Defining Uncertainty
A Conceptual Basis for Uncertainty Management
in Model-Based Decision Support

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ABSTRACT

The aim of this paper is to provide a conceptual basis for the systematic treatment of uncertainty in model-based decision support activities such as policy analysis, integrated assessment and risk assessment. It focuses on the uncertainty perceived from the point of view of those providing information to support policy decisions (i.e., the modellers' view on uncertainty) – uncertainty regarding the analytical outcomes and conclusions of the decision support exercise. Within the regulatory and management sciences, there is neither commonly shared terminology nor full agreement on a typology of uncertainties. Our aim is to synthesise a wide variety of contributions on uncertainty in model-based decision support in order to provide an interdisciplinary theoretical framework for systematic uncertainty analysis. To that end we adopt a general definition of uncertainty as being any deviation from the unachievable ideal of completely deterministic knowledge of the relevant system. We further propose to discriminate among three dimensions of uncertainty: location, level and nature of uncertainty, and we harmonise existing typologies to further detail the concepts behind these three dimensions of uncertainty. We propose an uncertainty matrix as a heuristic tool to classify and report the various dimensions of uncertainty, thereby providing a conceptual framework for better communication among analysts as well as between them and policymakers and stakeholders. Understanding the various dimensions of uncertainty helps in identifying, articulating, and prioritising critical uncertainties, which is a crucial step to more adequate acknowledgement and treatment of uncertainty in decision support endeavours and more focused research on complex, inherently uncertain, policy issues.

Keywords: uncertainty, ignorance, model-based decision support, policy analysis, integrated assessment, risk assessment, uncertainty management.

1. INTRODUCTION

The world is undergoing rapid changes. The future is uncertain. Even with respect to understanding existing natural, economic and social systems, many uncertainties have to be dealt with. Furthermore, because of the globalisation of issues and the interrelationships among systems, the consequences of making wrong policy decisions have become more serious and global – potentially even catastrophic. Nevertheless, in spite of the profound and partially irreducible uncertainties and serious potential consequences, policy decisions have to be made. Scientific decision support aims to provide assistance to policymakers in developing and choosing a course of action, given all of the uncertainties surrounding the choice.

That uncertainties exist in practically all policymaking situations is generally understood by most policymakers, as well as by the scientists providing decision support. But there is little appreciation for the fact that there are many different dimensions of uncertainty, and there is a lack of understanding about their different characteristics, relative magnitudes, and available means of dealing with them. Even within the different fields of decision support (policy analysis, integrated assessment, environmental and human risk assessment, environmental impact assessment, engineering risk analysis, cost-benefit analysis, etc.), there is
neither a commonly shared terminology nor agreement on a
generic typology of uncertainties.

The need for more constructive approaches to account-
ability about uncertainty and ignorance in regulatory
decisions has grown with the increasing attention to the
"precautionary principle." The principle has put uncertainty
even more firmly and explicitly on the political agenda,
because the principle deals with situations where uncertainty
prevails regarding decisions about activities potentially
generating harm. The key questions are: What level of
certainty is demanded to curtail or even ban an activity that
might be harmful? Who should bear the burden of proof?
Who should run the risks associated with making the
wrong decision? These, and similar questions are high-
lighted in a recent publication from the European Environ-
ment Agency [1].

The aim of this paper is to provide a conceptual
framework for the systematic treatment of uncertainty in
decision support in order to improve the management of
uncertainty in decisionmaking processes.

There are many good reasons to develop a typology of
uncertainties for model-based decision support. First and
foremost, it will provide for better communication among
policy analysts. In the current situation, different analysts
use different terms for the same kinds of uncertainty, and
some use the same term to refer to different kinds. This
makes it extremely difficult for those who have not
participated in the actual work to understand what has been
done. Defining uncertainty through a typology will also
provide for better communication among policy analysts,
policymakers and stakeholders. It is widely held that
policymakers expect scientists to provide certainties and
hence dislike uncertainty in the scientific knowledge base.
But, uncertainty is a fact of life and a better understanding of
the different dimensions of uncertainty and their implica-
tions for policy choices would be likely to lead to more trust
in the scientists providing decision support, and ultimately to
better policies. Finally, a better understanding of the
different dimensions of uncertainty and their potential
impact on the relevant policy issues at hand would help in
identifying and prioritising effective and efficient research
and development activities for decision support. For
example, it would help at the beginning of a project to
decide on the allocation of project resources. Knowing about
the relative differences in outcomes from better parameter
estimates for an assumed model, a more appropriate model
or better information on inputs might reveal the most
resource-effective strategy for carrying out the analysis.

2. MODEL-BASED POLICY ANALYSIS

2.1. The Policymaking Process
Policymaking processes involve policymakers, stakeholders
and scientists. The stakeholders communicate their goals,
objectives, and preferences to the policymakers who must
then decide on the policies to be adopted. Policies are the set
of forces within the control of the policymakers that affect
the structure and performance of the system of interest.
Loosely speaking, a policy is a set of actions taken by an
administration to control the system, to help solve problems
within it or caused by it, or to obtain benefits from it. In
public policy, the problems and benefits generally relate to
broad international, national or regional goals – for example,
tradeoffs among national environmental, social, and eco-
nomic goals. A goal is a generalized policy objective
(frequently non-quantitative, e.g., "reduce air pollution" or
"ensure traffic safety," and more rarely quantified, e.g., 80% reduction of nutrient discharge). Policies are intended to
help achieve the goals.

