Mathematical modelling of morphological changes and hyperconcentrated floods in the Yellow River
Mathematical modelling of morphological changes and hyperconcentrated floods in the Yellow River

Proefschrift

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof. ir. K.C.A.M. Luyben,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen op
dinsdag 8 april 2014 om 12.30 uur

door

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This research has been financially supported by “the China Scholar Council (CSC)”, “Het Lamminga Fonds”, and the Sino-Dutch collaboration project “Effects of human activities on the eco-morphological evolution of rivers and estuaries” (08-PSA-E-01) funded by the Royal Dutch Academy of Sciences (KNAW) and the Chinese Ministry of Science and Technology (MOST) within the framework of the Programme of Scientific Alliances between China and the Netherlands.

Copyright © 2014, Wei LI
Published by: VSSD, Delft, the Netherlands
ISBN 9789065623539

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Hyperconcentrated flow/flood is a water-driven sediment transport phenomenon, which is characterized by high sediment concentrations between normal sediment-laden flow and debris/mud flow. In a hyperconcentrated flow, strong interactions exist between water flow, sediment, and river bed, which may not only change the flow rheological properties, but also affect the characteristics of sediment transport and bed deformation.

In the lower reach of the Yellow River (China), hyperconcentrated flows/floods are frequently observed (with sediment concentrations > 200 kg/m^3) and primarily Newtonian, turbulent flows (van Maren et al., 2009a). This reach is characterized by two special, and possibly unique, hydrodynamic and sediment transport phenomena that are associated with hyperconcentrated flow: a downstream increasing peak discharge (at a rate far exceeding the contribution from tributaries) during hyperconcentrated floods, and a downstream decreasing runoff due to water diversions. Assuming spatially continuous diversions along a constant-width channel, previous studies suggest a longitudinally convex bed at the equilibrium state. However, the validity of a convex bed profile, for discrete diversions in natural channels of longitudinally varying width, remains to be justified. Also, such equilibrium analysis does not reveal the morphological time scale (MTS) associated with water diversions. Moreover, though many explanations have previously been proposed for the peak discharge increase, they have focused on only one possible mechanism (e.g., bed roughness change, bed erosion, floodplain influences) and no consensus has been achieved. The underlying physics still remain largely unknown. Research efforts are therefore needed to further investigate these two issues, which comprise the main work of the present PhD research and this thesis.

For the study of the downstream peak discharge increase phenomenon, mathematical modelling is the main research method, together with the field data analysis. High-resolution morphodynamic modelling of complex fluvial processes, such as in a hyperconcentrated flood, has so far been limited by model accuracy or computational efficiency. In order to account for the strong interactions during the hyperconcentrated flood and to acquire accurate and efficient solutions in the field scale, a fully coupled morphodynamic model has first been developed using the finite volume method for structured grids. Physically, this model is based on the concept of non-capacity sediment transport, and it incorporates the effects of sediment density and bed
deformation on the flow (both in mass and momentum), as well as the influences of turbulence and sediment diffusions. Numerically, this model combines the high accuracy of high-order upwind schemes and the efficiency of centered schemes by the extension of a recent upwind-biased centered (UFORCE) scheme (Stecca et al., 2010) originally developed for clear flow and scalar transport over a fixed bed, to sediment-laden flows over an erodible bed. For stability, a two-stage splitting approach together with a second order Runge-Kutta method is used for the source terms. Moreover, the full set of governing equations is solved at one time to obtain synchronous solutions in mathematics. The model is verified in a number of dam-break tests, covering a wide range of complex (sediment-laden) flows. It is demonstrated to accurately simulate shock waves and reflection waves, as well as rapid bed deformations at high sediment transport rates.

Using this model, the relative role of bed roughness change and bed erosion on the downstream peak discharge increase is then investigated in schematized 1-D channels for two hyperconcentrated floods. The results reveal that although erosion effects may contribute to the downstream discharge increase (especially in case of extreme erosion), for most cases the increase is mainly due to a reduction in bed roughness during peak discharge conditions. Additionally, based on the concept of channel storage reduction, the effects of decreasing bed roughness and (very strong) bed erosion can be integrated in the explanation of the peak discharge increase. Later, this model is also applied to reveal the floodplain influences on the peak discharge increase in schematized 2-D channel-floodplain reaches. The results indicate that the cross-sectional changes of channel erosion and floodplain deposition during hyperconcentrated floods are often limited and that it is difficult to drive a peak discharge increase in the downstream direction.

For the study of the water diversion impact, a general theoretical framework is proposed to predict the equilibrium state of the fluvial system, which is applicable to both continuous and discrete water diversions in a longitudinally width-varying channel. Numerical experiments by the SOBEK-RE software (version 2.52.005, Delft Hydraulics, 2005) complement the MTS studies for water diversions. The effects of diversion intensity, diversion placement (discrete and continuous) and diversion schemes (pure water and water-sediment mixture) are also systematically studied. The present work confirms the previous findings that water diversions lead to a decrease of the equilibrium depth with respect to natural conditions and a convex bed in a constant-width channel. Moreover, it reveals that in a widening channel a convex bed also develops under conditions of water diversions, while convex, concave or quasi-linear beds may occur in a narrowing channel. Non-monotonic beds may develop in a strongly narrowing channel, depending on the diversion schemes. On a large spatial scale, diversion placement is less important for the equilibrium development. The
MTS for water diversions and natural development are very similar and large, indicating considerable influences of water diversions on river morphology. The present thesis advances our understanding of the long-term impact of water diversions on the evolution of a river.
Samenvatting

Hypergeconcentreerde stroming/hoogwater is een door water aangedreven sedimenttransport fenomeen dat wordt gekenmerkt door hoge sediment concentraties tussen die in een normale sedimentvoerende stroming en die in een landverschuiving/modderstroming. In de hypergeconcentreerde stroming bestaat een sterke wisselwerking tussen waterstroming, sediment en rivierbodem, welke niet alleen de reologische eigenschappen van stroming beïnvloedt, maar de kenmerken van sedimenttransport en morfologische verandering van de rivier.

In de benedenloop van de Gele Rivier (China), worden vaak hypergeconcentreerde stromingen/hoogwatergolven waargenomen (met sedimentconcentraties tot meer dan 200 kg/m³), in combinatie met voornamelijk Newtoniaanse, turbulente stroming (van Maren et al., 2009a). Dit deel van de Gele Rivier wordt gekenmerkt door twee speciale - en mogelijk unieke - hydrodynamische en sedimenttransport verschijnselen die samenhangen met hypergeconcentreerde stroming: een stroomafwaartse toename van de piekafvoer (meer dan te verklaren is uit toevoer vanuit zijrivieren) tijdens hypergeconcentreerde hoogwatergolven, en een stroomafwaartse afname van de jaarafvoer als gevolg van wateronttrekkingen. Uitgaande van een ruimtelijk continue onttrekking langs een rivier met een constante breedte, suggereren eerdere studies een in langsrichting neerwaarts convex evenwichtsligging van de rivierbodem. Echter, de geldigheid van dit convex profiel voor praktijksituaties, met discrete onttrekkingen in rivieren met een in langsrichting variërende breedte, moet nog onderzocht worden. Dergelijke evenwichtsanalyses laten echter niet de tijdschaal (MTS) van de morfologische aanpassingen aan de wateronttrekkingen zien.

Hoewel in het verleden vele verklaringen zijn gegeven voor de stroomafwaartse toename van de piekafvoer tijdens hooggeconcentreerde hoogwatergolven, lag de nadruk steeds op slechts één mogelijk mechanisme (bijv. verandering van bodemruwheid, sterke erosie, invloed van uiterwaarden) en is men het er nog steeds niet over eens wat nu precies de oorzaak is. De onderliggende fysica is nog grotendeels onbekend.

Deze twee aspecten worden in dit promotieonderzoek nader onderzocht en vormen het hart van dit proefschrift.

Wiskundige modellering en analyse van velddata vormen samen de belangrijkste onderzoeksmethode bij het thema "stroomafwaartse toename van piekafvoer tijdens
een hoogwatergolf’. Morfodynamische modellering in hoge resolutie van complexe fluviatiele processen, zoals hypergeconcentreerde stroming, is tot nu toe beperkt gebleven door onnauwkeurigheid van het model of inefficiëntie van de berekeningen. Om de sterke interacties tijdens hypergeconcentreerde stroming te verklaren, en nauwkeurige en efficiënte oplossingen te verkrijgen, is voor het eerst een volledig gekoppeld morfodynamisch model ontwikkeld, met behulp van de ‘finite volume’ methode met gestructureerde roosters. Fysisch is dit model gebaseerd op het concept van niet-evenwicht sedimenttransport, en bevat het de effecten van zowel sediment dichtheid als bodemliggingsverandering op de stroming (beide in de massa- en de impulsiebalans), alsmede de effecten op turbulentie en diffusie. Numeriek, combineert dit model de hoge nauwkeurigheid van hoger-orde-‘upwind’ schema’s en de efficiëntie van gecentreerde schema’s door een recent ‘upwind-biased centered’ (UFORE) schema (Stecca et al., 2010), oorspronkelijk ontwikkeld voor stroming zonder sediment en scalair transport over vaste bodem, uit te breiden tot met een sedimentvoerende stroming over een bewegelijke bodem. Ten behoeve van de stabilititeit wordt een tweestaps ‘splitting’ aanpak gecombineerd met de tweede-orde Runge-Kutta methode gebruikt voor de bronterm. Bovendien is de volledige set van vergelijkingen simultaan opgelost om tot synchrone wiskundige oplossingen te komen. Het model is gecontroleerd aan de hand van een aantal dam-doorbraak testen met een breed scala van complexe (sedimentvoerende) stromingen. Daarbij is aangetoond dat het model niet alleen schokgolven en reflectiegolven, maar ook snelle bodemveranderingen bij hoge sedimenttransporten goed kan weergeven.

Met behulp van dit model wordt vervolgens de relatieve rol van veranderingen in bodemruwheid en bodemerosie bij de stroomafwaartse toename van de piekafvoer onderzocht voor twee hypergeconcentreerde hoogwatergolven in een geschematiseerd 1-D kanaal. De resultaten laten zien dat, hoewel erosie-effecten kunnen bijdragen aan de stroomafwaartse toename van de piekafvoer (met name in geval van extreme erosie), de toename in de meeste gevallen toch voornamelijk te wijten is aan een afname van de bodemruwheid tijdens piekafvoer omstandigheden. Op basis van het concept van afnemende berging kunnen de gevolgen van afnemende bodemruwheid en (zeer sterke) bodemerosie worden geïntegreerd in een verklaring van de stroomafwaartse toename van de piekafvoer. Later wordt dit model ook toegepast om de invloeden van de uiterwaarden op de piekafvoer toename te onderzoeken in een 2D rivierssectie met een hoofdgeul en uiterwaarden. De resultaten wijzen erop dat de veranderingen in dwarsprofiel door geulerosie en uiterwaarddepositie tijdens een hypergeconcentreerde hoogwatergolf vaak beperkt zijn en dat dit moeilijk de oorzaak kan zijn van een stroomafwaartse toename van de piekafvoer.

Om de impact van wateronttrekkingen te onderzoeken wordt een algemeen theoretisch kader voorgesteld dat de morfologische evenwichtstoestand van het
riviersysteem beschrijft en voor beide typen onttrekkingen (continu en discreet) in een rivier met variërende breedte toepasbaar is. Numerieke experimenten met de SOBEK-RE-software (versie 2.52.005, Delft Hydraulics, 2005) vormen een aanvulling op de MTS studies voor wateronttrekkingen. De effecten van intensiteit, plaatsing (discreet en continue) en vorm (zuiver water en water-sediment mengsel) van de onttrekking worden systematisch bestudeerd. Het huidige werk bevestigt de eerdere bevindingen dat water onttrekkingen tot een afname leiden van evenwichtsdiepte ten opzichte van natuurlijke staat en tot een neerwaarts convex langsprofiel van bodem in een rivier van constante breedte. Bovendien blijkt dat ook in een rivier met een in stroomafwaartse richting toenemende breedte een convex langsprofiel van bodem ontstaat bij wateronttrekkingen, terwijl concave, convexe of quasilineaire langsprofielen mogelijk zijn in een rivier met een stroomafwaarts afnemende breedte. In langsrichting niet-monotone rivier bodemhoogteveranderingen kunnen zich - afhankelijk van de onttrekkingen - ontwikkelen in een sterk vernauwende rivier. Op grote ruimtelijke schaal zijn aard en locatie (continu of discreet) van de onttrekkingen minder belangrijk voor de morfologische evenwichtstoestand. De morfologische tijdschalen met betrekking tot wateronttrekkingen en natuurlijke ontwikkeling zijn zeer vergelijkbaar en groot, wat een indicatie is dat wateronttrekkingen een aanzienlijke invloed op de riviermorfologie kunnen hebben. Het huidige werk heeft het inzicht in het lange-termijn effect van wateronttrekkingen op de morfologische ontwikkeling van rivieren verbeterd.
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Chapter 1

Introduction

1.1 Explanation of hyperconcentrated flow

Hyperconcentrated flow is a water-driven sediment transport phenomenon, which is characterized by high sediment concentrations. It can be considered to be intermediate between normal sediment-laden flow and debris/mud flow (Pierson, 2005). Normal sediment-laden flow is a two-phase Newtonian flow, in which the existence of sediment particles has little effect on the fluid properties, and turbulence is the principal mechanism to maintain sediment transport (Chien and Wan, 1983; Smith and Lowe, 1991; Wu, 2008). In this type of flow, relatively weak sediment transport and mild morphological changes are often observed. Debris flow and mudflow are pseudo-one-phase flow characterized by a water-sediment mixture with volumetric sediment concentrations (i.e., ratio of sediment mass concentration and sediment density) often exceeding 0.6 (Costa, 1984; Pierson and Costa, 1987; Wan and Wang, 1994). They usually behave as a non-Newtonian flow due to prodigious amounts of fine sediments (i.e. clay and silt). The debris flow has the ability to transport large cobbles and boulders, and is mostly distinct from other types of sediment-laden flow by strong grain-dispersion and poorly-sorted (or non-stratified) deposits (Chien and Wan, 1983; Smith and Lowe, 1991; Wan and Wang, 1994; Pierson, 2005). Hyperconcentrated flow is characterized by a highly concentrated water-sediment mixture (mainly consisting of clay, silt, sand and gravel) with sediment concentrations between the normal sediment-laden flow and debris/mud flow. The sediment is transported by a combination of (damped) turbulence, grain-dispersive pressure and fluid buoyancy (Smith, 1986; Smith and Lowe, 1991). Strong erosion and deposition (usually well-sorted deposits) have been often observed in hyperconcentrated flow (Chien and Wan, 1983; Wan and Wang, 1994; Xu, 2002; Pierson, 2005). The grain-size distribution and physical-chemical properties of the transported sediment, especially the amount of fine sediments, determine the physical mechanisms of
hyperconcentrated flow (Costa, 1988; Cao and Qian, 1990; Wan and Wang, 1994; Xu, 2003; Pierson, 2005).

The term ‘hyperconcentrated flow’ was first defined by Beverage and Culbertson (1964) as having a suspended volumetric sediment concentration between 20% and 60%. However, this definition is quite case-specific and gives rise to ambiguities when fluid rheology is used to categorize flow types. Regarding the sediment-induced rheological effects, hyperconcentrated flow has been defined as flow with sediment concentrations ranging from that at the onset of measurable yield strength (i.e. transition from Newtonian to non-Newtonian fluid) (Qian et al., 1981; Pierson and Costa, 1987; Xu, 2003) to that above which yield strength is nearly invariant (i.e. transition to debris flow or mudflow) (Pierson and Costa, 1987). These concentration limits vary considerably for different grain-size distribution and sediment types (Wang et al., 1994). A pure smectite clay suspension acquires yield strength at volumetric concentrations of only 1% (Hampton, 1975) while non-cohesive or coarse particles can maintain the Newtonian-fluid at volumetric concentrations up to about 30% (Fei, 1983; Wang et al., 1994). Flows in the Yellow River (China) with well-sorted fine clay-silt particles can transform to hyperconcentrated flow at volumetric concentrations of 8-11% and behave as debris/mud flow at concentrations of 19-37% especially in tributary channels in the Middle Yellow River basin on the Loess Plateau (Wan and Wang, 1994; Xu, 1999).

However, other classification methods suggest that hyperconcentrated flow is not strictly non-Newtonian. Chien and Wan (1983) classified hyperconcentrated flow into two types: a pseudo-one-phase flow with non-Newtonian properties, in which cohesive fine particles are dominant; and a two-phase flow maintaining Newtonian flow behavior, in which sediment is primarily non-cohesive. A more quantitative description of non-Newtonian/Newtonian hyperconcentrated/normal flow can be derived from the Richardson and Reynolds numbers. The Richardson number distinguishes normal sediment-laden flow from hyperconcentrated flow (Winterwerp, 2006). In the hyperconcentrated regime, laminar (non-Newtonian) flow can be separated from turbulent (Newtonian) flow using a Reynolds number (Wang et al., 1994). Combining both parameters implies that hyperconcentrated flow/floods in the Yellow River are primarily Newtonian, turbulent flow (van Maren et al., 2009a).

In nature, hyperconcentrated flows have been observed in a diversity of environmental settings. Many hyperconcentrated flows occur in volcanic terrains with recent eruptions by downstream diluting of debris flow (Pierson and Scott, 1985; Smith, 1986, 1987; Cronin et al., 2000; Lirer et al., 2001). Other initiation mechanisms include rainstorm-induced landslides and check-dam collapses in mountains (Pierson and Costa, 1987; Batalla et al., 1999; Sohn et al., 1999; Kostaschuk et al., 2003), breaking of debris-dammed lakes (Manville et al., 1999;
1.2 Yellow River basin

Thouret, 1999; Capra, 2007), glacier-outburst floods (Maizels, 1989; Breien et al., 2008), collapses of aeolian damming and flash floods in desert ephemeral rivers (Laronne and Reid, 1993; Svendsen et al., 2003; Cohen and Laronne, 2005), and submarine transition from debris flow (Sohn et al., 2002; Russell and Arnott, 2003).

In tributaries of the middle Yellow River in the highly erodible Loess Plateau (China), flow/floods with sediment concentrations higher than 400 kg/m³ are considered as hyperconcentrated flow (Xu, 1998) while in the Lower Yellow River this critical sediment concentration is considered to be 200 kg/m³ (Zhang and Xie, 1993; Wan and Wang, 1994; Zhao, 1996; Xu, 2004). The hyperconcentrated flow/floods in the Yellow River are usually treated as turbulent flow (Chien and Wan, 1983; Chien, 1989; Wang et al., 2002; van Maren et al., 2009a). In the next section, brief introduction of these floods and associated fluvial processes is provided.

1.2 Yellow River basin

1.2.1 Geography, runoff and sediment load

The Yellow River, the second largest river in China, has a length of 5464 km and a basin area of 795,000 km². Originating from the Bayankela Mountains in the Tibet highlands, it flows across the Loess Plateau and North China Plain before emptying into the Bohai Sea (Yang et al., 2004; Xu and Yan, 2005). Geographically the Yellow River is divided into three reaches. The Upper Yellow River (above Hekouzhen), having a length of 3472 km, is the main water resources area. In the 1206 km long middle reach (from Hekouzhen to Taohuayu), the Yellow River acquires 90% of its sediment load from tributaries on the Loess Plateau (Wang et al., 2007a) and is therefore characterized by highly sediment-laden flow. The tributaries in the area between Hekouzhen and Longmen, and the Manian and Beiluo Rivers (two tributaries of the Wei River) carry coarse sediment (i.e., $d_{50} > 0.05$ mm) to the Yellow River while the Wei and Fen Rivers transport fine sediment load (i.e., $d_{50} < 0.01$ mm) (Zhao et al., 1998; Xu and Yan, 2005). In the 786 km long Lower Yellow River, the sediment-laden flow from the upstream reaches causes complicated fluvial processes resulting in three sub-reaches of distinct morphological features. The upper section is a typically braided reach extending over 299 km from Mengjing to Gaocun. The channel width varies between 1 and 3.5 km with the bed slope ranging from $1.72 \times 10^{-4}$ to $2.65 \times 10^{-4}$ (Wu et al., 2003). The lower section (from Taochengpu to river mouth) is characterized by meandering channels of width 300-1000 m with a relatively mild slope of about $1 \times 10^{-4}$. Between Gaocun and Taochengpu, the river is
a transitional channel with bed slope of $1.1 \times 10^{-4} - 1.7 \times 10^{-4}$ (Long et al., 2002; Qi et al., 2010). Figure 1.1 shows a sketch of the Middle and Lower Yellow River.

![Figure 1.1: Map of the Middle and Lower Yellow River, China (adapted from source: Wu et al. (2008c)).](image)

Dam construction, water-soil conservation, and water diversions (in addition to climate changes) have considerably changed the flow regime and sediment load entering the Lower Yellow River, resulting in dramatic reduction of runoff and sediment load, as well as the modification of their seasonal distribution (i.e., reducing high flow discharges in the flood seasons and increasing low flow discharges in the dry seasons) (Yang et al., 2004; Xu, 2004; Wang et al., 2007a; Wu et al., 2008a,c). Prior to the construction of the Sanmenxia Dam in 1960, the average annual runoff and suspended sediment load entering the Lower Yellow River (measured at the Huayuankou station) amounted to $48.6 \times 10^9$ m$^3$ and $1.56 \times 10^9$ tonnes, respectively (Wu et al., 2008a). The high discharge flows occurred in the flood seasons (from June to October) transporting more than 80% of the annual total sediment load. However, in the 1990s, the average annual runoff and suspended sediment load reduced considerably to $25.7 \times 10^9$ m$^3$ and $0.7 \times 10^9$ tonnes at Huayuankou. Moreover, the decreasing ratio of flood season runoff to annual total runoff largely reduced the sediment transport capacity in sediment-rich flood seasons and thus caused continuous sedimentation in the Lower Yellow River (Xu, 2004). After the construction of Xiaolangdi Reservoir (in October 1999), the average annual runoff and suspended sediment load further reduced to $20.8 \times 10^9$ m$^3$ and $0.13 \times 10^9$ tonnes in the period 2000-2009 (MWRPRC, 2000-2009) while the sedimentation problem in the
Lower Yellow River was partly relieved due to the water-sediment regulating scheme operated by the Xiaolangdi Reservoir (Qi et al., 2010). Figure 1.2 shows the decreasing trend in the annual runoff and suspended sediment load at Huayuankou in the period of 1950-2009.

![Figure 1.2](image.png)

Figure 1.2: Yearly variations of annual runoff and suspended sediment load at Huayuankou in the period 1950-2009.

The Lower Yellow River is characterized by two special and possibly unique hydrodynamic and sediment transport phenomena: a downstream increasing peak discharge during hyperconcentrated floods, and a downstream decreasing runoff due to water diversions (Wang et al., 2008b; Wang et al., 2009; Jiang et al., 2008). Despite many advances in studying the impact of water diversions on the amount of sedimentation and erosion, predicting the long-term impact on morphological changes remains an important challenge. Moreover, though many explanations have been proposed previously for the peak discharge increase, no consensus has been achieved and its physics still remain largely unknown. Research efforts are therefore needed to further investigate these two issues, which comprise the main work of the present PhD research. Below both the water diversions and flood peak increase are introduced in more detail.

### 1.2.2 Impacts of water diversions in the Lower Yellow River

In the 1970s and 1980s, large improvements of irrigation systems were made in the Lower Yellow River (LYR) making it the largest irrigation area in the river basin (Yang et al., 2004). As a result, the (annual-mean) amount of water and sediment load
diverted from this reach increased significantly by 275% and 41.2% from the 1950s to 1980s (Zhang, 1995). In the 1980s, about 26% of the annual water discharge and 13.8% of the sediment load were diverted from this reach respectively (Zhang, 1992). Moreover, the spatial distribution of water-sediment diversions changed significantly during the above periods. The water-sediment diversion was mainly conducted in the upper braiding and transitional reaches (above Gaocun) of the LYR in the 1950s whereas the water and sediment diversions were 131% and 57.4% larger in the lower meandering reach (below Gaocun) than the upper reaches in the 1980s (Zhang, 1995). Combined with the effects of climate change (Yang et al., 2004; Wang et al., 2007a), the extensive water diversions have led to dry-up problems in the lower reach of the LYR since 1972. Infrequent dry-up events were observed in the 1970s and 1980s and the situation rapidly degenerated in the 1990s when a dry-up occurred every year. In the 1990s over half (about 53%) of the water discharge was diverted from the LYR resulting in 897 dry-up days (Wu et al., 2008c) and continuous sedimentation (Wang et al., 2008b). For the most severe event in 1997, the dry-up lasted for 226 days with its influence extending 704 km upstream from the river mouth. After the Xiaolangdi reservoir became operational (October 1999), the dry-up problem was relieved but considerable reduction of flow discharge along the lower reach was still observed. The average ratio between the total runoff at Lijin (near the river mouth) and that at Huayuankou (about 700 km upstream from the river mouth) was 0.59 in the period of 2000-2010, indicating a diversion rate of about 41%. Figure 1.3 shows (a) the spatial variations of runoff and sediment transport during 1960-2010 and (b) the dry-up conditions before 2000 in the LYR. As the water diversions continue, their impacts on discharge and sediment load reduction will inevitably affect the sediment transport processes and accordingly morphological evolution in the LYR in the long term.

Previous studies related to the morphological effects of water diversions mainly focus on the relations of the amount of channel deposition or erosion with the change of diversion rate and the previous channel condition (i.e., degradation, aggradation or equilibrium) (Zhang and Liang, 1995; Zhang, 1995; Liang et al., 1995, 1999). However, few studies have investigated the bed level change and the long-term development of the longitudinal bed profile under the impacts of water diversions, which are very important for the sustainable use of diversion facilities and river flood defense. Recently, a marked improvement in the understanding of these issues has been made (Wang and Hu, 2004; Wang et al., 2007b; Wang et al., 2008b). Assuming spatially continuous diversions along a constant-width channel, it has been shown that the bed evolves to a longitudinally convex equilibrium state. However, its validity for practically discrete diversions in natural longitudinally width-varying channels remains to be justified. Moreover, such equilibrium analysis cannot reveal the water diversion impacts on the morphological time scale (MTS). Therefore, this thesis aims
to study the roles of continuous/discrete diversions on the long-term fluvial processes and the MTS for channels of longitudinally varied widths.

Figure 1.3: Spatial variations of runoff and sediment transport, and dry-up conditions in the LYR.

1.2.3 Phenomenon of downstream peak discharge increase in hyperconcentrated floods

River floods are usually characterized by a downstream flattening of the discharge peak, resulting from friction-induced energy losses. However, in the Yellow River (Fig. 1.1), the peak discharge increases in the downstream direction at a rate exceeding the contribution of tributaries during many hyperconcentrated silt-laden floods. Since the Sanmenxia Dam commenced the operation of storing clear water (dry seasons) and releasing turbid flow (flood seasons) in 1973 (Wang et al., 2005), the peak discharge increase was observed in the Middle and Lower Yellow River in the period of 1970s-1990s. After 2000 when the Xiaolangdi Reservoir became operational, this increase frequently occurred in the Lower Yellow River (Li, 2008). During a recent event with peak discharge increase (in 2010) the measured peak
discharge nearly doubled between two stations spaced about 126 km apart (from 3490 m$^3$/s at Xiaolangdi to 6680 m$^3$/s at the Huayuankou hydrological station). The discharge from tributaries (Yiluo River and Qin River) was negligible (only 86 m$^3$/s). Figure 1.4 shows the magnitude of the peak discharge increase and the ratio of increase (to original discharge) for the flood events in the Yellow River.

From 1973 to the early 1990s, seven peak discharge increase event occurred in the Xiaolangdi – Huayuankou (X-H) reach of the Lower Yellow River, and one in the Xiaobeiganliu (from Longmen-Tongguan) of the Middle Yellow River. In these floods, the increasing peak discharge was usually observed with considerable overflowing floodplains and rapid channel deformation (from wide-shallow to narrow-deep) (Wang et al., 2009). However, an increase of peak discharge barely occurred in the late 1990s, probably resulting from the intensive water diversions (i.e., causing severe dry-ups in the downstream portion of the Lower Yellow River) and the shortage of water resources in the catchment (Yang et al., 2004). In the period of 2004-2010 when the water and sediment discharges were effectively regulated by the Xiaolangdi reservoir, five events with an average discharge increase of 50% occurred in the X-H reach. Comparing with the floods before 2000, these floods were mostly conveyed in the channel with smaller discharge and concentration.

Previous studies on the flood peak increase have focused on only one of the possible mechanisms (i.e., bed roughness reduction, strong bed erosion or channel-floodplain interactions) (Jiang et al., 2006; Cao et al., 2006, 2012; Li, 2008; Wang et al., 2009; Qi et al., 2010), but there has been no consensus on which mechanism most strongly contributes. Therefore, the relative roles of these mechanisms and their relations still need to be analyzed in detail. This thesis aims to fill this gap in knowledge.

Before proceeding, a review of the methodologies on the study of hyperconcentrated flow/flood is presented in the next section.
1.3 Survey of methodologies

Field observations, laboratory experiments and mathematical modeling have been the main methodologies for the study of hyperconcentrated flow/flood. Though this thesis primarily develops and applies mathematical modeling, a description of the state-of-the-art knowledge from the field and laboratory observations is also provided below.

1.3.1 Field observations

Field observation is the most straightforward way of studying natural phenomena. Many significant findings of hyperconcentrated flow are derived from field observations. In volcanic areas and submarine fans, the characteristics of the evolution of hyperconcentrated flow (viz., transition from debris flow to hyperconcentrated flow)
can be back-estimated through the resultant morphological features and deposit structure (Sohn et al., 2002; Russell and Arnott, 2003; Pierson, 2005). In natural rivers hyperconcentrated flow/flood is usually observed to be fully turbulent (e.g., in the Yellow River) while laminar flow may occur with non-Newtonian behavior at very high concentrations (e.g., in some tributaries of the Yellow River on the Loess Plateau) (Chien and Wan, 1983). Such laminar hyperconcentrated flow featuring a smooth and mirror-like surface, may result in flow instabilities, e.g., river clogging and intermittent flow (Wan and Wang, 1994). In addition, mass erosion, during which eroded large blocks of sediment emerge above the water surface and are transported by the ambient flow, is observed in hyperconcentrated floods of high flow energy. In this case, the maximal bed erosion can be up to 9 m in several hours as reported in a hyperconcentrated flood in the Middle Yellow River (Xu, 2000; Wan and Wang, 1994). As for the sediment transport characteristics, the sediment transport rates are much higher in a hyperconcentrated flow/flood than in a normal flow/flood of similar magnitude (Pierson and Scott, 1985; Dinehart, 1999; Xu, 2000). The flow resistance is observed to decrease in hyperconcentrated flow of moderate sediment concentrations and subcritical flow regime (due to turbulence damping), but increase in very high concentrations and supercritical flow regime due to bed-form development and an increased viscous resistance (Chien and Wan, 1983; Wang et al., 1994; Wang et al., 1998; Dinehart, 1999; Pringle and Cameron, 1999; Jiang et al., 2008).

The knowledge from field observations has been applied in the practice of dam operations. From the failure of effectively operating the Sanmenxia Dam (i.e., severe siltation in the reservoir), the approach of storing clear water flow in dry seasons and releasing turbid flow in flood seasons was first applied in 1973 and is now operated by all dams and reservoirs in the Yellow River and other rivers in China to maintain the reservoir storage capacity in highly sediment-laden flow settings (Zhao, 1996). To avoid the degeneration of the perched reaches in the Lower Yellow River and reduce high flood risk, the water and sediment regulating scheme has been operated by the Xiaolangdi Reservoir (since 2002). It regulates hyperconcentrated floods in the flood seasons causing scouring of the downstream channels of the Lower Yellow River (Jiang et al., 2008; Hu et al., 2012).

1.3.2 Laboratory experiments

Laboratory experiments provide another useful way to study the physical properties and mechanisms of hyperconcentrated flow. Many flume experiments have been conducted to investigate the rheological properties of fluids with hyperconcentration (Bagnold, 1954; Govier et al., 1957; Savage and McKeown, 1983; Englund and Wan,
1.3 Survey of methodologies

1984; O’Brien and Julien, 1988; Fei and Zhu, 1991; Rickenmann, 1991; Wang et al., 1994; Komatina and Jovanovic, 1997; Huang and Aode, 2009) and resultant flow instabilities such as river clogging and roll waves (Englund and Wan, 1984; Wang, 2002). The sediment transport characteristics (i.e., suspended load, bed load and their interactions) in hyperconcentrated flow (Wan and Shen, 1978; Wan, 1985; Wang, 1990; Wang et al., 1990; Rickenmann, 1991; Xu, 1993), the sediment settling behaviors (i.e., hindered settling effects) (Davies, 1968; Wang et al., 1995; Song and Chiew, 1997; te Slaa et al., 2013) and the morphological development (i.e., mass erosion and hindered erosion) (Wang et al., 1990; Winterwerp et al., 1990, 1992; Kuang et al., 1999; Wang et al., 2000) have also been systematically studied through flume experiments. In addition, special attention has been paid to the vertical distribution of flow velocity and sediment concentration, and the sediment effects on turbulence structure and intensity in hyperconcentrated flow (Lau and Chu, 1987; Wang and Qian, 1989; Winterwerp et al., 1990; Wang and Larsen, 1994; Wang and Plate, 1996; Cellino and Graf, 1999; Baas and Best, 2002). In studying the flow resistance in hyperconcentrated flow (Yang and Zhao, 1983; Winterwerp et al., 1990; Rickenmann, 1991; Komatina and Jovanovic, 1997; Wang et al., 1998), drag reduction has been frequently observed in many experiments and its mechanisms have been a topic of significant interest as well (Vanoni, 1946; Einstein and Chien, 1955; Wan, 1985; Lau and Chu, 1987; Best and Leeder, 1993; Wang et al., 1998; Cellino and Graf, 1999; Li and Gust, 2000; Shu et al., 2008; Zhu and Hao, 2008).

