalized cavity depth for a stand off distance of $1D_n$.](image)

In Figure 6.17 the normalized calculated cavity widths are compared with the measured ones. It seems that the failure mechanism of the soil for jet ratios smaller than about 7.3 can be predicted with the 1D-approach of the dispersing jet straight penetrating to the maximum depth, except the test where hydro fracturing occurred.\(^2\) For jet ratios larger than 7.3, the cavity depth is larger than the critical depth (the dispersing jet flow is not able to push the soil up to the surface). For these jet ratios the cavity width follows from the proposed modeling of the deflecting jet and is assumed to be $1.7D_n$, see Section 6.2.

\(^2\)The normalized measured cavity depth of 1.4 at a jet ratio of 5.6 was realized at a traverse velocity of 0.15 m/s, see Figure 6.17. It was determined that hydro-fracturing occurred. As a result the cavity depth was relatively large: $Z_c = 7.2D_n$. Because of this the jet flow could not disperse at the cavity bottom. This explains why the cavity width of this test was relatively small. Given the cavity depth this test must be approached as a penetrating jet, see the next chapter.
6.3. Dispersing jet flow

6.3.3 Gradual penetration

Jet tests with a TSHD showed trench widths at the original soil surface between 3.5 and 5 times the jet diameter at this surface \( D_{j,ss} \approx 1 \text{ m} \), see Section 5.2.4. These widths cannot be explained by a radial jet flow that only disperses at the bottom of the cavity, as discussed above. Therefore another mechanism is proposed. It is assumed that the jet flow continuously disperses on the top of the soil strip during the penetration, see Figure 6.1 (d).

The underlaying mechanism is still unknown. Initially, it was assumed that the vertical penetration velocity of the jet decreases with an increase in the scale of the soil strips, because of the inertia. The lower the vertical penetration velocity of the jet, the more the jet will disperse in radial direction during the penetration. However, in Subsection 7.4.3 it is shown that the vertical penetration velocity of the jet is independent of the scale of the soil elements (strips). Therefore this still needs further investigation.

**Modeling**  In the modeling, it is assumed that the final cavity depth and width are realized in multiple layers. The height of these horizontal layers (and thus the dimensions of the discrete soil elements), depends on the ratio between the vertical and radial propagation velocity of the jet front.

From Equation 6.22 it follows that, when the soil layer height is negligible compared to the radial distance \( Z_{rl} \ll r \), the required stagnation ratio for soil failure in radial direction becomes \( \pi \). Assuming that the jet flow density in radial direction does not increase significantly, the cavity width can be calculated by substituting the required stagnation pressure \( p_{stag,r} = \pi sup \) into the equation for the radial stagnation pressure (Equations 6.18 or 6.19):

for \( SOD + Z_c < D_n(1 + \sqrt{k/2}) \), \( r \geq r_{i0} \):

\[
\frac{W_c}{D_n} = 2 \sqrt{\frac{p_j}{sup} \frac{f_2}{\pi 8 f_1 \alpha_{mom}} - (N_2 + 1)^2 \frac{1/f_2 - 4 f_1 \alpha_{mom}}{8 f_1 \alpha_{mom}}} \hspace{1cm} (6.24)
\]

for \( SOD + Z_c \geq D_n(1 + \sqrt{k/2}) \), \( r \geq r_{i0} \):

\[
\frac{W_c}{D_n} = 2 \sqrt{\frac{p_j}{sup} \frac{f_2}{\pi 8 f_1 \alpha_{mom}} - (2N_3)^2 \frac{1/f_2 - 4 f_1 \alpha_{mom}}{8 f_1 \alpha_{mom}}} \hspace{1cm} (6.25)
\]

The calculated normalized cavity dimensions \( (Z_c/D_n \text{ and } W_c/D_n) \) of the large scale jet tests \((p_j/sup \approx 30, \ D_n = 0.5 \text{ m}, \ SOD = 3 \text{ m})\) are compared with the measured ones in Table 6.2. The cavity depths are calculated with the Equations 6.20 and 6.21.
The calculated normalized cavity depth is smaller than the measured one. Taken into consideration that the determination of the peak undrained shear strength \((su_p)\) was very inaccurate, it is also possible that the actual peak undrained shear strength \((su_p)\) was a little lower. The calculated and measured cavity depths will be similar when the peak undrained shear strength is decreased by 15\% \((p_j/su_p = 35)\).

Assuming for \(f_1\) and \(f_2\) a value of 1.5 and 0.7, respectively, the calculated cavity width is about 3.2 \(\sim\) 3.6 times the nozzle diameter \((\approx 1.6 \sim 1.8D_{j,ss})\). This is much smaller than the measured value: 7 \(\sim\) 10\(D_n\) \((3.5 \sim 5D_{j,ss})\). Assuming for \(f_1\) and \(f_2\) a value of 1 (theoretically possible), the calculated cavity width is more realistic, see Table 6.2.

Note that the cavity width can partly be explained by fluctuations in the position of the nozzle, which was not very accurate due to waves and currents.

Table 6.2: Comparison of the measured and calculated normalized jet cavity dimensions, according to the modeling of the dispersing jet flow, gradually penetrating \((D_n = 0.5 \text{ m}, SOD = 3 \text{ m})\).

<table>
<thead>
<tr>
<th>(p_j/su_p)</th>
<th>(f_1)</th>
<th>(f_2)</th>
<th>(Z_c/D_n)</th>
<th>(W_c/D_n)</th>
<th>(Z_c/D_n)</th>
<th>(W_c/D_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>calculated</td>
<td>measured</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.5</td>
<td>0.7</td>
<td>1.5</td>
<td>3.2</td>
<td>2</td>
<td>7 (\sim) 10</td>
</tr>
<tr>
<td>35</td>
<td>1.5</td>
<td>0.7</td>
<td>2.1</td>
<td>3.6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>35</td>
<td>1</td>
<td>1</td>
<td>2.1</td>
<td>6.9</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

6.4 Conclusions

With respect to the modeling of the deflecting jet and the dispersing jet the following conclusions can be drawn:

- If the cavity depth is smaller than about 2.5 times the jet diameter at the original soil surface \((D_{j,ss})\), the jet flow will deflect backwards directly after penetrating the soil. In this case the soil will fail in large curved strips. When the jet front is not supported by the jet flow, the soil is assumed to fail in small discrete soil elements. The transition between these two mechanisms, takes place at a jet ratio of 12 \((Z_c = 2.5D_{j,ss})\) for a stand off distance of 1 times the nozzle diameter \((SOD = 1D_n)\).
6.4. Conclusions

- For the failure of the soil strips a large part of the jet flow has to impinge on the soil surface. There exists a clear zone of impingement. As a result the cavity width is wider than the jet diameter.

- The calculated minimum stagnation ratio \( \frac{p_{stag}}{su_p} \) for the failure of these strips is about 6.2, assuming a cavity width of \( 1.4D_{j,ss} \). For small stand off distances the stagnation ratio at the original soil surface is about equal to the jet ratio \( \frac{p_j}{su_p} \). The measured minimum jet ratio for the laboratory jet test was about 5.4. The corresponding minimum cavity depth is about 1 times the nozzle diameter \( (1D_n) \).

- The jet flow will disperse in radial direction on the top of the soil strip. For the laboratory jet tests it is assumed that the jet flow will only disperse at the bottom of the cavity. When the required stagnation ratio for radial soil failure is smaller than the required stagnation ratio in axial direction, the radial flow will push up the soil to the surface and will widen the cavity. The required stagnation pressure for soil failure in axial direction is assumed to be equal to the bearing capacity \( (q_{bc}) \). This mechanism will occur for a stand off distance of \( 1D_n \), at jet ratios smaller than about 7.3.

- For large scale jets it is assumed that the jet flow continuously disperses on the top of the soil strip during penetration. This will widen the cavity significantly. This mechanism is probably the explanation for the large cavity widths, observed at the large scale jet tests.
6. Modeling deflecting jet and dispersing jet flow
Chapter 7
Modeling moving penetrating jet

7.1 Introduction

In this chapter the modeling of the penetrating jet type will be discussed. The jet ratio \( \frac{p_j}{su_p} \) of the penetrating jet is such, that the whole jet penetrates the soil and only after a certain jet distance deflects backwards. A clear non-deflection zone exists, in contrast with a deflecting jet which deflects backwards directly after penetrating the soil surface, see Section 5.3 and previous chapter. In the non-deflection zone, the jet front is not supported (in horizontal direction) by the jet flow. Therefore it is assumed that in this zone the soil will fail in small discrete soil elements. It has been concluded that this mechanism occurs, when the cavity depth becomes larger than about 2.5 times the jet diameter at the original soil surface \( Z_c > 2.5D_{j,ss} \).

For small stand off distances \( \approx 1D_n \) the required jet ratio to reach this depth is about 12. The cavity shape of the penetrating jet is rectangular, with a constant cavity width \( W_c \) of about 1 to 2 times the nozzle diameter. To predict the cavity dimensions three models are proposed:

- A simple analytical model (Section 7.2), only valid for low traverse velocities \( v_t < 0.1 \text{m/s} \). In this model the entrainment of soil is neglected.
- A 1D-approach (Section 7.3). In this approach the entrainment of soil is included.
- An advanced model (Section 7.4). In this model the jet cavity is divided into two zones: the non-deflection zone and the deflection zone. In the non-deflection zone the soil is assumed to fail in small discrete soil elements. In the deflection zone the curved jet front is supported by the deflecting jet flow and is assumed to fail in large strips. Both mechanisms are included in the model. With this model also the jet trajectory can be predicted.
7. Modeling moving penetrating jet

7.2 Slowly moving penetrating jet; analytical approach

In this section a simple calculation model is proposed to predict the cavity width and depth. This model is only valid for non-cavitating jets \((p_j < p_{cav})\) with low traverse velocities \((v_t)\). In Section 3.3 it was concluded that for traverse velocities \((v_t)\) smaller than 0.1 m/s the influence of the entrainment of soil at the front side can be neglected. The jet flow density is assumed to be constant.

The cavity width is assumed to be equal to the jet diameter of the jet at the original soil surface \((W_c = D_{j,ss})\), which can be calculated with the equations of a free jet. This will be elaborated firstly. Secondly, the calculation of the cavity depth \((Z_c)\) will be derived. To calculate the cavity depth, the stagnation pressure of the jet \((p_{stag})\) at the cavity bottom is assumed to be equal to bearing capacity of the soil \((q_{bc})\). The stagnation pressure is a function of the jet distance \((s)\) and can be derived from the flow velocity development in a confined jet. The calculated cavity depths will be compared with the measured ones.

7.2.1 Cavity width

The jet ratio of the tests with a penetrating jet was found to be larger than about 12. For these tests the jet ratio was about equal to the stagnation ratio at the original soil surface: \(p_j/su_p \approx p_{stag,ss}/su_p \geq 12\), because of the small stand off distances. This is significantly higher than the needed minimum stagnation pressure for soil failure, which is theoretically 6.2 times the undrained shear strength \((p_{stag,ss} \geq 6.2su)\), see Section 6.2 and Appendix A. Therefore the whole jet flow is assumed to be able to penetrate the soil surface and the cavity width is assumed to be equal to the jet diameter of the jet at the original soil surface \((W_c = D_{j,ss})\). This diameter follows from the equations for the free jet (Equation 3.23 and 3.25, \(D_j = 2ru\)):

\[
W_c = \sqrt{\frac{2}{k}} SOD + D_n, \text{ if } SOD < s_{dr}
\]

\[
W_c = \sqrt{\frac{8}{k}} SOD, \text{ if } SOD \geq s_{dr}
\]

where \(SOD\) is the stand off distance of the nozzle and \(s_{dr}\) is the length of the flow development region \((s_{dr} = \sqrt{k/2D_n})\). The cavity width is mainly determined by the nozzle diameter. At small stand off distances the cavity width is about equal to the nozzle diameter.

\[1\] At traverse velocities smaller than 0.15 m/s hydro-fracturing can occur. This mechanism will enlarge the cavity dimensions significantly. However, this phenomena is not taken into account in this simple analytical approach.
The influence of the stand off distance on the cavity width is limited. When the stand off distance is equal to the length of the flow development region ($SOD = s_{dr} \approx 6.2D_n$), the cavity width increases till two times the nozzle diameter. Note that in this approach, the cavity width is independent of the jet pressure and soil strength.

The calculated cavity widths for the jet tests, normalized by the original jet diameter ($D_{j0} = \sqrt{\mu_n D_n}$, where $\mu_n$ is the nozzle discharge coefficient), are listed in Table 7.1.

Table 7.1: Normalized calculated cavity widths, according to the analytical approach.

<table>
<thead>
<tr>
<th>$D_n$ [mm]</th>
<th>$D_{j0}$ [mm]</th>
<th>$SOD$ [mm]</th>
<th>$W_c/D_{j0}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.8</td>
<td>30</td>
<td>3.4</td>
</tr>
<tr>
<td>5</td>
<td>4.7</td>
<td>30</td>
<td>2.0</td>
</tr>
<tr>
<td>7</td>
<td>6.6</td>
<td>30</td>
<td>1.7</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>20</td>
<td>1.16</td>
</tr>
<tr>
<td>32</td>
<td>32</td>
<td>30</td>
<td>1.15</td>
</tr>
</tbody>
</table>

### 7.2.2 Cavity depth

At the cavity bottom, the stagnation pressure of the jet ($p_{stag}$) is assumed to be equal to bearing capacity of the soil ($q_{bc}$). From Appendix A it follows that the bearing capacity for cavities deeper than 1.6 times the jet load diameter is about $8.2s_{up}$. For the penetrating jet the jet load diameter is equal to the cavity width ($D_l = W_c = D_{j,ss}$). This results in the following condition for the cavity depth:

$$p_{stag}(SOD + Z_c) = q_{bc} \approx 8.2s_{up} \quad (7.3)$$

where the stagnation pressure is written as a function of ($SOD + Z_c$). The stagnation pressure as a function of the jet distance can be derived from the equations for the velocity development of a confined jet (Equation 3.39 and 3.40). These equations deal with a cavity width as described above. Substituting the stagnation pressure development into Equation 7.3 results in the following equation for the cavity depth:

$$\frac{Z_c}{D_n} = a_1 \frac{p_j}{s_{up}} + b_1 \quad (7.4)$$
It follows that the normalized cavity depth increases linearly with the jet ratio. The variables $a_1$ and $b_1$ are defined as follows:

for $Z_c + SOD < s_{dr}$:

$$a_1 = \sqrt{\frac{k}{2} \frac{1}{N_{bc}(N_1 + 1)}}$$  \hspace{1cm} (7.5)

$$b_1 = -\sqrt{\frac{k}{2}(N_1 + 1)}$$  \hspace{1cm} (7.6)

for $Z_c + SOD \geq s_{dr}$:

$$a_1 = \sqrt{\frac{k}{2} \frac{1}{N_{bc}(2N_1 + 2)}}$$  \hspace{1cm} (7.7)

$$b_1 = -\sqrt{\frac{k}{2}N_1}$$  \hspace{1cm} (7.8)

where $N_1$ is defined as: $SOD/s_{dr}$. For $SOD = 1D_n$ the relations for the cavity depth become ($N_1 = \sqrt{2/k} \cdot SOD/D_n \approx 0.16$):

$$\frac{Z_c}{D_n} = 0.65 \frac{p_j}{s_{up}} - 7.2, \text{ if } 15 < \frac{p_j}{s_{up}} < 19$$  \hspace{1cm} (7.9)

$$\frac{Z_c}{D_n} = 0.33 \frac{p_j}{s_{up}} - 1, \text{ if } \frac{p_j}{s_{up}} \geq 19$$  \hspace{1cm} (7.10)

When the jet ratio is smaller than 15, the cavity depth becomes shallower than $2.5D_{j,ss}$. In this case the jet must be considered as a deflecting jet, see Section 6.2.

This approximation is based on the following simplifications/assumptions:

- The cavity width is equal to the jet diameter at the soil surface.

- In the region of fully developed flow the entrainment coefficient is: $\alpha_{mom} = 1/\sqrt{2k} \approx 0.083$. The influence of cavitation is neglected.

- In the flow development region the entrainment coefficient is half the entrainment coefficient in the fully developed flow region ($\alpha_{mom,dr} = 1/2 \cdot \alpha_{mom} \approx 0.041$). Therefore the cavity depths of shallow cavities ($Z_c < \sqrt{k/2D_n - SOD}$) are underestimated.
7.2. Slowly moving penetrating jet; analytical approach

- The influence of the traverse velocity of the nozzle and the shear stresses exerted by the jet are neglected. This means that the jet flow density and jet momentum flux are constant. This results in an overestimation of the cavity depth. The influence of the traverse velocity of the nozzle will increase with cavity depth. As a result, for larger cavity depths this simple analytical approach can be considered as an upper limit.

In Figure 7.1 the normalized calculated cavity depths, according to the simple analytical approach, are plotted as a function of the jet ratio. The main input parameters are: $SOD = 1D_n$, $k = 77$. As a reference also the normalized measured cavity depths of the low pressure (non-cavitating) jet tests with a normalized cavity depth larger than $2.5D_n$ are plotted in this figure (except those tests where hydrofracturing occurred). As expected the analytical approach can be considered as an upper limit.

![Figure 7.1: Calculated (analytical approach) and measured cavity depths ($Z_c$) normalized by the nozzle diameter ($D_n$) as a function of the jet ratio ($p_j/su_p$).](image)

**Cutting equation of Hashish and duPlessis**  Hashish and duPlessis (1978) developed a non-dimensional theoretical equation for the penetration depth of a water jet in all kind of materials:

\[
\frac{Z_c}{D_n} = \frac{\sqrt{\pi}}{2c_f} \left(1 - \frac{\sigma_c}{2p_j}\right) \left[1 - e^{-4c_f p_j/\left(\sqrt{\pi} \nu v_t\right)}\right]
\]  

(7.11)

where $c_f$ is the friction coefficient, $\sigma_c$ is the compressive yield strength of the cutting material and $\nu$ is a damping coefficient. The last two parameters are used to characterize the material.
The principal assumptions are:

- The hydrodynamic jet force (jet load) at the bottom of the cavity equals the resistance force of the cutting material.
- The jet is only resisted by the friction of the side walls of the cut.
- The area of cut ($A_c$) equals the nozzle exit area ($A_n$) and is orientated perpendicularly to the main jet direction.
- The time-dependent stress-strain behavior of the material is described with a Bingham model. The resistance of the cutting material is modeled as:

$$F_r = \sigma_c A_c + \nu A_c \frac{dZ_c}{dt} \quad (7.12)$$

- The loading time of the jet is assumed to be: $D_n/v_t$, where $v_t$ is the traverse velocity of the nozzle.

The disadvantage of this equation is, that this equation is not applicable for submerged jets, because the main cause of deceleration of the jet flow (entrainment of ambient water and soil, see Section 3.3) is not included. Because of this the calculated penetration depth of the jet will be overestimated with this cutting equation. Therefore this cutting equation is not directly applicable.

### 7.3 1D-approach

In the 1D-approach the stagnation pressures are derived from the velocities calculated according to Section 3.3. In this approach the jet is in jet direction divided in infinitesimal thin slices with a thickness $ds$. The uniform jet velocity is obtained by solving the mass and momentum balance for each slice, see also Appendix C. Some simplifications of the analytical approach are eliminated or improved in this approach:

- The entrainment coefficient in the region of flow development increases gradually from zero to a constant value in the region of fully developed flow, see Section 3.2.2, Figure 3.4.
- The influence of cavitation is included. For jet pressures higher than the pressure for cavitation cone development, the entrainment coefficient decreases with jet pressure, according to Equation 4.14.
- The momentum flux of the jet decreases with distance, because of the shear stresses exerted by the jet. These shear stresses are calculated according to Equation 3.26. These calculations are based on the boundary layer development in an uniform flow along a rough plate, see Section 3.3.1.
7.3. 1D-approach

- The traverse velocity of the nozzle is only taken into account in the density distribution of the jet flow. The density of the jet flow increases by the entrainment of soil at the front side of the jet. The entrainment of soil and ambient water is calculated with Equation 3.30.

7.3.1 Cavity width

In the 1D-approach the cavity width is derived from the velocity profile of a free jet. At the edges of the cavity, the stagnation pressure must be large enough to penetrate the soil surface:

\[ p_{stag}(s = \text{SOD}, r = \frac{1}{2} W_c) = \frac{1}{2} \rho_w u_j(s, r)^2 \geq N_{bc} s u_p = 6.2 s u_p \]  

(7.13)

where \( N_{bc} \) is the bearing capacity factor (see Appendix A), \( \rho_w \) is the density of the jet water and \( u_j(s, r) \) is the jet velocity at a (axial) distance \( s \) and a radial distance \( r \). Assuming a half Gaussian velocity profile in the shear zone of the flow development region and in the region of fully developed flow (Albertson et al., 1950), the cavity width can be calculated with the following equations:

\[ W_c = 2 \sqrt{-\frac{\text{SOD}^2}{2k} \cdot \ln \left( \frac{N_{bc} s u_p}{p_j} \right) + D_n \left( 1 - \frac{\text{SOD}}{\sqrt{\frac{2k}{D_n}}} \right)}, \text{ if } \text{SOD} < s_{dr} \]  

(7.14)

\[ W_c = 2 \sqrt{-\frac{\text{SOD}^2}{2k} \cdot \ln \left( \frac{N_{bc} s u_p}{p_j} \right) \cdot \frac{2}{k} \left( \frac{\text{SOD}}{D_n} \right)^2}, \text{ if } \text{SOD} \geq s_{dr} \]  

(7.15)

In Figure 7.2 the calculated cavity widths are plotted as a function of the normalized stand off distance, for different jet ratios. By increasing the stand off distance, the minimum jet ratio required for a penetration depth of \( 2.5 D_{j,ss} \) (defined condition for a penetrating jet) also increases. For a stand off distance of \( 4.5 D_n \), the critical jet ratio is for example 50. For this reason the different width-lines in Figure 7.2 end at different stand off distances.

Verification with low pressure tests In Figure 7.3 the calculated cavity widths of the penetrating jet, normalized by the nozzle diameter, are compared with the measured ones. In this figure only the results of the low pressure jet tests with a cavity depth larger than \( 2.5 D_n \) are plotted. The stand off distance of these tests was about \( 1 D_n \). Except one test at a jet ratio of 17, the measured and calculated cavity widths correspond well.

The calculated cavity widths according to the 1D-approach are rather small. It must be noted that the used bearing capacity factor \( (N_{bc}) \) of 6.2 is too high. In
Subsection 6.2.1 it has been determined that the minimum bearing capacity factor, based on the test results, is about 5.5. However the influence of $N_{bc}$ on the cavity width is negligible.

Figure 7.2: Normalized calculated jet width for a penetrating jet as a function of the normalized stand off distance, for different jet ratios.

Figure 7.3: Comparison calculated and measured cavity widths of a penetrating jet.

**Verification with high pressure tests** In Figure 7.4 the measured and calculated cavity widths for the high pressure jet tests (high jet ratios: $p_j/su_p > 50$) are compared. The average stand off distance was 0.03 m. The measured cavity width of $6D_n$ seems to be in conflict with the calculated width. It should be noted that only in this first high pressure jet test the nozzle showed a little oscillation. After this test, this problem was resolved. The calculated cavity widths seem to be too small, as previously mentioned. Possibly, this can be explained by the limited accuracy of these tests, the high value of the used bearing capacity factor and the impact of cavitation, see also Section 5.2.4.

