How proper similitude principles could have improved our understanding about fatigue damage growth

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Abstract: This paper discusses similitude parameters for predicting fatigue damage growth, and demonstrates that proper principles exploiting the strain energy release could have improved our knowledge on fatigue damage growth. The paper demonstrates how the original Stress Intensity Factor concept introduced by Irwin has been introduced inconsistently for describing fatigue crack growth. Subsequently, it illustrates how this has led to misinterpretation of certain phenomena, like for example plasticity induced crack closure.

Using proper similitude in agreement with the physics of observed fatigue damage phenomena, it is demonstrated how the average strain energy release over a single load cycle should relate to the crack surface extension in that same load cycle. The paper concludes with illustrating how phenomena like for example plasticity and fibre bridging in composites can be understood and quantified.

INTRODUCTION

To describe fatigue in metallic structures and materials, different approaches are adopted. Fatigue life, or fatigue initiation life, is generally described based on stress amplitudes and stress concentration factors, whereas for crack propagation, methodologies adopting Linear Elastic Fracture Mechanics (LEFM) have been developed. Reviewing the literature illustrates that most methodologies have been developed in a rather empirical fashion. A parameter was taken to describe similitude and using this parameter, observed data was correlated and formulated as material characteristic. For stress based methods, these material characteristics are generally provided as S-N curves, while for the LEFM methods, these characteristics may be presented in the form of Paris-type relationships [1].

Despite the differences between these two categories, they both seem to have in common that various additional corrections or factors are required. Some corrections are attributed to geometry or finite width, others to the mean stress or stress ratio, and some corrections are specifically attributed to phenomena like crack closure or plasticity [2,3]. This paper discusses the meaning of the necessity of additional corrections in light of the similitude taken in these methodologies. The hypothesis here is that some corrections have been introduced with an explanation that considering the similitude principle adopted, is incorrect.

To illustrate these discrepancies, the present paper first discusses the origin of in particular the methodology using Stress Intensity Factors (SIF). The objective is to demonstrate how the original SIF concept introduced by Irwin [4] has been introduced inconsistently for describing fatigue crack growth. Then, an alternative methodology is proposed that utilizes strain energy release over individual load cycles as similitude parameter to describe fatigue damage growth.

PROBLEM STATEMENT

The correction factor that has received most attention within the literature on fatigue crack growth is the stress ratio correction. This correction has been explained by numerous authors to be caused by plasticity induced crack closure. In an earlier paper [5], the author has demonstrated that this correction to great extent can be described by considering cyclic energy rather than cyclic stress. A more thorough discussion on the nature of the stress ratio correction is given in [6], where the following observations are reported:

- Stress ratio effect is present in any material system, even brittle composites, not only ductile alloys
- Plasticity induced crack closure correction is similar to common mean stress corrections for the fatigue limit in S-N fatigue analyses
- Paris curves for different stress ratios seem to collapse when dividing the SIF by the Young's modulus
- Stress ratio dependency on threshold SIF vanishes when recalculated to threshold Strain Energy Release Rates (SERRs).

A straightforward demonstration that one can make is the second observation reported in [6]. Figure 1 (a) presents the correction proposed by Schijve for aluminium 2024-T3 [7,8]. This correction is generally plotted in terms of S_{op}/S_{max} versus stress ratio R. With the mechanical properties for this aluminium known, the correction factor can be recalculated to an amplitude stress S_a versus mean stress S_m, assuming that the effective stress cycle is between S_{op} and S_{max}. The

result of this recalculation is presented in Figure 1 (b) where the curve is compared with the Gerber mean stress correction for the fatigue limit.

The correlation between the crack closure correction and the mean stress correction is striking. In particular when one considers that one correction aims to correct for plasticity induced closure in the wake of macroscopically long cracks, while the other correction corrects for the fatigue limit, i.e. where the contribution of crack growth to the life is nil [8].
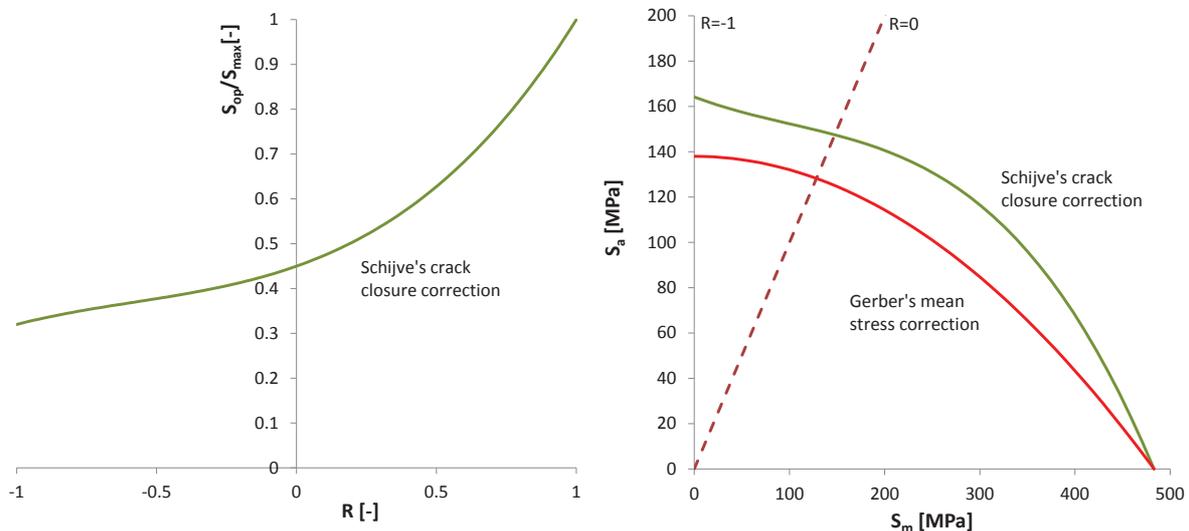
Figure 1. Correlation between Schijve's crack closure correction for aluminium 2024-T3 (a) and the mean stress correction for the fatigue limit for that alloy (b).

