Application of extreme value theory for the assessment of turret moored floating structures

De Bruyn, N.

Abstract— **In the design of mooring systems, it is a common practice to use a 100-year design environment to calculate extreme responses. Statistical inference is executed on the environmental data to produce a 100-year environment. This 100-year environment is then simulated to calculate the loads that occur when the vessel is under the influence of the 100-year environment. An alternative for this method is response based design. In response based design, measurements with a 3-hour interval of the environment over a long period of time are used to simulate the behaviour of the FPSO. The simulation provides a data-set of dominant loads for the mooring system design over the time period. With extreme value theory the tail of this data is fitted on a generalized Pareto distribution. With this distribution a 100-year extrapolation can be made that results in the 100-year extreme loads. It can be concluded that the response based approach can be used as a verification tool of the conventional method when the limitations of the method are known by the user.**

Index Terms— **extreme value theory · turret · floating structures · statistical inference ·response based approach · data fitting**

I. INTRODUCTION

Classification society requires that the design of a mooring system for long-term applications like FPSOs is based on not less than a 100-year recurrence interval of the environmental conditions (DNV.GL 2015). The common practice in the industry is to create a 100-year maximum for all environmental parameters. The combination of all of these 100-year environmental parameters form the design conditions. This is a misinterpretation of the reality and could lead to an overestimation of the 100-year loads on the mooring system, to deal with this more detailed assessments are performed to derive associated wind, waves and currents.

In this paper a different approach to obtain the 100-year loads is presented. This alternative is response based design. Extrapolation of measured or simulated loads on the mooring system will be executed with the use of extreme value theory. This is studied by (Oostra 2015) for one specific case and one load parameter.

The results of this study were promising but are only studied for one specific case. This paper is written after the study (Bruyn 2016) to the possibility to apply response based design for all dominant load parameters and for various design choices.

This will lead to conclusions about the possibility of using extreme value theory in a design stage to produce reliable and realistic 100-year return levels. First the research question will be formulated and the response based approach is explained. Further the robustness of the method is checked and a case study is executed to test the methodology during a realistic design process.

II. RESEARCH QUESTION

A research question is formulated after thorough analysis of the work done by (Oostra 2015), (Tromans et Vanderschuren 1995) and (Battjes 1979).

This previous work discusses the potential of the response based approach to incorporate the environment in a more elegant way. (Tromans et Vanderschuren 1995) present two alternatives to use response based methods in the calculation of extreme return levels. The first is a complex method to produce a more realistic 100-year environment by including joint statistics of all environmental variables and the other is to apply extreme value theory on load responses as studied in (Oostra 2015). In this technique statistical inference is performed on load data and not on environmental data.

This as a possible alternative and verification method for the conservative methodology that is a common practice in the industry. Considering the needs of Bluewater for a helpful tool that can be used in an efficient way during the design stage to calculate reliable return periods for load data oriented for single point mooring systems the following research question is formulated.

"Is it possible to apply extreme value theory in an efficient way during the design stage with the use of hindcasted metocean data for the assessment of mooring configurations?"

III. RESPONSE BASED DESIGN

A. Introduction

The conventional method to obtain extreme load events is to fit hindcast series to a distribution and extrapolate this to obtain a design sea-state. (Tromans et Vanderschuren 1995) For this sea-state the extreme loads can be calculated with a response model. With this method, to calculate extreme return levels, the influence of the present environmental conditions on the uncertainty of the extreme wave of a sea-state is neglected. An alternative to this method is response based design. In response based design two categories can be seen. The first is performing statistical inference on response to extrapolate a 100-year return

N. De Bruyn Delft University of Technology, Delft, Netherlands.
Offshore and Dredging engineering (e-mail: N.debruyn@student.tudelft.nl).

level. The second (Coles et Tawn 1994) is a complex method of performing joint statistics of all metocean variables to obtain a detailed design sea-state.

The conventional method will lead to an over estimation of the return level because it is assumed that extreme wind, extreme wave and extreme current occur at the same moment and come from the same direction. The first category of response based design, fitting responses to a distribution, will be studied in this paper. The steps that need to be made in a response based design process are listed below.

