Longitudinal Driving Behavior: Theory and Empirics

Saskia Ossen
The research presented in this dissertation thesis is part of the research program “Tracing Congestion Dynamics – with Innovative Traffic Data to a better Theory, sponsored by the Netherlands Organization for Scientific Research MaGW-NWO.”
Longitudinal Driving Behavior: Theory and Empirics

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Saskia Josephina Leontine OSSEN

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TRAIL
P.O.Box 5017
2600 GA Delft
The Netherlands
Phone: +31 (0) 15 278 6046
Fax: +31 (0) 15 278 4333
E-mail: info@rsTRAIL.nl

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Preface

In May 1955 for the first time congestion occurred in The Netherlands (see cover illustration). People were excited about this interesting phenomenon and climbed a bridge to get a better overview of what was going on on the roadway. Nowadays congestion has become a serious problem significantly influencing the daily pattern of a lot of drivers. Consequently a lot of effort is put in taking measures to reduce congestion. Whether such measures lead to the desired effect appears to be largely dependent on the driving behavior of individual drivers. And, although the public opinion about congestion changed considerably, the best method for obtaining the required insights into driving behavior is still the one used by the excited people in 1955, i.e. by observing the traffic from a position above the roadway.

This thought motivated the “Tracing Congestion Dynamics: With Innovative Microscopic Data to a Better Theory” research program in which observations made by a helicopter are used to gain new insights into driving behavior. In the first subpart of the project a dedicated data collection method is developed to make it possible to derive trajectories from images collected by a digital camera attached to a helicopter. This part of the project is carried out at the “Optical & Laser Remote Sensing” department of TU Delft. In the second part of the project the resulting trajectory observations are used to perform detailed analyses on the behavior of individual drivers during congestion. This part of the project, to which this thesis belongs, is performed at the Transport & Planning department of TU Delft.

At the end of my Ph.D. research I want to thank the organizations that made my work possible. I especially want to acknowledge the Netherlands Organization for Scientific Research MaGW-NWO for sponsoring the “Tracing Congestion Dynamics” project. I also want to acknowledge DVS for allowing us to use several of their trajectory datasets.

I also want to express my gratitude to all those people who contributed to this thesis in many ways. I first of all want to thank my promoter Serge Hoogendoorn who gave me the opportunity to perform my Ph.D. research within the “Tracing Congestion Dynamics” project. He provided me with excellent opportunities for making a very instructive period of the past
four years. In his everlasting enthusiasm he came up with several interesting ideas of which this thesis certainly benefited.

Special thanks go to Kees Landman and Peter van der Vlist who actually collected the trajectory observations used within this thesis. This often required brave behavior as they had to spend hours in a shaking helicopter having only minimum leg space and several computers and computer screens surrounding them. By repairing my bike every now and then, they also made it possible for me to arrive everyday in time at my office.

In the months that I was writing this thesis a very important role was played by the members of my “reading committee”, Piet Bovy and Henk van Zuylen. I would like to thank them for all the effort they have put in reading all chapters and providing me with very valuable comments. Thanks to these comments I have been able to improve my thesis significantly. In addition I would like to thank the members of my promotion committee for reading the draft manuscript and providing me with very instructive comments. I am also grateful to Fatemeh Karimi for reviewing my appendix on the data collection method.

As working hard for a prolonged time is only possible in a nice working environment, I furthermore would like to express my gratitude to my colleagues at the Transport & Planning Department. Special thanks go to Victor, Winnie, Geertje and Huizhao. Geertje, I want to thank you for helping me to unravel the many secrets of “MS WORD”. Huizhao I really appreciated that you were always just ahead of me in writing your thesis, I really learned a lot from your experiences. Winnie, thanks for being a very nice roommate during our trips to TRB. Also thanks for being always available for answering my questions and most important for having just a talk. Victor, thanks for our many useful and useless discussions. Our “stroopwafelpauzes” and walks around the building always gave me just the positive energy I needed for spending again hours behind my computer writing this thesis. You certainly are my “hero at the scooter”.

I would also like to express my gratitude to Arne Kesting and his colleagues from Dresden University of Technology. I sincerely appreciated the warm welcome they gave to me when I came to Dresden.

And last but certainly not least I want to use the language of the region I was born in to say thanks to my boyfriend and my family for always supporting me. Pap, mam, Marjolyn en de res van mieng famillieë. Ich ken hei wal enne janse hoof jroeësse woad joa schrieve uvver wat uur vuur miech betzegent, mar ejentlich zunt het jüs al die kling dinger die uuch zoe wiechtig vuur miech mache. Der is volgens miech nuuks wiechtigers dan tse wisse dat me urje e jans sjun en jemuutlieg heem hat mit lü woa tse unmer tsereët kens. Lü die e keëtsje vuur diech brene went tse examen has en die diech zage dat tse diech net jek mots losse maache. Danke doa vuur.

Camiel, in de iesjte plaatsj wil iech diech bedanke dat ste mit miech mit jekoame bis noa het hollandsj, angesj woar iech warschienlich nit ens an dit buchsje bejonne. Tsesame hant vuur de afjeloove veer joar nit alleng enne janse hoof jeliërd mar ôch enne janse hoof sjapas jehat. Der is jeweun nuuks zoë sjuin wie noa sjiech noa heem joa en diech doa vrundlig laachend op d’r balkon op miech zie winke. Doe bis mienge sjat.

To all those people who contributed in one way or the other to this thesis, also the ones not mentioned by name, Thank you!
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Notation

This section provides an overview of variables and model parameters frequently used throughout this thesis.

**List of frequently used variables**

\[ t \quad = \quad \text{time instant} \]
\[ n \quad = \quad \text{vehicle index} \]
\[ K_n \quad = \quad \text{total number of time instants at which follower } n \text{ is observed} \]
\[ a_n \quad = \quad \text{acceleration of vehicle } n \]
\[ v_n \quad = \quad \text{speed of vehicle } n \text{ (indices decrease in driving direction)} \]
\[ v_{*n} \quad = \quad \text{desired speed of vehicle } n \]
\[ \Delta v_{n-j,n} \quad = \quad v_n - v_{n-j} \]
\[ \Delta v_{n,n-j} \quad = \quad v_n - v_n \]
\[ v_{opt} \quad = \quad \text{optimal velocity (optimal velocity models)} \]
\[ x_n \quad = \quad \text{x-position of vehicle } n \]
\[ \Delta x_{n-j,n} \quad = \quad x_n - x_{n-j} \]
\[ \Delta x_{n-j,n}^* \quad = \quad \text{desired value for } x_n - x_{n-j} \text{ by vehicle } n \]

**List of frequently used model parameters**

\[ c_{1,n-j} \quad = \quad \text{sensitivity to } \Delta v_{n-j,n} \]
\[ c_2 \quad = \quad \text{sensitivity to } \Delta x_{n-1,n} - \Delta x_{n-1,n}^* \]
\[ c_{2,n-j} \quad = \quad \text{sensitivity to } \Delta x_{n-j,n} - \Delta x_{n-j,n}^* \]
\[ c_3 \quad = \quad \text{sensitivity to } v_n^* - v_n \]
\[ c_4 \quad = \quad \text{sensitivity to } \Delta v_{n-1,n} / \Delta x_{n-1,n} \]
\[ c_5 \quad = \quad \text{sensitivity to } (\Delta x_{n-1,n} - \Delta x_{n-1,n}^*)^3 \]
\( c_{n-j} \) = sensitivity to \( V^{\text{opt}} - v_n \) regarding leader \( n-j \)

\( T_r \) = reaction time

\( d \) = desired distance at standstill

\( \gamma \) = desired increase of distance for a 1 m/s speed increase

\( a^{\text{max}} \) = maximum desired acceleration of vehicle \( n \)

\( b^{\text{max}} \) = maximum desired deceleration of vehicle \( n \)

\( b^{\text{max abs}} \) = \( |\text{maximum desired deceleration of vehicle } n| \)

\( \theta \) = safety reaction time

\( T_{\text{safe}} \) = safe time headway

\( \delta \) = acceleration component

\( m \) = slope of optimal velocity function at inflection point

\( b_f \) = distance headway at inflection point of optimal velocity function

\( m_1 \) = number of leaders to which a driver responds with respect to relative speed

\( m_2 \) = number of leaders to which deviations from the desired following distance are considered

\( m_3 \) = number of leaders to which deviations from the optimal velocity are considered
1 Introduction

1.1 Introduction

Especially in dense conditions, traffic flows on freeways are characterized by highly complex interactions between individual traffic participants and between individual traffic participants and the roadway system. These interactions occur at the lane level as well as between lanes. All interactions together determine the state of the traffic flow. For example, an unexpected lane change of a driver can cause a follower on the target lane to brake, requiring a reaction of the drivers driving behind this follower. In this situation even a slight overreaction of one of these drivers can lead to a traffic breakdown.

The complexity of these interactions becomes even clearer when we consider that all individual traffic participants are different and accordingly react in their own way to disturbances in the traffic flow. In addition, the driving behavior of a single driver may change over time, for example depending on the mental state (e.g. activation or attention level) of the driver or on the prevailing traffic conditions.

Given this complexity, a thorough understanding of the behavior of individual traffic participants is a fundamental requisite for taking successful (dynamic) traffic management measures leading to a more efficient use of existing infrastructure or to predict the effects of future changes in the infrastructure. For example, to predict whether a specific measure will have a desired effect on traffic flow, often microscopic simulation studies are performed. This entails that important and costly decisions rely on the adequacy of the behavioral assumptions made by microscopic simulation tools. The practical relevance of taking successful measures is emphasized when we consider the high congestion level on freeways in many (European) countries (Schallaböck et al., 1999). For example, in the Netherlands congestion leads both to a considerable loss of time of drivers (44 million vehicle lost hours) and thereby to economic losses (700 million euro) (Adviesdienst Verkehr en Vervoer, 2007) as well as to negative effects on the environment.

Profound knowledge on how traffic participants interact is furthermore indispensable in the development of Advanced Driver Assistance Systems (ADAS), like Adaptive Cruise Control.
(ACC), i.e. a system that automatically maintains a specified speed taking into account a minimal distance with respect to the leader (Van Arem et al., 1997). As drivers are more skeptical in accepting supporting systems influencing their control task than systems pertaining to, for example, the navigation task (Groeger et al., 1993), it is important that drivers feel comfortable with the “driving style” imposed by the automated system. In this context it is also valid to state that the impact of these systems on traffic flow characteristics can only be predicted when it is exactly known on which aspects they perform their tasks differently than human drivers would do.

In this thesis we present extensive empirical analyses on interactions between drivers moving on the same lane, i.e. on how individuals execute their so-called longitudinal driving task. We aim at increasing the fundamental knowledge on longitudinal driving behavior as well as at improving mathematical models describing this type of behavior. To this end detailed microscopic trajectory observations collected by means of a helicopter are analyzed providing a complete view of the dynamics of all drivers present on a given roadway stretch.

In the remainder of this introductory chapter we motivate our choice for an empirical study on longitudinal driving behavior further (section 1.2). We show that a lack of appropriate microscopic observations, so far, caused several important questions on longitudinal driving behavior to remain unanswered. In section 1.3 we discuss the context of this research. In section 1.4 we introduce our main research questions, followed by a presentation of the research approach by which the research questions are tackled in section 1.5. In section 1.6 we explain the scope of the research. We finally summarize the main contributions of our work in section 1.7.

1.2 Motivation for a trajectory based research on longitudinal driving behavior

The longitudinal driving behavior of individual drivers determines to a large extent the equilibrium as well as the dynamical characteristics of traffic flow. In illustration, equilibrium properties are mainly influenced by the distances drivers want to keep to their leaders, while the dynamics of traffic flows are largely governed by the way in which drivers react to disturbances in the dynamics of their leader(s) on the same lane. This important role of longitudinal driving behavior is among others reflected in the large variety of mathematical models describing this behavior (see for example (Brackstone and McDonald, 1999a) or chapter 3 of this thesis).

Gaining insight into the longitudinal driving behavior of a driver in real traffic conditions requires detailed observations on the dynamics of the driver himself as well as on the dynamics of cars in his direct neighborhood. First of all, observations need to be available on the dynamics of driver/vehicle combinations driving on the same lane to analyze how the driver reacts to them. Second of all, observations of driver/vehicle combinations on other lanes are required as they can also possibly influence the longitudinal driving dynamics of the driver. Figure 1-1 shows an example of consecutive images taken from above at a high frequency. These images contain information on the dynamics of all driver/vehicle combinations driving at the observed roadway stretch.

Collecting observations on the dynamics of all vehicles on a roadway stretch having a high spatial and temporal resolution is technically very demanding. Such detailed observations were therefore not available for a long time. Consequently a lot of important questions on the
longitudinal driving behavior of drivers under real traffic conditions remained unresolved till now.

For example, the extent of heterogeneity in longitudinal driving behavior can not be determined from local measurements. This holds even when these measurements are available on the microscopic level. Of course, it is possible to measure differences between time headways at a detector location. It is however unclear whether these differences are caused by heterogeneity or by differences in the dynamic situation of drivers. For instance, when passing the detector some drivers are satisfied with their current distance, while other ones are busily involved with increasing their distance to their leader after a lane change, and so forth.

Another open question in this research area is the degree of multi-anticipation of a driver, i.e. the number of leaders a driver considers in his longitudinal driving behavior. Answering this question is essentially equal to detecting the leaders whose dynamics a driver reacts to. This can only be done using detailed observations on the dynamics of the driver himself as well as the dynamics of his leaders.

![Figure 1-1 Consecutive images from a given roadway stretch. The line behind each vehicle is indicative for the momentary speed of the vehicle.](image)

### 1.3 Context of research

The research presented in this thesis is part of a larger research project called “Tracing Congestion Dynamics: With Innovative Microscopic Data to a Better Theory” sponsored by the Netherlands Organization for Scientific Research MaGW-NWO. The aim of this research project is twofold.

The first part of the project aims at developing a data collection method able to collect observations on the dynamics of all vehicles driving on a roadway stretch, i.e. the observations needed to gain insight into individual driving behavior. To reach this goal a data collection method based on remote sensing has been developed with which raw data are
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collected by a digital camera attached to a helicopter. These raw data are input to dedicated software giving as output the trajectories of all observed vehicles as illustrated in Figure 1-2 (for more details we refer to Appendix A). This subproject is carried out at the “Optical and Laser Remote Sensing” department of the faculty of Aerospace Engineering of Delft University of Technology.

Figure 1-2 Raw data consist of images taken by a digital camera attached to a helicopter. These raw data are input to dedicated software giving as output the trajectories of all observed vehicles. These trajectories serve as input for the empirical analyses performed in this thesis.

In the second part of the project detailed empirical analyses on individual driving behavior just before, during and after congestion are performed based on the microscopic trajectory observations obtained in the first subproject. Apart from increasing the fundamental knowledge on individual driving behavior, these analyses also aim at improving and introducing microscopic (mathematical) models describing this behavior. This part of the research program is carried out at the Transport & Planning Department of the Faculty of Civil Engineering and Geosciences of Delft University of Technology. The research presented in this thesis, focusing at longitudinal driving behavior during congested conditions, is part of this subproject.
1.4 Research questions

The main objective of the research presented in this thesis is to obtain better understanding of, and new insights into the longitudinal driving behavior of individual traffic participants based on detailed analyses of microscopic trajectory observations of all vehicles driving on selected roadway stretches. We thereby focus on two aspects of longitudinal driving behavior, namely heterogeneity and multi-anticipation, so far largely neglected in scientific research.

From every day experience it is clear that there exist differences in the driving behaviors of drivers, but no information is available yet on how large and how important these differences actually are. The general research question on heterogeneity that will be addressed is therefore,

- What is the extent of heterogeneity in longitudinal driving behavior in real traffic?

In answering this research question differences between the driving behaviors of individual traffic participants will not only be identified but also be quantified.

In our theoretical analysis of the longitudinal driving task (chapter 2) we show that heterogeneity can have different causes. To further increase the insight into heterogeneity we decompose the aforementioned research question into the following sub questions,

- To what extent do the longitudinal driving behaviors of drivers differ due to differences in personal characteristics, like different driving objectives?
- To what extent do the longitudinal driving behaviors of drivers differ due to different car characteristics, i.e. person cars versus trucks?

In our empirical analysis of multi-anticipative longitudinal driving behavior, i.e. longitudinal driving behavior in which drivers also consider vehicles driving in front of their direct leader, we start by finding evidence for the presence of multi-anticipation. Thus by asking the question,

- To what extent are drivers multi-anticipative, i.e. do drivers consider multiple leaders in their longitudinal driving behavior?

The motivation behind this question is that, although the existence of multi-anticipation in longitudinal driving behavior is often hypothesized (for example, (Bexelius, 1968, Lenz et al., 1999, Treiber et al., 2006a)), to our best knowledge no convincing empirical evidence has been provided yet.

Once we may have established the presence of multi-anticipation in longitudinal driving behavior from our empirical analyses, we examine this multi-anticipative behavior in more depth. Examples of research questions that will be addressed are,

- How many leaders does an observed driver react on?
- Which stimuli regarding these leaders are of influence and to what extent?
- Do differences exist between the multi-anticipative behaviors of drivers?

All research questions related to heterogeneity and multi-anticipation will be tackled by calibrating mathematical longitudinal driving models. That is, a broad range of different
models aiming at describing the longitudinal driving behavior of drivers are calibrated for all observed drivers separately, after which the research questions are answered by analyzing the calibrated parameter values and comparing the model performances.

This approach necessarily yields a derived research objective as due to the lack of microscopic trajectory observations only little methodological background information is available on the topic of calibrating car-following models using trajectory data. To fill this gap, we will address the following research questions:

- How can inferences be drawn from calibration results? That is, how can the reliability of parameter estimates be determined and how can performances of models having different complexities be compared? This question is especially relevant as microscopic calibration procedures commonly result in autocorrelated error terms (as will be discussed in chapter 4).
- What is the influence of methodological choices in the development of a calibration procedure, like the definition of the calibration objective function, on parameter estimates?
- What is the influence of practical issues in the use of real-life microscopic trajectory observations, like measurement errors, on parameter estimates?

The insights obtained in answering these methodological research questions enable us to design a calibration procedure that is less sensitive to measurement errors. Next to that we are better able to evaluate calibration results, which is an absolute requisite for reliably answering the aforementioned research questions on longitudinal driving behavior.

The previous research questions all concentrate on increasing the empirical insights into heterogeneity and multi-anticipation in longitudinal driving behavior and the thereto required calibration method. Apart from these theoretical contributions of our work, we recognize that an important application of mathematical models describing how drivers perform their longitudinal driving task is in microscopic simulation tools. As assumptions on longitudinal driving behavior have a large influence on the reliability of the predictions made by these tools, we complete this thesis by considering the following research question:

- How does consideration of the empirical findings on heterogeneity and multi-anticipation presented in this thesis influence predicted traffic flow characteristics?

In sum, in this thesis we try to obtain new empirical insights into heterogeneity and multi-anticipation in longitudinal driving behavior. As our approach is based on calibrating mathematical models, we also perform an in-depth methodological study into microscopic calibration. We finally consider the impact of our empirical findings on predictions made by microscopic simulation tools.

1.5 Research approach

To answer these research questions this thesis is organized according to the scheme presented in Figure 1-3.
Theory on longitudinal driving behavior

Chapter 2
Analysis of longitudinal driving behavior

Chapter 3
Evaluation of assumptions on human behavior made by mathematical models describing longitudinal dynamics

Methodology for calibrating longitudinal driving models

Chapter 4
Introduction and validation of a microscopic trajectory data based approach for calibrating longitudinal driving models

Empirical analysis of longitudinal driving behavior

Chapter 5
Empirics on heterogeneity in longitudinal driving behavior

Chapter 6
Empirics on multi-anticipation in longitudinal driving behavior

Impact of empirical findings on prediction

Chapter 7
Impact of empirical findings on predicted equilibrium and dynamical traffic flow characteristics

Chapter 8
Conclusions and recommendations

Figure 1-3 Schematic presentation of the thesis outline.

In the upcoming we discuss the different components of the scheme in more detail.

Theory on longitudinal driving behavior

Before we can start with performing empirical analyses on longitudinal driving behavior, we need to investigate this task in more detail. Based on existing literature, we will in chapter 2 discuss how the longitudinal driving task fits within the driving task as a whole. Consecutively we show how a driver is expected to execute this task and which typical human characteristics are thought to influence this execution. This task analysis reveals several possible causes for differences between the longitudinal driving behaviors of driver/vehicle combinations. For instance, different drivers may have different objectives, depending not only on personal and car characteristics but also on the purpose of their trip and their mental state. We will furthermore illustrate the large impact of longitudinal driving behavior on traffic flow characteristics.

Having established how the longitudinal driving task is expected to be performed by drivers, we will discuss in chapter 3 which assumptions on human behavior current mathematical models describing this task make. We thereby concentrate on the models that will be calibrated and analyzed in the empirical analyses. It will be shown that there exist considerable differences between the assumptions these models make on longitudinal driving behavior. For instance, some models assume that drivers consider only their direct leader in choosing an appropriate control action, while according to other models also stimuli regarding leaders driving further downstream influence the longitudinal behavior of a driver.

This in-depth discussion of the behavioral assumptions made by these models assists us in drawing inferences on human behavior in the empirical analyses.
Methodology for calibrating longitudinal driving models

In order to be able to calibrate the models described in chapter 3, we propose and subsequently assess a calibration method in chapter 4. In this chapter, it is especially important that insights are gained into the sensitivity of the calibration method to methodological choices, like the definition of the calibration objective function, and practical issues, like the influence of measurement errors.

Also heuristics will be proposed for determining the reliability of parameter estimates and comparing performances of models having different complexities. These heuristics are based on existing statistical tests, while taking the problem of autocorrelated error terms into account.

This knowledge is an absolute requisite for adequately performing the empirical analyses and correctly interpreting the corresponding calibration results.

Empirical analyses of longitudinal driving behavior

In chapters 5 and 6 we present our empirical analyses on longitudinal driving behavior, which consist of estimating the behavioral parameters of driver models using our trajectory observations.

In chapter 5 we show that heterogeneity regarding longitudinal driving behavior is highly present in real traffic. The driving styles of different drivers turn out to be inherently different, i.e. different drivers appear to react to different stimuli.

In chapter 6 we show that multi-anticipation is clearly present in real traffic. Also with respect to multi-anticipation differences between drivers are identified. For instance, the number of leaders a driver reacts to differs between drivers.

Impact of empirical findings on predicted traffic flow characteristics

In chapter 7 we assess the impact of the empirical findings on predicted traffic flow properties. We consider both equilibrium properties as well as dynamical properties of traffic flow. To this end, we develop a dedicated microscopic simulation tool providing the opportunity to assign different longitudinal driving models and parameter values to each individual driver. Using this tool we can incorporate heterogeneity and multi-anticipation in our simulations.

1.6 Research scope

The previous sections discussed the research questions that will be tackled throughout this thesis. The aim of this section is to discuss and motivate the research scope.

The analyses presented in this thesis focus on the longitudinal driving behavior of drivers at freeways. This choice can for one be motivated by the fact that traffic flow dynamics on freeways is mainly determined by interactions between vehicles, while on urban roads the interaction with the environment (for example traffic lights) is at least as important as the interactions between vehicles (Brackstone and McDonald, 1996). Another reason for focusing on freeways is the important role these roads play in network performance as a whole. For research on car-following behavior in urban traffic we refer to (Bleile, 1997, 1999).

The analyses will furthermore concentrate on the constrained driving component of the longitudinal driving task, i.e. longitudinal driving behavior in which a driver is constrained by
his leader. When drivers are not constrained, their longitudinal driving behavior is largely
determined by their desired speeds. These desired speeds can be derived from loop detector
measurements (more information can be found in (Hoogendoorn, 2005b, Hoogendoorn,
2005a)).

Although changes in the longitudinal driving behavior of drivers over time deserve certainly
empirical investigation, we will not consider this in detail in the empirical analyses. The
practical reason for this is that such an analysis requires that drivers are monitored for a
longer period of time. Given our observation method this would require that the helicopter
flies along with a group of cars instead of staying at a fixed position. At the moment we are
not able to process data of a helicopter flying along with cars.

We will finally concentrate on how drivers react to disturbances in the speed(s) of their
leader(s) on the same lane. For instance, we do not consider how a driver adapts his
longitudinal driving behavior for allowing a merging vehicle to enter the main road. To be
able to concentrate on these specific disturbances we will only consider vehicle pairs whose
composition did not change during the period of observation.

1.7 Main research contributions

The contributions of the research presented in this thesis can be divided in
theoretical/scientific contributions and (possible) practical contributions. In this section we
will consider both, i.e. subsection 1.7.1 discusses the main scientific contributions, while
section 1.7.2 presents the possible practical contributions of our findings.

1.7.1 Scientific contributions

For the first time large scale microscopic trajectory based studies are performed on two
important facets of longitudinal driving behavior, namely heterogeneity and multi-
anticipation. The results of these analyses add considerably to existing fundamental
knowledge on longitudinal driving behavior. Furthermore contributions are made to
mathematical models describing this behavior. For example behavioral parameters of existing
car-following models are estimated and the ability of these models in predicting the behavior
of individual drivers is examined. Based on the obtained knowledge new car-following
models are proposed. We now shortly summarize the main scientific contributions.

With respect to heterogeneity we establish in chapter 5 that there exist clear differences
between the driving behaviors of driver/vehicle combinations, i.e. we show that different
behavioral rules are needed to adequately describe the behavior of different drivers. As we
perform the research on heterogeneity by calibrating mathematical models, we are able to
 quantify the extent of heterogeneity present in real traffic. We furthermore relate differences
between the behaviors of driver/vehicle combinations to causes. That is, a distinction is made
between heterogeneity caused by personal characteristics and heterogeneity related to vehicle
characteristics.

In chapter 6 we provide empirical (and statistical) evidence for the presence of multi-
anticipative behavior in real-life longitudinal driving behavior. This multi-anticipative behavior is
quantified for all observed drivers separately. It turns out that especially the relative speed
regarding leaders further downstream is often of influence to the longitudinal driving behavior
of a driver. In comparing the multi-anticipative driving behaviors of drivers we also identify
differences between drivers. Different drivers appear to consider, for example, different
numbers of leaders.
The analyses on both aspects of longitudinal driving behavior are performed for observations referring to two roadway stretches having different prevailing traffic conditions. One measurement site is characterized by stop-and-go traffic, while the other one is characterized by heavy congestion. By performing our analyses for these two different sets of observations, we increase the generality of our conclusions. A comparison of the results for the two measurement sites in Appendix E suggests furthermore a relation between longitudinal driving behavior and traffic conditions as drivers seem to be less sensitive to stimuli when driving for a prolonged time in heavy congestion.

Another indirect contribution is the calibration of several commonly used car-following models. We also compare the performances of these models in predicting the car-following dynamics of individual drivers. Although car-following models are at the core of all microscopic simulation tools, such detailed analyses of car-following models became only recently possible thanks to technological developments (for an overview of other methods and data types used for calibrating longitudinal driving models we refer to Appendix B). Other examples of recent work on the evaluation of car-following models at the microscopic level are (Brockfeld et al., 2004, Ranjitkar et al., 2004, Punzo and Simonelli, 2005).

Apart from these contributions stemming from our empirical analyses, this dissertation thesis also provides new insights into the process of automated calibration of longitudinal driving models when using microscopic trajectory observations. We show, for instance, to which extent the bias of parameter estimates increases in case of different types of measurement errors. We furthermore draw the important conclusion that it is possible to draw inferences about longitudinal driving behavior from calibration results even when the calibrated model is not the “perfect” model, i.e. the model fully representing the behavior of the driver. This finding is particularly important as for real trajectory observations it will most likely not be possible to identify a mathematical model fully describing the behavior of a real driver.

Next to these calibration issues directly relating to the use of real-life trajectory observations, we also consider the influence of the choice of the calibration objective function and the variable(s) in this calibration objective. It is shown that especially the choice of the variable in the objective can have a strong influence on calibration results.

The simulation results on the impact of our empirical findings finally add to existing knowledge on the effects of heterogeneity on the dynamics of simulated traffic flows. Till now the impact of differences between desired speeds of driver/vehicle combinations was mainly considered in discussions on the impact of heterogeneity on traffic flow dynamics. In chapter 7, we show however that differences between the longitudinal driving behaviors of drivers driving in congested traffic can be clearly of influence on how a disturbance propagates through a flow of vehicles driving on the same lane.

1.7.2 Practical relevance

The obtained knowledge on human longitudinal driving behavior can be used to improve the way in which longitudinal driving behavior is modeled in microscopic simulation tools. Our exploratory simulations provide preliminary evidence that incorporating our findings in microscopic simulation tools changes equilibrium properties as well as dynamical properties of predicted traffic flows, although more research is needed on this topic. In general it can be stated that when assumptions on longitudinal driving behavior in microscopic simulators become more realistic, the effects of, for example, (dynamic) traffic management measures
can be better predicted providing opportunities for a more efficient use of existing infrastructures.

Also in the development and assessment of systems supporting the driver, like ACC, the empirical results are useful as they provide insights into how human drivers perform their longitudinal driving task. In designing an ACC which will be accepted by a large share of drivers, it is important that the system behaves such that drivers feel comfortable. Our research, for example, shows that different drivers keep different time headways, this most probably entails that different drivers would adopt different time headway values when driving with ACC. The findings on multi-anticipation are also particularly interesting in this respect as our findings show that humans consider vehicles further downstream, while existing systems supporting the driver only react to the direct leader.

The findings on the sensitivity of the calibration results to methodological choices and practical issues related to the use of real trajectory observations are of importance to the growing group of people calibrating longitudinal driving models using such observations. The analyses presented in chapter 4 can firstly be used in the development of more robust calibration procedures. They are furthermore important in correctly interpreting calibration results.
2 The longitudinal driving task: description, analysis, importance and reasons for modeling

2.1 Aim and structure of this chapter

In this thesis extensive data analyses are presented to gain new insights into longitudinal driving behavior and to suggest improvements to models describing this behavior. Before these analyses are presented, it is important to explain what is exactly meant by the longitudinal driving task and to set up a framework describing how we hypothesize this task is executed by drivers. It furthermore needs to be motivated why we are interested in how people perform this driving task and what the motivations are for modeling it.

The chapter is structured as follows. Sections 2.2 and 2.3 discuss the longitudinal driving task in detail. Section 2.2 shows how the longitudinal driving task fits within the driving task on freeways as a whole, while section 2.3 deals with the actual execution of this driving task by drivers. We present the driver as a feedback controller, i.e. a driver monitors his current state and takes corrective actions when needed. Also several typical human characteristics related to the tasks of perception, selecting, and performing an appropriate control will be discussed.

The motivation for studying this particular component of the driving task will be the topic of section 2.4. From this section it will become clear that the longitudinal driving behavior of traffic participants determines to a large extent the characteristics of traffic flow. Next to that it will be argued that profound knowledge about how humans execute this task is a critical component for developing systems supporting the driver, like ACC, that are accepted by users and to predict the impact of these systems on traffic flow characteristics.

Section 2.5 will finally focus on the reasons for modeling this driving subtask. More specific, modeling offers unique opportunities for testing hypotheses about the longitudinal driving behavior from observations. Another important application of models describing the longitudinal driving behavior is in prediction, i.e. these models are the core of all microscopic simulation tools predicting traffic flows on the roads.
2.2 Position of the longitudinal driving subtask within the overall driving task

The driving task is a comprehensive term that consists of all tasks a driver must execute to reach his travel destination safely, comfortably and timely. For example, a driver must keep a safe distance to the vehicle in front, follow the desired route, conform to prevailing traffic rules, use turn indicators timely, keep the vehicle on the road etc. (Minderhoud, 1999).

The driving task can be categorized in several ways (for an overview we refer to (Hoedemaeker, 1999)). To gain a better insight into how the longitudinal driving subtask fits into the driving task as a whole, we will introduce three of these categorizations, i.e. the action based categorization, the task based categorization, and the task execution based categorization. We will furthermore discuss for each of these categorizations how the longitudinal component of driving fits into it.

2.2.1 Action based categorization of the driving task

The action based categorization presented in (Janssen et al., 1993) distinguishes the navigation subtask, the maneuvering subtask and the control subtask. In the navigation subtask drivers prepare their journey, while in the maneuvering subtask drivers are primarily concerned with interacting with other traffic and the road system. The control level finally involves the elementary tasks that have to be performed to enable maneuvering the vehicle (Hoedemaeker, 1999).

The longitudinal driving subtask contains both a maneuvering component as well as a control component. It deals with the interaction with other traffic and the road system, while performing this task requires control as will be shown in section 2.3. Typical examples of control actions performed in longitudinal driving are braking, accelerating or decelerating by adjusting the throttle position, changing gear, and changing foot from one pedal to another (Minderhoud, 1999).

2.2.2 Task based categorization of the driving task

To position longitudinal driving behavior relative to other maneuvering/control tasks and to distinguish its two components we use the task categorization introduced in (Minderhoud, 1999). This categorization discerns the following subtasks within the maneuvering/control tasks:

- **Roadway subtask**, defined as the collection of decisions of the driver needed to guide the vehicle properly and comfortably over the available infrastructure and its elements such as driving lanes, curves and on- or off-ramps. This subtask can be further decomposed in a *longitudinal* component and a *lateral* component.
- **Vehicle interaction subtask**, defined as the collection of decisions of the driver needed to guide the driver/vehicle combination properly and comfortably around vehicles and possibly other traffic participants actually present on the roadway. This subtask can also be further decomposed in a *longitudinal* component and a *lateral* component.

These subtasks are schematized in Figure 2-1, in which a clear distinction is made between the maneuvering/control tasks we will deal with in this thesis and the higher level navigation subtask.
As the main aim of this thesis is to gain a better insight into longitudinal driving behavior in dense traffic conditions, the interaction part of longitudinal driving behavior is most interesting to us. In analyzing this component of driving we will when needed take care of the characteristics of the considered roadway. For example, in interacting with other vehicles, drivers still need to adhere to prevailing traffic regulations.

**Navigation subtask**

![Diagram of navigation subtask]

**Figure 2-1 Classification of driving tasks based on (Minderhoud, 1999). The route navigation is separated from the maneuvering/control tasks shown in the right-hand side of the figure.**

### 2.2.3 Task execution based categorization of the driving task

The previous shows that “the driving task” comprises a lot of different subtasks on different levels, i.e. from planning a route to changing gears. The way in which a driver executes these subtasks depends on the skills/experience of a driver and his familiarity with the environment in which he needs to perform the subtask. In (Rasmussen, 1983) the following ways to execute a task are distinguished:

- **Knowledge-based level.** In performing an unfamiliar subtask and being faced with an environment for which no know-how or rules are available from previous encounters, a driver needs to develop new ways of problem solving. Accordingly a driver needs to pay a lot of attention and effort.

- **Rule-based level.** When a driver becomes more familiar with the subtask he is performing he can use stored rules to determine an appropriate action. The process of choosing a rule may be more or less conscious, but once a rule is chosen the actions are carried out automatically; so less attention is required compared to the knowledge-based level (Hoedemaeker, 1999).

- **Skill-based level.** At the lowest attention level actions take place as smooth, automated and highly integrated patterns of behavior without conscious control.
For an experienced driver, driving in a familiar car and under normal conditions the longitudinal driving task is performed mostly on the rule/skill-based level, implying that a driver often performs this task more or less unconsciously. The level of attention a driver needs to pay is higher when he is not familiar with his car, is less experienced, or when external conditions like the weather complicate longitudinal driving (Hale et al., 1990).

2.3 Execution of the longitudinal driving task

The previous subsection explained how the longitudinal driving subtask fits within the driving task as a whole. In this subsection we analyze the actual execution of the longitudinal driving subtask. As a base we use the feedback oriented control system of (Minderhoud, 1999) presented in Figure 2-2.

Although always care has to be taken in presenting human behavior in a single scheme, the concept is useful for our purpose as it gives insights into the different components of performing the longitudinal driving task and especially the different human attributes (errors and time delays) influencing it. The scheme is thus clearly a simplification suited for the intended analysis. As the longitudinal driving task is mostly skill-based we recognize that a driver performs most of the components of the scheme unconsciously and as less as possible to minimize the effort needed (Hale et al., 1988).

In the scheme presented in Figure 2-2 the longitudinal driving task is described as a repeated sequence of state observations and state estimations, followed by predictions of the expected future states and a control decision to best achieve the objective, after which the control actions are carried out and the state may be changed. A control action can in this sequence be described as the employment of a suitable skill to bring the vehicle within a limited amount of time into an acceptable state that is expected to remain acceptable for some time (Boer, 1999).

In fact, it is assumed that a driver performs as a kind of feedback controller in that he is repeatedly monitoring his current state and takes corrective actions when needed.

The different components of the scheme will now be discussed in more detail.
Chapter 2 - The longitudinal driving task: description, analysis, importance and reasons for modeling

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(t)$</td>
<td>vector of actual state attributes at instant $t$</td>
</tr>
<tr>
<td>$y(t)$</td>
<td>vector of state observations (e.g., speeds, relative speeds, distances)</td>
</tr>
<tr>
<td>$\hat{x}(t)$</td>
<td>vector of estimated state attributes</td>
</tr>
<tr>
<td>$x^p(t)$</td>
<td>vector of predicted future state attributes</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>vector of feasible control actions</td>
</tr>
<tr>
<td>$u^*(t)$</td>
<td>vector of selected control actions</td>
</tr>
<tr>
<td>$\bar{u}(t)$</td>
<td>performed control actions</td>
</tr>
<tr>
<td>$\varepsilon_{\text{obs}}$</td>
<td>vector of perception errors</td>
</tr>
<tr>
<td>$\varepsilon_{\text{est}}$</td>
<td>vector of estimation errors</td>
</tr>
<tr>
<td>$\varepsilon_{\text{dec}}$</td>
<td>vector of prediction/decision errors</td>
</tr>
<tr>
<td>$\varepsilon_{\text{act}}$</td>
<td>vector of actuator errors</td>
</tr>
<tr>
<td>$H$</td>
<td>time horizon considered for control action</td>
</tr>
<tr>
<td>$\tau_{\text{obs}}$</td>
<td>state observation time delay</td>
</tr>
<tr>
<td>$\tau_{\text{est}}$</td>
<td>state estimation time delay</td>
</tr>
<tr>
<td>$\tau_{\text{dec}}$</td>
<td>decision time delay (state prediction and control decision)</td>
</tr>
<tr>
<td>$\tau_{\text{act}}$</td>
<td>actuator time delay</td>
</tr>
</tbody>
</table>

**Figure 2-2** Control loop of the (longitudinal) driving task execution of a single actor (adapted from (Minderhoud, 1999)).
State observation and estimation
A driver uses his senses to make state observations. By looking, hearing, and feeling (such as pedal pressures, steering forces) information is collected and parts of the state are monitored. A state observation consists of several attributes. Typical examples that are possibly of influence on the execution of the longitudinal driving task are:

- The own speed, and the speed(s) of the leader(s).
- The distance to the leader(s).
- The speed of a vehicle preparing to enter the main road in front of the driver under consideration.
- The lane-change status of the direct leader.

The driver uses these observations together with earlier observations and past experiences to estimate the state he is in.

A human needs some time to perform the perception and estimation tasks. These time delays are denoted by $\tau_{obs}$ and $\tau_{est}$ in Figure 2-2. The vectors $\epsilon_{obs}$ and $\epsilon_{est}$ indicate that drivers are likely to make errors in perceiving and estimating the true state. For example, drivers face problems in estimating distances, especially longitudinally, absolute velocities and accelerations of other surrounding objects (Boer, 1999). For a good understanding of the longitudinal driving task it is therefore important to recognize that drivers use perceptual variables in performing this task.

State prediction and control decision
Based on the estimated state attributes $\hat{X}(t)$ a driver “selects” an appropriate control action in order to best achieve his objective. The word “selects” is written in quotation marks here to stress that the control decision is for most of the drivers performed automatically and unconsciously.

What exactly is meant by an appropriate control depends on the objective of a driver. Typical components of the objective are travel time, risk, comfort, and energy consumption.

The actual objective of a driver depends on the characteristics of the driver and the car he is driving in. It can furthermore vary over time depending on the purpose of the trip, the mental state of the driver, and external conditions. For example, in (Van Der Hulst, 1999) evidence is shown that drivers change their driving style when they become tired and when visual conditions get worse due to fog. Given these differences between the objectives of drivers and changes in the objective of a single driver, it seems justified to conclude that longitudinal driving behavior differs between drivers and even for a single driver.

But given that a driver has an objective how does he select an appropriate control?

In (Minderhoud, 1999) it is assumed that a driver makes a control decision by maximizing his own individual utility or minimizing his disutility. In doing so it is assumed that the driver considers a limited time horizon $H$ in determining the optimal decision taken at an instant $t$, where the decision refers to a trajectory of control actions to be taken over time period $H$. In other words, the driver predicts the expected impact of possible control actions on the future state and chooses the best alternative. In doing so, for example, expected changes of the
traffic situation, lane configuration, vehicle positions and speeds, and oncoming lane-changes are taken into account.

Comparable approaches in which a driver is assumed to optimize his utility function can be found in (Peppard and Gourishankar, 1972, Burnham et al., 1974, Lubashevsky et al., 2003). A good example of how such a utility maximization theory can be operationalized, i.e. how the optimal decision variables can be determined, is given in (Hoogendoorn and Bovy, 2003).

The reasoning that a driver seeks for an optimal control action is often rejected in psychological literature. In (Boer, 1999) for example, it is stressed that drivers satisfice rather than optimize. This alternative approach is motivated by the fact that drivers are constrained by bounded rationality implying that they are limited in their ability to evaluate all possible alternatives. Moreover, it is argued that if the current alternative is acceptable there is no need to look for and evaluate other alternatives.

Especially regarding the latter argument it can of course be argued that it can be incorporated in an objective function by attaching a disutility to the effort needed in examining more alternatives, i.e. transaction disutility, such that the optimization approach is defendable again. Nevertheless for the sake of gaining insight into the execution of the longitudinal driving task, it seems correct to conclude that regardless of the approach taken it is important to consider that drivers are not able/ not willing to evaluate all possible alternatives.

For any of the two points of view it is also valid to state that a human needs time to make a decision as indicated by parameter \( \tau_{\text{dec}} \) in Figure 2-2. Furthermore next to errors in state perception and estimation it can also be expected that the actual decision making process of drivers is prone to errors \( \varepsilon_{\text{dec}} \).

### Control action

The control action selected by the driver is performed by applying the brake, gas pedal, or even by maintaining all pedals in the same positions as before (“do nothing”, “change nothing”). The control action results in a new state that is also influenced by the actions of other traffic participants in the neighborhood and changed road and weather conditions.

Also in performing the actual control action drivers are not perfect as denoted by variable \( \varepsilon_{\text{act}} \), as they can for example not handle their pedals completely precise. Furthermore another time delay will occur in executing the control action. For instance, a driver has to move his foot to the pedal. This time delay is in Figure 2-2 denoted by parameter \( \tau_{\text{act}} \).

### 2.4 Relevance of insights into longitudinal driving

The aim of the previous sections was to gain better insights into the execution of the longitudinal driving task by drivers and its position within the driving task as a whole. The aim of this section is to show the importance of this particular subtask of driving. It will be shown that the longitudinal driving behavior of individual traffic participants determines the characteristics of a traffic flow to a large extent, stressing the importance of having good insights into this behavior. Also the importance of having knowledge about this particular type of human behavior in the development and evaluation of systems supporting the driver, like ACC, will be discussed.
2.4.1 Longitudinal driving behavior and the fundamental diagram
A clear relation exists between longitudinal driving behavior and the fundamental diagram representing the equilibrium relation between flow, density, and space mean speed of a traffic flow. The density \( k \) can for example be directly derived from the gross distance headways between vehicles using eq.(2.1):

\[
k = \frac{1}{\bar{s}}
\] (2.1)

in which \( \bar{s} \) represents the mean gross distance headway between successive vehicles. This relation will be used in chapter 7 to explore the impacts of heterogeneity and multi-anticipation on the fundamental diagram.

Also macroscopic traffic flow dynamics are strongly influenced by the longitudinal driving behavior of individual drivers on the road (Tampère et al., 2005b). To illustrate this, we first discuss the relation between longitudinal driving behavior and traffic flow stability. After this we show how two important phenomena in traffic, namely the capacity drop and hysteresis, can be directly explained by considering longitudinal driving behavior.

2.4.2 Longitudinal driving behavior and stability of traffic flows
A lot of (small) disturbances can be observed in real-life traffic flows caused by for example vehicles changing lanes, merging vehicles, a vehicle braking unexpectedly etc.. For the dynamics of a traffic flow it is therefore important whether such a disturbance will grow, or fade away, i.e. whether the traffic flow is stable.

In studies on longitudinal driving behavior in general two types of stability are distinguished (Leutzbach, 1988). The first one is concerned with how a single following vehicle reacts to a disturbance in the speed of his leader. When such a disturbance does not die out but increases with time the longitudinal driving behavior of the follower is called locally unstable.

The second type of stability refers to the propagation of a disturbance in a platoon of vehicles. More specific, when a disturbance grows in magnitude as it propagates from vehicle to vehicle the car-following behavior is called asymptotically unstable.

Both types of instability are illustrated in Figure 2-3 in which we show the simulated reactions of followers to a disturbance in the speed of the leader. Also an example of a stable platoon is given. It is important to mention that in these cases the propagation of the disturbance is completely determined by the longitudinal driving behavior of drivers.

Whether or not real-life traffic flows are stable depends, next to the longitudinal driving behavior of the individual drivers, also on the sizes of the platoons present in the traffic flow and the gaps between consecutive platoons (Tampère, 2004). For example, when the car-following behavior of drivers driving in a very small platoon is asymptotically unstable, while there is a large gap between this platoon and the next platoon, then the next platoon will most probably not be influenced by small disturbances occurring in the former platoon.

Related to traffic flows, therefore, another type of stability can be defined, namely metastability. A traffic flow is more specifically considered to be metastable if it is stable with respect to disturbances with small amplitude, but unstable with respect to larger disturbances (Kerner, 2001).
Metastability can also be used to explain the stochastic nature of capacity as described by (Bovy and Thijs, 1998, Minderhoud et al., 1998, Kerner, 2004, Brilon et al., 2005, Kühne et al., 2007). For a given flow level, traffic will only breakdown when a large enough disturbance occurs. Such a disturbance occurs with a certain probability. When the flow level increases, the magnitude of the disturbance needed for causing a breakdown will decrease and correspondingly the probability that a large enough disturbance occurs increases (Kerner, 2004). Also in this property of traffic flows the longitudinal driving behavior of individual drivers thus plays a central role.

Figure 2-3 Examples of (a) a stable platoon, (b) an asymptotically unstable platoon and (c) locally unstable behavior of a following vehicle (simulation results).

Despite the fact that traffic flow stability is influenced by more factors than platoon stability, it is still completely valid to state that also this type of stability is largely determined by the longitudinal driving behavior of the individual drivers composing the traffic flow. The strong relation between longitudinal driving behavior and platoon/flow stability will be further illustrated in chapter 7 in which we explore the impacts of our empirical findings on heterogeneity and multi-anticipation on platoon stability and flow stability.

2.4.3 Longitudinal driving behavior, capacity drop and hysteresis

To stress the relation between the macroscopic characteristics of traffic flow and the longitudinal driving behavior of individuals further, the phenomena of hysteresis and capacity drop are discussed in the upcoming.

Hysteresis refers in this context to the observation that so-called transient states, i.e. changes from congestion to free-flow or vice versa, are generally not on the fundamental diagram and
follow different paths. The capacity drop furthermore implies that the maximum flow rate achievable in congested traffic is lower than in free flow traffic (Zhang and Kim, 2005).

Regarding hysteresis two facets of longitudinal driving behavior can be mentioned causing the observed difference between the phase trajectory during acceleration and the phase trajectory during deceleration.

![Diagram showing hysteresis loops](image)

**Figure 2-4 Example of different types of hysteresis loops (based on (Zhang, 1999)).**

The first aspect is the aforementioned *time delay* in the response of a driver. This time delay causes that a reaction is delayed, implying in the case of acceleration that a follower does not immediately start to accelerate when his leader accelerates such that the time headway temporally increases. This means that, given the speed, a driver follows his leader at a larger distance than he would do in equilibrium conditions. In macroscopic terms this leads temporarily to a lower density (eq. (2.1)). The opposite holds for deceleration. Loop two in Figure 2-4 represents an example of a hysteresis loop caused by time delay.

On the other hand drivers can also possibly *anticipate* future traffic conditions by looking further ahead. As a consequence drivers can already adjust their speed to future conditions that are not yet applicable to them. They can start braking, for example, before their direct leader starts to brake by observing that the vehicle in front of their leader starts to brake. This has an effect opposite to the one mentioned before in that this implies for accelerating that the time headway is temporally smaller, while it is temporally larger during deceleration. Loop one in Figure 2-4 represents an example of a hysteresis loop caused by multi-anticipation.

Also for the capacity drop it is possible to point at at least two causes pertaining to longitudinal driving behavior. Firstly, a small capacity drop can in fact be explained by hysteresis by considering that drivers at the head of a queue are constrained and cannot anticipate to conditions ahead.

\[\text{Equation 2.1}\]

\[\text{Equation 2.1}\]

\[\text{Equation 2.1}\]

---

1The special shape of the “composed” hysteresis loop presented in Figure 2-4 can be explained by reasoning that traffic conditions are of influence to the component of behavior that is most applicable for real drivers. To get this specific shape it is more specifically reasoned that in denser traffic, behavior will more likely be delayed as drivers can not see the disturbance until it reaches them, while in dilute traffic drivers have much more opportunity to anticipate (Tampère, 2004).
Empirical findings suggest however another important cause for the capacity drop as measurements provide evidence that drivers follow their leaders with smaller headways in non-congested flow than in congested flow (Dijker et al., 1997, Dijker et al., 1998). A psychological explanation for this is that there is no advantage for a driver to take more risk, i.e. to follow with a smaller time headway, once congestion has set in as this does not lead to a gain in travel time (Van Toorenburg, 1983a, b). This change in the mental state of a driver is also discussed in for example (Daganzo, 2002, Kerner, 2004, Tampère, 2004).

2.4.4 Adaptive Cruise Control and human longitudinal driving behavior
Apart from the strong influence on traffic flow dynamics, another reason for studying the longitudinal driving behavior of drivers is in the development of systems supporting the driver, like ACC (Brackstone and McDonald, 1999a). As drivers are more skeptical in accepting supporting systems influencing their control task than systems pertaining to for example the higher level navigation task (Groeger et al., 1993), it is important to ensure that drivers feel comfortable with “the driving style” imposed by the automated system. This thus requires thorough insights into human driving behavior.

Also detailed knowledge about the human driving behavior is needed to assess the impact of ACC systems on the properties of traffic flow. As discussed before people react to changes in their environment with a certain time delay, but they can compensate for this delay to a certain extent by anticipating the future by looking further ahead. Existing autonomously operating ACC systems on the other hand face a smaller time delay but are not able to consider the car in front of their direct leader (Kesting et al., 2007b). For predicting the effects of ACC and other systems supporting the longitudinal driving task, by performing for example microscopic simulation studies (Van Arem et al., 1997, Minderhoud, 1999, Hoogendoorn et al., 2007c, Van Driel, 2007, Kesting, 2008), it is necessary to know exactly to which extent the driving style of human drivers differs from the automated style. The empirical analyses performed in chapters 5 and 6 can therefore particularly contribute to such studies on the impact of ACC as they provide important insights into the level of heterogeneity and the degree of multi-anticipation in human longitudinal driving behavior.

2.5 Motives for modeling longitudinal driving behavior
The previous section showed the relevance of analyzing longitudinal driving behavior. In this section two motivations for modeling longitudinal driving behavior mathematically will be discussed, namely gaining insights into this type of behavior and prediction.

2.5.1 Modeling to gain insights into longitudinal driving behavior
An important aim of mathematical modeling is to gain thorough quantitative insights into relations present in real-world behavior. More technically stated, to identify dependent and independent variables, to specify a functional form, and in most cases, to make at least a qualitative statement about effects that occur when independent variables in the model change (Greene, 2000). In the research presented here this implies that models are used to gain insights into factors influencing longitudinal driving dynamics, as well as to quantify the extent to which these factors are actually of influence.

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2 In the future, technological developments like vehicle-to-vehicle communication can make it possible that also ACC systems become able to anticipate (Van Arem et al., 2006).

3 Although it can not be established from helicopter images whether drivers use a system like ACC, it is justifiable to assume that under the observed traffic conditions almost all drivers are performing the longitudinal driving task without ACC.
In this thesis models quantitatively describing longitudinal driving dynamics will be calibrated using trajectory observations describing the dynamics of real driver/vehicle combinations. The resulting performance measures and parameter estimates will be used to draw inferences about how drivers execute their longitudinal driving task. For example, by fitting models to empirical observations insight is obtained into the number of leaders drivers consider in their longitudinal driving behavior.

2.5.2 Modeling to predict traffic flow

Another important motive for modeling longitudinal driving dynamics is for predictive reasons. That is, models describing longitudinal driving dynamics are critical components in any microscopic traffic simulation tool, i.e. a simulation tool in which all traffic participants are simulated separately (Dijker and Knoppers, 2004, SIAS Limited, 2005, PTV, 2006, TSS-Transport Simulation Systems, 2006).

These simulation tools have the advantage that they incorporate many parameters that can be used to describe the individual features of driver behavior thereby enabling even investigation of the influence of subtle changes in this behavior (Brackstone and McDonald, 1996). Modern computers are becoming more and more able to handle the related high computational burden increasing, for example, the size of the network that can be handled.

Since the longitudinal driving behavior of traffic participants determines to a large extent the equilibrium and dynamical properties of traffic flows it is clear that an adequate modeling of this behavior is critical for the validity of the simulation outcomes. The previously mentioned goal of modeling, namely gaining insights into longitudinal behavior of real drivers using observations, is thus unambiguously the first and main step towards successful prediction.

This does not necessarily imply that the model best describing human behavior is the most suitable model for a simulation tool. As running time is also an important component in microscopic simulation also the efficiency of a model plays its role. Next to that the extent to which the parameters of a given model can be identified from observations needs to be considered. The question that needs to be asked in this sense is whether the applied models are an acceptable tool for the job they are used for (Brackstone and McDonald, 1999b). That is, models used in simulation tools need to be as simple as possible while still being able to make predictions that correctly answer the research question at hand.

In this thesis therefore also attention will be paid to the impact of the established empirical findings regarding longitudinal driving behavior on simulation outcomes (chapter 7). For example, to which extent do predicted traffic flow dynamics change when models are made more complex by incorporating driver heterogeneity?

2.6 Summary

This chapter first aimed at gaining theoretical insights into the longitudinal driving task. To this end, it has been shown how the longitudinal driving subtask fits within the driving task as a whole. Also the execution of this driving task by real drivers has been discussed.

The chapter starts from the widely accepted hypothesis that a driver can be seen as a feedback controller, i.e. a driver monitors his current state and takes corrective actions when needed. How a driver exactly executes the longitudinal driving task is not only dependent on his personal objectives and characteristics, and the car he is driving in, but also on his mental
state and the prevailing external conditions. Longitudinal driving behavior differs thus not only between drivers but also for a single driver.

It has furthermore been shown how several human attributes may influence the execution of the longitudinal subtask. For example, drivers face difficulties in estimating absolute velocities of surrounding vehicles and they need time to process information.

The second aim of the chapter has been to motivate why profound knowledge about how people perform the longitudinal subtask is important and what are the reasons for modeling this behavior.

It became clear that the longitudinal driving behavior of individual traffic participants determines to a large extent the characteristics of traffic flow as a whole. The fundamental diagram is directly related to the longitudinal behavior of drivers and also the stability of traffic flow is mainly determined by this behavior. Next to that the phenomena of hysteresis and capacity drop can be well explained from a longitudinal driving point of view.

Two important reasons for modeling the longitudinal driving task have been mentioned. Models offer first of all unique opportunities for testing hypotheses about longitudinal driving behavior from observations. This property of models will be used throughout this thesis to gain new insights into longitudinal behavior from trajectory data. Another important application of models on longitudinal driving behavior is in prediction, i.e. these models are the core of all microscopic simulation tools. In chapter 7 also attention will be paid to this use of models as it will be analyzed how the established empirical findings influence predictions made by simulation tools.

Given the important role of mathematical models describing the longitudinal driving task in this thesis, the next chapter will introduce and critically consider a range of different longitudinal driving models. To assess the models, we use the control scheme presented in Figure 2-2, i.e. we compare the control objectives assumed by the models and we consider how the models deal with time delays and human errors. Next to that we describe how modelers currently deal with differences between the longitudinal driving behaviors of drivers and changes in the longitudinal driving behavior of a single driver. Especially gaining insight into the way in which heterogeneity is currently incorporated in microscopic simulation tools is interesting, as it provides the opportunity to compare our empirical findings on heterogeneity presented in chapter 5 to current practice.
3 Behavioral assumptions in mathematical models of longitudinal driving dynamics

3.1 Aim and structure of this chapter

In the previous chapter we analyzed the longitudinal driving behavior of drivers. We presented the driver as a feedback controller and we considered typical human characteristics influencing the execution of this control task. We also discussed the reasons for modeling longitudinal driving behavior, i.e. testing hypotheses from observations and predicting.

In this thesis both applications of mathematical models (i.e. behavioral analysis and prediction) play an important role. First of all, we use models to gain new insights into longitudinal driving behavior from observations. Second of all, we assess whether and how these findings influence predictions made by these models.

In order to be able to test behavioral hypotheses by calibrating longitudinal driving models, we need to have detailed knowledge about the behavioral assumptions made by these models. The first aim of this chapter is therefore to present and compare the behavioral assumptions made in the longitudinal driving models used throughout this thesis. We more particularly show how these models fit within the control scheme presented in the previous chapter. The second aim of this chapter is to show how modelers currently deal with differences between the longitudinal behaviors of drivers and changes in the longitudinal behavior of a single driver. In chapter 5 we can use this knowledge on the state of the art in modeling heterogeneity to assess whether the level of heterogeneity currently assumed in microscopic models is in line with the level of heterogeneity present in real traffic.

Sections 3.2 to 3.5 focus on the first aim. In section 3.2 we start by giving a general overview of the main behavioral assumptions made by the different models used throughout this thesis, i.e. we summarize the control objectives and indicate whether the models take care of time delays and human limitations. After the general overview the assumptions made by the models on these three features will in more detail be discussed in sections 3.3 (control objectives), 3.4 (time delays), and 3.5 (human limitations).
In section 3.3 we will show that the models make considerably different assumptions on the stimuli a driver considers in determining an appropriate control action. For example, some models assume that a driver takes only his first leader into account, while other models assume that drivers multi-anticipate, i.e. consider also leaders further downstream in selecting a control action. Although these multi-anticipative models do not give profound suggestions on the number of leaders a driver considers.

In section 3.4 we show that not all models take care of the time delay between the actual occurrence of a stimulus and the reaction of the driver to that stimulus. In section 3.5 we conclude that most models ignore that people make errors in perceiving a state, deciding on an appropriate control action and applying this action.

In section 3.6 we change the topic from what different models assume about how a driver handles his longitudinal driving task at a given time, to a discussion on how modelers deal with differences between the car-following behaviors of drivers and changes in the behavior of a single driver. Since heterogeneity plays a dominant role in this thesis, the main finding from this section is that modelers in general agree on the importance of heterogeneity but that empirical insight into the level of heterogeneity present in real traffic is lacking. Regarding the method of incorporating heterogeneity in microscopic simulation tools, it can in general be concluded that heterogeneity is most of the times modeled by assuming that the same longitudinal driving model is valid for all drivers, while behavioral parameters differ.

This chapter does not aim to give a complete overview of existing models describing longitudinal driving dynamics. It will nevertheless become clear that there exist a lot of such models, which all make different assumptions on longitudinal driving behavior. Since all these models are simplified representations of complex human behavior, we conclude the chapter by a discussion on model validity. We conclude that it does not necessarily hold that a more complex model containing more human characteristics is the best model when it comes, for example, to predicting the influences of traffic management measures using microscopic simulations. The decision that a simpler representation of human behavior may suffice, can however only be made based on detailed insights into the behavior of real drivers, as only with a profound understanding of human behavior the influence of making simplifications can be evaluated.

3.2 Overview of assumptions on longitudinal behavior made by models

In the previous chapter we presented the driver as a feedback controller, i.e. a driver monitors and estimates his current state, evaluates this state based on his control objective and takes corrective actions when needed. We found that two important human characteristics influence this control process:

- Drivers are not able to react instantaneously on unexpected stimuli surrounding them. People need time to perceive stimuli, to make a decision on an appropriate control action and to actually apply this control action.
- Drivers face problems in perceiving stimuli exactly and without error. Also in evaluating alternative control actions, selecting an appropriate control action and carrying out this action errors are likely to occur.

