Analytical Approach to Grid Operation
With Phase Shifting Transformers

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Abstract—Analytical expressions are derived to gain insight in the operating principles of phase shifting transformers (PSTs) in a highly meshed grid. To this extent, the dc load flow algorithm is adapted to account for such devices. This leads to a linear expression for the relation between PST settings and the active power flow in the lines. Based on these equations, the total transfer capacity (TTC) can be described mathematically, which allows for optimization. Furthermore, the linear least squares method is used to distribute a cross-border transport evenly over the interconnectors. Both criteria are demonstrated by two examples.

Index Terms—dc load flow, load flow control, phase shifting transformer, total transfer capacity.

I. INTRODUCTION

In the current framework of the liberalization of electrical energy markets, active power flow control is becoming increasingly important. This is especially the case when the grid is highly meshed, such as in Western Europe. As phase shifting transformers (PSTs) are a proven technology [1], they are often chosen as power flow controlling devices. Several examples are described in the literature [2]–[6].

If several PSTs are installed in a highly meshed grid and not too far from each other, the control problem can become very complex. The danger of counter-regulation has to be taken into account, not only in order to avoid that the investments are not used to their full potential but, above all, because overloading may result, threatening security of supply. This means that a local control strategy is not sufficient or at least not optimal.

The operation principles of a PST in a single line are simple, and the load flow equations can be adapted accounting for the device in an easy way. If the device is modeled as an extra phase angle and a fixed reactance, the ac load flow algorithm does not even has to be adapted. However, this kind of approach does not result in analytically closed equations, which makes thorough insight in the working principles of a PST in the grid harder to achieve, and most importantly, it complicates optimization techniques. If a certain parameter is to be optimized, but there is no closed expression for the objective function, simulation optimization is the only option [7].

If dc load flow is used, simpler analytical expressions can be derived, which are easier to comprehend and to handle. There is of course some loss of accuracy by making certain assumptions, but it has been shown that the error lies within reasonable limits, as long as the phase shift is not extremely large [8].

In this paper, the analytical expressions are applied to investigate how PSTs must be controlled in order to make optimal use of the transmission capacity. Of course, the definition of “optimal use” has to be determined carefully. Two different criteria are used for that and are described in Section II. Subsequently, the theoretical principles of these criteria and how they can be calculated are discussed in Section III. In Section IV, the theory is put into practice by giving two examples.

II. OPTIMALITY CRITERIA

The organization of European Transmission System Operators (ETSO) provides procedures on how to calculate transfer capacities between countries [9]. The maximum amount of power that can be transferred between countries A and B without violating any security criterion is called the total transfer capacity (TTC) between A and B.

However, the exact working conditions cannot be predicted with full accuracy. The information that a transmission system operator (TSO) receives (be it from market players or measurements) and that it uses to predict future conditions is mostly uncertain and hard to guarantee. Hence, a security margin is introduced: the transmission reliability margin (TRM). The TRM is determined by the TSOs. It is mostly a fixed value, but it can be adapted according to seasonal variations or network configuration changes.

The transfer capacity that can be offered to the market is the net transfer capacity (NTC). It is the maximum amount of power that can be transferred over a border without violating any security constraints and taking into account the uncertainty in the planning process as follows:

\[
NTC = TTC - TRM. \tag{1}
\]

It is important to maximize the capacity that can be offered to the market by the TSO. Hence, a first optimality criterion is to maximize the NTC by choosing the best settings for the PSTs. However, one of the problems with the use of the NTC...
is the fact that its value depends on the TRM. A TSO of one country can decide to make the TRM 300 MW, but another TSO can use a value of 500 MW. This is the reason why the TTC is more appropriate to make transfer capacity calculations. Of course, in practice, the NTC should be used for operation, but for optimization, the TTC is a more objective parameter.

For the calculation of the TTC, a base case is used, corresponding to a certain amount of base case exchange (BCE) between the two areas considered. In order to increase the power flow on the border, the generation in one country is increased and decreased by the same amount in the other. This process is repeated until a security constraint is violated. The maximum change in generation in a given area is designated as $\Delta G_{\text{MAX}}$. The maximum decrease is $\Delta G_{\text{MIN}}$. A graphical representation of the transfer capacities can be seen in Fig. 1.

Another optimality criterion is chosen based on the equal loading of interconnectors. The PSTs are used to divert power in such a way that the relative loadings (in p.u.) of all interconnectors on the border are equal.

