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1aBA3. A contrast source inversion method for breast cancer detection

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Tomogrophic ultrasound imaging is gaining popularity in breast cancer detection. Reconstructing the acoustic properties of a breast from the ultrasound measurements is stated as a nonlinear inverse problem, which is usually solved by linearized methods because of computational efficiency. However, linearization of the problem reduces the quality of the reconstruction. To improve the accuracy, we developed and tested a three-dimensional nonlinear inversion method that allow for three-dimensional reconstruction of the breast in terms of speed of sound. The method, referred to as contrast source inversion (CSI), uses an integral equation formulation to describe the inverse acoustic scattering problem. The resulting integral equation is solved to reconstruct the unknown contrast (speed-of-sound profile of the breast). The contrast and contrast sources (the product of the contrast with the total field) are iteratively updated by minimizing a cost functional using conjugate gradient directions. In this study, we tested the CSI method on synthetic data retrieved form full-wave simulations for a realistic three-dimensional cancerous breast model. Results show that the CSI method outer performs other conventional methods as it yields speed-of-sound reconstructions that are akin to the model. This shows that the approach offers a contribution to the detection of breast cancer.

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INTRODUCTION

Ultrasound breast scanning is gaining popularity due to its low cost and high efficiency. In addition, recent studies have shown that it can successfully detect tumors in dense breasts whereas mammography can miss them. Therefore, researchers are developing fully automated three-dimensional ultrasound breast screening systems [1]. These systems allow for advanced ultrasound imaging techniques to improve the detection. Though imaging of breasts is a nonlinear problem, in many areas, it is linearized by applying the Born approximation, and hence neglecting the multiple scattering of the acoustic field within the breast. As a result of this approximation, images get blurred and incorrect reconstructions are obtained. Our study aims to give a detailed image of the breast by reconstructing the speed of sound profile. Next, the profile may be used to differentiate a tumor from other tissues. To solve the actual nonlinear inverse scattering problem we use a contrast source inversion (CSI) method [2]. In our conjugate gradient based method, a cost functional is minimized iteratively, and contrast sources and contrast functions are updated within each iteration. The following sections give the integral equation representation of the nonlinear inverse problem, and reconstruction examples obtained with the CSI method.

PROBLEM STATEMENT

To describe the nonlinear inverse scattering problem, we consider a model which is given by a bounded, simply connected, inhomogeneous object domain $\mathbb{D}$ in an unbounded homogeneous background medium. Scattering object (or objects) $\mathbb{B}$ of which contrast and location are unknown is placed in the object domain $\mathbb{D}$. The sources and receivers are located in a domain (or on a curve) $\mathbb{S}$ surrounding $\mathbb{D}$ (See Figure 1).

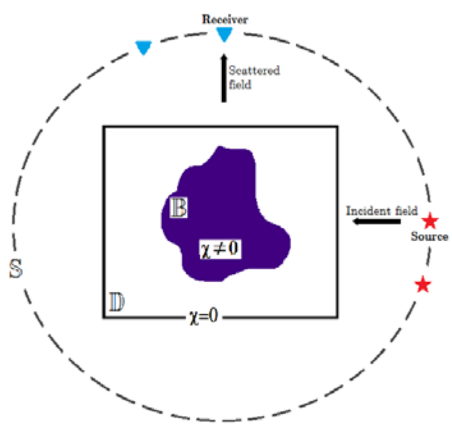

FIGURE 1: Symbolic representation of the object domain $\mathbb{D}$, which includes the scattering object $\mathbb{B}$, and the source (or data) domain $\mathbb{S}$.

The integral equation formulation of the total pressure wave field $\hat{p}^{tot}$ in $\mathbb{D}$ is given in the frequency domain as

$$\hat{p}^{tot}(\boldsymbol{x}) = \hat{p}^{inc}(\boldsymbol{x}) + \int_{\mathbb{D}} \omega^2 \hat{G}(\boldsymbol{x},\boldsymbol{x}')\chi(\boldsymbol{x}')\hat{p}^{tot}(\boldsymbol{x}')\mathrm{d}V(\boldsymbol{x}') \qquad \boldsymbol{x},\boldsymbol{x}' \in \mathbb{D}, \tag{1}$$

where ω is the angular frequency, $\hat{G}$ is the Green's function, and $\hat{p}^{inc}$ is the incident field, which is the actual pressure wave field in the absence of any contrast [3, 4]. Here, the contrast function χ is defined as

$$\chi(\boldsymbol{x}') = \frac{1}{c^2(\boldsymbol{x}')} - \frac{1}{c_{bg}^2}, \tag{2}$$

where c and c_{bg} are the speed of sound of the object and background domain, respectively. This equation refers to an inverse problem due to two unknowns inside the integral: i.e., the contrast function χ, and the total field $\hat{p}^{tot}$. Because of the χ dependency of the total field, this problem is a nonlinear problem. Note that, the symbol ˆ indicates that those functions are dependent on the angular frequency. Our solution method for this integral equation will be discussed in the next section.