In the upcoming we will analyze how these different aspects of human longitudinal driving behavior are incorporated in different mathematical models describing longitudinal driving
dynamics. These models have in common that they aim at quantitatively describing the longitudinal driving dynamics of a driver/vehicle combination based on the known dynamics of the leading car(s). It is important to stress that these models do not aim at describing all facets of the decision making process of a driver, nor at describing how a driver interacts with his car, they only consider the resulting dynamics of a following driver/vehicle combination.

We focus on the eight models that will be empirically investigated in this thesis. This chapter will illustrate one of the main reasons for selecting these models in our empirical analyses, i.e. it will be shown that these models make considerably different assumptions on the control objectives of drivers. As a consequence of this, the models are particularly suited for gaining insights into the stimuli and mechanisms governing the longitudinal driving behavior of individual drivers. A second reason for choosing these models is that it seems likely that the parameters of these models can be identified using our trajectory observations referring to congested conditions.

Before entering in detail, Table 3-1 gives an overview of the main characteristics of the considered models.

**Table 3-1 Overview of how several facets of human longitudinal driving behavior are incorporated in models aiming at describing longitudinal driving dynamics (X=included in the model, x=included to a limited extent, v=speed, a=acceleration).**

<table>
<thead>
<tr>
<th>Control objective</th>
<th>Gipps</th>
<th>CHM</th>
<th>Bexelius</th>
<th>Tampère</th>
<th>Addison &amp; Low</th>
<th>IDM</th>
<th>OVM</th>
<th>Lenz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaching a safe distance</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reaching a desired distance to the direct leader</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Synchronizing speed with direct leader</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Synchronizing speed difference with leaders further downstream</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reaching a speed in line with distance to direct leader</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Reaching a speed in line with distance to leaders further downstream</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reaching the desired free speed</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Control variable</td>
<td>v</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>Inclusion of time delay</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inclusion of human limitations</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-1 indicates clear differences between the control objectives of the models. The CHM model of (Chandler et al., 1958) assumes that a driver only aims at synchronizing his speed with the one of his leader. The model of (Tampère, 2004) assumes that also the distance to the leader and the desired speed of the follower play a role in choosing an appropriate control
action. Next to that the control objectives of the models differ regarding their assumptions on
the number of leaders a driver considers in his decision making process.

The table also shows that time delays are only incorporated in a part of the models, while
almost all models refrain from modeling human errors. To give an impression of how these
aspects of human driver behavior not incorporated in the empirically investigated models can
be included in modeling, we also shortly refer to other modeling approaches.

In the upcoming section 3.3 we consider the control objectives assumed by the empirically
investigated models. In sections 3.4 and 3.5 we discuss respectively how the models deal with
time delays and human errors.

### 3.3 Control objectives assumed by models

In this section we discuss the control objectives of the models in more detail. Regarding the
applied notation it needs to be mentioned that the vehicle indices decrease in the driving
direction. We furthermore stress that we assume all behavioral parameters to be driver
specific, for presentation clarity we refrain however from adding vehicle indices to all
parameters.

#### Gipps model

Intuitively the most important task of a driver in following another car is to avoid collisions,
thus to keep a safe distance to the direct leader. The model proposed in (Gipps, 1981)
correspondingly assumes that a driver controls his speed such that he is able to keep the
minimum distance at standstill whenever his leader brakes at the maximum desired
deceleration rate. This is expressed in the following control law:

\[
v^\text{con}_n(t + T_r) = b^{\text{max}} \left( \frac{T_r}{2} + \theta \right) + \sqrt{\left( \frac{b^{\text{max}}}{2} \right)^2 \left( \frac{T_r}{2} + \theta \right)^2 - b^{\text{max}} \left[ 2 \Delta x_{n-1,n}(t) - d - v_n(t) T_r - \frac{v_{n-1}(t)^2}{b^{\text{max}}_{n-1}} \right]}
\]

where,
- \(v^\text{con}_n\) = maximum speed follower \(n\) can choose to be able to keep a safe distance in
case of a sudden stop of leader \(n-1\) (m/s)
- \(t\) = time instant (s)
- \(v_n\) = speed of follower \(n\) (m/s)
- \(v_{n-1}\) = speed of leader \(n-1\) (m/s)
- \(\Delta x_{n-1,n}\) = gross distance headway between leader \(n-1\) and follower \(n\) (m)
- \(b^{\text{max}}\) = most severe braking the driver of the following car wishes to undertake
\(<0\) (m/s²)
- \(d\) = gross distance headway at standstill (m)
- \(b^{\text{max}}_{n-1}\) = estimate (made by the driver of the following car) of the maximum desired
braking of the leading car \(<0\) (m/s²)
- \(T_r\) = reaction time (s)
- \(\theta\) = safety reaction time (s)

The safety reaction time \(\theta\) can be interpreted as a further safety margin introduced by the
driver allowing for a more relaxed driving style.
When the driver is not constrained in his longitudinal driving behavior by the car directly in front, his control objective changes from keeping a safe distance to accelerating to or driving at his desired speed $v^*$ (m/s). This so-called free driving regime is modeled by eq. (3.2).

$$v_{n}^{\text{free}}(t + T_{r}) = v_{n}(t) + 2.5 \cdot a_{\text{max}}^* T_{r} \left( 1 - \frac{v_{n}(t)}{v^*} \right) \sqrt{0.025 + \frac{v_{n}(t)}{v^*}}$$  \hspace{1cm} (3.2)

Parameter $a_{\text{max}}^*$ (m/s$^2$) represents the maximum acceleration the driver of the following car wishes to undertake and $v_{n}^{\text{free}}$ (m/s) denotes the speed the driver chooses in the free driving regime.

In line with this composed control objective of the follower, the final speed of the follower is according to the Gipps model equal to:

$$v_{n}(t + T_{r}) = \min(v_{n}^{\text{con}}(t + T_{r}), v_{n}^{\text{free}}(t + T_{r}))$$  \hspace{1cm} (3.3)

**CHM model**

The model proposed in (Chandler et al., 1958) assumes that a follower controls his acceleration $a_{n}$ such that the difference between his own speed and his leaders speed, denoted by $\Delta v_{n-1,n}$ vanishes. This objective leads to the following control law:

$$a_{n}(t + T_{r}) = c_{1,n-1} \cdot \Delta v_{n-1,n}(t)$$  \hspace{1cm} (3.4)

where,

$$c_{1,n-1} = \text{sensitivity to speed difference with leader } n-1 \text{ (1/s)}$$

This control objective is clearly too simple for representing the decision making process of a real driver. Suppose for example that a driver feels that he is driving too close to his leader while both vehicles drive at the same speed, in this case no action would be taken by the follower according to this control objective. It is also assumed that the extent to which a driver reacts to speed differences is independent of the distance to the direct leader. To overcome this last criticism (Gazis et al., 1959) proposed to divide the speed difference $\Delta v_{n,n-1}$ by the distance headway $\Delta x_{n-1,n}$.

The control objective is furthermore only applicable to the constrained driving regime in which a driver is following another car.

**Bexelius model**

This CHM control objective of eq. (3.4) is extended in (Bexelius, 1968) to include stimuli from leaders further downstream. The corresponding control law is given by:

$$a_{n}(t + T_{r}) = \sum_{j=1}^{n} c_{1,n-j} \cdot \Delta v_{n-j,n}(t)$$  \hspace{1cm} (3.5)

where,

$$\Delta v_{n-j,n} = \text{speed difference between leader } n-j \text{ and follower } n \text{ (m/s)}$$

$$c_{1,n-j} = \text{sensitivity to speed difference with leader } n-j \text{ (1/s)}$$
Longitudinal driving behavior: theory and empirics

$m_1 =$ number of leaders considered regarding relative speed

A driver driving in line with this control objective thus anticipates changes in the behavior of his direct leader by considering the dynamics of vehicles driving in front of the direct leader. Unfortunately information on how many leaders a follower considers is lacking so far, stressing the importance of the empirical analyses presented in chapter 6.

Like for the CHM model it also holds for this model that this control objective disregards important stimuli influencing the behavior of real drivers.

**Tampère model**

Like the control objective of the Gipps model, the control objective of the model proposed in (Tampère, 2004) consists of two parts. One that is applicable when a driver is constrained by his leader (this objective is equal to the one proposed by Helly (Brackstone and McDonald, 1999a)), while the other one applies when the driver is free to accelerate to or drive at his desired speed.

In the constrained driving regime the driver is assumed to react to two stimuli, namely the speed difference with his direct leader and the difference between the actual distance to the leader and the corresponding desired distance $\Delta x_{n-1,a}^*$.

$$a_n(t + T_r) = c_{1,n-1} \Delta v_{n-1,a}(t) + c_2 \left( \Delta x_{n-1,a}(t) - \Delta x_{n-1,a}^*(v_n(t)) \right) \quad (3.6)$$

where,

$c_{1,n-1} =$ sensitivity to speed difference with leader $n-1 \ (1/s)$

$c_2 =$ sensitivity to difference between real distance and desired distance $(1/s^2)$

Thus even when there is no speed difference with the direct leader, a driver has an incentive to change his speed when he is not satisfied with his current distance. Alternatively, a driver does not immediately start braking when he drives faster than his leader and his current distance is larger than the desired distance. By assuming this control objective the model is thus able to explain the phenomena of closing in and shying away.

The desired distance headway is assumed to increase linearly with speed:

$$\Delta x_{n-1,a}^*(v_n(t)) = d + \gamma v_n(t) \quad (3.7)$$

Thus at zero speed the desired distance is as in the Gipps model equal to $d \ (m)$, while it increases linearly with $\gamma \ (s)$ for every $1 \ m/s$ increase of speed.

In the free driving regime the driver changes his objective to reaching the desired speed. The composed control law is given by the following equation.

$$a_n(t + T_r) = \min \left( c_{1,n-1} \Delta v_{n-1,a}(t) + c_2 \left( \Delta x_{n-1,a}(t) - \Delta x_{n-1,a}^*(v_n(t)) \right), c_3 (v^*(t) - v_n(t)) \right) \quad (3.8)$$

where,

$c_3 =$ sensitivity to difference between actual speed and desired speed $(1/s)$
Addison and Low

Also in the control objective proposed by (Addison and Low, 1998) the difference between the actual distance and the desired distance is included. Contrary to the model of Tampère it is assumed that the larger the difference between the actual and the desired distance, the more the driver tries to reach the desired distance, either by accelerating or by decelerating. The influence of speed differences increases furthermore when the distance between the leader and the follower decreases.

The control law corresponding to this model is given by:

\[
a_n(t + T_r) = c_4 \frac{\Delta v_{n-1,n}(t)}{\Delta x_{n-1,n}(t)} + c_5 \left( \Delta x_{n-1,n}(t) - \Delta x^*_n(v_n(t)) \right)^3
\]

\[
(3.9)
\]

where,

- \( c_4 \) = sensitivity to \( \Delta v_{n-1,n}/\Delta x_{n-1,n} \) (m/s)
- \( c_5 \) = sensitivity to difference between real distance and desired distance to power three \((1/m^2 s^2)\)

**IDM**

The control law of the Intelligent Driver Model (IDM) presented in (Treiber et al., 2000) is applicable to both the constrained driving regime and the free driving regime:

\[
a_n(t) = a_{\text{max}} \left[ 1 - \left( \frac{v_n(t)}{v^*_n} \right)^\delta - \left( \frac{\Delta x_{\text{min}}^*(v_n(t), \Delta v_{n,n-1}(t))}{\Delta x_{n-1,n}^*(t)} \right)^2 \right] \]  

\[
(3.10)
\]

The minimum desired distance headway \( \Delta x_{\text{min}}^* \) is calculated as follows:

\[
\Delta x_{\text{min}}^*(v_n(t), \Delta v_{n,n-1}(t)) = d + v_n(t) T_{\text{safe}} + \frac{v_n(t) \Delta v_{n,n-1}(t)}{2 \sqrt{a_{\text{max}} b_{\text{max abs}}}}
\]

\[
(3.11)
\]

where,

- \( \Delta v_{n,n-1} \) = speed difference between follower \( n \) and leader \( n-1 \) (m/s)
- \( d \) = distance at standstill (m)
- \( T_{\text{safe}} \) = safe time headway (s)
- \( a_{\text{max}} \) = maximum desired acceleration of the following car \((m/s^2)\)
- \( b_{\text{max abs}} \) = \(|\text{maximum desired deceleration of the following car}| (>0) \ (m/s^2)\)
- \( \delta \) = acceleration component

When the distance headway between leader and follower is clearly larger than the minimum desired distance \( \Delta x_{\text{min}}^* \) the follower accelerates to his desired speed. When the distance headway decreases, thus when the follower becomes more and more constrained by his leader, the interaction term becomes more important.
The speed difference between follower and leader also plays its role in this control law. It is assumed that the minimum desired distance is larger when the follower drives with a higher speed than his leader than when the follower has a speed lower than his leader.

This single-leader IDM control objective is extended to a multi-leader objective in the Human Driver Model (HDM) presented in (Treiber et al., 2006a). However, also for the HDM model like for the Bexelius model no empirical evidence is provided so far on the number of leaders a driver considers.

**OVM**

The Optimal Velocity Model (OVM) proposed in (Bando et al., 1995a, Bando et al., 1995b) assumes that a driver determines an appropriate speed based on his current distance to the leader in front. This is expressed in the following control law:

\[
a_{n}(t) = c_{6,n-1} \left( V_{opt}(\Delta x_{n-1,n}(t)) - v_{n}(t) \right)
\]

where,

- \( V_{opt} \) = optimal velocity, i.e. the speed the follower wants to drive with given the distance headway (m/s)
- \( c_{6,n-1} \) = sensitivity to difference between the current speed and \( V_{opt} \) (1/s)

The relation between the current distance and \( V_{opt} \) is in line with (Bando et al., 1995a) as follows:

\[
V_{opt}(\Delta x_{n-1,n}(t)) = V_0 \left[ \tanh m \left( \Delta x_{n-1,n}(t) - b_f \right) - \tanh m \left( d - b_f \right) \right]
\]

where,

- \( d \) = distance at standstill (m)
- \( b_f \) = distance headway corresponding to the inflection point of the optimal velocity function (m)
- \( m \) = the slope of the optimal velocity function at the inflection point (1/m)
- \( V_0 \) = maximum speed at large enough headways minus the speed at the inflection point (m/s)

This relation is illustrated in Figure 3-1, from which it can be concluded that this control law is both applicable to free driving as well as to constrained driving. When the distance headway is small the optimal velocity is smaller than the desired speed, while the optimal velocity is always equal to the maximum speed at large enough distance headways. Thus when the follower is not influenced by his direct leader he wants to drive with his desired speed.

**Lenz**

The single-leader control objective of the OVM is extended in the model proposed by (Lenz et al., 1999) to include multiple leaders:

\[
a_{n}(t) = \sum_{j=1}^{m} c_{6,n-j} \left( V_{opt} \left( \frac{\Delta x_{n-j,n}(t)}{j} \right) - v_{n}(t) \right)
\]
where,
\[ c_{6,n-j} = \text{sensitivity to difference between the current speed and } V_{opt} \text{ regarding leader } j \text{ (1/s)} \]
\[ m_3 = \text{number of leaders considered} \]

Also regarding this multi-anticipative model information is lacking on an appropriate number of leaders.

![Graph](image)

**Figure 3-1** Relation between distance headway \( \Delta x_{n-1,n} \) and optimal speed \( V^{opt} \) for different values for the parameters \( m \) and \( b_f \) (eq. (3.13)).

### 3.4 Inclusion of time delays in models

In several of the model representations provided above, a parameter \( T_r \) was included representing a reaction time. In the model of Tampère, for example, it is assumed that a driver observes stimuli on time instant \( t \), while he responds to these stimuli on time instant \( t + T_r \).

When using the notation of the control scheme presented in the previous chapter (Figure 2-2) this reaction time \( T_r \) consists of the following components:

\[ T_r = \tau_{obs} + \tau_{est} + \tau_{dec} + \tau_{act} \] \hspace{1cm} (3.15)

where,
\[ \tau_{obs} = \text{state observation time delay (s)} \]
\[ \tau_{est} = \text{state estimation time delay (s)} \]
\[ \tau_{\text{dec}} = \text{decision time delay (s)} \]
\[ \tau_{\text{act}} = \text{actuator time delay (s)} \]

The reaction time \( T_r \) should thus not be confused with the time needed to reach the desired state (frequently referred to as the relaxation time or acceleration time). For instance, in the model of Tampère it does not necessarily hold that the speed difference has vanished after the reaction time. Also the sensitivity parameters determine how quickly this occurs.

The model representations show that the IDM model, the OVM model and the Lenz model ignore a time delay. The influence of incorporating a reaction time in the OVM is studied in (Bando et al., 1998). Also the IDM has been extended to include a reaction time in the earlier mentioned HDM (Treiber et al., 2006a). For reasons discussed later in this chapter the developers of these models nevertheless prefer the use of the simpler IDM, lacking a reaction time, above the use of the HDM in most cases.

Whether models not including a reaction time can adequately predict the dynamics of human drivers will become clear from the empirical analyses presented in chapters 5 and 6, in which the performances of models including a reaction time are compared to models not including a reaction time.

### 3.5 Inclusion of human limitations in models

All model representations presented so far ignore that drivers face problems in:

- Perceiving stimuli exactly.
- Evaluating alternative control actions.
- Selecting an appropriate control action.
- Carrying out this action.

Only the Gipps model is an exception to this general observation as it takes care of the fact that the parameter \( b_{n+1}^{\text{max}} \), representing the maximum deceleration of the leading car, is not known to the driver of the following car and therefore needs to be estimated. Nevertheless also the Gipps model assumes that a driver is able to perceive \( v_{n+1} \) and \( \Delta x_{n+1,n} \) exactly (for \( v_n \) this is justifiable as the driver can read this value from his speedometer).

In literature several proposals are made to also incorporate these human characteristics in longitudinal driving behavior models. An important example is the action point concept. This concept, that can be incorporated in car-following models in many ways, incorporates findings from perceptual psychology showing that there are limits to the stimuli to which drivers respond (Michaels, 1963). It is assumed that the driver of a following vehicle is not influenced by speed differences with his leader when the distance between the two is large. Also for smaller distances there are combinations of relative speeds and distance headways for which there is no response of the following vehicle because the relative motion is small (Leutzbach, 1988). This is illustrated in Figure 3-2.

In Figure 3-2 the following vehicle first approaches his leader. As the distance to this leader is larger than the perceptual threshold the driver will not react. At a certain moment the perceptual threshold is reached and the follower starts to lower his speed. The driver is however not able to make his speed exactly equal to the speed of the leader. Possible reasons
are that the driver is not able to perceive small relative speeds, or that the driver is not able to control his braking pedal exactly enough. When the driver enters again the “zone with reaction” the opposite occurs. This in fact implies that the distance to the leader fluctuates over time even when the leader drives with a constant speed. These ideas are, for example, incorporated in the models presented in (Wiedemann, 1974) and (Fritzsche, 1994).

![Figure 3-2 Sketch of the behavioral assumptions made by action point models.](image)

Furthermore, in recent years techniques from fuzzy logic have been proposed to bridge the gap between the imprecise way in which people categorize observations (i.e. distance is too close to leader, instead of distance is 10.34 m.) and classical logic using mathematical techniques. Examples of fuzzy approaches for modeling longitudinal driving behavior are (Rekersbrink, 1994, 1995, Chakroborty and Kikuchi, 1999, Brackstone, 2000, Chakroborty and Kikuchi, 2003, Wu et al., 2003, González-Rojo et al., 2006, Harding, 2007).

Adding more “human aspects” to longitudinal driving models, in general leads to an increase in model complexity making the model more difficult to calibrate. For example, an empirical analysis of the action point model of Fritzsche (1994) would require calibration of a considerable number of thresholds (perception threshold for $\Delta V_{n-1,n} > 0$, perception threshold for $\Delta V_{n-1,n} < 0$, desired distance, risky distance, safe distance, braking distance). As these thresholds separate different regimes in longitudinal driving, this would call for sufficient data on all these regimes. Due to the fact that vehicles are only observed during a relatively short time period when using our data collection method (as will be shown in chapter 5), these thresholds will either be not attained during the observation or not long enough making calibration results very unreliable. The same applies to the well-known model of Wiedemann (Wiedemann, 1974). For this reason we decided not to include these more complex models in our empirical analyses.

### 3.6 Modeling differences between drivers and within drivers

In sections 3.3, 3.4, and 3.5 we considered how the different facets of the control loop presented in the previous chapter are incorporated in models quantitatively describing longitudinal driving dynamics. We thus focused at how models assume that a single driver
executes the longitudinal driving task. In this discussion we ignored the following two important aspects of human longitudinal driving behavior:

- Driver/vehicle combinations differ regarding their way of executing the longitudinal driving task.
- The longitudinal driving behavior of a single driver might change over time depending, for example, on the traffic conditions, the mental state of the driver and so on.

These aspects will be the topic of this section. In subsection 3.6.1 we discuss how driver/vehicle combination heterogeneity is currently incorporated in models, while we consider how modelers deal with changes in the behavior of a single driver in subsection 3.6.2.

### 3.6.1 Driver/vehicle combination heterogeneity

In modeling driver/vehicle combination heterogeneity, roughly stated, two options can be identified:

- It can be assumed that all driver/vehicle combinations apply the same objective, i.e. react to the same stimuli, while only the extents to which these respective stimuli play a role and to which the driver is able to react differ. In modeling language this means that only parameter values are assumed to differ between individual driver/vehicle combinations.
- The extent of heterogeneity may also be assumed more substantial, such that there are also differences between the objectives of driver/vehicle combinations. In this case models following different logics are assumed to apply to different driver/vehicle combinations.

In general it can be stated that most microscopic simulation tools apply the first option as comes clear from the following examples.

In the simulations performed in (Kesting et al., 2007a) only a distinction is made between cars and trucks. Both types of vehicles are assumed to have comparable objectives represented by the IDM longitudinal driving behavior rule. The main difference is the desired speed. This rather limited extent of heterogeneity is introduced to the simulations to induce lane changes leading to realistic lane distributions.

In the microscopic simulation program FOSIM (Dijker and Knoppers, 2004) five different types of driver/vehicle combinations are distinguished when the default settings are used, i.e. three types of person cars and two types of trucks. Again all different types of driver/vehicle combinations are assumed to apply the same control objective, while differences between these groups are expressed in different parameter values.

Also in commercial microscopic simulation tools as S-Paramics (SIAS Limited, 2005), VISSIM, (PTV, 2006) and AIMSUN (TSS-Transport Simulation Systems, 2006) heterogeneity of traffic flows is explicitly considered by allowing for different parameter values. For example, in S-Paramics several different types of vehicles are simulated all having their own specific dynamic properties regarding, for example, acceleration, deceleration and maximum speed. Next to that different driver behavioral characteristics are represented by aggression and awareness factors.
In VISSIM next to different parameter values, two different specifications of the Wiedemann model can be assigned to drivers pointing in the direction of a stronger degree of heterogeneity, i.e. differences between control objectives. This conclusion is however only partly true as one of these control objectives is mainly suited for urban traffic, while the other one is mainly suited for freeway traffic.

The examples presented in this subsection illustrate that modelers seem to agree on the fact that heterogeneity can not simply be ignored in microscopic modeling. Driver heterogeneity has a clear impact on the flow specific lane distributions, especially desired speeds are a key point in this sense. Also the role of heterogeneity with respect to other aspects of car-following can not simply be neglected. In dense traffic for example, lane changing possibilities decrease considerably forcing vehicles to drive behind each other on the same lane. In this situation it seems reasonable to assume that car-following heterogeneity can influence the resulting traffic flow dynamics (chapter 7 will provide clear support for this assumption).

The question is therefore not whether heterogeneity should be taken into account, but to which extent and how it should be modeled.

3.6.2 Adaptive changes in the longitudinal behavior of a single driver

In modeling changes in the longitudinal behavior of a driver again the same options can be identified, i.e. the control objective of a driver can either be assumed to change over time or to remain the same.

Like in the case of heterogeneity also changes in the behavior of a single driver are in general modeled by adjusting the parameter values of the same control objective as can be seen from the following examples. In (Brackstone and McDonald, 1999a) an overview is given of different studies in which the parameters of the model presented in (Gazis et al., 1961) are estimated. This overview shows that parameters are often separately estimated for accelerating and decelerating cars. In such a study presented in (Treiterer and Myers, 1974) it is found that drivers tend to place more emphasis on both their own velocity and the spacing between themselves and the leading vehicle while decelerating than while accelerating. Intuitively this can be easily interpreted by considering that drivers decelerate to avoid dangerous situations, while drivers accelerate to improve their situation.

In models applying the action point concept (section 3.5) often care is taken of intra-driver differences by varying the applied perception thresholds (Leutzbach, 1988). Perception thresholds are, for example, assumed to be smaller in the case of decreasing distances, than in the case of increasing distances. Thus it is assumed that people are able to perceive smaller relative speeds when they are coming closer to their leader then when they are increasing the distance. It is furthermore proposed to replace the perception thresholds by “perception areas” reflecting a random variation in the exact location of the perception threshold of a single driver.

Another example is the VDT (Variance Driven Time headways) concept proposed in (Treiber et al., 2006b). This concept proposes a variance-driven adaptation mechanism of the safety time gap of, for example the IDM, according to which drivers increase their gaps when the local traffic dynamics are largely varying.
An exception to the rule that changes in the longitudinal driving behavior of a driver are modeled by changing parameter values of the same control objective can be found in (Zhang and Kim, 2005). In this paper the functional form of the objective is assumed to vary depending on gap distance and traffic phase (acceleration, deceleration, and coasting).

3.7 Discussion on model validity

In this chapter we compared, among others, the control objectives assumed by different models. By considering the stimuli of the simplest models, we could already state that they were too simple, i.e. ignored important stimuli. For more elaborated models it is not possible to simply indicate by looking at the model assumptions which models best approximate the longitudinal driving behavior of real drivers. For several stimuli, like the number of leaders considered in executing the longitudinal driving task, no empirical knowledge is available on whether they play a role and related to this, whether they need to be incorporated in longitudinal driving models.

For this reason the models need to be compared to the dynamics of real drivers. In this way it can be established which stimuli play a role and which models best reflect the driving behavior of real drivers.

When longitudinal driving models are applied in microscopic simulations for the sake of prediction, the consideration of chapter 2 is also important. That is, models have to be as complex as needed for getting realistic simulation results, but they should not be over complex as this results in an increase of the computational burden and calibration problems.

For example, in this chapter we saw that the IDM assumes that a driver considers only one leader in his car-following behavior. The model furthermore ignores the existence of a time delay in the reaction of a driver to changes in stimuli. The more elaborated version of this model, i.e. the HDM, considers multiple leaders ahead and incorporates a reaction time. By adding these features of human behavior, it seems intuitively justified to conclude that the model resembles a real driver closer.

After performing simulations with this extended model incorporating several human aspects it was however found that the negative effects of finite reaction times could be compensated for to a large extent by multi-anticipation and temporal anticipation. That is, the qualitative macroscopic dynamics predicted by the HDM turned out to be almost similar to the qualitative macroscopic dynamics predicted by the simpler IDM not including these aspects. For this reason often the simple IDM is used in simulations instead of the more complex HDM although this last model incorporates more human characteristics, see for example (Kesting et al., 2007b).

In this case the different additions thus seem to outweigh each other, motivating the use of a simpler model in simulations. The question that should be posed in this context is of course in how far this conclusion depends on the specific model assumptions made and whether these assumptions are realistic. That is, when multi-anticipation is incorporated in another way in the model is the conclusion that the time delay and multi-anticipation outweigh each other still valid?

Again we can conclude that to assess whether the assumptions are realistic or not thorough analyses of detailed microscopic observations are needed. Only such analyses can give
insights into the number of leaders a driver considers in his longitudinal driving behavior and to which extent.

After this thorough analysis of the observations it deserves serious recommendation to reconsider, like mentioned before, whether some factors are indeed outweighing each other and thus whether a simpler model suffices. To actually make this decision also real-life macroscopic observations are needed giving profound insights into, for instance, traffic phases and phase transitions to which the model predictions can be compared.

### 3.8 Summary and conclusions

In this chapter we used the insights obtained in chapter 2 regarding the way humans execute their longitudinal driving task, to compare existing mathematical models describing the longitudinal dynamics of a driver/vehicle combination based on the dynamics of the leading car(s). We thereby focused on the models that will be used throughout this thesis for testing hypotheses on longitudinal driving behavior.

We showed that the control objectives assumed by these models differ considerably. For example, different models make different assumptions on the number of leaders considered by the follower. This particular disagreement is mainly caused by a lack of empirical evidence on multi-anticipation in longitudinal driving behavior.

We also saw that only part of the models include time delays people face in executing their control task, while human limitations are in most cases neglected completely.

Next to that we considered how modelers currently deal with differences between the longitudinal driving behaviors of drivers and changes in the longitudinal driving behavior of single drivers. We found that, roughly stated, both features can be modeled in two ways. In the first one, only parameter values of the same model are changed, thus in fact the control objective is assumed to remain the same, while in the stronger case also the control objective itself is assumed to alter. We concluded that in modeling differences between the behaviors of drivers as well as differences within the behavior of a single driver, in most cases the weakest option was taken, i.e. the control objective was assumed to stay the same.

In the sequel of this thesis, we will calibrate the different models discussed in this chapter using trajectory observations for a large sample of drivers. In comparing the model performances, we gain insights into the appropriateness of the assumed control objectives in describing the longitudinal dynamics of real drivers. By analyzing the estimated parameter values, we obtain information on the extents to which the various stimuli incorporated in these objectives are considered. Based on the insights obtained from these analyses, we also propose new longitudinal driving models.

As calibration results play a dominant role in our empirical analyses, we will in the next chapter propose a microscopic calibration method and thoroughly examine the sensitivity of the calibration results returned by this method to, for example, measurement errors. This analysis of the calibration method provides important insights into how we can make the calibration method more robust against measurement errors and how to interpret calibration results.
4 Validity of trajectory based microscopic calibration approach

4.1 Aim and structure of this chapter

The previous chapter introduced several different longitudinal driving models, each making different assumptions on longitudinal driving behavior. In the sequel of this thesis we calibrate these models and compare their performances with the main aim to gain new empirical insights into longitudinal driving behavior. The advantage of this model based approach is that we are able to quantitatively express our findings on longitudinal driving behavior. For example, an estimate of the sensitivity of a driver to a stimulus expresses the extent to which this stimulus influences the behavior of this driver.

Given this important role of calibration results in our empirical analyses, it is clear that the reliability of our conclusions on longitudinal driving behavior is directly related to the validity of the applied calibration approach. In developing such an approach, knowledge from other research fields can advantagely be used. This pertains mostly to algorithms developed for efficiently searching parameter space for (locally) optimal parameters. Nevertheless several important questions regarding calibration of longitudinal driving models using microscopic trajectory observations and regarding inferences drawn from calibration results on longitudinal driving behavior remain unresolved till now.

For instance, commonly used microscopic calibration approaches result in autocorrelated error terms making standard statistical tests inapplicable. Given this problem, several important questions arise, for example, how can the reliability of parameter estimates be assessed? And, how can models having different complexities be compared?

Also the influences of methodological choices such as the functional specification of the calibration objective and the selected variable(s) in the calibration objective on calibration results are not yet clear. Although several empirical studies indicated differences in parameter values obtained using, for instance, the speed or the distance headway in the calibration objective (Ranjitkar et al., 2004), no detailed background research has been performed on this
topic so far. In our studies this background information is indispensable, as different parameter values lead to different conclusions on longitudinal driving behavior.

Next to that, a better understanding of the impact of several fundamental issues related to the use of empirical observations is needed. For example, how valid and reliable are estimated parameter values in the case of measurement errors? And, what do parameter estimates tell about the driving behavior of an individual given that the calibrated model is only a (rough) approximation of the complex real driving behavior?

In this chapter we aim at answering these questions in order to develop a robust calibration approach providing us with reliable calibration results in our empirical analyses.

This chapter consists of two related parts. In the first part we start by introducing a generic calibration framework (section 4.2). Based on this generic framework we propose heuristic methods for determining the reliability of parameter estimates and comparing models of different complexities (section 4.3).

In the second part, we examine the impact of both methodological factors and observational characteristics on parameter estimates using a systematic experimental design based on synthetic trajectory data. We first give an overview of these factors in section 4.4. In sections 4.5 and 4.6 we consecutively introduce our research questions on the impacts of these factors and discuss the synthetic data based approach for answering these research questions. The results are presented in sections 4.7, 4.8, 4.9 and 4.10. In each of these sections we show the impact of methodological choices on estimated parameter values when using microscopic trajectory observations having specific characteristics. That is, we consider four scenarios in which observations are either clean or noisy and in which we do either have full information or missing information on the longitudinal driving model describing the observed longitudinal driving behavior. In section 4.11 we complete this analysis by considering the influences of characteristics of trajectory observations on the reliability of parameter estimates.

Our analyses show a clear negative impact of measurement errors on both estimated parameter values as well as on the reliability of these values. We therefore conclude the chapter by proposing two methods for reducing the negative impact of measurement errors in section 4.12.

### 4.2 Generic microscopic calibration framework

In this section we provide a generic description of a trajectory based microscopic calibration framework of longitudinal driving models.

Let $z_n(t)$ denote the actual state of driver $n$ at time instant $t$. As we discussed in chapter 3 the specification of the state depends on the longitudinal driving model that is considered. In most cases, the state consists of the position $x_n(t)$ and the speed $v_n(t)$ of the driver. Let the vector $\xi_n(t)$ denote the time-dependent traffic state to which driver $n$ may react or anticipate. This vector may, for example, contain (estimates of) positions and speeds of driver/vehicle combinations driving directly in front of driver $n$ on the same lane. It always contains the state $z_n(t)$ of the driver $n$ himself. For the models that are calibrated in this thesis, the traffic state can be defined completely by:

$$\xi_n(t) = (z_{n-1}(t), \ldots, z_n(t))$$

(4.1)
where \( j \) is the number of direct leaders considered.

The longitudinal driving models of chapter 3 can then be described in either of the following generic forms:

\[
\frac{d}{dt} x_n(t) = f(\xi_n(t), \xi_n(t - T_r) | \beta_n) \tag{4.2}
\]

when the model predicts the speed of driver \( n \) like in the Gipps model or

\[
\frac{d}{dt} x_n(t) = f_1(\xi_n(t)) = v_n(t) \tag{4.3}
\]

\[
\frac{d}{dt} v_n(t) = f_2(\xi_n(t), \xi_n(t - T_r) | \beta_n) \tag{4.4}
\]

when the model predicts the acceleration of driver \( n \) like in the model of Tampère.

In these equations \( \beta_n \) denotes the set of parameter values describing the longitudinal driving behavior of driver \( n \), including the reaction time \( T_r \). More generally, we can define the state \( z_n(t) \) of driver \( n \) as either \( z_n(t) = x_n(t) \) or \( z_n(t) = (x_n(t), v_n(t)) \) and write:

\[
\frac{d}{dt} z_n(t) = f(\xi_n(t), \xi_n(t - T_r) | \beta_n)) \tag{4.5}
\]

The calibration approach applied in this thesis entails comparing the state predictions \( \hat{z}_n(t_k) \) of the longitudinal driving model when using parameter set \( \hat{\beta}_n \) at discrete time instants \( t_k \) with \( k = 1, \ldots, K \) with the available microscopic trajectory observations, denoted by \( y(t_k) \). To obtain the predictions of the longitudinal driving model, eq. (4.5) is integrated given the initial conditions \( (y_n(t_1)) \), i.e.:

\[
\hat{z}_n(t_k) = \begin{cases} 
  y_n(t_1) & \text{for } k = 1 \\
  \hat{z}_n(t_{k-1}) + \int_{t_{k-1}}^{t_k} f(\xi_n(s), \xi_n(s - T_r) | \hat{\beta}_n) \, ds & \text{otherwise}
\end{cases} \tag{4.6}
\]

Mathematically the calibration process can be described as finding the parameters \( \beta^*_n \) satisfying,

\[
\beta^*_n = \arg\min_{\beta_n} g(y_n, \hat{z}_n) \tag{4.7}
\]

where \( g \) is the calibration objective quantifying the difference between the observed state and the predicted state.

In the remainder we will, like in the previous chapter, loose the subscript \( n \) from \( \beta_n \) for notational convenience.
4.3 Drawing inferences from calibration results

In our empirical analyses, we draw inferences on longitudinal driving behavior from estimated parameter values and comparing the performances of mathematical models describing this behavior. Before an estimated parameter value can be used in these analyses, we first need to determine whether the estimate is reliable, i.e. whether the applied trajectory data contained enough information to identify the parameter.

To obtain new insights into longitudinal driving behavior from comparing model performances, we furthermore need to specify a method for comparing models having different complexities. This holds especially for the analyses on multi-anticipative car-following behavior. In these analyses we need to establish whether a model including an additional stimulus regarding a leader further downstream performs significantly better than a simpler model not including this stimulus.

A problem in determining the reliability of parameter estimates and comparing model performances is that the calibration method specified above results, like other commonly used methods, in autocorrelated error terms. With autocorrelated error terms we mean that differences between model predictions and observations at subsequent time instants are not independent (see for example (Hoogendoorn and Ossen, 2006)). As a result of this, standard statistical tests can not be directly applied. We therefore propose heuristics for determining the reliability of parameter estimates and comparing models of different complexities in respectively subsection 4.3.2 and 4.3.3. In subsection 4.3.1, we explain the causes for autocorrelated error terms in more detail.

4.3.1 Causes for autocorrelated error terms

Common sources for autocorrelated error terms are the omission of explanatory variables, misspecification of the mathematical model or interpolation of observations (e.g. due to smoothing). In microscopic calibrations based on the framework above two additional sources for autocorrelation can be pointed at.

Firstly, in the specified calibration framework, we obtain the trajectory predicted by the model by simulating the trajectory of the follower for the whole period of observation in the same way as would be done in a microscopic simulation tool (eq. (4.6)). This means, for instance, that the predicted position of driver \( n \) at time instant \( t_{k+1} \) is determined using the predicted speed of driver \( n \) and the predicted position of driver \( n \) at time instant \( t_k \). A deviation of the predicted position at time instant \( t_k \) from the real position at time \( t_k \) is thus due to the applied integration also directly of influence to the predicted position at time \( t_{k+1} \).

Another complicating factor is that the dynamics of the follower are updated at every time step based on the model, the observed state of the leader(s), and the current predicted dynamics of the follower. This has the disadvantage that deviations between the observations and predictions not only influence the error term at a given time instant, but are also introduced indirectly into the next prediction step as it is assumed that followers update their dynamics based on their simulated current dynamics instead of their observed current dynamics.

4.3.2 Reliability of parameter estimates

As we will discuss later on in this chapter, it is not always possible to identify all model parameters from observations (subsection 4.4.5). Since we use parameter estimates in our
empirical analyses to gain insights into observed longitudinal driving behavior, we need to be able to determine whether a parameter estimate is reliable.

A commonly used method in statistics for determining the reliability of a parameter estimate is to consider the variance of the estimator. It generally holds that the smaller this variance, the more reliable the parameter estimate is. However, as we need to deal with autocorrelated error terms, conventional methods to determine this variability most probably underestimate it (Greene, 2000).

To be nevertheless able to assess the reliability of parameter estimates in our empirical analyses, i.e. to identify parameter identification problems, we consider the second-order partial derivative of the calibration objective \( g(y_n, z_n) \) with respect to the model parameter under consideration \( \beta_i \) at the obtained calibrated parameter value \( \beta_i^* \):

\[
\frac{\partial^2 g(y_n, z_n)}{\partial \beta_i^2} \bigg|_{\beta_i = \beta_i^*} \quad (4.8)
\]

We select the second-order partial derivative of the calibration objective as a measure for the reliability of a parameter estimate, due to its relation with the variance of the estimator. This is illustrated in Intermezzo I for maximum likelihood estimators.

Although the sensitivity of the objective to small changes in parameter values will, because of autocorrelated error terms, most likely be overestimated compared to the case with no autocorrelation, it still provides useful insights into the reliability of the estimate. That is, the larger the sensitivity of the objective function to a small change in the estimated parameter value, the more pronounced the minimum is and the more weight can be attached to the parameter estimate. In setting a threshold for the minimum required sensitivity of the objective to a parameter estimate before considering it as reliable, we can take the overestimation problem into account by increasing the threshold.

In using the second-order partial derivative as a measure for the reliability of a parameter estimate, we need to take care that without normalization the relative size of a small parameter change \( \partial \beta_i \) depends on the range of the parameter. In order to be able to compare the second derivatives between parameters having considerably different ranges, we therefore normalize the obtained values. To normalize the second-order partial derivatives, we define a new scale \( \phi \) with:

\[
\phi_i = \frac{\beta_i}{m} \quad (4.9)
\]

where \( m \) is equal to the median value of the parameter estimates for all considered followers for a specific parameter. Within this new scale a change \( \partial \phi_i \) is of relatively the same size for all parameters. In eq. (4.10) we derive the relation between the ‘unscaled’ second derivates determined using eq. (4.8) and the corresponding ‘scaled’ values.

This derivation thus shows that the ‘unscaled’ second derivatives obtained using eq. (4.8) can be scaled by multiplying them with the square of the corresponding median parameter estimate.
\[
\frac{\partial^2 g}{\partial \phi_i^2} = \frac{\partial}{\partial \phi_i} \frac{\partial g}{\partial \phi_i} = \frac{\partial}{\partial \phi_i} \left( \frac{\partial g}{\partial \beta_i} \frac{\partial \beta_i}{\partial \phi_i} \right) = \frac{\partial}{\partial \phi_i} \left( \frac{\partial g}{\partial \beta_i} \right) = m \left( \frac{\partial}{\partial \phi_i} \left( \frac{\partial g}{\partial \beta_i} \right) \right) = m \left( \frac{\partial}{\partial \beta_i} \left( \frac{\partial g}{\partial \beta_i} \right) \right) = m^2 \frac{\partial^2 g}{\partial \beta_i^2}
\]

(4.10)

### Statistical interpretation of the variance of a point estimator and its relation with the second derivative of the calibration objective

Two important criteria for evaluating an estimator of a parameter \( \beta \) are the bias of the estimator and the variance of the estimator. An estimator with a lower variance is preferred to an estimator with a higher variance (ceteris paribus).

The Cramer-Rao Lower bound states that the variance of an unbiased estimator of a parameter \( \beta \) will always be at least as large as:

\[
\left( E \left[ \left( \frac{\partial \ln L(\beta)}{\partial \beta} \right)^2 \right] \right)^{-1}
\]

In which the function, denoted \( L(\beta) \) is the likelihood function for \( \beta \) given the data \( X \). It can be proven that the negative of the expected second derivative equals the expected square of the first derivative.

For an estimator as the Maximum Likelihood Estimator (MLE) it can be shown that it is asymptotically efficient, that is the variance goes to the Cramér-Rao Lower Bound as \( n \) (sample size) \( \rightarrow \infty \). This result can be used to estimate the asymptotic variance of the maximum likelihood estimator (Greene, 2000). That is, if the form of the expected values of the second derivatives of the loglikelihood is known,

\[
\left( E \left[ \left( \frac{\partial \ln L(\beta)}{\partial \beta} \right)^2 \right] \right)^{-1} = \left( -E \left[ \frac{\partial^2 \ln L(\beta)}{\partial \beta \partial \beta'} \right] \right)^{-1}
\]

can be evaluated at \( \hat{\beta} \) to estimate the covariance matrix of the MLE. This estimator will however be rarely available. That is the second derivatives of the log-likelihood will almost always be complicated nonlinear functions of the data, whose exact expected values will be unknown. An alternative estimator for the covariance matrix of the MLE is therefore to evaluate the actual second derivative matrix of the log-likelihood function at the maximum likelihood estimates:

\[
\left( -\frac{\partial^2 \ln L(\hat{\beta})}{\partial \hat{\beta} \partial \hat{\beta'} \hat{\beta'}} \right)^{-1}
\]

This last result points at the relation between the variance of an estimate and the second derivative.

**Intermezzo I** Illustration of relation between the variance of a point estimator and the second derivative of the calibration objective (based on (Greene, 2000)).
4.3.3 Comparison of models having different complexities

In our empirical analyses on multi-anticipation we compare the performances of models making different assumptions on the number of leaders influencing the longitudinal driving behavior of the follower. We more specifically consider a single leader model, like the Tampère model and establish for all drivers whether the performance of this model significantly improves when an additional stimulus referring to a leader further downstream is added.

In statistical terms this means that the following hypothesis needs to be tested:

\[ H_0: \beta^*_i = 0 \]

where \( \beta^*_i \) is an estimated model parameter referring to a stimulus related to a leader further downstream. Stated differently, it needs to be tested whether the model performs significantly better in its unrestricted form, including the additional stimulus referring to the leader further downstream, than in its restricted form in which the sensitivity corresponding to the additional stimulus is assumed to be equal to zero.

Due to the aforementioned autocorrelated error terms, we do not fulfill all conditions necessary for testing this hypothesis using regular statistical tests. We will therefore develop a heuristic approach to establish whether an unrestricted model performs better than a restricted model. To this end, we will first discuss conventional statistical methods for dealing with different numbers of parameters and we explain why we can not use them. We then continue by proposing our heuristic approach based on these existing statistical tests.

In (Greene, 2000) the following two methods for testing the hypothesis are discussed for the case in which maximum likelihood estimation is applied:

- **The likelihood ratio test**: This test requires estimation of both the restricted model and the unrestricted model. The basic idea is that if the restriction is valid, then imposing it should not lead to a large reduction in the log-likelihood function.

  \[ \lambda = \frac{\hat{L}_{\text{restricted}}}{\hat{L}_{\text{unrestricted}}} \]

  Where \( \hat{L}_{\text{restricted}} \) is the value of the likelihood function for the restricted, single-leader model and \( \hat{L}_{\text{unrestricted}} \) refers to the value for the unrestricted, multi-leader model. An important property of this ratio is that the large sample distribution of \(-2\ln \lambda\) is chi-squared with degrees of freedom equal to the number of restrictions. Based on this property it is possible to determine exactly for which values of \( \lambda \) the hypothesis can (not) be rejected. Stated differently, \( \lambda \) specifies the minimum improvement in the likelihood the unrestricted model needs to yield before it is accepted as being the better model.

- **The Wald test**: Contrary to the likelihood ratio test, this test is based on the estimates for the unrestricted model only. The basic notion is that if the restriction is valid, then the maximum likelihood estimate of \( \beta^*_i \) should be close to zero. The Wald test is
consequently based on measuring the extent to which the unrestricted estimate fails to satisfy the hypothesized restriction that $\beta_i$ is zero.

There are two problems in applying these tests to our calibration results. Firstly, due to autocorrelated error terms, the usual method for determining the likelihood of the trajectory of a single follower cannot be applied. The likelihood of a trajectory, i.e. the joint probability of the consecutive observations, cannot be determined by taking the product of the probabilities of the observations at single time steps due to the fact that these probabilities are correlated.

The second problem is that to perform the significance test belonging to the Wald test, we need to determine the variance of estimators. However, as stated before we cannot use conventional methods to determine this variance as they most probably underestimate it (Greene, 2000). Consequently, we cannot apply standard statistical significance tests. Thus both the likelihood ratio test as well as the Wald test cannot directly be used in our empirical analyses.

In traditional regression analyses the number of model parameters, i.e. the model complexity, is also often taken into account by evaluating the adjusted r-squared ($R^2_{adj}$) instead of the r-squared ($R^2$), where the relation between the two measures of performance is as described in eq. (4.12).

$$R^2_{adj} = 1 - \frac{\omega - 1}{\omega - J} \left( 1 - R^2 \right)$$  \hspace{1cm} (4.12)

![Figure 4-1](image)

*Figure 4-1 In traditional regression analyses the performance measure $R^2$ is adjusted for the number of parameters, i.e. for every additional parameter a penalty is introduced.*

In this formula, $\omega$ represents the number of observations, while $J$ is the number of regressors. It can be roughly stated that also in this approach, like in the likelihood ratio test, a model having more parameters is penalized as can be seen from Figure 4-1. A more complex model is thus only accepted as being better when the resulting improvement in the performance compared to the simpler model is large enough as can be seen from Figure 4-1.

We follow the same reasoning in our heuristic approach. That is, in our analyses on multi-anticipation, we only accept a multi-leader model specification to be better than a single leader model when its performance in predicting the movements of the individual driver clearly improves. The only problem is that since we do not fulfill the requirements posed by standard statistical techniques, like the aforementioned likelihood ratio test and the adjustment of the $R^2$ value, we are not able to exactly specify the improvement that is needed by a more
complex model before it is accepted as being better than the corresponding restricted model specification. Stated differently, we are not able to specify exactly which penalty needs to be added to the performance of a multi-leader extension of a single leader model in order to correct for the larger number of parameters.

To handle this problem, we introduce strong penalties on the performances of models when using more parameters. That is, we compute for all models and for all drivers penalized error terms using:

\[ \varepsilon^{\text{pen}} = \varepsilon(1 + \omega)^p \]  

In which \( \omega \) corresponds to the relative penalty imposed to the performance for every additional parameter \( p \).

In the analyses on multi-anticipation, we assume the parameter \( \omega \) to be equal to 0.2 meaning that we require an improvement in the predictive performance of 20\% (at least) for each additional parameter.

4.4 Overview of factors presumably influencing parameter estimates

In the previous sections, we introduced a generic microscopic calibration framework and we proposed heuristics for drawing inferences from calibration results.

In the upcoming, we will consider the calibration process itself in more depth. We will more specifically, discuss a broad range of factors that are expected to influence the calibration process and thereby calibration results. We first identify categories of factors possibly influencing calibration results in subsection 4.4.1. After which we consider these categories of factors in more depth in separate subsections.

4.4.1 Four categories of factors presumably influencing the calibration process

To identify categories of factors influencing the calibration process, we reconsider the generic calibration framework presented in section 4.2. Based on this framework we can distinguish four categories of factors possibly influencing calibration results as illustrated in Figure 4-2.
The four categories refer to:

- The definition of the calibration objective $g$ (subsection 4.4.2).
- The optimization algorithm (subsection 4.4.3).
- The method for obtaining model predictions (subsection 4.4.4).
- The characteristics of state observations (subsection 4.4.5).

In the upcoming subsections these categories of factors are discussed in more detail.

### 4.4.2 Definition of calibration objective

In defining a calibration objective $g$ two decisions need to be made. Firstly the functional form of the calibration objective needs to be specified. Secondly, it needs to be decided which element(s) of the state of driver $n$ need to be considered by the calibration objective. Both topics are discussed in the upcoming.

#### Functional specification of calibration objective

The function $g$ in eq. (4.7) quantifies the difference between the observed state $y_n$ of driver $n$ and the state of driver $n$ as predicted by the longitudinal driving model, i.e. $\hat{z}_n$. The function $g$ can be specified in several different ways. Below we briefly describe the functional forms which are most commonly applied in microscopic calibration studies.

The performance measures given by eq. (4.14) and eq. (4.15) minimize an absolute difference between the observed state $y_n$ and the predicted state $\hat{z}_n$. The difference is that eq. (4.15) considers the squared difference such that in the optimization process larger weight is assigned to larger differences, while all differences get the same weight in eq. (4.14). In the sequel of this chapter eq. (4.14) will be referred to as mae (mean absolute error) and eq. (4.15) will be referred to as rmse (root mean squared error).

$$ g(y_n, \hat{z}_n) = \frac{1}{K} \sum_{k=1}^{K} |y_n(t_k) - \hat{z}_n(t_k)| $$

(4.14)

$$ g(y_n, \hat{z}_n) = \sqrt{\frac{1}{K} \sum_{k=1}^{K} (y_n(t_k) - \hat{z}_n(t_k))^2} $$

(4.15)

The objective value calculated by eq. (4.14) is often divided as a whole by the mean of $y_n$ (see for example (Brockfeld et al., 2004)). For optimization purposes this division is irrelevant as it simply implies that the value calculated by eq. (4.14) is at the end always divided by a constant term.

Another possibility in specifying $g(y_n, \hat{z}_n)$ is to consider relative differences between the observed state $y_n$ and the predicted state $\hat{z}_n$ (eq. (4.16) and eq. (4.17)), i.e. to divide the difference at every considered discrete time instant $t_k$ by $y_n(t_k)$. These specifications consequently place relatively more weight on differences for which the values representing the observed state are small. In the sequel these relative performance measures will respectively be labeled mrae (mean relative absolute error, eq. (4.16)), and rmsre (root mean squared relative error, eq. (4.17)).
Chapter 4 - Validity of trajectory based microscopic calibration approach

\[ g(y_n, \hat{z}_n) = \frac{1}{K} \left( \sum_{k=1}^{K} \frac{|y_n(t_k) - \hat{z}_n(t_k)|}{|y_n(t_k)|} \right) \]  
\[ (4.16) \]

\[ g(y_n, \hat{z}_n) = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \left( \frac{y_n(t_k) - \hat{z}_n(t_k)}{y_n(t_k)} \right)^2} \]  
\[ (4.17) \]

A last possibility that will be considered is Theils’u (Theil, 1958),

\[ g(y_n, \hat{z}_n) = \sqrt{\frac{1}{K} \sum_{k=1}^{K} (y_n(t_k) - \hat{z}_n(t_k))^2} \]  
\[ \sqrt{\frac{1}{K} \sum_{k=1}^{K} (y_n(t_k))^2} + \sqrt{\frac{1}{K} \sum_{k=1}^{K} (\hat{z}_n(t_k))^2} \]  
\[ (4.18) \]

The outcome of this calibration objective is, except from the trivial case where all elements of \( y_n \) and \( \hat{z}_n \) are zero, confined to the closed interval between zero and one. In general, it can be stated that the smaller the outcome of this objective the better the match between the observed states \( y_n \) and the predicted states \( \hat{z}_n \).

All specified functional forms for the calibration objective share the common characteristic that negative differences and positive differences between the predicted states and the observed states are treated equally.

In (Punzo and Simonelli, 2005) the influence of the functional form of the calibration objective on calibration results is discussed. It is suggested that different specifications of the functional form lead to different parameter estimates in the case that calibrated longitudinal driving models are only approximations of the driving rule governing the observed longitudinal driving behavior of driver \( n \). According to this reasoning both model imperfections as well as measurement errors can result in differences between calibration results obtained when using different functional specifications for \( g \). Unfortunately, no detailed background analyses were performed to gain insight into this suggested influence of the objective and to prove these statements. In this chapter we therefore aim, among others, at establishing whether the functional specification of the calibration objective affects calibration results and if so, which specification can best be used in our empirical analyses.

**State variable in calibration objective**

The previous related to the functional specification of the calibration objective quantifying the difference between the observed state \( y_n \) and the predicted state \( \hat{z}_n \). We showed before that the state of driver \( n \) can consist of several elements like the speed \( v_n \) or the position \( x_n \). Consequently it needs to be specified which state element(s)/variable(s), should be incorporated in the calibration objective \( g \).

Variables like the speed and the position are clearly strongly related (see eq. (4.2), eq. (4.3) and eq. (4.4)). The empirical analyses presented in (Ranjitkar et al., 2004) reveal, however, differences between parameter estimates that are obtained using the speed and the distance headway in the calibration objective. This may have several explanations.
Firstly, suppose that a longitudinal driving model predicts the speed. According to eq. (4.2) and eq. (4.6) the predicted positions of driver $n$ are calculated based on the initial position of driver $n$ and the speed pattern of driver $n$. For parameter estimation this implies that when the speed $v_n$ is chosen in the calibration objective, the optimization algorithm matches the predicted speeds with the observed speeds. However, when the position $x_n$ is chosen, the algorithm matches the predicted positions with the observed positions. In order to do so, the speed can be used to compensate for errors made earlier. That is, when the predicted position of the vehicle is smaller than the observed position at a certain time instant it is favorable for the calibration objective when the predicted speed is at the next time instant larger than the observed speed.

Also measurement errors can cause differences between calibration results obtained using different elements of the state of driver $n$ in the calibration objective (Ranjitkar et al., 2004). One reason for this is that measurement errors on different state representatives can be of different size. For instance, when differentiating observed positions containing measurement errors to obtain speeds, errors can be amplified.

In this chapter we use a systematic experimental design based on synthetic trajectory observations to investigate how the state variable selected in the calibration objective affects calibration results. We will repeat these analyses for observations having different characteristics (sections 4.7, 4.8, 4.9, 4.10, 4.11). Based on these analyses, we decide which state variable(s) will be used in the empirical analyses of chapters 5 and 6.

4.4.3 Optimization algorithm

Once the calibration objective is defined, an appropriate method for minimizing it needs to be selected. This subject will be treated in this subsection.

Optimization algorithm

In choosing an algorithm to minimize the calibration objective, it has to be decided whether it is possible to enumerate over all possible solutions or whether the space of solutions contains too many elements to make this feasible.

The longitudinal driving models considered in this thesis contain up to nine parameters. All these parameters, except from the reaction time $T_r$, are assumed to be continuous and can consequently be equal to an infinite number of values making it practically impossible to enumerate over all solutions. Consequently an algorithm needs to be applied approximating the global minimum.

Suitable optimization algorithms developed in other research fields are mostly used for efficiently searching parameter space. Typical examples are the simplex method (Brockfeld et al., 2004, Ossen et al., 2006) and genetic algorithms (Ranjitkar et al., 2004). In this thesis, we will apply the simplex method. The use of this method is validated in Appendix F.

Use of prior information

In developing an optimization algorithm it needs to be decided whether the algorithm uses prior information on parameter values or not. Reasons for using such information can be to guarantee that the estimated parameter values fulfill certain requirements regarding for example the stability characteristics of the longitudinal driving model or the characteristics of the corresponding fundamental diagram.
For a more detailed discussion on the use of prior information we refer to (Hoogendoorn et al., 2007b). This topic will not be discussed in the remainder of this chapter.

### 4.4.4 Method for obtaining model predictions

Like the definition of the calibration objective and the optimization algorithm, also the method used for obtaining model predictions is expected to influence calibration results.

As stated by eq. (4.6) the states predicted by the model are generally obtained by simulating the movements of the following vehicle $n$ based on the driver dependent model parameters $\beta$, the initial conditions of driver $n$ and the observed traffic state $\xi_n$. This approach resembles the approach taken by a microscopic simulation tool.

Another possibility is to equalize the predicted state $\hat{z}_n$ and the observed state $y_n$ at given time instants belonging to the vector $t_{\text{reset}}$.

$$z_n(t_k) = \begin{cases} y_n(t_k) & \text{for } t_k \in t_{\text{reset}} \\ z_n(t_{k-1}) + \int_{t_{k-1}}^{t_k} f(\xi_n(s), \xi_n(s - T_k) | \beta) \, ds & \text{otherwise} \end{cases} \quad (4.19)$$

The reason for resetting the simulations can, for example, be to ensure that the predicted state of driver $n$ does not deviate too much from the corresponding observed state. In this chapter we explore whether estimated parameter values become less biased when predictions and observations are equalized at specified time instants.

### 4.4.5 Characteristics of state observations

Till now several facets have been discussed referring to the methodological part of a calibration process, i.e. the definition of the calibration objective, the optimization algorithm and the method used for obtaining model predictions. In the upcoming we consider several characteristics of microscopic trajectory observations that are expected to influence the calibration process.

**Measurement errors**

Despite all technological improvements it can be stated that trajectory observations always contain measurement errors. These measurement errors are often even increased when indicators are derived from the measured variable by means of taking the derivative.

Measurement errors can occur in many different ways. They can, for example, be symmetrically distributed around the correct values, or they can have a systematic component. Depending on the measurement method observations can be more precise for the following vehicle than for the leading vehicle (equipped vehicle, active mode), or the other way around (passive mode) (Appendix C).

These measurement errors enter the calibration procedure as follows. Let $\varepsilon_n(t_k)$ denote the measurement error at a given state observation of driver $n$ at time instant $t_k$. By using the state observations $y_n$ of driver $n$ in the optimization process as proposed in eq. (4.7), we ignore that $y_n$ is not equal to the true state of driver $n$ denoted by $z_n$, i.e.,

$$y_n(t_k) = h(z_n(t_k)) + \varepsilon_n(t_k) \quad (4.20)$$
In which \( h \) denotes the measurement equation (Haykin, 2001).

When also the trajectory observations of the direct leaders contain measurement errors and we assume these observations to represent the true states of these drivers, we can denote the ‘traffic state’ used in the optimization as:

\[
\tilde{\xi}_n(t) = (z_{n-1}(t) + \varepsilon_{n-1}(t), \ldots, z_n(t) + \varepsilon_n(t))
\]

(4.21)

This means that measurement errors enter the calibration process in three ways:

- Measurement errors with respect to the true states of the leading vehicles affect the predicted state of driver \( n \), i.e. \( \hat{z}_n(t_{k+1}) \) as can be seen from eq. (4.6).
- A measurement error with respect to the initial state of driver \( n \) influences all future state predictions of driver \( n \) (eq. (4.6)). When observations and model predictions are equalized at specified time instants \( t_k \) as proposed in eq. (4.19), the same reasoning can be applied to measurement errors regarding the states of driver \( n \) at time instants belonging to \( t_{reset} \).
- In optimizing, the predictions \( \hat{z}_n \) made by the longitudinal driving model are compared to erroneous state observations of driver \( n \) (eq. (4.7)).

