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ESTIMATION OF THE AIRCRAFT STATE
IN NON-STEADY FLIGHT

by

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SUMMARY

Kalman filtering and smoothing and maximum likelihood estimation techniques have been applied to the problem of estimating the aircraft state in non-steady flight from onboard noisy inertial and barometric measurements. Applied to actual flight test data the estimation schemes yielded similar results.

1. INTRODUCTION

In many flight test problems airspeed and angle of attack must accurately be known. Airspeed can be determined from barometric measurements. This, however, introduces measurement errors due to the limited accuracy of the barometric transducers.

The angle of attack is usually measured by means of a vane. The vane is positioned on a boom some distance ahead of the wing. This reduces the magnitude of local aircraft induced air velocities. Nevertheless in many cases the difference between the local direction of the air flow and the direction of the undisturbed flow is not negligible. Thus the necessity arises for the calibration of the vane in a series of steady straight flights. In steady straight flight conditions the aircraft pitch angle can be measured by means of a pendulum while the flightpath angle follows from the change of altitude during a given time interval. The difference between the pitch angle and the flightpath angle is considered to represent the "real" value of the angle of attack and thus a calibration of the vane is possible.

The situation becomes more complex in case the local direction of the airflow is significantly influenced by engine power or engine thrust. This may be experienced particularly when flight testing STOL or V/STOL aircraft.

Airspeed and angle of attack are of prime importance when measuring aircraft performance in quasi-steady and non-steady flight conditions. In those flight conditions several corrections have to be applied additionally to the measured value of the angle of attack, Ref. 1. Besides it remains at least questionable whether the calibrations in steady straight flight conditions apply equally well to quasi-steady and non-steady flight conditions.

The problems mentioned above can be circumvented when in addition to the barometric variables several inertial variables are measured in flight. Airspeed and angle of attack may then be derived from the components of the aircraft state vector which can be extracted from the measurements by applying so called state trajectory estimation techniques as described in Ref. 4, 5 and 6.

The possibility of circumventing the task of directly measuring the angle of attack by means of a vane has first been mentioned in Ref. 2. In Ref. 3 the trajectory of the state vector has been calculated by applying regression analysis. In Ref. 4 least squares estimation has been used to this end. The application of an identical algorithm, maximum likelihood estimation, has been reported in Ref. 5. In Ref. 6 the state trajectory estimation problem has been solved by applying the Kalman filtering and smoothing algorithms.

In this paper the estimation techniques described in Ref. 5 and Ref. 6, i.e. maximum likelihood estimation and Kalman filtering and smoothing have been applied to measurements in one non-steady flight test manoeuvre and the results have been compared.

The shape of the nominally symmetric flight test manoeuvre is the subject of Ref. 10. Starting from an approximately steady straight and horizontal flight condition the aircraft, Fig. 1, is accelerated quasi-steady through the speed range of interest.

Once in every period of 30 seconds a dynamic manoeuvre is executed after which the acceleration proceeds in quasi-steady flight. Applying the techniques described in Ref. 7 both performance and handling characteristics are to be derived from the measurements during one manoeuvre.

The paper is organized as follows. In Chapter 3 the mathematical model is presented of the motion of an aircraft in an atmosphere which has been assumed to move uniformly with respect to earth. Some aspects of the instrumentation system are described in Chapter 4. Chapter 5 provides a brief discussion of the estimation schemes applied to the state trajectory estimation problem. In Chapter 6 the experimental results are discussed. Final conclusions are drawn in Chapter 7.
Reference frames

The origin of $F_Y$, $F_T$ and of $F_B$ is in the center of gravity of the aircraft. $F_Y$, $F_T$ and $F_B$ are rectangular and righthanded, see Fig. 2.

$F_Y$ The axis $X_Y$ and $Z_Y$ are in the plane through the aircraft's center of gravity and the earth's axis of rotation. The positive direction of $Z_Y$ is to the earth's center. The positive direction of the $X_Y$ axis is to the North.

$F_T$ $Z_T$ coincides with the $Z_Y$ axis. $X_T$ is in the plane through $X_Y$ and $Z_T$.

$F_B$ Body fixed reference frame. $X_B$ and $Z_B$ are in the plane of symmetry of the aircraft. The positive direction of $Y_B$ is to starboard. $X_B$ is parallel to the mean aerodynamic chord.

