Optimising the throughput for an integrated urban-freeway network by adapting variable speed limits and green time shares with an MPC controller

Annexes MSc Thesis Report (CIE5060-09)

MSc Civil Engineering track Transport & Planning

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The figures on the front page were edited from the originals obtained from the following sources:
Top figure: Chriszwolle, 2009
Bottom figure: LennartBolks, 2003
Full references can be found in the MSc Thesis report.

Contents

Annex 1: Preliminary literature review

Annex 1 on preliminary literature review gives an overview of the results of literature review that did not fit the goal that was formulated for literature review, but was nevertheless informative for the MSc Thesis project. Due to editing, the associated literature research had taken place before the final version of the MSc Thesis report was written; therefore this literature review is referred to as preliminary literature review.

The annex is structured by subject. In section A1.1 a review of control theory is given. In section A1.2 it is shown why it is shown why non-linear optimisation is much more difficult than linear optimisation. In section A1.3 a review of fundamental diagram concepts is given. Section A1.4 gives an overview of traffic flow models. The annex is concluded with references.

A1.1 A review of control theory

Control theory is about the dynamic behaviour of controlled input/output phenomena (Hegyi, 2014 [1]). Central elements in control theory are (Hegyi, 2014 [2]):

- The time t measured from a fixed origin
- The state of the process x
- The control input of the process u
- The uncontrolled input of the process, commonly referred to as the disturbance d
- The output of the process y

In this general formulation t is either a continuous or a discrete scalar, whereas x, u, d and y may be vectors or scalars that are functions of t (Verhaeghe, 2007). Depending on t being discrete or continuous, different formulations for the relations between x, u, d and y exist. However, they all fit the general framework depicted in figure 1.

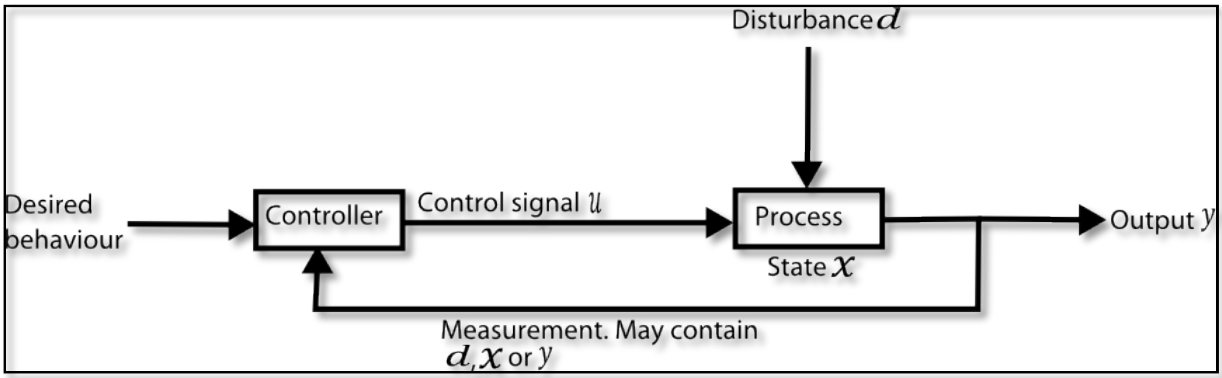

Figure 1 General framework control theory (Adapted from Hegyi, 2014 [2])

When t is continuous, the following relations between x, u, d and y exist (Verhaeghe, 2007), (Hegyi, 2014 [1]), (Hegyi, 2014 [2]):

$$\dot{x} = f_1(t; x; u; d)$$
$$y = f_2(t; x; u; d)$$
$$u = f_c(t; d; y)$$
$$f_3(t; x; u; y) \leq c$$

Where:

$\dot{x}$: The derivative of x with respect to t

$f_1(t; x; u; d)$: The relation between $\dot{x}$ on one side and $(t; x; u; d)$ on the other side

$f_2(t; x; u; d)$:	The relation between y on one side and $(t; x; u; d)$ on the other side
$f_c(t; d; y)$:	The control rule, defined by the desired behaviour
$f_3(t; x; u; y) \leq c$:	The constraints due to physical limitations of the process or the controller defined in its most general form; may contain any constraints from equality constraints to soft constraints (the latter may be violated, albeit at a cost (Pekař, 2010))

When t is discrete, the time is generally split into discrete intervals $[Tk; T(k+1))$ where T is the length of the individual discrete intervals and k is a natural number; for instance Van de Weg et al (2015) and Van de Weg et al (2016) use this method. In that case, the derivative $\dot{x}$ is replaced with X, which is x at the time step $k+1$; x, u, d and y are evaluated at time step k (Hegyi, 2014 [2]). Then the following relations between x, u, d and y exist (Hegyi, 2014 [1]), (Hegyi, 2014 [2]):

$$X = f_1(k; x; u; d)$$
$$y = f_2(k; x; u; d)$$
$$u = f_c(k; d; y)$$
$$f_3(k; x; u; y) \leq c$$

Where:

$f_1(k; x; u; d)$:	The relation between X on one side and $(k; x; u; d)$ on the other side
$f_2(k; x; u; d)$:	The relation between y on one side and $(k; x; u; d)$ on the other side
$f_c(k; d; y)$:	The control rule, defined by the desired behaviour
$f_3(k; x; u; y) \leq c$:	The constraints due to physical limitations of the process or the controller defined in its most general form; may contain any constraints from equality constraints to soft constraints (the latter may be violated, albeit at a cost (Pekař, 2010))

Hegyi (2014 [1]) gives five examples of control rules:

- Feed-forward Control
- Feedback Control
- Predictive Control
- Optimal Control
- Model Predictive Control

These examples will now be discussed.

Feed-forward Control

Feed-forward Control aims at controlling the process by measuring the disturbances d (Hegyi, 2014 [2]). As a result, the control signal is independent of the output of the process y.

Advantages of this method are that the control signal is determined more easily and that the combination of control signal and process is guaranteed to be stable if both are stable (Hegyi, 2014 [2]). Stability in this sense means that the state x converges to a certain equilibrium point x_e if the inputs (d for the process, u for the control signal and both for the combination) are constant after the initial state x_0 was a finite perturbation δ_0 away from x_e (Hegyi, 2014 [1]), (Hegyi, 2014 [2]):

$$x_0 = x_e + \delta_0$$

Thus given an initial state and constant input, stability yields:

$$\lim_{t \to \infty} x = x_e$$

Disadvantage of the feed-forward method is that if the desired behaviour is not achieved, the control signal does not change to accommodate the resulting problems and may even compound them (Hegyi, 2014 [2]).

Feedback Control

Feedback Conrol aims at controlling the process by measuring both the disturbance d and the output of the process y (Hegyi, 2014 [2]). Advantage of this method is that if the desired behaviour is not achieved, the control signal is changed to accommodate the resulting problems (Hegyi, 2014 [2]). Disadvantages are that it is harder to determine the control signal and that the combination of control signal and process may be unstable even if both the process and the control signal are stable. However, feedback controllers may stabilize unstable processes.

Predictive Control

Predictive Control is characterised by the fact that not only the current output of the process y and the current disturbance d is taken into account, but also a prediction of the future state of the process x_p (Hegyi, 2014 [1]). This prediction is made by a forecasting model which either produces a finite prediction of future states or a continuous function.

An example of a forecasting model which produces a finite prediction of future states is the linear state-space model, which is of the form (Papageorgiou et al, 2003):

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{C}\mathbf{d}(k)$$

Where the first state vector $\mathbf{x}(k_0)$ is based on the output vector $\mathbf{y}(k_0)$, where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are matrices, where the future disturbances $\mathbf{d}(k)$ need to be predicted separately and where the control measure $\mathbf{u}(k)$ needs to be determined separately.

This example shows that there is a wide range of prediction models. Advantage of this method is that the effects of control measures later in the future are explicitly taken into account. Disadvantage is that it may be difficult to find reliable prediction models. For example, currently potential applications of predictive control to traffic processes are currently limited to travel time information provision, decision support systems and network optimisation due to lack of accurate traffic prediction tools (Hegyi, 2014 [1])

Optimal Control

Optimal control is a form of Predictive Control where an objective function J is explicitly optimised (Hegyi, 2014 [1]). This objective function is commonly used to describe the predicted performance of the process over a certain time interval. This performance can be expressed in many ways; examples for traffic control are total queue lengths, total time spent, total vehicle loss hours, total emissions etc. Constraints for the states, controls and performance can also be taken into account.

The predicted performance over the time interval is calculated from the state prediction over this time interval. Optimising the predicted performance can be quite challenging. However, if the prediction model is the linear state-space model and the objective function J can be expressed in one of the two ways denoted here, optimising the predicted performance can be done by either linear or quadratic programming (Van de Weg, 2015), (Papageorgiou et al, 2003).

$$J = \sum_{k=k_0}^{K} \left(\mathbf{p}^T\mathbf{x}(k) + \mathbf{q}^T\mathbf{u}(k) + \mathbf{r}^T\mathbf{d}(k)\right)$$

$$J = \frac{1}{2}\sum_{k=k_0}^{K} \left(\|\mathbf{x}(k)\|_{\mathbf{P}}^2 + \|\mathbf{u}(k)\|_{\mathbf{Q}}^2 + \|\mathbf{d}(k)\|_{\mathbf{R}}^2\right)$$

Where $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are vectors and $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ are matrices which all 6 may or may not have different entries per time step. The squared vector-matrix norm $\|\mathbf{v}\|_{\mathbf{M}}^2$ is defined as follows:

$$\|\mathbf{v}\|_{\mathbf{M}}^2 = \left(\sqrt{\mathbf{v}^T\mathbf{M}\mathbf{v}}\right)^2 = \mathbf{v}^T\mathbf{M}\mathbf{v}$$

Once the predicted performance has been optimised and the associated control scheme has been found, optimal control implements it for the time interval used for the optimisation. Advantages of this method are that the effects of the control measures later in the future are explicitly taken into account and that the control is sought to be optimal. Disadvantages of this method is that it may be hard to optimise the control and that only once every interval optimisation takes place, whereas measurements may be available more frequently and help steer the control in case of unexpected disturbances.

Model Predictive Control (MPC)

Model Predictive Control (MPC) is a form of Optimal Control where instead of implementing the one optimal control scheme for an entire interval for which it was optimised, implementation only lasts until new measurements become available, which are subsequently used for optimising a new control scheme (Hegyi, 2014 [1]). This method is called rolling horizon prediction or receding horizon prediction. For this reason, MPC is sometimes referred to as Receding Horizon Control (Pekař, 2010). Advantages of this method are that the effects of the control measures later in the future are explicitly taken into account and that the control is sought to be optimal at the same frequency measurements become available. Disadvantages of this method are that it may be hard to optimise the control; particularly within the small time of measurements becoming available.

A1.2 Non-linear optimisation: why it is harder than linear optimisation

It is a well-known fact that non-linear optimisation is harder than linear optimisation, but the reasons behind this fact are less well-known. Chinneck (2012) lists twelve reasons why non-linear optimisation is much more difficult than linear optimisation:

1) Distinction between a local optimum and a global optimum is difficult
2) Unlike linear optimisations where optima only appear along the edges of the feasible region, optima for non-linear optimisations may appear anywhere within the feasible region on top of along the edges
3) The non-linearity of constraints may result in multiple, disconnected feasible regions
4) Different starting points in non-linear solvers may lead to different final solutions
5) Finding starting points that meet inequality constraints can be difficult
6) Keeping equality constraints satisfied may be difficult
7) Distinction between a very optimal value of the objective function and an unbounded value is difficult for a non-linear solver
8) There exist so many non-linear solvers for non-linear optimisation problems that choosing a non-linear solver for a given non-linear optimisation is difficult in and of itself
9) It is difficult to determine whether or not the conditions on which a non-linear solver is based are met
10) If multiple non-linear solvers (that are a good choice to apply to the optimisation at hand) are applied to an optimisation, the results may differ
11) Different but equivalent formulations of the optimisation may be handled differently by a non-linear solver. As an example Chinneck (2012) gives a constraint $x^2 + y^2 \leq 25$ which may be differently formulated as $a = x^2$, $b = y^2$ and $a + b \leq 25$. Some solvers will find the second formulation easier to work with by first dealing with a simple linear inequality and later solving for the values for x and y, which may result in following different trajectories through the feasible regions which in turn may result in arriving at different solutions
12) Finding the correct input parameters for the non-linear solver may be difficult

A1.3 A review of fundamental diagram concepts

The fundamental diagram is a relation between the density ρ $[\mathrm{veh/km}]$ and the speed v $[\mathrm{km/h}]$ based on the reasoning that under stationary traffic flow drivers will on average keep the same distance to the preceding vehicle (Hoogendoorn, 2007 [2]). Given the definitions by Hoogendoorn (2007 [1]) and the fundamental relation on the next page, it is possible to derive three fundamental diagrams:

$$v = f_1(\rho)$$
$$q = f_2(\rho)$$
$$q = f_3(v)$$

Figure 2 shows how the fundamental diagrams are typically presented. Notice that for the relation between q and v, the roles are swapped, resulting in a relation that is not a function, as there are two values for v for most values of q. However, one of these values corresponds to free flow conditions and the other to congested conditions, which can be readily distinguished.