To aid in the decisionmaking process, applied scientists
are frequently called upon to assess the outcomes of
alternative policies. In this paper, the scientists acting in
this capacity will be referred to as policy analysts and the
task they perform will be referred to as decision support. A
common approach to decision support is to create a model of
the system of interest that defines the boundaries of the
system and its structure – i.e., the elements, and the links,
flows, and relationships among these elements [2]. In this
case the analysis is referred to as being model based. The
system model is usually, but not necessarily, a computer-
based model. This paper will focus on model-based decision
support.

Each policy goes through its own unique process of
development and implementation. In practice, the involve-
ment of policy analysts, stakeholders and policymakers in
the process can take different forms. However, the simpli-
ified and idealized multi-stage iterative process shown in Figure 1 captures many of the important elements of the interactions
between the policy analysis process and the policymaking
process and is a sufficient conceptual basis for the purposes
of this paper.

The first stage of the process is the problem identification
and framing stage. This stage is ideally conducted in the
form of a dialogue among policymakers, stakeholders, and

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Fig. 1. The policymaking process viewed as a multi-stage iterative process.
scientists. It may be that it is not possible to agree on a single definition of the problem that is not possible to agree on a single definition of the problem and that rival problem framings must be explored in the analysis. In model-based policy analysis, criteria are used to measure the degree to which alternative policy actions can help to reach the goals. These criteria are used to determine the outputs that should be produced by the model. Those model outcomes that are related to the policy goals and objectives are termed outcomes of interest. The problem identification and framing stage is instrumental in determining the structure of the system model and identifying the outcomes of interest.

In the second stage, policy analysts assess the information available to produce the knowledge required to support a policy decision according to a range of plausible circumstances and developments, and according to the uncertainty involved. In principle, both expert and lay knowledge should be included in the analysis and the assessment process. This process should ideally be accompanied by a quality control stage, e.g., in terms of a peer review or a critical self-reflection, which makes explicit the underlying assumptions, underpinnings, and quality of the performed analysis, thereby increasing confidence in the obtained results.

In the next stage the results of the analysis are discussed by the policymakers and stakeholders. If the information provided does not adequately match the information needs agreed upon in the problem identification and framing stage, or if the review by peers or stakeholders indicates that the assessment is inadequately framed, too uncertain, too unreliable, too biased, or excessively value laden, the process can be returned to the problem framing stage. If the model structure and the results of the peer review are acceptable, the policymakers and stakeholders can develop their perspective on the results based on their values and interests. Although a policy action may be designed with a single goal in mind, it will seldom have an effect on only one outcome of interest. Policy choices, therefore, depend not only on estimating the outcomes of interest relative to the policy goals and objectives, but identifying the preferences of the various stakeholders, and identifying tradeoffs among the outcomes of interest given these various sets of preferences.

In the final stages of the policy process, a policy is chosen, implemented, and communicated to the public. The impacts of the policy can then be monitored in order to see whether the objectives are being achieved or not, to identify new problems, and to assess whether identified uncertainties have been resolved or new ones are emerging.

2.2. The System Model

Decision support activities must often explore the effect of alternative policies on the full range of outcomes of interest under a variety of scenarios, and examine the tradeoffs among different policies. This exploration requires a structured analytical process. Because of the complexity of the system being studied and the wide range of scenarios to be considered, a system model is a useful and often indispensable tool in this process.

A system model is an abstraction of the system of interest – either the system as it currently exists, or as it is envisioned to exist for purposes of evaluating policies in a different (e.g., future) context. Here it is important to note that we employ a broad interpretation of the term “model,” including both a conceptual formulation and/or a mathematical model (algorithm), frequently found in the form of a computer programme. A conceptual model may be as simple as a line and box diagram of the structure of the system, with lines representing more or less well known relationships, varying from facts to beliefs. A widespread conceptual model is that of modelling the concept of risk as a function of probability and consequence. This conceptual model has been adapted to fit the various fields of risk assessment, for example the expression of risk as a function of exposure and effect in human and environmental risk assessment.

The system model represents the cause-effect relationships characteristic of the system. In a mathematical model, the relationships among the various components of the system are expressed as functions. Although formulated in mathematical terms, these models usually contain inherent components of subjectivity. Subjectivity manifests itself already in the conceptual phase when decisions are made concerning which elements will be included in the analysis and which will be left out. Subjectivity affects the manner in which modellers translate the conceptual model into mathematical equations.

A computer program is a translation of the mathematical model into computer code. Typically the resulting system model represents a compromise between desired functionality, plausibility, and tractability, given the resources at hand (data, time, money, expertise, etc.).

In decision support activities, the focus of a modelling exercise is typically on the response of a system to outside forces (external changes or policy changes) and the system’s performance (i.e., the resulting values of the outcomes of interest) in these future contexts. A much-used analytical tool to deal with the deep uncertainties of the unknown (and unknowable) future is to use scenarios as plausible descriptions of how the system and its driving forces may develop. A scenario is based on a coherent and internally consistent set of assumptions about key relationships and driving forces (e.g., technology changes, prices, etc.).

