Meander dynamics: A reduced-order nonlinear model without curvature restrictions for flow and bed morphology

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[1] Reduced-order models remain essential tools for meander modeling, especially for processes at large length scales and long time scales, probabilistic simulations, rapid assessments, or when input data are scarce or uncertain. Present reduced-order meander models consider their dependent variables either as small-amplitude variations compared to a basic state (linearity) or as varying gradually in a spatial sense (gradual variation). In a prequel, Blanckaert and de Vriend (2010) derived a nonlinear reduced-order hydrodynamic model without curvature restrictions and showed that linearity or gradual curvature variations assumptions do not hold in strongly curved channels. Moreover, in strongly curved channels, a nonlinear feedback mechanism causes the secondary flow strength to be smaller than its linear mild-curvature equivalent. In the limit of mild-amplitude variations and mild curvature, their nonlinear meander flow model simplifies to a well-known linear formulation. The present paper extends this nonlinear modeling to the bed morphology in strongly curved bends, making use of Exner’s sediment conservation principle. Furthermore, the model quantifying the relative influence of the downslope gravitational force is refined by considering nonlinear effects. The coupled nonlinear flow and bed morphology model yields satisfactory results for the bed topography, whereas the corresponding linear model strongly overpredicts the magnitude of the transverse bed slope. Analysis of the forcing mechanisms indicate that this erroneous behavior is caused by an overestimation of the upslope drag force due to the secondary flow.


1. Introduction

[2] Meandering rivers are landscape features which have interested scientists, civil engineers, geomorphologists, and decision makers for centuries. Although great progress has been made in the study of meandering rivers over the last few decades, their behavior remains to be fully understood.

[3] Meander models generally consider three interconnected processes: the hydrodynamics, bed morphodynamics, and bank morphodynamics (cf. Figure 1 and Camporeale et al. [2007, Figure 2] for a graphic overview of these interconnected processes). The flowing water induces shear stresses on the river bed which move the bed sediments, ultimately leading to a pattern of stable and migrating morphological features. The flow of water also induces shear stresses on the banks which cause them to erode, whereas banks in zones of low shear stress may accrete, thus changing the river’s course. As the banks are generally made of more cohesive material or bound by riparian vegetation, the time scales of the bank adaptation are generally longer than those of the bed adaptation. This implies that the bed and bank morphodynamics can be considered separately. The morphological changes of the bed and the banks in their turn influence the hydrodynamics, thus giving rise to further morphological changes, and so on. An alluvial river is thus a complex adaptive dynamic system. As a step toward unraveling this system, this article focuses on the bed morphodynamics.

[4] At present, it is possible to model meandering channels in great detail. Rüther and Olsen [2007] simulated the 3 day meander experiment by Friedkin [1945] with a three-dimensional meander model, solving the hydrodynamics by means of a Reynolds-averaged Navier Stokes model with a linear $k$ – $\epsilon$ turbulence closure. Typical length and time scales for meander evolution are much larger than the 72 h and the 40 m of the Friedkin [1945] experiment. The simulation of longer time and length scales motivates the use of reduced-order models (obtained by depth-averaging (and width-averaging) operations), which are generally an order
of magnitude faster than their three-dimensional counterparts. Moreover, reduced-order models require less input, which is useful when data are scarce or uncertain.

[5] Present reduced-order meander models are either based on the linearity or gradual variation assumption. The linearity assumption implies that the variables are assumed to be small with reference to a basic state, a condition which is often justified in mild-curvature bends. The gradual variation assumption allows large variations of the variables as long as they vary gradually in space, a condition which is generally justified in mild-curvature bends [cf. discussion in Bolla Pittaluga et al., 2009].

[6] Models which are based on the linearity assumption include Struiksm [1983a], de Vriend and Struiksm [1984], Odgaard [1989], Lancaster and Bras [2002], Abad and Garcia [2006], and Crosato [2008], whereas models based on the gradual variation assumption include Zolezzi and Seminara [2001] and Bolla Pittaluga et al. [2009].

[7] In strongly curved channels, the linearity and gradual variation assumptions no longer hold. Furthermore, in strongly curved bends, the frequently used linear parameterization of secondary flow [Rozovskii, 1957; Engelund, 1974; de Vriend, 1977], which is proportional to the depth to radius of curvature ratio 1/R of the bend, is no longer valid. This is due to a non-linear feedback mechanism which causes the reduction of the secondary flow strength compared to its linear equivalent [Blanckaert and de Vriend, 2003, 2010; Ottevanger et al., 2012]. These issues motivate the development of non-linear reduced-order meander models which also allow sudden variations in curvature and include a nonlinear treatment of the secondary flow (in short, nonlinear without curvature restrictions). Blanckaert and de Vriend [2003, 2010] extended the hydrodynamic modeling to strongly curved bends. In this paper, the morphodynamics will be extended in a similar manner.

[8] Morphological development is related to the sediment transport field. In the cases considered herein, this transport field can be described by a sediment transport formula relating the transport rate and direction to local flow properties. Most sediment transport formulae are derived from straight flume measurements and are related to the drag force imposed by the fluid on the sediment particles. In curved channels, a transverse bed slope ∂z_b/∂n (the gradient of the bed level z_b in transverse direction n) exists with a bed that usually deepens in outward direction (cf. Figure 1). Gravity exerts a downhill force on sediment particles positioned on the transversally inclined bed. The combination of these forces allows us to determine the sediment transport direction on sloping beds as found in curved channels.

[9] Fargue [1868] [cf. Hager, 2003] was the first to report that the magnitude of the transverse bed slope in a river bend correlates well with the inverse radius of curvature 1/R of the bend. Later research showed the correlation of the transverse bed slope in a bend to the water depth H as well

\[ \frac{\alpha}{R} = \frac{f(c, \frac{1}{R}, H, \frac{\alpha}{R}, B)}{R} \]

\[ \psi = f(c, \frac{1}{R}, H, \frac{\alpha}{R}, B) \]

\[ \frac{\alpha}{R} = \frac{f(c, \frac{1}{R}, H, \frac{\alpha}{R}, B)}{R} \]

\[ \frac{\alpha}{R} = \frac{f(c, \frac{1}{R}, H, \frac{\alpha}{R}, B)}{R} \]

\[ M = \frac{\alpha}{R} = \frac{f(c, \frac{1}{R}, H, \frac{\alpha}{R}, B)}{R} \]
However, not easily be expressed in terms of equation (1). Their results can, based on the force balance on a particle in an implicit theoretical model for bed load transport at low motion along arbitrarily sloping beds. Later, Rozovskii [1957] independently found a balance on a stationary sediment particle. Later, Rozovskii [1957] independently found A = 11, which he also justified by comparison to field and laboratory data. Engelund [1974, 1975] obtained A = 21 for an annular flume with a moving lid and A = 7 for open-channel field conditions. Ikeda et al. [1981] and Odgaard [1981] reported values of the scour factor A between 2.5 and 6 for alluvial rivers. Zimmerman and Kennedy [1978] found an expression for A related to the friction factor (based on an empirical relation found by Nunner [1956]), a ratio relating the projected surface area of a non-spherical particle to its volume and the particle densimetric Froude number ($U \sqrt{g \Delta D}$), where $U$ is the bulk velocity, $g$ is the gravitational constant, $\Delta (\approx 1.65)$ is the relative density of sediment, and $D$ is a characteristic sediment diameter. The above mentioned findings are all based on the assumption of mildly sloping streamwise bathymetries. Recently, Seminara et al. [2002] and Parker et al. [2003] developed an implicit theoretical model for bed load transport at low shield stresses, based on the force balance on a particle in motion along arbitrarily sloping beds. Their results can, however, not easily be expressed in terms of equation (1).