The defenitions by Hoogendoorn (2007 [1]):

$$q = \frac{1}{\frac{1}{n}\sum_{i=1}^{n} h_i}$$

$$\rho = \frac{1}{\frac{1}{m}\sum_{i=1}^{m} s_i}$$

$$v = \frac{1}{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{v_i}}$$

Where:

n:	The number of vehicles that passed a certain location during a time period T [h]
h_i [h]:	The time headway between vehicle i and vehicle $i+1$
m:	The number of vehicles in a certain road stretch with length X [km]
s_i [km]:	The space headway between vehicle i and vehicle $i+1$
v_i [km/h]:	The speed of vehicle i

The fundamental relation:

$$q = \rho v$$

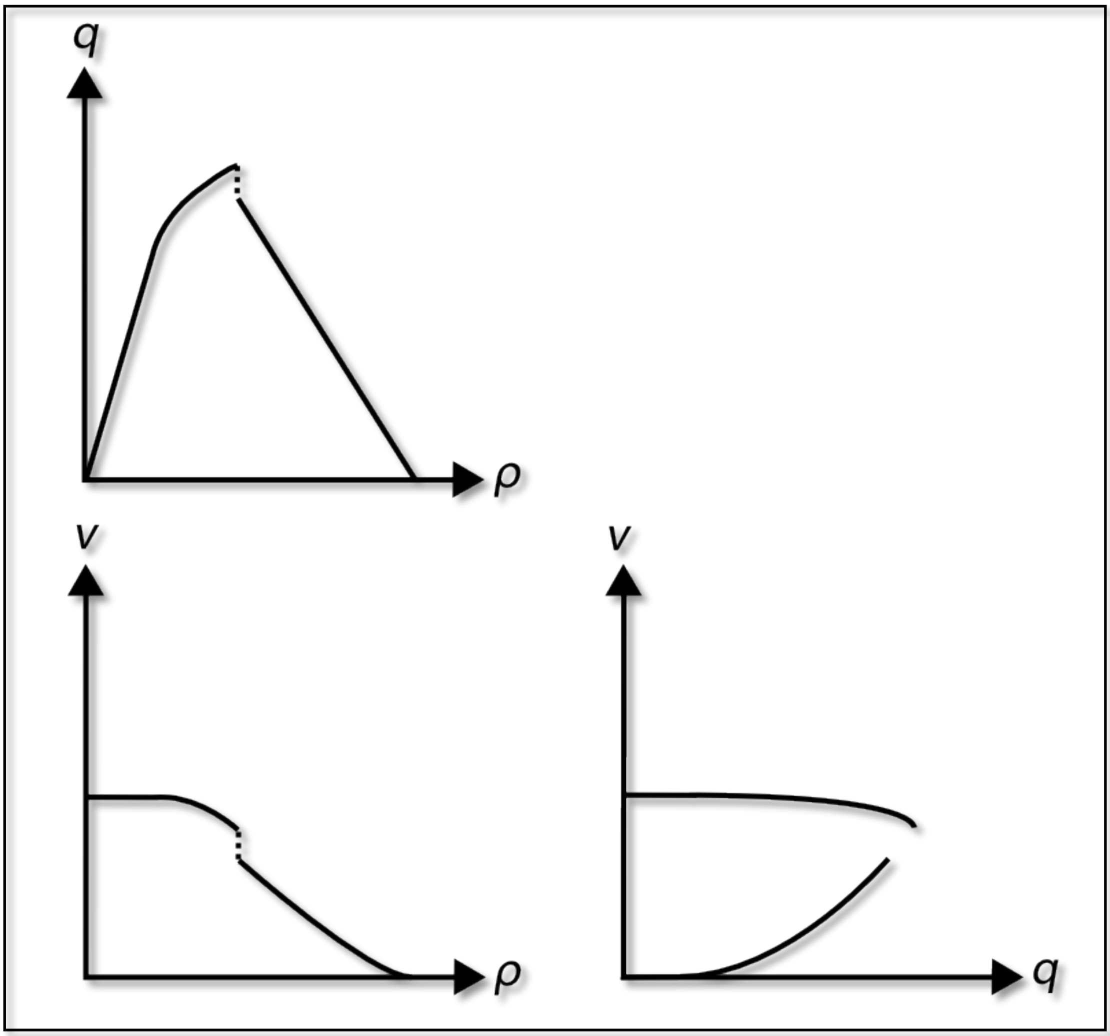

Figure 2 Typical representation of fundamental diagrams

Typical points in the fundamental diagram are:

- Free flow speed v_0; the speed corresponding to $q = \rho = 0$
- Capacity q_{cap}; the maximum flow
- The critical density ρ_{crit}; the density corresponding to $q = q_{cap}$
- The critical speed v_{crit}; the speed corresponding to $q = q_{cap}$
- The jam density ρ_{jam}; the density corresponding to $q = v = 0$

Basically the fundamental diagram shows that when the inflow into a road stretch increases, the outflow will increase as well, up to capacity. When capacity is reached, breakdown may occur and the outflow will decrease due to the onset of congestion. This observation has been noted to be valid for areas as well (Knoop, 2014). This led to the formulation of the generalised network fundamental diagram: the network relation between the speeds on individual links v [km/h], the average density μ_ρ [veh/km] in the network and the standard deviation σ_ρ [veh/km] of the density in the network. Figure 3 shows the network fundamental diagram of a network containing the ring road of Amsterdam, the A10, with the speeds for the A10. Also, it shows how during a certain day the speeds evolved over the day.

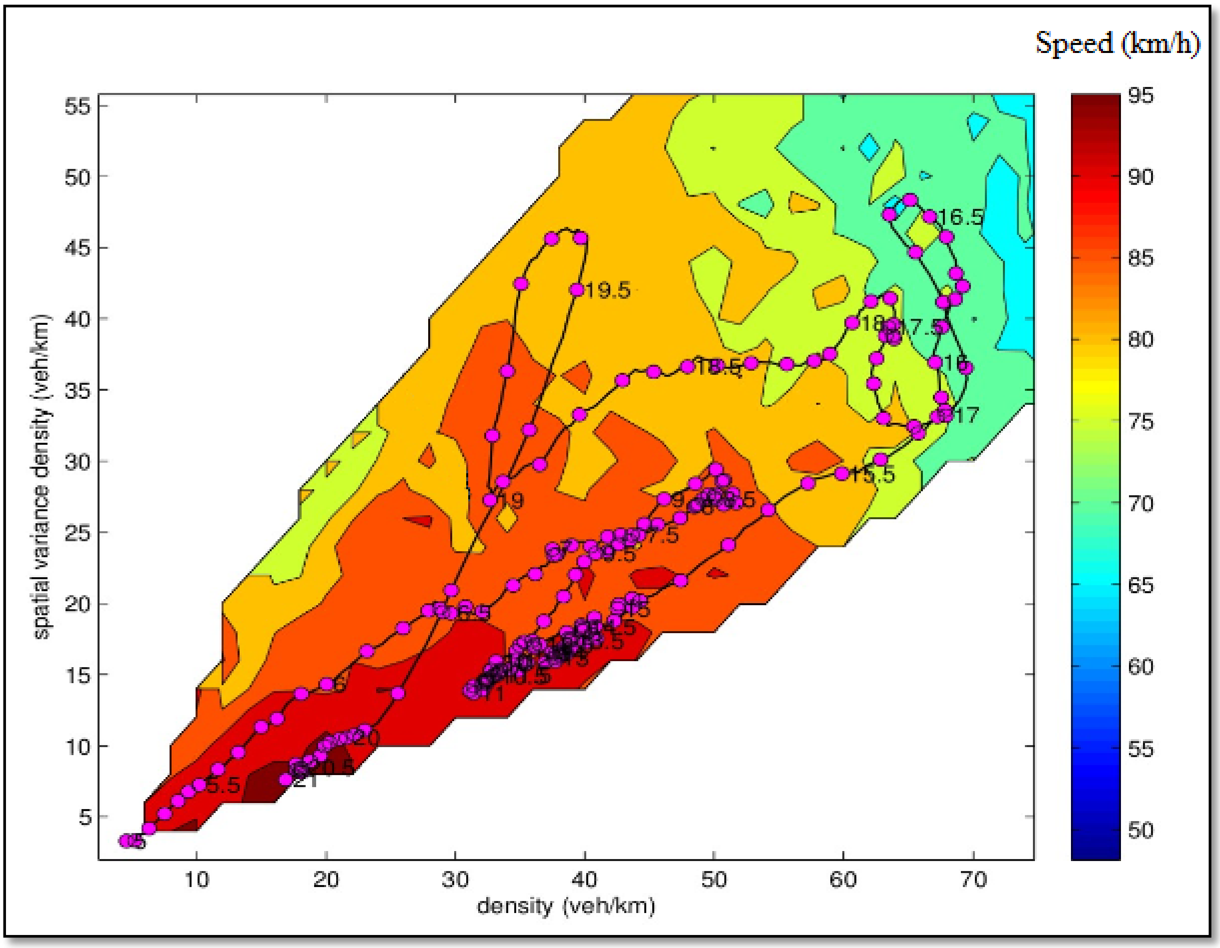

Figure 3 the generalised network fundamental diagram depicting the average speed on the A10. To give an impression on how the speed might vary over the day, a trajectory for the measurements of a particular day with the time in hours since midnight is depicted (Adapted from Knoop, 2014)

A1.4 Traffic flow models: an overview

Traffic flow models come in two different categories: freeway traffic flow models and urban traffic flow models. In this section, first an overview is given of freeway traffic flow models before the same is done for urban traffic flow models.

Freeway traffic flow models

The freeway traffic flow models that have been found in the literature are (Pel, 2013 [1]):

- Static assignment models
- Link performance models
- Vertical queue models
- Horizontal queue models
- Fundamental diagram models
- The METANET model

These traffic flow models will now be discussed.

Static assignment models

Static assignment models have been traditionally used to determine the interaction between route choice and congestion (Pel, 2013 [2]) As such, they consider periods of time with constant flow in which all the traffic has to move from the origins to the destinations. Based on the travel times, traffic is distributed over routes, causing flows on links. These flows affect the travel time on the links, which is used to adapt the traffic distribution until sufficient convergence has been established. Static assignment models differ in the way they determine how the flows affect the travel time; this is done with travel time functions (Pel, 2013 [1]). For instance, the travel time τ_i $[\mathrm{h}]$ of link i as a function of the flow q_i $[\mathrm{veh/h}]$ in link i:

$$\tau_i = f_i(q_i)$$

Example functions are the BPR function (BPR, 1964) and the Davidson function (Davidson, 1966), respectively:

$$f_i(q_i) = \tau_i^0 \left(1 + \alpha_i \left(\frac{q_i}{q_{cap;i}} \right)^{\beta_i} \right)$$

$$f_i(q_i) = \tau_i^0 \left(1 + \gamma_i \frac{\left(\frac{q_i}{q_{cap;i}} \right)}{1 - \left(\frac{q_i}{q_{cap;i}} \right)} \right) \quad \text{for } q_i \in [0; q_{cap;i}]$$

Where τ_i^0 $[\mathrm{h}]$ is the free flow travel time, α_i, β_i and γ_i are parameters and $q_{cap;i}$ $[\mathrm{veh/h}]$ is the capacity of link i. Notice that for the Davidson function $f_i(q_i) \to \infty$ for $q_i \to q_{cap;i}$ whereas the BPR function accepts $q_i \geq q_{cap;i}$. This results in the behaviour of the functions depicted in figure 4.

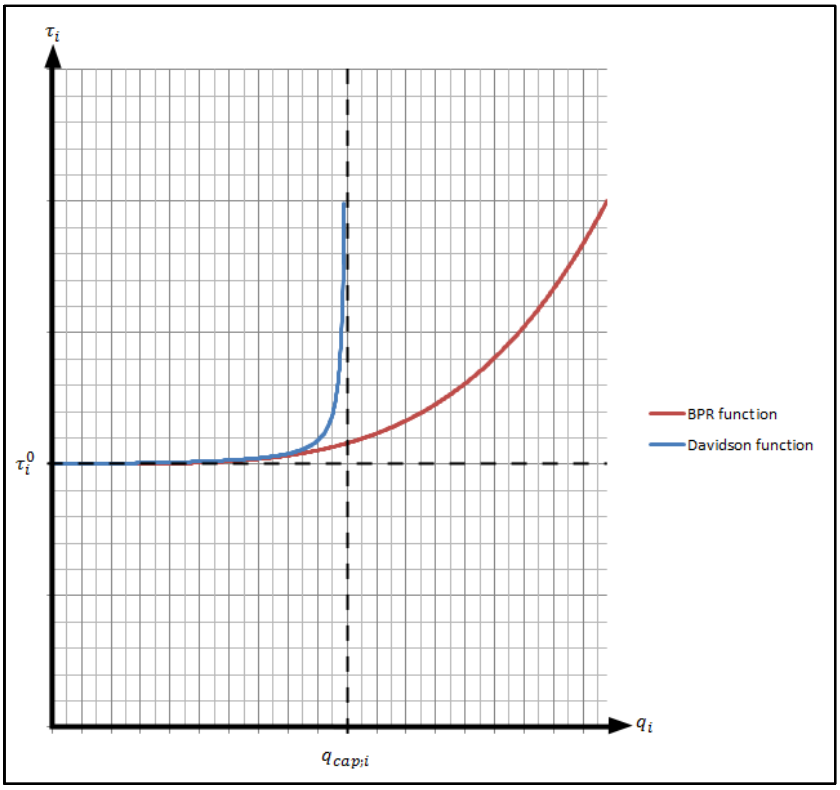

Figure 4 Behaviour of the BPR function and Davidson function

Once the travel time τ_i is known, static assignment models use it to determine the speeds v_i $[\mathrm{km/h}]$ in the links, as low speeds are typical of congestion. Static assignment models assume constant uniform speeds, thus for a link length ℓ_i $[\mathrm{km}]$ it follows:

$$v_i = \frac{\ell_i}{\tau_i}$$

The speeds are not used for assignment, but are generally presented by static assignment models to highlight the bottlenecks (i.e. the places where the flow approaches or surpasses capacity) as these are classically considered the main cause of congestion.