Different scenarios reflect the variety of alternative economic, environmental, social, and technological conditions that may be present in reality, including variations in the behaviour of people. These conditions act on the system, which leads to changes in the system and, ultimately, changes in the outcomes of interest. Within the decision support exercise, alternative scenarios may manifest themselves as alternative model formulations, as alternative sets of input data, or as both. Policies represent the alternative mechanisms for affecting the system that are under the
In order to encompass all dimensions of uncertainty, we adopt due to variability inherent to the system under consideration. Between uncertainty due to lack of knowledge and uncertainty more complex than thought before [5]. Our understanding is more limited or that the processes are understated. In this way, more knowledge illuminates that knowledge on complex processes may reveal the presence of inadequate information, which can be of three sorts: inexactness, unreliability, and border with ignorance. Uncertainty is not simply the absence of knowledge. Funtowicz and Ravetz [3] describe uncertainty as a situation of inadequate information, which can be of three sorts: inexactness, unreliability, and border with ignorance. However, uncertainty can prevail in situations where a lot of information is available [4]. Furthermore, new information can either decrease or increase uncertainty. New knowledge on complex processes may reveal the presence of uncertainties that were previously unknown or were understated. In this way, more knowledge illuminates that our understanding is more limited or that the processes are more complex than thought before [5].

As will be elaborated further on in the paper, we distinguish between uncertainty due to lack of knowledge and uncertainty due to variability inherent to the system under consideration. In order to encompass all dimensions of uncertainty, we adopt a general definition of uncertainty as being any departure from the unachievable ideal of complete determinism.

There have been many uncertainty typologies developed for many purposes. Few have claimed to be comprehensive, and even fewer have had model-based decision support as their point of departure.¹ Our framework for uncertainty in model-based decision support is consistent with most of them, but is comprehensive within its context. Others are more general, not targeted specifically on model-based decision support (such as [3,11]) or apply to a specific context, such as water management (e.g., [8]). Classifications that are model-oriented either focus on a single dimension of uncertainty (e.g., Alcamo and Bartnicki [7], who focus on the location of uncertainty), reduce uncertainty to error (e.g., [6]), or do not discriminate explicitly between the level and the nature of uncertainty [10]. Within the context of model-based decision support, therefore, it can easily be concluded that there is neither a commonly shared terminology nor agreement on a generic typology of uncertainties. The aim of this paper is to highlight the agreements in order to provide a conceptual basis for the systematic treatment of uncertainty in policy analysis and integrated assessment.

Our major challenge was to find a categorization such that all of the different kinds of uncertainty found in the literature can be mapped into the categories that we propose. In doing that, the resulting synthesis should then be comprehensive and complete. A second challenge was to be specific to model-based decision support – removing categories unrelated to this context and clustering the remaining notions. Finally, labels had to be found for our categories that were at least supported by our group of authors.

Those discussing uncertainty in scholarly fora (journals, conferences), referred to in this paper as uncertainty experts, agree that it is important to distinguish between what can be called the modellers’ view of uncertainty and the decision-makers’/policymakers’ view of uncertainty. The modellers’ view focuses on the accumulated uncertainties associated with the outcomes of the model and the (robustness of) conclusions of the decision support exercise; the policymakers’ view includes uncertainty about how to value the outcomes in view of his/her portfolio of goals and possibly conflicting objectives, priorities, and interests. For example, what are the current or future societal values related to environmental impacts versus economic costs and benefits? This paper focuses on the uncertainty perceived from the point of view of those providing information to support policy decisions (i.e., the modellers’ view on uncertainty) – uncertainty regarding the analytical outcomes and conclusions of the decision support exercise.

Uncertainty experts agree that there are different dimensions of uncertainty related to model-based decision support exercises. Through a process of consultation and discussion, the authors of this paper have chosen to distinguish three dimensions of uncertainty:

(i) the location of uncertainty – where the uncertainty manifests itself within the model complex;
(ii) the level of uncertainty – where the uncertainty manifests itself along the spectrum between deterministic knowledge and total ignorance;
(iii) the nature of uncertainty – whether the uncertainty is due to the imperfection of our knowledge or is due to the inherent variability of the phenomena being described.

¹Among recent papers and books directly or indirectly addressing the issue of characterizing uncertainty in model-based decision support are: [3–15].
In the following sections, we present the three dimensions of uncertainty in more detail.

4. THE LOCATION OF UNCERTAINTY: IDENTIFIED BY THE LOGIC OF THE MODEL FORMULATION

Location of uncertainty is an identification of where uncertainty manifests itself within the whole model complex. This dimension refers to the logical structure of a generic system model within which it is possible to pinpoint the various sources of uncertainty in the estimation of the outcomes of interest.

The description of the model locations will vary according to the system model in question. Ideally, the location should be characterised in a way that is operationally beneficial to understanding where in the model the uncertainty associated with the outcome is generated. To this end, we identify the following generic locations with respect to the model:

- **Context** is an identification of the boundaries of the system to be modelled, and thus the portions of the real world that are inside the system, the portions that are outside, and the completeness of its representation. The model context is typically determined in the problem framing stage and is crucial to the decision support exercise as it clarifies the issues to be addressed and the selection of the outcomes of interest to be estimated by the model.