The question is, given these influences of measurement errors, is it still possible to identify the parameters best describing the longitudinal driving behavior of the observed driver? If not, how large is the bias of the estimated parameters and how reliable are the estimated parameter values? These questions will be handled in this chapter.

**Missing information about the underlying longitudinal driving model**

It is impossible to capture all aspects of complex human behavior in equations. This means that longitudinal driving models that are calibrated only approximate human longitudinal driving behavior. When taking this into account a fundamental question is, what do estimated parameter values tell about the real underlying driving mechanisms of the observed driver? Stated differently, is it possible to derive meaningful conclusions about the longitudinal driving behavior of an observed driver given that the calibrated longitudinal driving model is not the model fully describing the observed behavior? This question will be handled in sections 4.9 and 4.10.

**Richness of the trajectory observations**

A final point that is considered here in using empirical trajectory observations for estimation is whether the observations contain enough information to estimate all model parameters. Suppose, for example, that the leading and following car drive with (almost) constant speed during the observation, then it becomes almost impossible to obtain reliable estimates for parameters pertaining to the dynamical part of car-following. To estimate these parameters the observed state variables need to show enough variability.

Another possibility is that only model parameters can be estimated referring to a specific traffic regime. Suppose, for example, that a car-following model distinguishes between a free-driving regime and a car-following regime. When the observation was performed during congestion it is generally not possible to reliably estimate the free-driving parameters, simply because this regime was not observed. Consequently, the (normalized) sensitivity of the calibration objective to small changes in the values of the parameters referring to the free-
driving regime will, under these circumstances, generally be low. To exclude behavioral parameters that can not be well identified from our empirical analyses, we do not consider behavioral parameters for which the normalized sensitivity is low in chapters 5 and 6.

A detailed overview of which traffic conditions need to be observed in order to be able to estimate specific parameters is provided in Appendix D.

4.5 Research questions on influences of factors on parameter estimates

In the previous, four categories of factors presumably influencing parameter estimates were identified. To be able to use estimated parameter values for obtaining insights into longitudinal driving behavior in our empirical analyses, it needs to be established how large these influences actually are. We will in the remainder of this chapter therefore aim at answering the following research questions regarding the impacts of methodological choices in developing a calibration procedure:

- What is the influence of the functional specification of the calibration objective on parameter estimates?
- What is the influence of the variable chosen in the calibration objective on parameter estimates?
- What is the influence of the method used for obtaining model predictions on parameter estimates, i.e. what is the influence of equalizing model predictions and observations at given time instants during the simulation?

As we use real-life trajectory observations in our empirical analyses, we will furthermore consider the following research questions:

- What is the influence of measurement errors on parameter estimates?
- Is it possible to draw inferences on observed longitudinal driving behavior by calibrating a longitudinal driving model only approximating complex human longitudinal driving behavior?

In answering these research questions, we need to take care of the in section 4.4 hypothesized relation between the characteristics of the trajectory observations used in calibrating and the influence of methodological choices. It was, for instance, suggested that measurement errors could partly explain the difference between parameter estimates obtained using different variables in the calibration objective.

To analyze this hypothesized correlation between the influences of methodological factors and the characteristics of the applied trajectory observations, we will vary the assumptions on the characteristics of the observations in answering the research questions on methodological factors.

4.6 Experimental design

This section describes the approach used for answering the research questions. We will first give a general overview of the approach (subsection 4.6.1) after which we will discuss several subparts in more detail (subsection 4.6.2).
4.6.1 Overview of research approach

As stated before we will answer the research questions on the influences of methodological factors, while making different assumptions on the characteristics of the available trajectory observations. More specific, we consider four possible scenarios with respect to the available trajectory observations (Table 4-1):

1. The trajectory observations used in calibrating do not contain measurement errors and the true longitudinal driving model governing the longitudinal behavior of the observed follower is known.
2. The trajectory observations used in calibrating do contain measurement errors, while the true longitudinal driving model governing the longitudinal behavior of the observed follower is known.
3. The trajectory observations used in calibrating do not contain measurement errors but the true longitudinal driving model governing the longitudinal behavior of the observed follower is unknown.
4. The trajectory observations used in calibrating do contain measurement errors and the true longitudinal driving model governing the longitudinal behavior of the observed follower is unknown.

Table 4-1 Overview of considered scenarios regarding observational characteristics.

<table>
<thead>
<tr>
<th>Measurement errors</th>
<th>True longitudinal driving model known</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>yes</td>
<td>no</td>
</tr>
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</table>

scenario 1 (section 4.7)                     scenario 3 (section 4.9)
scenario 2 (section 4.8)                     scenario 4 (section 4.10)

To perfectly control the characteristics of the trajectory observations used in all scenarios, we create synthetic trajectory observations for a leading car and a following car. This approach has the additional advantage that we have full knowledge about the driving rule governing the behavior of the following car.

To study the influences of the aforementioned methodological factors on parameter estimates, we perform for all four scenarios a large number of calibrations while varying the functional specification of the calibration objective, the variable in the calibration objective and the time interval at which observations and predictions are equalized. Table 4-2 provides a summary of the calibrations performed for each scenario. The table indicates, for example, that the calibration objective is set to Theils’u when the impact of the variable in the objective is investigated.

For scenario 1 and 2 in which the longitudinal driving rule describing the observed behavior of the follower is assumed to be known, we evaluate the estimated parameter values returned by the calibration algorithm by comparing them to the parameter values used in creating the synthetic trajectory observations.

For scenario 3 and 4 in which we assume the driving rule governing the behavior of the observed follower not to be known, we derive the characteristics of the modeled longitudinal driving behavior using information on the calibrated model and the estimated parameter values. We compare these characteristics to the known characteristics of the longitudinal driving behavior of the observed follower.
Table 4-2 Summary of experimental design for handling methodological research questions.

<table>
<thead>
<tr>
<th>Research question</th>
<th>Methodological choices in calibration procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration objective</td>
<td>Variable in objective</td>
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<tr>
<td>Impact of calibration objective</td>
<td>1. Theils‘u</td>
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<tr>
<td></td>
<td>2. mae</td>
</tr>
<tr>
<td></td>
<td>3. rmse</td>
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<tr>
<td></td>
<td>4. mrae</td>
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<td>5. rmsre</td>
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<tr>
<td>Impact of variable in objective</td>
<td>Theils‘u</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Impact of reset period</td>
<td>Theils‘u</td>
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4.6.2 Approach details

In the upcoming we discuss details of several subparts of the approach.

Synthetic trajectory observations

The method for creating the synthetic trajectory observations is described in detail in Appendix F, in this chapter we only summarize the components of it needed in understanding our analyses.

To ensure that the synthetic trajectory observations resemble empirical observations as much as possible the dynamics of the leading car are derived from the empirical BOSCH trajectory observations (Deutsches Zentrum für Luft- und Raumfahrt (DLR)). We use these trajectory observations instead of our helicopter observations as these observations cover a clearly longer time period. The BOSCH trajectory observations are furthermore collected in urban traffic such that they show a larger variation in speed over time.

Given the dynamics of the leader, we create 25 different trajectories for the following car by performing simulations using the Gipps model. The values of the behavioral parameters of the Gipps model used in creating a trajectory for the following car are randomly drawn from the following uniform distributions (we refer to section 3.3 for more details on the meanings of the individual parameters):

- Gipps: $a_{\text{max}} \sim \text{uniform}(1,2), b_{\text{max}} \sim \text{uniform}(-3.5,-1), \theta \sim \text{uniform}(0,1), d \sim \text{uniform}(0.5,5), b_{n-1,\text{max}} \sim \text{uniform}(-3.5,b_{\text{max}})$.

The desired speed is assumed to be equal to 30 m/s. while the reaction time is equal to 1 s. for all drivers. The 25 sets of parameter values used in creating the trajectories of the follower are saved.

To study the impact of measurement errors on parameter estimates, we introduce measurement errors to these trajectory observations. We thereby consider two possibilities:
• Measurement errors are added to the trajectory of the leading car only, to resemble data collected by an equipped car (active mode) in which the observations on the dynamics of the following car are often much more accurate than observations referring to the leading car.

• Measurement errors are (independently) added to both the trajectory of the leader and the trajectory of the follower. This scenario resembles, for example, remote sensing data collection methods.

In both cases errors are introduced first to the trajectories resembling the practical situation in which positions are directly measured. For the sake of consistency speeds are derived afterwards from these noisy position measurements using eq. (4.22).

\[ v_k(t_k) = \frac{x_n(t_{k+1}) - x_n(t_k)}{t_{k+1} - t_k} \]  

(4.22)

As we assume the time interval between two consecutive observations, i.e. \( t_{k+1} - t_k \), like in our helicopter observations to be equal to 0.1 s., applying eq. (4.22) implies that measurement errors on positions are amplified in deriving speeds.

Measurement errors are created by performing independent random draws from statistical distributions. In order to be able to investigate the influence of the specific appearance of measurement errors several different distributions will be used. First a normally distributed error term with a mean \( \mu \) equal to 0 will be added to the trajectories, in this context especially the influence of the standard deviation \( \sigma \) of the error term is interesting. For this sake the following two error distributions will be examined, \( \varepsilon \sim N(0; 0.05) \) and \( \varepsilon \sim N(0; 0.1) \). Next to that also the impact of a systematic component in the error term will be studied by applying the following distributions, \( \varepsilon \sim N(0.05; 0.05) \), \( \varepsilon \sim N(0.05; 0.1) \), \( \varepsilon \sim N(0.1; 0.05) \), \( \varepsilon \sim N(0.1; 0.1) \). Finally a non-symmetric error will be simulated based on the assumption that the error term is exponentially distributed with parameter 0.05.

In total we create thus 400 different synthetic trajectories for the following car (25 different clean trajectories*7 error distributions + the reference case without measurement errors)*2 possibilities for introducing measurement errors, i.e. only leading car or both leading and following car). All information on applied parameters and error distributions is saved for all these 400 trajectories of the following driver.

To investigate the scenarios in which it is assumed that the longitudinal driving rule governing the behavior of the observed follower is unknown, no new synthetic data need to be created. These scenarios are imitated by calibrating the Tampère model based on trajectories created by the Gipps model. The Tampère model is selected because we need a model capturing both the free-driving regime and the congested-driving regime like the Gipps model. Another similarity of the models is that drivers want to drive at a specified distance. Although both models share some common characteristics it is also important to stress that the driving dynamics are different for both models. For more details on both models we refer to chapter 3.

Table 4-3 summarizes the experimental design for all scenarios.
Table 4-3 Summary of experimental design for assessing the impact of the trajectory characteristics on parameter estimates.

<table>
<thead>
<tr>
<th>Model used for creating synthetic trajectories</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated model</td>
<td>Gipps</td>
<td>Gipps</td>
<td>Tampère</td>
<td>Tampère</td>
</tr>
<tr>
<td>Error types introduced to clean trajectories</td>
<td>-</td>
<td>1. $\varepsilon_t \sim N(0;0.05)$</td>
<td>-</td>
<td>1. $\varepsilon_t \sim N(0;0.05)$</td>
</tr>
<tr>
<td></td>
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<td>2. $\varepsilon_t \sim N(0;0.1)$</td>
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<td>2. $\varepsilon_t \sim N(0;0.1)$</td>
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<td>3. $\varepsilon_t \sim N(0.05;0.05)$</td>
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<td>3. $\varepsilon_t \sim N(0.05;0.05)$</td>
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<td>4. $\varepsilon_t \sim N(0.05;0.1)$</td>
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<td>4. $\varepsilon_t \sim N(0.05;0.1)$</td>
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<td>5. $\varepsilon_t \sim N(0.1;0.05)$</td>
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<td>5. $\varepsilon_t \sim N(0.1;0.05)$</td>
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<td>6. $\varepsilon_t \sim N(0.1;0.1)$</td>
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<td>6. $\varepsilon_t \sim N(0.1;0.1)$</td>
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<tr>
<td></td>
<td></td>
<td>7. $\varepsilon_t \sim \text{Exp.}(0.05)$</td>
<td></td>
<td>7. $\varepsilon_t \sim \text{Exp.}(0.05)$</td>
</tr>
<tr>
<td>Trajectories errors are added to</td>
<td>-</td>
<td>1. only leader</td>
<td>-</td>
<td>1. only leader</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. follower and leader</td>
<td>-</td>
<td>2. follower and leader</td>
</tr>
</tbody>
</table>

**Performance measures**

To evaluate the estimates returned by the calibration procedure for the scenarios in which the longitudinal driving rule governing the behavior of the driver is known, two criteria will be applied. Both these criteria refer to the relative difference ($\Delta(\beta^*_i, \beta_i)$) between the estimated parameter value, and the ‘real’ parameter value (the parameter value used in generating the synthetic trajectories) as represented in eq. (4.23). This value is interpreted as a measure for the bias of parameter estimates.

$$\Delta(\beta^*_i, \beta_i) = \frac{\beta^*_i - \beta_i}{\beta_i}$$  \hspace{1cm} (4.23)

In this equation $\beta^*_i$ is a vector containing the values for the five estimated parameters obtained in a calibration study based on a specific trajectory of the follower, while $\beta_i$ is a vector containing the corresponding parameter values used for creating the trajectories. More specifically:

$$\beta = \begin{bmatrix} a_{\text{max}} \\ b_{\text{max}} \\ \theta \\ d \\ b_{\text{max}} \end{bmatrix}$$ \hspace{1cm} (4.24)

These analyses result thus at the end in 25 values for $\Delta(\beta^*_i, \beta_i)$ for every considered error distribution and for all parameters $i$ of the model. Note that the parameters referring to the reaction time $T_r$ and the desired speed $v^*$ are not considered here, as they are in this chapter assumed to be fixed for all drivers.
First the median of $\Delta(\beta_1', \beta_1)$ obtained for the 25 trajectories having the same error distribution will be assessed following the simple logic that the closer this value to zero the closer the estimated parameter values are in general to the true parameter values. Apart from the median also the spread in the values for eq. (4.23) is considered as this spread indicates how sure one can be about the relation between an obtained parameter estimate and the corresponding true value. The larger the spread is, the larger the uncertainty becomes.

In the scenarios in which we assume the longitudinal driving rule governing the behavior of the observed follower not to be known, we apply a different approach as only the parameter values of the model used for creating the synthetic trajectories are known, while another driving rule is calibrated. To solve this problem we analyze the correlations between the parameters of the model used for creating the trajectories (Gipps model), and the parameters that are obtained when calibrating the Tampère model. The parameters used for creating the trajectories after all give information about the characteristics of the “true longitudinal driving behavior” of the observed driver. By studying the correlations it can be established whether the combination of the calibrated model and the corresponding estimated parameter values resembles this “true longitudinal driving behavior”.

The correlation coefficient between the estimated parameter values of the Tampère model for a given parameter and the corresponding parameter values of a given parameter of the Gipps model used in creating the synthetic trajectories is determined as follows (for a given error distribution). Firstly, a vector containing the values for the parameter of the Gipps model used for creating the different trajectories is composed (for illustrations aims this vector is called $\mathbf{v}$). Next a vector of the corresponding estimated parameter values of the Tampère model obtained when calibrating this model using the trajectories with measurement errors in line with the specified distribution is composed (vector $\mathbf{s}$). The correlation is then obtained by applying the usual formula (Casella and Berger, 1990):

$$\rho_{vs} = \frac{\text{Covariance}(\mathbf{v}, \mathbf{s})}{\sigma_v \sigma_s}$$

Optimization algorithm

In the analyses a constrained nonlinear optimization algorithm based on the simplex method will be applied. The motivation for introducing constraints is twofold. The constraints ensure for one that the estimated parameter values are reasonable. In this sense the constraints can be interpreted as using prior information. Special care is however taken of not constraining the values too much as deviating parameter values indicate estimation problems. The second aim of the constraints is to search the solution space more efficiently.

More specifically, the following constraints are applied:

- **Gipps**: $a_{\text{max}} \in [0, 4]$, $b_{\text{max}} \in [-6, 0]$, $\theta \in [0, 4]$, $d \in [0, 15]$, $b_{n-1}^{\text{max}} \in [-6, 0]$.
- **Tampère**: $c_{1,n-1} \in [0, 1]$, $c_2 \in [0, 1]$, $c_3 \in [0, 3]$, $d \in [0, 15]$, $\gamma \in [0, 4]$ (these constraints are applicable when the influence of missing information on the car-following model is examined, thus the scenarios in which the Tampère model is calibrated).

The bounds imposed to the parameter values in calibrating are thus clearly less strict than the earlier presented ranges of the parameter values used in creating the synthetic trajectories.
As the proposed algorithm does not necessarily end up in a global minimum, it is repeated five times using different starting conditions.

### 4.7 Impacts of methodological factors without measurement errors

In this section the research questions on the influences of methodological choices will be examined for the case in which observations are free of measurement errors and full information is available about the car-following model governing the driving behavior of the observed driver. To improve the clarity of the presentation we do in this section and the upcoming sections not aim at discussing all results for all possible methodological choices, only the most important results are presented (for more details we refer to (Ossen and Hoogendoorn, 2007)).

#### Impact of functional specification of calibration objective

In the first part of the analysis the influence of the functional specification of the calibration objective is examined. This is done by estimating the parameters of the Gipps car-following model for all 25 trajectories of the follower for the five structures of the calibration objective (see subsection 4.4.2 and Table 4-2), while the distance headway between the cars $\Delta x_{n-1,n}$ is always selected as the variable in the objective.

The results of this analysis are provided in Figure 4-3. To be able to visualize both the median value for $\beta^*_i$, $\beta_i^*$ and the spread in $\Delta(\beta^*_i, \beta_i)$, the results are represented in boxplots. The horizontal black lines refer to the median value of the 25 values for $\Delta(\beta^*_i, \beta_i)$ while the upper and lower boundaries of the boxes indicate respectively the upper quartile and the lower quartile. The black lines on top and below the boxes furthermore show the values outside the quartiles and extreme outliers are indicated by $*$. Every subplot in Figure 4-3 represents the results for the five aforementioned structures of the objective for one specific parameter.

The general conclusion drawn from the figure is that all parameters can be well identified for all objective specifications, justifying the conclusion that for this ‘ideal’ data situation the choice of the specification of the calibration objective is not significantly affecting the results.

---

4 Not all outliers are presented in the boxplots presented in this chapter as the range of the values on the y-axis is chosen such that the main parts of the figures are well visible, and distinguishable.
Impact of variable in calibration objective
When performing the same analysis taking the speed as variable in the calibration objective again all parameters could be well identified such that again no important differences between the different specifications of the calibration objective could be revealed. Also the variable in the objective did not significantly affect the parameter estimates.

Impact of reset period
In answering the research question on the impact of equalizing observed states and predicted states at given time intervals, we will assume that there are fixed intervals between equalizations and we indicate these intervals as “reset period”.

The reset period is implemented as follows, when the reset period is smaller than the total duration of the observation, the time series is subdivided in time series covering the same period as the reset period. For instance, when an observation covers a time period of 200 sec. while the reset period is 75 sec., the original time series is subdivided in 3 parts: two parts of 75 sec., and one part covering the final part of the time series having a length of 50 sec. (200-75*2). In predicting, all these separate parts are predicted separately, after which they are concatenated. The result is then, like for the case without equalizing observations and predictions, evaluated by the calibration objective after which parameters are adapted when needed.

To assess the influence of the reset period, the functional form of the calibration objective and the variable in the objective are assumed to be respectively Theils’u, and the speed. Given these settings regarding the calibration objective and a certain length of the reset period the estimation procedure is again applied to all 25 trajectories of the follower. This procedure is first performed for the reference case without resetting, and afterwards repeated for reset periods of 50 seconds, 25 seconds and 10 seconds.

For all different reset periods, we established that parameters could be well identified, such that no compelling motivation is found for taking the additional effort of equalizing observations and predictions at regular time intervals. The only exception is the smallest reset period for which the parameters $\theta$ and $d$ can be slightly better identified.

This can be explained by taking into account that different parameter values result in different assumptions on the distance considered to be safe by driver $n$. The estimated parameter values will therefore be those parameters leading to a safe distance best reflecting the observed distance headway. When this is not the case the simulated dynamics of driver $n$ will consist of reaching the assumed safe distance implying that, depending on the difference between the observed distance headway and the assumed safe distance headway, the simulated driver will brake or accelerate harshly.

In sum, when observations do not contain measurement errors and full information is available about the car-following model governing the behavior of the observed following driver, there seems to be no large influence of making different methodological choices. Thus the general answer to all research questions on the impacts of methodological choices on parameter estimates is that all parameters can be well identified independent of the specification of the calibration objective, the variable in the objective and the reset period.
4.8 Impacts of methodological factors with measurement errors

These answers change when measurement noise is added to the observations, while the car-following rule governing the behavior of the observed follower is still known (scenario 2).

Before assessing the impact of methodological choices for scenario 2, we first show that measurement errors lead to a considerable bias of estimated parameter values. This bias is illustrated in Figure 4-4 showing boxplots for all separate parameters while the distribution of the measurement errors introduced to the trajectory observations is varied. All these results are obtained by using Theil’s u as functional form for the calibration objective, taking the speed of the follower $v_n$ as variable in the objective and without resetting. The synthetic observations used for obtaining this figure do furthermore only contain measurement errors on the observations of the leader. It needs to be stressed however that comparable results are obtained when both the measurements of the leader and follower contain noise.

![Figure 4-4 Influence of measurement errors on calibration results for the model of Gipps. Explanation of references to error distributions: 1) no measurement errors (reference case), 2) $\varepsilon_t \sim \mathcal{N}(0;0.05)$, 3) $\varepsilon_t \sim \mathcal{N}(0;0.1)$, 4) $\varepsilon_t \sim \mathcal{N}(0.05;0.05)$, 5) $\varepsilon_t \sim \mathcal{N}(0.05;0.1)$, 6) $\varepsilon_t \sim \mathcal{N}(0.1;0.05)$, 7) $\varepsilon_t \sim \mathcal{N}(0.1;0.1)$, 8) $\varepsilon_t \sim \text{exp}(0.05)$.]

For the parameters $a_{\text{max}}^\text{max}$, and $b_{n-1}^\text{max}$ the figure shows that the median values of $|\Delta(\beta^*_i, \beta_i)|$ increase when the standard deviation of the measurement errors introduced to the synthetic trajectories increases. The related increase in the spread of $\Delta(\beta^*_i, \beta_i)$ is even more pronounced.

When considering the results for these two parameters more closely it appears that the standard deviation of the measurement errors has a larger influence than the systematic component of the noise. A plausible explanation for this is as follows; the noise is first introduced to the trajectories and the speeds are derived afterwards. This implicitly causes that the speeds are less influenced by the systematic component of the measurement noise than by the standard deviation. That is, the speed is derived from the distance traveled in one time step, implying that noise enters when this distance contains noise. A systematic error in the observed positions however implies that all subsequent positions are generally overestimated/underestimated. This has clearly a smaller impact on the distance traveled in one time step than the standard deviation of the errors. As an extreme example, suppose that a systematic component of measurement errors entails that all positions are increased by 10 m., the derived speed is in this case not influenced at all.
The estimates for the parameters $\theta$, $d$ and $b_{n-1}^{\text{max}}$ show the largest bias. For the parameters $\theta$ and $d$ almost all relative differences between the real parameter values and the estimated parameter values are equal to -1. A value of -1 for the relative difference implies in this sense that the estimates for these parameters are equal to 0. For the parameter $b_{n-1}^{\text{max}}$ also a remarkably large bias can be observed, especially when the standard deviation of the error on the positions is equal to 0.1. A detailed check of the results showed that the estimates for $b_{n-1}^{\text{max}}$ were very often equal to the applied lower bound. The related spread in the corresponding results can be understood by considering that the real parameter values differ between the 25 datasets while the imposed lower bounds (and in this light also the estimated parameters) are part of the optimization algorithm.

In the upcoming, we answer the research questions regarding the impacts of methodological choices for scenario 2. The most important finding will be that the bias of the parameter estimates for the Gipps model can be considerably reduced by selecting the distance headway $\Delta x_{n-1,n}$ in the objective instead of the speed $v_n$. This result turns out to be model dependent.

**Impact of functional specification of calibration objective**

The impact of choosing another functional specification of the calibration objective is studied for three different types of measurement errors. That is, the analysis is based on the 25 different synthetic trajectories for the follower to which measurement errors following the following three distributions are added: $\epsilon_t \sim N(0;0.05)$, $\epsilon_t \sim N(0;0.1)$ and $\epsilon_t \sim N(0.1;0.1)$. In all cases the error is introduced to the measurements of both the leader and the follower. To be able to focus at the influence of the functional specification of the calibration objective only, the variable in the objective is always the speed and predictions and observations are not equalized in between.

The results are not shown here, as these analyses in general revealed that it is not possible to select one structure of the calibration objective always outperforming the other ones. It seems therefore justified to conclude that the large bias of the parameter estimates after the introduction of measurement noise can not be attributed to the selected specification of the calibration objective.

**Impact of variable in calibration objective**

The impact of the variable in the calibration objective on parameter estimates is studied for the same trajectories as the ones used to analyze the influence of the functional specification of the calibration objective. The only difference in this analysis is that the variable in the calibration objective is now varied while Theils’ $u$ is continuously used as functional specification of the calibration objective.

The results are provided in Figure 4-5. From this figure it is obvious that the estimates for the parameters showing a large bias before ($\theta$, $d$, $b_{n-1}^{\text{max}}$) considerably improve when the speed is not selected in the calibration objective.

These findings can be understood when considering Figure 4-6. Figure 4-6 shows predicted speeds for the follower using three different parameter values of $b_{n-1}^{\text{max}}$, as well as the observed speed pattern of the follower. For the sake of simplicity, it is assumed in drawing this figure that only the measurements of the leader contain errors. In the upper plot $b_{n-1}^{\text{max}}$ is equal to -2.2, in the middle one $b_{n-1}^{\text{max}}$ is equal to -4, and in the lower one $b_{n-1}^{\text{max}}$ is equal to -6. All other parameter values are equal in all three cases.
Figure 4-5 Influence of choosing another variable in the calibration objective on estimated parameter values of the Gipps model in the presence of measurement errors.

The figure shows clearly that the deviating parameter estimates for $b_{n-1}^{\text{max}}$ when using the speed as variable in the calibration objective are caused by a non-trivial side effect. A lower value for $b_{n-1}^{\text{max}}$, given the other parameter values, results in this figure in a relatively smoother predicted reaction of the following driver to speed changes of the leading car. Thus when the leader is assumed to be willing to brake harder the follower is predicted to react less heavily as he keeps a larger distance to his leader. The key point is that noise in the observed speed of the leader is due to this same effect much more transferred to the predicted speed of the follower when the parameter values used in predicting the dynamics of the follower are such that he is assumed to react more accurately to speed changes in the speed of the leader. In the calibration process this effect is applied as a kind of smoother. That is, the lower $b_{n-1}^{\text{max}}$ (ceteris paribus) the smoother the predicted dynamics of the following vehicle become.

The speed dynamics of the follower are in fact predicted less accurately when $b_{n-1}^{\text{max}}$ is decreased. From an estimation point of view the value returned by the calibration objective, Theils’ $u$, should thus increase. The problem is however that this increase in Theils’ $u$ due to the larger difference between observed and predicted dynamics is to a certain extent compensated for by the smaller amplitude of the transferred measurement errors. Theils’ $u$ (denoted by $g(y_n, \hat{z}_n)$) in fact decreases when $b_{n-1}^{\text{max}}$ is decreased to -4 (middle figure), although the predicted dynamics resemble the observed dynamics clearly less. When $b_{n-1}^{\text{max}}$ is decreased to -6, Theils’ $u$ increases again due to the even worse match between observed and predicted dynamics. When the safe distance is however decreased by setting the parameters $\theta$ (safety reaction time) and $d$ (distance at rest) to 0, the dynamics are improved so far that the
smoothing effect becomes more important again than the worse dynamics, leading to parameter estimates with $b_{n-1}^{\text{max}}$ equal to the lowerbound, and $\theta$ and $d$ equal to 0.

Exactly the same reasoning can be followed when both the trajectories of the leader and follower contain noise (even if the errors on the observations for the leader and the follower would be the same, there is still a time delay in the reaction of the follower caused by the reaction time).

When the relative distance $\Delta x_{n-1,n}$ is selected for calibration this side effect vanishes. Figure 4-7 shows that the lower $b_{n-1}^{\text{max}}$ is, the larger the predicted distance between the vehicles is, as stated before. This effect causes Theil’s $\text{u}$ to increase obviously, making it clearly less attractive for the optimization algorithm to select smaller values for $b_{n-1}^{\text{max}}$. In the same way the better estimates for the other parameters can be explained.

When we performed the analyses presented in this chapter for the model of Tampère this finding appeared to be model dependent. For the model of Tampère in fact better results were obtained when selecting the speed as variable in the objective, although the differences between the different variables in the objective were clearly smaller than the ones found for the model of Gipps (for details see (Ossen and Hoogendoorn, 2007)).
\[ b_{n-1}^{\text{max}} = -6, \ g(y_n, z_n) = 0.27448 \]

\[ b_{n-1}^{\text{max}} = -4, \ g(y_n, z_n) = 0.21477 \]

\[ b_{n-1}^{\text{max}} = -2.2, \ g(y_n, z_n) = 0.029545 \]

Figure 4-7 Explanation for improvements of estimates for \( b_{n-1}^{\text{max}} \) when \( \Delta x_{n-1,n} \) is used as variable in the calibration objective instead of \( v_n \).

Impact of reset period
We also verified whether the results can be improved further by resetting the observations and the predictions in between. Theil's \( u \) was used as specification of the calibration objective and the distance headway \( \Delta x_{n-1,n} \) was selected as variable in the objective. The same reset periods as before were examined, thus the reference case without resetting and reset periods of 50 seconds, 25 seconds and 10 seconds.

This analysis showed that the parameter related to the free driving regime \( a_{\text{max}} \) can be clearly less well identified when the smallest reset period is selected. The only parameter for which there seems to be a general trend of improved results when observations and predictions are equalized at the highest frequency is the parameter \( b_{n-1}^{\text{max}} \).

In sum, it can be concluded that measurement errors can lead to a strong bias in parameter estimates. The analyses showed that this bias could be reduced by changing the variable chosen in the calibration objective.

4.9 Impacts of methodological factors when behavioral rule is unknown and trajectories do not contain measurement errors

The previous sections on scenarios 1 and 2 shared the common characteristic of full information on the longitudinal driving model describing the behavior of the observed follower. In this section and the next section this will no longer be the case (scenarios 3 and
4). This section handles scenario 3 in which observations do not contain measurement errors, while the next section treats scenario 4 in which observations do contain measurement errors.

In performing these analyses the following research question plays a central role, what do estimated model parameters tell about the observed car-following behavior of a driver? To answer this question the same approach will be taken as in the preceding analyses except from the fact that the Tampère model will be calibrated based on trajectories of the follower created by the Gipps model.

The results provide in general evidence that the characteristics of the following behavior of the follower can be recovered by calibrating a car-following model even when this model is not the real model behind the observations as long as enough insight is present into the calibrated car-following model. This is an important finding as the real longitudinal driving model governing the behavior of a driver will in reality never be known due to the complexity of human longitudinal driving behavior.

This finding is illustrated in Table 4-4 showing the correlation matrix obtained after repeatedly calibrating the Tampère model to observations created using the Gipps model. Theils’u is selected as functional specification of the calibration objective and the speed is used as variable in the objective.

Apart from the separate model parameters of the Gipps model also the ratio $\frac{b_{n-1}^{\text{max}}}{b_{\text{max}}}$ is considered as this ratio was found to have a clear impact on the predicted behavior of the follower (distance kept to the leader, magnitude of reaction of follower to changes in the speed of the leader, and so on). The larger the ratio the more severe the leader wants to brake relatively to the follower and thus the larger the distance the follower keeps to his leader.

<table>
<thead>
<tr>
<th>Table 4-4 Example of correlations between the parameter values of the Gipps model used in creating the synthetic trajectories of the follower and the parameter values obtained when calibrating the Tampère model based on these trajectories.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gipps</strong></td>
</tr>
<tr>
<td>$a_{\text{max}}$</td>
</tr>
<tr>
<td>$b_{\text{max}}$</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$d$</td>
</tr>
<tr>
<td>$b_{n-1}^{\text{max}}$</td>
</tr>
<tr>
<td>$b_{\text{max}}$</td>
</tr>
</tbody>
</table>

There is a strong correlation between the ratio $\frac{b_{n-1}^{\text{max}}}{b_{\text{max}}}$ and the estimates of the different parameters of the Tampère model. That is, a clear negative correlation between the ratio, and the sensitivity parameters $c_{1,n-1}$ and $c_2$ of the Tampère model is found. Thus the larger the ratio in the Gipps model is, the smaller the estimated sensitivities of the follower are. This seems reasonable as it was shown before that in the Gipps model the reaction of the follower to a speed change of the leader is relatively smaller when the ratio becomes larger. This is thus in line with the observed decrease of the estimated sensitivities of the Tampère model.

On the other hand a strong positive correlation can be observed between the ratio and the parameters $d$ and $\gamma$. This implies that the predicted desired distance headway of the Tampère
model increases with an increase in the ratio. This is also perfectly in line with the expectations. The same relations (only less strong) can be found between the safety reaction time $\theta$ and the estimated parameters of the Tampère model. The only parameter that does not show a strong correlation with any of the parameters is the distance at standstill. This even holds for the parameter $d$ of the Tampère model.

Based on these results it seems justified to conclude that the true car-following behavior of the observed follower can be identified by calibrating an “imperfect” model.

Also for this scenario the impacts of choosing another functional specification of the calibration objective, another variable in the objective and another reset period were assessed. Most effects were only minor or already discussed before and will not be treated in detail here. One conclusion from the analysis in which the variable in the calibration objective is varied deserves nevertheless attention.

**Impact of variable in calibration objective**

As the influence of the variable chosen in the calibration objective is examined, the functional specification of the calibration objective is kept fixed to Theils’u and no in between resetting occurs.

The estimated parameter values obtained when performing all calibrations using respectively the speed and the distance headway in the objective are presented in Figure 4-8. When comparing the estimated values it turns out that there seems to be a generic difference between the values obtained when using respectively the speed in the calibration objective and the estimates obtained using the distance headway between the vehicles.

![Figure 4-8 Histograms of the parameter estimates for the different variables in the objective.](image)

To gain insight into the reason for this finding, a detailed analysis of the predicted dynamics was performed for the different variables in the objective. It was found that when the speed is selected in the objective there is a clear difference between the predicted distance headway between the cars and the observed distance headway. This entails in fact that the model in this case is best able to mimic the speed pattern of the follower when the predicted distance headway deviates from the observed one.

The opposite occurs when the distance between the cars is used for calibration. In that case a clearly better match between the predicted and the observed distance headway between the cars is found. The match between the true speed pattern and the predicted speed pattern of the follower is however worse. The predicted speeds start in fact to become unstable (they amplify
the disturbance in the speed of the leader), however, only in such a modest way that the integrated speeds resulting in the positions of the follower do not suffer so heavily that the solution becomes non-optimal.

A conclusion that can be drawn from these particular findings is that the variable that can best be chosen in the calibration objective is dependent on the research objective. That is, when a car-following model is calibrated with the aim to predict travel times it is most important that the calibrated model can best resemble the speed pattern. This can be a motivation for selecting the speed in the calibration objective. However, when the model is calibrated with the aim to draw conclusions about, for example, the impact of ACC on road capacity, choosing the distance headway in the calibration objective seems to be more appropriate.

4.10 Impacts of methodological factors when behavioral rule is unknown and observed trajectories contain measurement errors

The final scenario that will be considered is scenario 4 in which observations do contain measurement noise and in which the car-following model governing the observed behavior of the following driver is assumed to be unknown. This is the most realistic scenario.

The corresponding analyses showed that it can roughly be stated that the conclusions belonging to this scenario are a combination of the conclusions in the scenario with only measurement errors and the scenario with no measurement errors but lack of information about the driving rule behind the observed car-following behavior.

No considerable differences between the parameter estimates obtained using the different structures of the calibration objective could be noticed. Furthermore, the strongest correlations found in the previous subsection (about trajectory observations not containing measurement noise and lack of information about the underlying car-following model) were also observed in the analyses for this scenario.

Also an analysis on the influence of choosing different variables in the calibration objective showed the same results as before; the sensitivity parameter estimates for $c_1,n-1$ and $c_2$ obtained when selecting the speed as variable in the objective function differed from the parameter values obtained when using the distance headway in the objective function.

4.11 Influence of observational characteristics on reliability of parameter estimates

In the previous sections we considered among others the relation between observational characteristics and parameter estimates. In evaluating these parameter estimates we only considered possible biases, while we ignored the reliability of the parameter estimates (see subsection 4.3.2).

To complete the analysis on the impact of various observational characteristics on parameter estimates, we focus in this section on the sensitivity of the calibration objective to small changes in parameter values for the four scenarios. In all scenarios, we assume the calibration objective to be given by Theil's $u$, while the distance headway $\Delta x_{n-1,n}$ is selected as variable in the objective. Measurement errors are furthermore introduced to both the trajectories of the leader and the trajectories of the follower.
Table 4-5 Medians of sensitivities of parameter estimates of Gipps model for observations having different characteristics.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$d_{max}$</th>
<th>$b_{max}$</th>
<th>$\theta$</th>
<th>$d$</th>
<th>$b_{n-1}^{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True model, no error</td>
<td>2.12</td>
<td>161.07</td>
<td>76.05</td>
<td>0.42</td>
<td>38.69</td>
</tr>
<tr>
<td>True model, $\varepsilon_t \sim N(0,0.1)$</td>
<td>0.33</td>
<td>16.52</td>
<td>7.77</td>
<td>0.04</td>
<td>4.97</td>
</tr>
<tr>
<td>Incorrect model, no error</td>
<td>2.22</td>
<td>2.41</td>
<td>1.88</td>
<td>0.01</td>
<td>3.31</td>
</tr>
<tr>
<td>Incorrect model, $\varepsilon_t \sim N(0,0.1)$</td>
<td>2.69</td>
<td>5.61</td>
<td>2.49</td>
<td>0.01</td>
<td>6.34</td>
</tr>
</tbody>
</table>

Table 4-5 provides an overview of the sensitivity of the calibration objective to small changes in parameter values for observations having different characteristics. More specific, every row of the table presents the median sensitivity of the calibration objective obtained when calibrating the Gipps car-following model based on all 25 trajectories of the follower having the same characteristics. For the scenarios assuming that the rule governing the longitudinal driving behavior of the observed driver is not known, the Gipps model is calibrated on observations created using the Tampère car-following model. As we are only interested in comparing the sensitivity of the calibration objective to changes in the same parameter under different observational characteristics and we do not aim at comparing sensitivities between parameters, we did not normalize the sensitivities.

The calibration objective is generally most sensitive to small changes in parameter values when observations do not contain measurement errors and when the driving rule governing the behavior of the observed driver is known.

The smaller sensitivities of the calibration objective in the presence of measurement errors can be explained as follows; when observations do not contain measurement errors differences between predictions and observations occur completely due to wrong parameter settings. When the observations contain measurement errors this is no longer the case, a considerable part of the difference between the predictions and the observations can be attributed to the measurement errors contained in the observations. Thus small changes in the values of the estimated parameters do not have such clear impacts on values returned by the calibration objective as was the case in the scenario in which observations did not contain measurement errors.

The same reasoning can be applied to explain the decrease in sensitivity for the scenarios in which the rule governing the behavior of the observed driver is unknown.

### 4.12 Reduction of negative impact of measurement errors

In reconsidering all previous outcomes the most important conclusion is on the large negative influence of measurement errors on parameter estimates. Measurement errors can not only result in a large bias in parameter estimates, but also in a decreased reliability of these estimates.

As the aim of this chapter is to develop a solid base for our empirical analyses, we will conclude the chapter with examining two methods for reducing the negative impact of measurement noise. First of all the use of a multi-criterion objective will be discussed.

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5 In this section we decided to calibrate the Gipps model on trajectories created using the Tampère model to enable a comparison between the different scenarios.
Secondly the influence of pre-processing observations is considered. To be best able to evaluate the estimated parameters it will be assumed that the car-following model is known, such that the bias of the parameter estimates can be determined.

4.12.1 Multi-criterion objective

In our analyses, we showed that the bias in estimated parameter values depends on the variable used in the calibration objective. That is, the bias in the parameter estimates of the Gipps model is considerably lower when using the distance headway as variable in the calibration objective instead of the speed. The opposite result holds for the model of Tampère, although the difference in the bias was less profound for this model.

Therefore, in this subsection the use of a multi-criterion objective will be discussed, implying that an objective is applied containing both the speed and the distance headway \((eq. \ (4.26))\). The parameters \(\lambda_1\) and \(\lambda_2\) define the weights assigned to both variables.

\[
g(y_n, \hat{y}_n, \lambda_1, \lambda_2) = \frac{\sqrt{\frac{1}{K} \sum_{k=1}^{K} (\Delta x_{n-1,n}(t_k) - \Delta \hat{x}_{n-1,n}(t_k))^2}}{\sqrt{\frac{1}{K} \sum_{k=1}^{K} \Delta x_{n-1,n}(t_k)^2}} + \frac{\sqrt{\frac{1}{K} \sum_{k=1}^{K} (\hat{v}_n(t_k) - \hat{v}_n(t_k))^2}}{\sqrt{\frac{1}{K} \sum_{k=1}^{K} \Delta v_{n-1,n}(t_k)^2}}
\]

(4.26)

The motivation for analyzing a multi-criterion objective is to develop a calibration objective performing well for a range of longitudinal driving models. This is a necessary condition for being able to compare model performances in our empirical analyses. As both variables in the multi-criterion objective performed best for one of the considered longitudinal driving models, we assume the weights to be equal in the remainder of this section, i.e. \(\lambda_1=\lambda_2=1\).

To illustrate the results, Figure 4-9 shows an example of calibrations performed using the 25 trajectories of the follower to which measurement errors are added distributed as \(\varepsilon_t \sim N(0;0,1)\). These errors are added to both the trajectories of the leader and the follower. Theil's \(u\) is always selected as the calibration objective, while the variable in the calibration objective is varied. Apart from the results for the objectives containing only one variable, the results for the multi-criterion objective (multi) are presented as well.

![Figure 4-9 Influence of choosing another variable in the calibration objective on the estimates of the parameters for the Gipps model in the case of measurement errors. Also the results for a multi-criterion objective including both the speed and the distance headway are shown.](image-url)
The results for the multi-criterion objective are mostly in between the results for the calibration objective including only the speed and the calibration objective containing only the distance headway. They furthermore appear to be closer to the results obtained when using only the distance headway as variable in the objective function.

Using the analyses and considerations from the previous subsections this can be explained as follows; when calibrating the Gipps model on speed it turned out that the optimal parameter values were those best smoothing the predicted speeds. The predicted distance headways differed in this case however remarkably from the observed ones. This strong deviation was however completely ignored by the calibration objective considering only the speed. The bias in the parameter estimates became therefore obviously less when the distance headway was used as variable in the calibration objective. The same in fact happens in the case of the multi-criterion objective function as the large differences between the predicted and real relative distances are also penalized by the multi-criterion objective.

For the model of Tampère the decrease in the bias of the estimated parameter values when using a multi-criterion objective function instead of an objective function considering only the distance headway $\Delta x_{n-1,n}$ appeared to be only minor.

Nevertheless it can be concluded that the use of a multi-criterion objective is preferred above the use of a single-criterion objective when calibrating a broad range of different car-following models. By using a multi-criterion objective function the strong bias of estimated parameter values of the Gipps model obtained when using an objective function considering only the speed $v_n$ could be avoided. For the Tampère model the additional bias obtained when using the distance headway $\Delta x_{n-1,n}$ in the objective function instead of the speed $v_n$ decreased slightly by using a multi-criterion objective function.

4.12.2 Smoothing microscopic trajectory observations

In this subsection the influence of smoothing observations will be studied. To this end the synthetic trajectories used in the previous analyses containing measurement errors are smoothed using a moving average filter having a time span of 9 observations. This time span is carefully selected to assure that the observations indeed become smoother, while the original dynamics of the follower are not affected. The corresponding speeds are afterwards derived from these smoothed trajectories.

These speeds are illustrated in Figure 4-10 showing the speed observations containing noise, the smoothed speed observations and the synthetic speed observations before the introduction of measurement errors. Note that the smoothed speed observations are still not completely free of measurement errors; a thorough visual inspection showed however that when we would increase the time span, the real dynamics of the follower that should be preserved in order to be able to estimate the parameters, would be affected.

To assess the impact of smoothing the same operating procedure is used as in the previous subsection, i.e. the analyses for the Gipps model showing the strongest bias of parameter estimates are performed again using different variables in the calibration objective. An example of the results is provided in Figure 4-11 showing the values for $\Delta(\beta^*, \beta)$ for the case in which trajectories having for both the leader and the follower measurement errors distributed as $\epsilon \sim N(0;0.1)$ are smoothed.
Figure 4-10 Example of pre-processed observations on speed. For the sake of comparison also the non-smoothed speeds are shown and the speed observations before the introduction of measurement errors.

The results are promising as they show that after smoothing especially the parameters obtained when selecting the speed in the objective are improved noticeably. Despite the improvements it still seems however better to use either a multi-criterion objective or an objective incorporating the distance headway. A possible explanation for this is that also the smoothed observations contain some noise.

Figure 4-11 Influence of smoothing observations before using them for calibration of the Gipps model. Results are shown for different variables in the objective. Also the results for a multi-criterion objective are shown.

The moving average filter used in this example is rather basic, for more advanced smoothing methods we refer to (Punzo et al., 2005, Toledo et al., 2007, Thiemann et al., 2008). It needs to be noticed that although the results shown here are promising, some care needs to be taken when considering smoothing as a solution for removing measurement noise. To successfully smooth the observations it is very important to select the filter in such a way that errors are removed as much as possible, while the real dynamics of the leading and following car are preserved in order to be able to identify the parameters best describing observed longitudinal driving dynamics. When this is not the case another source for deviating parameter estimates is created. Also the type of measurement noise of course influences the effect of smoothing. For example, when measurement errors at subsequent time steps are correlated the effect of smoothing is expected to be less positive as it is very likely that the information on the dynamics of the leading car and following car present in the data is affected after smoothing.
4.13 Summary and conclusions

In this dissertation thesis, we will perform empirical analyses based on trajectory observations of consecutive cars to increase the insights into longitudinal driving behavior. We thereto calibrate mathematical models and use estimated parameter values and model performances to draw inferences on observed behavior. This model based approach has the advantage that we can quantitatively express our findings.

To use calibration results for obtaining insights into observed behavior, a thorough understanding is needed regarding the calibration process and its influence on parameter estimates. Obtaining this understanding has been the aim of this chapter. This chapter can therefore be seen as a solid preparation for our empirical analyses presented in the remainder.

We showed that in using our microscopic calibration framework we need to deal with autocorrelated error terms, i.e. differences between model predictions and observations at subsequent time instants are not independent. This has the disadvantage that standard statistical tests for drawing inferences from calibration results become inapplicable. We therefore proposed heuristics for determining the reliability of estimated parameter values and comparing model specifications having different complexities. These heuristics were derived from standard statistical tests, while taking the problem of autocorrelated error terms into account.

We furthermore gave an overview of several factors presumably influencing calibration results. We distinguished four categories, namely the specification of the calibration objective, the applied optimization algorithm, the method used for obtaining model predictions and the characteristics of the trajectory observations.

Based on this overview, we performed a synthetic trajectory based experimental study on the influences of methodological factors, like the specification of the calibration objective and the variable in the calibration objective, on estimated parameter values. We performed these analyses for trajectory observations having different characteristics in order to answer the following research questions:

- What is the influence of measurement errors on parameter estimates?
- Is it possible to draw inferences on observed longitudinal driving behavior, by calibrating a longitudinal driving model only approximating complex human longitudinal driving behavior?

A large influence of measurement errors on parameter estimates was found, i.e. the bias of estimated parameter values increased in the presence of measurement errors and the reliability of estimates decreased significantly. This negative influence of measurement errors was dependent on the variable used in the objective function. It furthermore turned out that the variable that could best be used in the calibration objective depends on the model to be calibrated. This is a clear disadvantage for the empirical analyses in chapters 5 and 6 in which we calibrate a broad range of longitudinal driving models and compare their performances as a requisite for comparing performances is that for all models the same performance measure is applied. To handle this problem, we proposed and examined a multi-criterion objective considering both the speed and the distance headway.
The bias of estimated parameters in the presence of measurement errors could be reduced by smoothing the observations before using them for calibration. Also the use of a multi-criterion objective turned out to be favorable for the parameter estimates of both considered models, implying that a multi-criterion objective is most appropriate for the empirical analyses in chapters 5 and 6.

Furthermore, evidence was obtained that the characteristics of observed longitudinal behavior of a follower can be identified by calibrating a car-following model, even when this model is not the model fully describing the dynamics of the observed driver. This however requires insight into the characteristics of the calibrated car-following model.

Based on this chapter, we are first of all able to determine the reliability of parameter estimates and to compare models having different complexities by using the proposed heuristics. Second of all, we will make our calibration algorithm more robust against measurement errors by using a multi-criterion objective considering both the speed and the distance headway in our empirical analyses. Given our findings on the positive impact of smoothing observations before using them in calibrations, we will also spent effort in carefully pre-processing our helicopter observations (Appendix A).

We continue by using this dedicated calibration framework for performing empirical analyses on heterogeneity (chapter 5) and multi-anticipation (chapter 6) in longitudinal driving behavior.
5 Theory and empirics of heterogeneity in car-following

5.1 Aim and structure of this chapter

From everyday experience it is intuitively clear that different drivers behave differently under the same conditions. For instance, some people do have a more relaxed driving style while other ones prefer a more sportily one. Next to these individual aspects also differences related to car characteristics are likely to have their influence on longitudinal driving behavior. A heavily loaded truck can, for example, brake less severe than a person car.

But how large are these differences between different driver/vehicle combinations in real traffic? This question is both relevant from a behavioral point of view as from a simulation point of view. All commercial microscopic simulation tools take care of heterogeneity but due to a lack of observations no profound knowledge is available about how heterogeneity exposes itself in real traffic.

In this chapter we will therefore perform detailed analyses on heterogeneity caused by driver characteristics as well as heterogeneity caused by car characteristics. We will use two datasets collected by aerial observation comprising trajectory observations for all vehicles driving on the observed roadway stretches. These observations offer excellent opportunities to perform a large sample based study on heterogeneity. The high spatial and temporal resolution of the observations furthermore enables quantification of heterogeneity. Heterogeneity is established by estimating multiple individual driver models and comparing these models within and between individuals.

We will show that heterogeneity caused by driver characteristics is larger than what currently is assumed in microscopic simulation tools. Different behavioral rules are needed to describe the behavior of different drivers. Furthermore clear differences are identified between drivers adopting a comparable driving style.
Also the extent of heterogeneity caused by car characteristics turns out to be considerable. Not only dissimilarities between driving styles of truck drivers and person car drivers are found, but also several significant differences are established between drivers of person cars and trucks having a similar driving style.

The chapter is structured as follows. In section 5.2 heterogeneity will be defined and several causes and types of heterogeneity will be discerned. Section 5.3 introduces our hypotheses on heterogeneity caused by driver characteristics and car characteristics that will be tested throughout this chapter. In sections 5.4, 5.5, and 5.6 the approach adopted for testing the hypotheses will be detailed and the observations to which the approach is applied are discussed.

In section 5.7 the general performances of eight different car-following rules in describing the longitudinal driving behavior of individual drivers will be analyzed. In sections 5.8 and 5.9 the hypotheses on heterogeneity caused by driver characteristics will be tested. Section 5.8 will focus on the strongest type of heterogeneity caused by driver characteristics, namely heterogeneity revealed in different driving styles, while section 5.9 will concentrate on heterogeneity within the group of drivers having a comparable driving style. Sections 5.10 and 5.11 perform comparable analyses for heterogeneity caused by car characteristics.

5.2 Heterogeneity: definition, causes and types

This section presents theoretical insights into heterogeneity in longitudinal driving behavior being the foundation of the empirical analyses performed in this chapter. The term heterogeneity will be defined, and the different causes and types of heterogeneity will be discussed.

5.2.1 Definition of heterogeneity

Throughout this chapter we will define heterogeneity as differences between the longitudinal driving behaviors of driver/vehicle combinations driving under exactly the same conditions, i.e. the same stretch of road, same traffic conditions, same weather conditions and so on. This longitudinal driving behavior refers to distance keeping, speed adaptation, reaction time, etc (see also chapter 2).

Using this definition, heterogeneity is a comprehensive term as it can have many different causes and can reveal itself in several ways. To be able to perform the proposed analysis on heterogeneity it is therefore important to discern on the one hand two main causes of heterogeneity and on the other hand two ways in which heterogeneity can be present in real traffic, i.e. types of heterogeneity.

In the upcoming the main causes and types of heterogeneity will be discussed.

5.2.2 Causes of heterogeneity

In chapter 2, the driver was presented as a feedback controller monitoring his current state and taking corrective actions when needed. This representation implicitly yields several potential causes of heterogeneity related to current driver characteristics, for example:

- Even when driving under the same conditions, different drivers will have different objectives (in terms of safety, efficiency, etc.).
- Different drivers do have different driving capabilities. When considering the different components of the execution of the longitudinal driving task, i.e. state
observation/estimation, future state prediction, control decision, and control action, these differences can both be reflected in different time delays, and different types and sizes of errors.

Next to these causes of heterogeneity related to the characteristics of the driver, also the car a driver is driving in can have its influence on the longitudinal movements of a driver/vehicle combination. For example, a heavily loaded truck can brake less severely than a normal person car and also the speed limits on freeways for both types of cars differ.

5.2.3 Types of heterogeneity

Clearly several causes for heterogeneity can be pointed at. But given these causes, how large are the actual differences between the driving behaviors of different driver/vehicle combinations in real-life? To be able to answer this question we will distinguish two types of heterogeneity, namely, driving style heterogeneity and heterogeneity within a driving style.

Driving style heterogeneity refers to heterogeneity in which the driving styles of drivers are inherently different. In this case different drivers have a different objective and react, for example, to different stimuli or use a different driving rule for determining an appropriate control action. The second type of heterogeneity, i.e. heterogeneity within a driving style, is less strong and refers to differences in degree between the group of drivers reacting to the same stimuli and applying the same driving rule. This last type of heterogeneity is assumed in existing microscopic simulation tools (chapter 3).

5.3 Hypotheses on heterogeneity

Based on these theoretical considerations, we can split the original research question posed in the introduction, namely, to which extent is heterogeneity present in real traffic?, into the following two subquestions:

1. To which extent is heterogeneity caused by driver characteristics present in real traffic?
2. To which extent is heterogeneity caused by different car characteristics present in real traffic?

This subdivision of the original research question based on the different causes of heterogeneity enables us to get not only insight into the extent of heterogeneity but also in the causes of heterogeneity. This additional insight is also very useful from a simulation point of view as it provides information on how to extrapolate the empirical findings to simulation studies referring to other traffic compositions than the ones present in the observations.

We answer these research questions using hypotheses. To improve the clarity of the presentation these ‘main’ hypotheses answering the central research questions of the chapter are numbered throughout the thesis, while other ‘sub’ hypotheses are not numbered. To answer the first research question the following two hypotheses will be tested:

Hypotheses on driver heterogeneity within the group of person cars

H1. Drivers of person cars differ with respect to their longitudinal driving styles.
H2. Within the group of drivers of person cars having a similar longitudinal driving style differences in the degree of how they exert this style exist.
These hypotheses aim at establishing the extent of heterogeneity in real traffic related to
differences between drivers of person cars. Based on the theoretical insights the term ‘extent’
is refined by making the distinction between driving style heterogeneity and heterogeneity
within a driving style.

In using these hypotheses to answer the first research question it is assumed that within the
group of person cars heterogeneity is mainly caused by driver characteristics, while
heterogeneity related to car properties plays only a minor role.

To answer the second research question relating to heterogeneity caused by different car
properties the following two hypotheses are introduced:

**Hypotheses on heterogeneity related to different car characteristics**

H3. Driving styles of trucks and person cars are different.

H4. Significant differences exist between trucks and person cars having a similar driving
style.

The introduction of these hypotheses is natural as the distinction between person cars and
trucks is the first one made in studies on traffic flow taking care of heterogeneity. However, in
using the results of testing these hypotheses to answer the second research question, it is
assumed that the characteristics of drivers of person cars and drivers of trucks are the same.
This assumption is to a certain sense contestable as truck drivers are professional drivers
having, for example, more experience than a part of the group of drivers of person cars. To
make this assumption as plausible as possible we will only consider peak periods during
normal working days such that most of the drivers of person cars present on the road are
commuters who are also at a regular base present on the road.

5.4 Experimental design

This section will describe the approach for testing these hypotheses. We will first give a
general overview of the approach after which several subparts are treated in more detail.

5.4.1 Overview of research approach

To test the hypotheses, we start by selecting eight different car-following rules and estimate
their parameter values using microscopic trajectory observations of individual following cars.
These different car-following rules represent different driving styles. For all individual
followers we also determine the performances of the respective rules in predicting the
dynamics of the individual.

By comparing the performance levels of the respective car-following rules between drivers
the hypotheses on driving style heterogeneity can be tested (H1 and H3). When there are no
differences between the driving styles of drivers it will be possible for us to select one
particular car-following rule that performs well for all drivers. When differences can be
identified between drivers regarding appropriate driving rules evidence is found for driving
style heterogeneity.

To test the hypotheses on heterogeneity within a driving style (H2 and H4), first all drivers
are grouped having a comparable driving style. Within these groups we compare the
parameter values of the different drivers, showing for example to which extent different
drivers differ in reacting to a certain stimulus.
A schematic overview of the approach is provided in Figure 5-1.

![Figure 5-1 Overview of research approach for testing hypotheses on heterogeneity.](image)

5.4.2 Approach details
The remainder of this section is used to provide some details for subcomponents of the approach, which are necessary for the sake of understanding. The models representing different driving styles will be introduced, additional information on the calibration approach will be given and the approaches for comparing model performances and estimated parameter values between drivers will be described.

Models representing the different driving styles
The different driving styles will in the upcoming analyses be represented by the eight different car-following models handled in detail in chapter 3:

1. The model of Chandler, Herman and Montroll (referred to as CHM model in the remainder) (Chandler et al., 1958).
2. The model of Bexelius, i.e. the two leader version of the CHM model (Bexelius, 1968).
3. The model of Tampère (Tampère, 2004).
4. The model of Addison and Low (Addison and Low, 1998).
5. The model of Gipps (Gipps, 1981).
6. The Intelligent Driver Model (referred to as IDM model) (Treiber et al., 2000).
7. The Optimal Velocity Model (referred to as OVM model) (Bando et al., 1995b).
8. The two leader version of the model of Lenz (Lenz et al., 1999).
These models are selected because they differ considerably with respect to the assumptions made on car-following behavior. For example, the models of Bexelius and Lenz assume that people consider also their second leader in executing the longitudinal driving task, while according to the other models only the direct leader is of influence. Also important differences can be observed between what the different driving rules assume about how drivers handle their distance to the driver in front. These differences refer both to the assumptions on the distance the driver exactly wants to keep as well to the importance the driver attaches to actually reaching this distance (for more details on the models we refer to chapter 3).

In selecting models we also considered whether it is likely that the available observations contain sufficient information to calibrate the corresponding model parameters. For the selected models the number of parameters is relatively limited. Most model parameters refer furthermore to the car-following regime instead of the free driving regime.

**Parameter estimation approach**

The *driver specific* model parameters \( \beta \) are calibrated using the simulation approach validated in chapter 4. The hereto required knowledge on the dynamics of the first leader and the second leader is extracted from the observations and serves as input for the different car-following models. Based on these observations the models simulate the movements of the following car. As the “real” observations for the following car are also available we can compare them to the results of the simulation.

In performing these optimizations the following multi-criterion calibration objective is used to represent the performance level of the driving rule:

\[
g(y_n, \hat{z}_n, \lambda_1, \lambda_2) =
\]

\[
\lambda_1^* \left[ \frac{1}{K_n} \sum_{k=1}^{K_n} (\Delta z_{n-1,n}(t_k) - \Delta \hat{y}_{n-1,n}(t_k, \beta))^2 \right]^{\lambda_1} + \lambda_2^* \left[ \frac{1}{K_n} \sum_{k=1}^{K_n} v_n(t_k) - \hat{v}_n(t_k, \beta))^2 \right]^{\lambda_2}
\]

In this equation \( t_k \) denotes the time instant, while \( K_n \) is the total number of time instants for which a specific follower \( n \) is observed. The time interval between two consecutive time instants is taken equal to the time interval of the observations, namely 0.1 seconds. The choice for the multi-criterion objective is based on the analyses performed in chapter 4 (see eq. (4.26)). The parameters \( \lambda_1 \) and \( \lambda_2 \) are set equal to respectively 1 and \( \frac{1}{2} \).

