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DOI
10.1016/j.coastaleng.2020.103835

Publication date
2021

Document Version
Accepted author manuscript

Published in
Coastal Engineering

Citation (APA)
Etemad-Shahidi, A., Bali, M., & van Gent, M. R. A. (2021). On the toe stability of rubble mound structures. *Coastal Engineering*, *164*, Article 103835. https://doi.org/10.1016/j.coastaleng.2020.103835

On the toe stability of rubble mound structures

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The Netherlands

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Abstract

Toe design is an important task for coastal engineers as it ensures the stability of the main armor layer and prevents scour in front of the armor slope. Several laboratory experiments have been conducted to investigate the toe stability using different testing approaches, i.e. damage due to a single test condition and cumulative damage due to a number of conditions. In addition, the methods of measuring and reporting damage to the toe are not the same as some researchers have counted only stones that were washed away from the toe; while others counted all the stones that have been displaced. Several formulas with different levels of success have been developed based on these studies. However, the scatter in the results is large and effects of some parameters are disregarded. The aims of this study are (a) to briefly review the abovementioned differences and exiting formulas, and (b) to develop a and physically sound formula for common design conditions, which considers the effect of all governing parameters. To achieve this, first a comprehensive data base from existing reliable studies was collected. Then nondimensional parameters which capture effects of governing parameters such as wave height, wave period, water depth, toe depth, toe width and foreshore slopes were deployed to develop a stability formula using physical reasoning and regression analysis. The new formula outperforms existing formulae for toe stability. The coefficient of variation of the formula was also determined to be used for probabilistic design applications. Finally, some design hints are provided for practitioners.

Keywords: toe berm design; breakwaters; rock; scour protection; physical model tests; design formula

Introduction

Coastal structures such as breakwaters and seawalls usually have a toe. The functions of a toe are (a) to support the main armor layer by restraining its sliding and providing a stable footing and (b) to prevent undermining of the structure due to scour as the wave action is enhanced due to wave reflection. In breaking wave conditions where the water depth, h, is less than two times of the significant wave height, H_s, support of the armor layer at the toe can be conservatively ensured by placing one or two extra rows of main armor units at the toe of the slope.

Several studies have been carried out to derive a formula for the estimation of required toe rock size (e.g. Gerding, 1993; Van der Meer, 1995; Van der Meer et al., 1998; Muttray, 2013; Van Gent and Van der Werf, 2014; Herrera et al., 2016). However, existing formulas show large scatter when compared with measurements outside their range of validity (Muttray, 2013). This is mainly because these formulas are generally validated by using a specific data set and not with other tests. Muttray (2013) and Muttray et al. (2014) attempted to derive a formula using a larger data base including both regular and irregular tests. It should be mentioned that results of regular tests (e.g. Markle, 1989) are different from those of irregular ones, which better represent the real world. In addition, there are different ways of conducting tests and different ways to define damage. Most of the early laboratory studies (e.g. Gerding, 1993, Docters van Leeuwen, 1996) used single test/rebuilt approach where the damage was repaired, and model was rebuilt after each test. However, more recent studies (e.g. Ebbens, 2009; Van Gent and Van der Werf, 2014; Herrera and Medina, 2015; Herrera et al., 2016) have been conducted using the cumulative approach, where the model is rebuilt only after each test series. Another difference is the way that toe damage is quantified. In early studies (e.g. Gerding, 1993; Docters van Leeuwen, 1996) the "washed away method" has been used, i.e. only rocks that have been totally washed away from the toe were counted. While more recent ones (e.g. Ebbens, 2009; Van Gent and Van der Werf, 2014; Herrera and Medina, 2015) used the "displacements method" where all the rocks that have been moved (within the toe or washed away) were considered, similar to the approach adopted for armor layers. Hence, presumably their reported damage based on the "displacements method" should be higher than those of the "washed away method". There are some other differences such as the used wave height and period characteristics that will be discussed later. A summary of existing studies, their measurement method and some notes are given in Table 1.

The aim of this study is to develop a robust formula which is both physically sound and accurate for the estimation of the required toe size in design conditions with a wider range of validity than that of existing formulae. To achieve this, first a comprehensive database of existing studies is collected. Then, using scaling argument and physical reasoning, a design formula is developed based on the cumulative tests where the number of displaced stones is reported. The formula is modified for a probabilistic design approach and validated using existing single tests. Finally, for the sake of consistency with the design of armor layers, the proposed formula is modified to include damage level based on the erosion profile, S_d, rather than the number of displaced units.

Table 1. Summary of the exiting experimental studies, their test type and measurement method

Study	Test type	Counting method	Number of waves	Notes
Gerding (1993)	single	washed away	1000	$1.6 < h/H_s < 3.3$ $0.5 < h_t/h < 0.8$ m=1:20
Docters van Leeuwen (1996)	single	washed away	2000	used material other than rock
Ebbens (2009)	cumulative	displaced	1000	$0.7 < h/H_s < 6.1$ $-0.1 < h_t/h < 0.8$ 1:50 <m<1:10
Van Gent and Van der Werf (2014)	cumulative	displaced	1000	$1.2 < h/H_s < 4.5$ $0.7 < h_t/h < 0.9$ m=1:30
Herrera and Medina (2015)	cumulative	displaced	500	very shallow water and steep foreshore $-9.9 < h/H_{so} < 10.1$, m=1:10

Background

Coastal structures such as rubble mound breakwaters, dikes and seawalls are mostly located in depth limited water conditions with a toe (CEM, 2011). Fig. 1 displays cross section of a typical conventional rubble mound breakwater with a toe. A ratio of $0.3 < h_t/h < 0.5$ means that the toe is relatively high and close to the still water level (SWL). In this case, the toe structure acts more or less like a berm or a stepped structure. Large values of h_t/h (say $h_t/h > 0.9$) means that the toe is positioned relatively deep and hence may not be influenced by the wave action (Rock Manual, 2007). The toe structure damage is conventionally quantified by N_{od}, the number of displaced/washed away armour stones within a strip of width D_{n50} across the structure. N_{od} = 0.5 means start of damage which is generally considered as acceptable, N_{od} = 2 means acceptable damage with some flattening out of the toe, and N_{od} = 4 corresponds to failure and complete flattening out of the toe (Gerding, 1993). Fig. 2 shows the levels and damage evolution of toe. As seen, the response of toe to the incident waves is similar to that of reshaping berm breakwaters when the berm is eroded and the toe profile reshapes to a more stable profile (see also Ebbens, 2009). It is noteworthy that these levels are valid for standard toes which are 3 to 5 stones wide and 2 to 3 stones thick (Rock Manual, 2007). A thicker toe may not reduce the damage, but for a wider toe structure, a higher damage level may be acceptable (see also CEM 2011).

In addition, the definition of damage and its quantification is a subject of debate, as different researchers have used different counting approaches. As discussed by Van der Meer (1992), moving stones (N_{om}) can be rocking (N_{or}) or be completely displaced (N_{od}) and hence $N_{om} = N_{or} + N_{od}$. As mentioned above, in the earlier studies only stones that are displaced out of the toe (washed away) were considered, while in more recent ones (e.g. Ebbens, 2009, Van Gent and Van der Werf, 2014, Herrera and Medina, 2015), all the stones that have been displaced (within the toe or out of the toe) are reported and used for formula development.

