A Calculation of Ship Turning Motion Taking Coupling Effect Due to Heel into Consideration

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Summary

This paper presents a method to calculate ship turning motion taking the coupling effect due to heel into consideration for a ship which performs large heel in her turning motion. The turning motion is calculated being based upon the coupled equations of surge, sway, yaw and heel. For two kinds of roll-on/roll-off ships computations are made, and the computed results are compared with the results of model experiment which was carried out using 5.0 m long and self-propelled models. Both the computed and the experimental results are in satisfactory agreement, and furthermore the computed results explain very well the speed effect on the turning ability, namely the reduction of the turning radius according to the increase of the ship speed. The major conclusions obtained in the study of this paper are that the turning motion of a ship with large heel, such as a roll-on/roll-off ship, should be treated together with the motion of heel simultaneously, and that the calculation method proposed here is effective and useful for the analysis of the turning motion of such a ship.

1. Introduction

When a ship enters into turning motion in calm water, she generally performs motions in four degrees of freedom. They are three components of the motion in horizontal plane, namely surge, sway and yaw, and heel in addition. In the studies of ship turning motion, most of works have been made on the assumption that the horizontal motions could be separated from other types of motion such as heel. Namely neglecting the coupling effect due to heel the turning motion has been calculated being based upon the coupled equations of the horizontal motions. The motion of heel has been treated alone, although the effect due to the turning motion has been considered in the equation of heel. The assumption and the normal practice described above, which have been employed in the analysis of the ship turning motion, can be understood to be quite reasonable and acceptable for most of ships in general, because the horizontal motions are predominant and the motion of heel is usually small in the turning motion of most of ships. But some ships, such as a roll-on/roll-off ship and a high speed container carrier, perform considerable heel in their turning motion. In relation to this fact there arises a question about the adequacy of the assumption and the normal practice, described before, to these kinds of ships. The study in this paper started from this question.

The authors recently had a chance to investigate the maneuverability of roll-on/roll-off ships. In connection to this investigation, a model experiment of the turning motion was carried out using two kinds of 5.0 m long and self-propelled roll-on/roll-off ship models. In this experiment an interesting fact was found that the turning radius decreased with large heel according to increase of the ship speed within the approach speed range of \( F \) (Froude number): 0.2-0.3.

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while it is generally understood that the turning radius hardly changes with change of the ship speed in such a speed range as that under the approach speed of \( F_s \): 0.3. Similar result to that of roll-on/roll-off ships was obtained experimentally by Mori et al.\(^9\) on a high speed container carrier with quadruple screw propellers. Both roll-on/roll-off ships and a container carrier, described above, have similar characteristics of hull geometry, in which large beam to draft ratio and large KG (high position of the center of gravity) may be one of major causes of the large heel in the turning motion. The speed effect on the turning ability, namely the reduction of the turning radius according to the increase of the ship speed, could be attributed to the large heel caused by centrifugal force, as pointed out by Mori et al.\(^9\). This fact clearly suggests that the turning motion of a ship with large heel, such as a roll-on/roll-off ship, should be treated together with motion of heel simultaneously.

In this paper an attempt is made to calculate the ship turning motion on the basis of the coupled equations of the horizontal motions and the motion of heel. The computed results are compared with the experimental results, and the validity of the method proposed here is examined. In the turning motion of some naval ships with such very high speed as that above \( F_s \): 0.3, another type of speed effect on the turning ability, namely the increase of the turning radius according to the increase of the ship speed, can be seen due to the phenomenon of wave-making at bow and due to considerable changes in trim and sinkage\(^n\). But in this paper this kind of problem is not included limiting the application range of the calculation to conventional merchant ships with approach speed under \( F_s \): 0.3 such as a roll-on/roll-off ship and a container carrier.