Static assignment models assume constant flows q_i as well, which means that the fundamental relation dictates that the density ρ_i $[\mathrm{veh/km}]$is constant as well, and can be calculated by:

$$\rho_i = \frac{q_i}{v_i}$$

Which is subsequently used to determine the amount of vehicles N_i in the link:

$$N_i = \rho_i \ell_i$$

These values are not used for assignment, but are very useful for comparison between static assignment models and other models.

Advantage of static assignment models is the ease at which the calculations can be done. Disadvantages are the dubious behaviour of the travel time around capacity and the fact that static assignment models predict congestion at bottlenecks rather than in front of bottlenecks (Pel, 2013 [1])

Link performance models

Link performance models are models based on cumulative curves (Pel, 2013 [1]) Cumulative curves represent the number of vehicles that have passed a cross section x from an arbitrary starting moment t_0 (Hoogendoorn, 2007 [1]). Usually, cumulative curves are used as smooth functions $N(x;t)$ for which it holds that the flow $q(x;t)$ $[\mathrm{veh/h}]$ and density $\rho(x;t)$ $[\mathrm{veh/km}]$ are given by:

$$q(x;t) = \frac{\partial N(x;t)}{\partial t}$$

$$\rho(x;t) = -\frac{\partial N(x;t)}{\partial x}$$

Link performance models use the cumulative curves at the entrance and exits of a link to calculate the number of vehicles in the link (Pel, 2013 [1]). From this, they calculate the density in the link. They assume the speed in the link is a function of the density. A sketch of such a function is given in figure 5.

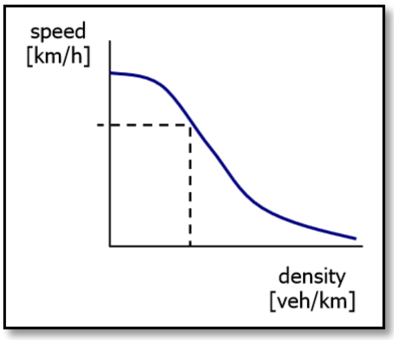

Figure 5 Sketch of a function connecting speed and density in a link performance model (source: Pel, 2013 [1])

Based on the speed, link performance models calculate the uniform travel time, which they use to check the inflow from that long ago compared to the capacity of the link; the lowest of these two becomes the outflow of the link and is used to update the cumulative outflow curve and the cumulative inflow curve of the next link (Pel, 2013 [1]).

Advantage of link performance models is the ease at which the calculations can be done, while the outflow of links is limited. Disadvantage of link performance models is that the inflow is not limited, which still causes the models to predict congestion at bottlenecks rather than in front of bottlenecks (Pel, 2013 [1]).

Vertical queue models

Vertical queue models are, like link performance models, based on cumulative curves (Pel, 2013 [1]). Vertical queue models use the fact that the derivative of the cumulative curve at the exit of a link is equal to the outflow of the link. Subsequently, they limit this outflow such that it is at most equal to the capacity of the next link. Excess inflow is temporarily stored in a vertical queue at the exit of the link. Advantage of vertical queue models is the ease at which the calculations can be done, while both inflow and outflow of links are limited. Disadvantage of vertical queue models is that in reality there is no such thing as a vertical queue; as a result, vertical queue models predict vertical queues just upstream of bottlenecks, whereas in reality the queue will spread out horizontally.

Horizontal queue models

Horizontal queue models are similar to vertical queue models (Pel, 2013 [1]). The difference is that where vertical queue models let the traffic that cannot enter the next link wait in a vertical queue at the exit of the link, horizontal queue models describe a horizontal queue. Outflow of this horizontal queue is equal to the capacity of the next link, and thus happens at the exit of a link. This means that, while horizontal queue models describe queues that spread out horizontally over the network, they keep the head of the queues fixed at the location where a queue started to form, which does not realistically describe queue dissipation, as realistic queue dissipation looks rather more like it is sketched in figure 6.

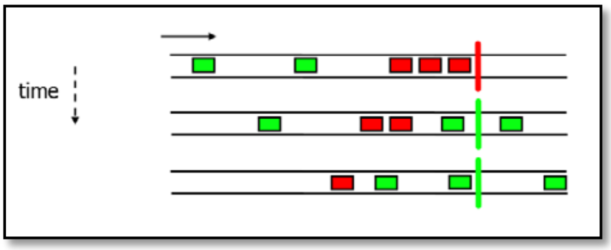

Figure 6 Sketch of realistic queue dissipation; the queue head moves against the flow of traffic (source: Pel, 2013 [1])

Fundamental diagram models

Fundamental diagram models assume that traffic flow on a roadway stretch is build up out of several shorter stretches where flow q [veh/h], density ρ [veh/km] and speed v [km/h] are constant (Pel, 2013 [1]). Between those shorter stretches, moving shocks exist, of which the speeds can be determined by taking the difference in flow and dividing it by the difference in density; when considering the shock between stretch i with constant q_i, ρ_i and v_i and stretch j with constant q_j, ρ_j and v_j, it moves at a speed ω_{ij} [km/h]:

$$\omega_{ij} = \frac{q_j - q_i}{\rho_j - \rho_i}$$

Notice that this speed corresponds to the slope in the q-ρ fundamental diagram. Negative speeds correspond to shocks that move against the direction of traffic, whereas positive speeds correspond to shocks that move in the direction of traffic. When the two shocks separating a stretch from other stretches meet, that stretch will disappear and a new shock between the other stretches will be formed. This way, the fundamental diagram models come quite close to reality; fundamental diagram models allow the heads of queues to move during queue dissipation. The only significant difference between fundamental diagram models and reality is that fundamental diagram models do not take into account that vehicles need to accelerate or decelerate to drive at a different speed. Another disadvantage of fundamental diagram models is that it can be quite challenging to keep track of all the moving stretches in a network.

The METANET model

The METANET model is an extension of fundamental diagram models, as it takes acceleration and deceleration of vehicles into account (Pel, 2013 [1]). It was proposed by Kotsialos et al (2002) and it does so by dividing the freeway network in links which are subdivided in segments (Hegyi et al, 2005). For each segment, the flow is calculated from the current speed and density with the fundamental relation, and the evolutions of the density and speed are calculated. The evolution of density is a result of the current difference between inflow and outflow and the evolution of speed is a result of the desired speed given the current density, the inflow of vehicles with a different speed and the reaction of drivers to density changes downstream. Advantage of the METANET model is that it practically takes every traffic flow aspect into account. Disadvantage is its computational complexity.

Urban traffic flow models
The urban traffic flow models that have been found in literature are:

- The store- and forward model (Aboudolas et al, 2009)
- Adapted freeway link flow models combined with intersection models (Sander-Smits, 2013)
- Link transmission model (Yperman, 2007)
- Extended Kashani model (Van den Berg et al, 2007)

These traffic flow models will now be discussed.

<u>The store- and forward model</u>
The store- and forward traffic flow model stores traffic in a link if it cannot forward all of its inflow and internal departures through outflow and internal arrivals (Aboudolas et al, 2009). The storage capacity of a link is bounded, however, and this puts a constraint on the inflow of a link, which puts a constraint on the outflow of a previous link. The outflow of the previous link is constrained in two more ways: first it must be so that it does not cause negative link storage and at its maximum it can be the average flow during a traffic signal cycle (or similar constraints for intersections that are not controlled with a signal). Advantage of the store- and forward model are that it basically is a vertical queue model with a constraint on queue height which allows for a linear state-space description for networks of arbitrary size, topology and characteristics (Papageorgiou et al, 2003). Disadvantages are that spillback effects in the model jump in a shock-like fashion from link to link, whereas in reality they move more continuously, and that queue heads do not move.

<u>Adapted freeway link flow models combined with intersection models</u>
Freeway link flow models can easily be adapted by changing parameter values in order to be combined with intersection models if they do not contain nodes (Pel, 2013 [1]). This is the case for:

- Static assignment models
- Link performance models
- Vertical queue models
- Horizontal queue models
- Fundamental diagram models

Intersection models do then model what happens with the traffic flow when links end or begin at an intersection. There are two types of intersection models (Sander-Smits, 2013):

- Node models
- Spatial intersection models

Node models model the intersection as one node, whereas spatial intersection models model the intersection such that each conflict point and diverging point in the intersection has one node, for which simple rules for continuing flows can be formulated (Sander-Smits, 2013). For the node models, there exist several models can be distinguished:

- Capacity proportional model
- Capacity consumption equivalence model
- Single server model
- Equal outlink delay model

Thus there are 5 link flow models that can be combined with 5 intersection models. This means that in total, there are 25 combinations possible. For all combinations, the following basic notions hold (Sander-Smits, 2013):

- Input variables:
    - o Set of ingoing links (inlinks) I
    - o Set of outgoing links (outlinks) J
    - o Set of turns $\mathcal{T} \subseteq \{ij | i \in I; j \in J\}$
    - o Demand from inlinks (Sending flow) S_i [veh/h] for $\forall i \in I$
    - o Supply at outlinks (Receiving flow) R_j [veh/h] for $\forall j \in J$
    - o Link capacity Q_i [veh/h] for $\forall i \in I \cup J$
    - o Turn fractions $\alpha_{ij} \in [0; 1]$ for $\forall ij \in \mathcal{T}$
- Input constraint: the sum of all turn fractions must be 1 for all inlinks i:
$$\sum_{j \in J} \alpha_{ij} = 1 \quad \text{for } \forall i \in I$$
- Output variables:
    - o The flows in inlinks i f_i [veh/h] are the maximum value they can have given the model constraints:
$$f_i = \max(f_i) \quad \text{for } \forall i \in I$$
    - o The demand on turn ij S_{ij} [veh/h] is equal to:
$$S_{ij} = \alpha_{ij} S_i$$
    - o The flow on turn ij f_{ij} [veh/h] is equal to:
$$f_{ij} = \alpha_{ij} f_i$$
    - o The total demand towards outlink j S_j [veh/h] is equal to:
$$S_j = \sum_{i \in I} \alpha_{ij} S_i$$
    - o The total flow into outlink j f_j [veh/h] is equal to:
$$f_j = \sum_{i \in I} \alpha_{ij} f_i$$
- Model constraints:
    - o Flows are non-negative:
$$f_i \geq 0 \quad \text{for } \forall i \in I$$
    - o Flows are never larger than demand:
$$f_i \leq S_i \quad \text{for } \forall i \in I$$
    - o Supply constraint:
$$f_j \leq R_j \quad \text{for } \forall j \in J$$

- Invariance principle: if the flow in inlink i is smaller than the demand at inlink i, it cannot be dependent on the demand. This constraint can be put in a basic notion formula as follows:

$$\frac{\partial f_i}{\partial S_i} = \begin{cases} 1 & \text{if } f_i = S_i \\ 0 & \text{if } f_i < S_i \end{cases} \quad \text{for } \forall i \in I$$

Spatial intersection models have one node for each conflict point (Sander-Smits, 2013). Thus there are by definition at most two inlinks and two outlinks, and inlinks and outlinks that are connected by turn fractions that are equal to 1 in case of conflicts and by splitting fractions in case of diverging points. Depending on the type of intersection, one of the flows can have priority (which means the other flow is determined by the gaps in the priority flow) or the flows can be controlled by a signal, which means that both flows can pass at certain times in case of a diverging point, either one of the flows can pass at certain times in case of a conflict, or neither of the flows.

The capacity proportional model distributes the supply R_j proportional to directed capacities D_i (Sander-Smits, 2013):

$$D_i = \alpha_{ij} Q_i$$
$$f_j = R_j$$
$$f_j = P_j \sum_{i \in I} D_i$$

Where P_j is the proportionality constant the capacity proportional model uses. The capacity proportional model searches for each inlink for the most limiting supply constraint.

The capacity consumption equivalence model models occupation times for vehicles in inlinks (Sander-Smits, 2013). Based on the available supply in outlinks, the occupation times in inlinks is modelled. Based on the occupation times, the flows are calculated.