The problem here is that an apparent effect has been observed: the Paris relationship between *da/dN* and *ΔK* depends on the stress ratio *R*. But the physics of that effect is not well understood. Hence, hypotheses have been proposed that could have been falsified if data had been considered beyond the limited scope of fatigue crack growth in ductile alloys.

To fully understand the mechanism of fatigue crack propagation in metallic alloys, one has to understand the physics of fatigue damage growth in engineering materials in general. Because all mechanisms in these materials are subject to the first and second law of thermodynamics, this implies understanding the nature of energy dissipation in the process of fatigue. Any intrinsic or extrinsic shielding mechanism, like plasticity, crack closure, fibre bridging (in fatigue delamination), etc. should then be described with similar energy balance considerations. However, this requires the establishment of proper principles of similitude that reflect both the physics and mechanics of fatigue damage growth. It seems that despite the claims in the literature, such principles have not yet been well established.

STRESS INTENSITY FACTOR FOR CRACK GROWTH

In 1921, Griffith [9] proposed considering strain energy, rather than using stress based methods, as was common in those days. In his hypothesis, the prediction of crack growth is based on the balance of energy required to break atomic bonds, and the energy that is available. This concept was further developed by Irwin [4] who quantified the available energy, defined as the SERR. For fixed grip conditions in which a crack increment does not impose application of extra work, the release in strain energy as result of the crack increment can be compared with the energy required to further increment the crack. Hence, the SERR could be used as a quasi-static failure (or stability) criterion, in which the SERR constitutes a 'driving force' for crack growth.

Furthermore, Irwin related the SERR to the SIF describing the singular stress field at a crack tip. This relationship is generally accepted to be valid, in particular for the quasi-static fixed grip conditions that Irwin studied. The release of elastic strain energy as result of an instantaneous infinitesimal increment of crack growth is thus considered to be equivalent to the elastic stress field at the crack tip for the applied load prior to that crack increment.

This relationship was used by Paris [1] who demonstrated that the crack growth rate, *da/dN*, plotted against the SIF range, *ΔK = K_{max} – K_{min}*, could be described by a power law

$$\frac{da}{dN} = C\Delta K^n \tag{1}$$

In this equation, *C* and *n* are curve-fitting parameters. The argument of Paris for using *K* was that the crack tip stress field controls the rate of crack extension, and that in *K* the effects of external load and configuration were reflected. At present, this 'Paris relationship' is the basis for nearly all crack growth prediction models. Later, modifications to equation (1) have been proposed to include a critical K_c to capture the asymptotic behaviour at high ΔK [10], and/or a threshold ΔK to capture the lower asymptote [11,12,13].

STRAIN ENERGY VERSUS STRESS INTENSITY

For the growth of disbonds in adhesive joints and for delaminations in composites, the 'Paris relationship' is also commonly used. Because these cracks are often at an interface between two dissimilar materials, the SERR is easier to compute than the SIF. Hence, rather than ΔK, G_{max} or ΔG are used in equation (1), but as parameter to represent the stress state at the crack tip, rather than to represent strain energy release as proposed by Griffith.

Within LEFM this relationship between *K* and *G* is taken generally for plane stress conditions as

$$G = \frac{K^2}{E} \tag{2}$$

It seems that most people who apply LEFM to fatigue crack propagation consider *G* equivalent to *K* in that it is a driving force, rather than the consequence of crack growth. However, equation (2) was derived for fixed-grip conditions where an instantaneous and infinitesimal crack increment releases strain energy that can be related to the stress field in the crack tip vicinity. That does not necessarily imply that the release in strain energy by crack extension during a single fatigue cycle relates to ΔK with equation (2)!

This misunderstanding is illustrated by Paris [14] when he refers to earlier observations by Anderson, and Harris and himself [15], who reported that measured fatigue crack growth rates (in inert environments) were identical for various base materials and their alloys, if *K* was normalized with the elastic modulus *E*. The implication of this observation, i.e. fatigue crack growth is governed by energy rather than stress, was not considered.

SHORTCOMING OF SIF RANGE ΔK

In the quasi-static analysis of Irwin, the correlation between the SIF and SERR according to equation (2) is valid. However, due to the cyclic nature of the fatigue crack extension process, this relation requires some elaboration. Either the process is governed by stress, and then $\Delta S = S_{max} - S_{min}$ suffices, or the process is governed by energy and then, to describe the crack driving force one should use

$$\Delta U = \frac{P_{max}^2}{2E} - \frac{P_{min}^2}{2E} = \frac{V}{2E}\left(S_{max}^2 - S_{min}^2\right) \tag{3}$$

In this equation *V* is the volume of the sample or system. The correlation between strain energy and load (and thus stress) is illustrated in Figure 2 (a). In this illustration it is evident that in case the mean stress increases, substantially more work is applied to the specimen, which is not considered with ΔS. Hence, a stress ratio effect is to be expected [5].