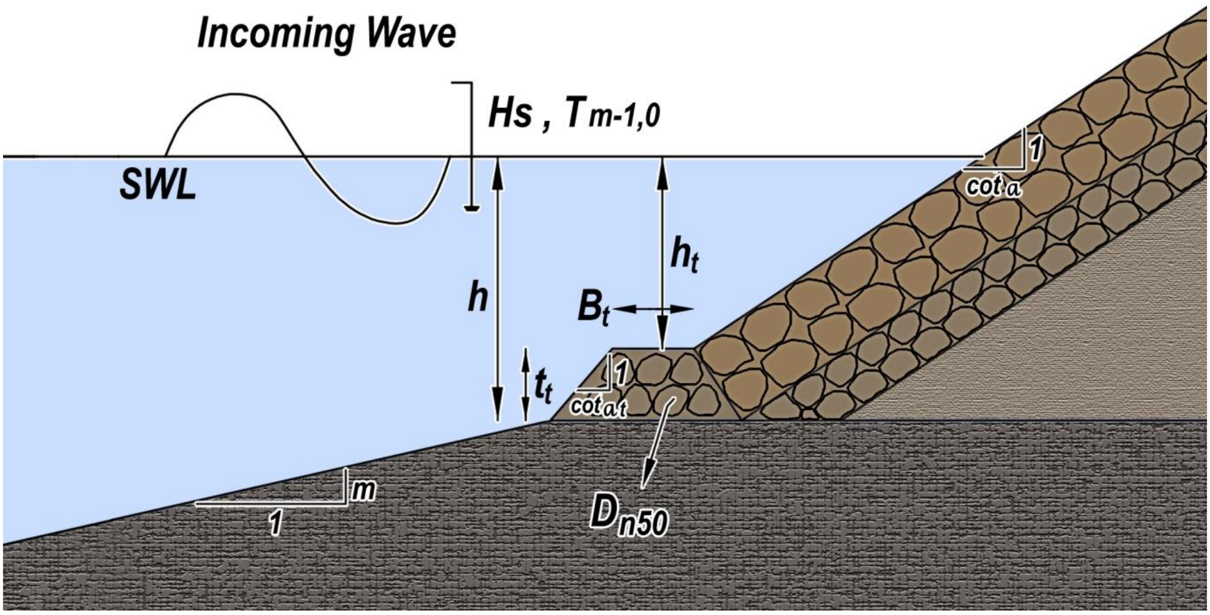


Fig. 1 Typical cross section of a conventional breakwater with toe modeled in the laboratory

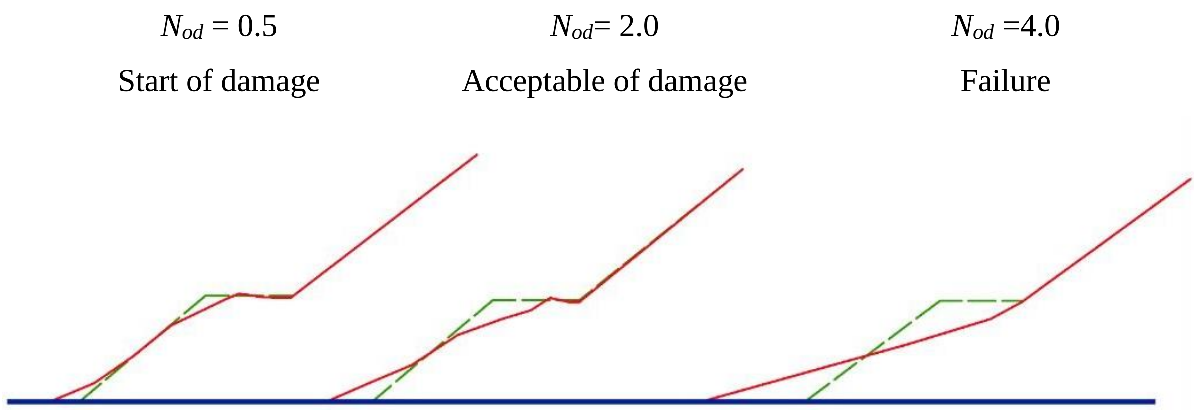


Fig. 2. Different level and progress of toe structure damage (after Docters van Leeuwen, 1996).

Similar to the stability formulas for armor layer, the design formulas for toe stability are based on the stability number, $N_s = H_s/(\Delta\ D_{n50})$. The stability number is simply the ratio between driving and resisting forces where Hs is the (measured) significant wave height at the toe of structure using several wave probes (in this paper, $H_s= H_{m0}$), $\Delta= \rho_a /\rho_w$ -1 is relative buoyant density, ρ_a is the rock armor density, ρ_w is water density and D_{n50} is the median toe rock size. Gerding (1993) derived a formula (see Eq.1 in Table 2) based on the number of washed away stones. In this formula, which is suggested in the Rock Manual (2007) and CEM (2011), the stability number is a linear function of toe depth (normalized by the rock size) and a power function of the damage level. It also implies that when N_s = 1.6, the damage values are very low. The effects of other important parameters such as water depth and wave period /steepness were not captured by Gerding (1993). This is probably due to limited ranges of variables as tests were mostly conducted in shallow water and long waves (Table 3). Burcharth et al. (1995), as cited in CEM (2001), suggested that the stability number depends on $h_t/\Delta D_{n50}$ (Eq. 2). However, use of this parameter was debated by Muttray et al. (2014) as they noticed a strong correlation between h_t and H_s when $0.5 < N_{od} < 1.5$. Van der Meer (1998) noticed that the proposed formula by Gerding (1993) yields unrealistic rock sizes for deep (low) toes and recommended a different formula. The formula has a different constant (Eq. 3) and includes water depth parameter (as h_t/h) rather than the rock size. Compared to previous formulas, the power of h_t increased from 1 to 2.7 while the power of N_{od} remained the same.

The effects of wave steepness (period) and foreshore slope were first noticed by Sayao (2007) by reanalyzing others' data. It was shown that the stability number is inversely related to the foreshore slope and directly to the wave steepness, which was confirmed in later studies (e.g.

Etemad-Shahidi et al. 2020). Surprisingly, the damage level was not included in the formula by Sayao (2007) and hence did not receive much attention. Effects of the wave steepness and foreshore slope were investigated in more detail by Ebbens (2009) through conducting tests with different foreshore slopes, water and wave periods in different water depths. Ebbens (2009) found that the stability is inversely related to the (square root of) foreshore slope and directly to the wave steepness (raised to the power of ¼). It should be mentioned that Ebbens (2009), defined a new damage parameter called $N_{\%}$ which is roughly equal to 0.1 N_{od} (Herrera and Medina, 2005). Based on limited data, Ebbens (2009) also stated that the results obtained from cumulative and single tests are close.

Muttray (2013) attempted a more theoretical approach based on the force balance and critical velocity concept. Applying his approach to data collected from regular wave tests of Markle (1989) and irregular tests of Gerding (1993) and Ebbens (2009), resulted in a new formula (Eq. 4). In this formula, the stability number was more dependent on the damage level (compared to the previous formulas) and was inversely correlated to the water depth (normalized by the wave height) above the toe. In addition, the stability number threshold for the start of damage was specified as 1.4. The formula was limited to cases with $h_t/H_s < 3$ and was conservative to be used in practice.