### III. Theory

#### A. Linearized Power Flow and PSDFs

In the dc load flow approach, all active line powers are proportional to the phase angle differences. If the power balance on all buses is taken into account, the power flow equations can be written as follows:

$$P = Y \cdot \delta$$

where $P$ is the vector of net injected bus powers, $Y$ the admittance matrix of the system, and $\delta$ the phase angle vector.

If a single PST with phase shift $\alpha$ is put in series with a line, the power through that line can be written as follows:

$$P_{ij} = y_{ij}(\delta_{ij} + \alpha).$$

If this modification is taken into account, (2) becomes

$$P = Y \cdot \delta + Y_\alpha \cdot \alpha.$$

$Y_\alpha$ is a vector with value $y_{ij}$ at position $i$, $-y_{ij}$ at position $j$, and zero elsewhere. The indices $i$ and $j$ refer to the bus numbers between which the PST is installed.

If this matrix equation is solved, the following results:

$$\delta = Y^{-1} \cdot (P - Y_\alpha \cdot \alpha).$$

If element $(i, j)$ of matrix $Y^{-1}$ is referred to as $c_{ij}$, the angles $\delta_i$ and $\delta_j$ can be written as follows:

$$\delta_i = c_{i1} P_1 + \cdots + c_{ii} P_i - y_{ij}(\alpha) + \cdots$$
$$+ c_{ij} (P_j + y_{ji}(\alpha)) + \cdots + c_{i(n-1)} P_{n-1}$$

with both equations and the knowledge that $Y^{-1}$ is a symmetrical matrix ($c_{ij} = c_{ji}$), the difference of the angles can be calculated as follows:

$$\delta_{ij} = \delta_i - \delta_j = \sum_{k=1}^{n-1} P_k (c_{ik} - c_{jk}) + y_{ji}(\alpha)(2c_{ij} - c_{ii} - c_{jj}).$$

By combining (3) and (8), the active power through line $ij$ as a consequence of the PST in that line becomes

$$P_{ij} = y_{ij} \left( \sum_{k=1}^{n-1} P_k (c_{ik} - c_{jk}) + \alpha y_{ji}(\alpha)(2c_{ij} - c_{ii} - c_{jj}) + 1 \right).$$

In a similar way, the power through a certain line $pq$ as a function of the setting of the PST in line $ij$ can be written as follows:

$$P_{pq} = y_{pq} \left( \sum_{k=1}^{n-1} P_k (c_{pk} - c_{qk}) + \alpha y_{jq}(\alpha)(c_{pj} - c_{pi} + c_{qi} - c_{qj}) \right).$$

Since $Y$ is a highly sparse matrix, $Y^{-1}$ is typically very dense [10]. The result is that the linear term in $\alpha$ in (10) is nonzero for every line. This illustrates the fact that the PST influences the entire network. Obviously, the influence can be very small for distant lines.

The derivative of the power through the line with respect to $\alpha$ is referred to as the phase shifter distribution factor (PSDF), and it can be expressed as follows:

$$\xi_{ij}^{ij} = \frac{\partial P_{ij}}{\partial \alpha} = y_{ij}(1 + y_{ij}(\alpha)(2c_{ij} - c_{ii} - c_{jj}))$$

$$\xi_{pq}^{pq} = \frac{\partial P_{pq}}{\partial \alpha} = y_{pq} y_{jq}(\alpha)(c_{pj} - c_{pi} + c_{qi} - c_{qj}).$$

This result is consistent with results reported in the literature [11]. These factors only depend on the network configuration and not on generation or demand values.

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1 This is true in the case where the slack bus is assumed to be bus $n$. However, if this is not the case, the indexing of the $\varepsilon$-elements is no longer correct. The most easy way to deal with this is to assume that $c_{ij}$ is element $(i, j)$ of the augmented matrix of $Y^{-1}$, i.e., with a dummy row and column added at the position of the slack bus.
By using this notation, the line powers can be approximated in the following way:

\[ P_{ij} = P_{ij}^0 + \alpha c_{ij} \]  
(13)

\[ P_{pq} = P_{pq}^0 + \alpha e_{pq} \]  
(14)

where \( P_{ij}^0 \) and \( P_{pq}^0 \) indicate the line powers at zero phase shift. They represent the regular dc load flow expressions without PSTs.

