Stellingen

(Propositions going with the Ph.D. thesis: Modeling of Indoor Thermal Conditions for Comfort Control in Buildings)

Xiudong Peng

1. Niets is volmaakt, maar onvolmaakt is beter dan niets.

Nothing is perfect, but imperfect is better than nothing.

2. Voor onderzoekers op het gebied van de regeltheorie is een klimaat-regelingsinstallatie een "black box". Voor onderzoekers op het gebied van de klimaatregeling is CFD een black box. Voor onderzoekers op het gebied van de Computational Fluid Dynamics is de regeltheorie een black box. In dit proefschrift worden de drie black boxes met elkaar verbonden en transparant gemaakt.

For control theory researchers, an air-conditioning system is a black box. For air-conditioning researchers, CFD is a black box. For CFD researchers, control theory is a black box. The three black boxes are linked together and made transparent in this thesis.

3. In China heeft het verbeteren van de kwaliteit van de gebouwen de eerste prioriteit, ook al wordt er steeds meer air-conditioning toegepast.

In China, although there is an increasing demand for air-conditioners, the improvement of building quality is of primary importance.

4. In de wereld van vandaag zijn wapens eenvoudig te maken, maar moeilijk te vernietigen.

In today’s world, weapons are easy to make but difficult to destroy.

5. De taak van de promovendus (of onderzoeker) is niet zelf een probleem voor de werkelijkheid te verzinnen en dit vervolgens op te lossen, maar een probleem uit de werkelijkheid af te lezen en dit op te lossen.
The task of a Ph.D. candidate (or researcher) is not to imagine a problem for reality and then solve it, but to discover a problem from reality and then solve it.

6. Principieel is natuurlijke koeling beter dan mechanische koeling, zoals Chinese medicijnen beter zijn dan westerse. Helaas is in de praktijk de kracht van zowel natuurlijke koeling als Chinese medicijnen beperkt of nog niet voldoende onderzocht.

Principally, natural cooling is better than mechanical cooling just as Chinese medicines are better than western medicines. Unfortunately in practice, the power of both natural cooling and Chinese medicines is limited, or perhaps has not been fully explored.

7. Als u het gedrag van een ander niet kan verdragen zult ook u dit gedrag niet vertonen (Zhu Xi)

If you can not tolerate the behaviour of somebody else, you should not behave like that yourself — Zhu Xi

8. Tirannie is meedogenlozer dan een tijger. (Confucius)

Tyranny is fiercer than tiger — Confucius

9. Modellen zijn noodzakelijk voor onderzoekers, maar gevaarlijk voor ontwerpers.

Models are necessary for researchers but dangerous for designers.

10. De bevolkingsaanwas in China onder controle brengen of dit juist achterwege laten zijn beide in strijd met de rechten van het Chinese volk.

Either controlling or not controlling the population of China violates the human rights of the Chinese people.
Modeling of Indoor Thermal Conditions for Comfort Control in Buildings

Key words: air-conditioning, indoor climate, air flows, heat transfers, computational fluid dynamics (CFD), state estimators, control

Xiudong Peng
Modelvorming Binnenluchtcondities voor
de Regeling van het Comfort in Gebouwen

PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus Prof. ir. K.F. Wakker,
in het openbaar te College van Dekanen aangewezen,
op dinsdag 12 november 1996 te 13:30 uur
door

Xiudong PENG

MSc. in Automatic Control, Harbin Institute of Technology, China
geboren te Changyuan, Henan, China
Dit proefschrift is goedgekeurd door
de promotor: Prof. ir. H. van der Ree

Samenstelling promotiecommissie:

Rector Magnificus, voorzitter
Prof. ir. H. van der Ree, Technische Universiteit Delft, promotor
Dr. ir. A.H.C. van Paassen, Technische Universiteit Delft, toegevoegd promotor
Prof. ir. O.H. Bosgra, Technische Universiteit Delft
Prof. dr. ir. F.T.M. Nieuwstadt, Technische Universiteit Delft
Prof. dr. ir. P.G. Luscuere, Technische Universiteit Delft
Prof. Dipl.-Ing. F. Steimle, Universität Essen, Duitsland
Prof. ir. P.G.H. Rutten, Technische Universiteit Eindhoven

To my wife Baorong and my daughter Chengcheng
To my parents
Summary

One of the signs of raised living standards is the improvement of indoor thermal comfort conditions. As more and more air-conditioning units are installed in residential and office buildings, there is also an increasing demand for better control of indoor temperature distributions and air movements.

It is well-known that to design a control system, a mathematical model of the controlled plant in the form of either transfer function or state space is often required. The reliability and control accuracy of the control system are strongly dependent on those of the model. Until present, when the indoor thermal conditions are modeled, the indoor air is almost always considered to be well-mixed and only one temperature is assigned to it. This over-simplification violates the reality of nature since there exist indoor temperature distributions and air flows. Such phenomenon makes the local thermal comfort conditions vary from place to place in a room, which the well-mixed-air model can seldom predict accurately. In addition to this, the model often leads to the bad performance of the control system and thus the complaints of the room occupant.

But if we go one step forward to take the dynamic indoor temperature distributions and air flows into consideration, the modeling process becomes very complicated and time-consuming since the theory of computational fluid dynamics (CFD) must be involved and hundreds of thousands of equations based on finite air volumes have to be solved iteratively. The computing time taken by the dynamic CFD method on the present-day widely available computers such as PC-486s, PC-pentiums and workstations is so long that it is absolutely impractical to use CFD as a dynamic model for simulating indoor temperature responses or designing control systems. A compromise model between the simple but dynamic well-mixed-air model and the complicated but more realistic CFD model is needed for the analysis and design of control systems.

This thesis studies two simplified modeling methods of indoor dynamic temperature distributions. The modeling procedures still rely on the CFD theory but only solves the energy equations. The fundamental assumption for the modeling methods is that the indoor air flow field can be kept constant. It is validated to be true by using smoke visualization in an air-conditioned room with forced air flows. The air flow field and effective exchange coefficients can be precalculated with the steady-state CFD calculation.
The first model is the direct solving of the energy equations for temperatures by using the same grid as that used by the CFD calculation. The comparisons between the calculated results and measured results under the same boundary conditions show that such modeling method not only gives realistic predictions of dynamic indoor temperature distributions but also greatly reduces computing time.

Although we need to know the detailed temperature distributions, over-detailed information of temperatures is really not necessary since most people are insensitive to minor temperature differences (e.g. < 0.5 °C) inside the room. Generally, there exist significant temperature differences between the working zone where the room occupant sits and works and the heat/cold source zone where the air-conditioning unit is located. Another obvious phenomenon is the temperature gradient of indoor air, which influences the local thermal comfort conditions and the energy consumption of the room. This indicates that the room can be divided into several air zones. A model built with this approach would give enough temperature information and accuracy for simulations and control system designs. According to the CFD theory, if the air mass is balanced for every single finite air volume of the grid, then the mass is also balanced for any arbitrary group of finite air volumes. Starting from this point, we get the second model by reorganizing the thousands of finite volumes used by CFD simulations and transforming them into several air zones. Each air zone is assumed to be well mixed and is depicted by one temperature. Such a zonal model is validated to give equally good results as the first model but is much faster than it since only several temperatures need to be solved.

Another big advantage of the zonal model is that it can be easily transformed into a state space model with which modern control theory can find better applications in the control of indoor thermal conditions.

In many practical control systems, there are usually some variables that need to be controlled or monitored but are either impossible to measure or too expensive to measure. The normal practice in the control of indoor thermal conditions is that only one temperature sensor is mounted in each room. Therefore, the temperatures of places other than the one where the sensor is located need to be estimated in some way. This thesis discusses the estimation methods in such respect by introducing the state estimators of modern control theory. Design procedures of the state estimators are given and some case studies are carried out for real-time estimations of temperature responses.
Summary

In traditional control systems, the measured variable is also the controlled variable. In many air-conditioning control systems, the temperature sensor is mounted to measure the temperature of either the recirculated air or the exhaust air. Since the temperature of the working zone in a room is significantly different from either one of these two temperatures and is usually inconvenient to measure, it has to be estimated, feedback to the controller and then controlled. Otherwise if the measured temperature is directly controlled, the controller will give wrong actions and the working zone can not reach the comfortable state. This aspect is discussed in Chapter 6 of this thesis. Simulation results shows that much better control results can be obtained if, instead of the measured temperature of the recirculated air, the estimated temperature of the working zone is controlled. With this "estimation-feedback-control" scheme, the displayed result of the controller is consistent with the human sensation of indoor thermal conditions.
Summary
Samenvatting

Het verbeteren van het binnenklimaat duidt op een toename van de levensstandaard. Steeds meer air conditioning systemen worden in woningen en kantoorgebouwen toegepast. Dit doet ook de vraag naar een betere regeling van de luchtbeweging en de verdeling van de luchttемperatuur toenemen.

Voor het ontwerp van een regelsysteem is meestal een mathematisch model van de geregelde installatie nodig in de vorm van overdrachtsfuncties of toestandsvergelijkingen. De betrouwbaarheid en regelzuivereigheid van het regelsysteem zijn sterk afhankelijk van die van het gebruikte model. Tot op heden wordt voor de modelvorming van de thermische binnencondities verondersteld dat de lucht in een ruimte volledig gemengd is en met één temperatuur gekarakteriseerd kan worden. Deze grove vereenvoudiging komt bepaald niet overeen met de werkelijkheid, want er komen wel degelijk temperatuurverdelingen en gerichte luchtstromingen voor. Door deze fenomenen varieert het comfort van plek tot plek in een ruimte. Hierdoor levert het model een regeling op, die niet voldoet en aanleiding geeft tot klachten.

Gaan we een stap verder door de dynamica van de temperatuur en snelheidsverdeling in beschouwing te nemen dan wordt de modelvorming zeer gecompliceerd en tijdzorgvuldig omdat de theorie van de stromingsdynamica (CFD) gebruikt moet worden en duizenden balans vergelijkingen van eindige volume elementen iteratief opgelost moeten worden. De rekenseenheid, die de huidige computers zoals PC-486, PC-pentiums en werkstations nodig hebben voor een dynamische berekening is zo lang, dat het niet practisch is om CFD te gebruiken voor het simulieren van de responsie van de binnentemperatuur en het ontwerpen van regelsystemen. Een dynamisch model, dat een compromis vormt tussen het goed gemengde model en het realistischer maar gecompliceerde CFD model, is nodig voor het analyseren van regelsystemen.

In dit proefschrift zijn twee vereenvoudigde modellen voorgesteld en geanalyseerd. De modelvorming is wel gebaseerd op de CFD theorie, maar los alleen de energievergelijking op. De fundamentele aannames hierbij is dat het stromingsveld in dynamische situaties constant blijft ook al verandert het temperatuur veld. De juistheid van deze aannames is aangetoond met roofproeven in een geconditioneerde ruimte. Het luchtsnelheidsveld en de uitwisselingscoëfficiënten kunnen van te voren worden bepaald met een statische CFD berekening.
Bij het eerste model worden de energievergelijkingen opgelost door hetzelfde discretisatie rooster te gebruiken als bij de CFD berekening is toegepast. Een vergelijking van de hiermee berekende temperaturen en de gemeten waarden toonde aan dat deze methode niet alleen een realistische voorspelling van de binnentemperaturen oplevert, maar ook de rekentijd enorm reduceert.

Ofschoon we de temperatuurverdeling gedetailleerd willen kennen, moeten we niet overdrijven wat de nauwkeurigheid aangaat. De meeste mensen zijn ongevoelig voor kleine temperatuurverschillen (<0,5 °C). In het algemeen bestaat er een groot temperatuurverschil tussen de werkzone waar de mensen zich bevinden en de locatie waar de toe- en afvoer van warmte door de klimaateenheid plaatsvindt. Een ander voor de hand liggend fenomeen is de temperatuurgradient van de binnenlucht, die de lokale comfort condities en het energiegebruik beïnvloeden. Dit geeft aan, dat de ruimte opgedeeld kan worden in een aantal geschikt gekozen zones, waarin de lucht-temperatuur overal dezelfde is. Deze aanpak levert modellen die voldoende informatie verschaffen voor simulaties en het ontwerp van regelsystemen.

Volgens de CFD theorie geldt het volgende. Klopt de massa balans voor ieder lucht element van het rooster dan geldt dit ook voor ieder willekeurige groep van elementen. Hiervan uitgaand is het tweede model opgezet door de duizende eindige elementen van de CFD simulatie te reorganiseren en te transformeren in een beperkt aantal zones. Iedere zone wordt verondersteld ideaal gemengd te zijn en wordt gekarakteriseerd met één temperatuur. Een dergelijk zone-model geeft dezelfde goede resultaten als het eerste model, maar is veel sneller omdat slechts een beperkt aantal temperaturen moeten worden bepaald.

Een ander voordeel van het zone-model is, dat het kan worden geschreven in een stelsel toestandsvergelijkingen zodat het goed aansluit bij de moderne regeltheorie en gebruikt kan worden voor het ontwerp van betere regelingen voor het binnenklimaat.

Bij zeer veel regelsystemen in de praktijk komt het voor dat eigenlijk meerdere variabelen zouden moeten worden geregeld of geobserveerd. Dit gebeurt echter niet omdat het onmogelijk is of te duur. Bij de regeling van de thermische binnencondities wordt slechts één temperatuur opnemer gemonteerd op een praktische plaats in de ruimte. Daarom zal op andere plaatsen het klimaat niet altijd comfortabel zijn. Dit proefschrift toont hoe het zone-model gebruikt kan worden om de temperaturen op andere plaatsen te schatten, zodat ook hierdoor een betere regeling mogelijk wordt. De schattingsmethode, die dit mogelijk maakt wordt hier geïntroduceerd door gebruik te maken van toestandschatters van de moderne regeltheorie. Ontwerpmethoden voor de
Samenvatting

schatters worden voorgesteld en enkele case studies worden uitgevoerd voor de real time schatting van de temperatuurreponsie.

In de praktijk wordt de temperatuurenpnemer op een zodanige wijze in de klimaateenheid geplaatst dat hij de temperatuur meet van de aangezogen vertreklucht of nog erger, hij wordt eenvoudigweg naast de deur gemonteerd. Beide plaatsen zijn verre van ideaal, omdat de temperatuur in de leefzone aanzienlijk kan verschillen. Om dit probleem op te lossen moet de temperatuur in de leefzone geschat worden met de toestandschatter en de onjuist gemeten waarde. In hoofdstuk 6 wordt dit besproken. Resultaten van de simulatie tonen aan dat betere resultaten kunnen worden verkregen als de binnentemperatuur wordt geregeld op basis van de geschatte temperatuur in de leefzone in plaats van met het signaal van de opnemer. Met dit schattings-terugkoppelingelsysteem komt de temperatuurregeling meer overeen met de wijze waarop mensen een comfortabel klimaat ervaren.
# Contents

Summary

Samenvatting

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Introduction</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>§1.1 Types of Air-conditioning Systems</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>§1.2 Problems in Modeling of Indoor Thermal Conditions for Comfort Control</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>§1.3 Outline of This Thesis</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>A Brief Description of Computational Fluid Dynamics</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>§2.1 Conservation Laws and Turbulence Modeling</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>§2.2 Boundary Conditions</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>§2.3 Numerical Method in Computational Fluid Dynamics</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>§2.4 Discussion and Summary</td>
<td>31</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Visualization and Prediction of Indoor Air Flows</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>§3.1 Test Room</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>§3.2 Visualization and Prediction of Indoor Air Flows</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>§3.3 Discussion and Summary</td>
<td>46</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Simplified Dynamic Modeling of Indoor Temperature Distributions</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>§4.1 Modeling with Fixed Air Flow Fields</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>§4.2 Effect of Wall Radiation on Temperature Measurement</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>§4.3 Model Validations</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>§4.4 Air-Zone (Zonal) Model</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>§4.5 State Space Representation of the Air-Zone Model</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>§4.6 Validations of the State Space Model</td>
<td>86</td>
</tr>
<tr>
<td></td>
<td>§4.7 Discussion and Summary</td>
<td>92</td>
</tr>
</tbody>
</table>
Contents

Chapter 5  Real-time Estimations of Indoor Temperatures 97
§5.1 State Estimator for Temperature Estimations 98
§5.2 A Case Study 103
§5.3 Reduced-order Estimator for Temperature Estimations 106
§5.4 Kalman Filter for Temperature Estimations 115
§5.5 Discussion and Summary 117

Chapter 6  Estimation and Control of the Temperature of the Working Zone 119
§6.1 Dynamic Simulation Scheme in Matlab/Simulink 120
§6.2 Controlling the Measured Temperature 123
§6.3 Controlling the Temperature of the Working Zone 125
§6.4 Discussion and Summary 128

Chapter 7  Conclusions and Recommendations 129
§7.1 Conclusions 129
§7.2 Recommendations for Future Research 131

References 133

Appendix A  Some Basic Concepts in State Space Control Theory 143

Appendix B  Equations for Heat Transfer Calculations in Buildings 145
§B.1 Heat Balance at the Outside Surface of a Building Wall 145
§B.2 Solar Heat Gain through a Window 148
§B.3 Heat Conduction through Walls 149
§B.4 Radiative Heat Exchanges among Inside Wall Surfaces 151

Acknowledgment 155

Biography 157
Chapter 1

Introduction

The further improvement of indoor thermal conditions and the reduction of energy consumption in building rooms ask for better modeling and control of indoor temperature distributions and air movement. While over simplified models may lose valuable information about temperature differences at different room locations, complicated models are often time consuming and difficult to use for control system designs. This chapter describes the problems existing in the modeling of indoor thermal conditions for comfort control and outlines what is done in the whole thesis.

People spend approximately 80% of their life inside buildings. A good indoor thermal environment not only means comfortable living conditions in residences but also increases the productivity and efficiency of people in manufacturing workshops and office buildings. In recent decades, air-conditioning systems as well as heating and ventilating systems are widely installed in dwellings, commercial and public buildings, vehicles, etc.. Consequently, the influence of the changing outdoor climate on indoor thermal environment is counteracted. Odors from human bodies and furniture, pollutants from building materials, decoration materials, computers, copying machines, etc. are greatly removed.
§1.1 Types of Air-Conditioning Systems

To meet different performance, capacity, spatial and cost requirements, various air-conditioning systems are developed and manufactured. Air-conditioning systems are categorized by how they control heating, cooling and ventilating in the conditioned area. Generally speaking they can be classified as all-air systems, air-and-water systems and unitary refrigerant-based systems. All-air systems can be further divided as single duct systems and dual duct systems, both of which can be either CAV (Constant Air Volume) systems or VAV (Variable Air Volume) systems. Air-and-water systems include fan-coil + primary air systems, induction unit systems and radiant panel + fresh air systems. Window-mounted and through-the-wall mounted air conditioners and heat pumps belong to unitary refrigerant-based systems. Figure 1.1 (van Paassen 1993) explains how the temperature is controlled in the above systems. In all-air systems, the supply air is first conditioned at a central air handling unit and is then supplied to different rooms. In a CAV system, the room temperature is controlled by the supply air temperature which is changed by adjusting the reheating unit. In a VAV system, the room temperature is controlled through the change of the amount of the supply air. In an induction unit system, centrally conditioned air is supplied to the unit plenum at medium to high pressure. A balancing damper adjusts the primary air quantity. When the primary air flows through the induction nozzles, the secondary air from the room is induced into the unit through a heating/cooling coil and is mixed with the primary air. The mixed air is then discharged into the room. The room temperature is controlled by the heating or cooling of the secondary air. In a fan-coil unit system, the primary air is mixed with the secondary (recirculated) air from the room first. Then the mixed air is blown to the heating/cooling coil where it is heated or cooled. After this, the mixed air is discharged into the room. The room temperature is influenced by the supply air temperature which can be controlled by the water flow valve of the coil, and the supply air quantity which can be controlled by the fan speed and the positions of the dampers of the primary air and secondary air. The primary air can be either from a central air handling unit or directly from outdoor air.

It should be pointed out that traditionally, when air-conditioning systems are designed, sized and installed for buildings, the outdoor climate is often treated as a disturbance that must be offset by mechanical heating and cooling. This is obviously not an energy conservation idea. Apparently, natural ventilation can remove the excess heat of a room.
In mild weather countries such as the Netherlands, the outdoor air temperature is almost always lower than the indoor air temperature. Thus natural ventilation can be used as a cooling source for buildings. Further, for building envelopes of high heat capacities, natural ventilation can be used to cool down the envelopes at night so that the cooling energy is partly or even completely saved during the day. The shading device such as awning or venetian blinds prevents the direct solar radiation from entering the room and thus reduces the cooling load. Buildings that are installed with controllable natural ventilation windows, shading devices and insulating shutters in order to reduce cooling energy are called passive indoor climate buildings. Such buildings have been systematically studied by van Paassen (1988, 1989, 1990, 1992, 1993) and Lute (1990, 1992). Their results show that natural ventilation windows and
Chapter 1 Introduction

shading devices, when adequately controlled, supply one of the biggest possibilities to save air-conditioning energy while keeping the indoor thermal comfort unchanged.

§1.2 Problems in Modeling of Indoor Thermal Conditions for Comfort Control

Whenever a control system is designed or tested based on simulation, a mathematical model of the controlled plant in the form of either transfer functions or state space or differential equations is often required. If a mathematical model is not available, at least the dynamic response of the plant should be known.

Traditionally and presently, when the control system of indoor thermal conditions is designed, and/or the energy consumption of a building is evaluated, and/or the indoor thermal comfort level is evaluated, the indoor air is almost always considered to be well-mixed. That is, one single temperature is assumed for the whole air volume of the room. The mathematical model developed based on this assumption of “homogeneous temperature distribution” is usually called one-point model. Such a model is still very popular and useful for cooling/heating load calculations. But the control system of indoor thermal conditions designed according to this model often causes complaints from the room occupant. This means that the control system does not function properly. When the reason of the malfunction is checked, it is not difficult to discover that the assumption that the indoor temperature distribution is homogeneous is incorrect and contradictory to reality, and must be corrected in the modeling. In fact, there are significantly non-uniform temperature distributions in rooms (Figure 1.2). This phenomena should be reflected in a model that is used for evaluating and/or controlling local thermal comfort conditions.

Due to the dynamic temperature distributions in a room, the temperature of the working location (zone) where the room occupant is seated is almost always different from that of the recirculated or exhaust air. The latter is usually measured by a temperature sensor and controlled by the controller in practice as can be seen in Figure 1.1. Here we see a problem: the temperature that is measured and controlled is not the temperature of the working zone that needs to be controlled. There are two ways to solve this problem. One is to move the temperature sensor to the working zone of the room occupant. The other is to develop a method to estimate the temperature of the working zone from the measured temperature of the recirculated or exhaust air. Since the first method causes new problems such as the sensor may easily be damaged by the
careless activity of the occupant, the wires of the sensor bring some inconvenience to the occupant, etc., we believe it is not a practical solution. The second method needs some estimation theory and a proper model of the indoor temperature distributions. But once it is built, it can be easily implemented with computer software.

State-of-the-art of Research on the Modeling

When indoor temperature distributions and air flows are taken into consideration, the modeling of the indoor thermal environment becomes very complicated and difficult since the knowledge of fluid dynamics must be involved due to the fact that air is a type of fluid in nature. The fundamental differential equations of fluid dynamics depicting turbulent flows (air flows are turbulence) are the continuity, momentum conservation and energy conservation equations. Since there has been no analytical solution to these equations, the numerical solving method is employed, which leads to the Computational Fluid Dynamics (CFD) theory. The basic idea of CFD is that the flow domain is first divided into thousands of finite volumes by setting a grid (Figure 1.3). For every finite volume of the grid, the conservation equations are solved iteratively until the solutions of the equations for all finite volumes have converged.

With the advent of faster and faster personal computers and work-stations since the late 1980s, the CFD method has been extensively used to predict indoor temperature distributions and air flows (Awbi 1991, Chen 1988, 1990, 1992, 1995, Li, 1993, Matsumoto 1987, Murakami 1987, 1988, Nielsen 1974, 1989, Niu 1992, 1993, 1994, Schaelin 1992, Zhang 1992, etc.). But in contrast to the one-point model, it goes to the other way round — too complicated theory and too time consuming computation. Until recently, most published CFD results for predicting indoor temperature distributions and air flows are steady state simulations. Due to the speed limitation of the present-day widely available computers such as work stations, PC-486s, PC-pentiums, etc., dynamic CFD simulations for buildings are seldom seen in literature. This makes it impossible to use CFD as a dynamic model for simulating indoor dynamic temperature responses or designing control systems. A trade-off model between the simple but dynamic one-point model and the complicated but more realistic CFD model should be found out at the present time in order to predict and control the detailed indoor thermal conditions. It could be anticipated that during the search of such a dynamic model, some assumptions will have to be made and some information will have to be sacrificed.
Figure 1.2 Temperature distribution in a building room
Hemmi (1967) developed a model which divided the room air volume into several ideally mixed zones that were connected both in series and parallel with respect to the flow in the room. van der Kooi and Förch (1985) presented a model in which the room center (in the middle of the ceiling and floor) was assigned a temperature $T_a$. Since there was indoor temperature gradient, the air temperature near the ceiling was represented with $T_a + \Delta T_c$ and the air temperature near the floor was represented with $T_a + \Delta T_f$. $\Delta T_c$ and $\Delta T_f$ were measured first. The room was then represented with three indoor temperatures at different heights, which could easily be calculated with fixed values of $\Delta T_c$ and $\Delta T_f$ since only one temperature $T_a$ is unknown. In the same paper, van der Kooi presented a more-point model in which the room air volume was divided into several air zones, each of which was represented with one temperature.

The precondition for the models of Hemmi and van der Kooi was that the air mass flow between one zone and another could be prescribed. But both Hemmi and van der Kooi did not give a theoretical method of how to prescribe these mass flows. The method they gave was measurement, i.e. the air mass flows were estimated according to the visualization of indoor air flows and the measurement of air speeds.
Chapter 1 Introduction

Hill (1985) described a model based on the one-point model of Borresen (1981). Although the total air mass was represented with two temperatures, the room air was still treated as fully mixed and represented with one temperature. The second temperature was that of the air in the air duct.

Dalicieux (1992) presented a simplified model to represent indoor air motions based on the degradation of fluid mechanics equations. In his paper, the room air volume was divided into two parts: the plume areas where the supply air inlet or heating panel was located and the standard areas which were the remaining parts and where the air movement was slow. The mass flow between one part and an adjacent part is calculated based on Bernoulli's law.

Chen and van der Kooi (1987, 1988) presented the idea of combining a cooling load program and an air flow program in order to find better agreement between computed and measured results of annual heating/cooling loads.

Peng, et al (1994) studied the prediction of indoor dynamic temperature distributions by using a fixed flow pattern. A good agreement between calculations and measurements was obtained.

State-of-the-art of Research on the Temperature Estimation for Comfort Controls

Over the past years, although many people have noticed the temperature difference between the recirculated or exhaust air and the working zone in the room, few people have really taken the problem seriously in spite of the fact that it is this temperature difference that leads to the bad performance of the air-conditioning control system. Until present, we have not seen any literature investigating the dynamic estimation of the working-zone temperature based on the measured temperature of the recirculated or exhaust air. Neither have we seen any publication on the indirect control of the temperature response of the working zone.

§1.3 Outline of This Thesis

For the purpose of thermal comfort control in buildings, this thesis studies the modeling of dynamic indoor temperature distributions and the estimation of the working-zone temperature based on the measured temperature of the recirculated or exhaust air.