The relationship of $N_s \sim N_{od}^{1/3}$ was later confirmed by experiments of Van Gent and Van der Werf (2014). They investigated the effect of toe height specifically and used linear wave theory to derive a characteristic velocity (over the toe) to be used in the stability formula for the range of $0.7 < h_t/h < 0.9$ (Table 3). They noticed that by increasing the toe thickness, toe width and wave steepness, the stability decreases. These findings are justifiable as by increasing the toe thickness, h_t decreases and toe rocks become more exposed to wave action. Similarly, in a wider toe, larger number of rocks are exposed to the wave loading and hence more rocks will be moved. Regarding the effect of wave steepness, it can be argued that steeper waves have a higher orbital velocity and hence more power to destabilize the rocks of the toe. They could reduce the scatter in the predicted values by including the abovementioned parameters (Eq. 5). However, they did not resolve the effect of foreshore slope as it was fixed in their experiments. They also suggested that wider toes can have a higher acceptable damage level by a factor of $(B_t / 3D_{n50})^{1/2}$ for toes up to 12 D_{n50} wide (see also Gerding 1993).

Table 2. Summary of the most common formulas for toe stability

Eq.	Reference	Formula	notes
(1)	Gerding (1993)	$N_s = N_{od}^{0.15} (0.24\, h_t/ D_{n50} + 1.6)$	Based on $H_{1/3}$ $0.4 < h_t/h < 0.9$ $0.28 < H_s/h < 0.8$ $3 < h_t/D_{n50} < 25$
(2)	Burcharth et al. (1995)	$N_s = N_{od}^{0.15} (0.4\, h_t/\Delta D_{n50} + 1.6)$	Based on $H_{1/3}$ $0.4 < h_t/h < 0.9$ $0.28 < H_s/h < 0.8$ $3 < h_t/D_{n50} < 25$
(3)	Van der Meer (1998)	$N_s = N_{od}^{0.15} [6.2(h_t/h)^{2.7} + 2]$	Based on $H_{1/3}$ $0.4 < h_t/h < 0.9$ $3 < h_t/D_{n50} < 25$
(4)	Muttray (2013)	$N_s = 6\, N_{od}^{1/3} / (3.5 - h_t/H_s)$	Based on $H_{1/3}$ $-0.1 < h_t/h < 0.84$ $0.17 < H_s/h < 1.4$
(5)	Van Gent and Van der Werf (2014)	$N_s = 3.15 N_{od}^{1/3} (t_t / H_s)^{-1/3} (B_t / H_s)^{-1/10} (\hat{u}_\delta / \sqrt{gH_s})^{-1/3}$ $\hat{u}_\delta = (\pi H_s / T_{m-1,0}) / \sinh kh_t$	$0.7 < h_t/h < 0.9$ $7 < h_t/D_{n50} < 25$ $0.22 < H_s/h < 0.81$ m=1:30
(6)	Herrera and Medina (2015)	$N_s = S_{op}^{1/2} [5.5 + N_{od} \{(1.4 - 0.2\, h/D_{n50}).$ $\exp(0.25\, h/D_{n50} - 0.65)\}^{0.15}]$	Based on the deep water wave characteristics $-0.5 < h/D_{n50} < 5.01$

It should be mentioned that the original formula of Van Gent and Van der Werf (2014) was given for the damage level (without any threshold) and it has been rearranged to be comparable with previous ones. Herrera and Medina (2015) conducted extensive tests in very shallow waters and steep foreshore (Table 1) with emerged and submerged toes. They reported that the most critical condition occurs when the water depth above the toe is equal to D_{n50} and developed a formula to estimate the damage level. Their method was different to other as their formula (Eq. 6) was based on the deep water wave height and the damage was independent of toe depth and the numbers of waves were limited to 500. They also noticed the effect of wave period and provided a conservative formula for estimation of damage level. Their formula is rearranged in Table 2 to be used for estimation of stability no (Eq. 6). As seen, the stability number has a threshold, and linearly related to the damage level. In addition, it implies that N_s depends on the square root of wave steepness, showing the importance of the wave period. The formula was later modified by Herrera et al. (2016) to include the effect of toe width with steep foreshore slope (m= 1:10) in shallow water. The concept of using a wider toe was also studied by Herrera et al. (2016) where they suggested the use of sacrifice toe and stated that the increase in the acceptable damage level depends on the foreshore slope and water depth as well as the toe width. They found that the size of required rock can be reduced in wide toe by a factor of $(B_t / 3D_{n50})^{-0.4}$.

The used data sets and modeling

The cumulative data sets of Ebbens (2009) and Van Gent and Van der Werf (2014) were used for developing the formula. Although based on a different damage definition, the data by Gerding (1993) were also used for comparison. Following the Rock manual (2007) and CEM (2011), tests with damage levels between 0.5 and 4 and h_t/h between 0.4 and 0.9; which are more important in practice were selected for further processing. Table 3 displays the range of parameters and number of records of used data sets. As seen, the range of the used data base is wide. For example, $1.2 < h/H_s < 4.5$ (very shallow to deep water) , $0.4< h_t/h < 0.9$, (shallow toe to deep toe) and $0.01 < s_{om-1,0} < 0.06$ (seas and swell conditions). H_{m0} and $T_{m-1,0}$ values were not reported by Gerding (1993), and hence they were estimated (see details below).

Formula development

Noting the difference between the experiments, the formula was developed based on the datasets where N_{od} based on all displaced stones was reported from cumulative tests, i.e. Ebbens (2009) and Van Gent and Van der Werf (2014), and then it was compared to data by Gerding (1993) where N_{od} was based on washed away stones. In this way, some insight regarding the difference between these two types of damage level recording can be provided. As discussed before, there are two types of formulas developed for toe rock. In the first type which is more common and useful for design purposes, the required parameter is the rock size (nondimensionalized as stability number). In the second type of formulas, which is more suitable for assessment of existing toe structures, damage level is the required parameter. Hence, first the formula is developed for the stability number and then rearranged for estimation of damage level. The ratio between driving and resisting forces depicted as N_s depends on both environmental conditions and toe rock characteristics.

Table 3. Parameters' ranges and number of records used for formula development. Estimated values are denoted by *.

parameter	Van Gent and Van der Werf (2014)	Ebbens (2009)	Total
H_s *(m)*	0.071-0.264	0.058-0.122	0.058-0.264
T_p (s)	1.09-3.36	1.14-3.05	1.09-3.36
$T_{m-1,0}$ (s)	1.15-2.89	0.95-2.42	0.95-3.25
N_w	1000	1000	1000
h (m)	0.200-0.400	0.133-0.253	0.133-0.400
h_t *(m)*	0.142-0.353	0.053-0.173	0.053-0.353
D_{n50} *(m)*	0.015-0.023	0.018-0.027	0.015-0.027
B_t *(m)*	0.044-0.210	0.10	0.044-0.210
t_t *(m)*	0.029-0.058	0.08	0.029-0.08
cot α	2	1.5	1.5-2
cot α_t	1	1.5	1-1.5
m	0.033	0.02-0.1	0.02-0.1
Δ	1.7	1.65-1.75	1.65-1.75
$s_{m-1,0}$	0.012-0.042	0.009-0.061	0.009-0.061
h/H_s	1.23-3.0	1.22-2.48	1.22-3.0
h_t/H_s	0.87-2.58	0.48-1.69	0.48-2.58
B_t/H_s	0.17-1.92	0.81-1.72	0.17-1.92
h_t/h	0.71-0.88	0.39-0.68	0.39-0.88
h_t/D_{n50}	6.5-23.40	1.97-8.04	1.97-23.40
N_{od}	0.51-3.79	0.5-3.01	0.5-3.79
N_s	2.75-10.0	1.58-3.93	1.58-10.0
Number of tests	93	47	140