- Metocean data is obtained from measurements with 3 hourly intervals.
- defining high Hs threshold that results in a I.I.D (Independent and Identically Distributed) data set
- using environmental parameters above threshold in the load/response model
- fit response values to a Generalized Pareto Distribution
- set a second threshold above where the parameters become stable
- verification of the distribution (visual comparison, bootstrap, goodness-of-fit)
- extrapolation to the required return period

The conventional method results in more conservative extrapolations while the proposed method will give a better representation of the reality, since the metocean data applied represents measured combinations of wind, waves and current for the 3hour intervals.

B. *Pre-processing*

Pre-processing includes all steps needed to select the appropriate environmental data that can be used as an input for the response model.

Because it is not practical to simulate all sea-states that are measured for a hindcast (5-50 years) a selection of the hindcast data is made which is responsible for the high loads that are in the tail of the distribution. To achieve this, it is of importance to get a good insight in the environmental phenomena that result in the highest loads. (Oostra 2015) studied which sea-states result in the highest loads.

Besides sea-states with high waves, sea-states with the following characteristics are typically responsible for also high loads:

- Sea-states with considerable angles between wind and waves, resulting in an angle between equilibrium-heading and waves. (directional-cases)
- Sea-states with two wave systems, resulting in an angle between equilibrium-heading and the dominant wave-system. (swell-cases)

With this information the data-set with the environmental conditions that lead to high loads in the mooring system is produced.

The dataset is formed by first setting a wave-height threshold. This results into a number of sea-states that are simulated in the response model with 25 realizations. The data points that resulted in the highest maximum line tension in a storm cluster are selected as input values for the response model. A storm cluster is defined as a series of consecutive measurements that are above the set threshold.

Figure III-1 gives a visual representation of the selection procedure.

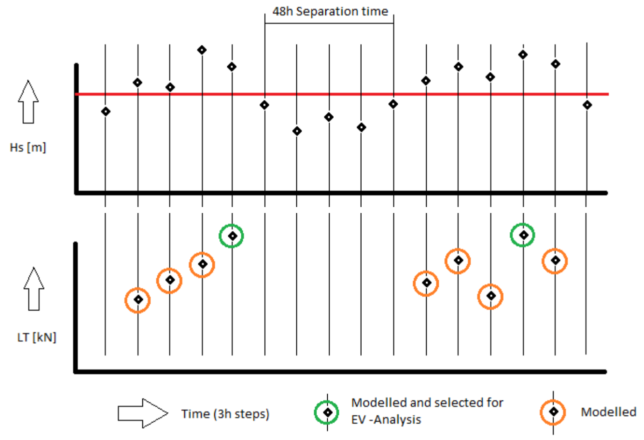


Figure III-1 Visual representation of the selection procedure (Oostra 2015)

C. *Simulation*

The simulations are executed with a time domain simulation program that analyses the dynamic behaviour of a moored FPSO in an ocean environment.

Coupled analysis is used to calculate the dynamic loads in the mooring system, the wave frequency and low frequency vessel motions. A block diagram of this coupling is given in Figure III-2.

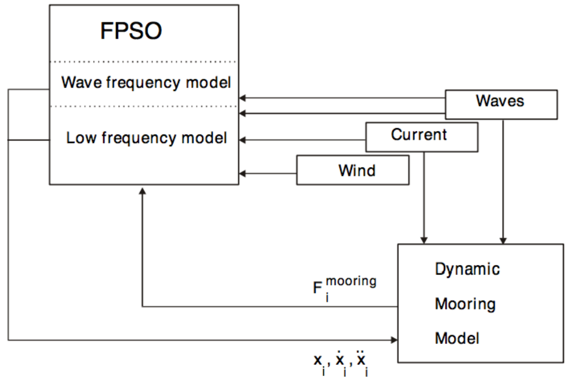


Figure III-2 Numerical model (Marin 2008)

D. *Post-processing*

In the post-processing part of the method, the output generated with the simulation will be fitted on an extreme value distribution. With this distribution, return levels for a desired return period can be calculated.

1) *Fitting realizations to a Generalized Pareto Distribution*

The peak-over threshold approach is chosen because it makes better use of data in comparison with the block maxima approach that can be a waste of date. Since the metocean data is available as a time-series of measured data points with 3-hourly intervals peak-over threshold approach and fitting this data to a generalized Pareto distribution (GPD) will make better use of the available data.