- **Model uncertainty** is associated with both the conceptual model (i.e., the variables and their relationships that are chosen to describe the system located within the boundaries and thus constituting the model complex) and the computer model. Model uncertainty can, therefore, be further divided into two parts: model structure uncertainty, which is uncertainty about the form of the model itself, and model technical uncertainty, which is uncertainty arising from the computer implementation of the model.

- **Inputs** to the model are associated with the description of the reference system, which is often the current system, and the external forces that are driving changes in the reference system. It is sometimes useful to divide the inputs into controllable and uncontrollable inputs, depending on whether the decisionmaker has the capability to influence the values of the specific input variables.

- **Parameter uncertainty** is associated with the data and the methods used to calibrate the model parameters.

- **Model outcome uncertainty** is the accumulated uncertainty associated with the model outcomes of interest to the decisionmaker.

The following paragraphs describe each of the locations in more detail.

4.1. Context

The “context” refers to the conditions and circumstances (and even the stakeholder values and interests) that underlie the choice of the boundaries of the system, and the framing of the issues and formulation of the problems to be addressed within the confines of those boundaries.

Context uncertainty includes uncertainty about the external economic, environmental, political, social, and technological situation that forms the context for the problem being examined. The context could fall within the past, the present, or the future. Uncertainties are often introduced in framing a decision situation because the context of the decision support is unclear. Actors in a decision situation often have different perceptions of reality, which are related to their different frames of reference or views of the world (see [16, 17]). That is why it is important to involve all stakeholders from the very beginning of the process of defining what the issue is. In recent years, expert groups have been accused increasingly of framing problems such that the context fits the tacit values of the experts and/or fits the tools, which the experts can use to provide a “solution” to the problem. The public is better educated today and may identify such “decision support” as biased and manipulative. Deciding on a proper framing of context is a significant part of the problem and should be given attention to such an extent that reasonable alternative framings are incorporated in the analysis. The concept and methodology of context validation proposed by Dunn [18] can help to avoid problems arising from incorrect problem framing.

4.2. Model

There are two major categories of uncertainty within this location of uncertainty: (1) model structure uncertainty, and (2) model technical uncertainty.

**Model structure uncertainty** arises from a lack of sufficient understanding of the system (past, present, or future) that is the subject of the policy analysis, including the behaviour of the system and the interrelationships among its elements. Uncertainty about the structure of the system that we are trying to model implies that any one of many model formulations might be a plausible representation of the
system, or that none of the proposed system models is an adequate representation of the real system. We may be uncertain about the current behaviour of a system, the future evolution of the system, or both. Model structure uncertainty involves uncertainty associated with the relationships between inputs and variables, among variables, and between variables and output, and pertains to the system boundary, functional forms, definitions of variables and parameters, equations, assumptions and mathematical algorithms.

Model technical uncertainty is the uncertainty generated by software or hardware errors, i.e. hidden flaws in the technical equipment. Software errors arise from bugs in software, design errors in algorithms and typing errors in model source code. Hardware errors arise from bugs, such as the bug in the early version of the Pentium processor, which gave rise to numerical error in a broad range of floating-point calculations performed on the processor [5].

4.3. Input

Input is associated primarily with data that describe the reference (base case) system and the external driving forces that have an influence on the system and its performance. The "input" location, therefore, includes two sub-categories:

1. Uncertainty about the external driving forces that produce changes within the system (the relevant scenario variables and policy variables) and the magnitude of the forces (the values of the scenario and policy variables). The external forces driving system change (FDSCs) that are not under the control of the policymakers are of particular importance to policy analyses, especially if they affect the outcomes of interest. Not only is there great uncertainty in the FDSCs and their magnitudes, there is also great uncertainty in the system response to these forces. This is one of the factors that may lead to significant model structure uncertainty (see above).

2. Uncertainty about the system data that ‘drive’ the model and typically quantify relevant features of the reference system and its behaviour (e.g. land-use maps, data on infrastructure (roads, houses)). Uncertainty about system data is generated by a lack of knowledge of the properties (including both the deterministic and the stochastic properties) of the underlying system and deficiencies in the description of the variability that can be an inherent feature of some of the phenomena under observation. These uncertainties are discussed in the ‘nature’ dimension below.
4.4. Parameters

Parameters are constants in the model, supposedly invariant within the chosen context and scenario. There are the following types of parameters:

- **Exact parameters**, which are universal constants, such as the mathematical constants π and e.
- **Fixed parameters**, which are parameters that are so well determined by previous investigations that they can be considered exact, such as the acceleration of gravity (g) at a particular location in earth.
- **A priori chosen parameters**, which are parameters that may be difficult to identify by calibration and are chosen to be fixed to a certain value that is considered invariant. However, the values of such parameters are associated with uncertainty that must be estimated on the basis of a priori experience.
- **Calibrated parameters**, which are parameters that are essentially unknown from previous investigations or that cannot be transferred from previous investigations due to lack of similarity of circumstances. They must be determined by calibration, which is performed by comparison of model outcomes for historical data series regarding both input and outcome. The parameters are generally chosen to minimise the difference between model outcomes and measured data on the same outcomes.

