Dike raising in a seismic, subsiding area 3

"two studies on liquefaction"

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This report is written as an additional study to my final master's thesis on the design of the coastal dike at the Costa Oriental del Lago de Maracaibo.
It was carried out at the Delft University of Technology, Faculty of Civil Engineering, Geotechnics section, under supervision of professor dr. ir. A. Verruijt.
This additional study consists of two parts, one finished in May of this year and one in December:

1) AN INTRODUCTION TO EARTHQUAKE INDUCED LIQUEFACTION

2) EVALUATION OF LIQUEFACTION RESISTANCE AND THE IMPLICATIONS FOR THE DESIGN OF DIKES

Delft, December 1987

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AN INTRODUCTION TO

EARTHQUAKE INDUCED LIQUEFACTION

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LIQUEFACTION

1. INTRODUCTION.

Cohesionless soils have a shear strength \( \tau = \sigma' \tan \phi \), with \( \sigma' = \sigma - u \).

Liquefaction of saturated soil (usually sand) can generally be defined as a condition in which the soil has lost its shear strength completely due to pore water pressure rise or sudden drop of the confining stress. In both cases the effective stress goes down to zero. The sand starts to flow like a thick, viscous fluid and offers little or no resistance to landsliding.

Liquefaction can be of static or dynamic origin. Casagrande treated liquefaction under unidirectional static loading in 1936, but only the Niigata (Japan) and Anchorage (Alaska) earthquakes of 1964 gave the onset for investigation into earthquake induced liquefaction. Other types of cyclic loading like wave action, machine vibrations and traffic are also dynamic causes for liquefaction. We restrict ourselves to earthquake induced liquefaction in saturated soil layers, usually sand. (see section 2)

It is important to find out whether a construction is threatened by liquefaction. The liquefaction potential of soil layers is affected by the characteristics of these layers and the earthquake. (section 3)

In the following section several methods to evaluate liquefaction potential are laid down. In the last section attention is paid to propagation of initial liquefaction.
Seepage, Boils, Quick Condition, Slope Failure Slip Circle, Dike Sliding Along the Base

Residual high U

Liquefaction

Cyclic liquefaction

Initial liquefaction

\[ \Delta U = \sigma_{v0}^v \rightarrow \sigma_{v}^v = 0 \]

Residual pore water pressure rise, decrease of effective stress

Soil volume reduction, effective stress transfer to water

Slip at intergranular surfaces

Acceleration of and shearing forces on soil particles

Shear waves from bedrock

Soil conditions earthquake characteristics
2. PHENOMENON OF EARTHQUAKE INDUCED LIQUEFACTION

In an idealized situation liquefaction occurs in horizontal saturated sand strata overlying bedrock. This implies that no static shear stresses apply to the soil particles under consideration. (Static shear stresses increase the resistance to liquefaction.) Soil particles in these strata are affected by the shear waves that propagate upwards from the bedrock. [see figure 1] Assumed is that there is no lateral force except the lateral pressure at rest, $K_0\sigma'$. During one loading cycle the soil particle is acted upon by a shear force. This shear force causes slip at the intergranular surfaces. Thus there is a tendency in the soil to settling, to a decrease in volume. However, one cycle is short and the little compressible water does not get the chance to flow out (undrained situation). Part of the confining stress must then be transferred from the soil particle contacts to the water: after one stress cycle there is a residual pore water pressure rise, $\Delta u$. This process repeats itself as long as the earthquake lasts and initial liquefaction occurs when all the confining effective stress is transferred from the grains to the water: $\sigma' = 0$, $u = \sigma$.

In addition to the general definition for liquefaction Seed (lit. 1) distinguishes also:

Initial liquefaction: During the application of cyclic stresses the pore water pressure $u$ reaches the confining stress $\sigma$. It is not yet sure what the consequences will be as for the resulting deformation of the soil.

Initial liquefaction with limited strain potential or cyclic mobility or cyclic liquefaction: A situation where initial liquefaction occurs, but the soil deformation is limited because of remaining soil strength or because dilation of the soil with subsequent pore water pressure decrease and recovery of the shear strength (effective stress).

The importance of this distinction is that, if liquefaction with subsequent landsliding does not occur, the residual increased pore water pressure may give rise to liquefaction in overlying
layers, to sand boils, upbursting or seepage after the earthquake, i.e. after the application of cyclic stresses has stopped.
3. FACTORS AFFECTING LIQUEFACTION POTENTIAL

Whether liquefaction may occur, depends on many factors. The chief factors are:

1. type of soil, grain characteristics
2. relative density, void ratio
3. initial confining stress
4. aging
5. strain history
6. lateral pressure coefficient and overconsolidation ratio
7. duration of shaking
8. intensity of shaking

-1. type of soil
Tests and field experience show that liquefaction occurs more easily for a) cohesionless soil (sands), rather than gravels, silt and clay.
   b) uniformly graded sands, rather than well graded sands.
   c) sands with round shaped grains, rather than angular grains.
   d) fine sands, rather than coarse sands.

Some soils have a marked tendency for spontaneous liquefaction. These are soils with high porosity (n > 44%), well rounded grains, and a very uniform grain size distribution: (u < 5, u = d_{60} / d_{10}).

-2. relative density
Loose sand is more susceptible for liquefaction than the same sand when more compacted. Loose sands tend to decrease in volume when subjected to shear forces, so that the pore water pressure rises as described in the above section. Denser sands however, are not able to undergo such deformations (or at least to a lesser extent) or even dilate, resulting in a pore water pressure decrease so that the sand develops enough resistance to the applied stress. [see figure 2]
-3. initial confining pressure

For a given initial density, sands subjected to higher initial confining stresses (σ and σ') show higher required stresses to initiate liquefaction. This is logic in the sense that the higher the confining stress, the more stress must be accumulated in the pore water during the earthquake before liquefaction occurs. This is opposite to the static situation.