Besides the flume experiments, large-scale physical models that take account of the sinuosity of natural channels and features of floodplains are also used in the study of hyperconcentrated flow. In the Yellow River Institute of Hydraulic Research (YRIHR) in China, a laboratory-scale physical model (800 m long) for the Xiaolangdi-Taochengpu reach (1480 km long in situ) has been constructed and used to understand the characteristics of hyperconcentrated flood propagation and its morphological processes in the Lower Yellow River (Jiang et al., 2008). Additionally, experiments of hyperconcentrated flow/flood in reservoir settings have also been carried out in the physical models of Xiaolangdi Reservoir and Sanmenxia Dam.

1.3.3 Mathematical modeling

In addition to field observations and laboratory experiments, mathematical modeling is becoming increasingly popular as a means to study hyperconcentrated flow. Many mathematical models have been developed under distinct frameworks from classical fluid dynamics theory to new computational paradigms, among which the models based on fluid dynamics theory have been the most common methods for the simulation of hyperconcentrated flow. As hyperconcentrated flow is a special case of
sediment-laden flow, its mathematical modeling is more complex than that of normal sediment-laden flow:

(1) Fluid properties of hyperconcentrated flow
(2) Sediment-turbulence interactions
(3) Sediment density effects and rapid bed deformation
(4) Numerical schemes

These issues will be explained in more detail below.

The first issue to be considered is the fluid properties of hyperconcentrated flow. As mentioned above, hyperconcentrated flow can occur in Newtonian or non-Newtonian fluids. For non-Newtonian hyperconcentrated flow, the rheological effects of hyperconcentration should be incorporated in the estimation of the friction slope through the shear stress in mathematical models. Generally a Bingham rheological model has been used to account for the influences of yield stress and viscous stress on shear stress (Schamber and MacArthur, 1985; O’Brien, 1986; Fei, 1991; Pastor et al., 2004; Quecedo et al., 2004; Dewals et al., 2011). Excluding the viscous effects, a frictional fluid model that considers the effective pressure contribution has also been applied to modeling waste dump failure (Pastor et al., 2002). Additionally, a dilatant-fluid model derived from Bagnold’s dispersive stress theory has been proposed when the turbulent-dispersive effects between sediment particles are dominant (Takahashi and Tsujimoto, 1985). The combination of rheological properties of Bingham fluid and turbulent-dispersive flow in hyperconcentrated mudflows has been suggested by O’Brien and Julien (1988), Major and Pierson (1990), and Julien and Lan (1991). A quadratic shear stress model including the effects of yield stress, viscous stress and turbulent-dispersive stress has been proposed and applied to modeling hyperconcentrated mudflow in a 2-D finite difference framework (O’Brien and Julien, 1985; O’Brien et al., 1993). However, this model is unable to capture shock waves and only valid for a rigid bed.

Hyperconcentrated floods in the main stem of the Yellow River (i.e., especially in the Lower Yellow River) have been demonstrated to be fully turbulent flows retaining Newtonian fluid behaviors (Chien and Wan, 1983; Zhou et al., 1983), and therefore the rheological properties of non-Newtonian fluids have often been neglected in mathematical modeling (Zhang et al., 2001; Ni et al., 2004; Wu et al., 2004; Wang et al., 2008a; van Maren et al., 2009b; Xia et al., 2012). In this thesis, the mathematical modeling of the Yellow River hyperconcentrated floods follows this treatment of Newtonian turbulent flow.

The second issue concerns the influences of hyperconcentration on turbulence. In hyperconcentrated flow, turbulence (including intensity and structure) has been
1.3 Survey of methodologies

demonstrated to be affected by highly concentrated suspended load and thus the profiles of flow velocity and sediment concentration are seen to deviate from those of normal flows (Wang and Larsen, 1994; Winterwerp, 2001). In depth-resolving models (i.e., 1DV or 2DV models and fully 3-D models), it is possible to incorporate this hyperconcentration effect in turbulence closure models (e.g., $k-\varepsilon$ turbulence models) by a modified Schmidt number larger than unity (Winterwerp, 2006; van Maren et al., 2009b; Jha and Bombardelli, 2010). In depth-averaged models, this effect can be considered by a modified bed roughness coefficient in the calculation of friction slope (Zhang et al., 2001; Wang et al., 2008a; Winterwerp et al., 2009).

The third issue focuses on the effects of sediment density and bed deformation on the flow. For hyperconcentrated flows with rapid morphological changes, the sediment-induced density effects and bed deformation influences are so important that a fully coupled modeling approach is required in the mathematical modeling of hyperconcentrated flow/flood. Here, the term ‘fully coupled’ means that the effects of sediment density and bed deformation are completely incorporated in the flow mass and momentum conservation (Cao et al., 2012). In addition, as the sediment transported during the flood in the Yellow River usually consists of fine sediments, a non-capacity modeling framework is often used to account for the spatial and temporal lag effects of sediment transport (Zhang et al., 2001; Wu et al., 2004; Wang et al., 2008a; van Maren et al., 2009b; Cao et al., 2012; Xia et al., 2012). However, though a number of fully coupled non-capacity models have been recently developed for sediment-laden flow (Egashira et al., 2001; Cao et al., 2004; Simpson and Castelltort, 2006; Wu and Wang, 2008; Hu and Cao, 2009; Xia et al., 2010; Li and Duffy, 2011; Hu et al., 2012), few have been applied in the modeling of hyperconcentrated flow/flood.

The fourth issue on the mathematical modeling of hyperconcentrated flow/flood is the numerical scheme used for solving the governing equations. In general, the complexity of the fluvial processes (i.e., highly unsteady flow, high sediment concentration and rapid morphological change) requires robust and accurate numerical schemes. This includes tracking of the wet/dry front, capturing shock waves between supercritical and subcritical flow regimes and contact discontinuities (i.e. steep sediment concentration gradients), and the accurate conservation of water and sediment mass. In this regard, the Total Variation Diminishing (TVD) versions of the finite volume method (FVM) (Toro, 2001), which are capable of automatically capturing shock waves and easily incorporating a wetting/drying procedure, could be a good choice. Moreover, the hyperconcentrated flood in natural rivers often requires a large computational demand and thus an accurate but also efficient numerical scheme is needed. Fortunately, a newly developed second-order version of the upwind-biased first order centered (UFORCE) scheme (Stecca et al., 2010) that
combines the high accuracy of (high-order) upwind methods and the efficiency of centered methods in the FVM framework provides a promising tool to cover these points. As this method has been initially developed for an idealized rigid bed (Stecca et al., 2010), efforts are required to extend its application to mobile beds. Additionally, a synchronous solution procedure for the full set of governing equations (e.g., Li and Duffy, 2011) is also needed to achieve a numerically fully coupled approach.

1.4 Research objectives

The main objectives of this research are:

1. to reveal the long-term morphological impacts of water diversions in the Lower Yellow River;
2. to unravel the physical mechanisms underlying the downstream peak discharge increase in hyperconcentrated floods.

For the first end, a general theoretical framework is proposed to predict the equilibrium state of the fluvial system, which is applicable to both continuous and discrete water diversions in a longitudinally width-varying channel. For the second issue, a fully coupled morphodynamic model, which aims to cover the above critical modeling issues, is developed and validated by a series of laboratory experiments. Using this newly developed model, the relative contribution of bed erosion, bed roughness change, and channel-floodplain morphological change to the peak discharge increase will be analyzed and explained. Efforts will also be made to physically integrate these distinct mechanisms in explaining this increase.

1.5 Thesis outlines

In Chapter 1, a review of the explanation and research methodology of hyperconcentrated flow was presented. Two major issues related to the Yellow River hyperconcentrated flow, i.e., the long-term impact of water diversions and the short period phenomenon of downstream peak discharge increase in hyperconcentrated floods, were addressed and comprised the main research questions of this PhD thesis.

In Chapter 2, the depth-averaged 2-D fully coupled model using the concept of non-capacity sediment transport is presented in terms of the governing equations, the empirical relationships and the numerical algorithm. The high efficiency and accuracy of the model are achieved by the second order extension of the upwind-biased first order centered (UFORCE) scheme (Stecca et al., 2010) to a mobile bed (Li et al., 2013). A modified characteristic method is developed to couple the morphological update and flow variations at open boundaries.
In Chapter 3, the fully coupled model is tested against a series of dam-break cases characterized by rapidly changing flows and intensive sediment transport over fixed or erodible beds. The model’s abilities to capture shock waves (e.g., interchange from supercritical to subcritical flow regimes) and contact discontinuities (e.g., a strong spatial sediment concentration gradient), as well as its stability on rapid bed deformation at high sediment transport rates are demonstrated.

In Chapter 4, the mechanisms of the downstream peak discharge increase in hyperconcentrated floods are studied. First, the historical peak discharge increase events in the Yellow River are briefly analyzed. Then the two peak discharge increase events, the 2004 flood (moderate concentration) and the 1977 flood (high concentration), are numerically modeled. The relative importance of morphological change and bed roughness change to peak discharge increase without floodplain influences is quantitatively analyzed and explained using the newly developed model (in Chapter 2). Finally, the concept of channel storage reduction is proposed to integrate these two different mechanisms.

In Chapter 5, the fully coupled model is applied to hyperconcentrated floods in a channel-floodplain system. The typical morphological features of channel erosion and floodplain deposition in hyperconcentrated floods are numerically reproduced. The influence of such morphological change on hyperconcentrated flood propagation is investigated. The effects of different floodplain widths and channel-floodplain types, bed roughness change and parameter settings on discharge and morphological evolution are systematically studied.

In Chapter 6, a general theoretical framework is proposed for predicting the equilibrium depth and bed slope in the longitudinal direction, which is applicable to both continuous and discrete water diversions in a longitudinally width-varying channel. The importance of water diversions to the evolution of alluvial rivers is also demonstrated through the morphological time scale studies by numerical experiments of schematic Lower Yellow River cases.

Conclusions and discussions are presented in Chapter 7.
Chapter 2

Fully coupled morphodynamic model

2.1 Introduction

Turbulent flows (including wind, open-channel flow, pipe flow and subaqueous flow) over an erodible bed generate sediment transport and morphological changes, which in turn influence the flow. In addition to laboratory experiments and field surveys, mathematical modeling is becoming increasingly popular as a means to study morphological processes. This chapter describes the development of a morphological model for open channel flow, taking into account the latest progresses in both the understanding of the physical processes of sediment transport and numerical methods to solve the equations.

Morphological models can be coupled or decoupled, with a capacity or non-capacity transport formulation (e.g. Armanini and Di Silvio, 1988; Rahuel et al., 1989; Holly and Rahuel, 1990a,b; Crave and Davy, 2001; Cui and Parker, 2005; Simpson and Castelltort, 2006; Garegnani, 2011; Cao et al., 2012). In a capacity model, sediment transport is assumed to be always in equilibrium with the local flow conditions, whereas a non-capacity model includes spatial and temporal lag effects. By the term ‘coupled’ and ‘decoupled’, we follow the philosophy of Cao et al. (2012). In a decoupled model (e.g. Cunge et al., 1980; Cui et al., 1996; Sieben, 1999; Di Cristo et al., 2006), the effects of sediment density and bed deformation rate on the flow are neglected, whereas a fully coupled model incorporates those effects into the flow mass and momentum conservation. A partially coupled model (Phillips and Sutherland, 1989; Capart and Young, 1998; Fracarollo and Capart, 2002; Lesser et al., 2004; Rosatti and Fracarollo, 2006; Canestrelli et al., 2010; Garegnani, 2011) falls in between the decoupled and fully coupled models. A decoupled or partially coupled

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1 This chapter is based on a journal paper published in Water Resources Research:
model is applicable only in the cases for which the timescale of morphological change is much longer than that of the flow adaptation to changing bed topography (Cao et al., 2007; Hu and Cao, 2009). Otherwise a fully coupled model is required. In the present work, a fully coupled non-capacity model is employed to incorporate as much physics as possible. This is important because for highly concentrated flows with rapid morphological changes, the sediment-induced density effects and bed deformation influences are so important that the decoupled or partially coupled models cannot adequately represent all important physical processes. This is exemplified by a number of recently developed fully coupled non-capacity models (Egashira et al., 2001; Cao et al., 2004; Simpson and Castelltort, 2006; Wu and Wang, 2008; Xia et al., 2010; Li and Duffy, 2011; Hu et al., 2012).

Another key issue on resolving the complex morphological processes in highly concentrated flows is the numerical scheme used for solving the governing equations. In general, the complexity of the fluvial processes requires robust and accurate numerical schemes. This includes tracking of the wet/dry front, capturing shock waves between supercritical and subcritical flow regimes and contact discontinuities (i.e. steep sediment concentration gradients), and the accurate conservation of water and sediment mass. Total Variation Diminishing (TVD) versions of the finite volume method (FVM) have been widely used in mathematical modeling of shallow water flows in the last two decades (Alcrudo and Garcia-Navarro, 1993; Toro, 2001; Bradford and Sanders, 2002; Cao et al., 2004; Hu and Cao, 2009; Xia et al., 2010; Wu et al., 2012). They are attractive because of 1) their ability to automatically capture shock waves and 2) the incorporation of a simple wetting/drying procedure enabling to track wet/dry fronts. Most TVD algorithms, however, take either an upwind approach represented by Godunov type methods (Godunov, 1959), such as the widely used Roe approximate solver, HLL, HLLC method (Roe, 1981; Toro, 2001), or a centered approach based on the Lax-Friedrichs method (Lax, 1954), such as FLIC or SLIC (Toro, 2001). High-order upwind methods produce highly accurate numerical results as they use local flow information, but they are complex and computationally expensive. Centered methods, on the other hand, can enhance the computational efficiency considerably and are much simpler to apply to complicated set of equations, but they may significantly compromise accuracy, especially in case of small Courant numbers and contact discontinuities. Fortunately, the high accuracy of an upwind scheme and the computational efficiency of a centered scheme were recently combined in an ‘upwind-biased FORCE (First Order Centered) scheme (UFORCE)’ based on the FVM for structured meshes (Stecca et al., 2010). So far, this method has not been applied to the fully coupled non-capacity model for mobile bed processes.

The aim of this chapter is to extend the 2nd order version of the upwind-biased FORCE method (UFORCE) to sediment-laden flows over a mobile bed, solving the
fully coupled non-capacity equations. If successful, this would be an important step forward on the way to efficient high-accuracy modeling of complex morphodynamics. Firstly, in natural rivers the computational domain is usually large, hence requires a highly efficient computational procedure. Secondly, the effects of high sediment concentrations and rapid bed deformations (such as in mobile-bed dam-break flows and during highly sediment-laden floods in the Yellow River, China) require a numerical accuracy that is often insufficient in most existing morphological models. Thirdly, the spatial and temporal lag effects of sediment transport can be important to morphological changes, thus need a non-capacity modeling framework to incorporate as much physics as possible.

### 2.2 Mathematical model

#### 2.2.1 One-dimensional (1-D) governing equations

Based on the concept of non-capacity sediment transport, the non-linear system of governing equations for the 1-D fully coupled mathematical model consists of the mass and momentum conservation equations for sediment-laden flow, the mass conservation equation for the sediment, and a bed update equation (Cao et al., 2004):

\[
\frac{\partial (\rho h)}{\partial t} + \frac{\partial (\rho hu)}{\partial x} = -\frac{\partial (\rho_0 z)}{\partial t} + (\rho_s - \rho_w) \Phi_{xx} \tag{2.1}
\]

\[
\frac{\partial (\rho hu)}{\partial t} + \frac{\partial (\rho hu^2 + 0.5 \rho gh^2)}{\partial x} = \rho gh (S_{0x} - S_{f}) \tag{2.2}
\]

\[
\frac{\partial (hc)}{\partial t} + \frac{\partial (huc)}{\partial x} = E - D + \Phi_{xx} \tag{2.3}
\]

\[
(1 - p) \frac{\partial z}{\partial t} = D - E \tag{2.4}
\]

where \( t \) = time; \( x \) = horizontal coordinate; \( h \) = water depth; \( u \) = depth-averaged flow velocity in \( x \) direction; \( c \) = depth-averaged volumetric sediment concentration; \( z \) = bed elevation; \( E, D \) = sediment entrainment and deposition fluxes respectively; \( S_{0x} \) = bed slope in \( x \) direction, expressed as \( S_{0x} = -\hat{c}z/\hat{c}x \); \( S_{f} \) = friction slope in \( x \) direction; \( \Phi_{xx} = \frac{\partial (h\varepsilon_s \hat{c}c / \hat{c}x)}{\partial x} \) = sediment dispersion/diffusion in \( x \) direction; \( \varepsilon_s \) = sediment dispersion/diffusion coefficient; \( \rho = \rho_s (1 - c) + \rho_s c \) = density of sediment-laden flow; \( \rho_s \) = sediment density; \( \rho_w \) = water density; \( \rho_0 = \rho_s p + \rho_s (1 - p) \) = density of saturated bed; \( p \) = bed porosity; \( g \) = acceleration of
gravity. Formally there should be a coefficient (near 1) in the advection term (the second term) of equation (2.2) resulting from depth-averaging the flow velocity, while it is set to 1 for simplicity in this thesis.

2.2.2 Two-dimensional (2-D) governing equations

For 2-D conditions, the governing equations also include the influences of flow turbulence, as

\[
\frac{\partial (\rho h)}{\partial t} + \frac{\partial (\rho hu)}{\partial x} + \frac{\partial (\rho hv)}{\partial y} = - \frac{\partial (\rho v_z)}{\partial t} + (\rho_s - \rho_w) \Phi_s
\]  

(2.5)

\[
\frac{\partial (\rho hu)}{\partial t} + \frac{\partial (\rho h u^2 + 0.5 \rho g h^2)}{\partial x} + \frac{\partial (\rho h uv)}{\partial y} = \rho g h (S_{oy} - S_{fy}) + \rho \frac{\partial (h \tau_{sx})}{\partial x} + \rho \frac{\partial (h \tau_{sy})}{\partial y}
\]  

(2.6)

\[
\frac{\partial (\rho hv)}{\partial t} + \frac{\partial (\rho hu v)}{\partial x} + \frac{\partial (\rho v^2 + 0.5 \rho g h^2)}{\partial y} = \rho g h (S_{oy} - S_{fy}) + \rho \frac{\partial (h \tau_{sy})}{\partial x} + \rho \frac{\partial (h \tau_{sy})}{\partial y}
\]  

(2.7)

\[
\frac{\partial (hc)}{\partial t} + \frac{\partial (huc)}{\partial x} + \frac{\partial (hvc)}{\partial y} = (E - D) + \Phi_s
\]  

(2.8)

\[
(1 - p) \frac{\partial z}{\partial t} = D - E
\]  

(2.9)

where \( y \) = horizontal coordinate; \( v \) = depth-averaged flow velocity in \( y \) direction; \( S_{oy} \) = bed slope in \( y \) direction, expressed as \( S_{oy} = -\partial z / \partial y \); \( S_{fy} \) = friction slope in \( y \) direction; \( \tau_{sx}, \tau_{sy}, \tau_{sy} \) = depth-averaged Reynolds stresses, expressed as \( \tau_{sx} = 2 \nu_t \partial u / \partial x, \tau_{sy} = 2 \nu_t \partial v / \partial y, \tau_{sy} = \tau_{sx} = \nu_t (\partial u / \partial y + \partial v / \partial x) \); \( \nu_t \) = turbulent eddy viscosity; \( \Phi_s = \partial (h e, \partial c / \partial x) / \partial x + \partial (h e, \partial c / \partial y) / \partial y \) = sediment dispersion/diffusion terms. Other parameters are the same as those specified above.

Physically, sediment dispersion exists due to the non-uniform distributions of flow velocity in vertical and horizontal directions (i.e., in both turbulent and laminar flows), and sediment diffusion occurs in turbulent mixing processes (Julien, 2010). Together with the advection influences, these processes may contribute to the sediment mass variation, and therefore to the mass variation of the sediment-laden flow. In some multiphase models, the sediment dispersion/diffusion effect has been considered in the mass conservation of sediment load, though neglected in that of the sediment-laden flow (Lesser et al., 2004; Xia et al., 2010). Its complete absence in most other models (in the mass conservation of both the moving sediment and the water-
sediment mixture) is due to its negligible influence in most conditions for which advection always dominates. Yet dispersion/diffusion would be important in conditions such as overbank flows. To incorporate more well-understood physical processes in the mathematical formulations, the sediment dispersion/diffusion effect is fully considered in the present model in the mass conservation of not only the moving sediment, but also the water-sediment mixture.

2.3 Auxiliary relationships

To close the governing equations, empirical formulations are used for a set of physical variables. Generally, these include the bed friction and the sediment exchange (entrainment and deposition) between flow and river bed.

2.3.1 Flow resistance

For the mathematical modeling throughout the thesis, the friction slope is estimated by Manning roughness coefficient \( n \), as

\[
S_{fx} = \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}}, \quad S_{fy} = \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}}
\]  

(2.10a,b)

In normal conditions, the roughness coefficient is considered as a constant. Under hyperconcentrated flow conditions (e.g. during floods in the Yellow River), however, the bed roughness would change considerably with the variation of sediment concentration and flow regime (Gust, 1976; Zhang and Xie, 1993; Zhao and Zhang, 1997; Chien and Wan, 1998; Wang et al., 1998; Zhu and Hao, 2008; Jiang et al., 2008; Winterwerp et al., 2009; van Maren et al., 2009a, b; Qi et al., 2010). In order to account for this effect, a varied bed roughness should be used. In this thesis, two formulations are used for the estimates of Manning roughness in hyperconcentrated flood modeling. See details in Chapter 4.

2.3.2 Sediment entrainment and deposition

The mass exchange of sediment between flow and river bed has been a fundamental issue for the study of non-capacity sediment transport (Chien and Wan, 1998; Cao and Carling, 2002; van Rijn, 2007). It constitutes two distinct mechanisms, viz. sediment entrainment due to turbulence and deposition due to gravity. For the deposition, there is little dispute to calculate it by the product of the effective sediment settling velocity and the near-bed sediment concentration. However, the mechanism for sediment entrainment is still poorly understood. A traditional way for the estimation of
sediment entrainment is based on the assumption that the entrainment usually occurs at the same rate as it does under equilibrium conditions.

Following Cao et al. (2004)’s approach, an empirical coefficient $\alpha$, representing the ratio of the near-bed and the depth-averaged concentration (generally $\alpha > 1$), is used to estimate the sediment exchange flux with the bed (see Eq. (2.11)). Though differing in physical meaning and mathematical expressions of the adaptation parameter measuring the rapidity of sediment exchange between flow and bed, the method of $\alpha$ is essentially equivalent to the method of the adaptation length for sediment lag effects in previous studies (Galappatti and Vreugdenhil, 1985; Armanini and Di Silvio, 1988; Phillips and Sutherland, 1989; Rahuel et al., 1989; Wu et al., 2000). A large value of $\alpha$ or a small value of the adaptation length represents a rapid sediment adaptation while a small $\alpha$ or a large adaptation length means a slow adaptation.

The sediment entrainment and deposition are estimated as

$$E = \alpha \omega_c c_e, \quad D = \alpha \omega_c c$$  \hspace{1cm} (2.11a,b)

where $c_e$ = depth-averaged sediment transport capacity, (-); $\omega_c$ = effective sediment settling velocity (m/s); $\alpha =$ coefficient denoting the ratio between the near-bed and the depth-averaged concentrations. In this thesis, the sediment settling velocity in clear and still water is estimated by the Zhang and Xie (1993) formula, unless specified otherwise:

$$\omega_b = \sqrt{(13.95v / d)^2 + 1.09(\rho_s - \rho_w)gd / \rho_w - 13.95v / d}$$  \hspace{1cm} (2.12)

where $\omega_b$ = settling velocity of a single particle in clear and still water (m/s); $d$ = sediment diameter (m); $v$ = kinematic viscosity of water.

For bed-load transport (see the mobile-bed dambreak tests in Chapter 3), constant values of the coefficient $\alpha$ are specified in the simulation but they differ from case to case. In those test cases, the sediment is relatively coarse (viz. natural sand, or artificial pearls) and the hindered settling effect is negligible. This means that the formulation of the sediment settling velocity in clear and still water (Eq. (2.12)) can be used. The depth-averaged sediment transport capacity in these cases is calculated by a modified Meyer-Peter and Müller (1948) (MPM) formula with an adjustment parameter $\phi$, as indicated in Eq. (2.13a).

$$q_b = \phi 8 \sqrt{sgd^2(\theta - \theta_s)^{1.5}}, \quad c_e = q_b / (h\sqrt{u_*^2 + v^2})$$  \hspace{1cm} (2.13a,b)

where $q_b$ = bed-load transport rate under capacity state (m$^2$/s); $\phi$ = adjustment parameter; $s$ = submerged specific gravity of sediment = $\rho_s / \rho_w - 1$ ;
2.4 Numerical algorithm

\[ \Theta = u^2 / sgd = \text{Shields parameter}; \; u_* = \text{friction velocity}; \; \Theta_t = \text{threshold Shields parameter}. \]

In case of suspended load transport, such as the hyperconcentrated flood modeling in Chapter 4, the estimation for \( \alpha, c_e, \omega_j \) is more complicated. Different approaches are used to calculate these parameters, see details in Chapter 4.

2.4 Numerical algorithm

In this section, the numerical algorithm applied to the 2-D model is elaborated as an example. Using a finite volume method, the full set of the governing equations is rewritten in a conservative form and solved explicitly by a synchronous solution for structured grids. The key problem for computing advection of strong spatial gradients in sediment-laden flows is tackled with a version of the 2nd order UFORCE scheme (Stecca et al., 2010) extended from an idealized frictionless-fixed bed to a movable bed (Li et al., 2013). For stability, a two-step splitting approach (Toro, 2001) is used in combination with a 2nd order Runge-Kutta method for the source terms (Toro, 2009). The model is 2nd order accurate in space and time. The time step is restricted by the Courant-Friedrichs-Lewy (CFL) condition and additional conditions relevant to the sediment transport and bed change computations. Wetting and drying fronts are treated using two tolerance depths for the cell type judgment (Zhao et al., 1994), one for a wet bed and one for a dry bed. For open boundaries, a modified characteristic method is adopted, which uses the above Runge-Kutta method and considers morphological update in each iteration cycle. This permits a synchronous solution for the entire computational domain. A free-slip and non-permeable condition is used for closed walls.

2.4.1 Conservative form of the governing equations

Following Cao et al. (2004), the governing equations for the sediment-laden flow are manipulated in a conservative form where the contributions of flow density variation and bed deformation to the flow appear in the source terms. It results in a simple format of the advection terms that enables an easy extension of any high-resolution FVM previously developed for shallow water flows over a fixed bed to a mobile-bed situation. To achieve a synchronous solution for the full equation set, further mathematical treatment (Zhang and Xie, 1993; Li and Duffy, 2011) is used in deriving the conservative form. The bed update equation is replaced by a newly constructed equation (i.e. the 5th equation of the vector form of the governing equations (2.14-15)), obtained by summation of Eqs. (2.8) and (2.9). Physically it indicates that the
variation of sediment mass (including the sediment in motion and the static sediment in bed) in time depends on the effects of advection and diffusion. Furthermore, it ensures consistency in the numerical solution of the sediment-laden flow and the bed deformation, avoiding possible mass balance errors when applying inconsistent numerical method to solve the sediment transport and the bed deformation equations. Thus, the vector form of the 2-D governing equations reads

\[ \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S}_0 + \mathbf{S}_t + \mathbf{S}_i \]  

(2.14)

\[
\mathbf{U} = \begin{bmatrix} h \\ hu \\ hv \\ hc \\ \phi \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} hu \\ hu^2 + 0.5gh^2 \\ hvu \\ hv^2 + 0.5gh^2 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} hv \\ hvu \\ hv^2 + 0.5gh^2 \end{bmatrix},
\]

(2.15a,b,c)

\[ \mathbf{S}_0 = \begin{bmatrix} 0 \\ ghS_{0x} \\ ghS_{0y} \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{S}_t = \begin{bmatrix} E - D \\ -ghS_{px} - (\rho_s - \rho_w)gh^2 \frac{\partial c}{\partial x} - \frac{(\rho_0 - \rho)(E - D)u}{\rho(1 - p)} \\ -ghS_{py} - (\rho_s - \rho_w)gh^2 \frac{\partial c}{\partial y} - \frac{(\rho_0 - \rho)(E - D)v}{\rho(1 - p)} \\ E - D \end{bmatrix}, \quad (2.15d,e)
\]

\[ \mathbf{S}_i = \begin{bmatrix} \frac{\partial (h\tau_{sx})}{\partial x} + \frac{\partial (h\tau_{sy})}{\partial y} \frac{\rho_s}{\rho} \Phi_s \\ \frac{\partial (h\tau_{sx})}{\partial x} + \frac{\partial (h\tau_{sy})}{\partial y} \frac{\rho_s - \rho_w}{\rho} \Phi_s \\ \Phi_s \\ \Phi_s \end{bmatrix} \]

(2.15f)

where \( \mathbf{U} = \) vector of conservative variables; \( \mathbf{F}, \mathbf{G} = \) vectors of flux variables; \( \mathbf{S}_0, \mathbf{S}_t, \mathbf{S}_i = \) vectors of source terms; \( \phi = (1 - p)z + hc \) = newly constructed conservative variable. For this fully coupled model, the feedback of the bed deformation on the sediment-laden flow is represented by the source term involving \( (E - D)/(1 - p) \) in the mass and momentum conservation equations, i.e. in term \( \mathbf{S}_t \) of Eq. (2.15e).

Here, two issues need to be clarified.
2.4 Numerical algorithm

First, from a mathematical point of view, Eq. (2.14) written in terms of the conservative variables of Eq. (2.15a), is not in a perfect conservation form due to the existence of the spatial gradients of concentration and bed elevation on the right hand side (see Eq. (2.15e)). In order to satisfy the well-balance property, non-conservative methods are required. However, the available non-conservative methods, for either weakly coupled models (Castro et al., 2009; Siviglia et al., 2013) or strongly coupled models (Rosatti and Fraccarollo, 2006; Armanini et al., 2009; Rosatti and Begnudelli, 2013a,b), are limited and currently based on the capacity models. Their applications to the non-capacity model (of open channel flows) are rare. While viewing these spatial gradients (non-conservative terms) as source terms in the momentum equation, many non-capacity models show satisfactory performance in strongly coupled and hyperconcentration conditions (Cao et al., 2004, 2006; Simpson and Castelltort, 2006; Xia et al., 2010; Li and Duffy, 2011; Wu et al., 2012). Therefore, in this thesis, these non-conservative terms are treated as source terms for model applications to most simulation cases (see details in Chapter 3 and 4), and accordingly a DGM version of the UFORCE scheme (see section 2.4.4) is used for the computation of advection fluxes. While in conditions of rather complex topography and drying-wetting processes (e.g., hyperconcentrated flood simulations in channel-floodplain reaches in Chapter 5), the spatial gradient of bed elevation is moved to the left hand side of the momentum equation, and a SGM version of the UFORCE scheme (see section 2.4.4) is developed and further applied to obtain more accurate results especially for shock wave capturing.

Second, the source terms still contain some unknowns depending on the primary variables (i.e., h, u, v, c). Therefore, a predictor-corrector scheme (i.e., a splitting approach by Toro (2001)) is adopted to explicitly solve the equations. Firstly, the homogeneous partial derivative equations are solved by a conservative method (the DGM version of the UFORCE scheme), which is an extension of the second order UFORCE scheme (Stecca et al., 2010) for mobile beds (Li et al., 2013). Alternatively, the SGM version of the UFORCE scheme is used when the spatial gradient of bed elevation is moved to the left hand side of the momentum equation. Secondly, the ordinary derivative equations (containing source terms) are solved via a second order Runge-Kutta method (Toro, 2009) using the intermediate solutions from the first step. See details in sections 2.4.3 and 2.4.4.

2.4.2 Discretization of the governing equations

For solving the governing equations, the computational domain is divided into a structured set of rectangular cells. A collocated cell-centered FVM is adopted in the model, with the average values of conserved variables being defined at the centre.
Figure 2.1 shows a sketch of a control volume \((i,j)\) bounded by four sides in 2-D conditions. Integrating Eq. (2.14) within this control volume and applying the divergence theorem, we can get the discretized equation in the Cartesian \(x, y\)-coordinates:

\[
\frac{\partial U_{i,j}}{\partial t} + \frac{1}{\Delta x} \left( F_{i+1/2,j}(U) - F_{i-1/2,j}(U) \right) + \frac{1}{\Delta y} \left( G_{i,j+1/2}(U) - G_{i,j-1/2}(U) \right) = R(U_{i,j}) \tag{2.16}
\]

where \(\Delta x, \Delta y\) = spatial steps in \(x\) - and \(y\) - directions; \(i, j\) = cell indices in \(x\) and \(y\) directions; \(U_{i,j}\) = vector of cell-averaged conservative variable at the center; \(R = S_0 + S_t + S_s\) = vector of total source terms; \(F_{i+1/2,j}(U), F_{i-1/2,j}(U), G_{i,j+1/2}(U), G_{i,j-1/2}(U)\) = vectors of normal flux at interfaces \((i + 1/2, j), (i - 1/2, j), (i, j + 1/2), (i, j - 1/2)\). The fluxes are computed using the 2nd order UFORCE scheme in the \(x\)- and \(y\)-directions respectively (see Section 2.4.4). The spatial gradients of bed level and sediment concentration in the source terms are discretized by centered difference scheme.