Another aspect that is apparent from Figure 2 (a) is that the minimum stress represents part of the total applied work to the specimen following $U_{tot} = U_{mon} + U_{cyc}$. Hence, deducting the minimum stress (or minimum strain energy) from the maximum stress (or total strain energy) may be deemed inappropriate as it constitutes a contribution to the crack driving force. In particular when high stress ratios are considered, this contribution may be significant.
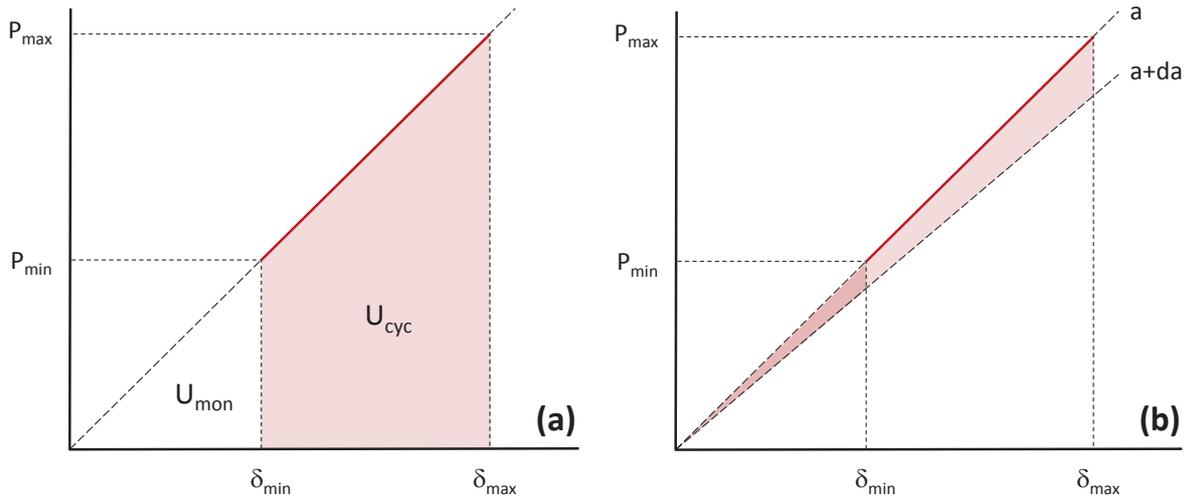
Figure 2. Illustration of linear elastic strain energy applied when loading from P_{min} to P_{max} (a) and the strain energy release due to crack extension *da* (b).

Another reason for considering the strain energy below the minimum load P_{min} is that in case of a crack increment *da*, strain energy is released even below P_{min}, see Figure 2 (b). In that respect it indeed seems more appropriate to consider G_{max} to describe fatigue delamination in composites, rather than *ΔG* [16]. However, when taking data from the literature on aluminium 7075-T6 [17] it appears that when plotting *da/dN* against *ΔG* rather than G_{max} a good correlation is obtained. In particular for higher crack growth rates, there is substantially less difference between the curves compared to the conventional Paris relationship. This is illustrated in Figure 3.
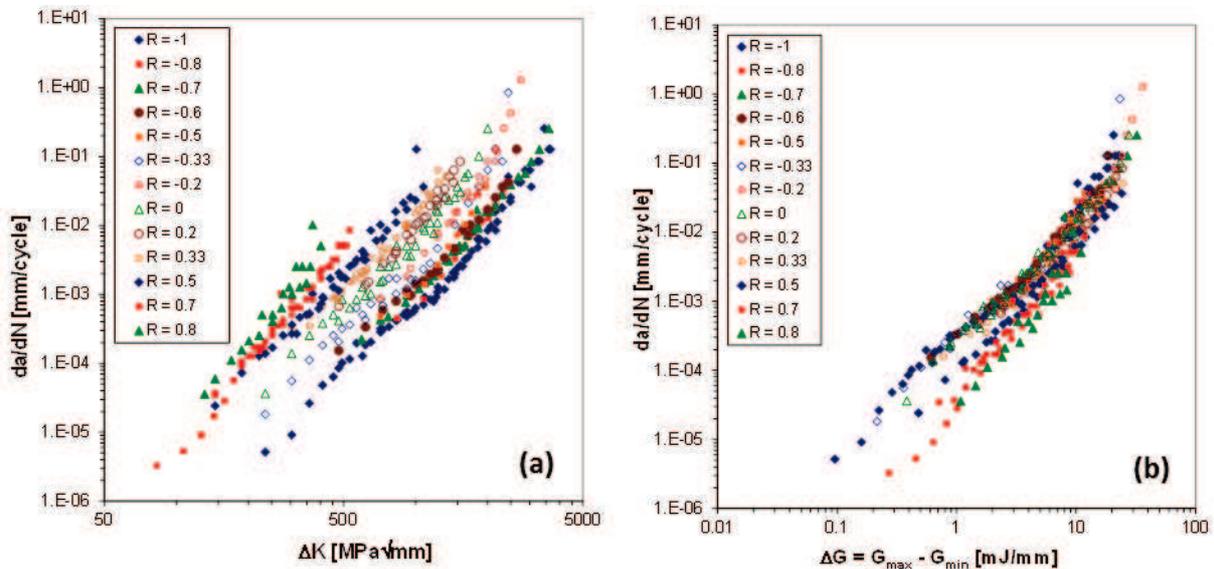

Figure 3. *da/dN* versus *ΔK* for different stress ratio for aluminium 7075-T6 (a), the same data plotted as *da/dN* versus *ΔG* (b); data from [17].

Evidence as provided with Figure 3, and observations reported in [6] indicate that the SIF range *ΔK* does not represent a proper parameter to describe similitude in fatigue. The arithmetic difference between K_{max} and K_{min} does not reflect the cyclic nature of the energy dissipation process that takes place during fatigue crack propagation. Hence, a better concept is needed to describe fatigue damage growth in line with the energy balance originally proposed by Griffith, but based on energy physically released rather than artificial SERR values.