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**Nomenclature**

- \( A_r \) = rudder area
- \( a_r \) = ratio of hydrodynamic force, induced upon ship hull by rudder action, to rudder force
- \( d \) = draft of ship (molded)
- \( d_h \) = maximum draft at heeled condition (see Fig. 1(b))
- \( F_s \) = rudder normal force
- \( GZ(\phi) \) = restoring moment lever of heel
- \( h \) = vertical distance between calm water surface and point upon which lateral force \( Y_{null} \) acts (see Fig. 1(b))
- \( I_{xx}, I_{yy}, I_{zz} \) = moment of inertia of ship with respect to \( x, y \) and \( z \)-axes respectively
- \( J_{ee}, J_{zz} \) = added moment of inertia of ship with respect to \( x \) and \( z \)-axes respectively
- \( J_{uu} \) = dimensionless added moment of inertia of ship with respect to \( x \)-axis (\( = J_{uu}/J_{uu} \))
- \( J_{zz} \) = advance coefficient (\( = u(1-w)/nD_0 \))
- \( K \) = total heel moment
- \( L \) = length of ship (between perpendiculars)
- \( m \) = mass of ship
- \( m_x, m_y \) = added mass in \( x \) and \( y \)-axes direction respectively
- \( m_x', m_y' \) = dimensionless added mass in \( x \) and \( y \)-axes direction respectively (\( = m_x, m_y, 1/2D_0 \))
- \( N \) = total yaw moment
- \( N(\phi) \) = heel damping moment
- \( R \) = turning radius
- \( r \) = turning rate
- \( r \) = dimensionless turning rate (\( = rL/V = L/R \))
- \( T_s \) = natural period of roll
- \( t \) = thrust deduction coefficient
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\[ u = \text{ship speed in } x\text{-axis direction} \]
\[ V = \text{ship speed } \left(\sqrt{u^2 + v^2}\right) \]
\[ V_e = \text{effective rudder inflow speed} \]
\[ v = \text{ship speed in } y\text{-axis direction} \]
\[ v' = \text{dimensionless ship speed in } y\text{-axis direction } \left(= v/V\right) \]
\[ W = \text{displacement of ship} \]
\[ X = \text{total force in } x\text{-axis direction} \]
\[ x_e = \text{x-coordinate of point upon which rudder force } Y_{\text{rudder}} \text{ acts} \]
\[ Y = \text{total force in } y\text{-axis direction} \]
\[ z = \text{z-coordinate of point upon which lateral force } Y_{\text{null}} \text{ acts} \]
\[ z_e = \text{z-coordinate of point upon which rudder force } Y_{\text{rudder}} \text{ acts} \]
\[ \alpha = \text{effective rudder inflow angle} \]
\[ \delta = \text{rudder angle} \]
\[ \epsilon = \text{dimensionless linear damping coefficient of heel} \]
\[ l = \text{aspect ratio of rudder} \]
\[ \sigma = \text{heel angle} \]

2. Mathematical Model

2.1. Equation of Motion

A set of coordinate axes with origin fixed at the center of gravity of the ship, as shown in Figs. 1(a) and 1(b), is used to describe the ship turning motion. Longitudinal and transverse horizontal axes are represented by the \( x \) and \( y \)-axes respectively. By reference to this coordinate system, the equations of the turning motion, in general, can be written in the following form, considering the coupling effects between the horizontal motions and the motion of heel.

\[
\text{Surge: } m\left(\dot{u} - \sigma\right) = X
\]
\[
\text{Sway: } m\left(\dot{v} + \omega r\right) = Y
\]
\[
\text{Yaw: } [I_{xx} - (I_{yy} - I_{zz})\sin^2 \phi] \ddot{\phi} + (I_{yy} - I_{zz})\omega \sin(2\phi) = V
\]
\[
\text{Heel: } I_{xx} \ddot{\phi} - (I_{yy} - I_{zz})\omega^2 \sin(2\phi) = K
\]

Assuming \( I_{rr} = I_{xx} \) then equations (1) become

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Fig. 1(a) Coordinate System (1)

Fig. 1(b) Coordinate System (2)
Surge: \( m_2 \dot{u} - u_2 \dot{u} = X \)
Sway: \( m_2 \dot{v} + u_2 \dot{u} = Y \)
Yaw: \( I_{zz} \dot{\phi} = N \)
Heel: \( I_{yy} \dot{\phi} = K \)

where \( X, Y, N \) and \( K \) denote total hydrodynamic forces and moments generated by ship motions, propeller and rudder. The hydrodynamic terms can be expressed in the form

\[
X = X_{\text{HULL}} + X_{\text{PROPELLER}} + X_{\text{RUDDER}}
\]
\[
Y = Y_{\text{HULL}} + Y_{\text{RUDDER}}
\]
\[
N = N_{\text{HULL}} + N_{\text{RUDDER}}
\]
\[
K = K_{\text{HULL}} + K_{\text{RUDDER}}
\]

where the term with subscript HULL represents hydrodynamic forces and moments produced by motions of ship hull (without propeller and rudder) and acting upon it, and the term with subscript RUDDER represents rudder forces and moments including hydrodynamic forces and moments induced upon ship hull by rudder action.