The *driver specific* optimal parameters \( (\beta^*) \) are those parameters that minimize the objective, i.e.

\[
\beta^* = \arg \min_{\beta} g(y_n, \hat{z}_n(\beta), \lambda_1, \lambda_2)
\]

Like in chapter 4 the parameters are constrained in order to avoid unrealistic values and to be able to explore parameter space more efficiently. An overview of all the parameters that need to be calibrated for the different models and the applied constraints is given in Table 5-1.
Table 5-1 Overview of driver specific parameters to be calibrated and applied constraints by model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Param</th>
<th>Range</th>
<th>Short Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CHM</td>
<td>$c_{1,n-1}$</td>
<td>[0,2]</td>
<td>reaction time</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>[0.5,3]</td>
<td>sens. to speed difference with leader n-1</td>
</tr>
<tr>
<td>2. Bexelius</td>
<td>$c_{1,n-1}$</td>
<td>[0,2]</td>
<td>reaction time</td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-2}$</td>
<td>[0,2]</td>
<td>sens. to speed difference with leader n-2</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>[0.5,3]</td>
<td>sens. to speed difference with leader n-1</td>
</tr>
<tr>
<td>3. Tampère</td>
<td>$c_{1,n-1}$</td>
<td>[0,2]</td>
<td>reaction time</td>
</tr>
<tr>
<td></td>
<td>$c_2$</td>
<td>[0,1]</td>
<td>sens. to difference real distance and desired distance</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>[0.05,3]</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>[4,15]</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>[0.5,3]</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td>4. Addison &amp; Low</td>
<td>$c_4$</td>
<td>[0.30]</td>
<td>div. by distance</td>
</tr>
<tr>
<td></td>
<td>$c_5$</td>
<td>[0.1]</td>
<td>(sens. to difference real distance and desired distance)$^3$</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>[4,15]</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>[0.5,3]</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td>5. Gipps</td>
<td>$a_{max}$</td>
<td>[0.5,4]</td>
<td>maximum desired acceleration following car</td>
</tr>
<tr>
<td></td>
<td>$b_{max}$</td>
<td>[-6,0]</td>
<td>maximum desired deceleration following car</td>
</tr>
<tr>
<td></td>
<td>$\theta$</td>
<td>[0,4]</td>
<td>safety reaction time</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>[4,15]</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$b_{n-1}^{max}$</td>
<td>[-6,0]</td>
<td>maximum desired deceleration leading car</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>[0.5,3]</td>
<td>reaction time</td>
</tr>
<tr>
<td>6. IDM</td>
<td>$d$</td>
<td>[4,15]</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$T_{safe}^{max}$</td>
<td>[0.2,4]</td>
<td>safe time headway</td>
</tr>
<tr>
<td></td>
<td>$b_{max abs}$</td>
<td>[0.5,4]</td>
<td>maximum desired acceleration following car</td>
</tr>
<tr>
<td></td>
<td>$m$</td>
<td>[1,\infty)</td>
<td>acceleration component</td>
</tr>
<tr>
<td>7. OVM</td>
<td>$c_{6,n-1}$</td>
<td>[0,15]</td>
<td>sens. to difference real speed and optimal velocity leader n-1</td>
</tr>
<tr>
<td></td>
<td>$m$</td>
<td>[0.0,3]</td>
<td>slope of optimal velocity function at inflection point</td>
</tr>
<tr>
<td></td>
<td>$b_f$</td>
<td>[4,75]</td>
<td>distance headway at inflection point of optimal velocity</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>[4,15]</td>
<td>distance at standstill</td>
</tr>
<tr>
<td>8. Lenz</td>
<td>$c_{6,n-1}$</td>
<td>[0,15]</td>
<td>sens. to difference real speed and optimal velocity leader n-1</td>
</tr>
<tr>
<td></td>
<td>$c_{6,n-2}$</td>
<td>[0,10]</td>
<td>sens. to difference real speed and optimal velocity leader n-2</td>
</tr>
<tr>
<td></td>
<td>$m$</td>
<td>[0.0,3]</td>
<td>slope of optimal velocity function at inflection point</td>
</tr>
<tr>
<td></td>
<td>$b_f$</td>
<td>[4,75]</td>
<td>distance headway at inflection point of optimal velocity</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>[4,15]</td>
<td>distance at standstill</td>
</tr>
</tbody>
</table>

As the available trajectory observations all pertain to very busy traffic, the desired speeds of the individual drivers will not be estimated from the observations as the observations do not contain enough information. To make nevertheless sure that the calibration algorithm can evaluate all different modeling regimes the desired speed will be assumed to be equal to 30 m/s in all calibrations in which the following car is a person car and 22.22 m/s in the cases that the following car is a truck.
All parameters except reaction time are treated as continuous variables. The reaction time $T_r$ is treated as a discrete variable with value steps of 0.1 seconds.

By applying our optimization approach to the whole observed time series of an individual driver at once, it is implicitly assumed that the driving style does not differ during the time period of observation. This assumption can be justified by the relatively short periods for which single drivers are observed. It does not necessarily imply that there are no intra-driver differences perse.

**Approach to identify driving style heterogeneity**

To identify differences in driving styles of drivers, we compare corresponding model performances between drivers where the model performances are represented by the minimized values of eq. (5.1). When a given reference model performs best for part of the drivers, while for another part other driving rules show clearly better results, we argue that evidence is found that differences exist between the driving styles of drivers.

It needs to be stressed that in performing the analyses in this chapter mostly driving rules are compared having a different logic. This is particularly important as not all models contain the same number of parameters.

In case a given model is extended to a model having more parameters most probably a better model fit can be obtained as discussed in chapter 4. Thus when both the CHM model and the Bexelius model are fitted to the observations of the same triplet it will most probably hold that for the Bexelius model a better fit can be obtained. Therefore, when all driving rules would follow the same logic, applying our approach for identifying driving style heterogeneity would result in the conclusion that there is only one driving style, namely the one represented by the most elaborated model.

When models cannot be regarded as extensions of each other, like is most of the times the case in this chapter, it does not necessarily hold that a larger number of parameters results in an improved model performance. Also the appropriateness of the logic behind the model, i.e. the extent to which a model reflects the actual execution of the car-following task of a follower, determines the degree of fit in this case. Nevertheless, a model with more parameters will of course provide better opportunities for a better fit.

**Approach to identify ‘within’ driving style heterogeneity**

To identify ‘heterogeneity within a driving style’ we consider the spread in the estimated parameter values of drivers having a comparable driving style. To make this approach valid it is, firstly, important to distinguish variability present in the calibrated parameters due to ‘within driving style heterogeneity’ and variability caused by problems in identifying a parameter. We hereto determine the reliability of estimated parameter values by calculating the normalized sensitivity of the calibration objective to a small change in the estimated parameter value as explained in chapter 4.

Secondly, we need to take care of the constraints imposed during optimization. When too many parameter estimates of a specific parameter are equal to the lower- or upper bound, the spread in the parameter estimates is not a good indicator for ‘within driving style heterogeneity’ as the magnitude of the spread of the parameter estimates is in this case determined to a large extent by the choices made in the calibration procedure. Furthermore, as
Chapter 5 - Theory and empirics of heterogeneity in car-following

the imposed bounds are in general not very strict, too many parameter estimates equal to the imposed bounds can be seen as problems in identifying a parameter.

5.5 Observation requirements

In this section we discuss the requirements the observations need to fulfill in order to be useful input to the procedure introduced in the previous section.

Based on the insights obtained in chapter 4 and in (Ossen and Hoogendoorn, 2007), we compose the following list of requirements pertaining to model calibration:

- Microscopic trajectory observations are needed of three consecutive vehicles (the models assume up to two leaders implying that for calibration observations are needed for the following car and its two direct leaders).
- These trajectory observations need to show sufficient variability in the dynamics of the follower, i.e. contain enough information for calibrating the different model parameters.
- Considered following drivers need to be mainly occupied with their longitudinal driving task.
- Observations need to have a high spatial as well as a high temporal resolution.

An important additional requirement related to the definition of heterogeneity and thus the topic of this chapter is that all followers that are compared are driving under the same conditions. When this is not the case no valid conclusions can be made on heterogeneity as established differences between drivers can be caused by different external conditions as well.

For testing the hypotheses related to heterogeneity caused by car characteristics furthermore large enough samples of both vehicle types need to be available.

5.6 Characteristics of selected observations

Based on these requirements we decided to use microscopic trajectory observations derived from digital pictures taken by a helicopter. This data collection method is described in Appendix A. These observations are particularly useful for a study on heterogeneity as trajectory data are available for all vehicles driving on the roadway stretch observed by the helicopter. In this way the requirement that all drivers are driving under comparable conditions is fulfilled (Appendix C shows that other methods for collecting trajectory observations mostly do not fulfill this criterion). The condition that observations need to be available for three consecutive vehicles is also easily fulfilled. The observations furthermore show a high spatial and temporal resolution as will be discussed in the upcoming.

In the analyses, we will use observations pertaining to two different measurement sites. One dataset has been collected at the three-lane A2 motorway near the Dutch city of Utrecht (referred to as the ‘Everdingen site’). The other dataset has been collected at the three-lane A15 motorway south of the Dutch city of Rotterdam (referred to as the ‘Waalhaven site’). Pictures of both measurement sites are provided by Figure 5-11 (at the end of this chapter). Although observations are made for both driving directions, for each dataset only one driving direction will be considered (these driving directions are indicated by rectangles in Figure 5-11). In the other driving directions, traffic turned out to be free such that these observations are no useful input to a study on car-following behavior.
Both datasets are collected during the afternoon peak hour. The observed traffic conditions differ however considerably between the datasets. The measurements of the Everdingen site are characterized by stop-and-go traffic conditions, while for the Waalhaven site congestion was quite heavy during the entire period of observation. These different traffic conditions can be clearly observed from Figure 5-2 showing the space mean speeds for both sites while making a distinction between the different lanes.

From the individual driver perspective this implies that the speed of a driver changes generally more during the time period of observation for the Everdingen measurement site than for the Waalhaven measurement site.

For both measurement sites longitudinal positions of all observed vehicles are collected at 0.1 sec. intervals. From these positions speeds are derived. Also the lengths and widths of the observed vehicles are obtained. The spatial accuracy of the observations is less than 40 cm.

During both observations the helicopter hovered over a fixed part of the road. For both measurement sites, the length of the considered stretch is approximately 450 meters. The Everdingen measurement site is observed for approximately 4.5 minutes, while for the Waalhaven measurement site observations are available for approximately 10 minutes.

![Figure 5-2 Lane specific space mean speeds for (a) Everdingen measurement site, (b) Waalhaven measurement site.](image)

5.6.1 Selecting observations for model calibration
The models considered in this chapter assume up to two leaders, implying that we have to select triplets of three consecutive vehicles (follower, first leader, second leader) from the
observations fulfilling the aforementioned requirements. This is done using the following criteria.

The first set of criteria is introduced to ensure a high enough level of information contained in the selected observations. To be included in the calibration study a following vehicle should have experienced a speed change of at least 5 m/s during the observation period. Next to that the three consecutive cars need to be observed jointly during a period of at least 15 seconds. These constraints refer to the information contained in the observations for calibrating model parameters belonging to the constrained driving regime. Since observations are gathered during congestion, parameters describing behavior in the free driving regime can only be calibrated in occasional cases.

The second criterion in selecting observations is that the composition of the triplets does not change during the considered period. This criterion is introduced to justify the assumption that the following car is mainly occupied with the car-following task.

Table 5-2 provides an overview of the numbers of selected triplets and properties of these triplets for both datasets. A driver/vehicle combination can be part of more than one triplet having different roles, i.e. as a follower, as first leader, or as second leader.

Although the Waalhaven dataset covers a clearly longer time period it contains fewer triplets fulfilling the speed variation requirements. This can be explained by the in general high level of congestion during the observation period, causing that the speeds of vehicles in the Waalhaven site are in general much more constant than the speeds of cars observed at the Everdingen measurement site.

<table>
<thead>
<tr>
<th>Total number of appropriate triplets</th>
<th>Average speed change follower (m/s)</th>
<th>Average duration of observation (sec.)</th>
<th>Number of times that the following vehicle is a person car</th>
<th>Number of times that the following vehicle is a truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement site</td>
<td>Everdingen Waalhaven</td>
<td>124 117 11.5 6.4 22.7 45.8 119 112 5 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5-2 also shows the number of times that the following vehicle was a truck. Given the applied criteria only such a limited number of triplets are available for which the following car is a truck that the sample size is not large enough to perform the analysis on heterogeneity caused by car characteristics.

An analysis of all available observations showed that the sample size of trucks for the Waalhaven site can be considerably increased when the criteria on the speed variability are weakened. Trucks show a more constant speed during the observation than person cars.

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6 To make the distinction between person cars and trucks the lengths of the observed vehicles are considered. More specific, all vehicles larger than 10 meters are marked as trucks while all other vehicles are identified as person cars. This criterion is mainly motivated by studying the helicopter pictures and the available data on the lengths of the observed vehicles.
In testing the hypotheses on heterogeneity caused by car characteristics (H3 and H4) therefore triplets will be considered for which the speed change of the follower is between 2.5 m/s and 5 m/s. In these analyses the same constraints will be applied to person cars and trucks to ensure that it is justified to compare these two types of cars. To lower the probability of confusing differences related to different car types with differences due to different driving conditions of the driver further, the upper limit to the speed change is imposed.

Table 5-3 gives selective properties of the observations used in testing the hypotheses on heterogeneity caused by car characteristics.

**Table 5-3 Overview of properties of selected triplets for the comparison of trucks and person cars (section 5.10).**

<table>
<thead>
<tr>
<th>Measurement site</th>
<th>Total number of appropriate triplets</th>
<th>Average speed change follower (m/s)</th>
<th>Average duration of observation (sec.)</th>
<th>Number of times that the following vehicle is a person car</th>
<th>Number of times that the following vehicle is a truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waalhaven</td>
<td>196</td>
<td>3.7</td>
<td>39.4</td>
<td>167</td>
<td>29</td>
</tr>
</tbody>
</table>

In testing the hypotheses on heterogeneity caused by driver characteristics, we will perform separate analyses for both measurement sites. By comparing our conclusions for both sites more general conclusions can be drawn.

In testing the hypotheses on heterogeneity caused by car characteristics only the Waalhaven measurement site will be considered as the Everdingen site does not provide enough useful observations relating to trucks.

### 5.7 General model performances for drivers of person cars

Before we test the hypotheses on the different types of heterogeneity caused by driver characteristics (H1 and H2), we first consider the overall performances of the different longitudinal driving rules and how they are correlated.

By comparing the overall performances of the different longitudinal driving rules we can identify already those behavioral rules that show in general good performances. This knowledge can later on be used for testing the hypothesis related to the presence of driving style heterogeneity caused by driver characteristics; when there is no driving style heterogeneity it is by definition possible to select one driving rule showing good results for all drivers.

The correlations between the performances for the respective models are furthermore analyzed in order to establish whether the different behavioral rules show good performances for the same followers or for different followers. As will be explained later on, this can also give first insights into the presence of driving style heterogeneity caused by driver characteristics.
5.7.1 Comparison of general performances of driving rules for drivers of person cars
To assess the general performances of the models in predicting the movements of individual
drivers of person cars, the empirical cumulative distribution functions (c.d.f.) of the
performance levels are shown for all models. To simplify the notation the performance level,
i.e. the minimized value of the multi-criterion calibration objective represented by eq. (5.1),
for a given follower and a given model is denoted by $\varepsilon_{model}^*$. 

The empirical c.d.f. $(F_{\varepsilon_{model}^*}(e))$ for a given model is determined as follows:

$$F_{\varepsilon_{model}^*}(e) = \frac{\text{total number of followers with } \varepsilon_{model}^* < e}{\text{total number of followers}}$$  \hspace{1cm} (5.3)

The empirical c.d.f. thus shows for every value of $e$ for which fraction of the observed drivers
the model under consideration resulted in a lower value of $\varepsilon_{model}^*$ than $e$. The resulting c.d.f.’s
for the Everdingen dataset are provided in Figure 5-3a, while the c.d.f.’s for the Waalhaven
measurement site are presented in Figure 5-3b. Table 5-4 furthermore shows some descriptive
statistics of the values obtained for $\varepsilon_{model}^*$ for the Everdingen measurement site, while Table
5-5 shows some descriptive statistics corresponding to the Waalhaven measurement site.

In Figure 5-3a, the error term $\varepsilon_{CHM}^*$ is smaller than or equal to approximately 0.04 for 50%
(corresponding to the median) of the drivers if it is assumed that all followers drive according
to the CHM model. If the Tampère model is used to simulate their movements, the median is
equal to approximately 0.016.

In general, the better the model reflects the observations of all individual drivers, the more the
line lies to the left, i.e. the larger the fraction of drivers with $\varepsilon_{model}^*$ smaller than $e$ is for every
value of $e$. It can be seen that the model with the lowest number of parameters, i.e. the model
of CHM performs clearly worse than the more elaborated models. The two-leader model of
Bexelius yields better results than the one-leader CHM model but it does not perform as well
as the models of Tampère, Lenz, Gipps, Addison and Low, and the IDM.

For the models with the largest number of parameters, i.e. the Gipps model and the Tampère
model having 6 parameters, it can be seen that the Tampère model performs in general better
than the Gipps model. For the models having 5 parameters especially the Lenz model shows
good results for the Everdingen dataset, even slightly better than the results for the Gipps
model. The obtained fits of the corresponding one leader OVM model show however not such
a good resemblance between the observations and the simulations providing first evidence
that followers multi-anticipate, i.e. consider more than one leader. For the Waalhaven site the
results for the one-leader OVM model are even comparable to the results obtained with the
simpler Bexelius model.

Another conclusion that can be drawn from Figure 5-3 is that the model performances are in
general better for the Everdingen measurement site than for the Waalhaven measurement site.
A possible explanation for this is the on average longer period of observation for triplets from
the Waalhaven dataset. The larger this period the more difficult it will be to fit a car-following model that is only a rough approximation of the real car-following behavior of the driver. When a model does not describe the longitudinal driving behavior of a driver perfectly it is well possible that parameters can for a short period be tuned such that a good model fit is obtained. When the length of the period increases this will become more difficult as parameters that are tuned to fit part of the time series face problems in predicting other parts. (Note that the performance indicator represented by eq. (5.1) corrects for differences between durations of observation periods, so the presented performances are ‘normalized’)

Figure 5-3 Empirical c.d.f.’s of $\epsilon^*_{\text{model}}$ for all eight models for (a) the Everdingen dataset (b) for the Waalhaven dataset.

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7 When a model does not describe the longitudinal driving behavior of a driver perfectly it is well possible that parameters can for a short period be tuned such that a good model fit is obtained. When the length of the period increases this will become more difficult as parameters that are tuned to fit part of the time series face problems in predicting other parts. (Note that the performance indicator represented by eq. (5.1) corrects for differences between durations of observation periods, so the presented performances are ‘normalized’)

5.7.2 Correlations between model performances of different driving rules

Table 5-4 and Table 5-5 show the correlations between the error terms for the different models. A high positive correlation (near one) between the error terms of two models generally means that a follower with a relatively small error for one model most probably also has a relatively small error for the other model. A low negative correlation (near minus one) between two models means in this sense that followers having a low error term for one model most probably have a high error for the other model and vice versa.

Table 5-4 Descriptive statistics of values obtained for $\varepsilon^*_{\text{model}}$ (Everdingen site, N=119).

<table>
<thead>
<tr>
<th>Model</th>
<th>Median $\varepsilon^*_{\text{model}}$ (std. dev.)</th>
<th>Nr. of times best</th>
<th>CHM</th>
<th>Bex.</th>
<th>Tamp.</th>
<th>Lenz</th>
<th>Gipps</th>
<th>Add. &amp; Low</th>
<th>IDM</th>
<th>OVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHM</td>
<td>0.0396 (0.0404)</td>
<td>0</td>
<td>1</td>
<td>0.858</td>
<td>0.732</td>
<td>0.717</td>
<td>0.715</td>
<td>0.772</td>
<td>0.669</td>
<td>0.836</td>
</tr>
<tr>
<td>Bexelius</td>
<td>0.0317 (0.0337)</td>
<td>2</td>
<td>1</td>
<td>0.766</td>
<td>0.814</td>
<td>0.726</td>
<td>0.786</td>
<td>0.729</td>
<td>0.842</td>
<td></td>
</tr>
<tr>
<td>Tampère</td>
<td>0.0162 (0.0285)</td>
<td>41</td>
<td>1</td>
<td>0.862</td>
<td>0.847</td>
<td>0.901</td>
<td>0.888</td>
<td>0.864</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lenz</td>
<td>0.0170 (0.0297)</td>
<td>32</td>
<td>1</td>
<td>0.765</td>
<td>0.798</td>
<td>0.878</td>
<td>0.906</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gipps</td>
<td>0.0195 (0.0233)</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.867</td>
<td>0.809</td>
<td>0.769</td>
</tr>
<tr>
<td>Addison &amp; Low</td>
<td>0.0196 (0.0313)</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>0.825</td>
<td>0.841</td>
</tr>
<tr>
<td>IDM</td>
<td>0.0207 (0.1208)</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td>0.797</td>
</tr>
<tr>
<td>OVM</td>
<td>0.0230 (0.0323)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5-5 Descriptive statistics of values obtained for $\varepsilon^*_{\text{model}}$ (Waalhaven site, N=112).

<table>
<thead>
<tr>
<th>Model</th>
<th>Median $\varepsilon^*_{\text{model}}$ (std. dev.)</th>
<th>Nr. of times best</th>
<th>CHM</th>
<th>Bex.</th>
<th>Tamp.</th>
<th>Lenz</th>
<th>Gipps</th>
<th>Add. &amp; Low</th>
<th>IDM</th>
<th>OVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHM</td>
<td>0.0630 (0.0387)</td>
<td>0</td>
<td>1</td>
<td>0.867</td>
<td>0.693</td>
<td>0.775</td>
<td>0.709</td>
<td>0.746</td>
<td>0.696</td>
<td>0.88</td>
</tr>
<tr>
<td>Bexelius</td>
<td>0.0524 (0.0342)</td>
<td>4</td>
<td>1</td>
<td>0.757</td>
<td>0.865</td>
<td>0.793</td>
<td>0.794</td>
<td>0.753</td>
<td>0.840</td>
<td></td>
</tr>
<tr>
<td>Tampère</td>
<td>0.0398 (0.0277)</td>
<td>19</td>
<td>1</td>
<td>0.820</td>
<td>0.868</td>
<td>0.918</td>
<td>0.909</td>
<td>0.815</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lenz</td>
<td>0.0399 (0.0276)</td>
<td>22</td>
<td>1</td>
<td>0.836</td>
<td>0.851</td>
<td>0.858</td>
<td>0.903</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gipps</td>
<td>0.0434 (0.0264)</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.868</td>
<td>0.890</td>
<td>0.815</td>
</tr>
<tr>
<td>Addison &amp; Low</td>
<td>0.0398 (0.0286)</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>0.92</td>
<td>0.847</td>
</tr>
<tr>
<td>IDM</td>
<td>0.0415 (0.0276)</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td>0.823</td>
</tr>
<tr>
<td>OVM</td>
<td>0.0540 (0.0324)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As the calibrated models differ regarding their underlying assumptions about car-following behavior, these correlations implicitly provide some indications whether the driving styles of different drivers differ (the topic discussed in the next section). Accordingly, we state that the
lower the correlation the higher the differences in driving styles of drivers, given at least that both models show good results for a part of the drivers. On the other hand, low correlations seem to be unrealistic for the problem at hand as most considered car-following models share common stimuli while only the exact relations between the stimuli and the response differ.

The tables show that the correlations are indeed all positive implying that the behavior of some of the followers can in general be better explained than the behavior of the others. The values are nevertheless also all clearly smaller than one giving a first indication that the driving styles of different drivers differ in that the performances of the respective car-following models differ between drivers.

5.8 Driving style heterogeneity caused by different driver characteristics

The final consideration of the previous subsection pointed at differences between the driving styles of drivers of person cars, i.e. driving style heterogeneity caused by driver characteristics. The aim of this section is to further explore this type of heterogeneity, i.e. to test the hypothesis:

H1. Drivers of person cars differ with respect to their longitudinal driving styles.

A first step in testing this hypothesis is to determine for each driver the model that performed best in simulating his longitudinal movements by looking at the $\varepsilon^\ast_{model}$ values. The results for the Everdingen site are presented in Table 5-4 (third column), while the results for the Waalhaven site are presented in Table 5-5 (third column).

The most important observation from these outcomes is that the most applicable driving rule differs for different drivers. For example, for the Everdingen site the model of Tampère appears best for 41 drivers, while for 32 followers the Lenz model performed best.

This finding induces the following question, how large are the differences in performances of the different driving rules? Stated differently, suppose we want to select one driving rule to describe the car-following behavior of all drivers, while taking heterogeneity caused by driver characteristics into account by allowing different parameter values for different drivers, what will then happen to the error term? When we are able to select from this analysis a driving rule for which no followers can be found for whom another driving rule performs clearly better, the hypothesis can be rejected.

This question is also relevant from a simulation point of view as most microscopic simulation models use a single model to describe car-following behavior and only distinguish drivers by applying different parameter values (chapter 3).

In order to answer this question, we have calculated the difference in the optimal value of the calibration objective when model B is used instead of model A (for all possible pairs of A and B with A $\neq$ B):

$$\Delta \varepsilon_{A \rightarrow B} = \varepsilon^\ast_{model A} - \varepsilon^\ast_{model B}$$  (5.4)

Let us assume that all drivers are driving according to the Tampère model, with each driver having his own best parameter values. In this case, model A is the Tampère model and
formula (5.4) calculates the improvement ($\Delta e_{A \rightarrow B} > 0$) or the deterioration ($\Delta e_{A \rightarrow B} < 0$) that is obtained when another model (model B) is used instead of the Tampère model.

For both measurement sites we repeat these analyses several times using respectively the model of Tampère, the model of Gipps, the model of Lenz, the model of Addison and Low and the IDM model as reference model, i.e. model A. The motivation for taking these models as reference models is that they performed well in the analyses on the general performances such that whenever it is possible to select a ‘universal model’ representing the driving style of all drivers it will be one of these.

Figure 5-4 and Figure 5-5 show the results for the analyses in which the best performing models\(^8\) for the Everdingen site and the Waalhaven site, respectively the model of Tampère and the model of Addison and Low, are taken as reference models. The results are represented by their empirical c.d.f.’s ($F_{\Delta e_{A \rightarrow B}}(\Delta e_{A \rightarrow B})$) calculated as follows:

$$F_{\Delta e_{A \rightarrow B}}(\Delta e_{A \rightarrow B}) = \frac{\text{total number of followers with } \Delta e_{A \rightarrow B} < 0}{\text{total number of followers}}$$

(5.5)

The c.d.f. $F_{\Delta e_{A \rightarrow B}}(\Delta e_{A \rightarrow B})$ thus shows for every value of $\Delta e_{A \rightarrow B}$ which fraction of the drivers has a value $\Delta e_{A \rightarrow B}$ smaller than $\Delta e_{A \rightarrow B}$. In illustration, by assuming $\Delta e_{A \rightarrow B}$ to be equal to 0 the c.d.f. shows for which fraction of the drivers model B yields an improvement compared to the reference model A ($\Delta e_{A \rightarrow B} > 0$) and vice versa.

![Model A: Tampère](image)

**Figure 5-4** Empirical c.d.f.’s of improvements that can be obtained when another model is used instead of the model of Tampère (Everdingen site).

---

\(^8\) Based on the number of followers for whom a given model is best.
Figure 5-4 indicates that when the model of Tampère is the reference model the real driving behavior can be better described for a considerable number of drivers when another model is used instead. For example, when the Tampère car-following model is replaced by the Lenz model this yields an improvement for 42% of the drivers despite the fact that the Tampère model appeared as best model for 41 drivers. The mean improvement is 0.01 while the maximum improvement that can be obtained when replacing the Tampère model with the Lenz model is 0.038. When these improvements are expressed in percentual changes of the error term (eq. (5.6)) it is found that the mean improvement is 28% while the maximum improvement is 83%.

\[
\% \Delta \varepsilon_{A \rightarrow B} = - \left( \frac{\varepsilon_{model B}^* - \varepsilon_{model A}^*}{\varepsilon_{model A}^*} \right) \times 100\% \quad (5.6)
\]

Comparable conclusions can be drawn from Figure 5-5 corresponding to the model of Addison and Low showing best results for the Waalhaven measurement site. Also the analyses of the other models show that there are always a considerable number of drivers for whom another driving rule is better than a given reference rule, performing well for a part of the drivers.

From these findings, we conclude that the hypothesis that heterogeneity caused by driver characteristics reveals itself in different driving styles (H1) can be accepted.
5.9 Heterogeneity within driving styles caused by different driver characteristics

In this section we test the second hypothesis related to heterogeneity caused by driver characteristics, namely:

\[ H2. \text{Within the group of drivers of person cars having a similar longitudinal driving style differences in the degree of how they exert this style exist.} \]

To test this hypothesis we consider the variability in the distributions of the parameter estimates for the generally best performing models, i.e. the models of Tampère, Gipps, Lenz, Addison and Low, and the IDM model. As we want to compare drivers having a comparable driving style, we compose these empirical distributions by selecting for each of these models and for each measurement site separately the followers having the lowest $\varepsilon^{*}_\text{model}$ values. More specific, based on Figure 5-3 we select for the Everdingen measurement site all followers with $\varepsilon^{*}_\text{model}$ smaller than 0.0175 and for the Waalhaven measurement site all followers with $\varepsilon^{*}_\text{model}$ smaller than 0.035.

To justify the approach of considering the variability in the estimated parameters as an indicator for ‘within driving style heterogeneity’, as discussed in section 5.4.2, we only consider parameters having for both measurement sites a normalized sensitivity larger than 0.4 and for which more than 75% of the estimates is not equal to one of the bounds. Table 5-6 and Table 5-7 hereto show the sensitivities (i.e. the second derivatives), the normalized sensitivities, and the percentages of parameters not equal to the imposed bounds, for respectively the Everdingen site and the Waalhaven site.

The parameters relating to the desired distance at standstill ($d$) are relatively often equal to the imposed bounds. A detailed check of the corresponding results reveals that a part of these estimates is equal to the lowerbound and a part is equal to the upperbound. This possibly points at problems in identifying these values. These identification problems can be explained by the fact that almost no followers are considered coming to a complete stop due to which the available observations contain insufficient information for deriving the desired distance at standstill.

Low normalized sensitivities can be observed for the parameter $c_3$ of the Tampère model referring to the sensitivity of the driver to the difference between the desired speed and the actual speed. This is most probably due to the lack of observations referring to the free driving regime. The same holds to a lesser extent for the parameter $a^{max}$ of the Gipps model.
Table 5-6 Descriptive statistics of the reliability analysis of the estimates of the model parameters for the Everdingen site. The highlighted parameters can be used in the analysis on ‘within driving style heterogeneity’.

<table>
<thead>
<tr>
<th>Model</th>
<th>Param.</th>
<th>Median sens.</th>
<th>Norm. sens.</th>
<th>% not equal to bound</th>
<th>Short interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tampère</td>
<td>c₁₁,n-1</td>
<td>6.08</td>
<td>0.78</td>
<td>92</td>
<td>sens. to speed difference with leader n-1</td>
</tr>
<tr>
<td></td>
<td>c₂</td>
<td>148.83</td>
<td>0.83</td>
<td>95</td>
<td>sens. to difference real distance and desired distance</td>
</tr>
<tr>
<td></td>
<td>c₃</td>
<td>1.31</td>
<td>0.01</td>
<td>83</td>
<td>sens. to difference real speed and desired speed</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>0.57</td>
<td>20.62</td>
<td>43</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>γ</td>
<td>3.44</td>
<td>6.75</td>
<td>85</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td>Addison &amp; Low</td>
<td>c₄</td>
<td>0.01</td>
<td>1.81</td>
<td>89</td>
<td>sens. to speed difference with leader n-1 div. by distance</td>
</tr>
<tr>
<td></td>
<td>c₅</td>
<td>1.9x10⁶</td>
<td>0.18</td>
<td>100</td>
<td>sens. to difference real distance and desired distance</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>1.00</td>
<td>20.62</td>
<td>45</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>γ</td>
<td>7.44</td>
<td>14.24</td>
<td>94</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td>Gipps</td>
<td>aₘₘₜₜ</td>
<td>0.34</td>
<td>0.45</td>
<td>94</td>
<td>maximum desired acceleration following car</td>
</tr>
<tr>
<td></td>
<td>bₘₘₜₜ</td>
<td>29.89</td>
<td>110.37</td>
<td>79</td>
<td>maximum desired deceleration following car</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>2.13</td>
<td>3.20</td>
<td>57</td>
<td>safety reaction time</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>0.29</td>
<td>4.88</td>
<td>42</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>bₙ₋₁ₘₘₜₜ</td>
<td>18.33</td>
<td>36.27</td>
<td>91</td>
<td>maximum desired deceleration leading car</td>
</tr>
<tr>
<td>Lenz</td>
<td>c₆ₙ₋₁</td>
<td>5.74</td>
<td>0.22</td>
<td>84</td>
<td>sens. to difference real speed and optimal velocity leader n-1</td>
</tr>
<tr>
<td></td>
<td>c₆ₙ₋₂</td>
<td>8.92</td>
<td>0.17</td>
<td>77</td>
<td>sens. to difference real speed and optimal velocity leader n-2</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>7690.4</td>
<td>8.26</td>
<td>97</td>
<td>slope of optimal velocity function at inflection point</td>
</tr>
<tr>
<td></td>
<td>bₗₜ</td>
<td>0.03</td>
<td>24.02</td>
<td>87</td>
<td>distance headway at inflection point of optimal velocity</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>0.03</td>
<td>2.89</td>
<td>51</td>
<td>distance at standstill</td>
</tr>
<tr>
<td>IDM</td>
<td>d</td>
<td>0.56</td>
<td>106.7</td>
<td>46</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>Tₛₙₑₐₑ</td>
<td>2.18</td>
<td>3.38</td>
<td>87</td>
<td>safe time headway</td>
</tr>
<tr>
<td></td>
<td>aₘₘₜₜ</td>
<td>0.25</td>
<td>0.70</td>
<td>78</td>
<td>maximum desired acceleration following car</td>
</tr>
<tr>
<td></td>
<td>bₘₘₜₜabs</td>
<td>0.10</td>
<td>0.27</td>
<td>70</td>
<td>maximum desired deceleration following car</td>
</tr>
<tr>
<td></td>
<td>δ</td>
<td>0.08</td>
<td>18.56</td>
<td>78</td>
<td>acceleration component</td>
</tr>
</tbody>
</table>
Table 5-7 Descriptive statistics of the reliability analysis of the estimates of the model parameters for the Waalhaven site. The highlighted parameters can be used in the analysis on ‘within driving style heterogeneity’.

<table>
<thead>
<tr>
<th>Model</th>
<th>Param.</th>
<th>Median sens.</th>
<th>Norm. sens.</th>
<th>% not equal to bound</th>
<th>Short interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tampère</td>
<td>$c_{1,n-1}$</td>
<td>6.05</td>
<td>0.40</td>
<td>100</td>
<td>sens. to speed difference with leader $n-1$</td>
</tr>
<tr>
<td></td>
<td>$c_2$</td>
<td>109.23</td>
<td>0.16</td>
<td>89</td>
<td>sens. to difference real distance and desired distance</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>sens. to difference real speed and desired speed distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>0.03</td>
<td>2.50</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>0.92</td>
<td>3.07</td>
<td>83</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td>Addison</td>
<td>$c_4$</td>
<td>0.02</td>
<td>0.58</td>
<td>95</td>
<td>sens. to speed difference with leader $n-1$ div. by distance</td>
</tr>
<tr>
<td>&amp; Low</td>
<td>$c_5$</td>
<td>18620</td>
<td>0.06</td>
<td>91</td>
<td>sens. to difference real distance and desired distance distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>0.03</td>
<td>1.49</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>1.27</td>
<td>3.02</td>
<td>91</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td>Gipps</td>
<td>$a^{\text{max}}$</td>
<td>0</td>
<td>0</td>
<td>89</td>
<td>maximum desired acceleration following car</td>
</tr>
<tr>
<td></td>
<td>$b^{\text{max}}$</td>
<td>6.24</td>
<td>8.83</td>
<td>95</td>
<td>maximum desired deceleration following car</td>
</tr>
<tr>
<td></td>
<td>$\theta$</td>
<td>0.90</td>
<td>0.01</td>
<td>43</td>
<td>safety reaction time distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>0.02</td>
<td>0.39</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b_{n-1}^{\text{max}}$</td>
<td>13.05</td>
<td>22.19</td>
<td>78</td>
<td>maximum desired deceleration leading car</td>
</tr>
<tr>
<td>Lenz</td>
<td>$c_{6,n-1}$</td>
<td>1.74</td>
<td>0.04</td>
<td>79</td>
<td>sens. to difference real speed and optimal velocity leader $n-1$</td>
</tr>
<tr>
<td></td>
<td>$c_{6,n-2}$</td>
<td>2.39</td>
<td>0.08</td>
<td>81</td>
<td>sens. to difference real speed and optimal velocity leader $n-2$</td>
</tr>
<tr>
<td></td>
<td>$m$</td>
<td>15800</td>
<td>6.53</td>
<td>98</td>
<td>slope of optimal velocity function at inflection point</td>
</tr>
<tr>
<td></td>
<td>$b_f$</td>
<td>0.02</td>
<td>0.38</td>
<td>42</td>
<td>distance headway at inflection point of optimal velocity distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>0.03</td>
<td>0.65</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>IDM</td>
<td>$d$</td>
<td>0.03</td>
<td>2.06</td>
<td>67</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{safe}}$</td>
<td>1.06</td>
<td>2.01</td>
<td>91</td>
<td>safe time headway</td>
</tr>
<tr>
<td></td>
<td>$a^{\text{max}}$</td>
<td>0.15</td>
<td>0.09</td>
<td>67</td>
<td>maximum desired acceleration following car</td>
</tr>
<tr>
<td></td>
<td>$b^{\text{max abs}}$</td>
<td>0.08</td>
<td>0.11</td>
<td>73</td>
<td>$\text{maximum desired deceleration following car}$</td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td>0.00</td>
<td>0.00</td>
<td>56</td>
<td>acceleration component</td>
</tr>
</tbody>
</table>

The parameters fulfilling the criteria, and thus the parameters considered in the analysis on ‘within driving style’ heterogeneity are highlighted in Table 5-6 and Table 5-7. In Figure 5-6
the empirical c.d.f.’s of the corresponding parameters are shown, while Table 5-8 gives an overview of the corresponding coefficients of variation.

![Empirical c.d.f.'s of parameter estimates](image)

**Figure 5-6** Empirical c.d.f.’s of the parameter estimates of the best performing models.

**Table 5-8** Coefficients of variation for the parameter estimates of the best performing models.

<table>
<thead>
<tr>
<th></th>
<th>Coefficient of variation</th>
<th>Everdingen</th>
<th>Waalhaven</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tampère: $c_{1,n-1}$</td>
<td>0.97</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>Tampère: $\gamma$</td>
<td>0.69</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>Lenz: $m$</td>
<td>1.1</td>
<td>1.53</td>
<td></td>
</tr>
<tr>
<td>Gipps: $b_{\text{max}}^{\text{n-1}}$</td>
<td>-0.87</td>
<td>-0.97</td>
<td></td>
</tr>
<tr>
<td>Gipps: $b_{n-1,\text{max}}$</td>
<td>-0.9</td>
<td>-0.92</td>
<td></td>
</tr>
<tr>
<td>Addison &amp; Low: $c_4$</td>
<td>0.59</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>Addison &amp; Low: $\gamma$</td>
<td>0.66</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>IDM: $T_{\text{safe}}$</td>
<td>0.62</td>
<td>0.60</td>
<td></td>
</tr>
</tbody>
</table>

For both measurement sites a considerable spread in the optimal parameter values can be observed. As the shown empirical distributions fulfill the strict conditions on model performances and reliability of parameter estimates, it seems justified to conclude that not only driving styles differ between drivers but that also behavioral parameter values for drivers having a comparable driving style show a large spread. For example, for the followers having
a driving style that can be well approximated by the Tampère car-following rule, the degree to which drivers react to the relative speed (expressed in the parameter $c_{1,n-1}$) shows large variations between drivers.

These findings support the hypothesis that also clear differences exist between drivers of person cars having comparable driving styles (H2).

Next to these differences between person cars having a comparable driving style and driving at the same measurement site also indications are found that the execution of the longitudinal driving task differs between drivers driving at the different sites. These differences are the topic of Appendix E.

To complete the analysis on within driving style heterogeneity within the group of person car drivers Table 5-9 shows the descriptive statistics of the discrete reaction times corresponding to the best performing models. For these models again only reaction times are considered of drivers of person cars yielding the lowest error terms, where the same thresholds for $\varepsilon^*_{mode}$ are used as before.

The optimal reaction times also clearly show a considerable spread as can be seen from the coefficients of variation. These findings thus also support the hypothesis on within driving style heterogeneity caused by driver characteristics (H2).

<table>
<thead>
<tr>
<th>Measurement site</th>
<th>everdingen</th>
<th>Waalhaven</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tampère</td>
<td>1.28</td>
<td>0.93</td>
</tr>
<tr>
<td>Addison &amp; Low</td>
<td>1.32</td>
<td>0.92</td>
</tr>
<tr>
<td>Gipps</td>
<td>1.75</td>
<td>1.55</td>
</tr>
</tbody>
</table>

5.9.1 Relation between driver characteristics and lane choice

The analysis on heterogeneity caused by driver characteristics concentrated up till now on longitudinal driving behavior only. Given the empirical evidence that people execute this task in different ways, an interesting question is whether these differences in longitudinal driving styles are reflected in the lane choice of a driver. Thus are different lanes characterized by different longitudinal driving styles?

Especially during free-flow conditions considerable differences between the driver populations at different lanes are assumed to exist. Slow vehicles like trucks and slow person cars will in general be driving on the shoulder lane, while the passing lanes are used by faster vehicles passing these slow vehicles (Hoogendoorn, 1999, Daganzo, 2002, Kerner, 2004). In congestion, when speed differences between lanes become smaller, differences between the populations of drivers of person cars driving at the different lanes will also become smaller.

The aim of this section is to establish whether we can identify from our calibration results statistically significant differences between the longitudinal driving behaviors of drivers of person cars at different lanes. To perform this analysis we test for the parameters considered in the previous section, the following hypotheses using a Kolgomorov-Smirnov test with a significance level of 5% (Chakravarti et al., 1967).
In these hypotheses the vector $\beta_{\text{lane A}}$ contains the estimated parameter values for a specific behavioral parameter for all considered triplets for which the following car was driving on lane A, while $\beta_{\text{lane B}}$ contains the estimated values belonging to followers driving on lane B. The hypotheses are tested for all parameters separately.

For both measurement sites the median lane is compared to the middle lane. The reason for this is that for the peak hour lane of the Everdingen site, as well as for the shoulder lane of the Waalhaven site not enough followers are available fulfilling all requirements. We select again for every lane and for every model separately the followers showing the best results.

The results show only one significant difference between the parameter values of person car drivers driving on the different lanes. It turns out that the estimates for the parameter $b_{\text{max}}$ of the Gipps model differ significantly for drivers of person cars driving at the middle lane and drivers of person cars driving at the median lane of the Everdingen measurement site. When we consider the corresponding c.d.f.’s of both lanes it can be seen that drivers driving at the middle lane often consider their own braking capabilities to be worse than drivers driving at the median lane. Ceteris paribus this can be interpreted as a more risk prone attitude of drivers driving at the median lane.

A reason for the low number of parameters for which a significant difference can be established is that the observations of both measurement sites refer to congested conditions for which the speed differences between the different lanes are relatively small (Figure 5-2), i.e. synchronized flow conditions.

It furthermore needs to be stressed that in comparing the results between the lanes we focused on the longitudinal driving behavior of drivers of person cars only. Differences between the longitudinal behaviors of drivers on the different lanes caused by different truck percentages were thus not considered as differences between longitudinal driving behaviors caused by car characteristics will be discussed in detail in the next sections.
5.10 Driving style heterogeneity caused by car type

In the previous sections we focused on the extent of heterogeneity within the group of drivers of person cars. Based on the assumption that car characteristics only have a minor effect on differences between drivers within this group, we concluded from the results that different driver characteristics lead both to differences between driving styles as well as to differences between drivers having a comparable driving style.

In the upcoming subsections we concentrate on heterogeneity caused by different car types. In this section we test the hypothesis:

H3. Driving styles of trucks and person cars are different.

while we test the hypothesis:

H4. Significant differences exist between trucks and person cars having a similar driving style.

in section 5.11. To test these hypotheses, observations are selected using the weakened selection criteria specified in section 5.6.1, implying that we only consider the Waalhaven measurement site. This need for adjusted criteria possibly already indicates a difference between trucks and persons cars as a possible interpretation is that trucks drive with a more constant speed than person cars. This might be caused by the larger weight of trucks making them less maneuverable. It is also well imaginable that truck drivers can adopt a more robust driving style as they can, for example, better anticipate future traffic conditions due to a better view and/or more driving experience.

Table 5-10 Differences in model performances between trucks and person cars (Waalhaven only).

<table>
<thead>
<tr>
<th>Model</th>
<th>% times best (absolute value)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>truck</td>
</tr>
<tr>
<td>CHM</td>
<td>0% (0)</td>
</tr>
<tr>
<td>Bexelius</td>
<td>7% (2)</td>
</tr>
<tr>
<td>Tampère</td>
<td>28% (8)</td>
</tr>
<tr>
<td>Addison &amp; Low</td>
<td>3% (1)</td>
</tr>
<tr>
<td>Gipps</td>
<td>10% (3)</td>
</tr>
<tr>
<td>IDM</td>
<td>28% (8)</td>
</tr>
<tr>
<td>OVM</td>
<td>0% (0)</td>
</tr>
<tr>
<td>Lenz</td>
<td>24% (7)</td>
</tr>
</tbody>
</table>

To test the hypothesis on generic differences between the driving styles of driver/person car combinations and driver/truck combinations (H3), we compare the performances of the different driving rules between person cars and trucks. Table 5-10 provides an overview of the number of times that each of the considered models is best for each vehicle class. For trucks the model of Tampère, the IDM and the Lenz model show good results. For person cars the models of Lenz, Addison & Low and Tampère perform best. These differences between the performances of the driving rules can also be clearly observed in Figure 5-8 showing the empirical c.d.f.’s of $\hat{e}_{model}$ for both trucks and person cars. Especially the difference between the performances of the model of Addison and Low is profound. The assumption made by this driving rule that followers are more eager in restoring larger deviations from their desired
distance compared to the Tampère model seems more appropriate for person cars than for trucks.

Evidence is thus provided supporting the hypothesis that optimal models do not only differ between drivers of the same vehicle class, but that also the best performing models differ between vehicle classes (H3).

![Figure 5-8 Empirical c.d.f.’s of $\varepsilon_{model}^*$ for all eight models for (a) trucks (b) person cars.](image)

### 5.11 Heterogeneity within driving styles caused by car type

In this section we establish whether we can identify differences between the driving behaviors of driver/person car combinations and driver/truck combinations applying a comparable driving rule (H4). We test for statistically significant differences between the parameter distributions of both groups of driver/vehicle combinations.

We perform these tests like before for the in general best performing models, i.e. the models of Tampère, Gipps, Lenz, Addison and Low and the IDM. As we want to compare drivers having a comparable driving style, we compose the empirical distributions by selecting for each of these models separately driver/person car combinations as well as driver/truck combinations having a value for $\varepsilon_{model}^*$ smaller than 0.035 (the same value as previously used in analyses corresponding to the Waalhaven measurement site). We will furthermore consider the same behavioral parameters as in the previous analyses.

Figure 5-9 shows the empirical c.d.f.’s for the parameter estimates for both types of driver/vehicle combinations. For the Lenz model the slope of the optimal velocity function...
seems to be a little smaller for trucks than for person cars. For the Gipps model the estimates for $b^{\text{max}}$ seem to be more or less comparable. Regarding the parameter $b_{n-1}^{\text{max}}$ however a difference between both vehicle types can be observed. The parameter estimates belonging to truck drivers are in generally namely lower, implying that truck drivers make a more risk-averse estimate of their leaders braking capacity than drivers of person cars. The plot referring to $T_{\text{safe}}$ of the IDM model shows that this value is larger for trucks than for person cars.

These visually established differences are analyzed in more depth by performing Kolgomorov-Smirnov tests with a significance level of 5%. To do so, the vector $\beta_{\text{person cars}}$ contains the estimated parameter values for a specific parameter for all considered triplets for which the following car was a person car, while $\beta_{\text{trucks}}$ contains the estimates belonging to trucks. For every separate parameter the following hypotheses are tested:

$$H_0: \beta_{\text{person cars}} \text{ and } \beta_{\text{trucks}} \text{ are not statistically different.}$$

$$H_A: \beta_{\text{person cars}} \text{ and } \beta_{\text{trucks}} \text{ are statistically different.}$$

The corresponding results are provided in Table 5-11. Two significant differences are found. One of them refers to the earlier mentioned difference between $T_{\text{safe}}$ for trucks and person cars for the IDM model. This finding is in line with the default parameter values suggested in, for example (Kesting et al., 2007b).

The other one corresponds to the model of Addison and Low and refers to the sensitivity of a driver to the ratio of the relative speed and the gross relative distance. At least two plausible
explanations can be given for this finding. The first one is that gross distances are considered. This means that the denominator of the ratio is larger for trucks than for person cars (trucks are longer) even when the net distance between the leader and the follower is the same.

Secondly, the conclusion drawn in the previous section that the assumption that a driver more eagerly restores larger differences between the desired distance and the real distance is often not appropriate for truck drivers, can also be used here. Figure 5-10 shows that the parameter estimates corresponding to the sensitivity to the difference between the desired distance and the real distance are in general lower for trucks than for person cars. This means that in simulating, the dynamics of a truck are often fully determined by the other stimulus considered in the model of Addison and Low, i.e. the ratio of the relative speed and the gross relative distance.

![Figure 5-10 Empirical c.d.f.’s of the parameter estimates for the parameter \( c_5 \) of the model of Addison and Low. Results are both shown for trucks and person cars.](image)

The assertions about the directions of the deviations are confirmed by performing one-sided Kolmogorov-Smirnov tests in which the alternative hypothesis is depending on the assumption about the direction of the deviation replaced by either:

\[ H_a: F(\theta_{\text{trucks}}) < F(\theta_{\text{person cars}}). \]

or

\[ H_a: F(\theta_{\text{trucks}}) > F(\theta_{\text{person cars}}). \]

In these hypotheses the functions F refer to the c.d.f. ‘s.

| Table 5-11 Results for testing the hypothesis that \( \theta_{\text{person cars}} \) and \( \theta_{\text{trucks}} \) are not statistically different. |
|---------------------------------|---------------------------------|
| Model                          | Test results for relevant parameters                     |
| Tampère                        | \( c_1, n^{-1} \): not reject \( H_0 \) | \( \gamma \): not reject \( H_0 \) |
| Lenz                           | \( m \): not reject \( H_0 \)                              |
| Gipps                          | \( b_{\text{max}} \): not reject \( H_0 \) | \( b_{n-1}^{\text{max}} \): not reject \( H_0 \) |
| Addison & Low                 | \( c_4 \): reject \( H_0 \) | \( \gamma \): not reject \( H_0 \) |
| IDM                           | \( T_{\text{safe}} \): reject \( H_0 \)                     |
From these empirical analyses it can be concluded that for several parameters significant differences exist between driver/person car combinations and driver/truck combinations having a comparable driving style.

Also within the groups of driver/person car combinations and driver/truck combinations adopting comparable driving styles large differences are identified. This conclusion is further supported by Table 5-12, showing the coefficients of variation belonging to the parameters of the best performing models for both driver/person car combinations as well as for driver/truck combinations.

Table 5-12 Coefficients of variation for the parameter estimates for trucks and person cars of the best performing models.

<table>
<thead>
<tr>
<th></th>
<th>Coefficient of variation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trucks</td>
<td>Person cars</td>
</tr>
<tr>
<td>Tampère: $c_{1,n-1}$</td>
<td>1.04</td>
<td>0.77</td>
</tr>
<tr>
<td>Tampère: $\gamma$</td>
<td>0.81</td>
<td>0.73</td>
</tr>
<tr>
<td>Lenz: $m$</td>
<td>0.76</td>
<td>1.40</td>
</tr>
<tr>
<td>Gipps: $b_{max}$</td>
<td>-0.96</td>
<td>-1.12</td>
</tr>
<tr>
<td>Gipps: $b_{n-1, max}$</td>
<td>-0.80</td>
<td>-1.07</td>
</tr>
<tr>
<td>Addison &amp; Low: $c_4$</td>
<td>0.78</td>
<td>0.69</td>
</tr>
<tr>
<td>Addison &amp; Low: $\gamma$</td>
<td>0.68</td>
<td>0.73</td>
</tr>
<tr>
<td>IDM: $T_{safe}$</td>
<td>0.54</td>
<td>0.63</td>
</tr>
</tbody>
</table>

5.12 Summary and conclusions

The aim of this chapter has been to gain insights into the extent of heterogeneity in longitudinal driving behavior present in real traffic. Based on a theoretical discussion on heterogeneity, two main causes for heterogeneity were distinguished: driver characteristics and car characteristics. We also considered two ways in which heterogeneity can reveal itself. The ‘strongest’ type of heterogeneity is that the longitudinal driving styles of drivers differ inherently, i.e. different drivers react to different stimuli or use a different driving rule for determining an appropriate control action. The second type of heterogeneity, i.e. heterogeneity within a driving style, is less strong and refers to differences between the group of drivers reacting to the same stimuli and applying the same driving rule.

Using these theoretical insights, the following hypotheses were introduced and consequently tested using dedicated microscopic trajectory observations:

**Hypotheses on driver heterogeneity caused by driver characteristics:**

H1. Drivers of person cars differ with respect to their longitudinal driving styles.

H2. Within the group of drivers of person cars having a similar longitudinal driving style differences in the degree of how they exert this style exist.

**Hypotheses on heterogeneity related to car characteristics**

H3. Driving styles of trucks and person cars are different.

H4. Significant differences exist between trucks and person cars having a similar driving style.

Regarding heterogeneity most likely caused by driver characteristics, we showed that different behavioral rules are needed to describe the behavior of different drivers. Furthermore
clear differences have been identified between drivers adopting a comparable driving style. Also the extent of heterogeneity caused by car characteristics turned out to be considerable. Not only dissimilarities between driving styles of trucks and person cars were found also several significant differences were established between drivers of person cars and trucks having a similar driving style.

These findings are important from a behavioral point of view as well as from a simulation point of view. The contribution to the insight into longitudinal driving behavior is that such an extensive analysis on heterogeneity as the one presented here has never been performed before. The reason for this is that till recently no appropriate observations have been available. Consequently, important new behavioral insights have been obtained.

The contribution to the microscopic simulation field is that this study shows that in real traffic heterogeneity is present to a larger extent than currently assumed in most microscopic simulation tools. Our empirical results show that real traffic flows are characterized by both driving style heterogeneity and within driving style heterogeneity, while most microscopic simulations tools only assume within driving style heterogeneity to exist (chapter 3). As the introduction of driving style heterogeneity to microscopic simulation tools increases the complexity of these tools, we will examine in chapter 7 how sensitive predicted traffic flow properties are to the assumed level of heterogeneity.
Figure 5-11 Helicopter pictures of measurement locations (a) Everdingen, (b) Waalhaven.
6 Theory and empirics of multi-anticipation in car-following

6.1 Aim and structure of this chapter

The aim of this chapter is to gain empirical insights into multi-anticipative car-following behavior of person car drivers, i.e. car-following behavior in which drivers anticipate traffic conditions further downstream by also considering vehicles driving in front of their direct leader in the same lane. Although it is often assumed that a driver looks further ahead than his direct leader, large scale observations needed for proving this empirically have not been available so far.

In this chapter we use, like in the previous chapter, our microscopic trajectory data collected by means of helicopter observations, to answer the following research questions:

- Do drivers consider more than one leader in their longitudinal driving? If so,
- How, and to which extent do these leaders influence the longitudinal behavior?
- Are there differences between the multi-anticipative behaviors of drivers?

By calibrating multi-anticipative longitudinal driving models and comparing their performances, we will show that drivers of person cars indeed often consider more than one leader in performing their longitudinal driving task. In line with the previous chapter, it turns out that also regarding multi-anticipative car-following behavior driving style heterogeneity as well as within driving style heterogeneity exist. Driving style heterogeneity reveals itself among others in which leaders are considered by a driver in selecting an appropriate control action, while within driving style heterogeneity is reflected in the extent to which the considered leaders actually influence the car-following behavior of drivers adopting a comparable driving style.

The chapter is organized as follows. We start by exploring the presence of multi-anticipation in real-traffic by considering time series of speeds of several groups of consecutive vehicles. We then continue with a more formal analysis on multi-anticipation. In section 6.3 we
introduce the hypotheses on multi-anticipation that will be tested throughout the chapter, while we discuss the approach and observations used for testing these hypotheses in respectively section 6.4 and section 6.5.

The actual findings on multi-anticipation are presented in sections 6.6 to 6.8. Section 6.6 shows that multi-anticipation is in general clearly present in real traffic. Sections 6.7 and 6.8 show that driving style heterogeneity as well as within driving style heterogeneity can also be identified regarding multi-anticipative car-following behavior.

### 6.2 Preliminary exploration of multi-anticipation

To perform an empirical study on multi-anticipative car-following behavior, microscopic trajectory observations need to be available for groups of at least three consecutive cars. Such observations have for long been very difficult to collect and for this reason only very little empirical research has been reported on multi-anticipative car-following behavior. We refer to Appendix C for a discussion on the appropriateness of several contemporary trajectory data collection methods in collecting observations needed for analyzing multi-anticipative longitudinal driving behavior.

One of the earliest (and also one of the latest) empirical studies on this type of behavior is described in (Herman and Rothery, 1963). In this study, three car-following experiments were conducted in which the dynamics of the first and second leader were obtained by linking the consecutive cars using steel wires. The authors conclude that although the results do not conclusively indicate that a driver follows only the lead vehicle directly ahead, they do indicate that the direct leader plays the most significant role in the actual execution of the car-following task by the follower.

Despite this conclusion, later on several existing models were extended to include multi-anticipative behavior (for example, (Bexelius, 1968, Lenz et al., 1999, Treiber et al., 2006a)). These extensions are however not motivated by direct observations on the dynamics of consecutive drivers. For example, in (Lenz et al., 1999) the extension is motivated by the statement: ‘from everyday experience one knows that drivers often observe two or more nearest vehicles ahead’. In (Treiber et al., 2006a) the multi-leader extension is supported by microscopic loop detector data (Tilch and Helbing, 2000, Knope et al., 2002) showing that drivers often adopt time headways much smaller than their reaction times. A possible explanation for the fact that despite of these short time headways drivers are in most cases able to avoid collisions, is that they can react earlier because they predict changes in the dynamics of their leader by looking further downstream. However, this can not be seen as direct evidence for the presence of multi-anticipation as also other explanations are possible, like the fact that local measurements do not provide insights into the “dynamic state of vehicles”. For example, it can well be that a follower just changed lanes before the measurement location and will increase his time headway shortly afterwards. This type of information can not be derived from local measurements.

In (Herman and Rothery, 1963) an approach related to this reasoning is suggested for getting a first indication on the presence of multi-anticipation taking care of the “dynamic state of vehicles”. This approach is based on the time a follower needs to react to changes in the dynamics of his direct leader. The underlying idea is that if a driver indeed uses information from his second leader, he will probably react more quickly to a change in the dynamics of his leader since he can anticipate the movements of the vehicle directly ahead.
We use this reasoning to explore a subset of our trajectory observations regarding the presence of multi-anticipation. These preliminary analyses provide an intuitive feeling for the implications of multi-anticipation. To this end, Figure 6-1 shows three randomly selected examples of time series of three consecutive vehicles collected at the Everdingen measurement site. The arrows approximately indicate the time instants at which individual vehicles start or stop accelerating or decelerating. For the sake of simplicity, we assume in this exploration that the car-following behavior is mainly determined by the speed(s) of the leader(s). To enable approximation of the time a driver needs to react, the vertical grid lines are 1 second apart. We define the time a driver needs to react as the time it takes before a following driver changes his speed in the same direction as his leader.