The wave loading conditions are characterized by the water depth, foreshore slope, wave height and wave period (Fig. 1). The wave height is commonly characterized by spectral significant wave height, H_s, and recent studies have shown that $T_{m-1,0}$ should be used as the governing period parameter. The characteristics of the toe are the rock density, toe width, toe height and slope. Hence, for a head on wave $D_{n50}= f\,(H_s, T_{m-1,0}, \cot\alpha_t, \cot\alpha, h, h_t, B_t, \rho_a, \rho_w, m)$. In terms of dimensionless variables, several forms of normalizing the right-hand side variables (e.g. by using H_s, D_{n50}) were investigated and the following form was selected:

$$N_s = f\,(h_t/h,\ N_{od},\ s_{m-1,0},\ m,\ \cot\alpha,\ \cot\alpha_t,\ m,\ B_t/H_s) \quad (7)$$

where $s_{m-1,0} = 2\,\pi\,H_s/\,g\,T_{m-1,0}^2$ is the wave steepness based on incident waves at the toe. The advantage of this form is that the parameter to be determined in the design, i.e. D_{n50}, only appears on the left side of the equation. It should be mentioned that parameters such as the breakwater slope was considered in the formula development. The results, however, showed that the toe stability is nearly independent of it. This is mainly due to the limited range of breakwater slopes ($\cot\alpha = 1.5$ to 2). Finally, different functional forms were investigated and considering three factors i.e. physical justification, accuracy and simplicity the following one was selected:

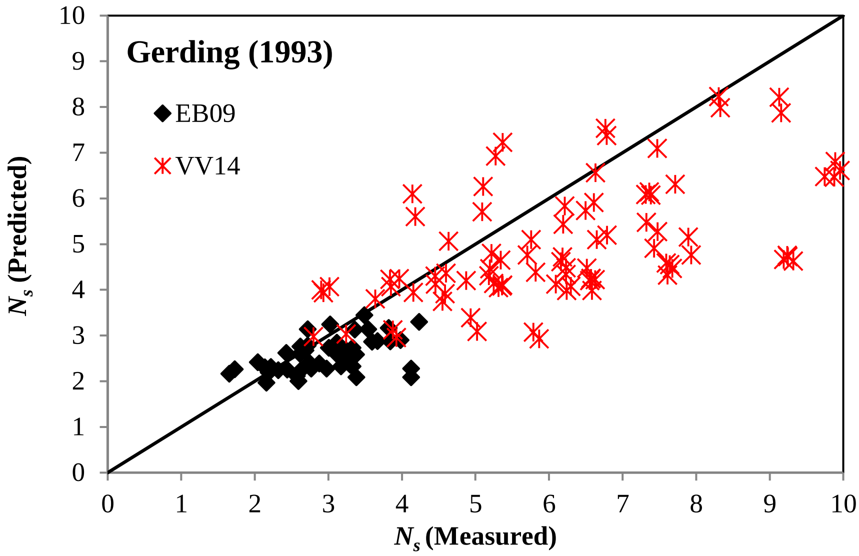
$$N_s = 1.2 + 11.2\,(h_t/h)^{7/4} s_{om}^{1/6}\, N_{od}^{2/5} (B_t/H_s)^{-1/10} (1-3.7\,m) \quad (8)$$

The coefficient of variation (*C.V.*) of Eq. (8) is 13.5% and its range of validity (in terms of dimensionless numbers) is given in Table 3. As seen, the formula implies that for the initiation of damage, a minimum wave height /stability number is required (see also Baart, 2008). In addition, it shows that by increasing the toe depth, wave steepness, damage level and having a narrower toe, smaller stones can be selected. In addition, a toe structure located at the end of a steep foreshore is less stable and hence a larger rock size is required. Interestingly, the foreshore reduction factor is similar to the one suggested for armor stability in shallow waters (Etemad-Shahidi et al. 2020) although the influence of the foreshore is somewhat stronger than for armor layer. In addition, the power of wave steepness is similar to the one suggested for armor stability in surging wave condition which is the case for all used tests. The relationship between N_s and the relative toe width is the same as the range as suggested by Van Gent and Van der Werf (2014) and lower than suggested Herrera et al. (2016). Eq. 8 also implies that $N_s \sim T_{m-1,0}^{-1/3}$, showing that longer waves result in a smaller stability number and hence require a larger rock size, a result in line with the previous findings. The relatively low value of the power (1/3) could be the reason for ignoring the wave period effect in some of the earlier studies (e.g. Gerding 1993) noting that his tests were mostly conducted with a limited and narrow range of wave steepness (based on the peak wave period). The power of N_{od} in the developed formula is also within the range of reported values in the literature, where powers from 0.15 to 1.0 are suggested (Table 2). The developed formula also shows the importance of relative water depth above the toe as $N_s \sim (h_t/h)^{7/4}$. This is also in line with previous findings as a shallow toe is more exposed to the wave action and higher orbital velocities (see also Nammuni-Krohn 2009).

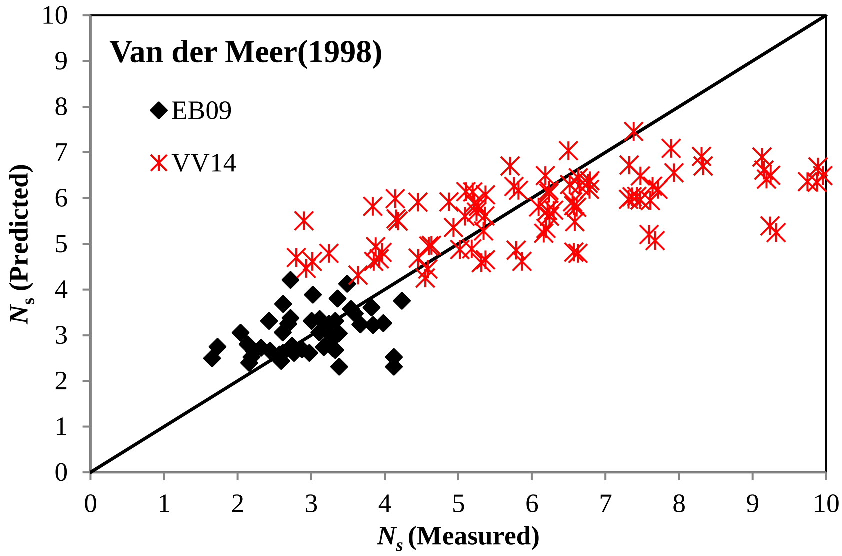
Evaluation of formulas

The qualitative evaluation of different formulas, including the developed one is shown in Fig. 3 where the predicted stability numbers are plotted against measured ones. As seen, Gerding (1993) formula (Fig. 3a) underestimated the measurements specially the high N_s number tests by Van Gent and Van der Werf (2014). On average, the performance of the formula by Van der Meer (1998) (Fig. 3b) is more or less the same as the one by Gerding (1993) but with less bias. The behaviour of the formula by Burcharth et al. (1995) is more similar to that of Gerding (1993). Comparing the existing formulas, the performance of Muttray (2013) formula (Fig. 3d) is the lowest, and it generally underestimates the measured values except for the tests with stability numbers less than 2.5. This is somehow unexpected as this formula is developed for design purposes and should be conservative. The formula by Van Gent and Van der Werf (2014) (Fig. 3e) mimics the trend of measured data with less scatter and bias, especially those of their own. This not surprising as this formula has been calibrated with that data, and Ebbens (2009) measurements were similar to their experiments, except with lower stability numbers. Finally, the last panel (Fig. 3f) shows that the developed formula is well calibrated both for low and high stability numbers. It should be noted that Ebbens (2009) and Van Gent and Van der Werf (2014) tests were conducted with breakwater slopes of 1.5 and 2, respectively; Hence, Fig. 3f shows that the developed formula has a good performance for both breakwater slopes, and the toe stability is nearly independent of cot α in this study.