The single server model states that the delay τ_i^{del} [h] for inlink i is inversely proportional to its capacity (Sander-Smits, 2013):

$$\tau_i^{del} = \frac{P_i}{Q_i}$$

Where P_i is the proportionality constraint the single server model uses. Based on this information, the single server model searches for flows that match this delay.

The equal outlink delay model states that the experienced delay for all vehicles heading towards the same outlink is equal (Sander-Smits, 2013). Based on this information, the equal outlink delay model searches for flows that match this delay.

Advantages of combining adapted freeway link flow models with intersection models is that this allows for application of freeway link flow models in an urban context and that intersection models take all typical aspects of intersections into account. Disadvantage is that intersection models often produce results that can differ from reality quite drastically due to the non-unique nature of intersections; take for instance the left-turning behaviour of a regular intersection with no turning lane. Through traffic has priority, so if the first vehicle in one direction wants to turn left, it will block the intersection as long as there is traffic from the other direction, whereas opposing traffic can freely turn left at any time. However, the directions can freely be swapped, showing the non-uniqueness.

Link transmission model

The link transmission model consists of a link model for updating the link dynamics and a node model to connect the links (Yperman, 2007). The link dynamics are described by cumulative curves of the link inflow and outflow, similar to a collection of freeway link flow models called link performance models. The difference is that the link transmission model restricts the outflow in three ways:

- The outflow cannot be larger than the inflow a free flow travel time ago
- The outflow cannot be larger than the saturation flow multiplied by the fraction of a cycle time green is given to a link
- The outflow cannot be larger than the minimum of these two multiplied by a reduction factor ensuring that the outflow of a link can flow into a next link based on the storage space in the next link and the time it takes for a shockwave to propagate through the next link.

The link transmission model allows for different node models to connect the links. One could, for instance, use a capacity-proportional node model like the node model by Tampère et al (2011). Van de Weg et al (2016) propose to use a demand-proportional node model.

Advantages of the link transmission model are that it models traffic jams similar to horizontal queue models, but additionally models spillback effects and the way they affect traffic waiting to enter links that are filled and that it allows for a linear state-space description (Van de Weg et al, 2016). Disadvantage is that signal plans cannot be taken explicitly into account; only the average flow during a signal cycle is considered by multiplying the saturation flow with the green time share of the considered stage. The green time share of a stage is defined as the share of the cycle time that a stage (a collection of approaches with no mutual conflicts (Salomons, 2014)) has green.

Extended Kashani model

The extended Kashani model is based on the Kashani model (Kashani & Sardis, 1983) and explicitly models intersections with green phases and red phases (Van den Berg et al, 2007) During each time step, it calculates what numbers of vehicles can be moved into a link, from one end of a link to the end of a queue at the other end of the link, from the end of this queue in a queue waiting for a green phase and from this queue into the next link. Advantage of the extended Kashani model is that it models traffic jams similar to the link transmission model, but additionally models intersection dynamics more realistically. Disadvantage is that the model only contains one congested traffic state, whereas fundamental diagram models can contain more congested traffic states.

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Annex 2: Deriving a formula for the number of integrations

Annex 2 on deriving a formula for the number of integrations gives the derivation of said formula.

In subsection 2.2.5 of the main report it had been established that the possible integrations are combinations of at least one urban algorithm and at least one freeway algorithm. The algorithms that are considered are:

- Signal control algorithms; these are urban algorithms
- Ramp metering algorithms; these are freeway algorithms
- Speed limit algorithms; these are freeway algorithms
- Route guidance algorithms; these algorithms control both the urban and the freeway network, thus any combination involving them is a possible integration
- Combined algorithms; there is only one combined algorithm considered in the main report, and it combines a ramp metering algorithm and a speed limit algorithm, thus it is a freeway algorithm

Now, given the above definition, a possible integration can consist of 4, 3 or 2 algorithms. Combinations of the same type of algorithms are not possible, as they will attempt to control the same control variables in a different manner. As a result, the possible integrations sorted by the number of algorithms they consist of are:

- 4 algorithms:
    - The possible integrations consist of signal control algorithms, ramp metering algorithms, speed limit algorithms and route guidance algorithms. When the number of signal control algorithms is denoted by a, the number of ramp metering algorithms is denoted by b, the number of speed limit algorithms is denoted by c and the number of route guidance algorithms is denoted by d, the number of possible integrations described this way is equal to $abcd$.
- 3 algorithms:
    - The possible integrations consist of signal control algorithms, route guidance algorithms and combined algorithms. When the number of combined algorithms is denoted by e, the number of possible integrations described this way is equal to ade.
    - The possible integrations consist of signal control algorithms, ramp metering algorithms and speed limit algorithms. The number of possible integrations described this way is equal to abc.
    - The possible integrations consist of signal control algorithms, ramp metering algorithms and route guidance algorithms. The number of possible integrations described this way is equal to abd.
    - The possible integrations consist of signal control algorithms, speed limit algorithms and route guidance algorithms. The number of possible integrations described this way is equal to acd.
    - The possible integrations consist of ramp metering algorithms, speed limit algorithms and route guidance algorithms. The number of possible integrations described this way is equal to bcd.

- 2 algorithms:
    - The possible integrations consist of signal control algorithms and combined algorithms. The number of possible integrations described this way is equal to ae.
    - The possible integrations consist of route guidance algorithms and combined algorithms. The number of possible integrations described this way is equal to de.
    - The possible integrations consist of signal control algorithms and ramp metering algorithms. The number of possible integrations described this way is equal to ab.
    - The possible integrations consist of signal control algorithms and speed limit algorithms. The number of possible integrations described this way is equal to ac.
    - The possible integrations consist of signal control algorithms and route guidance algorithms. The number of possible integrations described this way is equal to ad.
    - The possible integrations consist of ramp metering algorithms and route guidance algorithms. The number of possible integrations described this way is equal to bd.
    - The possible integrations consist of speed limit algorithms and route guidance algorithms. The number of possible integrations described this way is equal to cd.

Now, denoting the total number of possible integrations with N, the total number of integrations can be calculated with the formula:

$$N = abcd + ade + abc + abd + acd + bcd + ae + de + ab + ac + ad + bd + cd$$

Annex 3: Determining simulation aspects

Annex 3 on determining simulation aspects gives an overview of the more complicated determinations of simulation aspects. These simulation aspects include the disturbances and the initial control signals. The disturbances will be determined in section A3.1 and the initial control signals will be determined in section A3.2. The annex is concluded with references.

A3.1 Disturbances

Disturbances include demands, turn fractions, exit capacities and downstream densities. Determining them has been simplified in subsection 4.1.3 of the main report to determining them during the peak period such that the demands from the freeway network and the urban network for freeway link 3 in figure 1 are such that together they surpass the capacity of 4000 veh/h. This creates the congestion due to high demand of traffic moving from the urban to the freeway network described in the problem description of this MSc Thesis project.

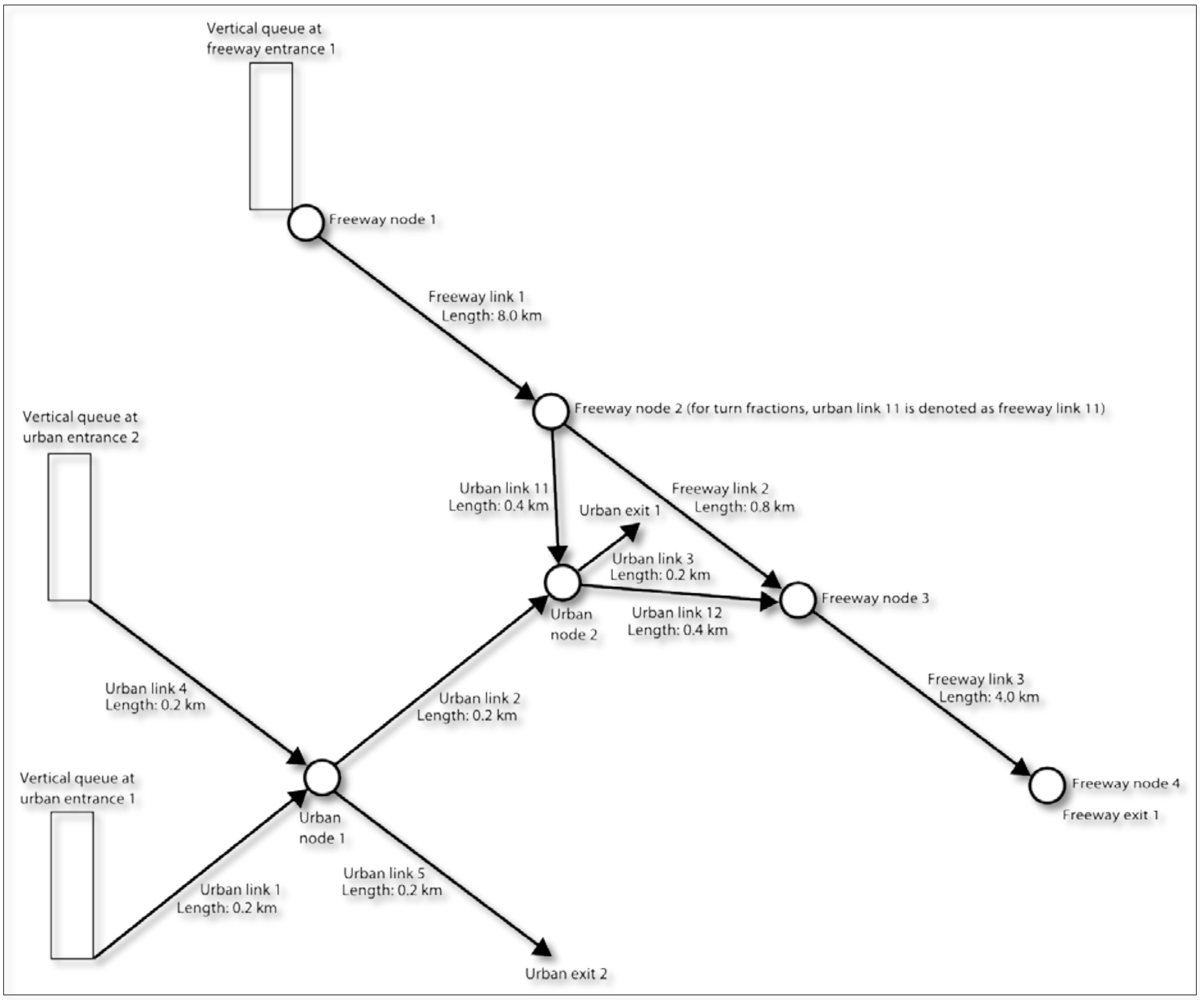

Figure 1 The simulation network (figure inspired by Van de Weg et al (2015) and Van de Weg et al (2016))

Overseeing the effects of choosing values for the disturbances is not easy. Therefore, it has been chosen to set up a simplified analytical calculation to determine the effects of choosing the values. Once the effects can be determined, one can optimise them such that the values of the disturbances reflect the traffic situation best.

Simplified analytical calculation

Given that it is hard to oversee the effects of choosing values for the disturbances is not easy, it is difficult as well to determine what the effects are. However, it is reasonable that the effects are at least related to the total time spent. Therefore, the simplified analytical calculation will be focussed on determining the total time spent. The analytical approximation that can be derived will be referred to as the comparative total time spent. Before considering the various control actions, a general formula needs to be determined. Based on this, the comparative total time spent can be determined analytically for the various control actions, albeit expressed in parameters, disturbances and control variables. Given that the controller developed in the main report chooses the control variables to optimise the total time spent according to its objective function, it makes sense to try and optimise the effects of the disturbances such that the control variables can be determined in the same optimisation. Therefore, only the parameters need to be determined before the optimisation can start.