To fit the data to the generalized GPD the extreme value procedure described by (S. Coles 2001) is used. Equation III-1 describes the GPD.

$$\lim_{u\to x_F} P\left\{\frac{X-u}{g(u)} \le x \middle| X>u\right\} = \begin{cases} 1-e^{-\frac{(x-\mu)}{\sigma}} & for\ \xi = 0, x > 0 \\ 1-\left(1+\frac{\xi(x-\mu)}{\sigma}\right)^{\frac{-1}{\xi}} & for\ \xi \neq 0, 1+\xi\frac{(x-\mu)}{\sigma}, x > 0 \end{cases}$$

Equation III-1 General Pareto Distribution (GPD)

With X a random variable with continuous distribution function F, x_F being the endpoint of F ($x_F \le +\infty$) such that $F(x_F) = 1$ and $F(x) < 1$ for $x < x_F$. Where variable $\frac{X-u}{g(u)}$ is a scaled excess and scale parameter δ dispenses the necessity to have prior knowledge of function g. This means that (conditionally on an observation being high enough) the probabilistic behaviour can be described by the Generalized Pareto Distribution function. The scale- ($\tilde{\sigma}$) and shape-parameters (ξ) of the GPD for the POT (Peak Over Threshold) describe the distribution for a given set of empirical data.

First the parameters of the distribution need to be estimated. Parameter estimation is done by maximum likelihood (Dekking, et al. 2005). The values $y_1, \ldots, y_k$ are the k excesse above a threshold u. for $\xi \neq 0$ the log-likelihood function (Equation III-2) is described below.

$$l(\sigma,\xi) = -k\ log\sigma - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{k}\log{(1+\frac{\xi y_i}{\sigma})}$$

Equation III-2 Log-likelihood function for parameter estimation

If this equation is maximized for a range of threshold values a plot can be made that illustrates the selected threshold and the parameter estimation for that threshold. Such a plot can be seen in Figure III-3. The point where the plot becomes stable will be used as the threshold chosen for the final distribution. The empirical data behind this stable point can be represented by the same GPD fit parameters. In the example this threshold will be 2900kN.

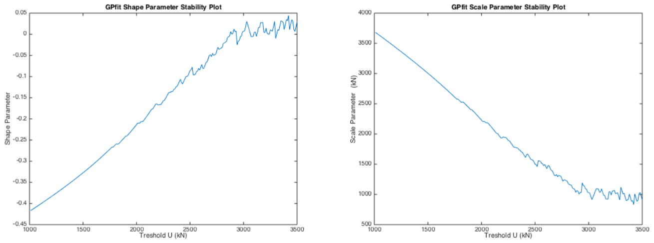


Figure III-3 Parameter stability

2) *Goodness-of-fit tests*

After this step the distribution, with the corresponding fit parameters, is known that describes the initial extreme load data. To check that the distribution is actually a good representation of the reality, some goodness-of-fit tests needs to be done.

There are a couple methods to assess the goodness-of-fit of the distribution to the empirical data. These tests underpin a claim of the form "the empirical distribution of the data matches a certain theoretical distribution". This can be checked visually and numerically. The two numerical tests used are the Chi-squared goodness-of-fit test and the Kolmogorov Smirnov test (Klemens 2009).

Three visual methods are presented. The first two are visual comparisons of the empirical data and the distribution. In the first method the theoretical distribution density function is compared with histograms of the empirical data. In the second method both the cumulative distribution functions are compared. If the distribution is a good representation of the empirical data, both curves will be close to each other.

The third method of visual comparison is a quantile-quantile plot of the parameter estimates from bootstraps of the empirical data. The parameter estimates are normal distributed by definition when the distribution is a good representation of the empirical data. The quantiles of the estimates are plotted to the standard normal quantiles. A straight line is observed if the parameter estimates show asymptotically standard normal behaviour.

3) *Extrapolation*

If the GPD is a good representation of the empirical data, a return level can be calculated for a desired return period N (100, 1000,... year). With Equation III-3 the N-year return period can be calculated. This equation uses the fit parameters of the GPD.

$$z_u = u + \frac{\sigma}{\xi}\left[\left(N * n_y * \xi_u\right)^{\xi} - 1\right]$$

Equation III-3 N-year return period value

In which u is the treshold value, σ the GPD scale parameter, ξ the GPD shape parameter, N the return year, n_y the number of observations in a year and ξ_u the probability of a threshold exceedance for an arbitrary observation.

