Static stability of loose materials under wave attack New design formulae and probabilistic approach

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by : Msc M.J.Koster
Rijkswaterstaat, Road and Hydraulic Engineering Division
Van Der Burghweg 1, P.O. box 5144
2600 GA Delft, The Netherlands

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1. Introduction

Coastal protection or harbour entrance works like breakwaters, jetties, groins etc. can consist of loose materials used as a top layer. Loose materials are considered those materials that are not interlocking with each other. Loose materials can range from materials having a small diameter (like gravel) up to materials having a relative large diameter (like large concrete blocks).

A breakwater consist of a core material. If this core material is too fine (i.e. too light) then the core material will be protected by a secondary layer that consists of heavier material. If this material is also not stable enough under the design conditions, then another layer should be applied. The top layer is called the primary armor layer.

If, under design storm conditions, the waves are such that the primary armor layer will suffer from severe erosion, then most certainly the secondary armor layer will also erode, as well as the core material. The breakwater will then collapse completely. The event "failure of the breakwater" will cause direct damage to the breakwater itself. But also secondary damage will be caused by failure of any coastal protection works. In the case of a breakwater this might be : damage caused to ships and their cargo, that are in the sheltered area behind the breakwater.

A good probabilistic approach should take into account both the probability of failure of the considered construction and the direct and secondary damage caused by failure of the construction. From such studies an acceptable level of risk can be determined. The next task then is : to design a breakwater that meets the required level of risk.

In this paper new developed design formulae for the static stability of loose materials as primary layer under wave load are considered. Two simple formulae will be given. It will be shown how design graphs can be used in a simple strait forward design of a breakwater. Also the probabilistic design of a breakwater will be subject of this paper.

2. Study of Van der Meer

Until some years ago the design of structures like breakwaters etc., consisting of loose materials, had to be performed with traditional design formulae. The most common used formulae in the Netherlands were : the Hudson formula and the Iribarren formula.

Because these formulae did not take into account some parameters that really appeared to be of influence, the public works department of the Netherlands (the Rijkswaterstaat) decided to contract Delft Hydraulics to carry out an extensive study on the stability of loose materials.

The results of these studies are now available. The results are now known as the "Van der Meer formulae". The results are published in several papers and communications. This paper summarizes the main results for the statically stable structures. For more details reference is made to the literature list. The purpose of this paper is to show and discuss the main results and to show how an engineer can work using the latest know how on this topic.

3. Static stability versus dynamic stability

Constructions consisting of loose materials can be classified into static and dynamic stable constructions.

Statically stable structures are structures where no or minor damage is allowed under design conditions. Damage is defined as displacement of armor units. The mass of individual units must be large enough to withstand the wave forces during design conditions. Caissons and traditionally designed breakwaters belong to the group of statically stable structures.

Dynamically stable structures are structures where profile development is concerned. Units (stones or gravel) are displaced by wave action until an equilibrium profile is reached. Once this profile is reached, the units will still move, but the profile will not change any more.

Although Van der Meer also derived empirical formulae for dynamically stable structures, this paper only takes into account : the statically stable structures.

4. Limitations of the classical Hudson formula

Although several traditional formulae exist for the static stability of loose materials, here the Hudson formula will be considered as an example. Other traditional formulae will also have one or more of the limitations as those that will be considered here.

The traditional Hudson formula can be re-arranged to :

$$\frac{H}{\Delta . Dn50} = [Kd.Cotg(\alpha)]^{1/3} \quad [1]$$

where :

H = The wave height in [m]
Δ = The relative buoyant density = ($\rho r/\rho w$) - 1
ρr = The mass density of the rock [kg/m3]
ρw = The mass density of the water [kg/m3]
Dn50 = The nominal grain diameter [m]
Kd = A stability coefficient
Cotg(α) = The cotangent of the slope

The relation between M50 and D50 is :

$$Dn50 = (M50/\rho r)^{1/3} \quad [2]$$

where :

M50 = median mass of unit given by 50% on mass distribution curve in [kg]

The Kd value to be applied, can be found in literature, for example the Shore Protection Manual. Kd values suggested for design correspond to a "no damage" condition, where up to 5 % of the units may be displaced. Also the Kd values should take into account all variables that are not in the stability formula and that are of importance.

The main advantage of the Hudson formula is its simplicity and the wide range of armor units and configurations for which values of Kd have been derived. The Hudson formula has many limitations . Briefly they include :

- Scale effects due to small scale test.
- The use of regular waves only.
- Wave period and storm duration do not appear in the formula.
- No description of the damage level.
- The use of non overtopped and permeable core structures only.