The range of the scour factor in strongly curved bends remains to be investigated, but measurements in the field by Nanson [2010] and Schnaude and Sukhodolov [2012] and measurements in laboratory flumes by Blanckaert [2010] and Abad and Garcia [2009] reveal scour factors of $A = 0.1$, 0.35, 2, and 2, respectively, which suggest that in sharp bends significantly lower scour factors apply than those typically found in more mildly curved channels.

Allen [1978] reported that van Bendegom [1947] derived a first estimate of the scour factor $A = 10$ from the force balance on a stationary sediment particle. Later, Rozovskii [1957] independently found $A = 11$, which he also justified by comparison to field and laboratory data. Engelund [1974, 1975] obtained $A = 21$ for an annular flume with a moving lid and $A = 7$ for open-channel field conditions. Ikeda et al. [1981] and Odgaard [1981] reported values of the scour factor $A$ between 2.5 and 6 for alluvial rivers. Zimmerman and Kennedy [1978] found an expression for $A$ related to the friction factor (based on an empirical relation found by Nunner [1956]), a ratio relating the projected surface area of a non-spherical particle to its volume and the particle densimetric Froude number ($U \sqrt{g \Delta D}$), where $U$ is the bulk velocity, $g$ is the gravitational constant, $\Delta (\approx 1.65)$ is the relative density of sediment, and $D$ is a characteristic sediment diameter. The above mentioned findings are all based on the assumption of mildly sloping streamwise bathymetries. Recently, Seminara et al. [2002] and Parker et al. [2003] developed an implicit theoretical model for bed load transport at low shield stresses, based on the force balance on a particle in motion along arbitrarily sloping beds. Their results can, however, not easily be expressed in terms of equation (1).

The objectives of the present paper are (i) to investigate the mechanisms responsible for the generation of the bed morphology in sharp river bends; (ii) to develop a nonlinear model for the bed morphology without curvature restrictions, encompassing existing linear models; (iii) to improve the model of the gravitational pull on the sediment particles and to analyze the sensitivity of results to this model; and (iv) to analyze the importance of nonlinear effects in the prediction of the bed morphology.

2. Model Description

As stated in section 1, meander models consist of three interacting components that account for the flow, the bed morphology, and the planform, respectively. Blanckaert and de Vriend [2010] presented a meander model framework of which the different components are summarized in Figure 1 for a schematized river bend.

2.1. Meander Flow Submodel

Because morphological processes in rivers are mainly driven by the flow, the dominant flow variables playing a role in the hydrodynamic submodel will first be described briefly. A detailed description is reported in Blanckaert and de Vriend [2003], Blanckaert [2009], and Blanckaert and de Vriend [2010].

The hydrodynamic submodel computes the cross-stream distribution of the depth-averaged downstream velocity $U_\psi$ via the parameter

$$\frac{\alpha_\psi}{R} = \frac{1}{U} \frac{\partial U_\psi}{\partial n}$$

When $\alpha_\psi = -1$, the downstream velocity distribution corresponds with a potential vortex distribution, and when $\alpha_\psi = 1$, it corresponds to a forced vortex distribution [Vardy, 1990].

In curved open channels, $\alpha_\psi$ is the result of different processes described in detail by Johannesson and Parker [1989] and Blanckaert and de Vriend [2010]. A dominant factor is the curvature-induced secondary flow which

Figure 2. Solutions of Blanckaert and de Vriend’s [2003] nonlinear meander flow model without curvature restrictions. (a) The direction of the bed shear stress parameterized by $\alpha_\psi$. The solution is given as a correction factor to the linear model solution $\alpha_{\psi,0}$ (equation (4)). When plotted against the bend parameter $\beta$, all solution points nearly collapse on a single curve. (b) The curvature-induced energy loss factor $\psi$ (equation (5)). Minimum scatter around a single curve is obtained when expressing $\psi$ as a function of $\beta C_{F,15}$.

\[ \alpha_\psi = \left( \frac{C_r}{\beta} \right)^{2.275} \left( \frac{H/R}{\alpha_{\psi,0}} \right)^{0.35} (\alpha_{\psi,0})^{0.25} \]

\[ \beta = \left( \frac{C_t}{\alpha_{\psi,0}} \right)^{2.275} \left( \frac{H/R}{\alpha_{\psi,0}} \right)^{0.35} (\alpha_{\psi,0})^{0.25} \]
requires adequate representation in reduced-order hydrodynamic models [Johannesson and Parker, 1989; Finnie et al., 1999; Blanckaert and Graf, 2004].

[16] The fluid motion normal to the primary flow $v_n$ is referred to as spanwise flow $v_n$, which is the sum of the depth-averaged cross-flow $U_n$ and the secondary flow $v_n^*$. In the paper, we consider the linear secondary flow model by de Vriend [1977] and the nonlinear secondary flow model by Blanckaert and de Vriend [2003].

[17] The nonlinear flow model of Blanckaert and de Vriend [2003, 2010] also computes the direction of the bed shear stress, which may be written as the ratio of the transverse and streamwise components of the bed shear stress $(t_{bn}, t_{bs}$, respectively), which is known to be dominant in the formation of the transversally inclined bed [e.g., van Bendegom, 1947; Engelund, 1974; Camporeale et al., 2007; Blanckaert and de Vriend, 2010]. The transverse component of the bed shear stress is composed of two components, $t_{bn}^*$ and $t_{bs}^*$. The former is related to the cross-flow $U_n$, and the latter is the bed shear stress induced by the secondary flow (cf. Figure 1).

$$
\frac{t_{bn}}{t_{bn}^*} = \frac{t_{bn}}{t_{bn}^*} = \frac{g_t}{Hg_t} \frac{\alpha_t \infty}{R} \ \text{Blanckaert and de Vriend [2003]}
$$

[18] The direction of the bed shear stress induced by secondary flow can be approximated as follows:

$$
\frac{t_{bn}^*}{t_{bs}^*} = \left[ \frac{\alpha_t \infty}{\alpha_t \infty} \right]^{g_t} = Hg_t \left[ \frac{\alpha_t \infty}{\alpha_t \infty} \right]^{g_t} = Hg_t \left[ \frac{\alpha_t \infty}{\alpha_t \infty} \right]^{g_t} = Hg_t \left[ \frac{\alpha_t \infty}{\alpha_t \infty} \right]^{g_t}
$$

where $g_t(n)$ is the distribution function of the bed shear stress direction over the channel width, and $\kappa (\approx 0.4)$ is the von Kármán constant. The contributions $\alpha_t \infty / R$ and $\alpha_t \infty / R$ represent the solution of the bed shear stress direction as computed by the nonlinear and linear models, respectively. The present paper adopts the linear model solution of de Vriend [1977] for $\alpha_t \infty / R$, which is included in equation (4).