### 7.3.2 Cavity depth

In Figure 7.5 the calculated cavity depths, according to the 1D-approach, for the penetrating jet are compared with the measured ones. The cavity depth is calculated for two traverse velocities: 0 and 1 m/s. The used input parameters are: $SOD = 1D_n$, $k = 77$, $\rho_b = 1900 \text{ kg/m}^3$ (situ density of the soil bed) and $k_s = 110 \mu\text{m}$ (bed roughness height). The jet tests were carried out with a traverse velocity of the nozzle between 0.15 and 2 m/s. The average traverse velocity was about 1 m/s. The calculated cavity depths according to the 1D-approach, based on a traverse velocity of 1 m/s correspond well with the actual test data.
7.3. 1D-approach

Figure 7.4: Comparison calculated and measured cavity widths, for high jet ratios.

Figure 7.5: Calculated and measured cavity depths normalized by the nozzle diameter as a function of the jet ratio.

To compare the 1D-approach with the analytical approach (Figure 7.1), the cavity depth is also calculated for the theoretical traverse velocity of 0 m/s. At low jet ratios, the depths calculated according to the 1D-approach are larger than the analytically calculated ones. This can be explained by the different entrainment coefficients in the region of flow development. Therefore the jet ratio at the transition between the deflecting jet and the penetrating jet is also lower: 11.5 (instead of 15 in the analytical approach). At higher jet ratios the entrainment coefficient for both calculations becomes equal. The difference in cavity depth at higher jet ratios, is because of the difference in cavity width and the influence of shear stresses.

In Figure 7.6 the calculated cavity depths, according to the 1D-approach, are compared with the measured ones at high jet ratios. The cavity depths are calculated
7. Modeling moving penetrating jet

for a non-cavitating jet with a jet pressure of 0.1 MN/m² and for a cavitating jet with a jet pressure of 10 MN/m². The stand off distance is 0.03 m. The calculated depths show clearly the influence of cavitation. Without the influence of cavitation the realized normalized cavity depths are quite underestimated.

![Figure 7.6: Comparison of the calculated and measured cavity depths at high jet ratios (SOD = 0.03 m, non-cavitating: \(p_j = 0.1\) MN/m², cavitating: \(p_j = 10\) MN/m²).](image)

![Figure 7.7: Comparison between calculated cavity depths according to the 1D-approach and the measured ones.](image)

In Figure 7.7 for all tests with a cavity depth larger than 2.5\(D_{j,ss}\), the calculated cavity depths according to the 1D-approach are compared with the measured ones. For these calculations the actual input parameters were used. The standard deviation (\(\sigma_{sd}\)) of the differences between the measured and calculated cavity depths is 25.5%, except those tests where hydro-fracturing occurred. Given the assumed bandwidth in undrained shear strength (\(s_u\)) of \(\pm 20\%\) (see Section 5.1.3), it can be concluded that this calculation method is a good approach to estimate the cavity depth.

Because the jet trajectory as a function of the traverse velocity of the nozzle is not taken into account, most cavity depths, realized with a low pressure jet, are slightly overestimated. In the 1D-approach the traverse velocity of the nozzle (\(v_t\)) is only taken into account in the jet flow development (entrainment of soil at the front side of the jet). The jet trajectory as a function of the traverse velocity of the nozzle is modeled in the advanced approach, see the next section. As a result the calculated cavity depths according to the advanced approach are smaller, compare the Figures 7.7 (1D-approach) and 8.6 (advanced approach). For the high pressure jets the jet trajectory is of less importance, because the non-deflection zone is much larger than the deflection zone. Most cavity depths realized with the high pressure jets are even slightly underestimated. It is difficult to determine
the cause of this underestimation. Several explanations are possible. Probably
the used entrainment coefficient is too high. Because the jet cavities of the high
pressure jets are relatively narrow and deep, less ambient water is available to
be entrained. Therefore it is possible that the jet starts cavitating at a lower jet
pressure.

7.4 Advanced approach

In the 1D-approach the cavity depth is only based on the condition that the
stagnation pressure at the cavity bottom is equal to the bearing capacity at that
level; The actual soil failure mechanisms of the soil are not taken into account
and the traverse velocity of the nozzle is only taken into account in the jet flow
development (entrainment of soil at the front side of the jet).

In the advanced approach the cavity is divided in two zones: the non-deflection
zone and the deflection zone, see Figure 5.8. In the non-deflection zone the soil is
assumed to fail by the shearing of small discrete soil elements (Subsections 7.4.1 up
to 7.4.4) and in the deflection zone by the shearing of small soil strips (Subsection
7.4.6).

The vertical penetration velocity of the jet in the soil (Subsection 7.4.7) and the jet
trajectory (Subsection 7.4.7) can also be predicted, by modeling these soil failure
mechanisms.

Note that the advanced approach is not applicable for a deflecting jet \(Z_c <
2.5D_{j,ss}\). The deflecting jet will deflect backwards directly after penetrating the
original soil bed, no clear non-deflection zone occurs, see Section 6.2. The ad-
vanced approach is also not applicable when hydro-fracturing occurs, because this
phenomena is not included.

7.4.1 Modeling failure mechanism in non-deflection zone

It is assumed that the soil, in the non-deflection zone of the penetrating jet, fails
by the shearing of small discrete soil elements. To model this, the soil is divided
into elements of the same dimension and shape. The main characteristic of these
elements is the circular shear surface at the backside, see Figures 7.8 and 7.9. The
main dimensions are: the radius of the curved shear surface \(R_{de}\), the length of
the frontal surfaces \(L_{de}\) and the width of the element \(W_{de}\), see Figure 7.8 (plain
2D strain approach).

Based on the high speed recordings and photographs of the jet cavity walls (dis-
tances between nerves, see e.g. Figures 5.9 and 5.26), the main dimension of the
soil elements \(L_{de}\) in the non-deflecting zone seems to be, for all tests, in the order
of magnitude of 2 to 5 mm. A clear explanation for this main dimension cannot
be given. No correlation was found between the characteristic length \((L_{de})\) and the traverse velocity of the nozzle \((u_t)\) or the stagnation ratio at the original soil surface \((p_{stag,ss}/u_p)\).

A element starts to shear/rotate along the curved shear surface when the jet load on the top of a soil element exceeds the resistance in the shear surfaces. The mobilized shear stresses \((\tau_s)\) in the schematized shear surfaces are a function of the peak undrained shear strength \((s_u)\). The strains \((\epsilon)\) before reaching the peak undrained shear strength are about 5-10\% (see Table 5.8). It is assumed that these deformations are realized instantaneously after loading a soil element; The soil is modeled as a rigid-plastic material.

After reaching the peak, the deformations localize and will form a shear band, as a result the resistance in the shear band decreases. It is assumed that the soil in the shear band reaches instantaneously the critical state after starting to rotate: \(\tau_s = s_{u_{cs}}, \) see Section 2.5.1. The undrained shear strength at the critical state is correlated to the peak undrained shear strength by the sensitivity \((S_s)\): \(s_{u_{cs}} = s_u/S_s.\) For the used cohesive soil samples the estimated values of the sensitivity range between 1.2 and 1.6, see Table 5.10.

Two mechanisms can occur:

1. The resistance in the shear surfaces remains intact during the rotation of the soil element and increases due to an increase in deformation rate.

2. Jet water is penetrating the shear band by fracturing which will release the resistance in the shear band.
7.4. Advanced approach

**Strain rate dependency of the shear stresses** Because of the viscous resistance, the shear stresses are strain-rate dependent, see Section 2.5.5. Assuming, that the thickness of the shear band is independent of the radius of this shear surface ($R_{de}$). The strain rate, and as a result the shear stresses, become a function of the tangential velocity of the curved shear surface ($v_{de,t}$), namely (see also Equation 2.55):

\[
\frac{\tau_s}{su_{cs}} = \left( \frac{v_{de,t}}{v_{ref,t}} \right)^{0.1} = \left( \frac{v_\omega R_{de}}{v_\omega ref R_{vane}} \right)^{0.1}
\]  

(7.16)

in which $su_{cs}$ is the undrained shear strength at critical state, determined with a shear vane test at angular velocity of $v_\omega ref$. This function is only validated up to a tangential velocity of about 18 mm/s (see Figure 2.7). This tangential velocity is orders of magnitude lower than the expected tangential velocities of the soil elements ($v_{de,t}$), see also Section 7.4.3. Therefore in the modeling the shear stresses are calculated in two ways:

1. According to Equation 7.16, assuming that this relation for the strain-rate dependency is also valid for higher tangential velocities, see Figure 7.10: upper limit.

2. Assuming that determined strain-rate dependency is limited, the strain-rate dependency is stopped when $\tau_s$ exceeds a value of $2su_{cs}$, see Figure 7.10: Limited strain-rate dependency of $\tau_s$.

![Figure 7.10: Schematic depiction of the development of shear stresses ($\tau_s$) as a function of the tangential velocity ($v_{de,t}$) of the schematized shear surfaces.](image-url)
7. Modeling moving penetrating jet

Fracturing of shear band  During the forming of a shear band mini-cracks are formed. These mini-cracks can coalesce and form a macro-crack. This process will be speed up by the jet water that is penetrating the shear band. In the literature on fracture mechanics no unambiguous calculation model is present to predict this propagation speed.

Hydro-fracturing was no part of this study, therefore only a rough estimate of the propagation speed of the crack \( u_{cr} \) was made, see Appendix H. It was concluded that the propagation velocities of the crack normalized by the uniform jet velocity just above a soil element \( u_u \) is a function of the stagnation ratio \( (p_{stag}/su_p) \). For a stagnation ratio between 8 and 50, the estimated normalized propagation velocities of the crack \( u_{cr}/u_u \) range from 0.65 to 0.86. Otherwise for fracturing the micro-cracks has to coalescence and form macro-cracks. This process takes time and can be considered as a consolidation process, as discussed in Section 2.6.1. A rough estimate of the duration of this process suggests that fracturing will not occur, see Appendix H.

Because these are only rough estimations, it is recommended to investigate the propagation velocity of a crack more in detail. In the present study only an lower limit is considered assuming that the duration of cracking (penetrating time of the jet in the shear band) is negligible, with respect to rotation time of a soil element. In this case the resistance in the shear band is fully released instantaneously after exceeding the peak resistance, see Figure 7.10: lower limit. At the upper limit the resistance in shear surfaces remains intact during rotation.

End of failure process  After a certain rotation the soil element will be fully dislodged and entrainment in the jet flow. Three possible criteria are defined, see Figures 7.11 to 7.13. The soil element starts to rotate, with respect to the horizontal rotation axis RP I, when the jet load on the top of a soil element exceeds the resistance in the shear surfaces. After a certain rotation the soil element:

1. slides off and is entrained in the jet flow (mechanism I, see Figure 7.11),

2. tilts around the axis of rotation RP II before it is entrained in the jet flow (mechanism II, see Figure 7.12) or

3. the underneath located element already starts to rotate (mechanism III, see Figure 7.13).

At the upper limit (resistance in shear surfaces remains intact during rotation), the duration of the failure process depends strongly on the stagnation ratio \( (p_{stag}/su_p) \) at the top of a discrete soil element. The higher this ratio, the shorter the duration of this process. At the lower limit (resistance in shear surfaces disappears instan-
7.4. Advanced approach

Figure 7.11: Mechanism I: After a certain rotation, with respect to the rotation axis RPI, the element slides off.

Figure 7.12: Mechanism II: After a certain rotation the element is torn off the soil bed by the resulting moment around the horizontal axis of rotation RPII.

Figure 7.13: Mechanism III: After a certain rotation, with respect to the horizontal rotation axis RPI, the next element under the rotating element already starts to rotate.

Immediately after exceeding the peak resistance) the failure process only depends on the stagnation ratio \( p_{stag} \).

7.4.2 Minimum stagnation ratio for failure

The minimum stagnation ratio for failure depends on the actual geometry of the discrete soil elements and can be derived from the equilibrium of moments with respect to the horizontal axis of rotation RPI, see Figure 7.11. The driving jet
7. Modeling moving penetrating jet

moment \((M_j)\), can be calculated with the following equation, see Figure 7.14:

\[
M_j = L_{Mj}A_s p_{stag} = L_{de} \cos (\alpha_{de}) L_{de} W_{de} p_{stag} = L_{de}^2 W_{de} \cos (\alpha_{de}) p_{stag}
\]  

(7.17)

where \(L_{Mj}\) is the moment arm of the jet load, \(A_s\) is the upper surface of the discrete soil element and \(\alpha_{de}\) is a characteristic angle, defined as:

\[
\sin \left( \frac{1}{2} \alpha_{de} \right) = \frac{L_{de}}{2R_{de}}
\]  

(7.18)

![Figure 7.14](image)

Figure 7.14: Driving jet moment with respect to axis of rotation RP I. (a) Exerted stagnation pressure (jet load). (b) Definition of the characteristic angle \(\alpha_{de}\) and characteristic isosceles triangles. (c) Definition of the moment arm of the jet load \((L_{Mj})\).

The resisting soil moment \((M_r)\) consists of two parts: the resisting soil moment from the curved shear surface \((M_{rb})\) and the resisting soil moment from the flat shear surfaces on both sides of the element \((M_{rs})\). These moments with respect to
the horizontal axis of rotation RPI can be calculated with the following equations:

\[
    M_r = M_{rb} + M_{rs} \\
    M_{rb} = 2\alpha_{de} W_{de} R_{de}^2 \tau_s \\
    M_{rs} = M_{rs,a} - M_{rs,b} + M_{rs,c} \\
    M_{rs,a} = \frac{4}{3} \alpha_{de} R_{de}^3 \tau_s \\
    M_{rs,b} = \frac{4}{3} R_{de}^3 \cos^2(\alpha_{de}) \sin(\alpha_{de}) \tau_s \\
    M_{rs,c} = L_{de}^3 \cos\left(\frac{1}{2}\alpha_{de}\right) \left[ \cos(\alpha_{de}) - \frac{1}{6} \frac{L_{de}^2}{R_{de}} \right] \tau_s
\]

in which \( \tau_s \) is shear stress in the shear surfaces. The calculation of \( M_{rs} \) consists of three terms (\( M_{rs,a}, M_{rs,b} \) and \( M_{rs,c} \)), related to the surfaces as depicted in Figure 7.15. In the special case that the radius of the circular shear surface and the characteristic length of the soil element are equal (\( R_{de} = L_{de} \)), the last two terms are equal and can be omitted.

![Figure 7.15: Dividing of the shear surfaces on both sides of the soil element in standard shapes to calculate the resisting moment (\( M_{rs} \)). For the calculation of the polar moment of inertia a similar division is made, see Section 7.4.3.](image)

The minimum stagnation ratio for soil failure follows from the equilibrium between the driving jet moment and the resisting soil moments, assuming that the shear stress in the shear surfaces is equal to the peak undrained shear strength (\( su_p \)). In the simple case \( R_{de} = L_{de} \) (illustrated in Figure 7.16), the minimum stagnation ratio becomes:

\[
    \frac{p_{stag}}{su_p} = \frac{2\alpha_{de} W_{de} R_{de}^2 + \frac{4}{3} \alpha_{de} R_{de}^3}{L_{de}^2 W_{de} \cos(\alpha_{de})} = \pi \left( \frac{4}{3} + \frac{8}{9} \frac{R_{de}}{W_{de}} \right)
\]

For the simple case \( W_{de} = R_{de} = L_{de} \), the minimum stagnation ratio is about 7. By increasing the ratio between the radius of the circular shear surface and
the characteristic length of the soil element, the axis of rotation ($RPI$) shifts to a position outside the element. As a result the minimum stagnation ratio decreases. The lowest value is obtained when $R_{de} \approx 1.3L_{de} = 1.3W_{de}$, see Figure 7.9. In this case the stagnation ratio becomes 6.4.

7.4.3 Calculation method of the rotation time of a discrete soil element

When the jet moment on a discrete soil element ($M_j$) exceeds the resisting soil moment of the shear surfaces ($M_r$), the element starts to accelerate and rotate around the axis of rotation RP I. The angular acceleration ($a_\omega$) is determined by the resulting moment ($M_j - M_r$) and the polar moment of inertia ($Ip$) of the soil element, namely:

$$a_\omega = \frac{M_j - M_r}{Ip}$$  \hspace{1cm} (7.21)

The angular velocity ($v_\omega$) and rotation angle ($\omega_{de}$) as a function of angular acceleration ($a_\omega$), can be calculated with the following integrals:

$$v_\omega = \int_0^t a_\omega \, dt$$  \hspace{1cm} (7.22)

$$\omega_{de} = \int_0^t v_\omega \, dt$$  \hspace{1cm} (7.23)
7.4. Advanced approach

**Polar moment of inertia**  The calculation of the polar moment of inertia \((Ip)\) can be divided in three terms \((Ip_a, Ip_b \text{ and } Ip_c)\), related to the shapes as depicted in Figure 7.15:

\[
Ip = Ip_a - Ip_b + Ip_c
\]  

(7.24)

The first term \((Ip_a)\) can be calculated with the following double integral, see Figure 7.17 (a):

\[
Ip_a = \rho_b W_{de} \int_0^{\alpha_{de}} \int_0^{R_{de}} r^2 r d\alpha dr
\]

\[
= \rho_b W_{de} \frac{1}{2} \alpha_{de} R_{de}^4
\]  

(7.25)

*Figure 7.17: Definition sketch for the calculation of the polar moments of inertia with respect to RPI (a) \(Ip_a\), (b) \(Ip_b\): definition of axis xx and yy, (c). \(Ip_c\): definition of axis xx, yy and yy'.*

The polar moment of inertia of the triangle \((Ip_b)\), see Figure 7.15 (b), can be calculated by summing the linear moments of inertia with respect to the axis xx and yy, respectively, see Figure 7.17 (b).

\[
Ip_b = I_{xx,b} + I_{yy,b}
\]  

(7.26)

\[
I_{xx,b} = \rho_b W_{de} \frac{1}{6} R_{de}^4 \cos(\alpha_{de}) \sin^3(\alpha_{de})
\]

\[
I_{yy,b} = \rho_b W_{de} \frac{1}{2} R_{de}^4 \cos^3(\alpha_{de}) \sin(\alpha_{de})
\]

Similar to the calculation of \(Ip_b\), \(Ip_c\) can be calculated, using the theorem of
7. Modeling moving penetrating jet

Steiner, see Figure 7.15 (c):

\[ I_{p,c} = I_{xx,c} + I_{yy',c} + L_{Ip,c}^2 m_c \]  \hspace{1cm} (7.27)

\[ I_{xx,c} = \rho_b W_{de} \frac{1}{6} L_{de}^4 \sin(\frac{1}{2} \alpha_{de}) \cos^3(\frac{1}{2} \alpha_{de}) \]

\[ I_{yy',c} = \rho_b W_{de} \frac{1}{18} L_{de}^4 \cos(\frac{1}{2} \alpha_{de}) \sin^3(\frac{1}{2} \alpha_{de}) \]

\[ L_{Ip,c} = R_{de} \cos(\alpha_{de}) - \frac{1}{3} L_{de} \sin(\frac{1}{2} \alpha_{p}) \]

\[ m_c = \rho_b W_{de} L_{de}^2 \cos(\frac{1}{2} \alpha_{de}) \sin(\frac{1}{2} \alpha_{de}) \]

In which \( I_{yy',c} \) is the linear moment of inertia with respect to the axis yy' and \( L_{Ip,c} \) is the distance between the axes yy and yy' and \( m_c \) is the mass of the considered element. In the special case that the radius of the circular shear surface and the characteristic length of the soil element are equal (\( R_{de} = L_{de} \)), the terms \( I_{p,b} \) and \( I_{p,c} \) are equal and can be omitted.

**Driving jet moment and resisting soil moment are constant**

The difficulty in the calculation of the angular acceleration of the soil elements is, that the moments change with the rotation angle. The driving jet moment \((M_j)\), decreases because of a resisting jet load on the surface of the soil element that rotates into the jet flow. At high angular velocities the effective jet load at the top of the soil element will also decrease a little. The resisting soil moment \((M_r)\) changes, because of a decrease in shear surface and an increase in (dynamic) shear stresses \((\tau_s)\) acting on the shear surfaces. In the theoretical case that the moments are constant, and consequently the angular acceleration \((a_\omega)\) is constant (see Equation 7.21), and \(W_{de} = L_{de} = R_{de}\) (illustrated in Figure 7.16), the required time to rotate to a defined rotation angle \((\omega_{de})\) can be calculated simply:

\[ t_{de} = \sqrt{\frac{2\omega_{de}}{a_\omega}} = \sqrt{\frac{2\omega_{de} I_{p}}{M_j - M_r}} \]  \hspace{1cm} (7.28)

The characteristic angle \(\alpha_{de}\) is \(1/3\pi\), when \(L_{de} = R_{de}\), see Equation 7.18. Substituting this angle in the equations for the resulting jet moment (7.17), the resisting soil moment (7.19) and the polar moment of inertia (7.25) results in the following equations:

for the driving jet moment:

\[ M_j = p_{stag} \frac{1}{2} L_{de}^3 \]  \hspace{1cm} (7.29)
for the resisting soil moment \((\tau_s = su_{cs})\):

lower limit: \(M_r = 0\) \hspace{1cm} (7.30)

upper limit: \(M_r = su_{cs} \left(\frac{2}{3} \pi L_{de}^3 + \frac{4}{9} \pi L_{de}^3\right) = \frac{10}{9} \pi su_{cs} L_{de}^3\) \hspace{1cm} (7.31)

and for the polar moment of inertia:

\[Ip = \rho_b \frac{1}{6} \pi L_{de}^5\] \hspace{1cm} (7.32)

Substituting the Equations 7.29 to 7.32 into Equation 7.28 gives for the required time to rotate to a defined rotation angle \((\omega_{de})\):

lower limit: \(t_{de,\omega} = L_{de} \sqrt{\frac{2}{3} \cdot \frac{\omega_{de} \rho_b}{P_{stag}}}\) \hspace{1cm} (7.33)

upper limit: \(t_{de,\omega} = L_{de} \sqrt{\frac{\omega_{de} \rho_b}{su_{cs} \left(\frac{3}{2} \cdot \frac{P_{stag}}{su_{cs}} \cdot \frac{1}{\pi} - \frac{10}{3}\right)}}\) \hspace{1cm} (7.34)

The rotation angle \((\omega_{de})\) just before the soil element is entrained in the jet is about: \(\alpha_{de}\). Substituting this angle in the Equations 7.33 and 7.34 gives for the rotation time:

lower limit: \(t_{de} = L_{de} \pi \sqrt{\frac{2}{9} \cdot \frac{\rho_b}{P_{stag}}}\) \hspace{1cm} (7.35)

upper limit: \(t_{de} = L_{de} \sqrt{\frac{\rho_b}{su_{cs} \left(\frac{9}{2} \cdot \frac{P_{stag}}{su_{cs}} \cdot \frac{1}{\pi^2} - \frac{10}{\pi}\right)}}\) \hspace{1cm} (7.36)

Note that the rotation time is linear proportional to the characteristic length of the discrete soil element \((L_{de})\). The angular velocity \((v_{\omega} = t_{de} a_{\omega})\) of the element and the corresponding tangential velocity in the schematized curved shear surface \((v_{de,t} = v_{\omega} R_{de})\) just before entrainment in the jet flow can also be calculated with the equations above.
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The tangential velocity just before entrainment in the jet is \((\omega_{de} = \alpha_{de})\):

\[
\begin{align*}
\text{lower limit: } v_{de,t} &= \frac{R_{de}}{L_{de}} \sqrt{\frac{6}{\pi} \frac{p_{stag} \alpha_{de}}{\rho_b}} = \sqrt{\frac{2p_{stag}}{\rho_b}} \\
\text{upper limit: } v_{de,t} &= \sqrt{2p_{stag} - \frac{40}{9} \pi su_{cs}} \\
\end{align*}
\]

Note that the angular velocity \((v_\omega)\) depends on the characteristic length of the discrete soil element \((L_{de})\), whereas the tangential velocity \((v_{de,t})\) does not.