For forced convection flows, the indoor air flow pattern changes little if the conditioning air is supplied at constant speed. Thus the fundamental idea of this thesis is to calculate indoor dynamic temperature distributions by using fixed air flow fields.
Although the dynamics of air flows are neglected, the fixed air flow fields can still indicate the rough values of the air speeds at various room locations so that local thermal comfort levels can be evaluated. The estimation method used in this thesis is the state estimators in modern control theory. The (test) room that is investigated resembles many typical office building rooms installed with a fan-coil unit system. The outline of this thesis is as follows.

Since CFD constitutes one of the fundamental theories on which this thesis is based, a brief introduction is given to it. Visualization of various indoor air flows are carried out and compared with the CFD results in the test room to validate the reliability of CFD when it used for predicting indoor air flows. Air flow fields (velocity components of the finite volumes in the flow domain) and effective exchange coefficients corresponding to various air flow patterns are extracted from the CFD results. By fixing the flow field, the indoor dynamic temperature distributions are calculated. A simplified zonal model based on indoor air flow patterns is presented and validated. Based on this zonal model, a state space model is constructed. The real-time prediction of the indoor dynamic temperature responses is carried out by employing the state estimator theory. The temperature of the working zone is predicted based on the temperature of the recirculated air. A better control of the thermal conditions of the working zone is achieved by controlling the estimated temperature.

Chapter 1: This chapter depicts the existing problems in the modeling of indoor thermal conditions for comfort control in buildings. It explains the necessity of the modeling of dynamic indoor temperature distributions and the necessity of a method to predict the working-zone temperature. The contents of the thesis is outlined here.

Chapter 2: CFD constitutes one of the fundamental theories on which this thesis is based. This chapter briefly introduces the conservation laws, turbulence modeling, boundary conditions and the numerical method related with the CFD theory.

Chapter 3: To validate the reliability of CFD when it used for predicting indoor air flows and to have a scenario of the indoor air flow patterns under different heating and cooling situations, typical air flow patterns are calculated with the CFD code, PHOENICS, and validated with visualized air flows in the test room.

Chapter 4: Since CFD takes too much computing time especially when it is used for dynamic calculations. It is impractical to use it either for designing control systems of indoor thermal conditions or for investigating the dynamic indoor temperature distributions. This chapter gives the simplified modeling methods of dynamic indoor temperature distributions to meet the above dual requirements. The models are
constructed based on fixed air flow fields. The first model is direct calculations of temperatures of all finite air volumes used by the CFD calculations. To further reduce the computing time, a simplified zonal model of temperature distributions is introduced and validated. This zonal model can further speed up the simulation of dynamic indoor thermal responses without losing too much temperature information. Finally, a state space model with a rather simple form is presented based on the zonal model. The advantage of a state space model lies in that it is in line with the requirement of the modern control theory and can be directly used for controller designs.

Chapter 5: In many practical control systems, there are usually some variables that need to be controlled or monitored but are either impossible to measure or too expensive to measure. The reality in the control of indoor thermal conditions is that only one temperature sensor is mounted in a building room. Therefore, the temperatures of places other than the one where the sensor is located need to be estimated in some way. This chapter discusses the estimation methods in such respect by introducing the state estimators of modern control theory. This measured temperature can be used as feedback information to update the estimated values of other temperatures. The estimation results are validated with measurements.

Chapter 6: In most air-conditioning control systems, the temperature sensor is mounted to measure and control either the temperature of the recirculated air or the temperature of the exhaust air. Since the temperature of the working zone in a room is significantly different from either one of these two temperatures, the controlled temperature is not the temperature that needs to be controlled. With the estimator theory introduced in Chapter 5, the temperature of the working zone can be estimated, used as feedback information, and controlled. This chapter illustrates the implementation of this "estimation-feedback-control" scheme. The thermal inertia of walls and the disturbances from outdoor weather conditions are also considered. The simulation is realized in the Matlab/Simulink programming environment.

Chapter 7: This chapter gives conclusions of this thesis and points out what should be done next.
Chapter 2

A Brief Description of Computational Fluid Dynamics

To further improve indoor thermal comfort, detailed indoor temperature distributions and air flows need to be investigated and modeled. Computational Fluid Dynamics (CFD) is the best tool to carry out such studies. This chapter briefly introduces the conservation equations, turbulence modeling, boundary conditions and numerical method of solving the differential equations regarding turbulent flows. The contents described here constitute one of the theoretical bases of the following chapters.

In recent years, mathematical (or theoretical) modeling is becoming very popular thanks to the advent of faster and faster computers that can be used to solve differential equations numerically. The advantages of computer simulations are apparent: fast, low cost, relatively complete prediction of the features and responses of the physical process, able to investigate the influence of different parameters easily, etc.

To further improve the indoor thermal comfort, detailed indoor temperature distributions and air flows need to be investigated and modeled. Computational fluid dynamics (CFD) has been proved to be the best tool to carry out such studies until now. Since CFD is one of the fundamental theories on which this thesis is based, it is well worth to give a brief introduction to it.
§2.1 Conservation Laws and Turbulence Modeling

Just as other fluid flows, air flows of practical relevance are almost always turbulence. Turbulent flows are characterized by randomness, three dimensional, unsteady, diffusivity, dissipation and continuity. The mathematical equations describing turbulent flows are well known. They are the energy, mass and momentum conservation equations. The derivation of these equations can be found in most fluid dynamics books. Here we will only introduce some of the final results governing turbulent flows.

For incompressible flows, the partial differential equations of conservation can be represented in tensor notation:

Mass conservation (continuity equation):

\[ \frac{\partial \tilde{u}_i}{\partial x_i} = 0 \]  \hspace{1cm} (2.1)

Momentum conservation (Navier-Stokes equation):

\[ \frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho_r} \frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} + \beta g_i \left( \theta_r - \tilde{\theta} \right) \]  \hspace{1cm} (2.2)

Thermal energy conservation:

\[ \frac{\partial (C_p \rho \tilde{\theta})}{\partial t} + \tilde{u}_i \frac{\partial (C_p \rho \tilde{\theta})}{\partial x_i} = \lambda \frac{\partial^2 \tilde{\theta}}{\partial x_i \partial x_i} + S_H \]  \hspace{1cm} (2.3)

Species concentration conservation:

\[ \frac{\partial (\rho \tilde{c})}{\partial t} + \tilde{u}_i \frac{\partial (\rho \tilde{c})}{\partial x_i} = \gamma \frac{\partial^2 (\rho \tilde{c})}{\partial x_i \partial x_i} + S_C \]  \hspace{1cm} (2.4)

where the physical meaning of the variables and parameters, as well as the properties of the fluid in the above equations are as follows:
Conservation Laws and Turbulence Modeling

\( \vec{\nabla} \cdot = \text{three coordinate axes in space} \ [m] \),

(customarily, \( x_1, x_2 \) and \( x_3 \) are represented with \( x, y \) and \( z \))

\( t = \text{time coordinate} \ [s] \),

\( \vec{u}_i \) (or \( \vec{u}_j \)) = instantaneous velocity component in direction \( x_i \) (or \( x_j \)) \ [m/s],

(customarily, \( \vec{u}_1, \vec{u}_2 \) and \( \vec{u}_3 \) are represented with \( \vec{u}, \vec{v} \) and \( \vec{w} \))

\( \rho = \text{local density} \ [\text{kg/m}^3] \),

\( \overline{p} = \text{instantaneous static pressure} \ [\text{Pa}] \),

\( g_i = \text{gravitational acceleration in direction} \ x_i \ [\text{m/s}^2] \),

\( \nu = \text{(molecular) kinematic viscosity} \ (\text{or laminar viscosity}) \ [\text{m}^2/\text{s}] \),

\( \overline{\theta} = \text{instantaneous temperature} \ [\text{K or } ^\circ \text{C}] \),

\( \Theta_r = \text{reference (or mean) temperature} \ [\text{K or } ^\circ \text{C}] \),

\( \beta = \text{volumetric thermal expansion coefficient} \ (1/\text{K}) \), its relation with \( \rho \) and \( \overline{\theta} \) is:

\[ \beta = (\rho - \rho_r) / [\rho_r (\Theta_r - \overline{\theta})] \],

\( \rho_r = \text{reference (or mean) density} \ [\text{kg/m}^3] \),

\( C_p = \text{specific heat at constant pressure} \ [\text{J/kg-K}] \),

\( \lambda = \text{thermal conductivity} \ [\text{W/m-K}] \),

\( S_H = \text{heat generation rate per unit volume} \ [\text{J/s-m}^3] \),

\( \overline{c} = \text{species concentration} \ [\text{m}^3/\text{m}^3] \),

\( \gamma = \text{molecular diffusivity of concentration} \ [\text{m}^2/\text{s}] \),

\( S_C = \text{mass generation rate of the species per unit volume} \ [\text{kg/s-m}^3] \).

The subscript \( i \) and \( j \) can take the values 1, 2, 3, denoting the coordinates of the three dimensional space. When a subscript is repeated in a term of an equation, a summation of three terms is implied. For example,

\[ \frac{\partial \vec{u}_i}{\partial x_i} = \frac{\partial \vec{u}_1}{\partial x_1} + \frac{\partial \vec{u}_2}{\partial x_2} + \frac{\partial \vec{u}_3}{\partial x_3} \]

in equation (2.1);

\[ \vec{u}_j \frac{\partial \vec{u}_i}{\partial x_j} = \vec{u}_1 \frac{\partial \vec{u}_i}{\partial x_1} + \vec{u}_2 \frac{\partial \vec{u}_i}{\partial x_2} + \vec{u}_3 \frac{\partial \vec{u}_i}{\partial x_3} \quad \text{and} \]

13
\[\nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} = \nu \frac{\partial^2 \bar{u}_i}{\partial x_1 \partial x_1} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_2 \partial x_2} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_3 \partial x_3}\]

in equation (2.2).

The fluctuating body force (the buoyant force) caused by fluctuating density which in turn is caused by temperature fluctuation is represented by the last term on the right-hand side of the momentum balance equation (2.2). Here the Boussinesq approximation is used so that the variable density only appears in the buoyant force term. Together with a definition equation of \( \beta \) relating the local values of \( \rho \) and \( \bar{\theta} \), equations (2.1) through (2.4) form a close set of equations describing all the details of the air flows.

These equations have not been solved with an exact mathematical method until now. Although numerical procedures are available to solve these equations, the storage capacity of the present day computers are still not sufficient to allow a solution of these equations for any practical relevant turbulent flow. The reason is that turbulent motion contains elements of high fluctuating frequencies such as tiny eddies which are much smaller than the extent of the flow domain, typically of the order of \( 10^3 \) times smaller (Rodi 1984). To resolve the motion of these elements in a numerical procedure, the mesh size of the numerical grid points would have to be even smaller. Therefore at least \( 10^9 \) grid points would be necessary to cover the flow domain in three dimensions. Take the calculation of 6 variables: temperature, concentration, pressure and the three velocity components in three dimensions, as an example, if one variable takes 4 bytes of computer memory, then at least \( 6 \times 4 \times 10^9 \) bytes or 24000 mega bytes of computer memory are needed to store the variables, let alone that a great many auxiliary variables are needed when the equations are being solved. This requirement on memory is far beyond the capacity of most computers. What is more is that the huge number of arithmetic operations together with the large number of iterations and time steps would further make such a calculation impossible.

Therefore, a statistical approach has to be taken to only study the mean turbulent flows. As suggested by Osborne Reynolds, the instantaneous values of velocity \( \tilde{u}_i \), pressure \( \tilde{p} \) and the scalar quantity \( \bar{\theta} \) and \( \bar{c} \) are separated into mean and fluctuating quantities:

\[
\tilde{u}_i = U_i + u_i, \quad \tilde{p} = P + p, \quad \bar{\theta} = \theta + \theta', \quad \bar{c} = C + c
\]  

(2.5) 

where the mean quantities are defined as
\[ U_i = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \bar{u}_i \, dt, \quad P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \bar{p} \, dt, \quad \theta = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \bar{\theta} \, dt, \quad C = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \bar{c} \, dt \]  
\[ \text{(2.6)} \]

and the means of the fluctuating terms \( u_i, p, \theta' \) and \( c \) are assumed to be zero, i.e.

\[ \bar{u}_i = 0, \quad \bar{p} = 0, \quad \theta' = 0, \quad \bar{c} = 0 \]
\[ \text{(2.8)} \]

The averaging time \( t_2 - t_1 \) is long compared with the time scale of the turbulent motion and small compared with the time scale of the mean flow in transient problems.

Assume that the local density \( \rho \) is a variable only in the body force (buoyant force) term of equation (2.2), we may rewrite equations (2.3) and (2.4) as follows:

\[ \frac{\partial \tilde{h}}{\partial t} + \bar{u}_j \frac{\partial \tilde{h}}{\partial x_i} = a \frac{\partial^2 \tilde{h}}{\partial x_i \partial x_j} + \frac{S_{\tilde{h}}}{\rho} \]
\[ \text{(2.9)} \]

\[ \frac{\partial \tilde{c}}{\partial t} + \bar{u}_j \frac{\partial \tilde{c}}{\partial x_i} = \gamma \frac{\partial^2 \tilde{c}}{\partial x_i \partial x_j} + \frac{S_{\tilde{c}}}{\rho} \]
\[ \text{(2.10)} \]

where

\[ a = \lambda / C_p \rho \] is the thermal diffusivity \([m^2/s]\). 

\[ \tilde{h} = C_p \tilde{\theta} = C_p (\theta + \theta') = H + h \] is the enthalpy \([\text{J/kg}]\).

Due to definitions (2.5) through (2.8) and the fact that the averaging value of \( U_n, P, H \) and \( C \) is equal to itself, we have the following averages over time \( t_2 - t_1 \):

\[ \frac{(U_j + u_j)}{\partial x_j} \frac{\partial (U_i + u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \frac{(U_j + u_j)(U_i + u_i)}{\partial x_i} \]
\[ = \frac{\partial}{\partial x_j} (U_i U_j) + \frac{\partial}{\partial x_j} (u_i u_j) = U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial}{\partial x_j} (u_i u_j) \]
\[ \text{(2.11)} \]
\[ (U_i + u_i) \frac{\partial (H + h)}{\partial x_i} = U_i \frac{\partial H}{\partial x_i} + \frac{\partial}{\partial x_i} (u_i h) \] (2.12)

\[ (U_i + u_i) \frac{\partial (C + c)}{\partial x_i} = U_i \frac{\partial C}{\partial x_i} + \frac{\partial}{\partial x_i} (u_i c) \] (2.13)

Now we introduce (2.5) into (2.1), (2.2), (2.9) and (2.10), then average each equation over time \( t_2 - t_1 \). Noticing (2.6) through (2.8) and (2.11) through (2.13), we get the following equations:

Continuity equation:

\[ \frac{\partial U_i}{\partial x_i} = 0 \] (2.14)

Momentum equation:

\[ \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho_r} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( v \frac{\partial U_i}{\partial x_j} - u_i u_j \right) + \frac{\beta g_i}{C_p} (H_r - H) \] (2.15)

Energy equation:

\[ \frac{\partial H}{\partial t} + U_i \frac{\partial H}{\partial x_i} = \frac{\partial}{\partial x_i} \left( a \frac{\partial H}{\partial x_i} - u_i h \right) + \frac{S_H}{\rho} \] (2.16)

Concentration equation:

\[ \frac{\partial C}{\partial t} + U_i \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \gamma \frac{\partial C}{\partial x_i} - u_i c \right) + \frac{S_C}{\rho} \] (2.17)

These are the equations governing the mean flow quantities \( U, P, H \) and \( C \). But they no longer form a closed set since the averaging process introduces the unknown correlations between fluctuating velocities, \( \overline{u_i u_j} \), and between velocity and scalar.
fluctuations, \( \overline{u_i h} \) and \( \overline{u_i c} \). Physically, these correlations, multiplied by density \( \rho \), represent the transport of momentum, heat and mass due to the fluctuating (i.e. turbulent) motion. \(-\rho u_i u_j\) is the transport of \( x_i \) - momentum in the direction of \( x_j \). It acts as a stress on the fluid and is called turbulent or Reynolds stress. \(-\rho u_i h\) and \(-\rho u_i c\) are the transports of heat and mass in the direction of \( x_i \). They are heat and mass fluxes. In most flow regions, the turbulent stresses and fluxes are much larger than their laminar counterparts \( \nu \partial U_j / \partial x_j \), \( a \partial \theta / \partial x_i \) and \( \gamma \partial C / \partial x_i \) which are therefore often negligible.

Equations (2.14) through (2.17) can not be solved until the turbulence correlations \( \overline{u_i u_j} \), \( \overline{u_i h} \) and \(-\overline{u_i c}\) can be determined in some way. In fact, the determination of these correlations forms the main problem of turbulence modeling which is essentially a closure problem of simultaneous equations.

**Eddy-viscosity concept.** This concept was proposed by Bousinesq (1877) and forms the basis of most turbulence models. It assumes, similar to the viscous stresses in laminar flows, the turbulent stresses \(-\rho u_i u_j\) are proportional to the mean velocity gradient, i.e.

\[
-\overline{u_i u_j} = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} \quad (2.18)
\]

where \( \nu_t \) is the turbulent or eddy viscosity \( [m^2/s] \). Unlike the molecular kinematic viscosity \( \nu \), \( \nu_t \) is not a fluid property but depends strongly on the state of turbulence. It may vary significantly from one point in the flow to another and also from flow to flow. \( \delta_{ij} \) is the Kronecker delta whose value is 1 when \( i = j \) and 0 when \( i \neq j \). \( k \) is the kinetic energy of the turbulent motion \( [J/kg] \). Let \( i = j = 1, 2 \) and 3 respectively in equation (2.18) will yield three equations representing normal stresses. Then add the three equations together and notice the continuity equation (2.14), we have:

\[
k = \frac{1}{2} \left( \overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2} \right) \quad (2.19)
\]

The eddy-viscosity concept was conceived by an analogy between the molecular motion and the turbulent motion. The turbulent eddies were thought of as lumps of fluid
which, like molecules, collide and exchange momentum. The molecular viscosity $v$ is proportional to the average velocity and mean free path of the molecules. Accordingly, the eddy viscosity $v_i$ is considered proportional to a velocity, $\dot{V}$, characterizing the fluctuating motion, and to a typical length of this (large-scale) motion which Prandtl called "mixing length", $L$. That is:

$$v_i \propto \dot{V}L$$  \hspace{1cm} (2.20)

It should be pointed out that the assumption of (2.18) is not always correct and realistic. But in spite of all the controversy, the eddy-viscosity concept has proved successful in many practical applications.

**Eddy-diffusivity concept.** The eddy-diffusivity concept assumes that the turbulent heat and mass transport is related to the gradient of the transported quantity, i.e.

$$- u_i h = \Gamma_H \frac{\partial H}{\partial x_i}$$  \hspace{1cm} (2.21)

$$- u_i c = \Gamma_C \frac{\partial C}{\partial x_i}$$  \hspace{1cm} (2.22)

where $\Gamma_H$ and $\Gamma_C$ are the turbulent diffusivities of heat and mass $[m^2/s]$. Like the turbulent viscosity, $\Gamma_H$ and $\Gamma_C$ are not fluid properties but dependent on the state of the turbulence. Likewise, the eddy-diffusivity concept is not valid in certain regions. $\Gamma_H$ and $\Gamma_C$ depend on the direction of the heat and mass fluxes. Experiments have shown that the ratios between $v_i$ and $\Gamma_H$ and between $v_i$ and $\Gamma_C$ change very little across any flow and from flow to flow, i.e.

$$\sigma_H = \frac{v_i}{\Gamma_H}$$  \hspace{1cm} (2.23)

* The mixing length $L$ is defined in the following way (Rodi 1984). When a fluid lump traveling at its original mean velocity is displaced due to the turbulent motion in the transverse direction from $y_1$ to $y_2$, its velocity differs from the surrounding mean velocity at $y_2$ by $\Delta U$. The distance $y_2 - y_1$ at which $\Delta U$ is equal to the mean transverse fluctuation velocity is the mixing length.
and

$$\sigma_C = \frac{v_i}{\Gamma_C}$$  \hspace{1cm} (2.24)

can be assumed to be constants. $\sigma_H$ and $\sigma_C$ are the turbulent Prandtl number and Schmidt number (dimensionless).

Substitute (2.18) into (2.15), (2.21) into (2.16), (2.22) into (2.17), and notice (2.14) we have:

Momentum equation:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial}{\partial x_i} \left( p + \frac{2}{3} \rho k \right) + (v + v_i) \frac{\partial U_i}{\partial x_j} \frac{\partial x_j}{\partial x_i} + \frac{\beta g_i}{C_p} (H_r - H)$$  \hspace{1cm} (2.25)

Energy equation:

$$\frac{\partial H}{\partial t} + U_i \frac{\partial H}{\partial x_i} = \left( \alpha + \Gamma_H \right) \frac{\partial^2 H}{\partial x_i \partial x_i} + \frac{S_H}{\rho}$$  \hspace{1cm} (2.26)

Concentration equation:

$$\frac{\partial C}{\partial t} + U_i \frac{\partial C}{\partial x_i} = \left( \gamma + \Gamma_C \right) \frac{\partial^2 C}{\partial x_i \partial x_i} + \frac{S_C}{\rho}$$  \hspace{1cm} (2.27)

Now four new variables $v_i$, $k$, $\Gamma_H$ and $\Gamma_C$, but only two additional equations (2.23) and (2.24) are introduced. In order to make the above equations be a closed set, two more independent equations are needed. In fact $k$ is absorbed by pressure $P$ so that a new unknown pressure variable $P + 2 \rho k / 3$ is formed. Therefore it is the distribution of the eddy viscosity $v_i$ that has to be determined now. The popular $k-\varepsilon$ model just came out of the study process of $v_i$.

The $k-\varepsilon$ model:

We know from equation (2.19) that $k$ is a direct measure of the intensity of the turbulence fluctuations in the three dimensional space. As $k$ is contained mainly in the
large-scale fluctuations, \( \sqrt{k} \) is a velocity scale for the large-scale turbulent motion [m/s]. According to Kolmogorov-Prandtl, equation (2.20) can be written as:

\[
v_t = C'_\mu \sqrt{k} L \tag{2.28}
\]

where \( C'_\mu \) is an empirical constant (dimensionless).

According to its definition, \( k \) can be found to satisfy the following conservation equation through mathematical manipulations of the Navier-Stokes equation (2.2) and the Reynolds time-mean equation (2.15) for high Reynolds numbers:

\[
\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \nu_t \frac{\partial k}{\partial x_i} \right) + G - \varepsilon \tag{2.29}
\]

where

\[
G = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j} + \frac{\beta g_i}{C_p} \frac{\partial H}{\partial x_i} \tag{2.30}
\]

\( \varepsilon \) is the dissipation rate of turbulence energy [W/kg] and it is found to satisfy the following equation:

\[
\frac{\partial \varepsilon}{\partial t} + U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \nu_t \frac{\partial \varepsilon}{\partial x_i} \right) + C_\varepsilon \frac{\varepsilon}{k} G - C_\varepsilon^2 \frac{\varepsilon^2}{k} \tag{2.31}
\]

\( \varepsilon \) is modeled by the expression:

\[
\varepsilon = C_D \frac{k^{3/2}}{L} \tag{2.32}
\]

where \( C_D \) is an empirical constant (dimensionless). Combine equations (2.28) and (2.32) to eliminate the length scale variable \( L \) we have:
Conservation Laws and Turbulence Modeling

\[ \nu_t = C_\mu \frac{k^2}{\varepsilon} \]  \hspace{1cm} (2.33)

where \( C_\mu = C_\mu' C_D \).

Until now all the unknown variables are modeled and the equations describing them form a closed set again. Due to the similarity in forms of the above conservation equations, we can write equations (2.14), (2.25), (2.26), (2.27), (2.29) and (2.31) as a general equation:

\[ \frac{\partial \phi}{\partial t} + \text{div} \left( \rho \vec{V} \phi - \Gamma_{\phi,\text{eff}} \text{grad} \phi \right) = S_\phi \]  \hspace{1cm} (2.34)

where \( \vec{V} \) is the velocity vector, \( \text{div} \) represents divergence, dependent variable \( \phi \) and the corresponding effective exchange coefficient \( \Gamma_{\phi,\text{eff}} \) as well as the source term \( S_\phi \) are listed in Table 2.1

| Table 2.1 Variables, Coefficients and Source Terms in equation (2.34) |
|-----------------|---|-----------------|---|
| Equation        | \( \phi \) | \( \Gamma_{\phi,\text{eff}} \) | \( S_\phi \) |
| Continuity      | 1  | 0               | 0  |
| Momentum        | \( U_i \) | \( \rho (\nu + \nu_t) \) | \(-\partial P/\partial x_i + \rho \beta g_i (H_i - H)/C_p \) |
|                 | \( k \)  | \( \rho \nu_t/\sigma_k \) | \( G_k - \rho \varepsilon \) |
|                 | \( \varepsilon \) | \( \rho \nu_t/\sigma_\varepsilon \) | \( \varepsilon (C_1 \varepsilon_k - C_2 \rho \varepsilon)/k \) |
| Enthalpy        | \( H \)  | \( \lambda/C_p + \rho \nu_t/\sigma_H \) | \( S_H \) |
| Concentration   | \( C \)  | \( \rho \gamma + \rho \nu_t/\sigma_C \) | \( S_C \) |

\[ \beta = (\rho - \rho_t)/(\rho(\theta_i - \theta)) \],  \hspace{1cm} \nu_t = C_\mu k^2/\varepsilon \,

\[ G_k = \rho G = \rho \nu_t \left( \frac{\partial U_i}{\partial x_i} + \frac{\partial U_j}{\partial x_j} \right) + \rho \beta g_i \nu_t \frac{\partial H}{\partial x_i}, \]

\[ C_1 = 1.44, C_2 = 1.92, C_\mu = 0.09, \sigma_k = 1.0, \sigma_\varepsilon = 1.3, \sigma_H = 0.9, \sigma_C = 1.0 \]

In Table 2.1, there are 10 variables to be solved. They are \( U_i \) \((i = 1, 2, 3)\), \( k \), \( \varepsilon \), \( \rho \), \( P \), \( H \), \( C \) and \( \nu_t \). For the momentum, enthalpy and concentration equations, the effective exchange coefficients consist of both the laminar exchange coefficients \( (\rho \nu, \lambda/C_p \) and
\( \rho \gamma \) and the turbulence exchange coefficients \((\rho v_c, \rho v_H/\sigma_H \text{ and } \rho v_C/\sigma_C)\). In most turbulent flows, the laminar exchange coefficients can be neglected.