-4. aging

Laboratory studies and field experience show that sands that have been subjected to sustained loads for a long period, prior to the earthquake, show more resistance to liquefaction. For very long periods (centuries) this increase may be as much as 75%. This increase must be attributed to a sort of cementation or welding at the intergranular contact points.

-5. strain history

Previous earthquakes that did not lead to liquefaction, have, when strains were small enough, a favourable effect on the resistance to liquefaction. This may be attributed to an increase in the lateral pressure coefficient $K_0$.

-6. lateral pressure coefficient and overconsolidation ratio

A higher overconsolidation ratio - defined as the ratio of the maximum past consolidation stress and the effective consolidation stress - implies a higher lateral pressure coefficient $K_0$. This reflects an increase in the mean confining pressure:

$$\sigma_{m0}' = \sigma_{v0}' \left(1 + 2K_0 \right)/3$$  \hspace{1cm} (1)

The higher this $\sigma_{m0}'$, the higher the resistance to liquefaction. [see figure 3]

Both sustained strains and higher overconsolidation ratios have certainly an effect on the grain structures, so that it is not so likely that slip between the particles will occur.

-7. duration of shaking

The application of cyclic stresses must be long enough for the stress to accumulate in the pore water for liquefaction. A minimum of stress cycles is required for $u$ to reach $\sigma$. 

7
8. intensity of shaking

The intensity of the earthquake is reflected in the simple relationship for the applied shear stress on a soil column over a soil particle at depth $d$:

$$\tau_n = C \times (\gamma d / g) \times a_{\text{max}}$$

(2)

$$\tau_n = C \times (\gamma d) \times n$$

where

- $\gamma$ = unit weight of the soil,
- $g$ = acceleration of gravity,
- $a_{\text{max}}$ = maximum acceleration of the earthquake,
- $n$ = seismic coefficient of the earthquake,
- $C$ = constant $< 1$. 


4. METHODS TO EVALUATE LIQUEFACTION POTENTIAL

In general there are two methods to evaluate the liquefaction potential of saturated sand layers.

1. Methods based on the performance of sand deposits during previous earthquakes, involving in-situ testing.

For these methods measurements are required at sites where liquefaction is known to have taken place, during or just after an earthquake.

Usually a relationship is found between the stress ratio $\tau/\sigma'_0$ and $N$, the number of blows in the Standard Penetration Test (SPT), which is an indication for the relative density $D$, of the soil.

The cyclic stress ratio that is required for liquefaction is then:

$$\frac{\tau}{\sigma'_0} = 0.65 \times \frac{a_{\text{max}}}{g} \times \frac{\sigma'_0}{\sigma'_0} \times r_d$$  \hspace{1cm} (3)

where $a_{\text{max}}$ = maximum earthquake acceleration at the surface

$\sigma'_0$ = total overburden pressure on considered sand layer, before cyclic stresses are applied

$\sigma'_0$ = effective overburden pressure, before cyclic stresses

$r_d$ = a stress reduction factor varying from 1 at the surface to 0.9 at 10 m depth

This is essentially the same formula as (2), given in the previous section 'FACTORS'. The values are plotted versus $N$ (corrected for the overburden pressure). [see figure 4]

A disadvantage is that there are not sufficient reliable data available yet, especially for high stress ratios. This method does not count for the duration of the earthquake, which is certainly a factor to be considered.

Although the penetration resistance (relative density) may not be an appropriate value to evaluate liquefaction potential of soil layers, an increasing value of the stress ratio $\tau/\sigma'_0$ has the same effect on penetration resistance as on other factors [see table 1.

The parameter $\tau/\sigma'_0$ can be correlated with other soil parameters. Arulmoli et al. (1981) investigated on electrical characteristics of soil, Marchetti used a so called flat dilatometer test. More
recent is the correlation with CPT-values instead of SPT-values. The CPT has some advantages over the SPT. It provides data more rapidly, it provides a continuous record of penetration resistance and its data are more reliable for various reasons. The main disadvantage is however that CPT data to predict resistance to liquefaction are much scarcer than SPT data. Correlations between SPT- and CPT-values may be used for the evaluation of liquefaction potential using CPT-values. [see figure 5] [see lit. 4]

-2. Methods based on evaluation of stress conditions causing liquefaction in the field and in laboratory tests.
These methods are more challenging, they involve two independent determinations:
- evaluation of the cyclic stresses to which the soil is subjected during an earthquake
- evaluation of the cyclic stresses that cause liquefaction in laboratory tests

There is a general working procedure in these methods:
a. Evaluation of earthquake induced stresses.
b. Conversion of irregular stress application into an equivalent uniform stress application.
c. Evaluation of factors affecting liquefaction potential.
d. Development of laboratory test procedures.
e. Evaluation of the effect of sampling.

-a. Evaluation of earthquake induced stresses.
Methods that consider the pore water pressure rise lead to direct evaluation of the liquefaction potential (Martin, 1975 and Finn, 1976). These methods require more material properties. In general though, the build-up can be disregarded without doing harm to the computed stresses in a soil deposit and this gives the opportunity to use more simplified methods (Seed and Idriss, 1967 and 1971).
b. Conversion of irregular stress application into an equivalent uniform stress application. There are three methods:

- visual inspection (with experience a surprisingly good method)
- cumulative damage approach, a black box method (Donovan and Valera 1976)

Valera and Donovan show that whatever method is used, it has little effect on the final result.

-c. Evaluation of factors affecting the liquefaction potential.
At first only the density of the deposits was considered to affect the liquefaction potential, but it is recognised that there are many more factors. [see section 3]

d. Development of laboratory test procedures.
Because of the lack of sufficient field data there is a need to generate more stress conditions for liquefaction by laboratory tests. A lot of difficulties must be overcome to transfer test results to the actual situation in the field.
Two types of test are common, (1) the cyclic triaxial test and (2) the simple shear tests.