There is a relationship between model structure uncertainty and calibrated parameter uncertainty. A simple model with few parameters that does not simulate reality well may be calibrated with data obtained for both input and output under well-known conditions. In this case, model structure uncertainty will most likely dominate the result. In the case of a more complicated model with many parameters, the parameters may be manipulated to fit the calibration data beautifully, but the result may be dominated by parameter uncertainty. This would happen if the calibration data did not contain sufficient information to allow for the calibration of some parameters with an adequate degree of certainty. This could be revealed by attempting to validate the model using a different set of data. There is in principle an optimum combination of model complexity and number of parameters as a function of the data available for calibration and the information contained in the data set used for calibration. Increased model complexity with an increased number of parameters to be calibrated may in fact increase the uncertainty of the model outcomes for a given set of calibration data. This has been described in detail (see [19]). The calibration data must contain variations of information fit to deal with all parameters chosen for calibration. Otherwise the parameter estimates become very uncertain and the model outcomes become uncertain accordingly. Finally, even when the parameters are well calibrated, a residual uncertainty will often remain, and is usually treated as a parameter in itself.

4.5. Model Outcome Uncertainty

This is the accumulated uncertainty caused by the uncertainties in all of the above locations (context, model, inputs, and parameters) that are propagated through the model and are reflected in the resulting estimates of the outcomes of interest. It is sometimes called prediction error, since it is the discrepancy between the true value of an outcome and the model’s predicted value. If the true values are known (which is rare, even for scientific models), a formal validation exercise can be carried out to compare the true and predicted values in order to establish the prediction error. However, practically all policy analysis models are used to extrapolate beyond known situations to estimate outcomes for situations that do not yet exist. For example, the model may be used to explore how a policy would perform in the future or in several different futures. In this case, in order for the model to be useful in practice, it is necessary to (1) build the credibility of the model with its users and with consumers of its results (see, for example, [20]), and (2) describe the uncertainty in the model outcomes using the typology of uncertainties presented in this paper.

5. LEVELS OF UNCERTAINTY: A PROGRESSION FROM “KNOW” TO “NO-KNOW”

Contrary to the common perception, an entire spectrum of different levels of knowledge exists, ranging from the unachievable ideal of complete deterministic understanding at one end of the scale to total ignorance at the other. In many cases, decisions must be taken when there is not only a lack of certainty about the future situation or about the outcomes from policy changes, but also when some of the possible changes themselves remain unknown. Here, decisionmaking is faced with the continual prospect of surprise. It is in this grey area between the well known and what is not known that the degree of uncertainty and ignorance ought to affect the approach to decisionmaking. The ultimate goal of decisionmaking in the face of uncertainty should be to reduce the undesired impacts from surprises, rather than hoping or expecting to eliminate them [21]. Many different approaches are used in practice to cope with uncertainty. It is useful to try to match the approach to the level of uncertainty. For example, Schlesinger [22] distinguishes between Captain Cook’s tour planning for circumnavigating the globe and Lewis and Clark’s tour planning for exploring the previously unexplored western United States. In Cook’s case, the future was sufficiently certain that one could chart a straight course years in advance. By contrast, Lewis and Clark’s planning “acknowledges that many alternative course of action and forks in the road will appear, but their precise character and timing cannot be anticipated.” Thus, very uncertain situations call for robust plans (which will succeed in a variety of situations) [23] or adaptive plans...
Fig. 5. The progressive transition between determinism and total ignorance.

(\textit{which can be easily modified to fit the situations encountered}) \cite{24}. For example, in the case of applying the precautionary principle, the level of uncertainty and ignorance should be accounted for by deciding on an appropriate level of proof as the basis for decisions to act or not act, if there is potential for large-scale and/or irreversible harm from an activity or a chemical \cite{1}.

To distinguish between the various levels of uncertainty, we employ the following terminology: determinism, statistical uncertainty, scenario uncertainty, recognised ignorance and total ignorance. This is illustrated in Figure 5.

Determinism is the ideal situation in which we know everything precisely. It is not attainable, but acts as a limiting characteristic at one end of the spectrum.

\textbf{Statistical uncertainty} is any uncertainty that can be described adequately in statistical terms. Statistical uncertainty can apply to any location in the model, even to model structure uncertainties, as long as the deviation from the true value can be characterised statistically.

Statistical uncertainty is what is usually referred to as “uncertainty” in the natural sciences. An exclusive focus on statistical uncertainty, however, implicitly assumes that the functional relationships in the given model are reasonably good descriptions of the phenomena being simulated, and the data used to calibrate the model are representative of circumstances to which the model will be applied. If this is not the case, deeper forms of uncertainty supersede statistical uncertainty, and statistical uncertainty should not be accorded as much attention as other levels of uncertainty in the uncertainty analysis.

The most obvious example of statistical uncertainty is the \textit{measurement uncertainty} associated with all data. Measurement uncertainty stems from the fact that measurements can practically never precisely represent the “true” value of that which is being measured. Measurement uncertainty in data can be due to \textit{sampling error, or inaccuracy or imprecision} in the measurements.