§2.2 Boundary Conditions

In the preceding section, the differential transport equations describing turbulent flows can be solved only when the turbulent boundary conditions are specified. The most commonly encountered boundaries in studying indoor air flows are wall boundaries. The wall boundary layer is composed of three regions: the viscous sublayer adjacent to the wall, where the viscous effects are predominant \((0 < y^+ \leq 5)\), the inertial sublayer where the turbulent effects are overwhelming \((30 \leq y^+ \leq 130)\), and the buffer layer in between them where neither the viscous stress nor the Reynolds stress can be neglected \((5 < y^+ < 30)\). At the wall boundary surface, the no-slip condition applies so that both mean and fluctuating velocities are zero. Since the viscous effects become important near the wall and the local Reynolds number \((Re = Uy/\nu \text{ where } U \text{ is the velocity parallel to the wall at a distance } y \text{ from it})\) is very small, the above \(k-\varepsilon\) turbulence model for high Reynolds numbers is no longer applicable. Fortunately empirical laws of sufficient generality are available that connect the wall conditions (e.g. wall shear stress, heat flux and temperature) to the dependent variables in the inertial sublayer. The typical expression of the velocity for the wall boundary with zero pressure gradients and constant (turbulent) shear stress is the log-law wall function:

\[
 u^+ = \frac{1}{\kappa} \ln(Ey^+) 
\]  

(2.35)

for \(30 < y^+ < 130\), where \(u^+ = \text{a dimensionless velocity and } u^+ = U/u_\tau\), \(U = \text{velocity parallel to the wall [m/s], } u_\tau = \text{friction velocity [m/s], } y^+ = yu_\tau/\nu = \text{a dimensionless wall distance, } y = \text{distance from } U \text{ to the wall [m], } \kappa = \text{the von Kármán constant (} \kappa = 0.41\), and \(E = \text{a roughness parameter (} E = 9 \text{ for hydraulically smooth walls). } u_\tau \text{ is defined as}

\[
 u_\tau = \sqrt{\tau/\rho} 
\]  

(2.36)

where \(\tau = \text{viscous shear stress at the leading edge of the wall [N/m}^2\), i.e.
\[ \tau = \mu \frac{\partial U}{\partial y} \bigg|_{y=0} \]  

(2.37)

where \( \mu \) is the dynamic viscosity of the fluid [N·s/m²]. The boundary condition for temperature is

\[ \theta^* = 2.195 \ln \left( y^+ \right) + 13.2 \sigma - 5.66 \]  

(2.38)

where \( \sigma = \frac{v}{\nu} \) is the laminar Prandtl number (for air at room temperatures \( \sigma = 0.71 \)),

\[ \theta^* = \frac{\left( \theta_w - \theta \right)}{\theta_{\tau}}, \quad \theta_{\tau} = -\frac{v}{\sigma u_{\tau}} \left( \frac{\partial \theta}{\partial y} \right) \bigg|_{y=0} \]  

(2.39)

where \( \theta^* \) is a dimensionless temperature, \( \theta_w \) is the wall surface temperature [K or °C].

The boundary conditions for \( k \) and \( \varepsilon \) are:

\[ k = \frac{u_{\tau}^2}{\sqrt{C \mu}} \quad \text{and} \quad \varepsilon = \frac{u_{\tau}^4}{k \nu y^*} \]  

(2.40)

§2.3 Numerical Method in Computational Fluid Dynamics

Since there is not an exact mathematical solution for equation (2.34), numerical methods have to be introduced to get the discret solution. Details about various numerical schemes were given by Patankar (1980). Here we only introduce some of the basics and conventions that are often used in CFD.

1. For the dynamic part of equation (2.34), the fully implicit method (the backward difference method) is used to reduce the possibility of the divergence of numerical computations, and all fluid properties are presumed to be independent of position within a grid. For example, the time dependent term for grid P in Figure 2.1 is:

\[ \frac{\partial (\rho \phi)}{\partial t} = \frac{(\rho \phi)_p(t) - (\rho \phi)_p(t-1)}{\Delta t} \]  

(2.41)
Chapter 2 A Brief Description of Computational Fluid Dynamics

Figure 2.1 The Two Dimensional View of Some Grids

The convective and diffusive parts in equation (2.34) can be written as:

\[
\frac{\partial (\rho u \phi)}{\partial x} + \frac{\partial (\rho v \phi)}{\partial y} + \frac{\partial (\rho w \phi)}{\partial z} = \frac{\partial}{\partial x} \left( \Gamma \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma \frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left( \Gamma \frac{\partial \phi}{\partial z} \right)
\]

(2.42)

where \( \Gamma \) is the simplified notation of \( \Gamma_{\text{eff}} \). Discretize the above equation at grid P in Figure 2.1, we have:

\[
\frac{1}{(\Delta x)_P} \left[ (\rho u \phi)_e - (\rho u \phi)_w \right] + \frac{1}{(\Delta y)_P} \left[ (\rho v \phi)_n - (\rho v \phi)_s \right] + \frac{1}{(\Delta z)_P} \left[ (\rho w \phi)_i - (\rho w \phi)_b \right]
\]
\[
\frac{1}{(\Delta x)_p} \left[ \left( \Gamma \frac{\partial \phi}{\partial x} \right)_e - \left( \Gamma \frac{\partial \phi}{\partial x} \right)_w \right] + \frac{1}{(\Delta y)_p} \left[ \left( \Gamma \frac{\partial \phi}{\partial y} \right)_n - \left( \Gamma \frac{\partial \phi}{\partial y} \right)_s \right] \\
+ \frac{1}{(\Delta z)_p} \left[ \left( \Gamma \frac{\partial \phi}{\partial z} \right)_t - \left( \Gamma \frac{\partial \phi}{\partial z} \right)_b \right]
\]

\[
= \frac{1}{(\Delta x)_p} \left[ \Gamma_e \left( \frac{\partial \phi}{\partial x} \right)_e - \Gamma_w \left( \frac{\partial \phi}{\partial x} \right)_w \right] + \frac{1}{(\Delta y)_p} \left[ \Gamma_n \left( \frac{\partial \phi}{\partial y} \right)_n - \Gamma_s \left( \frac{\partial \phi}{\partial y} \right)_s \right] \\
+ \frac{1}{(\Delta z)_p} \left[ \Gamma_t \left( \frac{\partial \phi}{\partial z} \right)_t - \Gamma_b \left( \frac{\partial \phi}{\partial z} \right)_b \right] \\
\] (2.43)

where e (east), w (west), n (north), s (south), t (top) and b (bottom), represent the interfaces between grid P and grids E, W, N, S, T and B. Grids P, E, W, N, S, T and B represents both a cell and the cell center. \( u, v \) and \( w \) denote the velocity components in the directions of \( x, y \) and \( z \) respectively. Variables with e, w, n, s, t and b as subscripts are the interface values. For simplicity, Figure 2.1 only shows the 2 dimensional view on the \( x-y \) plane.

Equation (2.43) can be further written as

\[
\frac{1}{(\Delta x)_p} \left[ \rho_e u_e \phi_e - \rho_w u_w \phi_w \right] + \frac{1}{(\Delta y)_p} \left[ \rho_n v_n \phi_n - \rho_s v_s \phi_s \right] + \frac{1}{(\Delta z)_p} \left[ \rho_t w_t \phi_t - \rho_b w_b \phi_b \right]
\]

\[
= \frac{1}{(\Delta x)_p} \left[ \Gamma_e \left( \frac{\partial \phi}{\partial x} \right)_e - \Gamma_w \left( \frac{\partial \phi}{\partial x} \right)_w \right] \\
+ \frac{1}{(\Delta y)_p} \left[ \Gamma_n \left( \frac{\partial \phi}{\partial y} \right)_n - \Gamma_s \left( \frac{\partial \phi}{\partial y} \right)_s \right] \\
+ \frac{1}{(\Delta z)_p} \left[ \Gamma_t \left( \frac{\partial \phi}{\partial z} \right)_t - \Gamma_b \left( \frac{\partial \phi}{\partial z} \right)_b \right] \\
\] (2.44)
where \( \phi \) with subscripts P, E, W, N, S, T and B represents value of \( \phi \) at the corresponding grid centers.

We define:

\[
F_e = \rho_e u_e, \quad F_w = \rho_w u_w, \quad F_n = \rho_n v_n, \quad F_s = \rho_s v_s, \quad F_t = \rho_t w_t, \quad F_b = \rho_b w_b \tag{2.45}
\]

\[
D_e = \frac{\Gamma_e}{(\delta x)_e}, D_w = \frac{\Gamma_w}{(\delta x)_w}, D_n = \frac{\Gamma_n}{(\delta y)_n}, D_s = \frac{\Gamma_s}{(\delta y)_s}, D_t = \frac{\Gamma_t}{(\delta z)_t}, D_b = \frac{\Gamma_b}{(\delta z)_b} \tag{2.46}
\]

(2.44) becomes

\[
\frac{F_e}{(\Delta x)_p} \phi_e - \frac{F_w}{(\Delta x)_p} \phi_w + \frac{F_n}{(\Delta y)_p} \phi_n - \frac{F_s}{(\Delta y)_p} \phi_s + \frac{F_t}{(\Delta z)_p} \phi_t - \frac{F_b}{(\Delta z)_p} \phi_b
\]

\[
= \frac{D_e}{(\Delta x)_p} \phi_E + \frac{D_w}{(\Delta x)_p} \phi_W + \frac{D_n}{(\Delta y)_p} \phi_N + \frac{D_s}{(\Delta y)_p} \phi_S + \frac{D_t}{(\Delta z)_p} \phi_T + \frac{D_b}{(\Delta z)_p} \phi_B
\]

\[
- \left( \frac{D_e}{(\Delta x)_p} + \frac{D_w}{(\Delta x)_p} + \frac{D_n}{(\Delta y)_p} + \frac{D_s}{(\Delta y)_p} + \frac{D_t}{(\Delta z)_p} + \frac{D_b}{(\Delta z)_p} \right) \phi_p \tag{2.47}
\]

If the central difference scheme which assumes

\[
\phi_e = \frac{1}{2}(\phi_E + \phi_P), \quad \phi_w = \frac{1}{2}(\phi_P + \phi_W), \quad \phi_n = \frac{1}{2}(\phi_N + \phi_P), \tag{2.48}
\]

\[
\phi_s = \frac{1}{2}(\phi_P + \phi_S), \quad \phi_t = \frac{1}{2}(\phi_T + \phi_P), \quad \phi_b = \frac{1}{2}(\phi_P + \phi_B) \tag{2.49}
\]

is used, the numerical computation procedure might become divergent when the convective terms are much larger than the diffusive terms. Therefore the following upwind (or upstream) difference scheme is usually recommended.

2. The upwind difference is used in the convective terms and the “new” (i.e. end-of-time interval \( \Delta t \)) values are supposed to prevail throughout the time interval. That is:

26
\[ \phi_e = \phi_p \text{ and } \rho_e = \rho_p \text{ when } u_e \geq 0, \quad \phi_e = \phi_E \text{ and } \rho_e = \rho_E \text{ when } u_e < 0, \quad (2.50) \]
\[ \phi_w = \phi_w \text{ and } \rho_w = \rho_w \text{ when } u_w \geq 0, \quad \phi_w = \phi_p \text{ and } \rho_w = \rho_p \text{ when } u_w < 0, \quad (2.51) \]
\[ \phi_n = \phi_p \text{ and } \rho_n = \rho_p \text{ when } v_n \geq 0, \quad \phi_n = \phi_N \text{ and } \rho_n = \rho_N \text{ when } v_n < 0, \quad (2.52) \]
\[ \phi_s = \phi_s \text{ and } \rho_s = \rho_s \text{ when } v_s \geq 0, \quad \phi_s = \phi_p \text{ and } \rho_s = \rho_p \text{ when } v_s < 0, \quad (2.53) \]
\[ \phi_t = \phi_p \text{ and } \rho_t = \rho_p \text{ when } w_t \geq 0, \quad \phi_t = \phi_T \text{ and } \rho_t = \rho_T \text{ when } w_t < 0, \quad (2.54) \]
\[ \phi_b = \phi_B \text{ and } \rho_b = \rho_B \text{ when } w_b \geq 0, \quad \phi_b = \phi_p \text{ and } \rho_b = \rho_p \text{ when } w_b < 0, \quad (2.55) \]

Define MAX[a, b] equal to the larger one between a and b. From equations (2.50) - (2.55) we have:

\[ F_e \phi_e = \rho_p \phi_p \text{MAX}[u_e, 0] - \rho_E \phi_E \text{MAX}[-u_e, 0] \quad (2.56) \]
\[ F_w \phi_w = \rho_w \phi_w \text{MAX}[u_w, 0] - \rho_p \phi_p \text{MAX}[-u_w, 0] \quad (2.57) \]
\[ F_n \phi_n = \rho_p \phi_p \text{MAX}[v_n, 0] - \rho_N \phi_N \text{MAX}[-v_n, 0] \quad (2.58) \]
\[ F_s \phi_s = \rho_s \phi_s \text{MAX}[v_s, 0] - \rho_p \phi_p \text{MAX}[-v_s, 0] \quad (2.59) \]
\[ F_t \phi_t = \rho_p \phi_p \text{MAX}[w_t, 0] - \rho_T \phi_T \text{MAX}[-w_t, 0] \quad (2.60) \]
\[ F_b \phi_b = \rho_B \phi_B \text{MAX}[w_b, 0] - \rho_p \phi_p \text{MAX}[-w_b, 0] \quad (2.61) \]

Substitute equations (2.56) - (2.61) into (2.47) we get:

\[ a_p \phi_p = a_E \phi_E + a_W \phi_W + a_N \phi_N + a_S \phi_S + a_T \phi_T + a_B \phi_B \quad (2.62) \]

where
Chapter 2 A Brief Description of Computational Fluid Dynamics

\[ a_E' = \frac{1}{(\Delta x)_p} (D_e + \rho E \text{MAX}[-u_e, 0]), \quad a_W' = \frac{1}{(\Delta x)_p} (D_w + \rho W \text{MAX}[u_w, 0]), \]

\[ a_N' = \frac{1}{(\Delta y)_p} (D_n + \rho N \text{MAX}[-v_n, 0]), \quad a_S' = \frac{1}{(\Delta y)_p} (D_s + \rho S \text{MAX}[v_s, 0]), \]

\[ a_T' = \frac{1}{(\Delta z)_p} (D_t + \rho T \text{MAX}[-w_t, 0]), \quad a_B' = \frac{1}{(\Delta z)_p} (D_b + \rho B \text{MAX}[w_b, 0]), \]

\[ a_P' = a_E' + a_W' + a_N' + a_S' + a_T' + a_B' \]

\[ + \frac{\left( \rho_p \text{MAX}[u_e, 0] - \rho E \text{MAX}[-u_e, 0] \right) - \left( \rho W \text{MAX}[u_w, 0] - \rho_p \text{MAX}[-u_w, 0] \right)}{(\Delta x)_p} \]

\[ + \frac{\left( \rho_p \text{MAX}[v_n, 0] - \rho N \text{MAX}[-v_n, 0] \right) - \left( \rho S \text{MAX}[v_s, 0] - \rho_p \text{MAX}[-v_s, 0] \right)}{(\Delta y)_p} \]

\[ + \frac{\left( \rho_p \text{MAX}[w_t, 0] - \rho T \text{MAX}[-w_t, 0] \right) - \left( \rho B \text{MAX}[w_b, 0] - \rho_P \text{MAX}[-w_b, 0] \right)}{(\Delta z)_p} \]

Since the discretization of the continuity equation

\[ \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \quad (2.63) \]

will yield the same sum as the last three terms in \( a_P' \), we obtain:

\[ a_P' = a_E' + a_W' + a_N' + a_S' + a_T' + a_B' \quad (2.64) \]

3. Staggered grids are used for velocity components. That is:

\[ u_e = u_P, \quad u_w = u_W, \quad (2.65) \]
\[ v_n = v_P, \quad v_s = v_S, \quad (2.66) \]
\[ w_t = w_P, \quad w_b = w_B, \quad (2.67) \]

4. For the diffusion terms, the transport properties are arithmetic or harmonic averages of those of two adjacent cells, i.e.

\[ \Gamma_e = \frac{\Gamma_E + \Gamma_P}{2}, \quad \Gamma_w = \frac{\Gamma_P + \Gamma_W}{2}, \quad (2.68) \]
\[ \Gamma_n = \frac{\Gamma_N + \Gamma_P}{2}, \quad \Gamma_s = \frac{\Gamma_P + \Gamma_S}{2}, \quad (2.69) \]
\[ \Gamma_t = \frac{\Gamma_T + \Gamma_P}{2}, \quad \Gamma_b = \frac{\Gamma_P + \Gamma_B}{2} \quad (2.70) \]

or

\[ \frac{(\Delta x)_E + (\Delta x)_P}{\Gamma_e} = \frac{(\Delta x)_E}{\Gamma_E} + \frac{(\Delta x)_P}{\Gamma_P}, \quad \frac{(\Delta x)_P + (\Delta x)_W}{\Gamma_w} = \frac{(\Delta x)_P}{\Gamma_P} + \frac{(\Delta x)_W}{\Gamma_W}, \quad (2.71) \]
\[ \frac{(\Delta y)_N + (\Delta y)_P}{\Gamma_n} = \frac{(\Delta y)_N}{\Gamma_N} + \frac{(\Delta y)_P}{\Gamma_P}, \quad \frac{(\Delta y)_P + (\Delta y)_S}{\Gamma_s} = \frac{(\Delta y)_P}{\Gamma_P} + \frac{(\Delta y)_S}{\Gamma_S}, \quad (2.72) \]
\[ \frac{(\Delta z)_T + (\Delta z)_P}{\Gamma_t} = \frac{(\Delta z)_T}{\Gamma_T} + \frac{(\Delta z)_P}{\Gamma_P}, \quad \frac{(\Delta z)_P + (\Delta z)_B}{\Gamma_b} = \frac{(\Delta z)_P}{\Gamma_P} + \frac{(\Delta z)_B}{\Gamma_B} \quad (2.73) \]

Having the above discussions, we can get the final discret form of equation (2.34):

\[ a_P \phi_P(t) = a_E \phi_E(t) + a_W \phi_W(t) + a_N \phi_N(t) + a_S \phi_S(t) + a_T \phi_T(t) + a_B \phi_B(t) + b \quad (2.74) \]

where

\[ a_E = (\Delta y)_P(\Delta z)_P \left( D_e + p_E \text{MAX}[-u_P, 0] \right), \quad (2.75) \]
\[
\begin{align*}
    a_W &= (\Delta y)_p (\Delta z)_p \left( D_w + \rho_w \text{MAX}[u_w, 0] \right), \\
    a_N &= (\Delta x)_p (\Delta z)_p \left( D_n + \rho_N \text{MAX}[-v_p, 0] \right), \\
    a_S &= (\Delta x)_p (\Delta z)_p \left( D_s + \rho_S \text{MAX}[v_S, 0] \right), \\
    a_T &= (\Delta x)_p (\Delta y)_p \left( D_t + \rho_T \text{MAX}[-w_p, 0] \right), \\
    a_B &= (\Delta x)_p (\Delta y)_p \left( D_b + \rho_B \text{MAX}[w_B, 0] \right), \\
    a_p^0 &= \frac{\rho_p (\Delta x)_p (\Delta y)_p (\Delta z)_p}{\Delta t}, \\
    b &= S_{\phi} (\Delta x)_p (\Delta y)_p (\Delta z)_p + a_p^0 \phi_p (t - 1), \\
    a_p &= a_E + a_W + a_N + a_S + a_T + a_B + \frac{\rho_p (\Delta x)_p (\Delta y)_p (\Delta z)_p}{\Delta t}
\end{align*}
\]

In coefficient \(a_p\), the continuity equation is implicitly supposed to be satisfied so that the Scarborough criterion [the sufficient (but not necessary) condition for the convergence of the Gauss-Seidel (GS) method for solving algebra equations]:

\[
\sum |a_{nb}|/|a_p| \begin{cases} \leq 1 & \text{for all equations} \\ < 1 & \text{for at least one equation} \end{cases} \quad (2.84)
\]

is always satisfied at all grids, where the subscript of \(a_{nb}\), nb, represents E, W, N, S, T and B.

When equation (2.74) is solved, the relaxation technique is often used as follows:

\[
\phi_p (t)^{i+1} = \omega \left[ \phi_p (t)^{i+1} \right]_{\text{GS}} + (1 - \omega) \phi_p (t)^i
\]  

(2.85)
where $\omega$ is the relaxation coefficient and $0 < \omega < 2$. $0 < \omega < 1$ is known as under relaxation and $1 < \omega < 2$ is termed as over relaxation. $\phi_l(t)$ is the value at iteration $l$.

For the final computation of the flow field and temperature distribution, the SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) (Patankar and Spalding 1972) and the SIMPLER (SIMPLE Revised) (Patankar 1979) are extensively used.

§2.4 Discussion and Summary

To improve indoor thermal conditions, detailed indoor temperature distributions and air flows have to be predicted and modeled. There seems to be no other better way to make such predictions than the method of Computational Fluid Dynamics (CFD) until now.

Since the CFD theory is based on the numerical solution of turbulence models, this chapter gives a brief introduction of the conservation laws of mass, momentum and energy, turbulence modeling, boundary conditions and the numerical method. The widely used $k-\varepsilon$ turbulence model is emphasized here.

There is nothing new in the contents described in this chapter, but they constitute one of the theoretical bases of the following chapters that are devoted to the simplified modeling of dynamic indoor temperature distributions.
Chapter 2 A Brief Description of Computational Fluid Dynamics
Chapter 3

Visualization and Prediction of Indoor Air Flows

In this chapter, an air-conditioned test room that is used to make measurements and validate the results of theoretical calculations is introduced. By using this test room as a reference room in air-conditioned office buildings, the steady-state air flows are calculated with CFD method. Indoor air flows under typical heating and cooling situations are visualized by using smoke in the test room and compared with the CFD results.

Indoor temperature distributions are strongly influenced by air flows. The correct prediction of air flows will guarantee the correct modeling of indoor temperature distributions. Therefore, it is necessary to validate the CFD results with measurements for some typical heating and cooling situations. To carry out CFD calculations, we need to know the geometry of the room, furniture locations in the room, boundary conditions of walls, conditions of indoor heat sources, location of the supply air inlet, velocity and temperature of the supply air, location of the exhaust air outlet, etc. Finally, we need a CFD computer software (code) to carry out the calculations. The CFD software used in this thesis is the commercially available CFD code, PHOENICS from Cham Company.
§3.1 Test Room

To carry out various research related with the modeling and control of indoor thermal conditions and building energy consumption, a test room was built at the Laboratory of Refrigeration and Indoor Climate Technology, Delft University of Technology. The test room resembles typical air-conditioned rooms in some often seen office buildings. Data concerning the test room (Figure 3.1) is as follows:

Location and orientation:

\[
\begin{align*}
\text{Latitude} &= 52^\circ \quad \text{(North)} \\
\text{Longitude} &= 4^\circ 6' \quad \text{(East)} \\
\text{Facade (window surface) azimuth} &= 30^\circ \quad \text{(East)}
\end{align*}
\]

Measures:

\[
\begin{align*}
\text{Internal} &= 4.1 \text{ m} \times 3.1 \text{ m} \times 2.7 \text{ m} \\
\text{External} &= 4.4 \text{ m} \times 3.4 \text{ m} \times 3.0 \text{ m}
\end{align*}
\]

Window (double glazing):

\[
\begin{align*}
\text{Glass area} &= 3.1 \text{ m} \times 1.9 \text{ m} \\
\text{Height of window sill} &= 0.8 \text{ m} \\
\text{Double glazing thickness} &= 5\text{-}12\text{-}5 \text{ mm} \\
\text{Double glazing U-factor} &= 2.7 \text{ W/m}^2\text{-K}
\end{align*}
\]

Construction of walls, roof and floor:

- **Walls and roof:** Light-weight sandwich plates.
  - **Material:** Polystyrene with steel plate layers on both sides.
  - **Floor material:** Soil, polystyrene, plywood with aluminum plate on the top.

Heat source box:

\[
0.50 \text{ m} \times 0.60 \text{ m} \times 0.40 \text{ m}
\]

The heat source is to emulate the heat generated from a computer and a seated person. An incandescent lamp of 150 W is mounted inside the heat source box.


**Test Room**

**Table 3.1** Physical properties of the construction materials of the test room

<table>
<thead>
<tr>
<th>Type of envelope</th>
<th>Thickness [m]</th>
<th>Conductivity [W/m·K]</th>
<th>Density [kg/m³]</th>
<th>Specific heat [J/kg·K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall and roof:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>0.0006</td>
<td>52</td>
<td>7800</td>
<td>500</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.148</td>
<td>0.034</td>
<td>20</td>
<td>1300</td>
</tr>
<tr>
<td>Floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.0025</td>
<td>200</td>
<td>2800</td>
<td>880</td>
</tr>
<tr>
<td>Plywood</td>
<td>0.018</td>
<td>0.17</td>
<td>700</td>
<td>1880</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.15</td>
<td>0.034</td>
<td>20</td>
<td>1300</td>
</tr>
</tbody>
</table>

![Figure 3.1 Dimension of the test room and locations of some thermocouples](image-url)

Figure 3.1 Dimension of the test room and locations of some thermocouples
Figure 3.2 Locations of some thermocouples in the test room

Figure 3.3 Fan-coil unit
Figure 3.4 Air-conditioning control system in the test room

Thermocouples:

There 26 thermocouples mounted in the test room to measure indoor air temperatures (9 thermocouples which are located on the imagined plane in the middle of the room), inside surface temperatures of room envelopes (7 thermocouples), supply air temperature (4 thermocouples), recirculated air temperature (3 thermocouples), outdoor air temperature (1 thermocouple), heating water temperature (1 thermocouple) and cooling water temperature (1 thermocouple). Figures 3.1 and 3.2 shows some of the thermocouples.

One air speed sensor specifically designed to investigate thermal comfort levels is used to measure low air speeds at various places.
Chapter 3 Visualization and Prediction of Indoor Air Flows

Fan-coil unit: \[ 0.45 \text{ m} \times 1.00 \text{ m} \times 0.80 \text{ m} \]

The fan-coil unit is installed below the window (Figures 3.1 and 3.2). Hot water and chilled water are supplied to the coil (heat exchanger). The flow rates of the hot and chilled water can be changed by the control device (controller) through two proportional solenoid valves (Figures 3.3 and 3.4). In Figure 3.4, OA = Outside Air, RA = Recirculated Air, SA = Supply Air, CW = Chilled Water, HW = Hot Water. The size and shape of the louver at the top of the fan-coil unit is shown in Figure 3.5. The direction of the louver can be changed so that the supply air from the fan-coil unit can be guided either toward the room or toward the window.

![Figure 3.5 Louver of the fan-coil unit](image)

§3.2 Visualization and Prediction of Indoor Air Flows

By using the test room as a reference room, some CFD calculations are carried out. The grid number of $31 \times 26 \times 30$ is used for the room. The $k$-$\epsilon$ turbulence model is used for all calculations. The assumed boundary (wall surface) temperatures are 27 °C for cooling situations and 18 °C for heating situations. The supply air temperature is fixed at 17 °C for cooling situations and 30 °C for heating situations. The fan has three speeds: low, mid and high, that can be selected. Since the high fan speed is seldom used, or only used during extreme situations, we will not study the cases of high fan
speed. The supply air leaves the louver at about 1.6 m/s and 2.2 m/s for low and mid fan speeds respectively. The damper for recirculated air has the same size as the louver. The number of sweeps for each CFD calculation is set at 1000 ~ 2000. Boundary conditions are used for the variables $k$, $\varepsilon$ and three speed components ($u$, $v$ and $w$) in the directions of $x$, $y$ and $z$. For the calculation of enthalpy (or temperature) close to boundary walls, the convective heat transfer coefficient of a value of 3.0 W/m² K is used. This is a typical value often used for the simulation of building thermal responses by lots of people at the interior surfaces of rooms (van Paassen 1981).

The CFD calculations are three dimensional. But it is not necessary to show the air flow at every place of the room. In this thesis, we suppose that the room occupant is seated in the middle of the room. Therefore, we will concentrate on the air flow patterns at the imagined center plane of the room (see Figure 3.1), where the human body is most sensitive to local air temperature and speed. The air flow patterns is measured with smoke at this plane too.