Contrast Source Inversion Method

In scattering theory, the difference between the total field, recorded by the receivers, and the incident field, generated by the sources, equals the scattered pressure field. The CSI method assumes that the two unknowns, contrast function and total field in $\mathbb{D}$, act together as a contrast source that produces the scattered field. This method does not include any approximation, and updates the unknown total field and the contrast within each iteration [5, 6]. The contrast source term $\hat{w}_j$ is defined as

$$\hat{w}_j(\boldsymbol{x}') = \chi(\boldsymbol{x}')\hat{p}_j^{tot}(\boldsymbol{x}'), \tag{3}$$

where the subscript $j = 1,2,...$ indicates the source number. Since the scattered field can be measured in the domain $\mathbb{S}$ where all sources/receivers are located, one can write

$$\hat{f}_j(\boldsymbol{x}) = \int_{\mathbb{D}} \omega^2 \hat{G}(\boldsymbol{x},\boldsymbol{x}')\chi(\boldsymbol{x}')\hat{p}_j^{tot}(x')\mathrm{d}V(\boldsymbol{x}') \qquad \boldsymbol{x} \in \mathbb{S},\ \boldsymbol{x}' \in \mathbb{D}, \tag{4}$$

where $\hat{f}_j$ is the measured scattered field. Rewriting Eq. (1) and Eq. (4) in operator notation, we obtain

$$\hat{p}_j^{tot} = \hat{p}_j^{inc} + L^{\mathbb{D}}\hat{w}_j, \tag{5}$$

and

$$\hat{f}_j = L^{\mathbb{S}}\hat{w}_j. \tag{6}$$

The superscripts $\mathbb{D}$ and $\mathbb{S}$ on the operators indicate the location of the point $\boldsymbol{x}$ as defined in Eq. (1) and Eq. (4). Multiplying Eq. (5) with χ yields

$$\chi\hat{p}_j^{inc} = \hat{w}_j - \chi L^{\mathbb{D}}\hat{w}_j. \tag{7}$$

Eq. (6) and Eq. (7) can be solved together for the unknown $\hat{w}_j$. So, the cost functional for both equations can be defined as

$$E(\hat{w}_j,\chi) = \eta_{\mathbb{S}} \left\|\hat{f}_j - L^{\mathbb{S}}\hat{w}_j\right\|_{\mathbb{S}}^2 + \eta_{\mathbb{D}} \left\|\chi\hat{p}_j^{inc} - \hat{w}_j + \chi L^{\mathbb{D}}\hat{w}_j\right\|_{\mathbb{D}}^2, \tag{8}$$

with the normalization factors

$$\eta_{\mathbb{S}} = \left(\left\|\hat{f}_j\right\|_{\mathbb{S}}^2\right)^{-1}, \quad \eta_{\mathbb{D}} = \left(\left\|\chi\hat{p}_j^{inc}\right\|_{\mathbb{D}}^2\right)^{-1}, \tag{9}$$

and

$$\|u\|_{\mathbb{D},\mathbb{S}}^2 = \langle u,u\rangle_{\mathbb{D},\mathbb{S}} \quad u \in \mathbb{D} \quad \text{or} \quad u \in \mathbb{S}. \tag{10}$$

The normalization is chosen so that both terms are equal to one when $\hat{w}_j = 0$. The CSI method computes the contrast sources by minimizing the cost functional using a steepest descent method. In each iteration step, the total field is updated using Eq. (5) followed by an update of the contrast via Eq. (3). Table 1 shows the detailed scheme of the CSI method.

TABLE 1: Contrast source inversion scheme. The symbols * and ¯ indicate the adjoint of the operator, and the complex conjugate of the variable, respectively.

$$\hat{w}_{j,0} = \frac{\|L^{\mathbb{S}*}\hat{f}_j\|^2_{\mathbb{D}}}{\|L^{\mathbb{S}}L^{\mathbb{S}*}\hat{f}_j\|^2_{\mathbb{S}}} L^{\mathbb{S}*}\hat{f}_j$$

$$\hat{p}^{tot}_{j,0} = \hat{p}^{inc}_j + L^{\mathbb{D}}\hat{w}_{j,0}$$

$$\chi_0 = \frac{\sum \bar{\hat{p}}^{tot}_{j,0}\ \hat{w}_{j,0}}{\sum |\hat{p}^{tot}_{j,0}|^2}$$

$$\hat{\rho}_{j,0} = \hat{f}_j - L^{\mathbb{S}}\hat{w}_{j,0}$$

$$\hat{r}_{j,0} = \chi_0 \hat{p}^{tot}_{j,0} - \hat{w}_{j,0}$$

$$\eta^{\mathbb{S}} = \left(\|\hat{f}_j\|^2_{\mathbb{S}}\right)^{-1}$$

for n = 1,2,...