2.1. Longitudinal Force Acting upon Ship Hull and Propeller Thrust

According to the mathematical model proposed by the Group-MMG, the longitudinal force \( X_{\text{HULL}} \) and the propeiler thrust \( X_{\text{PROPELLER}} \) can be written

\[
X_{\text{HULL}} = -m_2 \dot{u} - (m_v + K_{uv}) u_v - X(u) + X_v, \phi^2 + x_2 r^2
\]
\[
X_{\text{PROPELLER}} = (1 - C) T(J) \]

where \( X(u) \) represents ship resistance as a function of \( u \) and \( T(J) \) can be obtained by making use of propeller characteristic in open water as a function of \( J \).

2.3. Lateral Force and Yaw Moment Acting upon Ship Hull

Introducing dimensionless forms such as

\[
Y_{\text{HULL}} = \frac{Y}{k_{\text{HULL}}}, \quad N_{\text{HULL}} = \frac{N}{k_{\text{HULL}}}
\]

then the lateral force \( Y_{\text{HULL}} \) and the yaw moment \( N_{\text{HULL}} \) excluding the added inertia terms, are considered to be functions of \( \psi, \phi \) and \( \psi \), in general, taking the coupling effect due to heel into consideration. But there is no well-established mathematical models for the expression of \( Y_{\text{HULL}} \) and \( N_{\text{HULL}} \) in forms with inclusion of the heel effect. Hence the following approach is attempted in this paper. Let us express \( Y_{\text{HULL}} \) and \( N_{\text{HULL}} \) in the following form

\[
Y_{\text{HULL}} = -m_2 \dot{\psi} - m_2 \dot{\psi} - Y_{\text{HULL}}(\psi, r^2) + Y_{\text{HULL}}(\psi, r^2) + Y_{\text{HULL}}(\phi, r', \phi)
\]
\[
N_{\text{HULL}} = -l_2 \dot{\psi} + N_{\text{HULL}}(\psi, r') + N_{\text{HULL}}(\psi, r', \phi)
\]

\( Y_{\text{HULL}}(\psi, r') \) and \( N_{\text{HULL}}(\psi, r') \) in equations (7) represent the hydrodynamic force and moment of maneuvering motion without the heel effect. Many mathematical models are available for the expression of \( Y_{\text{HULL}}(\psi, r') \) and \( N_{\text{HULL}}(\psi, r') \). Referring to the extensive studies made by Prof. S. Inoue, the following model is employed here.

\[
Y_{\text{HULL}}(\psi, r') = Y_{\text{HULL}}(\psi, r') + Y_{\text{HULL}}(\psi, r') - Y_{\text{HULL}}(\psi, r') - Y_{\text{HULL}}(\phi, r') - Y_{\text{HULL}}(\phi, r')
\]
\[
N_{\text{HULL}}(\psi, r') = N_{\text{HULL}}(\psi, r') - N_{\text{HULL}}(\psi, r') + N_{\text{HULL}}(\psi, r') + N_{\text{HULL}}(\phi, r') - N_{\text{HULL}}(\phi, r')
\]

Now \( Y_{\text{HULL}}(\psi, r', \phi) \) and \( N_{\text{HULL}}(\psi, r', \phi) \) in equations (7) represent the added terms due to inclusion of the heel effect. Referring to the force measurement test result of a container carrier model, an assumption is made that the linear terms with respect to \( \phi \) and the variation of the
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so-called linear terms, such as \( Y_1, Y_1', N_i, \) and \( N_i' \), due to heel are important in the expression of \( Y_1'(x', r', \phi) \) and \( N_i'(x', r', \phi) \). \( Y_1'(x', r', \phi) \) and \( N_i'(x', r', \phi) \) then can be expressed in the form

\[
Y_1'(x', r', \phi) = Y_1' + [Y_1'(\phi = 0) - Y_1'(\phi = 0)] x' + [Y_1'(\phi = 0)] r' + [Y_1'(\phi = 0)] \phi
\]

\[
N_i'(x', r', \phi) = N_i' + [N_i'(\phi = 0) - N_i'(\phi = 0)] x' + [N_i'(\phi = 0)] r' + [N_i'(\phi = 0)] \phi
\]

1.4. Heel Moment Acting upon Ship Hull

The heel moment \( K_{\text{null}} \) can be written

\[
K_{\text{null}} = -I_{xx} \dot{\phi} - NW_z G Z - Y_{\text{null}} x_z
\]

where the coupling effects due to the horizontal motions are described in the form of \( Y_{\text{null}} x_z \).