The general formula

Determining a general formula for the total time spent can be done by considering the simulation network as a whole. Denote the number of vehicles in the network at time step k as $N(k)$, the total outflow as $q^{out;tot}(k)$ and the total inflow as $q^{in;tot}(k)$. The amount of vehicles in the network at time step $k+1$ is then equal to the amount of vehicles at time step k added to the inflow at time step k multiplied by the duration of a time step T minus the outflow at time step k multiplied by the duration of a time step T:

$$N(k+1) = N(k) + q^{in;tot}(k)T - q^{out;tot}(k)T$$

In order to calculate the total time spent J^{TTS}, one needs to sum the number of vehicles over all time instances from a first time step $k = k_1$ until a final time step $k = k_1 + N_s$ (where N_s is the number of time steps taken from the first time step until the final time step) and multiply by the duration of a time step T. In formula form:

$$J^{TTS} = T \sum_{k=k_1}^{k_1+N_s} N(k)$$

In order to simplify this analytical calculation, assume that at time step $k = k_1$ all effects of the control actions that need to spread over the network, already have spread and that the inflow has a constant value $q^{in;tot}$ over the time steps. Given that the other disturbances remain constant over the peak period considered here, the outflow $q^{out;tot}(k)$ has a constant value $q^{out;tot}$ and one can write for the number of vehicles at time step $k = k_1 + \tilde{k}$:

$$N\left(k_1 + \tilde{k}\right) = N(k_1) + \tilde{k} q^{in;tot} T - \tilde{k} q^{out;tot} T$$

Which can be simplified to:

$$N\left(k_1 + \tilde{k}\right) = N(k_1) + \tilde{k} T\left(q^{in;tot} - q^{out;tot}\right)$$

Which allows to write:

$$\sum_{k=k_1}^{k_1+N_s} N(k) = \sum_{\tilde{k}=0}^{N_s} \left(N(k_1) + \tilde{k} T\left(q^{in;tot} - q^{out;tot}\right)\right)$$

Which can be recognized as an arithmetic series. Using the formula for arithmetic series (WolframAlpha, 2016) one can write:

$$\sum_{\tilde{k}=0}^{N_s} \left(N(k_1) + \tilde{k} T\left(q^{in;tot} - q^{out;tot}\right)\right) = (N_s + 1)N(k_1) + \tfrac{1}{2} T (N_s + 1) N_s \left(q^{in;tot} - q^{out;tot}\right)$$

When comparing the effects of control actions on the total time spent via simplified analytical calculation, one would be interested in the second term of this sum, as that term shows how the total time spent is affected by the control actions once their effect has spread over the network. Therefore, the first term is omitted and the comparative total time spent J^{CTTS} is given by:

$$J^{CTTS} = \frac{1}{2}T^2(N_s + 1)N_s\left(q^{in;tot} - q^{out;tot}\right)$$

The inflow is given by the demand at urban entrance 1 d_1^U, the demand at urban entrance 2 d_2^U and the demand at freeway entrance 1 d_1^F:

$$q^{in;tot} = d_1^U + d_2^U + d_1^F$$

The outflow is given by the outflow out of urban link 3 q_3^U, the outflow out of urban link 5 q_5^U and the outflow out of freeway link 3 q_3^F:

$$q^{out;tot} = q_3^U + q_5^U + q_3^F$$

The values of these outflows are dependent on the control actions taken.

<u>The control actions</u>

The control actions considered here are:

- No control: no control actions are undertaken. There are no variable speed limits on the freeway and the green time shares for all urban links are set to 50% (except urban link 12, which has a green time share of 100%, as no ramp metering is active). This value for the green time shares was inspired by the fact that all conflicts in the network are the results of two conflicted approaches at an intersection.
- Coordinated controllers active: there are two controllers active: an urban controller that attempts to minimise the total time spent in the urban network (urban links 1-5, 11 and 12 and urban entrances 1 and 2) and a freeway controller that attempts to minimise the total time spent in the freeway network (freeway links 1-3 and freeway entrance 1).
- Integrated controller active: there is one controller that attempts to minimise the total time spent by all vehicles in the combined networks.

Now, the comparative total time spent for these control actions can be found by determining the values of their outflows.

For the *No control* action a traffic jam forms in freeway link 3 that spills back onto the freeway and affects the outflow on urban link 11 and 3. Test simulations have shown that spillback on the on-ramp does not occur given the parameter values used in the on-ramp model and the green time shares used in the urban network. Therefore, the outflows for the no control action are as follows:

- The outflow out of freeway link 3 is equal to the jam dissipation rate q_{dis}. Thus $q_3^F = q_{dis}$
- The outflow out of urban link 3 is equal to the turn fractions towards link 3 multiplied by the outflows out of urban links 2 and 11, in formula form $q_3^U = \eta_{2;3}q_2^U + \eta_{11;3}q_{11}^U$. The outflow out of urban link 2 is equal to the turn fractions towards link 2 multiplied by the outflows out of link 1 and 4, in formula form $q_2^U = \eta_{1;2}q_1^U + \eta_{4;2}q_4^U$. The outflows q_1^U and q_4^U are equal to the demands from urban entrances 1 and 2, $q_1^U = d_1^U$ and $q_4^U = d_2^U$. The outflow out of urban link 11 is equal to the turn fraction from freeway node 2 to urban link 11 multiplied by the inflow into freeway node 2, in formula form $q_{11}^U = \beta_{2;11}q_1^F$. The inflow into freeway node 2 can be determined by the fact that it is equal to the outflow, i.e. $q_1^F = q_{11}^U + q_2^F$. q_2^F can be determined from the fact that $q_2^F + q_{12}^U = q_{dis}$, where

$q_{12}^U = \eta_{2;12} q_2^U = \eta_{2;12}\left(\eta_{1;2} q_1^U + \eta_{4;2} q_4^U\right) = \eta_{2;12}\left(\eta_{1;2} d_1^U + \eta_{4;2} d_2^U\right)$. This means that $q_2^F = q_{dis} - \eta_{2;12}\left(\eta_{1;2} d_1^U + \eta_{4;2} d_2^U\right)$. Now the following equation is found for q_1^F:

$$q_1^F = \beta_{2;11} q_1^F + q_{dis} - \eta_{2;12}\left(\eta_{1;2} d_1^U + \eta_{4;2} d_2^U\right)$$

The solution is:

$$q_1^F = \frac{1}{1-\beta_{2;11}}\left(q_{dis} - \eta_{2;12}\left(\eta_{1;2} d_1^U + \eta_{4;2} d_2^U\right)\right)$$

Therefore:

$$q_{11}^U = \frac{\beta_{2;11}}{1-\beta_{2;11}}\left(q_{dis} - \eta_{2;12}\left(\eta_{1;2} d_1^U + \eta_{4;2} d_2^U\right)\right)$$

$$\begin{aligned} q_3^U &= \eta_{2;3}\left(\eta_{1;2} d_1^U + \eta_{4;2} d_2^U\right) + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}\left(q_{dis} - \eta_{2;12}\left(\eta_{1;2} d_1^U + \eta_{4;2} d_2^U\right)\right) \\ &= \left(\eta_{2;3} - \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}\eta_{2;12}\right)\left(\eta_{1;2} d_1^U + \eta_{4;2} d_2^U\right) + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} q_{dis} \\ &= a_{3;1} d_1^U + a_{3;2} d_2^U + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} q_{dis} \end{aligned}$$

Where:

$$a_{3;1} = \eta_{1;2}\eta_{2;3} - \eta_{1;2}\eta_{2;12}\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}$$

$$a_{3;2} = \eta_{4;2}\eta_{2;3} - \eta_{4;2}\eta_{2;12}\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}$$

- The outflow out of urban link 5 is equal to the turn fractions towards link 5 multiplied by the outflows out of link 1 and 4, which are the demands of urban entrances 1 and 2. In formula form $q_5^U = \eta_{1;5} d_1^U + \eta_{4;5} d_2^U$

The differences between inflows and outflows are now given by:

$$\begin{aligned} q^{in;tot} - q^{out;tot} &= d_1^U + d_2^U + d_1^F - a_{3;1} d_1^U - a_{3;2} d_2^U - \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} q_{dis} - \eta_{1;5} d_1^U - \eta_{4;5} d_2^U - q_{dis} \\ &= a_1 d_1^U + a_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} + 1\right) q_{dis} \end{aligned}$$

Where:

$$a_1 = 1 - a_{3;1} - \eta_{1;5}$$
$$a_2 = 1 - a_{3;2} - \eta_{4;5}$$

Which means the comparative total time spent is equal to:

$$J_{No\ control}^{CTTS} = \tfrac{1}{2} T^2 (N_s + 1) N_s \left(a_1 d_1^U + a_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} + 1\right) q_{dis}\right)$$

When *coordinated controllers* are active, the freeway controller places a speed-limited area on the freeway to limit the outflow out of freeway link 2 such that the inflow into freeway link 3 from freeway link 2 and urban link 12 add up to the capacity of freeway link 3. This affects the outflow on urban link 11 and 3. As the urban controller has no way to improve the total time spent in the urban network by varying the green time shares, it lets them stay at 50%. Therefore, the effects on the outflows are as follows:

- The outflow out of freeway link 3 is equal to the capacity q_{cap}. Thus $q_3^F = q_{cap}$
- The outflow out of urban link 3 is calculated similar to how it was calculated for the no control action. Since the outflow of urban link 2 is not affected by the control actions, it

remains the same. The outflow out of urban link 11 is once again determined by the outflow out of freeway link 1, which is in turn determined by the outflow out of freeway link 2, which can now be determined from the fact that $q_2^F + q_{12}^U = q_{cap}$. Therefore, the only thing that changed is that q_{dis} has been replaced with q_{cap} and one can write the following formula:

$$q_3^U = a_{3;1} d_1^U + a_{3;2} d_2^U + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} q_{cap}$$

- The outflow out of urban link 5 is not affected by the control actions, therefore it remains the same. Thus $q_5^U = \eta_{1;5} d_1^U + \eta_{4;5} d_2^U$

The differences between inflows and outflows are now given by:

$$\begin{aligned} q^{in;tot} - q^{out;tot} &= d_1^U + d_2^U + d_1^F - a_{3;1} d_1^U - a_{3;2} d_2^U - \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} q_{cap} - \eta_{1;5} d_1^U - \eta_{4;5} d_2^U - q_{cap} \\ &= a_1 d_1^U + a_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} + 1\right) q_{cap} \end{aligned}$$

Where:

$$a_1 = 1 - a_{3;1} - \eta_{1;5}$$
$$a_2 = 1 - a_{3;2} - \eta_{4;5}$$

Which means the comparative total time spent is equal to:

$$J_{Coordinated}^{CTTS} = \tfrac{1}{2} T^2 (N_s + 1) N_s \left(a_1 d_1^U + a_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} + 1\right) q_{cap} \right)$$

Which has a lower value than the value for the no control action, provided $q_{cap} > q_{dis}$

When the *integrated controller* is active, the outflow out of freeway link 2 resulting from applying a speed-limited area on the freeway can be higher, because the integrated controller also limits the outflow out of urban link 2. This causes urban link 2 to fill and at some point the integrated controller will limit the outflow out of links 1 and 4 in order to maintain a higher outflow at link 5 than when spillback would have limited these outflows. Therefore, the effects on the outflows are as follows:

- The outflow out of freeway link 3 is equal to the capacity q_{cap}. Thus $q_3^F = q_{cap}$
- The outflow out of urban link 3 is dependent on the outflows of urban link 2 and 11 similar to the no control action. Given reduction factors f_1, f_2 and f_4, the outflow out of link 2 is now given by $q_2^U = f_1 f_2 \eta_{1;2} d_1^U + f_4 f_2 \eta_{4;2} d_2^U$. The outflow out of link 11 is determined similar to the no control action via the outflow out of freeway link 1, freeway link 2 and $q_2^F + q_{12}^U = q_{cap}$, where q_{12}^U is now given by $q_{12}^U = f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U + f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U$. Therefore $q_2^F = q_{cap} - f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U$ and:

$$q_{11}^U = \frac{\beta_{2;11}}{1-\beta_{2;11}} \left(q_{cap} - f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U \right)$$

$$\begin{aligned} q_3^U = {} & f_1 f_2 \eta_{1;2} \eta_{2;3} d_1^U + f_4 f_2 \eta_{4;2} \eta_{2;3} d_2^U \\ & + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} \left(q_{cap} - f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U \right) \end{aligned}$$

$$= \left(f_1 f_2 \eta_{1;2} \eta_{2;3} - \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} f_1 f_2 \eta_{1;2} \eta_{2;12} \right) d_1^U$$
$$+ \left(f_4 f_2 \eta_{4;2} \eta_{2;3} - \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} f_4 f_2 \eta_{4;2} \eta_{2;12} \right) d_2^U + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} q_{cap}$$
$$= f_1 f_2 a_{3;1} d_1^U + f_4 f_2 a_{3;2} d_2^U + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} q_{cap}$$

- The outflow out of urban link 5 is equal to the turn fractions towards link 5 multiplied by the reduced outflows out of link 1 and 4. Thus, the outflow out of urban link 5 is given by the formula $q_5^U = f_1 \eta_{1;5} d_1^U + f_4 \eta_{4;5} d_2^U$

The differences between inflows and outflows are now given by:

$$q^{in;tot} - q^{out;tot} = d_1^U + d_2^U + d_1^F - f_1 f_2 a_{3;1} d_1^U - f_4 f_2 a_{3;2} d_2^U - \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} q_{cap}$$
$$- f_1 \eta_{1;5} d_1^U - f_4 \eta_{4;5} d_2^U - q_{cap}$$
$$= b_1 d_1^U + b_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} + 1 \right) q_{cap}$$

Where:

$$b_1 = 1 - f_1 f_2 a_{3;1} - f_1 \eta_{1;5}$$
$$b_2 = 1 - f_4 f_2 a_{3;2} - f_4 \eta_{4;5}$$

Which means the comparative total time spent is equal to:

$$J_{Integrated}^{CTTS} = \tfrac{1}{2} T^2 (N_s + 1) N_s \left(b_1 d_1^U + b_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}} + 1 \right) q_{cap} \right)$$

Which has a lower value than the value for coordinated controllers, provided $b_1 < a_1$ or $b_2 < a_2$ (the one that is not smaller should be equal). Which is possible, as calculations have shown that it is possible to have $a_{3;1} < 0$ and $a_{3;2} < 0$.