\textit{Sampling error} is the error associated with the degree to which the sample is representative. The location, the time and the circumstances at which the sample has been taken may not be completely representative of those of the “true” value. \textit{Inaccuracy} is the deviation from the “true” value; i.e., it refers to how close a measured value is to the value considered “true”. \textit{Imprecision} reflects variation of measurements around a mean value, which may or may not be the “true” value because of sampling error or inaccuracy. This is in fact a measure of the reproducibility of the result. These terms belong to a well-established vocabulary that can be found in most textbooks on physical and chemical experimentation. A good primer on measurement uncertainty is \cite{25}.

“Statistical uncertainty” may also relate to uncertainty in measuring the probabilities in a stochastic model (see section on variability below).

5.1. \textbf{Scenario Uncertainty}

The use of scenarios is one approach used in policy analysis to deal with uncertainty related to the external environment of a system (usually its future environment) and its effects on the system (see, for example, \cite{26, 27}). A scenario is a plausible description of how the system and/or its driving forces may develop in the future. To be plausible, it should be based on a coherent and internally consistent set of assumptions about key relationships and driving forces (e.g., technology changes, prices). Scenarios do not forecast what will happen in the future; rather they indicate what might happen (i.e., they are plausible futures). Because the use of scenarios implies making assumptions that in most cases are not verifiable, the use of scenarios is associated with uncertainty at a level beyond statistical uncertainty.

Contrary to statistical uncertainty, where the functional relationships are well described and a statistical expression of the uncertainty present can be formulated, scenario uncertainty implies that there is a range of possible outcomes, but the mechanisms leading to these outcomes are not well understood and it is, therefore, not possible to formulate the probability of any one particular outcome occurring. There is a demarcation in the transition from statistical uncertainty to scenario uncertainty at the point where a change occurs from a consistent continuum of outcomes expressed stochastically to a range of discrete possibilities, where choices must be made with respect to the options to analyze without allocation of likelihood.

Scenario uncertainty can manifest itself in various ways – for example, (a) as a range in the outcomes of an analysis due to different underlying assumptions, (b) as uncertainty about which changes and developments (e.g., in driving forces or in system characteristics) are relevant for the outcomes of interest, or (c) as uncertainty about the levels of these relevant changes.

\textbf{Recognised ignorance} is fundamental uncertainty about the mechanisms and functional relationships being studied. We know neither the functional relationships nor the statistical properties and the scientific basis for developing scenarios is weak.
Uncertainty due to ignorance can further be divided into reducible ignorance and irreducible ignorance. Reducible ignorance may be resolved by conducting further research, which implies that it might be possible to somehow achieve a better understanding. Irreducible ignorance applies when neither research nor development can provide sufficient knowledge about the essential relationships. Irreducible ignorance is also called indeterminacy.

Total ignorance is the other extreme from determinism on the scale of uncertainty, which implies a deep level of uncertainty, to the extent that we do not even know that we do not know. In Figure 5, the continuing arrow at this end of the scale is used to indicate that we have no way of knowing the full extent of our ignorance.

The rationale for our categorisation of the levels of uncertainty we have presented is to establish a scale of graduation from determinism to total ignorance. We argue that this characterisation scheme provides a complete logical structure of the level of uncertainty for uncertainty analysis.

6. THE NATURE OF UNCERTAINTY: INHERENT VARIABILITY OR LACK OF KNOWLEDGE?

In the above we have focused on locations within models where uncertainty may manifest itself. We have also discussed the level of uncertainty as being an expression of the scale of the uncertainty we are faced with. We would now like to introduce the third dimension of the concept of uncertainty: the nature of uncertainty. An important feature of the nature of uncertainty is the distinction between two extremes:

- **Epistemic uncertainty**: The uncertainty due to the imperfection of our knowledge, which may be reduced by more research and empirical efforts.
- **Variability uncertainty**: The uncertainty due to inherent variability, which is especially applicable in human and natural systems and concerning social, economic, and technological developments.

Assessing the nature of uncertainty may help to understand how specific uncertainties can be addressed. In the case of epistemic uncertainty, additional research may improve the quality of our knowledge and thereby improve the quality of the output. However, in the case of variability uncertainty, additional research may not yield an improvement in the quality of the output.

Although the terminology used may differ, the above distinction in the nature of uncertainty is well recognised in the literature about uncertainty. For example, the terms epistemic or epistemological uncertainty have been used to refer to imperfection of knowledge, while the terms ontic or ontological uncertainty, derived from philosophy, or aleatory uncertainty, derived from physical science, have been used to describe uncertainty due to variability. An overview of terms used to characterise the nature of uncertainty is given in [28]. They stipulate that it is not always easy to clearly distinguish between these categories of uncertainty; it often remains a matter of convenience and judgement linked up to features of the problem under study as well as to the current state of knowledge or ignorance.