**Driving jet moment as a function of the rotation angle**

The total jet moment \((M_j)\) consists of two components: the driving jet moment, caused by the exerted stagnation pressure on the top of the soil element and the resisting jet moment, caused by the jet load on the surface of the soil element that rotates into the jet flow, see Figure 7.18. The driving jet moment, is assumed to

\[
M_j = L_{de}^2 W_{de} \cos (\alpha_{de}) (p_{stag} - p_b)
\]

where \(p_b\) is the pressure acting on the surface that rotates into the jet flow.

Figure 7.18: Definition of pressures exerted by the jet on a discrete soil element.

\(^2\)The decrease in stagnation pressure as a result of the increasing angular velocity of a soil element is initially neglected.
7.4. Advanced approach

To calculate the pressure $p_b$ the following assumptions are made:

- The jet will incline slightly at the top of a soil element, due to the soil reaction force ($F_{rt}$), see also Figure 7.19 (b). This reaction force is provided by the top of the discrete soil element and the soil wall.

The horizontal component of the soil reaction force above a rotating soil element ($F_{rt,h}$) is assumed to be equal to the frontal surface projected in the horizontal plane just before rotation times the stagnation pressure ($p_{stag}$) exerted by the jet at that location. The vertical component of the soil reaction force above a rotating soil element ($F_{rt,v}$) is assumed to be equal to the horizontal component.

The inclination angle of the jet ($\beta_t$) at the top of a soil element, can be calculated with the following equation, see also Figure 7.19 (b):

$$\tan(\beta_t) = \frac{F_{rt,v}}{I_j - F_{rt,h}} = \frac{L_{de,f}W_{de}p_{stag}}{2L_j W_{de}p_{stag} - L_{de,f}W_{de}p_{stag}} = \frac{L_{de,f}}{2L_j - L_{de,f}}$$

(7.40)

in which $I_j$ is the jet momentum flux per width of the soil element ($W_{de}$), $L_j$ is length of the jet cross section and $L_{de,f}$ is the thickness of a soil element in the horizontal plane, see Figure 7.19 (a).

- As long as the rotation angle of the soil element ($\omega_{de}$) is smaller than the inclination angle of the jet ($\beta_t$), the pressure on the soil surface of the element that rotates into the jet flow ($p_b$) is neglected.

- The jet momentum flux that is acting frontal on the soil surface that rotates into the jet flow ($I_{bi}$) is deflected to an angle parallel to that surface, see Figure 7.19 (c).

- The pressure $p_b$ is constant over the whole surface with a maximum equal to the stagnation pressure.

---

Note that in reality this is only the case for $R_{de} = L_{de}$. For $R_{de} \neq L_{de}$ the frontal surface projected in the horizontal plane, and as a result the horizontal component of the soil reaction force will increase during rotation. This increase is neglected.
This results in the following equations for the pressure \( p_b \):

\[
\begin{align*}
\omega_{de} < \beta_t: \quad p_b &= 0 \quad \text{(7.41)} \\
\omega_{de} \geq \beta_t: \quad p_b &= \frac{F_{de,b}}{L_{de}W_{de}} \\
&= \frac{\sin(\omega_{de} - \beta_t)I_{bi}}{L_{de}W_{de}} = \rho_m u_b^2 \sin^2 (\omega_{de} - \beta_t) \quad \text{(7.42)}
\end{align*}
\]

where \( F_{de,b} \) is the soil reaction force on the surface that rotates into the jet flow, \( u_b \) is the uniform jet velocity after deflecting on the top of the element, which can be calculated with the following equation:

\[
u_b = u_u \frac{I_b}{I_j} = u_u \frac{I_j - F_{rt,v}}{I_j \cos(\beta_t)} = u_u \frac{1 - \frac{L_{de,f}}{2L_j}}{\cos(\beta_t)} \quad \text{(7.43)}
\]

in which \( I_b \) is the jet momentum flux per width of the element after impinging on the top of the element.

From Equation 7.40 it follows that, the smaller the ratio between the length of the jet cross section \( L_j \) and the thickness of the soil elements in the horizontal plane
(L_{de,f}), the larger the rotation angle of the soil element before the jet moment starts to decrease (\omega_{de} = \beta_t). In the special case that the thickness of the soil element equals the jet flow thickness (L_j = L_{de,f}), the inclination angle of the jet (\beta_t) is 45°. In this case the element is probably already dislodged before the resisting jet moment starts. On the other hand, when the ratio \( L_j/L_{de,f} \) is about 10, the inclination of the jet is almost negligible.

**Resisting soil moment as a function of the rotation angle**

The resisting soil moment \( (M_r) \) changes because:

1. The area of the shear surfaces decreases with the rotation angle (\omega_{de}). This influence can be included in the calculations by substituting \( \alpha_{de} \) in Equation 7.19 by \( \alpha_{de} - \omega_{de} \).

2. The shear stress (\( \tau_s \)) increases with the tangential velocity of the curved shear surface (\( v_{de,t} \)), according to Equation 7.16.

The resisting soil moment \( (M_r) \) as a function of the rotation angle (\( \omega_{de} \)) must be calculated numerically. Only, when \( R_{de} = W_{de} = L_{de} \) and the strain-rate dependency of the shear stress are neglected this can be done analytically:

\[
M_r = s u_{cs} (2\alpha_{de} - \omega_{de}) L_{de}^3 \left(1 + \frac{2}{3}\right)
\]

\( (7.44) \)

**7.4.4 Calculation results of the rotation time**

First, the rotation time of the discrete soil elements without soil resistance (lower limit: \( M_r = 0 \)) is considered. Subsequently, also the soil resistance will be taken into account (upper limit: \( M_r > 0 \)) in the calculations of the rotation time. The rotation time \( (t_{de}) \) is defined as the time that it takes for a discrete soil element to rotate to an angle \( \omega_{de} \) at which the soil element is fully dislodged and entrained in the jet. This angle is assumed to be about \( \alpha_{de} \). This corresponds with Mechanism I, see Figure 7.11.

**Lower limit**

In Figure 7.20 for two theoretical situations and one practical situation the calculated rotation times \( (t_{de}) \) are plotted as a function of the stagnation pressure \( (p_{stag}) \) at the top of an element. The following cases are calculated:

- Driving jet moment is constant and \( R_{de} = W_{de} = L_{de} \) (see Figure 7.20: \( R_{de} = L_{de}, M_j = \text{const.} \)). In this theoretical case the rotation time \( (t_{de}) \) can be calculated with Equation 7.35.
7. Modeling moving penetrating jet

- Driving jet moment is constant and the dimension of the soil element is optimal: $R_{de} \approx 1.3L_{de}$ (see Figure 7.20: $R_{de} \approx 1.3L_{de}, M_j = \text{const.}$). In this theoretical case the jet moment increases with respect to the previous case due to a larger moment arm and must be calculated with Equation 7.17, assuming that $\alpha_{de} = 1/4\pi$. Also the polar moment of inertia ($I_p$) changes and must be calculated with the Equations 7.24 to 7.27.

The rotation time decreases significantly by changing the ratio between the radius of the curved shear surface ($R_{de}$) and the characteristic length of the soil element ($L_{de}$).

- Driving jet moment decreases due to a resisting jet load on the surface of the soil element that rotates into the jet flow and the dimension of the element is optimal: $R_{de} \approx 1.3L_{de}$ (see Figure 7.20: $R_{de} \approx 1.3L_{de}, M_j = \text{var.}$). In this practical case the jet moment changes with the rotation angle and must be calculated using the Equations 7.39 to 7.43. Because the jet moment is not constant the angular acceleration ($a_\omega$) also changes in time. Therefore the rotation is calculated numerically.

It can be concluded that the impact of the resisting jet load is limited.

In the calculations the following parameters are used: characteristic length of soil element ($L_{de}$) is 2 mm, density of! soil element ($\rho_b$) is 1900 kg/m$^3$, length of jet cross section is $10L_{de}$ m. Note that the rotation time is linear proportional to the characteristic length of the soil element ($L_{de}$).

Figure 7.20: Calculated rotation time ($t_{de}$) as a function of the stagnation pressure ($p_{stag}$) for the lower limit ($M_r = 0$).

Figure 7.21: Calculated rotation time ($t_{de}$) as a function of the stagnation ratio ($p_{stag}/s_{up}$) for the upper limit ($s_{up} = 40$ kN/m$^2$, $S_s = 1.4$).
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Upper limit

When the resisting soil moment is taken into account (upper limit), the rotation time will increase. In Figure 7.21 for two theoretical and two practical situations the calculated rotation time \( t_{de} \) is plotted as a function of the stagnation ratio \( p_{stag}/su_p \). The corresponding tangential velocity \( v_{de,t} \) and the ratio between the dynamic shear stress and the undrained shear strength at critical state \( \tau_s/su_{cs} \) just before the element will be entrained in the jet are plotted in the Figures 7.22 and 7.23, respectively. In the calculations the following parameters are used: characteristic length of soil element \( L_{de} \) is 2 mm, density of soil element \( \rho_b \) is 1900 kg/m\(^3\), length of jet cross section is 10\( L_{de} \), the peak undrained shear strength \( su_p \) is 45 kN/m\(^2\) and the sensitivity \( S_s \) is 1.4.

The following cases are calculated:

- Driving jet moment and resisting soil moment are constant and \( R_{de} = W_{de} = L_{de} \). In this theoretical case the rotation time \( t_{de} \) can be calculated with Equation 7.36. Note that according to Equation 7.36 the rotation time depends also on the undrained shear strength at the critical state \( su_{cs} \). Therefore the calculated values of the rotation time are only valid for this specific case: \( su_{cs} = su_p/S_s = 45/1.4 \approx 32 \) kN/m\(^2\) (see Figure 7.21: \( R_{de} = L_{de}, M = \text{const.} \)).

- The driving jet moment is constant, the resisting soil moment increases only due to the dynamic shear stress and \( R_{de} = W_{de} = L_{de} \). At low stagnation
ratios ($p_{stag}/su_p$), the rotation time ($t_{de}$) increases significantly (see Figure 7.21: $R_{de} = L_{de}, M_r = \text{var}$). This can be explained as follows:

The maximum tangential velocity in the curved shear surface can be estimated by substituting the relation for the dynamic shear stress (Eq. 7.16) in the equation for the resisting soil moment (Eq. 7.31) and equating this moment with the driving jet moment (Eq. 7.29):

$$M_j = M_r$$

$$p_{stag} \frac{1}{2} L_{de}^3 = \frac{1}{9} \pi su_{cs} \left( \frac{v_{de,t}}{v_{ref,t}} \right)^{0.1} L_{de}^3$$

$$v_{de,t,max} = v_{ref,t} \left( \frac{9}{20} \frac{p_{stag}}{\pi su_{cs}} \right)^{10} \approx 4.5 \cdot 10^{-13} \left( \frac{p_{stag}}{su_{cs}} \right)^{10}$$

in which $v_{ref,t}$ is the reference tangential velocity at which the undrained shear strength at critical state was determined (0.02 [rad/s]·6.35 [mm] $\approx 1.25 \cdot 10^{-3}$ [m/s]). Assuming a stagnation ratio ($p_{stag}/su_p$) of 12 and a sensitivity of 1.4, the maximum tangential velocity will be about 0.8 m/s, see also Figure 7.22: $R_{de} = L_{de}, M_r = \text{var}$. The maximum tangential velocity without the influence of the dynamic shear stress ($\tau_s = su_{cs}$) can be calculated with Equation 7.38 and will be about 18 m/s. The difference between both tangential velocity is significantly. According to Equation 7.45 the maximum tangential velocity increases rapidly with the stagnation pressure and is soon no longer limiting. This explains the explosive growth of the rotation time with a decreasing stagnation ratio.

- Both the driving jet moment and the resisting soil moment change with rotation angle and the dimension of the soil element is optimal: $R_{de} \approx 1.3 L_{de}$ (see Figures 7.21, 7.22 and 7.23: $R_{de} \approx 1.3 L_{de}, M = \text{var}$). In this case the resisting soil moment decreases with rotation angle $\omega_{de}$, despite an increase in dynamic shear stress, because of a decrease in shear surface. As a result the soil element can still accelerate at low stagnation ratios. This shortens the rotation time with respect to previous case, see Figure 7.21.

Note that in this practical case the dynamic shear stress will increase to a value of 3.5 times the undrained shear strength at critical state, see Figure 7.23.

- Both the driving jet moment and the resisting soil moment change with rotation angle and the dimension of the soil element is optimal: $R_{de} \approx \frac{7.184}{0.58}$

\[^{4}\text{In this case the ratio between the dynamic shear stress and the undrained shear strength at critical state (}$\tau_s/su_{cs}$\text{) is 2.4, see Figure 7.23. This corresponds with the minimum stagnation ratio of 7, which was derived in Section 7.4.2: } p_{stag}/\tau_s = p_{stag}/(2.4 \cdot su_{cs}) = p_{stag}S_s/(2.4 \cdot su_p) \approx 0.58p_{stag}/su_p = 0.58 \cdot 12 = 7.\]
7.4. Advanced approach

1.3\(L_{de}\). The only difference with the previous case is that the dynamic shear stress \(\tau_s\) is limited to a value of two times the undrained shear strength at critical state: \(\tau_s/su_{cs} < 2\) (see Section 7.4.1 and Figure 7.10). By limiting the dynamic shear stress also the resisting soil moment and as a result the rotation time \(t_{de}\) decreases significantly (see figure 7.21: \(R_{de} \approx 1.3L_{de}, M = \text{var}, \tau_s/su_{cs} < 2\)).

7.4.5 Vertical propagation velocity of the soil failure front

The shearing process of the discrete soil elements was not visible on the high speed recordings, see Section 5.3.3 and (Nobel et al., 2010). In most cases only a vertical thin front was visible, below which deforming material occurs and above which the material has been removed already. The vertical propagation velocity of this front \(u_{f,v}\) can be estimated, assuming that the deforming soil below this front consists of separately dislodging discrete soil elements (see Figure 7.24), by dividing the characteristic length of the soil elements by the rotation time:

\[
 u_{f,v} = \frac{L_{de}}{t_{de}} \tag{7.46}
\]

Because the rotation time of the discrete soil elements \(t_{de}\) is linearly proportional to the characteristic length of these elements \(L_{de}\), see e.g. Equations 7.35 and 7.36, the vertical propagation velocity of the soil front \(u_{f,v}\) becomes independent of the characteristic length of the elements.

Figure 7.24: Definition of the vertical propagation velocity of a thin front \(u_{f,v}\), below which deforming material occurs and above which the material has been removed already and the definition of the horizontal propagation velocity of the jet front just below the original soil surface \(u_{f,h}\).
7. Modeling moving penetrating jet

Figure 7.25 shows the upper (resisting soil moment is neglected: $M_r = 0$) and lower limit (resisting soil moment is calculated assuming an unlimited dynamic shear stress) of the vertical propagation velocity ($u_{f,v}$), for a soil with a peak undrained shear strength ($su_p$) of 70 and 20 kN/m$^2$. These velocity ranges are independent of the characteristic length of the soil elements ($L_{de}$).

![Figure 7.25: Calculated upper and lower limit of the vertical propagation velocity of a thin front below which continuously soil is removed ($u_{f,v}$) for a soil with a peak undrained shear strength ($su_p$) of 70 and 20 kN/m$^2$.](image)

**Verification**  For three tests it was possible to determine the vertical propagation velocity of the thin front $u_{f,v}$ from the high speed recordings. In Table 7.2 the observed and calculated vertical propagation velocities are compared. The vertical propagation velocities were determined at a location of about 50 mm below the original soil surface. The stagnation ratio at this position was calculated with the 1D-approach of a confined jet, see Section 3.3.3 ($D_n = 18$ mm, $SOD = 20$ mm).

The calculated propagation velocities ($u_{f,v}$) according to the upper limit of the soil resistance ($M_r > 0$) correspond quite well with the observed ones. The differences between the observed and calculated values are smaller than 15%. The calculated propagation velocities are greatly overestimated, when the soil resistance is not taken into account ($M_r = 0$). In Figure 7.26 the calculated and observed vertical propagation velocities are plotted as a function of the stagnation ratio. In this figure the peak undrained shear strength is assumed to be 33 kN/m$^3$. This figure shows clearly the overestimation of the propagation velocity when the soil resistance is not taken into account ($M_r = 0$). The propagation velocity is also
7.4. Advanced approach

Table 7.2: Comparison of observed and calculated vertical propagation velocity \( (u_{f,v}) \) of the thin front below which continuously soil is removed \( (S_s = 1.2) \).

<table>
<thead>
<tr>
<th>( p_j ) ( (\text{Exp}) ) [MN/m²]</th>
<th>( s_{up} ) ( (\text{Calc}) ) [kN/m²]</th>
<th>( p_{stag} ) [MN/m²]</th>
<th>( p_{stag}/s_{up} ) [-]</th>
<th>( u_{f,v} ) ( (\text{Exp}) ) (M_r &gt; 0) [m/s]</th>
<th>( u_{f,v} ) ( (\text{Exp}) ) (M_r = 0) [m/s]</th>
<th>( u_{f,v} ) ( (-13%) )</th>
<th>( (+42%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.06</td>
<td>28</td>
<td>0.66</td>
<td>23.5</td>
<td>12</td>
<td>10.4</td>
<td>17</td>
<td>(+42%)</td>
</tr>
<tr>
<td>1.06</td>
<td>33</td>
<td>0.7</td>
<td>21</td>
<td>10</td>
<td>9.6</td>
<td>17.5</td>
<td>(+75%)</td>
</tr>
<tr>
<td>1.06</td>
<td>39</td>
<td>0.7</td>
<td>18</td>
<td>8.3</td>
<td>7.8</td>
<td>17.6</td>
<td>(+112%)</td>
</tr>
</tbody>
</table>

overestimated when the dynamic shear stress is limited to 2 times the undrained shear strength at critical state \( (\tau_s < 2s_{cs}) \).

It can be concluded that the propagation velocity can be estimated with the proposed calculation method taken into account the dynamic soil resistance.

**Influence of turbulence on the rotation time** When the timescale of the turbulent pressure fluctuations is of the same order, or larger than the rotation time of the discrete soil elements \( (t_{de}) \), this will influence the soil failure process in the non-deflection zone. The largest timescale of the pressure fluctuations is assumed to be in the order of magnitude \( D_n/u_0 \approx 1 \cdot 10^{-3} \) s. From the Figures 7.20 and 7.21 it follows that the rotation time is probably smaller than the timescale of the pressure fluctuations. Consequently, turbulence will extend and shorten the rotation time of the discrete soil elements. Because of the non-linear dependency between the rotation time and the stagnation pressure (or stagnation ratio, see Figure 7.21), the height averaged vertical propagation \( (u_{f,v}) \) will decrease a little. This influence is assumed to be negligible.

### 7.4.6 Curvature of the jet in the deflection zone

From a certain depth the jet starts deflecting backwards. The deflecting jet flow supports the soil. This prevents the shearing of small discrete soil elements in the deflection zone. The soil will fail in large curved strips. This failure process is modeled in this section. The model comprises a relation between the thickness of the strip \( (L_{st}, \text{see figure 7.27}) \) and the stagnation ratio \( (p_{stag}/s_{up}) \) at the start of the deflection zone. An estimation is made of the stagnation ratio at the transition between the non-deflection zone and the deflection zone, based on measured values of the strip thickness.
The thickness of the soil strips follows from the equilibrium of moments. The following assumptions are made:

1. At large cavity depths the length of the jet cross section ($L_j$) just before the curvature is much larger than the cavity width ($W_c$). This allows a 2D approximation with rectangular jet and soil strip cross sections, see Figure 7.27 (b).

2. The pressure at the top of the strip is equal to the stagnation pressure ($p_{stag}$) of the jet at that location.

3. Over the full height of the strip the jet velocity and jet curvature are constant. This results in a constant pressure increase ($\Delta p_{st}$) over the total curvature. This pressure increase can be estimated, realizing that the total vertical jet momentum ($I_j$) has to be converted, with the following equation:

$$\Delta p_{st} = \frac{I_j - L_{st}W_c p_{stag}}{R_{st}} = \frac{2W_c L_{ja} - L_{st}W_c}{R_{st}} \cdot p_{stag}$$

(7.47)

in which $R_{st}$ is the radius of the curved strips, $W_c$ is the cavity width and $L_{ja}$ is the length of the rectangular jet cross section, defined as $A_j/W_c$.

4. The mobilized shear stress ($\tau_s$) in the shear surfaces is equal to the peak undrained shear strength ($s_u$).
7.4. Advanced approach

The driving jet moment, with respect to the axis of rotation \( RP \), can be calculated with the following equation:

\[
M_j = p_{stag} L_{st} W_c \left( R_{st} - \frac{1}{2} L_{st} \right) - \Delta p_{st} W_c \left( R_{st} L_{st} - \frac{3}{8} L_{st}^2 \right) \tag{7.48}
\]

Note that only in the theoretical case that the strip thickness is zero, the second term can be omitted.

The resisting soil moment \( (M_r) \) with respect to the axis of rotation \( RP \) consists of two parts, the resisting soil moment of the curved shear surface \( (M_{rb}) \) and the resisting soil moment of the flat shear surfaces on both sides of the strip \( (M_{rs}) \). The resisting soil moment of the curved shear surface can be calculated with the following equation:

\[
M_{rb} = \left( \frac{1}{2} \pi + \xi \right) R_{st}^2 W_c \tau_s \approx \left( \frac{1}{2} \pi R_{st}^2 + \frac{1}{2} L_{st} R_{st} \right) W_c \sigma_p \tag{7.49}
\]

where \( \xi \) is an auxiliary angle, see Figure 7.27. The resisting soil moment of the shear surfaces \( (M_{rs}) \) on both sides of the strip is more difficult to calculate, because of the shape of these surfaces, see also Section 6.2.1. Therefore, this moment is calculated numerically.

In Figure 7.28 the calculated normalized soil strip thickness \( (L_{st}/L_{ja}) \) is plotted as a function of the stagnation ratio just above the curvature. In these calculations the cavity width \( (W_c) \) and the length of the rectangular jet cross section \( (L_{ja}) \) are assumed to be \( 1.5D_n \) and \( 2D_n \), respectively. The strip thickness is calculated with and without the influence of the shear stresses on both sides of the strip \( (M_{rs}) \). Because of 3D-effects, the expected value of the soil strip thickness \( L_{st} \) will be somewhere between these two values.