![Figure 2.1: Sketch of 2-D structured finite volume mesh.](image)
2.4 Numerical algorithm

2.4.3 Method of time integration

A standard splitting approach (Toro, 2001) is deployed to solve the inhomogeneous system of Eq. (2.16). In each time step, the homogeneous equation (Eq. (2.17a)) is solved first and the result is used to evaluate a predictor of the source terms. Subsequently, a 2nd order Runge-Kutta method (Toro, 2009) is used to determine the definitive solution after that time step. The overall accuracy in time is supposed to be 2nd order by approximating the source term and the advection fluxes with 2nd order schemes. The solution procedure for the splitting approach can be summarized as follows:

\[
U_{i,j}^* = U_{i,j}^n - \frac{\Delta t}{\Delta x} \left( F_{i+1/2,j}^n(U^n) - F_{i-1/2,j}^n(U^n) \right) - \frac{\Delta t}{\Delta y} \left( G_{i,j+1/2}^n(U^n) - G_{i,j-1/2}^n(U^n) \right) \quad (2.17a)
\]

\[
U_{i,j}^{n+1} = U_{i,j}^* + 0.5 \left( K_1 + K_2 \right) \quad (2.17b)
\]

with \( K_1 = \frac{\Delta t}{\Delta x} R(U_{i,j}^*), \quad K_2 = \frac{\Delta t}{\Delta y} R(U_{i,j}^* + K_1) \quad (2.18a,b) \)

where \( U_{i,j}^* \) = vector of predictor variables at the intermediate stage; \( U_{i,j}^n, U_{i,j}^{n+1} \) = vectors of conservative variables at the time step \( n, n+1 \); \( \Delta t \) = computational time step. Equation (2.17a, b) is termed the original version of time integration, which is applied with the depth gradient method (DGM) for flux computation (see 2.4.4). A modified version, which moves the bed slope effect to the homogeneous part, is further introduced in section 2.4.4, together with a different treatment of the flux computation, the so-called surface gradient method (SGM).

Because the solution procedure is explicit, the computational time step should be restricted by stability conditions. For flow computation, the time restriction mainly results from the advection term and the diffusion term. The former can be described by the Courant-Friendrichs-Lewy (CFL) condition and the latter by the stability condition related to the Peclet number. For advection-dominated problems, the CFL condition plays a major role in controlling the flow computation, whereas in diffusion-dominated problems with very fine computational grids the Peclet number may strictly restrict the time step.

The Courant condition reads

\[
\Delta t \leq \frac{Cr}{\max \left[ \left( \left| u \right| + \sqrt{gh} \right)/\Delta x, \left| v \right| + \sqrt{gh} \right)/\Delta y \right] \quad (2.19)
\]

where \( \Delta t \) = maximum time step related to CFL condition; \( Cr \) = critical Courant number for stability \( (0 < Cr \leq 1) \).

The Peclet-condition reads
Fully coupled morphodynamic model

\[ \Delta t_g \leq \frac{1}{\max \left( 2\nu_x / \Delta x^2, 2\nu_y / \Delta y^2 \right)} \]  \hspace{1cm} (2.20)

where \( \Delta t_g \) = maximum time step according to the Peclet-condition.

For sediment transport and bed change, an additional condition applies, ensuring that the amount of sediment deposited within a time step is less than the amount carried by the flow (i.e. \( \Delta t_s D \leq hc \)), which is similar to that by Heng et al. (2009),

\[ \Delta t_s \leq \frac{h}{\alpha \omega_s} \]  \hspace{1cm} (2.21)

where \( \Delta t_s \) = time step related to the mobile bed condition.

Exact expressions of stability conditions for the whole complex system are difficult to derive. Therefore the most restrictive of the above conditions is applied (see Eq. (2.22)). The applicability of this approach is demonstrated by the satisfactory results in the model testing in Chapter 3.

\[ \Delta t = \min(\Delta t_f, \Delta t_g, \Delta t_s) \]  \hspace{1cm} (2.22)

It should be pointed out that in addition to the stability condition for the exponential adaptation process of the concentration in time (Eq. (2.21)), a similar condition should apply in space:

\[ \Delta x \leq \alpha 2hu / \alpha \omega_x, \hspace{0.5cm} \Delta y \leq \alpha 2hv / \alpha \omega_y \]  \hspace{1cm} (2.23a,b)

### 2.4.4 UFORCE scheme for computing fluxes

The UFORCE scheme (Stecca et al., 2010) is an upwind-biased version of the FORCE scheme (Toro, 2009), which requires a primary (Cartesian) mesh to compute cell-averaged variables and four staggered (non-Cartesian due to the upwinding) meshes for computing interface fluxes in 2-D conditions as shown in Fig. 2.2. An upwind bias parameter is introduced by modifying the shape of the staggered mesh from the original Cartesian shape of the FORCE scheme in the \( x \)- and \( y \)-directions respectively. The UFORCE scheme has been demonstrated to be capable of accurately capturing shock waves and contact discontinuities for shallow water flow and scalar transport over a rigid bed without source terms. Here, the 2\(^{nd}\) order version of this scheme, constructed through the MUSCL (Monotonic Upstream-Centered Scheme for Conservation Laws) -Hancock approach (van Leer, 1979), is extended to sediment-laden flows over a mobile bed (Li et al., 2013). Three steps are involved: 1) spatially 2\(^{nd}\) order accuracy by data reconstruction; 2) temporally 2\(^{nd}\) order accuracy by state evolution; 3) computation of UFORCE flux (viz. solve Riemann problem).
Here, two versions of the UFORCE scheme are introduced, which differ from each other in the steps of data reconstruction and state evolution. The first version belongs to the class of depth gradient methods (DGM), which uses the spatial gradient of water depth. The second one belongs to the class of surface gradient methods (SGM), which uses the spatial free surface gradient.

With the DGM, the UFORCE scheme is robust and stable when high water level gradients occur with small water depth gradients over regular bed topography. It is also capable of tracking dry-wet fronts if a tolerance depth is specified. The version is verified by the dam-break cases in Chapter 3 and applied to the 1-D hyperconcentrated flood modeling in Chapter 4. However, the DGM version does not yield a stable solution for static flow over irregular bed topography (usually called that the C-property is not satisfied), due to the imbalance between the advection flux and the bed slope (Hubbard and Garcia-Navarro, 2000; Zhou et al., 2001; Audusse et al., 2004; Aureli et al., 2008; Liang and Marche, 2009). This may produce considerable numerical errors in practical applications with highly irregular beds. Therefore, the SGM developed by Zhou et al. (2001) is adopted to modify the original UFORCE scheme in order to satisfy the C-property (retaining the initial static flow

Figure 2.2: Sketch of mesh configuration: P denotes the centre of a cell in the primary Cartesian mesh (solid lines); the circles indicate the upwind-biased cell-centres; S denotes the centres of the four surrounding non-Cartesian staggered cells (dashed lines).
over irregular beds). This well-balanced version is deployed in the model of a hyperconcentrated flood in a channel-floodplain system described in Chapter 5.

In the following, the DGM and SGM versions of the UFORCE scheme are introduced for the computation of the advection flux in \( x \)-direction. In \( y \)-direction, the method is analogous.

- **DGM version of the UFORCE scheme**

**Step 1: MUSCL reconstruction**

In order to achieve 2\(^{nd}\) order accuracy in space, the cell interface values are extrapolated from the cell center values by a limited slope,

\[
U_{i+1/2,j}^L = U_{i,j}^n + 0.5\overline{A}, \quad U_{i+1/2,j}^R = U_{i+1,j}^n - 0.5\overline{A}_{i+1}
\]

where \( U_{i+1/2,j}^L, U_{i+1/2,j}^R \) = vectors of extrapolated variables at the left and right sides of the interface between cells \((i, j)\) and \((i+1, j)\); \( \overline{A}, \overline{A}_{i+1} \) = vectors of limited slope.

Many methods have been introduced to construct the limited slope and slope limiters for TVD schemes. Here it is computed by the ENO (Essentially Non Oscillatory) approach (Harten et al., 1987) to avoid spurious oscillations near large gradients (Stecca et al., 2010),

\[
\overline{A} = \begin{cases} 
U_{i+1,j}^n - U_{i,j}^n & \text{if } |U_{i+1,j}^n - U_{i,j}^n| \leq |U_{i,j}^n - U_{i-1,j}^n| \\
U_{i+1,j}^n - U_{i-1,j}^n & \text{otherwise}
\end{cases}
\]

**Step 2: State evolution by \( \Delta t/2 \)**

The cell interface variables are further evolved over a half time step to reach 2\(^{nd}\) order accuracy in time, as

\[
U_{i+1/2,j}^{*L} = U_{i+1/2,j}^L + \frac{\Delta t}{2\Delta x}\left[F(U_{i+3/2,j}^R) - F(U_{i+1/2,j}^L)\right] + \frac{\Delta t}{2\Delta y}\left[G(U_{i,j+1/2}^R) - G(U_{i,j+1/2}^L)\right]
\]

\[
(2.26a)
\]

\[
U_{i+1/2,j}^{*R} = U_{i+1/2,j}^R + \frac{\Delta t}{2\Delta x}\left[F(U_{i+1/2,j}^R) - F(U_{i+3/2,j}^L)\right] + \frac{\Delta t}{2\Delta y}\left[G(U_{i+1,j+3/2}^R) - G(U_{i+1,j+1/2}^L)\right]
\]

\[
(2.26b)
\]

**Step 3: UFORCE flux computation** (i.e. Riemann problem)
At the interface \((i+1/2, j)\), a pair of constant states \((U_{i+1/2, j}^{L*, R*})\) has been obtained from the above steps. The advection flux can now be computed by the UFORCE scheme, which is essentially an average of the upwind-biased versions of the Lax-Friedrichs flux and the two-step Lax-Wendroff flux, as

\[
F_{i+1/2, j}^{\text{UFORCE}} = 0.5 \left( F_{i+1/2, j}^{\text{ul,W}} + F_{i+1/2, j}^{\text{ul,F}} \right)
\]  

(2.27)

where \(\alpha_D\) = number of dimensions, i.e. = 1 for 1-D case, = 2 for 2-D case; \(F_{i+1/2, j}^{\text{ul,W}}\) = upwind-biased version of the two-step Lax-Wendroff flux; \(F_{i+1/2, j}^{\text{ul,F}}\) = upwind-biased version of the Lax-Friedrichs flux; \(F_{i+1/2, j}^{\text{UFORCE}}\) = UFORCE flux.

The upwind-biased Lax-Wendroff flux is estimated by introducing an upwind bias parameter \(\beta\) in the original Lax-Wendroff flux (Toro, 2009), like

\[
F_{i+1/2, j}^{\text{ul,W}} = F \left( U_{i+1/2, j}^{\text{ul,W}} \right)
\]  

(2.28a)

\[
U_{i+1/2, j}^{\text{ul,W}} = \frac{1}{2(1-(\beta_x)_{i+1/2, j}+(\beta_x)_{i+1/2, j})} \left[ (1+2(\beta_x)_{i+1/2, j})U_{i+1/2, j}^{L*} + (1-2(\beta_x)_{i+1/2, j})U_{i+1/2, j}^{R*} - \frac{\alpha_D \Delta t}{\Delta x} \left( F(U_{i+1/2, j}^{R*}) - F(U_{i+1/2, j}^{L*}) \right) \right]
\]  

(2.28b)

where \(U_{i+1/2, j}^{\text{ul,W}}\) = vector of newly constructed conservative variable in the two-step Lax-Wendroff flux; \(\beta_x\) = upwind bias parameter in \(x\) direction. The upwind-biased Lax-Friedrichs flux is estimated by (Stecca et al., 2010)

\[
F_{i+1/2, j}^{\text{ul,F}} = \frac{1}{2(1-(\beta_x)_{i+1/2, j}+(\beta_x)_{i+1/2, j})} \left[ (1+2(\beta_x)_{i+1/2, j})F(U_{i+1/2, j}^{L*}) + (1-2(\beta_x)_{i+1/2, j})F(U_{i+1/2, j}^{R*}) \right]
\]  

\[
- \frac{\Delta x}{\alpha_D \Delta t} (1+2(\beta_x)_{i+1/2, j})(1-2(\beta_x)_{i+1/2, j})(U_{i+1/2, j}^{R*} - U_{i+1/2, j}^{L*})
\]  

(2.29)

The critical issue in the extension to the mobile bed case therefore relates to the estimation of the upwind bias parameter. In shallow water flows, this parameter is computed separately in \(x\) and \(y\) directions using the same method (Stecca et al., 2010). For the parameter in the \(x\) direction,

\[
(\beta_x)_{i,j} = \text{SIG}_{i,j} \left( 0.5 - \lambda_{x,i,j} \frac{\Delta t}{\Delta x_{i,j}} \right)
\]  

(2.30)

with function
Fully coupled morphodynamic model

\[
SIG_{i,j} = \begin{cases} 
\text{sign}(u_{i,j}) & \text{if } (u_{i,j} \neq 0) \\
\text{sign}(\lambda_{i-1,j} + (\lambda_{i+1,j}) & \text{if } (u_{i,j} = 0) \text{ and } (\lambda_{i-1,j} + (\lambda_{i+1,j}) \neq 0) \\
0 & \text{if } (u_{i,j} = 0) \text{ and } (\lambda_{i-1,j} + (\lambda_{i+1,j}) = 0)
\end{cases}
\]

(2.31)

\[
\text{sign}(u_{i,j}) = \begin{cases} 
1 & \text{if } u_{i,j} > 0 \\
-1 & \text{if } u_{i,j} < 0
\end{cases}
\]

(2.32)

where \( \lambda_{sx} = \) absolute value of the maximum local characteristic speed in shallow water flows; \( \lambda_{i,j} = \) characteristic speed related to flow part. For the fixed bed case, the values of the characteristic speed are (Stecca et al., 2010)

\[
\lambda_{sx} = \left( u + \sqrt{gh} \right)_{i,j}, \quad \lambda_{1} = u - \sqrt{gh}, \quad \lambda_{2} = u + \sqrt{gh}
\]

(2.33a,b,c)

In the extension to a mobile bed, the values of the characteristic speed can be estimated by rewriting the equation system in a non-conservative form. In the 1-D problem, the non-conservative vector form of the governing equations for the present model reads (i.e. in \( x \) direction)

\[
\frac{\partial W}{\partial t} + A \frac{\partial W}{\partial x} = R
\]

(2.34)

\[
W = \begin{bmatrix} h \\ u \\ c \\ (1-p)z+c \end{bmatrix}, \quad A = \begin{bmatrix} u & h & 0 & 0 \\ g & u & a_{23} & a_{24} \\ 0 & 0 & u & 0 \\ 0 & 0 & u & 0 \end{bmatrix}
\]

(2.35a,b)

\[
R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = -gS_f - \frac{\rho_g u(E-D)}{\rho h(1-p)} - \frac{(\rho_s - \rho_w)u}{\rho h} \Phi_{sx} + \frac{1}{h} \partial(\tau_{sx}) + \frac{E-D}{1-p} + \frac{1}{h} \Phi_{sx}
\]

(2.36)

where \( a_{23} = (\rho_s - \rho_w)g/2 \rho - g/(1-p) \), \( a_{24} = g/(1-p) \), \( \Phi_{sx} = \partial(hc/\partial x)/\partial x \), \( \tau_{sx} = 2 \nu \partial u/\partial x \). Other variables are the same as specified above. It should be pointed out that in the present non-conservative equation form, the diffusion terms containing high order derivatives are still put in the source term as in the conservative equation form. This treatment neglects the influences of diffusion terms on the characteristic...
2.4 Numerical algorithm

celerities as it does in a fixed bed situation. Therefore, the characteristic celerities of the model are determined by the advection part. By solution of the eigenvalues of the Jaccobian matrix $A$ through $|A - \lambda I| = 0$, the characteristic speeds for mobile bed case are obtained, as

$$\lambda_{1,3} = u \pm \sqrt{gh}, \quad \lambda_2 = u, \quad \lambda_4 = 0$$

(2.37a,b,c)

where $\lambda_{2,4}$ = sediment and bed related characteristic speed. Thus for the present model of non-capacity sediment transport (Eq. (2.34)), the flow-related eigenvalues $\lambda_{1,3}$ and the maximum local characteristic speed $\lambda_{\text{max}}$ in the mobile bed condition are the same as those for the fixed bed case. This is also demonstrated by Cao et al. (2007) using a similar equation system. Note that the characteristic analysis is a way of revealing certain aspects of the behaviour of the solution, but does not describe the entire solution. In the present work employing the physically more realistic non-capacity modeling approach, the characteristic speeds are only determined by the flow condition. This should not be confused with the characteristic analysis for a capacity sediment transport model that the characteristic speeds are also affected by sediment concentration (De Vries, 1969; Morris and Williams, 1996).

- SGM version of the UFORCE scheme

**Step 1: MUSCL reconstruction**

The water depth is obtained by free surface reconstruction, as

$$\xi_{i+1/2,j}^L = \xi_{i,j}^n + 0.5 \overline{\Delta \xi}, \quad \xi_{i+1/2,j}^R = \xi_{i+1,j}^n - 0.5 \overline{\Delta \xi}$$

(2.38a,b)

$$h_{i+1/2,j}^L = \xi_{i+1/2,j}^L - z_{i+1/2,j}, \quad h_{i+1/2,j}^R = \xi_{i+1/2,j}^R - z_{i+1/2,j}$$

(2.39a,b)

with

$$\overline{\Delta \xi} = \begin{cases} \xi_{i+1,j}^n - \xi_{i,j}^n & \text{if } |\xi_{i+1,j}^n - \xi_{i,j}^n| \leq |\xi_{i,j}^n - \xi_{i-1,j}^n| \\ \xi_{i,j}^n - \xi_{i-1,j}^n & \text{else} \end{cases}$$

(2.40)

$$z_{i+1/2,j} = (z_{i,j} + z_{i,j+1}) / 2$$

(2.41)

where $\xi$ = water level; $\overline{\Delta \xi}$ = limited slope for water level; $z_{i+1/2,j}$ = bed elevation at the cell interface. Other conservative variables are reconstructed in the same way as in the DGM-method (see Eq. (2.24)).

**Step 2: State evolution by $\Delta t/2$**
Differing from the DGM version, the bed slope effect is included in this step for state evolution:

\[
U_{i+\frac{1}{2},j}^{L,*} = U_{i+\frac{1}{2},j}^L + \frac{\Delta t}{2\Delta x} \left[ F(U_{i+\frac{1}{2},j}^R) - F(U_{i+\frac{1}{2},j}^L) \right] + \frac{\Delta t}{2\Delta y} \left[ G(U_{i+\frac{1}{2},j-\frac{1}{2}}) - G(U_{i+\frac{1}{2},j+\frac{1}{2}}) \right] + \frac{\Delta t}{2} S_{0,i,j} \tag{2.42a}
\]

\[
U_{i+\frac{1}{2},j}^{R,*} = U_{i+\frac{1}{2},j}^R + \frac{\Delta t}{2\Delta x} \left[ F(U_{i+\frac{3}{2},j+\frac{1}{2}}^R) - F(U_{i+\frac{3}{2},j-\frac{1}{2}}^L) \right] + \frac{\Delta t}{2\Delta y} \left[ G(U_{i+\frac{3}{2},j+\frac{1}{2}}) - G(U_{i+\frac{3}{2},j-\frac{1}{2}}) \right] + \frac{\Delta t}{2} S_{0+i+1,j} \tag{2.42b}
\]

with

\[
S_{0,i,j} = \begin{bmatrix}
0 \\
- \frac{g}{2} \frac{h_{i+1/2,j}^L + h_{i+1/2,j}^R}{\Delta x} \frac{z_{i+1/2,j} - z_{i-1/2,j}}{\Delta x} \\
- \frac{g}{2} \frac{h_{i,j+1/2}^L + h_{i,j+1/2}^R}{\Delta y} \frac{z_{i,j+1/2} - z_{i,j-1/2}}{\Delta y} \\
0 \\
0
\end{bmatrix} \tag{2.43}
\]

**Step 3: UFORCE flux computation**

This step is the same as that in the DGM-method described above.

Note that the time integration of the SGM-version should include the effect of the bed slope in the homogeneous part, whereas the DGM-version considers it as a source term. Deploying a 2nd order Runge-Kutta method (Toro, 2009), the time integration of the SGM version reads

\[
U_{i,j}^{n} = U_{i,j}^{n,*} - \frac{\Delta t}{\Delta x} \left( F_{i+1/2,j}(U^n) - F_{i-1/2,j}(U^n) \right) - \frac{\Delta t}{\Delta y} \left( G_{i,j+1/2}(U^n) - G_{i,j-1/2}(U^n) \right) + \Delta \tilde{S}_{0,i,j} \tag{2.44a}
\]

\[
U_{i,j}^{n+1} = U_{i,j}^{n,*} + 0.5 \times \left( K_i + K_j \right) \tag{2.44b}
\]

with
2.4 Numerical algorithm

\[
S_{0i,j} = \begin{bmatrix}
0 \\
- \frac{g h^{L*}_{i/2,j} + h^{K*}_{i/2,j}}{2} \frac{z_{i+1/2,j} - z_{i-1/2,j}}{\Delta x} \\
- \frac{g h^{L*}_{i,j/2} + h^{K*}_{i,j/2}}{2} \frac{z_{i+j+1/2} - z_{i,j-1/2}}{\Delta y} \\
0 \\
0
\end{bmatrix}
\]  

\[
K_1 = \Delta t (S_t(U^+_{i,j}) + S_t(U^-_{i,j})) , \quad K_2 = \Delta t (S_t(U^+_{i,j} + K_t) + S_t(U^-_{i,j} + K_t))
\]  

(2.45)

2.4.5 Estimation of turbulent eddy viscosity

Unlike unbounded 2-D flows, the flow pattern in shallow water is strongly influenced by the depth and bed friction (Chen and Jirka, 1995, 1997; Lloyd et al., 2001). Thus for 2-D shallow water flows, the turbulence model should consider both turbulence generated at the bed and turbulence associated with 2-D horizontal eddies (Uijttewaal and Tukker, 1998). Following this concept, the turbulent eddy viscosity accounting for both the horizontal and vertical production of turbulence is calculated from (Jia and Wang, 1999; Cea et al., 2007)

\[
v_t = l_s^2 \sqrt{2S_\eta S_\eta + \left(\frac{\kappa u_* h}{6l_s^2}\right)^2}
\]  

(2.47)

with

\[
S_\eta = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)/2 , \quad l_s = C_s \sqrt{\Delta x \Delta y}
\]  

(2.48)

where \( S_\eta \) = horizontal mean strain-rate tensor; \( u_i, u_j \) = depth-averaged velocities; \( x_i, x_j \) = horizontal coordinates; \( l_s \) = characteristic horizontal turbulent length scale; \( C_s \) = Smagorinsky constant \( \approx 0.16 \) for isotropic turbulent flow (Lilly, 1967); \( \kappa = 0.41 \) = von Karman constant. The diffusion coefficient for suspended load is simply assumed to be the same as the turbulent eddy viscosity, whereas it is set equal to zero for bed-load transport.

2.4.6 Treatment of wetting and drying fronts

When solving the constituting equations with a FVM, it is critical to predict the evolution of wetting and drying fronts in cases such as flood inundation and dam-break over a dry bed. In practice, a small value of water depth is usually used to simulate the wetting and drying fronts. Following Zhao et al. (1994), two tolerance
depths $h_{tol1}$ and $h_{tol2}$ ($h_{tol1} > h_{tol2}$) are introduced for the cell type judgment: dry, partially dry and wet cells. For a wet cell with the water depth being larger than $h_{tol1}$, the governing equations (Eqs. (2.14-15)) are used. For a partially dry cell, the water depth is between the two specified tolerance depths with the velocity, sediment concentration and bed level change being set to zero. For a dry cell, the water depth is even smaller than $h_{tol2}$ and set to zero together with the velocity, sediment concentration and bed level change. At the interface of the wet cells, the normal flux is automatically computed by the second order UFORCE scheme (see section 2.4.4).

At the interface of wet and dry cells, two conditions are considered. If the water surface of wet cell is higher than the bed elevation of dry cell, the normal flux from the wet to dry cells is computed by the second order UFORCE scheme. If the water surface of wet cell is not higher than the bed elevation of dry cell, the normal flux should be balanced by the bed slope term to achieve zero velocity. In this case, the interface bed elevation is set at the level of the (extrapolated) free surface of the adjacent wet cell to avoid numerical instability (Brufau et al., 2002; Aureli et al., 2008). Between dry cells the normal flux is zero.

For a new time interval, a cell changes from dry to wet if water can flow into it from surrounding cells, and from wet to dry if the computed depth is smaller than $h_{tol2}$. The equations for the above treatment of a partially dry cell and a dry cell are:

$u = v = c = \Delta z = 0$ (2.49)

$h = u = v = c = \Delta z = 0$ (2.50)

, respectively, where $\Delta z =$ bed change over a time increment.

### 2.4.7 Boundary conditions

The present model includes two types of boundary conditions, for closed and open boundaries, respectively. At a closed boundary, a free-slip and non-permeable condition is used. For Cartesian coordinates of $x$ and $y$, this condition reads

$h_b = h_L, \quad c_b = c_L, \quad \Delta z_b / \Delta t = \Delta z_L / \Delta t$ (2.51a,b,c)

where the subscripts $b,L$ denote the positions of the closed boundary and the adjacent inner cell respectively. If the closed boundary is along the $x$ direction, we set $u_b = u_L$, $v_b = 0$. Otherwise $v_b = v_L$, $u_b = 0$.

At an open boundary, different methods are used based on the directions of the characteristics. When the characteristics enter the model domain (e.g., supercritical flow conditions), boundary conditions should be given. Usually, the time variations of flow depth, velocity and sediment concentration are imposed at the boundary. When
the characteristics leave the model domain (e.g., subcritical flow conditions), the boundary variables can be computed through the method of characteristics. In this thesis, a modified characteristic method is developed to compute the boundary variables for flow and sediment, together with a second order Runge-Kutta scheme (see Eqs. (2.17-18)). The bed level is computed using the bed update equation (Eq. (2.4)).

Unlike the traditional characteristic method for clear water flow, in which the bed level update is decoupled from the flow computation, the modified version updates the bed level while computing the flow and sediment simultaneously. For a general form of the compatibility equation along the characteristics, such as

\[
\frac{dw}{dt} = f(w)
\]  

(2.52)

where \( w \) = non-conservative variable (i.e. \( u \) or \( c \)) along the characteristics; \( f \) = source term in the compatibility equation. An example for the exact formulation of \( f \) in the sediment-laden flow is given by Cao et al. (2007). The modified characteristic method within an iterative step reads

\[
w^{n+1} = w^n + 0.5(k_1 + k_2)
\]  

(2.53)

with \( k_1 = \Delta t f(w^n) \), \( k_2 = \Delta t f(w^n + k_1) \)  

(2.54a,b)

where \( w^{n+1} \) = estimate of the variable at the boundary for the new time step; \( w^n \) = estimate of the variable along the characteristics (inside the computational domain) for the previous time step. During one iterative step, the bed level is updated by Eq. (2.4) using the newly-obtained flow and sediment variables after the computation of Eq. (2.53). The iteration continues until the difference between two consecutive iterative steps is negligible. Therefore the computed variables from the last iterative step are used to approximate the boundary conditions for the new time step. This coupling treatment at the open boundaries facilitates a completely synchronous solution of the sediment-laden flow and river bed in the whole computational domain. Furthermore, it enhances the numerical stability at open boundaries in the case of rapid bed deformation, such as hyperconcentrated floods and mobile bed dam-break.

### 2.5 Summary

An accurate and efficient 2-D depth-averaged model has been developed solving the fully coupled non-capacity equations. This has been achieved by the successful extension of the 2\(^{nd}\) order UFORCE method (developed by Stecca et al. (2010)) from
the idealized frictionless fixed bed case to the mobile bed case. The combination of
the upwind and centered methods in the UFORCE scheme yields high accuracy and
computational efficiency, which is an improvement compared with previous fully
coupled and non-capacity models.

In solving the equations, flow, sediment concentration and bed level changes are
computed simultaneously. An explicit two-stage splitting approach (Toro, 2001)
together with a 2\textsuperscript{nd} order Runge-Kutta method is used for the inhomogeneous system.
The model is 2\textsuperscript{nd} order accurate in space and time. At the open boundaries, a modified
characteristic method is used, which incorporates the above Runge-Kutta method and
the morphological update in each iterative cycle of flow (sediment) computation. This
treatment enhances the model stability and permits a completely synchronous and
fully coupled solution in the entire computational domain.

The DGM and SGM versions of the present model will be verified by a series of dam-
break tests in Chapter 3, and then applied to the 1-D hyperconcentrated floods in
Chapter 4 (DGM version), and to the 2-D hyperconcentrated floods in channel-
floodplain system in Chapter 5 (SGM version) respectively.
3.1 Introduction

The peculiar features of hyperconcentrated floods entail great challenges to the existing mathematical modeling, due to 1) highly unsteady hydrographs and sediment processes; 2) intensive sediment transport and rapid bed deformation; 3) strong interactions between flow, sediment and morphological changes. The development of the fully coupled mathematical model introduced in Chapter 2 aims to provide a refined resolution for hyperconcentrated flood propagation over an erodible bed using physically more advanced mathematical descriptions and deploying fully synchronous numerical solutions. Also, it will facilitate better understanding of the dynamic mechanisms of hyperconcentrated floods.

Tests of the present model are necessary before it is used for application and research purpose. Yet field-scale observations of the hyperconcentrated flood cannot provide sufficient information for the test, because of 1) their incomplete data such as event-specific or discontinuous records; 2) accuracy restricted by the field measuring techniques; 3) many uncertainties from the complex field conditions. Instead, the dam-break flow, which may be also characterized by rapidly changing flows and intensive sediment transport over erodible beds, is served as an alternative for the model test. Plenty of laboratory experimental data, analytical and numerical solutions for dam-break problems can be found in literature. The model’s abilities to capture shock waves (e.g., interchange from supercritical to subcritical flow regimes) and contact discontinuities (e.g., strong spatial sediment concentration gradient), as well as its stability on rapid bed deformation at high sediment transport rates and C-property will be tested through dam-break cases.

In this chapter, six test cases are selected to verify the DGM and SGM versions of the present model (see Chapter 2), including Cases 1-5 for DGM and Cases 4 and 6 for SGM. They are: (1) 2-D idealized dam break over a fixed dry bed; (2) a 2-D dyke-breach experiment on a wet fixed bed; (3) idealized dam-break flows over mobile beds; (4) dam-break flow in an abruptly widening mobile-bed channel; (5) a 2-D partial dam-break in a straight mobile-bed channel; (6) preservation of still water at a 1-D surface-piercing hump.

3.2 Model results of DGM version

3.2.1 2-D idealized dam-break over fixed dry bed

Many laboratory experiments have been performed for dam-break flows over fixed beds (Fraccarollo and Toro, 1995; Stelling and Duinmeijer, 2003; Ferrari et al., 2010). To verify the model’s ability for flooding over a fixed dry bed, the experiment conducted by Fraccarollo and Toro (1995) is simulated. The experimental flume consisted of an upstream reservoir of 1×2 m and a downstream floodplain of 2×2 m (see Fig. 3.1). A 0.4 m wide breach was located in the middle of the downstream boundary of the reservoir. For the present test, the flume was horizontal and the bottom friction was negligible. The dam-break was initiated with upstream 0.6 m deep water flowing over a downstream dry bed. Measuring gauges were installed at distinct locations (see Fig. 3.1) with coordinates shown in Table 3.1.

![Figure 3.1: Sketch of the experimental flume (unit: m).](image)
3.2 Model results of DGM version

Table 3.1 Coordinates of measuring gauges (unit: m)

<table>
<thead>
<tr>
<th>gauge</th>
<th>O</th>
<th>-5A</th>
<th>8A</th>
<th>C</th>
<th>5B</th>
<th>-5B</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0</td>
<td>-0.82</td>
<td>0.722</td>
<td>-0.52</td>
<td>0.454</td>
<td>-0.845</td>
</tr>
<tr>
<td>y</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.6</td>
<td>0.25</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

The computational domain is divided by grid cells of 0.02×0.02 m. Closed boundary conditions are imposed at all solid edges of the reservoir and free outflow at the three open boundaries of the downstream floodplain. The two tolerance depths for judging the dry and partially dry cells are set to 0.0001 m and 0.0002 m. The computation lasts for 10 s under the CFL condition of \( Cr = 0.45 \).

Figure 3.2: Comparison of computed (solid lines) and measured (circles) water level history at distinct gauges.

Figure 3.2 shows good agreement between the computed and measured water level above the fixed bed at distinct gauges. At the breach (i.e. Gauge O), the model captures the phenomenon of falling and rising stages at the beginning of the dam break. Inside the reservoir, the lowering of water level due to a depression wave is also well reproduced by the present model (see Gauges -5A, -4B, C). In the downstream region, the model satisfactorily predicts the wave front at Gauge 8A.
while it computes much lower water level in the early 2 s at Gauge 5B. This could be caused by the measuring method that the water level was obtained from pressure meters at Gauge 5B while it was directly measured by wave height meters at the other gauges (Fraccarollo and Toro, 1995). In general, the model is capable of capturing the key features of 2-D dam-break flow over a dry bed.