THEORY VERSUS PREDICTION MODEL

In the following discussion, one has to acknowledge the fundamental difference between a theory describing the physics of fatigue, and a prediction model. A scientific theory constitutes the body of knowledge on how processes are understood and described consistent with certain laws. A prediction model is an engineering tool that basically relates input parameters that an engineer can work with to the consequence, like failure, or in this particular case: crack growth.

This distinction is important. When characterizing materials for their response to cyclic loading, one must be careful that the format of representation comprises material behaviour only. In particular this aspect is generally neglected when interpreting the Paris relationship. In that relationship, material response (da/dN) is plotted against a model (ΔK). Hence, any trend observed in that representation could either be attributed to some physical mechanism, or to the adopted model. With the example of the stress ratio effect, generally plasticity induced crack closure was considered as a physical mechanism to explain the trend, where it would have been more appropriate to reconsider the use of ΔK for describing similitude.

PHYSICS OF STRAIN ENERGY RELEASE IN FATIGUE

If one considers the concept of strain energy balance to the problem of fatigue crack growth, several distinct differences with the quasi-static approach of Griffith and Irwin can be identified. First, the crack does not increment instantaneously under a quasi-static load, but it progresses incrementally with every load cycle. In fact, with the increase in load during the load cycle, the crack gradually develops, and is generally considered to have reached its maximum extension at the maximum load of the cycle.

This implies that strain energy release dU should be related to the crack increment da generated in a particular cycle N. Thus dU/dN should physically correlate to da/dN in the form

$$\frac{dU}{dN} = \frac{dU}{da}\frac{da}{dN}$$

(4)

where dU/da is the effective or average strain energy release rate G_{eff} per unit width for that particular load cycle. If crack growth is the only strain energy dissipating mechanism present, then one would expect this G_{eff} to be constant and independent of either a or da/dN.

The second observation relates to the applied work W. Instead of linear-elastically increasing the load to a maximum value and subsequent instantaneous load reduction induced by crack extension, OAB in Figure 4, the force displacement curve gradually develops from O to B, because the crack gradually develops.
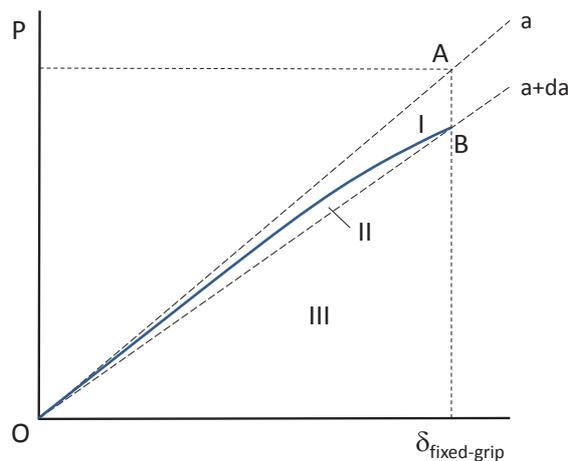

Figure 4. Illustration of energy partitioning of a load-displacement curve.

In general the SERR for this fixed grip condition is taken equal to the area I+II, while the applied work to the specimen W is characterised by the area I+II+III. However, during fatigue crack growth, the effective SERR is equal to area II, while the effective work applied is equal to area II+III. Area I represents strain energy that never has been applied to,

nor released from the specimen. Note that this effectively changes the ratio of energy dissipated (dU) over the work applied (W).

Another observation is that SERR in equation (3) includes all dissipating mechanisms that may occur during that load cycle. In particular plasticity has been observed to provide a major contribution to the strain energy dissipation [18]. In fact, strain energy may be released by plasticity even before a crack may increment. This implies that equation (4) is part of

$$\frac{dU}{dN} = \frac{dU_a}{dN} + \frac{dU_{pl}}{dN}$$

(5)

where dU_{pl}/dN is the energy released by plasticity in a particular cycle, and dU_a/dN the energy released by crack extension. Note that dU_{pl}/dN may be substantially greater than dU_a/dN [18]. In addition, dU_a/dN mostly relates to the decohesion of material and formation of crack surfaces. Thus only an increase in effective crack surface per projected crack area, equivalent to for example an increase in roughness, may alter the magnitude of dU_a/dN. If the roughness would not change, the value for dU_a/dN would remain constant, independent of the crack length. This is not the case for dU_{pl}/dN, which seems to be linearly related to the crack length in case of force controlled tests [6]. Hence, the ratio of energy dissipated by crack extension to the energy dissipated by plasticity is not constant along the crack length, which was the implicit assumption in the application of LEFM to fatigue crack growth.