2. Rudder Force and Moment

The rudder forces and moments including the hydrodynamic forces and moments induced upon ship hull by rudder action, namely \( X_{\text{rudder}}, Y_{\text{rudder}}, N_{\text{rudder}} \), and \( K_{\text{rudder}} \), can be written in the following form according to the MDG mathematical model\(^7\),

\[
X_{\text{rudder}} = -F_s \sin \delta
\]

\[
Y_{\text{rudder}} = -(1 + a_P) F_s \cos \delta
\]

\[
N_{\text{rudder}} = -(X_d + a_s X_d) F_s \cos \delta
\]

\[
K_{\text{rudder}} = X_d + a_s F_s \cos \delta
\]

where the hydrodynamic force induced upon ship hull by rudder action is described in the form of \( a \), \( F_s \), \( \cos \delta \). The rudder normal force \( F_n \) can be expressed in the form\(^9\)

\[
F_n = \frac{1}{2} \frac{8.131}{1.25} \frac{a_2}{A} \frac{V}{2} \sin \alpha
\]

where the study made by Dr. Y. Yoshimura et al.\(^{10}\) is referred in the calculation of the effective rudder inflow speed \( V_z \).

3. Numerical Calculations

3.1. Ship Model

Two kinds of roll-on/roll-off ships, Ship A (with single screw propeller and single rudder) and Ship B (with twin screw propellers and twin rudders), are used in this study. The principal particulars of hull, propeller and rudder of the self-propelled models of both ships, with which the model experiment of the turning motion was carried out, are shown in Table 1.

3.2. Hydrodynamic Forces and Moments

Force measurement test was not carried out for both Ship A and Ship B. Therefore the hydrodynamic force and moment derivatives etc. in the equations of the turning motion were obtained by estimate calculation. Results are summarized in Table 2. The methods of estimation of the derivatives etc. employed in this paper are as follows.

1. The added inertia terms, namely \( m_t, m_r, \) and \( J_{xx} \), are determined by the estimate charts given by Prof. S. Motoura.

2. The derivatives in equations (3) are estimated being based upon the results of studies made by Prof. S. Inoue\(^6\). The derivatives in equations (9) are estimated using the force measurement test result\(^{10}\), including the heel effect of a container carrier model, which has length to beam ratio of...
beam to draft ratio of 2.97 and block coefficient of 0.57. The same quantity as that obtained of the container carrier is used for \( Y' \) and \( N \). In Figs. 2(a)-2(d), the results of the force measurement test of the container carrier mentioned above are shown in the form of the ratio of the so-called linear term at \( \varphi = 0 \) to that at \( \varphi = 0 \), namely \( Y'(\varphi = 0) \), etc., taking heel angle \( \varphi \) in abscissa.

The derivatives \( Y' \), \( Y'' \), \( Y''' \), and \( Y'''' \) are determined using the slope at the straight lines drawn in Figs. 2(a)-2(d).

The heel damping moment \( N' \) can be expressed

\[
N(\varphi) = \alpha \varphi = \frac{c}{L_d} (L_d - h_d) \alpha \varphi
\]

The result obtained by free roll test at running condition is utilized for the heel damping coefficient \( c \), and shown in Table 2.

The following relation can be written for the \( z \)-coordinate of the point \( z_y \), upon which the lateral force \( Y_{\text{null}} \) acts.

\[
z_y = OG - h_y = \frac{GZ(\varphi)}{Y'_{\varphi}}
\]
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From equation (14), \( h_r \) can be obtained using measured quantities of \( \phi, r \) and \( \varphi \) at steady turning. The results obtained in this manner, which are mean value about the turning motions of several kinds of rudder angle, are shown in Table 2 in the forms of \( h_r/d \) and \( h_r/d_1 \) for both ships. On the other hand, some estimate charts are available for estimation of \( h_r \). Referring to these charts, the result of Ship A obtained by equation (14) and shown in Table 2 is considered to be quite adequate. But \( h_r \) of Ship B seems abnormally large, namely it has the value more than 1.0 even in the form of \( h_r/d_1 \). The large beam to draft ratio (=4.59) of Ship B might be one of major causes of that, but further discussion about that is not attempted here. The results shown in Table 2 are used in the numerical calculations for both ships.

3.3 Numerical Results and Discussion

At first the computed results of Ship A are shown in Figs. 3-7. The results of the model experiment, which was carried out by the authors at the square basin of Ship Research Institute of Japan (S. R. I.) using the before-mentioned self-propelled model, are also shown in these figures.

The dimensionless turning rate at steady turning, \( \tau_c \), is shown in Fig. 3 taking rudder angle \( \beta \) in abscissa for three cases of the approach speed of \( F_s = 0.21, 0.26 \) and 0.30. It can be seen from Fig. 3 that the computed results agree satisfactorily with the experimental results, and explain very well the speed effect on the turning ability described in the Introduction. The computed result being based upon the coupled equations of only the horizontal motions with exclusion of the heel effect, namely by the first three of equations (1), is also shown in Fig. 3 for the case of \( F_s = 0.26 \) for reference. Without consideration of the coupling effect due to heel...
the computed $r_t$ shows lower value than that of the experimental result, in other words larger turning radius would be computed without the heel effect. The speed effect on the turning ability can be interpreted as

(1) According to the increase of the ship speed, heel angle increases due to the centrifugal force.