### 3.2.2 2-D dyke-breach over horizontal fixed bed

Case 2 is the wet bed dyke-breach experiment undertaken in the Fluid Mechanics Laboratory at Delft University of Technology (Stelling and Duinmeijer, 2003). In this test case, the influences of the gate opening and the effects of breach contraction are not negligible, which causes more complexity and difficulty in numerical modeling.

The test flume contained a flat fixed bed with an area of 31.4×8 m. The dam was a solid wall at \( x = 2.5 \) m and had a 0.4 m wide gate opening in the middle (see Fig. 3.3). The initial water depth inside the upstream reservoir was 0.6 m, and 0.05 m in the downstream region. Measuring gauges were installed along the flume centre line and numbered in sequence (G0-G5) by specific distances from the gate opening, -1, 1, 6, 9, 13, 17 m. At the start of the experiment, the gate was opened by lifting the door at a speed of 0.16 m/s.

The relatively slow movement of the gate door requires a special treatment until the gate is fully opened. Following Liang et al. (2004), the discharge at the gate is calculated using the formula for submerged culvert flow:

\[
q = C_w h_w \sqrt{2g \left( H - C_w h_w \right)}
\]  

(3.1)

where \( q \) = flow discharge, m\(^2\)/s; \( H \) = headpond water depth (taken equal to the water depth of the grid cell immediately upstream of the gate); \( h_w \) = gate opening height; \( C_w \) = contraction coefficient = 0.9. When the gate is completely opened, the local head loss due to the sudden contraction is taken into account by incorporating an extra resistance stress (Zhou et al., 2002) immediately upstream of the opening:

\[
\tau_{\xi_x} = -\xi u_x \sqrt{u_x^2 + v_y^2}, \quad \tau_{\xi_y} = -\xi u_y \sqrt{u_x^2 + v_y^2}
\]

(3.2a,b)

where \( \tau_{\xi_x(y)} \) = resistance stress in \( x (y) \) directions; \( \xi = 0.8; u_x, v_y = \) velocities in the \( x \) and \( y \) directions at the breach.

The computational domain is divided by grid cells of 0.1×0.1 m. The model runs for 25 s under the CFL condition of \( Cr = 0.45 \). Closed boundary conditions are imposed at all solid edges. The Manning roughness coefficient is set at 0.012 for the wet bed (Stelling and Duinmeijer, 2003).
The numerical results of the present 2-D coupled model agree closely with those published by others (Stelling and Duinmeijer, 2003; Liang et al., 2004; Liang et al., 2006; Cui et al., 2010). Figure 3.4 shows good agreement between the computed and measured wave front positions at \( t = 1, 2, 3 \) and \( 4 \) s. The wave front is predicted slightly faster near the downstream side wall of the dyke and along the flume center line. This is probably because the wall friction and the influence of a piece of plastic along the center line are not considered in the current modeling.
Figure 3.5 shows 3-D view and the contour plots of the computed water surface at distinct instants. In the early stage of the dyke-breach (e.g. $t=4$ s, see Fig. 3.5a), a shock wave is observed propagating downstream with a semi-circular wave front heading a water mass of almost constant depth. This water mass is then followed by a hydraulic jump, which is pushed downstream by the strength of high-speed flow from the gate opening. Reflective waves are observed when the wave front hits the lateral walls. As time proceeds, the wave front becomes nearly straight with curved bore-reflection waves behind as shown in Fig. 3.5b, due to the complex interactions between the reflective waves and the initial waves. The hydraulic jump still exists in the later stage but migrates downstream at a much lower speed than the wave front.
3.2 Model results of DGM version

As shown in Fig. 3.6, the computed water depth agrees generally well with the experiment data at various gauges along the center line. Particularly the magnitude and arrival time of the first flood wave are well simulated at all gauges downstream of the breach. Yet less accurate results are predicted for the second flood wave. In the immediate downstream of the breach (i.e. Gauge 1), the water level is overestimated after the first flood wave. This probably causes the underestimation of the magnitude and speed of the second flood wave in the downstream. Moreover, the treatment of the gate opening and contraction effects is quite empirical in the model, uncertainties are inevitable. With increasing distance, the accumulative effect of bed friction further slows down the second wave propagation. But in general, the model result is satisfactory and in similar trend of accuracy with other published models (Stelling and Duinmeijer, 2003; Liang et al., 2004; Liang et al., 2006).

3.2.3 Idealized dam-break flows over mobile beds

To verify the model’s hydro-morphological modeling performance, two sets of idealized tests on dam-break flow over a mobile bed are numerically investigated.
These tests comprise the experiments carried out in Taipei (University of Taiwan) and Louvain-la-Neuve (Université Catholique de Louvain), reported by Capart and Young (1998) and Fraccarollo and Capart (2002) respectively. Both experiments were performed in horizontal prismatic flumes of rectangular cross-sections, but differ primarily in the sediment material used. In the Taipei experiment, the flume was 1.2 m long, 0.2 m wide and 0.7 m deep. It was initially covered by a 5-6 cm thick layer of light artificial pearls (not natural sand), with a diameter of 6.1 mm, specific gravity of 1.048 and settling velocity of 0.076 m/s. In the Louvain experiment, the flume was 2.5 m long, 0.1 m wide and 0.35 m deep. Cylindrical PVC pellets (not natural sand) having a diameter of 3.2 mm, height of 2.8 mm (hence, an equivalent spherical diameter of 3.5 mm), specific gravity of 1.54 and settling velocity of 0.18 m/s constituted an initial sediment layer of 5-6 cm thick over the fixed bottom. In both experiments, an idealized dam was located in the middle of the flume separating an upstream static flow region of 10 cm deep from a dry downstream part. At \( t = 0 \) s, the dam was lifted rapidly to create the dam break flow.

For the numerical simulation of both tests, the domain is divided into grid cells of 0.0025×0.0025 m\(^2\) and the Courant number in the CFL condition is 0.3 based on a convergence study. The simulation time is 0.6 s and 1.2 s for the Taipei and Louvain tests respectively. Following Wu and Wang (2007), the bed porosity is set at 0.28 and 0.3 in the Taipei and Louvain tests respectively while the Manning roughness coefficient of 0.025 is used for both cases. For the sediment entrainment/deposition, Equations (2.11-2.13) are used (see Chapter 2) with relevant parameters specified below. The threshold Shields parameter \( \theta_c \) is set at 0.15 and 0.05 in the Taipei and Louvain tests respectively (Li and Duffy, 2011). The submerged specific gravity of sediment \( s \) is calculated as 0.048 and 0.54 for the Taipei and Louvain tests. Trial runs give values of other parameters: \( \alpha = 3, \varphi = 6 \) in the Taipei test and \( \alpha = 3, \varphi = 3 \) in the Louvain test.

Based on a number of assumptions (i.e. capacity sediment transport, a constant concentration in sediment moving layers and zero momentum loss due to bed friction etc.), pseudo-analytical solutions of free surface and bed evolution were derived by Fraccarollo and Capart (2002) for both tests. In Figs. 3.7 and 3.8 the computed results at varying times are compared with those analytical solutions in non-dimensional self-similar coordinates together with the experimental data. For both tests, good agreement is generally observed between the simulation and measurement while to some parts the analytical solution exhibits qualitative discrepancies from them.

For the Taipei test (see Fig. 3.7), the agreement between the simulation, analytical solution and measurement is fairly good for the wave front location, the erosion magnitude and the downstream (adverse) slope of the scour hole. Yet considerable differences appear in the upstream (downward) slope of the scour hole and the water
3.2 Model results of DGM version

surface. Along the upstream slope of the scour hole, erosion is somewhat underestimated by the numerical simulation while excessively overestimated by the analytical solution, as compared to the experiment. The hydraulic jump occurring near the dam-break location can be qualitatively captured by the simulation, though with less accurate prediction of its location. In contrast, the analytical solution shows a constant water level and fails to predict this phenomenon. For the front water surface, the computed and analytical results are very close and agree well with the measured data. The back water surface can be satisfactorily predicted by the simulation but less well by the analytical solution.

Figure 3.7: Comparison of computed water surface and bed level with pseudo-analytical solution (Fraccarollo and Capart, 2002) and the Taipei experimental data.

For the Louvain test (see Fig. 3.8), better agreement is observed between the computed, analytical and measured results. The measured water surface and bed level are satisfactorily predicted in the numerical simulation. The location of the hydraulic jump is well reproduced by the numerical model for this test, where both computed and measured results show the hydraulic jump propagates upstream. The location of wave front and erosion magnitude are also modeled well in the simulation. Though the analytical solution shows great resemblance to the simulation and measurement in
the erosion magnitude as well as in the front- and back- water surface, it again cannot predict the hydraulic jump and obviously overestimates the wave front propagation.

Figure 3.8: Comparison of computed water surface and bed level with pseudo-analytical solution (Fraccarollo and Capart, 2002) and the Louvain experimental data.

To understand the causes of those differences, some critical issues should be addressed. Firstly, comparing with the simulation based on the non-capacity sediment transport, the failure to reproduce the hydraulic jump with the analytical solution may be mainly caused by the assumption of the capacity sediment transport. This echoes the previous suggestions by Capart and Young (1998) and Fraccarollo and Capart (2002) that the central wave region is the most sensitive to non-equilibrium behavior. Moreover, the non-hydrostatic pressure, which is not considered in both numerical simulation and analytical analysis, may also have some influence (Fraccarollo and Capart, 2002). Secondly, the faster propagation of the wave front in the analytical solution for late times (see the Louvain test in Fig. 3.8) is probably due to the assumption of zero momentum loss from bed friction. Practically the dissipative effect due to bed friction becomes important and slows down the bore movement when time evolves (Fraccarollo and Capart, 2002). Thirdly, the less satisfactory analytical solution in the Taipei test is attributed to the use of light particle material, which
makes the theory of capacity sediment transport less appropriate for this case (Fraccarollo and Capart, 2002). Fourthly, the assumption of a constant concentration in the sediment moving layer is another flaw in the analytical derivation (Cao et al., 2004; Wu and Wang, 2007), which could result in less accurate predictions. But in general, the pseudo-analytical solution agrees qualitatively well with the measurement for the idealized tests and can serve as an alternative reference for the model test.

To shed light on the model’s capacity of modeling high sediment concentrations, the computed longitudinal concentrations in the Taipei test are drawn for distinct time instants (see Fig. 3.9). The model is stable at very high sediment concentrations (e.g. $c_{\text{max}} \approx 0.64$ at $5t_0$) under relatively strong erosive conditions. Steep concentration gradient can be well captured by the model. The computed profile and magnitude at $4t_0$ are very close to those simulated by Wu and Wang (2007).

![Figure 3.9: Computed volumetric sediment concentration for the Taipei experiment.](image)

### 3.2.4 Dam-break flow in an abruptly widening mobile-bed channel

Laboratory-scale experiments of dam-break over an erodible bed were conducted at the Civil Engineering Laboratory of the Université Catholique de Louvain (UCL), Belgium (Spinewine and Zech, 2007; Palumbo et al., 2008; Zech et al., 2008; Goutière et al., 2011). The experiment considered here was conducted in a 6 m long flume, with a sudden asymmetric enlargement from 0.25 m to 0.5 m in the downstream reach (see Fig. 3.10). A 0.1 m high layer of saturated sand with diameter of $d = 1.82$ mm and density of $\rho_s = 2680$ kg/m$^3$ constituted the initial flat bed with bed porosity of $p = 0.47$. At the downstream end of the flume, a weir was installed to allow a free outflow. The dam-break was initiated instantaneously by rapidly moving down a thin gate at the middle of the flume, allowing the water contained in the upstream reservoir ($h_0 = 0.25$ m) pouring into the downstream dry bed. Measurements
of water level history and final bed topography were carried out at specific gauges and cross-sections (Palumbo et al., 2008). Gauges 1-4 were located along the line \( y = 0.125 \) m, with \( x = 3.75, 4.2, 4.45 \) and \( 4.95 \) m respectively. Gauges 5, 6 were in line of \( y = 0.375 \) with \( x = 4.2 \) and \( 4.95 \) m. The two cross-sections are along \( x = 4.1 \) m (CS1) and \( x = 4.4 \) m (CS2).

Based on a convergence study, the computational domain is discretized into grid cells of \( \Delta x = 0.025 \) m and \( \Delta y = 0.005 \) m, and the Courant number in the CFL condition is 0.5. The total computational time is 12 s with the time step controlled by the stability condition Eq. (2.22). Free outflow condition is imposed at the downstream boundary, while the closed boundary condition is used for all side walls. For sediment entrainment and deposition computation, equations (2.11-2.13) are used with specified parameters: \( \theta_c = 0.04 \), \( \alpha = 10 \), \( \varphi = 0.22 \), \( s = 1.68 \). The Manning roughness coefficient is 0.024 m\(^{1/3}\)/s, and this value refers to those used by others (Zech et al., 2008; Xia et al., 2010; Wu et al., 2012). The two tolerance depths in dry/wet front treatment are set to 0.0001 m and 0.0002 m respectively.

The present 2-D coupled model yields results of similar accuracy as those predicted by triangular/rectangular meshes and upwind methods such as Roe-MUSCL or HLLC, HLL schemes (Palumbo et al., 2008; Xia et al., 2010; Soare-Frazão and Zech, 2011; Wu et al., 2012). Figure 3.11 shows the comparison between the computed and measured water level histories at distinct gauges. The computed results agree well with the measurement though at Gauge 1 the water level is slightly over-predicted after the shock wave.
3.2 Model results of DGM version

Figure 3.11: Comparison between computed (thick line) and measured (circles) water level at distinct gauges.

Figure 3.12: Comparison of computed (thick line) and measured (circles) bed profiles at (a) CS1 and (b) CS2.
Figure 3.12 shows the computed final bed changes in the immediate downstream of the sudden enlargement. At the cross section CS1, the model can well reproduce the bed erosion at the center of the flume near the corner of the enlargement, but underestimates the deposition at the enlarged zone (Fig. 3.12a). At CS2, the measured bed profile of moderate erosion in the narrow part and strong deposition in the enlarged zone (Fig. 3.12b) is also satisfactorily reproduced. Near the sidewall of the narrow part (at the left side of Fig. 3.12b) the erosion magnitude is slightly over-predicted. In the enlarged zone the predicted highest deposition point shifts a bit to the sidewall compared with the measurement. The local erosion near the sidewall of the enlarged zone (at the far right sides of Figs. 3.12a and b) is most probably caused by vertical re-circulation (observed by Palumbo et al. (2008)), which cannot be adequately represented by a depth-averaged model.

### 3.2.5 Partial dam-break in a straight erodible channel

Case 5 is a mobile-bed dam-break experiment that was also conducted in UCL-Belgium (Wu et al., 2012; Soares-Frazão et al., 2012). The flume was 3.6 m wide and about 36 m long (Fig. 3.13). A 1-m wide gate between two impervious blocks represented the dam-break location, with the gate center denoting the coordinate (0,0). The mobile bed had a 0.085 m thick layer of saturated sand with diameter of $d = 1.61$ mm and density of $\rho_s = 2630$ kg/m$^3$, which extended from 1 m upstream of the gate to 9 m downstream with bed porosity of $p = 0.42$. The downstream end of the flume consisted of a weir and sediment entrapment system. The initial water level is 0.47 m (above the fixed flat bed) upstream of the gate and 0 m in the downstream region (dry bed case). The partial dam break was modeled by rapidly lifting the gate and lasted for 20 s. Measurements of water surface and bed level were conducted in two repeated experiment runs for the above experimental set-up. Water surface measurements were undertaken at 8 gauges during both experiment runs. Gauges 1-4 were located along $x = 0.64$ m, with $y = -0.5$, -0.165, 0.165 and 0.5 m. Gauges 5-8 were along $x = 1.94$ m, with $y = -0.99$, -0.33, 0.33 and 0.99 m. Bed measurements were carried out at the end of the two experiments ($t = 20$ s), with data available for three longitudinal lines ($y = 0.2$, 0.7 and 1.45 m).
Based on a convergence study, the computational domain is discretized into grid cells of $\Delta x = \Delta y = 0.025$ m and the Courant number in the CFL condition is 0.45. The total computational time is 20 s with the time step controlled by the stability condition Eq. (2.22). Free outflow (i.e., zero spatial gradient in flow direction) is assumed at the downstream boundary due to the relatively short experiment period, and closed boundaries at walls are free-slip and non-permeable. The sediment entrainment and deposition are computed by Eqs. (2.11-2.13), with specified parameters: $\theta_c = 0.04$, $\alpha = 5$, $\varphi = 1$, $s = 1.63$. The Manning roughness coefficient is 0.0165 for sand layer and 0.01 for fixed bed portion (Wu et al., 2012). The tolerance depths in dry/wet front treatment are 0.001 m and 0.0005 m respectively.

Figure 3.14 shows the comparison of the computed and measured water level histories (above the fixed flat bed) at 8 gauges. The difference between the two measurements for the same experimental set-up indicates the difficulty of repeating such experiments. At the symmetric gauges (viz., 1(4), 2(3), 5(8), 6(7)), the computed water level changes are exactly the same indicating the model’s accuracy in the symmetric property of $y$ direction. At gauges 5(8) and 6(7), the computed results agree well with the experiment data. In the near downstream region of the dam, less accurate results are predicted. At gauges 1 and 4, the water level is under-predicted in the early stage of the dam break while it is over-predicted at gauges 2 and 3 near the center line.
Model performance

Figure 3.14: Comparison of computed and measured water levels at distinct gauges.

Figure 3.15 shows the predicted bed profiles against the experiment data along three longitudinal lines ($y = 0.2, 0.7$ and $1.45$ m) at the end of the experiment ($t = 20$ s). Considerable discrepancies between the two groups of measured bed profile (with the same experimental set-up) are observed especially in the near downstream of the breach. It indicates that the bed change near the dam region is very sensitive and unstable even to trivial disturbances in doing the experiments. In this region, the predicted bed erosion along $y = 0.2$ and $0.7$ m is between the two measurements while the deposition along $y = 1.45$ m is under-predicted. In the far downstream region where close agreement is observed between the two measurements, the accuracy of the predicted bed level is also much improved.
3.3 SGM version versus DGM version

The well-balanced property, i.e., the C-property, is associated with the model ability to maintain the initially static water conditions over a non-flat topography. This property of the present model is achieved by developing the SGM version (see Chapter 2). In this section, the satisfaction of C-property in the SGM version is tested by a static water case. The accuracy of the SGM version for mobile bed conditions is

Figure 3.15: Comparison between computed and measured bed profiles along different longitudinal lines.

The less accurate prediction for both water level and bed profile in the near downstream of the breach may relate to the strong 3D flow features around the corners of the expansion which cannot be fully considered in a depth-averaged model. At the early stage, less flow is computed to expand laterally (see gauges 1 and 4) while more flow concentrates along the center line (see gauges 2 and 3). The uncertainty in bed change prediction for this sensitive area may affect the water level computation as well. Yet considering the complexity of the problem, the model results are generally good and should be considered acceptable.
also verified by revisiting a dam-break case illustrated above (see Case 4). In addition, the results of DGM version for these tests are given to facilitate comparisons.

### 3.3.1 Preservation of static water at a 1-D surface-piercing hump

Case 6 is a hypothetical 1-D test containing dry-wet interface to examine the well-balanced property of the present model. It consists of a surface-piercing hump at the center of a 1 m long domain with the bed elevation defined by

\[ z(x) = \max[0, 0.25 - 5(x - 0.5)^2] \]  \hspace{1cm} (3.3)

Initially the hump is partially submerged by water at rest. The water level is determined by \( \zeta = \max(0.1, z) \). Closed walls are imposed at the boundaries and bed friction is assumed negligible. Based on a convergence study, the computational domain is discretized by \( \Delta x = 0.01 \) m and the Courant number in the CFL condition is 0.3. The total computational time is 200 s.

The accuracy of the present model to keep the initially static water conditions is illustrated by the error norms:

\[ L_{\infty}(h) = \max_i | h_i - h_i |, \quad i \in [1, N] \]  \hspace{1cm} (3.4a)

\[ L_{\infty}(q) = \max_i | q_i - q_i |, \quad i \in [1, N] \]  \hspace{1cm} (3.4b)

\[ L_1(h) = \frac{1}{N} \sum_{i=1}^{N} | h_i - h_i | \]  \hspace{1cm} (3.4c)

\[ L_1(q) = \frac{1}{N} \sum_{i=1}^{N} | q_i - q_i | \]  \hspace{1cm} (3.4d)

where \( N = \) number of computational cells; \( h_i, q_i = \) computed water depth and discharge \( (q = hu) \); \( h_i, q_i = \) analytical solutions of water depth and discharge. For water at rest, the analytical water depth equals to the initial water depth and the analytical discharge should be zero.

As shown in Table 3.2, the SGM version is demonstrated to keep perfectly the water at rest during the whole computational time. However, the DGM version shows an error range of \( 10^{-4} \sim 10^{-3} \) in water depth and discharge. Such discrepancies generally decrease in time but hardly diminish. The computed water level and discharge at two distinct time instants are also shown in Figs. 3.16 and 3.17 for the SGM and DGM versions respectively. Figure 3.16 shows the initially horizontal water surface can be exactly preserved by the SGM version with zero flow discharge. However, the DGM version shows small undulations in water surface at locations of non-flat bed (see Fig.
3.3 SGM version versus DGM version

Accordingly, flow movement initiates in such areas and further influences adjacent field.

The above results suggest that the SGM version of the present model is well-balanced and thus should be applicable to field scales of irregular topographies especially for the static water conditions.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Model</th>
<th>( L_1(h) )</th>
<th>( L_1(q) )</th>
<th>( L_\infty(h) )</th>
<th>( L_\infty(q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>SGM</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>DGM</td>
<td>( 2.75 \times 10^{-4} )</td>
<td>( 3.78 \times 10^{-4} )</td>
<td>( 3.23 \times 10^{-3} )</td>
<td>( 4.33 \times 10^{-3} )</td>
</tr>
<tr>
<td>200</td>
<td>SGM</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>DGM</td>
<td>( 1.76 \times 10^{-4} )</td>
<td>( 3.23 \times 10^{-4} )</td>
<td>( 3.24 \times 10^{-3} )</td>
<td>( 3.75 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

Figure 3.16: Computed water depth and discharge at distinct instants by SGM version.
Figure 3.17: Computed water depth and discharge at distinct instants by DGM version.

3.3.2 2-D mobile bed dam-break flows

The applicability of the SGM version to mobile bed conditions is tested by revisiting Case 4 of dam-break flow in an abruptly widening mobile-bed channel. In the computation, the Courant number is set to 0.3 for the CFL condition and the tolerance depths for dry-wet treatment are 0.0002 and 0.0003 m. Other parameters and empirical relations are the same as used in Case 4 by the DGM version.

The experimental data from the locations in Case 4 (see Fig. 3.10) are used again for the present comparison. The computed results of water level histories and final bed elevation by the SGM version are compared with the experimental data in Figures 3.18 and 3.19 respectively together with those of the DGM version. It shows that the SGM version predicts satisfactorily the water level and bed elevation for this test. With nearly identical parameter settings, the SGM and DGM versions result in almost the same order of accuracy for the water level computations. For the bed elevation, similar results are also observed between the two versions. Overall, the SGM version of the present model gives satisfactory solution and should be applicable to mobile bed conditions.
3.3 SGM version versus DGM version

Figure 3.18: Comparison of the SGM (red dashed line), DGM (black thick line) computed water level and measured (circles) data at distinct gauges.

Figure 3.19: Comparison of the SGM (red dashed line), DGM (black thick line) computed final bed elevation and measured (circles) data at (a) CS1 and (b) CS2.
3.4 Summary

In this chapter, the capabilities of the 2-D coupled model presented in Chapter 2 have been verified against six test cases, which cover a wide range of complex flow and sediment transport conditions. The model is demonstrated to be capable of simulating not only shock waves and reflection waves, but also rapid bed deformations under highly active sediment transport conditions. The SGM version of the present model can well preserve the static water conditions satisfying the C-property and has the same order of accuracy as the DGM version for mobile bed conditions. Numerically, the extension of the UFORCE scheme to the sediment-laden flow over a mobile bed is less computationally demanding than most upwind methods, while offering attractive efficiency and high accuracy. The use of physically complete description for the interactions between the sediment-laden flow and river bed, as well as the fully synchronous solutions for the inner domain and boundaries, are critical for the model to accurately handle the complex flows over mobile beds.

In practical applications, such as flood forecasting, flood risk and river basin management, river channel restoration, efficient and accurate solutions are favored in order to cope with the large spatial scales involved. The combination of high numerical accuracy and computational efficiency by the present work makes the present model a promising tool for such applications. Yet it should be pointed out that for highly irregular bed topographies and complex boundaries, the modeling accuracy and efficiency depend on both the numerical method and the computational grid. For such conditions, the use of rectangular grid in the present model may reduce the modeling accuracy and efficiency even though the well-balanced SGM version is deployed. Further development to incorporate a more flexible mesh-cell like triangular cell or adaptive quadtree grid is part of future work.

While uncertainty related to empirical and numerical parameters of the model is inevitable, reasonable agreement between numerical solutions and experimental data can be obtained if those parameters are sensibly specified. Also, the values of the parameters used in the present numerical cases may be only applicable to the experimental configurations of the specific case. For further applications of morphological models, it is important to collect as much information as possible for the targeted configuration so that sensible values of parameters can be given.
Peak discharge increase in hyperconcentrated floods

4.1 Introduction

River floods are usually characterized by a downstream flattening of the discharge peak resulting from friction-induced energy losses. However, in the Yellow River, China (see Fig. 1.1 for location), the peak discharge increases in the downstream direction at a rate exceeding the contribution of tributaries during many hyperconcentrated silt-laden floods. To account for the peak discharge increase, many explanations have been proposed, which mainly attribute the responsible mechanisms to changes in morphology or in bed roughness, or to bed erosion related increasing flow volume.

In a hyperconcentrated flood, the morphology often changes from wide and shallow to narrow and deep accelerating flow propagation (Wang et al., 2009). As it occurs consecutively from upstream to downstream, the flood wave from upstream would overtake that at the downstream thus possibly causing a downstream increased peak discharge (Wang et al., 2009). This effect may be enhanced by storage on the floodplains: after a deep channel has been formed, water stored on the floodplains rapidly flows back to the channel and is conveyed downstream (Chien and Wan, 1983; Qi, 1992; Qi and Zhao, 1993; Qi and Li, 1996).

In addition, erosion rates are thought to be so large in hyperconcentrated flood that it has been hypothesized that the increasing volume of the water–sediment mixture by erosion may be responsible for the peak discharge increase (Qi et al., 2010; Cao et al., 2006, 2012).

Another process explaining the peak discharge increase is the bed roughness change. In highly concentrated sediment-laden flow, sediment concentration has a strong influence on bed roughness. Sediment-turbulence interactions may result in a collapse of the turbulence and vertical concentration (Winterwerp, 2001, 2006), profoundly reducing the bed roughness (Winterwerp et al., 2009; van Maren et al., 2009b). Secondly, the thickness of the viscous sub layer near the bed increases, up to a factor 2-5 (Gust, 1976), resulting in a decreasing bed roughness. Thirdly, in the lower flow regime bed forms are usually smaller in hyperconcentrated flows compared to similar flow conditions of clear water (Wan, 1982; Jiang et al., 2008), thus contributing to bed roughness decrease. The decreasing bed roughness may accelerate flow propagation and probably cause a downstream increased peak discharge.

Though each of the above explanation appears reasonable, to date, there has been no consensus on which mechanism contributes most to the phenomenon. Because the amount of data available on the peak discharge increase events is rather limited, and associated processes are very complex and non-linear, numerical models provide useful tools to better understand the underlying physical mechanisms and their relative roles. However, previous modeling studies for the peak discharge increase have only concentrated on one of the mechanisms, which renders it difficult to unravel their relative roles. Cao et al. (2006, 2012) numerically reproduced the downstream peak discharge increase by incorporating the bed erosion effects into the flow for hyperconcentrated flood modeling. Jiang et al. (2006, 2008) modeled the peak discharge increase of the 2004 flood in the Lower Yellow River using an empirical bed roughness formula (Zhao and Zhang, 1997) with the hydraulic and sedimentary effects included.

Recently, Li et al. (2013) developed a model aiming at simulating rapid morphological change, such as what typically occurs during hyperconcentrated floods in the Yellow River. The present work aims to quantitatively analyze and explain the relative importance of morphological change and bed roughness change without floodplain influences, using this newly developed model. Here, two critical issues should be pointed out. First, hyperconcentrated floods in the Yellow River are mostly turbulent Newtonian flow (Zhang and Xie, 1993; Wang et al., 2002; van Maren et al., 2009a), thus the rheology-related flow resistance change in Bingham fluid (Wan and Wang, 1994) is not studied. Second, the numerical analyses are carried out in schematic one-dimensional (1-D) channels of constant width, because (1) the focus is put on floods (and associated processes) that are mostly conveyed in the channel. These dynamics are well captured by a 1-D model, which also allows a more detailed process analysis compared to 2D / 3D models. (2) the topography data in the Yellow River is usually insufficient for high-resolution hyperconcentrated flood modeling.
4.2 Analysis on field observations

In this chapter, the historical peak discharge increase events in the Yellow River are briefly analyzed first. Then, the relative role of bed erosion and bed roughness change to two floods (the 2004 flood in the Lower Yellow River and the 1977 flood in the Middle Yellow River) is numerically studied with the newly developed morphodynamic model (see Chapter 2). Finally, the concept of channel storage reduction is proposed to integrate the effects of bed erosion and bed roughness reduction on the peak discharge increase.

4.2 Analysis on field observations

Since 1973, 13 flood events with downstream increasing peak discharge (see Table 4.1) have been observed, of which 12 occurred in the Xiaolangdi – Huayuankou (X-H) reach of the Lower Yellow River, and one in the Xiaobeiganliu (from Longmen-Tongguan) of the Middle Yellow River (see Fig. 1.1 for location). Using the field data of the 13 floods, this section presents a simple theoretical analysis on the bed erosion role for the peak discharge increase.

Table 4.1 Peak discharge increase events in the Yellow Rivera

<table>
<thead>
<tr>
<th>Reach</th>
<th>Year</th>
<th>Qp (m³/s)</th>
<th>Sp (kg/m³)</th>
<th>Tributary Q (m³/s)</th>
<th>ΔQp (m³/s)</th>
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</thead>
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<td></td>
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<td>XLD HYK</td>
<td>XLD HYK</td>
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<td>4710</td>
<td>110</td>
<td>150</td>
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<td>5020</td>
<td>509</td>
<td>450</td>
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<tr>
<td></td>
<td>1973.9</td>
<td>4400</td>
<td>5890</td>
<td>338</td>
<td>348</td>
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<td></td>
<td>1977.8</td>
<td>10100</td>
<td>10800</td>
<td>941</td>
<td>809</td>
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<tr>
<td></td>
<td>1992.8(1)</td>
<td>2930</td>
<td>3260</td>
<td>193</td>
<td>133</td>
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<td></td>
<td>1992.8(2)</td>
<td>3780</td>
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<td>6260</td>
<td>535</td>
<td>488</td>
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<td></td>
<td><strong>2004.8</strong></td>
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<td><strong>343</strong></td>
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<td>3530</td>
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<td>2008.6-7</td>
<td>4050</td>
<td>4600</td>
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<td>3490</td>
<td>6680</td>
<td>288</td>
<td>152</td>
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<td>Xiao-beiganliu</td>
<td><strong>1977.8</strong></td>
<td><strong>12700</strong></td>
<td><strong>15400</strong></td>
<td><strong>821</strong></td>
<td><strong>911</strong></td>
</tr>
</tbody>
</table>

aQp and Sp are discharge peak and sediment peak; XLD, HYK, LM, TG denote the Xiaolangdi, Huayuankou, Longmen and Tongguan stations respectively. Events marked bold are numerically reproduced in Sections 4.4-4.5. The data is compiled from China Hydrology Annual, Li (2008), Qi et al. (2010).
Any flow volume mixing with an eroded bed will increase relative to the original flow, together with an increase in sediment concentration. If the increased discharge is fully caused by bed erosion, the relation between the discharge increase and concentration increase may be quantified by:

\[ V_i = V_c + (1 - p)\phi V \]  \hspace{1cm} (4.1)

\[ \Delta c = \frac{V_i}{V} (1 + \phi) - c_u = \frac{(1 - p - c_u)\phi}{1 + \phi} \]  \hspace{1cm} (4.2)

where \( V \) = original discharge-related flow volume; \( c_u \) = original volumetric sediment concentration (-); \( \phi \) = ratio of discharge increase (relative to original discharge) (-); \( \Delta c \) = volumetric concentration increase due to bed erosion (-); \( V_i \) = sediment volume after bed erosion; \( p \) = bed porosity (-), here a value of 0.45 is used (representative for the Yellow River). Based on Eq. (4.2), a discharge increase should be accompanied by a concentration increase which depends on the ratio of discharge increase, original flow concentration and bed porosity. This concentration increase increases with the ratio of discharge increase. For a specific discharge increase, the flow with (originally) low concentration requires more increase in concentration (implying more bed erosion) than the flow with higher original concentration.