The equation shows that the reduction of the secondary flow strength due to the nonlinear feedback mechanism also reduces the difference between the transverse and streamwise components of the bed shear stress vector. Blanckaert and de Vriend’s [2003] nonlinear meander flow model quantifies this reduction by means of the correction function $\alpha_t \infty / \alpha_t 0$, which is also included in equation (4). Their nonlinear meander flow model provides this correction factor as a function of the parameter $\beta = C_f 0.275(H/R)^{-\frac{5}{2}}(\alpha_t 1)^{0.25}$. Figure 2 graphically shows how this factor varies with $\beta$.

[19] The hydrodynamic submodel also computes the magnitude of the bed shear stress vector as follows:

$$
\| \vec{t}_b \| = \rho C_f U_{tot}^2 = \rho \psi C_f U_{tot}^2,
$$

where $C_f$ is the local channel friction factor, and $U_{tot}$ is the magnitude of the depth-averaged velocity vector. The factor $\psi$ accounts for additional energy losses in a curved open-channel flow as compared to a straight open-channel flow. These curvature-induced energy losses are related to increased near-bed gradients of the deformed velocity profiles caused by the nonlinear secondary flow effects, the additional transverse component of the bed shear stress, and additional curvature-induced turbulence production (for details, see Blanckaert and de Vriend [2003] and Blanckaert [2009]). These additional curvature-induced energy losses are neglected in mildly curved bends, implying that $\psi = 1$. They are important, however, in strongly curved bends.

Figure 2b graphically shows the value of $\psi$ as a function of $\beta C_f 0.15$ according to the nonlinear meander secondary flow model of Blanckaert and de Vriend [2003]. Other sources of increases energy loss in the bend are also accounted for in the hydrodynamic submodel (for details, see Blanckaert [2009]). The magnitude and direction of the bed shear stress, as computed from (4) and (5), are important ingredients to the morphology submodel described below.

2.2. Morphology Submodel

2.2.1. Basic Governing Processes and Equations

[20] The bed morphology in a bend is determined by the conservation of sediment mass, as described by Exner’s balance equation:

$$
\frac{1}{1 - \epsilon_p} \frac{\partial s_{bn}}{\partial t} + \frac{1}{1 + n R} \frac{\partial s_{bn}}{\partial s} + \frac{1}{1 + n R} \frac{s_{bn}}{R} = 0,
$$

where $s_b$ denotes the bed level, $s_{bn}$ is the sediment transport capacity, and $\epsilon_p$ denotes the porosity of the bed. Typical values of $\epsilon_p$ for clean uniform sand range between 0.29 and 0.50 [Lambe and Whitman, 1979, p. 31].

[21] The transport is assumed to be at capacity always and everywhere. Moreover, it is assumed that the transport rate and direction can be expressed in terms of local flow properties, such as the flow velocity. Mosselman [2005] gave an overview of many such sediment transport formulae, one class of which is the so-called Engelund and Hansen [1967] type formula given by

$$
\frac{s_{bn}}{s_{b}} = \alpha \sqrt{g D^b \theta} = \alpha \sqrt{g D^b \left( \frac{t_{bn}}{\rho g D} \right)} = \alpha \sqrt{g D^b \left( \frac{\psi C_f U_{tot}^2}{g D} \right)} = \alpha \sqrt{g D^b \left( \frac{\psi C_f U_{tot}^2}{g D} \right)} = \alpha \sqrt{g D^b \left( \frac{\psi C_f U_{tot}^2}{g D} \right)}.
$$

where $\theta$ denotes the dimensionless bed shear stress, $\alpha$ is a nondimensional O(1) calibration coefficient, and $b$ is an exponent indicating the nonlinearity of the sediment transport formula (e.g., for Engelund and Hansen [1967], $b = 5$).

[22] Although Engelund-Hansen’s sediment transport formula is a formula for the total sediment load, i.e., bed load and suspended load, it should only be applied in the present model to configurations where bed load transport is dominant. The dimensionless Rouse number $P = w_s / (\kappa \sqrt{C_f U})$ represents the ratio between the settling velocity $w_s$ and the shear velocity multiplied by the von Kármán constant. A Rouse number larger than 2.5 is often used as approximate criterion for the predominance of bed load transport [Fryirs and Brierly, 2013]. Both Zeng et al. [2005] and Wu et al. [2000] have shown that ignoring the suspended sediment transport does not change significantly the equilibrium bed morphology predictions in meander bends if the
bed load accounts for more than 75% of the total sediment load. Moreover, for cases in which the suspended load is negligible, the model of Engelund and Hansen [1967] was shown to give comparable or slightly better predictions than the van Rijn [1984a] model in moderately curved meander bends [e.g., see Zeng, 2006]. It was also shown to satisfactorily predict the bed morphology in Blanckaert’s [2010] bed load-dominant strongly curved M89 experiment [Zeng et al., 2008], which will also be simulated in the present paper.

[22] The direction of the bed load transport is given by the following [cf. van Bendegom, 1947; Engelund, 1974]:

$$\frac{s_{bh}}{s_{hs}} = \frac{u_{bh}}{u_{hs}} - G \frac{\partial z_e}{\partial n}.$$  

For mildly sloping streamwise slopes, a simplified expression given by Olesen [1987] may be used, which makes an error of at most 10% for $u_{bh}/u_{hs} < 0.5$:

$$\frac{s_{bh}}{s_{hs}} = \frac{u_{bh}}{u_{hs}} - G \frac{\partial z_b}{\partial n}.$$  

[24] This expression shows that the direction of the bed load transport vector deviates from the direction of the depth-averaged velocity, $U_b/U_s$, due to two effects. First, the transverse component of the bed shear stress induced by the secondary flow, $H\alpha_t z_s/R$ (equation (4)) exerts a drag force on the sediment particles which is typically uphill. Second, gravity exerts a downhill force on the sediment grains, parameterized by the last term in equation (9). This gravitational force is modeled as being proportional to the transverse bed slope, with a factor of proportionality that is commonly called the gravitational pull $G$.