5. The Van der Meer formulae for static stability.

An extensive series of model tests were conducted at Delft Hydraulics. The tests included a wide range of core and/or underlayer permeabilities and a wide range of wave conditions. Based on these tests and on earlier tests of others, Van der Meer derived two formulae for plunging and surging waves respectively. These formulae may be written as :

for plunging waves :

$$\frac{Hs}{\Delta . Dn50} = a.P^{0.18} . (S/\sqrt{N})^{0.2} . K^{-0.5} \quad [3]$$

for surging waves :

$$\frac{Hs}{\Delta . Dn50} = b.P^{-0.13} . (S/\sqrt{N})^{0.2} . \sqrt{cotg(\alpha)}^{P} . K \quad [4]$$

where :

Hs = The significant wave height
a = A coefficient a = 6.2
b = a coefficient b = 1.0
P = The permeability of the structure.
S = The damage level
N = The number of waves
K = The surf similarity parameter

A criterion on when to use formula [3] or [4] is given by the following :

$$Kc = [a.P^{0.31} . \sqrt{\tan(\alpha)}\,]^{1/(P+0.5)} \qquad [5]$$

If K < Kc then [3] must be used, else [4] must be used.

6. The surf similarity parameter K.

The most useful parameter describing wave action on a slope is the surf similarity parameter, K, which relates the slope angle to the wave steepness, and which gives a classification of breaker types, see table below :

Surf similarity K	Breaker type
5	surging
3	collapsing
1.5	plunging
0.5	plunging
0.2	spilling

Table 1

The applied formula for the surf similarity parameter is :

$$K = \frac{\tan(\alpha)}{\sqrt{(Hs/Lo)}} \qquad [6]$$

where :

Hs/Lo = the wave steepness.
Lo = the wave length in deep water, and :

$$Lo = gT^2/(2\Pi) \qquad [7]$$

7. The damage level S

Van der Meer introduced a dimensionless damage level. The damage level S is the erosion area around the still water level, divided by the area of the stones :

$$S = Ae/Dn50^2 \qquad [8]$$

where :

Ae = Erosion area around still water level

This damage level is independent of the size (slope and height) of the structure. A perhaps more comprehensive explanation of S is :

- the number of squares with a side Dn50 which fit into the erosion area.
- or : the number of cubic stones with a side of Dn50 eroded within a Dn50 wide strip of the structure.

Generally the actual number of stones eroded in a Dn50 wide strip will be 0.7 to 1 times the damage level S.

Table 2 can be used as a guide in proper design of a structure. Initial damage (or: start of damage), corresponds with a value for S of 2 to 3. The damage thus calculated can be compared with the 5% damage criterion in the classical Hudson formula. Complete failure means that the filter layer will be exposed. For S values higher than 15-20 the structure is not statically stable any more, deformation of the top layer will be such that an S shaped profile will develop. In that case the structure must be calculated according to the formulae for dynamically stable structures. The table was set up for a two diameter thick rock armor layer.

Cotg(α)	Initial	Intermediate	Failure
1.5	S = 2	3 < S < 5	S = 8
2	S = 2	4 < S < 6	S = 8
3	S = 2	6 < S < 9	S = 12
4	S = 3	8 < S < 12	S = 17
6	S = 3	8 < S < 12	S = 17

Table 2

8. The permeability P

The permeability of the structure has influence on the stability of the armor layer. The size of the filter layers and the core material will influence the permeability. The permeability of the structure is represented by the parameter P. Van der Meer gives the following examples of constructions with their P values :