When applying this theoretical framework to the flood events in Table 4.1, the observed peak discharges (which define the flow volume) and the corresponding concentrations (measured simultaneously with the peak discharges) at the up- and downstream reaches are used to compute \( \phi \) and the concentration increase \( \Delta S = \rho_s \Delta c \), \( \rho_s = 2650 \text{ kg/m}^3 \) is the sediment density) (see Table 4.2). In contrast to the positive values for the required concentration increase, during most floods the sediment concentration is observed to decrease. In such cases, an increasing flow volume by erosion obviously cannot explain the increasing peak discharge. For the minor events in which the concentration does increase, the concentration increase is far too low to explain the discharge increase (see Table 4.2).

Despite the simplicity of this approach, the results strongly indicate that the peak discharge increase is not merely attributed to the increase in volume due to erosion. In the following, the governing equations and empirical relations of the 1-D coupled morphodynamic model are introduced. This model is used to further analyze the responsible mechanisms, focusing on the 2 floods in Table 4.1 marked bold.
Table 4.2 Computed and measured concentration increase for flood events in Table 4.1 (unit: kg/m³)

<table>
<thead>
<tr>
<th>Reach</th>
<th>Time</th>
<th>ϕ</th>
<th>S_{XLD}</th>
<th>S_{HYK}</th>
<th>ΔS_{compute}</th>
<th>ΔS_{measure}</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-H</td>
<td>1973.8(2)</td>
<td>0.383</td>
<td>360</td>
<td>230</td>
<td>304</td>
<td>-130</td>
</tr>
<tr>
<td></td>
<td>1973.9</td>
<td>0.339</td>
<td>325</td>
<td>330</td>
<td>286.7</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1977.8</td>
<td>0.069</td>
<td>840</td>
<td>437</td>
<td>39.8</td>
<td>-403</td>
</tr>
<tr>
<td></td>
<td>1992.8(1)</td>
<td>0.113</td>
<td>193</td>
<td>110</td>
<td>128.4</td>
<td>-83</td>
</tr>
<tr>
<td></td>
<td>1992.8(2)</td>
<td>0.08</td>
<td>389</td>
<td>258</td>
<td>79.1</td>
<td>-131</td>
</tr>
<tr>
<td></td>
<td>1992.8(3)</td>
<td>0.370</td>
<td>535</td>
<td>200</td>
<td>249.1</td>
<td>-335</td>
</tr>
<tr>
<td></td>
<td>2004.8</td>
<td>0.483</td>
<td>170</td>
<td>100</td>
<td>419.3</td>
<td>-70</td>
</tr>
<tr>
<td></td>
<td>2005.7</td>
<td>0.395</td>
<td>138</td>
<td>30.0</td>
<td>373.6</td>
<td>-108</td>
</tr>
<tr>
<td></td>
<td>2006.8</td>
<td>0.608</td>
<td>28.9</td>
<td>18.3</td>
<td>540.2</td>
<td>-10.6</td>
</tr>
<tr>
<td></td>
<td>2008.6-7</td>
<td>0.136</td>
<td>108.5</td>
<td>24.7</td>
<td>161.5</td>
<td>-83.8</td>
</tr>
<tr>
<td></td>
<td>2010.7</td>
<td>0.914</td>
<td>1.09</td>
<td>16.53</td>
<td>695.5</td>
<td>15.49</td>
</tr>
<tr>
<td>Xiaobeiganliu</td>
<td>1977.8</td>
<td>0.212</td>
<td>481</td>
<td>911</td>
<td>171</td>
<td>430</td>
</tr>
</tbody>
</table>

aS_{XLD}, S_{HYK}, S_{LM}, S_{TG} are concentrations measured simultaneously with the peak discharges (see Tab. 4.1) at Xiaolangdi, Huayuankou, Longmen and Tongguan (data from Qi et al. (2010)).

4.3 Coupled mathematical model

The numerical analyses are done with the morphodynamic model developed by Li et al. (2013). Here, the basic formulations for hyperconcentrated flood modeling are provided, see Chapter 2 for details on the numerical method for solution.

4.3.1 Governing equations

The governing equations for a 1-D coupled morphodynamic model of non-capacity sediment transport consist of the mass and momentum conservation equations for sediment-laden flow, the mass conservation equation for sediment, and a bed update equation. Here, two model versions are used as a comparison to reveal how large the peak discharge increase is related to the increasing flow volume by bed erosion. The first one is a fully coupled model, which fully considers the effects of sediment density and bed deformation on the flow (Cao et al., 2004; Li et al., 2013). The vector form of the fully coupled governing equations reads
\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = R
\]

(4.3)

\[
U = \begin{bmatrix} h \\ hu \\ hc \\ \phi \end{bmatrix}, \quad F = \begin{bmatrix} hu \\ hu^2 + 0.5gh^2 \\ huc \\ huc \end{bmatrix}
\]

(4.4a,b)

\[
R = \begin{bmatrix} E-D \\ 1-p \\ gh(S_o - S_f) - \frac{(\rho_s - \rho_w)gh^2}{2\rho} \frac{\partial c}{\partial x} - \frac{(\rho_0 - \rho)(E-D)u}{\rho(1-p)} \\ E-D \\ 0 \end{bmatrix}
\]

(4.5)

where \( U \) = vector of conservative variables; \( F \) = vector of flux variables; \( R \) = vector of source terms; \( t \) = time; \( x \) = horizontal coordinate; \( h \) = water depth; \( u \) = depth-averaged flow velocity in \( x \) direction; \( c \) = depth-averaged volumetric sediment concentration; \( z \) = bed elevation; \( \phi = (1-p)z + hc \) = newly constructed conservative variable; \( E,D \) = sediment entrainment and deposition fluxes respectively; \( S_o \) = bed slope in \( x \) direction, expressed as \( S_o = -\partial z / \partial x \); \( S_f \) = friction slope in \( x \) direction; \( \rho_s = 2650 \text{ kg/m}^3 \) is the sediment density; \( \rho_w = 1000 \text{ kg/m}^3 \) is the water density; \( \rho = \rho_w (1-c) + \rho_s c \) = density of sediment-laden flow; \( \rho_0 = \rho_w p + \rho_s (1-p) \) = density of saturated bed; \( p \) = bed porosity; \( g \) = 9.8 m/s\(^2\) is the acceleration of gravity.

The other is a partially coupled model, which ignores the bed erosion effect in the mass conservation equation for sediment-laden flow (also corresponding change of the bed erosion related term in the momentum equation). The governing equations of the partially coupled model differ from the fully coupled model in the source terms:

\[
R = \begin{bmatrix} 0 \\ gh(S_o - S_f) - \frac{(\rho_s - \rho_w)gh^2}{2\rho} \frac{\partial c}{\partial x} - \frac{(\rho_s - \rho_w)(E-D)u}{\rho} \\ E-D \\ 0 \end{bmatrix}
\]

(4.6)

4.3.2 Empirical relationships

The friction slope is estimated from the Manning roughness coefficient \( n \), as

\[
S_f = n^2 u^2 / h^{4/3}
\]

(4.7)
Since the majority of the sediment transported in the Middle and Lower Yellow River is composed of suspended load (Long and Zhang, 2002), only the suspended load exchange with the bed (see Eq. (4.8)) is computed using an empirical adaptation coefficient \(\alpha\) (the ratio of near-bed to depth-averaged concentrations) (Cao et al., 2004). Though differing in physical meaning and mathematical expressions of the adaptation parameter measuring the rapidity of sediment exchange between flow and bed, the method of \(\alpha\) is essentially equivalent to the method of the adaptation length for sediment lag effects in previous studies (Galappatti and Vreugdenhil, 1985; Armanini and Di Silvio, 1988; Phillips and Sutherland, 1989; Rahuel et al., 1989; Wu et al., 2000). A large value of \(\alpha\) or a small value of the adaptation length represents a rapid sediment adaptation while a small \(\alpha\) or a large adaptation length means a slow adaptation.

\[
E = \alpha \omega_c c_e, \quad D = \alpha \omega_c c 
\]  
(4.8a,b)

where \(c_e\) = depth-averaged sediment transport capacity (\(\cdot\)); \(\omega_c\) = effective sediment settling velocity (m/s). An empirical formula that has been widely used in the Yellow River (Wang and Xia, 2001) is deployed to estimate the coefficient \(\alpha\):

\[
\alpha = \begin{cases} 
0.001/ \omega_c^{0.3}, & c > c_e \\
0.001/ \omega_c^{0.7}, & c \leq c_e 
\end{cases} 
\]  
(4.9)

Bed roughness is an important physical parameter to quantify the bed resistance to the flow. Although it has been widely recognized that the bed roughness is affected by many inter-dependent factors (i.e., the hydraulic conditions, bed forms, bed material properties, and sediment transport), there has been no generic theoretical equation for it. Various empirical relations are frequently applied, among which quantitative uncertainty is inevitable. Here two approaches (termed as approach I and approach II) are adopted to estimate \(n, c_e, \) and \(\omega_c\). Approach II (see Eqs. (4.14-19)) is a set of empirical formulations that have been frequently used for modeling hyperconcentrated flood in the Yellow River (Zhang et al., 2001; Ni et al., 2004; Wang et al., 2008a; Xia et al., 2012). This approach incorporates the effects of sediment concentration \(c\), water depth \(h\) and friction thickness \(\delta^*\) to estimate of the Manning roughness. Since one of the aims of the present study is to unravel how sediment concentration affects flood propagation through roughness modulation, a simpler roughness formulation is also proposed (see approach I), only depending on sediment concentration.

In the following, the two approaches are introduced in detail:

(1) **Method of approach I**

The sediment transport formula by Wu et al. (2008b) reads:
Peak discharge increase in hyperconcentrated floods

\[ c_e = \frac{K}{\rho_i} \left( \frac{\rho}{\rho_i - \rho} \frac{u^3}{gh\omega_i} \right)^m \]  

(4.10)

where \( K = 0.4515 \) (kg/m³) and \( m = 0.7414 \) (-) are empirical parameters. The hindered settling effect in hyperconcentration (Richardson and Zaki, 1954) is accounted for by

\[ \omega_i = \omega_0 (1 - \frac{c}{1 - p})^5 \]  

(4.11)

with \( \omega_0 \) calculated by Zhang and Xie (1993)’s formula:

\[ \omega_0 = \sqrt{(13.95v/l \, d)^2 + 1.09(\rho_s - \rho_w)gd/l \, \rho_w} - 13.95v/l \, d \]  

(4.12)

where \( d \) = sediment diameter (m); \( \omega_0 \) = settling velocity of a single particle in clear and still water (m/s); \( \nu = 10^{-6} \) is kinematic viscosity of water. Other parameters are the same as those specified above. Based on the previous experimental findings (Li et al., 2008; Zhu and Hao, 2008), a power law relation is proposed to estimate the Manning roughness:

\[ n = n_r (1 + c_r - c)\beta \]  

(4.13)

where \( n_r \) = reference Manning roughness; \( c_r \) = reference sediment concentration; \( \beta \) = power exponent (>0). The reference roughness and concentration are set to the initial values \((n_0, c_0)\) for each numerical case. Eq. (4.13) computes a decreasing roughness with concentration increase. Although it has not been validated by field data, it is physically conceptual. Thus a sensitivity analysis has been done for values of \( \beta \) (1 ≤ \( \beta \) ≤ 5), revealing that qualitatively the downstream peak discharge increase is reproduced in a similar way for the 2004 flood. The middle value \( \beta = 3 \) best approximates observations, and therefore all simulations are done with \( \beta = 3 \). It is important to realize that the experiments by Li et al. (2008) and Zhu and Hao (2008) cover sediment concentrations up to ~300 kg/m³, during which \( n \) decreases with \( c \). However, there are strong indications that the bed roughness increases at higher sediment concentrations (Jiang et al., 2008), which is implemented in approach II.

(2) Method of approach II

The empirical formulation by Zhang and Zhang (1992) is used to compute the sediment transport capacity:

\[ c_s = 2.5 \frac{(0.0022 + c)u^3}{\kappa \rho_s - \rho \rho_i gh \omega_i} \ln\left(\frac{h}{6D_{s0}}\right)^{0.62} \]  

(4.14)
4.3 Coupled mathematical model

where $D_{50}$ = median diameter of bed material (m); with the Von Karman coefficient $\kappa$ in turbid flow and the effective sediment settling velocity $\omega_s$ calculated by Eq. (4.15) and Eq. (4.16) respectively:

$$\kappa = 0.4 - 1.68(0.365 - c)\sqrt{c}$$ (4.15)

$$\omega_s = \omega_b(1 - 1.25c)(1 - \frac{c}{2.26\sqrt{d_{50}}} )^{3.5}$$ (4.16)

where $d_{50}$ = median diameter of suspended load (mm). In Eq. (4.15), the Von Karman coefficient $\kappa$ is close to 0.4 at very low concentration.

The Manning roughness is estimated using the empirical formula by Zhao and Zhang (1997):

$$n = h^{1/6} c_n \frac{\delta_s}{h} \left[ 0.49 \left( \frac{\delta_s}{h} \right)^{0.77} + \frac{3\pi}{8} (1 - \frac{\delta_s}{h}) \left[ \sin \left( \frac{\delta_s}{h} \right) \right]^{\frac{5}{2}} \right]^{-1}$$ (4.17)

with

$$c_n = 0.375\kappa$$ (4.18)

$$\delta_s = D_{50} \left( 1 + 10^{8.1 - 13Fr^{0.5}(1-Fr^{0.5})} \right)$$ (4.19)

where $\delta_s$ = friction thickness; $Fr =$Froude number; $c_n =$vortex coefficient, related to concentration through the Von Karman coefficient $\kappa$ in Eq. (4.15).

Eq. (4.17) incorporates both hydraulic and sedimentary effects. Sensitivity analysis (see Figs. 4.1 and 4.2) reveals that the roughness in Eq. (4.17) is considerably affected by the variations of friction thickness and sediment concentration while it is not so sensitive to water depth changes. The variation of roughness with concentration is similar to that of the vortex coefficient (Eq. (4.15) and (4.18)). The roughness increases with friction thickness, representing a bed form effect through the bed material properties ($D_{50}$) and the flow regime indicated by Froude number (Eq. (4.19), see also Fig. 4.2). Yet for very small and large Froude numbers, Eq. (4.19) may compute a very large friction thickness (up to the water depth in some shallow water conditions), thus causing instability and inaccuracy of the bed roughness estimates. In order to avoid such problems, the Froude number in Eq. (4.19) is often limited to a certain range (Wang et al., 2008a), and therefore a validity range for the Froude number of $Fr \in [0.19, 0.8]$ is defined; larger and smaller Froude numbers are reset to 0.8 and 0.19 respectively.
Figure 4.1: Variations of Manning roughness (in Eq. (4.17)) with concentration for different water depth at (a) Fr=0.25, (b) Fr=0.5, and (c) Fr=0.75. The sediment median diameter is 0.02 mm.

Figure 4.2: Variations of Manning roughness (in Eq. (4.17)) with $\delta_*$ for different water depth (volumetric concentration is 0.1, sediment median diameter is 0.02 mm).
It is important to realize that Eq. (4.17) predicts a decreasing bed roughness until a sediment concentration around ~300 kg/m³, after which it increases with c (see also Fig. 4.1). This concentration coincides with the supersaturated flow regime in the Yellow River (van Maren et al., 2009a) during which sediment-induced buoyancy effects lead to low bed roughness (Winterwerp, 2006). Alternatively, bedforms may be responsible for the roughness variation – see the discussion section later in this chapter.

4.4 The 2004 flood in the Lower Yellow River

During a hyperconcentrated flood that occurred in August 2004, the measured flood peak was 2690 m³/s at the Xiaolangdi station. About 16 hrs later, the peak discharge was 3990 m³/s at the Huayuankou station (125.8 km downstream of Xiaolangdi). The sediment peak (around 350 kg/m³ at both stations) occurred after the flood peak, with the time lag increasing from 15.4 hrs at Xiaolangdi to 18hrs at Huayuankou.

4.4.1 Model setup

The Xiaolangdi-Jiahetan reach (see Fig. 1.1 for location) is schematized as a 1-D channel of 226.6 km long, 1000 m wide with a bed slope of \( S_0 = 2.55 \times 10^{-4} \). Uniform sediment is considered: \( D_{50} = d_{50} = 0.02 \) mm, \( p = 0.45 \). The discharge and concentration measured at Xiaolangdi (Fig. 4.3) are prescribed at the upstream boundary and the stage-discharge relationship at Jiahetan is for the downstream boundary. Using the stage-discharge relationship and bed elevation at Jiahetan, the initial flow conditions (\( h_0 = 0.96 \) m, \( u_0 = 0.6688 \) m/s) are obtained from \( q_0 = 0.642 \) m²/s. Under the steady uniform flow condition, \( n_0 = 0.02328 \) can be back-estimated. Initial equilibrium sediment transport is assumed, with \( c_s = 0.00438 \) for approach I (see Eq. (4.10)) and 0.00023 for approach II (see Eq. (4.14)). It should be realized that with this initial flow and sediment condition, a roughness value can also be computed by approach II (Eq. (4.17)), which might be different from the value back-estimated. To avoid the discrepancy, the roughness by approach II would be multiplied by a modified coefficient. A value of 1.5 is used for the 2004 flood. To analyze the relative importance of bed erosion (related to flow volume increase) and bed roughness reduction, cross-comparisons of five cases (Table 4.3) are conducted.
Figure 4.3: Discharge hydrograph and sediment concentration measured at Xiaolangdi.

Table 4.3 Summary of numerical simulations for the 2004 flood

<table>
<thead>
<tr>
<th>Case</th>
<th>Bed roughness change</th>
<th>Bed erosion effect on increasing flow volume</th>
<th>Sediment transport capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Considered, Eq. (4.13)</td>
<td>Considered, Eq. (4.5)</td>
<td>Eq. (4.10)</td>
</tr>
<tr>
<td>2</td>
<td>Ignored, n= constant</td>
<td>Considered, Eq. (4.5)</td>
<td>Eq. (4.10)</td>
</tr>
<tr>
<td>3</td>
<td>Considered, Eq. (4.13)</td>
<td>Ignored, Eq. (4.6)</td>
<td>Eq. (4.10)</td>
</tr>
<tr>
<td>4</td>
<td>Ignored, n= constant</td>
<td>Ignored, Eq. (4.6)</td>
<td>Eq. (4.10)</td>
</tr>
<tr>
<td>5</td>
<td>Considered, Eq. (4.17)</td>
<td>Considered, Eq. (4.5)</td>
<td>Eq. (4.14)</td>
</tr>
</tbody>
</table>

4.4.2 Model results

Halfway the model domain at the Huayuankou station (x=125.8 km, where observed discharge and concentration data is available), the model reproduces the flood peak increase well when a decreasing bed roughness with concentration is modeled (using approach I, see Cases 1 and 3 in Fig. 4.4a). Yet with a constant bed roughness (Cases 2 and 4), it fails to predict the discharge increase (see Figs. 4.4a and 4.5b). The effect of bed erosion (increasing flow volume) appears negligible (the differences between Cases 1 and 3, and between Cases 2 and 4 are small). The downstream discharge increase is computed in the upper reach (i.e., x ≤ 125.8 km, see Case 1 in Fig. 4.5a), consistent with field observations (Jiang et al., 2008). The computed sediment concentration at Huayuankou compares well with the observations: here the different model settings do not differ substantially (see Cases 1-4 in Fig. 4.4b). For the second flood peak after 120 h, relatively low concentration occurs with slow variations (comparing with the first flood peak), which contributes to a weak concentration effect on decreasing Manning roughness and a slow roughness change (Figs. 4.6 and 4.7). Therefore, an obvious downstream discharge increase is difficult to observe (Fig. 4.5).
Applying the approach II, the magnitude of the peak discharge increase at Huayuankou is reproduced (Case 5, Fig. 4.4a), similar to Case 1 and Case 3. However, the computed flood propagation by approach II is substantially faster than both the observed and calculated (in Case 1 using approach I). This is not surprising when comparing the different Manning roughness equations (see Eq. (4.13) and Eq. (4.17)).

In approach I, the Manning roughness depends on the sediment concentration and its effect travels with concentration related celerity which is equal to the flow velocity \((u)\). In approach II, the roughness coefficient not only depends on the concentration but additionally on the Froude number and water depth. The travel celerity of the roughness effect is a combination of many factors, including the concentration related celerity \((u)\) and the water depth related celerity which is larger than \(u\). It should be realized that the interrelation between water depth, flow velocity and sediment concentration is not straightforward (related to e.g. the adaptation for sediment transport (erosion/deposition) and the relation between velocity and water depth) and therefore the downstream propagation of the roughness is more diffuse than approach I (Eq. (4.13)) and approach II (Eq. (4.17)) suggest.

![Figure 4.4: Computed and observed discharge and sediment concentration at the Huayuankou hydrological station. See Table 4.3 for case descriptions.](image)
Comparing Fig. 4.6 and Fig. 4.7, the magnitude of roughness using approach II is lower (as low as 0.01 from 60 hr to 240 hr) than that for approach I (a minimum value of ~0.016, which lasts for very short time) during the flood. Therefore faster propagation is computed by approach II. The earlier start of roughness variation by approach II is influenced by the flow variation before concentration changes (Fig. 4.7). However, the bed roughness computed with approach I (Fig. 4.6) is closer to the observed roughness by Qi et al. (2010) than the roughness computed with approach II (Fig. 4.7), further suggesting that approach I is better suited for the 2004 flood.

In summary, the effect of bed roughness reduction with concentration is more important than bed erosion effect (of increasing flow volume) to the peak discharge increase of the 2004 flood. Additionally, approach I better reproduces the observations than approach II.
Figure 4.6: Time variations of Manning roughness and concentration at distinct locations for Case 1 (using approach I).
Figure 4.7: Time variations of Manning roughness, concentration and Froude number at distinct locations for Case 5 (using approach II).

4.5 The 1977 flood in the Middle Yellow River

Three hyperconcentrated floods occurred in 1977 in the Xiaobeiganliu of the Middle Yellow River. For the first two floods (on 6th July and 3rd August respectively), the advance of the flood waves on the wide floodplains of this reach (especially along the channel width direction) is important to the evolution of flow discharge and morphology, which however cannot be fully covered by a 1-D model but requires 2-D/3-D simulation. After these floods, a narrower and deeper channel was shaped, and the third flood (on 6th August) was mostly confined inside the channel without floodplain inundation. Therefore, only the third flood is investigated in this chapter using a 1-D modelling approach.

In this flood, the peak discharge of 12700 m$^3$/s occurred at Longmen with a corresponding sediment concentration of 480 kg/m$^3$, which was 9 hour later than the sediment peak of 821 kg/m$^3$. After 7.5 hour propagation, an enlarged peak discharge of 15400 m$^3$/s was observed at Tongguan (132.5 km downstream of Longmen) with the corresponding concentration peaking at 911 kg/m$^3$. 
4.5.1 Model setup

The Xiaobeiganliu section (between Longmen and Tongguan, see Fig. 1.1 for location) is schematized as a 1-D channel of 132.5 km long. Uniform sediment is considered: $D_{50} = d_{50} = 0.086$ mm, $p = 0.45$. A bed slope of $S_0 = 3.82 \times 10^{-4}$ is specified based on the bed elevations at Longmen and Tongguan stations, and a channel width of 550 m is used as a mean of the width at these two stations. The discharge and concentration measured at Longmen (Fig. 4.8) are prescribed at the upstream boundary and the stage-discharge relationship at Tongguan is for the downstream boundary.

For the 1977 flood, the sediment concentration is very high (up to ~ 900 kg/m$^3$) and far beyond the concentration (up to ~ 300 kg/m$^3$) of the 2004 flood. While approach I implies a decrease of roughness when concentration increases, field observation indicates that at very high concentrations (greater than ~ 300 kg/m$^3$) the bed roughness may increase, which is implemented in approach II. Therefore, despite the good performance of approach I for the 2004 flood, only the results of approach II for the 1977 flood are presented. To reveal the relative role of bed roughness reduction and bed erosion in the 1977 flood, four numerical cases (Table 4.4) are designed. Using the same method as for the 2004 flood, the initial conditions for the 1977 flood are computed: $h_0 = 1.42$ m, $u_0 = 1.17$ m/s, $n_0 = 0.021$; $c_0 = 0.001$ for Cases 6-8 and 0.005 for Case 9; and the roughness of approach II (Eq. (4.17)) is also multiplied by a modified coefficient of 1.6 for the 1977 flood.

![Figure 4.8: Discharge and sediment concentration measured at Longmen hydrological station.](image)
Table 4.4 Summary of numerical cases for the 1977 flood

<table>
<thead>
<tr>
<th>Case</th>
<th>Bed roughness change</th>
<th>Bed erosion effect on increasing flow volume</th>
<th>Sediment transport capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Considered, Eq. (4.17)</td>
<td>Considered, Eq. (4.5)</td>
<td>Eq. (4.14)</td>
</tr>
<tr>
<td>7</td>
<td>Considered, Eq. (4.17)</td>
<td>Ignored, Eq. (4.6)</td>
<td>Eq. (4.14)</td>
</tr>
<tr>
<td>8</td>
<td>Ignored, n = constant</td>
<td>Considered, Eq. (4.5)</td>
<td>Eq. (4.14)</td>
</tr>
<tr>
<td>9</td>
<td>Ignored, n = constant</td>
<td>Considered, Eq. (4.5)</td>
<td>3×Eq. (4.14)</td>
</tr>
</tbody>
</table>

4.5.2 Model results

At the downstream end (Tongguan, x=132.5 km) the increasing discharge peak is well reproduced in cases with variable bed roughness (Cases 6 and 7, see Fig. 4.9a). Incorporating both effects of bed erosion (related to increasing flow volume, see Eq. (4.5)) and roughness change (Eq. (4.17)), Case 6 reproduces most accurately the peak discharge increase at Tongguan. In this case, the computed bed level change (-2.7 m) also agrees reasonably with observations (-2.1 m from Wan and Song (1991)) at the Longmen station although the flood propagation velocity is slightly overestimated. The effect of only erosion (Case 8) is much smaller than only roughness change (Case 7). Only when the sediment transport capacity is exaggerated by a factor 3 (Case 9, for which the bed is strongly eroded (-5 m) at Longmen), the increasing peak discharge is also reproduced with a constant roughness. The faster flood propagation in Cases 6 and 7 (using approach II for a variable bed roughness) is because of the faster traveling of the Manning roughness effect that is related to both hydraulic and concentration changes. The decreasing bed roughness may in turn accelerate the flow velocity, thus faster concentration propagation is also computed for Cases 6 and 7 (Fig. 4.9b).

Even though the magnitude of sediment peak is well predicted in most of the cases, the variation of the sediment concentration over time not necessarily so (Fig. 4.9b). Especially when exaggerating sediment transport, concentration after the sediment peak is substantially enlarged. The poor prediction of the temporal variation of sediment concentration may be caused by (1) the complexity of sediment transport processes at very high sediment concentration, and (2) the large degree of topography schematization.
4.5 The 1977 flood in the Middle Yellow River

The relative importance of roughness reduction and bed erosion on flood propagation can be further assessed in Fig. 4.10. With the effect of roughness change (Cases 6 and 7), the computed peak discharge increases substantially from upstream to downstream (Fig. 4.10a,b); while for only erosion effect (Case 8) it gradually decreases (Fig. 4.10c). Only by exaggerating bed erosion (Case 9) increasing peak discharge is computed along the reach (Fig. 4.10d) while it is much smaller than that with roughness effect.

These numerical results indicate that very strong bed erosion in hyperconcentrated floods may lead to downstream peak discharge increase. However, floods during which the river bed is eroded sufficiently deep are very rare. During most hyperconcentrated floods (including very high concentration case like the 1977 flood), an increasing peak discharge is probably mainly caused by bed roughness reduction.
Figure 4.10: Discharge hydrographs at distinct locations for Cases 6-9. See Table 4.4 for case descriptions.

4.6 Discussion

4.6.1 Channel storage

The modeling results clearly show that inclusion of bed roughness reduction or very strong bed erosion leads to a downstream flood peak increase. In both cases, the flood peak increase should be accompanied by a change in channel storage (which is always implicitly modeled). A relationship between the spatial variation of discharge and the time variation of channel storage (Eqs. (4.21-22)) can be obtained by spatially integrating the mass conservation equation for sediment-laden flow (Eq. (4.20)) from the distance of \( x = x_1 \) (upstream) to \( x = x_2 \) (downstream):

\[
\frac{\partial \zeta}{\partial t} + \frac{\partial hu}{\partial x} = 0 \tag{4.20}
\]

\[
\frac{\partial}{\partial t} \int_{x_1}^{x_2} \zeta(x,t) dx + \int_{x_1}^{x_2} \frac{\partial hu}{\partial x} dx = 0 \tag{4.21}
\]
thus \[ q_2 - q_1 = - \frac{\partial V_{sr}}{\partial t} \] (4.22)
where \( \zeta(x,t) = z + h = \) water level; \( q_1 = \) upstream flow discharge ( \( x = x_1 \) ); \( q_2 = \) downstream flow discharge ( \( x = x_2 \) ); \( V_{sr} = \int_{x_1}^{x_2} \zeta(x,t)dx = \) channel storage in the river section between \( x_1 \) and \( x_2 \); other variables are the same as those specified above. With respect to the initial state, Eqs. (4.21-22) can be rewritten as follows

\[ \frac{\partial}{\partial t} \int_{x_1}^{x_2} (\zeta(x,t) - \zeta(x,0))dx + \int_{x_1}^{x_2} \frac{\partial h u}{\partial x} dx = 0 \] (4.23)

\[ q_2 - q_1 = - \frac{\partial V'_{sr}}{\partial t} \] (4.24)

where \( \zeta(x,0) = \) initial water level; \( V'_{sr} = \int_{x_1}^{x_2} (\zeta(x,t) - \zeta(x,0))dx = \) relative channel storage with respect to the initial channel storage.

These equations indicate that the simultaneous downstream discharge (i.e., \( q_2 \)) is higher than the upstream discharge (i.e., \( q_1 \)) when the (relative) channel storage decreases in time (\( \frac{\partial V_{sr}}{\partial t} < 0 \) or \( \frac{\partial V'_{sr}}{\partial t} < 0 \)). Therefore for an increasing peak discharge event, a reduction of (relative) channel storage should occur in the period between the upstream and downstream flood peaks.

### 4.6.2 Relation between channel storage, bed roughness, and bed erosion

For the 2004 flood simulations including roughness change and bed erosion effects (Case 1), the downstream peak discharge increase (Fig. 4.5a) is accompanied by a considerable (relative) channel storage reduction in the period between the upstream and downstream flood peaks (Figs. 4.11a and b). In comparison, the (relative) channel storage for simulations using a constant bed roughness (Case 2, for which no increasing peak discharge occurs (Fig. 4.5b)) hardly decreases (Fig. 4.11). As a result, the bed roughness reduction decreases the (relative) channel storage, which then leads to the peak discharge increase.

For simulations of the 1977 flood, including both roughness change and bed erosion effects (Case 6, for which an increasing peak discharge is observed (Fig. 4.10a)), the (relative) channel storage decreases rapidly and considerably in the period between the upstream and downstream flood peaks (Fig. 4.12). Similar to the 2004 flood, the bed roughness reduction is mainly responsible for the channel storage reduction. However, for simulations with constant bed roughness (Case 8, for which no peak
discharge increase is observed (Fig. 4.10c)) the (relative) channel storage hardly decreases when the flood peak propagates from the upstream to downstream (Fig. 4.12). Exaggerating sediment transport (Case 9) does decrease the channel storage (Fig. 4.12) due to very strong bed erosion and hence reproduces the downstream peak discharge increase (Fig. 4.10d).

Figure 4.11: Time variation of the relative channel storage for Cases 1-2. The storage in the period between the upstream and downstream flood peaks in each sub-reach is highlighted by solid thick line in (a) red for Case 1 and (b) blue for Case 2.
4.6 Discussion

Although having distinct mechanisms, both bed roughness reduction (accelerating flow propagation) and (very strong) bed erosion (lowering water level and increasing flow volume) may lead to channel storage reduction and thus the increase of flow conveyed downstream during the peak period. As a result, a downstream increasing discharge peak is probably observed under the two conditions. Additionally, the two mechanisms are physically coupled. Bed erosion increases the sediment concentration, which may then strengthen bed roughness effects. A decrease in bed roughness would in turn reduce the flow energy loss and favor potential bed erosion.

4.6.3 Current findings and previous studies

Previous studies on the flood peak increase focused on only one possible mechanism (Jiang et al., 2006; Cao et al., 2006, 2012; Li, 2008; Wang et al., 2009; Qi et al., 2010), while the present work combines the relative role of bed roughness change, bed erosion and their relations to channel storage in hyperconcentrated floods. The current

Figure 4.12: Time variation of the relative channel storage for Cases 6, 8, 9. The storage in the period between the upstream and downstream flood peaks in each sub-reach is highlighted by solid thick line in (a) red for Case 6, (b) black for Case 8 and (c) blue for Case 9.
findings show that for the hyperconcentrated flood at moderate concentrations (up to several 100 g/l, such as the 2004 flood in the Lower Yellow River) the roughness reduction effect due to hyperconcentration is the most important mechanism generating a peak discharge increase. The importance of bed roughness was previously addressed through numerical simulations (Jiang et al., 2006) and analytical analysis (Li, 2008). Flume experiments and field observations in the Yellow River confirm that for concentrations up to several 100 g/l, the bed roughness decreases with increasing sediment concentration (Zhang and Xie, 1993; Li et al., 2008; Zhu and Hao, 2008; Jiang et al., 2008). For hyperconcentrated floods with very high concentrations (e.g., the 1977 flood in the Xiaobeiganliu), the current study indicates the bed roughness change is the dominant mechanism as well, even though previous studies concluded that the peak discharge increase might be attributed to intensive bed erosion (Cao et al., 2006, 2012; Qi et al., 2010). However, the present work suggests that bed erosion mainly contributes to the effects of bed roughness change, and that bed roughness is the most influential parameter. Even if very strong bed erosion may increase the peak discharge, the conditions during which the bed is eroded sufficiently deep are rare for most hyperconcentrated floods. Additionally, the present work might be the first to integrate the two distinct mechanisms of bed roughness change and (very strong) bed erosion by the concept of channel storage reduction to explain the downstream peak discharge increase.