3. THE SYSTEM AND OBSERVATION MODEL

The kinematical relations of the aircraft motion relative to a spherical and rotating earth are well known, Ref. 11. The following set of simplified relations have been used in the present formulation of the state trajectory estimation problem.

$$V_{ST}^a = a_{ST} - \frac{\omega}{R} \cos \phi \sin \lambda \cos \lambda$$

$$\dot{V}_{ST} = a_{ST} - (2 \omega \sin \phi \cos \lambda + \frac{1}{R} (V_{XT}^a + u_{XT})) (V_{XT}^a + u_{XT}) - \frac{\omega}{R} \cos \lambda$$

$$\dot{\lambda} = \dot{V}_{ST}$$

$$\dot{\phi} = q_B \cos \phi - r_B \sin \phi + w \sin \phi \cos \lambda + \frac{1}{R} (V_{XT}^a + u_{XT})$$

The kinematical accelerations $a_{ST}$ and $a_{XT}$ can be derived from:

$$a_{ST} = A_{XB} \cos \theta + A_{YB} \sin \theta \sin \phi + A_{ZB} \sin \theta \cos \phi$$

$$a_{XT} = -A_{XB} \sin \theta + A_{YB} \cos \theta \sin \phi + A_{ZB} \cos \theta \cos \phi$$

The set of first order differential equations represent a system:

$$\dot{x} = f(x, u, \beta)$$

(3-1)

in which the state vector $x$ is defined by:

$$x = \text{col} \{V_{ST}^a, V_{XT}^a, \Delta x_T, \Delta y_T, \theta\}$$

and the input vector $u$ by:

$$u = \text{col} \{A_{XB}, A_{YB}, A_{ZB}, q_B, r_B, \phi\}$$

$\beta$ denotes a vector of unknown parameters which is defined in Chapter 4.

In the derivation of (3-1) from the general expressions in Ref. 11 the following assumptions were made.

a) In the course of a non-steady manoeuvre as described in the Introduction the variations of $R$, $\lambda$ and $\mu$ are small enough as to permit the substitution of average values $R$, $\lambda$ and $\mu$.

b) $\omega_T$, the velocity of the atmosphere relative to the earth can be assumed to be constant along the trajectory of a non-steady manoeuvre.

c) As mentioned in the Introduction, the non-steady manoeuvre is nominally symmetric. This may be deduced also from the time histories of the "asymmetric components" $A_{YB}, r_B$ and $\phi$ of the input vector $u$, Fig. 3. It is therefore reasonable to assume that $V_{ST}^a$ is small compared to $V^a$ and changes of $\phi$ are small enough as to permit the substitution of an average heading angle $\psi$.

d) For reasons to be stated below, the trajectory of the manoeuvre is selected to be parallel to the atmospheric isobars, hence:

$$\omega_T = 0$$

The observation model of (3-1) can be written as:

$$y = h(x)$$

(3-2)

In (3-2) $y$ denotes an observation vector defined by

$$y = \text{col} \{V^a, \Delta y_T\}$$
These zero shifts have therefore to be modelled as the components of an unknown parameter vector \( \beta \) defined as:

\[
\beta = \text{col} \{ \Delta x_\text{p}, \Delta q_\text{r} \}
\]

in order to avoid unallowable model errors.

5. SOLUTION OF THE STATE TRAJECTORY ESTIMATION PROBLEM

The state trajectory estimation problem can be formulated as a statistical estimation problem. The nonlinear dynamic system (3-1) is observed according to the observation model (3-2). At discrete instants of time \( t_i, i = 0, \ldots, N \), measurements are made of the components of the input vector \( u \) and the observation vector \( y \). These measurements are corrupted by measurement errors which are assumed to be additive.

\[
\begin{align*}
\bar{u}_m(i) &= u(i) + w(i) \\
\bar{y}_m(i) &= y(i) + v(i)
\end{align*}
\]

The assumption is made that \( w(i) \) and \( v(i), i = 0, \ldots, N \), can adequately be represented by zero mean gaussian random sequences according to:

\[
\begin{align*}
E \{w(i)\} &= 0 \\
E \{w(i) \, w^T(i)\} &= \delta_{ij} Q \\
E \{v(i)\} &= 0 \\
E \{v(i) \, v^T(i)\} &= \delta_{ij} R
\end{align*}
\]

The components of \( x \) at times \( t_i \) and additionally the components of the parameter vector \( \beta \) have to be estimated in some optimal way from the noisy measurements of \( u(i) \) and \( y(i) \).