$$\eta^{\mathbb{D}}_n = \left(\left\|\chi_{n-1}\hat{p}^{inc}_j\right\|^2_{\mathbb{D}}\right)^{-1}$$

$$\hat{g}_{j,n} = -\eta^{\mathbb{S}} L^{\mathbb{S}*}\hat{\rho}_{j,n-1} - \eta^{\mathbb{D}}_n \left(\hat{r}_{j,n-1} - L^{\mathbb{D}*}\bar{\chi}_{n-1}\hat{r}_{j,n-1}\right)$$

$$\gamma_n = \|\hat{g}_{j,n}\|^2_{\mathbb{D}} / \|\hat{g}_{j,n-1}\|^2_{\mathbb{D}}$$

$$\hat{d}_{j,n} = \hat{g}_{j,n} + \gamma_n \hat{d}_{j,n-1}$$

$$\alpha_n = \frac{-\mathrm{Re}\langle \hat{g}_{j,n}, \hat{d}_{j,n}\rangle_{\mathbb{D}}}{\eta^{\mathbb{S}}\|L^{\mathbb{S}}\hat{d}_{j,n}\|^2_{\mathbb{S}} + \eta^{\mathbb{D}}\|\hat{d}_{j,n} - \chi_{n-1}L^{\mathbb{D}}\hat{d}_{j,n}\|^2_{\mathbb{D}}}$$

$$\hat{w}_{j,n} = \hat{w}_{j,n-1} + \alpha_n \hat{d}_{j,n}$$

$$\hat{p}^{tot}_{j,n} = \hat{p}^{inc}_j + L^{\mathbb{D}}\hat{w}_{j,n}$$

$$\chi_n = \frac{\sum \bar{p}^{tot}_n w_n}{\sum |p^{tot}_n|^2}$$

$$\hat{\rho}_{j,n} = \hat{f}_j - L^{\mathbb{S}}\hat{w}_{j,n}$$

$$\hat{r}_{j,n} = \chi_n \hat{p}^{tot}_{j,n} - \hat{w}_{j,n}$$

$$E^{\mathbb{S}}_n = \eta^{\mathbb{S}} \|\hat{\rho}_{j,n}\|^2_{\mathbb{S}}$$

$$E^{\mathbb{D}}_n = \eta^{\mathbb{D}}_n \|\hat{r}_{j,n}\|^2_{\mathbb{D}}$$

$$E_n = E^{\mathbb{S}}_n + E^{\mathbb{D}}_n$$

if $E_n < \epsilon$ stop

end

RESULTS

The CSI method is tested on synthetic data which is obtained by solving the forward problem. The forward problem refers to the situation where the unknown total field is computed for known contrast function and known incident field. Using a full wave method [7, 8], the scattered field is computed for a three-dimensional breast model which is built in terms of speed of sound inhomogeneities from a MRI scan of a real cancerous breast [9]. The speed of sound parameters for this model are given in Table 2. The spatial domain is defined as 16 cm × 16 cm × 16 cm. The incident field is modulated by a Gaussian pulse with 0.08 MHz center frequency. First, we run

TABLE 2: Speed of sound parameters for different tissues used in the model.

Tissue	c [m/s]
Water	1524
Muscle	1532
Fat	1478
Breast	1485
Tumor	1500

several iterations using only the center frequency, and 160 sources/receivers distributed equally (i.e., 10 rings with 16 sources/receivers in each ring). The speed of sound profile of the breast is reconstructed using the CSI method. The xy-plane and xz-plane in the middle of the spatial domain for different number of iterations are shown in Figure 2 and Figure 3, respectively. As can be seen in the figures, increasing the number of iterations beyond 256 did not yield a significant improvement. Next, we tested the effect on the reconstruction of adding more