If multiple PSTs are installed in the system, the equations for the line power must be adapted consequently. This is fairly straightforward and is not elaborated here. If the power flow through a line with a PST is to be calculated with other PSTs in lines \((m,n)\), the following equation results:

\[ P_{ij} = P_{ij}^0 + \alpha c_{ij} + \sum_{(m,n)} \alpha_{mm} c_{mn} \cdot \alpha_{mn}, \]  
(15)

It can be seen that every PST contributes an extra term to the equation. The power flow through the line is now a linear function of the different phase shifter settings.

The power through a line without a PST, but with PSTs in lines \((m,n)\), is

\[ P_{pq} = P_{pq}^0 + \sum_{(m,n)} \alpha_{mm} e_{pq} \cdot \alpha_{mn}. \]  
(16)

B. Total Transfer Capacity

In the analytic approach, first the \( n \)-secure TTC is considered, i.e., without contingency cases. When power exchange between the considered areas is only possible via direct transmission across the examined border (i.e., not via parallel paths through other areas), the BCE is equal to the sum of the power flows in the interconnectors between the systems. The \( n \)-secure TTC can then be written as follows:

\[ \text{TTC}^{n-sec} = \sum_{\text{interconn}} P_{k0} + \Delta E_{\text{max}}. \]  
(17)

The question is now what the relation is between the line powers and the power shift. Every line power changes by a certain amount related to the power shift. This is called the sensitivity of the line and is described by the sensitivity factor

\[ s_l = \frac{dP_l}{d\Delta E}. \]  
(18)

In practical calculations, this is taken as a constant, as shown in Fig. 2. The sensitivity factors are calculated once by applying a certain power shift (typically 100 MW). The linear power curves are then extrapolated until one line reaches its maximum power.

The fact that \( s_l \) is taken as a constant can be validated by considering (9) or (10). If a power shift is applied, these equations are influenced, as the injected powers at the different buses change. This effect can be described by

\[ P_k = P_{k0} + f_k \cdot \Delta E \]  
(19)

in which \( f_k \) is a participation factor for bus \( k \). If this equation is combined with (9) or (10), and the derivative with respect to the power shift is taken, the following expression can be written for an arbitrary interconnector \( g_h \) (with or without PST and assuming that the PST settings do not change):

\[ s_{gh} = \frac{dP_{gh}}{d\Delta E} = y_{gh} \sum_{k=1}^{n_l} f_k (c_{gk} - c_{hk}). \]  
(20)

The maximum power shift is determined by the line overloaded first. For every line, there is the following constraint that the maximum line power may not be exceeded:

\[ P_{k0} + s_l \cdot \Delta E \leq \text{sgn}(s_l) \cdot P_{k\text{max}}, \quad \forall l \]  
(21)

where \( \text{sgn} \) is the sign function.

This results in the following expression for the maximum power shift:

\[ \Delta E_{\text{max}} = \min_{\text{lines}} \left( \frac{\text{sgn}(s_l) \cdot P_{k\text{max}} - P_{k0}}{s_l} \right) \]  
(22)

where \( \text{lines} \) are all the lines monitored for the calculation of the TTC.

In (22), only \( P_{k0} \) is a function of \( \alpha \), as described by (9) and (10). Since these relations are linear, according to (22), \( \Delta E_{\text{max}} \) is piecewise linear with regard to \( \alpha \). In the same way, when more PSTs are involved, \( \Delta E_{\text{max}} \) is piecewise linear with regard to every \( \alpha \). The optimal TTC can now be found using conventional optimization techniques. This is not discussed in detail.

The extension to the \((n-1)\)-secure TTC is pretty straightforward

\[ \text{TTC}^{(n-1)-sec} = \min_{\text{cont}} (\text{TTC}_{\text{cont}}). \]  
(23)

Every contingency results in a new system for which the TTC as a function of \( \alpha \) can be calculated. The \((n-1)\)-secure TTC is the minimum of all TTCs under contingency conditions. Obviously, the \((n-1)\)-secure TTC is also a piecewise linear function of every \( \alpha \).