As a reference also all measured strip thicknesses are plotted as a function of the calculated stagnation ratio \( (p_{stag}/\sigma_p) \) just above the soil strip.\(^5\)

From Figure 7.28 it follows that the model overestimates the strip thickness. It must be noted that the accuracy of the measured values of the soil strip thickness is limited. In addition the measured thicknesses are normalized by the calculated values of the length of the rectangular jet cross section \( (L_{ja}) \) and plotted as a function of the calculated values of the stagnation ratio \( (p_{stag}/\sigma_p) \) just above the curvature. These calculations also introduce some uncertainties. This makes it difficult to draw a clear conclusion about the stagnation pressure at the transition

\(^5\)The stagnation ratio and the length of the rectangular jet cross section just above the soil strip are calculated according to the 1D-approach of a confined jet, see Section 3.3.3, assuming that the strip starts at a depth: \( Z_c - R_{cu} \), in which \( Z_c \) is the measured cavity depth and \( R_{cu} \) is the measured radius of the curvature, see Figure 5.21.
7. Modeling moving penetrating jet

Figure 7.28: Calculated and measured soil strip thickness ($L_{st}$) normalized by the length of the rectangular jet cross section ($L_{ja}$), as a function of the stagnation ratio just above the strip ($p_{stag}/su_p$), assuming: $L_{ja} = 2D_n$, $R_{st} = 2.5L_{ja}$ and $W_c = 1.5D_n$.

between the non-deflection zone and the deflection zone. For the time being, it is assumed, based on Figure 7.28, that the transition takes place at a stagnation ratio ($p_{stag}/su_p$) of about 15. It is recommended to investigate the stagnation pressure at the transition and the soil strip thickness in more detail.

In the simple theoretical case that the strip thickness and the radius of the curvature are two times the average jet thickness ($L_{st} = R_{st} = 2L_{ja}$, see Figure 7.29), the total jet impinges on the soil strip and will be deflected horizontally ($\Delta p_{st} = 0$).

In this case the required stagnation ratio is equal to that of a discrete soil element for which $L_{de} = R_{de}$, see Section 7.4.2 and Figure 7.30:

\[
\frac{p_{stag}}{su_p} = \pi \left( \frac{4}{3} + \frac{8}{9} \frac{R_{st}}{W_c} \right) \tag{7.50}
\]

Assuming that the cavity width ($W_c$) is $1.5D_n$ and the radius of the curvature ($R_{st}$) is $2L_{ja} = 4D_n$, the minimum stagnation ratio becomes about 11.6. This stagnation ratio is a little lower than the minimum value in Figure 7.28 ($L_{st} = 2L_{ja}$), because the radius of the curvature is smaller ($R_{st} = 2L_{ja}$ instead of $2.5L_{ja}$).
7.4.7 Modeling Jet trajectory

The jet flow of a moving jet will deflect backwards. The trajectory of the flow depends mainly on the stagnation ratio at the original soil surface \((p_{stag,ss}/su_p)\) and the traverse velocity of the nozzle \((v_t)\). In this section a calculation model is described to predict the jet trajectory as a function of these two parameters.

The coordinates of the jet trajectory are defined by a moving right-handed Cartesian coordinate system, with the center of the nozzle as origin and the positive x-direction opposite the nozzle traverse direction and the positive z-direction opposite the initial jet direction (note that the cavity depth \(Z_c\) is defined positive).

The jet trajectory of the penetrating jet is divided into two zones: the non-deflection zone and the deflection zone.

Non-deflection zone When the traverse velocity of the nozzle is very low, almost all discrete soil elements in a vertical column are dislodged before a new column of soil elements starts to dislodge, see Figure 5.25 (a). Increasing the traverse velocity results in more and more columns that are dislodging at the same time, see Figure 5.25 (b). Therefore it is possible that the jet already gets a small inclination in the non-deflection zone.

The x- and z-coordinates of the jet trajectory in the non-deflection zone can be calculated, assuming that the time after passing the nozzle, equals the sum of the rotation times \((t_{de})\) of the different discrete soil elements in a column.

Figure 7.29: Simple theoretical case that the whole jet impinges on the top of the strip: \(L_{st} = 2L_{ja} = R_{st}\).

Figure 7.30: Comparison of the vertical longitudinal cross sections of a discrete soil element and a soil strip. The cross section of a soil element is rotated by \(90^\circ\), with respect to that of a soil strip.
7. Modeling moving penetrating jet

It follows that, see Figure 7.31:

\[
\frac{x}{v_t} = \sum_{1}^{n} t_{de}(z) \quad (7.51)
\]

\[
z = -nL_{de} \quad (7.52)
\]

where \(n\) is the number of dislodged elements in a column, and \(L_{de}\) is the characteristic length of a discrete soil element. The rotation time of the discrete soil elements at every position can be calculated, by substituting the calculated stagnation ratios into the equations for the rotation time, see Section 7.4.3. The stagnation ratio at every position can be calculated according to the 1D-approach of a confined jet, see Section 3.3.3.

![Diagram of jet trajectory](image)

Figure 7.31: Definition of the jet trajectory in the non-deflection zone.

**Deflection zone** The curvature of the jet trajectory in the deflection zone is assumed to be 2.5 times the length of the rectangular jet cross section (\(L_{ja}\)) just before the curvature. The inclination angle of the jet at the beginning of this zone can be approximated with the following equation, see figure 7.32:

\[
\tan(\beta_{dz0}) = \frac{x_{dz0}}{-z_{dz0}} \quad (7.53)
\]

where \(x_{dz0}\) and \(z_{dz0}\) are the coordinates at the beginning of the deflection zone.
7.4. Advanced approach

Non-deflection zone Deflection zone

Figure 7.32: Definition of the jet trajectory in the deflection zone.

Verification jet trajectory

For three jet tests at different traverse velocities of the nozzle \(v_t\), the jet trajectories are calculated, see Figure 7.33. All these tests were carried out on the same soil sample. In these calculations, the deflecting zone is assumed to start when the stagnation ratio \(p_{stag}/su_p\) is decreased to a value of 15. The following main parameters are used: \(D_j = 18\) mm, \(p_j = 1.06\) MN/m\(^2\), \(SOD = 0.02\) m, \(su_p = 40\) kN/m\(^2\), \(S_s = 1.2\) (GD-sample), \(L de = 2\) mm.

Figure 7.33: Calculated jet trajectory for the jet tests on the GD-samples \((p_j/su_p = 26.5)\). As a reference the measured end of the jet front (asterisks) and the calculated cavity depths according to the 1D-approach are plotted (triangles).

The influence of the traverse velocity of the nozzle is clearly visible. By increasing
the traverse velocity from 0.15 to 1.4 m/s, the calculated cavity depth decreases from about 11 to 8.25 cm. The measured cavity depths are of the same order, they show a decrease from 10.5 to 8.25 cm (asterisks), see also Figure 7.34. For all tests the horizontal length of the curvature is underestimated.

Figure 7.34: Influence of traverse velocity on the cavity depth (a) $v_t = 0.15$ m/s (b) $v_t = 1$ m/s (c) $v_t = 1.4$ m/s. Other test conditions: $D_j = 18$ mm, $p_j/su_p = 26.5$.

The calculated vertical length of the non-deflection zone at a traverse velocity of 0.15 m/s is about 0.06 m. The measured length of this zone is about 0.04 m, see Figure 7.34 (a). For the tests with a traverse velocities of 1 and 1.4 m/s the transition between the non-deflection zone and the deflection zone is not clearly visible in Figure 7.34, because the deflection of the jet in the non-deflection zone is already significant. This is in good agreement with the pictures of the cavity wall structure, see Figures 7.34 (b) and (c). It can be concluded that the jet trajectory can be well predicted. However, a lot of assumptions have to be made. Not all these assumptions are unambiguous. This still needs further investigation.

It follows that at high traverse velocities the inclination of the jet flow already can be significantly in the non-deflection zone. As a result the transition between the two zones blurs and possibly will take place at another stagnation ratio.

As a reference also the calculated cavity depths according to the 1D-approach are plotted in Figure 7.33 (triangles). In this approach the traverse velocity of the nozzle is only taken into account in the jet flow development (entrainment of soil at the front side of the jet), see Section 7.3.2. Also with this simple approach a reasonable prediction of the cavity depth ($Z_c$) can be made.
7.4. Advanced approach

Verification influence traverse velocity of nozzle on penetration depth

In Figure 7.35 for different jet ratios \( p_j/su_p \approx p_{stag,ss}/su_p \), see Table 7.3) the calculated cavity depths, according to the advanced approach, are plotted as a function of the traverse velocity of the nozzle.

As a reference the measured cavity depths of the laboratory jet tests from 2007 are plotted. For the jet ratios of 29.5 and 26.5 the results are in agreement with the experiments. However for the jet ratio of 19.5 the calculations underestimate the cavity depth, especially at low traverse velocities. A clear explanation for this deviation cannot yet be given. Possibly the stagnation ratio at the transition between the non-deflection zone and the deflection zone is not a constant, but depending on the traverse velocity. More tests are needed to confirm this and to verify the calculation model in more detail.

![Figure 7.35: Measured and calculated (according to the advanced approach) cavity depths \( (Z_c) \) as a function of the traverse velocity of the nozzle \( (v_t) \), for different jet ratios \( (p_j/su_p) \).](image)

Table 7.3: Input for calculations to verify the influence of the traverse velocity of nozzle on the penetration depth, see Figure 7.35.

<table>
<thead>
<tr>
<th>( p_j/su_p ) [-]</th>
<th>( p_j ) [MN/m²]</th>
<th>( su_p ) [kN/m²]</th>
<th>( S_s ) [-]</th>
<th>(see Table 5.10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.5</td>
<td>1.06</td>
<td>36</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>26.5</td>
<td>1.06</td>
<td>40</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>19.5</td>
<td>1.06</td>
<td>54</td>
<td>1.6</td>
<td></td>
</tr>
</tbody>
</table>
7. Modeling moving penetrating jet

7.5 Conclusions

With respect to the modeling of the penetrating jet type, the following conclusions can be drawn:

- For a penetrating jet (with a clear non-deflection zone; \( Z_c > 2.5D_{j,ss} \)), the minimum stagnation ratio at the original soil surface (\( p_{stag,ss}/su_p \)) must be larger than 12.

- For a slowly moving penetrating jet (\( v_t < 0.1\text{m/s} \)) the cavity depth and width can be calculated analytically. The cavity width is assumed to be equal to the jet diameter at the original soil surface (\( W_c = D_{j,ss} \)). The cavity depth is based on the condition that the stagnation pressure (\( p_{stag} \)) at the bottom is 8.2 times the peak undrained shear strength (\( su_p \)).

- For a moving jet the cavity dimensions can be calculated with the 1D-approach. In this approach the cavity width is based on the condition that the stagnation pressure at the original soil surface must be at least \( 6.2su_p \). The cavity depth is based on the condition that the stagnation pressure at the bottom is \( 8.2su_p \). In this approach the influence of the traverse velocity of the nozzle is only taken into account in the jet flow development (entrainment of soil at the front side of the jet).

- The cavity of a penetrating jet can be divided into two zones, the non-deflection zone and the deflection zone. In the non-deflection zone the soil is assumed to fail in small discrete soil elements. In the deflection zone the soil fails in large curved strips. The radius of the curvature is assumed to be 2.5 times the length of the rectangular jet cross section just above the curvature. The transition between both zones is assumed to take place at a stagnation ratio of about 15. Both failure mechanisms are considered in the advanced calculation model.

- The soil in the non-deflection zone can be modeled as small discrete soil elements with a curved shear surface at the backside. The typical size of these elements ranges from 2 to 5 mm. A clear explanation for the size of the elements cannot be given.

- A soil element starts to rotate when the stagnation pressure exerted by the jet on the top of this element is larger than 6.4 times the peak undrained shear strength (\( p_{stag}/su_p > 6.4 \)).

- The mobilized shear stress (\( \tau_s \)) in the shear surfaces of a soil element is assumed to decrease to the undrained shear strength at critical state (\( su_{cs} \))
Conclusions

instantaneously after reaching the peak undrained shear strength and starting to rotate). The shear stress increases with the tangential velocity of the curved shear surface ($v_{de,t}$).

- The required time to dislodge a soil element completely (rotation time $t_{de}$), depends strongly on the actual stagnation ratio ($p_{stag}/su_p$) and the dynamic shear stress. The larger the stagnation ratio and the lower the dynamic shear stress, the quicker a soil element will be dislodged.

- On the high speed recordings the shearing process of small soil elements was not visible. The recordings only showed a thin horizontal front below which soil is removed continuously. Based on the rotation time of the discrete soil elements, the vertical propagation velocity of this front ($u_{f,v}$) can be estimated. Assuming that the resistance in the shear surfaces remains intact and increases during rotation, the difference between the calculated and observed vertical propagation velocity is less than 15%.

- With the advanced approach the jet trajectory and the cavity depth, both as a function of the jet ratio and the traverse velocity of the nozzle can be well predicted.
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Chapter 8

Conclusions and recommendations

8.1 Main results

In this section the main results of modeling the excavation process of a moving vertical jet in cohesive soil are summarized. At first the main dependencies of the excavation process are discussed with a description of the expected jet trajectory. Secondly, the different calculation results of the cavity width ($W_c$) will be described, followed by the calculations of the cavity depth ($Z_c$).

8.1.1 Jet trajectory

The main parameters, which determine the excavation process of a moving vertical jet in cohesive soil are: the stagnation ratio $p_{stag,ss}/su_p$, the jet diameter $D_{j,ss}$ (both at the original soil surface) and the traverse velocity of the nozzle ($v_t$). Note that both, the stagnation ratio $p_{stag,ss}/su_p$ and the jet diameter $D_{j,ss}$ are functions of the stand off distance ($SOD$). For small stand off distances these parameters are more or less equal to the jet ratio ($p_{stag,ss}/su_p \approx p_j/su_p$) and the nozzle diameter ($D_{j,ss} \approx D_n$), respectively.

Stagnation pressure

For low traverse velocities ($v_t < v_{t0}$, see Equation 8.1), the cavity depth ($Z_c$) and the jet trajectory are mainly determined by the stagnation ratio ($p_{stag,ss}/su_p$) and the jet diameter ($D_{j,ss}$), as depicted in Figure 8.1 (a). The jet trajectory scales with the jet diameter at the soil surface ($D_{j,ss}$). When the stagnation ratio at the original soil surface is larger than 12, the jet will penetrate the soil nearly vertically. When the stagnation ratio at the original soil surface is smaller than 12, the jet will deflect backwards directly after penetrating the soil; a clear non-deflection zone is missing. For stagnation ratios smaller than about 5.4, the jet will not penetrate the soil any longer.
Jet traverse velocity

In Figure 8.1 (b) the influence of the traverse velocity of the nozzle on the jet trajectory is depicted for a stagnation ratio which is much larger than 12 \( (p_{stag,ss}/su_p \gg 12) \). The influence of the traverse velocity on the jet trajectory is limited for traverse velocities smaller than \( v_{t0} \), which is defined as:

\[
v_{t0} = \frac{\bar{u}_{f,v} L_{de,f}}{Z_c - R_{cu}} \tag{8.1}
\]

in which \( \bar{u}_{f,v} \) is the height averaged value of the vertical propagation velocity of a thin vertical front below which continuously discrete soil elements will be removed (see Equation 7.46), \( L_{de,f} \) is the characteristic width of the soil elements in traverse direction (see Figure 7.24) and \( Z_c - R_{cu} \) is an estimate of the length of the non-deflection zone (see Figure 5.8).

Below this traverse velocity, all discrete soil elements of a column in the non-deflection zone are removed before a new column made out of soil elements starts to dislodge, see Figure 5.24 (a). The traverse velocity only affects the density of
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the jet flow. The cavity depth will decrease slightly with an increase in traverse velocity of the nozzle.

Increasing the traverse velocity \( (v_t > v_{t0}) \), results in more and more columns that are dislodging simultaneously, see also Figure 5.24(b). As a result the jet already gets a small inclination in the non-deflection zone.

When the traverse velocity becomes higher than \( v_{t1} \), which is defined as:

\[
v_{t1} = \frac{\bar{u}_{f,v} D_{j,ss}}{Z_c - R_{cu}}
\]  

(8.2)
the length of the non-deflection zone \((Z_c - R_{cu})\) will decrease. The length of the non-deflection zone becomes zero, at a traverse velocity of:

\[
v_{t2} = u_{f,h} = \frac{L_{de,f}}{t_{de}}
\]  

(8.3)
in which \( u_{f,h} \) is the horizontal propagation velocity of the jet front just below the original soil surface (see Figure 7.24) and \( t_{de} \) is the rotation time of a soil element just below the soil surface.

By increasing the traverse velocity further, the jet will not be able to remove the strips in the deflecting region completely. As a result the cavity depth becomes irregular and decreases further. At a certain traverse velocity \((v_{tc})\) the jet cannot penetrate the soil any longer.

In Figure 8.2 the expected normalized cavity depth \((Z_c/D_{j,ss})\) is plotted as function of the traverse velocity of the nozzle \((v_t)\) for different stagnation ratios: \( p_{stag,ss1}/su_p > p_{stag,ss2}/su_p > p_{stag,ss3}/su_p \gg 12 \).

The lower limit of the traverse velocity of the nozzle \((v_{t0})\) will decrease, while the upper limits \((v_{t2} \text{ and } v_{tc})\) will increase with increasing the stagnation ratio at the original soil surface \((p_{stag,ss}/su_p)\). Five traverse velocity regimes can be distinguished, as earlier mentioned and listed in Table 8.1.

Note that the jet tests, carried out in the present study, do not cover the whole range of traverse velocities, as depicted in Figure 8.2. The traverse velocity of the nozzle was smaller than \( v_{t1} \) for most of the tests. Therefore this figure cannot be verified completely.

Jet diameter

In Figure 8.3 the jet trajectory is depicted for three different jet diameters \((D_{j,ss1} = 2L_{de} < D_{j,ss2} = 4L_{de} < D_{j,ss3} = 6L_{de})\) and two traverse velocities \((v_t < v_{t0} \text{ and } v_t = v_{t1})\). In this figure the stagnation ratio at the original soil surface is much larger than 12 \((p_{stag,ss}/su_p \gg 12)\). As previously mentioned, the jet trajectory
8. Conclusions and recommendations

Figure 8.2: Influence of the traverse velocity ($v_t$) on the normalized cavity depth ($Z_c/D_{j,ss}$) for three stagnation ratios at the original soil surface ($p_{stag,ss1}/su_p > p_{stag,ss2}/su_p > p_{stag,ss3}/su_p \gg 12$).

Table 8.1: Traverse velocity regimes.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Dependency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_t \leq v_{t0}$</td>
<td>$v_t$ influences $Z_c$ only indirect by the jet flow density.</td>
</tr>
<tr>
<td>$v_{t0} &lt; v_t \leq v_{t1}$</td>
<td>Jet already gets a small inclination in the non-deflection zone.</td>
</tr>
<tr>
<td>$v_{t1} &lt; v_t \leq v_{t2}$</td>
<td>$Z_c$ is highly dependent on $v_t$ (non-deflection zone decreases).</td>
</tr>
<tr>
<td>$v_{t2} &lt; v_t \leq v_{t,c}$</td>
<td>Soil strips in deflection zone are only partially removed.</td>
</tr>
<tr>
<td>$v_t &gt; v_{t,c}$</td>
<td>No penetration of the jet.</td>
</tr>
</tbody>
</table>

scales with the jet diameter at the soil surface ($D_{j,ss}$); The shape of the jet trajectory is similar for all nozzle diameters. The only difference is the number of columns made out of soil elements that are dislodging at the same moment. Because the main dimension of the soil elements is independent of the nozzle diameter, the number of the columns will increase with increasing jet diameter.

The critical traverse velocity below which hydro-fracturing may occur, probably depends on the jet diameter. The larger the jet diameter, the higher the traverse velocity at which hydro-fracturing may occur, see also Section 2.6.1.
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(a) \[ v_t = v_{t1} \]

(b) \[ v_t \leq v_{t0} = \frac{\bar{u}_{f,v} L_{de,r}}{(Z_c R_{cu})} \]

(c) \[ v_t = v_{t1} \]

Figure 8.3: Jet trajectory, assuming \( p_{stag,ss/su_p} \gg 12 \), for three jet diameters \( D_{j,ss1} = 2L_{de} < D_{j,ss2} = 4L_{de} < D_{j,ss3} = 6L_{de} \) at two traverse velocities \( (v_t < v_{t0} \text{ and } v_t = v_{t1}) \).

8.1.2 Modeling cavity width

In Table 8.2 the applicability range of the different calculation methods for the cavity width is listed. The applicability range is expressed in terms of the stagnation ratio at the original soil surface \( (p_{stag,ss/su_p}) \). The corresponding range in calculated cavity widths is normalized by the jet diameter at the original soil surface \( (D_{j,ss}) \). Note that the value of \( p_{stag,ss/su_p} \) at the transition between the deflecting- and penetrating jet will increase at higher traverse velocities of the nozzle than tested in the present study.

In Figure 8.4 the calculated cavity widths for low jet ratios \( (p_j/su_p < 50) \) normalized with the nozzle diameter \( (W_c/D_n) \), are compared with the measured ones (except those tests where hydro-fracturing occurred).

For the higher stagnation ratios the cavity widths are underestimated, see also Section 7.3.1, Figure 7.4. This still needs further investigation.
8. Conclusions and recommendations

Table 8.2: Applicability range of the different calculation methods for the cavity width ($W_c$).

<table>
<thead>
<tr>
<th>Method</th>
<th>$p_{stag,ss}/su_p$ [-]</th>
<th>$W_c/D_{j,ss}$ [-]</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dispersing jet flow (straight penetration)</td>
<td>5.4-7.3</td>
<td>2.45-1.7</td>
<td>6.3.2</td>
</tr>
<tr>
<td>Deflecting jet</td>
<td>7.3-12</td>
<td>1.7</td>
<td>6.2.1</td>
</tr>
<tr>
<td>Penetrating jet (analytical approach)</td>
<td>12-50</td>
<td>1.16</td>
<td>7.2.1</td>
</tr>
<tr>
<td>Penetrating jet (1D-approach)</td>
<td>$&gt;50$</td>
<td>-</td>
<td>7.3.1</td>
</tr>
</tbody>
</table>

Figure 8.4: Comparison calculated and measured cavity width for low pressure tests ($SOD \approx 1D_n$).

8.1.3 Modeling cavity depth

In Table 8.3 the proposed calculation methods for the cavity depth ($Z_c$) are listed. When the stagnation ratio at the original soil surface ($p_{stag,ss}/su_p$) is smaller than 12, the jet will deflect backwards directly after penetrating the soil (deflecting jet).

In Figure 8.5 the calculated and measured cavity depths for the deflecting jet type are compared with each other. These calculations are based on a cavity width of $1.7D_{j,ss}$. The correlation is disappointing, though the predicted trend is right.

When the stagnation ratio at the original soil surface is larger than 12, the jet will penetrate the soil almost vertically (penetrating jet). The cavity depth of the penetrating jet type can be calculated according to three methods.

For very small traverse velocities ($v_t < 0.1$ m/s) the cavity depth can be calculated analytically. However, hydro-fracturing can occur at these traverse velocities.

\footnote{Note that the value of $p_{stag,ss}/su_p$ at the transition between the deflecting- and penetrating jet will increase at higher traverse velocities of the nozzle than tested in the present study.}
Table 8.3: Applicability range of the different calculation methods for the cavity depth ($Z_c$).

<table>
<thead>
<tr>
<th>Method</th>
<th>$p_{stag,ss}/su_p$ [-]</th>
<th>$v_t$ [m/s]</th>
<th>$Z_c$ [m]</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflecting jet ($W_c = 1.7D_{j,ss}$)</td>
<td>6.2-12</td>
<td>-</td>
<td>&lt; 2.5$D_{j,ss}$</td>
<td>6.2.1</td>
</tr>
<tr>
<td>Analytical approach</td>
<td>&gt; 12</td>
<td>&lt; 0.1</td>
<td>≥ 2.5$D_{j,ss}$</td>
<td>7.2.2</td>
</tr>
<tr>
<td>1D-approach</td>
<td>&gt; 12</td>
<td>–</td>
<td>≥ 2.5$D_{j,ss}$</td>
<td>7.3.2</td>
</tr>
<tr>
<td>Advanced approach</td>
<td>&gt; 12</td>
<td>&lt; $u_{f,h}$</td>
<td>≥ 2.5$D_{j,ss}$</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Hydro-fracturing is observed at traverse velocities smaller than 0.15 m/s. This can deepen the cavity significantly.