The bed roughness reduction may be related to sediment-induced effects (increasing viscous sublayer or sediment-induced buoyancy destruction), but also to bed form changes. The bed form roughness probably decreases during a rising flood stage because uneven bed forms (i.e. dunes or ripples) may flatten (upper plane bed) (see stability diagram of Southard and Boguchwal (1990); Qi et al., 2010). However, the dynamics of bed forms in very fine-grained sediment, such as the silt-dominated Yellow River, are poorly known, and their dynamics under hyperconcentrated conditions are even less well understood. In comparison, the role of sediment-induced buoyancy destruction on roughness is better understood (Winterwerp, 2006; Winterwerp et al., 2009). At moderate sediment concentrations (up to several 100 g/l) the flow is supersaturated, and the vertical concentration gradient suppresses turbulence, leading to strong velocity shears and rapid settling from suspension (van Maren et al, 2009b). This results in very low bed roughness. At even higher sediment concentrations, hindered settling becomes dominant and the effective settling velocity of sediment particles becomes so low that sediment becomes uniformly distributed in vertical. This possibly contributes to increasing bed roughness, but the bedform type may change as well with the sedimentary and hydraulic changes. However, because of the logistic difficulties associated with data collection during hyperconcentrated floods, the evolution of bed forms and their influence on bed roughness are still poorly known.
In the present work, a varying Manning coefficient is used to estimate the roughness change. This coefficient is considered to depend on the rheological property and hydraulic conditions of the water-sediment mixture. It corresponds to a relationship between bed shear stress and velocity, which is not strictly quadratic. Therefore, this relationship is valid for any non-Bingham fluid, and the present modelling approach cannot exclude that the effect is purely rheological.

**4.7 Conclusions**

A data analysis of 13 historical hyperconcentrated floods reveals that the contribution of bed erosion to increasing flow volume may not be the main cause of the downstream peak discharge increase for most hyperconcentrated floods in the Yellow River. Two hyperconcentrated floods, a high concentration and a moderate concentration event, have been numerically investigated using a high-resolution, fully coupled morphodynamic model (Li et al., 2013). These studies reveal that a downstream increasing peak discharge can be qualitatively predicted when the effect of bed roughness change is properly taken into account. At specific stations (i.e. Huayuankou and Tongguan) where the measured data for peak discharge increase is available, the model can satisfactorily reproduce this increase. With a constant bed roughness the peak discharge increase can only be reproduced in case of very strong and probably unrealistic bed erosion.

Even though the effects of bed roughness change and bed erosion are very different, they are also mutually related to flood peak increase. Both the bed roughness reduction and (very strong) erosion result in a channel storage reduction, which then leads to a downstream discharge increase. Additionally, the two mechanisms are physically coupled. Bed erosion increases the sediment concentration, which may then strengthen bed roughness effects. A decrease in bed roughness would in turn reduce the flow energy loss and favor potential bed erosion.

Numerical uncertainty is inevitable when using empirical parameters (relations) and simple topography. Improvements can be made when more detailed topography data becomes available and our fundamental knowledge (supported by quantitative field observations) on bed roughness effects under such extreme conditions improves.
Peak discharge increase in hyperconcentrated floods
Floodplain influences on the hyperconcentrated flood propagation

5.1 Introduction

While the previous Chapter 4 deals with 1-D channel flood, it should be pointed out that in the periods 1970s - 1990s, hyperconcentrated floods usually occurred with overflowing the floodplain in the Lower Yellow River (Qi et al., 2010). A downstream increasing peak discharge was observed during some of these flood events (i.e., 1973.8 flood, 1977.8 flood, 1992.8 flood) with a maximal increase of about 30% between the Xiaolangdi hydrological station and the Huayuankou hydrological station (125.8 km downstream of Xiaolangdi). Based on the bathymetry measurement before and after the flood (usually done in May and October), a possible explanation for the discharge increase is the rapid morphological change (Wang et al., 2009; Qi et al., 2010). During the rising stage, the then wide and shallow channel easily overflows into the floodplains, which leads to floodplain sedimentation. This may enhance erosion of the main channel as the flow discharge increases. The flow velocity and the propagation speed of the flood wave are likely to increase when a narrow and deep channel develops. As a result, a downstream increasing peak discharge may occur when the morphological change is so large that the upstream part of the flood wave can overtake the downstream part. This effect may be further enhanced by a return flow from the floodplains: as the main channel deepens, water stored on the floodplains rapidly flows back into the channel (i.e. at the beginning of the waning stage) and is conveyed downstream (Chien and Wan, 1983; Qi, 1992; Qi and Zhao, 1993; Qi and Li, 1996).

As the amount of hydrographic and topographic data available on these events with an increasing peak discharge is rather limited, and the associated processes are very complex and non-linear, the effects of the floodplain-related morphological change
and floodplain storage remain poorly known. Even though the bathymetry measurement before and after the flood shows a considerable cross-sectional change from wide and shallow to narrow and deep, it is still unclear how the morphology changes during the flood and to what extent such changes affect the discharge increase. Moreover, the lack of sufficient data also hampers developing a predictive model that is well calibrated and validated by observations in the Yellow River. As an alternative, numerical experiments with the fully coupled morphodynamic model (see Chapter 2) are conducted in schematized channel-floodplain reaches in order to investigate this mechanism.

Consistently with the work in Chapter 4, the 2004 flood is first revisited by adding an assumed floodplain to the 1-D channel. Subsequently, a detailed investigation is carried out for the 1992 hyperconcentrated flood in the Lower Yellow River, for which downstream increasing peak discharge was observed in the field together with floodplain inundation. The influences of erosion-deposition settings, floodplain width, channel-floodplain types and bed roughness change on the morphological development and peak discharge evolution have been studied systematically. The roles of morphological change and roughness change in the peak discharge increase phenomenon in a channel-floodplain setting are discussed.

5.2 Method

The numerical analyses for the channel-floodplain systems are carried out with the newly developed 2-D fully coupled morphodynamic model (in Chapter 2). The SGM version of the UFORCE scheme is deployed to compute the numerical fluxes in the governing equations (Eqs. (2.14-2.15)); see section 2.4.4 for details. For the estimation of turbulent eddy viscosity and diffusion coefficient for suspended sediment, see section 2.4.5. For the CFL condition, a courant number of 0.45 is used. The two tolerance depths for judging the dry and partially dry cells are set to 0.01 m and 0.05 m, respectively.

It should be pointed out that for most of the simulated cases in this chapter a constant value of Manning roughness is used; only in the cases for investigating the effect of roughness change a varying bed roughness is calculated. In the 2004 flood simulations of moderate concentrations (up to ~ 300 kg/m$^3$), a simple power-law relation (Eq. (4.13)), which is demonstrated to perform well for the 1-D simulation of the 2004 flood (see Chapter 4), is used to represent a decreasing roughness with increasing concentration in the channel-floodplain system. For the 1992 flood, the sediment concentration (up to ~ 500 kg/m$^3$) is much higher than the concentration of the 2004 flood. Field observation shows that bed roughness may increase at very high concentrations (greater than ~ 300 kg/m$^3$). Therefore, the empirical formulation Eq.
(4.17), which implements this high concentration effect, is used to calculate bed roughness.

The sediment entrainment and deposition fluxes are estimated using the Partheniades-Krone formulations (Partheniades, 1965) for the current hyperconcentrated flood modeling in the channel-floodplain system. This formula is valid for very fine sediments, and has been satisfactorily applied to a detailed 3-D modeling of a hyperconcentrated flood in a channel-floodplain reach by the Delft3D software (van Maren et al. 2009b). The formulations are:

\[
E = \begin{cases} 
    \frac{M}{\rho_s} \left( \frac{\tau}{\tau_{cr,e}} - 1 \right) & \text{for } \tau > \tau_{cr,e} \\
    0 & \text{for } \tau < \tau_{cr,e} 
\end{cases} \quad (5.1a,b)
\]

\[
D = \begin{cases} 
    \omega_s c (1 - \frac{\tau}{\tau_{cr,d}}) & \text{for } \tau < \tau_{cr,d} \\
    0 & \text{for } \tau > \tau_{cr,d} 
\end{cases} \quad (5.2a,b)
\]

where \( M = \) erosion rate (kg/m²/s); \( \rho_s = 2650 \text{ kg/m}^3 = \) sediment density; \( c = \) volumetric concentration (-); \( \omega_s = \) effective sediment settling velocity (m/s); \( \tau = \) bed shear stress (Pa); \( \tau_{cr,e}, \tau_{cr,d} = \) critical bed shear stresses for erosion and deposition, respectively (Pa).

In the following, the value of \( M, \tau_{cr,e}, \tau_{cr,d} \) is specified case by case. For the setting of other related parameters, see section 4.3.

### 5.3 Test of the floodplain effects on the 2004 hyperconcentrated flood

Previously the 2004 hyperconcentrated flood has been simulated in a schematized 1-D channel of the Lower Yellow River to reveal the relative importance of bed roughness change and bed erosion to the phenomenon of downstream peak discharge increase. As the flood was observed to remain within the main channel, floodplains were not considered (see details in Chapter 4).

In order to have a first impression of the floodplain effects, we first revisit the 2004 hyperconcentrated flood, for which a downstream increasing peak discharge was satisfactorily reproduced with a varying bed roughness in a 1-D channel in Chapter 4. In this section, three numerical cases are conducted, one in a 1-D channel and two in 2-D channel-floodplain systems. In Case 1, the 2004 flood is first simulated with a varying bed roughness (Eq. (4.13)) in a 1-D channel of constant width 1000 m (the same as in Chapter 4). By adding an assumed floodplain in Case 1, Case 2 is conducted (with a varying bed roughness). To exclude the roughness effects, Case 3 is modified from Case 2 by using a constant bed roughness.
In the channel-floodplain reach (Cases 2 and 3), a floodplain of 550 m wide is added at one side of the main channel, which has a trapezoidal cross-section. The surface and bottom widths of the channel are 1050 m and 950 m, respectively, which correspond to an average channel width of 1000 m, as considered in Chapter 4. As the peak discharge of the 2004 flood is relatively small compared to those events with overflows on the floodplain, a shallow channel is assumed with depth 0.97 m, in order to make floodplain easily flooded during the 2004 flood simulation. Longitudinally the floodplain is assumed to have the same bed slope (i.e., $2.55 \times 10^{-4}$, see section 4.4.1) as the main channel. For the sediment entrainment and deposition, a sensitivity analysis shows that the trend of the downstream discharge evolution is not very sensitive to $M$ (in Eq. (5.1)). Therefore the simulation with $M = 1.0 \times 10^{-4}$ kg/m²/s is shown as an example, together with $\tau_{cr,d} = 1.0$ Pa, $\tau_{cr,s} = 0.9$ Pa (van Maren et al., 2009b). The initial Manning roughness is 0.02328. Other model parameters remain the same as those in the 1-D 2004 flood simulations in Chapter 4 (see section 4.4.1).

When considering a varying bed roughness, a downstream increasing peak discharge (Fig. 5.1) is computed both in a 1-D channel (see Case 1; also previous 1-D channel case of the 2004 flood with a varying roughness in Chapter 4) and in a channel-floodplain reach (see Case 2). Figures 5.1a and b show that the floodplain may have more damping effects on the flood waves (especially in the further downstream) and thus the downstream peak discharge increase in Case 2 is only observed above 80 km. When using a constant bed roughness, the peak discharge gradually decreases in the downstream direction in Case 3 (Fig. 5.1c). This is consistent with the previous 1-D channel case of the 2004 flood with a constant roughness in Chapter 4.

Halfway the model domain (x=120 km) near the Huayuankou hydrological station (x=125.8 km), Case 1 for which the conditions are closest to the practice (no overflows and a varying roughness), can satisfactorily reproduce the observed peak discharge increase (see Fig. 5.2). Though the peak discharge in Case 2 is close to the observed data, the first flood peak is smaller and arrives later because of the floodplain inundation which does not occur in practice. The peak discharge of the first flood peak in Case 3 is far below the observed data, possibly due to the floodplain inundation and the use of a constant bed roughness which are far from the practice. It indicates that in the assumed channel-floodplain reach, the deformation of the cross-sectional shape (shown in Fig. 5.3 at x = 40 km) cannot cause a downstream peak discharge increase due to the more damping effects of floodplains on flood waves. For the 2004 flood, the less the channel-floodplain interaction, the better the simulation is.

Subsequently, a more detailed study of floodplain effects on the downstream peak discharge evolution has been carried out for the 1992 hyperconcentrated flood in the LYR, during which a downstream peak discharge increase was observed together with floodplain inundation.
5.3 Test of the floodplain effects on the 2004 hyperconcentrated flood

Figure 5.1: Computed discharge hydrographs at different cross-sections for the 2004 flood.

Figure 5.2: Comparison of computed discharge hydrographs at x=120 km and observed data at Huayuankou for the 2004 flood.
5.4 Observations of the 1992 hyperconcentrated flood

During a hyperconcentrated flood event in the Lower Yellow River in August 1992, with four successive flood peaks, overflowing floodplains and increasing discharge peaks were observed. Figure 5.4 shows the discharge hydrograph and the sediment concentration during this event at the Xiaolangdi hydrological station. The four flood peaks occurred around \( t=34, 66, 110, 145.7 \) hr with the discharge of about 3560, 3050, 3600, 4550 m\(^3\)/s, respectively. The corresponding sediment peaks (in volumetric concentration) were observed to be about 0.0777 (206 kg/m\(^3\)), 0.078 (207 kg/m\(^3\)), 0.163 (432 kg/m\(^3\)), 0.198 (525 kg/m\(^3\)) at \( t=41.1, 66, 90.8, 145.7 \) hr, respectively. At the Huayuankou hydrological station (125.8 km downstream of Xiaolangdi), increased peak discharges were observed for the second to the fourth flood peaks, with respective discharge of 3230, 4080, 6430 m\(^3\)/s, whereas the sediment concentration measured at Huayuankou was (slightly) lower than that at Xiaolangdi.
5.4 Observations of the 1992 hyperconcentrated flood

Figure 5.4: Measured discharge hydrograph and sediment concentration during the 1992 hyperconcentrated flood event at the Xiaolangdi hydrological station.

Figure 5.5: Measured cross-sectional bed profiles at Huayuankou in (a) 1993, (b) 1994.
Due to the logistic difficulties in measuring topography during a hyperconcentrated flood with such high concentrations, bathymetric data for the 1992.8 flood are not available. As an alternative, the cross-sectional bed profiles at Huayuankou in 1993 and 1994 (topography data before the flood seasons available) are used to describe the main features of the channel-floodplain topography. Two typical channel-floodplain types were observed before the floods: an asymmetric channel-floodplain (Fig. 5.5a) and a symmetric channel-floodplain (Fig. 5.5b). In the asymmetric channel-floodplain case, the floodplain on the left hand side (LHS) is so high that overflowing cannot occur, whereas on the right hand side (RHS) the elevation of floodplain is sufficiently low to allow floodplain inundation. In the symmetric channel-floodplain case, the floodplains on either side are almost of the same height and both are flooded when overflowing occurs. In the present numerical study, these two channel-floodplain types are considered in separate cases for comparison.

5.5 Modeling of the 1992 hyperconcentrated flood

5.5.1 Schematic channel-floodplain reaches

By lack of sufficient data (both hydrographic and topographic) it is difficult to develop a predictive model that is well calibrated and validated by observations in the Yellow River. Yet, mathematical modeling can be applied to investigate the underlying mechanisms through numerical experiments. Therefore, schematized channel-floodplain reaches are used in the numerical study.

The computational reach consists of two sub-reaches: an upper narrow channel reach and a lower channel-floodplain reach.

The upper narrow channel reach is designed to be sufficiently long and composed of rectangular cross-sections of constant width, because: (1) it provides sufficient distance for flow adaptation so that a stable flow condition can be obtained at the start of the channel-floodplain reach; (2) it avoids uncertainties arising from giving empirical relations for the lateral distribution of the discharge and the sediment concentration if floodplain is present at the upstream boundary.

The narrow channel and the asymmetric and symmetric channel-floodplain configurations are shown in Fig. 5.6.
Figure 5.6: Schematized channel-floodplain reach (a) asymmetric, (b) symmetric.
5.5.2 Numerical cases and model set-up

The main objectives of studying the channel-floodplain reach for the 1992 flood are: (1) to reproduce the typical morphological changes of channel erosion and floodplain deposition during the hyperconcentrated flood; (2) to unravel the floodplain influences on the morphological change and flood propagation; (3) to reveal the effects of the typical morphological change and roughness change on the peak discharge evolution; and (4) to determine the difference between an asymmetric and a symmetric channel-floodplain type.

Therefore eight cases are considered in the numerical experiments (see Tab. 5.1). In these cases, the initial flow conditions are obtained by first running the model over a fixed bed with a constant small discharge (under which no overflows occur) until it reaches a steady state (which takes some 60 hr in the present simulations).

A sufficiently long reach of 160 km is schematized, which is composed of an upper narrow channel of 20 km long and a downstream channel-floodplain reach of 140 km. Based on limited topographic data at the Xiaolangdi and Huayuankou hydrological stations in 1993, a bed slope of $3.0 \times 10^{-4}$ is used for both the channel and the floodplain. The width of the upstream narrow channel (also the bottom width of the downstream main channel), denoted by $B_1$ in Fig. 5.6, is set at 300 m, which approximates the channel width at Xiaolangdi. In the downstream channel-floodplain reach, a total cross-sectional width of 1100 m is considered, which is the mean of the total widths at Xiaolangdi and Huayuankou (Cases 1-3, 6-8), respectively. For the asymmetric and symmetric types, the width of a single channel side slope ($B_2$ in Fig. 5.6) is 200 m and 100 m, respectively, while the same channel depth of 1.6 m is used (the cross-sectional areas of the main channel are the same in these two reach types). These settings result in a surface width of 500 m for the downstream main channel, and a single floodplain width ($B_3$ in Fig. 5.6) of 600 m and 300 m, respectively, for the asymmetric (Cases 1-3, and 8) and symmetric (Cases 6-7) configurations. Additionally, different floodplain widths, i.e., $B_3=400$ m for Case 4 and $300$ m for Case 5, are also simulated in the asymmetric cases.

For the sediment transport, uniform sediment is considered: $d_{50}=0.025$ mm, $p=0.45$. In most of the numerical cases, the critical bed shear stresses for erosion ($\tau_{cr,e}$) and deposition ($\tau_{cr,d}$) are set at 1.0 and 0.9 Pa, respectively, which are close to the values used by van Maren et al. (2009b) for the 1959 hyperconcentrated flood of the Lower Yellow River with a Delft3D model. Additionally, large ($\tau_{cr,e}=1.6$ Pa, $\tau_{cr,d}=1.5$ Pa in Case 7) values are also used in the symmetric reach for comparison. In order to reproduce the typical morphological feature of channel erosion and floodplain deposition, different erosion rates are tested by considering a wide range of values for
5.6 Results of the 1992 flood in the asymmetric channel-floodplain

the parameter M (in Eq. (5.1)). The results of M=1.0×10⁻⁴ (Case 2), 2.5×10⁻⁴ (Case 1), 5.0×10⁻⁴ (Case 3) are shown for the analysis. All the cases are simulated with a constant Manning roughness of 0.011 (typical value for the LYR), except for Case 8 in which a varying roughness is considered for comparison.

Table 5.1 Numerical cases and model settings

<table>
<thead>
<tr>
<th>Case</th>
<th>Reach type</th>
<th>Channel width (m)</th>
<th>Floodplain width (m)</th>
<th>Erosion and deposition parameters</th>
<th>Manning roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>300</td>
<td>200</td>
<td>600</td>
<td>2.5×10⁻⁴, 1.4, 0.9</td>
</tr>
<tr>
<td>2</td>
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<td>300</td>
<td>200</td>
<td>600</td>
<td>2.5×10⁻⁴, 1.0, 0.9</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>300</td>
<td>200</td>
<td>600</td>
<td>2.5×10⁻⁴, 1.0, 0.9</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>300</td>
<td>200</td>
<td>600</td>
<td>2.5×10⁻⁴, 1.0, 0.9</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>300</td>
<td>200</td>
<td>600</td>
<td>2.5×10⁻⁴, 1.0, 0.9</td>
</tr>
<tr>
<td>6</td>
<td>S</td>
<td>300</td>
<td>100</td>
<td>300</td>
<td>1.0×10⁻⁴, 1.6, 1.5</td>
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<td>100</td>
<td>300</td>
<td>1.0×10⁻⁴, 1.6, 1.5</td>
</tr>
<tr>
<td>8</td>
<td>A</td>
<td>300</td>
<td>200</td>
<td>600</td>
<td>1.0×10⁻⁴, 1.0, 0.9</td>
</tr>
</tbody>
</table>

*In the reach type, the abbreviation A means asymmetric, S means symmetric.

5.6 Results of the 1992 flood in the asymmetric channel-floodplain

5.6.1 Typical morphodynamics and peak discharge evolution

In the asymmetric channel-floodplain, the typical morphological change of channel erosion and floodplain deposition during the hyperconcentrated flood is reproduced if a suitable erosion rate (M) is chosen. Figure 5.7 shows the cross-sectional changes at x=40 km for Cases 1-3 with different erosion rates.

With M=2.5×10⁻⁴ kg/m²/s in Case 1, continuous erosion (-0.5~0.7 m) occurs in the channel and the channel-floodplain transitional area while sedimentation is observed in the middle of the floodplain (Fig. 5.7a). The sediment deposition is strongest during the second flood peak (t=60-90 hr), amounting to a deposit height up to 0.2 m. During the third and fourth flood peaks (t=90-150 hr), the deposits are slightly eroded, probably due to an increase in bed shear stress as the discharge increases. In the waning stage (after 150 hr), sediment deposits again on the floodplain following the decrease of flow discharge and sediment concentration.

Furthermore, the channel erosion may also affect the floodplain sedimentation. With a smaller erosion rate (M=1.0×10⁻⁴ kg/m²/s) in Case 2, the channel erosion becomes much weaker and floodplain deposition only occurs in the waning stage after 150 hr.
Floodplain influences on the hyperconcentrated flood propagation (Fig. 5.7b). The less channel deepening in Case 2 results in a higher flow velocity and a larger water depth on the floodplain (comparing the upper panels of Figs. 5.8-9 with the upper panels of Figs. 5.13 and 5.16), which corresponds with a relatively high bed shear stress. This yields no deposition or even minor erosion on the floodplain during the flood peaks. Moreover, the sediment available for deposition becomes less in the case of weaker channel erosion (see Fig. 5.10 for the sediment concentration distribution just before the occurrence of floodplain deposition), which also contributes to less floodplain sedimentation.

With a larger erosion rate \( M=5.0 \times 10^4 \text{ kg/m}^2/\text{s} \) in Case 3, the floodplain deposition is strengthened with stronger channel erosion (Fig. 5.7c). The deposits move more closely towards the channel with a maximal height of about 0.25 m at \( t=90 \) hr. The stronger channel erosion causes shallow flows of small velocity (see the lower panels of Figs. 5.8-9) together with more sediment available for deposition on the floodplain (see Fig. 5.10). As a result, relatively large deposition occurs on the floodplain due to a small bed shear stress and high sediment concentration. After 90 hr there is hardly morphological change on the floodplain though erosion continues in the channel and the transitional area. This indicates that the channel erosion and previous floodplain deposition are so large that not enough overflows can occur in the third and fourth flood peaks and the floodplain becomes partly dry during these periods.

In addition, the cross-sectional change also plays an important role in the downstream discharge evolution, as shown in Fig. 5.11. The peak discharge decreases uniformly in the downstream direction for both Case 1 and Case 2 where no dry cells are observed on the floodplain during the flood. The most distinguishable difference between these two cases lies in the second flood peak. For Case 1 the peak discharge decreases slightly in the downstream (Fig. 5.11a), when both the channel erosion and floodplain deposition occur between 60 and 90 hr (Fig. 5.7a). For Case 2 it decreases considerably (Fig. 5.11b) when only minor channel erosion occurs (Fig. 5.7b). However, in Case 3, the third and fourth peak discharges decrease significantly at \( x=40 \) km (compared with Cases 1 and 2) while the decrease becomes less significant and more uniform in the further downstream (Fig. 5.11c). This is mainly because the previously large deposition (between 60 and 90 hr, see Fig. 5.7c) in the upstream blocks part of the floodplain flow and causes considerable sediment loss, and a sudden discharge decrease therefore occurs. As the effect of floodplain deposition weakens further downstream, uniform discharge decrease is again observed. In the period of 60-90 hr, the rapid morphological change due to strong channel erosion and floodplain deposition also contributes to a downstream constant (or slightly increasing) second flood peak in Case 3.

Regarding the satisfactory modeling of the channel erosion and floodplain deposition patterns and the smooth computation of downstream discharge evolution, the erosion
rate of $M=2.5 \times 10^{-4}$ kg/m$^2$/s is used for further numerical study of the influence of floodplain width.

Figure 5.7: Cross-sectional morphological changes at $x=40$ km for (a) Case 1 with $M=2.5 \times 10^{-4}$, (b) Case 2 with $M=1.0 \times 10^{-4}$, (c) Case 3 with $M=5.0 \times 10^{-4}$. 
Figure 5.8: Contours of flow velocity (unit: m/s) at t=60 hr in the floodplain area near \( x=40 \) km, for Case 2 (upper) and Case 3 (lower).

Figure 5.9: Contours of water depth (unit: m) at t=60 hr in the floodplain area near \( x=40 \) km, for Case 2 (upper) and Case 3 (lower).
5.6 Results of the 1992 flood in the asymmetric channel-floodplain

Figure 5.10: Contours of volumetric sediment concentration (unit: \( g/L \)) at \( t=60 \) hr in the floodplain area near \( x=40 \) km, for Case 1 (upper), Case 2 (middle), and Case 3 (lower).
5.6.2 Influences of floodplain width

The floodplain width is an important parameter characterizing the floodplain geometrical features. To further investigate the floodplain influences on the hydro- and morphodynamic changes, simulations are carried out for different floodplain width. Using the same model parameters as in Case 1, Cases 4 and 5 consider a narrower floodplain by setting the floodplain width $B_3=400$ m and $B_3=300$ m respectively.

Figure 5.12 shows the computed cross-sectional changes at $x=40$ km for Cases 4 and 5. Compared to Case 1 (see Fig. 5.7a), the different floodplain widths marginally affect the erosion patterns in the main channel. However, different sedimentation patterns are observed on the floodplain for the different floodplain width. In Case 1 with a relatively wide floodplain, the sedimentation reaches the maximum in the middle of the floodplain (around $y=800$ m along the cross-section) and gradually decreases towards the channel (left) and the sidewall (right). The strong deposition on the middle floodplain is mainly related to small flow velocity in this area (Figs. 5.13-15). In Cases 4 and 5, the overflow movement may be strongly restrained by the narrow floodplain, and thus the strongest deposition occurs near the sidewalls where the flow velocity tends to be small (Figs. 5.13-15). However, the deposition height and area on the floodplain are much smaller for Cases 4 and 5 than those for Case 1.

Figures 5.19-20 show the discharge evolution for the different floodplain width. In the upstream reach, very small differences are observed for the computed discharges in Cases 1, 4 and 5 (Fig. 5.19). With increasing distance, the discrepancy increases and
becomes visible for the first flood peak. In the downstream reach (Fig. 5.20), the first flood peak is obviously more dampened in a wide floodplain than in a narrow floodplain, because the larger wetted area in the wide floodplain results in more energy loss by bed friction. This effect accumulates with distance and thus more significant discharge decrease is observed in the downstream of the wide floodplain reach when overflow first occurs during the first flood peak. The more energy loss also leads to a smaller flow velocity, which is related to the slower propagation of the first flood in the wide floodplain reach (i.e., Case 1 in Figs. 5.19-20). However, the different energy loss in different floodplain width does not yield different discharge evolution during the second to fourth flood peaks. This is possibly because after the first flood, the floodplains in Cases 1, 4 and 5 are fully inundated with sufficient water depth (Figs. 5.16-18). The successive floods further increase the flow depth on the floodplain, which may mitigate the effects of the difference of energy loss related to bed friction in the three cases. Moreover, the floodplain deposition is relatively small (compared to the channel erosion) for these cases, and the discharge evolution therefore mainly depends on the channel erosion patterns. Since almost identical channel erosion behavior is observed in Cases 1, 4 and 5, very small difference is observed in the discharge evolution during the second to fourth floods.

Figure 5.12: Cross-sectional changes for (a) Case 4 with B3=400 m, and (b) Case 5 with B3=300 m.
Figure 5.13: Contours of flow velocity (unit: m/s) at t=60 hr in the floodplain area near x=40 km, for Case 1 (upper), Case 4 (middle), and Case 5 (lower).

Figure 5.14: Contours of flow velocity (unit: m/s) at t=90 hr in the floodplain area near x=40 km, for Case 1 (upper), Case 4 (middle), and Case 5 (lower).
5.6 Results of the 1992 flood in the asymmetric channel-floodplain

Figure 5.15: Contours of flow velocity (unit: m/s) at t=150 hr in the floodplain area near x=40 km, for Case 1 (upper), Case 4 (middle), and Case 5 (lower).

Figure 5.16: Contours of water depth (unit: m) at t=60 hr in the floodplain area near x=40 km, for Case 1 (upper), Case 4 (middle), and Case 5 (lower).
Figure 5.17: Contours of water depth (unit: m) at $t=90$ hr in the floodplain area near $x=40$ km, for Case 1 (upper), Case 4 (middle), and Case 5 (lower).

Figure 5.18: Contours of water depth (unit: m) at $t=150$ hr in the floodplain area near $x=40$ km, for Case 1 (upper), Case 4 (middle), and Case 5 (lower).
5.6 Results of the 1992 flood in the asymmetric channel-floodplain

Figure 5.19: Computed discharge hydrographs for different floodplain width at (a) $x=40$ km, (b) $x=80$ km.

Figure 5.20: Computed discharge hydrographs for different floodplain width at (a) $x=120$ km, (b) $x=160$ km.
5.6.3 Influences of bed roughness change

During the hyperconcentrated flood the change of bed roughness due to the variations of sediment concentration and flow conditions may be significant, and thus influences the flood propagation. In the channel-floodplain reach, this roughness effect is investigated by comparing Case 8 and Case 2. Considering a varying bed roughness with hydraulic and sedimentary conditions (e.g., the Manning roughness decreases from 0.011 to 0.008), Case 8 computes a downstream increasing discharge of the second flood peak (Fig. 5.21); while Case 2 computes a considerably decreasing second flood peak with a constant roughness (Fig. 5.11b). Moreover, the roughness change in Case 8 also results in floodplain sedimentation (Fig. 5.22), which further deepens the channel and possibly leads to larger flow velocity compared to Case 2 where no sedimentation occurs (Fig. 5.7b). Therefore, a downstream increasing peak discharge is computed in Case 8 due to the flow acceleration during the second flood peak. However, this increase is not so large probably because of the relatively small change of bed roughness and the limited channel deepening.

For other flood peaks, there is no qualitative difference between Cases 2 and 8, which compute downstream decreasing discharge peaks (Figs. 5.11b and 5.21). During the third and fourth flood peaks (t=90-150 hr), the sediment concentration is so high (Fig. 5.4) that the bed roughness may not decrease but increase with concentration in Case 8. Also, as the flow discharge does not vary considerably in these two flood peaks, the bed roughness change due to flow variation may not be large. Moreover, the morphological change in the two flood peaks is only related to limited channel erosion (Figs. 5.7b and 5.22), which however cannot cause a flow acceleration that is sufficient for an increasing peak discharge during over-flooding.

Figure 5.21: Computed discharge evolution for Case 8 with a varying bed roughness.
5.7 Results of the 1992 flood in the symmetric channel-floodplain

As a comparison to the asymmetric channel-floodplain, numerical experiments are also conducted in a symmetric channel-floodplain. For a small erosion rate ($M=1.0 \times 10^{-4}$, comparing Cases 2 and 6), the floodplain of the symmetric reach is continuously eroded during the flood (Fig. 5.23a), while sedimentation occurs on the floodplain after 150 hr in the asymmetric reach (Fig. 5.7b). More erosion occurs at the channel-floodplain transitional areas in the symmetric reach, while the patterns of channel erosion are very similar in the asymmetric and symmetric reaches. For a larger erosion rate ($M=2.5 \times 10^{-4}$, see Case 7), sedimentation develops near the sidewalls of the floodplain while erosion occurs elsewhere (Fig. 5.23b). Compared to the asymmetric reach (Case 1, see Fig. 5.7a), the floodplains (especially in the transitional areas) are more likely to be eroded in the symmetric reach even though larger critical bed shear stresses are considered (in Case 7).

Due to more floodplain erosion, larger flow discharge is computed in the symmetric reach than the asymmetric reach (Comparing Cases 6 and 2, and Cases 7 and 1), especially during the third and fourth flood peaks (Figs. 5.24-25). However, a downstream increasing peak discharge is still difficult to observe (Fig. 5.26).
Floodplain influences on the hyperconcentrated flood propagation

Figure 5.23: Cross-sectional changes at x=40 km for (a) Case 6 with M=1.0 × 10^{-4}, (b) Case 7 with M=2.5 × 10^{-4}.