INTERPRETING STRAIN ENERGY RELEASE RATES IN FATIGUE

To fully understand the concept of strain energy release rates for the problem of fatigue damage growth, and to properly interpret the experimental data, one has to first look at what equation (4) represents. This equation can be illustrated with Figure 5 (a). In that illustration, the slope of the curve represents the magnitude of the effective SERR. It implies that indifferent of the rate with which the crack extends under the applied loading, the amount of energy dissipated by per unit of crack extension is the same. Obviously, this could only be true if the physical fracture surface at small da/dN and at large da/dN would be identical, which is not the case as numerous papers in the literature report.
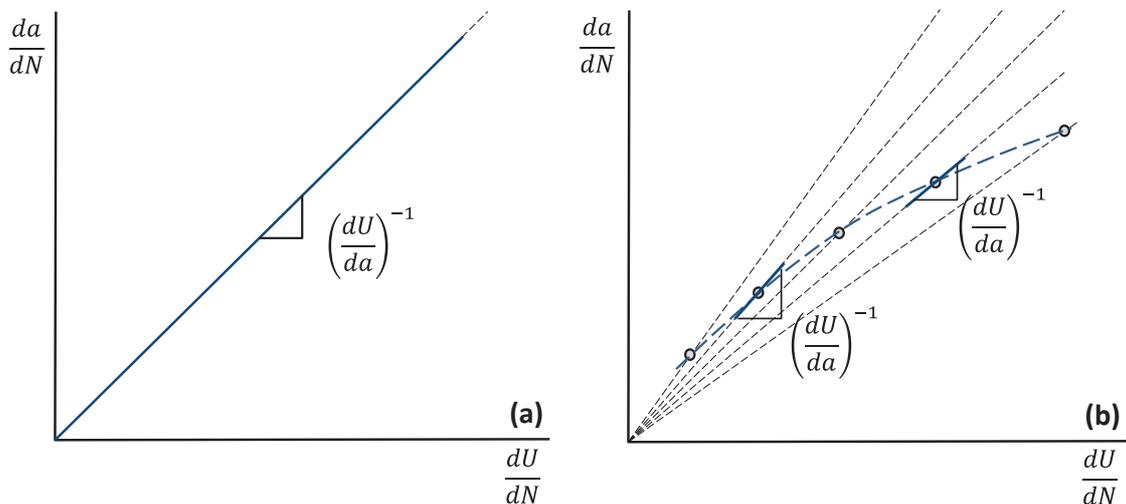

Figure 5. Theoretical constant SERR (a) and the definition of SERR in case the effective fracture surface increases with da/dN (b); note that both graphs contain linear scales.

Thus the fracture surface develops with crack extension, for example, by an increase in roughness or a transition from a mode I crack to a mixed mode crack by shear lip formation. The crack length a generally is a planar projection of the fracture surface. This increase in roughness in that case should be interpreted as an increase in effective fracture surface per crack area da (unit width). An increase in fracture surface constitutes an increase in decohesion of atomic bonds, and an increase in strain energy released. This is illustrated in Figure 5 (b), where at higher crack growth rates effectively more strain energy is released. Hence, dU/da is not constant and dependent on the crack growth rate da/dN.

But one has to be careful on the definition of the slope of the curve. Figure 5 (b) illustrates with the blue dashed line the curve that one may obtain with a single crack growth tests. With an increase in da/dN relatively more strain energy is

released per cycle. But the slope representing the SERR is not the tangent of the dotted line, but as illustrated in Figure 5 (b), it is the slope between the origin and the point on the curve. A decrease in that slope, is equivalent to an increase in SERR.

In the illustrated cases in Figure 5, the assumption is that all strain energy released is related to crack propagation and not to other dissipating mechanisms. With equation (5) it is stated that the strain energy that is released in an experiment comprises more energy dissipating mechanisms. A major contribution to energy dissipation for example is provided by plasticity. In fact, as mentioned before, strain energy may dissipate by plasticity, prior to any crack increment. The consequence of this is illustrated in Figure 6 where $dU/dN \neq 0$ at $da/dN = 0$, which constitutes a physical threshold.
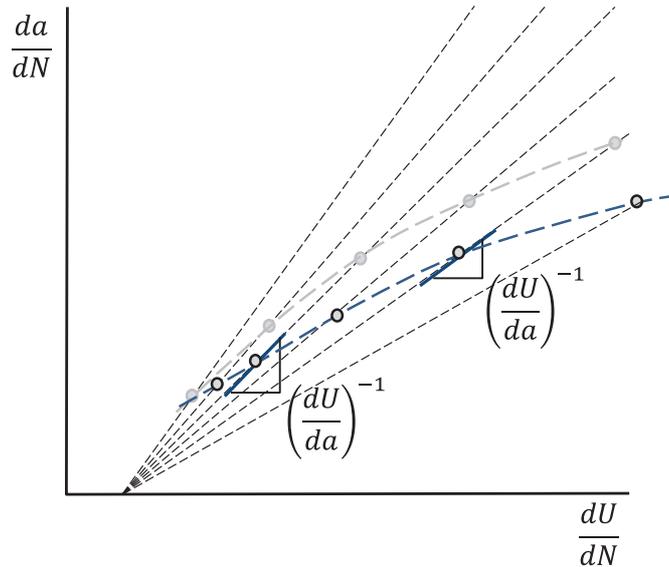

Figure 6. Definition of SERR in presence of plasticity: $dU/dN \neq 0$ at $da/dN = 0$ (a) and further shift in the curve with increasing plasticity at higher da/dN (b); note that both graphs contain linear scales.

In [6] is is explained that the energy dissipation due to plasticity increases approximately with crack length, whereas the energy dissipation due to crack formation remains approximately constant. Hence, the dissipation dU_{pl}/dN is greater for the case more plastic deformation occurs, than for the case little or no plastic deformation occurs. Thus, the amount of plastic deformation for small cracks is substantially less than the plastic deformation at large cracks. This explains the observation reported by various authors that for small cracks very small or even zero thresholds have been observed [20,21].

SHIELDING MECHANISMS

Various shielding mechanisms have been reported that improve the crack growth resistance. These mechanisms may be classified as intrinsic and extrinsic shielding mechanisms [19]. A very distinct extrinsic shielding mechanism in interlaminar delamination growth in composites is fibre bridging. The above described procedure applied to the problem of fatigue delamination has revealed that fibre bridging as shielding mechanism does not contribute to the position of the *da/dN* versus *dU/dN* curve. The elastic strain energy that these fibres carry is released upon unloading and is therefore not dissipated. Only when these fibres are pulled out of the matrix, or when they fail, is strain energy permanently dissipated [22].