[25] In the past, different investigations have expressed the dependency of the gravitational pull model on the dimensionless shear stress, based on either theoretical derivations, empirical evidence, or numerical particle simulations [cf. Mosselman, 2005]. van Bendegom [1947] was the first to do so and proposed $G = (1.5\theta)_1$. Engelund [1974] proposed $G \propto \theta^0$. Ikeda [1982], Hasegawa [1984], Parker and Andrews [1985], Olesen [1987], Struiksm [1988], and Talmon et al. [1995] proposed relations $G \propto \theta$ $^{0.5}$, and finally, Sekine and Parker [1992] proposed a similarity solution $G \propto \theta^{0.25}$ based on numerical simulations of the motion of saltating particles. To this end, three different gravitational pull models relating $G$ to $\theta$ to the power $-1$, $-0.5$, and $-0.25$ are introduced, which will be referred to as van Bendegom, Struiksm, and Sekine and Parker type, respectively.

[26] Experimental gravitational pull studies are either based on results in curved flumes (e.g., Engelund [1974], Zimmerman and Kennedy [1978] or in straight flumes [Talmon et al., 1995; Talmon and Wiesemann, 2003; Fracalanci and Solari, 2007]. The experiments of Zimmerman and Kennedy [1978] were performed in a circular flume which was long enough for the transverse slope to reach an equilibrium. The transverse slope angle in these experiments varied between 3 and 15°, which encompasses both mild and strong transverse slopes. Talmon et al. [1995] and Talmon and Wiesemann [2006] performed bed leveling experiments of mildly sloping transverse slopes from which they derived their gravitational pull model. Fracalanci and Solari [2007] performed experiments in a straight flume where the streamwise slope angle and transverse slope angle were varied. As the instantaneous behavior of the sediment particles was observed, the experimental setup was very suitable to study gravitational pull effects in strongly sloping beds. Fracalanci and Solari [2007] used steel discs with a major diameter of 3 mm and a minor diameter of 0.6 mm instead of almost spherical particles. It is questionable whether the theoretical model by Seminara et al. [2002]; Parker et al. [2003], derived for spherical particles, can be applied directly to such ellipsoidal particles without any adaptation. For example, the drag force upon such a particle could vary up to a factor 5 depending on the orientation of the particle in the flow. For that reason, the experimental data of Fracalanci and Solari [2007] have not been considered in the present study.

[27] Olesen [1987] and Struiksm [1988] analyzed the experiment of Zimmerman and Kennedy [1978] to compute the gravitational pull $G$. In these experiments, the flow and bed morphology reached a so-called fully developed state, which means that all longitudinal gradients vanish, $\partial/\partial x = 0$, and no depth-averaged transverse velocity exists, $U_n = 0$. Hence, equation (9) reduces to the following:

$$\frac{\partial z_{bh,e}}{\partial n} = -H \frac{\alpha_t}{G R} (z_s) = -H \frac{A_e}{R} g_t,$$  

where the subscript $e$ indicates fully developed conditions. The value of the gravitational pull $G$ can be obtained by measuring the transverse bed slope in the experiments and by

![Figure 3.](image-url)
estimating the direction of the bed shear stress according to a meander flow model.

[28] Olesen [1987] and Struikzma [1988] estimated \( \tau_{bs}/\tau_m \) according to the linear secondary flow model of de Vriend [1977] (cf. equation (4)). In the present paper, the gravitational pull \( G \) is reevaluated by estimating \( \tau_{bs}/\tau_m \) according to the nonlinear secondary flow model of Blanckaert and de Vriend [2003]. The three types (van Bendegom, Struikzma, and Sekine and Parker) are evaluated in Figure 3 using the above equation (10). Figure 3 also shows the newly computed values as well as the old fit by Struikzma [1988] and the model by van Bendegom [1947]. The newly computed values obtained for the gravitational pull parameter \( G \) are smaller than previously found by Olesen [1987]. This means that the effect of the gravitational pull in determining the direction of the bed load transport is weaker than previously derived and the equilibrium transverse bed slope steeper (see (10)). The performance of the three newly derived models will be evaluated in section 3.2.

### 2.2.2. Reduced-Order Bed Morphology Model Without Curvature Restrictions

[29] The steady state Exner equation ((6) with \( \partial U/\partial t = 0 \)) with equation (9) substituted in it, elaborated term by term and subsequently divided by \( s_{bs}/(1 + n/R) \), yields the following:

\[
\begin{align*}
\frac{b}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{b - 1}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{1}{\psi} \frac{\partial \psi}{\partial s} \left( \frac{U^2}{\psi} \right) + \left[ \frac{1}{n/R} b \frac{\partial U}{\partial s} + \frac{1}{R} \right] \frac{U_n}{U} \frac{h}{R} \frac{g_s}{R^2} - G \frac{\partial z_{bs}}{\partial n} \\
+ \left[ \frac{b - 1}{2} \frac{1}{n/R} \frac{\partial}{\partial s} \left( \frac{U^2}{\psi} \right) \right] \left[ \frac{U_n}{U} \frac{h}{R} \frac{g_s}{R^2} - G \frac{\partial z_{bs}}{\partial n} \right] \\
+ \left[ 1 + (n/R) \right] \frac{H}{R} \frac{\partial g_s}{\partial s} - G \frac{\partial z_{bs}}{\partial n} \right] + (1 + n/R) \frac{h}{R} \frac{\partial h}{\partial n} = 0,
\end{align*}
\]

(11)
in which the first line is related to downstream sediment transport gradient, and the second to fourth lines are related to the transverse transport.

[30] Using the equation of continuity for the flow

\[
\frac{1}{1 + n/R} \frac{\partial h U}{\partial s} + \frac{1}{1 + n/R} \frac{h U}{R} = 0,
\]

the term \( \partial h/\partial n \) in (11) can be rewritten to yield the following:

\[
\begin{align*}
\frac{b}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{b - 1}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{1}{\psi} \frac{\partial \psi}{\partial s} \left( \frac{U^2}{\psi} \right) + \left[ \frac{1}{n/R} b \frac{\partial U}{\partial s} + \frac{1}{R} \right] \frac{U_n}{U} \frac{h}{R} \frac{g_s}{R^2} - G \frac{\partial z_{bs}}{\partial n} \\
+ \left[ \frac{b - 1}{2} \frac{1}{n/R} \frac{\partial}{\partial s} \left( \frac{U^2}{\psi} \right) \right] \left[ \frac{U_n}{U} \frac{h}{R} \frac{g_s}{R^2} - G \frac{\partial z_{bs}}{\partial n} \right] \\
+ \left[ 1 + (n/R) \right] \frac{H}{R} \frac{\partial g_s}{\partial s} - G \frac{\partial z_{bs}}{\partial n} \right] + (1 + n/R) \frac{h}{R} \frac{\partial h}{\partial n} = 0,
\end{align*}
\]

(13)