To determine whether a driver reacts in reality quicker than we would expect in the case that he considered only his direct leader, we need to specify the reference reaction time. The problem here is that the reaction time differs between drivers, between expected and unexpected changes in the speeds, between (emergency) braking and decelerating and so forth (Ma and Andréasson, 2006). As we do not consider emergency stops, it seems reasonable from the overview presented in (Ma and Andréasson, 2006) to use 1 second as a reference value. In this sense it is no problem that real drivers may have larger reaction times since we only look for reaction times smaller than the reference reaction time.

Figure 6-1 Three randomly selected examples of time series of triplets of consecutive vehicles. The arrows indicate the time instants at which individual vehicles start or stop accelerating or decelerating.

When we consider the cases in Figure 6-1 we can identify several instances in which a following driver reacts quicker than this reference reaction time. When we consider, for example, the braking action indicated by number (3), it appears that follower \( n \) changes his speed in clearly less than a second after his leader \( n-1 \) applied the same speed change. The
same holds for the action indicated by number (2) in which the vehicles stop braking. That is, vehicle $n-1$ stops braking very quickly after the speed change of vehicle $n-2$.

Although these quick reactions can also have different causes, like for example changes in stimuli not considered here\(^9\), first indications are found for the existence of multi-anticipative car-following behavior.

The analyses on multi-anticipative car-following behavior will be formalized and extended to clearly larger samples of vehicles driving both at the Everdingen measurement site as well as at the Waalhaven measurement site in the remainder of this chapter.

### 6.3 Hypotheses on multi-anticipation

The research questions on multi-anticipative longitudinal behavior of drivers of person cars posed in the introduction, will be answered in this chapter by testing several hypotheses. These hypotheses will be presented in this section. Like in the previous chapter these ‘main’ hypotheses will be numbered.

Firstly, we want to establish whether multi-anticipation is present in real traffic. To that end, we will test the following hypothesis in section 6.6.

**Hypothesis on the presence of multi-anticipation**

H5. Most drivers consider at least two leaders in their longitudinal driving behavior.

Once we have found empirical evidence for the presence of multi-anticipation in car-following, we want to explore this behavior in more detail. That is, we want to derive from the observations which leaders are considered, which stimuli regarding these leaders are taken into account and to what extent. Based on the previous chapter, we also want to gain insight into possible differences between drivers regarding multi-anticipative car-following behavior. All this knowledge will be obtained by testing the following hypotheses in sections 6.7 and 6.8.

**Hypotheses on heterogeneity regarding multi-anticipative car-following behavior**

H6. Significant differences exist between the driving styles of multi-anticipative drivers.

H7. Significant differences exist between the car-following behaviors of drivers having a similar multi-anticipative driving style.

All desired knowledge can be obtained by testing these hypotheses as testing hypotheses on differences between multi-anticipative driving behaviors of drivers requires separate analyses of the behaviors of these drivers.

### 6.4 Experimental design

The upcoming section discusses the approach adopted in testing the hypotheses presented in the previous section. As will be shown, the approach used for testing the hypotheses regarding multi-anticipation strongly resembles the approach taken in the previous chapter on heterogeneity in longitudinal driving behavior.

\(^9\) An example of such a stimulus is that the driver is satisfied no longer with the distance headway to the direct leader.
6.4.1 Overview of research approach
We start by calibrating twelve different driving rules for each observed driver by using microscopic trajectory observations. These driving rules are derived from the generally best performing car-following rules of the previous chapter, i.e. the model of Tampère and the Lenz model. The driving rules differ regarding the assumptions on which leaders are considered as well as regarding the assumptions on how these respective leaders affect car-following behavior.

By comparing the model performances obtained after calibration the following insights are acquired:

- Which leaders do individual drivers consider in their longitudinal driving behavior? (hypothesis on the presence of multi-anticipation (H5))
- Can we establish differences between the longitudinal driving styles of individual drivers? (hypothesis on differences between multi-anticipative driving styles (H6))

By comparing the parameter values of drivers having a comparable longitudinal driving style, we test the hypothesis on heterogeneity within a driving style (H7).

6.4.2 Approach details
In this subsection we will discuss the approach in more detail, i.e. we introduce the models representing the driving styles, we present the calibration approach and we consider the methods used for identifying driving style heterogeneity and within driving style heterogeneity.

Models representing different multi-anticipative driving styles
The multi-anticipative models that will be considered are selected such that it is possible to derive for every observed follower which leaders he considers and which stimuli regarding these leaders influence his longitudinal driving behavior. In this first large sample based empirical analysis of multi-anticipative car-following behavior, we restrict ourselves to direct leaders. That is, when we state that a follower considers two leaders, we refer to the two drivers driving directly in front of the follower.

To determine the leaders a driver reacts to, models are considered having one, two, and three direct leaders respectively\(^{10}\). Furthermore, for models incorporating the same leaders the stimuli regarding the different leaders are varied between the models to enable answering the question of how the respective leaders influence the following behavior.

To apply these ideas to the constrained driving part of the model of Tampère, we introduce the following generalized version of the Tampère model, referred to as GT\((m_1, m_2)\) model (Hoogendoorn et al., 2006, Hoogendoorn et al., 2007a):

\[
a_n(t + T_r) = \sum_{j=1}^{m_1} c_{1,n-j} \Delta v_{n-j,n}(t) + \sum_{j=1}^{m_2} c_{2,n-j} (\Delta x_{n-j,n}(t) - \Delta x_{n-j,n}^*(t))
\]  

\[(6.1)\]

\(^{10}\) The maximum number of leaders is chosen to be equal to three for the following two reasons:
1) Given the difficulties of drivers in determining exact longitudinal distances and relative speeds it seems unlikely that drivers are able to determine the stimuli in the model for leaders even further downstream.
2) Given that the observed stretch has a fixed length, increasing the number of leaders further means that the time a follower can be observed together with its leaders decreases. Given the criteria this negatively influences the number of followers that can be considered in our analysis.
\[ \Delta x_{n-j,n}^* = j^* (d + \gamma^* v_n(t)) \]  \hspace{1cm} (6.2)

where,

- \( m_1 \) = number of leaders to which driver \( n \) responds with respect to relative speed
- \( m_2 \) = number of leaders to which deviations from the desired following distance are considered
- \( c_{1,n-j} \) = sensitivity of follower \( n \) to the relative speed with leader \( n-j \) (1/s)
- \( c_{2,n-j} \) = sensitivity of follower \( n \) to the difference between the real distance between leader \( n-j \) and the following car and the corresponding desired distance (1/s²)
- \( \Delta v_{n-j,n} \) = relative speed between leading car \( n-j \) and the following car \( n \) (m/s)
- \( \Delta x_{n-j,n} \) = actual (gross) distance between leader \( n-j \) and the following car \( n \) (m)
- \( \Delta x^*_{n-j,n} \) = desired (gross) distance between leader \( n-j \) and the following car \( n \) (m)
- \( T_r \) = reaction time (s)
- \( d \) = desired distance at standstill (m)
- \( \gamma \) = desired increase of distance for every 1 m/s increase of speed (s)

For clarity of the presentation, we added the subscript \( n-j \) to the variable \( c_2 \). The advantage of this minor notational change for the analysis presented here is that this new subscript shows clearly to which leader a given sensitivity parameter refers.

We will calibrate the following specifications of the GT\((m_1, m_2)\) model for all considered drivers: GT\((1,1)\), GT\((1,2)\), GT\((1,3)\), GT\((2,1)\), GT\((2,2)\), GT\((2,3)\), GT\((3,1)\), GT\((3,2)\), GT\((3,3)\). Note that we do not impose the restriction that \( m_1 = m_2 \) as only empirical evidence can show whether this restriction is valid. An overview of the considered specifications of the GT\((m_1, m_2)\) model is provided in Table 6-1.

As these specifications differ with respect to the assumptions made on the number of leaders considered regarding relative speed and regarding the difference between the desired distance and the relative distance, they provide us with the opportunity to estimate for each individual driver which stimuli of the respective leaders are considered by him. The sensitivity parameters \( c_{1,n-j} \) and \( c_{2,n-j} \) enable us furthermore to examine to which extent the respective stimuli play a role. To be able to compare the multi-anticipative longitudinal driving behaviors of drivers, we assume all behavioral parameter values to be driver specific.

| Table 6-1 Overview of considered specifications of the GT\((m_1, m_2)\) model. |
|-----------------|-----------------|-----------------|
| Number of leaders a driver is assumed to consider regarding distance headway | \( m_2 = 1 \) | \( m_2 = 2 \) | \( m_2 = 3 \) |
| Number of leaders a driver is assumed to consider regarding relative speed | \( m_1 = 1 \) | GT(1,1) | GT(1,2) | GT(1,3) |
| | \( m_1 = 2 \) | GT(2,1) | GT(2,2) | GT(2,3) |
| | \( m_1 = 3 \) | GT(3,1) | GT(3,2) | GT(3,3) |
For the “free-driving component” of the model of Tampère no multi-leader extensions are needed as it holds by definition that in the free-driving regime the behavior of a driver is not influenced by any leader.

Next to the GT\((m_1, m_2)\) model, we also consider the Lenz model (Lenz et al., 1999) in this analysis, assuming, in contrast with the Tampère model, a nonlinear relation between speed and desired distance, and without an explicit reaction time. The model is summarized in eq. (6.3) and eq. (6.4). For more detailed information on the model we refer to chapter 3.

\[
a_n(t) = \sum_{j=1}^{m_3} c_{6,n,j} \left( V_{opt} \left( \frac{\Delta x_{n,j,n}(t)}{j} \right) - v_n(t) \right) \tag{6.3}
\]

\[
V_{opt}(\Delta x(t)) = V_0 \left[ \tanh(\Delta x(t) - b_f) - \tanh(d - b_f) \right] \tag{6.4}
\]

where,
- \(m_3\) = number of leaders considered regarding optimal velocity
- \(c_{6,n,j}\) = sensitivity to difference between the current speed and \(V_{opt}\) regarding leader \(j\) (1/s)
- \(m\) = slope of the optimal velocity function at the inflection point (1/m)
- \(b_f\) = distance headway corresponding to the inflection point of the optimal velocity function (m)
- \(d\) = distance at standstill (m)
- \(V_0\) = maximum speed at large enough headways minus the speed at the inflection point (m/s)

The motivation for also considering this model is that the previous chapter showed that this model, like the Tampère model, performs very well in predicting the dynamics of part of the considered driver population.

Different from the Tampère model no adjustments are needed to the Lenz model as this model is already defined such that it is possible to vary the number of leaders considered. To distinguish in the upcoming between specifications of the Lenz model making different assumptions on the number of leaders, we will denote the Lenz model in the sequel as Lenz\((m_3)\). In the analysis we will consider the Lenz(1), Lenz(2) and Lenz(3) model as summarized in Table 6-2. Like for the Tampère model we assume the values of the behavioral parameters to be driver specific.

<table>
<thead>
<tr>
<th>Table 6-2 Overview of considered specifications of the Lenz((m_3)) model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of leaders a driver is assumed to consider regarding optimal velocity</td>
</tr>
<tr>
<td>(m_3=1)</td>
</tr>
<tr>
<td>Lenz(1)</td>
</tr>
</tbody>
</table>

In total we have now 12 different models, i.e. 9 different specifications of the GT\((m_1, m_2)\) model and 3 specifications of the Lenz\((m_3)\) model.
Calibration approach

To calibrate the parameters of the different models, we apply exactly the same approach as in the previous chapter (see section 5.4). The constraints imposed to the parameter values of the most extended models (GT(3,3) and Lenz(3)) are summarized in Table 6-3. The constraints imposed to the parameter values belonging to the less elaborated models can be derived from this table as all parameters relating to the same stimulus do have the same bounds.

Table 6-3 Overview of parameters to be calibrated and applied constraints.

<table>
<thead>
<tr>
<th>model</th>
<th>param.</th>
<th>range</th>
<th>Short interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>GT(3,3)</td>
<td>c_{1,n-1}</td>
<td>[0,2]</td>
<td>sens. to speed difference with leader n-1</td>
</tr>
<tr>
<td></td>
<td>c_{1,n-2}</td>
<td>[0,2]</td>
<td>sens. to speed difference with leader n-2</td>
</tr>
<tr>
<td></td>
<td>c_{1,n-3}</td>
<td>[0,2]</td>
<td>sens. to speed difference with leader n-3</td>
</tr>
<tr>
<td></td>
<td>c_{2,n-1}</td>
<td>[0,1]</td>
<td>sens. to difference real distance to leader n-1 and corresponding desired distance</td>
</tr>
<tr>
<td></td>
<td>c_{2,n-2}</td>
<td>[0,1]</td>
<td>sens. to difference real distance to leader n-2 and corresponding desired distance</td>
</tr>
<tr>
<td></td>
<td>c_{2,n-3}</td>
<td>[0,1]</td>
<td>sens. to difference real distance to leader n-3 and corresponding desired distance</td>
</tr>
<tr>
<td></td>
<td>c_{3}</td>
<td>[0.05,3]</td>
<td>sens. to difference real speed and desired speed</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>[4,15]</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>γ</td>
<td>[0.2,4]</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td></td>
<td>T_r</td>
<td>[0.5,3]</td>
<td>reaction time</td>
</tr>
<tr>
<td>Lenz(3)</td>
<td>c_{6,n-1}</td>
<td>[0,15]</td>
<td>sens. to difference real speed and optimal velocity leader n-1</td>
</tr>
<tr>
<td></td>
<td>c_{6,n-2}</td>
<td>[0,10]</td>
<td>sens. to difference real speed and optimal velocity leader n-2</td>
</tr>
<tr>
<td></td>
<td>c_{6,n-3}</td>
<td>[0,8]</td>
<td>sens. to difference real speed and optimal velocity leader n-3</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>[0,0.3]</td>
<td>slope of optimal velocity function at inflection point</td>
</tr>
<tr>
<td></td>
<td>b_f</td>
<td>[4,75]</td>
<td>distance headway at inflection point of optimal velocity</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>[4,15]</td>
<td>distance at standstill</td>
</tr>
</tbody>
</table>

Approach for identifying driving style heterogeneity

The driving styles considered in this chapter belong to either the GT(m_1,m_2) model family or the Lenz(m_3) model family. Driving styles belonging to the same model family differ regarding their assumptions on the number of leaders influencing the longitudinal driving behavior of the follower.

To identify driving style heterogeneity within a model family, i.e. to establish whether different drivers consider different number of leaders, it needs to be established for all drivers whether the performance of a model significantly improves when an additional stimulus referring to a leader further downstream is added. For comparing models of different complexities we use the heuristic proposed in chapter 4.

That is, we compute for all models and for all drivers penalized error terms using:

$$\varepsilon^{pen} = \varepsilon(1 + \omega)^p$$

(6.5)

In which \(\omega\) corresponds to the relative penalty imposed to the performance measure for every additional parameter \(p\). We assume the parameter \(\omega\) to be equal to 0.2 meaning that an additional parameter is only assumed to be significant when the corresponding performance is still better after the introduction of a penalty of 20%.
The reason for introducing such strong penalties is that the chapter mainly aims at providing empirical evidence of multi-anticipative behavior. When this behavior can be established even with these strong penalties, it will also be identified when weaker criteria are applied. This reasoning is also from a microscopic simulation point of view favorable as additional model complexity has a computational cost that can only be compensated for by clearly better results.

**Approach for identifying within driver heterogeneity**

To identify within driving style heterogeneity, we can apply the same approach as in the previous chapter (see section 5.4). We first select all drivers having a comparable driving style by considering the model performances. Afterwards we determine for all model parameters separately whether the variance in the parameter estimates can be used as an indicator for within driving style heterogeneity. To this end the same criteria are used, namely:

- The normalized sensitivity values should exceed the critical value of 0.4.
- At least 75% of the parameter estimates are not equal to either of the imposed bounds.

### 6.5 Observations on multi-anticipation

The requirements our trajectory observations need to fulfill in order to be useful input to the proposed study are comparable to the requirements used in selecting observations for the analysis on heterogeneity caused by driver characteristics as discussed in the previous chapter (see section 5.5). The only exception is that now car-following models having up to three leaders are considered, such that groups of four consecutive cars need to be selected instead of groups of three cars. As a result, the number of groups of consecutive vehicles meeting the requirements may be smaller than in the previous chapter.

Table 6-4 provides an overview of the number of selected quartets and properties of these quartets for both the Everdingen measurement site and the Waalhaven measurement site. As expected fewer vehicle groups are selected in this case than in the case of the previous chapter and also the average duration of observation is lower. This is also plausible as the time period during which four consecutive vehicles are jointly observed at a fixed stretch of road is shorter compared to the time that at least three of these four vehicles are observed.

<table>
<thead>
<tr>
<th>Measurement site</th>
<th>Total number of appropriate quartets</th>
<th>Average speed change follower (m/s)</th>
<th>Average duration of observation (sec.)</th>
<th>Number of times that the following vehicle is a person car</th>
<th>Number of times that the following vehicle is a truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Everdingen</td>
<td>96</td>
<td>11.96</td>
<td>21.9</td>
<td>91</td>
<td>5</td>
</tr>
<tr>
<td>Waalhaven</td>
<td>101</td>
<td>6.4</td>
<td>43.1</td>
<td>97</td>
<td>4</td>
</tr>
</tbody>
</table>

As, for the sake of performing reliable analyses, there are too few quartets having a truck as following vehicle, we do not consider these quartets in the analyses presented in the sequel of this chapter.
6.6 Presence of multi-anticipation in car-following

In this section we test the following hypothesis:

H5. Most drivers consider at least two leaders in their longitudinal driving behavior.

To this end, we need to show that the models considering more than one leader perform significantly better than the models assuming that drivers only react to their direct leader. In Figure 6-2 we therefore show the empirical c.d.f.’s of the ratios of the performances:

\[
\frac{\varepsilon_{\text{single leader}}}{\varepsilon_{\text{multi leader}}}
\]

in which \(\varepsilon_{\text{single leader}}\), in line with the likelihood ratio test, refers to the performance of the one leader model, while \(\varepsilon_{\text{multi leader}}\) refers to the more elaborate model considering stimuli regarding more than one leader. As it generally holds that \(\varepsilon_{\text{single leader}}\) is larger than \(\varepsilon_{\text{multi leader}}\), the ratio is generally larger than one. The larger the ratio, the larger the improvement in the performance of the model when adding the additional stimuli regarding leaders further downstream.

![Figure 6-2 Empirical c.d.f.’s of the ratio \(\frac{\varepsilon_{\text{single leader}}}{\varepsilon_{\text{multi leader}}\) for (left) the Everdingen measurement site and (right) the Waalhaven measurement site.](image-url)
It appears that the performances of the models clearly improve when more than one leader is considered, although there is especially for the Lenz model family a part of the driver sample for whom the inclusion of additional leaders does not result in a better model performance (in Figure 6-2d the ratio is for all considered extensions equal to one for part of the drivers).

The various specifications of the $\text{GT}(m_1, m_2)$ model differ regarding their assumptions on the stimuli related to drivers further downstream playing a role in the execution of the longitudinal driving task of a driver, i.e. relative speed or distance. An interesting question is therefore which stimulus improves the model performance most?

To answer this question, the ratio’s presented in Figure 6-2 for the following models belonging to the $\text{GT}(m_1, m_2)$ family are considered in more detail: $\text{GT}(1,2)$, $\text{GT}(2,1)$, $\text{GT}(1,3)$, $\text{GT}(3,1)$. For the clarity of the presentation these ratios are repeated in Figure 6-3.

It can be concluded that in general including the relative speed to a leader further downstream improves model performances more than including the difference between the desired distance and the actual distance to a leader further downstream. For example, when we consider the ratio’s corresponding to the $\text{GT}(2,1)$ and $\text{GT}(1,2)$ model for the Everdingen measurement site the c.d.f. corresponding to the $\text{GT}(2,1)$ model lies clearly more to the right. As both models do have the same number of parameters our conclusion is not influenced by a different number of parameters.

![Figure 6-3 Empirical c.d.f.’s of the ratio $\frac{e_{\text{single leader}}}{e_{\text{multi leader}}}$ for (a) the Everdingen measurement site and (b) the Waalhaven measurement site.](image-url)

To show that model performances significantly improve for part of the drivers when stimuli regarding leaders driving further downstream on the same lane are included, we again
compute the ratio’s presented in Figure 6-2 while harshly penalizing an increase in the number of parameters (section 6.4.2). As we assume \( \omega \) in eq. (4.13) to be equal to 0.2 the ratio,

\[
\frac{E_{\text{pen}}^{\text{single leader}}}{E_{\text{pen}}^{\text{multi leader}}} = \frac{E_{\text{single leader}}^{(1 + \omega)^{\text{single leader}}}}{E_{\text{multi leader}}^{(1 + \omega)^{\text{multi leader}}}}
\]

(6.7)

is only larger than one when it holds that:

\[
E_{\text{single leader}} \geq E_{\text{multi leader}}^{(1 + 0.2)^{\text{multi leader}} \cdot \omega^{\text{single leader}}}
\]

(6.8)

From Figure 6-4 it can be concluded that even after this severe correction for the number of parameters there is still a considerable part of the drivers for whom the ratio is larger than one. This justifies accepting the hypothesis that drivers often do consider more than one leader in their car-following behavior (H5).

Figure 6-4 Empirical c.d.f.’s of the ratio \( \frac{E_{\text{pen}}^{\text{single leader}}}{E_{\text{pen}}^{\text{multi leader}}} \) for (left) the Everdingen measurement site and (right) the Waalhaven measurement site.

### 6.7 Driving style heterogeneity of multi-anticipative drivers

Figure 6-4 incorporates evidence supporting the following hypothesis:
H6. Significant differences exist between the driving styles of multi-anticipative drivers.

For example, when we consider the GT(1,2) model we can see that after imposing the penalty for approximately 66% of the followers no improvement in model performance can be observed anymore compared to the simplest GT(1,1) model. For the remaining 34% the model considering two leaders regarding the relative distance performs clearly better than the simplest one leader model meaning that these drivers most probably adopt a different driving style. This is further supported when we select for every driver the best performing model after imposing the penalties. The results are shown in Table 6-5. Clear differences can be observed between the best performing models for different drivers. Different drivers appear, for example, to consider different leaders. This difference between the leaders considered by drivers can be caused by differences between the control objectives of drivers. We can however not disregard that also the characteristics of leading vehicles are likely to be of influence. For example, the characteristics of the first leading car (like the car size) are likely to determine whether and to which extent a driver is able to consider his second leader. Also a second leader making a lot of unexpected movements is likely to attract more attention than a second leader driving smoothly.

Another related interesting finding is that when we determine for all drivers next to the best performing model also the second best performing model, we find for the Everdingen site that in 73% of the cases both models belong to the same model family, while this holds for 58% of the drivers driving at the Waalhaven site. This provides especially for the Everdingen measurement site additional evidence that also inherent differences exist between, for example, the relations between speed and desired distance for different drivers, like we concluded from the previous chapter.

A minor remark to the conclusions on driving style heterogeneity drawn from Table 6-5 is that the number of times that the Lenz(1) model is selected as best is most likely overestimated. The reason for this is that this model has the least number of parameters and consequently the smallest penalty is introduced to the error terms corresponding to this model. A detailed check of the results revealed that the Lenz(1) model is therefore relatively often appointed as best model when there exist only minor differences between the performances of all models without penalties.

### Table 6-5 Differences in model performance between followers.

<table>
<thead>
<tr>
<th>Model</th>
<th>Everdingen (91 followers)</th>
<th>Waalhaven (97 followers)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% times best (absolute value)</td>
<td>% times best (absolute value)</td>
</tr>
<tr>
<td>GT(1,1)</td>
<td>29% (26)</td>
<td>21% (20)</td>
</tr>
<tr>
<td>GT(1,2)</td>
<td>5% (5)</td>
<td>7% (7)</td>
</tr>
<tr>
<td>GT(1,3)</td>
<td>3% (3)</td>
<td>1% (1)</td>
</tr>
<tr>
<td>GT(2,1)</td>
<td>15% (14)</td>
<td>12% (12)</td>
</tr>
<tr>
<td>GT(2,2)</td>
<td>3% (3)</td>
<td>2% (2)</td>
</tr>
<tr>
<td>GT(2,3)</td>
<td>1% (1)</td>
<td>3% (3)</td>
</tr>
<tr>
<td>GT(3,1)</td>
<td>8% (7)</td>
<td>9% (9)</td>
</tr>
<tr>
<td>GT(3,2)</td>
<td>1% (1)</td>
<td>3% (3)</td>
</tr>
<tr>
<td>GT(3,3)</td>
<td>1% (1)</td>
<td>0% (0)</td>
</tr>
<tr>
<td>Lenz(1)</td>
<td>21% (19)</td>
<td>25% (24)</td>
</tr>
<tr>
<td>Lenz(2)</td>
<td>7% (6)</td>
<td>11% (11)</td>
</tr>
<tr>
<td>Lenz(3)</td>
<td>5% (5)</td>
<td>5% (5)</td>
</tr>
</tbody>
</table>
This remark does not influence that the table provides additional evidence on the presence of multi-anticipation (H5). That is, even when applying very large penalties to multi-anticipative models it turns out that for 51% of the drivers driving at the Everdingen measurement site a multi-anticipative driving rule performs best, the same can be concluded for 55% of the drivers observed at the Waalhaven measurement site. For 20% of the drivers driving at the Everdingen measurement site and for 22% of the drivers driving at the Waalhaven measurement site even a model considering more than two leaders appears to be best. These percentages would only increase when we would decrease the penalties for additional model parameters.

Next to that, Table 6-5 also supports our earlier conclusion that the relative speed regarding leaders further downstream affects the longitudinal driving behavior of a driver often more than the distance to leaders further downstream. For example, for the Everdingen site the GT(1,2) model is selected as best model for only 5% of the drivers, while the GT(2,1) model is selected as best model for 15%.

The large resemblance between the results for the two measurement sites provides further confidence in the generality of our conclusions. For instance, only minor differences exist between the percentages of drivers considering more than one leader (51% for the Everdingen site versus 55% for the Waalhaven site). The same holds for the percentages of drivers considering more than two leaders (20% for the Everdingen site versus 22% for the Waalhaven site). These resemblances provide evidence that our findings are not strongly site specific.

6.8 Heterogeneity within driving styles of multi-anticipative drivers

The previous section discussed differences between the multi-anticipative driving styles of drivers. In this section we concentrate on differences between drivers having a comparable multi-anticipative driving style. In this analysis we will focus on the GT(2,1) model and the GT(3,1) model. The motivation for doing so is that the previous analyses showed that these models considering the relative speed of leaders further downstream in general showed a larger increase in model performance than models considering the distance to leaders further downstream. We do not consider the GT(1,1) and Lenz(2) models here further as these models were already discussed in detail in the previous chapter 5.

We use the same approach as in the previous chapter 5 to select for both models and for both measurement locations separately the followers for whom the models perform well. Thus we select every time the followers having an error term smaller than a given critical value. As we saw before that these models with multi-leader extensions do yield lower error terms than the one leader GT(1,1) model, both due to better performances and parsimony, we apply lower threshold values here than in the previous chapter. More specific, to keep the percentages of quartets fulfilling the requirement approximately in the same order, we changed the threshold of 0.0175 for the Everdingen site to 0.0125, while the value of 0.035 for the Waalhaven site is now set equal to 0.0275.
Table 6-6 Descriptive statistics of the reliability analysis of the estimates of the model parameters for the Everdingen site. The highlighted parameters can be used in the analysis on ‘within driving style heterogeneity’.

<table>
<thead>
<tr>
<th>model</th>
<th>param</th>
<th>Median sens.</th>
<th>Norm. sens.</th>
<th>% not equal to bound</th>
<th>Short interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(2,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-1}$</td>
<td>12.67</td>
<td>0.31</td>
<td>83</td>
<td>sens. to speed difference with leader 1</td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-2}$</td>
<td>98.95</td>
<td>1.14</td>
<td>74</td>
<td>sens. to speed difference with leader 2</td>
</tr>
<tr>
<td></td>
<td>$c_{2,n-1}$</td>
<td>112.91</td>
<td>0.57</td>
<td>95</td>
<td>sens. to difference real distance and desired distance</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>7.90</td>
<td>0.08</td>
<td>98</td>
<td>sens. to difference real speed and desired speed</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>0.87</td>
<td>33.17</td>
<td>50</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>5.91</td>
<td>11.94</td>
<td>88</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-1}$</td>
<td>16.24</td>
<td>0.35</td>
<td>76</td>
<td>sens. to speed difference with leader 1</td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-2}$</td>
<td>53.39</td>
<td>0.08</td>
<td>59</td>
<td>sens. to speed difference with leader 2</td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-3}$</td>
<td>119.98</td>
<td>0.12</td>
<td>59</td>
<td>sens. to speed difference with leader 3</td>
</tr>
<tr>
<td></td>
<td>$c_{2,n-1}$</td>
<td>157.92</td>
<td>0.75</td>
<td>94</td>
<td>sens. to difference real distance and desired distance</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>3.92</td>
<td>0.04</td>
<td>90</td>
<td>sens. to difference real speed and desired speed</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>0.97</td>
<td>147.12</td>
<td>49</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>6.26</td>
<td>8.78</td>
<td>80</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
</tbody>
</table>

Table 6-7 Descriptive statistics of the reliability analysis of the estimates of the model parameters for the Waalhaven site. The highlighted parameters can be used in the analysis on ‘within driving style heterogeneity’.

<table>
<thead>
<tr>
<th>model</th>
<th>param</th>
<th>Median sens.</th>
<th>Norm. sens.</th>
<th>% not equal to bound</th>
<th>Short interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(2,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-1}$</td>
<td>11.52</td>
<td>0.10</td>
<td>68</td>
<td>sens. to speed difference with leader 1</td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-2}$</td>
<td>40.64</td>
<td>0.48</td>
<td>84</td>
<td>sens. to speed difference with leader 2</td>
</tr>
<tr>
<td></td>
<td>$c_{2,n-1}$</td>
<td>175.03</td>
<td>0.41</td>
<td>95</td>
<td>sens. to difference real distance and desired distance</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>sens. to difference real speed and desired speed</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>0.06</td>
<td>3.69</td>
<td>68</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>1.46</td>
<td>4.26</td>
<td>86</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-1}$</td>
<td>8.21</td>
<td>0.05</td>
<td>62</td>
<td>sens. to speed difference with leader 1</td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-2}$</td>
<td>37.08</td>
<td>0.07</td>
<td>62</td>
<td>sens. to speed difference with leader 2</td>
</tr>
<tr>
<td></td>
<td>$c_{1,n-3}$</td>
<td>73.92</td>
<td>0.25</td>
<td>64</td>
<td>sens. to speed difference with leader 3</td>
</tr>
<tr>
<td></td>
<td>$c_{2,n-1}$</td>
<td>226.99</td>
<td>0.47</td>
<td>98</td>
<td>sens. to difference real distance and desired distance</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>0</td>
<td>0</td>
<td>98</td>
<td>sens. to difference real speed and desired speed</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>0.06</td>
<td>3.89</td>
<td>60</td>
<td>distance at standstill</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>1.31</td>
<td>2.49</td>
<td>78</td>
<td>desired increase of distance for a 1 m/s speed increase</td>
</tr>
</tbody>
</table>
To determine for which parameters it is valid to interpret the variability in the estimated parameter values as an indicator for within driving style heterogeneity, we show in Table 6-6 and Table 6-7 the median sensitivities, the normalized sensitivities and the percentages of parameters not equal to the imposed bounds for respectively the Everdingen measurement site and the Waalhaven measurement site. We highlight the parameters that fulfill our strict criteria.

Again we see that the estimates for the distances at standstill ($d$) are relatively often equal to the imposed bounds, thus indicating estimation problems. The normalized sensitivities regarding the relative speed appear often to be low. A reasonable explanation for this is that the parameter estimates for these sensitivities are for the multi-leader models in general clearly lower than the parameter estimates for the sensitivity of the follower regarding the speed difference with the direct leader in the previous chapter. This means that the sensitivities are multiplied by lower values than before.

Figure 6-5 shows the parameter estimates for the sensitivity parameters regarding the speed differences with the different leaders and the parameters fulfilling the imposed criteria. The corresponding coefficients of variation are shown in Table 6-8. When we consider the parameter estimates and the coefficients of variation for the parameters fulfilling the criteria, it seems valid to accept the hypothesis that there exist differences between drivers having a
comparable multi-anticipative driving style (H7). Thus even when drivers consider the same leaders, differences exist between the extents to which these different leaders are considered.

Table 6-8 Coefficients of variation for the parameter estimates for the GT(2,1) model and the GT(3,1) model for both the Everdingen measurement site and the Waalhaven measurement site (the parameters in gray fulfill our strict requirements as indicated in Table 6-6 and Table 6-7).

<table>
<thead>
<tr>
<th>Coefficient of variation</th>
<th>Everdingen</th>
<th>Waalhaven</th>
</tr>
</thead>
<tbody>
<tr>
<td>GT(2,1): $c_{1,n-1}$</td>
<td>1.24</td>
<td>1.24</td>
</tr>
<tr>
<td>GT(2,1): $c_{1,n-2}$</td>
<td>1.40</td>
<td>0.89</td>
</tr>
<tr>
<td>GT(2,1): $c_{2,n-1}$</td>
<td>0.90</td>
<td>0.92</td>
</tr>
<tr>
<td>GT(2,1): $\gamma$</td>
<td>0.61</td>
<td>0.57</td>
</tr>
<tr>
<td>GT(3,1): $c_{1,n-1}$</td>
<td>1.13</td>
<td>1.14</td>
</tr>
<tr>
<td>GT(3,1): $c_{1,n-2}$</td>
<td>1.40</td>
<td>1.07</td>
</tr>
<tr>
<td>GT(3,1): $c_{1,n-3}$</td>
<td>1.71</td>
<td>1.14</td>
</tr>
<tr>
<td>GT(3,1): $c_{2,n-1}$</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>GT(3,1): $\gamma$</td>
<td>0.76</td>
<td>0.71</td>
</tr>
</tbody>
</table>

When we consider the estimates for the reaction times for the GT(1,1) model, the GT(2,1) model and the GT(3,1) model (Figure 6-6), we can also see considerable variations between drivers. The most interesting observation that can be made in this context is however that the estimates for the reaction times increase in general with the number of considered leaders. For the one leader model a lot of reaction times are equal to the imposed lower bound of 0.5 seconds, this holds especially for the Waalhaven site. This number decreases considerably when more leaders downstream are assumed to be considered. These findings are in line with the assumptions made in the first explorative part of this chapter.

Figure 6-6 Empirical c.d.f.’s of the estimates for $T_r$ for the GT(1,1), GT(2,1), and GT(3,1) model for (a) the Everdingen site and (b) the Waalhaven site.

We extended these analyses a little bit further by obtaining a least squares fit for the following relation:
\[ T_{r,GT(1,1)} = a - b \frac{e_{GT(3,3)} - e_{GT(1,1)}}{e_{GT(1,1)}} \] (6.9)

The idea behind this relation is that the higher the increase in the relative model performance for a driver when introducing more leaders downstream, the lower the estimate of the reaction time of the one leader model for the respective driver. For both measurement sites indeed a weak negative relation \((b<0)\) was observed.

### 6.9 Summary and conclusions

The aim of this chapter has been to gain insights into multi-anticipative longitudinal behavior of drivers, i.e. car-following behavior in which drivers anticipate future flow conditions by also considering vehicles driving in front of their direct leader in the same lane. To this extent the following hypotheses have been tested:

- **H5.** Most drivers consider at least two leaders in their longitudinal driving behavior.
- **H6.** Significant differences exist between the driving styles of multi-anticipative drivers.
- **H7.** Significant differences exist between the car-following behaviors of drivers having a similar multi-anticipative driving style.

It has been shown that at least 51% of the observed drivers consider more than one leader in performing the longitudinal driving task. At least 20% of the considered drivers even appear to react to more than two leaders. We stress the words at least here as we applied very strong criteria for accepting a model containing stimuli to leaders further downstream to perform significantly better than a model considering only the direct leader. Especially the relative speed regarding leaders further downstream turns out to be of influence to the car-following behavior of drivers. The large resemblance between the results for the two measurement sites provides furthermore evidence that our findings are not strongly site specific. For instance, only minor differences exist between the percentages of drivers considering more than one leader (51% for the Everdingen site versus 55% for the Waalhaven site). The same holds for the percentages of drivers considering more than two leaders (20% for the Everdingen site versus 22% for the Waalhaven site).

Also with respect to multi-anticipative driving behavior heterogeneity was clearly present. First of all differences between multi-anticipative driving styles of drivers were identified. For example, the leaders considered varied between drivers. Next to that, evidence has been provided that different model types are needed to correctly explain driving behavior for specific drivers, i.e. differences between drivers can not be caught by only varying the leaders considered for a single behavioral rule rather different behavioral rules need to be applied.

Also differences were established between drivers adopting a comparable longitudinal driving style. Thus even when a driver considers the same leaders and when the same model type is valid, differences between the driving behaviors are present.

These findings do significantly contribute to fundamental knowledge on longitudinal driving behavior as so far no large scale trajectory based research was performed on multi-anticipative car-following behavior. Thus although the presence of multi-anticipation was often assumed, in this chapter for the first time empirical evidence has been provided.
Like for the results on heterogeneity provided in the previous chapter, it is hypothesized that also the results presented in this chapter do have important impacts on current microscopic simulation practice. They do not only stress further that heterogeneity is in real traffic much larger than assumed in existing microscopic simulation tools, but they indicate also that real drivers often do consider more than the direct leader in front, while most microscopic simulation tools include only the first leader.

Because of this hypothesized large influence of heterogeneity and multi-anticipation on the properties of predicted traffic flows, we will in chapter 7 explore the impacts of our empirical findings on the fundamental diagram, platoon stability and flow stability.
7 Including heterogeneity and multi-anticipation in traffic flow predictions

7.1 Aim and structure of this chapter

In the previous chapters two important contributions were made to existing knowledge on longitudinal driving behavior. Firstly, we showed that heterogeneity is to a large extent present in real traffic. It turned out that different behavioral rules are needed to satisfactorily describe the longitudinal driving behaviors of different driver/vehicle combinations. For driver/vehicle combinations adopting the same behavioral rule, behavioral parameter values appear to differ. Secondly, we established that more than half of the drivers consider more than one leader in performing the longitudinal driving task.

The level of heterogeneity assumed in current commercial microscopic simulation tools is not based on detailed analyses of trajectories of a large sample of drivers and accordingly is likely to differ from our empirical findings. For example, the level of heterogeneity assumed in these tools is in general smaller than the empirically established one, i.e. all drivers are assumed to apply the same behavioral rule while only behavioral parameter values are varied between drivers. Multi-anticipation is furthermore generally neglected in microscopic simulations. The question is however how does this affect simulated traffic flows? This question is especially relevant from a microscopic simulation point of view as model complexity increases by increasing the level of heterogeneity and including more leaders to models. Making these extensions is thus only worthwhile when the conclusions obtained from microscopic simulation studies improve.

In this chapter we explore whether and how our empirical findings on heterogeneity and multi-anticipation are expected to change predicted macroscopic traffic flow properties. This is an important step towards showing that traffic flow predictions made by microscopic simulation tools improve when incorporating our findings on heterogeneity and multi-anticipation. For instance, by performing microscopic simulations for single lane roads we will show that the level of heterogeneity assumed in microscopic simulations considerably influences platoon stability and flow stability. This finding motivates future research in which
the empirically established level of heterogeneity is incorporated in an existing microscopic simulation tool. Using this microscopic simulation tool a large number of simulations can be performed, focusing on a broad range of different road configurations. The resulting traffic flow predictions then need to be compared to, for example, microscopic double loop detector data to establish whether predictions improve.

This chapter is organized as follows, in section 7.2 we start by discussing the expected impacts of heterogeneity and multi-anticipation on the fundamental diagram, platoon stability and flow stability. As it appears that especially little information is available on the impacts of driving style heterogeneity and within driving style heterogeneity, we continue by performing exploratory simulations to establish whether we can support our expectations on the impacts of heterogeneity on traffic flow predictions. Table 7-1 provides an overview of this chapter.

Table 7-1 Overview of topics considered in this chapter.

<table>
<thead>
<tr>
<th></th>
<th>heterogeneity expectations</th>
<th>heterogeneity simulations</th>
<th>multi-anticipation expectations</th>
<th>multi-anticipation simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental diagram</td>
<td>7.2.1</td>
<td>7.3</td>
<td>7.2.1</td>
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<tr>
<td>Platoon stability</td>
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<td>Flow stability</td>
<td>7.2.3</td>
<td>7.5</td>
<td>7.2.3</td>
<td>-</td>
</tr>
</tbody>
</table>

The simulations in section 7.3 focus on the impacts of driving style heterogeneity and within driving style heterogeneity on the fundamental diagram. We show that driving style heterogeneity mainly causes the shape of the fundamental diagram to change. Within driving style heterogeneity mainly introduces scatter to the fundamental diagram, i.e. the density corresponding to a given equilibrium speed depends on the composition of the sample of driver/vehicle combinations currently present on the road.

The microscopic simulations in section 7.5 are on the stability of traffic flow for two different (single lane) roadway configurations. In the first configuration drivers have to lower their speed due to a speed limit, while in the second scenario traffic flow on the main road is disturbed by an on-ramp. The first main conclusion from this section is that in case of a heterogeneous traffic flow, flow dynamics are not only influenced by the longitudinal behavior of the average driver but that especially the level of heterogeneity in longitudinal behavior plays an important role. Another important contribution is that we show that also the flow composition, i.e. the order of driver/vehicle combinations having different longitudinal characteristics, can considerably influence the way in which a disturbance propagates in congested conditions.

7.2 Expected impacts of heterogeneity and multi-anticipation on traffic flow predictions

The aim of this section is to discuss possible influences of our empirical findings on heterogeneity and multi-anticipation on macroscopic traffic flow predictions. We thereby distinguish the expected impacts on the fundamental diagram, platoon stability and flow stability.

We start by discussing the expected impacts of heterogeneity and multi-anticipation on the fundamental diagram in section 7.2.1. In sections 7.2.2 and 7.2.3, we continue by considering the expected impacts on respectively platoon stability and flow stability.
7.2.1 Expected impacts on fundamental diagram

In presenting the expected impacts of the empirical findings on the fundamental diagram, we first consider the expected impact of heterogeneity after which we consider the expected impact of multi-anticipation.

**Expected impact of heterogeneity on fundamental diagram**

Most of the longitudinal driving models discussed in chapter 3 and calibrated in chapters 5 and 6 assumed that drivers want to keep a speed-dependent distance headway to their leader. The motivation for a driver to choose a particular distance headway can, for instance, be related to safety and comfort. The macroscopic traffic variable density, denoted by $k$, is directly related to the gross distance headways individual drivers keep to their leader as can be seen from eq. (7.1) (Leutzbach, 1988).

$$k(v) = \frac{1}{s(v)}$$

(7.1)

Where $s(v)$ denotes the speed-dependent average gross distance headway.

When we assume that all drivers want to keep exactly the same distance headway for a given speed $v$, for illustration aims denoted by $s(v)$, the speed-dependent density $k(v)$ is given by,

$$k(v) = \frac{1}{s(v)}$$

(7.2)

Accordingly, for this homogeneous reference case the fundamental diagram can be represented by a single line as illustrated in Figure 7-1 for the Lenz model and the Tampère model.

Let us now assume that longitudinal driving styles differ between drivers, while behavioral parameters are the same for all drivers applying the same driving style. In using the terminology of this thesis the driver population is in this case characterized by driving style heterogeneity. To assess the impact of this type of heterogeneity on the fundamental diagram, it is important to consider that the assumptions made on the shape of the relation between speed and desired distance headway $s(v)$ differ between different driving styles (chapter 3). Based on eq. (7.2), this means that the shape of the fundamental diagram corresponding to these individual driving styles differs. This can also be seen in Figure 7-1.

To illustrate what happens to the fundamental diagram when the traffic flow consists of a mixture of driving styles, assume that there are two different groups of drivers. For the first group of $r_1$ drivers the speed-distance headway relation is always given by $s_1(v)$, while for the other group consisting of $r_2$ drivers this relation is always given by $s_2(v)$. This means that the speed-density relation is given by,

$$k(v) = \frac{1}{s(v)} = \frac{1}{r_1s_1(v) + r_2s_2(v)} = \frac{r_1 + r_2}{r_1s_1(v) + r_2s_2(v)}$$

(7.3)
fundamental diagram corresponding to Tampère model

fundamental diagram corresponding to Lenz model

Figure 7-1 Illustration of different shapes of fundamental diagrams belonging to different driving styles, and horizontal scatter.

Under the assumption that $r_1$ and $r_2$ do not change, the resulting fundamental diagram will still be a single line. The shape of this one-dimensional speed-density relation $k(v)$ will be a mixture of the shapes of the speed-density relations $s_1(v)$ and $s_2(v)$.

The congested branch of the fundamental diagram will become a two-dimensional region rather than a line, when the density corresponding to a given speed is calculated for several random samples of limited size. That is, for each of these random samples the fractions of drivers driving according to either driving style will most likely vary, resulting in another density for the same speed. The larger the sample size, the smaller this variety due to different fractions of drivers applying a driving rule will become (law of large numbers (Greene, 2000)).

In the presence of driving style heterogeneity, the fundamental diagram will also become a two-dimensional region when we assume the driving style adopted by a driver to be dependent on the characteristics of his neighboring vehicles as suggested in (Hoogendoorn and Bovy, 2000). That is, once the driving styles of consecutive drivers are not independent, the numbers of drivers $r_1$ and $r_2$ having respectively speed-distance headway relation $s_1(v)$ and $s_2(v)$ will depend on the prevailing order of driver/vehicle combinations.

Now let us assume that all drivers adopt the same driving style, while behavioral parameters differ between drivers. The driver population is thus characterized by within driving style heterogeneity only. In that case the speed-distance headway relation of driver $n$ can be written
as \( s(v; \beta_n) \), in which \( \beta_n \) is a vector containing the behavioral parameter values of driver \( n \). The corresponding speed-density relation is given by,

\[
k(v) = \frac{1}{s(v)} = \frac{1}{\sum_{n=1}^{r} s(v; \beta_n)} = \frac{r}{\sum_{n=1}^{r} s(v; \beta_n)}
\]

(7.4)

Where \( r \) denotes the total number of drivers in a sample. When for every speed \( v \) the density is determined based on a fixed sample of vehicles the corresponding fundamental diagram will be a line again.

The fundamental diagram will become two-dimensional when densities for a given speed are determined using several different finite size random samples of vehicles. The range of different densities that can be obtained for a given speed can be determined when considering the speed dependent distance headways of all individual drivers in the population and the sample size \( r \). That is, the maximum density for a given speed will be obtained when the density is calculated based on a sample consisting of the \( r \) drivers adopting the smallest headways for a given speed. The opposite holds for the minimum density. The probability that these maximum and minimum densities actually occur, decreases with the sample size \( r \).

In illustration, suppose that the total driver population consists of 1000 drivers having different behavioral parameters. In the case that these parameters are independent of the driver/vehicle combinations in the direct neighborhood of the driver and the sample size \( r \) is equal to 1, the probability that we determine the density based on a sample consisting of the driver having the smallest headway for that speed is 1/1000. When the sample size \( r \) becomes equal to 2, the probability that a random sample consists of the two drivers adopting the smallest time headway for a given speed is 1/1000*1/999.

In general, the larger the sample size \( r \) the smaller the variance in the average headway \( \bar{s} \) calculated based on different samples will become. For illustration aims, suppose that the distance headways of all drivers in a population for a given speed \( v \) follow a normal distribution with population mean \( \mu_{pop} \) and population standard deviation \( \sigma_{pop} \). Assume furthermore that \( \bar{s} \) is the average headway of drivers in a sample with size \( r \). When we calculate \( \bar{s} \) based on different samples most likely different values will be obtained. From statistics, we know that these sample averages \( \bar{s} \) are also normally distributed. The standard deviation \( \sigma_{av} \) of this distribution of sample averages is given by,

\[
\sigma_{av} = \frac{\sigma_{pop}}{\sqrt{r}}
\]

(7.5)

Thus the larger the sample size \( r \) becomes, the smaller the standard deviation \( \sigma_{av} \) of the distribution of sample averages \( \bar{s} \) becomes. Based on eq. (7.1) this implies that a larger sample size \( r \) results in a smaller spread in the density values for a given speed calculated based on different samples.

The general conclusion that can be drawn is that within driving style heterogeneity results in ‘horizontal scatter’ in the density-speed plane, where we mean with ‘horizontal scatter’ that for a fixed equilibrium speed, different densities are predicted (Figure 7-1). The variability in
these densities is according to eq. (7.5) dependent on the variability of the desired headways of drivers in the population and the sample size $r$.

This suggestion on the impact of within driving style heterogeneity is confirmed in (Treiber and Helbing, 1999), in which simulations are performed distinguishing two types of vehicles, namely trucks and person cars. It is shown that even when distinguishing only two vehicle types, scatter is introduced to the fundamental diagram.

An interesting consequence of these considerations is that heterogeneity is expected to contribute to the stochasticity of capacity as the maximum attainable flow depends on the prevailing composition of traffic flow.

Expected impact of multi-anticipation on the fundamental diagram

In the previous chapter, we referred to (Treiber et al., 2006a), who used multi-anticipation to explain measurements of double loop detector data showing that drivers often adopt time headways much smaller than their reaction times. The logic behind was that by considering more leaders downstream, drivers are able to react more quickly to a change in the dynamics of the direct leader as this change can be predicted (Herman and Rothery, 1963). This probably allows drivers to follow their direct leader with a smaller time headway.

In illustration, suppose that the leading vehicle and the following vehicle do have the same maximum braking rate. This means that they both travel the same distance before coming to a complete stop when they start braking at the same speed. When the following driver is not able to predict a sudden braking action of his leader at time $t$ by considering the dynamics of drivers in front of the leader, he will only react on this sudden braking action at time $t + T_r$. This means that in order to avoid a collision, the follower has to keep a time headway to his leader at least equal to his reaction time $T_r$. When a driver looks further ahead than his direct leader, he might reason that he can predict a sudden braking action of his leader and consequently will be able to react at an earlier time instant than $t + T_r$. This would mean that the driver can keep a time headway smaller than $T_r$ while still being able to avoid collisions. Of course, it can be argued that looking further ahead does not allow a driver to predict sudden braking actions of his leader in case that the braking action of the leader is not caused by braking actions of the leaders further ahead. This does however not imply that multi-anticipating drivers do not accept smaller time headways. They might, for example, consider such a sudden braking action of the direct leader without any for them observable reason, very unlikely when driving at freeways.

Following this reasoning the main expected impact of multi-anticipation on equilibrium conditions is that multi-anticipation might allow for a larger capacity compared to the situation in which drivers only look one vehicle ahead. This increase of capacity can both be attributed to the hypothesized possibility to follow with a smaller time headway, as well as to the increase in stability caused by multi-anticipation as will be discussed in the next subsection.

7.2.2 Expected impacts on platoon stability

In the previous subsection the expected impacts of heterogeneity and multi-anticipation on the fundamental diagram were discussed. In this subsection and subsection 7.2.3, we consider the expected impacts of heterogeneity and multi-anticipation on dynamic characteristics of traffic flow. We first consider the expected effects on platoon stability, referring to how a
disturbance in the dynamics of the platoon leader propagates through the platoon. Based on these insights for single platoons, we continue by broadening the view to flow stability.

**Expected impact of heterogeneity on platoon stability**

In literature on longitudinal driving models only little information can be found on the impact of heterogeneity in longitudinal driving behavior on platoon stability (we use the term platoon stability here as a synonym for asymptotical stability). Most studies on stability of platoons in which all drivers drive according to a given longitudinal driving rule are performed under the assumption of a homogeneous driver population.

The most well-known example of a longitudinal driving rule for which analytical analyses have been performed to determine the boundaries for platoon stability in the case of a homogeneous driver population is the CHM model presented in (Chandler et al., 1958). As shown in chapter 3, this model assumes that a follower controls his acceleration $a_n$ such that the difference between his own speed and his leaders speed, denoted by $\Delta v_{n-1,n}$ vanishes,

$$a_n(t + T_r) = c_{1,n-1} \Delta v_{n-1,n}(t)$$

(7.6)

A homogeneous platoon driving in line with this longitudinal driving rule is found to be stable when,

$$c_{1,n-1} * T_r \leq \frac{1}{2}$$

(7.7)

An intuitive interpretation of this analytically derived threshold for platoon stability is that groups of consecutive driver/vehicle combinations having behavioral parameters $c_{1,n-1}$ and $T_r$ satisfying $c_{1,n-1} * T_r \leq 1/2$ smooth a disturbance, while groups of driver/vehicle combinations having behavioral parameters for which $c_{1,n-1} * T_r > 1/2$ amplify a disturbance. Following this reasoning, the extent to which the disturbance is smoothed or amplified depends on $|c_{1,n-1} * T_r - 1/2|$. In the case of heterogeneous platoons this would intuitively mean that a disturbance in the dynamics of the platoon leader amplifies when proceeding through a part of the platoon with $c_{1,n-1} * T_r > 1/2$ and smoothes when proceeding through a part of the platoon with $c_{1,n-1} * T_r \leq 1/2$.

This intuitive reasoning is supported by (Mason and Woods, 1997). In (Mason and Woods, 1997) analyses are performed on the stability characteristics of the longitudinal driving rule presented in (Bando et al., 1995b) under the assumption of heterogeneous platoons. It is concluded that disturbances grow when proceeding through groups of driver/vehicle combinations having behavioral parameters causing platoons to become unstable and smooth when proceeding through groups having behavioral parameters resulting in stable platoons. The amplitude of the disturbance the last vehicle in the platoon faces compared to the original disturbance in the dynamics of the platoon leader, is found to be dependent on the number of consecutive stabilizing and destabilizing driver/vehicle combinations.

In (Treiber et al., 2007) stability analyses are performed for the HDM ((Treiber et al., 2006a), chapter 3) in which the reaction times of drivers are uniformly distributed within a range of ±30% around the mean value. It is concluded that distributed reaction times can stabilize platoons compared with the situation in which drivers have identical reaction times that are equal to the mean. Generally, however, the effect of distributed reaction times is found to be
small. Unfortunately, the dependence of this conclusion on the assumed level of heterogeneity is not examined.

Even from these few studies on the impact of heterogeneity in longitudinal driving behavior, it seems valid to state that heterogeneity affects platoon stability. This conclusion in combination with our empirical findings, indicating large differences between the longitudinal behaviors of driver/vehicle combinations in real-life traffic, stresses the importance of more research on this topic.

**Expected impact of multi-anticipation on platoon stability**

Whereas the previous focused on the impact of heterogeneity on platoon stability, we now continue by considering the impact of multi-anticipation on platoon stability.

An important influence of multi-anticipation on driving dynamics is that by multi-anticipating drivers are able to react earlier to disturbances occurring downstream. The question is how this affects platoon stability.

In (Lenz et al., 1999, Treiber et al., 2007) this has been examined for respectively the Lenz car-following model and the HDM. For both models it is shown that multi-anticipation increases the stability of platoons and thereby the stability of traffic flows.

In (Treiber et al., 2007) support is found for the earlier statement that multi-anticipation can explain the observation that drivers follow their leaders often with smaller headways than their reaction time. That is, simulations based on the HDM show that multi-anticipating drivers can adopt reaction times exceeding the safe time headway $T_{safe}$ while the resulting traffic flow is still stable.

### 7.2.3 Expected impacts on flow stability

In the previous, we considered the impacts of heterogeneity and multi-anticipation on platoon stability. Real-life traffic flows typically consist of consecutive platoons of different lengths led by slower vehicles (e.g. trucks). As traffic flows are disturbed frequently by, for example, merging vehicles entering the main road, we now extent the discussion on the impacts of our empirical findings to flow stability.

**Expected impacts of heterogeneity and multi-anticipation on flow stability**

The in literature most discussed impact of heterogeneity on simulated traffic flows driving in line with European legislation, is related to the lane choices of drivers having different desired speeds. By making a distinction between driver/vehicle combinations having a low desired speed and driver/vehicle combinations having a high desired speed, more realistic flow dependent distributions of fast vehicles and slow vehicles over the lanes are obtained. Accordingly also lane changes are induced causing disturbances in traffic flow.

This impact of “desired speed” heterogeneity is illustrated in the discussion presented in (Kerner and Klenov, 2004, Kerner, 2004) about the main effects of heterogeneity in simulations distinguishing slow, fast, and long vehicles\(^{11}\):

\[^{11}\text{Fast and slow vehicles have the same vehicle length that is lower than the length of long vehicles. The maximum vehicle speed in free flow of fast vehicles is higher than the one for slow and long vehicles.}\]
Different driver/vehicle combination characteristics lead to the well-known lane specific behavior in free traffic flow, i.e. fast vehicles use mostly the left (passing) lane, whereas slow and long vehicles use mostly the right freeway lane. As a result, the average speed in free flow in the left lane is higher than the average speed in the right lane.

When there is a low percentage of slow and/or long vehicles, large amplitude disturbances can appear in free flow associated with passing of fast vehicles when they approach slow (long) vehicles in the right lane. At a high enough flow rate in free flow these large amplitude disturbances can be comparable with disturbances at bottlenecks.

The type of heterogeneity we established in our empirical analyses clearly differs from this “desired speed” heterogeneity as it refers to differences between the longitudinal dynamics of driver/vehicle combinations while driving in congested conditions (due to this we did even refrain from calibrating the desired speed in chapters 5 and 6). Like in the discussion on platoon stability, we are therefore interested in the impact of differences between the car-following behaviors of drivers on traffic flow stability.

As traffic flows consist of platoons, it seems valid to state that the expected impacts of heterogeneity and multi-anticipation on flow stability are strongly related to the earlier discussed expected impacts on platoon stability. However, whether platoon instability results in flow instability depends on the lengths of instable platoons as well as on the gaps between consecutive platoons (Tampère, 2004).

Summary of expected impacts of empirical findings

The aim of this section has been to discuss the main expected impacts of heterogeneity and multi-anticipation on traffic flow properties. These impacts are summarized in Table 7-2.

From this section we can also conclude that in literature particularly little information is available on the impacts of driving style heterogeneity and within driving style heterogeneity on the characteristics of predicted flows. In the sequel of this chapter we therefore perform microscopic simulations to explore the impacts of heterogeneity on the fundamental diagram (section 7.3), platoon stability (section 7.4) and flow stability (section 7.5). Because of the strong correlation between platoon stability and flow stability, we will only summarize the main conclusions on platoon stability in this chapter. For more details on the impact of heterogeneity on platoon stability, we refer to Appendix G.
Table 7-2 Summary of main expected impacts of heterogeneity and multi-anticipation on predicted traffic flow characteristics.

<table>
<thead>
<tr>
<th></th>
<th>heterogeneity</th>
<th>multi-anticipation</th>
</tr>
</thead>
<tbody>
<tr>
<td>fundamental</td>
<td>• Driving style heterogeneity is mainly expected to change the shape of the fundamental diagram.</td>
<td>• Multi-anticipation possibly leads to an increase in capacity as drivers might reason that they can choose time headways smaller than their reaction time as they can predict changes in the dynamics of the direct leader by looking further ahead.</td>
</tr>
<tr>
<td>diagram</td>
<td>• Within driving style heterogeneity mainly causes the density belonging to a given speed to become stochastic, i.e. the fundamental diagram will become a two-dimensional region instead of a single line.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• The variability of densities belonging to a given speed decreases when sample sizes increase.</td>
<td></td>
</tr>
<tr>
<td>platoon stability</td>
<td>• Disturbances in the dynamics of the platoon leader are expected to grow when proceeding through groups of driver/vehicle combinations having behavioral parameter values causing homogeneous platoons to become unstable. Similarly, disturbances are expected to smooth when proceeding through groups having behavioral parameter values resulting in stable homogeneous platoons.</td>
<td>• Multi-anticipation is expected to lead to an increased stability of platoons.</td>
</tr>
<tr>
<td>flow stability</td>
<td>As traffic flows consist of platoons, the expected impacts of heterogeneity and multi-anticipation on flow stability are strongly related to the expected impacts on platoon stability. However, whether platoon instability results in flow instability depends on the lengths of instable platoons as well as on the gaps between consecutive platoons.</td>
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</table>

7.3 Simulation study on impact of heterogeneity on the fundamental diagram

The aim of this subsection is to further explore the impacts of driving style heterogeneity and within driving style heterogeneity on the fundamental diagram. In line with our expectations, it will be shown that driving style heterogeneity mainly causes the shape of the fundamental diagram to change, while within driving style heterogeneity only will introduce scatter to the fundamental diagram. Although this last mentioned impact of within driving style heterogeneity was already discussed partly in (Treiber and Helbing, 1999), we perform these
analyses here since our empirically established extent of heterogeneity was clearly larger than the distinction between person cars and trucks made by (Treiber and Helbing, 1999).