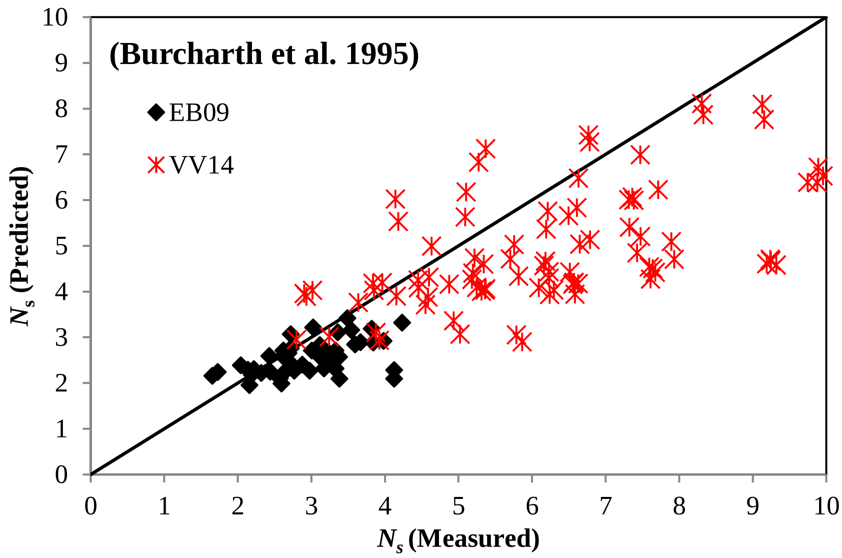
(a)


Gerding (1993)
EB09
VV14
N_s (Predicted)
N_s (Measured)

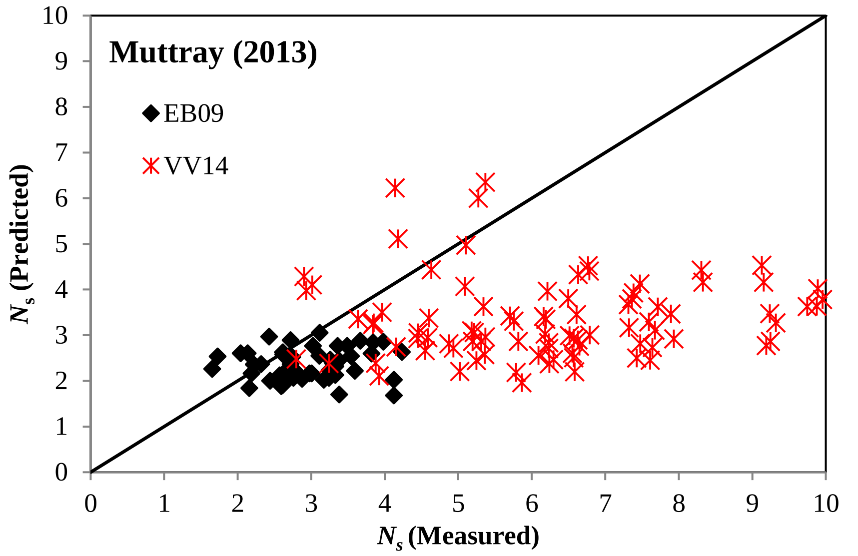
(b)


Van der Meer(1998)
EB09
VV14
N_s (Predicted)
N_s (Measured)

(c)


(Burcharth et al. 1995)
EB09
VV14
N_s (Predicted)
N_s (Measured)

(d)


Muttray (2013)
EB09
VV14
N_s (Predicted)
N_s (Measured)
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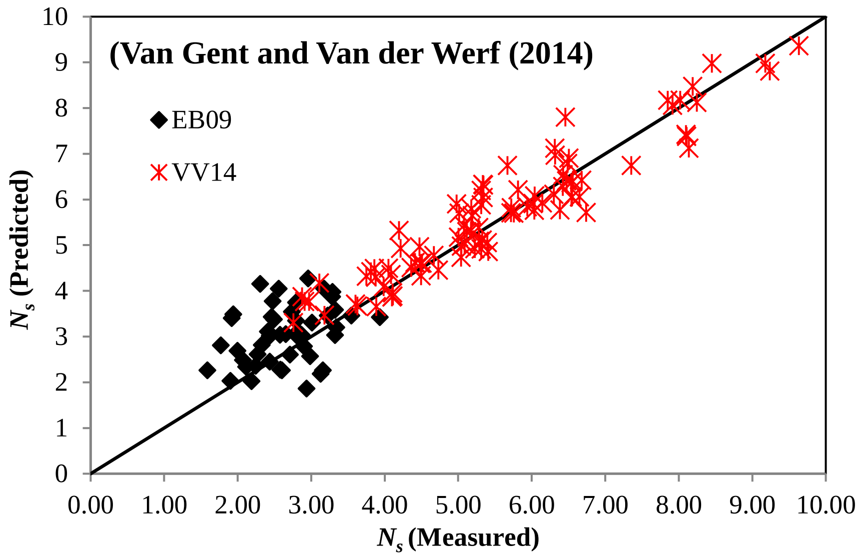
1
2
3
4
5
6
7
8
9
10

(e)


(Van Gent and Van der Werf (2014)
EB09
VV14
N_s (Predicted)
N_s (Measured)
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00

(f)

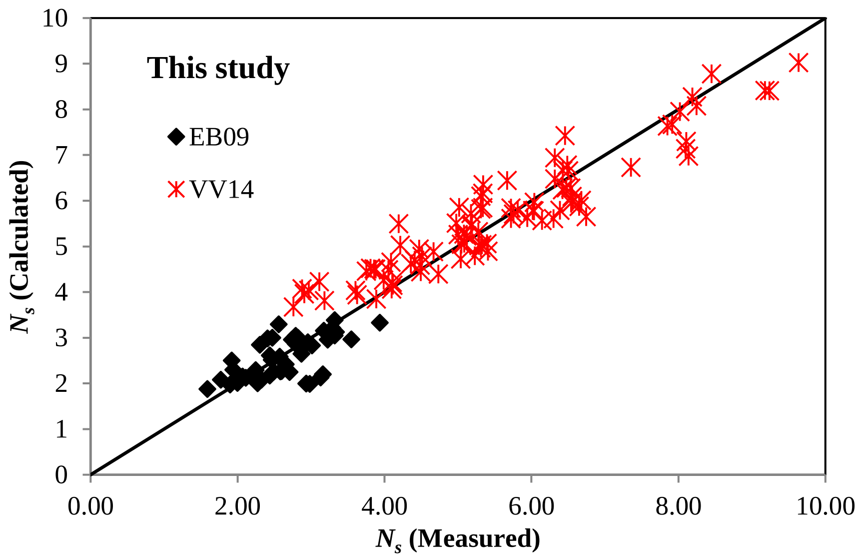

This study
EB09
VV14
N_s (Calculated)
N_s (Measured)
0.00
2.00
4.00
6.00
8.00
10.00

Fig. 3 Comparison between the measured and predicted stability numbers using different formulas, tests of Ebbens (2009) and Van Gent and Van der Werf (2014) with $0 \leq N_{od} \leq 4$ and $0.4 \leq h_t/h \leq 0.9$ (a) Van der Meer et al. (1995), (b) Van der Meer (1998), (c) Burcharth et al. (1995); (d) Van Gent and Van der Werf (2014), (e) Muttray (2013) and (f) this study.