Figure 3.6(a) Predicted air flow pattern (3D view)
(cooling situation, low fan speed, SA toward window)
Figure 3.6(b) Predicted air flow pattern (2D view) (cooling situation, low fan speed, SA toward window)

Figure 3.6(c) Measured air flow pattern with smoke (cooling situation, low fan speed, SA toward window)
Visualization and Prediction of Indoor Air Flows

Figure 3.7(a) Predicted air flow pattern
(cooling situation, low fan speed, SA toward room inside)

Figure 3.7(b) Measured air flow pattern with smoke
(cooling situation, low fan speed, SA toward room inside)
Figure 3.8(a) Predicted air flow pattern
(cooling situation, medium fan speed, SA toward window)

Figure 3.8(b) Measured air flow pattern with smoke
(cooling situation, medium fan speed, SA toward window)
Figure 3.9(a) Predicted air flow pattern
(cooling situation, medium fan speed, SA toward window)

Figure 3.9(b) Measured air flow pattern with smoke
(cooling situation, medium fan speed, SA toward window)
Figure 3.10(a) Predicted air flow pattern
(heating situation, low fan speed, SA toward window)

Figure 3.10(b) Measured air flow pattern with smoke
(heating situation, low fan speed, SA toward window)
Figure 3.11(a) Predicted air flow pattern
(heating situation, low fan speed, SA toward room inside)

Figure 3.11(b) Measured air flow pattern with smoke
(heating situation, low fan speed, SA toward room inside)
Chapter 3 Visualization and Prediction of Indoor Air Flows

The predicted and measured cases include: (1) cooling situations with low and medium fan speeds and with supply air (SA) guided toward the window and the room inside by the louver, and (2) heating situations with low fan speed and with supply air guided toward the window and the room inside.

Figures 3.6 through 3.11 show the predicted and measured air flow patterns at the center plane. Compare the predicted and visualized air flow patterns, we may find that the CFD result is in good agreement with the measurement.

It should be pointed out that the calculations are steady-state. Our measurements at 10 minutes intervals for one hour indicate that the air flow pattern for one situation almost does not change. The computing time is about 6 CPU hours on a SUN-Sparc (IPX) station.

For heating situations, since the low fan speed is very often used in practice, other fan speeds are not considered in this thesis.

§3.3 Discussion and Summary

In this chapter, a test room that resembles typical air-conditioned rooms in some often seen office buildings is introduced. The steady-state indoor air flow patterns in the test room are predicted with the CFD method. Visualization of air flow patterns with smoke are carried out in the test room. The calculated results are in good agreement with visualized results.

The most important phenomenon that we observe from the visualization is that the indoor air flow pattern for a fixed fan speed almost does not change.

From calculations we also discover that the CFD calculation takes considerable computing time. One steady-state simulation takes at least 6 CPU hours on a SUN-Sparc (IPX) station. Thus it is too expensive to use CFD for any dynamic simulation in building rooms.
Chapter 4

Simplified Dynamic Modeling of Indoor Temperature Distributions

It is widely admitted now that there exist dynamic temperature distributions in building rooms, which may influence the better control of indoor thermal conditions and may lead to more energy consumption. Since complete CFD simulation is too complicated and too time consuming for the prediction of such dynamic temperature responses, this chapter discusses some simplified modeling methods that can be used for control system designs and simulations. The models presented here are validated with experimental results. The effect of wall radiation on temperature measurement and on the conditions of thermal comfort is also considered.

CFD is perhaps the best method that can give good predictions of indoor temperature distributions and air flows. However from Chapters 1 and 2, we know that a room has to be divided into thousands of small finite volumes first. Then, the mass, momentum and energy balance equations of every finite volume has to be solved in an iterative way. Since the iteration has to be continued adequately until all dependent variables converge to some satisfactory extent, the CFD calculation takes so much computing time that it is only suitable for steady state simulations. If it is used to simulate the indoor dynamic temperature distributions and air flows of one hour on the basis of a time step of one or
two minutes, the computing time on the widely available personal computers such as PC-486 or PC-pentium or work stations like Sun-Sparc-IPX would be several days or even weeks. Obviously, this is unacceptably time consuming and expensive. We can not afford to carry out such dynamic simulations with the complete CFD method for the purpose of simulating indoor dynamic temperature responses.

On the other hand, the better control of the indoor thermal conditions requires the information of indoor dynamic temperature distributions and air flows. If we try to simulate the indoor dynamic temperature distributions under different control strategies, it seems the combination of CFD and automatic control theory is inevitable, which is in fact not feasible at present. Thus to solve this contradictory, a trade-off model of indoor temperature distributions and air flows has to be found. It can be imagined that in the process of searching such a model, some simplifications will have to be made and some information will have to be sacrificed. Since temperature is of primary importance for the control of indoor thermal conditions, its dynamics obviously can not be ignored. Then we are compelled to make some simplifications on the dynamics of indoor air flows.

§4.1 Modeling with Fixed Air Flow Fields

We may have noticed such a phenomenon from our experience of daily life and experiments that for a typical heating (or cooling) situation, although the indoor air flow in some room locations may vary with time due to the existence of turbulence and disturbances, the prevailing air flow field that has dominant effects on air mass transportation and temperature distribution is relatively stable (conclusion of Chapter 3, Peng 1994, Chen 1995). This phenomenon is especially apparent when the heating (or cooling) energy is supplied through an air-conditioning unit with forced air flows.

From the experience of using CFD codes, we also know that due to the nonlinearity of the momentum equations, under-relaxation has to be introduced in order to get converged results. This means that we intentionally slow down the solving process in order to get the correct air velocities. In fact, it is the solving process of the momentum equations (the velocity components) that takes most of the computing time of a CFD simulation.

What we face now is such a situation: on the one hand, the solving process of velocities takes huge amount of time; on the other hand, the indoor air flow pattern does not change much for forced convection air flows. Then, why should we spend so much computing time on calculating the air velocities at every time step. If we could use a fixed
air flow field for dynamic temperature calculations, then only the energy equation is left to be solved and a great of amount of computing time should be saved.

Three questions will have to be answered with this approximation: (1) Can this method give satisfactory predictions of indoor dynamic temperature responses? (2) How much computing time could be saved, or if the computing time can satisfy our requirement of dynamic control system designs and simulations? (3) Since this method still needs the solving of thousands of equations, is it possible to get any further simplified model?

From equation (2.32) in chapter 2 we know that the energy equation is:

$$\frac{\partial \rho H}{\partial t} + \text{div}(\rho \vec{V} H - \Gamma_{H,\text{eff}} \text{grad} H) = S_H$$ (4.1)

where $\Gamma_{H,\text{eff}} = \lambda/C_p + \rho \nu / \sigma_H$. If $\vec{V}$ and $\nu$ are calculated with a CFD code (PHOENICS is used in this thesis), it would be much easier to solve the energy equation. Suppose the temperature $\theta$ is to be solved, equation (4.1) becomes:

$$\frac{\partial \rho \theta}{\partial t} + \text{div}(\rho \vec{V} \theta - \Gamma_{H,\text{eff}} \text{grad} \theta) = \frac{S_H}{C_p}$$ (4.2)

The numerical method described in chapter 2 can be used to discretize the above equation. The final difference equation is (see Figure 2.1):

$$a_F \theta_p (t) = a_E \theta_E (t) + a_W \theta_W (t) + a_N \theta_N (t) + a_S \theta_S (t) + a_T \theta_T (t) + a_B \theta_B (t) + b$$ (4.3)

where

$$a_E = C_p (\Delta y)_p (\Delta z)_p \left[D_e + \rho \text{MAX}[-u_p, 0]\right]$$ (4.4)

$$a_W = C_p (\Delta y)_p (\Delta z)_p \left[D_w + \rho \text{MAX}[u_w, 0]\right]$$ (4.5)

$$a_N = C_p (\Delta x)_p (\Delta z)_p \left[D_n + \rho \text{MAX}[-v_p, 0]\right]$$ (4.6)

$$a_S = C_p (\Delta x)_p (\Delta z)_p \left[D_s + \rho \text{MAX}[v_s, 0]\right]$$ (4.7)
Chapter 4 Simplified Dynamic Modeling of Indoor Temperature Distributions

\[ a_T = C_p (\Delta x)_p (\Delta y)_p \left( D_t + \rho \text{MAX}[-w_p, 0] \right) \]  
(4.8)

\[ a_B = C_p (\Delta x)_p (\Delta y)_p \left( D_b + \rho \text{MAX}[w_B, 0] \right) \]  
(4.9)

\[ a_p^0 = \frac{C_p \rho (\Delta x)_p (\Delta y)_p (\Delta z)_p}{\Delta t} \]  
(4.10)

\[ b = S_H (\Delta x)_p (\Delta y)_p (\Delta z)_p + a_p^0 \theta_p (t-1) \]  
(4.11)

\[ a_p = a_E + a_W + a_N + a_S + a_T + a_B + a_p^0 \]  
(4.12)

\[ D_c = \frac{\Gamma_c}{(\delta x)_c}, D_w = \frac{\Gamma_w}{(\delta x)_w}, D_n = \frac{\Gamma_n}{(\delta y)_n}, D_s = \frac{\Gamma_s}{(\delta y)_s}, D_t = \frac{\Gamma_t}{(\delta z)_t}, D_b = \frac{\Gamma_b}{(\delta z)_b} \]  
(4.13)

The determination of the diffusion terms should be consistent with that used in the air velocity calculations in CFD. Here the arithmetic averages is used:

\[ \Gamma_c = \frac{\Gamma_E + \Gamma_p}{2}, \quad \Gamma_w = \frac{\Gamma_p + \Gamma_W}{2} \]  
(4.14)

\[ \Gamma_n = \frac{\Gamma_N + \Gamma_p}{2}, \quad \Gamma_s = \frac{\Gamma_p + \Gamma_S}{2} \]  
(4.15)

\[ \Gamma_t = \frac{\Gamma_T + \Gamma_p}{2}, \quad \Gamma_b = \frac{\Gamma_p + \Gamma_B}{2} \]  
(4.16)

If cell P is represented with coordinate \((i, j, k)\), then cell E = \((i+1, j, k)\), cell W = \((i-1, j, k)\), cell N = \((i, j+1, k)\), cell S = \((i, j-1, k)\), cell T = \((i, j, k+1)\), cell B = \((i, j, k-1)\). Therefore,

\[ (\Delta x)_p = x_i - x_{i-1}, \quad (\Delta y)_p = y_j - y_{j-1}, \quad (\Delta z)_p = z_k - z_{k-1} \]  
(4.17)

\[ (\delta x)_c = \frac{x_{i+1} - x_{i-1}}{2}, \quad (\delta x)_w = \frac{x_i - x_{i-2}}{2} \]  
(4.18)
(δy)_n = \frac{y_{j+1} - y_{j-1}}{2}, \quad (δy)_s = \frac{y_j - y_{j-2}}{2} \tag{4.19}

(δz)_i = \frac{z_{k+1} - z_{k-1}}{2}, \quad (δz)_b = \frac{z_k - z_{k-2}}{2} \tag{4.20}

Γ_p = Γ_{i,j,k} \tag{4.21}

Γ_E = Γ_{i+1,j,k}, \quad Γ_W = Γ_{i-1,j,k} \tag{4.22}

Γ_N = Γ_{i,j+1,k}, \quad Γ_S = Γ_{i,j-1,k} \tag{4.23}

Γ_T = Γ_{i,j,k+1}, \quad Γ_B = Γ_{i,j,k-1} \tag{4.24}

u_p = u_{i,j,k}, \quad v_p = v_{i,j,k}, \quad w_p = w_{i,j,k} \tag{4.25}

u_w = u_{i-1,j,k}, \quad v_s = v_{i,j-1,k}, \quad w_b = w_{i,j,k-1} \tag{4.26}

If we let \(a_{E,i,j,k} = a_E, a_{W,i,j,k} = a_W, a_{N,i,j,k} = a_N, a_{S,i,j,k} = a_S, a_{T,i,j,k} = a_T, a_{B,i,j,k} = a_B, D_{E,i,j,k} = D_E, D_{W,i,j,k} = D_W, D_{N,i,j,k} = D_N, D_{S,i,j,k} = D_S, D_{T,i,j,k} = D_T, D_{B,i,j,k} = D_B\), then equations (4.4) through (4.12) can be written as:

\[a_{E,i,j,k} = C_p (A_γΔz)_{i,j,k} \left(D_{E,i,j,k} + ρMAX[-u_{i,j,k}, 0]\right) \tag{4.27}\]

\[a_{W,i,j,k} = C_p (A_γΔz)_{i,j,k} \left(D_{W,i,j,k} + ρMAX[u_{i-1,j,k}, 0]\right) \tag{4.28}\]

\[a_{N,i,j,k} = C_p (A_γΔz)_{i,j,k} \left(D_{N,i,j,k} + ρMAX[-v_{i,j,k}, 0]\right) \tag{4.29}\]

\[a_{S,i,j,k} = C_p (A_γΔz)_{i,j,k} \left(D_{S,i,j,k} + ρMAX[v_{i-1,j,k}, 0]\right) \tag{4.30}\]

\[a_{T,i,j,k} = C_p (A_γΔz)_{i,j,k} \left(D_{T,i,j,k} + ρMAX[-w_{i,j,k}, 0]\right) \tag{4.31}\]
Chapter 4 Simplified Dynamic Modeling of Indoor Temperature Distributions

\[ a_{b,i,j,k} = C_p(\Delta t \Delta y)_{i,j,k}(D_{b,i,j,k} + p\text{MAX}[w_{i,j,k-1}, 0]) \]  
\[ a_{i,j,k}^0 = \frac{C_p \rho(\Delta t \Delta y \Delta z)_{i,j,k}}{\Delta t} \] 
\[ b = S_H(\Delta t \Delta y \Delta z)_{i,j,k} + a_{i,j,k}^0 \theta _{i,j,k}(t-1) \] 
\[ a_{i,j,k} = a_{E,i,j,k} + a_{w,i,j,k} + a_{N,i,j,k} + a_{s,i,j,k} + a_{T,i,j,k} + a_{B,i,j,k} + a_{i,j,k}^0 \]

Finally, equation (4.3) becomes:

\[ a_{i,j,k} \theta_{i,j,k}(t) = a_{E,i,j,k} \theta_{i+1,j,k}(t) + a_{w,i,j,k} \theta_{i-1,j,k}(t) + a_{N,i,j,k} \theta_{i,j+1,k}(t) \]
\[ + a_{s,i,j,k} \theta_{i,j-1,k}(t) + a_{T,i,j,k} \theta_{i,j,k+1}(t) + a_{B,i,j,k} \theta_{i,j,k-1}(t) + b \]  

With this equation, the dynamic temperature responses can now be calculated. It can be noted that the calculation needs to know the boundary surface temperatures, i.e. the temperatures of the internal surfaces of walls which are actually not known. This problem can be solve in two ways:

(1) List the heat balance equations of each wall, solve the equations together with the above equations for air temperature calculations.

(2) Due to their much higher heat capacities, the responses of most walls are much slower than that of the room air. Then, the solution of the heat balance equations of walls could be separated from the solution of the equations of the air. That is, as long as the time step is not too big (say less than 5 minutes), the wall surface temperatures at the previous time instant could be used for the solutions of the air temperatures at the present time instant. Then, the solved temperatures of the air adjacent to a wall can in turn be used to solve the surface temperature of the wall. If one wants to increase the accuracy of the solution, he may use the solved wall surface temperatures to calculate the air temperatures again, and proceed with several iterations until a satisfactory result is obtained. But generally speaking, such iteration is not necessary.
Effect of Wall Radiation on Temperature Measurement

For window surface temperature calculations, only the first method should be used since the response speed of the window glass can be comparable to that of the air.

Some useful formulas and results on the heat conduction inside walls, radiative heat transfer among inside wall surfaces and the influence of outdoor weather are listed in Appendix B.

§4.2 Effect of Wall Radiation on Temperature Measurement

It should be noted that the measured indoor temperature is almost always a combination of the local air temperature and the radiant temperatures of the surrounding walls if the temperature sensor is not well shielded. This is similar to the phenomenon that a person feels both the air temperature and the radiant temperatures of walls. Therefore, it is necessary to discuss the influence of wall radiation on the temperature measurement before we proceed to the validation of the modeling method in the previous section.

The total effect of the wall radiation is often described with a mean radiant temperature which is defined as the uniform radiant temperature of an imagined black enclosure in which the human body will give the same amount of radiant heat as in the actual enclosure (Fanger 1970, ASHRAE Handbook: Fundamentals 1985). Defined for human body, such a concept can equally be used for the temperature measuring element. Obviously, the mean radiant temperature is a function of the angle factors (or shape factors, view factors, configuration factor as used in some books) between the temperature sensor (or the human body) and the surrounding walls, and the surface temperatures of those walls. Two approximation formulas of its estimation are (Fanger 1970, ASHRAE Handbook: Fundamentals 1985):

\[ T_{mr}^4 = \sum_{i=1}^{N} T_i^4 F_{s,i} \]  

(4.37)

where \( T_{mr} = \theta_{mr} + 273.15 \) [K] and \( T_i = \theta_i + 273.15 \) [K] \((i = 1, 2, \ldots, N)\) are absolute mean radiant temperature and wall surface temperatures, \( N \) is the number of walls, \( \theta_i \) \((i = 1, 2, \ldots, N)\) is the surface temperature of wall \( i \) [°C]. \( F_{s,i} \) is the angle factor of the temperature sensor (or the human body) in relation to wall \( i \), and
\[ \theta_{\text{mr}} = \sum_{i=1}^{N} \theta_i F_{s-i} \]  
\[ (4.38) \]

In the above two formulas

\[ \sum_{i=1}^{N} F_{s-i} = 1 \]  
\[ (4.39) \]

Equation (4.38) always gives a slightly lower estimation of the mean radiant temperature than equation (4.37). But the difference can generally be ignored and equation (4.38) is more convenient to use than equation (4.37).

![Diagram of room dimensions and heat flow](image)

**Figure 4.1** Unfolded plane view of the surroundings walls of the test room in relation to the position of “low” thermal couple
Fanger (1970) gave the diagrams of calculating the angle factors of a seated and a standing person in relation to the vertical and horizontal walls. Using his diagrams, we can calculate the angle factors of the sensors at three measuring points (Figures 4.1, 4.3 and 4.4) in relation to the surrounding walls in the test room (Table 4.1). The radiation from the surfaces of the heat source and the fan-coil unit is neglected.

The formula for calculating the angle factors of a sphere such as a thermometer or a person’s head in relation to a finite rectangle (Figure 4.2) is (Mackey et al 1943, Jakob 1957, Billington 1967):

$$F_{dA_1-A_2} = \frac{1}{4\pi} \sin^{-1} \frac{XY}{\sqrt{1 + X^2 + Y^2 + X^2 Y^2}}$$  \hspace{1cm} (4.40)

where $X = b/c, Y = a/c$. Using this formula, the angle factors can be calculated again (Table 4.2).

**Table 4.1** Angle factors of the measuring points in relation to walls
[using Fanger’s diagrams]

<table>
<thead>
<tr>
<th>Angle factors of sensors</th>
<th>Floor</th>
<th>Ceiling</th>
<th>Window</th>
<th>Remaining walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor “low”</td>
<td>0.466</td>
<td>0.084</td>
<td>0.046</td>
<td>0.404</td>
</tr>
<tr>
<td>Sensor “mid”</td>
<td>0.214</td>
<td>0.170</td>
<td>0.060</td>
<td>0.556</td>
</tr>
<tr>
<td>Sensor “high”</td>
<td>0.143</td>
<td>0.268</td>
<td>0.064</td>
<td>0.525</td>
</tr>
</tbody>
</table>
Table 4.2 Angle factors of the measuring points in relation to walls  
[Calculated according to equation (4.40)]

<table>
<thead>
<tr>
<th>Angle factors of sensors</th>
<th>Floor</th>
<th>Ceiling</th>
<th>Window</th>
<th>Remaining walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor “low”</td>
<td>0.474</td>
<td>0.102</td>
<td>0.041</td>
<td>0.383</td>
</tr>
<tr>
<td>Sensor “mid”</td>
<td>0.237</td>
<td>0.195</td>
<td>0.061</td>
<td>0.507</td>
</tr>
<tr>
<td>Sensor “high”</td>
<td>0.161</td>
<td>0.289</td>
<td>0.064</td>
<td>0.486</td>
</tr>
</tbody>
</table>

Compare the corresponding values in Table 4.1 and Table 4.2, we may find that there are only small differences (the maximum relative difference is smaller than 15%). This means, the mean radiant temperature to a person can be well represented with the mean radiant temperature to a thermometer or thermocouple. Or in other words, there is no significant difference between them. But, formula (4.40) is much easier to use than diagrams.

The heat balance equation of a temperature sensor can be approximately written as:

\[
\frac{\partial C_{ps} V_s \rho_s \theta_s}{\partial t} = A_s h_{cs} (\theta_a - \theta_s) + A_s \varepsilon_s \varepsilon_w \sigma (T_{mr}^4 - T_s^4) \tag{4.41}
\]

where \( C_{ps} \), \( V_s \), \( \rho_s \), \( \theta_s \), \( A_s \) and \( \varepsilon_s \) are the specific heat [J/kg·K], volume (m³), density [kg/m³], temperature [K], surface area [m²] and emissivity of the sensor. \( h_{cs} \) is the convective heat transfer coefficient [W/m²·K] between the sensor and the air. \( T_s = \theta_s + 273.15 \) is the absolute temperature of the sensor [K]. \( \varepsilon_w \) is the averaged emissivity of walls. \( T_{mr} = \theta_{mr} + 273.15 \) is the absolute mean radiant temperature [K]. \( \sigma = 5.67 \times 10^{-8} \) is the Spexan-Boltzmann constant [W/m²·K⁴]. Suppose the shape of the sensor is a sphere and its size is very small (say its diameter = 2 × 10⁻³ m), then the left hand side of equation (4.41) can be assumed to be zero, i.e. the heat capacity of the sensor itself is neglected. The second part on the right hand side of equation (4.41) can be normally approximated as follows (Jakob 1957, Kimura 1974):

\[
A_s \varepsilon_s \varepsilon_w \sigma (T_{mr}^4 - T_s^4) = 4 A_s \varepsilon_s \varepsilon_w \sigma T^3 (\theta_{mr} - \theta_s) = A_s h_{fs} (\theta_{mr} - \theta_s) \tag{4.42}
\]
Effect of Wall Radiation on Temperature Measurement

where

\[
T = \frac{1}{2} (T_{mr} + T_s)
\]  \hspace{1cm} (4.43)

\[
h_{rs} = 4 \varepsilon_s \varepsilon_w \sigma T^3
\]  \hspace{1cm} (4.44)

\(h_{rs}\) is called the radiative heat transfer coefficient [W/m\(^2\).K]. Now we have:

\[
A_s h_{cs} (\theta_a - \theta_s) + A_s h_{rs} (\theta_{mr} - \theta_s) = 0
\]  \hspace{1cm} (4.45)

or

\[
\theta_s = \frac{h_{cs}}{h_{cs} + h_{rs}} \theta_a + \frac{h_{rs}}{h_{cs} + h_{rs}} \theta_{mr}
\]  \hspace{1cm} (4.46)

Let

\[
a = \frac{h_{cs}}{h_{cs} + h_{rs}}
\]  \hspace{1cm} (4.47)

the we have

\[
\theta_s = a \theta_a + (1 - a) \theta_{mr}
\]  \hspace{1cm} (4.48)

This is the normally used formula to describe the relation of the measured temperature, the air temperature and the mean radiant temperature of the surrounding walls. For the study of thermal comfort in building rooms, it is also used to depict the operative temperature (\(\theta_s\)) of the air-conditioning system in relation to the air temperature and the mean radiant temperature of the surrounding walls if the heat capacity of the human body clothing is neglected.

For nonmetallic materials (including glass) that are often used to construct walls, \(\varepsilon_w = 0.95\) at room temperature. For sensors composed of metals, their emissivities are influenced by the smooth conditions of their surfaces and materials, and may vary
greatly between 0.02 and 0.95. For the thermocouple (copper/constantan thermocouple) that we use, an approximate value of 0.25 can be used for \( \varepsilon_s \) (ASHRAE Handbook Fundamentals 1985 says the emissivity of tarnished copper is 0.20–0.30), i.e. \( \varepsilon_s = 0.25 \). Since the measurement is done at room temperature, \( T \) can be assumed to be 296.15 K (23.0 °C) without causing too much error. Thus for copper/constantan thermocouple:

\[
h_{rs} = 4\varepsilon_s\varepsilon_w\sigma T^3 = 4 \times 0.95 \times 0.25 \times 5.67 \times 10^{-8} \times 296^3 = 1.40
\]

For glass bulb thermometers:

\[
h_{rs} = 4\varepsilon_s\varepsilon_w\sigma T^3 = 4 \times 0.95 \times 0.95 \times 5.67 \times 10^{-8} \times 296^3 = 5.31
\]

The above two equations indicate that the radiative heat transfer coefficient for a glass bulb thermometer could be 4 times of that for a metallic temperature sensor.

If we suppose that the temperature sensor is spherical or cylindrical, then the convective heat transfer coefficient \( h_{cs} \) between metallic sensors and the room air under free convection conditions is (Hausen 1924, Jakob 1957):

\[
h_{cs} + h_{rs} = 7.44 + 5.54 \times 10^{-3} / D \tag{4.51}
\]

and for nonmetallic sensors such as glass bulb thermometers:

\[
h_{cs} + h_{rs} = 12.43 + 5.54 \times 10^{-3} / D \tag{4.52}
\]

where \( D \) is the diameter of the sensor [m].

Suppose \( D = 0.002 \) m, then for copper/constantan thermocouples:

\[
h_{cs} + h_{rs} = 10.21 \tag{4.53}
\]

\[
h_{cs} = 10.21 - h_{rs} = 10.21 - 1.40 = 8.81 \tag{4.54}
\]

\[
a = \frac{h_{cs}}{h_{cs} + h_{rs}} = \frac{8.81}{10.21} = 0.86 \tag{4.55}
\]
\[ \theta_s = 0.86 \theta_a + 0.14 \theta_{mr} \quad (4.56) \]

for nonmetallic sensors such as glass bulb thermometers:

\[ h_{cs} + h_{rs} = 15.20 \quad (4.57) \]
\[ h_{cs} = 15.20 - h_{rs} = 15.20 - 5.31 = 9.89 \quad (4.58) \]

\[ a = \frac{h_{cs}}{h_{cs} + h_{rs}} = \frac{9.89}{15.20} = 0.65 \quad (4.59) \]
\[ \theta_s = 0.65 \theta_a + 0.35 \theta_{mr} \quad (4.60) \]

Compare equations (4.49) with (4.50) and (4.54) with (4.58), we may discover that the convective heat transfer coefficient between the metallic sensor and air is not much different from that between the glass bulb thermometer and air, but the radiative heat transfer coefficients related with the sensors are quite different due to the big difference of their emissivities. Equations (4.56) and (4.60) explain that the metallic sensor measures about 86% local air temperature and 14% local mean radiant temperature, while the glass bulb thermometer of the same size only roughly measures 65% local air temperature and 35% local mean radiant temperature.

To reduce the radiative effect of walls on temperature measurement, shield should be used around the sensor, the air speed should be deliberately increased near the sensor, the surface of the sensor (only metallic sensor) should be polished, and the size of the sensor should be reduced.