1. Cyclic triaxial tests with controlled deviator stress. These tests are still widely used but have considerable limitations.
A sample is saturated under isotropic confining stress. [see figure 6] During the test the deviator stress is cycled sinusoidally, resulting in the following stress conditions in the sample:

\[ \sigma_1 = \sigma_o + \sigma_0 \sin(2\pi N) \]  
\[ \sigma_2 = \sigma_3 = \sigma_o \]

where \( \sigma_1 \) = principal stress \( \sigma_0 \) = deviator stress
\( \sigma_o \) = initial octahedral stress \( \sigma_{oct} \) = octahedral stress during the test
\( N \) = load cycle


Limitations of this test method include:

- a. reproduction of level ground condition
- b. stress concentrations at the top and the base of the sample
- c. contraction (necking) of the sample must be considered
- d. principal stresses rotate during the two halves of the loading cycle
- e. the intermediate principle stress \( \sigma_{\text{int}} \) is not constant

2. Torsional simple shear test.

This test permits a sample to develop shear strains \( \gamma \) on top at linear rotation \( \theta \). [see figure 7]

The confining stresses \( \sigma_h \) and \( \sigma_v \) are not necessarily isotropic. The shearing stress \( \tau_{\text{hv}} \) is applied independently. In this simple shear test the objections a, b and e to triaxial tests have been overcome. Objections c and d are not applicable. More general difficulties however, remain:

- a. preparation of representative samples
- b. development of uniform shear stresses and strains

Instead of a torsional shear device, it is also possible to use a device with long shallow samples.

-e. Evaluation of the effect of sampling.

Undisturbed samples don't exist. The main problem in sampling seems to be the changing density when taking and transporting the sample. Loose sands will densify and yield higher resistance to liquefaction; for dense sands the opposite effect occurs. Strength increases due to strain history seem to vanish when sampling, also cementation effects from sustained loading gets lost and also the effect of increased \( K_0 \) disappears when sampling. Altogether relatively dense samples will show lower resistance to liquefaction than when still in-situ. [see figure 8]
5. PROPAGATION OF LIQUEFACTION

When, during an earthquake, initial liquefaction occurs in a certain soil layer this liquefaction must propagate throughout the layer or to other overlying layers, before actual failure of the dike occurs. This propagation needs time and may continue even after the earthquake shaking has stopped.

a. Propagation during the earthquake

When a sand layer in the base of a dike liquefies, the process will start near the centre of the base, where the earthquake induced shear stresses are much stronger than elsewhere along the base. [see figure 9] The liquefaction progresses somewhat faster in upstream direction than downstream. [see lit. 2, pg. 260] Hence, on the downstream side is the largest non-liquefied zone to maintain dike stability. [see figures 10 and 11]

There are two ways now in which the dike may fail. Initially the water pressure on the relatively impervious upstream side of the dike may maintain the stability of the outer slope, but if this pressure is not sufficient, the outer slope may still fail along a slip circle that passes through the liquefied layer. [see figure 12]

The second way of failure is that outer slope stability is maintained until the non-liquefied zone on the downstream side has become so small that the entire dike section slides downstream.

b. Propagation after earthquake

An earthquake may cause initial liquefaction in somewhat deeper layers, not causing damage to the construction immediately, due to rapid dissipation of the earthquake induced pore water pressures. However, high pore water pressures in deeper layers, produced by an earthquake, may result in upward flow of water, causing liquefaction in overlying layers, minutes after the earthquake shaking has stopped. This happened e.g. in the Niigata earthquake of 1964 where initial liquefaction during the earthquake occurred in layers between 15 and 40 feet depth and caused liquefaction in layers of 3 feet and 1 foot depth at 3 and 12 minutes after the earthquake had stopped. [see figure 13]
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   New York, U.S.A., 1973

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   A. Kézdi
   Amsterdam, Netherlands, 1974

4. Evaluation of the dynamic characteristics of sands by in-situ testing techniques
   H.B. Seed, Revue française de géotechnique, no. 23
   Paris, France, 1983
fig. 1 Idealized earthquake loading on soil particle

fig. 2 Influence of density on liquefaction potential

fig. 3 Influence of overconsolidation ratio on liquefaction
Figure 4: Correlation between stress ratio causing liquefaction in the field and penetration resistance of sand (after Seed et al.).

Figure 5: Proposed correlation between liquefaction resistance of sands for level ground conditions and cone penetration resistance.
**fig. 6** STRESS CONDITION DURING CYCLIC TRIAXIAL TEST
SHAPES OF SAMPLE, TORSIONAL SIMPLE SHEAR

\[ \gamma = \frac{T_z}{2 \pi r} \theta = \frac{3.55}{2 \pi} \theta \]

**Fig. 7** STRESS CONDITION, TORSIONAL SIMPLE SHEAR
Poorly graded sand; test fill compacted with vibratory roller

- Piston samples
- Hand-trimmed block samples (G.E.I. method)

All samples tested with confining pressure of 2 to 3 tons/sq ft

fig. 8a Influence of method of sampling on cyclic loading resistance of dense sand

fig. 8b Effect of sample disturbance on cyclic loading resistance of dense sand
Layer susceptible to liquefy during earthquake under a dike

Progressive liquefaction analysis. Determination of zone of liquefaction after 2 seconds of ground motion. (a) Initial vertical normal stress, \( \sigma' \). (b) Initial shear stress \( \tau_0 \). (c) Ratio: \( \tau_0 / \sigma' \). (d) Liquefaction analysis.
fig. 11  Progressive liquefaction
fig. 12  Outer slope failure due to liquefaction

fig. 13  ANALYSIS INDICATING PROGRESSIVE CHANGES IN PORE WATER PRESSURE WITH TIME FOLLOWING EARTHQUAKE
Table 1 Factors affecting soil liquefaction, stress ratio causing liquefaction and penetration resistance $N$

<table>
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<th>effect on stress ratio required for liquefaction</th>
<th>effect on penetration resistance</th>
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<td>higher relative density</td>
<td>increases</td>
<td>increases</td>
</tr>
<tr>
<td>increased stability of structure</td>
<td>increases</td>
<td>increases</td>
</tr>
<tr>
<td>increase in time under pressure</td>
<td>increases</td>
<td>probably increases</td>
</tr>
<tr>
<td>increase of lateral pressure coefficient</td>
<td>increases</td>
<td>increases</td>
</tr>
<tr>
<td>prior seismic strains</td>
<td>increases</td>
<td>probably increases</td>
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EVALUATION OF LIQUEFACTION RESISTANCE AND THE IMPLICATIONS FOR THE DESIGN OF DIKES

M. Tonneijck
December 1987
EVALUATION OF LIQUEFACTION RESISTANCE AND THE IMPLICATIONS FOR THE DESIGN OF DIKES

This report is written as preliminary study for my final thesis on the design of the coastal dike at the Costa Oriental de Lago Maracaibo.