### 2.2.2.1. Expressing the Bed Shear Stress Angle as Function of the Dependent Variables

[31] Equation (13) is further elaborated by expressing the bed shear stress angle and the bed level as functions of the water depths. The difference between the fully developed and the developing transverse bed slope is approximately equal to the difference in fully developed and the developing transverse water depth gradients:

\[
\frac{\partial z_{bs}}{\partial n} - \frac{\partial z_{bs}}{\partial n} \approx \frac{\partial h}{\partial n} + \frac{\partial h}{\partial n}.
\]

(14)
The above equation in combination with equation (10) leads to the following:

\[
- H \frac{\partial z}{\partial R} \frac{\partial z}{\partial R} - \frac{\partial z}{\partial n} \approx \frac{\partial h}{\partial n} + \frac{\partial h}{\partial n}.
\]

(15)

[32] If \( U_v/U_s \) is expressed as its centerline value multiplied by a transverse distribution function \( g_r(n) \), the substitution of (15) into (13) yields the following:

\[
\begin{align*}
&\frac{b}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{b - 1}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{1}{\psi} \frac{\partial \psi}{\partial s} \left( \frac{U^2}{\psi} \right) + \left[ \frac{1}{n/R} b \frac{\partial U}{\partial s} + \frac{1}{R} \right] \frac{U_n}{U} \frac{h}{R} \frac{g_s}{R^2} - G \frac{\partial z_{bs}}{\partial n} \\
&+ \left[ \frac{b - 1}{2} \frac{1}{n/R} \frac{\partial}{\partial s} \left( \frac{U^2}{\psi} \right) \right] \left[ \frac{U_n}{U} \frac{h}{R} \frac{g_s}{R^2} - G \frac{\partial z_{bs}}{\partial n} \right] \\
&+ \left[ 1 + (n/R) \right] \frac{H}{R} \frac{\partial g_s}{\partial s} - G \frac{\partial z_{bs}}{\partial n} \right] + (1 + n/R) \frac{h}{R} \frac{\partial h}{\partial n}.
\end{align*}
\]

(16)

Blanckaert and de Vriend [2010] derived the transverse velocity component at the centerline:

\[
U_{n0} = \frac{U_v}{n_{l=0}} = \frac{U_v^2}{8} \frac{\partial \psi}{\partial s} \left( \frac{\alpha_n}{R} + \frac{A + Fr^2}{R} \right).
\]

(17)

### 2.2.2.2. Choice of Profile Distribution Functions

[33] To simplify the depth-averaged model, transverse profile distributions with one degree of freedom are assumed for the depth-averaged streamwise velocity \( U_v \), water depth \( h \), and equilibrium water depth \( h_e \). In the derivation of meander models, different assumptions have been made regarding the transverse profile functions for the bed level and the downstream velocity. Oogaaard [1989] and Johannessen and Parker [1989] considered a linear profile approach; Struikzma [1983a], Struikzma et al. [1985], Struikzma and Crosato [1989], and Crosato [2008] considered sinusoidal profiles; and Blanckaert and de Vriend [2010] considered an exponential approach for their nonlinear hydrodynamic submodel. These profiles, shown in Figure 4, all represent first-order approximations of the real bed morphology, which is sufficient because the smallest relevant length scales in a reduced model are of the order of the channel width. The three types of profiles are quite similar in representing the main features of the bed topography and have comparable accuracy [Blanckaert and de Vriend, 2010, Figure 4].

[34] From now on, sinusoidal profile functions will be adopted:

\[
h = H \left( 1 + \frac{B}{\pi} \sin \left( \frac{\pi n}{B} \right) \right) = H \left( 1 + \frac{B}{\pi} \sin \left( \frac{\pi n}{B} \right) \right).
\]

(18)
Figure 4. Exponential, linear, and sinusoidal bed level distribution functions for a scour factor $A = 2$, a radius of curvature $R = 5$ m, a channel width $B = 2$ m, and a mean water depth $H = 0.2$ m.

$$h_s = H \left( 1 + \frac{B A_e}{\pi R} \sin \left( \frac{\pi n}{R} \right) \right) = H \left( 1 + \frac{B A_e}{\pi R} f_s \right),$$

(19)

$$U_s = U \left( 1 + \frac{B A_e}{\pi R} \sin \left( \frac{\pi n}{R} \right) \right) = U \left( 1 + \frac{B A_e}{\pi R} f_s \right),$$

(20)

$$g_c = \cos \left( \frac{\pi n}{R} \right) = \frac{B \partial f_s}{\pi \partial n} = f_c,$$

(21)

where $f_s$ and $f_c$ are shorthand notations for the sines and cosines. Substituting profiles (18)–(21) into (16) leads to the following:

$$\frac{b}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{b - 1}{b + \frac{U}{U f_s}} \frac{\partial (\frac{\alpha s}{R})}{\partial s} = \frac{1}{2} \left( 1 + \frac{U^2}{U f_s} \right) \frac{\partial}{\partial s} \left( \frac{U_0^2}{U^2} (f_s)^2 \right),$$

(22f)

$$+ \left( \frac{1 + n/R}{1 + n/R} \right) \frac{b}{2} \left( \frac{1}{1 + \frac{U}{U f_s}} \right) \frac{\partial}{\partial s} \left( \frac{U_0^2}{U^2} \right) \left( \frac{a}{U f_s} \right)^2,$$

(22g)

$$+ \left( \frac{1 + n/R}{1 + n/R} \right) \frac{b - 1}{2} \left( \frac{1}{1 + \frac{U}{U f_s}} \right) \left( \frac{U_0^2}{U^2} \right) \left( \frac{a}{U f_s} \right)^2,$$

(22h)

$$+ \left( \frac{1 + n/R}{1 + n/R} \right) \left( \frac{G H}{R} A e f_s - \frac{G H}{R} A e f_s \right) - \frac{1}{\left( 1 + \frac{n}{R} f_s \right)^2} \frac{B}{\pi R} \frac{\partial}{\partial s} \left( \frac{A}{R} f_s \right),$$

(22i)

$$- \frac{U_0}{U} \frac{f_s}{\left( 1 + n/R \right)} \left( \frac{1 + n/R}{1 + n/R} \right) \frac{\alpha s}{R} f_c + \frac{1}{1 + n/R} \left( 1 + n/R \right) \frac{A}{R} f_c = 0.$$ (22j)

Previous linear meander bed morphology models rely on the assumptions of mild-curvature and mild-amplitude perturbations, which means all denominators in curly brackets in equation (22) are approximately equal to 1. For the strongly curved mobile bed experiments in the strongly curved flumes of Abad and Garcia [2009] and Blanckaert [2010], the value of $B/\pi \cdot A/R$ is about 0.9, so these contributions to the denominators can no longer be neglected. The denominators in curly brackets in equation (22) are therefore retained.