Therefore, the hypothesis currently is that intrinsic (shielding) mechanisms directly influence the *da/dN* versus *dU/dN* curves, whereas extrinsic shielding mechanisms will only do that once they fail in the course of fatigue damage propagation. An example of intrinsic mechanisms [19] is for example plasticity in ductile alloys. Strain energy dissipates in each load cycle due to additional plastic deformation, in addition to dissipation due to the crack incrementing. However, one has to keep in mind that the intrinsic mechanism does not necessarily relate to crack propagation linearly, as illustrated by the difference between small cracks and large crack. Theoretically, both a small crack and large crack could experience the same SIF, but due to the different levels of plasticity yield different values for *dU/da*.

RELATION BETWEEN QUASI-STATIC AND FATIGUE CRACK GROWTH

With the introduction of the Paris relationship, the observed asymptotic behaviour of the relationship at high ΔK has been related to a quasi-static SIF K_c. Here, the idea is that once the maximum SIF is equal to the monotonic value of K_c, failure occurs in a single cycle. With modifications to equation (1), often forms have been proposed that normalise by this quasi-static K_c [10]. Similarly, researchers proposed to normalise G_{max} or ΔG by a critical G_c to capture the asymptotic behaviour [23].

The appropriate method to relate quasi-static SERRs with fatigue SERRs is demonstrated by Amaral [24] for delamination growth in composite laminates. The concept is illustrated in Figure 7 (a). Amaral shows how quasi-static crack growth data can be discretised and analysed as if it constitutes low cycle fatigue data. Depending on the level of discretization, multiple points can be derived from the same data set, that linearly line up as indicated with the solid line in Figure 7 (b).
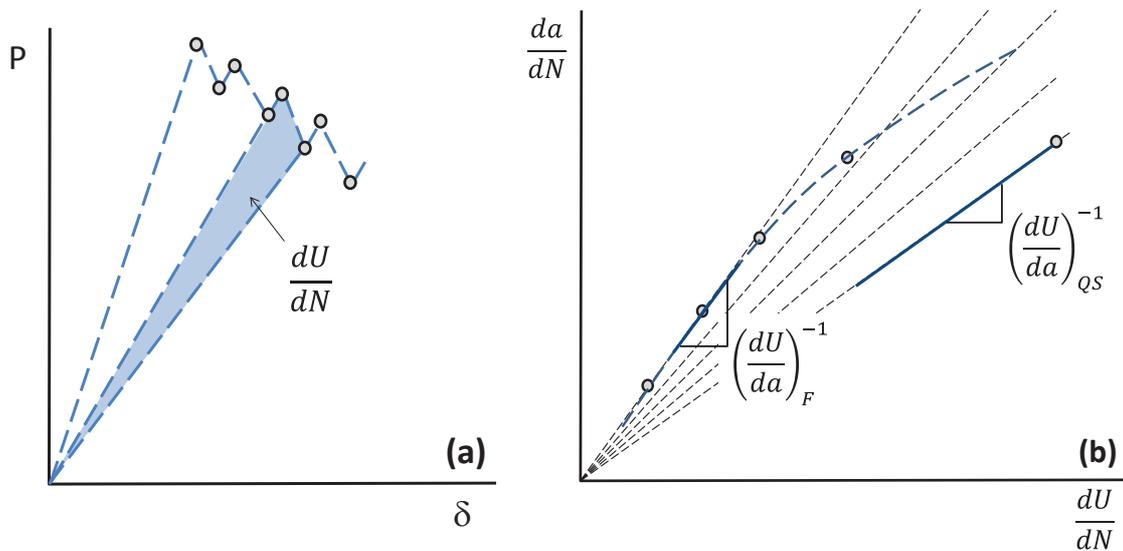

Figure 7. Discretization of the quasi-static force-displacement data equivalent to low-cycle fatigue (a), relation between fatigue SERRs and quasi-static SERRs (b); after [24].

Hence, the SERR obtained from fatigue tests can be correlated to the quasi-static SERR, in which the quasi-static curve represents the highest strain energy release per crack increment. Physically, quasi-static crack growth may be considered the least efficient form of crack propagation, i.e. per crack increment it dissipates the most strain energy.
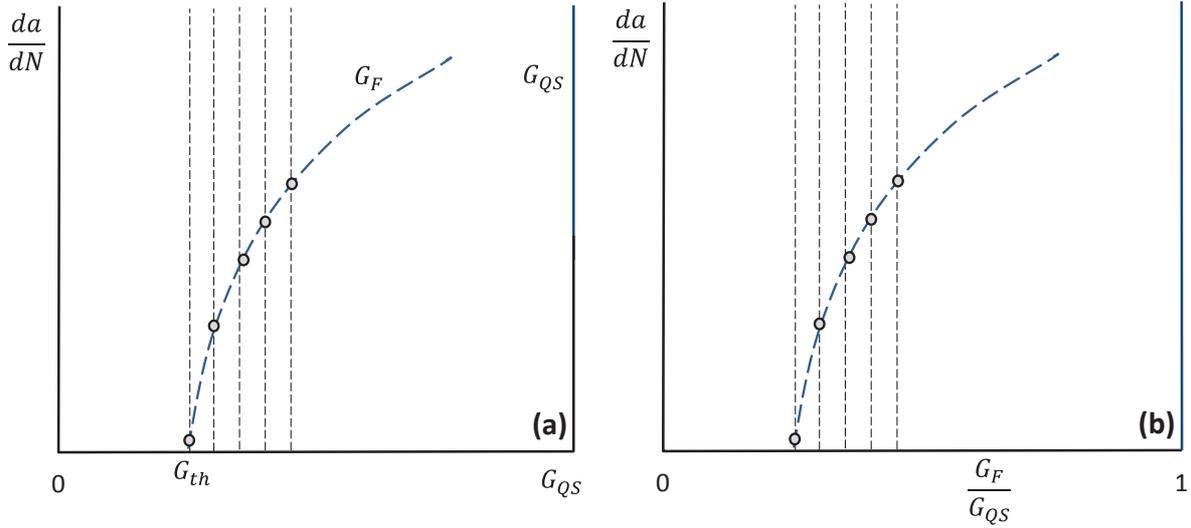
Figure 8. Correlation between the fatigue SERR G_F and the quasi-static SERR G_{QS} (a) and the same trend normalised with G_{QS} (b).