It was carried out at the Delft University of Technology at the faculty of Civil Engineering, Department of Geotechnics, under supervision of Prof. Dr. A. Verruijt.

An introduction to this preliminary study was already written earlier this year [1].

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EVALUATION OF LIQUEFACTION RESISTANCE AND
THE IMPLICATIONS FOR THE DESIGN OF DIKES

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EVALUATION OF LIQUEFACTION RESISTANCE AND THE IMPlications FOR THE DESIGN OF DIKES

1. INTRODUCTION
Earthquakes develop cyclic stresses in ground, such that soil particles are subjected to additional shear stresses. These additional shear stresses cause slip of soil particles and a subsequent tendency to volume decrease, that results into a pore water pressure rise. This procedure repeats every stress cycle that is developed by the earthquake until all the effective stress has been transformed to pore water pressure: the soil has lost its shear strength and behaves as a thick viscous liquid, in other words, liquefaction. [see lit. 1, pg.3]

It is thus clear that soil layers in a seismic area are subjected to the danger of possible liquefaction. This danger concerns relatively shallow fine sand or silty sand layers. The main factors that influence the liquefaction potential are:

1. magnitude of applied shear stress, compared to the available shear strength (earthquake intensity, magnitude)
2. duration of the earthquake in the number of applied stress cycles (earthquake magnitude involves the duration)
3. relative density, D,
4. particle size

In the procedure of the evaluation of the liquefaction potential of certain soil layers in a project area the first steps must be in the determination of the earthquake magnitude and the expected intensity in the project area. This determination should yield a certain ground acceleration from which the applied shear stresses can be estimated. The definition of earthquake magnitude includes the duration of the earthquake and thus the number of cycles.

These procedures deal with grounds under level conditions. In many engineering problems it is however the task to ensure the stability of slopes. Seed doesn't give explicitly a method for the design of such slopes, but his approach is adapted to yield certain requirements when building dikes in seismic areas.
2. EVALUATION PROCEDURE

2.1 DIRECT METHODS
In direct methods to evaluate the liquefaction potential the pore pressure build-up is followed till the residual pore pressure rise equals the initially present effective confining stress. These methods follow the real process of liquefaction closely (the tendency to volume decrease and the resulting pore pressure build-up) and lead to direct results. These methods give a good theoretical approach and understanding of the actual phenomenon and comply well with drained and undrained cyclic tests. (Martin, 1975) They remain however vulnerable to testing errors and they require ample knowledge of the soils concerned.

2.2 SIMPLIFIED METHODS
There are more simplified methods that disregard completely the process of pore pressure build-up, the best known still being the one developed by Seed and Idriss in 1971. This can be done easily as this pore pressure build-up does not have a significant influence on the computed cyclic shear stresses at the point of liquefaction. This is true assuming that the shear modulus of the ground \( G = \frac{\tau}{\gamma} \) (shear stress/shear strain) is a function of the shear strain only. Changes in the shear modulus, i.e. in the shear stiffness characteristics of the sand, when the pore water pressure rises progressively, are not taken into account.

This assumption is hazardous in the sense that close to the point of liquefaction, pore pressure rise suddenly stronger and the soil suddenly undergoes larger deformations (strains 20% or more). [lit. 2 and 3]

2.3 ASSESSMENT OF THE MAGNITUDE OF THE APPLIED SHEAR STRESSES
The most important simplified assessment of the magnitude of the applied shear stresses is still the one that was developed by Seed and Idriss in 1971. They compute the cyclic shear stress developed by an earthquake from:

\[
\tau_{eq} = 0.65 \times \frac{R_h}{g} \times a_{max} \times r_d
\]  

(1)
If there would be pure balance and if the considered column of soil would be rigid and if the maximum acceleration would occur at the ground surface this computed value of the cyclic shear stress would be \( \frac{\gamma h}{g} \times a_{max} \). For the equivalent uniform shear stress based on all wave components, 65% of the maximum shear stress is counted. The factor \( r_s \) is a reduction factor for the soil column not to be rigid. In fact these factors \( r_s \) and 0.65 are the litter boxes for all uncertainties in this formula. [see figures 2 and 3]

Before the applied shear stress can thus be computed seismic data and earthquake propagation characteristics of the area have to be evaluated. This evaluation should supply the \( a_{max} \) of formula (1). Direct methods use earthquake accelerograms directly to assess the applied shear stress per loading cycle and the subsequent pore pressure rise.

2.4 THE ACTUAL EVALUATION OF THE LIQUEFACTION POTENTIAL

To actually evaluate the liquefaction potential of a certain soil layer the applied cyclic shear stress is at first put over the initially present effective confining stress, \( \sigma'_0 \). Thus the extra applied shear stress is in fact compared to the actually present shear strength of which \( \sigma'_0 \) is a representative parameter. This stress ratio \( \sigma / \sigma'_0 \) will be the important parameter in the evaluation of the liquefaction potential. It has the advantage that it takes into account the depth of the soil layer and the depth of the water table. It is clear that the smaller the stress ratio, the greater the resistance to liquefaction.