We will first discuss the experimental design used for assessing both types of heterogeneity, after which we continue by showing the corresponding results.

### 7.3.1 Experimental design
For the clarity of the presentation we will only consider drivers driving in line with the GT(3,1) model and the Lenz(2) model. These models have been selected based on the results of chapters 5 and 6. The models make different assumptions on the relation between the desired distance headway and the speed. Consequently, the corresponding fundamental diagrams differ (Figure 7-1).

We will start by deriving the equilibrium relations for both model types in which we only consider the congested driving regime as our empirical findings refer to this regime. For more information on the impact of heterogeneity on the free-driving branch of the fundamental diagram, we refer to (Wu, 2002, Farzaneh and Rakha, 2006). After the derivation of the equilibrium relations for the congested regime, we introduce the experimental design used for considering both types of heterogeneity.

#### Equilibrium relation for the GT(3,1) model
A driver is in an equilibrium state at time $t$ when he does not have an incentive to change his speed, i.e. $a_x = 0$. When we assume that all 3 leaders are driving at the same speed as the follower $n$ this means for the GT(3,1) model that the following condition needs to be fulfilled for a driver driving in congested conditions:

$$\Delta x_{n-1,n}^*(t) - \Delta x_{n-1,n}(t) = 0$$

with

$$\Delta x_{n-1,n}^*(v_n(t)) = d + \gamma * v_n(t)$$

Thus given that all relative speeds $\Delta v_{n,i,n}$ are 0, a driver does not want to change his speed when his current distance to his first leader is equal to his desired distance.

#### Equilibrium relation for the Lenz(2) model
For the Lenz(2) model the following relation needs to be fulfilled for a driver to be in an equilibrium state:

$$c_{6,n-1} \left( V_{opt} \left( \frac{\Delta x_{n-1,n}(t)}{1} \right) - v_n(t) \right) + c_{6,n-2} \left( V_{opt} \left( \frac{\Delta x_{n-2,n}(t)}{2} \right) - v_n(t) \right) = 0$$

One possible solution to this relation is that:

$$V_{opt} \left( \frac{\Delta x_{n-1,n}(t)}{1} \right) = V_{opt} \left( \frac{\Delta x_{n-2,n}(t)}{2} \right) = v_n(t)$$
Eq. (7.11) is only fulfilled when the distances $\Delta x_{n-1,n}$, and $\Delta x_{n-2,n}$ are such that the current speed $v_n(t)$ of the follower is exactly in line with both distances.

Given that a driver can only regulate his distance to his direct leader and given the results of the previous chapters showing that there exist clear differences between the distances different drivers want to keep to their leaders, there is only a small probability that $\Delta x_{n-2,n}$ exactly fulfills this condition. The following more general solution to the equilibrium relation therefore needs to be considered:

\[
v_n(t) = \frac{c_{6,n-1}v_{\text{opt}} \left( \frac{\Delta x_{n-1,n}(t)}{1} \right) + c_{6,n-2}v_{\text{opt}} \left( \frac{\Delta x_{n-2,n}(t)}{2} \right)}{c_{6,n-1} + c_{6,n-2}} \tag{7.12}
\]

To get some feeling about how to interpret this equilibrium solution suppose that,

\[
c_{6,n-2} \left( V_{\text{opt}} \left( \frac{\Delta x_{n-2,n}(t)}{2} \right) - v_n(t) \right) > 0, \quad \text{with } c_{6,n-2} > 0 \tag{7.13}
\]

This means that the speed ($V_{\text{opt}}$) the follower wants to drive with based on $\Delta x_{n-2,n}$ is larger than his current speed $v_n$. To fulfill the equilibrium condition, it consequently has to hold that $V_{\text{opt}}$ regarding the first leader is smaller than the current speed of the follower.

Given that there exists a one-to-one relation between $V_{\text{opt}}$ and the distance headway in the constrained driving regime (chapter 3, Figure 3-1) this can also be more intuitively stated as, when the second leader drives at a distance larger than the distance $\Delta x_{n-2,n}$ satisfying:

\[
V_{\text{opt}} \left( \frac{\Delta x_{n-2,n}(t)}{2} \right) = v_n(t) \tag{7.14}
\]

then the driver keeps in equilibrium conditions a distance to his first leader smaller than the one satisfying:

\[
V_{\text{opt}} \left( \Delta x_{n-1,n}(t) \right) = v_n(t) \tag{7.15}
\]

As the driver can only regulate $\Delta x_{n-1,n}$, this in fact implies that the distance satisfying eq. (7.15) can be seen as median value for the distance $\Delta x_{n-1,n}$ fulfilling eq.(7.12). When the second leader is driving at a larger distance than the one satisfying eq. (7.14) the value for $\Delta x_{n-1,n}(t)$ resulting in $a_n(t)=0$ will be smaller than this median value, and vice versa.

**Approach for considering impact of within driving style heterogeneity**

In our analyses, we will first consider the impact of within driving style heterogeneity on the fundamental diagram. We thereto apply the following procedure to both models separately.
For a given speed $v$ we randomly select $m$ different samples of sets of parameter values from the parameter estimates of chapter 6, where a set of parameter values refers to the set of all behavioral parameters for a single driver. Every sample is of size $r$ and can be interpreted as a platoon of $r$ driver/vehicle combinations, where the composition of the platoon is based on the empirical findings.

Given the parameter values for all $r$ driver/vehicle combinations, the speed $v$ and the equilibrium relations derived before, we can determine for every vehicle in the platoon the equilibrium distance headway. Based on these equilibrium distances for all vehicles in the platoon we can, as discussed before, determine the density $k$ belonging to a sample. As we know that,

$$q = k \cdot v$$

we can also compute the corresponding flow $q$.

We repeat this procedure for a range of speeds between zero and 30 m/s (assumed free speed) with in between steps of 1 m/s. The outcomes enable us to draw fundamental diagrams.

We particularly stress the use of a fixed sample size $r$ in this analysis as the extent of variability between points on the fundamental diagram belonging to the same speed is, as discussed before, related to sample size. The smaller the sample size the larger the variability between samples will be. When allowing for different sample sizes for different speeds, such as in empirical studies using aggregate data of fixed time periods, it would not be possible to compare the impact of heterogeneity on the fundamental diagram between speeds.

The fixed sample size $r$ is based on (Tilch and Helbing, 2000) decided to be equal to 50. The number of samples $m$ considered for a given speed is furthermore assumed to be equal to 10. Given this sample size $r$ and the knowledge that empirical sets of parameter values are only representative in the case that a good model fit is obtained, we compose the empirical parameter distributions of the 50 driver/vehicle combinations yielding the lowest error term for the model under consideration. This means that the sample size $r$ is equal to the number of different parameter combinations in the empirical distribution. We therefore draw parameters from this distribution with replacement.

The previous chapters showed for both models difficulties in estimating the desired distance at standstill $d$, we therefore assume this parameter to be equal to 8.33 m. This implies that the jam density will always be equal to 120 veh/km in the upcoming analyses. This choice reduces the variability in densities corresponding to a given speed. However, the higher the speed the less important this influence of assigning a fixed value to $d$ will become.

**Approach for considering impact of both driving style heterogeneity and within driving style heterogeneity**

To analyze the combined impact of driving style heterogeneity and within driving style heterogeneity, we apply essentially the same approach. The only difference is that in this case not only the sets of parameter values of the driver/vehicle combinations in the platoon are randomly selected from the empirically estimated parameter values but also the models assigned to the drivers. The discrete empirical distributions according to which the models are assigned are derived from the percentages of times a model family appeared to be best in chapter 6 (see Table 7-3).
Like before we draw again for every speed 10 samples, each having a fixed sample size of 50 driver/vehicle combinations.

The details of the empirical designs for the analyses on within driving style heterogeneity only as well as for the analyses on the joint effect of driving style heterogeneity and within driving style heterogeneity are summarized in Table 7-4.

### Table 7-4 Overview of experimental designs for analyzing the impact of within driving style heterogeneity as well as the impact of a combination of driving style heterogeneity and within driving style heterogeneity.

<table>
<thead>
<tr>
<th>Influence of within driving style heterogeneity only</th>
<th>Influence of driving style heterogeneity and within driving style heterogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>model(s)</td>
<td>model(s)</td>
</tr>
<tr>
<td>driving style heterogeneity</td>
<td>GT(3,1)</td>
</tr>
<tr>
<td>within driving style heterogeneity</td>
<td>no</td>
</tr>
<tr>
<td>sample size $r$</td>
<td>yes</td>
</tr>
<tr>
<td>number of samples $m$</td>
<td>50</td>
</tr>
<tr>
<td>jam density</td>
<td>120 veh/km</td>
</tr>
<tr>
<td>free speed</td>
<td>30 m/s</td>
</tr>
<tr>
<td>remaining model parameters</td>
<td>empirical</td>
</tr>
<tr>
<td></td>
<td>GT(3,1), Lenz(2)</td>
</tr>
<tr>
<td></td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>120 veh/km</td>
</tr>
<tr>
<td></td>
<td>30 m/s</td>
</tr>
<tr>
<td></td>
<td>empirical</td>
</tr>
</tbody>
</table>

### 7.3.2 Impact of heterogeneity on the fundamental diagram

In the upcoming, we first show that within driving style heterogeneity causes the fundamental diagram to become a two-dimensional plane rather than a single line. This finding is in line with the empirical observations discussed in (Kerner, 2004). In the analyses of the joint impact of driving style heterogeneity and within driving style heterogeneity, we establish that driving style heterogeneity mainly causes the shape of the fundamental diagram to change.

**Impact of within driving style heterogeneity only**

Figure 7-2 and Figure 7-3 show the effects of within driving style heterogeneity on the fundamental density ($k$)-speed ($v$), density ($k$)-flow ($q$), and flow ($q$)-speed ($v$) relations for respectively the Everdingen measurement site and the Waalhaven measurement site. The fundamental relations shown in the upper parts of the figures are based on the empirical inputs for the GT(3,1) model, while the lower parts are based on the empirical inputs for the Lenz(2) model.

To facilitate a comparison between a homogeneous traffic flow and a heterogeneous traffic flow, the figures also show fundamental relations drawn on the assumption that all drivers are driving according to the mean values of the empirically estimated parameter values.

When comparing the density ($k$)-speed ($v$) relations for the homogeneous case and the heterogeneous case it turns out that heterogeneity introduces a lot of horizontal scatter in the fundamental diagram, i.e. the fundamental diagram becomes a two-dimensional region rather
than a single line. The density $k$ belonging to a given speed $v$ is stochastic and depends on the composition of the sample of driver/vehicle combinations present on the road.

![Graphs](image)

**Figure 7-2** Fundamental diagrams based on the calibrated parameters for (a) the GT(3,1) model and (b) the Lenz(2) model for the Everdingen measurement site.

![Graphs](image)

**Figure 7-3** Fundamental diagrams based on the calibrated parameters for (a) the GT(3,1) model and (b) the Lenz(2) model for the Waalhaven measurement site.
When comparing the fundamental relations belonging to the same model specification between the measurement sites, smaller capacity values can be observed for the Waalhaven site than for the Everdingen site. A plausible explanation for this is that the parameter estimates for the Waalhaven measurement site are obtained during heavy congestion. It is therefore well possible that drivers adjusted their driving styles to these conditions (for a closer investigation see Appendix E). This reasoning is supported by other empirical studies discussed in chapter 2 showing that the capacity drop, i.e. the phenomenon that the maximum flow rate achievable during congestion is lower than in free flow traffic, can be caused by drivers keeping larger time headways in congestion due to a loss of motivation. Thus when deriving a fundamental diagram from parameter values calibrated based on observations gathered during congestion lower capacity values are obtained than when deriving a fundamental diagram from parameter values calibrated based on observations obtained before the occurrence of congestion.

When comparing the capacities between the two model specifications for the same measurement location, it appears that the capacity values obtained using the GT model are plausible, while the values obtained using the Lenz model are significantly lower than in reality. These strong differences between the capacities of the two model specifications for the same measurement location can be explained in several ways. Firstly, there exist generic behavioral differences between the driving styles of drivers driving according to a model belonging to the GT\((m_1, m_2)\) family and drivers driving according to a model belonging to the Lenz\((m_3)\) model family.

Secondly, also two more technical explanations can be given possibly explaining part of the difference. The first one directly relates to the shape of the fundamental diagram belonging to the Lenz model. To explain this point Figure 7-4 shows 5 different flow \((q)\)-speed \((v)\) relations for homogeneous traffic flows in which drivers are assumed to be driving according to the Lenz\((2)\) model with parameter values equal to an estimated set of parameter values of a randomly selected driver driving at the Everdingen site.

![Figure 7-4](image_url) **Figure 7-4** Possible explanation for lower maximum flows of fundamental diagrams belonging to the Lenz\((2)\) model.
The maximum flow is for all these flow \((q)\) -speed \((v)\) relations obtained at a different speed level. This means that the maximum flow obtained for heterogeneous platoons when using our approach is smaller than,

\[
q_{\text{average}} = \frac{\sum_{\text{drivers}} q_{\text{max}}^{\text{driver}}}{\text{number of drivers}}
\]  

(7.17)

Where \(q_{\text{max}}^{\text{driver}}\) denotes the maximum flow obtained when drawing a fundamental diagram based on the set of estimated parameter values for a specific driver. This does not occur for the GT(3,1) model as the fundamental diagrams corresponding to the parameters of individual drivers all reach their maximum at the same speed.

Furthermore we need to take into account that by deriving fundamental diagrams from our empirically estimated parameter values, we assume that these parameters are transferable to all traffic conditions. Although the validity of this assumption is questionable in itself, it needs to be stressed that it does not affect the general conclusion that within driving style heterogeneity causes scatter in the fundamental diagram. The extent of scatter, i.e. the variability of densities for a given speed can however be affected.

**Impact of mixture of driving style heterogeneity and within driving style heterogeneity**

Figure 7-5 shows the results for the scenario with both within driving style heterogeneity and driving style heterogeneity. For the sake of comparison we also show the fundamental relations belonging to:

- Scenario 1: all driver/vehicle combinations are assumed to be driving in line with the GT(3,1) model having parameter values equal to the means of the corresponding empirically estimated parameter values. Traffic flow is thus completely homogeneous.
- Scenario 2: all driver/vehicle combinations are assumed to be driving in line with the Lenz(2) model having parameter values equal to the means of the corresponding empirically estimated parameter values. Traffic flow is thus completely homogeneous.
- Scenario 3: driving styles are randomly assigned to drivers in line with the earlier specified empirical distributions (Table 7-3). Drivers driving according to the same model specification are all having parameter values equal to the means of the empirically estimated parameter values. In terms of this thesis the traffic flow is characterized by driving style heterogeneity only.

When comparing the fundamental relations corresponding to scenario 1 and scenario 2, we can, as mentioned before, see that they differ regarding shape. Consequently, when we introduce driving style heterogeneity (scenario 3), we can see that the shape of the resulting fundamental relations is influenced by the fundamental relations belonging to scenario 1 and 2. We can conclude that driving style heterogeneity causes the shape of the fundamental relations to change.
7.4 Simulation results on impact of heterogeneity on platoon stability

In the previous section, we considered the impacts of driving style heterogeneity and within driving style heterogeneity on the fundamental diagram. We now consider the impacts of heterogeneity on platoon stability (section 7.4) and flow stability (section 7.5).

Because of the large correlation between platoon stability and flow stability, we only provide a summary of our exploratory results on platoon stability in this section (for the complete results we refer to Appendix G). These exploratory analyses focus on the relatively simple CHM longitudinal driving rule as for this model thresholds are derived analytically for platoon stability of homogeneous platoons as discussed in section 7.2.2. As the empirical analyses of chapter 5 showed that the CHM model is in general too simple for describing longitudinal driving dynamics reasonably well and as platoon stability is only one of the determinants determining flow stability, we use these analyses in fact as a preparation for the analyses on flow stability in section 7.5. That is, in section 7.5 we aim at establishing whether our main findings on stability referring to heterogeneous platoons driving in line with the CHM model are also applicable to flows driving in line with the GT(3,1) and GT(2,1) model.

In the exploratory analyses on the influence of within driving style heterogeneity on stability of platoons driving according to the CHM model, we mainly aim at verifying our expectation that groups of consecutive drivers having behavioral parameters $c_{1,n-1}$ and $T_r$ satisfying $c_{1,n-1} * T_r \leq 1/2$ smooth a disturbance, while groups of drivers having behavioral parameters for which $c_{1,n-1} * T_r > 1/2$ amplify a disturbance. We thereto assume the sensitivity parameter $c_{1,n-1}$
of eq. (7.6) to be independently and identically normally distributed, i.e. \( c_{x,1} \sim N(\mu, \sigma) \). The reaction time \( T_r \) is for all drivers assumed to be equal to 1 s. In the analyses we adopt two values for \( \mu \), namely 0.5 and 0.6. For both values for \( \mu \), we furthermore examine the stability results for standard deviations of 0.05, 0.1, 0.15 and 0.2. Because of the stochastic element, we perform 10 simulations for all different combinations of \( \mu \) and \( \sigma \).

The results show that the disturbance introduced to the speed of the platoon leader decays when proceeding through platoons in which the mean \( \mu \) of the applied normal distribution for \( c_{1,n-1} \) is equal to 0.5, independent of the applied standard deviation.

For platoons in which the mean value \( \mu \) of \( c_{1,n-1} \) is assumed to be equal to 0.6, the disturbance always amplifies for simulations with \( c_{1,n-1} \sim N(0.6, 0.05) \) and \( c_{1,n-1} \sim N(0.6, 0.1) \). In the cases that \( c_{1,n-1} \sim N(0.6, 0.15) \) and \( c_{1,n-1} \sim N(0.6, 0.2) \) the initial disturbance in the speed of the leader in general decays when proceeding through the platoon. This leads to the conclusion that platoon stability is in the case of heterogeneous drivers not only determined by the longitudinal driving behavior of the ‘average driver’, but that also the level of within driving style heterogeneity is important.

Another important observation is that especially in the cases with \( c_{1,n-1} \sim N(0.6, 0.15) \) and \( c_{1,n-1} \sim N(0.6, 0.2) \) clear differences between the way in which the disturbance propagates through the platoon can be observed between consecutive simulation runs. A more detailed look at the correlation between the propagation of the disturbance and the platoon composition reveals that this can be explained by our earlier expectation. That is, a disturbance smooths when proceeding through a small group of ‘stabilizing’ drivers and amplifies when proceeding through a group of ‘destabilizing’ drivers. The extent to which this happens is dependent on the specific parameter values, i.e. \( |c_{1,n-1} - 1/2| \). This leads to the conclusion that the platoon composition, i.e. the order of vehicles in the platoon, determines how the disturbance propagates in the case of heterogeneous platoons.

### 7.5 Simulation study on impact of heterogeneity on traffic flow stability

Based on the conclusions in our analyses on the impact of heterogeneity on the stability of platoons driving in line with the CHM model, we will particularly focus on two research questions regarding the propagation of a disturbance in heterogeneous traffic flows, namely,

- How does the level of within driving style heterogeneity influence the propagation of a disturbance in traffic flow?
- How does the flow composition, i.e. the order of vehicles having different characteristics, influence the propagation of a disturbance in traffic flow?

Before we handle these research questions, we perform simulations on the difference between the influences of driving style heterogeneity and within driving style heterogeneity on traffic flow dynamics. A difference that has, to our best knowledge, not obtained attention in literature so far.

Like in the previous section we start by discussing our experimental design.

#### 7.5.1 Experimental design

In the analyses we simulate single lane traffic flows that are externally disturbed by either a speed limit, due to which vehicles start braking, or an on-ramp forcing part of the merging
vehicles and vehicles on the main road to recover their headway. Both roadway configurations are presented in Figure 7-6.

![Figure 7-6 Road configurations applied in simulations (a) speed limit, (b) on-ramp.](image)

We mainly concentrate on the simulations with the speed limit as in these analyses no further assumptions on driving behavior need to be made apart from the empirically established ones. In the analyses on the impacts of the extent of heterogeneity and the order of vehicles on flow stability, we also consider the on-ramp scenario. This scenario has the disadvantage that additional assumptions need to be made referring, for example, to the merging behavior of drivers.

To perform the microscopic simulations, we developed a dedicated microscopic simulation tool programmed in C++ using object oriented programming (Friedman and Koffman, 2000). This microscopic simulation tool is developed in such a way that we can assign different longitudinal driving models and corresponding parameter values to all vehicles entering the main road and the on-ramp. For the clarity of the presentation we limit the range of available models to the GT(2,1) model and the GT(3,1) model in the upcoming analyses. The inflow to the main road and the on-ramp can be regulated by varying the time intervals at which new vehicles are inserted.

For the speed limit scenario, we consider three case studies as can be seen in Table 7-5. The aim of the first case study is to gain insight into the different impacts of driving style heterogeneity and within driving style heterogeneity on traffic flow. The values of the behavioral parameters used in this case study are based on the empirically estimated values. In the second case study, we concentrate on the impact of the extent of heterogeneity on flow dynamics. The behavioral parameter values selected in this study are, like in the analysis on platoon stability, close to the threshold of platoon stability for homogeneous platoons. In the third case study, we aim at determining the impact of the order of vehicles having different characteristics on flow stability. Vehicles are thereto inserted to the main road in systematically varied patterns.

To arrange that vehicles lower their speed in line with the speed regulations, we lower the desired speed of drivers from 30 m/s to 20 m/s when they pass the location of the speed limit sign.
Table 7-5 Overview of case studies performed for speed limit scenario.

<table>
<thead>
<tr>
<th></th>
<th>Case a</th>
<th>Case b</th>
<th>Case c</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>difference driving style and within driving style heterogeneity</td>
<td>level of within driving style heterogeneity</td>
<td>order of vehicles</td>
</tr>
<tr>
<td>models</td>
<td>GT(2,1), GT(3,1)</td>
<td>GT(3,1) synthetic</td>
<td>GT(3,1) synthetic</td>
</tr>
<tr>
<td>applied parameters</td>
<td>empirical</td>
<td>no</td>
<td>synthetic</td>
</tr>
<tr>
<td>driving style heterogeneity</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>within driving style heterogeneity</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>different levels of within driving style heterogeneity</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>order of vehicles combinations</td>
<td>random</td>
<td>random</td>
<td>systematically varied patterns</td>
</tr>
<tr>
<td>min. time headway between insertions</td>
<td>1.5 sec.</td>
<td>1.5 sec.</td>
<td>1.5 sec.</td>
</tr>
</tbody>
</table>

As stated before, we increase the insight into the impacts of the level of heterogeneity and the order of vehicles having different characteristics on flow stability further by also considering an on-ramp scenario. To model the behavior of drivers on the main road for whom a new vehicle merges and the behavior of merging vehicles themselves, we apply the “activation level” principle discussed in (Tampère, 2004, Tampère et al., 2005a). That is, we assume that both the main-lane drivers as well as the merging drivers temporarily accept smaller headways without braking after a merge. Thus they increase their activation level. Downstream of the on-ramp, when the higher activation-level is no longer useful, drivers relax to the normal activation level by increasing their headways to the normally acceptable ones. Due to this type of behavior, the onset of congestion caused by an on-ramp occurs usually downstream of the on-ramp itself.

In more detail, we model this type of adaptive behavior by lowering the behavioral parameter γ of affected vehicles directly after a merge such that the desired distance headways are in line with the prevailing headways. After the on-ramp the parameter value for γ is linearly increased by Δ γ at every simulation step till it becomes equal to the original level. Thus when driving at the location of the on-ramp, drivers affected by a merge are assumed to have a higher activation level, while they start recovering their desired distance after the on-ramp in order to adapt to a less demanding longitudinal driving behavior.

In the on-ramp scenario, we produce a flow on the main road close to capacity, i.e. every 1.5 sec. a new vehicle is inserted. We furthermore assume that merging vehicles accept all gaps that are at least as large as their own desired distance, implying that they can induce a rather large disturbance to traffic flow on the main road as they force drivers on the main road to brake in order to recover their desired headway to their direct leader. At the moment of merging, the speed of the entering vehicle is furthermore assumed to be equal to the speed of the new leader on the main road.

---

12 We take special care of not inserting vehicles at distance headways smaller than their desired distance headways as we do not want vehicles to start braking immediately after insertion because of a too small distance headway.

13 In an empirical study of (Sultan et al., 2002) on car-following behavior immediately after the cut-in of a new vehicle, indications are found that the average response to cut-in maneuvers is smoother than during undisturbed car-following.

14 The desired increase of desired distance for every 1 m/s increase of speed, see also eq. (7.9)
The case studies performed for the on-ramp scenario are summarized in Table 7-6.

### Table 7-6 Overview of case studies performed for on-ramp scenario.

<table>
<thead>
<tr>
<th>Case d</th>
<th>Case e</th>
</tr>
</thead>
<tbody>
<tr>
<td>level of within driving style heterogeneity</td>
<td>GT(3,1) synthetic</td>
</tr>
<tr>
<td>order of vehicles</td>
<td>no</td>
</tr>
<tr>
<td>models</td>
<td>yes</td>
</tr>
<tr>
<td>applied parameters</td>
<td>no</td>
</tr>
<tr>
<td>synthetic</td>
<td>yes</td>
</tr>
<tr>
<td>driving style heterogeneity</td>
<td>no</td>
</tr>
<tr>
<td>within driving style heterogeneity</td>
<td>systematically varied patterns</td>
</tr>
<tr>
<td>different levels of heterogeneity</td>
<td></td>
</tr>
<tr>
<td>order of driver/vehicle combinations</td>
<td>random</td>
</tr>
<tr>
<td>min. time headway between insertions at main road</td>
<td>1.5 sec.</td>
</tr>
<tr>
<td>min. time headway between insertions at on-ramp</td>
<td>40 sec.</td>
</tr>
<tr>
<td>$\Delta t^*$</td>
<td>0.01 sec.</td>
</tr>
<tr>
<td></td>
<td>0.005 sec.</td>
</tr>
</tbody>
</table>

For the clarity of the presentation, we provide the parameter values used in the different case studies, when showing and discussing the corresponding simulation results.

#### 7.5.2 Impact of heterogeneity for speed limit scenario

In this subsection, we consecutively show the simulation results for the case studies a, b and c referring to the speed limit scenario.

**Case study a: Difference between driving style heterogeneity and within driving style heterogeneity**

To obtain insights into the influences of driving style heterogeneity and within driving style heterogeneity, we perform several simulations for the speed limit scenario while varying the type of heterogeneity. We assume the parameter values of $c_{1,n,j}$ and $c_2$ to be equal to the means of the empirical parameter distributions for the Everdingen measurement site. An overview of all applied parameter values is provided in Table 7-7. The numbers in brackets present the standard deviations of the normal distributions applied in the simulations including within driving styling heterogeneity.

### Table 7-7 Overview of parameters used in case study a, mean parameter values (standard deviations).

<table>
<thead>
<tr>
<th></th>
<th>GT(2,1)</th>
<th>GT(3,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{1,n-1}$</td>
<td>0.23 (0.1)</td>
<td>0.20 (0.1)</td>
</tr>
<tr>
<td>$c_{1,n-2}$</td>
<td>0.17 (0.1)</td>
<td>0.13 (0.05)</td>
</tr>
<tr>
<td>$c_{1,n-3}$</td>
<td>-</td>
<td>0.09 (0.05)</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.11 (0.05)</td>
<td>0.1 (0.05)</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$d$</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>$T_r$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The resulting simulation outcomes are presented in Figure 7-7.
In Figure 7-7(a) and Figure 7-7(b), we assume traffic flow to be completely homogeneous. In Figure 7-7(a) all drivers are driving according to the GT(2,1) model having parameter values equal to the ones presented in Table 7-7, while in Figure 7-7(b) all drivers are driving in line with the GT(3,1) model.

The main difference between the simulation results for the two homogenous traffic flows is that drivers start lowering their speed earlier when they are driving in line with the GT(3,1) model than when they are driving in line with the GT(2,1) model. This can be understood when considering that drivers considering 3 leaders do earlier observe the disturbance and correspondingly react earlier as we assume the reaction time to be the same for both models.

In the simulations presented in Figure 7-7(c) and Figure 7-7(d), we leave the assumption of homogeneous traffic flows and instead introduce within driving style heterogeneity to the simulations. In Figure 7-7(c), we more specifically assume that all drivers are driving in line with the GT(2,1) model, while the parameters $c_{1,n-j}$ and $c_2$ are randomly assigned to drivers based on draws from normal distributions with means and standard deviations as presented in Table 7-7. Figure 7-7(d) is comparable to Figure 7-7(c) with the only difference that drivers are driving in line with the GT(3,1) model.

Like before we can observe that drivers start lowering their speed earlier when they look further ahead. The introduction of within driving style heterogeneity mainly causes the simulation results to become less smooth, i.e. different drivers adjust their speeds differently to the new speed limit. This can be understood as follows, by allowing sensitivity parameters to differ between drivers we actually change the extent to which drivers react to their respective leaders. For example, a low parameter value for $c_{1,n-2}$ in the GT(3,1) model implies that a driver does not very much consider his third leader in determining an appropriate control action, while a large setting for this parameter implies that the third leader is of more importance.

In Figure 7-7(e) we focus on the influence of driving style heterogeneity rather than the influence of within driving style heterogeneity. Accordingly we randomly assign driving styles to drivers, while the behavioral parameter values for all drivers adopting the same driving style are assumed to be the same.

The position on the road at which drivers start to lower their speed is now somewhere in the middle of the positions at which drivers started to lower their speed in Figure 7-7(a) in which all drivers were driving according to the GT(2,1) model, and Figure 7-7(b) in which all drivers were driving according to the GT(3,1) model. Also after introducing driving style heterogeneity, the adaptation to the new speed limit becomes less smooth than in the cases of homogeneous platoons. This effect seems to be less however than for the cases with only within driving style heterogeneity. A possible reason for this is in line with an earlier explanation. When the sensitivity parameters $c_{1,n-j}$ differ for different drivers this means that the extent to which people react to their respective leaders differs. This resembles to a certain level driving style heterogeneity in which only the degree of multi- anticipation (to how many leaders does a driver react) is varied between drivers.

Figure 7-7(f) finally shows the simulation results for a traffic flow containing both driving style heterogeneity as well as within driving style heterogeneity.
Figure 7-7 Impact of different types of heterogeneity. Figures (a) and (b) are the reference cases in which all vehicles drive respectively according to the GT(2,1) model and the GT(3,1) model. In figures (c) and (d) within driving style is added to the simulations of respectively the GT(2,1) model and the GT(3,1) model. Figure (e) refers to the case with only driving style heterogeneity, i.e. traffic flow is a mixture of drivers driving in line with the GT(2,1) model and the GT(3,1) model. In figure (f) traffic flow is characterized by both driving style heterogeneity as well as within driving style heterogeneity.

Case study b: Impact of level of within driving style heterogeneity

In the previous case study, we assumed the level of within driving style heterogeneity to be fixed, i.e. the standard deviations of the applied normal distributions were assumed to be fixed. In this case study, we explore the impact of the level of within driving style heterogeneity.

Like in the analyses of platoon stability, we select the parameters close to the stability boundary for which the disturbance induced by vehicles starting to adjust their speed to the speed limit grows when propagating through the simulated flow of vehicles. A summary of the applied parameter values is provided in Table 7-8. In this case study we only assume the
parameter $c_{1,n-1}$ to vary between drivers. The standard deviations assumed in the various simulation results are provided as well in Table 7-8.

### Table 7-8 Overview of parameter values used in case study b, mean parameter values (standard deviations).

<table>
<thead>
<tr>
<th>GT(3,1)</th>
<th>$c_{1,n-1}$</th>
<th>0.45 (a) 0.05, (b) 0.1, (c) 0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{1,n-2}$</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$c_{1,n-3}$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>$T_r$</td>
<td>1.1</td>
<td></td>
</tr>
</tbody>
</table>

To have a reference case we assume traffic flow to be homogenous in Figure 7-8(a). In this homogeneous case the speed limit has a clear impact on the traffic situation upstream of the speed limit. Disturbances created by drivers adjusting their speed to the new speed limit grow clearly when proceeding through traffic flow due to (over)reactions of following vehicles.

![Graphs](image)

**Figure 7-8** Impact of level of within driving style heterogeneity on traffic flow dynamics for the speed limit scenario. The parameter $c_{1,n-1}$ of the GT(3,1) model is assumed to be normally distributed, i.e. (a) $c_{1,n-1}=0.45$, (b) $c_{1,n-1}\sim N(0.45, 0.05)$, (c) $c_{1,n-1}\sim N(0.45, 0.1)$, (d) $c_{1,n-1}\sim N(0.45, 0.2)$.

In Figure 7-8(b), we introduce within driving style heterogeneity to the simulations by randomly assigning the parameter $c_{1,n-1}$ to vehicles entering the main road based on a normal
distribution with mean 0.45 and standard deviation 0.05. This minor level of variation does clearly not significantly affect traffic flow characteristics for the considered speed limit scenario.

When we increase the extent of within driving style heterogeneity further by increasing the standard deviation of the applied normal distribution to respectively 0.1 in Figure 7-8(c) and 0.2 in Figure 7-8(d), the traffic pattern clearly changes. That is, the larger the level of within driving style heterogeneity, the smaller the impact of the speed limit on traffic conditions upstream of the speed restriction. This is noticeable as the average value for the parameter $c_{1,n-1}$, and thus the average car-following behavior over the drivers is not altered, only the level of within driving style heterogeneity increased. This conclusion is in line with our earlier results on platoon stability.

Increasing the extent of heterogeneity to an even higher level leads to collisions due to, for example, drivers whose sensitivity to the speed difference with the first leader becomes too small.

The results show that flow predictions are sensitive for the assumed level of heterogeneity. This is an important finding as it underlines the importance of performing research on the level of heterogeneity in real-life traffic flows.

**Case study c: Impact of order of vehicles in heterogeneous traffic flows**

In the previous case study we showed a large influence of the assumed level of within driving style heterogeneity on our simulation results for the speed limit scenario. In this case study we want to establish whether the flow composition, i.e. the order of vehicles having different longitudinal driver behaviors, plays a role in the propagation of the disturbance created by the speed limit.

For the clarity of the presentation we restrict the number of driver types to two. More specific, we assume the sensitivity parameter $c_{1,n-1}$ to be equal to 0.5 for 2/3 of the drivers, while it is equal to 0.33 for the remaining drivers (a full overview of the applied parameters is provided in Table 7-9).

**Table 7-9 Overview of parameter values used in case study c,**

<table>
<thead>
<tr>
<th>mean parameter values</th>
<th>GT(3,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{1,n-1}$</td>
<td>0.33 or 0.5</td>
</tr>
<tr>
<td>$c_{1,n-2}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$c_{1,n-3}$</td>
<td>0.05</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.1</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.1</td>
</tr>
<tr>
<td>$d$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.4</td>
</tr>
<tr>
<td>$T_r$</td>
<td>1.1</td>
</tr>
</tbody>
</table>

To be better able to interpret the behavioral implications of the two parameter values for $c_{1,n-1}$, we start our analysis by showing in Figure 7-9(a) and in Figure 7-9(b) the simulation results for the speed limit scenario in which all drivers have respectively $c_{1,n-1}=0.33$ and $c_{1,n-1}=0.5$.  

Under the assumption of a homogeneous traffic flow in which \( c_{1,n-1} = 0.33 \), drivers smoothly adjust their speed to the speed restriction (Figure 7-9 (a)). For the homogeneous traffic flow scenario with \( c_{1,n-1} = 0.5 \), drivers clearly overreact to disturbances created by vehicles adjusting their speed to the speed limit, resulting in a clear negative effect on speed upstream of the speed limit (Figure 7-9 (b)).

In Figure 7-9(c),(d) and (e) we concentrate on the influence of the order of vehicles by systematically changing the order of drivers having a low and high value for \( c_{1,n-1} \) while keeping the fractions of drivers in traffic flow having low and high sensitivity values the same. In Figure 7-9(c) we control the inflow to the main road such that every time 20 vehicles...
having a value of $c_{1,n-1}$ equal to 0.5 enter the main road, after which 10 vehicles with $c_{1,n-1}$ equal to 0.33 are generated. In Figure 7-9(d) we assume a pattern of drivers in which every time a driver having a low value of $c_{1,n-1}$ is followed by 2 drivers with a high value for $c_{1,n-1}$. In Figure 7-9(e) we randomly assign the parameter $c_{1,n-1}$ to drivers based on the discrete distribution ($p(c_{1,n-1}=0.33)=1/3$ and $p(c_{1,n-1}=0.5)=2/3$).

The figures corresponding to heterogeneous traffic flows share the common characteristic that the impact of the disturbance created by the speed limit is larger than in the case with only drivers having a low sensitivity value (Figure 7-9(a)) and smaller than in the case with only drivers having a high sensitivity value (Figure 7-9(b)). This can be directly attributed to the composition of the heterogeneous traffic flows consisting of drivers having a low sensitivity value and drivers having a high sensitivity value.

However, between Figure 7-9(c), and Figure 7-9(d) a large difference between the traffic conditions upstream of the speed limit can be observed. In the case that every time 20 drivers having a high sensitivity are followed by 10 drivers having a low sensitivity, the boundary of the upstream area affected by the speed limit seems to stabilize after some period at an almost fixed position. In the case in which 2 drivers having a high sensitivity are alternated by 1 driver having a low sensitivity this boundary continuously propagates in upstream direction. The difference between the upstream traffic conditions in Figure 7-9(d), and Figure 7-9(e) in which sensitivity parameters are randomly assigned to drivers is obviously smaller. This can be understood when considering that in the case of a fully random assignment, the order of vehicles in traffic flow is most probably closer to the scenario in which 2 drivers with a high sensitivity are alternated by 1 driver with a low sensitivity, than to the scenario presented in Figure 7-9(c). Consequently, simulation results based on a random assignment will often show resemblance with the results obtained when alternating 2 drivers with high sensitivities with 1 driver with a low sensitivity.

Figure 7-9(c) and Figure 7-9(d) show that the order of vehicles can have a considerable influence on the propagation of a disturbance created by drivers lowering their speed due to a speed limit. It seems thus justified to conclude that although a lot of random simulations will often show only minor differences (see Figure 7-9(d) and Figure 7-9(e)), care needs to be taken of the stochasticity of simulation outcomes caused by differences between the orders of vehicles in different simulations.

7.5.3 Impact of heterogeneity for on-ramp scenario

In the previous case studies referring to the speed limit scenario, we showed that:

- The level of heterogeneity can considerably affect flow dynamics.
- The order of vehicles in the traffic flow is important for the way in which a disturbance propagates.

To gain insight into the generality of our conclusions, we repeat our analyses on the impacts of the level of within driving style heterogeneity and the flow composition for the on-ramp scenario while using the same parameter values as in the speed limit scenario.

Case study d: Impact of level of within driving style heterogeneity

Figure 7-10 shows the simulation results on the impact of the level of within driving style heterogeneity for the on-ramp scenario. For the sake of comparison we use the same
parameter values as for the speed limit scenario (see Table 7-8), although we limit our studies to the homogenous traffic flow case and the scenarios with standard deviations of 0.05 and 0.1.

Like for the speed limit scenario clear differences exist between scenarios having a different level of within driving style heterogeneity. Traffic conditions upstream of the bottleneck are clearly less influenced by the bottleneck when the level of within driving style heterogeneity increases. These simulation results for the on-ramp scenario thus also support the conclusion that the level of heterogeneity has a clear impact on the dynamics of predicted traffic flows.

Figure 7-10 Impact of level of within driving style heterogeneity on traffic flow dynamics for the on-ramp scenario. The parameter $c_{1,n-1}$ of the GT(3,1) model is assumed to be normally distributed, i.e. (a) $c_{1,n-1}=0.45$, (b) $c_{1,n-1} \sim N(0.45, 0.05)$, (c) $c_{1,n-1} \sim N(0.45, 0.1)$.

Case study e: Impact of order of vehicles in heterogeneous traffic flows

To examine the impact of the order of vehicles in a heterogeneous traffic flow, we repeat our simulations in which we systematically vary the order of vehicles while keeping the share of drivers having low ($c_{1,n-1}=0.33$) and high sensitivity values ($c_{1,n-1}=0.5$) the same for the on-ramp scenario.

The results are shown in Figure 7-11. Like for the speed limit scenario a considerable difference in traffic conditions upstream of the on-ramp can be observed between the simulation results referring to the homogeneous traffic flow characterized by low sensitivities (Figure 7-11(a)) and the simulation results referring to the homogeneous traffic flow characterized by high sensitivities (Figure 7-11(b)). Whereas drivers cause the disturbance to amplify in the case with $c_{1,n-1}=0.5$ no negative effects on traffic upstream of the on-ramp can be observed when we assume that $c_{1,n-1}=0.33$. 
When comparing the simulation results between heterogeneous traffic flows, we can again observe differences between the scenarios making different assumptions on the order of vehicles. For the scenario in which every time one driver having a low sensitivity value is followed by two having a high value, merging vehicles do only affect a small area around the location of the on-ramp. In the scenario in which 20 vehicles having a high sensitivity are followed by 10 drivers having a low sensitivity value the impact on traffic flow dynamics appears to be clearly larger.

Figure 7-11 Impact of flow composition on traffic flow dynamics for the on-ramp scenario in which all drivers drive according to the GT(3,1) model. Figures (a) and (b) are the references cases in which all drivers are driving respectively with $c_{1,n-1}=0.33$ and $c_{1,n-1}=0.5$. In figure (c) each time 20 drivers driving with $c_{1,n-1}=0.5$ are followed by 10 drivers with $c_{1,n-1}=0.33$. In figure (d) one driver with $c_{1,n-1}=0.33$ is followed by 2 drivers with $c_{1,n-1}=0.5$.

In the speed limit scenario the opposite effect was observed. That is, the impact of the disturbance caused by the speed limit was clearly larger for the scenario in which one vehicle having a low sensitivity value was alternated by two having a high value. A plausible explanation is the intrinsic difference between the scenario in which disturbances are externally caused by a speed limit and a scenario in which these disturbances are caused by an on-ramp. That is, in the case of a speed limit all vehicles are affected by the disturbance as they all have to lower their speed due to the new imposed speed. In the case of an on-ramp, however, not all vehicles on the main road are directly disturbed by a merging vehicle. This holds especially when the on-ramp inflow is low and the flow on the main road is high as we assume in our simulations. It therefore seems logical that the impact of a single merge is affected by the longitudinal driving behavior of the drivers directly affected by the merge.
This explanation is supported by Figure 7-12 showing the trajectories of vehicles driving at the simulated stretch of road during the time interval 1110 sec.-1200 sec.. Trajectories belonging to vehicles having a low sensitivity are represented by interrupted lines while trajectories of vehicles with a high sensitivity are represented by continuous lines. The ellipse in Figure 7-12(b) corresponding to the scenario in which 20 vehicles having a high sensitivity are followed by 10 drivers having a low sensitivity shows an example in which a vehicle coincidentally merges in a group of vehicles having a high sensitivity. The disturbance that occurs when the directly affected vehicles start to increase their distance, amplifies to a certain extent when proceeding through the upstream vehicles also having a high sensitivity.

Following the same reasoning it is also well imaginable that it matters for traffic flow dynamics whether drivers merge in front of drivers on the main road having a high sensitivity or drivers having a low sensitivity. In the latter case drivers recover their distance in a smoother way.

In case of a smaller flow on the main road also headway distributions can play a role. That is, a merge has ceteris paribus less negative influence on traffic flow on the main road when there appears to be a large headway on the main road at the moment of the merge.

The general conclusion is that also for the on-ramp scenario evidence is found supporting the conclusion that the order of vehicles in traffic flow matters for the propagation of disturbances.
Figure 7-12 Snapshot of trajectories for (a) the scenario in which 1 vehicle having a low sensitivity is followed by two vehicles having a high sensitivity value and (b) the scenario in which 20 vehicles with a high sensitivity are followed by 10 vehicles with a low sensitivity. Trajectories corresponding to vehicles having a high sensitivity are represented by continuous lines, while trajectories referring to vehicles with a low sensitivity are represented by interrupted lines.
7.6 Summary and conclusions

The microscopic trajectory based analyses presented in chapters 5 and 6 provided important new insights into longitudinal driving behavior. In chapter 5 the level of heterogeneity in real traffic has been quantified. We established among others that different longitudinal driving models are needed for adequately describing the dynamics of different driver/vehicle combinations. This type of heterogeneity is not incorporated in most microscopic simulation tools yet. In chapter 6 we provided empirical evidence for the presence of multi-anticipation in longitudinal driving behavior.

Implementing these empirical findings in microscopic simulation tools certainly increases the complexity of these tools. An important question from a microscopic simulation point of view is therefore whether and how this added complexity influences the quality of predicted traffic flow characteristics. The aim of this chapter has therefore been to explore the impacts of our empirical findings on macroscopic traffic flow predictions, i.e. the fundamental diagram, platoon stability and flow stability.

We started the chapter with a discussion on the main expected impacts of heterogeneity and multi-anticipation. As it appeared that especially little information is available on the impact of heterogeneity, we continued by performing simulations to explore the impacts of heterogeneity on respectively the fundamental diagram, platoon stability and flow stability.

Impacts of heterogeneity and multi-anticipation on fundamental diagram

With respect to the influence of heterogeneity on the fundamental diagram, we made a distinction between the influence of driving style heterogeneity and the influence of within driving style heterogeneity. Our simulations indicated that driving style heterogeneity mainly causes the shape of the fundamental diagram to change. Within driving style heterogeneity on the other hand introduces horizontal scatter to the density-speed plane, meaning that the density corresponding to a given speed becomes stochastic and depends on the composition of the sample of vehicles currently present on the roadway. In the presence of within driving style heterogeneity the fundamental diagram thus becomes a two-dimensional plane rather than a single line.

This finding that the fundamental diagram is a two-dimensional region instead of a single line is in line with the empirical observations discussed in (Kerner, 2004). Regarding the plausibility of the fundamental diagrams derived from the parameter values obtained in chapter 6, we found furthermore that the stochastic capacities obtained using the GT model were realistic (given that parameters were calibrated under congested conditions), while the stochastic capacities obtained using the Lenz model were clearly lower than observed in reality.

Based on literature, we concluded that the main expected impact of multi-anticipative car-following behavior on the fundamental diagram can be an increase in capacity compared to the situation in which drivers consider only one leader.

Impacts of heterogeneity and multi-anticipation on platoon/flow stability

Our simulations indicated that the assumed level of heterogeneity can considerably influence platoon stability. We showed, for instance, that a disturbance in the dynamics of the platoon leader amplified when propagating through a platoon characterized by a low level of heterogeneity, while it smoothed when propagating thorough a platoon characterized by a
high level of heterogeneity. How the disturbance propagated from one vehicle in the platoon to the corresponding following vehicle in the platoon, was found to be dependent on the characteristics of the following vehicle. More specific, the considered heterogeneous platoons consisted of “stabilizing” and “destabilizing” vehicles. Consequently, the propagation of the disturbance was found to be dependent on the platoon composition, i.e. the order of vehicles having different characteristics.

As flows typically consist of platoons, the simulation results on flow stability were in general in line with the results on platoon stability. That is, we showed that the impact of disturbances created by a speed-limit and an on-ramp decreased obviously when the level of within driving style heterogeneity was increased. We also found that the flow composition, i.e. the order of vehicles having different characteristics, was important for how the disturbance propagated through traffic flow.

Findings from literature supported our expectation that multi-anticipation increases the stability of traffic flows.

Conclusion

Findings from literature and our exploratory simulations show that heterogeneity and multi-anticipation are expected to influence the characteristics of simulated traffic flows considerably. Based on this chapter, we therefore strongly encourage future research in which our empirical findings on heterogeneity and multi-anticipation are incorporated in an existing microscopic simulation tool. By performing simulations with this tool for a broad range of different road configurations and comparing the corresponding simulation results to, for example, microscopic double loop detector data, it can be established whether, how and to what extent our findings improve traffic flow predictions.
8 Conclusions and recommendations for further research

8.1 Aim and structure of this chapter

The aim of this final chapter is to look back on the main achievements of this dissertation thesis. In section 8.2 a summary is given of the main research aims and the approach used to achieve these aims. This summary is followed by a discussion of the main research findings and the conclusions in respectively section 8.3 and section 8.4. Next, we discuss several practical implications of our work in section 8.5, followed by a reflection on the presented research in section 8.6. In section 8.7 we give recommendations for future research.

8.2 Summary

The driving task is a comprehensive task that consists of all tasks a driver must execute to reach his travel destination safely, comfortably, and timely. For example, a driver must keep a safe distance to the vehicle in front, follow the desired route, conform to prevailing traffic rules, use turn indicators timely, keep the vehicle on the road etc. (Minderhoud, 1999). In this dissertation thesis we concentrated on the longitudinal component of the maneuvering/control subtasks of a driver, i.e. we analyzed how drivers interact with other driver/vehicle combinations driving on the same lane.

The motivation for analyzing this particular subtask of a driver was the fact that the longitudinal driving behavior of individual drivers determines to a large extent the characteristics of traffic flow as a whole. In-depth knowledge on how drivers interact is therefore a fundamental requirement for taking successful (dynamic) traffic management measures leading to a more efficient use of existing infrastructure or to predict the effects of future changes in the infrastructure. Detailed insight into how humans execute their longitudinal driving subtask is furthermore a critical component for developing systems supporting the driver, like ACC, accepted by users.
8.2.1 Analysis of the longitudinal driving task
In our analysis of the longitudinal driving task, we started from the widely accepted hypothesis that a driver can be seen as a feedback controller, i.e. a driver monitors his current state and takes corrective actions when needed. Whether corrective actions are needed depends on the control objective of a driver. A driver may, for instance, aim at:

- Synchronizing his speed with his leader’s speed
- Keeping a desired distance to his leader
- Reaching his desired speed

The control objective is likely to be driver dependent. Different drivers do, for example, have different personal characteristics, which might be of influence to their control objective. Next to that the vehicle a driver is driving in may influence his control objective. For example, in selecting an appropriate control action a truck driver needs to take care that a heavily loaded truck can brake less severe than a person car. In actually executing a control action drivers might also differ, due to, for instance, differences between driving skills. All these hypothesized differences between the control objectives and driving skills of individual driver/vehicle combinations can cause differences between the longitudinal driving behaviors of drivers.

Given these possible causes for heterogeneity the question is, how large are the differences between the longitudinal driving behaviors of different driver/vehicle combinations in real-traffic? Is real-traffic characterized by driving style heterogeneity? In other words, do the longitudinal driving styles of driver/vehicle combinations differ inherently, i.e. do different driver/vehicle combinations react to different stimuli or use a different objective for determining an appropriate control action? Or is real-traffic characterized by within driving style heterogeneity meaning that different driver/vehicle combinations react to the same stimuli and apply the same behavioral rule, while only the extents to which stimuli influence the behavior differ?

Although almost all microscopic simulation tools take care of heterogeneity in some way and even provide the opportunity to users to decrease or increase the level of heterogeneity, no large sample trajectory based empirical investigations on the level of heterogeneity present in real-traffic were presented so far. This implies that developers of microscopic simulation tools seem to agree on the necessity of incorporating heterogeneity in simulation models, but that the level of heterogeneity assumed in these tools is not based on detailed empirical observations on the longitudinal driving dynamics of real drivers. The first aim of this thesis was therefore to use empirical observations on the dynamics of individual driver/vehicle combinations to quantify differences between the longitudinal driving behaviors of different driver/vehicle combinations. As we mentioned before that heterogeneity can have different causes we also aimed at relating the observed heterogeneity to causes, i.e. to establish to what extent heterogeneity was caused by driver characteristics and car characteristics.

The second aim of this thesis was to perform an empirical analysis on so-called multi-anticipative longitudinal driving behavior, i.e. longitudinal driving behavior in which drivers anticipate changes in the dynamics of their leader by considering the dynamics of drivers driving in front of their direct leader on the same lane. In literature several well-known longitudinal driving models were extended already to incorporate this type of behavior. Analytical and simulation-based investigations of these “multi-anticipative” models showed a clear effect of multi-anticipative behavior on flow dynamics, i.e. it has been shown that the
stability of traffic flow increases when drivers also consider leaders further downstream in determining an appropriate control action. Based on these findings, it is even suggested that drivers can compensate for their reaction time by looking further ahead. Note that this suggestion is particularly important in predicting the impact of existing autonomously operating ACC systems on traffic flow, i.e. these ACC systems have a clearly smaller reaction time than human drivers but are only able to consider the leader directly in front. Consequently, the effect of ACC on traffic flow can only be predicted based on a thorough understanding of multi-anticipative behavior of human drivers.

Despite the importance of a deep understanding of multi-anticipative behavior and the fact that well-known models have already been extended to take care of multi-anticipation, no large sample based empirical research on multi-anticipative behavior was performed so far. Even no empirical evidence for multi-anticipative behavior was provided yet. In this thesis we therefore aimed at establishing from observations, whether and how, drivers driving in front of the direct leader influence the longitudinal driving behavior of drivers.

8.2.2 Model calibration for driving task analysis

To enable large sample based empirical analyses on heterogeneity and multi-anticipation in longitudinal driving behavior, we used microscopic trajectory observations derived from helicopter images collected at two different motorway measurement sites. Both datasets were collected during the afternoon peak hour. The observed traffic conditions differed however considerably between the two measurement sites. The measurements of the ‘Everdingen site’ were characterized by stop-and-go traffic conditions, while for the ‘Waalhaven site’ congestion was quite heavy during the entire period of observation. The motivation for using this particular data collection method was that trajectories could be derived for all vehicles driving at the roadway stretch observed by the helicopter. For our analysis on heterogeneity this meant that we fulfilled the prerequisite that the driver/vehicle combinations whose longitudinal driving behaviors we compared were driving under approximately the same conditions. For our analysis on multi-anticipation, trajectories of several leaders ahead could be easily derived.

In both empirical analyses we took a model based approach meaning that we calibrated mathematical longitudinal driving models and analyzed parameter estimates and model performances to increase the insights into longitudinal driving behavior. The advantage of this approach was that our findings could be expressed quantitatively.

The empirical analysis on heterogeneity was performed by calibrating eight longitudinal driving models making different assumptions on the driving styles of drivers reflecting different driving objectives, etc.. These models made, for example, different assumptions on the speed-dependent distance headways desired by drivers and the stimuli considered by drivers. To be able to compare the longitudinal driving behaviors of individual drivers, all eight models were calibrated for all drivers separately. To identify driving style heterogeneity, we compared model performances between drivers driving at the same measurement site. To study within driving style heterogeneity, we compared estimated parameter values of drivers adopting the same driving style and driving at the same measurement site.

In analyzing the presence and degree of multi-anticipation in real-traffic, we calibrated twelve longitudinal driving models for all individual drivers separately. These twelve models were derived from the two driving styles best describing the longitudinal driving dynamics of drivers in the analysis of heterogeneity. The derived models made different assumptions on
the number of direct leaders considered by the follower. For models considering the same number of direct leaders, the considered stimuli regarding these leaders were varied. This approach provided us, firstly, with the opportunity to establish whether multi-anticipation was present in real traffic. Secondly, we could determine the number of leaders influencing the behavior of the follower. Thirdly, we were able to identify to which stimuli regarding these leaders the follower responded. By comparing model performances and estimated parameter values between drivers, we were furthermore able to identify differences between multi-anticipative longitudinal driving behaviors.

As estimated parameter values and model performances played a dominant role in our empirical analyses on longitudinal driving behavior, we thoroughly examined the properties of the applied calibration method. We performed a synthetic trajectory based study on the influences of methodological factors, like the specification of the calibration objective and the variable in the calibration objective on estimated parameter values. As real-life trajectory observations do always contain measurement noise, we repeated these analyses for synthetic trajectory observations containing seven different types of measurement errors in order to be also able to assess the influence of measurement errors on both estimated parameter values and the reliability of these values. We furthermore investigated whether it is possible to draw inferences on observed longitudinal driving behavior by calibrating a longitudinal driving model only approximating complex human longitudinal driving behavior. The motivation for this latter investigation was that the mathematical longitudinal driving models used in our empirical analyses are due to the complexity of human behavior only approximations of real behavior.

To be able to draw inferences on longitudinal driving behavior from our empirical analyses, we proposed heuristics to determine the reliability of parameter estimates and compare models of different complexities. These heuristics were needed as the proposed and examined calibration procedure had the problem of autocorrelated error terms, i.e. differences between model predictions and observations at subsequent time instants were not independent. Consequently, we could not use standard statistical tests to determine the reliability of parameter estimates and compare models of different complexities in our empirical analyses. The proposed heuristics were derived from standard statistical tests, while taking the problem of autocorrelated error terms into account.

### 8.2.3 Impacts of heterogeneity and multi-anticipation on predicted traffic flow characteristics

As an important application of longitudinal driving models is in microscopic simulation tools, we concluded our research by exploring the impacts of our empirical findings on heterogeneity and multi-anticipation on predicted macroscopic flow characteristics. That is, we examined the influences of heterogeneity and multi-anticipation on the fundamental diagram, platoon stability and flow stability. We thereto firstly performed a literature study. As this literature study showed that especially little information was available on the influences of heterogeneity, we also performed a microscopic simulation study in which we explored the impacts of heterogeneity on the fundamental diagram, platoon stability and flow stability.

For the analyses on the impacts of heterogeneity on platoon stability and flow stability, we developed a dedicated microscopic simulation tool providing us with the opportunity to vary the longitudinal driving characteristics between individual driver/vehicle combinations. To explore the impact of heterogeneity on flow stability, we considered two different scenarios,
namely a single lane road with a speed limit and single lane road with an on-ramp. For both scenarios we were particularly interested in the impact of the level of heterogeneity. That is, the larger this impact of the level of heterogeneity on traffic flow predictions, the more important it is for developers of microscopic simulation tools to determine an, for the sake of getting reliable predictions, adequate level of heterogeneity. We also considered the impact of the order of heterogeneous driver/vehicle combinations on the propagation of a disturbance.

8.3 Main findings

This section shortly summarizes the main findings presented in this thesis.

8.3.1 Main findings on heterogeneity and its influence on traffic flow predictions

In our empirical analyses on heterogeneity within the group of drivers of person cars, we showed that the driving styles of drivers differ considerably. For example, clear differences were identified between the speed-dependent distances drivers wanted to keep to the driver in front of them. For some drivers these desired distances increased approximately linearly with speed, while for other drivers a nonlinear relation turned out to be more appropriate. Also the importance person car drivers attached to actually reaching this distance appeared to be driver dependent. These behavioral differences most likely reflect differences between the control objectives of drivers. In terms of modeling, we showed that different models are needed for describing the dynamics of different driver/vehicle combinations. This conclusion applies to both measurement sites, the results of the two measurement sites differed however regarding the distributions of optimal behavioral models.

When comparing the longitudinal driving behaviors of drivers of person cars adopting a similar driving rule and driving at the same measurement site, also clear behavioral differences were identified. This applied to both measurement sites. For example, for those drivers for whom a linear relation between speed and desired distance was found to be appropriate, clear differences were observed with respect to the steepness of this relation, i.e. the desired increase in distance for a 1 m/s. increase in speed. In illustration, the coefficients of variation of the corresponding empirical parameter distributions were equal to respectively 0.69 and 0.63 for the considered measurement sites. These differences can, for example, be explained by differences between the risk-attitudes of drivers. A risk-averse driver most probably prefers a larger distance headway than a risk-prone driver.

Also other behavioral parameters showed a large variation between drivers adopting a comparable driving style. For instance, regarding the sensitivity of a driver to the difference between his own speed and his leaders speed the coefficients of variation were found to be equal to respectively, 0.97 and 0.75 for the considered measurement sites. The average coefficient of variation for both measurement sites and all behavioral parameters that could be reliably estimated, was found to be equal to 0.82. Again differences between risk-attitudes of drivers can be used to explain these behavioral differences. Also differences between the mental states of drivers are likely to cause longitudinal driving behavior heterogeneity. For example, a driver who is tired is likely to be less sensitive. Next to that differences between trip purposes of drivers can cause differences between driving behaviors.

When comparing the longitudinal driving behaviors of truck drivers and person car drivers, we found that truck drivers in general appear to drive with a more constant speed than drivers of person cars. This might be caused by the larger weight of trucks making them less maneuverable. It is also well imaginable that truck drivers can adopt a more robust driving style as they can, for example, better anticipate future traffic conditions due to a better view
and/or more driving experience. Person car drivers turned furthermore out to be more eager in restoring large deviations from their desired distance than truck drivers. When comparing the longitudinal driving behaviors of person car drivers and truck drivers adopting comparable driving styles, we found for one particular driving style (IDM) that the distance headway truck drivers assume to be safe is significantly larger than the distance headway drivers of person cars consider as safe. A plausible explanation is that in choosing an appropriate distance headway to their leader, truck drivers take care of their larger braking distance.