Figure 5.24: Comparison of discharge hydrographs at distinct locations for Cases 2 and 6.
5.7 Results of the 1992 flood in the symmetric channel-floodplain

Figure 5.25: Comparison of discharge hydrographs at distinct locations for Case 1 and 7.

Figure 5.26: Computed discharge evolution for (a) Case 6 and (b) Case 7 in the symmetric channel-floodplain reach.
5.8 Discussions and conclusions

In this chapter, the 2-D fully coupled morphodynamic model (in Chapter 2) is applied in the channel-floodplain system to study the floodplain influences on the hyperconcentrated flood propagation. For consistency to the work in Chapter 4, the 2004 flood is first revisited by adding an assumed floodplain to the previous 1-D channel. Then, a detailed investigation is carried out for the 1992 hyperconcentrated flood in the Lower Yellow River, for which downstream increasing peak discharge was observed in the field together with floodplain inundation.

The results of the 2004 flood show that a downstream increasing peak discharge (of the first flood peak) occurs in the channel-floodplain reach if a varying bed roughness is considered, whereas a downstream decreasing peak discharge is computed when a constant bed roughness is used. These findings are consistent with the previous 1-D channel results of the 2004 flood in Chapter 4. Moreover, compared to the 1-D channel case, the smaller peak discharge in the 2-D channel-floodplain case (when a varying roughness is used in both cases) also implies that the floodplain has more damping (rather than amplifying) effects on the flood waves. This is consistent with the field observations that the downstream peak discharge increase is more frequently observed in the Lower Yellow River after 2000 when the flood is mostly maintained within the channel.

For the 1992 flood, an obvious downstream peak discharge increase is difficult to reproduce in the schematized channel-floodplain reach. Only with a varying bed roughness or a relatively large erosion rate (i.e. a large value of M), a small discharge increase is found for the second flood peak. This is probably caused by the flow acceleration due to roughness reduction and channel deepening. However, as the roughness change is not so large and the morphological change is restrained by the shallow flows on the floodplain (due to the relatively small peak discharge), the floodplain effects are not strong enough to cause a (large) discharge increase. Even with a large erosion rate, a considerable decrease of the peak discharge (in the third and fourth flood peaks of the 1992 event) may occur in the downstream part, when severe sedimentation occurs on the floodplain.

The detailed floodplain study of the 1992 flood also shows that with suitable erosion and deposition settings, the model can satisfactorily reproduce the typical morphological change of channel erosion and floodplain deposition during the hyperconcentrated flood. A decreasing (or increasing) erosion rate (represented by a decrease (or increase) of the value of the coefficient M) does not simply yield less channel erosion and more floodplain deposition (or more channel and floodplain erosion), but may cause qualitatively different morphological behavior. Due to the interactions between the channel and the floodplain, both channel and floodplain can
be eroded with a decreasing erosion rate while the channel erosion and floodplain deposition can be strengthened with an increasing erosion rate. The different floodplain width affects the sedimentation patterns on the floodplain, whereas it hardly influences the channel erosion and discharge evolution, possibly because of the relatively small morphological change of the floodplain. During over-flow the floodplain (especially the channel-floodplain transitional areas) is more likely to be eroded in the symmetric channel-floodplain reach, leading to a larger flow discharge than in the asymmetric reach.

Uncertainties are inevitable due to the use of empirical formulations (parameters) and a simplified channel-floodplain reach. In nature, the conditions are very complex when over-flows occur on the floodplain. For the model of the 1992 flood, the absence of high-resolution topographic data on the floodplain (note that, many dikes and houses have been built on the floodplain) makes it difficult to reproduce a discharge evolution comparable to the field observation at Huayuankou. Moreover, the 3-D effects on the flow structures and concentration profiles may be strong when over-flow occurs in the channel-floodplain system, and these are not fully taken into account in the present depth-averaged model. Further improvement requires future research.
Floodplain influences on the hyperconcentrated flood propagation
Chapter 6

Long-term effects of water diversions on river morphology

6.1 Introduction

Along many alluvial rivers, lateral water diversions for irrigation, power generation and municipal water supply have commonly occurred (Klaassen and de Vries, 1977; Wang et al., 1996; Yang et al., 2004; Zamora et al., 2012). Such disturbances to the natural fluvial systems can considerably influence fluvial processes especially for highly sediment-laden rivers such as the Yellow River in China.

In the 1970s and 1980s, large improvements of irrigation systems were made in the Lower Yellow River (LYR) and as a result, the (annual-mean) amount of water and sediment load diverted from this reach increased significantly by 275% and 41.2% from the 1950s to 1980s (Zhang, 1995). In the 1980s, about 26% of the annual water discharge and 13.8% of the sediment load were diverted from this reach respectively (Zhang, 1992). Moreover, the spatial distribution of water-sediment diversions changed significantly during the above periods. The water and sediment diversions were mainly conducted in the upstream reaches of the LYR in the 1950s whereas they were 131% and 57.4% larger in the downstream in the 1980s (Zhang, 1995). Combined with the effects of climate change (Yang et al., 2004; Wang et al., 2007a), the extensive water diversions have led to dry-up problems in the lower reach of the LYR since 1972. Infrequent dry-up events were observed in the 1970s and 1980s and the situation rapidly degenerated in the 1990s when a dry-up occurred every year. In the 1990s over half (about 53%) of the water discharge was diverted from the LYR resulting in 897 dry-up days (Wu et al., 2008c), and continuous sedimentation

Long-term effects of water diversions on river morphology

therefore occurred in the main channel of the whole LYR (Wang et al., 2008b). For the most severe event in 1997, the dry-up lasted for 226 days with its influence extending 704 km upstream from the river mouth. After the Xiaolangdi reservoir became operational (October 1999), the dry-up problem was relieved but considerable reduction of flow discharge along the lower reach was still observed. The average ratio between the total runoff at Lijin (near the river mouth) and that at Huayuankou (about 700 km upstream from the river mouth) was 0.59 in the period of 2000-2010, indicating a diversion rate of about 41%. As the water diversions continue, their impacts on discharge reduction will inevitably affect the long-term sediment transport processes and morphological evolution in the LYR.

Previous studies on the water diversion may be categorized by the spatial scales of the concerned problems. On small scales, the local flow structure and morphological evolution at a diversion point (Neary et al., 1999; Roiser et al., 2010, 2011) as well as the designs for junctions of diverting canals and sediment controlling issues (Barkdoll et al., 1999; Wang et al., 1996) have attracted much attention. On large scales, studies mostly focus on the hydrological and morphological influences of a single water diversion point at the equilibrium state by 1-D theoretical or numerical investigations (Klaassen and de Vries, 1977; Kerssens and van Urk, 1986). The impacts of multiple water diversions on large spatial scales and long terms mainly focus on the water resources problems and the loss of sediment load (Liu and Xia, 2004; Yang et al., 2004; Wang et al., 2007a; Wu et al., 2008c), the ecological effects and water quality (Cheong, 1991; Kingsford, 2000; Chen et al., 2003). However, the long-term morphological effects of multiple water diversions along alluvial rivers remain poorly understood while the understanding of this morphological evolution is very important for the sustainable use of the river, such as diversion facilities and flood defense.

For the Lower Yellow River, previous studies related to the morphological effects of water diversions have mainly focused on the relations of the amount of channel deposition or erosion with the change of diversion rate and the previous channel state (i.e., degradation, aggradation or equilibrium) (Zhang and Liang, 1995; Zhang, 1995; Liang et al., 1995, 1999). The water diversions have been demonstrated to cause continuous sedimentation in this reach, especially in dry seasons (Liang et al., 1995; Zhang, 1995). However, few studies have investigated the bed level change and the long-term development of the longitudinal bed profile under the impacts of water diversions. Recently, a marked improvement in the understanding of these issues has been made. Based on the minimum stream power theory (Yang, 1972), Wang and Hu (2004) and Wang et al. (2007b) pointed out that the LYR would develop to a convex longitudinal bed profile in the long term due to the impacts of water diversions, which was manifested by the convex bed profile measured in the lower portion of the LYR in 2002 (Wang and Hu, 2004). These interesting findings are similar to those for the
study of the equilibrium bed profile in rivers having tributaries that a convex longitudinal bed profile may develop near the conjunction of tributary (Sinha and Parker, 1996; Rice and Church, 2001) while alluvial rivers typically tend to exhibit a longitudinal concave bed profile in nature (Leopold et al., 1964).

The convex longitudinal bed profile is also theoretically predicted by Wang et al. (2008b) when incorporating the continuous water diversion effects in the governing equations (including the mass and momentum conservations for flow and mass conservation for sediment load) for the equilibrium state (derivatives in time become zero). When employing a spatially varying channel width, Wang et al. (2008b) pointed out that the longitudinal shape should be related to the variation of the specific discharge along the river rather than the total discharge as suggested by Wang and Hu (2004). However, in their application to the LYR, the variation of the specific discharge along the river is simplified to be only affected by continuous water diversions. This may be inappropriate as the water diversion impact could also be strengthened or weakened by the longitudinal change of channel width through the specific discharge. Furthermore, Wang et al. (2008b) only consider the idealized scenario of continuous water diversions along the river while in reality water diversions are discrete. It is unclear how significantly the above simplifications have influenced the analysis.

The present work extends the previous theory by Wang et al. (2008b) to develop a general theoretical framework for predicting the equilibrium longitudinal flow and bed profiles under multiple water diversions, which is applicable to both continuous and discrete water diversions in a longitudinally width-varying channel. This theoretical work considers the silt to very fine-sand river beds in the water diversions without barrages or other structures. It is then applied to the conditions of the water diversions in the LYR. Differences in the equilibrium flow and bed profiles with and without width variations are discussed with respect to the changes of diversion intensities. The impacts of different diversion scenarios (continuous and discrete, pure water diversion and water-sediment diversion) are also investigated. Furthermore, numerical simulations are conducted for a schematic LYR to reveal the morphological time scale (MTS) at which the impact of water diversions will develop. The numerical results for the final equilibrium state are compared with those of the theoretical analysis to further verify the accuracy and applicability of the developed theory.
6.2 Theoretical framework

6.2.1 Governing equations

In the long term, the longitudinal profiles of flow and river bed will reach an equilibrium state at which the fluvial system becomes stable in time. Assuming constant flow and sediment load at the upstream boundary, the mass and momentum conservations of the flow and the mass conservation of the sediment are used to describe this equilibrium state. The governing equations for a 1-D channel with varying width are (Wang et al., 2008b)

\[ \frac{\partial Q}{\partial x} = F_{Qx} \quad (6.1) \]

\[ \left(1 - \alpha_u \frac{u^2}{gh} \right) h' + S_f - \alpha_u \frac{u^2}{gB} B' - S_{ox} = 0 \quad (6.2) \]

\[ \frac{\partial Q_s}{\partial x} = F_{sx} \quad (6.3) \]

where \( x \) = streamwise coordinate (m); \( Q = Bq \) = flow discharge (m\(^3\)/s); \( q \) = specific flow discharge (m\(^2\)/s); \( B \) = river width (m); \( u \) = depth-averaged streamwise flow velocity (m/s); \( h \) = flow depth (m); \( S_{ox} = -\partial z / \partial x \) = bed slope; \( z \) = bed level (m); \( F_{Qx}, F_{sx} \) = source/sink (+/-) per unit length, e.g. tributaries, confluences or withdrawal of water and/or sediment (m\(^2\)/s); \( Q_s = Bq_s \) = sediment transport rate (m\(^3\)/s); \( q_s \) = specific sediment transport rate (m\(^2\)/s); \( S_f \) = friction slope; \( B' = \partial B / \partial x \) = the downstream gradient of \( B \); \( h' = \partial h / \partial x \) = the downstream gradient of \( h \); \( \alpha_u \) = the coefficient resulting from depth-averaging the flow velocity, set to 1 in the present work; \( g \) = gravitational acceleration (m/s\(^2\)).

For equilibrium conditions, the sediment transport reaches the capacity state and can be estimated by a sediment transport formula. Following Wang et al. (2008b), a simple power-law sediment transport formula is used, allowing a straightforward application in theoretical analysis:

\[ Q_s = Bq_s = Bmu^n \quad (6.4) \]

in which \( m \) and \( n \) are coefficients. In the present study, the parameter \( n = 5 \) is used, which refers to the Engelund and Hansen formula (Engelund and Hansen, 1967). The justification of this choice (\( n = 5 \)) is provided in the Discussion Section. The friction slope is approximated by Chézy coefficient \( C \).
6.2 Theoretical framework

\[ S_f = \frac{u^2}{C^2 h} \]  

(6.5)

6.2.2 General formulations

Considering the losses of discharge and sediment load due to both continuous and discrete water diversions, the flow discharge and sediment transport rate can be expressed by integrating Eqs. (6.1) and (6.3) in distance respectively, as

\[ Q = Bq = Q_0 + \int_0^x F_{Qx} \, dx = Q_0 + f_{cq} + f_{dq} \]  

(6.6)

\[ Q_s = Bq_s = Q_{s0} + \int_0^x F_{sx} \, dx = Q_{s0} + f_{cs} + f_{ds} \]  

(6.7)

where \( Q_0, Q_{s0} \) = flow discharge and sediment transport rate at the upstream boundary (m\(^3\)/s); \( f_{cq} = \int_0^x f_q \, dx \) = water loss due to continuous water diversions; \( f_q \) = continuous water diversion rate (m\(^2\)/s); \( f_{cs} = \int_0^x f_s \, dx \) = sediment loss due to continuous water diversions; \( f_s \) = continuous diversion rate of sediment load (m\(^2\)/s); \( f_{dq} = \sum_{x_j} p_{1j} Q_0 \) = total water loss from discrete water diversions; \( p_{1j} \) = dimensionless discrete water diversion rate with respect to \( Q_0 \) at the distance \( x_j \); \( j \) = indices of discrete water diversion; \( f_{ds} = \sum_{x_j} p_{2j} Q_{s0} \) = total sediment loss from discrete water diversions; \( p_{2j} \) = dimensionless discrete sediment diversion rate with respect to \( Q_{s0} \) at the distance \( x_j \). In the formulations, the water and sediment diversion rates are treated as sink terms with negative values. Other parameters are the same as specified above.

6.2.3 Equilibrium longitudinal profiles

Based on the above governing equations and general formulations, the equilibrium longitudinal profiles of flow and river bed can be explicitly solved. The flow velocity and depth only depend on the specific discharge and sediment transport rate while the bed slope and water surface are also directly affected by the longitudinal gradient of channel width. Substituting Eq. (6.7) into Eq. (6.4) yields
where \( B_0, u_0 \) = channel width and flow velocity at the upstream boundary respectively; 
\( b_r = B_0 / B \) = inverse of relative channel width (i.e., \( b_r > 1 \) for streamwisely narrowing channel; \( b_r < 1 \) for streamwisely widening channel; \( b_r = 1 \) for channels of constant width). Then the water depth and its downstream gradient are given as

\[
h = \frac{q}{(q_s / m)^{1/n}} = h_0 b_r^{1-1/n} \left( 1 + f_{cq} / Q_0 + \sum_{j, x} p_{ij} \right)^{1/n},
\]

\[
h' = \frac{h_0 b_r^{1-1/n}}{\left( 1 + f_{cq} / Q_{s0} + \sum_{j, x} p_{2j} \right)^{1/n}} \times \left( 1/(n-1) \right) B' \left( 1 + f_{cq} / Q_0 + \sum_{j, x} p_{ij} \right) + f_q / Q_0 - \frac{1}{n} f_s / Q_{s0} \frac{1 + f_{cq} / Q_0 + \sum_{j, x} p_{ij}}{1 + f_{cq} / Q_{s0} + \sum_{j, x} p_{2j}}
\]

where \( h_0 \) = depth at the upstream boundary. When substituting Eq. (6.5) and Eqs. (6.8-10) into Eq. (6.2), the bed slope is obtained through the momentum equation,

\[
S_{0x} = S_{0x1} + S_{0x2} + S_{0x3}
\]

in which

\[
S_{0x1} = (1 - \alpha \frac{u^2}{gh}) h' = \left( 1 - \frac{\alpha}{g} (q_s / m)^{3/n} / q \right) \frac{\partial}{\partial x} \left( \frac{q}{(q_s / m)^{3/n}} \right)
\]

\[
S_{0x2} = \frac{u^2}{C^2 h} = \frac{1}{C^2} \frac{(q_s / m)^{3/n}}{q} = \frac{u_0^2}{C^2 h_0} b_r^{3/n-1} \frac{1 + f_{cq} / Q_{s0} + \sum_{j, x} p_{2j}}{1 + f_{cq} / Q_0 + \sum_{j, x} p_{ij}}
\]

\[
S_{0x3} = -\frac{\alpha u^2}{gB} B' = -\frac{\alpha (q_s / m)^{2/n}}{gB} B' = -\frac{\alpha u_0^2}{gB_0} b_r^{2/n+1} \left[ 1 + f_{cq} / Q_{s0} + \sum_{j, x} p_{2j} \right] B'
\]

(6.12a,b,c)

Here, the water surface slope \( J \) is defined by

\[
J = -\frac{\partial \zeta}{\partial x}
\]

(6.13)
where \( \zeta = z + h \) = water level. The water surface slope follows from Eqs. (6.10-11). In case that the upstream boundary and the water diversion conditions are known, the equilibrium longitudinal profiles can be predicted using the above equations for a specific channel.

### 6.3 Case study

#### 6.3.1 Schematic cases

Water diversions vary spatially, and may consist of pure water or water-sediment mixture. These factors may all influence the equilibrium profiles of flow and river bed, which will be systematically analyzed. Therefore, five scenarios of water diversions are designed in the case study according to different diversion placement and diversion schemes (see Table 6.1). The influences of channel-width variations have been incorporated in the above formulations.

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
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<tr>
<td>Diversion placement</td>
<td>No</td>
<td>UC</td>
<td>UC</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>Diversion scheme</td>
<td>No</td>
<td>PW</td>
<td>WSM</td>
<td>PW</td>
<td>WSM</td>
</tr>
</tbody>
</table>

*the abbreviations of UC, D, PW and WSM mean uniformly continuous, discrete, pure water and water-sediment mixture respectively.*

#### 6.3.2 Equilibrium flow state

The equilibrium states for the above five cases can be straightforwardly obtained using Eqs. (6.8-13) and therefore the derivation is omitted here for brevity. Assuming that the five cases occur in the same channel (with monotonic width variations) and have the same upstream boundary conditions, the impacts of different water diversions on the longitudinal equilibrium flow profile are investigated first. For simplicity, the sediment concentration in the channel is assumed not to be affected by water-sediment mixture diversions, which implies an identical diversion intensity of sediment load and water. For discrete water-sediment diversions, it indicates \( p_{ij} = p_{zj} \).

For uniformly continuous water-sediment diversions, it leads to \( Q_0 / f_q = Q_{10} / f_x = \lambda \).

The parameter \( \lambda \) is defined as the discharge vanishing length (Wang et al., 2008b) and has negative value according to the previous definitions (in water diversion...
conditions). Physically, its absolute value is the length over which the discharge decreases to zero.

Table 6.2 Local change of equilibrium depth due to water diversions

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>1</td>
<td>$1 + \frac{x}{\lambda}$</td>
<td>$(1 + \frac{x}{\lambda})^{1/n}$</td>
<td>$1 + \sum_{x_j \leq x} P_{ij}$</td>
<td>$(1 + \sum_{x_j \leq x} P_{ij})^{1/n}$</td>
</tr>
</tbody>
</table>

The impacts of water diversions on flow depth are shown in Table 6.2 for the equilibrium state. A dimensionless depth ($h$) is introduced by dividing the results of Cases 2-5 by Case 1. According to the previous definitions, the values of dimensionless depth for Cases 2-5 are smaller than one indicating that water diversions result in a reduction in water depth. This shallowing effect becomes stronger in the downstream direction as the dimensionless depth decreases by the accumulative impacts of water diversions. By comparing Cases 2 and 3 (or Cases 4 and 5), it indicates that pure water diversions cause more decrease of local depth than the diversions of water-sediment mixture do with the same diversion rates. Comparing Cases 2 and 4 (or Cases 3 and 5) reveals that with the same diversion amount, uniformly continuous and discrete water diversions result in the same depth at the distance where discrete diversion locates. At this location, depth changes gradually in uniformly continuous water diversions while decreases abruptly in discrete water diversions. However, such a local difference does not change the large-scale similarity of the equilibrium longitudinal profiles for these two diversion schemes.

Moreover, the downstream variation of equilibrium depth is not only affected by water diversions, but also by a change in channel width. In a widening channel the equilibrium depth decreases along the reach when water diversions occur, whereas its downstream variation is uncertain in a narrowing channel (see Table 6.3). The depth reduction ($\Delta h$) with respect to the nature condition (of no water diversions) is accordingly affected by these two factors as well (see Table 6.3). In a narrowing channel the depth reduction ($\Delta h$) increases downstream while it may decrease, increase or remain constant downstream in a widening channel. In Table 6.3, the parameter $h = h_2 / h_1$ shows the downstream variation of equilibrium depth, and $\Delta h = \Delta h_2 / \Delta h_1 = (h_2 - h_2^*) / (h_1 - h_1^*)$ shows the downstream variation of local depth reduction with respect to the natural condition, $^*$ denotes the natural condition (without water diversions). The subscripts 1 and 2 denote the locations at upstream and downstream. The symbols WC and NC are abbreviations for widening channel and narrowing channel respectively. As for the depth variation, the symbols $\sim$, $+$, $\sim$ mean decrease, increase, and uncertain in the downstream direction respectively.
Table 6.3 Downstream variations of equilibrium flow state (see the above text for explanations)

<table>
<thead>
<tr>
<th>Case</th>
<th>Formulation of $h$</th>
<th>Variation of $h$</th>
<th>Formulation of $\Delta h$</th>
<th>Variation of $\Delta h$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>WC</td>
<td>NC</td>
<td>WC</td>
</tr>
<tr>
<td>2</td>
<td>$1 + x_i / \lambda \frac{B_1}{B_2}$ \quad $1 + x_i / \lambda \frac{B_1}{B_2}$</td>
<td>-</td>
<td>-</td>
<td>$x_i \frac{B_1}{B_2}^{1-1/n}$</td>
</tr>
<tr>
<td>3</td>
<td>$\left(1 + x_i / \lambda \frac{B_1}{B_2}\right)^{1-1/n}$</td>
<td>-</td>
<td>-</td>
<td>$\frac{(1 + x_i / \lambda)^{1-1/n} - 1}{(1 + x_i / \lambda)^{1-1/n} - 1}$</td>
</tr>
<tr>
<td>4</td>
<td>$1 + \sum_{x_i \leq x_n} p_{ij} \left(\frac{B_1}{B_2}\right)^{1-1/n}$</td>
<td>-</td>
<td>-</td>
<td>$\sum_{x_i \leq x_n} p_{ij} \left(\frac{B_1}{B_2}\right)^{1-1/n}$</td>
</tr>
<tr>
<td>5</td>
<td>$\left(1 + \sum_{x_i \leq x_n} p_{ij} \left(\frac{B_1}{B_2}\right)^{1-1/n}\right)$</td>
<td>-</td>
<td>-</td>
<td>$\frac{(1 + \sum_{x_i \leq x_n} p_{ij})^{1-1/n} - 1}{(1 + \sum_{x_i \leq x_n} p_{ij})^{1-1/n} - 1}$</td>
</tr>
</tbody>
</table>

6.3.3 Equilibrium river morphology

Based on the same assumptions made above for the equilibrium flow state, Eqs. (6.11-12) are used to investigate the development of the longitudinal bed profile with the variations of channel width ratio and water diversion intensity. Here, the channel width ratio ($r_w$) is defined as the ratio of the downstream boundary width to the upstream boundary width of the channel, showing the plan view of the river. The water diversion intensity ($r_{wd}$) is defined by the rate of diverted discharge to the discharge at the upstream boundary and can be represented by $-l/l$ with $l$ being the channel length (i.e. uniformly continuous water diversions). In the following, only the uniformly continuous water diversions are investigated, as similar results can be expected for discrete water diversions in large spatial scales.

Based on the dimensions of the Lower Yellow River, a 1-D hypothetical channel of 700 km long is considered with downstream varying width (monotonically linear). The channel width is set to 2000 m at the upstream boundary and the channel width ratio ranges from 0.01 to 2. As the operation of the Xiaolangdi reservoir has highly regulated the flow and sediment transport in the Lower Yellow River since October 1999, the field data in the period 2000-2007 (MWRPRC, 2000-2007) are used to approximate the recent flow and sediment conditions. At the upstream boundary, a
Long-term effects of water diversions on river morphology

constant flow discharge of 3000 m$^3$/s and sediment transport rate 5.7 m$^3$/s are specified. The water diversion intensity ranges from 0 to 0.8. Other parameters are: $C = 50$ m$^{1/2}$/s, $g = 9.8$ m/s$^2$, $d_{so} = 0.022$ mm, $n = 5$ (Engelund-Hansen formula).

Using the above settings, the equilibrium longitudinal bed develops into six profiles given different channel width ratios and water diversion intensities (shown in Fig. 6.1). For the bed profiles of convex, concave and quasi-linear shapes, the bed slope monotonically increases, decreases or remains constant along the reach. A dimensionless parameter, being the ratio of the bed slopes at the downstream and upstream boundaries, is introduced to determine the shape of the monotonic bed profiles and quantify their magnitude. When this parameter is greater than one, it presents a convex bed profile. The larger the parameter, the more convex the bed profile develops. A concave bed is represented by the parameter value smaller than one; concavity increases with a decreasing value. A quasi-linear bed profile develops when the parameter is equal to one. The contours of this parameter are shown in Figs. 6.2 and 6.3. For the other three profiles, the bed slope has a non-monotonic variation in the downstream direction. The longitudinal bed profiles of convex, concave, quasi-linear and S-shape 1 are observed for the present water-sediment mixture diversions (see Fig. 6.2), while the S-shape 2 and the Wave-shape bed profiles are also discerned for pure water diversions (see Fig. 6.3).

![Figure 6.1: Patterns of longitudinal bed profiles: (a) Quasi-linear, (b) Concave, (c) Convex, (d) S-shape 1, (e) S-shape 2, (f) Wave-shape.](image)

In the case of monotonic bed profiles, water diversions result in a convex longitudinal bed in a widening or constant-width channel (see Figs. 6.2 and 6.3). The development of the convex bed profile should be due to the loss of flow energy and thus the decrease of flow intensity during the diversions, which lead to reduction of sediment transport capacity and thus sedimentation. The convexity increases with the channel width ratio and the water diversion intensity. Such a bed profile may also occur in a narrowing channel when the downstream increase of bed slope due to water diversions dominates over the bed slope decrease due to channel narrowing. By comparing Figs. 6.2 and 6.3, it illustrates that the possibility of developing a convex
bed profile is higher for pure water diversions than for water-sediment mixture diversions. When the channel narrowing plays a dominant role, the longitudinal bed develops to a concave profile. The concavity increases with decreasing channel-width ratio and diversion intensity. Between the convex and concave beds, there exists a transitional state of quasi-linear bed profile, which may develop in a narrowing channel when the impacts of water diversions and channel narrowing are in balance.

Figure 6.2: Equilibrium longitudinal bed profiles in the case of water-sediment mixture diversions; the dashed line separates the regions of monotonic and non-monotonic bed slope changes.

In the case of non-monotonic bed profiles, the S-shape 1 bed profile (upstream-concave and downstream-convex) may develop in a strongly narrowing channel (i.e., \( r_B < 0.2 \)) for the present water-sediment mixture diversions (see Fig. 6.2). This shape becomes more pronounced with decreasing channel-width ratio and diversion intensity. However, a more complex condition may occur for pure water diversions (see Fig. 6.3). Three types of non-monotonic bed profiles may develop in a strongly narrowing channel given different water diversion intensities. With a moderate or relatively small water diversion intensity (i.e., \( r_{wd} < 0.4 \)), the S-shape 1 bed profile
develops. However, for larger water diversions (i.e., $r_{wd} \geq 0.4$), the bed will develop to the S-shape 2 profile (of upstream-convex and downstream-concave) or the more complicated Wave-shape bed (of convex-concave-convex shape). When the channel narrowing is extremely strong, a convex bed may occur at high diversion intensities.

Figures 6.2 and 6.3 also show that in case of no water diversions ($r_{wd} = 0$), a quasi-linear bed profile develops in a constant-width channel. This is in line with expectations because a constant bed slope should develop under steady-uniform flow conditions along a constant-width channel. A narrowing channel ($r_B > 0.2$) exhibits a concave bed without water diversions, as usually observed in nature. However, a convex bed develops in a widening channel without water diversions. Physically, widening of the channel leads to decrease in flow intensity and thus decrease in sediment transport capacity, which further lead to sedimentation and form a convex bed. A natural case of the convex profile is the development of central bars/shoals in a widening channel. Note that in nature, widening of the channel may be also related to discharge increase due to tributary contributions, therefore a concave bed may be observed.

![Equilibrium longitudinal bed profiles in the case of pure water diversions](image_url)

Figure 6.3: Equilibrium longitudinal bed profiles in the case of pure water diversions; the dashed lines separate the regions of monotonic and non-monotonic bed slope changes.
6.4 Numerical simulations and morphological time scale (MTS)

The above theoretical analysis provides a way to predict the final equilibrium profiles of flow and morphology under the impacts of water diversions, while does not yield a morphological time scale (MTS) over which the equilibrium state develops. Therefore, a series of 1-D numerical experiments are conducted to reveal the impacts of water diversions on the MTS. These experiments are carried out with the SOBEK-RE model (version 2.52.005, Delft Hydraulics, 2005). The flow is computed through the 1-D Saint-Venant equations (Chow, 1959) for clear water flows and the morphology is updated using the mass conservation of sediment under the assumption of capacity sediment transport. The governing equations may approach those for the theoretical analysis at the equilibrium state (see Eqs. (6.1-3)). A decoupled numerical solution is used to first solve the flow conditions and then the bed level changes. Though decoupled in physics and mathematics, this software is adequate to simulate the long term evolution of the alluvial river, during which the feedbacks of bed deformation on the flow in each computational time step are so small as to be negligible.

6.4.1 Numerical cases and model set-up

To be able to reveal the relative importance of water diversions to the natural processes, the water diversions are assumed to occur after the natural equilibrium state is reached. In this way, the MTS for the impacts of water diversions and the natural development can be distinguished. However, it should be pointed out that in reality the water diversions and the natural development occur in concert, the MTS calculated by the current method therefore cannot accurately show the real time scale for the equilibrium development under such coupled situations but qualitatively shows the relative role of water diversions.

Six numerical cases are introduced here for the simulations (see Table 6.4). Case 1 is a reference case without water diversions. The MTS for reaching the natural equilibrium state in this case is defined as \( T_n \). Focusing on the water diversion impact, Cases 2-6 are conducted based on the results for Case 1. The MTS for the development of the equilibrium state under water diversions in Cases 2-6 is defined as \( T_d \). Uniformly continuous diversions are considered by Cases 2 and 4 for water-sediment mixture and by Cases 5 and 6 for pure water. As different diversion intensities do not affect the findings that on large longitudinal scales (uniformly) continuous and (evenly) discrete diversions have qualitatively similar results (not shown), only an (evenly) discrete diversion case of water-sediment mixture is shown in Case 3 (the same diversion intensity as Case 2) for the diversion placement effects.
The simulations are conducted in a schematic channel of the LYR. The channel length and width at the upstream boundary are identical to those for the above theoretical analysis of equilibrium morphology. The channel is assumed linearly narrowing in the downstream direction with a width of 620 m at the downstream boundary where a constant water level of 3.67 m is imposed with an initial bed elevation of zero. The upstream boundary conditions and the sediment transport formula are the same as the theoretical analysis. For Case 1, the initial bed level is a linear interpolation of 22 thalweg points along the LYR measured in 1997 (Wang et al., 2008b). Assuming the total flow discharge and sediment transport rate are constant along the channel, the initial water depth at each cross-section is calculated with the Chézy formula using the upstream boundary conditions. The final equilibrium profiles of flow and bed for Case 1 are used as the initial conditions for Cases 2-6. Two water diversion intensities ($r_{wd}$) are deployed, with respective values of 0.25 and 0.43 being the average discharge ratio of Lijin and Huayuankou in the periods of 2003-2007 and 2000-2007 (MWRPRC, 2000-2007). In Case 3 the total diverted volume of flow and sediments is evenly distributed over three discrete diversion sites (i.e., $x = 175, 350, 525$ km). A spatial step of $\Delta x = 1000$ m and a time step of $\Delta t = 6$ hr are used in all the simulations. Other parameters are the same as specified in the theoretical analysis.

### 6.4.2 Morphological time scale (MTS)

In the present numerical experiments for the MTS study, the river system is considered as equilibrium when the maximal bed level change in the study reach (i.e., at the upstream boundary) is less than 0.5 cm per year. The time to reach this equilibrium state is considered as the MTS.