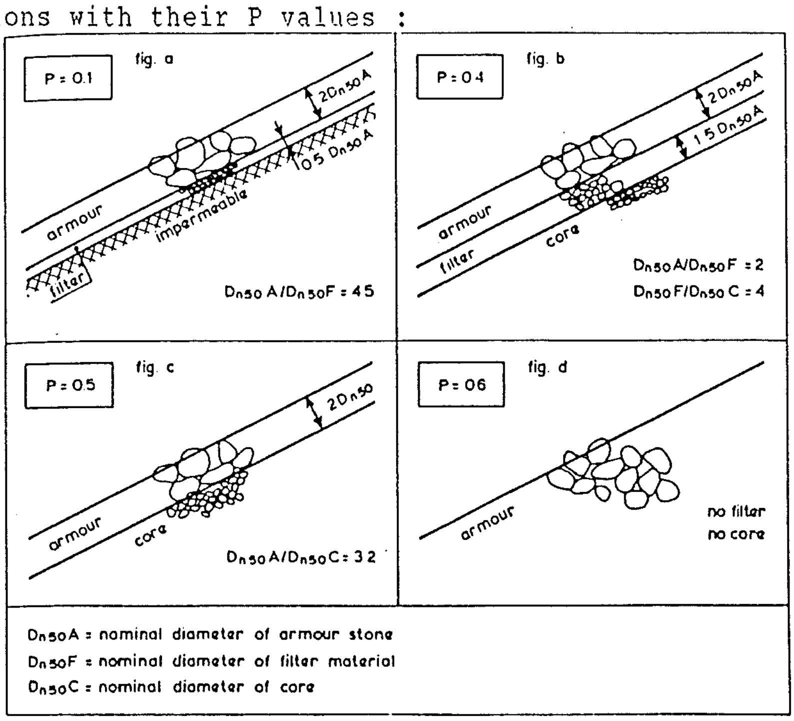

Fig a. The permeability P for various structures

The lower limit for P (P=0.1) is an armor layer with a thickness of two diameters on an impermeable core (sand or clay) and with only a thin filter layer. The upper limit of P (P=0.6) is given by a homogeneous structure which consists of only armor stones. Two other examples are given in figure a. Other structures have to be compared with the examples in order to come to a proper estimated value for the permeability parameter P.

Case Study Deterministic approach

The computer program MEERSTAB, that makes design graphs on a Personal Computer can be used for preliminary design of a construction.

Assume the following case :
Dn50 = 0.25 (m) , Cotg(α) = 3 , P = 0.4 , Δ = 1.7 , N = 3000 , Hs/Lo = 0.04

See figure a. for the kind of structure that corresponds with a permeability of P = 0.4

Assume the design wave, having a return period of on 50 years is Hs = 0.80 (m)

The calculated value for the surf similarity parameter is : K = 1.7

- See figure 1 : The damage level S for the design wave Hs = 0.8 (m) is less than S = 2, meaning : No damage. Start of damage will occur for a wave Hs = 0.90 (m).

- See figure 2. : If the design wave is Hs = 0.80 (m) and it is not desired to reach the start of damage S = 2, then the construction can withstand more than 5000 waves (N > 5000). If the design wave is Hs = 0.90 (m) the number of waves that the construction can withstand before start of damage occurs (S = 2) is N = 3000.

- See figure 3. : The effect of an alternative slope of respectively Cotg(α) = 1.5 , 2 , 3 can be seen from this figure. The K values for these slopes and the wave height for which start of damage will occur are :

Cotg(α)	K	Hs (m)
1.5	3.4	0.62
2	2.5	0.73
3	1.7	0.90

- See figure 4. : For the design wave Hs = 0.80 (m) the start of damage will occur for a construction having a permeability of P = 0.2 or higher.

- See figure 5. : Here the most worst assumption is made for K : K is such that the waves are just breaking. Start of damage (S=2) will occur for a wave height of Hs = 0.65 (m). For the design wave of Hs

= 0.80 (m) the start of damage (S=2) will occur for a Dn50 of 0.30 (m) in stead of Dn50 = 0.25 (m) . It must be understood that one of the advantages of the Van Der Meer formula is to take into account the expected value for the surf similarity parameter. From an engineering point of view it always interesting to see what happens if the surf similarity parameter differs considerably from the expected one.

- See figure 6. : For the design wave Hs = 0.80 (m) the start of damage
(S=2) will occur for a wave steepness of Hs/Lo > 0.025. Start of damage for a wave steepness of Hs/Lo = 0.04 will occur for a wave height of 0.90 (m).

The conclusion of all these figures is that the construction is well designed. Under the design conditions there will be no damage to the structure. A pessimistic assumption for the surf similarity parameter results in the conclusion that perhaps it is better to have a Dn50 of 0.30 (m).

10. Probabilistic approach

The reliability concept can be found in literature. Here only a brief summary will be given. It is customary to re-arrange a specific analytical problem into the following expression :

$$Z = R - S = Z\ (X_1, X_2, \ldots, X_n) \qquad [9]$$

where :

Z = the reliability function
R = the resistance or strength of the structure
S = the load of the structure
n = the number of random variables involved
$X_1 \ldots X_n$ = the random variables

The goal of a probabilistic calculation usually is to determine the probability of Z being less than zero :

$$Pr\ (\ Z<0\ ) \qquad [10]$$

meaning the probability that the load exceeds the strength of the structure. If the variables involved are statistically independent (not correlated), then a specific combination of the variables has a probability of occurrence of

$$f_1(X_1)\ .\ f_2(X_2)\ .\ \ldots\ .\ f_n(X_n) \qquad ss\ [11]$$

The probability of Z being less than zero is found by integration of the probability of occurrence (for a specific combination of the variables involved) over the complete range for each variable, in the area where Z is less than zero, or :

$$Pr\ (Z<0) = \iint_{Z<0} \cdots \int f_1(X_1).f_2(X_2)\ldots.f_n(X_n).dx_1.dx_2.\ \ldots dx_n \qquad [12]$$

Unfortunately in most cases this multi dimensional integral can not be solved explicitly. In that case alternative methods can be used, like

Monte Carlo simulation or the so called AFDA method (A Full Distribution Approach). However these methods are merely an approximation. The AFDA method was applied for the formulae of Van der Meer and the results where compared with the results of a full numerical integration. The AFDA approach proved to give satisfactory results. The reliability concept requires that all the probability density functions of the variables involved are known.