The graphs of da/dN versus dU/dN can be recalculated to graphs plotting da/dN versus $G_F = dU/da$, as illustrated in Figure 8 (a). In this illustration the SERR G_F increases from a threshold value for increasing da/dN, which could either relate to changes in effective fracture surface (roughness and changes in fracture mechanisms), or to additional strain energy dissipation mechanisms (plasticity). Note that the representation of Figure 8 (a) appears to be similar to common Paris relationships utilizing the SERR. However, instead of plotting da/dN against artificial ΔG or G_{max} values, the physical crack growth rate is plotted against the physical average strain energy released per load cycle. Consequently, the representation of Figure 8 (a) can be normalised by the quasi-static SERR G_{QS} as illustrated in in Figure 8 (b).

DISCUSSION

The representation of the crack growth resistance according to equation (4) relates the physical crack extension in a fatigue load cycle, to the physical strain energy released in that same load cycle. Hence, instead of plotting a measurement (da/dN) against a model (ΔK), a measured material characteristic is obtained. This characteristic can be interpreted physically for trends observed, because any trend in this representation must relate to a physical mechanism.

However, one has to keep in mind that both da/dN and dU/dN are consequences of the application of a load cycle. In fact, dU/dN could be the consequence of da/dN, if no other energy dissipating mechanisms are involved. Hence, in order to predict damage growth, a 'driving force' must be related to the consequence. Theoretically, the fatigue damage process can be described by

$$U_\uparrow = U_\downarrow + \frac{dU}{dN} \tag{6}$$

where $U_\uparrow$ is the work applied to the specimen or structure during loading, $U_\downarrow$ the work recovered during unloading, and dU/dN the strain energy dissipated in that load cycle according to equation (4).

Thus, the theory states that the strain energy dissipated should relate to the intrinsic fracture mechanisms involved, but it does not constitute a prediction model as it does not link a 'driving force' to damage growth. The appropriate parameters that represent the 'driving force', correlating to the physical material characteristic, still must be established.

Nevertheless, the benefit of plotting da/dN against either dU/dN or G_F is that the material characteristic is not polluted by modelling aspects, which currently is the case when da/dN is plotted against for example ΔK. Hence, any trend observed in such a representation can be related to physical mechanisms, and therefore contribute to understanding the problem of fatigue damage growth.
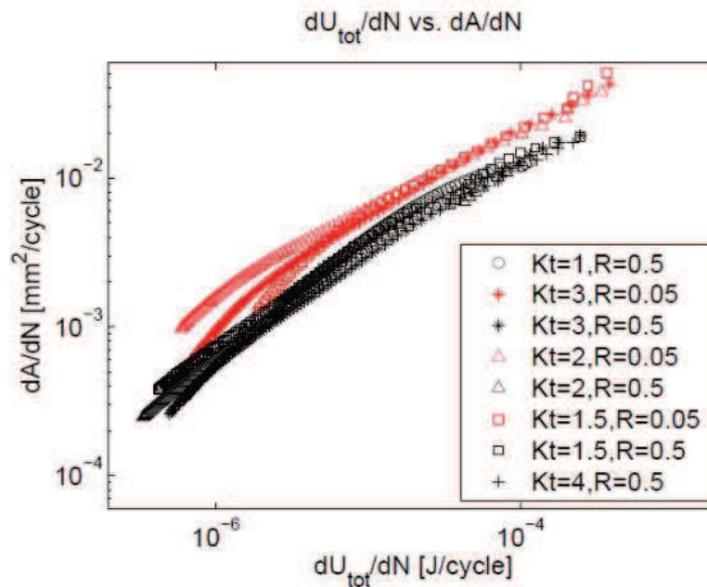
Figure 9. The total strain energy dissipated per load cycle versus the crack growth rate in a solid round metallic bars containing shoulder fillets [26].

For example, although the stress ratio effects greatly vanish when plotting data in the form of *da/dN* against *dU/dN*, still a stress ratio effect remains. This is illustrated by Pascoe for disbond growth in adhesively bonded joints [25], but recently also by Pasman [26] for fatigue crack growth in metallic round bars containing shoulder fillets. Because the material characteristic *da/dN* against *dU/dN* does not contain any implicit assumption related to a model, such stress ratio effect must relate to physical mechanisms.

Because extrinsic shielding mechanisms like crack closure (or fibre bridging in composites) are not present in the *da/dN* against *dU/dN*, the explanation should relate to intrinsic damage mechanisms. To obtain the same *da/dN* for both low and high stress ratios, different levels of maximum loading are applied yielding different fracture mechanisms and surfaces, and different levels of plasticity ahead of the crack tip. Hence, the strain energy dissipation at different stress ratios may yield different curves as illustrated in Figure 9.