The performances of the various formulas were also evaluated quantitatively using accuracy metrics such as the correlation coefficient (*CC*), normalized bias (*NBias*) and the scatter index (*SI*) defined below:

$$NBias = \frac{1}{n}\frac{\sum_{1}^{n}(p_i - m_i)}{\overline{m}_i} \times 100 \tag{9}$$

$$SI = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(p_i - m_i)^2}}{\overline{m}_i} \times 100 \tag{10}$$

where p_i and m_i denote the predicted and measured values, respectively. The number of measurements is n and the bar denotes the mean value.

The accuracy metrics of different formulas is given in Table 4. The metrics shows that the formulas calibrated based on data of Gerding (1993), i.e. Gerding (1993) and Van der Meer (1998), generally underestimate the measurements and the best one of the these two is the one by Van der Meer (1998) which has a lower bias than Gerding (1993). The observed variance was expected to some extend due to the difference in damage definition ("washed away" or "displaced") and test (single vs. cumulative) methods. Of the existing stability formulas, the one by Van Gent and Van der Werf (2014) shows the lowest bias, the lowest scatter and the highest correlation. The new formula shows an improvement for each of the three accuracy metrices.

Table 4. Accuracy metrics of different formulas, tests of Ebbens (2009) and Van Gent and Van der Werf (2014) with $0.5 < N_{od} \leq 4$ and $0.4 \leq h_t/h \leq 0.9$

Formula	*NBias (%)*	*SI* (%)	*CC*
Gerding (1993)	-19	31	0.82
Van der Meer (1998)	-8	27	0.83
Burcharth et al. (1995)	-20	32	0.82
Muttray (2013)	-42	58	0.46
Van Gent and Van der Werf (2014)	5	13	0.95
This study	0	11	0.96

The toe stability formulas were then compared to the data of Gerding (1993) (with $N_{od} > 0$), although the data by Gerding (1993) was based on a somewhat different definition of the damage. It should be mentioned that only $H_{1/3}$ and T_p were reported in Gerding (1993). Hence, methods of Hofland et al. (2017) and Muttray and Martinez (2017) were used to convert $H_{1/3}$ to H_{m0} and T_p to $T_{m-1,0}$, respectively before applying Eq. 8.

The accuracy metrics of all stability formulas (for data of Gerding, 1993) are shown in Table 5. As seen, the performance of the developed formula is superior or comparable to all others, except for the one by Burcharth et al. (1995) which especially shows a lower amount of scatter. The obtained *NBias* value implies that N_{od} (displaced, cumulative tests) is nearly equal to $1.26N_{od}$ (washed away, single tests), which is reasonable noting the mentioned differences between the experimental and counting methods. This comparison shows that even for the data of Gerding (1993), the performance of the new expression is reasonably good and comparable to the previous stability formulas that were mostly calibrated based on the data of Gerding (1993). Nevertheless, the scatter (*SI*=20%) is about a factor of two larger than for the data on which the new formula is calibrated (*SI*=11%).

Table 5. Accuracy metrics of different formulas, all tests of Gerding (1993) with non-zero damage level.

Formula	*NBias (%)*	*SI* (%)	*CC*
Gerding (1993)	5	18	0.92
Van der Meer (1998)	10	22	0.81
Burcharth et al. (1995)	10	13	0.92
Muttray (2013)	-12	24	0.68
Van Gent and Van der Werf (2014)	12	34	0.78
This study	6	20	0.82

Fig.4 displays the relevant scatter diagram. In general, there is a good agreement between predictions and observations, except that the model slightly overestimates the measurements. It was not possible to resolve the effects of number of waves, N_w, as it was mostly fixed (1000) in the available data sets and the used tests were cumulative. The only exception is the study of Docters van Leeuwen (1996) who used 2000 waves and studied the effects of rock density. Surprisingly, the observed damage levels in this study were comparatively lower than those of Gerding (1993). Baart (2008) argued that the damage was not counted properly in Docters van Leeuwen's experiments (see also Muttray et al. 2014). It can be speculated that N_w is not important as the toe behaves more or less like that of a berm breakwater where the reshaping improves the stability (Ebbens, 2009). Based on the data of Van der Meer (1988), Lykke Anderson and Burcharth (2010) suggested that the berm recession depends on $N_w^{(0.3-0.046\,N_s)}$.

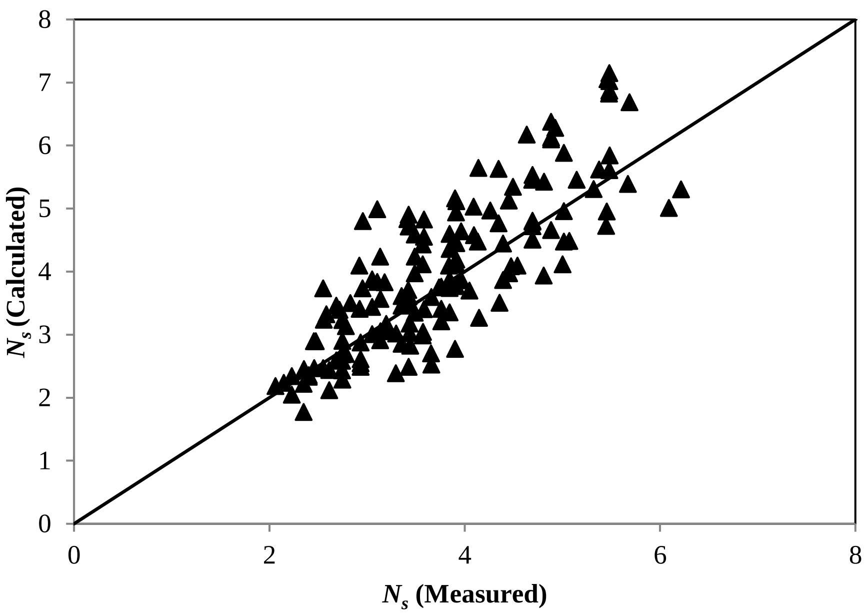


Fig. 4. Comparison between measured and predicted stability numbers, Gerding (1993) data.

The average N_s of the used datasets in this study is about 4. This means that according to Lykke Anderson and Burcharth (2010), by doubling the number of waves, the recession/damage will increase by about 8%, which is marginal. Investigation of the wave obliquity needs 3D experiments and disregarding its reduction effect is likely to be acceptable as oblique waves enhance longshore current (and hence erosion), on the other hand.