<u>Determining the parameters</u>

The comparative total time spent has been expressed in terms of the following parameters, disturbances and control variables:

T [h]:	The duration of a time step
N_s $[-]$:	The number of time steps taken from the first time step until the final time step
f_1 $[-]$:	Reduction factor for urban link 1
f_2 $[-]$:	Reduction factor for urban link 2
f_4 $[-]$:	Reduction factor for urban link 4
$\eta_{1;2}$ $[-]$:	Turn fraction from urban link 1 to urban link 2
$\eta_{2;3}$ $[-]$:	Turn fraction from urban link 2 to urban link 3
$\eta_{4;2}$ $[-]$:	Turn fraction from urban link 4 to urban link 2
$\eta_{2;12}$ $[-]$:	Turn fraction from urban link 2 to urban link 12
$\eta_{11;3}$ $[-]$:	Turn fraction from urban link 11 to urban link 3
$\eta_{1;5}$ $[-]$:	Turn fraction from urban link 1 to urban link 5
$\eta_{4;5}$ $[-]$:	Turn fraction from urban link 4 to urban link 5
$\beta_{2;11}$ $[-]$:	Turn fraction from freeway node 2 to urban link 11
d_1^U [veh/h]:	Demand at urban entrance 1
d_2^U [veh/h]:	Demand at urban entrance 2

d_1^F [veh/h]: Demand at freeway entrance 1
q_{dis} [veh/h]: Jam dissipation rate
q_{cap} [veh/h]: Capacity of freeway link 3
Out of these, T, N_s, q_{dis} and q_{cap} are parameters that are to be determined now. The value of q_{cap} has already been given in the introduction of this section: $q_{cap} = 4000$ veh/h. Given the model sampling time and controller sampling time of 10 seconds used in the main report, it makes sense to use $T = 10$ s. N_s needs to be such that the considered period is shorter than the peak period (as in the beginning of the peak period of the effects of control actions that need to spread over the network still need to spread over the network), but long enough to make a reasonable comparison between the control actions. As it is reasonable to have a peak period that takes longer than an hour, it makes sense to consider a one hour period, which amounts to setting $N_s = 360$. This leaves q_{dis} to be determined.

q_{dis} can be determined by running a test simulation on the freeway network, consisting of freeway links 1, 2 and 3, vertical queues at freeway entrance 1 and the on-ramp and destinations at freeway exit 1 and the off-ramp. This can be done by running the no control action in the script Main_VSL_PMPC.m and setting the demand (including the loading demand) at freeway entrance 1 to 3000 veh/h, the turn fraction at the off-ramp such that no traffic leaves the freeway there and the demand at the on-ramp such that it grows linear from 1000 veh/h to 2000 veh/h over the course of the simulation duration of 2 hours. The length of the simulation duration has been adopted from Van de Weg et al (2016). Doing so results in finding that once congestion has been formed, the outflow of link 3 drops until it reaches a constant value of 3712 veh/h. Therefore, $q_{dis} = 3712$ veh/h which enables coordinated controllers to reduce the comparative total time spent with respect to the no control action.

Optimisation
The optimisation needs to optimise the effects of choosing the values of disturbances and control variables such that reflecting the traffic situation described in the problem description of the MSc Thesis project coincides with optimising the total time spent. This can be done by maximising the difference in comparative total time spent between the coordinated controllers and the integrated controller.

This choice for the optimisation allows to determine the demands and turn fractions, but not the exit capacities and downstream densities. Given the parameter values chosen in section 4.1.2 in the main report, it makes sense to assume capacities of 2000 veh/h for all urban links, therefore it makes sense to set the exit capacities to 2000 veh/h. The downstream densities are determined from the fact that they should not cause congestion. Therefore, they should be below the critical density at all times. Given the critical density chosen in section 4.1.2 in the main report, the downstream densities are set to 25 veh/km/lane. Therefore the values for the disturbances that cannot be determined by optimisation are:
<u>Exit capacities</u>: 2000 veh/h
<u>Downstream densities</u>: 25 veh/km/lane

The aim is now to find:

$$\max\left(J_{Integrated}^{CTTS} - J_{Coordinated}^{CTTS}\right)$$

In doing so, there are constraints. Therefore, these constraints are now to be determined first. After determining the constraints, a MATLAB script is written to execute the optimisation. This MATLAB script is subject to the same software and hardware aspects as the MATLAB scripts used in the simulation. The only differences are that the fmincon options TolX and FinDiffRelStep are given the value 10^{-2} as the turn fractions are considered accurate if correct to two decimal places and that the initial values are based on test simulations and have the following values:

$$\eta_{1;2} = 0.7$$
$$\eta_{2;3} = 0.1$$
$$\eta_{4;2} = 0.7$$
$$\eta_{2;12} = 0.9$$
$$\eta_{11;3} = 1$$
$$\eta_{1;5} = 0.3$$
$$\eta_{4;5} = 0.3$$
$$\beta_{2;11} = 0.1$$
$$d_1^U = 530 \text{ veh/h}$$
$$d_2^U = 530 \text{ veh/h}$$
$$d_1^F = 3920 \text{ veh/h}$$
$$f_1 = 0.9$$
$$f_2 = 0.9$$
$$f_4 = 0.9$$

After writing the MATLAB script, the results are presented and rounded. As a check, the comparative total time spent for the rounded values will be calculated for the various control actions.

Determining the constraints

The constraints can be found by logically reasoning what the flows can be and where congestion can occur or be prevented by control measures.

The simplest constraints are formed by the bounds:

$$0 \le \eta_{1;2} \le 1$$
$$0 \le \eta_{2;3} \le 1$$
$$0 \le \eta_{4;2} \le 1$$
$$\eta_{2;12} = 1 - \eta_{2;3}$$
$$\eta_{11;3} = 1$$
$$\eta_{1;5} = 1 - \eta_{1;2}$$
$$\eta_{4;5} = 1 - \eta_{4;2}$$
$$0 \le \beta_{2;11} \le 1$$
$$0 \le d_1^U \le 2000$$
$$0 \le d_2^U \le 2000$$
$$0 \le d_1^F \le q_{cap}$$
$$0 \le f_1 \le 1$$
$$0 \le f_2 \le 1$$
$$0 \le f_4 \le 1$$

The more complicated constraints are that all calculated flows should be larger than zero:

$$q_2^U = f_1 f_2 \eta_{1;2} d_1^U + f_4 f_2 \eta_{4;2} d_2^U > 0$$
$$q_2^U = \eta_{1;2} q_1^U + \eta_{4;2} q_4^U > 0$$
$$q_{12}^U = f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U + f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U > 0$$
$$q_{12}^U = \eta_{1;2} \eta_{2;12} d_1^U + \eta_{4;2} \eta_{2;12} d_2^U > 0$$
$$q_2^F = q_{cap} - f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U > 0$$
$$q_2^F = q_{cap} - \eta_{1;2} \eta_{2;12} d_1^U - \eta_{4;2} \eta_{2;12} d_2^U > 0$$

$$q_1^F = \frac{1}{1-\beta_{2;11}} q_{cap} - \frac{1}{1-\beta_{2;11}} f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - \frac{1}{1-\beta_{2;11}} f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U > 0$$
$$q_1^F = \frac{1}{1-\beta_{2;11}} q_{cap} - \frac{1}{1-\beta_{2;11}} \eta_{1;2} \eta_{2;12} d_1^U - \frac{1}{1-\beta_{2;11}} \eta_{4;2} \eta_{2;12} d_2^U > 0$$
$$q_{11}^U = \frac{\beta_{2;11}}{1-\beta_{2;11}} q_{cap} - \frac{\beta_{2;11}}{1-\beta_{2;11}} f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - \frac{\beta_{2;11}}{1-\beta_{2;11}} f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U > 0$$

$$q_{11}^U = \frac{\beta_{2;11}}{1-\beta_{2;11}} q_{cap} - \frac{\beta_{2;11}}{1-\beta_{2;11}} \eta_{1;2} \eta_{2;12} d_1^U - \frac{\beta_{2;11}}{1-\beta_{2;11}} \eta_{4;2} \eta_{2;12} d_2^U > 0$$

$$q_3^U = f_1 f_2 a_{3;1} d_1^U + f_4 f_2 a_{3;2} d_2^U + \frac{\eta_{11;3} \beta_{2;11}}{1-\beta_{2;11}} q_{cap} > 0$$

$$q_3^U = a_{3;1} d_1^U + a_{3;2} d_2^U + \frac{\eta_{11;3} \beta_{2;11}}{1-\beta_{2;11}} q_{cap} > 0$$
$$q_5^U = \eta_{1;5} d_1^U + \eta_{4;5} d_2^U > 0$$
$$q_5^U = f_1 \eta_{1;5} d_1^U + f_4 \eta_{4;5} d_2^U > 0$$

Furthermore, the calculated flows should not exceed the capacity of the corresponding links. This results in the constraints:

$$q_2^U = f_1 f_2 \eta_{1;2} d_1^U + f_4 f_2 \eta_{4;2} d_2^U \leq 2000$$
$$q_2^U = \eta_{1;2} q_1^U + \eta_{4;2} q_4^U \leq 2000$$
$$q_{12}^U = f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U + f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U \leq 2000$$
$$q_{12}^U = \eta_{1;2} \eta_{2;12} d_1^U + \eta_{4;2} \eta_{2;12} d_2^U \leq 2000$$
$$q_2^F = q_{cap} - f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U \leq q_{cap}$$
$$q_2^F = q_{cap} - \eta_{1;2} \eta_{2;12} d_1^U - \eta_{4;2} \eta_{2;12} d_2^U \leq q_{cap}$$

$$q_1^F = \frac{1}{1-\beta_{2;11}} q_{dis} - \frac{1}{1-\beta_{2;11}} f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - \frac{1}{1-\beta_{2;11}} f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U \leq q_{cap}$$
$$q_1^F = \frac{1}{1-\beta_{2;11}} q_{dis} - \frac{1}{1-\beta_{2;11}} \eta_{1;2} \eta_{2;12} d_1^U - \frac{1}{1-\beta_{2;11}} \eta_{4;2} \eta_{2;12} d_2^U \leq q_{cap}$$
$$q_{11}^U = \frac{\beta_{2;11}}{1-\beta_{2;11}} q_{dis} - \frac{\beta_{2;11}}{1-\beta_{2;11}} f_1 f_2 \eta_{1;2} \eta_{2;12} d_1^U - \frac{\beta_{2;11}}{1-\beta_{2;11}} f_4 f_2 \eta_{4;2} \eta_{2;12} d_2^U \leq 2000$$

$$q_{11}^U = \frac{\beta_{2;11}}{1-\beta_{2;11}} q_{dis} - \frac{\beta_{2;11}}{1-\beta_{2;11}} \eta_{1;2} \eta_{2;12} d_1^U - \frac{\beta_{2;11}}{1-\beta_{2;11}} \eta_{4;2} \eta_{2;12} d_2^U \leq 2000$$

$$q_3^U = f_1 f_2 a_{3;1} d_1^U + f_4 f_2 a_{3;2} d_2^U + \frac{\eta_{11;3} \beta_{2;11}}{1-\beta_{2;11}} q_{cap} \leq 2000$$

$$q_3^U = a_{3;1}d_1^U + a_{3;2}d_2^U + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}q_{cap} \le 2000$$
$$q_5^U = \eta_{1;5}d_1^U + \eta_{4;5}d_2^U \le 2000$$
$$q_5^U = f_1\eta_{1;5}d_1^U + f_4\eta_{4;5}d_2^U \le 2000$$

Finally, the calculated flows should be such that without reductions a traffic jam should form in freeway link 3. This happens when the demand from freeway entrance 1 d_1^F multiplied by $1-\beta_{2;11}$ is larger than q_2^F and when this amount added to the unreduced version of q_{12}^U exceeds q_{cap} while the flows on urban links 1, 2, 3, 4, 5 and 11 remain below 1000 (50% of the capacity, as the green time shares are 50%). Thus these constraints are given by:

$$(1-\beta_{2;11})d_1^F - (q_{cap} - f_1f_2\eta_{1;2}\eta_{2;12}d_1^U - f_4f_2\eta_{4;2}\eta_{2;12}d_2^U) > 0$$
$$(1-\beta_{2;11})d_1^F - (q_{cap} - \eta_{1;2}\eta_{2;12}d_1^U - \eta_{4;2}\eta_{2;12}d_2^U) > 0$$
$$(1-\beta_{2;11})d_1^F + \eta_{1;2}\eta_{2;12}d_1^U + \eta_{4;2}\eta_{2;12}d_2^U - q_{cap} > 0$$
$$d_1^U \le 1000$$
$$q_2^U = \eta_{1;2}q_1^U + \eta_{4;2}q_4^U \le 1000$$

$$q_3^U = f_1f_2a_{3;1}d_1^U + f_4f_2a_{3;2}d_2^U + \frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}q_{cap} \le 1000$$

$$d_4^U \le 1000$$
$$q_5^U = f_1\eta_{1;5}d_1^U + f_4\eta_{4;5}d_2^U \le 1000$$
$$q_{11}^U = \frac{\beta_{2;11}}{1-\beta_{2;11}}q_{dis} - \frac{\beta_{2;11}}{1-\beta_{2;11}}\eta_{1;2}\eta_{2;12}d_1^U - \frac{\beta_{2;11}}{1-\beta_{2;11}}\eta_{4;2}\eta_{2;12}d_2^U \le 1000$$

Notice that the latter 6 constraints replace 6 earlier presented constraints.