(2) Consequently the moment derivative due to $\alpha' N_s$ increases and the moment derivative due to $r' N_s$ decreases as clearly can be seen from Figs. 2(b) and 2(d).

(3) Finally turning radius decreases by larger $N_s$ and smaller $N_s$.

The fact that considerable heel can be seen even at small rudder angle, shown in Figs. 5 and 6, could suggest a possibility of the decrease of the course keeping ability according to the increase of the ship speed. This problem is not treated here, but will be examined in another report.

The ratio of the speed reduction at steady turning, $V_t/V_s$, is shown in Fig. 4. Both the computed and the experimental results can be seen to be in fairly good agreement. The speed effect on the speed reduction at steady turning hardly can be found in Fig. 4, differing from that
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on the turning ability shown in Fig. 3.

The maximum heel angle $\varphi_{\max}$ is shown in Fig. 5. Quite satisfactory agreement between the computed and the experimental results can be seen for all the cases of the ship speed.

The heel angle at steady turning, $\varphi_s$, is shown in Fig. 6, and quite satisfactory agreement between the computed and the experimental results can be seen for all the cases of the ship speed just like as the maximum heel angle described above. An interesting result, that $\varphi_s$ has the maximum value at some amount of rudder angle between 10° and 20° for three cases of the approach speed, can be seen in Fig. 6, while $\varphi_{\max}$ increases with the increase of rudder angle as shown in Fig. 5. The rudder angle which gives the maximum value to $\varphi_s$ depends upon the ratio of magnitude between the ship speed and the turning rate at steady turning, as pointed out and discussed by Prof. S. Inoue.

Time histories of the turning rate and the heel angle from rudder execution to steady turning are shown in Fig. 7 as one of the example, for the case of the approach speed of $F_a=0.35$ and the rudder angle of $\delta=-35^\circ$ (the port turning). It can be seen from Fig. 7 that both the computed and the experimental results are in satisfactory agreement.

Next the computed results of Ship B are discussed. The computed results of the dimensionless turning rate at steady turning, the ratio of the speed reduction at steady turning, the maximum heel angle, the heel angle at steady turning, and time histories of the turning rate and

Fig. 8 Turning Rate at Steady Turning (Ship B)

Fig. 9 Speed Reduction at Steady Turning (Ship B)

Fig. 10 Maximum Heel Angle (Ship B)

Fig. 11 Heel Angle at Steady Turning (Ship B)
the heel angle are shown in Figs. 8, 9, 10, 11 and 12 respectively taking rudder angle in abscissa together with the experimental results at S. R. I. for two cases of the approach speed of $F_a = 0.22$ and $0.28$. It can be seen from these figures that the computed results generally give good agreement with the experimental results although some discrepancy is seen in the ratio of the speed reduction in Fig. 9. The same discussion as that of Ship A can be made of Ship B.

In spite of the fact that fairly simple mathematical model, shown in equations (9), is employed to describe the coupling effect due to heel, the computed results of both Ship A and Ship B show satisfactory agreement with the experimental results. It is realized that the turning motion of a ship with large heel, such as a roll-on/roll-off ship, should be treated together with the motion of heel simultaneously, and that the calculation method proposed here is effective and useful for the analysis of the turning motion of such a ship.

4. Conclusions

It is the purpose of this paper to calculate the turning motion taking the coupling effect due to heel into consideration for a ship which performs large heel in her turning motion. Computations were made for two kinds of roll-on/roll-off ships. The computed results were compared with the experimental results, and the validity of the calculation method proposed here was examined. Furthermore the speed effect on the turning ability etc. were discussed. The results obtained in the study of this paper are summarized as follows.

1. The turning motion of a ship with large heel, such as a roll-on/roll-off ship, should be treated together with the motion of heel simultaneously.
2. In spite of the fact that fairly simple mathematical model is employed to describe the heel effect, the computed results give satisfactory agreement with the experimental results.
3. The calculation method proposed here is effective and useful for the analysis of the turning motion of a ship with large heel.
4. In the turning motion of a ship with large heel, the speed effect on the turning ability, that the turning radius decreases according to the increase of the ship speed, is seen under the approach speed of $F_a = 0.3$, and this fact has already been found experimentally by Mori et al. The above-mentioned speed effect is assured from both aspects of the calculation and the model experiment in this paper.

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