For the assessment purposes, the developed stability formula can be rearranged for the damage level prediction and for investigating the role of different dimensionless parameters as:

$$N_{od} = 2.4\times10^{-3}\,[(N_s-1.2)/(1-3.7\,m)]^{5/2}\,(B_t/H_s)^{1/4}\,s_{m-1,0}^{\ -5/12}\,(h_t/h)^{-35/8} \qquad (11)$$

which show that N_{od} is the most sensitive to h_t/h and least sensitive to the relative toe width. The range of validity of this formula is given in Table 3 and as it has been obtained by rearranging the developed stability formula, reader may refer to the discussion given after Eq. (8) for its justification. In terms of design parameters, Eq. 11 becomes:

$$N_{od} \sim H_s^{\ 11/6}\,(\Delta D_{n50})^{-5/2}\,B_t^{\ 1/4}\,(1-3.7m)^{-5/2}\,T_{m-1,0}^{\ \ 5/6}\,h^{\ 35/8}\,h_t^{\ -35/8} \qquad (12)$$

The coefficient of variation (C.V.) of Eq. (12) is 74% (which is less than most of others) and the formula's range of validity is given in Table 3. This formula can be used for adaptation of existing coastal structures. For example, if due to climate change, the wave height increases by 10% (and assuming all other parameters constant), then the damage level will increase by 19%. For the design of the main armour layer, usually S_d or damage level is used in the stability formulas. Assuming a negligible rock settlement, the relationship between N_{od} and S_d can be written as $N_{od} = G\,(1-p)\,S_d$ (CEM, 2011). For rocks, p and G are in the range of 0.4-0.6 and 1.2-1.6, respectively. Assuming common values of p=0.4 and G= 1.2 values for toe rock, the relationship becomes N_{od}= 0.72 S_d or S_d =1.4 N_{od}. In that case, the toe stability formula can be written as:

$$N_s=1.2+9.82\,S_d^{2/5}\,(h_t/h)^{7/4}s_{m-1,0}^{1/6}\,(B_t/H_s)^{-1/10}(1-3.7\,m) \qquad (13)$$

and the acceptable damage levels become $S_d \leq 1$ (no damage), $S_d \leq 3$ (acceptable damage) and $S_d = 6$ (failure). Interestingly, these levels are close to those specified for armour stability with a slope of 1:1.5 (e.g. CEM , 2011).

Summary and conclusions

The aim of this study was to provide a design guideline to be used for the toe stability analysis. Hence, a brief overview of exiting knowledge, data sets and formulas was provided in order to provide some insight regarding the different approaches and methods used to investigate damage levels. Then, the existing data sets were collected, and filtered to be used for further processing. The selection of input variables and functional form has a large impact on the accuracy. Therefore, based on the existing knowledge about the role of geometrical and environmental parameters, different dimensionless numbers and functional forms were examined to derive an optimum formula.

A robust formula (Eq. 8) with a wide range of applicability (Table 3) and a low scatter (*C.V.*=13.5%) was developed for the toe stability number within the design range. The formula included the effects of governing parameters such as the damage level, wave steepness, toe depth, toe width and the foreshore slope in a compact and comprehendible way. It was argued that the effect of other parameters such as the storm duration can be disregarded due to their relatively minor role. The skills of the developed and previous formulas were compared, both qualifiedly and quantically, using different experimental data sets. The scatter plots and accuracy metrics, such as *NBias* and *SI* showed the good performance of the formula. The formula was also tested for a wider range of damage levels (see Appendix A3).

In summary, experimental results demonstrate the capability of the proposed formula to mimic the toe damage process and to predict its stability number. In order to be used for assessment purposes, an expression (Eq. 11) was provided to predict the damage level of existing structures. In addition, the toe stability formula was modified to be consistent with that of armor stability, and it was stated that their damage criteria levels are very similar. Finally, the coefficient of variation of Eq. 8 (13.5%) was provided along with some design hints and example (Appendices A1 and A2) to be used by end-users, i.e. coastal engineers and lecturers.

Acknowledgments

The authors thank Lisham Bonakdar and Mohammed Atia for their help in collecting the data.

Appendix A1. Practical notes

The design water level for the toe armor is usually at low tide as more force will be applied to the toe in this case. However, in shallow waters, selection of a lower design water level may result in lower design wave height (due to wav breaking). Hence, practitioners may need to consider different conditions when designing a toe and choose the most critical one.

The output of many numerical models is H_{m0}, the spectral wave height; that could be less than $H_{1/3}$, especially for swell conditions. If required, the method of Muttray and Martinez (2017) can be used to convert $H_{1/3,\mathrm{toe}}$ to H_{so} in depth limited waters. If wave transforming is not conducted numerically, then (for straight linear foreshore slopes and perpendicular wave attack) the approximate method of Hofland et al. (2017) can be used to estimate the spectral mean wave period at the toe ($T_{m-1,0}$):

$$\frac{T_{m-1,0}}{T_{m-1,0,o}} - 1 = 6\exp(-6\widehat{h}) + 0.25\exp(-0.75\widehat{h}) \qquad \text{(A.1)}$$

$$\widehat{h} = \frac{h}{H_{s,o}}\left(\frac{1}{100m}\right)^{0.2} \qquad \text{(A.2)}$$

where $T_{m-1,0,o} = T_{p,o}/1.1$. To transfer the wave height to shallow water manually, the method by Goda (2000) as described in Rock Manual (2007), can be used as the first estimate. Alternatively, the following rule of thumb, obtained from single variable regression analysis of Van Gent et al. 2003 data, can be used:

$$H_s/H_{so} \approx 0.4 + 0.2\,(h/H_{so}) \qquad \text{for } h/H_{so} < 3 \qquad \text{(A.3)}$$

Appendix A2. Worked example and design curve

The environmental conditions of Noshahr port in the Caspian Sea is used in this example. The design conditions are: H_s = 3.7 m, T_m = 10 s, $T_{m-1,0}$ =11.8 s, h= 7 m, m=1:50, ρ_s= 2700 kg/m^3, ρ_w= 1010 kg/m^3 , bedding layer thickness= 0.5 m and N_{od} = 0.5.

N_s=1.2+ [11.2 $N_{od}^{2/5}$ $(h_t/h)^{7/4}$ $s_{om-1,0}^{1/6}$ $(B_t/H_s)^{-1/10}$] (1-3.7m)

Δ= (2700-1010)/1010 = 1.67

$s_{om-1,0}$ = 3.7 / (1.56 ×11.8^2) = 0.017

First guess: D_{n50}= H_s /8 = 3.7/8= 0.46 m (two layers, three rocks wide):

h_t/h =(7-0.5-2×0.46)/7= 0.80, and B_t/H_s=3×0.46 /3.7= 0.37

N_s=1.2+(11.2×0.5$^{2/5}$×0.8$^{7/4}$×0.017$^{1/6}$× 0.38$^{-0.1}$)(1-3.7× 0.02)=4.16

D_{n50}= H_s /(Δ N_s) = 3.7 / (1.67×4.16)= 0.53 m

B_t/H_s= 3×0.53 /3.7= 0.43 and

h_t= 7.0-0.5-2 × 0.53 => h_t/h= 0.78

Retry: N_s=1.2+(11.2×0.5$^{2/5}$×0.78$^{7/4}$×0.017$^{1/6}$× 0.43$^{-0.1}$)(1-3.7× 0.02)=3.99

D_{n50}= H_s /(Δ Ns) = 3.7 / (1.67×3.99)= 0.554 m ~ 0.53 m, use 0.566 m (500 kg)