6.1. Epistemic Uncertainty

This form of uncertainty is related to many aspects of modelling and policy analysis – e.g., limited and inaccurate data, measurement error, incomplete knowledge, limited understanding, imperfect models, subjective judgement, ambiguities, etc. With their NUSAP method, Funtowicz and Ravetz [3] have introduced the concept of pedigree to systematically assess the imperfection in the knowledge base, thereby providing an indication of the degree to which uncertainty may be reducible. Pedigree conveys an evaluative account of the production process of information, and indicates different aspects of the underpinning of the numbers and scientific status of the knowledge used. Assessment of pedigree involves qualitative expert judgement. It should be noted that pedigree and degree of reducibility of uncertainty do not necessarily correspond to each other in a one-to-one fashion: increasing the pedigree by more research may either reduce or increase uncertainty. The latter can be the case if, for instance, unforeseen complexities are revealed by the research. Examples of pedigree analysis can be found in [29] and on the website www.nusap.net.

Related to the NUSAP method are methods being developed to rate the strength of scientific evidence that are grouped under the heading of “evidence-based practice” (see, for example, [30]). These methods, which are primarily used in the health care field, are designed to protect against the use of study results in individual and policy-level health care decisions that contain selection, measurement, and confounding biases.

6.2. Variability Uncertainty

Many empirical quantities (measurable properties of the real-world systems being modelled) vary over space or time in a manner that is beyond control, simply due to the nature of the phenomena involved. Variability uncertainty is defined here as the inherent uncertainty or randomness induced by variation associated with external input data, input functions, parameters, and certain model structures.

Different sources of variability uncertainty can be distinguished (see Fig. 6):²

- Inherent randomness of nature: the chaotic and unpredictable nature of natural processes – see also [16];

²See [4] or [15].
Human behaviour (behavioural variability): ‘non-rational’ behaviour, discrepancies between what people say and what they actually do (cognitive dissonance), or deviations of ‘standard’ behavioural patterns (micro-level behaviour);

Social, economic, and cultural dynamics (societal variability): the chaotic and unpredictable nature of societal processes (macro-level behaviour). The need to consider societal and institutional processes as a major contributor to uncertainty due to variability can be inferred from various papers of Funtowicz, Ravetz, and de Marchi (see, for example [31, 32]).

Technological surprise: New developments or breakthroughs in technology or unexpected consequences (‘side-effects’) of technologies.

These sources may contribute to variability uncertainty, but it may be difficult to identify precisely what is reducible through investigations and research, and what is irreducible because it is an inherent property of the phenomena of concern. However, it is important to make an assessment, because the information may be essential to the political process.

Models may use frequency distributions to represent variability uncertainty in case the property falls into the level of statistical uncertainty. From [10]: “It is possible to have a high degree of certainty about a frequency distribution. For example, it is not hard to imagine obtaining the statistics on the weights of all newborns in Washington, D.C. during 2000 and compiling a precise frequency distribution for the weights of newborn infants in Washington, D.C. during 2000. On the other hand, one may be quite uncertain about a frequency distribution, for example, the frequency distribution for newborn infants in Washington, D.C. during 2020”. Uncertainty about a frequency distribution may be represented by probability distributions about its various parameters, such as its mean, standard deviation, or median.

A common mistake is failure to distinguish between the uncertainty inherent in sampling from a known frequency distribution (variability uncertainty), and the uncertainty that arises from incomplete scientific or technical knowledge (epistemic uncertainty). For example, in throwing a fair coin, one knows that the outcome will be heads 1/2 the time, but one cannot predict what specific value the next throw will have (variability uncertainty). In case that the coin is not fair, there will also be epistemic uncertainty, concerning the frequency of the heads.

Similarly, input functions can exhibit variability that can be described as a mathematical relationship with an associated uncertainty. Such functions may be considered part of the model structure or separate as an external input function. An example is seasonal variation, which can be described functionally [15] or the variation in time and space of extreme rainfall, giving rise to flooding [33]. The location of this form of variability is in either the model structure or in input data. Input data can exhibit variability with an associated uncertainty. As with all locations of uncertainty, the uncertainty associated with variability of input data or model structure can fall into all four levels: Statistical uncertainty, scenario uncertainty, recognised ignorance, or total ignorance. If the model is used for extrapolation (e.g. projection into the future), the uncertainty associated with variability is also due to the application of the model to circumstances different from those associated with the experience upon which the model and data were developed.

7. THE UNCERTAINTY MATRIX

The purpose of an uncertainty matrix is to provide a tool by which to get a systematic and graphical overview of the essential features of uncertainty in relation to the use of models in decision support activities. The idea is to identify the location, level, and nature of the uncertainty associated with models, so that model developers and users will become aware of and address all of the important elements of uncertainty. The location, level, and nature of uncertainty can be combined to obtain an uncertainty matrix, as shown in Figure 7.

The vertical axis identifies the location of uncertainty – i.e., where the uncertainty is located in the framework shown in Figure 2. The first three columns of the horizontal axis cover the level of uncertainty in relation to all locations; the next two columns indicate the nature of uncertainty for each location. In both cases the columns can be interpreted as ‘brackets’ of characterisation:

- Level: statistical uncertainty, scenario uncertainty, and recognised ignorance.
- Nature: Epistemic and variability uncertainty.

The first three columns may also be interpreted as a continuum of uncertainty (based on the progressive transition from determinism to total ignorance depicted in Fig. 5).