IV. ROBUSTNESS CHECK

To ensure the method can be applied, the robustness shall be checked.

A. *Sensitivity to threshold choice*

In the process of fitting response data to a distribution some arbitrary decisions need to be made. The most important choice that needs to be made is the parameter threshold that defines the point where the fit parameters become stable. The data above this point can be fit on a distribution described by the same fit parameters.

This choice is a trade off between the amount of data to produce meaningful inferences and the quality of the fit. The sensitivity is illustrated by fitting a set of response data to a distribution for 3 different thresholds. The 100-year extrapolated load varied with 10% for the three thresholds. All three values where within the 95% reliability intervals of the other extrapolations.

B. *Parameter study*

A parameter study will be used to examine the sensitivity to variations in extrapolated load parameters and input parameters.

(Oostra 2015) applied the methodology for 1 set of input parameters. The extrapolation is executed for 1 load parameter (maximum line tension). For this specific case the methodology proved itself to be able to produce reliable 100-year extrapolations. The global characteristics of this case are listed below.

- ship: Bluewater's Haewene Brim
- mooring configuration: 8X1
- response parameter: line tension
- simulation method: time domain
- environment: central north sea

To gather an idea of the robustness of the method a limited parameter study of the underneath parameters is performed:

- change in mooring configuration
- change in response parameter
- change in environment

For all changes in the listed characteristics the method needs to be robust and must produce reliable extrapolations which can be be used with confidence during a design process.

In this parameter study 8 different cases are simulated.

1) Results parameter study

It can be concluded that for all parameter variations the proposed methodology performed well. All GPD representations of the different cases are good. All cases fitted reasonably good and all did pass the goodness-of-fit tests. However, some cases performed better than others. From the analysis of the results, three statements can be made regarding the less performing cases:

- The methodology performs better when more different wave realisations (wave seeds) are produced. This means that if the tail data is simulated by more random wave seeds a better representation of the load data is calculated. For this reason, a minimum of 10 seeds will be used in the successive work in this thesis.
- The performance of an asymmetric (3X3) mooring configuration is less good in comparison with a symmetrical (8X1) mooring configuration. In chapter V (case study) a 3X3 mooring configuration will be studied in depth and explanations for this behaviour will be presented.

C. GEV distribution

The GEV (Generalized Extreme Value) distribution can be used as less complicated alternative for the peak–over-threshold (POT) approach. The use of the POT (GPD), requires some experience from the user to evaluate the initial data and set a suitable threshold that considers both the most efficient use of data and the requirement of IID data points. According to (Méndez, et al. 2006) there are three main approaches to model extremes:

1) *monthly maxima series (MMS) (GEV),*

2) *exceedances over large thresholds, and*

3) *r-largest maxima within a year.*

The first approach (MMS) is studied. Two different block lengths are tested to fit the data to a GEV distribution. First a block length of 1 year is used. This was a waste of data and the extrapolation was 30% lower than the extrapolation made with the conventional method. Secondly a block length of 1 month is used. This resulted in an extrapolation that was significantly higher then the conventional method. In the illustrations some variability within a year can be observed. How these variations can be coped with is discussed in the next section.

4) Seasonality

In this section the influence of monthly seasonal variability of the maximum extrapolated loads will be discussed. Variation of the maximum loads can be seen on different time periods. (Menéndez, et al. 2009) These variations can be harmonics within a year, exponential long-term trend, El Niño covariate, etc. This chapter focusses on the variability that occurs within a year.

For every season of 3 months (winter(1), spring(2), summer(3), autumn(4)) a different fit is made. Figure IV-1 shows the GEV parameters and extrapolations for every season.