For the outer surface of human body clothing (Nielsen and Pedersen 1952, Fanger 1970):

\[ h_{cs} = h_{cclo} = 2.38 (\theta_{clo} - \theta_a)^{0.25} \quad (4.61) \]

where \( \theta_{clo} \) is the surface temperature of the human body clothing [K or °C] and \( h_{cclo} \) is the convective heat transfer coefficient between the clothing and the surrounding air [W/m²·K]. Suppose \( \theta_{clo} = 34 \degree C, \theta_a = 23 \degree C \), if we ignore the heat capacity of the clothing and use the value of \( h_{rs} \) given by (4.50) for the clothing, then from equations (4.61) and (4.47) we have:
Chapter 4 Simplified Dynamic Modeling of Indoor Temperature Distributions

\[ h_{cs} = 2.38 \times (34 - 23)^{0.25} = 4.33 \]  \hspace{1cm} (4.62)

\[ a = \frac{h_{cs}}{h_{ts}} = \frac{4.33}{4.33 + 5.31} = 0.45 \]  \hspace{1cm} (4.63)

This means that the operative temperature can be approximately taken as:

\[ \theta_o = 0.45\theta_a + 0.55\theta_{mr} \]  \hspace{1cm} (4.64)

which is close to the normally used formula (Billington 1967, Lute 1992, Niu 1994):

\[ \theta_o = 0.50\theta_a + 0.50\theta_{mr} \]  \hspace{1cm} (4.65)

Equation (4.65) or (4.64) is an easy way to roughly estimate the influence of the air temperature and the mean radiant temperature on indoor thermal comfort. But it does not include the effect of air speed on thermal comfort and the information of draught. To obtain such information, one can turn to Fanger’s PMV, PPD and PD indexes (Fanger 1970, 1982, 1986, 1988).

It can be seen from the above discussion that although the operative temperature is different from the measured temperature of the temperature sensor, the mean radiant temperatures of the surrounding walls to them are almost the same.

§4.3 Model Validations

To validate the modeling method in section 4.1, some dynamic measurements are made in the test room. 26 temperature sensors (thermocouples) are mounted in the test room for temperature measurement (see chapter 3).

Figure 4.3 shows again the locations of the three thermocouples in the central part of the test room where the room occupant usually sits and works. The temperatures at these positions have dominant effect on local thermal comfort. The heights of the thermocouples are arranged in such a way that the temperatures at the ankle and neck positions of the room occupant (when seated) and the temperature at the head position (when standing) should be measured. For clarity, the names “high”, “mid” and “low” are assigned to the thermocouples to reflect their heights.
Figure 4.3 Locations of the three thermocouples in the test room

Figure 4.4 Equipment for temperature measurements
Figure 4.4 shows the equipment for temperature measurements and data acquisitions. Signals from the thermocouples are first transmitted to the data-logger. Inside the data-logger, the signals are amplified and filtered. Then, the processed signals are sent via the IEEE-488 bus to the personal computer for display and storage. The outdoor air temperature, wind speed and direction and solar radiation are measured at the outdoor weather station which is connected with the computer with an RS-232 serial communication port.

Validation 1: Cooling Case

The first measurement was made on 9 November 1995. Since the outdoor climate had already been cold, the test room was first heated up with the fan-coil unit in order to emulate a hot room (say in a hot summer) that needs to be cooled down. Then cooling air was introduced from the fan-coil unit to cool down the room. (Here again we see the inconvenience of experiments due to seasonal and outdoor climatic changes and the necessity and usefulness of models). The cooling air was supplied at low fan speed (the averaged supply air speed = 1.64 m/s) and was directed toward the window. No outdoor air was used. The exhaust air outlet was closed. A heat source of 105 W/m² is put inside the room to emulate the heat generated from a person and a computer. The measured temperatures of the ceiling, floor, window, side wall, and the supply air are illustrated in Figure 4.5. Since the temperatures of the front wall, back wall and two side walls are not much different from one another, an average of them is drawn in the figure. The mean radiant temperatures (MRT) to the three sensors at every time step are calculated based on the surface temperatures of the surrounding walls, equation (4.38) and Table 4.2. They are displayed in Figure 4.6. The measured indoor temperature responses at the three thermocouple positions are shown in Figure 4.7 where “Mea” stands for “Measurement”. Please note that each measured temperature is 86% local air temperature plus 14% local mean radiant temperature.

Remember that we have assumed that the air flow field does not change as long as the cooling (or heating) situation and the fan speed are kept unchanged, and there is no sudden disturbance like the random opening of door or windows. Then the steady-state air flow field can be precalculated with CFD by using assumed steady-state boundary conditions.

Suppose cooling is needed in the test room on a hot summer day, the calculated steady-state air flow field has been shown in Figure 3.6 and will be used for dynamic temperature calculations here.
Figure 4.5 Measured boundary temperatures

Figure 4.6 Mean radiant temperatures (MRT) to the thermocouples
Figure 4.7 Measured temperature responses

Figure 4.8 Calculation vs measurement at “high” thermocouple position
Figure 4.9 Calculation vs measurement at “mid” thermocouple position

Figure 4.10 Calculation vs measurement at “low” thermocouple position
Chapter 4 Simplified Dynamic Modeling of Indoor Temperature Distributions

With the calculated velocity field and effective exchange coefficients, the coefficients of equation (4.36) are calculated with equations (4.27) through (4.35). The indoor dynamic air temperature distributions are then calculated with the energy equation (4.36). Again the value of 3.0 W/m²·K has been adopted for the convective heat transfer coefficient between walls and the adjacent air volumes.

Although the surface temperatures of the surrounding walls can be calculated with the method described in section 4.1, the measured dynamic wall surface temperatures are used here so that the boundary temperatures of the calculation can be kept exactly the same as those of the measurement. In this way, errors introduced from the calculation of the wall surface temperatures can be eliminated and we can focus on the consequence of using the fixed flow field for dynamic calculations.

The initial indoor air temperatures are set to be equal to the measured initial air temperatures of the room, which are calculated based on the measured initial temperatures (Figure 4.7), the initial mean radiant temperatures (Figure 4.6) and equation (4.56).

With equation (4.56) and the measured boundary temperatures (Figure 4.6), the calculated indoor air temperatures are finally converted to a kind of temperature that can incorporate the effect of the radiation of walls. Figures 4.8 through 4.10 show the comparisons between the calculated results and the measured results at the three thermocouple positions. “Cal” means “Calculation”.

Compare the measured and calculated responses, we see the indoor dynamic temperature distributions (temperature responses and gradient) are well predicted. The computing time for such a dynamic simulation is about 20 CPU minutes on a SUN-Sparc (IPX) station, less than 6% of the CPU time consumed by the steady-state CFD simulation, let alone the dynamic CFD simulation.

Validation 2: Cooling Case

A similar measurement was done on 16 November 1996. This measurement was simply a repeat of the above measurement. The measured boundary temperatures are shown in Figure 4.11. The calculated mean radiant temperatures are displayed in Figure 4.12. The measured temperature responses are given in Figure 4.13.

Since this is the same cooling situation as Validation 1, new CFD calculation is not necessary. The same flow field and effective exchange coefficients can be used for this the dynamic temperature calculations. The calculated temperature responses are shown in Figures 4.14 through 4.16 with comparisons with the measurements.
Model Validations

Figure 4.11 Measured boundary conditions

Figure 4.12 Mean radiant temperatures (MRT) to the thermocouples
Figure 4.13 Measured temperature responses

Figure 4.14 Calculation vs measurement at “high” thermocouple position
Figure 4.15 Calculation vs measurement at “mid” thermocouple position

Figure 4.16 Calculation vs measurement at “low” thermocouple position
Validation 3: Heating Case

The third validation is a heating case. The measurement was made on 23 November 1995. The heated air was supplied at low fan speed toward the room. The room heat load was 105 W/m².

The measured wall surface temperatures are shown in Figure 4.17. The calculated mean radiant temperatures are illustrated in Figure 4.18. The measured temperatures at the three sensor positions are given in Figure 4.19.

With the steady-state air flow field calculated with CFD (Figure 3.11) and the measured dynamic wall surface temperatures, the dynamic air temperature distributions were calculated. The air temperatures were then converted to temperatures that included the effect of wall radiation with equation (4.56) at the sensor positions. Compare the calculation results and the measurement results (Figures 4.23 through 4.25), we may see that the dynamic temperature responses are well predicted once again.

![Figure 4.17 Measured boundary temperatures](image-url)
Figure 4.18 Mean radiant temperatures (MRT) to the thermocouples

Figure 4.19 Measured temperature responses
Chapter 4 Simplified Dynamic Modeling of Indoor Temperature Distributions

Figure 4.20 Calculation vs measurement at “high” thermocouple position

Figure 4.21 Calculation vs measurement at “mid” thermocouple position
§4.4 Air-Zone (Zonal) Model

From the standpoints of detailed predictions of dynamic indoor temperature distributions, the method given in section 4.1 really gives more realistic results than a well-mixed-air model, and faster results than dynamic CFD simulations. However, since it still needs the calculation of hundreds of thousands of coefficients and temperatures, this method is inconvenient to use and some improvement should be made on it.

As a matter of fact, we are not interested in too detailed information of indoor temperature distributions since most people are insensitive to minor temperature changes (e.g., < 0.5 °C) inside the room. Generally, there exist significant temperature differences between the working zone where the room occupant sits and works and the heat/cold source zone where the air-conditioning unit is located. Another obvious phenomenon is the temperature gradient of indoor air, which influences the local thermal comfort and the energy consumption of the room.
Figure 4.23 Air zones for cooling case and their borders

Thus, the division of the room into several air zones would give enough temperature information and accuracy for simulations and control system designs. If this could be done, the computing time would be further reduced dramatically and a much simpler and easy to use dynamic model would be available. But the critical problem is how to find the mass and heat exchanges between one zone and another.

According to the CFD theory, if the mass is balanced for every single finite air volume of the grid, then the mass is also balanced for any arbitrary group of finite air volumes. Starting from this point, we may reorganize the thousands of finite volumes used by CFD simulations and transform them into several air zones. Figure 4.23 shows the classification of air zones for the cooling case of low fan speed in the previous section. By supposing each air zone is well mixed and is depicted with one temperature, the room thermal response model can be represented with several temperatures and thus be much simplified.

It is obvious that the border between two adjacent air zones is also the border of the grids that are located on both sides of the border. The velocities $\bar{V}$ and exchange coefficients $\Gamma_{H,\text{eff}}$ of these grids can be used to calculate the mass and heat exchanges between two air zones.
Suppose each air zone is well mixed and only has one temperature. For zone 1 which has air temperature $\theta_1$ and air volume $V_1$, we have

$$
\frac{\partial V_1 \rho C_p \theta_1}{\partial t} = q_1 + A_{sa} w_{sa} \rho C_p \theta_{sa} + \sum_{k=k_{ra1}}^{k_{ra2}} \sum_{j=j_{ra1}}^{j_{ra2}} \sum_{i=i_{ra1}}^{i_{ra2}} C_p (\Delta x \Delta y)_{i,j,k} \left[ (D_{t,i,j,k} + \rho \text{MAX}[-w_{i,j,k},0]) \theta_2 - (D_{t,i,j,k} + \rho \text{MAX}[w_{i,j,k},0]) \theta_1 \right]
$$

$$
+ \sum_{k=k_{a1}}^{k_{a2}} \sum_{j=1}^{n_y} \sum_{i=i_{a1}}^{i_{a2}} C_p (\Delta y \Delta z)_{i,j,k} \left[ (D_{c,i,j,k} + \rho \text{MAX}[-u_{i,j,k},0]) \theta_3 - (D_{c,i,j,k} + \rho \text{MAX}[u_{i,j,k},0]) \theta_1 \right]
$$

$$
+ \sum_{k=k_{a1}}^{k_{a2}} \sum_{j=1}^{n_y} \sum_{i=i_{a1}}^{i_{a2}} C_p (\Delta y \Delta z)_{i,j,k} \left[ (D_{c,i,j,k} + \rho \text{MAX}[-u_{i,j,k},0]) \theta_4 - (D_{c,i,j,k} + \rho \text{MAX}[u_{i,j,k},0]) \theta_1 \right]
$$

$$
+ \sum_{k=k_{a1}}^{k_{a2}} \sum_{j=1}^{n_y} \sum_{i=i_{a1}}^{i_{a2}} C_p (\Delta y \Delta z)_{i,j,k} \left[ (D_{c,i,j,k} + \rho \text{MAX}[-u_{i,j,k},0]) \theta_5 - (D_{c,i,j,k} + \rho \text{MAX}[u_{i,j,k},0]) \theta_1 \right]
$$

where $q_1$ is the summation of all heat flux from walls to zone 1. $w_{sa}$ is the speed of the supply air leaving the louver. $A_{sa}$ is the area of the louver from where the heating/cooling air is supplied. $\theta_{sa}$ is temperature of the supply air. $i_a$, $k_a$, $k_p$ and $k_c$ are the grid indexes of air zones. $i_{ra1}$, $j_{ra1}$, $j_{ra2}$, $k_{ra1}$ and $k_{ra2}$ are the grid indexes of the opening of the recirculation air (the air that leaves zone 1 at the recirculation opening).

The above equation can be written as:

$$
\frac{\partial V_1 \rho C_p \theta_1}{\partial t} + a_{p,1} \theta_1 = a_{T,12} \theta_2 + a_{E,13} \theta_3 + a_{E,14} \theta_4 + a_{E,15} \theta_5 + a_{sa} \theta_{sa} + q_1 \quad (4.66)
$$

where

$$
a_{T,12} = \sum_{k=k_{a1}}^{k_{a2}} \sum_{j=1}^{n_y} \sum_{i=i_{a1}}^{i_{a2}} C_p (\Delta x \Delta y)_{i,j,k} \left( D_{t,i,j,k} + \rho \text{MAX}[-w_{i,j,k},0] \right) \quad (4.67)
$$
\[ a_{E,13} = \sum_{k=1}^{k_o} \sum_{j=1}^{n_y} \sum_{i=i_{i_d}}^{i_{o}} C_p (\Delta \alpha \Delta \rho)_{i,j,k} \left( D_{e,i,j,k} + \rho \max[-u_{i,j,k},0] \right) \]  
\( (4.68) \)

\[ a_{E,14} = \sum_{k=k_o+1}^{k_a} \sum_{j=1}^{n_y} \sum_{i=i_{i_d}}^{i_{o}} C_p (\Delta \alpha \Delta \rho)_{i,j,k} \left( D_{c,i,j,k} + \rho \max[-u_{i,j,k},0] \right) \]  
\( (4.69) \)

\[ a_{E,15} = \sum_{k=k_o+1}^{k_a} \sum_{j=1}^{n_y} \sum_{i=i_{i_d}}^{i_{o}} C_p (\Delta \alpha \Delta \rho)_{i,j,k} \left( D_{e,i,j,k} + \rho \max[-u_{i,j,k},0] \right) \]  
\( (4.70) \)

\[ a_{s_a} = A_s w_{s_a} \rho C_p \]  
\( (4.71) \)

\[ a_{P,1} = a_{T,12} + a_{E,13} + a_{E,14} + a_{E,15} + a_{s_a} + \]

\[ \sum_{k=k_1}^{k_a} \sum_{j=1}^{n_y} \sum_{i=i_{i_d}}^{i_{o}} C_p (\Delta \alpha \Delta \rho)_{i,j,k} \rho \left( \max[w_{i,j,k},0] - \max[-w_{i,j,k},0] \right) \]

\[ + \sum_{k=1}^{k_a} \sum_{j=1}^{n_y} \sum_{i=i_{i_d}}^{i_{o}} C_p (\Delta \alpha \Delta \rho)_{i,j,k} \rho \left( \max[u_{i,j,k},0] - \max[-u_{i,j,k},0] \right) \]

\[ + \sum_{k=k_o+1}^{k_a} \sum_{j=1}^{n_y} \sum_{i=i_{i_d}}^{i_{o}} C_p (\Delta \alpha \Delta \rho)_{i,j,k} \rho \left( \max[u_{i,j,k},0] - \max[-u_{i,j,k},0] \right) \]

\[ - A_{s_a} w_{s_a} \rho C_p - \sum_{k=k_1}^{k_a} \sum_{j=1}^{n_y} \sum_{i=i_{i_d}}^{i_{o}} (\alpha \Delta \rho)_{i,j,k} u_{i,j,k} \rho C_p \]
Air-Zone (Zonal) Model

\[
= a_{T,12} + a_{E,13} + a_{E,14} + a_{E,15} + a_{sa} + \sum_{k=k_c}^{k_e} \sum_{j=1}^{n_y} \sum_{i=1}^{i_a} C_p (\Delta x \Delta y)_{i,j,k} \rho u_{i,j,k}
\]

\[
+ \sum_{k=1}^{k_e} \sum_{j=1}^{n_y} \sum_{i=i_a}^{i_a} C_p (\Delta y \Delta z)_{i,j,k} \rho u_{i,j,k} + \sum_{k=k_n+1}^{k_e} \sum_{j=1}^{n_y} \sum_{i=i_a}^{i_a} C_p (\Delta y \Delta z)_{i,j,k} \rho u_{i,j,k}
\]

\[
+ \sum_{k=k_n+1}^{k_e} \sum_{j=1}^{n_y} \sum_{i=i_a}^{i_a} C_p (\Delta y \Delta z)_{i,j,k} \rho u_{i,j,k} - A_{sa} w_{sa} \rho C_p - \sum_{k=k_n+1}^{k_e} \sum_{j=1}^{n_y} \sum_{i=i_a}^{i_a} (\Delta x \Delta y)_{i,j,k} u_{i,j,k} \rho C_p
\]

\[
= a_{T,12} + a_{E,13} + a_{E,14} + a_{E,15} + a_{sa} +
\]

\[
\sum_{k=k_c}^{k_e} \sum_{j=1}^{n_y} \sum_{i=1}^{i_a} C_p (\Delta x \Delta y)_{i,j,k} \rho w_{i,j,k} + \sum_{k=k_c}^{k_e} \sum_{j=1}^{n_y} \sum_{i=i_a}^{i_a} C_p (\Delta y \Delta z)_{i,j,k} \rho u_{i,j,k}
\]

\[-A_{sa} w_{sa} \rho C_p - \sum_{k=k_n+1}^{k_e} \sum_{j=1}^{n_y} \sum_{i=i_a}^{i_a} (\Delta x \Delta y)_{i,j,k} u_{i,j,k} \rho C_p
\]

Since the mass is balanced for zone 1, the sum of the last four terms of \( a'_{p,1} \) is zero. We have:

\[
a'_{p,1} = a_{T,12} + a_{E,13} + a_{E,14} + a_{E,15} + a_{sa} \quad (4.72)
\]

Compare equations (4.67) - (4.70) with (4.27) - (4.32) we have:

\[
a_{T,12} = \sum_{k=k_c}^{k_e} \sum_{j=1}^{n_y} \sum_{i=1}^{i_a} a_{T,i,j,k} \quad (4.73)
\]

\[
a_{E,13} = \sum_{k=1}^{k_e} \sum_{j=1}^{n_y} \sum_{i=i_a}^{i_a} a_{E,i,j,k} \quad (4.74)
\]

77
Chapter 4 Simplified Dynamic Modeling of Indoor Temperature Distributions

\[ a_{E,14} = \sum_{k=k_x+1}^{k_y} \sum_{j=1}^{n_y} \sum_{i=i_y}^{i_x} a_{E,i,j,k} \]  \hspace{1cm} (4.75)

\[ a_{E,15} = \sum_{k=k_x+1}^{k_y} \sum_{j=1}^{n_y} \sum_{i=i_y}^{i_x} a_{E,i,j,k} \]  \hspace{1cm} (4.76)

The heat flux from the boundary walls is:

\[ q_1 = A_{\text{flr}} h_{\text{cflr}} (\theta_{\text{flr}} - \theta_1) + A_{\text{win}} h_{\text{cwin}} (\theta_{\text{win}} - \theta_1) \]

\[ + A_{\text{fwall}} h_{\text{cfwall}} (\theta_{\text{fwall}} - \theta_1) + A_{\text{swall}} h_{\text{cswall}} (\theta_{\text{swall}} - \theta_1) \]  \hspace{1cm} (4.77)

where \( A_{\text{flr}}, A_{\text{win}}, A_{\text{fwall}} \) and \( A_{\text{swall}} \) are the contact surface areas of the floor, window, front wall and side walls with zone 1. \( h_{\text{cflr}}, h_{\text{cwin}}, h_{\text{cfwall}} \) and \( h_{\text{cswall}} \) are the convective heat transfer coefficients between the room air and the floor, window, front wall and side walls. \( \theta_{\text{flr}}, \theta_{\text{win}}, \theta_{\text{fwall}} \) and \( \theta_{\text{swall}} \) are the surface temperatures of the floor, window, front wall and side walls.

Substituting equation (4.77) into equation (4.66), we have

\[ \frac{\partial V_1 \rho C_p \theta_1}{\partial t} + a_{P,1} \theta_1 = a_{T,12} \theta_2 + a_{E,12} \theta_2 + a_{E,14} \theta_4 + a_{E,15} \theta_5 + b_1 \]  \hspace{1cm} (4.78)

where

\[ a_{P,1} = a'_{P,1} + A_{\text{flr}} h_{\text{cflr}} + A_{\text{win}} h_{\text{cwin}} + A_{\text{fwall}} h_{\text{cfwall}} + A_{\text{swall}} h_{\text{cswall}} \]  \hspace{1cm} (4.79)

\[ b_1 = a_{sa} \theta_{sa} + A_{\text{flr}} h_{\text{cflr}} \theta_{\text{flr}} + A_{\text{win}} h_{\text{cwin}} \theta_{\text{win}} \]

\[ + A_{\text{fwall}} h_{\text{cfwall}} \theta_{\text{fwall}} + A_{\text{swall}} h_{\text{cswall}} \theta_{\text{swall}} \]  \hspace{1cm} (4.80)

Likewise, for zone 2:

\[ \frac{\partial V_2 \rho C_p \theta_2}{\partial t} + a_{P,2} \theta_2 = a_{B,21} \theta_1 + a_{E,25} \theta_5 + b_2 \]  \hspace{1cm} (4.81)
where

\[ a_{B,21} = \sum_{k=k_1+1}^{k_2+1} \sum_{j=1}^{n_y} \sum_{i=i_1}^{i_y} a_{B_i, j, k} \]  \hspace{1cm} (4.82) \\

\[ a_{E,25} = \sum_{k=k_1+1}^{n_k} \sum_{j=1}^{n_y} \sum_{i=i_1}^{i_y} a_{E_i, j, k} \]  \hspace{1cm} (4.83) \\

\[ a_{p,2} = a_{B,21} + a_{E,25} + A_{2\text{ceiling}} h_{\text{ceiling}} + A_{2\text{win}} h_{\text{cwin}} + A_{2\text{swall}} h_{\text{cswall}} \]  \hspace{1cm} (4.84) \\

\[ b_2 = A_{2\text{ceiling}} h_{\text{ceiling}} \theta_{\text{ceiling}} + A_{2\text{win}} h_{\text{cwin}} \theta_{\text{win}} + A_{2\text{swall}} h_{\text{cswall}} \theta_{\text{swall}} \]  \hspace{1cm} (4.85) \\

For zone 3:

\[ \frac{\partial V_3 p C_p \theta_3}{\partial t} + a_{p,3} \theta_3 = a_{W,31} \theta_1 + a_{T,34} \theta_4 + b_3 \]  \hspace{1cm} (4.86) \\

where

\[ a_{W,31} = \sum_{k=k_1}^{k_2} \sum_{j=1}^{n_y} \sum_{i=i_1+1}^{i_y+1} a_{W_i, j, k} \]  \hspace{1cm} (4.87) \\

\[ a_{T,34} = \sum_{k=k_1}^{n_k} \sum_{j=1}^{n_y} \sum_{i=i_1}^{n_x} a_{T_i, j, k} \]  \hspace{1cm} (4.88) \\

\[ a_{p,3} = a_{W,31} + a_{T,34} + A_{3\text{flr}} h_{\text{cflr}} + A_{3\text{swall}} h_{\text{cswall}} + A_{3\text{bwall}} h_{\text{cbwall}} \]  \hspace{1cm} (4.89) \\

\[ b_3 = A_{3\text{flr}} h_{\text{cflr}} \theta_{\text{flr}} + A_{3\text{swall}} h_{\text{cswall}} \theta_{\text{swall}} + A_{3\text{bwall}} h_{\text{cbwall}} \theta_{\text{bwall}} \]  \hspace{1cm} (4.90) \\

where the subscript "bwall" means back wall.

For zone 4:
\begin{equation}
\frac{\partial V_4 \rho C_p \theta_4}{\partial t} + a_{P,4} \theta_4 = a_{W,41} \theta_1 + a_{B,43} \theta_3 + a_{T,45} \theta_5 + b_4
\end{equation} \tag{4.91}

where

\begin{equation}
a_{W,41} = \sum_{k=k_0+1}^{k_0+1} \sum_{j=1}^{ny} \sum_{i=i_0+1}^{i+1} a_{W,i,j,k}
\end{equation} \tag{4.92}

\begin{equation}
a_{B,43} = \sum_{k=k_0+1}^{k_0+1} \sum_{j=1}^{ny} \sum_{i=i_0+1}^{nx} a_{B,i,j,k}
\end{equation} \tag{4.93}

\begin{equation}
a_{T,45} = \sum_{k=k_0}^{k_0} \sum_{j=1}^{ny} \sum_{i=i_0+1}^{nx} a_{T,i,j,k}
\end{equation} \tag{4.94}

\begin{equation}
a_{P,4} = a_{W,41} + a_{B,43} + a_{T,45} + A_{4swall} h_{cswall} + A_{4bwall} h_{cbwall}
\end{equation} \tag{4.95}

\begin{equation}
b_4 = A_{4swall} h_{cswall} \theta_{swall} + A_{4bwall} h_{cbwall} \theta_{bwall} + q_{hl}
\end{equation} \tag{4.96}

where \(q_{hl}\) is the heat load in zone 4.

For zone 5:

\begin{equation}
\frac{\partial V_5 \rho C_p \theta_5}{\partial t} + a_{P,5} \theta_5 = a_{W,51} \theta_1 + a_{W,52} \theta_2 + a_{B,54} \theta_4 + b_5
\end{equation} \tag{4.97}

where

\begin{equation}
a_{W,51} = \sum_{k=k_0+1}^{k_0+1} \sum_{j=1}^{ny} \sum_{i=i_0+1}^{i+1} a_{W,i,j,k}
\end{equation} \tag{4.98}

\begin{equation}
a_{W,52} = \sum_{k=k_0+1}^{k_0+1} \sum_{j=1}^{ny} \sum_{i=i_0+1}^{i+1} a_{W,i,j,k}
\end{equation} \tag{4.99}
Air-Zone (Zonal) Model

\[ a_{B,54} = \sum_{k=k_y+1}^{k_y+1} \sum_{j=1}^{ny} \sum_{i=i_x+1}^{nx} a_{B,i,j,k} \]  \hspace{1cm} (4.100)

\[ a_{P,5} = a_{W,51} + a_{W,52} + a_{B,54} + A_{5\text{ceiling}}h_{\text{ceiling}} + A_{5\text{swall}}h_{\text{swall}} + A_{5\text{bwall}}h_{\text{bwall}} \]  \hspace{1cm} (4.101)

\[ b_5 = A_{5\text{ceiling}}h_{\text{ceiling}}\theta_{\text{ceiling}} + A_{5\text{swall}}h_{\text{swall}}\theta_{\text{swall}} + A_{5\text{bwall}}h_{\text{bwall}}\theta_{\text{bwall}} \]  \hspace{1cm} (4.102)

Since the air flow field of the heating situation is different from that of the cooling situation, the division of the room into zones should also be different. Figure 4.24 shows that air zones in the test room, which are classified based on the flow field.