For larger traverse velocities the cavity depth must be calculated according to the 1D-approach or the advanced approach. In Figure 8.6 the calculated cavity depths, according to the advanced approach, for the penetrating jet tests are compared with the measured ones, except those tests where hydro-fracturing occurred.

Figure 8.5: Comparison calculated and measured cavity depths of the deflecting jet type (LP = low pressure; $p_j < 3.5$ MN/m$^2$).

Figure 8.6: Comparison of the measured and calculated cavity depths for the penetrating jet type, according to the advanced approach.

The standard deviation ($\sigma_{sd}$) of the differences between the measured and calculated cavity depth is 22%. It can be concluded that the cavity depth can be well

\[^2\text{Note that this standard deviation is of the same order of magnitude as obtained with the 1D-approach, see Figure 7.7 (}\sigma_{sd} = 25.5\%\). However, the advanced approach is physically more substantiated and also applicable for higher traverse velocities of the nozzle.]
predicted with the advanced approach, assuming an accuracy in peak undrained shear strength \((s_{up})\) of ±20%.

### 8.2 Conclusions

It has been proved possible to model the jetting process of a moving vertical penetrating jet in cohesive soil. The main jet cavity dimensions and the jet trajectory can be calculated as function of the main jet settings and soil parameters.

The jet is simplified to have a uniform flow velocity \((u_u)\), which is defined as \(I/(\rho_m Q)\), where \(I\) is the momentum flux, \(\rho_m\) is the jet flow density and \(Q\) is the jet flow rate. The uniform jet velocity decreases with increasing distance to the nozzle \((s)\) mainly through the entrainment of soil at the front side of the jet and water at the rear side of the jet (interface between jet and ambient water).

The entrainment of ambient water can be approximated by multiplying the local uniform jet flow velocity with the surface area of the interface between the jet and ambient fluid and an empirical entrainment coefficient \((dQ_w/ds = \alpha_{mom}1/2\pi W_c u_u)\), where \(\alpha_{mom}\) is the entrainment coefficient. This coefficient increases gradually with increasing distance \(s\) to a constant value in the region of fully developed flow. For a non-cavitating jet this constant value is about 0.083. When the jet is cavitating this value decreases with the inverse of the quartic of the jet pressure \((p_j)\).

When the jet has penetrated the soil, the interface between the jet and ambient fluid is limited to the rear side of the jet. At the front side of the jet soil is entrained. This influences both the jet flow rate and the jet flow density. The entrainment of soil can be approximated by multiplying the traverse velocity of the nozzle \((v_t)\) with the frontal jet surface \((dQ_s/ds = v_t W_c)\).

The soil is modeled to fail along predefined shear surfaces. The soil fails when the jet load exceeds the peak resistance in these shear surfaces. The mobilized shear stresses are assumed to be equal to the peak undrained shear strength \((s_{up})\). Simultaneously after reaching the peak, the shear stresses are assumed to decrease to a value equal to the undrained shear strength at critical state \((s_{u cs})\).

The jet load at a certain distance \(s\) is function of the stagnation pressure \(p_{stag}\) which is defined as: \(p_{stag} = 1/2\rho_m u_u^2\) (known as the dynamic pressure). Therefore the most important parameter is the dimensionless stagnation ratio \((p_{stag}/s_{up})\).

From the jet tests it follows that for a large range of test settings \((5.4 < p_j/s_{up} < 200, 0.15 < v_t < 2 \text{ m/s})\) the cavity depth \((Z_c)\) normalized with the nozzle diameter.
(\(D_n\)) is linearly proportional to the jet ratio \((Z_c/D_n \propto p_j/su_p)\). For these tests the influence of the traverse velocity of the nozzle is limited. At high jet ratios the cavity depth is relatively large, as a result the entrained soil will also influence the jet flow density. Therefore the influence of the traverse velocity of the nozzle cannot be neglected at high jet ratios. At very low traverse velocities of the nozzle, the soil will fail along already existing planes of weaknesses (known as hydro-fracturing). Depending on the extent of these planes this can increase the dimensions of the cavity significantly. At high jet pressures the jet will cavitate, this will also influence the cavity depth.

### Deflecting jet

When the stagnation ratio at the original soil surface \((p_{stag,ss}/su_p)\) is smaller than 5.4 (theoretically 6.2), the jet cannot penetrate the soil. Up to a stagnation ratio of about 12, the jet will deflect backwards directly after penetrating the soil. The deflecting jet flow supports the curved soil front due to a pressure increase. The soil fails in large strips (lumps) by the pressure exerted by the jet on the top of these strips.

The jet flow will disperse in radial direction on the top of a shearing soil strip. When the required stagnation ratio for radial soil failure is smaller than the required stagnation ratio in axial direction, the radial flow will push up the soil to the surface and will widen the cavity. This mechanism will occur for stagnation ratios at the original soil surface between 5.4 and 7.3 \((Z_c \leq 1.7D_{j,ss})\).

The cavity depths of the deflecting jet range from about \(1D_{j,ss}\) to \(2.5D_{j,ss}\). The cavity width decreases with the cavity depth from about \(2.45D_{j,ss}\) for the shallowest cavities to \(1.7D_{j,ss}\) for the cavities with a depth between 1.5 and \(2.5D_{j,ss}\).

### Penetrating jet

When the stagnation pressure at the soil bed is larger than about 12 \((Z_c > 2.5D_{j,ss})\), the whole jet flow penetrates the soil nearly vertically, a non-deflection zone exists. Only after a certain jet distance the jet deflects backwards. It is assumed that in the non-deflection zone the soil fails by the shearing of very small discrete soil elements with a curved shear surface at the rear side. The typical size of these elements ranges from 2 to 5 mm.

When the exerted pressure on the top of a soil element \((p_{stag})\) is larger than 6.4 times the peak undrained shear strength \((su_p)\), the element starts to rotate. The time to dislodge a soil element completely (rotation time \(t_{de}\)) is independent of the main dimension of the soil elements \(L_{de}\) and depends strongly on the actual stagnation ratio and the dynamic shear stress \(\tau_s\) in the shear surfaces. It is assumed that the shear stresses decrease instantaneously after starting to rotate to the undrained shear strength at the critical state: \(\tau_s = su_{cs}\). The shear stresses
will increase with the tangential velocity \( (v_{de,t}) \) of the curved shear surface up to a factor of 3.5. This slows down the acceleration of the elements. The calculated rotation time of a soil element with a characteristic length \( (L_{de}) \) of 2 mm ranges typically from \( 1 \cdot 10^{-4} \) to \( 7 \cdot 10^{-4} \) s.

On high speed recordings the shearing process of small soil elements was not visible. The recordings only showed a thin horizontal front below which soil was removed continuously. Based on the rotation time of the discrete soil elements, the vertical propagation velocity of this front \( (u_{f,v}) \) can be estimated. Assuming that the resistance in the shear surfaces remains intact and increases during rotation, the difference between the calculated and observed vertical propagation velocity is less than 15%.

When the stagnation ratio in the jet cavity is decreased to about 15, the jet will deflect backwards (deflection region). The deflecting jet flow supports the soil. This prevents the shearing of small discrete soil elements in the deflection zone. The soil fails in large curved strips by the stagnation pressure exerted on the top of these strips. The curvature of these strips is about 2.5 times the length of the rectangular cross section of the jet \( (L_{ja}) \) just above the curvature.

### 8.3 Recommendations

To describe the jetting process of a moving vertical jet in cohesive soil a lot of subprocesses have to be modeled. It was not possible to investigate all subprocesses in detail. Therefore a lot of assumptions had to be made. All these assumptions involve their own uncertainties. For example, it is possible that an overestimation of the influence of the strain-rate on the shear stress is compensated by neglecting the influence of turbulence or the real jet velocity distribution. To enlarge the reliability and the accuracy of the model it is recommended to investigate the following topics/subprocesses in more detail, the:

- strain rate dependency of the shear stress.
- forming of a shear band and the probability of fracturing of this shear band, due to penetrating jet water.
- influence of the enclosed area behind the cavitating jet on the jet pressure for cavitation cone development.
- influence of the nozzle diameter on the jet pressure for cavitation cone development.
- failure mechanism of the soil at very low traverse velocities (hydro-fracturing).
- high plastic deformations before a discrete soil element or strip is removed.
- influence of the shear stresses exerted by the jet at high flow velocities.
8.3. Recommendations

- influence of the real jet flow velocity on the stagnation pressures exerted on the discrete soil elements.
- influence of turbulent pressure fluctuations on the jet production.
- determining parameters/processes for the minimum dimension of the discrete soil element.
- influence of the nozzle diameter on the shearing velocity of the soil strips of a deflecting jet and the occurrence of a radial dispersing jet flow.

Because the frame rate of the high speed recordings turned out to be too low, the real failure mechanism of the soil was not visible. For this reason it is recommended to repeat some jet tests and record them with a higher frame rate than was used. To see which mechanisms are responsible for the cavity width, it is also interesting to record the top of the original soil surface.

Another interesting topic to investigate is the transition between the drained jet erosion process of non-cohesive soils and the undrained jetting process of cohesive soils. For his topic the main research questions are: what is the critical water permeability of a non-cohesive soil in combination with the rate of loading at which the soil behaves undrained and how to derive the undrained shear strength of the soil?


Appendix A

Bearing Capacity Theory

For a vertical load on a horizontal soil surface the shear resistance is called the bearing capacity \( q_{bc} \), which can be calculated with the Bearing Capacity Theory. This theory is elementary formed by limit analysis, based on the plasticity theory. Initially a lower bound and upper bound are formulated. The true failure load is higher than the lower bound and lower than the upper bound. The lower bound is a static equilibrium system in which the yield condition is nowhere exceeded. The upper bound is a mechanism or a displacement field in combination with deformation field. Wherever deformations occur, the stresses satisfy the yield condition.

The main principles of the Bearing Capacity Theory are:

- The soil can be considered as perfectly plastic.
- The yield condition is the Mohr-Coulomb criterion.

For the simple case of a (2-dimensional) strip load Prandtl (1920) derived analytically that for purely cohesive material, the bearing capacity of the soil bed is \( (2 + \pi) s u_p \approx 5.14 s u_p \). This solution forms both the lower and the upper bound in the limit analysis.

For a circular loading at a certain depth below the soil surface no analytical solution has been found. Based on experiments, different correction factors have been proposed, see e.g. (Meyerhof, 1950), (Skempton, 1951), (Hansen, 1970) and (Salgado et al., 2004). For a purely cohesive material, the bearing capacity can be calculated with the following equation:

\[
q_{bc} = s_{bc} d_{bc} N_{bc} s u_p + \sigma'_v
\]  

(A.1)

Where \( N_{bc} \) is the bearing capacity factor, \( s_{bc} \) is a shape factor, \( d_{bc} \) is a depth factor and \( \sigma'_v \) is the vertical effective stress at load level.
The factors of Meyerhof are the most common:

\[
N_{bc} = 5.14 \\
s_{bc} = 1.2 \\
z/D_l < 2.5 : \quad d_{bc} = 1 + 0.2z/D_l \\
z/D_l \geq 2.5 : \quad d_{bc} = 1.5
\]

where \(D_l\) is the diameter of the load \((D_l = D_j)\) and \(z\) is the vertical distance between the upper surface of the soil and the load level. It follows that the bearing capacity at the soil surface for a circular load is \(6.2s_u\), see Figure A.1 (a).

![Diagram](image)

Figure A.1: Sketch of the shearing planes, according to the Bearing Capacity Theory.

The jet cavities are relatively shallow, therefore the influence of the vertical effective soil stress \(\sigma'_v\) can be neglected\(^1\), resulting in a maximum bearing capacity of about \(9.3s_u\) at depths larger than \(2.5D_j\) \((q_{bc} = s_{bc}d_{bc}N_{bc}s_u = 1.2 \cdot 1.5 \cdot 5.14s_u)\). At a cavity depth of \(2.5D_j\) the shear surfaces no longer rise up to the soil surface. A local mechanism occurs, see Figure A.1 (Skempton, 1951).

### A.1 Bearing capacity under penetrating jet

According to Meyerhof (1950) the bearing capacity factor of a circular load at depths larger than 2.5 times the load diameter is 9.3. In the case of a moving jet

\(^1\)Assuming a cavity depth of 0.3 m, a soil density of 1800 kg/m\(^3\) and a water density of 1000 kg/m\(^3\), the vertical effective stress at the cavity bottom level is 2.7 kN/m\(^2\). Assuming an undrained shear strength of 25 kN/m\(^2\), the vertical effective stress at the cavity bottom will increase the bearing capacity with only about 1\%.
penetrating the soil bed, the load base is not fully enclosed by soil. At the back side of the jet the soil is already removed, see Figure A.2. For this case an upper bound is found, which results in a lower value of the bearing capacity factor than the empirical value of Meyerhof.

This mechanism consists of a displacement field in which a circular base with a radius $r_l$ rotates around a horizontal axis which is positioned at the edge of the circle, see Figure A.3 (a). In the vertical plane this results in shear planes of semicircles with different radii $R_l$:

$$R_l = r_l[1 + \cos(\alpha_r)] \quad (A.6)$$

The bearing capacity (factor) can be calculated by considering the equilibrium of moments with respect to the axis of rotation. The resisting soil moment ($M_r$) can be calculated as follows:

$$M_r = su_p \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} 4r_l R_l^2 d\xi_r d\alpha_r$$

$$+ 2su_p \int_0^{\pi} \int_0^{r_l} R_l^2 d\xi_r dr \quad (A.7)$$
Substituting the Equation A.6 into A.7 and integrating to $\xi_r$ gives:

\[
M_r = su_p \int_0^{\frac{1}{2} \pi} 2\pi r_l^3 + 2r_l^3 \cos (\alpha_r) + \cos^2 (\alpha_r) \, d\alpha_r \\
+ 2su_p \pi \int_0^{r_l} R_l^2 \, dr
\] (A.8)

Integrating to $\alpha_r$ and $R_l$ results in the following equation for the resisting soil moment:

\[
M_r = su_p \left[ \pi^2 r_l^3 + 4\pi r_l^3 + \frac{1}{2} \pi^2 r_l^3 + 2 \frac{1}{3} \pi r_l^3 \right]
\] (A.9)

The driving jet moment is:

\[
M_j = p_{stag} \pi r_l^3
\] (A.10)
Equating the Equations A.9 and A.10 results in the following Equation for the bearing capacity factor:

\[ N_{bc} = \frac{p_{stag}}{s_{up}} = 1 \sqrt[2]{\frac{2}{\pi}} + 4 \sqrt[3]{\frac{2}{3}} \approx 9.4 \] (A.11)

A lower value of \( N_{bc} \) can be found by choosing the axis of rotation somewhat higher, see Figure A.3 (b). The resisting soil moment with the axis of rotation at a distance \( h_{ax} \) above the load base level can be calculated with the following equation:

\[
M_{f,r} = s_{up} \int_{0}^{\frac{\pi}{2}} \int_{\xi_{0}}^{\frac{\pi}{2}} 4r_{l}R_{l}^{2}d\xi_{r}d\alpha_{r} \\
+ 2s_{up} \left[ 2\pi\zeta_{s}\frac{1}{3}R_{s}^{3} - \frac{2}{3}h_{ax}^{2}\cos(\zeta_{s})R_{s} \right] 
\] (A.12)

where
\[
R_{0} = r_{l}[1 + \cos(\alpha_{r})] 
\] (A.13)
\[
\tan(\xi_{0}) = \frac{h_{ax}}{R_{0}} 
\] (A.14)
\[
R_{l} = \frac{R_{0}}{\cos(\xi_{0})} 
\] (A.15)
\[
\zeta_{s} = \tan\left(\frac{r_{l}}{h_{ax}}\right) 
\] (A.16)
\[
R_{s} = \frac{r_{l}}{\sin(\zeta_{s})} 
\] (A.17)

For definitions see Figure A.3 (b). By equating Equation A.12 and A.10, the bearing capacity factor can be calculated as function of the height of the axis of rotation above the load level \( (h_{ax}) \). In Figure A.4 the calculated values of bearing capacity factor \( (N_{bc} = p_{stag}/s_{up}) \) are plotted as function of the normalized height of the axis of rotation above the load level \( (h_{ax}/D_{j} = h_{ax}/2r_{l}) \). The lowest value of is 8.2, with the axis of rotation at a normalized height of 0.35. Therefore the bearing capacity factor at the cavity bottom is assumed to be limited to 8.2. Assuming the same linear relation as Meyerhof, the equations for the depth factor become:

\[
\frac{z}{D_{l}} < 1.6 : \quad d_{bc} = 1 + 0.2z/D_{l} 
\] (A.18)

\[
\frac{z}{D_{l}} \geq 1.6 : \quad d_{bc} = 1.33 
\] (A.19)
Figure A.4: Calculated bearing capacity factor of the soil beneath a moving penetrating jet as function of the normalized height of the axis of rotation above the load level ($h_{ax}/D_j$).
Appendix B

Entrainment

B.1 Rectangular jet

For a rectangular jet with a finite length $L_n$ and a nozzle height $2h_n$ (see Figure B.1), the velocity development in the center of the jet is found to be:

$$u_{rec}(s, 0) = f_1 u_0 \sqrt{\frac{2h_n}{s}}$$  \hspace{1cm} (B.1)

where $f_1$ is an empirical constant with a value between 2.37 and 2.45 (Fischer et al., 1979). It is assumed that the velocity profile can be described with an exponential function, similar to the circular jet:

$$u_{rec}(h, s) = f_1 u_0 \sqrt{\frac{2h_n}{s}} e^{-f_2 \left( \frac{h}{s} \right)^2}$$  \hspace{1cm} (B.2)

where $f_2$ is also an empirical constant.

By integrating the velocity profile over the height, the following equation for the jet flow can be derived:

$$Q_{rec}(s) = 2L_n \int_0^\infty u_{rec}(h, s)dh$$

$$= 2L_n f_1 u_0 \sqrt{\frac{2h_n}{s}} \frac{1}{2} \sqrt{\pi} \sqrt{\frac{s^2}{f_2}}$$

$$= \frac{f_1}{\sqrt{f_2}} L_n u_0 \sqrt{\pi} 2h_n s$$  \hspace{1cm} (B.3)
In the same way the equation for the momentum flux can be derived:

\[ I_{rec}(s) = 2L_n \rho_w \int_0^\infty u_{rec}(h, s)^2 dh \]

\[ = 2L_n \rho_w f_1^2 u_0^2 \frac{2h_n}{s} \frac{1}{2} \sqrt{\pi} \sqrt{\frac{s^2}{2f_2}} \]

\[ = \frac{f_1^2}{\sqrt{f_2}} L_n h_n \rho_w u_0^2 \sqrt{2\pi} \]  \hspace{1cm} (B.4)

Assuming preservation of momentum \( f_2 \) will be \( 1/2\pi f_1^4 \). Substituting the relations for the rectangular jet (Eq. B.3 and B.4) into Equation 3.2, results in the following equation for the uniform velocity development:

\[ u_{u,rec}(s) = f_1 u_0 \sqrt{\frac{h_n}{s}} = \frac{1}{2} \sqrt{2} u_{rec}(s, 0) \]  \hspace{1cm} (B.5)

Note that maximum flow velocity in the symmetry plane of the rectangular jet is \( \sqrt{2} \) larger than the uniform flow velocity, while for the circular jet the maximum flow velocity in the centerline is 2 times the uniform flow velocity.

Substituting the relation for the uniform velocity into Equation 3.14 results in the following equation for the entrainment of a rectangular wall jet:

\[ \frac{dQ_{rec}}{ds} = \alpha_{mom} 2L_n u_{u,rec}(s) = \alpha_{mom} 2L_n f_1 u_0 \sqrt{\frac{h_n}{s}} \]  \hspace{1cm} (B.6)
By equating this equation to the derivative of the jet flow (Equation B.3) the following relation for the entrainment coefficient can be derived:

\[ \alpha_{mom,rec} = \frac{1}{2f_1^2} \approx 0.085 \]  

(B.7)

This value is almost the same as for the circular submerged jet.

### B.2 Wall jet

The maximum velocity development in a smooth wall jet with a finite length \( L_n \) and a nozzle height \( h_n \), can be described with the following equation (see Figure B.2):

\[ u_{\text{max}}(s) = f_3 u_0 \sqrt{\frac{h_n}{s}} \]  

(B.8)

where \( f_3 \) is an empirical constant with a value of 3.5 (Rajaratnam, 1976). Assuming the wall as a symmetry plane it can easily be seen that the wall jet is half the rectangular jet (\( f_3 \approx \sqrt{2f_1} \)). Apparently the influence of the shear stresses on the maximum velocity development can be neglected, see also Appendix D.

Similar to the rectangular jet the entrainment coefficient of the smooth wall jet can be derived. It follows that the value of the entrainment coefficient of the smooth wall jet is the same as for the rectangular jet.

### B.3 Impinging jet

Poreh et al. (1967) published velocity profiles of a turbulent radial wall jet, produced by an impinging circular air jet, see Figure B.3. The velocity distribution in the mixing layer (\( h > \delta \)) can be described with the following equation:

\[ u_{rw}(r, h) = f_4 u_0 D_n \frac{D_n}{r} e^{-f_5 \frac{(h-\delta)}{h-\delta}} \]  

(B.9)
where $f_4$ and $f_5$ are empirical constants and $\delta$ is the height of the boundary layer. According to Rajaratnam (1976) $f_4 = 1.03$. The value of $f_5$ is found to be about 130. The boundary layer height increases with the radial distances and is about $0.021 r$ (Poreh et al., 1967). This equation is verified for stand off distances ($SOD$) between $8D_n$ and $24D_n$.

The velocity profile in the boundary layer can be described with a logarithmic function, see Appendix D. By integrating the velocity profiles the equation for the radial flow can be derived:

$$Q_{rw}(r) = \pi ru_{rw,max} \left[ \delta \left( 2 - \frac{2}{\ln \delta/h_0} \right) + \sqrt{\frac{\pi}{f_5}} r \right] \approx 0.6ru_0D_n \tag{B.10}$$

where $h_0$ is a integration constant, see Appendix D.1. The equation for momentum flux can also be derived:

$$I_{rw}(r) = \rho_w2\pi ru_{rw,max}^2 \left[ \delta \left( 1 - \frac{2}{\ln \delta/h_0} + \frac{2}{\ln \delta/h_0^2} \right) + \sqrt{\frac{\pi}{2f_5}} r \right] \approx 0.59\rho_w \frac{1}{4}\pi D_n^2 u_0^2 = 0.59M_0 \tag{B.11}$$

Apparently about 40% of the momentum flux is lost. Assuming that the loss of momentum flux in the flow developing region is about 10%, the loss of momentum in the zone of impingement is about 30%.