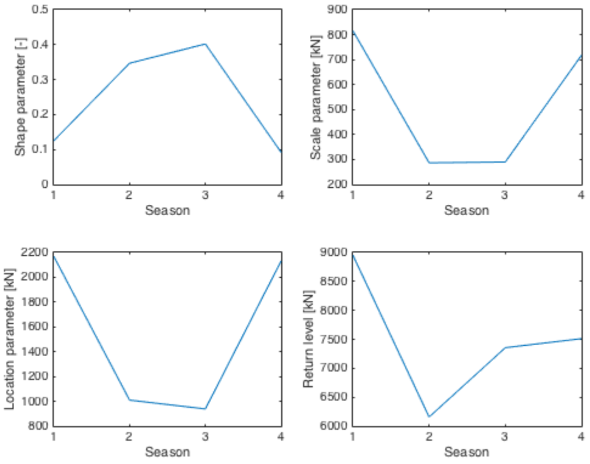


Figure IV-1 Seasonality of GEV parameters and extrapolation

To improve the fit of the empirical data on the distribution it is possible to include variable fit parameters. These fit parameters will be specific for every season. This will result in more reliable extrapolations. This is a complex process to determine the fit parameters for every season. The advantage of the simplicity of the GEV distribution is cancelled out when seasonality needs to be included. So the GPD distribution is preferred above the seasonal GEV distribution.

V. CASE STUDY

To illustrate a mooring design using extreme value theory to extrapolate 100-year return levels for the leading design parameters, a case study is performed. In the case study all conclusions made in the previous chapters will be implemented and an answer to the research question will be formulated.

The method discussed in the previous was validated for an existing system. For this case study a different mooring configuration is selected for which the results of the conventional method are not available.

A. Used methodology

Mooring design is based on three fundamental design aspects. First loading conditions and environmental conditions, secondly the analysis methodology and third the design criteria (Silva, et al. 2000).

The environmental conditions are in this case 30 years of metocean data in the North Sea. The metocean data points are the result of measurements performed with 3 hour intervals. This results in 2920 data points each year.

The second fundamental design aspect is the analysis methodology. Evaluating the proposed methodology, extrapolating 100-year return levels of different design parameters fitted on a generalized Pareto distribution is the main objective of this case study.

The third fundamental design aspects are the design criteria. These are the criteria that result in the limitations of the design of the mooring system. Figure V-1 illustrates the design parameters and where they act on the vessel.

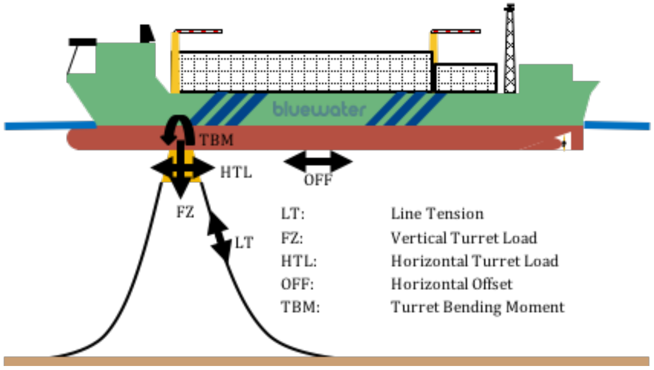


Figure V-1 Design parameters

The presented design parameters are not all parameters used in the design of mooring systems, but these parameters are the dominant load parameters that directly influence the mooring, turret and riser systems. Five different parameters are identified. The line tension that will result in the dimensions of the mooring lines, the turret loads divided in horizontal and vertical turret load and the turret bending moment, and the horizontal offset or surge/sway motions of the vessel.

B. Cases

The case study is focused on the applicability of the proposed methodology for the 5 load parameters. For the mooring lay-out a mooring system with 3 bundles of 3 mooring lines (3X3 clusters) is chosen. The mooring bundles are all separated 120° and the mooring lines within the bundles are separated 5°. The reason that a 3X3 mooring system is used is that it is a common used mooring system so it is likely that it is a design option that needs to be analysed with the proposed methodology in the future. Another reason for using a 3X3 mooring system is the asymmetrical lay-out that will possible expose the limitations of the methodology better then a symmetrical mooring lay-out. Four mooring design cases are analysed. First the base case. This is an over dimensioned system comparable with the dimensions of a symmetrical 8X1 configuration. Secondly an under dimensioned case, the light case, is evaluated. The two final cases are medium cases with mooring dimensions that are expected to be close to realistic design dimensions.