CONCLUSIONS

This paper discusses similitude parameters currently adopted to describe fatigue damage growth in various material systems. Where fatigue crack growth in metallic materials generally is described using the SIF, damage growth in adhesively bonded structures and composites often adopt the SERR. Either way, the method of characterization is highly empirical, and constitutes the correlation between a measurement (*da/dN*) and a model (*ΔK, ΔG, G_{max}*, etc).

In conjunction with earlier work, it is demonstrated that the current approaches lead to misinterpretation of phenomena. For example, attributing the stress ratio effect in the Paris relationship predominantly to plasticity induced crack closure is invalid. This has been demonstrated by correlating the corrections for plasticity induced crack closure to mean stress corrections in fatigue life analyses, and to the fact that the effect is also present in brittle material systems that do not exhibit closure in fatigue.

With the introduction of the average strain energy released over a full fatigue load cycle, *dU/dN* or G_F, different phenomena have been explained, like for example the contribution of plasticity to fatigue thresholds. The average or effective SERR can be correlated to the physical intrinsic damage mechanisms that contribute to damage growth in individual load cycles. In addition, a physical correlation has been established between fatigue damage growth and damage growth under quasi-static load conditions.

The author believes that developing prediction models that utilize physical material characteristics in the form of *da/dN* against G_F eventually leads to more understanding about fatigue phenomena, and to more efficient use of fatigue data.

REFERENCES

[1] Paris, P., Erdogan, F. (1963). *Journal of Basic Engineering*, vol. 85(4), p. 528-533.

[2] Elber, W. (1970), *Engineering Fracture Mechanics*, vol. 2, p. 37-45.

[3] Elber, W. (1971), *Damage tolerance in aircraft structures.* ASTM STP 486, p. 230–242.

[4] Irwin, G. R. (1957). *ASME Journal of Applied Mechanics*, vol. 24, p. 361-364.

[5] Alderliesten, R.C. (2014). *Advanced Materials Research,* vols. 891-892, p. 289-294.

[6] Alderliesten, R.C. (2015). *International Journal of Fatigue,* revision submitted.

[7] Schijve, J. (1981), Engineering Fracture Mechanics, vol. 14, p. 461-465.

[8] Schijve, J. (2009). *Fatigue of Structures and Materials,* 2nd ed., Springer Science+Business Media B.V.

[9] Griffith, A. A. (1921). *Philosophical Transactions of the Royal Society of London. Series A*, Containing Papers of a Mathematical or Physical Character, vol. 221, p. 163-198.

[10] Forman, R.G., Kearney, V.E., Engle, R.M. (1967), *Journal of Basic Engineering*, p. 459-463.

[11] Priddle, E.K. (1976), *Int. J. Pressure Vessels & Piping,* vol. 4, p. 89.

[12] Forman, R. G., Mettu, S. R. (1992). In: *Fracture Mechanics: Twenty-second Symposium*, vol. 1, ASTM STP 1131, p. 519-546, Ernst, H. A., Saxena, A., McDowell, D. L. (Eds), American Society for Testing and Materials, Philadelphia.

[13] Jones, R., Pitt, S., Brunner, A.J., Hui D. (2012). *Composite Structures*, vol. 94, p. 1343-1351.

[14] Paris, P.C. (1998), *Fatigue & Fracture of Engineering Materials & Structures*, vol. 21, p. 535-540.

[15] Paris, P.C., Harris, D.O. (1970). In: *Proceedings of the Beryllium Conference: A Report of the National Materials Advisory Board,* Vol. 1, p. 537–568, NMAB-272, NAS, Washington, D.C.

[16] Martin, R.H., Murri, B. (1990). In: *Composite Materials: Testing and Design (Ninth volume)*, ASTM STP 1059, pp. 251–270, Garbo, S.P. (Ed.), ASTM Philadelphia.

[17] Hudson, C.M., Scardina, J.T. (1969). *Engineering Fracture Mechanics,* vol 1, p. 429-446.

[18] Orowan, E. (1950), *Fundamentals of Brittle Behavior in Metals', Fatigue and Fracture of Metals*, Wiley, New York.

[19] Ritchie, R.O. (1990), *International Journal of Fracture*, vol. 100, p. 55-83.

[20] Wanhill, R.J.H. (2009), Characteristic stress intensity factor correlations of fatigue crack growth in high strength alloys: reviews and completion of NLR investigations 1985–1990. NLR-TP-2009-256.

[21] Jones, R., Molent. L., Walker, K. (2012), *International Journal of Fatigue*, vol. 40, p. 43-50.

[22] Yao, L., Alderliesten, R.C., Zhao, M., Benedictus, R. (2014), *Composites: Part A*, vol. 66, p. 65–72.

[23] Pascoe, J.A.P., Alderliesten, R.C., Benedictus, R. (2013), *Engineering Fracture Mechanics,* vol. 112, p. 72-96.

[24] Amaral. L., Alderliesten, R.C., Benedictus, R. (2015), In: Proceedings of the 28th ICAF Symposium, Siljander, A. (Ed.), International Committee on Aeronautical fatigue, Helsinki.

[25] Pascoe, J.A., Alderliesten, R.C., Benedictus, R. (2013), *Engineering Fracture Mechanics,* vol. 133, p. 1-13.

[26] Pasman, D.J. (2015), *Fatigue crack growth in solid round metallic bars with a shoulder fillet - An experimental research project*, MSc Thesis, Delft University of Technology.