Our analyses on the impact of heterogeneity on predicted traffic flows, stressed the importance of our efforts in quantifying the level of heterogeneity present in real-traffic.

We showed that our finding that different drivers adopt different driving styles clearly influences the shape of the fundamental diagram. Behavioral differences between drivers adopting a comparable longitudinal driving style furthermore cause the density corresponding to a given speed to become stochastic and dependent on the composition of the sample of vehicles currently present on the roadway. In the presence of heterogeneity the congested branch of the fundamental diagram thus becomes a two-dimensional region rather than a single line. The variability of the densities for a given speed was found to be closely related to the assumed level of heterogeneity. This finding that the congested branch of the fundamental diagram is a two-dimensional region instead of a single line is in line with the empirical observations discussed in (Kerner, 2004). Regarding the plausibility of the fundamental diagrams derived from the parameter values obtained in chapter 6, we found furthermore that the stochastic capacities obtained using the GT model were realistic (given that parameters were calibrated under congested conditions), while the stochastic capacities obtained using the Lenz model were clearly lower than observed in reality.

Our simulations furthermore indicated that the assumed level of heterogeneity can considerably influence platoon stability. We showed, for instance, that a disturbance in the dynamics of the platoon leader amplified when propagating through a platoon characterized by a low level of heterogeneity, while it smoothed when propagating thorough a platoon characterized by a high level of heterogeneity.

How the disturbance propagated from one vehicle in the platoon to the corresponding following vehicle in the platoon, was found to be dependent on the characteristics of the following vehicle. More specific, the considered heterogeneous platoons consisted of “stabilizing” and “destabilizing” vehicles. Consequently, the propagation of the disturbance was found to be dependent on the platoon composition, i.e. the order of vehicles having different characteristics.

The simulation results on flow stability also showed a large influence of the assumed level of heterogeneity. Like in the analyses on platoon stability, we showed that the impact of disturbances created by a speed-limit and an on-ramp decreased obviously when the level of within driving style heterogeneity was increased. We also found that the flow composition, i.e. the order of vehicles having different characteristics, was important for how the disturbance propagated through traffic flow.

8.3.2 Main findings on multi-anticipation and its influence on traffic flow predictions
In our empirical analyses on multi-anticipation, we showed that more than half of the considered person car drivers looked further ahead than their direct leader in performing the longitudinal driving task. At least 20% appeared to even consider more than two direct
leaders. Especially the relative speed regarding direct leaders further downstream turned out to be of influence to the dynamics of the following car. The large resemblance of the results for the two measurement sites provides evidence that these findings are not strongly site specific. For instance, only minor differences were established between the percentages of drivers considering more than one leader (51% for the Everdingen site versus 55% for the Waalhaven site). The same holds for the percentages of drivers considering more than two leaders (20% for the Everdingen site versus 22% for the Waalhaven site).

Also with respect to multi-anticipative driving behavior, heterogeneity was clearly present. First of all, differences between the multi-anticipative driving styles of drivers were identified. For example, the number of considered leaders differed between drivers. This difference between the leaders considered by drivers can be caused by differences between the control objectives of drivers. We can however not disregard that also the characteristics of leading vehicles can be of influence. For example, the characteristics of the first leading car (like the car size) are likely to determine whether and to which extent a driver is able to consider his second leader. Also a second leader making a lot of unexpected movements is likely to attract more attention than a second leader driving smoothly.

Like before evidence was provided that the relation between speed and desired distance could for part of the observed person car drivers better be described as linear, while for another part a nonlinear relation was more appropriate. This finding most likely indicates differences between the control objectives of drivers.

Also differences were established between drivers adopting a comparable multi-anticipative longitudinal driving style. Thus even when a driver considers the same number of leaders and when the same relation between speed and desired distance is appropriate, differences between driving behaviors can be pointed at. In illustration, the coefficients of variation regarding the sensitivity of drivers to the speed difference with their second leader were found to be equal to respectively 1.40 and 0.89 for the considered measurement sites. This implies that the extents to which the considered leaders actually influence the longitudinal behavior of the following vehicle are strongly driver/vehicle combination dependent. The average coefficients of variation of the empirical distributions of the behavioral parameters were equal to respectively 1.13 and 0.96 for the two considered measurement sites.

Our literature study revealed that also incorporating multi-anticipative behavior in microscopic simulation tools results most likely in different predicted properties of traffic flows. The main expected impact of multi-anticipative car-following behavior on the fundamental diagram might be an increase in capacity compared to the situation in which drivers consider only one leader. Multi-anticipative behavior is furthermore expected to increase platoon and flow stability.

8.3.3 Main findings on calibration of mathematical longitudinal driving models
Based on our synthetic trajectory based analysis of the microscopic calibration process, we made our microscopic calibration procedure more robust against measurement errors. This was needed as the synthetic trajectory based analysis of the calibration method revealed a large negative influence of measurement errors on parameter estimates. More specific, the bias of estimated parameter values was found to increase considerably in the presence of measurement errors and the reliability of parameter estimates appeared to decrease. This

15 These coefficients of variation are based on the estimated parameter values for all drivers for whom the GT(2,1) longitudinal driving model performed better than the specified criterion.
negative influence of measurement errors turned out to be dependent on the variable used in
the calibration objective. We furthermore established that the variable that could best be used
in the calibration objective was model dependent.

Motivated by these findings, we investigated two measures for reducing this negative
influence of measurement errors. We showed that the bias of estimated parameters in the
presence of measurement errors could considerably be reduced by smoothing the observations
before using them for calibration. We nevertheless warned that the applied smoothing
algorithm should be such that measurement errors are removed, while the dynamics of the
observed cars are preserved. Also the use of a multi-criterion objective turned out to be
favorable for parameter estimates. Both measures were consequently applied in the empirical
analyses presented above.

We also provided evidence that the characteristics of observed longitudinal behavior of a
follower can be recovered by calibrating a car-following model, even when this model is not
the model fully describing the dynamics of the observed driver. This however, requires a
thorough understanding of the characteristics of the calibrated car-following model. This
finding was particularly important, as it validated our approach used in performing the
empirical analyses.

8.4 Conclusions

For the first time large sample based empirical analyses have been performed on
heterogeneity and multi-anticipation in longitudinal driving behavior. We did not only show
that heterogeneity is to a large extent present in real traffic, but we also quantified the extent
of heterogeneity caused by driver characteristics and the extent of heterogeneity caused by car
characteristics. We indicated that different drivers apply different driving styles, i.e. different
models are needed for describing the dynamics of different driver/vehicle combinations. As
most microscopic simulation tools assume that all drivers adopt the same driving style, while
only behavioral parameters are different, this implies that we showed that differences between
longitudinal driving behaviors of driver/vehicle combinations are in real-life even larger than
assumed in most microscopic simulation tools.

Also for the first time empirical evidence was provided for the presence of multi-anticipation
in longitudinal driving behavior. Although this behavior was often assumed in, for instance,
mathematical models describing longitudinal driving dynamics no trajectory based research
on this topic was performed yet. To avoid that we wrongly indicated drivers considering only
one leader as being multi-anticipative, we applied very strict criteria for accepting multi-
anticipative driving rules as performing significantly better than single-leader driving rules.
But even given these strict criteria more than half of the drivers was found to adopt a multi-
anticipative longitudinal driving rule. This finding stresses that multi-anticipation is an
important component of human longitudinal driving behavior requiring serious attention in
modeling longitudinal driving behavior. Multi-anticipating drivers can, for example, predict
changes in the speed of their direct leader and consequently react quicker to speed changes
from their leader than seems plausible from a behavioral point of view. This entails that a
single-leader model enorporating a from a behavioral perspective “plausible” reaction time is
likely to overestimate the time real drivers need to react to speed changes of their leader.

The explorative simulations on the impacts of heterogeneity and multi-anticipation on
predicted traffic flows showed that our findings are expected to change traffic predictions
substantially. Our simulations showed, for instance, that the assumed level of heterogeneity clearly affects simulation results. This implies that the level of heterogeneity assumed in microscopic simulation tools needs to be carefully chosen in order to get adequate simulation outcomes. Our empirical findings on the level of heterogeneity can consequently not be neglected by developers of microscopic simulation tools, although more research is needed on this topic.

From our analyses of the calibration algorithm it can be concluded that calibration is far from trivial. Although the main challenge in calibrating longitudinal driving models was for long considered to be the collection of appropriate trajectory observations, it now turns out that also microscopic calibration methods deserve serious attention. Our analyses clearly revealed that to correctly interpret parameter estimates a profound insight into factors influencing the calibration procedure is required.

8.5 Practical application perspectives

The findings on heterogeneity and multi-anticipation provide an empirical foundation for modeling longitudinal driving behavior of drivers in microscopic simulation tools. This allows for a more realistic representation of human longitudinal behavior in microscopic simulation models, possibly leading to an increase in the predictive power of these tools. Our findings can therefore support microscopic simulation tools in becoming a more realistic means to perform evaluation studies on the effects of traffic management measures and adjustments of existing infrastructures.

Secondly, the new insights into human longitudinal driving behavior contain valuable information for developers of systems supporting the driver in performing his longitudinal driving task. Due to the collision risk involved in performing this task during congested conditions, it is likely that drivers will only accept these systems when they feel comfortable with the driving style the system imposes on them. Our findings can consequently be important as they provide insights into human longitudinal behavior. For instance, in this thesis we showed that the desired time headways of drivers differ, this most probably entails that different drivers would prefer different time headway settings when driving with ACC. The findings on multi-anticipation are furthermore particularly interesting in evaluating the impacts of ACC on traffic flow properties. For example, our findings show that more than half of the considered drivers consider vehicles further downstream, while existing autonomously operating ACC systems only react to the direct leader.

Furthermore, our methodological outcomes on microscopic calibration are useful to everybody involved in calibration studies based on microscopic trajectory observations. The obtained insights are not only useful in developing more robust calibration procedures to which standard statistical tests are applicable, but also in interpreting obtained calibration results to gain new insights into longitudinal driving behavior.

8.6 Reflections on the applied research approach

In this section we reflect on the work presented in this thesis.

To perform our analyses on longitudinal driving behavior, we applied a dedicated data collection method based on helicopter observations. Using this data collection method we could collect trajectory observations providing the opportunity to perform for the first time large sample based analyses on heterogeneity and multi-anticipation in longitudinal driving
behavior. In using these trajectories derived from images taken from a more or less stationary helicopter, we nevertheless also had to deal with two disadvantages of the applied data collection method.

Firstly, only limited information was available on the characteristics of the observed driver/vehicle combinations. That is, we could distinguish person cars and trucks but no information was available on, for example, the characteristics of the drivers in the observed vehicles. For our research this implied that we could not point at the causes for driver/vehicle combination heterogeneity at such a detailed level as desired.

Secondly, the stretch of road at which driver/vehicle combinations could be observed by the more or less stationary helicopter had a length of only 400 to 500 m. Because of this the duration of observation for a single driver/vehicle combination was limited. Using these observations it was therefore not possible to consider adaptive longitudinal driving behavior, i.e. changes in the longitudinal driving behavior of a driver over time. In the analysis of multi-anticipation we furthermore had to limit the maximum number of leaders to three.

In using the trajectory observations for our analyses on longitudinal driving behavior, we assumed that drivers were only occupied with their longitudinal driving task. In reality the driving task is of course a complex task consisting of a large number of subtasks (chapter 2). A driver might, for example, also be considering to change lanes. To justify our assumption, we only considered triplets (chapter 5) and quartets (chapter 6) of consecutive vehicles for which the composition did not change during the observation. Thus we excluded triplets and quartets for which either the follower or one of the leader(s) changed lanes. Despite this effort we can, using our data collection method, not be sure that the considered drivers were indeed ‘only’ occupied with the longitudinal driving task.

In carrying out the analyses we took a result oriented perspective. That is, our main interest was on getting a better understanding of heterogeneity and multi-anticipation in longitudinal driving behavior. The thereto applied calibration procedure had the disadvantage of autocorrelated error terms making standard statistical tests for testing hypotheses on longitudinal driving behavior inapplicable. It was outside the scope of this research to develop a completely new calibration method not having this disadvantage. We therefore proposed heuristics for drawing inferences from calibration results based on knowledge from statistics and taking the problem of autocorrelated error terms in mind. Despite their theoretical foundation, these heuristics only approximate the results from statistical tests.

In using this calibration method to perform the analysis on multi-anticipation, we restricted ourselves to the influences of direct leaders. In other words, when we assumed that a follower considered three leaders, we concentrated on the three leaders driving directly in front of the follower. In reality, it might be the case that a follower adjusts his behavior after a sudden brake of a leader driving further downstream, while not considering the behaviors of all drivers in between. Thus albeit our analysis adds considerably to fundamental knowledge on multi-anticipation in longitudinal driving behavior, it is not fully complete yet.

A more philosophical question stemming from our empirical analyses is to what level of detail we should analyze differences between drivers. The answer to this question is twofold. From a scientific point of view it seems completely valid to state that additional insight into differences between the behaviors of drivers adds to fundamental knowledge on longitudinal driving behavior. From a microscopic simulation point of view it might be argued that it is in
practice undesirable to model behavioral differences at such a detailed level (for example, different models for different drivers) as we did in this thesis. Taking this perspective, it might be much more convenient to distinguish a limited number of “driver groups” having comparable characteristics. However, a correct simplification of complex reality in modeling requires a deep understanding of this complex reality. It can therefore be concluded that our results are also important for practical applications for which the level of detail at which behavioral differences are considered in this thesis is too high. That is, our results provide important insights into the longitudinal driving behavior of real drivers.

To investigate the impact of our empirical results on heterogeneity and multi-anticipation on predicted macroscopic properties of traffic flows, we performed a microscopic simulation study in which we considered a single lane road. This study showed that the properties of simulated flows change when implementing our empirical findings. We did however not analyze whether, how and to what extent predictions of more complex microscopic simulation tools also modeling other subtasks of the driver, like lane-changing, improve when including our empirical findings. This would require large scale microscopic simulation studies for a broad range of bottlenecks and circumstances in which predictions were compared to, for instance, double loop detector measurements. Such an analysis was outside the scope of this thesis. As our explorative analyses show that the properties of predicted traffic flows do, for example, depend on the assumed level of heterogeneity, we nevertheless strongly recommend performing such an analysis in the future.

8.7 Recommendations for future research

In this final section we discuss future research directions naturally following from the research described in this thesis. More specific, we describe how our empirical research on longitudinal driving behavior needs to be expanded in the future by listing several important research questions.

- How do different driver and vehicle characteristics influence longitudinal driving behavior?
  As discussed in section 8.6, we only had limited information on driver/vehicle combination characteristics at our disposal. Once such information becomes available our large sample based empirical analyses on the causes for heterogeneity need to be performed at a more detailed level to further increase the fundamental knowledge on heterogeneity in longitudinal driving behavior. Having a more detailed insight into the causes of heterogeneity is also important from a practical point of view as it could, for example, assist the microscopic simulation community in transferring the empirical findings to traffic flows having a different composition than the observed one.

- How does the longitudinal driving behavior change over time depending on, for example, weather conditions and traffic conditions?
  The topic of adaptive driving behavior deserves more attention than was possible in this thesis. As the longitudinal driving behavior of drivers determines the properties of traffic flow to a large extent, it is important to know how this behavior depends on, for example, weather conditions and traffic state. Once appropriate trajectories become available to study this aspect of longitudinal driving behavior, the dynamic parameter identification method proposed in (Hoogendoorn and Ossen, 2005) can be used as a means for detecting changes in the longitudinal driving behavior of drivers over time.
• **Is there a correlation between the longitudinal driving behavior of the following driver and the characteristics of the leading driver/vehicle combination(s)?**

In (Hoogendoorn and Bovy, 2000) analyses of microscopic double loop detector measurements revealed a possible correlation between the time headway selected by the follower and the characteristics of the leading vehicle. This finding points at a possible correlation between the longitudinal driving behavior of the follower and the characteristics of the leader. Based on our finding that most drivers are multi-anticipative, we can also state that it is plausible that the number of leaders considered by a driver is dependent on the characteristics of the leader(s). When the first leader is a truck a driver is, for example, mostly not able to consider the second leader.

The existence of such a correlation would add another dimension to our conclusion that the propagation of a disturbance through a flow of vehicles is dependent on the order of vehicles having different characteristics. That is, our conclusion referred to the importance of the order of “stabilizing” and “destabilizing” drivers. The suggested correlation between longitudinal driving behavior and the composition of the leader-follower pair would imply that the driving behavior of a driver changed depending on the prevailing order of driver/vehicle combinations.

As local measurements do have serious drawbacks in studying longitudinal driving dynamics, these analyses on the relation between the longitudinal driving behavior of the follower and the characteristics of the leader(s) need to be repeated using microscopic trajectory observations.

• **How do events on other lanes affect longitudinal driving behavior?**

In this thesis we concentrated on the reactions of drivers on changes in the dynamics of leaders driving on the same lane. The trajectory based analysis on longitudinal driving behavior presented in (Knoop et al., 2008) suggests that also events on other lanes can seriously affect longitudinal driving behavior. More specific, in (Knoop et al., 2008) it is established that an accident in the opposite driving direction (where both driving directions are physically separated) can considerably affect the longitudinal driving behaviors of drivers at unaffected lanes in the other driving direction. More research is needed to identify which events on other lanes do affect longitudinal driving behavior and how these events are of influence.

• **Is there a correlation between the longitudinal and the lateral driving behavior of a driver in congested conditions?**

It is clear that driver/vehicle combinations having a larger desired speed often pass slower driver/vehicle combinations in free flow conditions. The relation between longitudinal driving behavior and lateral driving behavior becomes however less pronounced in congested conditions. For instance, do drivers showing a more aggressive longitudinal driving behavior change lanes more quickly, or select another lane when driving in congestion? Insight into these possible correlations would, for example, be important in linking models describing longitudinal and lateral driving behavior in microscopic simulation tools.

We already shortly addressed this topic in our empirical analyses in chapter 5 where we could only point at a minor difference between the longitudinal behaviors of drivers at different lanes. This matter however deserves certainly further research on a larger scale than was possible in this thesis.
• **How is the longitudinal driving behavior of a driver affected by vehicles not driving in front of the driver on the same lane?**

Given the driving conditions considered in this thesis the longitudinal driving behavior of a driver is likely to be most influenced by leaders driving on the same lane. This does however not entail that the behavior of other vehicles does not influence the driver. For instance, a vehicle driving shortly behind the driver under concern can make the driver feel uncomfortable resulting in a change in longitudinal driving behavior.

• **How does the longitudinal driving behavior of a driver depend on the road configuration?**

In (Ter Kuile, 2006b) evidence is presented that in case of a significant decrease in lane width due to roadworks also the dynamics of vehicles on neighboring lanes become of influence to the longitudinal driving dynamics of a driver. This finding stresses that to be able to predict the impacts of traffic management measures leading to changes in the road configuration, it is important to have a deep understanding of the influence of the road configuration on longitudinal driving behavior.

• **Are there generic differences between the longitudinal driving behaviors of drivers in different countries/different regions of the same country?**

Important causes for differences between the behaviors of driver/vehicle combinations driving in different countries/regions are different formal (and informal) traffic regulations. Next to that driver characteristics might be influenced by the country/region of origin of drivers. Obtaining insight into these possible differences between the longitudinal driving behaviors of drivers driving at different locations is important in increasing fundamental knowledge on longitudinal driving behavior.

This question is also especially relevant from a microscopic simulation point of view (Tapani et al., 2008). Commercial microscopic simulation tools are often applied to perform predictions for a broad range of different countries and regions. The question is however whether microscopic models that are developed for describing the dynamics of a driver in a given country, can be simply transferred to another country? This question is especially relevant when microscopic simulations are used to study, for example, the impact of systems supporting the driver and vehicle emissions. To adequately perform such studies, in which even subtle changes in the assumptions on behavior can lead to considerably different conclusions, the behavior of driver/vehicle combinations in the microscopic simulation tool should be very realistic.

• **Can longitudinal driving models used in microscopic simulation tools be adjusted such that they can predict accidents?**

An often posed question in assessing the impact of a (dynamic) traffic management measure is how the intended measure affects traffic safety. A problem in answering this question using a microscopic simulation tool is that these tools are mostly programmed such that the simulation results do not contain collisions. Consequently indirect indicators, like the time to collision (TTC), are mostly used to predict how the measure affects traffic safety. However, as developers of microscopic simulation tools program their software such that collisions are avoided, the question is whether risky situations that could possibly result in a collision are realistically handled in microscopic simulation tools. This reasoning also indirectly questions the validity of indicators like the TTC for assessing the impact of the traffic measure on traffic safety.
Recently several researchers therefore started to explore opportunities for developing longitudinal driving models able to predict collisions. In (Hamdar and Mahmassani, 2008b) existing longitudinal driving models are adjusted such that they are no longer collision free and simulations are performed to explore the properties of these adjusted models. As the simulations resulted either in no collisions or in an unrealistically high number of collisions, it is clear that it is very difficult to obtain a longitudinal driving model realistically predicting collisions by only adjusting the parts of the model that were originally introduced for avoiding collisions.

In (Xin et al., 2008a, Xin et al., 2008b) trajectory observations referring to ten real-life crashes are used to calibrate a longitudinal driving model able to predict crashes. As this model has not been used yet in microscopic simulations, it is not possible to assess whether the predicted number of accidents is realistic and whether the accidents occur under realistic conditions. Unfortunately, these trajectory observations have only been used to calibrate a predefined longitudinal driving model. No detailed analyses are performed to identify the causes for the collisions from the observations or the circumstances under which the collisions occurred. Such an analysis could provide very useful insights needed for developing a longitudinal driving model able to realistically predict accidents.

In general, the development of a longitudinal driving model able to realistically predict accidents, would require analyses of accidents to identify at a very detailed level under which conditions these accidents occurred. Based on these analyses a stochastic longitudinal driving model could possibly be introduced, specifying, among others, for a broad range of dynamic states of the driver probability distributions representing the chance of a driver to get involved in an accident.

- Can a control action of a driver always be modeled as an instantaneous action?

In the models considered in this thesis it was implicitly assumed that a driver selected a control action by considering his state at time $t$. In reality it might be imaginable that also previous control actions and the time that a driver is already driving in a “comparable state” play a role.

In (Hamdar and Mahmassani, 2008a) therefore a new approach for modeling car-following behavior is explored in which driving behavior is modeled as a sequence of “car-following” and “free-driving” episodes. The probability that a driver is willing to end such an episode is assumed to be dependent on the current duration of the episode and the current state of the driver. It can, for example, be that a driver becomes more impatient as he is following the same driver for prolonged time. Accordingly the driver will be increasingly considering terminating the corresponding “car-following” episode. Also the opposite is imaginable, i.e. after following the same leader for some time the driver gets used to his leader and does not aim at ending the “car-following” episode anymore.

Only very limited attention has been paid to this dependence of the control action selected by a driver at time $t$ on previous control actions and the time that a driver is already driving in a “comparable state”. This topic deserves clearly more research.
References


Appendix A  Description of helicopter based trajectory data collection method

A.1  Introduction

The trajectory data used in this thesis have been collected using a new data collection approach based on remote sensing (Hoogendoorn et al., 2004). Raw data consist of image sequences taken by a digital camera that is attached to a helicopter. After collecting the raw data all image frames are input to dedicated software, which is (after preprocessing) able to detect vehicles in pictures and to track them within consecutive pictures, resulting in trajectory data. The trajectories resulting from the software are checked manually.

Figure A-1 Schematic overview of the process for deriving trajectory data from a sequence of images.
The aim of this appendix is to provide insight into the applied process of deriving trajectories from an image sequence. A schematic overview of this process is given in Figure A-1. Steps 1 to 4 of this scheme will consecutively be handled in separate subsections. This appendix is, apart from the smoothing method, based on (Hoogendoorn and Schreuder, 2005).

### A.2 Preprocessing

The aim of step 1 of the procedure is to ‘standardize’ the images. This implies among others that the light-intensities in all the images need to be comparable, that all images refer to the same plane of observation and that all images give the same orientation.

To this end, the following techniques are used respectively:

- **Radiometric correction** ensures that the effects of changing lighting conditions (e.g. due to clouds, etc.) are reduced. In (Hoogendoorn et al., 2003) radiometric correction is discussed in detail.
- **Lens distortion correction** removes the so-called pincushion effect from the images. Even though this effect seems only moderate, removing it correctly is crucial to ensure correct stabilization.
- **Stabilization** of the images effectively corrects for the movement of the helicopter (changes in height, rotation, and so on).

#### A.2.1 Lens calibration and lens distortion correction

The lens of the applied camera yields a distortion that is commonly referred to as pincushion distortion. This effect is caused by the fact that light going through the lens bends in a certain way.

The distortion of the lens can adequately be described by a couple of parameters. To correct for the distortion, ideally these parameters are known so that the effect can be inverted. While in most cases standard parameter values are used, in the research presented here a dedicated effort was made to determine the parameters of the lens which were used to correct the data. This calibration revealed that the standard values used for lens distortion correction were considerably different from the calibrated values. As a result, the images could be corrected much more effectively and the resulting photogrammetric operations eventually resulting in vehicle detection and tracking could be performed more accurately.

#### A.2.2 Stabilization

In an aerial image of a rectangular object, the image of the object will only be rectangular (after correcting for lens distortion) if the camera is located exactly above the middle of the rectangle. Otherwise, the image will be distorted. The extent of which depends on the location and the rotations of the camera. On top of this, the size of the observed rectangle will depend on the height at which the images are collected.

During stabilization, projective transformation is removed such that the objects on the image are projected at the same location as the same objects in an extended reference image $R$. This reference image $R$ is determined from different images that are collected at different time instants. As such it reflects a larger part of the roadway than is observed in the individual images. These images are stitched together using dedicated image processing software to form the reference image.
Figure A-2 illustrates the large extent to which the helicopter is moving during an observation stressing the importance of this step of the procedure.

![Figure A-2 Example reflecting movement of helicopter and effect on collected images. The time gap between the two shown images equals 6 seconds.](image)

The stabilization process can be subdivided into two parts, i.e. coarse matching and fine matching. Both parts rely on so called control points, which are points in the image that are visible in both the reference image $R$ and the processed image $I$.

In coarse matching characteristic control points are used. These points are more or less unique for the entire image and can thus be used to match the images where the amount of perspective distortion is large (i.e. the objects on the processed image and the objects on the reference image are far apart). This is achieved using an iterative approach that maximizes the cross-correlation coefficient of pixels around the control points in the characteristic sets. The search window contains 50x50 pixels. The accuracy of coarse matching is approximately 1 pixel. Typical characteristic control points are gantries and lanterns.

In fine matching, road surface control points are used. These points are not unique and are used for fine matching only. Fine matching uses the same approach as coarse matching except from the differences that no unique points are used and that the search window is only 7x7 pixels large.

### A.3 Determining background image

A first step towards detecting vehicles is to determine a background image $B$. This is done based on the assumption that the road will be empty most of the time. Considering the median intensity of a pixel in a sequence of images thus reflects the intensity of the road surface, i.e. the background at that particular location. The final aim is thus to remove all moving objects from the scene. An example of a background image is provided in Figure A-3.

![Figure A-3 Example of a background image.](image)
Due to the movement of the helicopter and the use of an extended reference image, determination of the background image is done dynamically \((B_I)\). This implies that the approach described above is applied to a set of stabilized images around the current image \(I\). These images will approximately show the same part of the roadway and can thus be used to determine the background image \(B_I\) for image \(I\).

### A.4 Vehicle detection and tracking

Having determined the background images \(B_I\), the next step of the procedure is the detection and tracking of vehicles. During vehicle detection, the location of the vehicle and its dimensions are determined. Vehicle tracking entails recognizing a specific vehicle as being the same one in consecutive images. The two steps are described in more detail in this subsection.

#### A.4.1 Vehicle detection

For any image, vehicle detection is based on the difference between the current image \(I\) and the background image \(B_I\). A first heuristic is to use a threshold value to decide whether a pixel represents a vehicle or not. If so, neighboring pixels can either be identified as a vehicle or not. In practice, a number of complicating factors will occur:

- Both light and dark vehicles will cast shadows, which are generally darker than the roadway surface.
- Vehicles can have (approximately) the same intensity as the roadway
- Light vehicles have dark spots (windshields, etc.).
- On occasion, a small vehicle completely drives in the shadow of a large vehicle (truck or bus).

Different approaches have been implemented to resolve these issues (morphological grayscale operations, binary morphological operations, split and merge image segmentation, etc.). More details can be found in (Hoogendoorn et al., 2003).

#### A.4.2 Vehicle tracking

The aim of vehicle tracking is to follow the vehicles detected in an image, i.e. to determine their position in the other images. In most cases, tracking is done using an approach similar to the control-point approach used in the stabilization step (using both coarse matching and fine matching). Its application yields a unique label for all vehicles detected during the vehicle detection step, enabling determination of the vehicle trajectories. To improve the accuracy, the original subimage of the detected vehicle (i.e. determined during detection) as well as its subimage in the previous image \(I-1\) were jointly used to determine the position of the vehicle in the current image \(I\).

Vehicle detection is performed for each frame \(I\), that is, at each 10\(^{th}\) of a second. Vehicle tracking entails both forward and backward tracking of all detected vehicles in the next/previous 10 images. As a result, in each image, a vehicle may result in multiple data points: one from vehicle detection (given it is detected), and multiple data points from tracking. Redundant data is used to improve the accuracy of the traffic data.
Figure A-4 Vehicle tracking in four subsequent images. The lines behind the vehicles are indicative for the speed of the vehicles.

A.5 Data post-processing

In the upcoming section it will be explained how these detection and tracking results are used to obtain the trajectory data used in the analyses performed in this thesis.

A.5.1 Handling data redundancy

In theory, for each vehicle 21 data points are determined in the previous steps: one from detection, 20 from forward and backward tracking. Due to missing detection, in practical situations this number may be substantially less. Using dedicated cluster analysis techniques all available points are clustered, identifying eventual outliers. For the remaining points, the median value is determined and considered to be the best estimate for the vehicle position (lateral and longitudinal). For the vehicle dimensions, it turns out that the best estimate is the 85th percentile width and length measurement.

A.5.2 Data checking and correcting

A dedicated tool has been developed allowing the user to check the resulting detection and tracking results. False detections and incorrect tracking can be removed. The tool also offers the user the opportunity to manually detect vehicles. The software in turn takes care of the required vehicle tracking of the manually detected vehicles. Figure A-5 shows a screenshot of
the tool that has been developed to check, correct and complete the automatically collected dataset.

Practical applications show that on average 95% of all vehicles are detected and tracked automatically. This number depends on the quality of the images, the weather conditions during observation, the height of flying etc.

![Figure A-5 Viewer developed for checking and completing the automatically derived trajectory data.](image)

A.5.3 Georeferencing
Georeferencing entails transforming the data coordinates to real-life coordinates. This is roughly stated achieved by indicating in the stabilized images 4 points of which the real-life coordinates are known. Using the resulting two sets of coordinates, transformation parameters can be calculated that translate the data coordinates into real-life coordinates.

This step also enables to merge together different datasets collected at consecutive time periods afterwards.

A.5.4 Smoothing data
To post-process the data an adjusted from of the method proposed in (Toledo et al., 2007) is used. This method applies \textit{locally weighted} regression for smoothing vehicle trajectories. The term \textit{local} indicates in this setting that to estimate $x_{n\text{meas.}}(t)$ from observations of the positions of vehicle $n$, lets say $x_{n\text{noisy}}$, only observations are used belonging to a specified time window centered at $t$. \textit{Weighted} implies that in performing the regression different weights are assigned to the different observations.
Both these characteristics make the method particularly suited for the problem at hand. The local aspect has the advantage that it is often sufficient to fit a relatively simple function, like a low degree polynomial, to the observations in the specified time window while still being able to catch the dynamics sufficiently. Sufficient implies for the current problem that speeds that are derived from the x-positions later on reflect correctly the dynamics of the observed driver. This requirement is on its turn supported by assigning weights to observations. That is, observations closer (in time) to the time \( t \) for which the smoothed value is determined, contain naturally the most important information about the dynamics of the driver at time \( t \). In the applied weighted regression the weights therefore decrease when the distance (in time) between the observation and \( t \) increases.

Mathematically the applied locally weighted regression can be written as:

\[
\beta_{t_0}^* = \arg \min_{\beta_{t_0}} \left\{ \sum_{t \in \text{window}} w_{t_0}(t)\left( x_{n_{\text{meas}}}(t) - f(t, \beta_{t_0}) \right)^2 \right\}
\]  

(A.1)

where,  
\( t \) = time step id  
\( t_0 \) = time step id for which \( x_{n_{\text{meas}}}(t) \) is determined  
\( N \) = total number of surrounding observations considered in determining \( x_{n_{\text{meas}}}(t) \). This number is assumed to be odd.  
\( \beta_{t_0}^* \) = optimal parameters, the index \( t_0 \) indicates that these parameters are dependent on \( t_0 \)  
\( w_{t_0}(t) \) = weight assigned to time step \( t \), the index \( t_0 \) indicates that these parameters are dependent on \( t_0 \)  
\( f \) = local regression function

The weights are assigned according to the following tri-cube weight function:

\[
w(t_0, t) = \left(1 - u(t_0, t)^3\right)^3
\]  

(A.2)

In this formula \( u(t_0, t) \) is a normalized measure of the time difference between \( t \) and \( t_0 \) given by:

\[
u(t_0, t) = \frac{|t - t_0|}{d}
\]  

(A.3)

The value \( d \) is equal to the distance (in time) between \( t_0 \) and the nearest \( t \) not used in determining \( x_{n_{\text{meas}}}(t) \). Eq. (A.3) thus ensures that the further \( t \) from \( t_0 \) the smaller the weight the corresponding observation gets in the locally weighted regression.

So far the applied method is in line with the method proposed in (Toledo et al., 2007). The method applied in this thesis differs however regarding the choice for the regression function and the time window. These changes are necessary due to the smaller time intervals between consecutive observations in the datasets applied in this thesis than in the datasets used in (Toledo et al., 2007): 0.1 s. versus 1 s..
This first of all affects \( N \), i.e. the number of observations used in determining \( x_n^{\text{meas}}(t) \). In (Toledo et al., 2007) window sizes of \( N = 7-13 \) are considered, referring to time windows of 6-12 sec. As the time steps are in our case much smaller, \( N \) was decided to be equal to 35, i.e. 3.4 s. The larger value of \( N \) is needed because data need to be smoothed relatively more in order to keep the opportunity to later on derive useful estimates for the speeds from the data.

In (Toledo et al., 2007) furthermore polynomials up to order \( M \leq N-1 \) are considered. In this thesis only first order polynomials will be fitted to the observations. This choice is motivated by the smaller length of the time windows. When time periods of more than 10 s. are considered an important part of the dynamics present in the noisy data can be attributed to driving dynamics, i.e. dynamics that need to be preserved. This holds even more when the relatively large time steps of 1 s. are considered. In 1 s. clearly a larger distance is traveled than in 0.1 s. (ceteris paribus). This requires the use of larger order polynomials. Because of the high frequency with which data are collected in our case and the related smaller time windows such high order polynomials would be clearly inappropriate. A practical study of the observed data, smoothed data, and derived speeds showed that first order polynomials sufficed.

Figure A-6 provides an example of the trajectory data used throughout this thesis.

![Figure A-6 Part of the available trajectory data for the Everdingen measurement site.](image)

A.6 Current developments

Currently research is going on to refine the data collection method described in this appendix further and to increase the speed of the underlying algorithms.

The research described in (Gorte et al., 2005) aims at extracting the freeway under consideration from its surroundings having among others the advantage that the search space for detecting vehicles can be limited. In (Karimi Nejadasl et al., 2006a, Karimi Nejadasl et al., 2006b) an optical flow based method is suggested for improving the robustness of the tracking results in the case of for example cars having a low contrast with the road. Next to that effort is spent in increasing the robustness of the stabilization process and its further automatization.
Appendix B  Discussion of approaches used for calibrating longitudinal driving models

B.1 Introduction

An important component of this thesis is calibrating longitudinal driving models by matching simulated and observed dynamics of individual drivers. A lot of other approaches for calibrating longitudinal driving models using a broad range of different types of observations can be found in literature. In this appendix we aim at providing insight into these different methods for calibrating longitudinal driving models. In section B.2 we start by distinguishing different types of observations that can be used to calibrate longitudinal driving models. We then continue in section B.3 by discussing several in literature frequently used approaches for calibrating longitudinal driving models. In section B.4 we discuss which calibration approaches can be applied using which kind of observations.

B.2 Overview of different types of observations

Longitudinal driving models can be calibrated using completely different types of data. This is nicely illustrated in (Brockfeld and Wagner, 2006) describing calibration studies using respectively,

- Travel times of individual cars (Brockfeld et al., 2003).
- Aggregated double loop detector data collected at 3 detector stations positioned along a 1 km, 5 lane stretch (Brockfeld et al., 2005).
- Microscopic trajectory data of a platoon of ten vehicles driving at a test-track (Brockfeld et al., 2004).

In order to give a complete overview of possible types of observations that can be used for calibration two different properties of observations need to be considered (Figure B-1).
Figure B-1 Overview of main properties of observations.

The first one refers to the type of observation and focuses on the question: what is measured (see also (Bleile, 1999))? In this context a distinction needs to be made between local observations, instantaneous observations and time-space observations (Figure B-1). These different types of observations are defined as follows:

- **Local observation**: measurements are performed at a fixed measuring point \( x_i \).
- **Instantaneous observation**: measurements are performed at a fixed moment in time \( t_i \).
- **Time-space observation**: measurements covering a finite distance \( \Delta x \) during a finite time interval \( \Delta t \).

The double loop detector data applied in (Brockfeld et al., 2005) provide a typical example of a local observation, while the travel times and trajectory data of respectively (Brockfeld et al., 2003, Brockfeld et al., 2004) are instances of time-space measurements. Information on vehicles derived from a single image made by a camera attached to a helicopter can finally be considered as an instantaneous observation.

These different types of observations are further illustrated in Figure B-2.

The second criterion relates to the level of detail of observations. The highest level of detail that can be reached is that data are available on the individual driver level. Data satisfying this requirement are called *microscopic*. On the other hand data can also be aggregated over, for example, a specific time interval (local observation) or a specific stretch of the roadway (instantaneous observation) such data are referred to as *macroscopic* data.

In the example provided before the travel times and the trajectory data are microscopic, while the aggregated double loop detector data are macroscopic observations.
Figure B-2 Distinction between instantaneous, local and time-space measurements. The interrupted line shows an example of a trajectory, i.e. the data type used in this thesis.

To illustrate these two characteristic criteria of data further, Table B-1 provides for each possible combination an example of a well-known traffic variable.

<table>
<thead>
<tr>
<th>Level of detail</th>
<th>Microscopic</th>
<th>Macroscopic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous</td>
<td>time headway</td>
<td>flow</td>
</tr>
<tr>
<td>Local</td>
<td>distance headway</td>
<td>density</td>
</tr>
<tr>
<td>Time-space</td>
<td>trajectory</td>
<td>Average travel time for a group of vehicles</td>
</tr>
</tbody>
</table>

B.3 Overview of broad range of approaches used for calibrating longitudinal driving models

In the previous section, we gave an overview of different types of data that can be used to calibrate longitudinal driving models. We now continue by discussing several commonly used approaches for calibrating longitudinal driving models.

B.3.1 Matching fundamental diagrams
One way of calibrating longitudinal driving models is to consider steady state conditions, i.e. the fundamental diagram. This approach applies the knowledge that under steady state conditions most car-following models yield a relationship between the vehicle speed and the distance headway to the leader (Rakha and Crowther, 2003) that can easily be translated to a fundamental diagram.

In practice this approach proceeds as follows. First an empirical fundamental diagram is derived from empirical data. Then the parameters of the longitudinal driving model are tuned...
such that the corresponding fundamental diagram resembles the empirical one as closely as possible. As the fundamental diagrams can often be derived analytically this calibration approach is computationally clearly less demanding than the simulation based approaches we will consider in the next subsection.

Examples of such steady state approaches assuming completely homogeneous drivers are provided in (Rakha and Arafeh, 2007, Rakha et al., 2007). In (Punzo and Tripodi, 2007) a comparable approach is proposed with the difference that care is taken of heterogeneity in that cars and buses or trucks are distinguished. In (Farzaneh and Rakha, 2006) only the uncongested branch of the fundamental diagram is considered. Also here heterogeneity is taken into account as one of the parameters calibrated in this case, is the spread in the desired speed of drivers.

By only considering steady state conditions clearly not all parameters of dynamic longitudinal driving models can be calibrated. Reaction times can, for example, not be calibrated using this approach. This restriction makes this approach unsuitable for calibrating model parameters of a microscopic simulation tool aiming at predicting traffic flow dynamics.

B.3.2 Matching local dynamics using microscopic simulations
Another frequently used method for calibrating longitudinal driving models is to perform microscopic simulations. In this approach the measurement site is mimicked in a simulation tool and the observed traffic situation is compared to the simulated one. Although we will focus below at the verification of longitudinal driving models only, this method can also be applied to verify more complex microscopic simulation tools containing for example also models for lane changing, route choice, etc. Examples of such studies are (Gomes et al., 2004, Toledo et al., 2004, Ciuffo et al., 2007).

To verify car-following models using microscopic simulations two approaches can be distinguished, deterministic ones and stochastic ones. First the deterministic approach will be illustrated and then the stochastic approach will be discussed.

Calibration of longitudinal driving models based on deterministic microscopic simulations
In a deterministic approach microscopic simulations do not contain stochastic elements. Suppose, for example that empirical detector data are available for three neighboring detectors. Then a deterministic approach could have the following set up (Figure B-3):

- **Inflow regulation**, measurements from detector one could be used for inserting new vehicles in the simulation. Although care has to be taken at high flows as it needs to be avoided that vehicles are inserted at smaller distances than their desired ones as this creates a disturbance at the upstream boundary.
- **Outflow regulation**, speed measurements from detector three could be used to make sure that vehicles at the end of the simulated stretch have a speed equal to the measured one (Brockfeld et al., 2005). In this way for example congestion spilling back from downstream of the measurement site into the measurement site can be handled.
- **Comparison of simulated and real traffic dynamics**, measurements of detector two could be used to compare real and simulated traffic dynamics.

The term deterministic would in such simulations imply that all simulations using the same inflow and outflow data would result in the same simulation output. Calibration then entails that after every simulation run, parameters are tuned such that the difference between the real data and the simulated data becomes smaller. An important implication of the deterministic set up of the simulation is that the assignment of car-following parameters/characteristics to simulated cars should be done in a deterministic way.

A problem that needs to be considered in applying this approach for the separate verification of longitudinal driving models is that most real freeways consist of more than one lane. This implies that car drivers are not only occupied with performing their longitudinal driving tasks as assumed by longitudinal driving models, but that they also can change lanes when they, for example, want to improve on their current condition.

This problem is for example faced in (Brockfeld et al., 2005) were data of a five lane freeway are used for the separate verification of car-following models. To deal with this it is proposed to simplify the multi-lane reality by performing single-lane simulations. To do so the total flow at the detector used as input to the simulation is simply divided by the number of lanes.

Another way to deal with the problem could be to use measurements per lane as input to the simulations instead of dividing the total flow by the number of lanes, while still ignoring lane changes. In this approach at least care is taken of asymmetric lane distributions.

Of course it is possible to also include lane-changing in the microscopic simulations. Then it needs however to be considered that the lane-changing model also influences the simulation outcomes and that it thus becomes much more difficult to point at the cause(s) for deviations of the simulation outcomes from the measurements.

The importance of this multi-lane problem in calibrating longitudinal driving models might on the other hand be moderated by arguing that it is from a modeling perspective most challenging to model the longitudinal driving behavior in congestion. Under congested conditions often less lane changes will be observed as it becomes more difficult to change lanes.

**Calibration of longitudinal driving models based on stochastic microscopic simulations**
When microscopic simulations do contain stochastic elements, for example, when parameter values assigned to drivers are drawn from a probability distribution, calibration becomes even more difficult. Different simulation runs result in different simulation outcomes. Thus even in the hypothetical case that the models in the simulation tool represent reality completely correct and that the real parameter distributions are completely known, the empirical data represent just one particular random outcome. Calibration requires in this case thus multiple simulation runs.
Another influence of this can be that the same random outcome can be obtained using different parameter distributions. For example, it is possible that the same parameter values are assigned to drivers in different simulations even when different distributions are applied. Figure B-4 is meant to illustrate this in an intuitive way. Suppose that a longitudinal driving model is used having only one parameter. The pdf (probability density function) from which this parameter is drawn is in the first simulation different than in the second simulation. Despite this difference it is still possible that parameter $\alpha$ is assigned to the same driver in simulation one and in simulation two although the corresponding probabilities are different.

A way to handle this problem could be, for example, to identify the parameter distribution for which the empirical data are most likely. This problem of calibration of stochastic microscopic simulation tools is discussed in (Tian et al., 2002, Bayarri et al., 2004, Schultz and Rilett, 2004).

![Parameter value vs. probability density function](image)

**Figure B-4** Illustration explaining in an intuitive way that simulations based on different parameter distributions can result in different outcomes.

### B.3.3 Matching trajectories of individual vehicles

In the previously discussed microscopic simulation approaches, local measurements were used to calibrate longitudinal driving models. Using local measurements it is assessed whether the overall simulated traffic situation at the measurement location is in line with the measurements. Thus to check, for example, whether in the simulations congestion occurs during the same time period(s) as in the observations and whether the type of congestion is comparable (for overviews of different types of congestion see (Treiber et al., 2000, Kerner, 2004, Schönhof and Helbing, 2007)). We showed that this is often far from trivial as data, for example, often refer to real roads having multiple lanes such that it is difficult to consider only longitudinal driving behavior.

In a calibration study based on microscopic trajectory data, however, the ability of the model in predicting the longitudinal behavior of an individual driver is analyzed. A typical way of verifying a model in this way is to simulate the movements of the follower based on the measured data of the leader(s), a longitudinal driving model and the corresponding parameter values and to compare these simulated movements to the measured movements. Thus in calibrating a microscopic model using microscopic trajectory data the focus is in fact on the original aim of the model, namely its ability to mimic the behavior of a real driver.

Figure B-5 shows an example of the first step and the last step of a calibration process in which the parameter values of a car-following model are tuned such that they are able to mimic the observed longitudinal dynamics of the follower.
Assessing microscopic models on this detailed level can among others be useful in increasing the transferability of calibrated microscopic models to for example other measurement sites. For instance, it does not necessarily have to hold that a microscopic simulation tool that is able to satisfactorily match the traffic dynamics as measured by double loop detectors to simulation outcomes for a particular dataset, is making correct assumptions about the longitudinal driving behavior of individual drivers. Such a shortcoming can lead to an increased need for recalibrating the model each time that it is used in slightly different conditions.

Another motivation for choosing a microscopic approach for verifying car-following models is that it is in fact a lot easier to focus on the longitudinal driving task only. As the state of the considered vehicle (car-following versus lane-changing or preparing a lane-change) can often be derived from the observations those parts of the trajectories can be selected in which the driver is mainly occupied with his longitudinal driving task. Thus in this way data from multi-lane roads are clearly less problematic than in the microscopic simulation based approaches discussed before.

**B.4 Which types of observations are appropriate for which calibration approaches?**

The fundamental diagram calibration approach can be applied to all observations that can be used to compose empirical fundamental diagrams. For example, macroscopic local observations are often used to derive empirical fundamental diagrams although care needs to be taken of the fact that local measurements provide the time mean speed instead of the space mean speed. As microscopic local measurements can be aggregated to macroscopic local measurements, microscopic local measurements are suited as well. Also the microscopic
trajectory data used throughout this thesis referring to the behavior of all drivers on a fixed roadway stretch can of course be used to produce an empirical fundamental diagram.

The calibration approaches based on microscopic simulation studies can be designed such that they are suitable for all types of data. For example, when aggregated double loop detector data are available the calibration procedure has the objective to minimize the difference between measured and simulated aggregated double loop detector data. The same objective can be selected when microscopic double loop detector data are available, although it is also possible in this case to define an objective minimizing the gap between simulated and real traffic variables of individual vehicles. When travel times of vehicles are available the objective will focus on travel times.

Calibration of longitudinal driving models by matching observed and simulated dynamics of individual drivers is only possible when microscopic trajectory data are available. Only these microscopic trajectory data contain the thereto required highly detailed information on the dynamics of individual vehicles.
Appendix C  Description and comparison of trajectory data collection methods

C.1  Introduction

Despite the importance of calibrating and verifying longitudinal driving models using microscopic trajectory data, these tasks have been mostly performed using local measurements. The reason for this is that it is technically more demanding to collect detailed data having a high spatial and temporal resolution for a moving car than to collect local measurements. For long microscopic trajectory datasets have therefore been very scarcely available (this issue is, for example, discussed in (Brackstone and McDonald, 1996)). Some examples of early approaches for collecting microscopic data are the following.

In (Chandler et al., 1958) microscopic data have been collected for a following car driving on a test track at the General Motors Technical Center. In these experiments a wire was fastened to the rear bumper of the lead car and the front bumper of the following car in order to measure the distance between the cars. Also in (Rockwell et al., 1968) wire linked vehicles were used, these experiments were performed in both open road and urban expressway traffic.

In (Treiterer and Myers, 1974) data have been collected using a camera mounted in a helicopter. This helicopter followed a platoon of approximately 70 vehicles for 238 seconds in which approximately 5.3 kilometer was traveled. For the obtained pictures, taken at 1 second intervals, software was used to obtain a visual description of vehicle positions. Trajectories and afterwards speeds were derived from these data.

Recent technological developments have been an important stimulus in creating and improving several microscopic data collection methods. Distances between cars can now, for example, be measured using radar. Alternatively they can be derived from the x-positions of two consecutive cars equipped with GPS (Global Position System) receivers. Next to that automated remote sensing data collection methods become more widely available.
As these few examples already show the diversity of these modern approaches and as trajectory data play an important role in this thesis, we give in section C.2 an overview of contemporary microscopic data collection methods. In section C.3 we compare these methods regarding their usefulness in studying several different aspects of longitudinal driving behavior. This comparison is particularly useful in selecting an appropriate data collection method for a certain research question. Next to that it is also a helpful tool in improving the mentioned existing microscopic data collection methods.

To complete the overview on methods for collecting microscopic trajectory data, we also discuss the advantages and disadvantages of trajectory data collected in real traffic conditions and at test-tracks (section C.4).

C.2 Description of trajectory data collection approaches

Microscopic data collection methods can be roughly divided into two groups. Methods belonging to the first group perform field experiments to collect data, while methods belonging to the second group use a driving simulator to obtain data about the driving behavior of individuals. The group of field experiments can on its turn be subdivided into two different approaches. The first approach makes use of especially equipped cars being part of the traffic flow, while the second approach applies aerial observation methods in which data are collected from a position above the road. These three types of data collection methods will now be explained in more detail.

C.2.1 Field experiments: Instrumented vehicles

In general two methods can be distinguished for collecting data with instrumented vehicles. In the first method a vehicle is equipped with several devices measuring for example the distance between the vehicle itself and directly surrounding vehicles. The other type of methods employs GPS to monitor vehicles equipped with GPS data collection systems.

Equipped vehicle monitors surrounding vehicles

An illustrative example of a vehicle collecting data of surrounding vehicles is described in (Brackstone et al., 1999). The instrumented vehicle is in this study equipped with an optical speedometer and a radar rangefinder, i.e., a microwave radar unit capable of measuring the distance to and relative speed of, a number of immediately adjacent vehicles. Next to these measurement devices also a Video-Audio Monitoring System is installed in the car, making it for example possible to gain insights into macroscopic features undetectable for the detectors and to record comments made by drivers.

In collecting data in this way it is important to distinguish data collected in the active mode and data collected in the passive mode (Brackstone et al., 1999). When the behavior of a test driver within the instrumented vehicle is examined the active mode is applied, when the behavior of surrounding vehicles is studied the passive mode is adopted.

An important technological difference between both modes is that it can generally be stated that measurements are most precise for the instrumented vehicle itself, favoring the active mode. The accelerations of the surrounding vehicles, for example, often need to be derived from knowledge about the relative speed and the distance between the cars. A serious advantage of the passive mode is however that drivers can be monitored without being aware of taking part in an experiment. An example in which the passive mode is applied is described in (Kim et al., 2007).
Applications of GPS

The aforementioned impact of recent technological developments on collecting microscopic trajectory data becomes very clear when the second group of methods of collecting data via equipped cars is considered. In these methods the developments in the area of GPS are exploited. These methods all have in common that movements of vehicles equipped with GPS receivers are saved.

In (Hatipkarasulu et al., 2000) an exploration is made of the usefulness of DGPS (Differential GPS) in collecting microscopic data. Two vehicles were hereto equipped with GPS receivers, a differential correction unit and a notebook computer primarily used for data storage. The paper concludes that the employed techniques make it possible to inexpensively and accurately collect vehicle position and speed information under real traffic conditions.

In subsequent years the advantages of GPS have been recognized and utilized by more and more researchers. In (Gurusinghe et al., 2002) data are collected for a platoon of ten vehicles driving on a test-track in Japan applying RTK GPS (Real-Time Kinematic GPS). In these experiments the platoon leader was instructed to change his speed in line with several predefined patterns.

Apart from this application of GPS to collect data for drivers driving on a test-track, also other measurements are performed using comparable data collection methods collecting data for drivers driving in real traffic. For example, in (Punzo et al., 2005) an approach is described in which data are collected for a platoon of four vehicles driving in real traffic on urban as well as on extra-urban roads in Naples, Italy. A practical constraint of applying GPS in real traffic conditions is that special care needs to be taken in avoiding the intrusion of other cars in the platoon, as only data are available for the cars equipped with GPS receivers.

Another example of a comparable study in which data are collected in real traffic is (Basnayake and Lachapelle, 2006). More specifically, data are collected over two days, at the first day a platoon consisting of two vehicles is monitored at the second day data are collected for a platoon of three vehicles.

C.2.2 Field experiments: Aerial observation

All methods mentioned before share the common characteristic that data are collected via equipped cars being part of the traffic. Another approach is to observe traffic flow from a position above the road and to afterwards derive the trajectories of individual vehicles being part of that flow. This approach is taken in (Hoogendoorn et al., 2003, Hoogendoorn and Schreuder, 2005) and NGSIM (Next Generation Simulation (Kovvali et al., 2007, U.S. Department of Transportation- Federal Highway Administration, 2008)).

In (Hoogendoorn et al., 2003, Hoogendoorn and Schreuder, 2005) raw data are collected by a digital camera attached to a helicopter instructed to fly at a fixed position above the freeway. Dedicated software is used afterwards to derive the required microscopic trajectory data from these pictures. This approach is applied to collect the data used throughout this thesis.

The NGSIM approach differs from this helicopter approach in that camera’s are installed on the top of high buildings. The technological advantage of this approach above the helicopter approach is that buildings do not move themselves, so images need not to be corrected for movements of the platform to which the camera is fixed. A practical disadvantage is however that large enough buildings have to be available next to interesting measurement locations. In
the Netherlands this is for example not the case. Whenever buildings are too low the problem of occlusion occurs quite often as person cars driving on inner lanes are hid behind trucks driving on the outer lane.

C.2.3 Driving simulator
All aforementioned methods have in common that data are collected for cars that are physically driving either on the road or at a test-track. Another approach is to collect data for a driver driving in a simulated environment using a driving simulator. It has to be noted in this light that the term driving simulator is in fact used for all applications in which a driver is driving in some way in a simulated environment and the term is thus very broad. In (Kaptein et al., 1996) therefore three different types of simulators are distinguished, from very simple simulators to really advanced ones:

- Low-level simulators typically consist of a PC or graphics work station, a monitor and a simple cab with controls.
- Mid-level simulators include advanced imaging techniques, a large projection screen, a realistic cab and possible a simple motion base.
- High-level simulators typically provide close to a 360-degree field of view and an extensive moving base.

In using a simulator it is important to make sure that the behavior is realistic. In (Kaptein et al., 1996) two important types of validity are discussed, absolute and relative validity. The first one refers to the absolute size of the outcomes of the simulation study, while the second one relates to the relative size. Suppose for example that six possible measures to reduce speed are compared, then the driving simulator is relatively valid when the ranking of the six effects is the same as if it would have been in a field experiment.

Whether absolute or relative validity is required or obtained is strongly dependent on the research question. However, to calibrate car-following models absolute validity is an absolute requisite. That is, the reactions of the follower need to be of the same size as they would have been in reality.

C.3 Comparison of microscopic trajectory data collection approaches
All these microscopic data collection methods have their own possibilities/advantages, and impossibilities/disadvantages in studying longitudinal driving behavior. Some technological issues were already briefly mentioned, in the upcoming we will concentrate on the aspects of longitudinal driving behavior that can be analyzed using a certain data collection approach.

In comparing these methods we assume that the methods performing field experiments are applied in real traffic conditions. We furthermore assume that all data collection methods are able to provide data suited for calibrating existing one leader car-following models. The final assumption is that the data collection methods are applied in the same way as in the studies mentioned before. Thus when GPS is used often platoons of GPS equipped cars are studied, while the instrumented vehicle of (Brackstone et al., 1999) is used on his own.

Table C-1 gives an indication of the aspects of longitudinal driving behavior that can be studied using the different data collection methods. In the upcoming we explain this table by considering the characteristics of the respective data collection methods.
Table C-1 Cross-comparison of data collection methods with respect to their ability of studying specific aspects of car-following ((-) difficult, (o) possible in a limited way, (+) well possible).

<table>
<thead>
<tr>
<th></th>
<th>Equipped vehicles</th>
<th>Aerial observation</th>
<th>Driving Simulator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no GPS, active mode</td>
<td>no GPS, passive mode</td>
<td>GPS helicopter high building</td>
</tr>
<tr>
<td>Multi-anticipation</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Inter-driver differences</td>
<td>-</td>
<td>o</td>
<td>-</td>
</tr>
<tr>
<td>Intra-driver differences</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Driver characteristics</td>
<td>+</td>
<td>o</td>
<td>+</td>
</tr>
<tr>
<td>Car characteristics</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Ergonomic factors&lt;sup&gt;16&lt;/sup&gt;</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Influence of road characteristics</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Influence of prevailing traffic conditions</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Influence of weather conditions</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

C.3.1 Characteristics of instrumented vehicle without GPS, active mode
The advantage of using an instrumented vehicle without GPS in the active mode is that the characteristics of the following driver and his car are known. Next to that the driver can be monitored during the experiment, providing even more information. Another advantage of this method is that data can be gathered for a single driver over a longer time period enabling research into intra-driver differences.

A disadvantage is that only directly surrounding vehicles can be monitored, making it impossible to analyze multi-anticipative longitudinal driving behavior. The method is also unsuited for gathering data for a large sample of drivers under comparable driving conditions.

C.3.2 Characteristics of instrumented vehicle without GPS, passive mode
As the passive mode differs from the active mode in that the behavior of surrounding vehicles is monitored instead of the behavior of the driver of the instrumented vehicle, some of the advantages mentioned before do not apply anymore. Although more general characteristics of the observed following drivers, like car type and gender, can be noticed, considerably more additional effort afterwards is needed to derive more detailed information like years of driving experience.

An advantage of the passive mode compared to the active mode is that data can be gathered for a larger sample of following drivers as always other vehicles are in the surrounding of the

<sup>16</sup> Ergonomics is the study of the interaction of a person with a machine. Information derived from ergonomists contributes to the design and evaluation of tasks, jobs, products, environments and systems in order to make them compatible with the needs, abilities and limitations of people. Source: [http://en.wikipedia.org/wiki/Ergonomics](http://en.wikipedia.org/wiki/Ergonomics), accessed: April 20, 2007
instrumented vehicle. The disadvantage of this is that the datasets for all these drivers separately will cover a shorter period of time. That is, to make a study on car-following behavior possible the distance between the leader and the follower should be such that the follower is really following. This can be arranged by the leader by choosing a relatively low speed. The problem in doing so is however that the following driver then starts to look for an opportunity to pass (Brackstone et al., 1999). This reasoning is confirmed by the average observation time of 99 seconds reported in (Kim et al., 2007).

C.3.3 Characteristics of instrumented vehicle applying GPS
Using GPS based methods data are gathered for equipped cars only, thus in order to use these methods for a study on (one-leader) car-following at least platoons of two equipped vehicles are required. The characteristics of the drivers of these cars can be easily derived by the researchers. The more vehicles in the platoon the more aspects of car-following can be analysed, i.e. multi-anticipation, a small sample based study on inter-driver differences.

An important practical problem is that in real traffic conditions it is difficult to drive in a platoon consisting of a large number of vehicles equipped with GPS receivers as a lot of intrusions of other vehicles in the platoon are likely to occur.