### 2.2.2.3. A Nonlinear Meander Bed Morphology Equation Without Curvature Restrictions

Equation (22) still applies to all points $(s, n)$ of the river. In order to simplify it to a one-dimensional equation in $s$, the transverse dimension requires elimination. Since the bed level is approximated by a sine profile, the reduced version of (22) considers what each of the terms contributes to the sine profile. These contributions are obtained by projecting equation (22) onto a sine profile using an operation similar to a Fourier analysis:

$$\{\text{equation (24)}\} \approx \frac{1}{B} \int_{B/2}^{B/2} \{\text{equation (22)}\} f_s \, dn \cdot f_s.$$ (23)

Projecting equation (22) onto sine profiles then yields the following:

$$\left[ \frac{G H}{R} A e \frac{A}{R} - \frac{G H}{R} A \frac{A}{R} \right]$$

(22k)

$$+ \left( b - 1 \right) \frac{B}{\pi R} \frac{\partial}{\partial s} \left( \frac{A}{R} f_s \right) \frac{B}{\pi R} \frac{f_s}{f_s} - \frac{B}{\pi R} \frac{\partial}{\partial s} \left( \frac{B A}{R} f_s \right) \frac{B}{\pi R} \frac{f_s}{f_s} = 0,$$ (24)
The first few terms are given below:

\[ f_{p1} \left( \frac{BA}{\pi R} \right) = 1 + \frac{3}{4} \left( \frac{BA}{\pi R} \right)^2 + \frac{5}{8} \left( \frac{BA}{\pi R} \right)^4 + \frac{35}{64} \left( \frac{BA}{\pi R} \right)^6 + \cdots, \]

\[ f_{p2} \left( \frac{BA}{\pi R} \right) = \frac{8}{9\pi^2 R} \left( \frac{BA}{\pi R} \right) + \frac{105}{225\pi^2 R} \left( \frac{BA}{\pi R} \right)^2 - \frac{1}{8} \left( \frac{BA}{\pi R} \right)^3 + \cdots, \]

\[ f_{p3} \left( \frac{U_{00}^2}{U^2} \right) = \frac{1}{4} - \frac{1}{8} \left( \frac{U_{00}^2}{U^2} \right) + \frac{5}{64} \left( \frac{U_{00}^2}{U^2} \right)^2 - \frac{7}{128} \left( \frac{U_{00}^2}{U^2} \right)^3 + \cdots. \]

Figure 5 shows the distribution functions graphically. The coupled nonlinear meander flow and bed morphology model, which truncates the approximation functions \( f_{p1}, f_{p2}, \) and \( f_{p3} \) at 25 terms, predicts the transverse bed slope \( A/R \) with a precision of \( 10^{-5} \), and its simulation time takes eight times longer than the nonlinear flow model (in the case of the M89 experiment [Blanckaert and de Vriend, 2010]).

Division of (24) by \( GH \frac{\pi}{\theta} \) and rearranging the terms gives the following:

\[ \frac{A}{R} + \frac{1}{GH \frac{\pi}{\theta} f_{p1} \left( \frac{BA}{\pi R} \right)} \frac{\partial}{\partial s} \left( \frac{A}{R} \right) = \frac{A_k}{R} + (b - 1) \frac{1}{GH \frac{\pi}{\theta} f_{p1} \left( \frac{BA}{\pi R} \right)} \frac{\partial}{\partial s} \left( \frac{A_k}{R} \right) \]

\[ + \frac{1}{GH \frac{\pi}{\theta} f_{p2} \left( \frac{BA}{\pi R} \right)} \frac{\partial}{\partial s} \left( \frac{A_k}{R} \right) \frac{\partial}{\partial s} \left( \frac{A_k}{R} \right) \left[ (b - 1) \frac{U_{00}^2}{U^2} - bGH \frac{A_k}{R} + bGH \frac{A}{R} \right] \]

\[ - \frac{1}{GH \frac{\pi}{\theta} f_{p3} \left( \frac{BA}{\pi R} \right)} \frac{U_{00}^2}{U^2} \frac{\partial}{\partial s} \left( \frac{A_k}{R} \right) \frac{\partial}{\partial s} \left( \frac{A_k}{R} \right) \left[ (b - 1) \frac{U_{00}^2}{U^2} - bGH \frac{A_k}{R} + bGH \frac{A}{R} \right] \]

\[ - (b - 1) f_{p3} \left( \frac{U_{00}^2}{U^2} \right) \frac{U_{00}^2}{U^2} \left[ \frac{1}{GH \frac{\pi}{\theta} - \frac{A_k}{R} + \frac{A}{R}} \right] \]

which can be considered as a nonlinear reduced-order bed profile adaptation equation without curvature restrictions. The left-hand side of the equation describes the transverse slope evolution of the bed subject to a lag effect. The first term on the right-hand side represents the equilibrium profile for an infinitely long bend (equation (10)). The second term on the right-hand side is related to accelerations and decelerations of the flow field. The second to fourth lines relate to sediment transport caused by higher-order flow variations.
Table 1. Hydraulic and Morphodynamic Conditions in the Three Experiments

<table>
<thead>
<tr>
<th></th>
<th>T3</th>
<th>M89</th>
<th>Kinoshita</th>
</tr>
</thead>
<tbody>
<tr>
<td>B [m]</td>
<td>1.5</td>
<td>1.3</td>
<td>0.6</td>
</tr>
<tr>
<td>R&lt;sub&gt;0&lt;/sub&gt; [m]</td>
<td>12</td>
<td>1.7</td>
<td>0.72</td>
</tr>
<tr>
<td>arc length [°]</td>
<td>140</td>
<td>193</td>
<td>-</td>
</tr>
<tr>
<td>Q [l/s]</td>
<td>74</td>
<td>89</td>
<td>25</td>
</tr>
<tr>
<td>H [m]</td>
<td>0.091</td>
<td>0.141</td>
<td>0.15</td>
</tr>
<tr>
<td>C&lt;sub&gt;f&lt;/sub&gt; [-]</td>
<td>1.28e–002</td>
<td>1.26e–002</td>
<td>2.14e–02</td>
</tr>
<tr>
<td>D&lt;sub&gt;0&lt;/sub&gt; [mm]</td>
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<td>2</td>
<td>0.83</td>
</tr>
<tr>
<td>Fr [-]</td>
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<td>0.41</td>
<td>0.23</td>
</tr>
<tr>
<td>s&lt;sub&gt;b&lt;/sub&gt; [kg/s/m]</td>
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<td>0.023</td>
<td>0.0024</td>
</tr>
<tr>
<td>θ [-]</td>
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<td>0.12</td>
</tr>
<tr>
<td>w&lt;sub&gt;r&lt;/sub&gt; [m/s]</td>
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<td>0.17</td>
<td>0.1</td>
</tr>
<tr>
<td>P [-]</td>
<td>2.6</td>
<td>7.8</td>
<td>6.2</td>
</tr>
</tbody>
</table>

*R is the channel width, R<sub>0</sub> is the radius of curvature at the bend apex, Q is the flow discharge, H is the average water depth, C<sub>f</sub> is the friction factor as reported in the original article, D<sub>0</sub> is the 50th percentile of the cumulative sediment diameter distribution, Fr is the Froude number, θ is the dimensionless shear stress, w<sub>r</sub> is the settling velocity [van Rijn, 1984b], and P = w<sub>r</sub>/√(g C<sub>f</sub>) is the Rouse number.