In order to apply the reliability concept the stability formulae of Van der Meer were re-arranged in terms of resistance and load. The random variables that are taken into account are :

The resistance variables :

- P - the permeability
- Δ - the relative density
- Cotg(α) - the slope
- a - the calibration coefficient a in the formula
- b - the calibration coefficient b in the formula
- αDn50 - A coefficient representing the uncertainty in the Dn50

The load variables :

- N - the number of waves
- Hs/Lo - the wave steepness
- Hs - the significant wave height
- αHs - a coefficient representing the uncertainty in Hs

The variables a, b, αDn50 and αHs will be explained here.

- The coefficients a and b in the stability formulae [3] and [4] are the result of averaging, and performing best fit calculations on the results of all the tests. In fact the tests show some scatter, which can be taken into account by introducing a and b as random variables having a normal distribution ,a mean and a standard deviation. Van der Meer gives the following values for a and b :

$$\mu a = 6.2 \ , \ \sigma a = 0.4 \ , \ \mu b = 1.0 \ , \ \sigma b = 0.08 \qquad [13]$$

- An uncertainty coefficient αDn50 is introduced because the value for Dn50 is in practice not so clear. In stead of Dn50 in the formulae, the following expression was used :

$$\alpha Dn50.Dn50 \qquad [14]$$

For αDn50 a normal distribution can be used, having a mean of 1. So if it is believed that the Dn50 has an uncertainty of 10% then :

$$\mu(\alpha Dn50) = 1 \quad \text{and} \quad \sigma(\alpha Dn50) = 0.10 \qquad [15]$$

- An uncertainty coefficient αHs was introduced because normally the distribution function for the significant wave is not really known exactly. In stead of using Hs in the formulae, the following expression was used :

$$(1 + \alpha Hs).\ Hs \qquad [16]$$

Here also the normal distribution function can be used.
So : if it is believed that the significant wave for a certain return period has an inaccuracy of 25% , then :

$\mu(\alpha Hs) = 0$ and $\sigma(\alpha Hs) = 0.25$ [17]

Using probabilistic calculations makes the design of statically stable structures more easy and clear. It is the only way to determine the risk of failure of the structure. Also it is possible to get an answer on questions like :
- what Dn50 must be used in order to get a prescribed level of risk during the lifetime of the structure ?
- what is the probability that a certain damage level (S=2) would be exceeded during the lifetime of the structure, etc.
- what is the influence of the uncertainty of each random variable ?

11. Case study probabilistic approach

The program STATSTAB for Personal Computers was used for the case study. This program performs probabilistic and deterministic calculations for both the Hudson formula and the Van der Meer formulae.

Consider the same case as for the deterministic case study. For the random variables the following realistic normal distributions will be assumed :

Variabele	μ	σ
P	0.4	0.1
Δ	1.7	0.1
N	3000	1000
Hs/Lo	0.04	0.01
Cotg(α)	3	0.15
a	6.2	0.4
b	1.0	0.08
αDn50	1.0	0.1
αHs	0	0.25

For Hs the following exponential distribution is assumed :

$$Pr\ (Hs>Hs) = \exp\left\{ -\frac{(Hs-0.5)}{0.077} \right\}$$

Notice : Pr (Hs>0.80) = 1/50 [yr] being the design wave in the deterministic case study.

Question : what is the required Dn50 if the following criterion must be met :
the probability of failure (a damage level S > 2) during the lifetime of the structure must be less than 50%. . What is the answer for Dn50 for the following three lifetimes : 5, 20 and 50 years.
Question : what is the probability of failure (a damage level S > 2) if the lifetime of the structure is 50 years and the Dn50 = 0.25 (m) like in the deterministic case study.

See figure 7. : In this figure the probability of failure is shown as function of the Dn50 for the three considered lifetimes of 5, 20 and 50 years. From the figure it can be seen that if the criterion is that the probability of failure may not exceed 50% during the lifetime, that the following Dn50 must be used :

Lifetime (years)	Dn50 required (m)
5	0.25
20	0.32
50	0.36

If a Dn50 = 0.25 (m) is used, then for a lifetime of 50 years the probability of failure will almost be 1, meaning an almost certain failure. This can be seen in the figure 7.