To find out if a certain soil layer might liquefy the expected stress ratio, \( \sigma / \sigma'_0 \) must be compared to either laboratory cyclic test data on soil samples or to in-situ soil data of previous earthquakes with comparable conditions.
3. COMPARISON TO LABORATORY TEST DATA OR IN-SITU SOIL DATA

3.1 COMPARISON TO LABORATORY DYNAMIC TEST DATA

The stress ratio under field conditions \( \frac{\tau}{\sigma_0'} \), is to be compared to stress ratios measured in dynamic laboratory tests:

- The simple shear test yields:
  \[
  \left( \frac{\tau}{\sigma_0'} \right)_{ss}
  \]

- The cyclic triaxial test yields:
  \[
  \left( \frac{\frac{1}{2} \sigma_0'}{\sigma_3'} \right)_{ct}
  \]

Ample testing has rendered sufficient stress ratios \( \left( \frac{\tau}{\sigma_0'} \right)_{ss} \) and \( \left( \frac{\frac{1}{2} \sigma_0'}{\sigma_3'} \right)_{ct} \) that cause liquefaction in the soil samples. The value for the stress ratio that would cause liquefaction in the field \( \left( \frac{\tau}{\sigma_0'} \right)_{tl} \) can be estimated from these test data. [see figures 4 and 5]

In 1971 Seed already gave the factor \( c_r \) between \( \left( \frac{\tau}{\sigma_0'} \right)_{ss} \) of simple shear tests and \( \left( \frac{\frac{1}{2} \sigma_0'}{\sigma_3'} \right)_{ct} \) of cyclic tests, where the simple shear test was supposed to represent field conditions. This was confirmed by Finn (1971) and Castro (1975). Later studies show that the unidirectional shaking of simple shear test causes liquefaction at stress ratios that are in general about 10% higher than stress ratios causing liquefaction in the field. [see figure 6 and table 1] Thus,

\[
\left( \frac{\tau}{\sigma_0'} \right)_{tl} \approx 0.9 \times \left( \frac{\tau}{\sigma_0'} \right)_{ss} \approx c_r \left( \frac{\frac{1}{2} \sigma_0'}{\sigma_3'} \right)_{ct}
\]

Where \( r_d \) and the 0.65 are the litter boxes for the uncertainties in the developed shear stress in the field, this factor \( c_r \), that is directly dependent on the lateral pressure coefficient \( K_p \), hides all the effects of sampling, such as changes in density when sampling, loss of cementation, strain history. These uncertainties introduced by sampling [see also lit. 1, pg.12] are the main reasons why there is a need for comparison to in-situ test data.
3.1.1 PROCEDURE FOR COMPARISON TO LABORATORY TEST DATA

Ample testing with cyclic triaxial tests has yielded graphs as given in figures 4 and 5. The four main factors that affect the liquefaction potential are involved in these tests. The intensity, magnitude of the earthquake is imitated in the magnitude of the applied shear stresses. The other three are obvious.

Being interested in the liquefaction potential of a soil at a certain site with grain size $D_{50}$ and relative density $D_r$, soil samples should be taken and in cyclic triaxial tests it appears that liquefaction occurs when the stress ratio $(\frac{\sigma_c}{\sigma_3})_{1,50} = y$, this value $y$ to be determined from the figures 4 and 5. The stress ratio that would cause liquefaction in the field is:

$$\left(\frac{\tau}{\sigma_0'}\right)_{1,50} = \frac{\frac{1}{2} \sigma_{dc}}{\sigma_3} * c_r * \frac{D_r}{50} \quad (2)$$

From this follows that if an earthquake with magnitude $M$ ($\rightarrow N$ cycles) develops a cyclic stress

$$\tau_{IM} = y * c_r * \frac{D_r}{50} * \sigma_0' \quad (3)$$

the soil would liquefy.

You suspect a certain soil layer at depth $h$; from this depth the rigidity factor $r_d$ is determined. The earthquake develops:

$$\tau_{IM} = 0.65 * \gamma_h * \frac{a_{max}}{g} * r_d \quad (1)$$

$$\sigma_0 = \gamma_h = \gamma_d h_w + \gamma_n (h - h_w) \quad (4)$$

Then

$$\sigma_0' = \sigma_0 - u$$

$$= \gamma_d h_w + (h - h_w)(\gamma_n - \gamma_w) \quad (5)$$

Equating the stress developed by the earthquake (1) and the stress that causes liquefaction (3): the earthquake that the soil layer is able to withstand is

$$\frac{a_{max}}{g} = (c_r * D_r) * \left( \frac{y}{0.65 \times \gamma_d \times 50} * \frac{\sigma_0'}{\sigma_0} \right) \quad (6)$$
3.2 COMPARISON TO IN-SITU SOIL DATA

To circumvent the problems in obtaining truly undisturbed samples and all the consecutive problems in determining the true liquefaction potential of ground under certain site dependant conditions, there is a tendency now towards evaluation with help of in-situ soil data.

From the above it is still clear that the main factor of resistance to soil liquefaction is the relative density \( D_r \) of that soil. The principle of the evaluation of the liquefaction potential by in-situ testing is based on indirect measurement of this relative density and probably these in-situ tests involve at least partly effects of aging, strain history, the lateral pressure coefficient \( K_o \) and the overconsolidation ratio OCR as well.

3.2.1 SPT-PROCEDURES

The purpose of the Standard Penetration Test is to derive soil parameters from the number of blows \( N \) that is determined by this test, taking into account the type of soil. In particular the SPT is in use to determine the bearing capacity of soils. In this way, also the liquefaction potential of a certain soil can be evaluated as one of the parameters, using the \( N \)-value of that particular soil.

Since especially the Americans have investigated a lot on liquefaction a great data base exists for the purpose of the evaluation of liquefaction potential using SPT data. After the Niigata and the Alaska earthquakes in 1964 and really since 1969 this data base has grown.

In fact, in this evaluation the SPT-value \( N \) stands for the relative density \( D_r \) of the soil concerned; there is a direct relationship between relative density and liquefaction potential: the denser the soil (the higher the \( N \)-value), the more resistance to liquefaction.