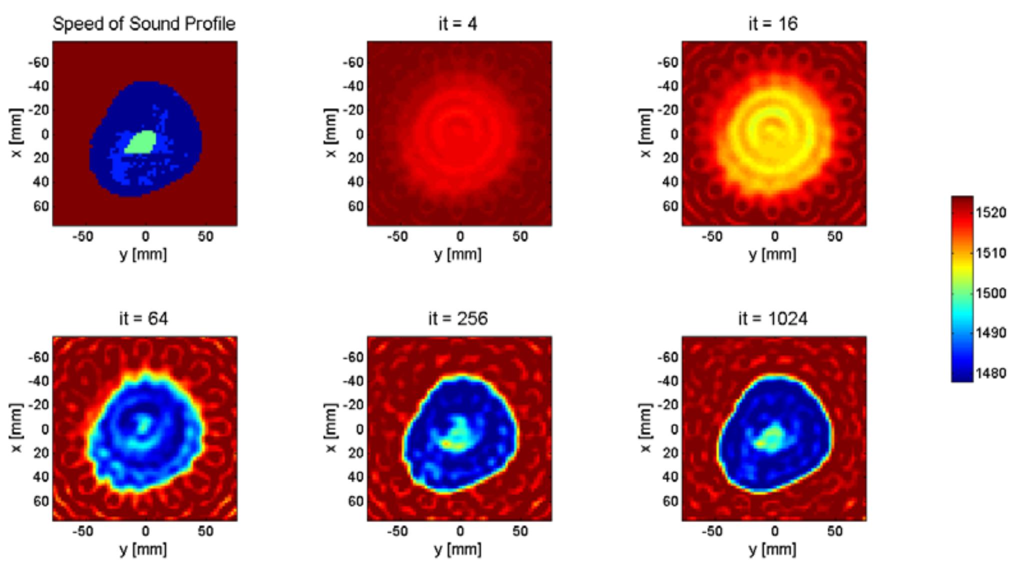

FIGURE 2: Reconstruction results in the xy-plane for different number of iterations, 'it' refers to the number of iterations. The left image in the first row shows the actual model.

frequencies. In principle, this should increase the available information. Three configurations are considered to investigate the frequency effect. We used 160 sources/receivers, and 256 iterations for one, three and five frequency components which are equally distrubuted over the spectrum. The results are shown in Figure 4. Although adding frequencies did not improve the results, it significantly increased the memory and computational time (See Table 3). Finally, we increased the number of sources/receivers from 160 to 200. As in the first configuration, we only used one frequency -center frequency- and 256 iterations. The results presented in Figure 5

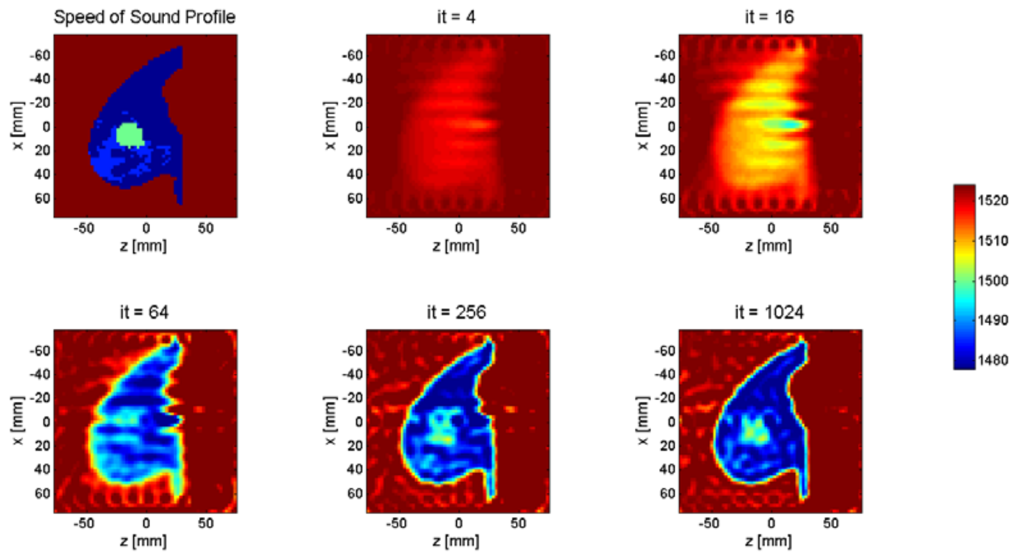

FIGURE 3: Reconstruction results in the xz-plane for different number of iterations, ‘it’ refers to the number of iterations. The left image in the first row shows the actual model.