<table>
<thead>
<tr>
<th>Case</th>
<th>Water diversions</th>
<th>Placement</th>
<th>Scheme</th>
<th>Intensity $r_{wd}$</th>
<th>MTS (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>no no</td>
<td>no no</td>
<td>0</td>
<td>130 ($T_n$)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>UC WSM</td>
<td>0.25</td>
<td>120 ($T_d$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>ED WSM</td>
<td>0.25</td>
<td>110 ($T_d$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>UC WSM</td>
<td>0.43</td>
<td>170 ($T_d$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>UC PW</td>
<td>0.25</td>
<td>105 ($T_d$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>UC PW</td>
<td>0.43</td>
<td>125 ($T_d$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a the abbreviations of UC, ED, WSM, PW mean uniformly continuous, evenly discrete, water-sediment mixture and pure water respectively.
The results of the MTS for the five cases are shown in Table 6.4. For the schematic LYR with the specified upstream boundary conditions, the morphological time scales for the natural development and the development under water diversions are in the order of 100~200 years. These results are twice as large as the MTS of 50~100 years suggested by Wang et al. (2008b) using the Lower Yellow River data of 1970s-1990s. As the MTS is inversely related to the specific sediment transport rate (de Vries, 1975), these differences can be understood. The sediment load used here is based on the data from 2000-2007, which is about half of the annual-average sediment load in the 1970s-1990s, thus the MTS of 100-200 years can be reasonable for the present conditions. The morphological time scales for Cases 2-6 ($T_d$) are close to that for Case 1 ($T_n$) indicating strong influences of water diversions on the MTS for reaching the equilibrium state.

Among different water diversion situations, the differences in the MTS are in the order of decades. Comparing Cases 2 and 4 (or Cases 5 and 6) shows the MTS increases with diversion intensity. This may be because high water diversion intensity brings stronger disturbance to the alluvial river and may lead to a new equilibrium state that is further altered from the natural equilibrium state. With the same flow diversion intensity, water-sediment mixture diversions require a larger MTS than pure water diversions, as shown by comparing Cases 2 and 5 (or Cases 4 and 6). For the former case the specific sediment transport rate reduces due to sediment diversions while it may increase or remain unchanged for the latter case. As the MTS is inversely related to the specific sediment transport rate (de Vries, 1975), smaller MTS is probably predicted for pure water diversions. The morphological time scales for Cases 2 and 3 are very close suggesting that the placement of water diversions (i.e., (evenly) discrete and uniformly continuous diversions) may only marginally affect the MTS (provided the water diversion intensity remains unchanged).

6.4.3 Numerically computed equilibrium state

The initial and final equilibrium states for Case 1 (without water diversions) are shown in Fig. 6.4. For the present conditions, continuous sedimentation occurs in the process of natural development until the equilibrium state is reached. The longitudinal flow and bed profiles are concave with downstream-increasing water depth. These equilibrium profiles for Case 1 are considered as the initial conditions for Cases 2-6 in the numerical study of water diversion impacts.
Long-term effects of water diversions on river morphology

Figure 6.4: Initial states and final equilibrium profiles for Case 1.

Figure 6.5 shows the longitudinal profiles (both theoretical and numerical) of water level, bed elevation, water depth and bed elevation increase (with respect to the initial bed elevation), at the equilibrium state for Cases 2-6. The theoretical profiles are obtained by the developed formulations (see Eqs. (6.8-13)) and boundary conditions following Wang et al. (2008b)’s method. Good agreement between the theoretical and numerical results is observed for all the cases, further demonstrating the accuracy and the applicability of the developed theoretical formulations.

For clear illustration, the water and bed levels in figs. 6.5 (a) and (b) are drawn with respect to a linear bed of constant slope $1.6 \times 10^{-4}$ from the downstream boundary bed elevation. The results show that a concave profile develops in Cases 2-5 and a convex profile in Case 6. It indicates that the equilibrium profiles of the schematic LYR may have qualitative differences for pure water and water-sediment mixture diversions when the water diversion intensity is sufficiently high (i.e., $r_{wd} > 0.4$). A convex bed develops for diversions with low sediment content while a concave bed forms when more sediment is diverted from the channel. In the 1990s, the high water diversion intensity ($r_{wd} > 0.5$) caused a concave and a convex bed in the upper and lower portions of the Lower Yellow River respectively (Wang and Hu, 2004). For the predicted concave bed conditions of Cases 2-5, increase in water diversion intensity
(comparing Cases 2 and 4) and decrease in sediment diversions (comparing Cases 2 and 5) are observed to reduce the concavity. On large spatial scales, similar equilibrium profiles of water level and bed elevation for Cases 2 and 3 indicate the possibility of using uniformly continuous water diversions to approximate the impacts of discrete water diversions when detailed information for the latter is missing.

Figure 6.5: Longitudinal profiles of (a) water level, (b) bed elevation, (c) water depth, and (d) bed elevation increase (with respect to the initial bed elevation) at the equilibrium state. The colors of black, red, blue, green and violet represent Cases 2-6 respectively; the dots represent the theoretical results and the solid lines represent the numerical results.

As shown in Fig. 6.5 (c), the water depth still increases in the downstream direction due to channel narrowing though water diversions occur. Comparing Case 2 with Cases 4-5 reveals that the deepening becomes smaller either if the water diversion increases or if the sediment diversion decreases. The evenly discrete water diversions in Case 3 cause locally saw tooth-shaped changes of water depth, but have an overall longitudinal trend similar to that for uniformly continuous water diversions in Case 2. In Fig. 6.5 (d), sedimentation is observed to increase in the upstream direction for Cases 2-6. The increase is significant in the downstream reach but less pronounced in
the upstream reach. For Case 6 where a convex bed develops, more sedimentation occurs in the upstream reach than for other diversion cases. By comparing Case 2 with Cases 4-5, it shows sedimentation increases with increasing water diversion and decreasing sediment diversion. The longitudinal profiles of sedimentation for Cases 2 and 3 are very similar though discrete diversions cause locally saw tooth-shaped changes.

6.5 Discussion

6.5.1 Sediment transport calculation

The present study uses a simple power-law formula (see Eq. (6.4)) to estimate the sediment transport for an easy application in the theoretical analysis. Using the parameter $n = 5$, the Engelund-Hansen (E-H) formula is adopted, which has been developed for fine sand-bed rivers. Quantitative uncertainties are inevitable when applying the E-H formula to the silt to very fine-sand beds of the Lower Yellow River. Since the key parameter $n$ is important in determining the equilibrium flow states and bed slope, the use of the E-H formula is justified based on the $n$ values.

Figure 6.6 shows the relations between the bankfull width and bed slope, i.e., $S_{0x} \propto B^{(n-3)/n}$ (Wang et al., 2008b), together with the measured data for the present focus, i.e., $B \leq 2000$ m. The results of $n=5$ (the E-H formula) show good agreement with the measured data and are as accurate as those of the best-fits ($n=5.1$). Therefore, $n=5$ (the E-H formula) is used to show the qualitative understanding of the water diversion impact in the present study reach of silt to very fine-sand beds, though there is indication of $n=4$ for sand-bed rivers (Crosato and Mosselman, 2009).

Although the present study aims at understanding the response of rivers to water diversions in general, it focuses on conditions typical for the Yellow River, and therefore the results are not representative for a gravel bed. For gravel-bed rivers, the discharge decrease due to water diversions may result in the stop of sediment transport because of (large) critical bed shear stress (or critical velocity) of gravel. The water diversion-induced sedimentation may only occur upstream of this transport threshold whereas there is no bed change in the downstream. A sharp bed transition may therefore develop around the transport threshold till sediment transport happens again by an increase of the local flow velocity. This pattern may repeat itself in the downstream direction.
6.5 Discussion

Figure 6.6: Relations of the bed slope and bankfull width in the LYR (B≤ 2000 m): the solid line denotes the power relation by $n = 5$ and the dashed line is the best-fit ($n = 5.1$).

6.5.2 Effects of barrages, dams and other structures

The effects of barrages, dams and other structures on the longitudinal bed profiles have not been considered in the present theoretical work for water diversions. A detailed investigation on their effects is difficult for the theoretical analysis but requires numerical simulations to solve.

In the study reach (lower reach of the Yellow River, downstream of the Huayuankou hydrological station), the river engineering works of bank protection and spur dikes have mainly regulated the channels in the planform (Wu et al., 2005). Upstream of the Huayuankou hydrological station, large dams have been constructed. Their influences on the longitudinal bed profiles may differ in different time scales (van Maren et al., 2011). In the short term, dams/barrages have more local effects. Reservoir sedimentation and river bed erosion will occur in the reaches immediately upstream and downstream of the dam respectively, which decrease the bed slopes. In the long term, decreasing bankfull discharge in the downstream reach of the dam leads to channel deposition, which may result in a bed slope increase when deposition rate decreases along the reach.
6.5.3 Temporally-constant channel width and discharge

Temporally-constant channel width and discharge are assumed in the present work. However, in practice, both channel depth and width may decrease in order to adapt to the reduction of discharge and sediment during water diversions. As the influence of water diversions accumulates with distance, a downstream decreasing width may occur, which in turn reduces the effect of diversion-induced depth decrease in the downstream. Moreover, a downstream decreasing width will reduce the possibility of a convex bed in water diversions so that a concave bed or more complex profiles (e.g., a longitudinal combination of concave and convex beds) may occur.

The simplification of a constant discharge does not account for seasonal discharge variations or water and sediment regulations by reservoirs, as practiced in the Yellow River (MWRPRC, 2000-2007; van Maren et al., 2011; Hu et al., 2012). However, when using the bankfull/dominant discharge, the resultant steady equilibrium state can qualitatively approximate the long-term dynamic equilibrium of the fluvial system. Experimenting with time-dependent discharges is part of future work.

6.6 Conclusions

The long-term impacts of water diversions on the longitudinal flow and bed profiles and the morphological time scales are investigated by a combination of 1-D theoretical framework and numerical simulations. The present theory has made a big improvement compared to previous studies by incorporating longitudinally varying channel widths and unifying the effects of continuous and discrete water diversions.

The theoretical work confirms previous findings that water diversions lead to a decrease of the equilibrium depth with respect to the natural conditions. It also shows that this shoaling effect becomes stronger with distance and is more pronounced for pure water diversions. While the equilibrium depth may longitudinally decrease, increase or be constant due to the interactions of water diversions and channel width variations. Besides a convex bed that has been predicted by previous studies for a constant-width channel, the present theory also predicts other bed profiles, given different diversion intensities and channel-width ratios. In a widening channel, a convex bed develops; in a narrowing channel, convex, concave or quasi-linear beds can develop while non-monotonic beds may also occur if channel narrowing is extremely strong.

For the concerned schematic LYR, numerical simulations show the morphological time scale from a natural equilibrium state to a new equilibrium state under water diversions is of the same order of magnitude (100~200 years) as that for developing the natural equilibrium state (without water diversions), which demonstrates the
significant effect of water diversions on the morphological time scale. An increase in water/sediment diversion intensity leads to a larger morphological time scale. When the diversion intensity is over 0.4, a convex bed will develop in pure water diversions while a concave bed still exists in water-sediment mixture diversions. In practice, the bed profile is probably in-between. On large spatial scales, the simple approach of (uniformly) continuous water diversions can qualitatively approximate the impacts of discrete water diversions when the latter lacks sufficient information.
Long-term effects of water diversions on river morphology
The Lower Yellow River (China) is characterized by two special and possibly unique hydrodynamic and sediment transport phenomena: a downstream increasing peak discharge during hyperconcentrated floods (typically with sediment concentrations > 200 kg/m$^3$), and a downstream decreasing runoff due to water diversions. The flood peak increase may be related to rapid morphological changes, to a modified bed roughness, or to bed erosion related an increasing flow volume. So far, there has been no consensus, however, on which mechanism contributes most to the phenomenon and the underlying physics are still largely unknown. As far as the water diversion effects are concerned, it has been shown that the bed evolves to a convex downward longitudinal equilibrium profile when assuming spatially continuous diversions along a constant-width channel. However, the validity of this result for discrete diversions in natural longitudinally width-varying channels remains to be justified. Moreover, an analysis of the equilibrium state does not reveal the impact of water diversions on the morphological time scale (MTS).

The present research aims to answer these questions and fill the principal knowledge gaps on morphological changes and hyperconcentrated floods in the Lower Yellow River. The mechanisms claimed to yield the peak discharge increase, i.e. bed roughness changes, bed erosion, and floodplain-related morphological changes, as well as their relative importance, have been investigated systematically, mainly through mathematical modeling with a newly-developed fully coupled morphodynamic model. Relevant data analysis on the Yellow River hyperconcentrated floods complements the study.

For the impact of water diversions, a general theoretical framework is proposed for predicting the equilibrium state of the fluvial system, which is applicable to both continuous and discrete water diversions in channels of varying width. Numerical
experiments are conducted to reveal the water diversion impact on the MTS. The conclusions of the study are summarized below.

### 7.2 Conclusions

**Fully coupled morphodynamic modeling**

Compared to the mathematical modeling of normal sediment-laden flows, the modeling of hyperconcentrated floods in the Yellow River is more complex, because: 1) the strong interactions of flow, sediment and river bed during hyperconcentrated floods require a coupled modeling approach that fully integrates the effects of sediment density and bed deformation on the flow; 2) the very fine sediments in the Yellow River need a non-capacity approach to account for the relatively large lag effects of sediment transport in spatial and temporal scales; 3) the large computational domain of the Yellow River requires an accurate and efficient numerical scheme.

In order to cover these issues, an accurate and efficient 2-D depth-averaged model has been developed solving the fully coupled non-capacity equations. This has been achieved by the successful extension of the second-order UFORCE method (developed by Stecca et al. (2010)) from the idealized frictionless fixed-bed case to the mobile-bed case. The combination of the upwind and centered methods in the UFORCE scheme yields high accuracy and computational efficiency, which is an improvement compared with previous fully coupled and non-capacity models. Moreover, a fully coupled solution in mathematics is also acquired by resolving the flow, the sediment concentration and the bed level at one time.

The model has been verified through a number of tests covering a wide range of complex flow and sediment transport conditions. It is demonstrated to be capable of simulating not only shock waves and reflection waves, but also rapid bed deformations under highly active sediment transport conditions. Nevertheless, uncertainty is inevitable because of the use of empirical parameters and relations in the model. For different applications, it is important to collect as much information as possible for the targeted configuration so that sensible parameter values can be given.

**Downstream peak discharge increase in the hyperconcentrated flood**

A data analysis of 13 historical hyperconcentrated floods suggests that bed erosion may not be the main cause of the downstream increase of the peak discharge during most hyperconcentrated floods in the Yellow River. Therefore, a more detailed numerical study, which aims to reveal the relative role of bed roughness change, bed
erosion and floodplain-related morphological changes on the peak discharge increase, has been carried out with the fully coupled morphological model mentioned above. First, two hyperconcentrated floods (a high-concentration (1977 flood) and a moderate-concentration (2004 flood) event), have been numerically investigated in schematized 1-D channels. The results show that a downstream increasing peak discharge can be obtained if the effect of bed roughness change is properly taken into account. At specific stations (i.e. Huayuankou and Tongguan) where measured data of the peak discharge increase are available, the model can satisfactorily reproduce this increase. With a constant bed roughness the peak discharge increase can only be reproduced in case of very strong and probably unrealistic bed erosion. In addition, the effects of bed roughness reduction and (very strong) bed erosion on the peak discharge evolution can be integrated by the effect of channel storage reduction. By adding an assumed floodplain to the previous 1-D channel (no overflows were observed in practice), the results of the 2004 flood confirm the previous roughness contribution to the peak discharge increase. Most importantly, it shows that the floodplain has more damping (rather than amplifying) effects on the flood waves and thus leads to smaller peak discharge compared to the previous 1-D channel case. This finding is consistent with the field observations that the downstream peak discharge increase is more frequently observed in the Lower Yellow River after 2000 when the flood is mostly maintained within the channel. Next, a detailed study of the floodplain effects is carried out for the 1992 flood in a schematized 2-D channel-floodplain reach, for which an increasing peak discharge was observed with floodplain inundation. With suitable erosion and deposition settings, the model can satisfactorily reproduce the typical phenomena of channel erosion and floodplain deposition during a hyperconcentrated flood, as reported in the literature. However, this morphological effect appears to be insufficient to explain the downstream increase of the peak discharge. This is probably because the morphological change does not cause sufficient flow acceleration for later parts of the flood wave to catch up with earlier parts. In addition, it also shows that with a decreasing erosion rate (a decrease of the coefficient M), both channel and floodplain can be eroded, while the channel erosion and floodplain deposition can be strengthened with an increasing erosion rate (an increase of M). The different floodplain widths primarily affect the floodplain sedimentation patterns and the floodplain types affect erosion especially in the channel-floodplain transitional areas. For the schematization of the floodplains, the present work only considers prismatic floodplains, whereas in reality they are far from prismatic. As a consequence, the water stored on the floodplain may be conveyed to a point further downstream and flow back into the main channel there. This may contribute to the peak discharge increase, but more research is needed on such complex configurations.
Conclusions and future work

Long-term impact of water diversions

The long-term impacts of water diversions on the morphological development in rivers are investigated with combination of a 1-D theoretical framework and numerical simulations. The effects of water diversions on the morphological equilibrium state are analyzed, as well as the morphological time scales for establishing this state.

The theoretical work confirms previous findings that water diversions lead to a decrease of the equilibrium depth with respect to the natural conditions. It also shows that this shoaling effect becomes stronger with distance along the river and is more pronounced for pure water diversions (i.e. without sediment). Besides a convex downward longitudinal bed profile, as predicted by previous studies for a constant-width channel, the present theory also predicts other bed profiles, given different diversion intensities and longitudinal variations of the channel width. In a widening channel, a convex downward bed develops; in a narrowing channel, convex downward, concave downward or quasi-linear beds can develop while non-monotonic beds may also occur if channel narrowing is very strong.

For a schematic Lower Yellow River, good agreement is obtained between the theoretical and numerical results for various water diversion conditions, further demonstrating the accuracy and the applicability of the developed theoretical formulations. Moreover, numerical simulations show the morphological time scale from a natural equilibrium state to a new equilibrium state under water diversions is of the same order of magnitude (100~200 years) as that for developing the natural equilibrium state (without water diversions), which demonstrates the significant effect of water diversions on the morphological time scale. An increase in water/sediment diversion intensity leads to a larger morphological time scale. The results also indicate that the simple approach of (uniformly) continuous water diversions can qualitatively approximate the impacts of discrete water diversions on large spatial scales.

7.3 Future work

Although the present work advances the understanding for the mechanisms of the peak discharge increase in hyperconcentrated floods and the long-term impact of water diversions on the flow and the bed morphology, it still contains limitations and needs to be improved in the future:

The modeling accuracy and efficiency depend on both the numerical method and the computational grid. For highly irregular bed topographies and complex boundaries, the use of rectangular grid in the present model may reduce the modeling accuracy and efficiency. Further development to incorporate a more flexible mesh-cell like
7.3 Future work

triangular cell (Siviglia et al., 2013) or adaptive quadtree grid (Liang et al., 2004) is part of future work.

For the present study on the mechanisms leading to the downstream peak discharge increase, the effect of bed roughness change is considered either by a simple conceptual relation valid for relatively low and moderate sediment concentrations or by an empirical formulation based on laboratory observations. An advance in the fundamental knowledge (supported by quantitative field observations) on bed roughness effects during hyperconcentrated floods is important for the development of a more physically and theoretically based bed roughness formulation and thus for the improvement of modeling such extreme conditions. Also, conditions in reality can be very complex, e.g. with highly non-prismatic floodplains which will not only affect the cross-sectional changes, but also the flow processes on the floodplain and the exchange of water with the main channel. Such complicated conditions require further research. Moreover, more detailed topographic data are necessary to improve the accuracy of model results. Additionally, 3-D effects on the flow structure and the concentration patterns may be strong when over-flow occurs in the channel-floodplain system. This cannot be fully reproduced in the present depth-averaged model. Efforts are needed to better parameterize these effects in depth-averaged models, or to further develop a depth-resolving module.

Although the present theoretical study aims at understanding the response of rivers to water diversions in general, it focuses on conditions typical of the Lower Yellow River, characterized by silt to very fine sand beds. Therefore, the results are not representative for rivers with a coarse sand or gravel bed. The discharge decrease due to water diversions may result in the stop of sediment transport in such rivers because the (large) critical bed shear stress is not exceeded. Diversion-induced sedimentation may only occur upstream of the location where this transport threshold is reached, whereas no bed changes occur further downstream. These effects should be carefully considered when extending the present theory to such rivers. Moreover, we assume the channel width and the discharge to be constant in time, whereas in reality both channel depth and width may decrease in order to adapt to the reduction of discharge and sediment load due to water diversions. As the influence of water diversions accumulates along the river, this decrease is more pronounced further downstream. This may reduce the possibility of a convex downward bed in reaches with water diversions, a concave downward bed or more complex profiles may occur. Taking these effects into account requires a more detailed modeling approach that can resolve channel width changes in time (e.g. 2D/3D models). Experimenting with time-dependent discharges is also part of future work on the dynamic equilibrium state of fluvial systems in response to water diversions.
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## Notations

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<td>A</td>
<td>Jacobian matrix.</td>
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<tr>
<td>F,G</td>
<td>vectors of flux variables.</td>
</tr>
<tr>
<td>K₁, K₂</td>
<td>vectors in Runge-Kutta method.</td>
</tr>
<tr>
<td>R,R</td>
<td>vector of total source terms.</td>
</tr>
<tr>
<td>S₀, Sᵢ</td>
<td>vectors of source terms.</td>
</tr>
<tr>
<td>U</td>
<td>vector of conservative variables.</td>
</tr>
<tr>
<td>W</td>
<td>vector of non-conservative variables.</td>
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<tr>
<td>B</td>
<td>river width.</td>
</tr>
<tr>
<td>B'</td>
<td>downstream gradient of river width.</td>
</tr>
<tr>
<td>bᵣ</td>
<td>inverse of relative channel width.</td>
</tr>
<tr>
<td>C</td>
<td>Chézy coefficient.</td>
</tr>
<tr>
<td>c, cₑ</td>
<td>depth-averaged volumetric sediment concentration and capacity, respectively.</td>
</tr>
<tr>
<td>cₘₕₓ</td>
<td>maximum sediment concentration.</td>
</tr>
<tr>
<td>Cₛ</td>
<td>Smagorinsky constant.</td>
</tr>
<tr>
<td>Cₑ</td>
<td>contraction coefficient.</td>
</tr>
<tr>
<td>Cr</td>
<td>Courant number.</td>
</tr>
<tr>
<td>cₙ</td>
<td>vortex coefficient.</td>
</tr>
<tr>
<td>dₜₕ₀</td>
<td>median diameter of suspended load.</td>
</tr>
<tr>
<td>Dₜₕ₀</td>
<td>median diameter of bed material.</td>
</tr>
<tr>
<td>d</td>
<td>sediment diameter.</td>
</tr>
<tr>
<td>E,D</td>
<td>sediment entrainment and deposition fluxes.</td>
</tr>
<tr>
<td>f</td>
<td>function in characteristic method.</td>
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Notations

\( f_q \)  continuous water diversion rate.
\( f_s \)  continuous sediment diversion rate.
\( f_{eq} \)  water loss due to continuous water diversions.
\( f_{es} \)  sediment loss due to continuous water diversions.
\( f_{dq} \)  water loss due to discrete water diversions.
\( f_{ds} \)  sediment loss due to discrete water diversions.
\( F_r \)  Froude number.
\( F_{Qx}, F_{sx} \)  source/sink (+:−) per unit length.
\( g \)  acceleration of gravity.
\( H \)  headpond water depth.
\( h, h_0 \)  water depth.
\( h' \)  downstream gradient of water depth.
\( h_w \)  gate opening height.
\( i, j \)  cell indices in the coordinate of \( x \) and \( y \) directions.
\( j \)  or index of discrete water diversion in Chapter 6.
\( J \)  water surface slope.
\( K \)  empirical parameter in Eq. (4.10).
\( k_1, k_2 \)  variables in characteristic method.
\( L_{e}, L_{q} \)  error norms.
\( l \)  channel length in Chapter 6.
\( l_s \)  characteristic horizontal turbulent length scale.
\( M \)  erosion rate in Eq. (5.1).
\( m \)  empirical parameter in Eq. (4.10).
\( N \)  number of computational cells.
\( n \)  Manning roughness coefficient, or power exponent in sediment transport formula in Chapter 6.
\( p \)  bed porosity.
\( P_e \)  Peclet number.
\( p_{1j} \)  dimensionless discrete water diversion rate.
\( p_{2j} \)  dimensionless discrete sediment diversion rate.
\( Q \)  flow discharge.
\( q \)  specific flow discharge.
\( q_b \)  bed load transport rate under capacity state.
\( Q_p \)  measured peak discharge.
Notations

\( Q \) sediment transport rate.
\( q \) specific sediment transport rate.
\( r_B \) channel width ratio.
\( r_{wd} \) water diversion intensity.
\( S \) sediment concentration in mass.
\( s \) submerged specific gravity of sediment.
\( S_p \) measured maximal sediment concentration in mass.
\( S_{0x}, S_{0y} \) bed slopes in \( x \) and \( y \) directions.
\( S_{fx}, S_{fy} \) friction slopes in \( x \) and \( y \) directions.
\( S_f \) horizontal mean strain-rate tensor.
\( SIG \) function defined in Eq. (2.31).
\( t \) time.
\( T_n \) morphological time scale for natural development.
\( T_d \) morphological time scale for water diversions.
\( u \) friction velocity.
\( u, v \) depth-averaged flow velocities in \( x \) and \( y \) directions.
\( u_g, v_g \) velocities in Eq. (3.2).
\( V, V_s \) flow and sediment volumes, respectively.
\( V_{str}, V_{str} \) (relative) channel storage.
\( w \) variable in characteristic method.
\( x, y \) horizontal coordinates.
\( z \) bed elevation.
\( \tau \) bed shear stress.
\( \tau_{cr,e}, \tau_{cr,d} \) critical bed shear stresses for erosion and deposition, respectively.
\( \tau_{xx}, \tau_{yy}, \tau_{xy}, \tau_{yx} \) depth-averaged Reynolds stresses.
\( \tau_{x(y)} \) resistance stress in \( x (y) \) directions.
\( \xi \) parameter in Eq. (3.2).
\( \zeta \) water level.
\( v_t \) turbulent eddy viscosity.
\( \Phi_s \) sediment diffusion term.
\( \varepsilon_s \) turbulent diffusion coefficient of sediment.
\( \rho_s, \rho_w \) densities of sediment and water respectively.
\( \rho \) density of sediment-laden flow.
\( \rho_0 \)  
density of saturated bed.

\( \varphi \)  
adjustment parameter in Eq. (2.13a), or relative ratio of discharge increase in Chapter 4.

\( \theta, \theta_c \)  
Shields parameter.

\( \alpha \)  
empirical coefficient.

\( \alpha_u \)  
coefficient related to depth-averaging flow velocity.

\( \beta \)  
power exponent in Eq. (4.13).

\( \beta_s \)  
upwind bias parameter in UFORCE scheme.

\( \kappa \)  
von Karman constant.

\( \lambda \)  
characteristic speed, or discharge vanishing length in Chapter 6.

\( \omega \)  
sediment settling velocity.

\( \phi \)  
newly constructed conservative variable in Eq. (2.15).

\( \delta_i \)  
friction thickness.

\( \Delta c \)  
volumetric concentration increase.

\( \Delta h \)  
deepth reduction in Chapter 6.

\( \Delta h \)  
downstream variation of depth reduction in Chapter 6.

\( h \)  
downstream variation of water depth in Chapter 6.

\( \Delta Q_p \)  
measured peak discharge increase.

\( \Delta S \)  
concentration increase in mass.

\( \Delta x, \Delta y \)  
spatial steps in \( x \)- and \( y \)-directions.

\( \Delta t \)  
time step.

\( \Delta z \)  
bed level change.

\( \Delta i \)  
vector of limited slope.
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<th>Description</th>
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<tr>
<td>MTS</td>
<td>Morphological Time Scale.</td>
</tr>
<tr>
<td>CFL</td>
<td>Courant-Friendrichs-Lewy condition.</td>
</tr>
<tr>
<td>ENO</td>
<td>Essentially Non Oscillatory.</td>
</tr>
<tr>
<td>FVM</td>
<td>Finite Volume Method.</td>
</tr>
<tr>
<td>TVD</td>
<td>Total Variation Diminishing.</td>
</tr>
<tr>
<td>DGM</td>
<td>Depth Gradient Method.</td>
</tr>
<tr>
<td>SGM</td>
<td>Surface Gradient Method.</td>
</tr>
<tr>
<td>HLL</td>
<td>Harten-Lax-van Leer scheme.</td>
</tr>
<tr>
<td>HLLC</td>
<td>Harten-Lax-van Leer-Contact scheme.</td>
</tr>
<tr>
<td>FLIC</td>
<td>Flux Limiter Centered scheme.</td>
</tr>
<tr>
<td>SLIC</td>
<td>Slope Limiter Centered scheme.</td>
</tr>
<tr>
<td>FORCE</td>
<td>First Order Centered scheme.</td>
</tr>
<tr>
<td>UFORCE</td>
<td>Upwind-biased First Order Centered scheme.</td>
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<td>MUSCL</td>
<td>Monotonic Upstream-Centered Scheme for Conservation Laws.</td>
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Acknowledgements

It has been a long journey to reach the end of the PhD research. At this moment, I would like to express my sincere gratitude to all the people who have made direct or indirect contributions to this doctoral dissertation.

First and foremost, I would like to thank my promoters, Prof. H. J. de Vriend and Prof. Z. B. Wang, for their expert guidance and continuous support through the whole research.

It is a great honor to be a PhD student of Prof. H. J. de Vriend, who provided me with ample freedom and encouragement throughout my research process. I appreciate him for his professional suggestions and the help on improving my written English. The most important thing that I have learnt from him is a rigorous attitude in scientific research.

I am also grateful for Prof. Z. B. Wang, who always inspires me by sparkling ideas and constructive suggestions. I acknowledge him for his excellent guidance on mathematical analysis, modeling and sediment transport issues, and for his patience during the difficult times of my research work. Also, I want to thank him for finding financial support to my extension period.

Next, with immense gratitude, I want to thank my supervisor Dr. D. S. van Maren. I have learnt a lot from him, not only for the specific knowledge of hyperconcentrated flow and fine sediments, but also for academic writing. I appreciate him for sharing his data and useful documents with me, and for his significant contributions to the publications that are related to my PhD work.

I also appreciate Prof. Baosheng Wu, who was my supervisor in Tsinghua University. I acknowledge him for supporting my PhD study at TU Delft. As an expert on the Yellow River, he gave constructive suggestions on my research and tried his best to help me collecting the necessary data. Prof. Jianjun Zhou from Tsinghua University is also acknowledged for his considerable help in my PhD application to TU Delft. Prof. Zhixian Cao and Prof. Zhilin Wei from Wuhan University are acknowledged as well for their insightful discussions and suggestions on the Yellow River research.
Additionally, Prof. Weiming Wu from Clarkson University is acknowledged for his guidance on the dam-break modelling.

My thanks should also be addressed to Prof. M. J. F. Stive, who is one of the kindest men I have ever met. He showed a keen interest in my research progress and my life in Delft. I am also grateful for his help in acquiring funding from the “Het Lamminga fonds” for the last half year of my research. My officemate, Dr. Howard Southgate, is acknowledged for his help with English corrections of the introduction chapter of my thesis. Mariette van Tilburg is acknowledged for her help with Dutch translation of the propositions and summary of my thesis.

Besides, my thanks also go to my colleagues and friends at TU Delft with a short and incomplete list: Min Su, Peng Yao, Hong Yan, Gensheng Zhao, Fan Li, Dongju Wu, Ao Chu, Leicheng Guo, Zhan Hu, Xuefei Mei, Hua Zhong, Huayang Cai, Yang Zhou, Lu Wang, Qian Ke, Mohamed Nabi, Mark Voorendt, Kees Sloff, Jaap, Constantine Memos, Xiangxiang Dang, Chen Li, Jie Lu.

I am deeply indebted to my parents for their love and unconditional support to my study; and to my boyfriend Peng Hu, with whom I share good times and bad, though we have often been apart during the last years.

Finally, in regard to the financial support on my PhD research, I wish to thank the sponsorships from “the China Scholar Council (CSC)”, “Het Lamminga Fonds”, and the Sino-Dutch collaboration project “Effects of human activities on the eco-morphological evolution of rivers and estuaries” (08-PSA-E-01) funded by the Royal Dutch Academy of Sciences (KNAW) and the Chinese Ministry of Science and Technology (MOST) within the framework of the Programme of Scientific Alliances between China and the Netherlands.

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February, 2014

Delft, the Netherlands
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Peer-reviewed international journal articles:


4. **Li, W., Wang, Z. B., van Maren, D. S., de Vriend, H. J., and B. S. Wu** (2014), Relative role of bed roughness change and bed erosion on peak discharge increase
in hyperconcentrated floods, *Advances in Geosciences*, EGU (Special issue: the 8th Symposium on river, coastal, and estuarine morphodynamics). [accepted]

**Peer-reviewed Chinese journal articles:**


**Articles in conference proceedings:**

7. **Li, W.,** Wang, Z. B., van Maren, D. S., de Vriend, H. J., and B. S. Wu (2013), Relative role of bed roughness, bed erosion and channel storage on peak discharge increase in hyperconcentrated floods, In: the 8th Symposium on river, coastal, and estuarine morphodynamics, Santander, Spain. [abstract and oral presentation]


9. **Li, W.,** Wang, Z. B., de Vriend, H. J., and D. S. van Maren (2011), Investigation to the mechanisms of the downstream increasing peak discharge in hyperconcentrated floods, In: the 7th Symposium on river, coastal, and estuarine morphodynamics, Beijing, China. [oral presentation]