Two different algorithms have been applied:

a) The extended Kalman filter followed by a fixed interval smoothing algorithm, corresponding to Table 9.4-3 and 9.5-3 of Ref. 15.
b) The maximum likelihood estimation scheme as described in Ref. 16.

Based on the review presented in Ref. 14 both algorithms are briefly discussed in Section 5.1 and 5.2.

5.1 EXTENDED KALMAN FILTERING AND FIXED INTERVAL SMOOTHING

The parameter vector \( \beta \) is assumed to be constant in the course of one flight test manoeuvre,

\[
\dot{\beta} = 0
\]

(3-1) and (3-1) can then be written as:

\[
\dot{x}^* = \Phi(x^*, u)
\]

in which \( x^* \) denotes an augmented state vector:

\[
x^* = \text{col} \{ x^T, \beta^T \}
\]

The time interval between two successive measurements, \( \Delta t \) is assumed to be small enough as to permit the discretization of (5-2) which may then be written as:

\[
x^*(i+1) = \Phi(x^*(i), u(i))
\]

(5-3)

(5-3) and (5-1) can then be written as:

\[
\begin{align*}
\dot{x}^* &= \Phi(x^*, u) \\
\dot{x}^* &= \Phi(x^*, u) \\
\dot{x}^* &= \Phi(x^*, u)
\end{align*}
\]

(5-4)

Apart from modelling the input vector measurement noise, the plant noise is considered to account also for differences between the model and the actual process.

When \( x^*(0) \) is assumed to be a gaussian random variable with variance matrix \( P^*(0|0) \) it is shown in Ref. 14 that following from the assumptions stated above the a posteriori probability density function is gaussian and can be written as:

\[
p(x^*(0), x^*(1), \ldots, x^*(N)|y_m(1), y_m(2), \ldots, y_m(N)) =
\]

\[
C.\exp\{-1/2|\sum_{i=1}^{N} [y_m(i) - h(x^*(i))]^T \Sigma^{-1}(i|0) [y_m(i) - h(x^*(i))] + \sum_{i=0}^{N} w^T(i)Q^{-1}w(i))\}
\]

in which \( C \) is a normalizing constant.
\[ v_{\text{in}}(0) = v_{m}(0) \]
\[ v_{\text{in}}(0) = (\Delta x_{m}(1) - \Delta x_{m}(0))/\Delta t_{c} \]
\[ \Delta x_{m}(0) = \Delta x_{m}(0) \]

in which \( \Delta t_{c} \) refers to the initial period of steady straight flight.

Nominal steady straight flight conditions imply the kinematical accelerations \( \Delta x_{m} \) and \( \Delta y_{m} \) to be approximately zero.

A reasonable estimate of \( \theta(0) \) may then be deduced from the relevant expressions in Chapter 3:
\[ \theta(0) = \arctan \left( \frac{\Delta x_{m}(0)}{\Delta y_{m}(0)} \right) \]

The zerothshifts \( \Delta x_{m} \) and \( \Delta y_{m} \) cannot be directly measured in flight prior to the execution of the flight test manoeuvre. Therefore, because no a priori information is available the initial estimates of \( \Delta x_{m0} \) and \( \Delta y_{m0} \) are put equal to zero:
\[ \Delta x_{m0}(0) = 0 \]
\[ \Delta y_{m0}(0) = 0 \]

The variance matrices \( P_{m}(0|0) \), \( Q \) and \( R \) have been assumed to be diagonal. Reasonable estimates of the diagonal elements of these matrices may be obtained by consulting the general description of the instrumentation system and the results of several laboratory calibrations in Ref. 8 and 9.

The resulting numerical values have been listed in Table 1.

When applying the maximum likelihood algorithm an initial augmented parameter vector \( \theta(0) \) and variance matrix \( R \) have to be specified.

From the discussion above and the definition of \( \theta(0) \) in Chapter 5 it follows \( \theta(0) \) can be put equal to \( x(0|0) \).