Since the penetration resistance reflects both the in-situ relative density and the effective stress, this \( N \)-value has to be corrected for the effective overburden pressure, the corrected value being called \( N_c \), the correction factor \( C_N \) :
For values of the effective overburden pressure dependant factor $C_N$ see figure 7.

application of the SPT procedure

This way of evaluation yields charts on which the N-value is plotted against the above-mentioned stress ratio $\tau / G'$. Either higher stress ratios or lower N-values (lower densities) give liquefaction, taking into account the type of soil ($D_{90}$) and the duration of the earthquake (M). [see figure 8] As a basis for the evaluation an $M = 7.5$ earthquake is taken, with 15 representative uniform cycles at a level of $0.65 * \tau_{\text{max}}$. Earthquakes with smaller magnitudes have a shorter duration and a smaller number of representative cycles, e.g. 10 for an $M = 6.75$ earthquake instead of the 15 for an $M = 7.5$ earthquake. In fewer cycles the pore pressure build-up will be smaller, so where an $M = 6.75$ earthquake may not achieve sufficient pore pressure build-up to cause liquefaction in 10 cycles, an $M = 7.5$ earthquake may do so in 15 cycles, even assuming that the intensity of the earthquakes (i.e. the applied cyclic shear stress) would be the same. [see figures 9 and 10] It is thus obvious that an earthquake with a smaller magnitude $M$ needs to develop a higher cyclic shear stress at a certain site to achieve the full pore pressure build-up, the intensity of the smaller earthquake must be greater because of a smaller focal distance or because of different damping characteristics of the interjacent ground.

Table 2 shows that an $M = 7.5$ earthquake needs to develop a shear stress that is $1 / 0.89 = 1.12$ times bigger than the cyclic shear stress of an $M = 8.5$ earthquake to cause liquefaction. This is the way figure 11 is constituted from the empirical graph of figure 8.

confirmations of the American SPT-investigations

Seed [lit. 5] reports numerous confirmations of his liquefaction chart [see figure 8] of $M = 7.5$ earthquakes. Earthquakes in China (Haicheng, 1974, $M = 7.3$ and Tangchan, 1976, $M = 7.6$), in Guatemala (1976, $M = 7.6$), in Argentina (1977, $M =
7.4) and in Japan (Miyagiken-Oki, 1978, M = 7.4) caused liquefaction. Places where liquefaction occurred or where it (apparently) didn’t occur are plotted in figures 12a, b, c, d. The proposed relationship between SPT and stress ratio holds firmly.

objections to SPT-procedures
Several objections to SPT-procedures exist; they can be split up into technical objections to the performance of the tests itself and to the subjective interpretation of the test results.

objections to technical performance
The SPT, an empirical dynamic penetration test, was developed in the 1920’s in the United States and became widely used. A standard split spoon soil sampler is being driven 12 inches deep into the bottom of a borehole by a freely falling hammer. The number of blows to drive the sampling tube into the ground is recorded as the penetration resistance N. The test is standardized as shown in figure 13. It is common practice to repeat the test at 1.50 m vertical intervals. The performance of this supposedly standardized test is however such that it allows seriously varying conditions. The most important variation is found in the way the hammer falls 'freely'. Usually the hammer is connected to a rope that is wound round a drum, leaving the hammer only 40 to 60% of the energy of a really freely falling hammer, depending on the rope and the number of turns round the drum. Various other factors that are not sufficiently standardized affect the test result:
1. the use of drilling mud or casing to support the walls of the borehole
2. the use of a hollow stem auger or casing plus water
3. the size of the drill hole
4. the length of the drive rods, loss of energy
5. the depth range where measured (i.e. 0-12in or 6-18in)

The test needs more standardization and older test results need to be adapted.
objections to subjective interpretation
The great experience and the enormous data base concerning penetration resistance and other soil parameters has led to a lot of procedures that try to derive all desirable soil parameters from this penetration resistance in stead of using the appropriate tests. This practice carries with it the danger of copying such procedures by not sufficiently experienced interpreters.
In spite of these objections it is in the above sufficiently explained why the SPT is considered to be useful to evaluate the liquefaction potential.

3.2.2 CPT-PROCEDURES
The purpose of the Cone Penetration Test or the Dutch cone test is basically the same as for the SPT: to derive soil parameters from its test results, in particular the bearing capacity and settlement characteristics. The CPT is a static penetration test that measures the penetration resistance of a small 60° cone with a cross section area of 10 cm. Above the cone a 150 cm² jacket is provided to measure side friction independantly from cone resistance.
Different from the SPT is that the CPT yields quick and continuous test results and is much better standardized. The major disadvantage in using CPT-data for the evaluation of liquefaction potential is the very limited data base, compared to the data base of SPT.
Also when using CPT-values for the evaluation of liquefaction potential it must be kept in mind that considering the type of soil the CPT-value represents the relative density, the most important factor in the resistance to liquefaction.
As long as this data base remains limited the liquefaction potential at new construction sites where only CPT's are made, can be estimated by using relationships between CPT and SPT. These relationships are relatively well known. The direct advantage of CPT can be used as well as the extended data base of SPT. It remains however undesirable to introduce another correlation into the evaluation procedure.

correlation between SPT-values and CPT-values
Schmertmann gives an approximate relationship between CPT-values
and SPT-values \( N \) for both clean sands (\( D_{50} > 0.25 \text{ mm} \)) and for sands that contain smaller silt particles (\( D_{50} < 0.15 \text{ mm} \)):

**clean sands:** \( q_e = 4 \text{ to } 5 \text{ N} \) \( (q_e \text{ in kg/cm}^2) \) \( (8) \)

**silty sands:** \( q_e = 3.5 \text{ to } 4 \text{ N} \) \( (q_e \text{ in kg/cm}^2) \) \( (9) \)

Like in the SPT, the penetration resistance of the Dutch cone reflects not only the relative density of the soil, but also the present effective stress. So, like the standard penetration resistance \( N \), the cone resistance \( q_e \) has to be corrected as well.