Applying the matrix is a means to make a complete inventory of where the uncertainties are located and how they can be typified in terms of uncertainty level and nature. In filling in the matrix, one should be aware that the level and nature of the uncertainty that occurs at any location can manifest itself in various forms simultaneously. For example, in a specific model input or driving force, part of the uncertainty can be due to statistical uncertainty, while another part can only be described by scenario uncertainty or
recognized ignorance. Similar divisions and overlaps hold with respect to the ‘nature’ dimension – part of the uncertainty can be of an epistemic character, and part due to variability. Attribution of the parts will not always be clear or unambiguous. In filling in the matrix, one can keep track of the various forms in which uncertainty in a certain box manifests itself by using indexes (e.g., index 1 might refer to one specific model input, index 2 to a different input, etc.).

Note that it is not necessarily true that the uncertainties located in a particular part of the matrix are more important than uncertainties in other parts of the matrix. Ignorance may be irrelevant in case it pertains to minor components, while in other cases ignorance may supersede statistical uncertainty. Further analysis is therefore necessary to assess the size of the various uncertainties and their influence on the outcomes of interest. Either a quantitative [10, 34] or a qualitative uncertainty or sensitivity analysis [5, 15] can be used to identify the uncertainty in the outcomes of interest induced by uncertainties in its inputs, as well as which uncertainties have the greatest effects on the outcomes of interest. The insights derived from the use of such techniques can help determine how best to allocate project resources to reduce uncertainty in the estimates of the outcomes of interest – e.g., would it be more worthwhile to focus on the structure of the model or to gather more information to estimate the model’s parameters?

The matrix looks conveniently small and handy as it is depicted in Figure 7, but in reality the level and nature of uncertainty have to be estimated for each location in the model structure. That can be a considerable endeavour in practise, if uncertainty has to be identified and estimated in detail. However, the effort invested to achieve this insight should vary according to the purpose of each particular exercise. The amount of effort to invest should therefore be chosen with care in order to provide an adequate combination of overview and detail. As well, in filling in the matrix, one should be aware of differences in the levels of quality and underpinnings of the information about the various uncertainties, as well as the presence of values and biases in the choices involved (e.g., concerning the way the scientific questions are framed, data are selected, interpreted, and rejected, methodologies and models are devised and used, and explanations and conclusions are formulated [29, 35]). These aspects will have important influences on the resulting uncertainties.

It should be noted that such a matrix may characterise the uncertainty associated with a particular issue only at a particular point in time. The matrix will change with more information and with the development of new circumstances.

The purpose of the matrix is to inspire model developers and users of models to make an explicit effort to identify, estimate, assess and prioritise all important contributions to uncertainty associated with the outcomes of interest in a systematic manner.

The uncertainty matrix can be applied at different stages in the decision support endeavour:

- as a heuristic during the preparatory pre-analysis phase (i.e., problem framing, determining system boundaries, and model-building);
- as a checklist during the analysis phase (i.e., model use, assessment of the results, reporting and communication);
- as a quality control checklist, used in peer review or for self-evaluation.

The uncertainty matrix has to be re-applied during the peer review because those performing the policy analysis may have overlooked some relevant uncertainties [15, 29]. In this way it can be tested whether personal and institutional lack of knowledge or overconfidence are associated with the uncertainty treatment. For example, a team of analysts may not be aware of the incompleteness of their model structure, which may be surfaced in a peer review or a self-evaluation. Finally, the matrix can be included in the reporting process, in order to make the results of the assessment more transparent to stakeholders and decisionmakers.
8. CONCLUSION

It is increasingly a requirement in model-based decision support that uncertainty has to be communicated in the science-engineering/policy-management interface. During the past decade several significant contributions to concepts, terminology, and typology have been proposed. However, there is no generally accepted approach to communication about uncertainty. The result is confusion and frequent lack of mutual understanding. This paper has attempted to condense and harmonise the terminology and typology as well as propose a tool – the uncertainty matrix – for identifying and characterising the potential uncertainty in model-based decision support. It suggests that uncertainty is a three dimensional concept defined by: the location in the analysis, the level of uncertainty, and the nature of the uncertainty. The uncertainty matrix can be combined with other tools – for example, sensitivity analysis and pedigree analysis – so that the most important locations of uncertainty can be identified and their influence on the results of the use of models in decision support can be identified, estimated, and assessed qualitatively or quantitatively. The intention is that such an approach be applied on a routine basis when communicating the results of decision support exercises to decisionmakers. We argue that harmonised terminology and a systematic use of the uncertainty matrix to identify, prioritise, and communicate uncertainty can substantially improve the quality of model-based decision support. Our first step has been to agree on dimensions of uncertainty and how to refer to them. This was a theoretical undertaking, which was, nonetheless, fed by our experiences with modelling and model-based decision-support. A next step would be to apply and test the uncertainty matrix in examples and case studies. The various authors intend to do that in their respective research, but a sensible application of the matrix was beyond the scope of our current effort. Note that we have focused ourselves on the modeller’s perspective of uncertainty in model-based decision-support, being aware that there is a decisionmaker’s perspective at the other end of decision-support. With our typology, we think modellers are well as propose a tool – the uncertainty matrix – for identifying and characterising the potential uncertainty in model-based decision-support, being aware that there is a decisionmaker

REFERENCES