For the uniform velocity development the following equation can be derived:

$$u_{u, rw}(r) = \frac{M_{rw}(r)}{\rho Q_{rw}(r)} \approx 0.73u_{rw,max} \tag{B.12}$$

Substituting the relation for the uniform velocity into Equation 3.14 results in the following equation for the entrainment of a radial wall jet:

$$\frac{dQ_{rw}}{dr} = \alpha_{mom, rw}2\pi ru_{u, wr} \tag{B.13}$$

By equating this equation to the derivative of the jet flow (Equation B.10), the entrainment coefficient can be calculated. It follows that the entrainment coefficient for the radial wall jet is about 0.128. This is more than 1.5 times the values of the entrainment coefficient of the other jets. The exact explanation for the difference in entrainment per unit surface can not be given.
Figure B.3: Definition sketch radial wall jet, produced by an impinging circular jet.
Appendix C

1D-approach

In the 1D-approach the jet is in the jet direction divided in infinitesimal thin slices with a thickness $ds$, see Figure C.1. For each slice a mass and momentum balance is solved.

**Mass balance**  The mass balance of slice number $k$ is:

$$\rho_{m,k} Q_j,k + \rho_w dQ_{w,k} + \rho_b dQ_{s,k} = \rho_{m,k+1} Q_{j,k+1}$$  \hspace{1cm} (C.1)

where $\rho_m$ is the jet flow density, $Q_j$ is the jet flow rate, $\rho_w$ is the density of the ambient water, $dQ_w$ is the entrainment of water over the slice, $\rho_b$ is the situ density of the soil and $dQ_s$ is the entrainment of soil over the slice.

The entrainment of water can be calculated with the following equation:

$$dQ_{w,k} = \alpha_{mom,k} u_{u,k} P_{w,k} ds$$  \hspace{1cm} (C.2)

where $\alpha_{mom}$ is the entrainment coefficient, $P_w$ is the perimeter of the interface between jet and ambient fluid and $u_u$ is the uniform jet velocity. The entrainment coefficient increases gradually with distance from 0 to a constant value at the beginning of the region of fully developed flow. For the non-cavitating jet this coefficient can be calculated with a simple transfer function, see Section 3.2.2. If the jet cavitates, this coefficient changes and can be calculated as proposed in Subsection 4.5.1.

The entrainment of soil is equal to the contact surface between jet and soil, projected in the horizontal plane times the traverse velocity of the nozzle ($v_t$):

$$dQ_{s,k} = W_{c,k} ds \cos (\beta_k) v_t$$  \hspace{1cm} (C.3)

where $W_c$ is the width of the jet cavity (=width of the slice) and $\beta_k$ is the angle between the jet direction and the vertical.
Momentum balance The momentum balance of slice number $k$ is:

$$I_k - F_{\tau,k} = I_{k+1}$$  \hspace{1cm} (C.4)

where $I$ is the momentum flux and $F_{\tau,k}$ is friction force exerted by the jet on the soil surface. The friction force can be calculated with the following equation:

$$F_{\tau,k} = \lambda \frac{1}{2} \rho_{m,k} u_{u,k}^2 P_{b,k} ds$$  \hspace{1cm} (C.5)

where $\lambda$ is a friction factor (for definition see Section 3.3.1), $\rho_{m,k}$ is the jet flow density and $P_{b,k}$ is the perimeter of the interface between jet and soil.
Uniform jet velocity and stagnation pressure  From these balances the uniform jet velocity and stagnation pressure at a certain jet distance can be derived:

\[
\begin{align*}
    u_u(s = k \cdot ds) &= u_{u,k} \frac{I_k}{\rho_{m,k+1}Q_{j,k}} \\
    p_{stag}(s) &= \frac{1}{2}\rho_{m,k}u_u(s)^2
\end{align*}
\] (C.6)

When the initial conditions for the jet momentum, the jet flow rate and the jet flow density are known, the jet behavior as function of the distance can be calculated. Assuming a jet pressure \(p_j\), a nozzle diameter \(D_n\) and a nozzle discharge coefficient \(\mu_n\) the initial conditions for a circular submerged jet are:

\[
\begin{align*}
    \rho_{m,0} &= \rho_w \\
    u_0 &= \sqrt{\frac{2p_j}{\rho_{m,0}}} \\
    Q_0 &= \mu_n \frac{1}{4} \pi D_n^2 u_0 \\
    I_0 &= \rho_{m,0} Q_0 u_0
\end{align*}
\] (C.8) (C.9) (C.10) (C.11)
Appendix D

Bed shear stresses

D.1 Normal boundary layer

The moving jet penetrating soil the soil bed is partially enclosed by it. At the interface between the jet and the soil a boundary layer is formed. In this boundary layer the no-slip condition results in a large velocity gradient and associated shear stresses perpendicular to the soil wall. The boundary layer thickness (δ) increases with distance. Initially the boundary layer thickness is negligible, resulting in high bed shear stresses.

Because the boundary layer is not fully developed in the area of interest it is not possible to use the well known Darcy-Weisbach friction factor for a closed pipeline or the Chezy friction factor for an open channel flow. These factors can only be used when the boundary layer in the jet is fully developed (δ ≈ 1/2\(W_c\)).

The velocity profile in a boundary layer is assumed to be logarithmic:

\[ u_{bl}(h) = \frac{u_*}{\kappa} \ln \frac{h}{h_0} \]  

(D.1)

where \(u_*\) is the bed shear velocity, defined as \(\sqrt{\tau_b/\rho_m}\), \(\kappa\) is the Kármán constant (≈ 0.41) and \(h_0\) is an integration constant to set the velocity somewhere near the bed at zero. The height averaged velocity over the boundary layer can be obtained by integrating the assumed logarithmic velocity profile over the boundary layer height (\(h = \delta\)), which is defined as the position above the bed where the flow velocity parallel to the bed is 99% of the outer flow velocity:

\[ u_{u,bl} = \frac{u_*}{\kappa} \left( \ln \frac{\delta}{h_0} - 1 \right) \]  

(D.2)
The integration constant $h_0$ depends on the bed roughness. For a smooth bed $h_0$ is experimentally found to be $0.11\nu/u_\ast$ and for a rough bed $k_s/30$.\footnote{A bed is considered as smooth when the roughness elements with an equivalent roughness height ($k_s$, see Section D.4.1) are submerged a viscous sub-layer. The height of this sub-layer is about $5\nu/u_\ast$, where $\nu$ is the kinematic viscosity of the jet fluid.

When $k_s$ is much larger then the height of the viscous sub-layer this layer does not play a significant role anymore. The flow separation behind the roughness elements dominates the velocity profile. This happens for $k_s > 70\nu/u_\ast$. The bed is considered as rough.}

The difficulty is to estimate the boundary layer thickness ($\delta$). The boundary layer thickness can be derived form the boundary layer development on a flat plate in an uniform flow and from the boundary layer development in a plane wall jet. These flow problems will be briefly discussed.

### D.2 Boundary layer development on a flat plate

The boundary layer thickness on a flat plate in an uniform flow with constant velocity has intensively been studied.

**Laminar Boundary layer development**

The boundary flow starts laminar and becomes turbulent after a certain distance. For highly turbulent flows it is assumed that the transition takes place at a local Reynolds number of about $5 \cdot 10^5$. The Local Reynolds Number ($Re_s$) is defined as $u_0s_{bl}/\nu$, where $s_{bl}$ is the distance to the origin of the boundary layer.

The equation for the normalized boundary layer thicknesses in the laminar zone is (White, 2009):

$$\frac{\delta}{s_{bl}} = \frac{5}{Re_s^{1/2}}$$

(D.3)

The corresponding friction coefficient is (White, 2009):

$$c_f = \frac{0.664}{Re_s^{1/2}}$$

(D.4)

where the friction coefficient is defined as follows:

$$c_f = \frac{2\tau_b}{\rho m u_{m,bl}^2}$$

(D.5)

where $u_{m,bl}$ is the maximum velocity at edge of the boundary layer. In the flow case of an uniform flow over a flat plate $u_{m,bl} \approx u_0$.  

\[234\]
Boundary layer development on a smooth flat plate

The accepted equations for the boundary layer thicknesses and corresponding friction coefficients on a smooth flat plate in the turbulent zone are (White, 2009)\textsuperscript{2}:

\[
\frac{\delta}{s} = \frac{0.16}{Re_s^{1/7}} \tag{D.6}
\]
\[
c_f = \frac{0.027}{Re_s^{1/7}} \tag{D.7}
\]

Boundary layer development on a rough flat plate

For the friction coefficient on a rough flat plate in the turbulent zone the following empirical relation was found (Schlichting, 1979):

\[
c_f = \left(2.87 + 1.58 \log \frac{s_{bl}}{k_s}\right)^{-2.5} \tag{D.8}
\]

Assuming a logarithmic velocity distribution over the boundary height (Equation D.1) and \(h_0 = k_s/30\), an estimation of the boundary layer height can be made:

\[
\delta = \frac{k_s}{30} e^{\kappa/\sqrt{1/2c_f}} \tag{D.9}
\]

There are two main differences between the flow case of a boundary layer development on a flat plate in an uniform flow and that of a moving jet penetrating cohesive soil:

1. The boundary layer in the jet is influenced by the decelerating outer flow, despite the boundary layer on a flat plate where the outer flow velocity is more or less constant.

2. The flow problem of a moving jet penetrating cohesive soil is more a three dimensional problem. While the jet is partly enclosed by soil the boundary layer development is hindered and limited.

\textsuperscript{2}for the boundary layer thickness in the turbulent zone also other more or less comparable equations are proposed: \(\delta/s = 0.21/\ln Re_s\) (\(10^5 < Re_s < 10^6\)) (Schlichting and Gersten, 2000) and \(\delta/s = 0.38/Re_s^{1/5}\) (\(10^5 < Re_s < 10^7\)) (Katz, 2010).
D.3 Boundary layer plane wall jet

In a plane wall jet adjacent to the boundary layer a mixing layer exist, see Figure B.2. The decelerating outer flow of the mixing layer influences the development of the boundary layer. For the moving jet penetrating soil this influence is probably limited, because the contact surface between the jet and the ambient fluid decreases with cavity depth relative to the contact surface between the jet and the soil wall.

D.3.1 Smooth bed

Many researchers have studied the boundary layer in the wall jet, see f.i. (Smith, 2008). They found comparable relations as for the boundary layer development in an uniform flow. The difficulty is that the empirical relations for the friction factor of a wall jet are only validated for a limited range of test settings, with relatively low Reynolds numbers. Table D.1 shows a summary of the found relations and their range of validity.

Table D.1: Summery different relations for the boundary thickness in a wall jet on a smooth plate and corresponding friction coefficient.

<table>
<thead>
<tr>
<th>Layer thickness [m]</th>
<th>Friction Coefficient [-]</th>
<th>Validity area</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>0.032 ( \left( \frac{\delta u_{m,bl}}{\nu} \right)^{-0.182} )</td>
<td>( h_n = 0.46 ) mm, ( Re = 0.6 \cdot 10^4 )</td>
</tr>
<tr>
<td>0.01s</td>
<td>-</td>
<td>( h_n = 20-54 ) mm, ( Re = 2 - 10 \cdot 10^4 )</td>
</tr>
<tr>
<td>0.23 ( \frac{s}{h_n} )^{0.88} ( \left( \frac{\nu}{u_{m,bl}} \right)^{0.24} )</td>
<td>-</td>
<td>( h_n = 10 ) mm, ( Re = 0.3 - 3 \cdot 10^{-3} )</td>
</tr>
<tr>
<td>0.013s + 0.056(h_n)</td>
<td>0.018 ( \left( \frac{\delta e u_{m,bl}}{\nu} \right)^{-0.113} )</td>
<td>( h_n = 9.6 ) mm, ( Re = 0.75 - 1.4 \cdot 10^4 )</td>
</tr>
<tr>
<td>0.21 ( \frac{s}{h_n} )^{0.91} ( \left( \frac{\nu}{u_{m,bl}} \right)^{0.18} )</td>
<td>-</td>
<td>( h_n = 12.7-25.4 ) mm, ( Re = 4 \cdot 10^4 )</td>
</tr>
</tbody>
</table>
Comparing the relations for the boundary layer development in a wall jet with the relation for the boundary layer development in an uniform flow, the differences are limited. For this reason, and given the limited validity of the relations for the wall jet, it is assumed that the relation for the boundary layer on a flat plate in an uniform flow also can be used for the flow case of a moving jet.

Because the laminar zone is very short, for the jets under investigation, the friction coefficient in the laminar zone is assumed to be constant and equal the start value of the friction coefficient in the turbulent zone. The start value of the friction coefficient in the turbulent zone is about $4 \cdot 10^{-3}$. The friction coefficient decreases gradually to a value of $2.5 \cdot 10^{-3}$ at a jet distance of $30h_n$.

Beltaos and Rajaratnam (1974) investigated the shear stress of an impinging jet ($D_n = 23.4 \text{ mm}, \, Re = 6.2 - 9.5 \cdot 10^4$) on a smooth surface (= radial wall jet). They found for the maximum shear stress on the smooth surface the following relation:

$$\tau_b = 0.016 \rho_m u_0^2 \left( \frac{D_n}{H} \right)^2$$  \hspace{1cm} (D.10)

Where $H$ is the height of nozzle above the surface. Assuming that the maximum radial flow velocity ($u_{mr}$) can be derived from the measured stagnation pressure in the center, the corresponding friction coefficient ($c_f = 2\tau_b/\rho_m u_{mr}^2$) is about 0.006. This value can be considered as a sort of upper limit.

### D.3.2 Rough bed

On the effects of the wall roughness on the plane wall jet flow less is published.

Table D.2 shows the found empirical relations for the boundary layer development in a wall jet on a rough plate. It is remarkable that the boundary layer thickness in these relations is independent of the equivalent roughness height, in the tested area.

Rajaratnam (1976) found that the maximum velocity at the edge of the boundary

$^3$Rajaratnam (1976) found for the maximum stagnation pressure on a plate perpendicular to the jet flow direction the following relation:

$$p_{stag} \approx 25\rho u_0^2 \left( \frac{D_n}{H} \right)^2$$  \hspace{1cm} (D.11)

The corresponding maximum radial flow velocity is:

$$u_{mr} = \sqrt{\frac{2p_{stag}}{\rho_m}} \approx \sqrt{50} u_0 \frac{D_n}{H}$$  \hspace{1cm} (D.12)
Table D.2: Summary of different relations for the boundary thickness in a wall jet on a rough plate and corresponding friction coefficient.

<table>
<thead>
<tr>
<th>Layer thickness [m]</th>
<th>Validity area</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.026s (Rajaratnam, 1976)</td>
<td>(h_n = 20-53.6 \text{ mm}, \text{Re} = 2 - 10 \cdot 10^4, 0.005 &lt; k_s/h_n &lt; 0.13)</td>
</tr>
<tr>
<td>(0.24 h_0^{0.89} \left( \frac{\nu}{u_{m,bl}} \right)^{0.22}) (Tachie et al., 2004)</td>
<td>(h_n = 10 \text{ mm}, \text{Re} = 0.6 - 1.2 \cdot 10^4, k_s/h_n = 0.12)</td>
</tr>
</tbody>
</table>

The boundary layer varies with the relative roughness height as follows:

\[
\frac{u_{m,bl}}{u_0} = K_1 - 0.54 \log \left( \frac{s}{k_s} \right) \tag{D.13}
\]

where \(K_1\) is an empirical constant, which can be approximated with the following empirical relation: \(K_1 = -0.29 \ln (k_s/h_n) + 1.13\). Assuming a logarithmic velocity profile over the boundary height, the bed shear stresses according to Rajaratnam can be calculated with Equation D.1 \((h_0 = k_s/30, \delta = 0.026s, u(\delta) = u_{m,bl})\).

In Figure D.1 the calculated boundary layer development in the wall jet, according to Rajaratnam (1976), is compared with the boundary layer development on a rough plate in an uniform flow. The boundary layer thickness is normalized with the assumed cavity width \((W_c = 2h_n = 0.02 \text{ m})\). The assumed roughness height \((k_s)\) is 110 \(\mu\text{m}\).

The boundary layer thickness in the uniform flow is about 40\% larger than in the wall jet. Over the first nozzle lengths the influence of the mixing layer in a wall jet should be negligible. This is not the case.\(^4\)

Because of the thinner boundary layer, the calculated friction coefficients \((c_f = 2\tau_b/\rho_m u_{m,bl}^2)\) for the wall jet are higher than for the uniform flow over a rough flat plate, see Figure D.2.

In the flow case of a moving jet penetrating cohesive soil, the boundary layer development is limited to \(0.5W_c\). Assuming that the boundary layer is fully developed

\(^4\)Note, Rajaratnam did not measure the boundary layer height in this area. The boundary layer thickness is based on measurements at normalized jet distances \(> 10h_n\).
at this point, the friction coefficient can also be derived from the Darcy-Weisbach friction factor\footnote{The Darcy-Weisbach friction factor \( f_0 = 8 \tau_b / \rho_m u_{u,p}^2 \) can be calculated with the Colebrook\&White equation (Colebrook, 1939): \[ \frac{1}{\sqrt{f_0}} = -5.6 \log \left( \frac{k_s}{3.7 D_p} + \frac{2.51}{R_{ep} \sqrt{f_0}} \right) \] where \( D_p \) is the pipe diameter and \( R_{ep} \) is the Reynolds number of the pipe, defined as \( u_{u,p} D_p / \nu \). The maximum velocity \( (u_{m,bl}) \) is about 18\% higher than the average flow velocity in the pipe, and can be approximated with the 1/7-law.} or the Chezy coefficient\footnote{The Chezy coefficient for a hydraulically rough flow \( (C_r = \sqrt{8g/L_{dw} = \sqrt{g/\lambda}}) \) can be calculated with the following equation: \( C_r = 18 \log \left( \frac{12 R_h}{k_s} \right) \) where \( R_{ec} \) is defined as: \( u_u R_h / \nu \), with \( u_u \) the average flow velocity and \( R_h \) the hydraulic radius. In the case of a moving jet penetrating soil, the hydraulic radius is about \( 1/2 W_c \).}

These values are also plotted in Figure D.2.

In Figure D.3 the calculated friction coefficients at a jet distance of \( 20h_n \) are plotted as function of the relative roughness height \( (k_s/W_c) \). Although the absolute differences, the influence of the roughness height is for both flow cases comparable.

Tachie et al. (2004) published friction coefficients for wall jets between 0.007 and 0.009, at relatively low Reynolds numbers \( (Re = 0.6 - 1.2 \cdot 10^4, k_s/h_n = 0.12, s = 20 \sim 80h_n) \). These values correspond well with the calculated friction coefficients.
Figure D.3: Calculated friction coefficients as function of relative roughness height at a jet distance of $20h_n$ ($h_n = 1/2W_c = 0.01$ m).

for an uniform flow on a flat plate with the same relative roughness height. The calculated friction coefficients at flow distances of $20h_n$ and $80h_n$ are 0.01 and 0.007, respectively.

Rajaratnam and Mazurek (2005) measured the maximum shear stresses on a rough plate perpendicular to the main jet flow direction at a distance of $12D_n$ ($H$). The maximum shear stresses occurred at a radial distance of $0.12H$. The derived values of the maximum friction coefficients varied between 0.025 and 0.028 ($D_n = 12.7$ mm, $u_0 = 45.7-68.6$ m/s, $k_s = 1.73$ mm).

The calculated friction coefficient on a rough plate with a relative roughness height of 0.32 in an uniform flow at a normalized distance of $0.12 \cdot 12D_n$ is 0.032. This value is in the same order of magnitude as the derived friction coefficients for the impinging jet.

Rajaratnam and Mazurek (2005) normalized the roughness height with the height of the nozzle above the surface ($H$). It seems more logical to normalize the roughness height with the minimum height of the radial jet flow. This height can be estimated with the following equation:

$$h \approx \frac{Q(H)}{umr \pi D_j(H)} = \sqrt{\frac{1}{800}} H$$  \hspace{1cm} (D.16)

where $Q(H)$ is the jet flow at a distance $H$ which can be calculated with Equation 3.8, $umr$ is the maximum radial flow velocity which can be approximated with Equation D.12 and $D_j(H)$ is the jet diameter of the jet at a distance $H$, based on a uniform flow velocity (Equation 3.16). The estimated radial height for $D_n = 12.7$ mm and $SOD = 12D_n$ is 5.4 mm. Consequently the relative roughness height is 0.32, assuming $k_s = 1.73$ mm.
D.4 Calculation of bed shear stresses

D.4.1 Equivalent roughness height

In general the bed roughness height of a flat and fixed surface is given in terms of the Nikuradse equivalent roughness height \( k_s \). Nikuradse’s equivalent sand-grain roughness is commonly taken to be proportional to a representative grain diameter: \( k_s = 1 \sim 5 \times D_{50}, D_{65}, D_{84} \text{ or } D_{90} \) according to the literature (Garcia, 2007).

The bed forms disappear in sand at high bed shear stresses. The sediment moves along the bottom, denoted as sheet flow. The presence of a relatively thin sheet flow layer with high sediment concentration markedly affects the flow above. One important aspect of the presence of a sheet layer is the increased roughness. Under sheet flow conditions the roughness height may be several orders of magnitude larger than for a fixed bed (Rijn, 1993).

Because of the high turbulent flow, the dislodged soil particles will be almost immediately entrained in the main flow. Therefore it is assumed that the surface of the jet cavity in cohesive soil can be modeled as a fixed bed. No sheet flow occurs.

The equivalent roughness height is assumed to be: \( 5D_{50} - 1D_{90} \) (25-110 µm). In the area of interest the flow case of a moving jet penetrating cohesive soil can be considered as rough \( (k_s > 70\nu/u_*) \). For the lowest jet pressures investigated \( (p_j = 0.4 \text{ MN/m}^2) \), the shear velocity is about 1.3 m/s, resulting in a transition roughness height in the order of \( 5 \cdot 10^{-5} \text{ m} \).

Friction coefficient

Because of the limited validity of the relations for the wall jet and the smaller expected influence of the mixing layer in the moving jet, in comparison with the wall jet, it is assumed that the boundary layer development in the jet is comparable to the development of a boundary layer in an uniform flow.

To determine the friction coefficient the following assumptions were made:

1. The length of the laminar zone of the boundary layer is independent of the roughness height and ends when the local Reynolds number is equal to \( 5 \cdot 10^5 \).

2. In the laminar zone the friction coefficient is constant and equal to the maximum value in the turbulent zone.

3. When the boundary layer thickness equals half the cavity width \( (\delta = 1/2W_c) \) the boundary layer is fully developed. Beyond this point the friction coefficient is kept constant.
4. The boundary layer thickness around the jet is constant at a certain jet distance, see Figure D.4 (a) and (b). In reality the boundary layer thickness decreases to zero at the position the jet flow releases the soil surface, see Figure D.4 (c).

![Figure D.4](image.png)

**Figure D.4:** Definition sketch boundary layer of moving jet partly enclosed by soil. (a) Velocity profile (b) Horizontal cross section jet (c) Expected boundary layer thickness.

It follows that the maximum friction coefficient, for small stand off distances is:

$$c_{f,max} = \left( 2.87 + 1.58 \log \frac{\nu Re_c}{u_0 k_s} \right)^{-2.5}$$  \hspace{1cm} (D.17)

The tested initially jet velocities ($u_0$) range from 28.6 to 176.6 m/s ($p_j = 0.41-15.6$ MN/m$^2$). Assuming a roughness height ($k_s$) of $0.11 \cdot 10^{-3}$ m, the maximum friction coefficients range from 0.01 to 0.017. The minimum friction coefficient occurs when the boundary layer is fully developed ($\delta = 1/2W_c$). Substituting this condition into Equation D.9 the minimum values can be calculated. It follows than the minimum friction coefficients range from 0.0075 to 0.0041.