The deterministic approach resulted in a Dn50 = 0.25 (m) for a design wave having a return period of 50 years. The probabilistic approach gives an answer of Dn50 = 0.36 (m) . The reason for such a difference can be explained. In this case an exaggerated uncertainty for Hs was used : 0.25 (m) for a design wave of 0.80 (m) is about 30 % uncertainty. So there is a non imaginary probability that the wave will be 0.90 or even 1.00 (m). A probabilistic calculation takes this into account.

A by-product of a probabilistic calculation is : the contribution of the uncertainty of each variable to the total probability of failure. In this case the following contributions where found :

variable	contribution to probability of failure
αHs	72%
αDn50	8%
Hs	7%
Hs/Lo	3%
a	3%
Other variables	negligible

As can be seen from the results, in this case the uncertainty in the wave height Hs dominates far over all other uncertainties. Also it can be seen that the uncertainty in the Dn50 is more important than for example the uncertainty in the permeability or any other variable (except αHs)

<u>Literature</u>

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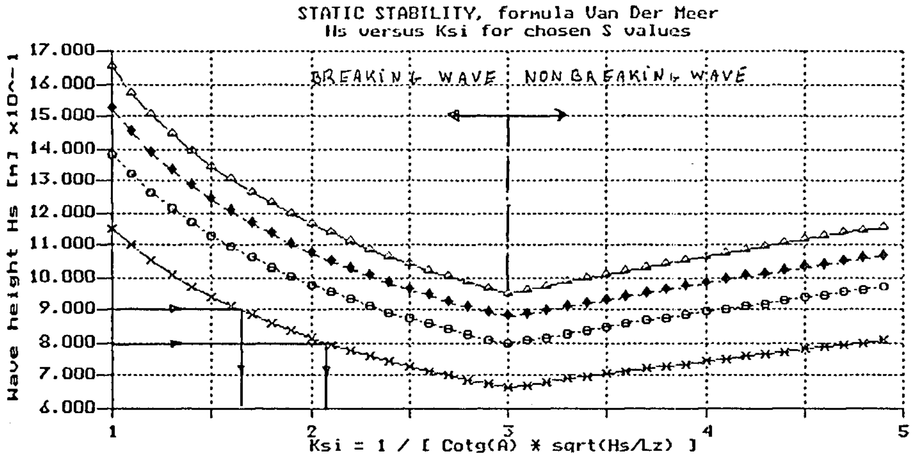
Licenced Company : ** DEMO-NAME **
Program MEERSTAB , Static stability v/d Meer formula
Koster Engineering



DELTA = 1.70 CotgA = 3.00
P = 0.40
N = 3000
Dn50 = 0.25

S = 12.00
S = 8.00
S = 5.00
S = 2.00

Fig. 1

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Program MEERSTAB , Static stability v/d Meer formula
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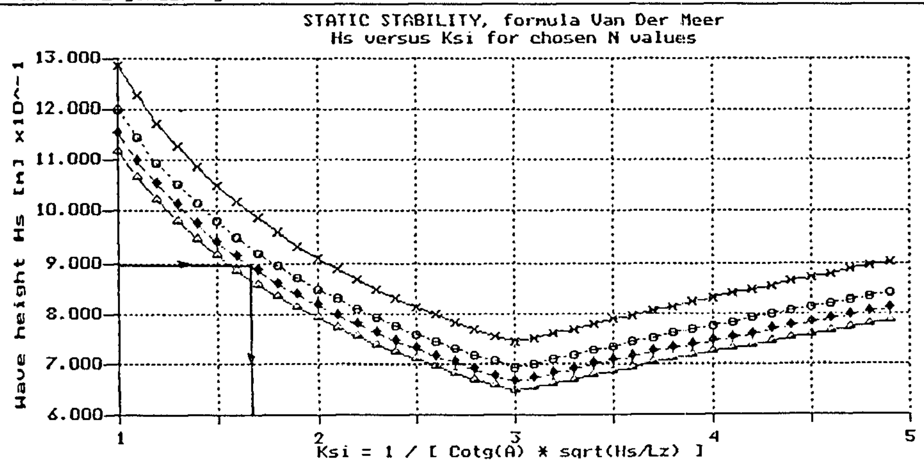