[36] The above equation in A/R is combined with the nonlinear flow equation in α<sub>a</sub>/R by Blanckaert and de Vriend [2010]. This yields a system of two coupled nonlinear equations with two unknowns. As the functions f<sub>r1</sub> and f<sub>r2</sub> depend nonlinearly on the variables A/R and α<sub>a</sub>/R, and f<sub>r3</sub> depends on U<sub>0</sub>/U, the solution to this system must be obtained by numerical iteration.

2.2.3. Mild-Amplitude and Mild-Curvature Limits

[40] Considering only the leading-order terms of equations (25)–(27), which corresponds to a mild-amplitude assumption, transforms the nonlinear bed adaptation equation (28) into

\[
\frac{A}{R} + \frac{1}{GH \pi^2} \frac{\partial}{\partial x} \left( \frac{A}{R} \right) = \frac{A_s}{R} + (b-1) \frac{1}{GH \pi^2} \frac{\partial}{\partial x} \left( \alpha_a \frac{A}{R} \right) + \frac{1}{GH \pi^2} \left( \frac{8}{5 \pi} \right) \left( \frac{\alpha_a}{R^2} \right) \left( \frac{b-1}{U_0} \frac{U_{0}}{U} - bGHA_s \frac{A}{R} + bGHA_s \frac{A}{R} \right) - \frac{1}{GH \pi^2} \left( \frac{8}{5 \pi} \right) \frac{U_{0}^2}{U} \frac{A}{R} \frac{A}{R} - \frac{b-1}{4} \frac{U_{0}^2}{U^2} \frac{1}{U} \frac{A_s}{R} \frac{A_s}{R} + \frac{A}{R} \frac{A}{R}.
\]  

(29)

With the mild-curvature assumption and U<sub>a</sub> << U, equation (29) reduces to

\[
\frac{A}{R} + \frac{1}{GH \pi^2} \frac{\partial}{\partial x} \left( \frac{A}{R} \right) = \frac{A_s}{R} + (b-1) \frac{1}{GH \pi^2} \frac{\partial}{\partial x} \left( \alpha_a \frac{A}{R} \right),
\]  

(30)

which is similar to the form found in Crosato [2008].

3. Model Validation and Analysis

3.1. The Experiments

[42] Three different experiments are considered with three different flow conditions (cf. Table 1) and flow geometries (cf. Figure 6).

[43] Figure 6 shows the mildly curved flume (R<sub>0</sub>/B = 8) at Delft Hydraulics (DHL). The 1.5 m wide flume consists of a straight inflow section, a bend with an arc length of approximately 140° with a radius of curvature of 12 m, followed by a straight channel outflow section [Struijsma et al., 1985].

[44] In experiment T3, the bed developed from an initially flat bed. The sediment was captured at the end of the flume in a sand trap and reintroduced at the upstream boundary [Struijsma, 1983b]. After the bathymetry reached its equilibrium, a transverse slope developed and a transverse equilibrium after some 5 m into the bend. Subsequently, the transverse slope gradually decreases, tending to a constant level. This feature at the beginning of the bend has been termed overshoot phenomenon [De Vriend and Struijsma, 1984]. At the bend exit, another slight increase in the transverse slope is observed.

[45] The Ecole Polytechnique Fédérale de Lausanne, Switzerland, has a sharply curved flume M89 (R<sub>0</sub>/B = 1.3). It is 1.3 m wide and consists of a 9 m straight inflow section, a 193° bend with a radius of curvature 1.7 m, and a 5 m
long outflow section (Blanckaert [2010], cf. Figure 6 and Table 1). At the upstream boundary, sediment was fed at a constant rate.

The bed in the M89 experiment in this flume exhibits a 0 transverse slope before the bend entrance (cf. Figure 7b). At the entrance of the bend, a slightly negative slope is observed, after which the transverse slope increases until some 2 m into the bend. Subsequently, the slope decreases toward the bend exit. At the exit it increases slightly, after which it exhibits an oscillatory decay in the downstream straight reach.

The Kinoshita flume is located in the Ven Te Chow Hydroystems Laboratory of the University of Illinois. The 0.6 m wide, sediment-recirculating flume consists of a sequence of alternating bends following a Kinoshita curve, which is an extension to a sine-generated curve including extra terms related to skewness and flatness [cf. Abad and Garcia, 2009].

The results from one of the experiments in the Kinoshita flume are shown in Figure 7c. Just downstream of the bend apex (s = 14.3 m), the largest transverse slope is found, after which it gradually decreases until s ≈ 16 m. Subsequently, the slope exhibits a second, slightly lower peak at s = 17 m, after which it decays strongly toward a minimum just after the apex of the next (opposite) bend (s = 19.3 m).

3.2. Assessment of the Gravitational Pull Models

Figure 7a shows the results of the T3 experiment in the DHL flume as simulated with the nonlinear model. When using the van Bendegom-type gravitational pull model (G ∝ θ^−1), the wavelength of the bar oscillations seems to be reproduced correctly, but the amplitude is overestimated. The nonlinear model combined with the Ikeda-type gravitational pull model (G ∝ θ^−0.5) underestimates the wavelength but reproduces the bar amplitude better than the van Bendegom-type model. The Sekine and Parker-type model (G ∝ θ^−0.25) yields slightly better results than the Ikeda-type model.

Figure 7b shows the nonlinear model results for the M89 experiment in the EPFL-flume are shown in Figure 7b. At the entrance of the bend, the nonlinear model reproduces the slight decrease in the transverse slope. All of the models underestimate the position of peak transverse slope. The Sekine and Parker-type model reasonably reproduces the magnitude of the transverse slope around the bend. The Ikeda and van Bendegom-type models, however, underestimate the slope by as much as 40% and 75%, respectively.

Figure 7c shows the nonlinear model solution for the sharply curved Kinoshita flume for each of the three fitted gravitational pull models. The Sekine and Parker-type model shows the best agreement. Again, the Ikeda and
van Bendegom-type models, however, significantly underestimate the transverse slope.

The evaluation of the performance of the nonlinear model with the three gravitational pull models reveals that the results are quite sensitive to the choice of this pull model. All in all, the Sekine and Parker-type gravitational pull model produces the best results for all three experiments.

### 3.3. Importance of Nonlinear Effects

The importance of nonlinear effects will be analyzed by comparing the results of the linear and nonlinear models, both with the gravitational pull model of Sekine and Parker [1992].