**Main flow velocity**

To calculate the bed shear stresses ($\tau_b = 1/2c_f \rho_m u_{m,bl}^2$), the flow velocity at the edge of the boundary layer ($u_{m,bl}$) must be known. This velocity can be derived from the known values of the jet flow ($Q$), the momentum flux ($I$), the mixture density ($\rho_m$) and the cavity width ($W_c$), with the following system of equations:
\[ Q(s) = \int_{\frac{1}{2}W_c - \delta(s)}^{\frac{1}{2}W_c} \pi ru(s, r) dr + 2L \int_{\frac{1}{2}W_c - \delta(s)}^{\frac{1}{2}W_c} u(s, r) dr \\
+ \pi \left( \frac{1}{2}W_c - \delta(s) \right)^2 u_u + 2L \left( \frac{1}{2}W_c - \delta(s) \right) u_u \]  
(D.18)

\[ I(s) = \rho_m \int_{\frac{1}{2}W_c - \delta(s)}^{\frac{1}{2}W_c} \pi hu(s, r)^2 dr + \rho_m 2L \int_{\frac{1}{2}W_c - \delta(s)}^{\frac{1}{2}W_c} u(s, h)^2 dr \\
+ \rho_m \pi \left( \frac{1}{2}W_c - \delta(s) \right)^2 u_u^2 + \rho_m 2L \left( \frac{1}{2}W_c - \delta(s) \right) u_u^2 \]  
(D.19)

\[ u(s, r) = \frac{u^*}{\kappa} \ln \frac{\frac{1}{2}W_c - r}{h_0} \\
= u_{m,bl} \sqrt{\frac{c_f(s)}{2}} \frac{1}{\kappa} \ln \frac{30(\frac{1}{2}W_c - r)}{k_s} \]  
(D.20)

where \( u(s, r) \) is the velocity in the boundary layer at a jet distance \( s \) and a (radial) distance \( r \) from the center axis, see Figure D.4 (a) and (b). The velocity in the center of the jet (outside the boundary layer) is assumed to be constant. The boundary layer thickness \( \delta(s) \), can be calculated with Equation D.9. The two unknown parameters in this system of equations are the flow velocity at the edge of the boundary layer \( (u_{m,bl}) \) and the length of the jet \( (L_{j,bl}) \). By solving this system these parameters can be determined.

The maximum ratio between the velocity at the edge of the boundary layer and the uniform flow velocity \( (u_{m,bl}/u_u) \) exist at the moment the boundary layer is fully developed. These values are listed for the ultimate test settings in Table D.3.

Table D.3: Maximum ratio between the maximum velocity and the average velocity \( (u_{m,bl}/u_u) \) for the ultimate test settings.

<table>
<thead>
<tr>
<th>( D_n ) [mm]</th>
<th>( p_j ) [MN/m^2]</th>
<th>( s/D_n ) [-]</th>
<th>( u_{m,bl}/u_u ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15.6</td>
<td>17.2</td>
<td>1.17</td>
</tr>
<tr>
<td>32</td>
<td>0.41</td>
<td>27.2</td>
<td>1.12</td>
</tr>
</tbody>
</table>

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Appendix E

Experimental setup (cavitating) jet tests

E.1 Pressure vessel of free cavitating jet tests

Figure E.1: Pressure vessel. (1) Measuring panel, (2) Jet pipe, (3) Jet inlet, (4) Linear table, 3 dimensions, (5) Inspection glasses.
E.2 Measuring panels of free cavitating jet tests

Figure E.2: Dimensions measuring panel I. The positions of the 9 measuring points were variable (topview).

Figure E.3: Dimensions measuring panel II in [mm] (a) topview, (b) frontview.

Figure E.4: Measuring panel I.

Figure E.5: Measuring panel II.
E.3 Jet pipe of free cavitating jet tests

![Diagram of jet pipe and nozzle's](image)

Figure E.6: (a) Jet pipe, dimensions in [mm]. (b) Nozzle’s.
E.4 Nozzles used for free cavitating jet tests

![Diagram of nozzles with dimensions](image)

Figure E.7: Dimensions used nozzles for free cavitating jet tests.

E.5 Nozzles used in the exploratory jet tests (2004)

![Diagram of circular nozzles with dimensions](image)

Figure E.8: Circular low pressure nozzles used in the exploratory jet tests (2004), dimensions in [mm] (a) Nozzle diameter = 32.5 mm (b) Nozzle diameter = 20 mm.
Figure E.9: Nozzles used in second test program (2007), dimensions in [mm]:
(a) nozzle with a diameter of 28 mm, (b) Nozzle with a diameter of 45 mm, (c) dimensions jet pipe.
Appendix F

Geotechnical tests

F.1 Oedometer test

To determine the main parameters related to the compressibility and rate of primary consolidation of the DH-samples, standard Oedometer tests were carried out (definitions and procedure according to NEN-5118). Cylindrical test specimens of ø65x20 mm were confined laterally and subjected to discrete increments of vertical axial loading. The specimens were allowed to drain axially from the top and bottom surfaces. The tests were started directly after preparing the specimens. In the Figures F.1 to F.6 the test results are plotted.\(^1\) In Table F.1 derived pre-consolidation pressures \((\sigma_v0)\), according to different methods, are listed. The bandwidth of the determined pre-consolidation pressures are also plotted in the Figures F.2, F.4 and F.6.

\(^1\)Unfortunately, the pore water pressures are unknown.
The vertical stress at which a soil sample starts to compress ($\sigma_{vs}$) is also of interest for the soil behavior. At this stress the initial pore water under pressure (suction) is largely eliminated. An exact value of this load cannot be determined, only a bandwidth, see Table F.1.

**Figure F.1:** Time settlement chart for load-controlled oedometer test on DH$_{20}$-sample.

**Figure F.2:** Linear strain as a function of the vertical stress for load-controlled oedometer test on DH$_{20}$-sample.
Figure F.3: Time settlement chart for load-controlled oedometer test on DH$_{45}$-sample.

Figure F.4: Linear strain as a function of the vertical stress for load-controlled oedometer test on DH$_{45}$-sample.
Figure F.5: Time settlement chart for load-controlled oedometer test on DH$_{70}$-sample.

Figure F.6: Linear strain as a function of the vertical stress for load-controlled oedometer test on DH$_{70}$-sample.
F.2 Constant Rate of Strain oedometer test (CRS-test)

The consolidation coefficient \(c_v\) and the vertical pre-consolidation pressure \(\sigma_{v0}\) of the DH\textsubscript{20}-sample were also determined, using continuous strain-controlled axial compression, referred to as constant rate-of-strain (definitions and procedure according to ASTM D4186-06). The specimen (ø65x22 mm) were restrained laterally and drained axially only at the top surface. The axial stress and base excess pore water pressure are measured during the deformation process.

Two tests, with a different strain-rate \((\dot{e}_v = 0.5 \text{ and } 0.1 \%/\text{hr})\), were conducted. In Figure F.7 the total vertical pressure \((\sigma_v)\) and the measured pore water excess pressure at the bottom \((\Delta p_w)\) are plotted as a function of the vertical displacement \((\Delta z)\). For the strain-rate of 0.1 \%/hr the measured excess pore water pressure is negligible with respect to the vertical total stress and can be considered as fully consolidated. However, for the strain-rate of 0.5 \%/hr during the early part of the test hardly any drainage seems to occur, with the excess pore pressure remaining only marginally smaller than the total vertical stress (this seems to be a measurement error).

Nevertheless for a displacement of \((\Delta z)\) larger than 3 mm, the excess pore water pressure suddenly reduces to negligible magnitude, which approximates fully consolidated behaviour.

Figure F.7: Vertical total stress \((\sigma_v)\) and excess pore water pressure \((\Delta p_w)\) as a function of the vertical displacement \((\Delta z)\) at two different strain rates for the DH\textsubscript{20}-sample.

Figure F.8: Vertical strain \((\varepsilon_v)\) as a function of the vertical effective stress \((\sigma'_v)\) at two different strain rates and the prediction of vertical pre-consolidation pressure \((\sigma_{v0})\) for the DH\textsubscript{20}-sample.
The average vertical effective stress ($\sigma'_v$) in the sample is assumed to be about: $\sigma'_v - 2/3 \Delta p_w$, because of excess pore water gradient over the height of the soil sample. The measured axial strain ($\varepsilon_v$) as a function of the estimated vertical effective stress is plotted in Figure F.8.\(^2\) From this figure an estimate of the vertical pre-consolidation pressure ($\sigma_{v0}$) and the compression index ($C_c$) can be derived.

For both tests the derived vertical pre-consolidation pressure ($\sigma_{v0}$) is about 65 kN/m\(^2\). This is about 30% higher than determined with the standard oedometer test (DH\(_{20}\): $\sigma_{v0} \approx 50$ kN/m\(^2\), see Table F.1).

For 1-dimensional compression the compression index ($C_c$), as defined in Equation 2.48, can also be written as, while $\varepsilon_v = \Delta e/(1 + e_0)$:

$$C_c = -\frac{\varepsilon_v(1 + e_0)}{\log \left(\frac{\sigma'_v}{\sigma_{v0}}\right)} \quad \text{(F.1)}$$

in which $e_0$ is the void ratio at $\sigma_{v0}$. For the DH\(_{20}\)-sample the value of $e_0$ is about 0.85 ($n_0 = 0.46$, see Table 5.5). At a vertical strain of 0.2, the vertical effective stress ($\sigma'_v$) is about 740 kN/m\(^2\), see Figure F.8. Substituting these values into Equation F.1 results in a compression index of 0.35.

In Figure F.9 the derived consolidation coefficient ($c_v$) is plotted as function of the vertical effective stress ($\sigma'_v$). The determined consolidation coefficient ($c_v$) is in the range: $1 \cdot 10^{-10} \sim 1 \cdot 10^{-9}$ m\(^2\)/s.

In the Figure F.10 the derived water permeability ($k_w$) is plotted as function of the vertical effective stress ($\sigma'_v$). The determined water permeability ($k_w$) is about $1 \cdot 10^{-10}$ m/s. However, it must be noted that the accuracy of the determined consolidation coefficient and water permeability is limited, because of the fluctuations in the measured excess pore water pressure ($\Delta p_w$).

\(^2\)The difference between the average vertical effective stresses at a strain-rate of 0.5 %/hr and 0.1 %/hr, for displacements smaller than 3 mm seems to be too small in this figure. However, this is not the case. For example, the calculated vertical effective stresses ($\sigma'_v$) for a displacement of 2 mm ($\varepsilon_v = 9$ %) are: $\varepsilon_v = 0.5$ %/hr: $\sigma_v - 2/3 \Delta p_w = 400 - 2/3 \cdot 360 = 160$ [kN/m\(^2\)], $\varepsilon_v = 0.1$ %/hr: $\sigma_v - 2/3 \Delta p_w = 230 - 2/3 \cdot 50 = 197$ [kN/m\(^2\)].
Figure F.9: Consolidation coefficient ($c_v$), derived from the CRS-tests, as a function of the vertical total stress ($\sigma_v$) at two different strain rates for the DH$_{20}$-sample.

Figure F.10: Water permeability ($k_w$), derived from the CRS-tests, as a function of the vertical total stress ($\sigma_v$) at two different strain rates for the DH$_{20}$-sample.
F.3 Labvane test

The lab-vane test on the DH$_{20}$ – sample was carried out with a vane diameter ($R_v$) of 25.4 mm. The vane diameter of the other two tests was 12.7 mm. In Table F.2 the determined peak strengths ($s_{up}$), the rotation angles at the peak ($\omega_p$) and the corresponding displacements in the shear zone ($\Delta d_p$) are listed.

Table F.2: Results of lab-vane tests.

<table>
<thead>
<tr>
<th></th>
<th>DH$_{20}$</th>
<th>DH$_{45}$</th>
<th>DH$_{70}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_v$ [mm]</td>
<td>25.4</td>
<td>12.7</td>
<td>12.7</td>
</tr>
<tr>
<td>$s_{up}$ [kN/m$^2$]</td>
<td>18</td>
<td>35</td>
<td>86</td>
</tr>
<tr>
<td>$\omega_p$ [$^\circ$]</td>
<td>48</td>
<td>90</td>
<td>116</td>
</tr>
<tr>
<td>$\Delta d_p$ [mm]</td>
<td>10.6</td>
<td>10.0</td>
<td>12.9</td>
</tr>
</tbody>
</table>
F.4 Unconsolidated Undrained Triaxial Compression tests

To determine the undrained shear strength \((s_u)\) and the stress-strain relationships of the DH-samples, unconsolidated undrained triaxial compression tests (UU-tests) were carried out. The used definitions and procedure were according to NEN-5117. These definitions and notations differ from the definitions used in the present thesis. In Table F.3 the differences are listed.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t) [kN/m(^2)]</td>
<td>(q/2) = ((\sigma_1 - \sigma_3)/2)</td>
</tr>
<tr>
<td>(s') [kN/m(^2)]</td>
<td>(\sigma'_p + \frac{1}{6}q) = ((\sigma'_1 + \sigma'_3)/2)</td>
</tr>
<tr>
<td>Axial strain</td>
<td>(\varepsilon_b[%]) = (\varepsilon_v)</td>
</tr>
<tr>
<td>Excess pore pressure</td>
<td>(\Delta u_b [kN/m^2]) = (\Delta p_w)</td>
</tr>
</tbody>
</table>

Table F.3: Definitions used in the UU-triaxial compression tests.

Figure F.11: Schematic depiction of the triaxial test apparatus.

**Procedure**

Cylindrical specimens (ø65x150 mm) were extracted from the clay tablets by penetrating a sharp sampling tube and cutting the samples at the required length. Then the specimens were placed on the pedestal of a triaxial apparatus, enclosed in a well fitting rubber membrane, see Figure F.11. Before the cell pressure was applied, the pore water pressure gauge was put at zero, by which the initial pore pressure...
water suction \( (p_{wi}) \) in the sample remained unknown. At this initial state the total stress was still zero. Consequently the isotropic mean stress \( (\sigma'_p = -p_{wi}) \) in the sample remained unknown as well, see Figure F.12 (a).

Then the confining fluid pressure \( (u_{cel}) \) in the triaxial chamber was applied and maintained for about 15 minutes, while all taps to the sample were closed (no drainage). During this undrained phase a pore pressure change of \( \Delta p_{w0} \) developed and was measured, see Figure F.12 (b). The initial effective isotropic stress \( (\sigma'_{p0}) \) before undrained standard triaxial compression is:

\[
\sigma'_{p0} = u_{cel} + \sigma'_p - \Delta p_{w0}
\]

(F.2)

Figure F.12: Stress states of soil sample during UU-triaxial compression test: (a) initial state, (b) state after application of cell pressure \( (u_{cel}) \) and before compression, (c) effective and total stress path during triaxial compression. Data of test on \( DH_{20} \)-sample, see Figure F.16 (a).
The specimens were sheared in compression without drainage (mass of specimen = constant) at a constant cell pressure \( (u_{cel} = \text{constant}) \) and a constant rate of axial deformation \( (\dot{\varepsilon}_v = 4 \% / \text{hr}) \). The applied total axial stress \( (\sigma_1) \) and radial stress \( (\sigma_3 = u_{cel}) \) and resulting pore water pressure increments \( (\Delta u_b) \) were measured during the tests.

During UU-triaxial compression the pore water stress \( (p_w) \), axial effective stress \( (\sigma'_1) \) and radial effective stress \( (\sigma'_3) \) in the specimen are:

\[
\begin{align*}
p_w &= p_{wi} + \Delta p_{w0} + \Delta u_b \\
\sigma'_1 &= \sigma_1 - p_w = \sigma_1 - p_{wi} - \Delta p_{w0} - \Delta u_b \\
\sigma'_3 &= \sigma_3 - p_w = u_{cel} - p_{wi} - \Delta p_{w0} - \Delta u_b
\end{align*}
\]

In terms of isotropic effective stress \( (\sigma'_p) \) and deviatoric stress \( (q) \), these stresses become:

\[
\begin{align*}
\sigma'_p &= \frac{\sigma'_1 + 2\sigma'_3}{3} = \frac{\sigma_1 + 2u_{cel}}{3} - p_{wi} - \Delta p_{w0} - \Delta u_b \\
q &= \sigma'_1 - \sigma'_3 = \sigma_1 - u_{cel}
\end{align*}
\]

In terms of stress quantities \( s' \) and \( t \), according to Table F.3, the effective stress path can be expressed by:

\[
\begin{align*}
s' &= \frac{\sigma'_1 + \sigma'_3}{2} = \frac{\sigma_1 + u_{cel}}{2} - p_{wi} - \Delta p_{w0} - \Delta u_b \\
t &= \frac{\sigma'_1 - \sigma'_3}{2} = \frac{\sigma_1 - u_{cel}}{2}
\end{align*}
\]

Note that in both \( \sigma'_p \) and \( s' \) the initial pore water stress \( p_{wi} \) remains unknown.

The experimental results of the UU-triaxial tests are illustrated in Figures F.13 to F.15 and involve:

(a) Effective stress path in terms of \( s' - t \), in which \( s' \) is calculated assuming \( p_{wi} = 0 \). Consequently the actual effective stress path will be the same as the depicted version but shifted by the unknown initial value of \( p_{wi} \) along the \( s' \)-axis.

(b) Deviatoric stress \( (q) \) as function of the axial strain \( (\varepsilon_b) \).

(c) Excess pore water pressure \( \Delta u_b \) during undrained triaxial compression versus strain \( (\varepsilon_b) \).
Figure F.13: Results of UU-triaxial compression test DH$_{20}$ – sample.
Figure F.14: Results of UU-triaxial compression test DH$_{45}$ – sample.

<table>
<thead>
<tr>
<th>$\varepsilon_b$ [%]</th>
<th>$s'$ [kN/m$^2$]</th>
<th>$t$ [kN/m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>42.6</td>
<td>0</td>
</tr>
<tr>
<td>2.0</td>
<td>39.7</td>
<td>25.6</td>
</tr>
<tr>
<td>5.0</td>
<td>45.4</td>
<td>31.2</td>
</tr>
<tr>
<td>9.6</td>
<td>55.0</td>
<td>32.6</td>
</tr>
<tr>
<td>10</td>
<td>58.4</td>
<td>32.6</td>
</tr>
<tr>
<td>16</td>
<td>68</td>
<td>29</td>
</tr>
</tbody>
</table>
Figure F.15: Results of UU-triaxial compression test DH$_{70}$ – sample.
Estimation of initial stress state

To analyze the behaviour of the soil samples, the actual stress state at failure \((\sigma'_{pf}, q_f)\) or \((s'_{f}, t_f)\) have to be known. Unfortunately, only the effective stress increments \((\Delta \sigma'_p, \Delta q')\) or \((\Delta s', \Delta t)\) during triaxial deformation were measured, see Figures F.13 to F.15. An estimate of the effective stresses, present before the UU-triaxial tests, namely \((\sigma'_{pi}, q_i = 0)\) or \((s'_{i}, t_i = 0)\), can be made using the test results of the Oedometer tests, see F.1.

The vertical effective stress after consolidation \((\sigma'_{vi})\) will lie somewhere between the vertical stress at which the sample stops to swell and starts to compress \((\sigma'_{vs})\) and the vertical pre-consolidation pressure \((\sigma'_{v0})\), see Table F.1. It is assumed that the vertical effective stress after consolidation \((\sigma'_{vi})\) lies between \(1/3 \sigma'_{v0}\) and \(2/3 \sigma'_{v0}\), see Table F.4. These stresses correspond on average with the range in \(\sigma'_{vs}\).

### Table F.4: Assumed average range of vertical effective stress after consolidation \((\sigma'_{vi})\) in [kN/m²].

<table>
<thead>
<tr>
<th></th>
<th>DH20</th>
<th>DH45</th>
<th>DH70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-consolidation pressure ((\sigma'_{v0}))</td>
<td>50</td>
<td>80</td>
<td>140</td>
</tr>
<tr>
<td>Vert. eff. stress after consolidation ((\sigma'_{vi}))</td>
<td>17-33</td>
<td>27-53</td>
<td>47-93</td>
</tr>
</tbody>
</table>

This vertical effective stress \((\sigma'_{vi})\) corresponds with an isotropic effective stress of \((q_i = 0)\):

\[
\sigma'_{pi} = \frac{\sigma'_{vi}(1 + 2K_0)}{3}
\]  \hspace{1cm} (F.10)

Assuming for \(K_0\) the relation of Jaky (Eq. 2.39), the initial isotropic effective stress \((\sigma'_{pi})\) can be written as:

\[
\sigma'_{pi} = \frac{\sigma'_{vi}(3 - 2 \sin \varphi_p)}{3}
\]  \hspace{1cm} (F.11)

in which \(\varphi_p\) is the mobilized peak friction angle. The corresponding initial stresses in terms of \(s'\) and \(t\) are:

\[
s'_i = \sigma'_{pi}; \quad t_i = 0
\]  \hspace{1cm} (F.12)

The stresses at the start of triaxial compression can be obtained by substituting \(\Delta u_b = 0, \sigma_1 = \sigma_{10} = u_{cel}\) and \(p_{wi} \approx -\sigma'_{pi}\) into the Equations F.3 to F.9.
Estimation of mobilized peak friction angle

The stress increments from start of triaxial compression up to failure are:

\[
\Delta p_w = p_{wf} - \Delta p_{w0} = \Delta u_b \tag{F.13}
\]
\[
\Delta \sigma'_p = \frac{\sigma_{1f} - \sigma_{10}}{3} - \Delta u_b \tag{F.14}
\]
\[
\Delta q = \sigma_{1f} - \sigma_{10} \tag{F.15}
\]
\[
\Delta s' = \frac{\sigma_{1f} - \sigma_{10}}{2} - \Delta u_b \tag{F.16}
\]
\[
\Delta t = \frac{\sigma_{1f} - \sigma_{10}}{2} \tag{F.17}
\]

in which the values at start and at failure are denoted with the subscripts 0 and \(f\), respectively. The experimental data of the undrained effective stress path during triaxial compression, illustrated in Figures F.13 to F.15, concern \(\Delta s'\) and \(\Delta t\), from which follows:

\[
\sigma_{1f} - \sigma_{10} = 2\Delta t \tag{F.18}
\]
\[
\Delta u_b = \frac{\sigma_{1f} - \sigma_{10}}{2} - \Delta s' = \Delta t - \Delta s' \tag{F.19}
\]

leading to:

\[
\Delta \sigma'_p = \frac{2\Delta t}{3} - \Delta t + \Delta s' = \Delta s' - \frac{1}{3}\Delta t \tag{F.20}
\]
\[
\Delta q = 2\Delta t \tag{F.21}
\]

Adding the stress increments to the values at the begin of the undrained triaxial compression, leads to the following expressions for the stresses at failure:

\[
\sigma'_{pf} = \sigma'_{p0} + \Delta \sigma'_p = u_{cel} + \sigma'_{pi} - \Delta p_{w0} + \Delta s' - \frac{1}{3}\Delta t \tag{F.22}
\]
\[
q_f = q_0 + \Delta q = 0 + 2\Delta t = 2\Delta t' \tag{F.23}
\]
\[
s'_f = s'_0 + \Delta s' = u_{cel} + \sigma'_{pi} - \Delta p_{w0} + \Delta s' \tag{F.24}
\]
\[
t_f = t_0 + \Delta t = 0 + \Delta t = \Delta t \tag{F.25}
\]

These stresses are related by the mobilized peak friction angle, namely (see also Equation 2.19):

\[
\frac{q_f}{\sigma'_{pf}} = \frac{6\sin \varphi_p}{3 - \sin \varphi_p} \tag{F.26}
\]
or in terms of $s'$ and $t$:

$$\frac{t_f}{s'_f} = \sin \varphi_p$$  \hspace{1cm} (F.27)

Combining the Equations F.11, F.22, F.23 and F.26 gives a quadratical polynomial of $\sin \varphi_p$, which can be solved analytically. The calculated values of the mobilized peak friction angles of the DH-samples are listed in Table F.5.