DELTA = 1.70 CotgA = 3.00
P = 0.40
S = 2.00
Dn50 = 0.25

N = 4000
N = 3000
N = 2000
N = 1000

Fig 2

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Program MEERSTAB , Static stability v/d Meer formula
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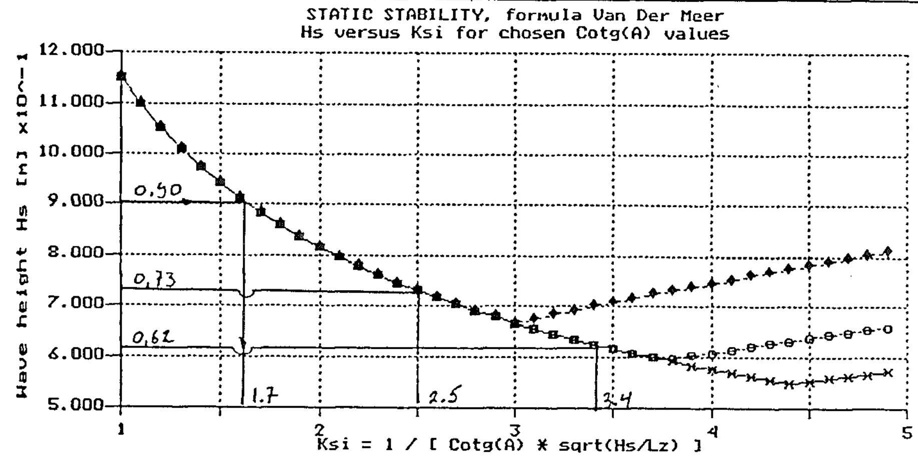

DELTA = 1.70 N = 3000
P = 0.40
S = 2.00
Dn50 = 0.25

COTG(A) = 3.00
COTG(A) = 2.00
COTG(A) = 1.50

Fig 3

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Program MEERSTAB , Static stability v/d Meer formula
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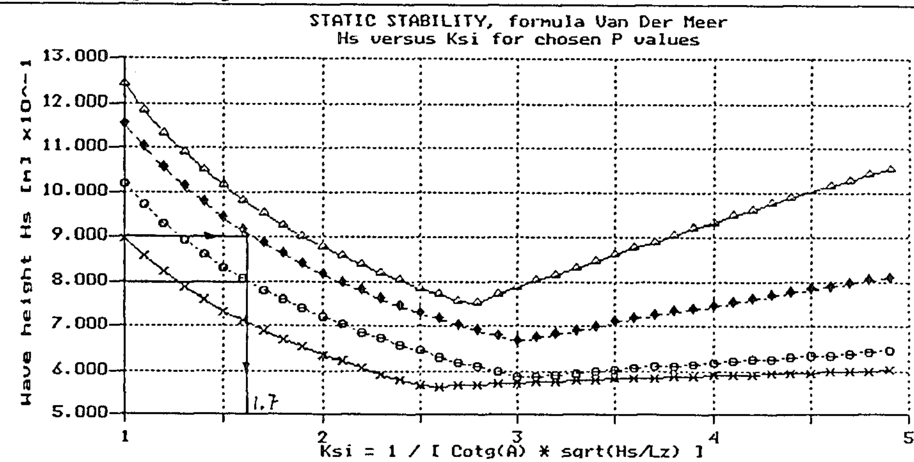
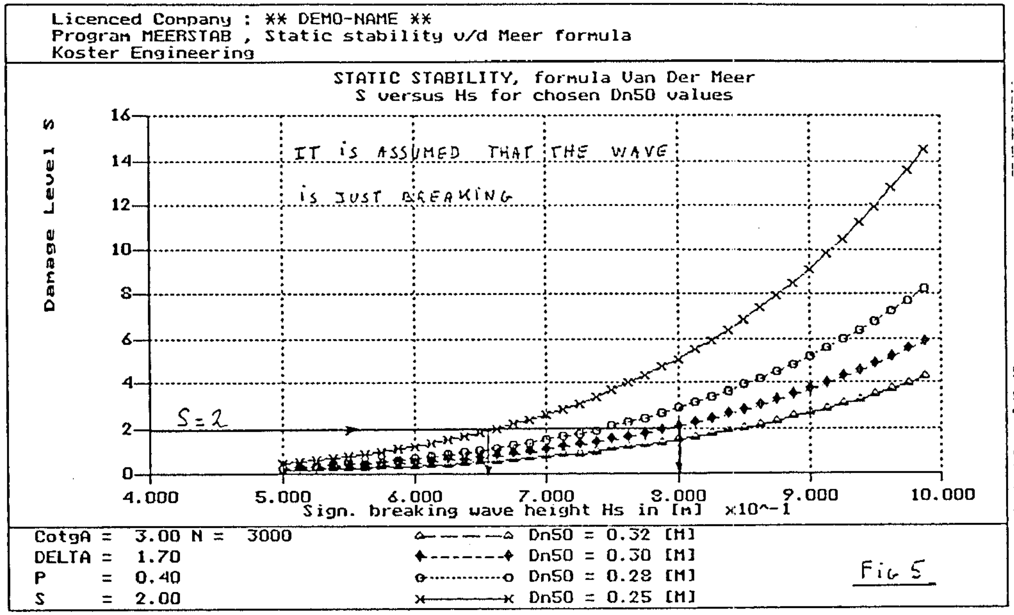