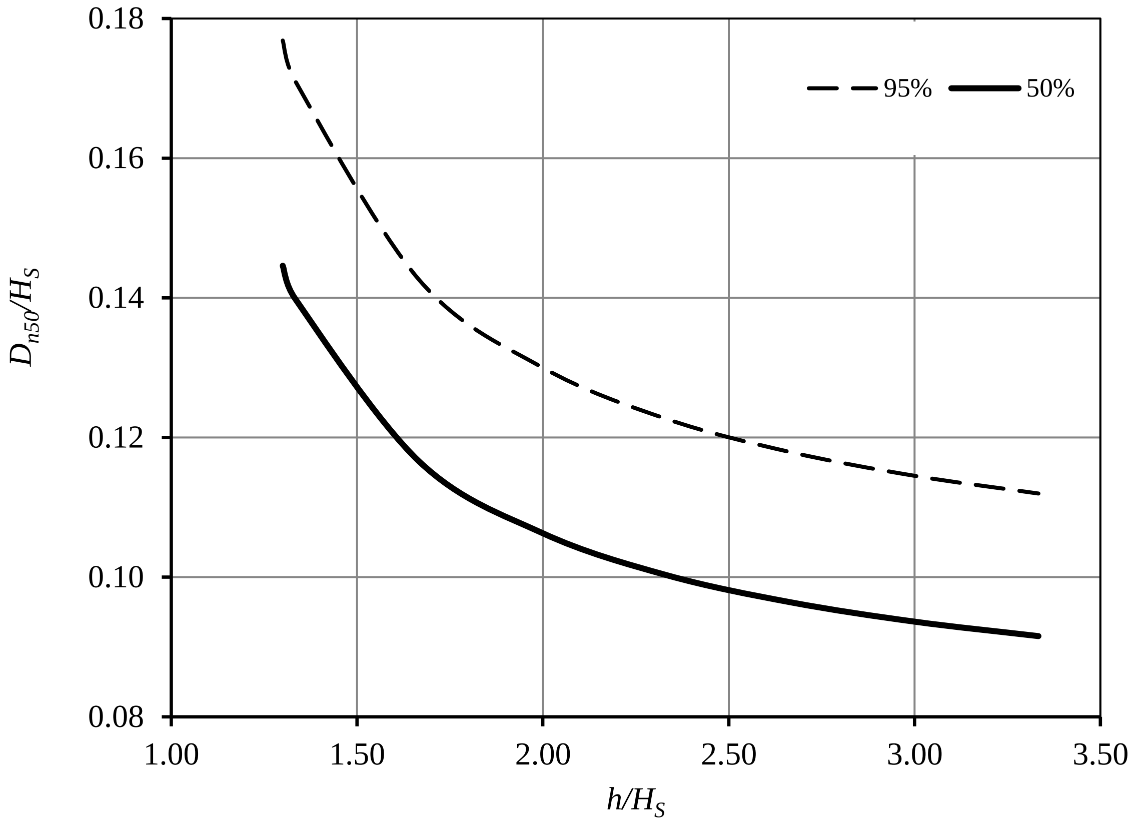
The estimated toe armor mass using different formulas are given below:

Formula	W_{50} (kg)
Gerding (1993)	1380
Burcharth et al. (1995)	1390
Van der Meer (1998)	260
Muttray (2013)	3390
Van Gent and Van der Werf (2014)	870
This study	500

A confidence interval of 5-95% will result in rock weight varying between 270 and 900 kg. In other words, to ensure having less than 5% chance of N_{od} > 0.5, M_{50} of the toe should be 900 kg or physical test be conducted. The armor and filter layers of Noshahr are 4-6 tons and 300-

1000 kg, respectively. In practice, toe size is greater or equal to that of filter. Hence, 600-1200 kg can be the selected rock grade.

The variation of relative rock size as a function of water depth for a typical condition is illustrated below.



Relative rock size as a function of relative water depth for N_{od} = 0.5, m=0.02, $s_{m-1,0}$ = 0.03, Δ=1.63, 0.5 m bedding layer, two layers thickness and three rocks wide.

Appendix A3.

Parameters' ranges and number of records with $0.4 \leq h_t/h \leq 0.9$ and a wider range of damage ($0 < N_{od} < 7.3$) . Estimated values are denoted by *.

parameter	Van Gent and Van der Werf (2014)	Ebbens (2009)	Gerding (1993)	Total
H_s (m)	0.046-0.296	0.053-0.131	0.138-0.23	0.046-0.296
T_p (s)	1.09-3.36	1.14-3.05	1.52-3.58	1.09-3.58
$T_{m-1,0}$ (s)	1.13-2.89	0.95-2.47	1.46-3.45*	0.95-3.45
N_w	1000	1000	1000	1000
h (m)	0.20-0.40	0.133-0.339	0.30-0.50	0.13-0.50
h_t (m)	0.142-0.353	0.053-0.259	0.15-0.42	0.053-0.42
D_{n50} (m)	0.015-0.023	0.018-0.027	0.017-0.04	0.015-0.04
B_t (m)	0.044-0.21	0.10	0.12-0.30	0.044-0.30
t_t (m)	2.9-5.8	8	8-22	2.9-22
cot α	2	1.5	1.5	1.5-2
cot α_t	1	1.5	1.5	1-1.5
m	0.033	0.02-0.1	0.05	0.02-0.1
Δ	1.7	1.65-1.75	1.68-2.18	1.65-1.75
$s_{om-1,0}$	0.012-0.042	0.008-0.061	0.01-0.044	0.008-0.061
h/H_s	1.23-4.5	1.22-3.98	1.30-3.61	1.22-4.50
h_t/H_s	0.87-3.63	0.48-3.04	0.71-2.8	0.48-3.63
B_t/H_s	0.17-2.91	0.77-1.88	0.52-1.95	0.17-2.91
h_t/h	0.71-0.88	0.39-0.76	0.45-0.84	0.39-0.9
h_t/D_{n50}	6.5-23.40	1.97-13.7	3.75-24.70	12.0-24.7
N_{od}	0.023-7.32	0.06-3.01	0.09-9.21	0.02-7.3
N_s	1.21-10.02	1.17-4.2	2.10-6.21	1.2-10.0
Number of tests	150	105	152	407

Accuracy metrics of different formulas, all tests with $0.4 \leq h_t/h \leq 0.9$

Formula	*NBias (%)*	*SI* (%)	*CC*
Gerding (1993)	-17	35	0.87
Van der Meer (1998)	-3	32	0.84
Burcharth et al. (1995)	-18	36	0.87
Muttray (2013)	-42	76	0.36
Van Gent and Van der Werf (2014)	-2	18	0.94
This study	-2	15	0.95

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Nomenclature

Symbol	Name	Unit
$\Delta = (\rho_s/\rho_w) - 1$	Relative buoyant density	[-]
B_t	Toe width	[m]
h	Water depth at the toe	[m]
h_t	Water depth above the toe	[m]
$H_{1/3}$	Significant wave height based on time domain analysis	
H_{m0}	Significant (spectral) wave height based on frequency domain analysis	[m]
H_{so}	Significant wave height in deep water	[m]
H_s	Significant wave height at toe of the structure	[m]
m	Foreshore slope	[-]
m_i	Measured values	[-]
n	The number of observations	[-]
N_w	Number of waves	[-]
N_s	Stability number	[-]
N_{od}	Damage level (number of displaced/washed away units within a strip width of D_{n50})	[-]
P_i	Predicted values	[-]
ρ_s	Rock density	[kg/m^3]
ρ_w	Water density	[kg/m^3]
$s_{m-1,0}$	Wave steepness using $T_{m-1,0}$	[-]
SI	Scatter index	[-]
$T_{m-1,0}=m_{-1}/m_0$	Mean energy wave period based on frequency domain	[s]
T_p	Peak wave period	[s]
t_t	Toe thickness	[m]