Where critical boundaries separating liquefiable from non-liquefiable conditions in terms of corrected standard penetration resistance \( N_l \), these boundaries must be transferred to conditions in terms of \( q_{el} \). Then, using for clean sands

\[ q_{el} = 4 \text{ to } 5 N_l \quad (q_{el} \text{ in kg/cm}^2) \] \( (10) \)

and for silty sands

\[ q_{el} = 3.5 \text{ to } 4.5 N_l \quad (q_{el} \text{ in kg/cm}^2) \] \( (11) \)

it follows then (from (7) to (11))

\[ q_{el} = C_N \times q_{el} \] \( (12) \)

Testing has shown that using relationships (10) and (11) instead of (8) and (9) is not entirely correct, because of a different combined influence of relative density and effective stress in the penetration resistance measured in SPT's and CPT's. This problem is met with by using a slightly different value for \( C_N \). [see figure 7]

Finally the chart of figure 14 show which grounds are susceptible to liquefaction when knowing cone penetration resistance and particle size, effective overburden pressure being in the order of 1 kg/cm\(^2\) or 100 kN/m\(^2\).
application of the CPT procedure

The actual evaluation of liquefaction potential by CPT is very much similar to the procedure with SPT.

Figure 14 shows the charts that are used to determine whether a certain sand (-silt) layer may liquefy or not for $M = 7.5$ earthquakes.

Having CPT-diagrams at your disposal, only, no ground samples from borings, the friction compared to the cone penetration resistance offers the possibility to determine whether a certain layer is susceptible to liquefaction at all. Sands and to a lesser extend silty sands tend to show high cone penetration resistances. A relatively low friction indicates a low density and thus possible liquefaction, which is clearly shown in figure 15.

Expected stress ratios $\tau / \sigma_0$ can be calculated with formula (1), the boundaries for other earthquake magnitudes with table 2.
4. IMPLICATION OF LIQUEFACTION DANGER FOR THE DESIGN OF SLOPES

4.1 SEED'S CONCEPT UNDER NON-LEVEL GROUND CONDITIONS

Suppose that in a situation without earthquake/liquefaction the failure circle ABCD of figure 16 would be the probable slip circle with the smallest factor of stability F, then the possible liquefaction in the sand layer may cause the slope to fail as a result of the loss of shear strength over BC.

Under level ground conditions the actually present strength of the soil, expressed as $(\tau / \sigma'_0)$, is to resist the earthquake induced $(\tau / \sigma'_0)_{eq}$. This is amply described in the above chapters, but a slope introduces an extra shear load, that is not counted for in Seed's conception.

Introducing the slope design in seismic areas into the evaluation the criterion must become:

$$\frac{\tau_{eq} + \tau_{slope}}{\sigma'_0} < \left(\frac{\tau}{\sigma'_0}\right)$$  \hspace{1cm} (13)

Thus, in fact it is so that, in presence of dikes, the soil strength $(\tau / \sigma'_0)$, should be reduced compared to its strength under level ground conditions. This reduction because of the actually present slope depends on the slope's angle and must be expressed in a shear stress - effective stress - ratio: $(\tau / \sigma')$. The shear stress along the slip circle [see figure 16] amounts:

$$\tau = \frac{1}{F} \left( c + \sigma'_0 \tan \varphi \right)$$  \hspace{1cm} (14)

Since we deal with sands, the cohesion may be assumed zero, leaving

$$\tau = \frac{1}{F} \sigma'_0 \tan \varphi$$  \hspace{1cm} (15)

Using the stability factor F of Fellenius in wet conditions:

$$\tau = \frac{1}{\left( \frac{\gamma' - \gamma_w}{\gamma} \tan \varphi \right) \tan \alpha} \sigma'_0 \tan \varphi$$  \hspace{1cm} (16)
Then
\[ \left( \frac{\tau}{\tau_0'} \right) = 2 \tan \alpha \]  
(17)

It is obvious that steep slopes (large \( \alpha \)) reduce the shear strength of the soil considerably:
\[ \left( \frac{\tau}{\tau_0'} \right)_{\text{eq}} < \left( \frac{\tau}{\tau_0'} \right)_{\text{dike,} - 2 \tan \alpha} \]  
(18)

In the above the \( \left( \frac{\tau}{\tau_0'} \right) \) is either equivalent to a (corrected) number of blows \( N_1 \) (in the SPT) or a cone penetration resistance in kg/cm\(^2\) (CPT).

Concentrating on CPT-tests where the magnitude of \( \left( \frac{\tau}{\tau_0'} \right) \) is put equivalent to a cone penetration resistance in kg/cm\(^2\), it should be possible to require a higher CPT-value in seismic areas when a dike slope is present.

Simplifying figure 14 for sands, a CPT-value of 50 kg/cm\(^2\) corresponds to a soil strength expressed as the angle or stress ratio \( \left( \frac{\tau}{\sigma_0'} \right) = 0.1 \), for silty sands this is 50 kg/cm\(^2\) = 0.13. [see figure 17]

4.2 REQUIREMENTS FOR SLOPE DESIGN

4.2.1 VARIATION ON SEED'S APPROACH
Looking into the consequences for the dike slope design, it is obvious that the commonly usual slope of 1:3 would be too pretentious if only Seed’s relationship under level ground conditions would be used.

A 1:3 slope means a reduction of \( 2 \tan(1/3) \approx 0.69 \) in the soil strength, which doesn’t make sense in the Seed’s concept, since the plot of figures 14 and 17 are no longer under 45° in that range.

Requiring a maximum soil strength reduction of ‘only’ 0.1 in terms of stress ratio (which is still much in the range under consideration) leads to dike slopes of 1:10, which doesn’t make much sense either in areas with limited space.

Hence, it is logic to require high values for the strength of the subsoils: sands in seismic areas with the possibility of liquefaction should have cone resistances of 170 kg/cm\(^2\) (17 MN/m\(^2\)) and silty sands 125 kg/cm\(^2\) (12.5 MN/m\(^2\)). [see figure 17] All
impurities of cohesive soils in the susceptible soil layer as well as the increase of the height of the dike (increase of effective stress in the susceptible soil layer) improve the situation considerably as these factors increase the resistance to liquefaction.