C. Equal Loading

Suppose a system of linear equations without an exact solution is given by

\[ Ax \approx b. \]  
(24)
In the linear least squares (LLS) approach, the aim is to find the \( x \) for which \( \| Ax - b \|^2 \) is minimal (hence the name). This can be written as follows:

\[
\min_x J = (Ax - b)^T(Ax - b).
\] (25)

It is straightforward to verify that this minimization problem has the following solution:

\[
x_0 = (A^T A)^{-1} A^T b.
\] (26)

If a border with all its interconnectors is considered, the active power flows can be described by the following matrix equation:

\[
P = P^l + \Xi \cdot \Delta \alpha \approx P_{\text{ref}}
\] (27)

where \( P^l \) is the vector of power flows with the PSTs set at their reference position (for example: all at zero degrees), \( \Xi \) is a matrix of PSDFs, and \( P_{\text{ref}} \) is a target power distribution which is to be approximated. In this approach, the relative loading of the interconnectors should be made equal. This can be expressed as follows:

\[
\frac{P_i}{P_{i,\text{r}}'} = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n P_{i,\text{r}}'} \iff P_i = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n P_{i,\text{r}}'} \cdot P_{i,\text{r}}'
\] (28)

where \( P_i \) is the power over one of the interconnectors and \( P_{i,\text{r}}' \) is the rating of that line.

If (27) is rearranged, it can be written as follows:

\[
\Xi \cdot \Delta \alpha \approx P_{\text{ref}} - P^l
\] (29)

which can be identified with (24). The (change in) phase shifter settings which results in the best approximation of an equal loading scenario can be found by using (26).

IV. EXAMPLES

A. Total Transfer Capacity

As an example, the New England 39-bus test system is used, with two PSTs added. One is located between buses 4 and 14 and the other between 17 and 18. The system is drawn in Fig. 3.

The system is divided into two areas. The power over the interconnectors (on a 100 MW base) is calculated using (15) and (16)

\[
P_{14-15} = -0.1729 + 2.8208 \cdot \alpha_{14-15} + 4.1834 \cdot \alpha_{17-18} \quad (30)
\]

\[
P_{17-18} = 1.3624 + 1.1230 \cdot \alpha_{14-15} + 8.1499 \cdot \alpha_{17-18} \quad (31)
\]

\[
P_{25-26} = -0.0087 - 1.3311 \cdot \alpha_{14-15} + 3.9665 \cdot \alpha_{17-18} \quad (32)
\]

where \( \alpha \) is in radians, and \( P \) in p.u.

The \( \Xi \)-secure import TTC of Area West is calculated opposed to Area East. Only the interconnectors are monitored in the process. All the rated line powers were set to 700 MW. The power shift is equally divided over all the generation buses in the area (except the swing bus, i.e., bus number 31). As the TTC is a function of both PST settings, a contour plot is used (see Fig. 4). It can be seen that a maximal TTC is obtained when PST 4–14 is at \(-10^\circ\) degrees and PST 17–18 at \(+10^\circ\).

Figs. 5 and 6 show the calculated results compared to those obtained from PSS/E (the built-in TTC calculation module has been used). Clearly, the deviations are small and can be attributed to the limitations of the dc load flow.

B. Equal Loading

As a second example, the Dutch–German border is considered (see Fig. 7). The aim is to use the linear least squares method in order to approximate an equal loading scenario.
PSTs are installed in Meeden and Gronau, as well as in some lines in Belgium (as will be the case in the near future) [12]. The LLS method is applied starting from four base cases, in which the settings of the Belgian PSTs are different. The optimum settings of the PSTs on the Dutch–German border and the corresponding line loadings are calculated by (26) and (27). The calculated loadings can be seen in Table I.² The optimum settings are applied to the simulation model in PSS/E and the line loadings are verified. They can also be seen in Table I.

From the results, the following conclusions can be drawn.

• The linear least squares method is able to equalize the border flows to a certain extent. However, perfect balancing is impossible as four line flows cannot be fully controlled by only two PSTs.
• The calculated and simulated results are slightly different due to the limitations of the dc load flow approach.

V. CONCLUSIONS

In this paper, analytical expressions have been derived to gain insight in the operating consequences of PSTs in a meshed grid. To this extent, the dc load flow equations are adapted to account for such devices. This leads to an expression for the relation between PST settings and the active power flow in the lines. Based on these equations, the TTC can be described mathematically, which allows for optimization. Furthermore, the LLS method is used to distribute a cross-border flow evenly over the interconnectors. Application to the New England test system shows that the TTC can be optimized for a specified zone by using PSTs, and that the deviations from the results calculated with dedicated simulation software are small. The equal loading challenge with LLS is illustrated by using real data from the Netherlands and its neighboring countries, and it is shown that although perfect balancing is not possible in general, the distribution over the Dutch–German interconnectors can be improved substantially by using the proposed approximation method.

REFERENCES


²Be aware that one interconnector has two lines, which means that three interconnectors result in four lines.


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