The elements of the variance matrix \( R \) indicate the accuracy of the barometric measurements of airspeed \( v_{\text{in}} \) and change of altitude \( \Delta z_{m} \) during quasi-steady flight conditions.

In the non-steady parts of the flight test manoeuvre these accuracies are reduced considerably because of

a) the dynamic response of the air pressure tubes,

b) the parasite sensitivity of the barometric transducers to accelerations.

The additional errors introduced into the barometric measurements in non-steady flight conditions depend on the non-steady motion of the aircraft and can therefore not be represented by independent random processes.

For these reasons it was decided to neglect the barometric measurements during the non-steady parts of the flight test manoeuvre.

Application of the extended Kalman filter, the fixed interval smoothing algorithm and the maximum likelihood estimation algorithm yield corresponding results i.e. estimates of the components of the state vector during the manoeuvre and in addition estimates of the unknown parameters.

The time history of the state vector \( x \) resulting from the application of the maximum likelihood estimation algorithm is presented in Fig. 4. Starting from the initial parameter vector \( \theta_{0} \), convergence was achieved within 10 iterations. The related residuals are shown in Fig. 5. Fig. 5 clearly illustrates the non-randomness of the errors introduced into the barometric measurements during the non-steady parts of the flight test manoeuvre. The extended Kalman filter yields a similar result, Fig. 6.

The results of the extended Kalman filter, the fixed interval smoothing algorithm and the maximum likelihood estimation algorithm have been compared in Fig. 7 and 8. With respect to Fig. 7 it should be remarked that because the zerothshifts have been assumed to be constant in the course of one flight test manoeuvre the final estimates of \( \Delta x_{m0} \) and \( \Delta y_{m0} \) resulting from the extended Kalman filter are not altered by subsequent application of the smoothing algorithm.

From the state vector airspeed \( v_{\text{in}} \), flightpath angle \( \gamma \) and angle of attack \( a \) can be derived by applying (3–3), (3–4) and (3–5). In Fig. 9 the resulting time histories are compared.

From Fig. 8 and Fig 9 it follows that subsequent application of the smoothing algorithm reduces considerably the differences between the results of the extended Kalman filter and the maximum likelihood estimation algorithm.

7. CONCLUSIONS

Two different estimation schemes i.e. the extended Kalman filter and fixed interval smoothing and the maximum likelihood estimation algorithm have been applied to the problem of estimating the aircraft state trajectory during a non-steady flight test manoeuvre.

When applying the extended Kalman filter and fixed interval smoothing algorithm the state trajectory estimation problem is linearized about a nominal trajectory. The result therefore constitutes an approximate solution to the problem.
\[
\begin{align*}
\text{diag } P^*(0|0) &= \{(1)^2, (2)^2, (1)^2, (1.10^{-1})^2, (2.10^{-2})^2, (1.10^{-4})^2\} \\
\text{diag } Q &= \{(4.10^{-3})^2, (2.10^{-2})^2, (6.10^{-3})^2, (5.10^{-5})^2, (1.10^{-4})^2, (5.10^{-6})^2\} \\
\text{diag } R &= \{(5.10^{-1})^2, (1)^2\}
\end{align*}
\]

Table 1. Numerical values of the elements of the variance matrices of the estimate initial augmented state, the plant noise and the measurement noise.

Fig. 1. The Hawker Hunter mk VII laboratory aircraft of the National Aerospace Laboratory (NLR). Note static pressure trailing cone.

Fig. 2. Definition of the vehicle carried vertical reference frame $F_T$ and body fixed reference frame $F_B$. 
$q_B$ (rad sec$^4$)

$A_{y_B}$ (msec$^2$)

$t$ (sec)

Fig. 3 continued.
Fig. 4. Maximum likelihood estimate of the state vector $\mathbf{x}$ during the non-steady manoeuvre.
Fig. 5. Residuals resulting from application of the maximum likelihood estimation algorithm.
Fig. 7. Difference between extended Kalman filter and maximum likelihood estimates (f) and fixed interval smoothing and maximum likelihood estimates (s) of the state vector $x$. 
Fig. 8. Extended Kalman filter estimates of the zero-shifts compared to the corresponding results from the maximum likelihood estimation algorithm.
Fig. 9 continued.