The heat balance equations can be obtained following the above equations. For example, for zone 1:

\[ \frac{\partial V_1 \rho C_p \theta_1}{\partial t} + a_{P,1} \theta_1 = a_{B,13} \theta_3 + a_{E,14} \theta_4 + a_{E,15} \theta_5 + b_1 \]  \hspace{1cm} (4.103)

where \( b_1 \) includes the heat flux into zone 1 from the window, ceiling and walls that have contact with zone 1 and from the air-conditioning unit.

Until now, we have obtained the simplified model that has only five temperatures to be solved. The coefficients in the model can be precalculated and later on be used many times for dynamic temperature calculations.

In the above modeling process, the heat flux from one air zone to a neighboring one is composed of two parts: 1. heat flux due to the air mass flows, and 2. heat flux due to the turbulent diffusivity and the thermal diffusivity of air. Therefore, unlike other simplified dynamic thermal models (chapter 1), this model does not ignore the very important physical phenomenon: turbulence and thermal conductivity of air. The prediction of indoor temperature distributions with this model is also more realistic.

It should be pointed out that, in the models of this section and the previous section, direct solar radiation is supposed to be completely shaded with a shading device such like an awning. Diffusion solar radiation entering the room can be distributed to every air zone, every wall, the ceiling and the floor. If Venetian blind is used as the shading device, solar radiation that hits the Venetian blind can be treated as a heat source added to zones 1 and 2 and the grid cells in which the blind is located.

The air zones are obtained based on the air flow pattern.
§4.5 State Space Representation of the Air-Zone Model

Whenever a control system is designed, a model of the form of either transfer function or state space is often needed. While transfer functions are suitable for SISO (Single Input Single Output) system designs with classical control theory, state space models are usually used for MIMO (Multi Input Multi Output) systems in modern control theory. When more than one indoor air temperature is to be predicted, the indoor thermal environment obviously becomes an MIMO system that has to be represented with a state space model. The air-zone model presented in the previous section can be just used to satisfy this requirement.

To be consistent with the modern control theory, we now change the definition of $x$ from coordinate of distance to states in a space and let it denote temperatures of air zones:
State Space Representation of the Air-zone Model

\[ x_1 = \theta_1, \ x_2 = \theta_2, \ x_3 = \theta_3, \ x_4 = \theta_4, \ x_5 = \theta_5, \]  
(4.104)

\[ \dot{x}_1 = \frac{\partial \theta_1}{\partial t}, \ \dot{x}_2 = \frac{\partial \theta_2}{\partial t}, \ \dot{x}_3 = \frac{\partial \theta_3}{\partial t}, \ \dot{x}_4 = \frac{\partial \theta_4}{\partial t}, \ \dot{x}_5 = \frac{\partial \theta_5}{\partial t}, \]  
(4.105)

\[ a_1 = V_1 \rho C_p, \ a_2 = V_2 \rho C_p, \ a_3 = V_3 \rho C_p, \ a_4 = V_4 \rho C_p, \ a_5 = V_5 \rho C_p \]  
(4.106)

For the cooling case, equations (4.78), (4.81), (4.86), (4.91) and (4.97) become:

\[ \dot{x}_1 = -\frac{a_{p,1}}{a_1} x_1 + \frac{a_{T,12}}{a_1} x_2 + \frac{a_{E,13}}{a_1} x_3 + \frac{a_{E,14}}{a_1} x_4 + \frac{a_{E,15}}{a_1} x_5 + \frac{b_1}{a_1} \]  
(4.107)

\[ \dot{x}_2 = -\frac{a_{p,2}}{a_2} x_1 - \frac{a_{p,2}}{a_2} x_2 + \frac{a_{E,25}}{a_2} x_5 + \frac{b_2}{a_2} \]  
(4.108)

\[ \dot{x}_3 = -\frac{a_{w,31}}{a_3} x_1 - \frac{a_{p,3}}{a_3} x_3 + \frac{a_{T,34}}{a_3} x_4 + \frac{b_3}{a_3} \]  
(4.109)

\[ \dot{x}_4 = -\frac{a_{w,41}}{a_4} x_1 + \frac{a_{B,43}}{a_4} x_3 - \frac{a_{p,4}}{a_4} x_4 + \frac{a_{T,45}}{a_4} x_5 + \frac{b_4}{a_4} \]  
(4.110)

\[ \dot{x}_5 = -\frac{a_{w,51}}{a_5} x_1 + \frac{a_{w,52}}{a_5} x_2 + \frac{a_{B,54}}{a_5} x_4 - \frac{a_{p,5}}{a_5} x_5 + \frac{b_5}{a_5} \]  
(4.111)

Representing equations (4.107) through (4.111) with a state space model we have:

\[ \dot{x} = Ax + Bu \]  
(4.112)

where

\[ x = [x_1, x_2, x_3, x_4, x_5]^T \]  
(4.113)

\[ u = [b_1, b_2, b_3, b_4, b_5]^T \]  
(4.114)
Chapter 4 Simplified Dynamic Modeling of Indoor Temperature Distributions

\[
\mathbf{A} = \begin{bmatrix}
- \frac{a_{p,1}}{a_1} & \frac{a_{T,12}}{a_1} & \frac{a_{E,13}}{a_1} & \frac{a_{E,14}}{a_1} & \frac{a_{E,15}}{a_1} \\
\frac{a_{B,31}}{a_2} & - \frac{a_{P,2}}{a_2} & 0 & 0 & \frac{a_{E,25}}{a_2} \\
\frac{a_{W,31}}{a_3} & 0 & - \frac{a_{P,3}}{a_3} & \frac{a_{T,34}}{a_3} & 0 \\
\frac{a_{W,41}}{a_4} & 0 & \frac{a_{B,43}}{a_4} & - \frac{a_{P,4}}{a_4} & \frac{a_{T,45}}{a_4} \\
\frac{a_{W,51}}{a_5} & \frac{a_{W,52}}{a_5} & 0 & \frac{a_{B,54}}{a_5} & - \frac{a_{P,5}}{a_5}
\end{bmatrix} \quad (4.115)
\]

\[
\mathbf{B} = \begin{bmatrix}
\frac{1}{a_1} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{a_2} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{a_3} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{a_4} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{a_5}
\end{bmatrix} \quad (4.116)
\]

where T represents transpose. \( \mathbf{A} \) is usually called system matrix. \( \mathbf{B} \) is named as input matrix. Vector \( \mathbf{u} \) contains the heat flux from the supply air.

The output vector is normally defined as:

\[
y = \mathbf{C} \mathbf{x} \quad (4.117)
\]

where

\[
y = [y_1, y_2, y_3, y_4, y_5]^T \quad (4.118)
\]
State Space Representation of the Air-zone Model

\[
C = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55}
\end{bmatrix}
\]  

(4.119)

The elements of the output (or measurement) matrix \( C \), \( c_{ij} \) (i, j = 1, 2, ..., 5), are determined according to the relation between the outputs and the states. For example, if the output vector \( y \) is equal to the state vector \( x \), then \( C = I \), where \( I \) is the unit matrix. If the system only has a single output, then \( C \) may be simply written as:

\[
C = \begin{bmatrix} c_1, c_2, c_3, c_4, c_5 \end{bmatrix}
\]  

(4.120)

If the system output is the air temperature of zone 4, then:

\[
C = [0, 0, 0, 1, 0]
\]  

(4.121)

If the output is the mean value of the air temperatures of zone 3 and zone 4, then:

\[
C = [0, 0, 0.5, 0.5, 0]
\]  

(4.122)

For the heating case, matrix \( A \) becomes

\[
A = \begin{bmatrix}
-a_{p,1} & 0 & a_{B,13} & a_{E,14} & a_{E,15} \\
a_1 & 0 & a_{a,1} & a_1 & 0 \\
0 & -a_{p,2} & a_{T,23} & 0 & 0 \\
a_{T,31} & a_{B,32} & a_{a,3} & a_{T,34} & a_{T,45}
\end{bmatrix}
\]  

(4.123)
§4.6 Validations of the State Space Model

With the formulas given in sections §4.4 and §4.5 and the flow fields calculated with CFD code, matrixes $A$ and $B$ can be easily obtained. For the cooling cases in the test room that is described in section §4.3, when the air zones are classified as follows (length by width by height):

Zone 1: $1.14 \text{ m} \times 3.1 \text{ m} \times 2.15 \text{ m}$  
Zone 2: $1.14 \text{ m} \times 3.1 \text{ m} \times 0.55 \text{ m}$  
Zone 3: $2.96 \text{ m} \times 3.1 \text{ m} \times 0.62 \text{ m}$  
Zone 4: $2.96 \text{ m} \times 3.1 \text{ m} \times 1.11 \text{ m}$  
Zone 5: $2.96 \text{ m} \times 3.1 \text{ m} \times 0.97 \text{ m}$

and the convective heat transfer coefficients between the air and the floor, ceiling, window and walls are assumed to be $3.0 \text{ W/m}^2\cdot\text{K}$, we may obtain:

$$A = \begin{bmatrix}
484.9 & 84.6 & 51.0 & 152.4 & 60.9 \\
49.1 & a_1 & a_1 & a_1 & a_1 \\
225.2 & 0 & -485.1 & 215.6 & 0 \\
64.1 & 0 & 390.0 & -532.3 & 48.2 \\
10.4 & 9.6 & 0 & 134.2 & -226.3
\end{bmatrix} \quad (4.124)$$

and $a_1 = 8685.7$, $a_2 = 2332.4$, $a_3 = 6826.9$, $a_4 = 12078$, $a_5 = 10681$.

Matrix $B$ is a function of the surface temperatures of boundary walls and can be obtained at every time step. With matrixes $A$ and $B$, the dynamic temperature responses of the air zones can be calculated for the cooling cases indicated in section §4.3. Since the thermocouples "low", "mid" and "high" are located in zones 3, 4 and 5 respectively, the air temperatures of the three zones should be combined with the mean radiant temperatures of the surrounding walls with equation (4.56) in order to compare with the
measured temperatures by the three thermocouples. The comparisons between the calculated results and the measured results for the two cooling cases are shown in Figures 4.25 through 4.30.

For the heating case, the test room is classified as follows:

Zone 1: 1.14 m × 3.1 m × 2.02 m
Zone 2: 4.10 m × 3.1 m × 0.16 m
Zone 3: 4.10 m × 3.1 m × 0.46 m
Zone 4: 2.96 m × 3.1 m × 1.11 m
Zone 5: 2.96 m × 3.1 m × 0.97 m

Then we have:

$$A = \begin{bmatrix}
\frac{368.1}{a_1} & 0 & \frac{45.6}{a_1} & \frac{122.6}{a_1} & \frac{54.7}{a_1} \\
0 & \frac{165.6}{a_2} & \frac{122.4}{a_2} & 0 & 0 \\
\frac{86.5}{a_3} & \frac{120.8}{a_3} & \frac{405.8}{a_3} & \frac{180.0}{a_3} & 0 \\
\frac{49.4}{a_4} & 0 & \frac{126.5}{a_4} & \frac{582.9}{a_4} & \frac{377.0}{a_4} \\
\frac{181.4}{a_5} & 0 & 0 & \frac{250.2}{a_5} & \frac{503.7}{a_5}
\end{bmatrix} \quad (4.125)$$

and $a_1 = 8723.7$, $a_2 = 2353.9$, $a_3 = 6767.5$, $a_4 = 12078$, $a_5 = 10681$.

The comparisons between the calculated temperatures of zones 2, 4, and 5 and the measured temperatures by the thermocouples “low”, “mid” and “high” in the heating case in section §4.3 are shown in Figures 4.31 through 4.33.

From the compared results, we may see that the indoor dynamic temperature distributions are well predicted. But since each air zone is assumed to be well mixed, it can be expected that the modeling discrepancy of this method might be bigger than that of the fixed-flow-field method occasionally. This can be seen from Figures 4.22 and 4.33. But the computing time has been reduced to less than 1 CPU second on a PC-486 (66 Mhz) computer.
Figure 4.25 Calculation vs measurement at zone 5 (cooling case 1)

Figure 4.26 Calculation vs measurement at zone 4 (cooling case 1)
Figure 4.27 Calculation vs measurement at zone 3 (cooling case 1)

Figure 4.28 Calculation vs measurement at zone 5 (cooling case 2)
Chapter 4 Simplified Dynamic Modeling of Indoor Temperature Distributions

Figure 4.29 Calculation vs measurement at zone 4 (cooling case 2)

Figure 4.30 Calculation vs measurement at zone 3 (cooling case 2)
Figure 4.31 Calculation vs measurement at zone 5 (heating case)

Figure 4.32 Calculation vs measurement at zone 4 (heating case)
§4.7 Discussion and Summary

Control system designs and simulations need a fast yet accurate enough model of the controlled plant. For the depiction of dynamic responses of indoor temperature distributions, CFD method should, theoretically, give the best prediction results. But it takes too much computing time and is not cost effective for the control study of indoor thermal conditions.

For rooms installed with air-conditioning units, the indoor air flow patterns are determined by the speed and direction of the supply air. If these two factors do not change, then the indoor air flow field can be thought of as time invariant (fixed). Based on this assumption, the air flow field needs to be solved only once. What is left to be solved is the energy equation. This forms the main idea of the simplified modeling methods of this chapter.

In this chapter, two simplified dynamic models are presented: the direct solution of the energy equations of all finite air volumes, and the air-zone model. The state space model is based on the air-zone model. Each of the models is validated with experimental results for both cooling and heating cases. The validations indicate that all
three models can give satisfactory and realistic predictions of the indoor dynamic
temperature distributions. The computing time used by the air-zone model and state
space model is less than 1 second on a PC-486-(66MHz) computer, or at least less than
0.0024% of that of the dynamic CFD simulation (the speed of a PC-486-(66MHz) is
comparable to that of a SUN-Sparc-1PX station).

The biggest advantage of the state space model is that it can be easily used for
control system designs. It can also be transformed into transfer functions so that the
frequency response analysis can be carried out. With a state space model, existing
modern control theories can find better applications in the prediction and control of the
indoor thermal conditions. Another advantage of the state space model is that it can be
easily used in the popular software MATLAB/Simulink for any control system
simulations.

Each element of matrix A contains the effects of both the heat transfer due to air
mass flows and the heat exchange due to turbulence and the free movement of air
molecules. Therefore, elements of matrix A can be artificially changed within a certain
extent without influencing the mass balance condition of each air zone. The noteworthy
point is that not one but two symmetric elements in matrix A should be changed
simultaneously. The equivalent result is that the turbulent effect is enhanced or
attenuated. The requirement of simultaneous change of two symmetric elements can be
seen from equations (4.13) through (4.16). Such changes might be necessary for model
fitting (curve fitting) and can be depicted with:

\[ a_{ij,\text{new}} = a_{ij,\text{old}} \pm \delta \]  (4.126)

\[ a_{ji,\text{new}} = a_{ji,\text{old}} \pm \delta \]  (4.127)

For example, in Figure 4.33, the temperature at “low” thermocouple is not predicted
as well as in the other figures. We may reduce the diffusion effect between zone 2 and
zone 3 by letting:

\[ \frac{a_{23,\text{new}}}{a_2} = \frac{a_{23,\text{old}} - 90.0}{a_2} \]  (4.128)
in matrix $A$ in equation (4.125), now we can get the new result which is much better than before (Figure 4.34). The temperatures of other air zones only change a little.

![Figure 4.34](image)

Figure 4.34 New calculation result vs measurement at “low” thermocouple after matrix $A$ is changed

For the commonly sized rooms with the air-conditioning unit installed at typical positions, the indoor air flow fields for cooling and heating situations can be precalculated with a CFD code. Matrix $A$ for those rooms and cooling and heating situations can also be obtained in advance. Thus an atlas can be made. Users can just pick a case from the atlas that can match his application.

For any model validations, it should be guaranteed that the calculated variable is validated with the same type of variable in measurement. As to the model validations of indoor thermal conditions, the temperatures in the models are air temperatures while the measured temperatures are combined effects of the air temperatures and the mean

\[
\frac{a_{32,\text{new}}}{a_3} = \frac{a_{32,\text{old}} - 90.0}{a_3} \quad (4.129)
\]
radiant temperatures of the surrounding walls. To compare the calculated results with
the measured results, the calculated temperatures must also be combined with the mean
radiant temperatures. The key point is what the mean radiant temperatures are and how
they influence the measured results. These questions are discussed in detail in this
chapter. The conclusion is that the measured temperature by a thermocouple is 86%
local air temperature plus 14% local mean radiant temperature. For human body
sensation of thermal comfort, the local air temperature and mean radiant temperature
contribute 50% each.

It should be known that there exist modeling errors for any kinds of models and
measurement errors for any types of measurements. The difference between one model
and another only lies in the variation of the degree of simplification and assumptions.
For the models of indoor dynamic temperatures, there are geometry errors, room
leakage errors, cut-off errors of algorithms, round-off errors of computers, etc.. Thus
error analysis of models is not an easy task to perform. This thesis will not proceed in
this aspect.
Chapter 5

Real-time Estimations of Indoor Temperatures

In real-time controls, there are usually some variables that need to be controlled or monitored but are either impossible to measure or too expensive to measure. The indoor thermal environment is just one of such cases. The unmeasured variables like temperatures should be estimated in some way so that they can be feedback to the controller in real time in case of necessity. This chapter discusses the estimation methods in such respect by introducing the state estimators of modern control theory. The designing procedures of the full-order estimator, reduced-order estimator and Kalman filter are explained. Some applications are illustrated by using the state space model developed in Chapter 4.

It has been pointed out in Chapter 4 that once the model of indoor dynamic temperature responses is written into a state space form, the modern control theories can find better applications in the prediction and control of the indoor thermal conditions. State estimator (also called observer) is just one of these applications.

It is self-evident that, in the real-time control of an industrial process, the more information is feedback to the controller, the better the system will be controlled. In classical control theory, only the system output (the measured variable) is used as
Chapter 5 Real-time Estimations of Indoor Temperatures

feedback. In modern control theory, not only the measured variable, but also other state variables can be used as feedbacks as long as they are known.

But a commonly seen problem in designing a controller is that some variables to be controlled or monitored are either impossible to measure or too expensive to measure. For instance, the idea of mounting 5 temperature sensors in an office room is obvious not feasible. The normal practice is that only one temperature sensor is used for one room. Thus it only measures the temperature of the air zone where it is located. If the room is depicted with several air zones and the temperatures of the zones other than the sensor-located zone need to be known, then they have to be estimated in some way.

Direct solution of the process model is a way of the estimations. However, there always exists modeling error in reality. If the error is small, the model is relatively accurate and can lead to good estimations of the process variables. This can be seen from the models in Chapter 4. But empirical knowledge indicates that the model used for designing a controller should not be too complicated, i.e., the order of the model should not be too high. This requirement inevitably limits the accuracy of the model. Thus the direct solution of the model in real-time controls may result in unfaithful estimations of the variables which, naturally, can not be used as real-time feedback information to the controller.

In real-time controls, there are always some measured variables. These measurements can be used not only as direct feedback information to the controller, but also to improve the reliability and accuracy of the estimations. It is obvious that the direct solution of the model does not make use of the measured variables. That is, no information from the real-time measurements is feedback to the estimations. We may expect that when such information is utilized, the real-time estimation results will be continuously updated and at least, unreasonable estimations can be prevented. This constitutes the main idea of state estimators.

§5.1 State Estimator for Temperature Estimations

From section 4.5 of chapter 4 we know that the state space model representing the indoor temperature distributions is

\[ \dot{x}(t) = Ax(t) + Bu(t) \quad (5.1) \]

\[ y(t) =Cx(t) \quad (5.2) \]
where the state vector $x(t)$ represents the temperatures of air zones. Output vector $y(t)$ is the measured temperature(s). To make the following results more general, the sizes of the vectors and matrixes in the system model (5.1) and (5.2) are defined as: $x \Rightarrow n \times 1$, $A \Rightarrow n \times n$, $B \Rightarrow n \times r$, $u \Rightarrow r \times 1$, $y \Rightarrow m \times 1$, $C \Rightarrow m \times n$, where $n$, $r$ and $m$ represent the number of indoor air zones, the number of inputs and the number of outputs respectively.

Let us assume that the system depicted by equations (5.1) and (5.2) is observable (see Appendix A), i.e., before the estimator is designed, the following observability condition is satisfied by the system:

$$\text{rank}\left[ C^T; A^T C^T; \ldots; (A^T)^{n-1} C^T \right] = n \quad (5.3)$$

where $T$ means transpose. Now, we wish to predict the real states with their estimates $\hat{x}$. To do so, information from the model as well as from the measurement should be utilized. The state estimator can be constructed by adding a correction term, $G[y(t) - \hat{y}(t)]$, to the state equation (5.1) and expressed as:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + G[y(t) - \hat{y}(t)] \quad (5.4)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (5.5)$$

or

$$\dot{\hat{x}}(t) = (A - GC)\hat{x}(t) + Bu(t) + Gy(t) \quad (5.6)$$

where $G$ is the feedback gain matrix of the estimator.

Let:

$$F = A - GC \quad (5.7)$$

$$H = B \quad (5.8)$$

The schematic diagram can be shown in Figure 5.1.
The design of a state estimator is actually the choice of matrix $G$. Matrix $G$ should at least be chosen in such a way that the real parts of the eigenvalues of matrix $F$ are negative, which guarantees the convergence of the estimator. Usually, $G$ is chosen to let the estimator give a much faster transient response or frequency response than the system.

The above obtained state estimator is for continuous systems. For the design of discrete state estimators, similar results can be obtained. If the system is continuous like the one depicted by equations (5.1) and (5.2), it can be discretized as:

$$x(k) = A_d x(k-1) + B_d u(k-1) \quad (5.9)$$

$$y(k) = C_d x(k) \quad (5.10)$$
where \( k \) is the sampling instant, \( A_d \) and \( B_d \) can be obtained from the following equations (see books on modern control theory, e.g., Ogata 1987)( suppose the system is time invariant):

\[
A_d = e^{A \Delta t} \tag{5.11}
\]

\[
B_d = A^{-1} \left( e^{A \Delta t} - I \right) B = \left( e^{A \Delta t} - I \right) A^{-1} B \tag{5.12}
\]

\[
C_d = C \tag{5.13}
\]

where \( \Delta t \) is the sampling period.

The observable condition for the system now becomes:

\[
\text{rank} \left[ C_d^T : A_d^T C_d^T : \cdots : (A_d^T)^{n-1} C_d^T \right] = n \tag{5.14}
\]

The discrete state estimator is written as follows:

\[
\hat{x}(k) = A_d \hat{x}(k-1) + B_d u(k-1) + G_d \left[ y(k-1) - \hat{y}(k-1) \right] \tag{5.15}
\]

\[
\hat{y}(k) = C_d \hat{x}(k) \tag{5.16}
\]

or

\[
\hat{x}(k) = (A_d - G_d C_d) \hat{x}(k-1) + B_d u(k-1) + G_d y(k-1) \tag{5.17}
\]

where \( G_d \) should be chosen in such a way that, at least, the magnitudes of all eigenvalues of matrix \( A_d - G_d C_d \) are less than unity. A common practice is to let the transient response of the estimator be 3 – 5 times faster than that of the system. Sometimes, deadbeat response may designed for the estimator.

When \( C_d \) is a \( 1 \times n \) vector, i.e., \( y(k) = y(k) \) is a scalar, a useful method of designing a state estimator is given by Ackermann’s formula:
\[ G_d = \phi(A_d) \begin{bmatrix} C_d & \vdots & 0 \\ C_dA_d & \vdots & 0 \\ \vdots & \vdots & \vdots \\ C_dA_d^{n-1} & \vdots & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \]  

(5.18)

where

\[ \phi(A_d) = A_d^n + \alpha_1 A_d^{n-1} + \cdots + \alpha_{n-1} A_d + \alpha_n I \]  

(5.19)

where \( \alpha_i \) \( (i = 1, 2, \cdots, n) \) are the coefficients of the desired characteristic equation of the estimator:

\[ |zI - A_d + G_d C_d| = \phi(z) = z^n + \alpha_1 z^{n-1} + \cdots + \alpha_{n-1} z + \alpha_n = 0 \]  

(5.20)

where \( z \) is the operator of the \( z \)-transformation.

The above discrete state estimator is the prediction-type estimator since the current measurements \( y(k) \) are not used to estimate the current states \( x(k) \) but the states at the next time instant \( x(k+1) \).

The current-estimation-type estimator can be obtained as follows:

\[ \hat{x}(k) = z(k) + G_{dc} [y(k) - C_d z(k)] \]  

(5.21)

\[ z(k) = A_d \hat{x}(k-1) + B_d u(k-1) \]  

(5.22)

or

\[ \hat{x}(k) = (A_d - G_{dc} C_d A_d) \hat{x}(k-1) + (B_d - G_{dc} C_d B_d) u(k-1) + G_{dc} y(k) \]  

(5.23)

where \( G_{dc} \) should be chosen in such a way that, at least, the magnitudes of all eigenvalues of matrix \( A_d - G_{dc} C_d A_d \) are less than unity.

Compare equation (5.23) with (5.17), we may find that \( C_d A_d \) takes the position of \( C_d \) while \( B_d - G_{dc} C_d B_d \) takes the position of \( B_d \) in equation (5.17). Thus, the current-estimation-type estimator can be designed also by using the Ackermann’s formula.
§5.2 A Case Study

For the cooling case in the test room shown in section §4.6 of chapter 4, we have:

\[
A = \begin{bmatrix}
-0.0558 & 0.0097 & 0.0059 & 0.0175 & 0.0070 \\
0.0210 & -0.0517 & 0 & 0 & 0.0193 \\
0.0330 & 0 & -0.0711 & 0.0316 & 0 \\
0.0053 & 0 & 0.0323 & -0.0441 & 0.0040 \\
0.0010 & 0.0009 & 0 & 0.0126 & -0.0212
\end{bmatrix}
\]  \hspace{1cm} (5.24)

\[
B = 10^{-3} \times \begin{bmatrix}
0.1151 & 0 & 0 & 0 & 0 \\
0 & 0.4287 & 0 & 0 & 0 \\
0 & 0 & 0.1465 & 0 & 0 \\
0 & 0 & 0 & 0.0828 & 0 \\
0 & 0 & 0 & 0 & 0.0936
\end{bmatrix}
\]  \hspace{1cm} (5.25)

A and B are all 5 × 5 matrixes. Suppose only one temperature is measured and the temperature sensor is mounted in the air zone 5, we have:

\[
C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]  \hspace{1cm} (5.26)

The eigenvalues of matrix A are -0.0856, -0.0800, -0.0072, -0.0425 and -0.0286 which are all real and negative. Thus the system is stable and has no overshoot. We also have:

\[
\text{rank}\left[ C^T; A^T C^T; \cdots; (A^T)^4 C^T \right] = 5 \hspace{1cm} (5.27)
\]

Thus the continuous system observable.