The results of the linear model for the T3 experiment are almost the same as those of the nonlinear model (cf. Figure 8a). This is not surprising as this is a case of mild curvature and mild amplitude. This is in line with our earlier observation that the nonlinear model encompasses the linear model as a limit case for mild curvature and mild amplitude.

Figure 8b shows the results for the strongly curved M89 experiment. The two models show no difference in the upstream reach and until about 1.5 m into the bend. The nonlinear model subsequently shows a decay in the normalized transverse slope $A/R$, whereas the linear model shows a slight increase and a leveling off of this quantity in the second half of the bend. Just before the bend exit, both the linear and nonlinear models show a slight increase in transverse slope, after which it decays rapidly toward the horizontal straight channel limit. The nonlinear model results are clearly superior to those of the linear model, which strongly overestimates the transverse slope in the second half of the bend by as much as 92%.

The linear and nonlinear model results for the Kinoshita experiment are shown in Figure 8c. The pattern of the transverse slope is approximately the same for both models, but the linear model overpredicts the magnitude of the transverse slope by as much as 100%.

The results for the M89 experiment and the Kinoshita flume reveal that nonlinear effects play a significant role in the formation of the transverse bed slope in strongly curved bends. Furthermore, the computations for the T3 experiment show that nonlinear effects are not important in mildly curved flumes with mild-amplitude variations and confirm that the nonlinear model reduces to the linear model in this limit case.

### 3.4. Dominant Mechanisms in the Formation of the Transverse Bed Slope

The mechanisms underlying the development of the bed morphology in a bend were described in section 2.2.2.3, and the corresponding terms in equations (28) to (30) are indicated by roman numerals. These mechanisms are compared in further detail using the linear and nonlinear model results.

The forcing mechanisms for the mildly curved T3 experiment are shown in Figure 9a. The first forcing term starts at 0 in the straight inflow reach and increases to the bend equilibrium value through the bend, and after the bend exit, it decreases again to 0. There is no difference between the linear and the nonlinear models. The second forcing term
is related to flow field accelerations and decelerations. At the bend entrance, the abrupt change of curvature causes the flow to accelerate near the inner bank and decelerate near the outer bank. Flow redistribution by topographic steering and secondary flow subsequently drives the downstream momentum toward the outer bank [cf. Blanckaert and de Vriend, 2010]. This effect is, however, too strong, and an overshoot of the equilibrium downstream velocity distribution for fully developed bend flow occurs. Subsequently, the velocity profile exhibits an oscillatory adaptation to the equilibrium state [cf. Struiksma, 1983a]. The linear and the nonlinear models (cf. Figure 9b) give comparable results. The third forcing term (sediment transport caused by higher-order flow variations), which is only included in the nonlinear model, has a negligible effect on the transverse bed slope.

[60] For mildly curved bends and mildly sloping transverse slopes, the balance of the secondary flow and the gravitational pull dominates the bed adaptation behavior throughout the domain. Local accelerations and decelerations of the flow may lead to a local maximum in the point bar height (e.g., to a navigation bottleneck).

[61] The forcing terms in the sharply curved M89 case are shown in Figures 9c and 9d. In the linear model case...
(Figure 9c), the first forcing term shows a similar behavior as in the mildly curved T3 experiment: 0 in the straight inflow and outflow reaches and constant around the bend. The nonlinear model results (Figure 9d) show a decay of the transverse slope in the second half of the bend, leading to a value at the bend exit that is almost half the maximum. Clearly, this decay is due to the nonlinear reduction of the secondary flow (Figure 2a). The corresponding reduction of the up slope drag force expresses itself in a smaller equilibrium bed slope. The second forcing term (flow field accelerations and decelerations) clearly shows the effect of changes in curvature at the bend entrance and exit in both the linear and nonlinear cases. The third forcing term still has a negligible effect on the transverse bed slope, be it that it is larger than in the T3 case.

[62] The conclusion to be drawn from the M89 case is that the balance of the secondary flow and the gravitational pull dominates the bed adaptation behavior throughout the domain but that the second forcing term can be locally important.

[63] For the strongly curved Kinoshita flume, the magnitude of the forcing terms according to the linear model is shown in Figure 9e. The forcing mechanism related to the equilibrium transverse slope is dominant, except at the transitions between the bends (s ≈ 12.8 m and s ≈ 17.8 m), where the flow accelerations and decelerations are dominant. The nonlinear model (cf. Figure 9f) shows a somewhat smaller contribution of the first forcing term (related to the equilibrium transverse slope) and behavior of the second forcing term similar to that in the linear model. Again, the third forcing term has a minor influence on the solution.

[64] In summary, the first forcing term is dominant in all three cases, with local effects of the second forcing term. These results confirm the findings of Camporeale et al. [2007]. In strongly curved bends, a downstream decrease of the transverse slope occurs in the second half of the bend, due to the nonlinear modeling of the secondary flow. The third forcing term plays a minor role in all cases investigated.

4. Conclusion

[65] Based on the Exner conservation principle and earlier work on reduced-order hydrodynamic modeling for sharp bends, a reduced-order nonlinear meander bed morphology model for strongly curved bends with large-amplitude bed level variations was derived. In its asymptotic formulation for mild-curvature and small-amplitude bed level variations, the nonlinear model reduces to a previously proposed linear model.

[66] The meander bed morphology model is quite sensitive to the parameterization of the downhill gravitational force on particles at a sloping bed. Three different gravitational pull models were considered, proposed by van Bendegom [1947], Ikeda [1982], and Sekine and Parker [1992], respectively. The Sekine and Parker [1992] type gravitational pull model was the only one that accurately reproduced the bed morphology in all cases investigated.

[67] The nonlinear model, without curvature restrictions, accurately reproduced the bed morphology in three investigated flume experiments: one with a mildly curved bend and two with strongly curved bends. The linear model significantly overestimated the transverse bed slopes, in one case by more than 100%.

[68] The smaller transverse bed slope resulting from the nonlinear model must be attributed mainly to a weaker curvature-induced secondary flow strength, due to a nonlinear feedback mechanism. The weaker secondary flow causes a weaker transverse component of the drag force on the particles. This explains why scour factors (the coefficient of proportionality in the relationship between the transverse bed slope in fully developed bend flow and the depth-curvature ratio) in strongly curved channels are significantly smaller than in mildly curved channels. Further investigation is recommended to distinguish the role of the aspect ratio on the scour factor.

[69] Overall, the balance between the uphill drag force exerted by the secondary flow on the sediment particles on the bed and the downhill gravitational force dominates the formation of the transverse bed slope. Only locally, e.g., near transitions in the channel curvature, flow accelerations and decelerations may be dominant.

[70] The present paper clearly shows the importance of including nonlinear effects when modeling the bed morphology in strongly curved channels. By coupling this model to a bank erosion model, a meander development model for the high-curvature range which allows sudden changes in curvature can be obtained.

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References


van Benedegom, L. (1947), Some considerations on river morphology and river improvement, *De Ingenieur*, 59(4), B1–B11.