Apart from bounds and constraints on the calculated flows, there is also a constraint on the result. The integrated controller should perform better than a controller that would only apply the reduction factors to the urban network, a reduction controller. Such a controller would influence the outflows as follows:

- The outflow out of freeway link 3 would remain equal to the capacity q_{cap}. Thus $q_3^F = q_{cap}$
- The outflow out of urban link 3 would change due to the reduction factors f_1, f_2 and f_4 on the outflows out of urban links 1, 2 and 4. The outflow out of link 2 would be given by $q_2^U = f_1f_2\eta_{1;2}d_1^U + f_4f_2\eta_{4;2}d_2^U$. Assuming the reduction controller would prevent congestion on the freeway, the outflow out of link 11 would be equal to $\beta_{2;11}d_1^F$. Therefore $q_{11}^U = \beta_{2;11}d_1^F$ and $q_3^U = f_1f_2\eta_{1;2}\eta_{2;3}d_1^U + f_4f_2\eta_{4;2}\eta_{2;3}d_2^U + \eta_{11;3}\beta_{2;11}d_1^F$
- The outflow out of urban link 5 would change due to the reduction factors f_1 and f_2 on the outflows out of urban links 1 and 2. It would be given by $q_5^U = f_1\eta_{1;5}d_1^U + f_4\eta_{4;5}d_2^U$

This would change the differences between inflows and outflows to:

$$\begin{aligned} q^{in;tot} - q^{out;tot} &= d_1^U + d_2^U + d_1^F - f_1f_2\eta_{1;2}\eta_{2;3}d_1^U - f_4f_2\eta_{4;2}\eta_{2;3}d_2^U - \eta_{11;3}\beta_{2;11}d_1^F \\ &\quad - f_1\eta_{1;5}d_1^U - f_4\eta_{4;5}d_2^U - q_{cap} \\ &= c_1d_1^U + c_2d_2^U + (1-\eta_{11;3}\beta_{2;11})d_1^F - q_{cap} \end{aligned}$$

Where:

$$c_1 = 1 - f_1f_2\eta_{1;2}\eta_{2;3} - f_1\eta_{1;5}$$
$$c_2 = 1 - f_4f_2\eta_{4;2}\eta_{2;3} - f_4\eta_{4;5}$$

Which means the comparative total time spent would be equal to:

$$J_{Reduction}^{CTTS} = \frac{1}{2}T^2(N_s+1)N_s\left(c_1 d_1^U + c_2 d_2^U + \left(1-\eta_{11;3}\beta_{2;11}\right)d_1^F - q_{cap}\right)$$

Since the difference with the isolated controllers is maximised, the difference with the reduction controller cannot be maximised. However, one could say for instance that the absolute difference between the coordinated controllers and the reduction controller should be smaller than or equal to half the absolute difference between the coordinated controllers and the integrated controller. In formula form:

$$\left|J_{Reduction}^{CTTS} - J_{Coordinated}^{CCTS}\right| \leq \frac{1}{2}\left|J_{Integrated}^{CCTS} - J_{Coordinated}^{CCTS}\right|$$

This forms the final constraint.

<u>The MATLAB script</u>

The MATLAB script will be written down here so it is clear what MATLAB script has run to produce the results of the optimisation. The MATLAB script contains two functions. First the main script, *Get_optimal_turn_fractions_and_demands.m* will be given and then the two functions *Get_TTS_negative_difference.m* and *Get_nonlinear_constraints.m* will be given.

Get_optimal_turn_fractions_and_demands.m

```
% Get_optimal_turn_fractions_and_demands is a script file that serves to
% obtain the optimal turn fractions and demands (and reduction factors)
% based on the simplified analytical calculation of the comparative total
% time spent in annex 3 of the MSc Thesis report 'Developing a novel
% integrated urban network controller'. Any further details can be found in
% this annex.

clear; close all; clc;
% Start the script with a clean slate; clear the workspace, close all
% figures and clear the command window.

% Set the values of the parameters
T = 10/3600; % Duration of a time step in hours
q_cap = 4000; % veh/h, capacity
N_s = 360; % Number of time steps over the prediction horizon
q_dis = 3712; % veh/h, jam discharge rate
parameters = [T;q_cap;N_s;q_dis]; % Collect all parameters in a vector

% Set initial values for the variables
% The initial values are based on a first attempt to create a situation
% where traffic from the urban network and freeway network together form a
% traffic jam on the freeway
eta_12_0 = 0.7;
eta_23_0 = 0.1;
eta_42_0 = 0.7;
eta_212_0 = 1 - eta_23_0; % Constraint
eta_113_0 = 1; % constraint
eta_15_0 = 1 - eta_12_0; % Constraint
eta_45_0 = 1 - eta_42_0; % Constraint
beta_211_0 = 0.1;
d_1_U_0 = 530; % veh/h
d_2_U_0 = 530; % veh/h
d_1_F_0 = 3920; % veh/h
f_1_0 = 0.9; % Reduction factor used in calculations to show that total
time spent for the integrated controller can be lower than when the
controllers are not integrated
f_2_0 = 0.9;
```

```
f_4_0 = 0.9;
x_0 =
[eta_12_0;eta_23_0;eta_42_0;beta_211_0;d_1_U_0;d_2_U_0;d_1_F_0;f_1_0;f_2_0;
f_4_0]; % Collect all initial values in a vector
y_0 = [eta_212_0;eta_113_0;eta_15_0;eta_45_0]; % Collect all values that
are determined by the constraints in a vector

% Set the lower bounds and upper bounds
l_b = zeros(1,10); % The lower bounds of all 10 variables are 0
u_b = [1;1;1;1;1000;1000;4000;1;1;1]; % The upper bounds for the turn
fractions are 1, the upper bounds for the urban demands are 1000 veh/h, the
freeway demand is 4000 veh/h and the upper bounds for the reduction factors
are 1

% Calculate the initial difference
TTS_negative_difference_0 = Get_TTS_negative_difference(x_0,parameters); %
Calculate the negative difference. This value should be minimised, as the
difference should be maximised
TTS_difference_0 = -TTS_negative_difference_0;
disp(['The initial difference in total time spent is
',num2str(TTS_difference_0),' veh-h']);

% Set parallel computing
use_parallel = 1; % Switch to 0 if you do not want to use parallel
computing
% You will need a recent version of Matlab to benefit from parallel
% computing
if use_parallel
    poolobj = gcp;
    if isempty(poolobj)
        parpool(6)
    end
else
    poolobj = gcp;
    if ~isempty(poolobj)
        delete(poolobj)
    end
end

% Set fmincon options
options = optimset('Algorithm','sqp', 'MaxIter',1000,...
    'UseParallel', use_parallel,'Display','final-detailed',...
    'TolFun',1e-1,'TolCon',1e-6,'TolX',1e-2,'FinDiffRelStep',1e-
2,'Diagnostics','off');

% Run fmincon i.e. the optimisation
[x_opt, f_k, exit_flag] = fmincon(@(x)
Get_TTS_negative_difference(x,parameters),...
    x_0,[],[],[],[],l_b,u_b,@(x)
Get_nonlinear_constraints(x,parameters),...
    options); % Equality constraints have been taken care of internally.
There are no linear inequality constraints
```

```
% Assemble and display the results
TTS_negative_difference_opt = f_k;
TTS_difference_opt = -TTS_negative_difference_opt;
eta_12 = x_opt(1);
eta_23 = x_opt(2);
eta_42 = x_opt(3);
eta_212 = 1 - eta_23;
eta_113 = 1;
eta_15 = 1 - eta_12;
eta_45 = 1 - eta_42;
beta_211 = x_opt(4);
d_1_U = x_opt(5);
d_2_U = x_opt(6);
d_1_F = x_opt(7);
f_1 = x_opt(8);
f_2 = x_opt(9);
f_4 = x_opt(10);
y_opt = [eta_212;eta_113;eta_15;eta_45];
disp(['Optimisation done exitflag ',num2str(exit_flag),', optimal
difference in total time spent ',num2str(TTS_difference_opt),' veh-h' ])
disp(['eta_12 is ',num2str(eta_12)])
disp(['eta_23 is ',num2str(eta_23)])
disp(['eta_42 is ',num2str(eta_42)])
disp(['eta_212 is ',num2str(eta_212)])
disp(['eta_113 is ',num2str(eta_113)])
disp(['eta_15 is ',num2str(eta_15)])
disp(['eta_45 is ',num2str(eta_45)])
disp(['beta_211 is ',num2str(beta_211)])
disp(['d_1_U is ',num2str(d_1_U), ' veh/h'])
disp(['d_2_U is ',num2str(d_2_U), ' veh/h'])
disp(['d_1_F is ',num2str(d_1_F), ' veh/h'])
disp(['f_1 is ',num2str(f_1)])
disp(['f_2 is ',num2str(f_2)])
disp(['f_4 is ',num2str(f_4)])
```

Get_TTS_negative_difference.m

```
function TTS_negative_difference =
Get_TTS_negative_difference(x,parameters)
%Get_TTS_negative_difference serves to calculate the negative difference in
%total tims spent between the integrated controller and the urban and
%freeway controllers that are not integrated.
%   In order to do so, first the variables are determined from the input.
%   Then the variables are used to calculate the TTS for both the
%   integrated and not-integrated controllers. Finally, the negative
%   difference is calculated by subtracting the not-integrated TTS from the
%   integrated TTS
% Determine the variables from the input
T = parameters(1);
q_cap = parameters(2);
N_p = parameters(3);
q_dis = parameters(4);
eta_12 = x(1);
eta_23 = x(2);
eta_42 = x(3);
eta_212 = 1 - eta_23;
eta_113 = 1;
eta_15 = 1 - eta_12;
eta_45 = 1 - eta_42;
beta_211 = x(4);
d_1_U = x(5);
d_2_U = x(6);
```

```
d_1_F = x(7);
f_1 = x(8);
f_2 = x(9);
f_4 = x(10);

% Calculate the TTS for the not-integrated controllers
a_31 = eta_12*eta_23 - eta_12*eta_212*eta_113*beta_211/(1-beta_211);
a_32 = eta_42*eta_23 - eta_42*eta_212*eta_113*beta_211/(1-beta_211);
a_1 = 1 - a_31 - eta_15;
a_2 = 1 - a_32 - eta_45;
TTS_not_integrated = 0.5*T^2*(N_p+1)*N_p*(a_1*d_1_U+a_2*d_2_U+d_1_F-
(eta_113*beta_211/(1-beta_211)+1)*q_cap);

% Calculate the TTS for the integrated controller
c_1 = 1 - f_1*f_2*a_31 - f_1*eta_15;
c_2 = 1 - f_4*f_2*a_32 - f_4*eta_45;
TTS_integrated = 0.5*T^2*(N_p+1)*N_p*(c_1*d_1_U+c_2*d_2_U+d_1_F-
(eta_113*beta_211/(1-beta_211)+1)*q_cap);

% Calculate the negative difference
TTS_negative_difference = TTS_integrated - TTS_not_integrated;

end
```