Take the sampling period Δt as 60 seconds, we can obtain the matrixes of the discrete system using equations (5.11) through (5.13):

103
Chapter 5 Real-time Estimations of Indoor Temperatures

\[
A_d = e^{A \Delta t} = \\
\begin{bmatrix}
0.1232 & 0.0388 & 0.0931 & 0.1706 & 0.1225 \\
0.0988 & 0.0697 & 0.0601 & 0.1296 & 0.2040 \\
0.1472 & 0.0325 & 0.1373 & 0.2259 & 0.0929 \\
0.1483 & 0.0273 & 0.1611 & 0.2709 & 0.0998 \\
0.0693 & 0.0170 & 0.0852 & 0.1862 & 0.3215 \\
\end{bmatrix}
\]

\[
B_d = A^{-1}(e^{A \Delta t} - I)B \\
= 10^{-3} \times \\
\begin{bmatrix}
2.3757 & 1.3855 & 0.6760 & 0.7687 & 0.5374 \\
0.8419 & 8.3833 & 0.2678 & 0.3769 & 1.0749 \\
1.2305 & 0.6157 & 2.8289 & 1.1459 & 0.2870 \\
0.8237 & 0.3792 & 1.6580 & 2.3676 & 0.3598 \\
0.2568 & 0.3006 & 0.4364 & 0.7277 & 3.2814 \\
\end{bmatrix}
\]

\[
C_d = C = [0 \ 0 \ 0 \ 0 \ 1] 
\]

The eigenvalues of matrix \( A_d \) are 0.6508, 0.1798, 0.0780, 0.0059 and 0.0082 which are less than unity. Thus the discretized system is stable. Since

\[
\text{rank} \left[ C_d^T \ A_d^T \ C_d^T : \cdots : (A_d^T)^4 \ C_d^T \right] = 5
\]

the discretized system is observable.

Let us design the discrete state estimator now. Suppose the estimator has deadbeat responses, i.e. all its eigenvalues (the roots of its characteristic equation) \( \mu_i \ (i = 1, 2, \ldots, 5) \) are zero. From equation (5.20) we have:

\[
|zI - A_d + G_d C_d| = \phi(z) = (z - \mu_1)(z - \mu_2)\cdots(z - \mu_5) = z^5 
\]

\[
= z^5 + \alpha_1 z^4 + \cdots + \alpha_4 z + \alpha_5 = 0
\]

Therefore, \( \alpha_i = 0 \ (i = 1, 2, \ldots, 5) \).
$$\phi(A_d) = A_d^5 + \alpha_1 A_d^4 + \cdots + \alpha_4 A_d + \alpha_5 I = A_d^5$$ (5.34)

The feedback gain matrix of the prediction-type estimator can be obtained from equation (5.18):

$$G_d = \phi(A_d) \begin{bmatrix} C_d & \cdots & 0 \\ C_d A_d & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C_d A_d^{d-1} & \cdots & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.8157 \\ 0.7760 \\ 0.9856 \\ 1.1079 \\ 0.9226 \end{bmatrix}$$ (5.35)

The responses of the prediction-type estimator can be obtained through equation (5.17) and are shown in Figure 5.2.

![Diagram of estimated air temperatures](image)

**Figure 5.2** Estimated air temperatures by the prediction-type estimator
Chapter 5 Real-time Estimations of Indoor Temperatures

For the current-estimation-type estimator, the feedback gain matrix is:

\[
G_{dc} = \phi(A_d) \begin{bmatrix}
C_d & A_d \\
C_d & A_d^2 \\
& \vdots \\
& C_d & A_d^5
\end{bmatrix}^{-1}
\begin{bmatrix}
0 \\
0 \\
\vdots \\
1
\end{bmatrix} = \begin{bmatrix}
1.4154 \\
1.3938 \\
1.6929 \\
1.7996 \\
1.0000
\end{bmatrix}
\] (5.36)

Using equation (5.23), we get the responses of the current-estimation-type estimator as shown in Figure 5.3.

![Figure 5.3 Estimated air temperatures by the current-estimation-type estimator](image)

§5.3 Reduced-order Estimator for Temperature Estimations

The state estimator developed in the above sections is called full-order estimators since all states of the system are estimated. Obviously, the measurement of a state is in
Reduced-order Estimator for Temperature Estimations

general more accurate than its estimate. Thus estimating all states is not necessary. We may take advantage of the already measured states to design a state estimator that estimates only those unmeasured states. Such an estimator is called a reduced-order estimator.

We consider the case in which only one state is measured. The rest states are to be estimated. Without losing generality, we assume that the last state, $x_n(t)$, is measured. The state vector can be partitioned as:

$$
\mathbf{x}(t) = 
\begin{bmatrix}
    \mathbf{x}_e(t) \\
    x_n(t)
\end{bmatrix} 
$$

(5.37)

where $\mathbf{x}_e(t) \Rightarrow n-1 \times 1$ is the states to be estimated and

$$
\mathbf{x}_e(t) = 
\begin{bmatrix}
    x_1(t) \\
    x_2(t) \\
    \vdots \\
    x_{n-1}(t)
\end{bmatrix} 
$$

(5.38)

The partitioned state equation is:

$$
\begin{bmatrix}
    \dot{x}_e(t) \\
    \dot{x}_n(t)
\end{bmatrix} = 
\begin{bmatrix}
    \mathbf{A}_{ee} & \mathbf{A}_{en} \\
    \mathbf{A}_{ne} & a_{nn}
\end{bmatrix} 
\begin{bmatrix}
    \mathbf{x}_e(t) \\
    x_n(t)
\end{bmatrix} + 
\begin{bmatrix}
    \mathbf{B}_e \\
    \mathbf{B}_n
\end{bmatrix} \mathbf{u}(t) 
$$

(5.39)

From this equation we can write the equation of the states to be estimated:

$$
\dot{x}_e(t) = \mathbf{A}_{ee} x_e(t) + \mathbf{A}_{en} x_n(t) + \mathbf{B}_e \mathbf{u}(t) 
$$

(5.40)

and the equation of the measured state:

$$
\dot{x}_n(t) = \mathbf{A}_{ne} x_e(t) + a_{nn} x_n(t) + \mathbf{B}_n \mathbf{u}(t) 
$$

(5.41)

Notice that $\mathbf{x}_e(t)$ is unknown and is to be estimated, $x_n(t)$ and $\mathbf{u}(t)$ are known.
Chapter 5 Real-time Estimations of Indoor Temperatures

Since we have derived the design equations of the full-order state estimator in the continuous time domain, if we can manipulate equations (5.40) and (5.41) into the same form as the system equations (5.1) and (5.2), we may easily get the design equations of the reduced-order estimator. Rewrite (5.40) as:

$$\dot{x}_e(t) = A_{ee} x_e(t) + \left[ A_{en} x_n(t) + B_e u(t) \right]$$ \hspace{1cm} (5.42)

and reorganize (5.41) as:

$$\left[ \dot{x}_n(t) - a_{nn} x_n(t) - B_n u(t) \right] = A_{ne} x_e(t)$$ \hspace{1cm} (5.43)

Comparing (5.1) with (5.42) and (5.2) with (5.43), we see that the equations are equivalent if we make the following substitutions:

$$x(t) \leftarrow x_e(t)$$

$$A \leftarrow A_{ee}$$

$$Bu(t) \leftarrow A_{en} x_n(t) + B_e u(t)$$

$$y(t) \leftarrow \dot{x}_n(t) - a_{nn} x_n(t) - B_n u(t)$$

$$C \leftarrow A_{ne}$$

Referring to the full-order estimator:

$$\dot{x}(t) = (A - GC)\dot{x}(t) + Bu(t) + Gy(t)$$

we can now obtain the reduced-order estimator:

$$\dot{x}_e(t) = (A_{ee} - G_e A_{ne})\dot{x}_e(t) + A_{en} x_n(t) + B_e u(t)$$

$$+ G_e \left[ \dot{x}_n(t) - a_{nn} x_n(t) - B_n u(t) \right]$$ \hspace{1cm} (5.44)
where \( G_e \) is the gain matrix of the reduced-order estimator. Likewise, \( G_e \) is chosen in such a way that the requirement on the dynamic response of the estimator is satisfied.

The characteristic equations of the estimator and of the estimation errors are:

\[
\alpha_e(s) = \left| sI - (A_{ee} - G_eA_{ne}) \right| = 0 \tag{5.45}
\]

Here one problem arises. As can be seen from equation (5.44), the reduced-order estimator requires the derivative of \( x_n(t) \) as an input. Although we can build a circuit or an algorithm that differentiates signals, the differentiator will amplify any high frequency noise in \( x_n(t) \). To solve this problem, we can introduce auxiliary variables so that the calculation of \( \dot{x}_n(t) \) is eliminated. Define:

\[
\hat{\chi}_e^*(t) = \hat{x}_e(t) - G_e x_n(t) \tag{5.46}
\]

or

\[
\hat{x}_e(t) = \hat{x}_e^*(t) + G_e x_n(t) \tag{5.47}
\]

Substituting (5.47) into (5.44) yields:

\[
\dot{\hat{x}}_e^*(t) + G_e \dot{x}_n(t) = \left( A_{ee} - G_e A_{ne} \right) \left[ \hat{\chi}_e^*(t) + G_e x_n(t) \right] \\
+ A_{en} x_n(t) + B_e u(t) + G_e \left[ \dot{x}_n(t) - a_{nn} x_n(t) - B_n u(t) \right]
\]

or

\[
\dot{\hat{\chi}}_e^*(t) = \left( A_{ee} - G_e A_{ne} \right) \hat{\chi}_e^*(t) \\
+ \left( A_{en} - G_e a_{nn} + A_{ee} G_e - G_e A_{ne} G_e \right) x_n(t) + \left( B_e - G_e B_n \right) u(t) \tag{5.48}
\]

With (5.48), \( \dot{\hat{\chi}}_e^*(t) \) can be solved. The estimated states, \( \hat{x}_e(t) \), are obtained through (5.47).

For simplification, denote

109
Chapter 5 Real-time Estimations of Indoor Temperatures

\[ A'_{ee} - G_e A'_{ne} = F'_e \]  
(5.49)

\[ A'_{en} - G_e a_{nn} + A'_{ee} G_e - G_e A_{ne} G_e = G'_e \]  
(5.50)

\[ B'_e - G_e B_n = H'_e \]  
(5.51)

Then (5.48) can be written as:

\[ \dot{x}'_e(t) = F'_e \dot{x}'_e(t) + H'_e u(t) + G'_e x_n(t) \]  
(5.52)

Figure 5.4 Block Diagram of Reduced-order Estimator
The block diagram of the reduced-order estimator is shown in Figure 5.4. According to equation (5.46), the initial condition of $\hat{x}_e^*(t)$ is:

$$\hat{x}_e^*(0) = \hat{x}_e(0) - G_e x_n(0)$$  \hspace{1cm} (5.53)

For the discrete system depicted by equations (5.9) and (5.10), we can similarly write:

$$x(k) = \begin{bmatrix} x_e(k) \\ x_n(k) \end{bmatrix}$$  \hspace{1cm} (5.54)

$$\begin{bmatrix} x_e(k) \\ x_n(k) \end{bmatrix} = \begin{bmatrix} A_{de} & A_{dn} \\ A_{dc} & a_{dn} \end{bmatrix} \begin{bmatrix} x_e(k-1) \\ x_n(k-1) \end{bmatrix} + \begin{bmatrix} B_{de} \\ B_{dn} \end{bmatrix} u(k-1)$$  \hspace{1cm} (5.55)

from which we obtain the state equation:

$$x_e(k) = A_{de} x_e(k-1) + [A_{dc} x_n(k-1) + B_{de} u(k-1)]$$  \hspace{1cm} (5.56)

and the measurement equation:

$$[x_n(k) - a_{dn} x_n(k-1) - B_{dn} u(k-1)] = A_{dn} x_e(k-1)$$  \hspace{1cm} (5.57)

Referring to the discrete full-order estimator:

$$\hat{x}(k) = (A_d - G_d C_d) \hat{x}(k-1) + B_d u(k-1) + G_d y(k-1)$$

and the above continuous estimator, we obtain the discrete reduced-order estimator:

$$\hat{x}_e(k) = (A_{de} - G_{de} A_{dn}) \hat{x}_e(k-1) + [A_{dc} x_n(k-1) + B_{de} u(k-1)]$$

$$+ G_{de} [x_n(k) - a_{dn} x_n(k-1) - B_{dn} u(k-1)]$$  \hspace{1cm} (5.58)
Chapter 5 Real-time Estimations of Indoor Temperatures

Remember that in the subsystem constructed by equations (5.56) and (5.57), $A_{de}$ is the system matrix and $A_{de}$ is the measurement matrix. The condition of observability now becomes:

$$\text{rank}\left[ A_{de}^T : A_{de}^T A_{de}^T : \ldots : (A_{de}^T)^{n-2} A_{de}^T \right] = n - 1$$

(5.59)

For the cooling case studied in section §5.2, $n = 5$, $x_5$ is the measured state, $x_1$ through $x_4$ are the states to be estimated. We have the following partition of the system matrices:

$$A_d = \begin{bmatrix} A_{de} & A_{de} \\ A_{de} & a_{dau} \end{bmatrix} = \begin{bmatrix} 0.1232 & 0.0388 & 0.0931 & 0.1706 & 0.1225 \\ 0.0988 & 0.0697 & 0.0601 & 0.1296 & 0.2040 \\ 0.1472 & 0.0325 & 0.1373 & 0.2259 & 0.0929 \\ 0.1483 & 0.0273 & 0.1611 & 0.2709 & 0.0998 \\ 0.0693 & 0.0170 & 0.0852 & 0.1862 & 0.3215 \end{bmatrix}$$

(5.60)

$$B_d = \begin{bmatrix} B_{de} \\ B_{dau} \end{bmatrix} = 10^{-3} \times \begin{bmatrix} 2.3757 & 1.3855 & 0.6760 & 0.7687 & 0.5374 \\ 0.8419 & 8.3833 & 0.2678 & 0.3769 & 1.0749 \\ 1.2305 & 0.6157 & 2.8289 & 1.1459 & 0.2870 \\ 0.8237 & 0.3792 & 1.6580 & 2.3676 & 0.3598 \\ 0.2568 & 0.3006 & 0.4364 & 0.7277 & 3.2814 \end{bmatrix}$$

(5.61)

From the partition, we have:

$$A_{de} = \begin{bmatrix} 0.1232 & 0.0388 & 0.0931 & 0.1706 \\ 0.0988 & 0.0697 & 0.0601 & 0.1296 \\ 0.1472 & 0.0325 & 0.1373 & 0.2259 \\ 0.1483 & 0.0273 & 0.1611 & 0.2709 \end{bmatrix}$$

(5.62)
Reduced-order Estimator for Temperature Estimations

\[
A_{den} = \begin{bmatrix}
0.1225 \\
0.2040 \\
0.0929 \\
0.0998
\end{bmatrix}
\] (5.63)

\[
A_{dne} = \begin{bmatrix}
0.0693 & 0.0170 & 0.0852 & 0.1862
\end{bmatrix}
\] (5.64)

\[a_{dnn} = 0.3215\] (5.65)

\[
B_{de} = 10^{-3} \times \begin{bmatrix}
2.3757 & 1.3855 & 0.6760 & 0.7687 & 0.5374 \\
0.8419 & 8.3833 & 0.2678 & 0.3769 & 1.0749 \\
1.2305 & 0.6157 & 2.8289 & 1.1459 & 0.2870 \\
0.8237 & 0.3792 & 1.6580 & 2.3676 & 0.3598
\end{bmatrix}
\] (5.66)

\[
B_{dn} = 10^{-3} \times \begin{bmatrix}
0.2568 & 0.3006 & 0.4364 & 0.7277 & 3.2814
\end{bmatrix}
\] (5.67)

Since

\[
\text{rank}\left[A_{dne}^T : A_{dne}^T A_{dne}^T : (A_{dne}^T)^2 : A_{dne}^T : (A_{dne}^T)^3 : A_{dne}^T\right] = 4
\] (5.68)

the subsystem is observable.

Let us now design a deadbeat estimator. The desired characteristic equation of the estimator is:

\[
\left|zI - A_{dne} + G_d A_{dne}\right| = \phi(z) = (z - \mu_1)(z - \mu_2)(z - \mu_3)(z - \mu_4) = z^4
\] (5.69)

\[
= z^4 + \alpha_1 z^3 + \alpha_2 z^2 + \alpha_3 z + \alpha_4 = 0
\] (5.70)

Therefore, \(\alpha_i = 0, (i = 1, 2, 3, 4)\).

\[
\phi(A_{dne}) = A_{dne}^4 + \alpha_1 A_{dne}^3 + \alpha_2 A_{dne}^2 + \alpha_3 A_{dne} + \alpha_4 I = A_{dne}^4
\] (5.71)
Chapter 5 Real-time Estimations of Indoor Temperatures

According to the Ackermann’s formula, the gain of the reduced-order estimator is:

\[
G_{\text{dec}} = \phi(\mathbf{A}_{\text{dec}}) \begin{bmatrix}
\mathbf{A}_{\text{dne}} \\
\mathbf{A}_{\text{dne}} \mathbf{A}_{\text{dec}} \\
\mathbf{A}_{\text{dne}} \mathbf{A}_{\text{dec}}^2 \\
\mathbf{A}_{\text{dne}} \mathbf{A}_{\text{dec}}^3 
\end{bmatrix}^{-1} \begin{bmatrix}
0 \\
0 \\
0 \\
1 
\end{bmatrix} = \begin{bmatrix}
1.4154 \\
1.3938 \\
1.6929 \\
1.7996 
\end{bmatrix}
\]

(5.72)

The responses of the reduced-order estimator can be calculated based on equation (5.58). They are shown in Figure 5.5. The measured air temperature of zone 5 is also shown in this figure.

Figure 5.5 Estimated air temperatures of zones 1 through 4 by the reduced-order estimator
§5.4 Kalman Filter for Temperature Estimations

Although it is possible to estimate the indoor temperatures (state variables) with the above designed state estimators and measurements, the assumed conditions in the estimator designs are that there are no modeling error, no system disturbance and no measurement noise, which are usually not true. In reality, no matter how good a model is, it is always an approximate representation of the physical system. There also exist various unmodeled factors and stochastic disturbance to the system. The measurements can not be noise-free too. If such errors, disturbances and noises are significant, the successful applications of the state estimators will be greatly limited.

If the modeling error, system disturbance and measurement noise can be taken into account by the estimator in an statistic manner, then more robust state estimations will be obtained. This problem is solved by the Kalman filter.

For the discretized state space model:

\[ x(k+1) = A_d x(k) + B_d u(k) \]  
\[ y(k) = C_d x(k) \]

(5.73) \hspace{1cm} (5.74)

suppose the modeling error and disturbance of the system can be represented with noise sequence \( w(k) \) and the measurement noise is sequence \( v(k) \), then the state space model can be written as:

\[ x(k+1) = A_d x(k) + B_d u(k) + w(k) \]
\[ y(k) = C_d x(k) + v(k) \]

(5.75) \hspace{1cm} (5.76)

We assume that the states and noises satisfy the following conditions:

\[ E[x(0)] = \bar{x}(0) \]
\[ E[w(k)] = 0 \]
\[ E[v(k)] = 0 \]

(5.77) \hspace{1cm} (5.78) \hspace{1cm} (5.79)
Chapter 5 Real-time Estimations of Indoor Temperatures

\[ E \left[ \left[ x(0) - \bar{x}(0) \right] \left[ x(0) - \bar{x}(0) \right]^T \right] = P_0 \]  \hspace{1cm} (5.80)

\[ E \left[ w(j)w^T(k) \right] = 0 \]  \hspace{1cm} (5.81)

\[ E \left[ w(j)w^T(k) \right] = Q \delta_{jk} \]  \hspace{1cm} (5.82)

\[ E \left[ v(j)v^T(k) \right] = R \delta_{jk} \]  \hspace{1cm} (5.83)

\[ E \left[ \left[ x(0) - \bar{x}(0) \right] w^T(k) \right] = 0 \]  \hspace{1cm} (5.84)

\[ E \left[ \left[ x(0) - \bar{x}(0) \right] v^T(k) \right] = 0 \]  \hspace{1cm} (5.85)

where \( \delta_{jk} = 1 \) when \( j = k \) and \( \delta_{jk} = 0 \) when \( j \neq k \). \( E \) and \( T \) are expectation and transpose.

The Kalman filter for state estimation is to minimize:

\[ P(k) = E \left[ \left[ x(k) - \hat{x}(k) \right] \left[ x(k) - \hat{x}(k) \right]^T \right] \]  \hspace{1cm} (5.86)

where \( \hat{x}(k) \) is the estimation of \( x(k) \).

It can be proved that the estimation-type Kalman filter has the following algorithm:

\[ \hat{x}(k) = z(k) + L_e(k) \left[ y(k) - C_d z(k) \right] \]  \hspace{1cm} (5.87)

\[ z(k) = A_d \hat{x}(k-1) + B_d u(k-1) \]  \hspace{1cm} (5.88)

\[ L_e(k) = P(k) C_d^T \left[ R + C_d P(k) C_d^T \right]^{-1} \]  \hspace{1cm} (5.89)

\[ P(k) = Q + A_d P(k-1) A_d^T - A_d P(k-1) C_d \left[ R + C_d P(k-1) C_d^T \right]^{-1} C_d P(k-1) A_d^T \]  \hspace{1cm} (5.90)

where \( P(0) = P_0 \).
\[ \dot{x}(k) = z(k) + L_e [y(k) - C_d z(k)] \]  
(5.91)

\[ z(k) = A_d \dot{x}(k-1) + B_d u(k-1) \]  
(5.92)

\[ L_e = PC_d^T \left[ R + C_d PC_d^T \right]^{-1} \]  
(5.93)

\[ P = Q + A_d PA_d^T - A_d PC_d^T \left[ R + C_d PC_d^T \right]^{-1} C_d PA_d^T \]  
(5.94)

where the last equation is the Riccati equation.

The key problem of designing a Kalman filter is to find the correct matrixes $Q$ and $R$ through experiments.

§5.5 Discussion and Summary

In real-time control systems of indoor thermal conditions, there are usually some temperatures that need to be controlled or monitored but are either impossible to measure or too expensive to measure. These temperatures of the places other than the one where the sensor is located need to be estimated in some way. This chapter discusses the estimation methods in such respect by introducing the state estimators of modern control theory.

The basic ideas of the estimators are illustrated. Design procedures of the estimators and some case studies are given in this chapter. The estimated temperatures can be used as feedback information for monitoring and controlling indoor thermal conditions.

From this chapter we know that many advantages can be taken of the state space model with the help of modern control theory.
Chapter 5 Real-time Estimations of Indoor Temperatures
Chapter 6

Estimation and Control of the Temperature of the Working Zone

In most air-conditioning control systems, the temperature sensor is mounted to measure and control either the temperature of the recirculated air or the temperature of the exhaust air. Since the temperature of the working zone in a room is significantly different from either one of these two temperatures, the controlled temperature is not the temperature that needs to be controlled. With the estimator theory introduced in chapter 5, the temperature of the working zone can be estimated, used as feedback information, and finally controlled. This chapter illustrates the implementation of this "estimation-feedback-control" scheme. The thermal inertia of walls and the disturbances from outdoor weather conditions are also considered. The simulation scheme is realized in the Matlab/Simulink programming environment.

In most air-conditioning control systems, the temperature sensor is mounted to measure and control either the temperature of the recirculated air or the temperature of the exhaust air. But the temperature of the working zone in a room is significantly different
Chapter 6 Estimation and Control of the Temperature of the Working Zone

from either one of these two temperatures. The problem we face now is that the controlled temperature is not the temperature that needs to be controlled.

Fortunately, we have the state estimators to estimate temperature responses that has been presented in chapter 5. That is, the temperature of the working zone can be estimated by a state estimator. The estimated temperature can then be used as feedback information the controller and be controlled.

Before we go to the estimation and control of the temperature, we need a simulation environment to realize relevant differential equations of walls and windows, the state space model and the control algorithms.

§6.1 Dynamic Simulation Scheme in Matlab/Simulink

In recent years, Matlab is becoming an increasingly popular programming environment for technical calculations. In many universities, it is the standard instructional tool for calculations in introductory and advanced courses. Matlab is matrix-based and integrates numerical analysis, matrix computation, control system design, signal processing and graphics into a user-friendly environment.

Simulink is a dynamic simulation environment based on Matlab. It realizes model definition, model analysis, control system design and grapher with block diagrams which can be easily connected with the mouse cursor.

Since building walls play an important role in damping and delaying the indoor temperature changes caused by heat gains or losses, the dynamic characteristics of walls should be taken into account by the simulation scheme. The influence of outdoor weather conditions should also be considered. Theoretical results and engineering methods in these respects can be found in many reference books such like the ASHRAE handbooks and are summarized in Appendix B.

Based on the equations described in Appendix B, a simulation scheme in the Simulink environment is designed to investigate the dynamic thermal responses in building rooms. Figures 6.1, 6.2 and 6.3 show some of the block diagrams. Detailed implementation and information can be obtained by double clicking on each icon. For example, Figure 6.2 is obtained by double clicking on the icon “House” in Figure 6.1. Figure 6.3 is obtained by double clicking on the icon “Room air model” in figure 6.2.

For a wall sublayer described by equations (B.15) and (B.17), the block diagram implemented in the Simulink environment is shown in Figure 6.4.
Dynamic Simulation Scheme in Matlab/Simulink

Figure 6.1 Dynamic simulation scheme in Matlab/Simulink

Figure 6.2 Sub-system for “House”
Figure 6.3 Sub-system for the “Room air model”

Figure 6.4 Block diagram implementation of the temperature calculation of a wall sublayer in Matlab/Simulink
Controlling the Measured Temperature

The parameters in Figure 6.4 have the following meaning:

For equation (B.15) in Appendix B,
\[
\text{import}_1 = \Theta(t, n-1), \text{import}_3 = \Theta(t, n+1), \text{outport}_2 = \Theta(t, n), \\
K_1 = 1, K_3 = 1, K_2 = K_1 + K_3 = 2, K = a/(\Delta t)^2.
\]

For equation (B.17) in Appendix B,
\[
\text{import}_1 = \Theta_{\text{air}}(t), \text{import}_3 = \Theta(t, 1), \text{outport}_2 = \Theta(t, 0), \\
K_1 = h_c, K_3 = \lambda/\Delta x, K_2 = K_1 + K_3 = h_c + \lambda/\Delta x, K = 2/(C_p \rho \Delta x)
\]

§6.2 Controlling the Measured Temperature

Like the practice of many air-conditioning control systems, we now control the measured temperature of the recirculated air. For the fan-coil installation and cooling situation indicated in Chapter 4, the temperature of the recirculated air can be approximated with the air temperature of zone 1 since the recirculated air comes from zone 1. The block diagram of the control system is shown in Figure 6.5 where \( \Theta_i \) = temperature of zone \( i \), \( \Theta_1 \) is also the measured temperature and \( \Theta_q \) is also the temperature of the working zone, \( \Theta_w \) is the outdoor air temperature, \( \Theta_{w_{i}} \) are the inside surface temperatures of walls, \( q_{\text{solar}} \) is the total solar radiation, \( \Theta_s \) is the temperature of the supply air.

The control algorithm is the PI controller
\[
u(t) = K_p \Delta e(t) + K_1 \int \Delta e(t) dt \tag{6.1}
\]

where
\[
\Delta e(t) = \Theta_{sp} - \Theta_1 \tag{6.2}
\]

and

\( u(t) \) = control action for the heating and cooling valve positions. \\
( 6 ) = setpoint temperature. \\
\( \Theta_1 \) = measured temperature of recirculated air (temperature of air zone 1).
Chapter 6 Estimation and Control of the Temperature of the Working Zone

\[ K_p = \text{proportional gain.} \]
\[ K_i = \text{integral gain} \]

Here the dynamics of the heat exchanger of the fan-coil unit is ignored. Thus the changes of the heating and cooling valve positions are supposed to be proportional to the temperature change of the supply air from the fan-coil unit.