CotgA = 3.00 N = 3000
DELTA = 1.70
S = 2.00
Dn50 = 0.25

P = 0.60
P = 0.40
P = 0.20
P = 0.10

Fig 4



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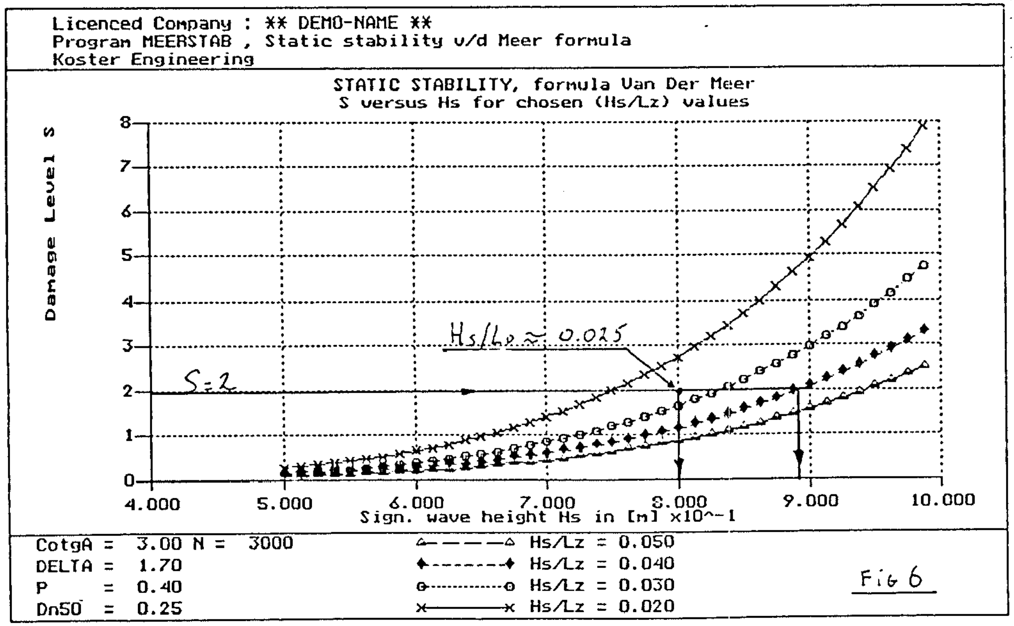
Program MEERSTAB , Static stability v/d Meer formula
Koster Engineering
STATIC STABILITY, formula Van Der Meer
S versus Hs for chosen Dn50 values
Damage Level S
IT IS ASSUMED THAT THE WAVE IS JUST BREAKING
S = 2
Sign. breaking wave height Hs in [m] x10^-1
CotgA = 3.00 N = 3000
DELTA = 1.70
P = 0.40
S = 2.00
Dn50 = 0.32 [M]
Dn50 = 0.30 [M]
Dn50 = 0.28 [M]
Dn50 = 0.25 [M]
Fig 5