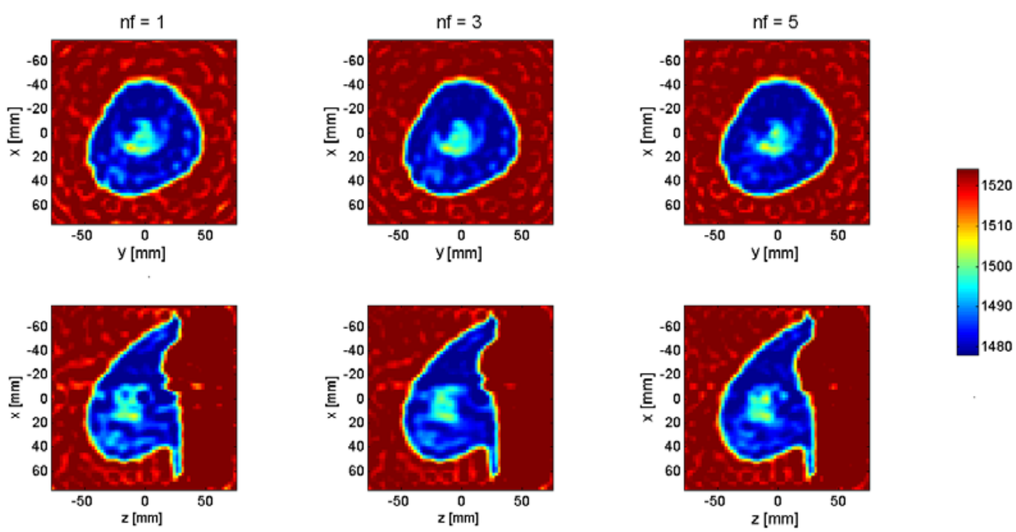

FIGURE 4: Reconstruction results by using several frequencies. The top row shows the xy-plane. The bottom row shows the xz-plane, ‘nf’ indicates the number of frequencies used for reconstruction.

indicate that increasing the number of sources/receivers yielded sharper reconstructions. Table 3 compares the memory stored for the reconstruction, and computational time for all configurations mentioned above. Increasing the number of sources not only gave a more detailed reconstruction but it also required less memory and computational time, as compared to using three or more frequencies.

CONCLUSIONS

We have implemented a three-dimensional contrast source inversion (CSI) method to identify breast cancer. First, synthetic data was computed using a full wave method. The data is based on a three-dimensional breast model obtained from a MRI scan of a real cancerous breast and shows all relevant wave phenomena such as multiple scattering, reflection, diffraction etc. Next,

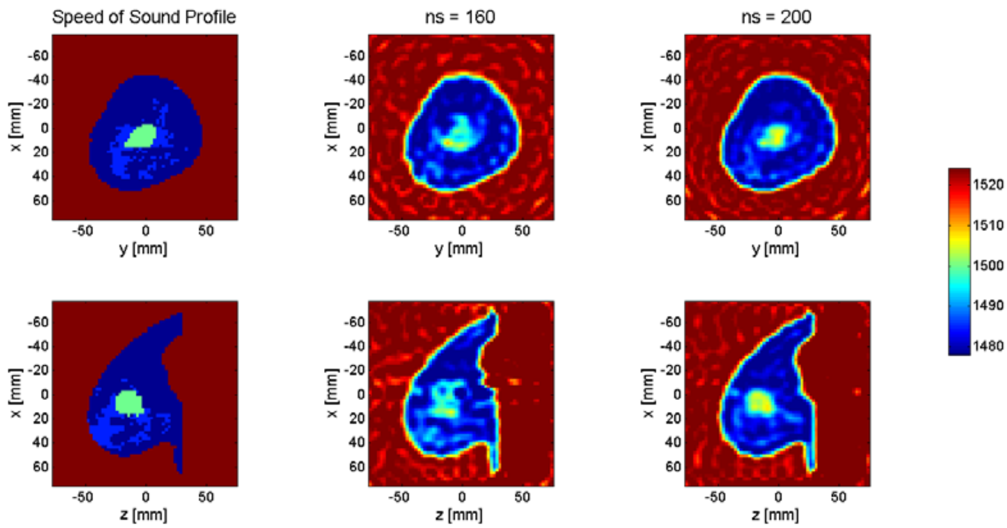

FIGURE 5: Reconstruction results for different number of sources. The top row shows the xy-plane. The bottom row shows the xz-plane, 'ns' refers to the number of sources/receivers used for reconstruction.

TABLE 3: Comparison of the computational time and memory for different settings.

Number of Sources	Number of Frequencies	Number of Iterations	Stored Memory [MB]	Time [h]
160	1	256	0.4	7
160	3	256	1.2	19
160	5	256	2	32
200	1	256	0.6	8

this data was used to test the three-dimensional CSI method. The performance of the method was investigated by means of number of iterations, frequencies and transducers. Results show that speed of sound profiles which are akin to the original model are obtained using the CSI method.

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