4.2.2 ALTERNATIVE APPROACH OF CASTRO

Castro has made investigations into 'undisturbed' soil samples, i.e. samples were taken and artificially put back into their local conditions. These samples were brought to liquefaction and the residual strength was measured. It appeared to be well possible that residual strengths of \( c = 20 \) kPa could be required. From the following calculations, it is clear that soils with a residual cohesional strength of \( c = 20 \) kPa are able to withstand an overburden in the form of a 1:3 dike slope [see figure 18].

\[
\begin{align*}
1/2 \gamma (h + d)^2 - 1/2 \gamma d^2 &= c.1 \\
1/2 \gamma h^2 + \gamma h d &= c.1 \\
1/h &= (\gamma h)/(2c) (1 + 2d/h)
\end{align*}
\]

For shallow sand layers at susceptible locations where the depth-height-ratio is very limited, the possible dike slope depends mainly on the height of the dike itself. For a 6m high dike and the above mentioned residual cohesional strength of \( c = 20 \) kPa a 1:3 slope is well possible:

\[
1/h = (20*6)/(2*20) = 3
\]

This approach of Castro is given here to stress, that in seismic areas with possible liquefaction, it is possible to construct dikes with commonly used slopes. Of course this approach brings up again the discussion on the topic of laboratory testing versus in-situ testing.

4.3 CONCLUSION

In the design of dikes in seismic areas with possible liquefaction it is possible to construct dikes with commonly used slopes of 1:3 under the condition that susceptible sand or silty
sand layers fulfill certain requirements. Relying on Seed's approach and using CPT, the susceptible layers should have a sufficiently high cone penetration resistance of 170 kg/cm$^2$ (17 MN/m$^2$) for pure sands and 125 kg/cm$^2$ (12.5 MN/m$^2$) for silty sands. The presence of more cohesive soils or an increased effective stress influence these figures favourably. If these figures can't be reached in the susceptible layers, considerably smoother slopes are necessary.
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Evaluation of the liquefaction potential using field performance data
**Fig. 1** Time History of Shear Stresses during Earthquake

**Fig. 2** Determination of Maximum Shear Stress

**Fig. 3** Range of Values of $r_d$ for Different Soil Profiles
fig. 4 STRESS CONDITIONS CAUSING LIQUEFACTION OF SANDS IN 10 CYCLES

fig. 5 STRESS CONDITIONS CAUSING LIQUEFACTION OF SANDS IN 30 CYCLES
fig. 6a COMPARISON OF SHAKING TABLE AND TRIAXIAL TEST RESULTS

fig. 6b RELATIONSHIP BETWEEN $c_r$ AND RELATIVE DENSITY
**fig. 7** Recommended Curves for Determination of $C_d$

**fig. 8a** Correlation between Field Liquefaction Behavior of Sands ($D_{50} > 0.25$ mm) under Level Ground Conditions and Standard Penetration Resistance (All Data)

**fig. 8b** Correlation between Field Liquefaction Behavior of Silty Sands ($D_{50} < 0.15$ mm) under Level Ground Conditions and Standard Penetration Resistance
fig. 9  REPRESENTATIVE CURVE FOR RELATIONSHIP BETWEEN CYCLIC STRESS RATIO AND NUMBER OF CYCLES TO LIQUEFACTION

fig. 10  Two earthquakes that possibly have the same intensity at the construction site.
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fig. 12a Comparison of Empirical Chart for Predicting Liquefaction with Data from Haicheng and Tengshan Earthquakes

fig. 12b Comparison of Empirical Chart for Predicting Liquefaction with Data from Guatemala Earthquake, 1976
fig. 12c  Comparison of Empirical Chart for Predicting Liquefaction with Data from Argentina Earthquake, 1977

fig. 12d  Correlation between Field Liquefaction Behavior of Sands ($D_{50} > 0.25$ mm) under Level Ground Conditions and Standard Penetration Resistance

fig. 13  Standard Penetration Test
fig. 14 Proposed Correlation between Liquefaction Resistance of Sands for Level Ground Conditions and Cone Penetration Resistance.

fig. 15 Soil Classification Chart for Electric Cone Showing Proposed Zone of Liquefiable Soils.
fig. 16 Dike slope over sand layer susceptible to liquefaction

fig. 17 Simplified relationship between liquefaction resistance and cone penetration resistance, with extra requirement for the presence of dike slopes.
fig. 18 Castro's approach with residual cohesional strength after liquefaction
Table 1 Various factors $c_r$

<table>
<thead>
<tr>
<th>Source</th>
<th>Equation</th>
<th>$c_r$ for $K_o = 0.4$</th>
<th>$c_r$ for $K_o = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finn et al. (1970)</td>
<td>$c_r = \frac{1 + K_o}{2}$</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Seed and Peacock (1971)</td>
<td>Varies</td>
<td>0.55 to 0.72</td>
<td>1.0</td>
</tr>
<tr>
<td>Castro (1975)</td>
<td>$c_r = \frac{2(1 + 2K_o)}{3\sqrt{3}}$</td>
<td>0.69</td>
<td>1.15</td>
</tr>
</tbody>
</table>

TABLE 2.—Ratios of the Ordinates of the Curve in Fig. 12, Relative to the Ordinate Corresponding to 15 Cycles

<table>
<thead>
<tr>
<th>Earthquake magnitude, $M$</th>
<th>Number of representative cycles at 0.65, $\tau_{max}$</th>
<th>$\frac{[\tau_{max}/\alpha']<em>{M-3.5}}{[\tau</em>{max}/\alpha']_{M-7.5}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-1/2</td>
<td>26</td>
<td>0.89</td>
</tr>
<tr>
<td>7-1/2</td>
<td>15</td>
<td>1.0</td>
</tr>
<tr>
<td>6-3/4</td>
<td>10</td>
<td>1.13</td>
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<td>6</td>
<td>5</td>
<td>1.32</td>
</tr>
<tr>
<td>5-1/4</td>
<td>2-3</td>
<td>1.5</td>
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