![Block diagram of the control system when the measured temperature is controlled](image)

The simulation scheme of the control system is shown in Figure 6.1. The setpoint temperature is 25 °C (for hot summer situation). The initial values for the temperatures of walls and air zones are 30 °C. The initial temperature of the supply air is 17 °C. The outdoor air temperature is assumed to be 28 °C for the simulation period. For simplicity, all walls (including the ceiling and floor) except the window and front wall are supposed to have the same thermal and air flow conditions on both sides. Only the window and front wall are exposed to the outdoor climate and receive solar radiation. The total solar radiation on their outside surfaces is assumed to be 550 W/m². The window is double
glazing with Venetian blinds at the outside. The wall (including ceiling and floor) materials are supposed to be brick. Figure 6.6 shows the simulation results.

From Figure 6.6 we can see that when the temperature of the recirculated air is controlled to the setpoint value of 25 °C, the air temperature of the working zone is about 27 °C, 2 K higher than the setpoint. This means that when the control system indicates satisfactory results, the room occupant may feel too hot in the room. Here we see the reason of the complaint of the room occupant and the problem of the control system. But we are unable to control the temperature of the working zone since we do not have the information of it.

![Simulation results when the measured temperature is controlled](image)

**Figure 6.6** Simulation results when the measured temperature is controlled

§6.3 Controlling the Temperature of the Working Zone

Although the temperature of the working zone can not be measured, it can be estimated from the information of the measured temperature of the recirculated air. This can done by using the state estimators discussed in Chapter 5.
Chapter 6 Estimation and Control of the Temperature of the Working Zone

The estimated temperature of the working zone can be feedback to the controller and be controlled. Figure 6.7 shows the block diagram of this “estimation-feedback-control” scheme. Figure 6.8 illustrates the simulation scheme. The estimator used here is the reduced-order estimator, i.e. the measured temperature is not estimated.

The values of parameters and all other conditions are the same as those in the simulation of the previous section.

From the simulation results shown in Figure 6.9, we see that the temperature of the working zone is well estimated and controlled. That is, it controlled to the setpoint of 25 °C. The feel of the room occupant about the local thermal conditions is now consistent with the indication of the control system.

Compared with the simulation results of the previous section, the temperature of the supply air is lower, which means that more cooling energy is now needed.

![Figure 6.7 Block diagram of the control system when the temperature of the working zone is estimated and controlled](image)
Figure 6.8 Simulation scheme of “estimation-feedback-control” system

Figure 6.9 Simulation results of the “estimation-feedback-control” system
§6.4 Discussion and Summary

In many air-conditioning control systems, it is the temperature of the recirculated or exhaust air that is controlled. The study of this chapter indicates that such control scheme is problematic since a satisfactory result maybe displayed by the control system while the temperature of the working zone of a room may be far from the setpoint temperature.

Although the temperature of working zone can not be measured, it can be estimated by using the state estimator. The results of this chapter shows that when the estimated temperature of the working zone is fed back to the controller and controlled, much better control results can be obtained. With this “estimation-feedback-control” scheme, the human sensation of indoor thermal conditions will be consistent with the indication of the controller.
Chapter 7

Conclusions and Recommendations

Conclusions have been given at the end of each chapter. This chapter reiterates some of the important points and recommends what should be done next.

§7.1 Conclusions

This thesis mainly investigates two problems, simplified modeling of dynamic indoor temperature distributions and the estimation of the working-zone temperature of a room based on the measured temperature. Both studies are for the better control of indoor thermal conditions so that the thermal comfort level can be improved.

The modeling procedure is as follows: (1) Steady-state CFD calculations are carried out for various heating and cooling situations in a room. (2) Air flow fields (velocity components of the finite volumes in the flow domain) and effective exchange coefficients corresponding a flow pattern are extracted from the CFD results. (3a). By fixing the flow field, the indoor dynamic temperature distributions are directly calculated. (3b). A simplified zonal model is constructed. (4). Based on the zonal model, a state space model is built. The modeling procedure and the structure of the thesis are shown in Figure 7.1.

Each of the models is validated with experimental results for both cooling and heating cases. The validations indicate that the models can give satisfactory and realistic predictions of the indoor dynamic temperature distributions. The computing
time used by the state space model is at least less than 0.0024% of that of the dynamic CFD simulation.

Figure 7.1 Modeling procedure and structure of the thesis
The state space model not only reduces the computing time significantly but also is a more detailed model than the well-mixed-air (one-point) model. The biggest advantage of the state space model is that it can be easily used for control system designs. It can also be transformed into transfer functions so that frequency response analyses can be carried out. With a state space model, existing modern control theories can find better applications in the prediction and control of the indoor thermal conditions.

Each element of the system matrix $A$ of the state space model contains the effects of both the heat transfer due to air mass flows and the heat exchange due to turbulence and the free movement of air molecules. Therefore, even if the parameters of the state space model are obtained through identification methods, they still have physical meanings.

In many practical control systems, there are usually some variables that need to be controlled or monitored but are either impossible to measure or too expensive to measure. The reality in the control of indoor thermal conditions is that only one temperature sensor is mounted in each room. Therefore, the temperatures of places other than the one where the sensor is located need to be estimated in some way. Chapter 5 of this thesis discusses the estimation methods in such respect by introducing the state estimators of modern control theory. Design procedures of the state estimators are given and some case studies are carried out for real-time estimations of temperature responses.

In many air-conditioning control systems, the temperature sensor is mounted to measure and control either the temperature of the recirculated air or the temperature of the exhaust air. Since the temperature of the working zone in a room is significantly different from either one of these two temperatures, it has to be estimated by using the state estimator and the measured temperature in order to control it. This aspect is discussed in Chapter 6. Simulation results shows that much better control results can be obtained if, instead of the measured temperature of the recirculated air, the estimated temperature of the working zone is controlled. With this "estimation-feedback-control" scheme, the displayed result of the controller is consistent with the human sensation of indoor thermal conditions.

§7.2 Recommendations for Future Research

This thesis is an interdisciplinary research work combining fluid dynamics, numerical method, modern control theory, air-conditioning systems and building physics. It may
be viewed as a first trial since we have not seen any publication of its kind. Thus the research is far from perfect. Much work needs to be done further and many aspects needs to be improved.

As the state space model presented in this thesis has been validated to be feasible, an atlas of air flow fields obtained from steady state CFD calculations should be made based on various room sizes of office buildings, fan speeds and locations of the air-conditioning unit. The matrixes in the state space model should then be calculated for each situation. With this atlas, the corresponding matrixes A and B can be found for a practical room.

After a room is divided into air zones, the form of the system matrix A in the state space model is determined. Thus as long as we have some measurement results of temperature distributions of the room, identification method can be utilized to find the values of the elements of matrix A. We suggest some work be done in this direction. The advantage of the identification method is that CFD calculations are no longer necessary. The disadvantage is that in-situ measurements are required.

Real-time estimation of indoor temperature distributions with state estimators is only one application aspect of state space models. Optimal controller and predictive controller can be designed based on a state space model to further improve indoor thermal conditions. The state estimation theory might also be used for fault diagnosis of any HVAC system as long as the system can be represented with a state space model.

Future work should include the dynamics of the air-conditioning unit which have not been taken into account by the simulations in this thesis.

The fundamental assumption for the modeling of dynamic indoor temperature distributions in this thesis is that the dynamic changes of indoor air flows are very small and can be neglected. This assumption is true only for forced air flows with constant fan speed. Thus the model developed in this thesis can only be used for mechanical ventilation rooms. To include dynamic air flows into the model of indoor thermal conditions, one has to look for new simplified modeling method and keeps constant scrutiny at new advances of turbulence modeling theory.
References

References


Clarke, J.A., Energy simulation in building design, Adam Hilger Ltd, Accord, MA, USA.


References


Hausen, H., Z. tech. physik 5, 169, 1924.


References


Lute, P.J., Liem, S.H. and van Paassen, A.H.C., Benefits of adjustable shutters, shading devices and vent windows in passive solar buildings, CIB International
References


References


Nielsen, P.V., Strommingforhold i luftkonditionerede lokaler (flow conditions in air-conditioned room), Ph.D. Thesis, Technical University of Denmark, Copenhagen, Denmark, 1974.


References


References


Xue, D., Air-conditioning, Tsinghua University, Beijing, China, 1991.


Appendix A

Some Basic Concepts in State Space Control Theory

State Variables, State Vector and State Space

State variables of a system are a set of variables which, together with their initial values and the inputs of the system, completely describe the dynamic behavior of the system.

State variables need not physically measurable or existing. But in practice, it is convenient to choose existing and easily measurable variables as state variables from the point of view of control system designs.

State variables can be put into a vector to form a state vector. All state variables constitute a state space whose dimension is the same as the number of state variables.

State Space Model

State space model is also called state space representation of a system. If a system can be described by the following first-order vector-matrix differential equation:

\[
\dot{x}(t) = Ax(t) + Bu(t) \tag{A.1}
\]

\[
y(t) = Cx(t) \tag{A.2}
\]

where \( x(t) \) is the state vector which contains state variables of the system, \( u(t) \) is the input vector which contains the system inputs, \( y(t) \) is the output vector which contains the system outputs, then the representation \( (A.1) \) and \( (A.2) \) is called a state space
model. Matrixes A, B and C are called system matrix, input matrix and output matrix respectively.

Just like a transfer function model can be obtained from the differential equations of a system, a state space model can be obtained in the same way. Theoretically, the state space model, transfer function model and differential equation model of a system can be deduced from one another.

Observability

The system described by the state space model (A.1) and (A.2) is said to be completely (state) observable if for any time $t_0$, there exists a finite time $t_1 > t_0$, such that the information of the output vector $y(t)$ and the input vector $u(t)$ in the time interval $t_0 \leq t \leq t_1$ is sufficient to uniquely determine the initial state $x(t_0)$.

The necessary and sufficient condition for a system to be observable is:

$$\text{rank} \begin{bmatrix} C^T & A^T C^T & \cdots & (A^T)^{n-1} C^T \end{bmatrix} = n$$  \hspace{1cm} (5.3)$$

where $T$ means transpose of a matrix, $n$ is the order of the system.

Observability is the precondition of designing state estimators (also called state observers).
Appendix B

Equations for Heat Transfer Calculations in Buildings

§B.1 Heat Balance at the Outside Surface of a Building Wall

The heat balance equation at a unit area of the outside surface of a wall can be expressed as (Kimura 1977):

\[ q_{h.o} + q_{l.o} + q_{c_v.o} = q_{e.o} + q_{c_d.o} \]  \hfill (B.1)

where \( q_{h.o} \) = solar (short wave) radiation absorbed by the outside surface of the wall, \( q_{l.o} \) = long wave radiation absorbed by the surface, \( q_{c_v.o} \) = convective heat transfer from the ambient air to the surface, \( q_{e.o} \) = long wave radiation emitted by the surface (i.e. wall emission), and \( q_{c_d.o} \) = heat conduction from the surface toward the inside of the wall. These heat flux quantities all have the unit of W/m².

B.1.1 Solar Radiation Absorbed by the Wall Surface: \( q_{h.o} \)

Solar radiation absorbed by the wall surface, \( q_{h.o} \), is influenced by the solar declination \( D \), solar altitude \( \beta \), solar azimuth \( \alpha \), cloud condition of the atmosphere, building location on the earth, orientation of the wall, material of the wall and smooth condition of the surface, etc. (Figure B.1). What is normally measured by the weather station is the global solar radiation \( q_{h.glob} \) on a horizontal surface, which is the sum of the direct solar radiation component and the diffuse radiation on the horizontal surface (van Paassen 1990, Lute 1992). For a horizontal wall such as a roof, \( q_{h.o} = \alpha_{a,w} q_{h.glob} \), where \( \alpha_{a,w} \) = absorptivity of the wall surface material (dimensionless).
For a vertical wall, the relation between \( q_{s,0} \) and \( q_{s,\text{glob}} \) can be calculated from the following equations:

\[
q_{s,0} = \alpha_{w,\text{w}} (q_{s,\text{dir}} + q_{s,\text{dif}} + q_{s,\text{grd}})
\]  \hspace{1cm} (B.2)

Figure B.1 Relative position between a vertical wall and the sun

where \( q_{s,\text{dir}} \) = the component of the direct solar radiation which is perpendicular to the surface, \( q_{s,\text{dif}} \) = diffuse component to the surface, \( q_{s,\text{grd}} \) = reflection component from the ground and surrounding surfaces to the surface. \( q_{s,\text{dir}} \), \( q_{s,\text{dif}} \) and \( q_{s,\text{grd}} \) can be calculated from the following equations:

\[
q_{s,\text{dir}} = \frac{\cos \theta}{\sin \beta} (1 - K)q_{s,\text{glob}}
\]  \hspace{1cm} (B.3)

\[
q_{s,\text{dif}} = F_{0,a} K q_{s,\text{glob}}
\]  \hspace{1cm} (B.4)
Heat Balance at the Outside Surface of a Building Wall

\[ q_{s,grd} = F_{o,g} \alpha_{s,grd} q_{s,glob} \]  \hfill (B.5)

where \( \theta \) = the angle between the incoming solar rays and the normal of the wall [rad] (Figure B.1), \( K \) is defined as the ratio between the diffuse radiation on a horizontal surface and \( q_{s,glob} \). \( \alpha_{s,grd} \) = reflectance of the ground (dimensionless), \( F_{o,a} \) = angle factor viewed from the wall surface to the atmosphere (sky) = 0.5 for a vertical wall, and \( F_{o,g} \) = angle factor viewed from the wall surface to the ground = 0.5 for a vertical wall. Detailed calculation of the above three terms and related parameters can be found in van Paassen (1990).

B.1.2 Long Wave Radiation Absorbed by the Wall Surface: \( q_{l,o} \)

Long wave radiation includes the radiation from the atmosphere \( q_{l,a} \) and the radiation from the ground and surrounding surfaces \( q_{l,g} \). According to Brunt (Kimura 1977),

\[ q_{l,a} = (1 - a_c K_c) \sigma T_a^4 Br + a_c K_c \sigma T_g^4 \]  \hfill (B.6)

where \( a_c \) = cloud cover ratio, \( K_c \) = reduction factor depending on the height of the cloud, \( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4 \) is the Stefan-Boltzmann constant, \( T_a \) = absolute temperature of the ambient air \( (T_a = \theta_a + 273.15) \) [K], \( Br \) = emissivity of the atmosphere and is a function of humidity, and \( T_g \) = absolute ground surface temperature \( (T_g = \theta_g + 273.15) \) [K]. Emission from the ground is:

\[ q_{l,g} = \varepsilon_g \sigma T_g^4 \]  \hfill (B.7)

where \( \varepsilon_g \) = emissivity of the ground surface. Therefore,

\[ q_{l,o} = \varepsilon_o F_{o,a} q_{l,a} + \varepsilon_o F_{o,g} q_{l,g} \]  \hfill (B.8)

where \( \varepsilon_o \) = emissivity of the wall surface.

B.1.3 Convective Heat Transfer from the Ambient Air to the Surface: \( q_{cv,o} \)

\[ q_{cv,o} = h_c (T_a - T_o) = h_c (\theta_a - \theta_o) \]  \hfill (B.9)
where $h_c$ = convective heat transfer coefficient between the surface and the air [W/m$^2$·K],
and $T_0$ = absolute temperature of the wall surface ($T_0 = \theta_0 + 273.15$) [K].

Kimura (1977) gave the method of determining the value of $h_c$ in relation to wind speed and direction, i.e.

(1) If the wall surface is windward:
   for $U > 2$ m/s $u = 0.25U$
   for $U \leq 2$ m/s $u = 0.5$

(2) If the wall surface is leeward:
   $u = 0.3 + 0.05U$

(3) The convective heat transfer coefficient: $h_c = 3.5 + 5.6u$

B.1.4 Long Wave Radiation Emitted by the Surface: $q_{e,0}$

$$q_{e,0} = \varepsilon_0 \sigma T_0^4$$ (B.10)

§B.2 Solar Heat Gain through a Window

Solar heat gain (SHG) through a window can be calculated based on the following formula
(ASHRAE handbook, fundamentals 1985):

$$\text{SHG} = F \, I_t$$ (B.11)

where $F = \text{solar heat gain coefficient which is equivalent to the ZTA used in the}$
Netherlands (ZonToetredingsfaktor Absoluut) (dimensionless), $I_t = q_{s,\text{dir}} + q_{s,\text{diff}} + q_{s,\text{grad}} =$
\text{solar radiation falling on the window surface. The relation between ZTA and the shading}$
\text{coefficient (SC) is:}$

$$\text{ZTA} = 0.87 \times \text{SC}$$ (B.12)

The solar heat gain includes two components: one (CF×SHG) goes directly into the
indoor air and becomes instantaneous cooling load, the other [(1-CF)×SHG] goes to the
inside walls of the room and raises the wall surface temperatures. Detailed values of ZTA
and CF for various window glazings and shadings were given by van Paassen (1993).
§B.3 Heat Conduction through Walls

The partial differential equation depicting the one-dimensional transient heat conduction through a homogeneous wall is given by:

\[
\frac{\partial \theta(t, x)}{\partial t} = a \frac{\partial^2 \theta(t, x)}{\partial x^2}
\]  

(B.13)

where

- \(a = \lambda/(C_p \rho)\) = the thermal diffusivity of the wall material [m²/s]
- \(\lambda\) = the thermal conductivity of the material [W/m·K]
- \(\rho\) = the density [kg/m³]
- \(C_p\) = the specific heat [J/kg·K]
- \(t\) = time [s]
- \(x\) = coordinate [m]
- \(\theta(t, x)\) = temperature of the wall at position \(x\) at time \(t\) [K or °C].

The well known methods to solve the above differential equation are the response factor method and the \(z\)-transfer function method presented by Mitilas and Stephenson (1967 and 1971). These methods were proposed because it was believed that it would take too much computing time to directly solve the equation using numerical calculation method due to the limited speed of computers of that time. However, the fast development of computer technology in recent years has made the one-dimensional calculation much easier.

A wall may be composed of more than one layers of different materials in reality. The basic idea of the numerical method is to first divide every layer of the wall into several sublayers (Figure B.2) and discretize equation (B.13), then calculate the temperatures of the sublayers.

Define \(\theta(t, n)\) as the temperature of sublayer \(n\), equation (B.13) can be discretized as:

\[
\frac{\partial \theta(t, n)}{\partial t} = a \frac{1}{\Delta x} \left[ \frac{\theta(t, n-1) - \theta(t, n)}{\Delta x} - \frac{\theta(t, n) - \theta(t, n+1)}{\Delta x} \right]
\]  

(B.14)

or
\[
\frac{\partial \theta(t,n)}{\partial t} = \frac{a}{(\Delta x)^2} \left[ \theta(t,n-1) + \theta(t,n+1) - 2\theta(t,n) \right] \tag{B.15}
\]

where \(\Delta x\) = the thickness of sublayers. For the boundary sublayer that has contact with the air, the heat balance equation is:

![Diagram showing temperatures of the sublayers of a multilayer wall at time step \(k\)]

\[
C_p \rho \frac{\Delta x}{2} \frac{\partial \theta(t,n)}{\partial t} = h_c \left[ \theta_{\text{air}}(t) - \theta(t,0) \right] - \lambda \frac{\theta(t,0) - \theta(t,1)}{\Delta x}\]

(B.16)

or
Radiative Heat Exchanges among Inside Wall Surfaces

\[
\frac{\partial \theta(t,n)}{\partial t} = \frac{2}{C_p \rho \Delta x} \left[ h_c \theta_{\text{air}}(t) + \frac{\lambda}{\Delta x} \theta(t,1) - \left( h_c + \frac{\lambda}{\Delta x} \right) \theta(t,0) \right]
\]  \hspace{1cm} (B.17)

where \( h_c \) = the convective heat transfer coefficient between air and the wall surface and \( \theta_{\text{air}}(t) \) = the temperature of the air close to the wall surface.

When the forward difference method is used to solve the above differential equation, the following numerical stability criterion must be satisfied:

\[
Fo = a \frac{\Delta t}{(\Delta x)^2} \leq 0.5
\]  \hspace{1cm} (B.18)

where \( Fo \) is called Fourier number. The above criterion means that \( \Delta t \) and \( \Delta x \) must be chosen carefully for the concerned wall material. If a large value can not be chosen for the thickness \( \Delta x \) of a sublayer, then the time step \( \Delta t \) must be very small and thereby the number of calculations will increase. Thus although smaller \( \Delta x \) can increase the calculation accuracy, it also increases the computing time. Condition (B.18) must be satisfied for all sublayers of the wall composed of multilayers of different building materials.

For two adjacent sublayers belonging to two layers with different thermal properties, the equivalent thermal conductivity \( \lambda \) can be determined from:

\[
\frac{\Delta x_1 + \Delta x_2}{\lambda} = \frac{\Delta x_1}{\lambda_1} + \frac{\Delta x_2}{\lambda_2}
\]  \hspace{1cm} (B.19)

where \( \Delta x_1 \) and \( \Delta x_2 \) are the thicknesses of the two adjacent sublayers, \( \lambda_1 \) and \( \lambda_2 \) are the thermal conductivity of the two adjacent layers.

§B.4 Radiative Heat Exchanges among Inside Wall Surfaces

The inside wall surfaces form an enclosure. Radiative heat exchanges happen among these surfaces. Here we introduce the method given in the "ASHRAE handbook, Fundamentals" to calculate the heat exchanges. Since multi-reflections are involved in the process of radiation exchanges, we need to define two new terms:

\[ G = \text{irradiation} = \text{total radiation rate incident upon a unit-area surface [W/m}^2\text{].} \]
Appendix B Equations for Heat Transfer Calculations in Buildings

\( J = \text{radiosity} = \text{total radiation rate that leaves the unit-area surface} \ [\text{W/m}^2]. \)

According to definition, radiosity \( J \) is the sum of the energy emitted and the energy reflected by the surface:

\[
J = \varepsilon \sigma T^4 + \rho G \tag{B.20}
\]

where \( \varepsilon \) := emissivity of the surface, \( T \) = absolute temperature of the surface [K], \( \rho \) = reflectivity of the surface. For long wave radiation, the window glass is opaque. Thus for every surface of the room,

\[
\rho = 1 - \alpha = 1 - \varepsilon \tag{B.21}
\]

where \( \alpha \) = absorptivity of the surface. Consider a room with \( n \) walls, the irradiation of wall \( i \) should be the sum of the radiosities of all \( n \) surfaces incident upon it, i.e.,

\[
G_i A_i = \sum_{j=1}^{n} F_{ij} A_j J_j = \sum_{j=1}^{n} F_{ij} A_i J_j = A_i \sum_{j=1}^{n} F_{ij} J_j \tag{B.22}
\]

or

\[
G_i = \sum_{j=1}^{n} F_{ij} J_j \tag{B.23}
\]

where \( F_{ij} \) = the angle factor viewed from surface \( i \) to \( j \), \( i = 1, 2, \ldots, n \). Thus equation (B.20) for surface \( i \) can be written as:

\[
J_i = \varepsilon_i \sigma T_i^4 + \rho_i \sum_{j=1}^{n} F_{ij} J_j \tag{B.24}
\]

With \( i = 1, 2, \ldots, n \), \( n \) equations can be obtained from (B.24) and \( J_1, J_2, \ldots, J_n \) can thus be solved.

The net energy lost by surface \( i \) is the difference between its radiosity and irradiation:

152
Radiative Heat Exchanges among Inside Wall Surfaces

\[ q_i = A_i (J_i - G_i) \quad (B.25) \]

where \( A_i \) = area of surface \( i \). From equations (B.20) and (B.21) we have:

\[ G_i = \frac{1}{1 - \varepsilon_i} \left( J_i - \varepsilon_i \sigma T_i^4 \right) \quad (B.26) \]

Thus:

\[ q_i = A_i \frac{\varepsilon_i}{1 - \varepsilon_i} \left( \sigma T_i^4 - J_i \right) \quad (B.27) \]
Appendix B Equations for Heat Transfer Calculations in Buildings
Acknowledgment

At the end of this thesis, I would like to express my sincere thanks to my supervisor (toegevoegd promotor), Dr. A.H.C. van Paassen, who not only had helped me to get the opportunity to carry out the research in this thesis, but also has constantly given me valuable ideas, suggestions and encouragement throughout the completion of the thesis. Without his generous help, I would not have been able to obtain the present research results.

I am very grateful to my advisor (promotor), Prof. H. van der Ree, for his concerns of the advance of my research and precious comments and suggestions on the contents of the thesis.

Without doubt, I can not forget the help of all my colleagues at the Laboratory of Refrigeration and Indoor Climate Technology. Special acknowledgments are due to Mr. R. Staal, Mr. M. Verwaal, Mr. H.F. Broekhuizen, Mr. J. Keuvelaar and Mr. A.C. van Geest. The help from Mr. H. Liem, Dr. J. van der Kooi, Mr. J.A. van der Kaaij and Mr. A.H. Visser is also appreciated.

I wish to thank Dr. J. Niu and Dr. Q. Chen for helping me start CFD simulations. Thanks are due to students, Mr. W.I.A. Jadoenathmisier for doing some measurements together with me, and Mr. J. Kalkman for letting me share some of his data on the performance of the fan-coil unit.

I am especially grateful to Mrs. G.C.M. Mollet for her help on so many daily matters which are so fragmented but closely related with the research work.

I feel very much indebted to my wife, Baorong, whose understanding and support lead not only to the final birth of this thesis, but also to the earlier birth of our daughter, Chengcheng.
Biography

Xiudong Peng was born on 30 October, 1962, in Henan Province of China. He received education from the primary and secondary schools in his hometown from 1970 to 1980. Upon graduation from a senior high school in 1980, we passed the national examinations and was admitted by Chengdu University of Science and Technology (now renamed as Sichuan Union University) where he studied in the Department of Electrical Engineering and Electronics for four years and was granted a Bachelor's Degree of Engineering at the end of the study.

In September 1984, after traveling 3 thousand kilometers by train from southwest China to northeast China, he came to Harbin Institute of Technology to continue his education as a postgraduate in the Department of Control Engineering. There he carried out extensive computer simulation work on adaptive control algorithms and was involved in the project of "Computer Control of a Cement Kiln" responsible for in-situ measurements. Three years' study and research led to the production a thesis for the university and a Master's Degree of Engineering for himself.

In 1987, he headed for Beijing to do research work at the Laboratory of Fluid Power Transmission and Control, Beijing Institute of Technology. He was a project leader responsible for research on the digital control of electro-hydraulic servo systems with heavily time-varying inertia load.

In 1991, he established contact with Dr. A.H.C. van Paassen and expressed his desire of coming to the Netherlands to study for a Ph.D. degree. Under the recommendation of Dr. van Paassen, he was appointed as an AIO (Assisten-in-Opleiding) by the Faculty of Mechanical Engineering and Marine Technology, Delft University of Technology, in May 1991 and was required to do research at the Laboratory of Refrigeration and Indoor Climate Technology. But the journey to the Netherlands turned out to be extremely difficult. He had to travel 5 thousand kilometers in China by train and 2 hundred kilometers in Beijing by bike to get at least 30 governmental stamps required for applying for a passport at that time. During this period, Dr. van Paassen showed his concerns and supplied all necessary help. When he finally got all necessary travel documents
including the visa of the Netherlands, it was already June 1992 from which Xiudong Peng started the research work of this thesis, Modeling of Indoor Thermal Conditions for Comfort Control in Buildings.