Get_nonlinear_constraints.m

```
function [ c,c_eq ] = Get_nonlinear_constraints(x,parameters)
%Get_nonlinear_constraints serves to obtain the nonlinear constraints for
%the optimisation of the turn fractions and demands.
%   In order to do so, first the variables are determined from the input.
%   Then the variables are used to calculate the value of each constraint.
% Determine the variables from the input
T = parameters(1);
q_cap = parameters(2);
N_s = parameters(3);
q_dis = parameters(4);
eta_12 = x(1);
eta_23 = x(2);
eta_42 = x(3);
eta_212 = 1 - eta_23;
eta_113 = 1;
eta_15 = 1 - eta_12;
eta_45 = 1 - eta_42;
beta_211 = x(4);
d_1_U = x(5);
d_2_U = x(6);
d_1_F = x(7);
f_1 = x(8);
f_2 = x(9);
f_4 = x(10);

% Calculate the TTS for the not-integrated controllers
a_31 = eta_12*eta_23 - eta_12*eta_212*eta_113*beta_211/(1-beta_211);
a_32 = eta_42*eta_23 - eta_42*eta_212*eta_113*beta_211/(1-beta_211);
a_1 = 1 - a_31 - eta_15;
a_2 = 1 - a_32 - eta_45;
TTS_not_integrated = 0.5*T^2*(N_s+1)*N_s*(a_1*d_1_U+a_2*d_2_U+d_1_F-
(eta_113*beta_211/(1-beta_211)+1)*q_cap);
```

```
% Calculate the TTS for the integrated controller
c_1 = 1 - f_1*f_2*a_31 - f_1*eta_15;
c_2 = 1 - f_4*f_2*a_32 - f_4*eta_45;
TTS_integrated = 0.5*T^2*(N_s+1)*N_s*(c_1*d_1_U+c_2*d_2_U+d_1_F-
(eta_113*beta_211/(1-beta_211)+1)*q_cap);

% Calculate the TTS for the reduction controller
k_1 = 1 - f_1*f_2*eta_12*eta_23 - f_1*eta_15;
k_2 = 1 - f_4*f_2*eta_42*eta_23 - f_4*eta_45;
TTS_reduction = 0.5*T^2*(N_s+1)*N_s*(k_1*d_1_U+k_2*d_2_U+(1-
eta_113*beta_211)*d_1_F-q_cap);

% Calculate inequality constraints (c <=0)
q_2_U_I = f_1*f_2*eta_12*d_1_U + f_4*f_2*eta_42*d_2_U;
q_2_U_N = eta_12*d_1_U + eta_42*d_2_U;
q_12_U_I = f_1*f_2*eta_12*eta_212*d_1_U + f_4*f_2*eta_42*eta_212*d_2_U;
q_12_U_N = eta_12*eta_212*d_1_U + eta_42*eta_212*d_2_U;
q_2_F_I = q_cap - f_1*f_2*eta_12*eta_212*d_1_U -
f_4*f_2*eta_42*eta_212*d_2_U;
q_2_F_N = q_cap - eta_12*eta_212*d_1_U - eta_42*eta_212*d_2_U;
q_1_F_I = 1/(1-beta_211)*q_cap - 1/(1-
beta_211)*f_1*f_2*eta_12*eta_212*d_1_U - 1/(1-
beta_211)*f_4*f_2*eta_42*eta_212*d_2_U;
q_1_F_N = q_cap - 1/(1-beta_211)*eta_12*eta_212*d_1_U - 1/(1-
beta_211)*eta_42*eta_212*d_2_U;
a_31 = eta_12*eta_23 - eta_12*eta_212*eta_113*beta_211/(1-beta_211);
a_32 = eta_42*eta_23 - eta_42*eta_212*eta_113*beta_211/(1-beta_211);
q_3_U_I = f_1*f_2*a_31*d_1_U + f_4*f_2*a_32*d_2_U + eta_113*beta_211/(1-
beta_211)*q_cap;
q_3_U_N = a_31*d_1_U + a_32*d_2_U + eta_113*beta_211/(1-beta_211)*q_cap;
q_5_U_I = f_1*eta_15*d_1_U + f_4*eta_45*d_2_U;
q_5_U_N = eta_15*d_1_U + eta_45*d_2_U;
demand_for_freeway_link_2 = (1-beta_211)*d_1_F;
demand_for_freeway_link_3 = (1-beta_211)*d_1_F + eta_12*eta_212*d_1_U +
eta_42*eta_212*d_2_U;
c(1) = -q_2_U_I;
c(2) = -q_2_U_N;
c(3) = -q_12_U_I;
c(4) = -q_12_U_N;
c(5) = -q_2_F_I;
c(6) = -q_2_F_N;
c(7) = -q_1_F_I;
c(8) = -q_1_F_N;
c(9) = -q_3_U_I;
c(10) = -q_3_U_N;
c(11) = -q_5_U_I;
c(12) = -q_5_U_N;
c(13) = q_2_U_I-2000;
c(14) = q_2_U_N-1000;
c(15) = q_12_U_I-2000;
c(16) = q_12_U_N-2000;
c(17) = q_2_F_I-q_cap;
c(18) = q_2_F_N-q_cap;
c(19) = q_1_F_I-q_cap;
c(20) = q_1_F_N-q_cap;
c(21) = q_3_U_I-2000;
c(22) = q_3_U_N-1000;
c(23) = q_5_U_I-2000;
c(24) = q_5_U_N-1000;
c(25) = q_2_F_I - demand_for_freeway_link_2;
c(26) = q_2_F_N - demand_for_freeway_link_2;
```

```
c(27) = q_cap - demand_for_freeway_link_3;
c(28) = abs(TTS_reduction-TTS_not_integrated)-0.5*abs(TTS_integrated-
TTS_not_integrated);

% Calculate nonlinear equality constraints
c_eq = []; % There are no nonlinear equality constraints

end
```

Results of running the MATLAB script

Running the MATLAB script gives the following results (with a slight constraint violation)[1]:

$$\eta_{1;2} = 0.84503$$
$$\eta_{2;3} = 0.060466$$
$$\eta_{4;2} = 0.63346$$
$$\eta_{2;12} = 0.93953$$
$$\eta_{11;3} = 1$$
$$\eta_{1;5} = 0.15497$$
$$\eta_{4;5} = 0.36654$$
$$\beta_{2;11} = 0.13519$$
$$d_1^U = 529.9698 \text{ veh/h}$$
$$d_2^U = 529.9692 \text{ veh/h}$$
$$d_1^F = 3919.9871 \text{ veh/h}$$
$$f_1 = 0.98938$$
$$f_2 = 0.84873$$
$$f_4 = 0.95973$$

Given the fact that this is only a simplified analytical calculation, the results can be rounded in order to simulate with values that are reasonably close to the actual optimum. The rounding can take place such that the jam as intended is created for the no control action. The values to be used are:

$$\eta_{1;2} = 0.9$$
$$\eta_{2;3} = 0.05$$
$$\eta_{4;2} = 0.7$$
$$\eta_{2;12} = 0.95$$
$$\eta_{11;3} = 1$$
$$\eta_{1;5} = 0.1$$
$$\eta_{4;5} = 0.3$$
$$\beta_{2;11} = 0.15$$
$$d_1^U = 530 \text{ veh/h}$$
$$d_2^U = 530 \text{ veh/h}$$
$$d_1^F = 3920 \text{ veh/h}$$
$$f_1 = 0.9$$
$$f_2 = 0.9$$
$$f_4 = 1$$

[1] It may also occur that the values for the pair $\eta_{1;2}$ and $\eta_{4;2}$, the pair $\eta_{1;5}$ and $\eta_{4;5}$ and the pair d_1^U and d_2^U switch places. Numerically this is an equivalent result, but given the network the results displayed here make more sense (less turning traffic).

Using these values, the calculations of the comparative total time spent for the various control actions give the following results:

$$J_{No\ control}^{CTTS} = \tfrac{1}{2}T^2(N_s+1)N_s\left(a_1 d_1^U + a_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}+1\right)q_{dis}\right) = 251.0\ \text{veh-h}$$

$$J_{Coordinated}^{CTTS} = \tfrac{1}{2}T^2(N_s+1)N_s\left(a_1 d_1^U + a_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}+1\right)q_{cap}\right) = 81.17\ \text{veh-h}$$

$$J_{Integrated}^{CTTS} = \tfrac{1}{2}T^2(N_s+1)N_s\left(b_1 d_1^U + b_2 d_2^U + d_1^F - \left(\frac{\eta_{11;3}\beta_{2;11}}{1-\beta_{2;11}}+1\right)q_{cap}\right) = 76.29\ \text{veh-h}$$

These results are reasonable, as the coordinated freeway controller vastly reduces the total time spent due to the elimination of the capacity drop, while the integrated controller improves slightly on this by letting the outflow out of the speed-limited area increase slightly more than the outflow out of urban links 1 and 2 is reduced. Therefore, the demands and turn fractions determined here will be used as the disturbances that characterise the traffic situation.

A3.2 Initial control signals

The initial control signals are based on the theoretical optima found in section A3.1. This means that the coordinated controllers have to work with initial control signals where the positions and speeds of head and tail of the speed-limited area prevent the capacity drop due to congestion onset in freeway link 3 while not hindering the urban traffic. The integrated controller has to work with an initial control signal where the outflow out of urban links 1 and 2 is hindered by 10% during the peak period (see the reduction factors in section A3.1) while the speed-limited area can allow larger outflows due to the positions and speeds of head and tail of the speed-limited area. It needs to be worked out what positions and speeds of head and tail lead to the desired result for both the coordinated controllers and the integrated controller. It can be reasoned from fundamental diagram concepts (see for instance the fundamental diagram used by the SPECIALIST algorithm in subsection 2.4.1 of the main report) that an expanding speed-limited area moving upstream (the tail moves faster upstream than the head) limits the outflow out of freeway link 2 such that the capacity drop can be prevented. Therefore several speed-limited areas defined by the initial position and speed of head and tail will be tried out via simulation. First to find out when the control should be started (notice the controller should start to optimise 60 seconds in advance). Then to find out what the initial position and speed of head and tail should be to approximate the outflow out of freeway link 2 following from the theoretical calculation on average as well as possible. Since the speed-limited area is expanding, the initial position of head and tail will be set to be the same to limit the number of simulations needed to determine the initial control signals. The simulations will be done with the MATLAB script Main_Generic_Integrated_Control.m with the control turned off and the variables describing the positions and speeds of head and tail set to their initial values (indicated by the variable followed by '_ini' in the script)

Determining the moment the control should be started

Determining the moment the control should be started is done by varying that moment and verifying whether or not the capacity drop has been prevented. Because the correct initial position and speed of head and tail are not yet known at this moment, the initial position is set to be the endpoint of freeway link 2 while the speeds vary as indicated in table 1. Based on table 1, only control started at 2710 with initial speeds of -5 km/h and -25 km/h (the negative value indicates upstream propagation) prevents the capacity drop.

Table 1 Determining the moment control should be started to prevent the capacity drop. The table indicates whether or not the capacity drop has been prevented based on the given variables

Speed (head;tail) (km/h) \ Starting point (s)	2170	2770	3370
(-5;-10)	No	No	No
(-5;-15)	No	No	No
(-10;-15)	No	No	No
(-5;-20)	No	No	No
(-10;-20)	No	No	No
(-15;-20)	No	No	No
(-5;-25)	Yes	No	No
(-10;-25)	No	No	No
(-15;-25)	No	No	No
(-20;-25)	No	No	No

Using the same speed of head and tail, table 2 further investigates the moment control should be started, as starting too early needlessly delays traffic[2] whereas starting too late does not prevent the capacity drop. Given the controller update time, one can conclude from table 2 that out of all the possible moments control should be started, control should be started at 2170 seconds. This means the controller should start to optimise at 2110 seconds.

Table 2 further investigation into the moment control should be started

Starting point (s)	Capacity drop prevented?
2170	Yes
2230	No
2290	No
2350	No
2410	No
2470	No
2530	No
2590	No
2650	No
2710	No
2770	No

Determining the initial position and speed of head and tail

Determining the initial position of head and tail is done by varying them as indicated in table 3. To prevent urban inflow to affect the outflow out of freeway link 2, outflow out of urban link 2 is blocked in these simulations. The results, the average outflows out of freeway link 2 during peak demands are presented in table 3. The theoretical outflows for the coordinated controllers and the integrated controller are respectively:

$$q_{cap} - \eta_{1;2}\eta_{2;12}d_1^U - \eta_{4;2}\eta_{2;12}d_2^U = 3194 \text{ veh/h}$$
$$q_{cap} - f_1 f_2 \eta_{1;2}\eta_{2;12}d_1^U - f_4 f_2 \eta_{4;2}\eta_{2;12}d_2^U = 3315 \text{ veh/h}$$

[2] At the rate the head moves, it does not reach the beginning of freeway link 1 before the end of the simulation duration of 2 hours (assuming starting point 2170 seconds or later). Therefore, there is no danger of high inflow from freeway link 1 causing a new traffic jam due to starting the control too early, therefore this disadvantage of starting the control too early is omitted from this discussion.

Table 3 The average outflow out of freeway link 2 during peak demands (veh/h)

Speed (head;tail) (km/h) \ Starting point (km)	8.8	8.0	6.0	4.0	2.0
(-5;-10)	3244	3247	3248	3239	3241
(-5;-15)	3182	3187	3199	3207	3229
(-10;-15)	3258	3260	3264	3279	3303
(-5;-20)	3147	3154	3175	3191	3221
(-10;-20)	3223	3227	3240	3265	3296
(-15;-20)	3276	3278	3290	3305	3319
(-5;-25)	3125	3134	3161	3183	3218
(-10;-25)	3201	3208	3226	3255	3292
(-15;-25)	3254	3259	3276	3294	3316
(-20;-25)	3291	3295	3306	3316	3326

Close inspection of table 3 shows that the best approximation for the coordinated controllers are found with speeds (-5;-20) and starting point 4.0 km and the best approximation for the integrated controller is found with speeds (-20;-25) and starting point 4.0 km. However, the third-best approximation for the coordinated controllers (-10;-25) and starting point 8.8 km did not prevent the capacity drop in table 1. When moving from this approximation to the approximation that did, one causes the speed-limited area to expand faster. Therefore, the results of table 3 are used with care: expansions need to be faster than the best approximations, the initial position and speeds found in table 1 are used for the coordinated controllers and when having the option to choose a more downstream starting point, that starting point is chosen. This results in the following initial position and speed for head and tail to be used in the initial control signal:

<u>Coordinated controllers</u>
Initial position of the head: 8.4 km
Initial speed of the head: -5 km/h
Initial position of the tail: 8.4 km
Initial speed of the tail: -25 km/h

<u>Integrated controller</u>
Initial position of the head: 4 km
Initial speed of the head: -15 km/h
Initial position of the tail: 4 km
Initial speed of the tail: -25 km/h

A2.3 References

Van de Weg, G.S., Hegyi, A., Hoogendoorn, S.P. & De Schutter, B., Efficient model predictive control for variable speed limits by optimizing parameterized control schemes, IEEE-ITSC, 18th International Conference on Intelligent Transportation Systems, 2015, pages 1137-1142

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