C.3.4 Characteristics of helicopter based aerial observation
In general we can state that the advantage of aerial observation methods is that a complete flow of vehicles driving on the monitored stretch is observed at once (Becker, 1988). Because of this, these methods are particularly suited to perform, for example, studies on inter-driver differences as a large sample of drivers driving under comparable traffic conditions is observed. Also multi-anticipation is an aspect for which these methods have clear advantages.

A disadvantage of these methods is that from above no information about the characteristics of the observed drivers can be collected. The height at which data are collected determines the level at which it is possible to distinguish different (types of) cars. This height also determines the length of the stretch that can be observed. In general we can say that the length of the observed stretch using a non-moving helicopter, while still being able to collect accurate data, is limited to approximately 500 m. This is a clear disadvantage of the method. Another disadvantage is that it is difficult and fatiguing for a helicopter pilot to stay at one position limiting the length of one time period for which data can be collected without a short interruption.

C.3.5 Characteristics of high building based aerial observation
This method shows a strong resemblance with the helicopter based method. There are however also some important differences. An advantage of the NGSIM approach above the helicopter based approach is that data collected from several neighboring buildings are joined, such that a clearly longer stretch can be observed. Also the problem of tiredness of the helicopter pilot is not faced. A disadvantage is however that the helicopter method is clearly more flexible in its use. Whenever an interesting traffic situation is observed, the helicopter can go there and collect data, while the positions of buildings are fixed.

C.3.6 Characteristics of a driving simulator
The clear advantage of a driving simulator is that the experiment is completely controllable. Different drivers can do the experiment on different times, while still facing exactly the same traffic situation. Another advantage of this perfect controllability is that the experiment can be designed such that the test person faces all situations important in answering the research question. It is also possible (Kaptein et al., 1996) to investigate the behavioral effects of non-
existing road elements, or to perform studies that would be too dangerous in reality or that seldom occur. Examples are the influence of alcohol on longitudinal driving behavior, or the influence of particular weather conditions.

The apparent disadvantage of a driving simulator is that the test person is aware of taking part in an experiment. This might seriously change his behavior and even to a larger extent than a driver, who is aware of being observed, driving in a real car. Accidents, for example, are no serious personal threat in a simulation, while a driver in a real car still seriously wants to avoid them even when he knows that he is monitored.

C.3.7 Short summary of practical abilities of the different methods

From the previous discussion it became clear that practical issues strongly correlate with the usefulness of data collection methods in studying certain aspects of longitudinal driving behavior. Often it is theoretically possible to perform a certain study using a data collection method; however it can be very difficult/impossible to practically carry out such an experiment. It is also important to consider the representativeness of the monitored behavior. Table C-2 provides for this reason an overview of several practical issues.

In this table the representativeness of a driving simulator study is especially highlighted to stress once more that this aspect might be a serious disadvantage of this method. This particularly holds for the case that the data are input to microscopic calibration studies as is the case in this thesis in which absolute validity of the trajectory data is an important requisite. Thus while almost all aspects of longitudinal driving behavior can be examined using a driving simulator as shown in Table C-1, it is very important to always ask how representative the results are.

Table C-2 Cross-comparison of data collection methods with respect to practical issues in using them ((-) disadvantage of method, (o) possible with method but considerable effort is needed, (+) advantage of method).

<table>
<thead>
<tr>
<th>Flexibility of approach</th>
<th>Equipped vehicles</th>
<th>Aerial observation</th>
<th>Driving Simulator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no GPS, active mode</td>
<td>no GPS, passive mode</td>
<td>GPS</td>
</tr>
<tr>
<td>Sample size</td>
<td>o</td>
<td>+</td>
<td>o</td>
</tr>
<tr>
<td>Sample size, comparable conditions</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Length of monitored stretch</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Representativeness</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

A final practical issue that can better be expressed in numbers is the accuracy of the different methods. The upcoming list therefore gives an overview of reported accuracies for several methods.

- The instrumented vehicle of (Brackstone et al., 1999) has an accuracy of 0.2 m with respect to relative distance and a accuracy of 0.4 m/s in relative speed.
• For the GPS based method proposed in (Hatipkarasulu et al., 2000) a 2-5 meters spherical error probability is reported. This implies that the obtained position is in 50 percent of the cases within a sphere with a radius of 2-5 meters from the true value. The mentioned accuracy of speeds is 0.16 km/h.

• For the RTK GPS based method proposed in (Gurusinghe et al., 2002) a positional accuracy of 10 mm+2 ppm17 is mentioned, while the accuracy of the speed is 0.16 km/h.

• The GPS system applied in (Punzo et al., 2005) is assumed to have an accuracy of 10 mm+1 ppm in the horizontal direction.

• The accuracy of the remote sensing based methods depends among others on the resolution of the used camera and the height of the camera. The lower the resolution, the larger the real-life area to which one pixel corresponds. According to (Kovvali et al., 2007) the accuracy of the NGSIM approach is 2 feet, corresponding to 0.61 meter. The spatial resolution of the helicopter method described in (Hoogendoorn et al., 2003, Hoogendoorn and Schreuder, 2005) is 40 cm (given the height of the helicopter during the described experiment).

C.4 Comparison of different environments: Real traffic versus test-track

In order to compare the different data-collection methods performing field experiments in a fair way, we assumed that data were collected under real traffic conditions. However, before also an example was given of a field experiment performed on a test-track. To complete the discussion on microscopic data collection, we therefore shortly compare these two conditions.

In this light it is obvious that the advantage of a test-track is that the experiment can be controlled to a large extent, while the disadvantage is that drivers are aware of taking part in an experiment, which can influence their behavior. When trajectory data are used to calibrate car-following models, it is important to stress the usefulness of being able to control a test-track experiment. In the process of calibrating it is namely important to have enough variability in the speeds of the monitored vehicles. If this is not the case the calibration objective may become relatively insensitive to changes in the parameter values and/or differences between model performances can become small. This advantage of a test-track is fully exploited by (Gurusinghe et al., 2002), as the platoon leader was in these experiments instructed to drive according to predefined speed patterns.

17 This value expresses the decrease in accuracy with distance to the base station and is known as “spatial decorrelation”. For example, if the distance is 200 kilometers, and the decorrelation of the applied GPS system is specified at 10 ppm a 2 meters accuracy degradation may be experienced (http://www.romdas.com/technical/gps/gps-acc.htm/).
Appendix D  Relation between traffic state during observation and information contained in trajectory observations

A practical point that needs to be considered in using real-life trajectory observations for calibration is whether the data contain enough information to calibrate all model parameters. Suppose for example that the leading and following car drive with (almost) constant speed during the observation, then it becomes impossible to estimate any parameters pertaining to the dynamical part of car-following.

It is also very well possible that only a specific part of the model can be calibrated fully. Suppose for example that a car-following model distinguishes a free-driving regime and a car-following regime. When the observation was performed during congestion it becomes impossible to reliably estimate the free-driving parameters, simply because of the fact that this regime was not observed.

In line with this reasoning several categories of parameters of car-following models can be distinguished:

- Parameters describing the dynamics of a following vehicle ($\beta_{\text{fol,dynamic}}$). Examples are parameters describing how a following vehicle synchronizes his speed to his leaders speed and parameters describing how a driver accelerates or decelerates to reach his desired distance.
- Parameters describing the desired (steady state) conditions of a following vehicle ($\beta_{\text{fol,steady}}$), like the parameters specifying the desired distance.
- Parameters describing the dynamics of free driving ($\beta_{\text{free,dynamic}}$). For instance, a parameter describing how a driver accelerates to his desired speed.
- Parameters describing the desired (steady state) conditions of a free driving ($\beta_{\text{free,steady}}$) driver. An obvious example of such a parameter is the free speed.
Longitudinal driving behavior: theory and empirics

Based on this subdivision of parameters it can be reasoned which driving conditions an observed driver should face such that the resulting microscopic trajectory data can be used to calibrate specific types of parameters. To do so, at least two criteria need to be specified. The first one relates to the headway the driver has to his leader. This headway can simply stated be smaller than the minimum headway for free driving or it can be larger. Secondly, the speed pattern of the driver is important, i.e. a driver can be driving with almost constant speed during an observation, next to that a from a calibration perspective more favorable situation can occur in which there are clear changes in the driving speed.

Table D-1 gives an overview of types of parameters that can be calibrated under specific individual driving conditions based on these criteria. From this table it can generally be concluded that when a driver is in a stationary state only steady state parameters can be estimated reflecting the desired conditions of a driver. When the driving speed varies however during the observation also the dynamic parameters can be estimated. Under these conditions also the steady state parameters can most probably be estimated as they describe the desired conditions of the driver and are therefore also influencing the dynamics.

Table D-1 Impression of the driving conditions an individual driver needs to face before the corresponding trajectory data can be used to calibrate a specific type of parameters.

<table>
<thead>
<tr>
<th>Distance headway</th>
<th>Stationary driving speed</th>
<th>Changes in driving speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum headway for free driving</td>
<td>β\text{fol,steady}</td>
<td>β\text{fol,dynamic}, β\text{fol,steady}</td>
</tr>
<tr>
<td>Minimum headway for free driving</td>
<td>β\text{free,steady}</td>
<td>β\text{free,dynamic}, β\text{free,steady}</td>
</tr>
</tbody>
</table>

In specifying criteria also a more macroscopic perspective can be taken, specifying which traffic states should be observed such that it becomes likely that the corresponding microscopic trajectory data can be used to calibrate specific types of parameters. This view is especially of interest when data are obtained using aerial observation, while the aforementioned view pertains most to data collected using instrumented vehicles.

To practically distinguish phases in free flow, a Level Of Service (LOS) approach can be taken (Transportation Research Board- National Research Council, 2000) in which the level of service ranges form A (Completely free flow) to E (Unstable flow), like illustrated in Figure D-1.

Figure D-1 Illustration of Level Of Service approach for distinguishing traffic phases in free flow.
Table D-2 indicates which parameters can most probably be calibrated given a certain LOS. The reasoning leading to this table is as follows; When the flow is very low drivers are generally driving at their desired speed such that only those parameters can be calibrated that refer to steady state free driving. The more the flow increases the more interactions between vehicles start to occur. That is, vehicles might be forced to follow a slower vehicle for some time because they have to wait for an opportunity to pass. The more the flow increases the sparser these opportunities will be. Because of this it becomes more and more likely that the parameters referring to car-following can be calibrated. After passing a slower vehicle at low flows it is again possible to accelerate to the desired speed such that the parameters referring to the dynamics of free driving can be estimated as well.

Table D-2 Indication of which parameters can be calibrated under which conditions in free flow (X=very likely, x=occasionally).

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{\text{free,dynamic}}$</th>
<th>$\beta_{\text{free,steady}}$</th>
<th>$\beta_{\text{fol,dynamic}}$</th>
<th>$\beta_{\text{fol,steady}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS: A</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOS: B</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOS: C</td>
<td>X</td>
<td>X</td>
<td>x</td>
<td>X</td>
</tr>
<tr>
<td>LOS: D</td>
<td>x</td>
<td>x</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>LOS: E</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

After a traffic breakdown when congestion occurs it becomes very unlikely that parameters of the type $\beta_{\text{free,dynamic}}$ and $\beta_{\text{free,steady}}$ can be calibrated because all vehicles will be constrained by other vehicles.

To indicate whether the parameters of type $\beta_{\text{fol,dynamic}}$ and $\beta_{\text{fol,steady}}$ can be calibrated, it needs to be considered again how much the speeds of single drivers change in general during the period that they are observed. That is, whenever the considered part of the freeway is heavily congested it is very well possible that drivers drive continuously with approximately the same speed. The smaller the speed differences drivers face the more unlikely it becomes that the parameters of the type $\beta_{\text{fol,dynamic}}$ can be estimated. However when clear speed changes can be observed, for example, when stop and go waves occur, the probability of being able to calibrate dynamic parameters increases. The parameters $\beta_{\text{fol,steady}}$ can in most of the cases independent of the type of congestion be estimated given that the modeled driving dynamics are approximately correct.
Appendix E  Comparison of parameter estimates for the Everdingen and Waalhaven measurement sites

In chapter 5 we compared parameter estimates between drivers of person cars having a comparable driving style. These analyses revealed differences between drivers of person cars related to driver characteristics. In performing these analyses for both the Everdingen site and the Waalhaven site, indications were found that optimal parameter values differ not only between drivers, i.e. within driving style heterogeneity, but also between different locations.

Under the assumption that the driver populations are comparable for both measurement sites these differences between both measurement locations can be interpreted as intra-driver differences, i.e. differences in the behavior of a single driver caused by different external conditions. A driver can adjust his driving style, for example, to different lane configurations (Ter Kuile, 2006a, b), to different weather conditions and to different traffic conditions. The aim of this appendix is to explore these possible behavioral differences between the measurement locations.

Figure E-1 hereto shows again the empirical c.d.f.’s of the parameters of the overall best performing models for both the Everdingen measurement site and the Waalhaven measurement site.

A visual comparison of the c.d.f.’s for both measurement locations gives the impression that the sensitivity parameter $c_{1,n-1}$ of the Tampère model is larger for the Everdingen measurement site as for the Waalhaven measurement site. The same holds for the sensitivity parameter $c_4$ of the model of Addison and Low. This gives the impression that drivers driving at the Everdingen measurement site are more sensitive than drivers driving at the Waalhaven measurement site.

This is also supported by the parameter estimates for the Gipps model, i.e. the estimates for $b^{\text{max}}$ seem to be smaller for the Everdingen measurement site than for the Waalhaven
measurement site. Under the assumption of equal driver populations this means that drivers driving at the Everdingen measurement site are more positive about their braking capabilities than drivers driving at the Waalhaven measurement site. This results, ceteris paribus, in a more risky driving style for drivers driving at the Everdingen site.

To formalize these visually established differences we verify whether they are statistically significant on the 5% level by performing Kolmogorov-Smirnov tests.

To this end, the estimates of the parameters presented in Figure E-1 are compared between person cars driving at the Waalhaven measurement site and person cars driving at the Everdingen measurement site. The estimated parameter values for a specific parameter for all considered triplets of the Waalhaven measurement site are represented in $\beta_{\text{Waalhaven}}$, while $\beta_{\text{Everdingen}}$ contains the estimates belonging to the Everdingen measurement site. For every separate parameter the following hypotheses are tested:

$$H_0: \beta_{\text{Everdingen}} \quad \text{and} \quad \beta_{\text{Waalhaven}}$$
$$H_1: \beta_{\text{Everdingen}} \quad \text{and} \quad \beta_{\text{Waalhaven}}$$

are not statistically different.

are statistically different.

The results of these tests are provided in Table E-1. The null hypothesis can be rejected for three parameters:
• The parameter $m$ of the Lenz model. It can be seen that the slope of the optimal velocity function of the Lenz model at the inflection point is larger for the Everdingen site than for the Waalhaven site, implying that in the neighborhood of this point the optimal speed increases more when the distance between the leader and the follower increases for the Everdingen site.
• The parameter $b_{max}$ of the Gipps model, as discussed before, the values for this parameter are in general lower for the Everdingen site than for the Waalhaven site implying that drivers driving at the Everdingen site are more optimistic about their braking capabilities than drivers driving at the Waalhaven site. Given that the estimates for $b_{n-1 \max}$ are not statistically different, this can be interpreted as a less stable driving style of drivers driving at the Everdingen site.
• The sensitivity parameter $c_4$ of the model of Addison and Low is in general smaller for drivers driving at the Waalhaven measurement than for drivers driving at the Everdingen measurement site.

The suggestions about the directions of the deviations between $\beta_{\text{Everdingen}}$ and $\beta_{\text{Waalhaven}}$ are statistically supported by the corresponding one-sided Kolmogorov-Smirnov tests.

**Table E-1 Results for testing the hypothesis that $\beta_{\text{Everdingen}}$ and $\beta_{\text{Waalhaven}}$ are not statistically different.**

<table>
<thead>
<tr>
<th>Model</th>
<th>Test results for relevant parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tampère</td>
<td>$c_{1,n-1}$: not reject H₀ \hspace{1cm} $\gamma$: not reject H₀</td>
</tr>
<tr>
<td>Lenz</td>
<td>$m$: reject H₀</td>
</tr>
<tr>
<td>Gipps</td>
<td>$b_{\max}$: reject H₀ \hspace{1cm} $b_{n-1 \max}$: not reject H₀</td>
</tr>
<tr>
<td>Addison &amp; Low</td>
<td>$c_4$: reject H₀ \hspace{1cm} $\gamma$: not reject H₀</td>
</tr>
<tr>
<td>IDM</td>
<td>$T_{\text{safe}}$: not reject H₀</td>
</tr>
</tbody>
</table>

Although these differences can be caused by a lot of differences regarding the external conditions, the different traffic conditions can well be used to explain these results. The results for the Tampère model and the Gipps model for example provide some evidence for a larger sensitivity of drivers during the stop-and-go conditions observed at the Everdingen measurement site than during the heavy congestion encountered at the Waalhaven measurement site. From a driver perspective this seems reasonable as the driving task is more demanding when speeds are changing rapidly than during prolonged congestion.

However, when we consider the discrete reaction times of the models of Tampère, Gipps and Addison and Low, it seems like they are in general larger for the Everdingen measurement site as for the Waalhaven site giving an indication that drivers react slower at the Everdingen measurement site. When performing Kolmogorov-Smirnov tests it turns out that the reaction times for the Everdingen measurement site are significantly larger at the 5% level than the reaction times at the Waalhaven measurement site for both the Tampère model and the model of Addison and Low. In this light it needs to be mentioned that for the reaction times no sensitivity analyses are performed due to their discrete character implying that less value can be attached to these findings than to the results for the other parameters.
Appendix F  Creating synthetic trajectory observations for analyzing the validity of the calibration procedure

F.1  Aim and structure

The aim of this appendix is to discuss the approach used for creating the synthetic trajectory observations for chapter 4. An overview of this approach is provided in Figure F-1.

Step 1: Define a realistic trajectory for the leader and create several different trajectories for the follower based on a given car-following model and given sets of parameter values.

Step 2: Prepare the trajectory observations for a study on the influences of the characteristics of the trajectory observations on parameter estimates (influence of measurement errors and missing information about the car-following model describing the dynamics of the following car).

Step 3: Verify whether the calibration procedure proposed in chapter 4 and the created synthetic trajectory observations are appropriate for performing the analyses in chapter 4.

Figure F-1 Overview of approach for creating synthetic microscopic trajectory observations.

In the upcoming we explain the subparts of the approach in more detail.
F.2  Step 1: Creating clean synthetic trajectory observations

To ensure that the synthetic trajectory observations resemble empirical observations as much as possible, the dynamics of the leading car are derived from the real-life BOSCH trajectory dataset (Deutsches Zentrum für Luft- und Raumfahrt (DLR)). We use these trajectory observations instead of our helicopter observations as these observations cover a clearly longer time period. The BOSCH trajectory observations are furthermore collected in urban traffic such that they show a larger variation in speed over time.

In our synthetic data set, we assume the speed pattern of the leader to be equal to the empirical pattern. The only difference is that we add 10 m/s to all speed measurements. This addition is made to avoid that the speeds of the leader become equal to 0 m/s for some periods. Based on these speeds the trajectory of the leading car is derived.

To enable also estimation of the parameters referring to the free driving regime, a jump of 100 m. is introduced in the trajectory of the leader. This jump can, for example, be interpreted as a lane change of the direct leading car, after which the driver follows his new leader. The reasoning behind this is that due to this sudden increase of the distance, the driver is free to accelerate to his desired speed before catching up with the new leader. Figure F-2 shows the speed pattern and the trajectory of the leading vehicle.

![Figure F-2 Synthetic data of the leading vehicle based on real-life data.](image)

After creating the data of the leader, data are created for the following car using the Gipps car-following model. To be able to draw more general conclusions from the analysis instead of conclusions only based on one particular dataset, 25 random datasets are created. That is, for each dataset parameter values are randomly drawn from uniform distributions using the following rules:
Appendix F - Creating synthetic trajectory observations for analyzing the validity of the calibration approach

- Gipps: $a_{\text{max}} \sim \text{uniform}(1,2)$, $b_{\text{max}} \sim \text{uniform}(-3.5, -1)$, $\theta \sim \text{uniform}(0, 1)$, $d \sim \text{uniform}(0.5,5)$, $b_{n-1\text{max}} \sim \text{uniform}(-3.5, b_{\text{max}})$.

The desired speed is furthermore assumed to be equal to 30 m/s, while the reaction time is equal to 1 sec. for all drivers.

For the analysis it is important that all parameter values used for creating the data are saved such that they can be used to judge the parameter values returned by the calibration procedure.

F.3 Step 2: Introducing measurement errors having known characteristics

In the previous step synthetic data for the leading vehicle were derived from real data and data for the following vehicle were created by assuming that the following driver reacted to the dynamics of his leader in line with a specified car-following model and in line with known parameter values. These data are in our analyses interpreted as clean microscopic trajectory observations reflecting the “real” driving behavior of the leader and the follower. To assess the influence of measurement errors, noise needs to be added to these clean data.

Based on the comparison of methods for collecting microscopic trajectory observations in Appendix C, measurement errors will be added to the observations in two different ways:

- Measurement errors will be added to the data of the leading car only, to resemble data collected by an equipped car (active mode), in which the data of the following car are often much more accurate than the data of the leading car.
- Measurement errors will be (independently) added to both the data of the leader as well as to the data of the follower. This scenario resembles the error made in the case of, for example, remote sensing data collection methods.

In both cases errors are introduced first to the x-positions resembling the practical situation in which x-positions are directly measured. For the sake of consistency speeds are derived afterwards from these noisy position measurements.

The errors themselves are created by performing independent random draws from statistical distributions. In order to be also able to investigate the influence of the specific appearance of measurement errors, several different distributions will be used. First a normally distributed error term with a mean $\mu$ equal to 0 will be added to the data, in this context especially the influence of the standard deviation $\sigma$ of the error term is interesting. For this sake the following two error distributions will be examined, $\varepsilon_t \sim \text{N}(0;0.05)$ and $\varepsilon_t \sim \text{N}(0;0.1)$. Next to that, also the impact of a systematic component in the error term will be studied by applying the following distributions, $\varepsilon_r \sim \text{N}(0.05;0.05)$, $\varepsilon_r \sim \text{N}(0.05;0.1)$, $\varepsilon_r \sim \text{N}(0.1;0.05)$, $\varepsilon_r \sim \text{N}(0.1;0.1)$. Finally a non-symmetric error will be simulated based on the assumption that the error term is exponentially distributed with parameter 0.05.

For all separate datasets, we save all information on the applied error distributions.
F.4 Step 2: Preparing the synthetic trajectories for analyzing the influence of missing information on the underlying behavioral rule

To investigate the impact of lacking information on the correct car-following model, in fact no special data operations need to be performed. This situation will namely be imitated by fitting the Tampère model to trajectory observations created by the Gipps model. The Tampère model is selected because we need a model capturing both the free-driving regime and the congested regime like the Gipps model. Another similarity of the models is that drivers want to drive at a specified distance. Although both models share some common characteristics, it is also important to stress that the driving dynamics are different for both models (chapter 3).

F.5 Step 3: Validity of proposed optimization approach

To analyze the impact of, for example, measurement errors on parameter estimates we need to verify whether the optimization approach proposed in chapter 4 is appropriate and whether the synthetic data contain enough information to estimate all individual parameter values.

In order to verify the optimization approach, three different (clean) synthetic datasets are created using the specified method and it is examined whether the selected algorithm can identify the parameters used in creating the data. For this sake the relative difference (\( \Delta(\beta_i^*, \beta_i) \)) between the estimated optimal parameter (\( \beta_i^* \)) and the known (saved) real parameters (\( \beta_i \)) is computed (eq. (F.1)).

\[
\Delta(\beta_i^*, \beta_i) = \frac{\beta_i^* - \beta_i}{\beta_i}
\]  

(F.1)

The corresponding results are provided in Table F-1. This table shows that the parameter values returned by the specified optimization algorithm are for all datasets and for all parameters very close to the real values. From this it can be concluded that the chosen optimization approach is ready for use in the proposed analyses.

| Table F-1 Verification of the optimization algorithm for the Gipps model. |
|-------------------|---------|---------|---------|---------|---------|
| \( \Delta(\beta^*(i), \beta(i)) \) | \(a_{max}^{\max}\) | \(b_{max}^{\max}\) | \(\theta\) | \(d\)   | \(b_{n-1}^{\max}\) |
| Data set 1        | -0.0016 | -0.0020 | -0.0006 | 0.0009  | -0.0019 |
| Data set 2        | -0.0009 | -0.0015 | -0.0036 | 0.0035  | -0.0013 |
| Data set 3        | -0.0014 | -0.0009 | 0.0057  | -0.0344 | -0.0016 |

It was also established that the algorithm was able to identify the parameters of the Tampère model when data created based on this model served as input.

F.6 Step 3: Validity of the synthetic trajectory observations

Although Table F-1 shows that the optimization method is able to estimate all parameters (also the ones referring to the free driving regime) quite closely, also a sensitivity analysis of the estimated parameters is needed to definitely proof that the synthetic data are appropriate for the analyses. Suppose for example that the optimization algorithm returned a parameter closely to the real one, while the sensitivity of the parameter to small changes is 0 or very small. In that case, it would have been just a matter of luck that the correct parameter values
were found as other values would have been as likely. Figure F-3 therefore shows an example of the sensitivities of all parameters of the Gipps model.

From this figure (and resembling ones not shown here) it can be concluded, that all parameters are sensitive to small changes, showing that the data contain enough information to identify the parameters of both regimes.

Thus when the calibration procedure in the analyses in chapter 4 fails to identify the known saved parameters, this can almost be fully attributed to the characteristics of the trajectory observations, or the methodological choice under concern.
Appendix G  Impact of ‘within driving style’ heterogeneity on platoon stability

G.1  Aim of analysis

The aim of the analysis presented in the upcoming is to gain insight into the effects of ‘within driving style’ heterogeneity on platoon stability. We hereto compare well-known results on platoon stability of homogeneous platoons for the CHM model of (Chandler et al., 1958) to stability characteristics of various types of heterogeneous platoons.

The results show profound differences between homogenous platoons and heterogeneous platoons. They demonstrate, for example, that it can not simply be stated that a heterogeneous platoon becomes unstable when the mean of the product of the parameters $c_{l,n-1}$ and $T_r$ is larger than ½ (the threshold for homogeneous platoons). It turns out that also the standard deviation plays a role, so how many drivers are able to compensate. Next to that the platoon composition turns out to have an impact on how the disturbance propagates through the simulated platoon.

G.2  Stability concept

The aim of a stability analysis is to determine how an initial disturbance in the speed of the platoon leader propagates through a platoon of vehicles. Roughly stated, instability refers to the situation in which such an initial disturbance amplifies (either in time or space), whereas such a disturbance smoothes in a stable situation. We can distinguish two types of instability (Leutzbach, 1988):

- **Local instability**, defined as the situation in which a disturbance does not die out but rather increases with time. This type of stability is concerned with the car-following behavior of one follower (May, 1990).
- **Asymptotic instability**, defined as the situation in which a disturbance grows in magnitude as it propagates from vehicle to vehicle.
In the investigation of stability characteristics of car-following models, analytical (Chandler et al., 1958, Bexelius, 1968, Lenz et al., 1999) and simulation based approaches (Lenz et al., 1999, Treiber et al., 2006a) can be adopted.

G.2.1 Analytical stability analysis
In the analytical approach, mathematical techniques from systems control theory are used. The most well-known example of a car-following model that has been examined analytically is the model of (Chandler et al., 1958) given by (for more details see chapter 3):

\[ a_n(t + T_r) = c_{1,n-1} \cdot \Delta v_{n-1,n}(t) \]  

where,

- \(a_n\) = acceleration of vehicle \(n\) (m/s²)
- \(\Delta v_{n-1,n}\) = speed difference with leader \(n-1\) (m/s)
- \(c_{1,n-1}\) = sensitivity to speed difference with leader \(n-1\) (1/s)
- \(T_r\) = reaction time (s)

For this model the following thresholds have been found applicable for local stability (Leutzbach, 1988):

- \(c_{1,n-1} \cdot T_r \leq 1/e\): In general stable, the response to a pulse input is non-oscillatory and damped.
- \(1/e < c_{1,n-1} \cdot T_r < \pi/2\): In general stable, the response to a pulse is oscillatory and damped.
- \(c_{1,n-1} \cdot T_r = \pi/2\): Stability boundary.
- \(c_{1,n-1} \cdot T_r > \pi/2\): The response is unstable, the response to a pulse is an oscillation with growing amplitude.

In this model asymptotical stability requires that \(c_{1,n-1} \cdot T_r \leq 1/2\). These well-known results refer to homogeneous platoons, i.e. all drivers are driving according to the same behavioral parameters. For an overview of stability criteria of other car-following models we refer to (Holland, 1998).

The advantage of an analytical approach is that its results reach the highest level of preciseness for small disturbances, whereas a simulation based approach is an approximation of the analytical results. Its disadvantage however is that an analytical approach only is possible for simple models under simplistic (and unrealistic) assumptions.

G.2.2 Simulation based stability analysis
For the reasons mentioned above a simulation approach is needed to analyze the stability characteristics of more complicated models or under more complicated assumptions. In such an approach the speed pattern of the leader (or multiple leaders in case of multi-anticipation) is specified by the researcher and simulations are performed in which all vehicles driving behind the platoon leader(s) adapt their speed to that of the leader(s) on the basis of the car-following model that has been assigned to each of them.

G.3 Experimental design
Chapter 3 showed that the assumptions made on longitudinal driving behavior in the CHM model of (Chandler et al., 1958) are rather simplistic. This model is nevertheless used in our
analyses on the impact of heterogeneity on platoon stability, as thanks to this simplicity detailed analytical stability results are available under the assumption of *homogeneous* drivers. These well-known results for homogeneous platoons provide an excellent base for a study on the impact of heterogeneity.

To assess the impact of heterogeneity, we use a simulation based approach. In all simulations we consider a platoon of 250 vehicles. This platoon consists of one platoon leader, who drives with a constant speed of 15.33 m/s and then after 10 sec. starts to decelerate with 0.7 m/s² till he reaches a speed of 14 m/s. The dynamics of all vehicles driving behind this platoon leader are determined using the CHM model. In these analyses we assume that the stability characteristics for our finite length platoon of 250 vehicles can approximate the stability characteristics of an infinite length platoon as implicitly assumed in analytical analyses on asymptotic stability. This assumption will be validated in our analyses.

### Approach for determining the impact of heterogeneity on platoon stability

A natural way to investigate the impact of parameter heterogeneity, is to assign the parameters $c_{l,n-1}$ and $T_r$ in such a way that a fraction of the drivers is driving according to a parameter combination of $c_{l,n-1}$ and $T_r$ belonging to the stable regime (thus with $c_{l,n-1} \cdot T_r \leq \frac{1}{2}$), while the other drivers are driving according to a parameter combination belonging to the unstable regime.

Based on these considerations, we performed in (Ossen and Hoogendoorn, 2006) an explorative analysis in which $c_{l,n-1}$ was normally distributed with mean 0.5 and standard deviation 0.1. Sensitivity parameters were assigned to all drivers in the platoon by performing independent random draws from this normal distribution. By choosing $T_r$ equal to 1 s. for all drivers we could regulate the product of $c_{l,n-1}$ and $T_r$.

This analysis showed that the disturbance smoothed in all performed simulations using these randomly assigned parameter values. This is interesting as half of the drivers in the platoon were due to the symmetric shape of the normal distribution driving with parameters belonging to the unstable regime. Motivated by this, we extend the analysis here by investigating what happens if we increase the mean of the normal distribution, such that more than half of the drivers is driving according to parameter values belonging to the unstable regime for homogeneous platoons. We also analyze how the results are related to the extent of parameter heterogeneity, i.e. the applied standard deviation.

We hereto adopt two values for the mean of $c_{l,n-1}$: 0.5, and 0.6. For both means we examine the stability results for standard deviations of 0.05, 0.1, 0.15, and 0.2. In drawing the random variables we do not allow for sensitivity parameters smaller than 0.1 and larger than 1.5. Figure G-1 shows examples of distributions used in the analyses.
All platoons are simulated for 1500 seconds. Because of the stochastic element we repeat the simulations for all combinations of the mean and standard deviation ten times.

G.4 Results on the impact of heterogeneity on platoon stability

Figure G-2 shows the results of the simulations performed for all different combinations of the mean and standard deviation. In these figures for all drivers in the platoon (x-axis) the absolute value of the maximum performed deceleration (acceleration) during the simulation (y-axis) is plotted. The lines in the plots increase therefore when a disturbance amplifies and decrease when it smoothes. Thus when the disturbance smoothes this implies that drivers driving further behind the platoon leader adjust their speed in a smoother way to the new speed than drivers driving shortly behind the platoon leader. For Figure G-2 it has to be mentioned that the y-axis is often cut off when the values of max|a| become too large, this is done to make it possible to concentrate on the interesting parts of these figures.

Apart from the results of the heterogeneous platoons (continuous lines) also the results for the homogeneous platoons are plotted (interrupted lines), both to enable a comparison of both types of platoons, and to enable the verification of the simulation approach and the aforementioned assumption. The thick interrupted lines show the results for a homogeneous platoon in which all sensitivity parameters $c_{1,n-1}$ are equal to the mean plus one standard deviation, while the dotted lines represent the results for the case that all sensitivity parameters are equal to the mean minus one standard deviation. For example, the thick interrupted line in Figure G-2 (a) shows what happens when the sensitivity parameters $c_{1,n-1}$ of all drivers are equal to 0.55 (0.5+0.05), while the dotted line refers to the case that all sensitivity parameters are equal to 0.45 (0.5-0.05).
Figure G-2 The maximum values of $|a_n|$ for all 250 vehicles in the platoon for 10 simulation runs with (a) $c_{1,n-1} \sim N(0.5;0.05)$, (b) $c_{1,n-1} \sim N(0.5;0.1)$, (c) $c_{1,n-1} \sim N(0.5;0.15)$, (d) $c_{1,n-1} \sim N(0.5;0.2)$, (e) $c_{1,n-1} \sim N(0.6;0.05)$, (f) $c_{1,n-1} \sim N(0.6;0.1)$, (g) $c_{1,n-1} \sim N(0.6;0.15)$, (h) $c_{1,n-1} \sim N(0.6;0.2)$.

Figure G-2 shows that the disturbance smoothes in all simulations referring to heterogeneous platoons when the means of the applied normal distributions are equal to 0.5. This holds even
when we set the standard deviation of the normal distribution equal to 0.2. This despite the fact that again approximately half of the drivers is driving according to a parameter combination of \(c_{1,n-1}\) and \(T_r\) exceeding the threshold for asymptotical stability for homogeneous platoons. For homogeneous platoons we can see that the disturbance amplifies for all cases when the sensitivity parameter is equal to 0.5+one standard deviation. This is in line with the analytical results.

When the mean of the normal distribution is increased to 0.6, such that more than half of the drivers is driving in a way that is harmful for the stability of the platoon as a whole, the disturbance does not longer smooth for all heterogeneous platoons (Figure G-2 (e-h)). In fact the disturbance grows for all cases with \(c_{1,n-1} \sim N(0.6,0.05)\) and \(c_{1,n-1} \sim N(0.6,0.1)\). This amplification of the disturbance for the cases with small standard deviations can easily be explained. If we consider for example the case with \(c_{1,n-1} \sim N(0.6,0.05)\) then there is only a very small probability for a driver \(n\) that \(c_{1,n-1}\) is smaller than or equal to 0.5 (Figure G-1). This implies that there are only a few drivers that can compensate for the unstable driving behavior of the others. In line with this it is also clear that given these settings for the mean and standard deviation not only few drivers are assigned sensitivity parameters lower than or equal to 0.5 but also those values that are smaller are not low enough to stabilize the platoon again.

When we increase the standard deviation further (Figure G-2 (g,h)) the disturbance seems to smooth more or less again, although we can also see that the further the standard deviation is increased the more variable the results become. Figure G-2 (h) for example shows that the disturbance smoothes for some simulation runs quite quickly while it first amplifies and then smoothes for other simulation runs.

![Figure G-3](image)

**Figure G-3** Correlation between the platoon composition reflected by the different values for the sensitivity parameters \(c_{1,n-1}\) for the drivers, and the propagation of the disturbance when proceeding through the platoon.

To draw a link between the platoon composition and the development of the disturbance, Figure G-3 shows the parameter values for the first 150 drivers and the course of the disturbance when propagating through the platoon. The figures (a) and (b) show a clear...
difference in the platoon composition that can explain the difference between how the disturbance propagates. In Figure G-3 (b) a number of sensitivity parameter values for the first vehicles behind the platoon leader are found that are smaller than 0.5, causing that the disturbance smooths so far that larger values later on cannot really amplify it. The opposite holds for (a) where in the beginning a lot of drivers are found driving in an unstable way.
Summary

Longitudinal driving behavior: theory and empirics

Saskia Ossen

The driving task is a comprehensive task consisting of all tasks a driver must execute to reach his travel destination safely, comfortably, and timely. For example, a driver must keep a safe distance to the vehicle in front, conform to prevailing traffic rules, use turn indicators timely, and keep his vehicle on the road. In this dissertation thesis we concentrate on the so-called longitudinal component of the maneuvering/control subtasks of a driver, i.e. we analyze how drivers interact with other driver/vehicle combinations driving on the same lane. We both aim at increasing the fundamental knowledge on longitudinal driving behavior, and at improving mathematical models describing this behavior type.

The motivation for analyzing this particular subtask of a driver is that the longitudinal driving behavior of individual drivers determines to a large extent the characteristics of traffic flow as a whole. In-depth knowledge on how drivers interact is therefore a fundamental requirement for taking successful (dynamic) traffic management measures leading to a more efficient use of existing infrastructure, or to predict the effects of future changes in the infrastructure. Detailed insight into how humans execute their longitudinal driving subtask is furthermore a critical component for developing systems supporting the driver, like Adaptive Cruise Control (ACC)\(^\text{18}\).

We focus on two particular aspects of longitudinal driving behavior:

- Heterogeneity, i.e. differences between the longitudinal driving behaviors of driver/vehicle combinations driving under exactly the same conditions (the same stretch of road, same traffic conditions, same weather conditions and so on).

\(^{18}\) This system maintains a by the user specified speed. When it is not possible to drive with this specified speed, the system regulates the speed such that the vehicle maintains a by the user defined headway to the leader.
• *Multi-anticipation*, i.e. car-following behavior in which drivers anticipate traffic conditions further downstream by considering the dynamics of more than one leader on the same lane.

Both these aspects of longitudinal driving behavior are expected to have a large influence on traffic flow characteristics implying that a profound knowledge on these aspects is needed for understanding and modeling traffic flows. However, till now no appropriate data were available for analyzing heterogeneity and multi-anticipation empirically. In illustration, local measurements that are commonly used in traffic research are not appropriate to derive the level of heterogeneity. Using local measurements it is possible to measure differences between time headways at a detector location. It is however unclear whether these differences are caused by heterogeneity or by differences in the dynamic situation of drivers. This is why we apply a dedicated data collection method in this thesis providing us with a large sample of trajectory observations offering unique possibilities for analyzing longitudinal driving behavior.

The first contribution of this thesis is that we use these trajectory observations to quantify the level of heterogeneity present in real-traffic. We both quantify the extent of heterogeneity caused by driver characteristics and the extent of heterogeneity caused by car characteristics. The second contribution is that we obtain new insights into multi-anticipative longitudinal driving behavior. We firstly show for the first time empirical evidence for the presence of multi-anticipation in real-traffic. Secondly we determine to which leaders a driver responds and in which manner.

As an important application of longitudinal driving models is in microscopic simulation tools, we also provide new insights into the impacts of our empirical findings on heterogeneity and multi-anticipation on the predicted macroscopic characteristics of traffic flows, i.e. the fundamental diagram\(^{19}\), platoon stability and flow stability.

**Research approach**

To enable the proposed empirical analyses on heterogeneity and multi-anticipation in longitudinal driving behavior, we use a large sample of microscopic trajectory observations derived from helicopter images collected at two different measurement sites. The motivation for using this particular data collection method is that trajectories can be derived for all vehicles driving at the roadway stretch observed by the helicopter. For our analysis on heterogeneity this means that all driver/vehicle combinations whose longitudinal driving behaviors we compare are driving under approximately the same conditions meaning that behavioral differences are not likely to be caused by different external conditions. For our analysis on multi-anticipation, we can easily derive trajectories of several leaders ahead.

In both empirical analyses we take a *model based approach* implying that we calibrate mathematical longitudinal driving models and analyze parameter estimates and model performances to increase the insights into longitudinal driving behavior. The advantage of this approach is that our findings can be expressed quantitatively.

Due to the lack of microscopic trajectory data so far, only little information is present on calibrating longitudinal driving models at the microscopic level. In this thesis we therefore thoroughly examine the properties of the applied automated calibration procedure to obtain

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\(^{19}\) Equilibrium relation between speed, flow and density.
the methodological insights needed for performing our empirical analyses. We examine the influences of methodological factors, like the specification of the calibration objective and the variable in the calibration objective on estimated parameter values. We also consider the influences of measurement errors on both estimated parameter values and the reliability of these values. We finally investigate whether it is possible to draw inferences on observed longitudinal driving behavior by calibrating a longitudinal driving model only approximating complex human longitudinal driving behavior. Based on the knowledge obtained from these analyses of the calibration method, we adjust our automated calibration procedure used in the empirical analyses such that the quality of its results becomes more robust in the presence of measurement errors.

To investigate the influences of heterogeneity and multi-anticipation on the fundamental diagram, platoon stability and flow stability, we firstly perform a literature study. As this literature study shows that in particular little information is available on the influences of heterogeneity, we also perform microscopic simulation studies in which we explore the impacts of heterogeneity on the fundamental diagram, platoon stability and flow stability.

**Empirical findings on heterogeneity in longitudinal driving behavior**

In our empirical analyses on heterogeneity within the group of drivers of person cars, we show that the driving styles of drivers differ considerably. For example, clear differences are identified between the speed-dependent distances drivers want to keep to the driver in front of them. For some drivers these desired distances increase approximately linearly with speed, while for other drivers a nonlinear relation turns out to be more appropriate. Also the importance person car drivers attach to actually reaching this distance appears to be driver dependent. In terms of modeling, we show that different models are needed for describing the dynamics of different driver/vehicle combinations.

When comparing the driving behaviors of drivers of person cars adopting a similar driving rule, also clear differences are identified. For example, for those drivers for whom a linear relation between speed and desired distance is found, clear differences are observed with respect to the steepness of this relation, i.e. the desired increase in distance for a 1 m/s increase in speed.

In our analysis on differences between the longitudinal driving behaviors of truck drivers and person car drivers, we show that truck drivers in general appear to drive with a more constant speed than drivers of person cars. This might be caused by the larger weight of trucks making them less maneuverable. Person car drivers turn furthermore out to be more eager in restoring large deviations from their desired distance than truck drivers. When comparing the longitudinal driving behaviors of person car drivers and truck drivers adopting comparable driving styles, we find for the group of drivers whose behavior can best be modeled with the Intelligent Driver Model that the distance headway truck drivers assume to be safe is significantly larger than the distance headway drivers of person cars consider as safe. A plausible explanation is that in choosing an appropriate distance headway to their leader, truck drivers take care of their larger braking distance.

**Empirical findings on multi-anticipation**

In our empirical analyses on multi-anticipation we show that more than half of the considered person car drivers looked further ahead than their direct leader in performing the longitudinal driving task. At least 20% appears to consider even more than two direct leaders. Especially
the relative speed regarding direct leaders further downstream turns out to be of influence to
the dynamics of the following car.

Also with respect to multi-anticipative driving behavior, heterogeneity is clearly present. For
example, the leaders considered differ between drivers. Also differences are established
between drivers adopting a comparable multi-anticipative longitudinal driving style. Thus
even when drivers consider the same leaders it turns out that the extents to which these
leaders actually influence the longitudinal behavior of the following vehicle are strongly
driver/vehicle combination dependent.

**Impact of empirical findings on traffic flow predictions**

Our analyses on the impacts of our empirical findings on predicted macroscopic traffic flow
characteristics show that both heterogeneity and multi-anticipation are expected to influence
the characteristics of simulated traffic flows considerably. We show, for instance, that
behavioral differences between drivers adopting a comparable longitudinal driving style cause
the density corresponding to a given speed to become stochastic and dependent on the
composition of the sample of vehicles currently present on the roadway. The variability of the
densities for a given speed is found to be closely related to the assumed level of heterogeneity.

Our simulations furthermore indicate that the assumed level of heterogeneity considerably
influences platoon/flow stability, stressing the importance of our efforts in quantifying the
level of heterogeneity present in real traffic. We show, for instance, an example in which a
disturbance in the dynamics of the platoon leader amplifies when propagating through a
platoon characterized by a low level of heterogeneity, while it smoothes when propagating
thorough a platoon characterized by a high level of heterogeneity. How the disturbance
propagates from one vehicle in the platoon to the corresponding following vehicle in the
platoon is furthermore found to be dependent on the characteristics of the following vehicle.
Consequently, the propagation of the disturbance is found to be dependent on the platoon
composition, i.e. the order of vehicles having different characteristics.

**Conclusions**

In this Ph.D. research for the first time large sample based empirical analyses are performed
on heterogeneity and multi-anticipation in longitudinal driving behavior. Our explorative
simulations on the impacts of heterogeneity and multi-anticipation on predicted traffic flow
characteristics show that our findings are expected to change traffic predictions substantially.
We therefore strongly recommend developers of microscopic simulation tools to explore our
findings further to establish whether and how they can use them to improve the predictive
power of these tools.

Our findings are furthermore particularly useful in developing systems supporting the driver
in his driving task, like ACC. For instance, in this thesis we show that the desired time
headways of drivers differ, this most probably entails that different drivers would prefer
different time headway settings when driving with ACC. The findings on multi-anticipation
are furthermore important in evaluating the impacts of ACC on traffic flow properties as they
give important insights into human driving behavior. For example, based on this thesis we can
conclude that real drivers are often able to anticipate traffic conditions further downstream by
looking further ahead. We also establish that real drivers need some time to react.

Autonomously operating ACC systems on the other hand can only consider the direct leader
in determining an appropriate control action, while their reaction time is likely to be smaller
than the one of human drivers. The question how these factors counterbalance, and thus what
the net behavioral effect of ACC is, can only be answered based on a profound knowledge regarding the longitudinal driving behavior of “real” drivers.
Samenvatting

Longitudinaal rijgedrag: theorie en empirie

Saskia Ossen

De rijtaak is een veelomvattende taak, die bestaat uit alle taken die een bestuurder moet uitvoeren om zijn bestemming veilig, comfortabel en tijdig te bereiken. Zo moet een bestuurder een veilige afstand tot zijn voorligger houden, zich houden aan de heersende verkeersregels, zijn richtingaanwijzers tijdig gebruiken en zijn voertuig op de rijbaan en liefst binnen de rijstrook houden. In dit proefschrift richten we ons op de zogenaamde longitudinale component van de manoeuvreer/controle subtaken van een bestuurder. Dit wil zeggen dat we de interacties tussen bestuurder/voertuig combinaties op dezelfde rijstrook analyseren. We hebben daarbij zowel ten doel om de fundamentele kennis op het gebied van longitudinaal rijgedrag te vergroten, als om wiskundige modellen die dit gedrag beschrijven te verbeteren.

De reden om deze specifieke subtaak van een bestuurder te bestuderen is dat het longitudinale rijgedrag van bestuurders voor een groot deel bepalend is voor de karakteristieken van de verkeersstroom in zijn geheel. Diepgaande kennis met betrekking tot het longitudinale rijgedrag van bestuurders is daarom noodzakelijk om succesvolle (dynamische) verkeersmanagement maatregelen te kunnen nemen, die moeten leiden tot een efficiënter gebruik van bestaande wegen, of om het effect te voorspellen van toekomstige veranderingen aan de infrastructuur. Inzicht in het rijgedrag van bestuurders is verder belangrijk bij de ontwikkeling van systemen, die de bestuurder moeten ondersteunen bij het uitvoeren van zijn rijtaak, zoals Adaptive Cruise Control (ACC)\(^20\).

We richten ons op twee specifieke aspecten van het longitudinale rijgedrag:

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\(^{20}\) Een systeem dat indien mogelijk een door de gebruiker ingestelde snelheid aanhoudt, wanneer dit niet mogelijk is zorgt het systeem er automatisch voor dat het voertuig een door de gebruiker ingestelde afstand tot de leider aanhoudt.
• **Heterogeniteit**, verschillen in het longitudinale gedrag van bestuurder/voertuig combinaties, die onder exact dezelfde condities rijden (zelfde wegconfiguratie, zelfde verkeerscondities, zelfde weerscondities enz.).

• **Multi-anticipatie**, longitudinaal rijgedrag waarbij bestuurders anticiperen op verkeerscondities stroomafwaarts door te kijken naar het rijgedrag van meer dan één leider op dezelfde stroom.

Van beide aspecten wordt verwacht dat ze een grote invloed hebben op verkeersstroomkarakteristieken, wat inhoudt dat goed inzicht in deze aspecten van het longitudinale rijgedrag onontbeerlijk is voor het begrijpen en modelleren van verkeersstromen. Tot nu toe waren er echter geen geschikte empirische data beschikbaar voor empirisch onderzoek naar heterogeniteit en multi-anticipatie. Ter illustratie, met lusdetector data is het mogelijk om de verschillen te meten tussen de volgtijden die verschillende bestuurders aanhouden. Het is echter onduidelijk of deze verschillen veroorzaakt worden door heterogeniteit of door verschillen in de dynamische situatie van bestuurders. In dit proefschrift passen we daarom een innovatieve data collectiemethode toe, die ons in staat stelt om een grote steekproef van microscopische trajectorie observaties te verzamelen. Deze data bieden unieke mogelijkheden om het longitudinale rijgedrag te analyseren.

De eerste bijdrage van dit proefschrift is dat we deze trajectorie observaties gebruiken om de mate van heterogeniteit in het dagelijkse verkeer te kwantificeren. We maken daarbij onderscheid tussen heterogeniteit veroorzaakt door verschillen tussen bestuurders en heterogeniteit veroorzaakt door verschillen tussen de voertuigen waarin deze bestuurders rijden. Daarnaast vergroten we de huidige kennis met betrekking tot multi-anticipatief rijgedrag. We laten niet alleen zien dat multi-anticipatie een belangrijke component is van het menselijke rijgedrag, maar we bepalen ook op welke leiders bestuurders reageren en hoe ze dat doen.

Omdat wiskundige modellen die het longitudinale rijgedrag beschrijven de spil vormen van microscopische simulatie software, onderzoeken we ook hoe onze empirische bevindingen met betrekking tot heterogeniteit en multi-anticipatie de voorspellingen, die worden gemaakt met deze software, beïnvloeden. We kijken meer specifiek naar het effect op het fundamenteel diagram\(^{21}\), colonne stabiliteit en stabiliteit van de verkeersstroom.

**Onderzoeksmethode**

Om de voorgestelde grootschalige empirische analyses naar heterogeniteit en multi-anticipatie mogelijk te maken, gebruiken we trajectorie data, die afgeleid zijn uit helicopterbeelden, verzameld op twee verschillende meetlocaties. Het grote voordeel van deze datacollectiemethode is dat trajectorieën afgeleid kunnen worden voor alle voertuigen die op het stuk weg rijden dat geobserveerd is door de helikopter. Voor ons onderzoek naar heterogeniteit betekent dit dat alle bestuurder/voertuig combinaties waarvan we het gedrag bestuderen onder dezelfde externe condities reden waardoor de kans dat gevonden verschillen inderdaad door heterogeniteit veroorzaakt worden aanzienlijk vergroot wordt. In ons onderzoek naar multi-anticipatie kunnen we voorts eenvoudig de trajectorieën van stroomafwaarts rijdende leiders bepalen.

In beide empirische analyses gebruiken we een modellmatige aanpak. Dit houdt in dat we wiskundige modellen kalibreren en dat we de resulterende parameterschattingen en

\(^{21}\) Evenwichtsrelatie tussen snelheid en intensiteit (verkeersdrukte).
modelprestaties analyseren om inzicht te krijgen in het longitudinale rijgedrag. Het voordeel van deze methode is dat we onze bevindingen kunnen kwantificeren.

Door het gebrek aan trajectorie observaties in het verleden, blijkt er slechts weinig informatie beschikbaar met betrekking tot het kaliberen van voertuigvolgmodellen op basis van dergelijke data. Omdat microscopische kalibraties noodzakelijk zijn voor de voorgestelde empirische analyses, voeren we in dit proefschrift een gedetailleerd onderzoek uit naar de eigenschappen van de gebruikte kalibratieprocedure. We bepalen de invloed van methodologische keuzes, zoals de specificatie van de doelfunctie en de variabele in de doelfunctie, op geschatte parameter waardes. Ook onderzoeken we hoezeer meetfouten van invloed zijn op de geschatte parameters. Tenslotte bekijken we of het mogelijk is om conclusies de trekken over het gedrag van bestuurders door middel van het kaliberen van een wiskundig model dat het complexe gedrag van echte bestuurders slechts benadert. De opgedane kennis gebruiken we om de kalibratieprocedure, die in de empirische analyses gebruikt wordt, beter bestand te maken tegen de negatieve invloed van meetfouten.

Om de invloeden van heterogeniteit en multi-anticipatie op het fundamentele diagram, colomne stabiliteit en verkeersstroom stabilité te bepalen, voeren we eerst een literatuurstudie uit. Omdat deze literatuurstudie aantoont dat er slechts weinig informatie beschikbaar is met betrekking tot de invloeden van heterogeniteit, voeren we tevens een microscopische simulatie studie uit waarin we verkennen hoe heterogeniteit de genoemde macroscopische verkeerskenmerken beïnvloedt.

**Empirische bevindingen met betrekking tot heterogeniteit in longitudinaal rijgedrag**

In ons empirische onderzoek naar heterogeniteit binnen de groep van bestuurders van personenauto’s tonen we aanzienlijke verschillen tussen de rijstijlen van bestuurders aan. Zo blijken de snelheidsafhankelijke afstanden, die mensen tot hun voorligger houden, behoorlijk te verschillen. Voor sommige bestuurders neemt de gewenste afstand ongeveer lineair toe met de snelheid, voor anderen geldt eerder een niet-lineair verband. Ook het belang dat bestuurders hechten aan het daadwerkelijk bereiken van de wensafstand varieert. Wat betreft het modelleren van longitudinaal rijgedrag laten we zien dat er verschillende modellen nodig zijn om het gedrag van verschillende bestuurders adequaat te beschrijven.

Ook wanneer we het gedrag vergelijken van mensen met een overeenkomstige rijstijl blijken er behoorlijke verschillen te bestaan. Wanneer we bijvoorbeeld alleen maar kijken naar bestuurders van wie de gewenste afstand een lineaire functie van de snelheid is, zien we dat de helling van deze functie, d.w.z. de gewenste toename van de afstand voor elke 1 m/s toename van de snelheid, sterk bestuurdersafhankelijk is.

Wanneer we kijken naar verschillen tussen de gedragingen van bestuurders van personenauto’s en bestuurders van vrachtwagens, blijkt ten eerste dat de snelheid van vrachtwagens over het algemeen constant is dan de snelheid van personenauto’s. Dit wordt mogelijk veroorzaakt door het grotere gewicht van vrachtwagens waardoor deze minder geschikt zijn voor veel snelheidsvariaties. Bestuurders van personenauto’s blijken verder meer belang te hechten aan het terugbrengen van grote verschillen tussen de daadwerkelijke afstand tot hun voorligger en de corresponderende gewenste afstand. Ook tussen de gedragingen van bestuurders van personenauto’s en bestuurders van vrachtwagens met een vergelijkbare rijstijl blijken verschillen te bestaan. Zo vinden we bijvoorbeeld voor de groep van bestuurders wier gedrag het beste met het Intelligent Driver Model benaderd kan worden dat de volgafstand, die bestuurders van vrachtwagens als veilig beschouwen, groter is dan de volgafstand, die
bestuurders van personenauto’s als veilig beschouwen. Een verklaring hiervoor is dat bestuurders van vrachtwagens bij het bepalen van een veilige volgafstand rekening houden met hun grotere remafstand.

**Empirische bevindingen met betrekking tot multi-anticipatie**

In ons empirische onderzoek naar multi-anticipatie laten we zien dat meer dan de helft van de geobserveerde bestuurders meer dan één leider beschouwt bij het uitvoeren van de longitudinale rijtaak. Tenminste 20% blijkt zelfs meer dan twee leiders te beschouwen. Vooral het snelheidsverschil met de verschillende leiders blijkt in hoge mate van invloed op het gedrag van de volger.

Ook met betrekking tot multi-anticipatief rijgedrag blijkt heterogeniteit duidelijk aanwezig te zijn. Niet alle bestuurders blijken bijvoorbeeld dezelfde voorliggers te beschouwen. Ook wanneer bestuurders dezelfde leiders beschouwen blijkt de mate waarin deze leiders het gedrag beïnvloeden sterk bestuurdersafhankelijk.

**Effect van empirische bevindingen op verkeersvoorspellingen**

In ons onderzoek naar het effect van de empirische bevindingen op macroscopische verkeersvoorspellingen tonen we aan dat zowel de gemaakte aannames betreffende heterogeniteit als de gemaakte aannames betreffende multi-anticipatie van groot belang zijn voor het beschrijven en het voorspellen van verkeersdynamica. We laten bijvoorbeeld zien dat verschillen tussen het longitudinale gedrag van bestuurders ervoor zorgen dat de snelheidsafhankelijke dichtheid een stochastische variabele wordt, waarvan de waarde afhankelijk is van de toevallige samenstelling van de verkeersstroom op het moment van meting. De mate van variabiliteit van de dichtheid blijkt sterk gecorreleerd met de aangenomen mate van heterogeniteit.

Onze simulaties laten voorts zien dat de aangenomen mate van heterogeniteit een sterke invloed heeft op de stabiliteit van colonnes/verkeersstromen. Dit onderstrept het belang van het kwantificeren van de mate van heterogeniteit in het dagelijkse verkeer. We geven een voorbeeld waarin een verstoring in de dynamiek van de colonneleider groeit wanneer deze propageert door een colonne met een lage mate van heterogeniteit. Dezelfde verstoring dempt echter wanneer hij propageert door een colonne met een hoge mate van heterogeniteit. Hoe de verstoring van het ene voertuig in de colonne naar het volgende voertuig propageert, blijkt afhankelijk van het longitudinale rijgedrag van de volger. Dientengevolge blijkt de manier waarop een verstoring propageert door een colonne/verkeersstroom afhankelijk te zijn van de volgorde van de voertuigen.

**Conclusies**

In dit promotieonderzoek is voor de eerste keer grootschalig empirisch onderzoek gedaan naar heterogeniteit en multi-anticipatie in longitudinaal rijgedrag. Onze verkennende microscopische simulaties naar het effect van de aangenomen mate van multi-anticipatie en heterogeniteit tonen aan dat onze bevindingen verkeersvoorspellingen vermoedelijk aanzienlijk veranderen. Derhalve raden we ontwerpers van microscopische simulatie software nadrukkelijk aan om verder te onderzoeken of en hoe onze empirische bevindingen verkeersvoorspellingen kunnen verbeteren.

Onze bevindingen zijn verder belangrijk bij het ontwerpen van systemen, die de bestuurder moeten helpen bij het uitvoeren van de longitudinale rijtaak, zoals ACC. In dit proefschrift laten we bijvoorbeeld zien dat verschillende bestuurders verschillende afstanden tot hun
Curriculum Vitae

Saskia J.L. Ossen was born in 1981 in Kerkrade. In 1999 she started her study in econometrics at Maastricht University, faculty of Economics and Business Administration. In 2003, she moved to The Hague for an internship at TNO (Netherlands Organization for Applied Scientific Research) during which she wrote her Master’s thesis on the development of a cost efficient and customer friendly method for supplying products ordered via the internet to customers. In the same year Saskia received her Master’s degree in econometrics with a specialization in Operations Research/ Information Science.

Since 2004, Saskia is affiliated with the Transport & Planning Department of the Faculty of Civil Engineering and Geosciences of Delft University of Technology. She conducted her Ph.D. research on individual driving behavior during congestion within the research program “Tracing Congestion Dynamics: With Innovative Microscopic Data to a Better Theory”.

Next to her Ph.D. research Saskia has been involved in two projects for the Traffic Research Centre of the Dutch Ministry of Transport, Public Works and Water Management. The first project, “Hiaatdetectie”, was on detecting incidents using microscopic double loop detector data. The second project, “Full traffic”, concerned a large field study on the impacts of Adaptive Cruise Control and Lane Departure Warning on individual driving behavior. In the last project Saskia cooperated with Dresden University of Technology in a microscopic simulation study to predict the influences of Adaptive Cruise Control on traffic flow characteristics.

Saskia has also been involved in several teaching activities. She organized, for instance, a TRAIL course on “Three-phrase traffic theory”. Furthermore, she supervised several MSc students.
Selection of author’s publications

Journal papers


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