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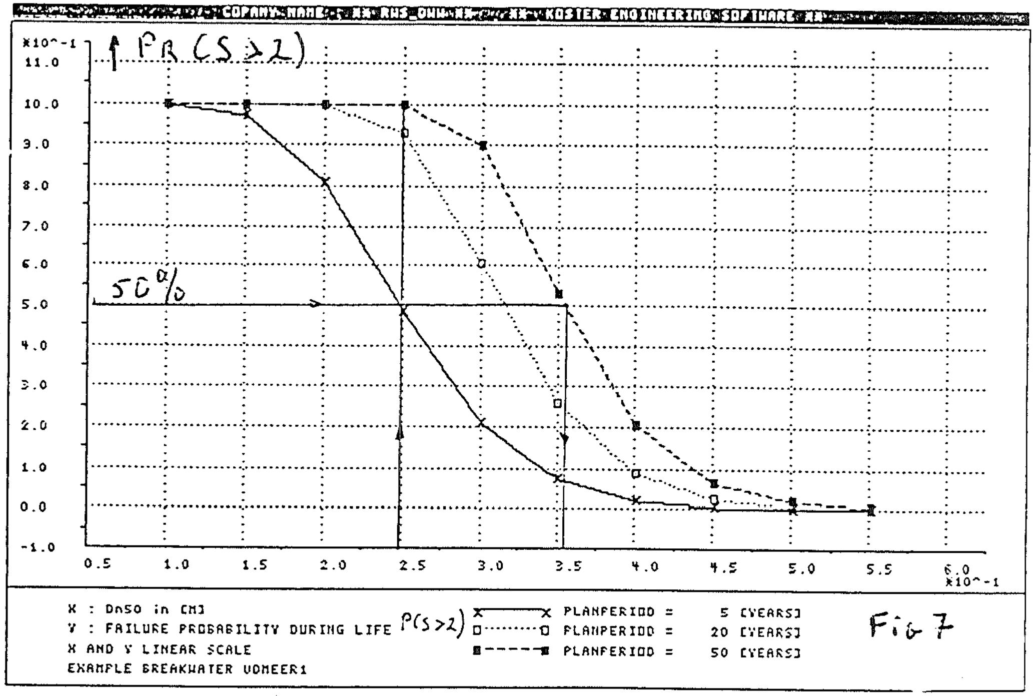
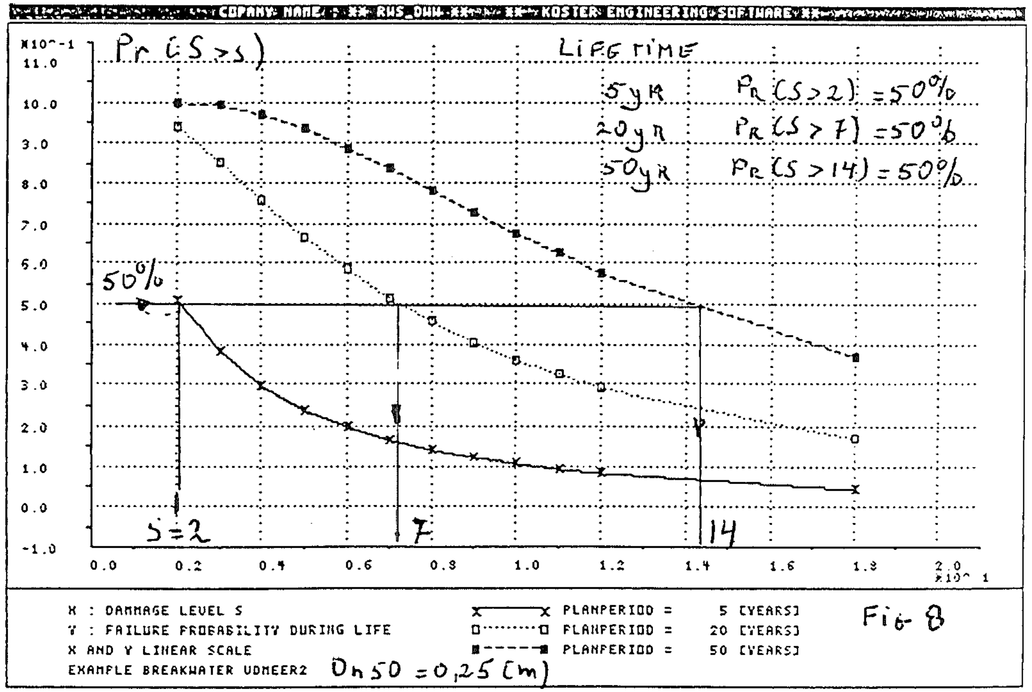
Program MEERSTAB , Static stability v/d Meer formula
Koster Engineering
STATIC STABILITY, formula Van Der Meer
S versus Hs for chosen (Hs/Lz) values
Damage Level S
Hs/Lp ≈ 0.025
S = 2
Sign. wave height Hs in [m] x10^-1
CotgA = 3.00 N = 3000
DELTA = 1.70
P = 0.40
Dn50 = 0.25
Hs/Lz = 0.050
Hs/Lz = 0.040
Hs/Lz = 0.030
Hs/Lz = 0.020
Fig 6



COMPANY NAME : ** RHS_DHW ** ** KOSTER ENGINEERING SOFTWARE **
Pr (S > 2)
50%
x10^-1
X : Dn50 in [M]
Y : FAILURE PROBABILITY DURING LIFE P(S>2)
X AND Y LINEAR SCALE
EXAMPLE BREAKWATER VDMEER1
PLANPERIOD = 5 [YEARS]
PLANPERIOD = 20 [YEARS]
PLANPERIOD = 50 [YEARS]
Fig 7



COMPANY NAME : ** RHS_DHW ** ** KOSTER ENGINEERING SOFTWARE **
Pr (S > s)
LIFETIME
5 yR Pr (S > 2) = 50%
20 yR Pr (S > 7) = 50%
50 yR Pr (S > 14) = 50%
50%
S = 2
7
14
X : DAMMAGE LEVEL S
Y : FAILURE PROBABILITY DURING LIFE
X AND Y LINEAR SCALE
EXAMPLE BREAKWATER VDMEER2 Dn50 = 0,25 [m]
PLANPERIOD = 5 [YEARS]
PLANPERIOD = 20 [YEARS]
PLANPERIOD = 50 [YEARS]
Fig 8