The mobilized peak friction angles can also be determined using the equation for the deviatoric stress at failure, assuming that the cohesion ($c'$) is negligible, see Section 2.4:

$$q_f = \frac{6\sigma_{p0}' \sin \varphi_p}{3 - [1 + 6(1 - \beta_h)\zeta - 6\beta_h \alpha_h \sin \varphi_p]} = 2\Delta t$$  \hspace{1cm} (F.28)

The parameter $\beta_h$ is equal to the measured value of $B_w$ (see Table F.5) and $\zeta$ is 1/3.

Parameter $\alpha_h$ follows from the Equation of Henkel (Eq. 2.27):

$$\alpha_h = \frac{\Delta p_w - \zeta \beta_h \Delta q}{\beta_h \Delta q} = \frac{\Delta t(1 - 2\zeta \beta_h) - \Delta s'}{2\beta_h \Delta t}$$  \hspace{1cm} (F.29)

In Table F.5 the different parameters of Equation F.28 are listed. In Figure F.16 the followed stress paths to failure in terms of deviatoric and isotropic stress ($\sigma_p - q$ diagram) are plotted, assuming a range in initial vertical effective stresses ($\sigma_{vi}'$) as listed in Table F.4.
Table F.5: Calculated values of the different stress states before and during triaxial compression of the DH-samples.

<table>
<thead>
<tr>
<th></th>
<th>DH\textsubscript{20}</th>
<th>DH\textsubscript{45}</th>
<th>DH\textsubscript{70}</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial state before UU-test</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical initial eff. stress</td>
<td>$\sigma'_v$ [kN/m\textsuperscript{2}]</td>
<td>Tbl. F.4</td>
<td>17-33</td>
</tr>
<tr>
<td>Initial isotropic eff. stress</td>
<td>$\sigma'_i$ [kN/m\textsuperscript{2}]</td>
<td>Eq. F.11</td>
<td>13-27</td>
</tr>
<tr>
<td><strong>State at start of UU-test</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cell pressure</td>
<td>$u_{cel}$ [kN/m\textsuperscript{2}]</td>
<td>(Greeuw, 2004)</td>
<td>308</td>
</tr>
<tr>
<td>Initial measured water press.</td>
<td>$\Delta p_{w0}$ [kN/m\textsuperscript{2}]</td>
<td>(Greeuw, 2004)</td>
<td>256</td>
</tr>
<tr>
<td>Initial isotropic eff. stress</td>
<td>$\sigma'_{p0}$ [kN/m\textsuperscript{2}]</td>
<td>Eq. F.2</td>
<td>65-79</td>
</tr>
<tr>
<td><strong>State at failure</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial strain</td>
<td>$\epsilon_v$ [%]</td>
<td>Fig. F.13-F.15</td>
<td>6.4</td>
</tr>
<tr>
<td>Absolute displacement</td>
<td>$\Delta d_p$ [mm]</td>
<td>Fig. F.13-F.15</td>
<td>13.6</td>
</tr>
<tr>
<td>Stress increments</td>
<td>$\Delta s'$ [kN/m\textsuperscript{2}]</td>
<td>Fig. F.13-F.15</td>
<td>-11</td>
</tr>
<tr>
<td></td>
<td>$\Delta t$ [kN/m\textsuperscript{2}]</td>
<td>Fig. F.13-F.15</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>$\Delta \sigma'_{p'}$ [kN/m\textsuperscript{2}]</td>
<td>Eq. F.20</td>
<td>-17</td>
</tr>
<tr>
<td></td>
<td>$\Delta q$ [kN/m\textsuperscript{2}]</td>
<td>Eq. F.21</td>
<td>37</td>
</tr>
<tr>
<td>Isotropic total stress</td>
<td>$\sigma_{pf}$ [kN/m\textsuperscript{2}] = $u_{cel} + \frac{2}{3} \Delta t$</td>
<td></td>
<td>320.2</td>
</tr>
<tr>
<td>Parameters of Henkel (1960)</td>
<td>$\beta_h$ [-]</td>
<td>$= B_w$; (Greeuw, 2004)</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>$\alpha_h$ [-]</td>
<td>Eq. F.29</td>
<td>0.62</td>
</tr>
<tr>
<td>Isotropic eff. stress</td>
<td>$\sigma'_{pf}$ [kN/m\textsuperscript{2}]</td>
<td>Eq. F.22</td>
<td>48-62</td>
</tr>
<tr>
<td>Deviatoric stress</td>
<td>$q_f$ [kN/m\textsuperscript{2}]</td>
<td>Eq. F.23</td>
<td>37</td>
</tr>
<tr>
<td>Mobilized peak friction angle</td>
<td>$\varphi_p$ [°]</td>
<td>Eq. F.26</td>
<td><strong>16-20</strong></td>
</tr>
<tr>
<td>Coeff. of earth pressure at rest</td>
<td>$K_0$ [-]</td>
<td>Eq. 2.39</td>
<td><strong>0.66-0.73</strong></td>
</tr>
</tbody>
</table>
Figure F.16: Total and estimated range of effective stress paths. Deviatoric total and effective stress \((q)\) as function of the isotropic total and effective stress \((\sigma_p, \sigma_p')\): (a) DH\(_{20}\)-sample, (b) DH\(_{45}\)-sample, (c) DH\(_{70}\)-sample.
Estimation of undrained shear strength at the critical state

Similar to the determination of the mobilized peak friction angles, also the mobilized friction angles at the end of the tests (\(\varphi_{end}\)) can be determined, as an indication of the mobilized friction angles at the critical state (\(\varphi_{cs}\)). Unfortunately, in no test the soil reached the critical state, see Figures F.13 (b), F.14 (b) and F.15 (b); the deviatoric stresses still decrease at the end of all tests. Only the DH20-sample almost reaches the critical state. Therefore the mobilized friction angle at the critical state (\(\varphi_{cs}\)) is assumed to range from 12° to 15° for all samples. In Figure F.17 the undrained shear strength at the critical state in terms of deviatoric stress (\(q_{cs} = 2s_{u_{cs}}\)) is estimated for the DH-samples. In Table F.6 the main parameters at the critical state are listed for the different samples are listed. Also the sensitivity (\(S_s\)), defined as the ratio between the undrained shear strength at peak and at critical state (\(s_{up}/s_{u_{cs}}\)).

Table F.6: Calculated values of the different stress states before and during triaxial compression of the DH-samples.

<table>
<thead>
<tr>
<th></th>
<th>DH20</th>
<th>DH45</th>
<th>DH70</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State just before stopping triaxial compression, see Fig. F.13-F.15</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stress increments</td>
<td>(\Delta s') [°]</td>
<td>-7</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>(\Delta t) [°]</td>
<td>15</td>
<td>29</td>
</tr>
<tr>
<td>Mobilized friction angle</td>
<td>(\varphi_{end}) [°]</td>
<td>12-15</td>
<td>15-19</td>
</tr>
<tr>
<td><strong>Estimated values at critical state, see Fig. F.17</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mobilized friction angle</td>
<td>(\varphi_{cs}) [°]</td>
<td>12-15</td>
<td>12-15</td>
</tr>
<tr>
<td>Isotropic eff. stress</td>
<td>(\sigma_{p,cs}) [kN/m²]</td>
<td>53-67</td>
<td>82-105</td>
</tr>
<tr>
<td>Deviatoric stress</td>
<td>(q_{cs}) [kN/m²]</td>
<td>30</td>
<td>46</td>
</tr>
<tr>
<td>Undrained shear strength</td>
<td>(s_{u_{cs}}) [kN/m²]</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>(S_s) [-]</td>
<td>1.2</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Figure F.17: Estimation of the undrained shear strength at the critical state in terms of the deviatoric stress \( q_{cs} = 2su_{cs} \), assuming that the mobilized friction angle at the critical state \( \phi_{cs} \) ranges from 12° to 15°: (a) DH\(_{20} \) – sample, (b) DH\(_{45} \) – sample, (c) DH\(_{70} \) – sample.
Appendix G

Consolidation process GD-sample

G.1 Apparatus consolidation cell

The diameter of the consolidation cell was 1.25 m, see Figures G.1 and G.2. The cell was drained on both sides.

Figure G.1: GD-sample before consolidation.

Figure G.2: Consolidation cell GD-sample.

G.2 Pore-water pressure and settlement time development

The initial height of the mud sample was 690 mm with a water content of 82%. The mud was consolidated under a vertical consolidation pressure of 330 kN/m². In Table G.1 the measured pore-water pressure \( (p_w) \) and vertical settlements \( (\Delta h_v) \)
during the consolidation process are listed. In the figures G.3 and G.4 the time-development of both are plotted. The pore-water pressure was measured at a constant position, 135 mm above the bottom plate of the consolidation cell. Therefore the relative position of the sensor \((z_{sensor}/0.5h_{sample})\) changed during the consolidation process. As a result the slope of the pore water pressure is slightly flatter than if the sensor had been positioned at a constant relative height.

Table G.1: Measured vertical settlements \((\Delta h_v)\) and pore-water pressures \((p_w)\) during the consolidation process.

<table>
<thead>
<tr>
<th>Time [hr]</th>
<th>(\Delta h_v) [mm]</th>
<th>(p_w) [kN/m(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>330.0</td>
</tr>
<tr>
<td>100</td>
<td>47</td>
<td>328</td>
</tr>
<tr>
<td>338</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>174</td>
<td>300.0</td>
</tr>
<tr>
<td>1416</td>
<td>208</td>
<td>285.0</td>
</tr>
<tr>
<td>1752</td>
<td>226</td>
<td>270.0</td>
</tr>
<tr>
<td>1800</td>
<td>232</td>
<td>265.0</td>
</tr>
<tr>
<td>2256</td>
<td>262</td>
<td>240.0</td>
</tr>
<tr>
<td>3432</td>
<td>302</td>
<td>138.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time [hr]</th>
<th>(\Delta h_v) [mm]</th>
<th>(p_w) [kN/m(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3972</td>
<td>310</td>
<td>113.6</td>
</tr>
<tr>
<td>4104</td>
<td>314</td>
<td>92.1</td>
</tr>
<tr>
<td>4272</td>
<td>316</td>
<td>77.9</td>
</tr>
<tr>
<td>4440</td>
<td>318</td>
<td>64.3</td>
</tr>
<tr>
<td>4608</td>
<td>320</td>
<td>51.9</td>
</tr>
<tr>
<td>4776</td>
<td>321</td>
<td>45.0</td>
</tr>
<tr>
<td>4944</td>
<td>320</td>
<td>40.5</td>
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<td>5112</td>
<td>323</td>
<td>28.2</td>
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<td>5280</td>
<td>324</td>
<td>26.4</td>
</tr>
<tr>
<td>5472</td>
<td>325</td>
<td>14.3</td>
</tr>
<tr>
<td>5616</td>
<td>325</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Figure G.3: Time-development of the measured vertical settlements during the consolidation process.

Figure G.4: Time-development of the measured pore water pressure during the consolidation process.
Figure G.5: Vertical strain ($\epsilon_v$) as function of the vertical effective stress for the GD-sample.

From Figure G.5 the compression index $C_c$, defined as in Equation F.1 can be derived. The compression index is about 0.54, assuming an initial void ratio ($e_0$) of 1.8 at a vertical effective stress ($\sigma'_v = \sigma_{v0}$) of 2 kN/m$^2$. This value is significantly higher, than the values determined by means of the CRS-tests ($C_c = 0.35$), see Appendix F.2. It can be concluded that the GD-sample behaves softer than the DH$_{20}$-sample.

### G.3 Prediction undrained shear strength

Beforehand the expected undrained shear strength ($s_u$) of the GD-sample was about 75 kN/m$^2$, assuming a vertical consolidation pressure ($\sigma_{v0}$) of 330 kN/m$^2$.

The undrained shear strength ($s_u$) of the GD-sample after consolidation can be estimated with Equation 2.42. The initial vertical effective stress ($\sigma'_{v0}$) was:

$$\sigma'_{v0} = \sigma_{v0} - p_{w0} = 330 - 9.6 \approx 320 \text{ [kN/m}^2]\] \tag{G.1}$$

in which $p_{w0}$ is the measured pore water pressure at the end of the consolidation process, see Table G.1. The estimated undrained shear strength is about 60 kN/m$^2$, assuming the following parameters: internal friction angle ($\varphi$) is $18^\circ$ (see Table F.5: DH$_{20}$ - sample), Henkel’s parameter $\alpha_h$ is 0.65 (normally consolidated, see Table 2.4), Henkel’s parameter $\beta_h$ is 1 (fully saturated sample) and $K_0 = 1 - \sin \varphi$ (Jaky, 1944), see Figure G.6.\footnote{For the aimed undrained shear strength of 75 kN/m$^2$, the parameters were: $\sigma'_{v0} = 330$ kN/m$^2$, $\varphi = 22.5^\circ$, $\alpha_h = 0.65$ and $\alpha_h = 1$.}

This is significantly higher than the undrained shear strength of 40 kN/m$^2$ determined by means of a pocket-penetrometer and a
hand-torvane. The Liquidity index of the GD-sample \((LI = 0.24)\) also suggests a higher undrained shear strength. According to the relation of Wroth (Eq. 2.50), the expected remoulded undrained shear strength is about 56 kN/m².

Figure G.6: Estimated range of effective stress path in terms of \(\sigma'_p\) and \(q\) for the DH\(_{20}\)- and GD-sample.

To become a shear strength of about 40 kN/m\(^2\), the mobilized peak friction angle \((\varphi_p)\) in Equation 2.42 must be about 10\(^o\), see Figure G.6. This is not quite realistic. Possible explanations are:

- Cavitation; the pore water suction pressure after unloading would be significantly lower than the absolute vacuum \((p_w \ll 0 \text{ kN/m}^2)\).

The pore water under pressure corresponding to an undrained shear strength of 40 kN/m\(^2\) \((p_{w40})\) is related by the initial isotropic effective stress \((\sigma'_{p0})\), by:

\[
p_{w40} = p_{abs} + p_{w0} - \sigma'_{p0} \quad \text{(G.2)}
\]

The initial isotropic effective stress \((\sigma'_{p0})\) can be calculated by rewriting Equation 2.34:

\[
\sigma'_{p0} = \frac{3su - [1 + 6(1 - \beta_h)\zeta - 6\beta_h\alpha_h] \sin \varphi su}{3\sin \varphi} \quad \text{(G.3)}
\]

Assuming, \(c' = 0\), \(q_0 = 0\), \(\zeta = 1/3\), \(\beta_h = 1\) \(\alpha_h = 0.65\), \(\varphi = 18^o\) and \(su = 40 \text{ kN/m}^2\), the initial isotropic effective stress \((\sigma'_{p0})\) becomes about 168
kN/m². Substituting this value in Equation G.2 gives a pore water pressure ($p_w$) of about -60 kN/m². Because of the colloidal size of the pores, the soil can withstand much higher under pressures. Therefore cavitation is not a plausible explanation.

- The used relation of Jaky (Equation 2.39) does not fit. A lower value of $K_0$ during the consolidation process will result in a lower isotropic stress level ($\sigma'_p$) after consolidation, see Equation F.10. As a result the undrained shear strength will also be lower (assuming $K_0 = 0.5$ and $\varphi = 16^\circ$, the calculated undrained shear strength becomes 46 kN/m²).

### G.4 Analysis consolidation process

The final settlement of the primary consolidation process ($\Delta h_\infty$) and the consolidation coefficient ($c_v$) can be determined according to different methods. The most common methods are the Log(t)-method of Casagrande and the $\sqrt{t}$-method of Taylor. For a description of these methods, see (Verruijt, 2006). It is known that the determined values of $\Delta h_\infty$ and $c_v$ can differ significantly (Leroueil et al., 1990). Another simple less known method, also used in practice is the Asaoka-method (Asaoka, 1978). In Table G.2 the predicted final settlements ($\Delta h_{v,\infty}$) of the GD-sample, determined with the different methods are listed.

#### Table G.2: Soil properties GD-sample determined from consolidation process.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta h_{v,\infty}$ [mm]</th>
<th>$c_v$ [m²/s]</th>
<th>$m_v$ [m²/N]</th>
<th>$k_w$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(t)</td>
<td>325</td>
<td>$7.2 \cdot 10^{-9}$</td>
<td>$1.43 \cdot 10^{-6}$</td>
<td>$1.0 \cdot 10^{-10}$</td>
</tr>
<tr>
<td>Osaoka</td>
<td>340</td>
<td>$8.4 \cdot 10^{-9}$</td>
<td>$1.49 \cdot 10^{-6}$</td>
<td>$1.3 \cdot 10^{-10}$</td>
</tr>
<tr>
<td>$\sqrt{t}$</td>
<td>366</td>
<td>$6.8 \cdot 10^{-9}$</td>
<td>$1.61 \cdot 10^{-6}$</td>
<td>$1.1 \cdot 10^{-10}$</td>
</tr>
</tbody>
</table>

The measured final water pressure of 9.6 kN/m² indicates that the consolidation process was almost finished. According to the Log(t)-method this was indeed the case. However, according to Asaoka and Taylor the soil sample was not yet fully consolidated. The measured undrained shear strength of 40 kN/m² and the final rate of decrease in pore-water pressure (compare the figures G.3 and G.4) also indicates that the consolidated process was not fully finished. In Table G.2 also the determined consolidation coefficient ($c_v$), compressibility ($m_v$) and water permeability ($k_w$) of the GD-sample are listed. The higher value determined with the Asaoka-method is consistent with the findings of other researchers (Dykstra and Joling, 2001).
In this thesis the consolidation coefficient, compressibility and permeability of the GD-sample are assumed to be, $c_v = 7 \cdot 10^{-9} \, \text{m}^2/\text{s}$ and $k = 1.1 \cdot 10^{-10} \, \text{m/s}$, respectively.
Appendix H

Estimation of crack propagation speed

During the forming of a shear band mini-cracks are formed. These mini-cracks can coalesce and form a macro-crack. This process will be speed up by the jet water that is penetrating the shear band. In this Appendix a rough estimate of the propagation speed of the crack ($u_{cr}$) in the shear band is made.

For fracturing micro-cracks present in the soil have to coalesce and form macro-cracks. This process takes time. Also a rough estimate of the duration of this process is given. In the estimation of the propagation speed of the crack the duration of this process is not taken into account.

Crack propagation speed

Winterwerp and van Kesteren (2004) suggest that the fracture energy for undrained tensile loading necessary to create 1 m$^2$ of new crack surface area ($G$ [N/m]), scales with the undrained shear strength:

$$G = W_{cr}su$$  \hspace{1cm} (H.1)

in which $W_{cr}$ can be interpreted as the crack tip opening displacement prior to the crack tip, see Figure H.1. Reported values of $W_{cr}$ range from 100 to 300 µm (Winterwerp and van Kesteren, 2004). The hydraulic power ($P_{cr}$ [Nm/s]) necessary to create a crack with a tip propagation speed $u_{cr}$ over a length $L_{cr}$ is:

$$P_{cr} = u_{cr}L_{cr}G = u_{cr}L_{cr}W_{cr}su$$  \hspace{1cm} (H.2)
The hydraulic power of the jet penetrating the crack can be predicted with the following relation:

\[ P_{j,cr} = p_{cr} Q_{cr} \approx \frac{1}{2} \rho_m (u_u - u_{cr})^2 L_{cr} W_{cr} u_{cr} \]  

(H.3)

in which \( p_{cr} \) is the pressure in the crack, caused by the penetrating jet water, \( Q_{cr} \) is the flow rate of the jet that is penetrating the crack, \( u_u \) is the uniform jet velocity at the top of the soil element/crack opening and \( \rho_m \) is the density of the jet flow. Equating the power consumption in the crack (\( P_{cr} \)) and the delivered jet power (\( P_{j,cr} \)), results in the following relation of the propagation speed of the crack tip, normalized by the uniform jet velocity at the top of a soil element:

\[ \frac{u_{cr}}{u_u} \approx 1 - \sqrt{\frac{su}{p_{stag}}} \]  

(H.4)

where \( p_{stag} \) is the stagnation pressure at the top of the soil element, which is defined as \( 1/2 \rho_m u_u^2 \). In this relation the normalized propagation speed is a function of the stagnation ratio (\( p_{stag}/su \)). For different stagnation ratios the calculated propagation velocities of the crack are listed in Table H.1. The estimated normalized propagation velocities of the crack (\( u_{cr}/u_u \)) range from 0.66 to 0.86.

**Coalesce of mini-cracks**

For fracturing, micro-cracks present in the soil have to coalescence and form macro-cracks. This process takes time and can be considered as a consolidation process, as discussed in Subsection 2.6.1.
Table H.1: Estimation of the propagation velocity of a crack in the shear band caused by penetrating jet water.

<table>
<thead>
<tr>
<th>$su_{cs}$ [kN/m$^2$]</th>
<th>$p_{stag}$ [kN/m$^2$]</th>
<th>$p_{stag}/su$ [-]</th>
<th>$u_w$ [m/s]</th>
<th>$u_{cr}/u_w$ [-]</th>
<th>$u_{cr}$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>560</td>
<td>28</td>
<td>33</td>
<td>0.81</td>
<td>27</td>
</tr>
<tr>
<td>70</td>
<td>560</td>
<td>8</td>
<td>33</td>
<td>0.65</td>
<td>22</td>
</tr>
<tr>
<td>20</td>
<td>1000</td>
<td>50</td>
<td>45</td>
<td>0.86</td>
<td>38</td>
</tr>
<tr>
<td>70</td>
<td>1000</td>
<td>14</td>
<td>45</td>
<td>0.74</td>
<td>33</td>
</tr>
</tbody>
</table>

A rough indication of the minimum duration of this process of coalescence, can be estimated, using Equation 2.56, assuming that for fracturing 10 to 60% of the pore water pressure increment in a crack induced by the jet has to be dissipated ($\Delta p_{w,mc} / \Delta \sigma_t = 0.4 – 0.9$). The unknown parameters are the radius ($R_{mc}$) and the drainage length ($h_{rad}$) of the mini-cracks present in the soil. Assuming a mini-crack radius of 0.1 mm, mentioned by Winterwerp and van Kesteren (2004), a drainage length of 0.15 mm and a consolidation coefficient ($c_v$) of $10^{-9}$ m$^2$/s, the prediction minimum duration ($t_{10}$) is in the order of one-tenth of a second, see also Table 2.6. The expected duration of the failure process of one soil element is possibly an order of magnitude shorter, see Section 7.4.3. In this case fracturing seems not to occur.

Therefore it is recommended to investigate the propagation velocity of a crack in more detail.
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Curriculum Vitae

Arno Nobel was born on 10 July 1976 in Vlaardingen, the Netherlands. After finishing high school in 1994 he studied Civil Engineering at Delft University of Technology. He graduated in 2000 at the section of Design and Construction.

After his study he worked at Heerema Infrastructure in Rotterdam, where he was involved in the development of an innovative Jet Excavation Tunneling System.

Since Heerema stopped this activity in 2002 Arno is working as a research engineer at Boskalis, dredging and marine experts in Papendrecht. He is involved in all kind of jetting problems, all over the world.

In 2004 he started part-time (two days a week) a Ph.D-study, commissioned by the Dutch dredging companies Boskalis and van Oord, at the Delft University of Technology on the excavation process of a moving vertical jet in cohesive soil.