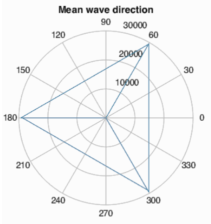
C. Results

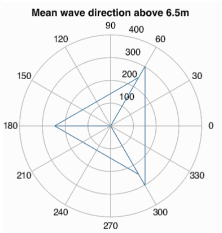
All cases performed well with the exception of the directional cases and for all cases the design parameter offset didn't perform well. These cases that didn't perform good are discussed in the following sections.

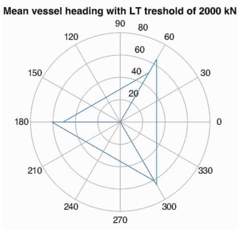
1) Directional cases

When an environment contains multimodal directional waves the design environment for the vessel will be more severe in comparison with unimodal directional waves. This is due to the steep nonlinear wave groups that occur in multimodal directional wave systems. In (Beal 1991) after a study about the practical value of directional ocean wave spectra, a conclusion is formulated. Critical wave conditions in local areas, based on calculated critical threshold values for wave direction and wave height are an improvement to unimodal descriptions of the ocean climate.

The most severe environmental conditions do not always occur when a storm is at its peak. Before a storm is fully developed large mooring forces already can occur. The reason for this is the poor alignment of the mean wind speed and significant wave height during the build up of a storm. This misalignment can be as large as 60°. Such conditions of already large waves and high wind speeds with a significant misalignment may result in the greatest mooring forces on a single point moored system (Bowers, Morton et Mould 2000).







2) Offset cases

The second type of cases that do not perform well are cases where maximum offset is the extrapolated parameter of interest. First the offset samples are plotted against the significant wave height. This is illustrated in Figure V-2 for two mooring configurations.

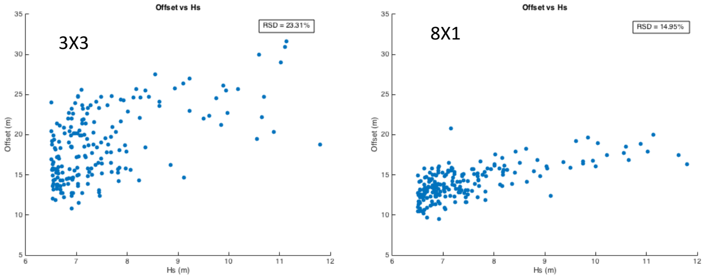


Figure V-2 Scatter plot of offset vs significant wave height

It can be seen that the spread of the plot offset vs Hs for the 3X3 mooring configuration is larger than for 8x1 mooring configuration. Also the relative standard deviation (RSD) is given. It is expected that the fit to the Generalized Pareto Distribution is better if the spread and so the relative standard deviation is smaller. This is showed in Figure V-3.

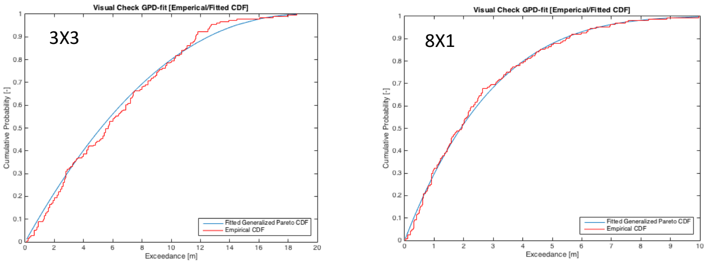


Figure V-3 Cumulative probability of the empirical data and the GPD distribution

It can be observed that indeed the fit of the data that has a smaller spread does fit better to the GPD than the data with a larger spread. So it can be concluded that the fit to the GPD has a relation to the spread of the data points plotted against the significant wave height.

This spread is the result of the difference in stiffness of the entire mooring system when the vessel experiences a displacement in line with a mooring line bundle or in between two bundles of mooring lines. Figure V-4 shows the difference in restoring force for in-line and in between line offset for the 3X3 and 8X1 mooring configuration. If the differences for in-line and in-between line tension for the 3X3 mooring configuration are compared with these differences in tensions for the 8X1 mooring configuration it can be seen that restoring force for a certain offset depends on the direction of this offset. For the 8X1 case it can be seen that the restoring force only depends on the offset and not on the direction of this offset. The dependence of the restoring force on the direction of the offset on a 3X3 mooring configuration will induce deviation in the restoring forces. This will have an adverse effect on the fit to the distribution.

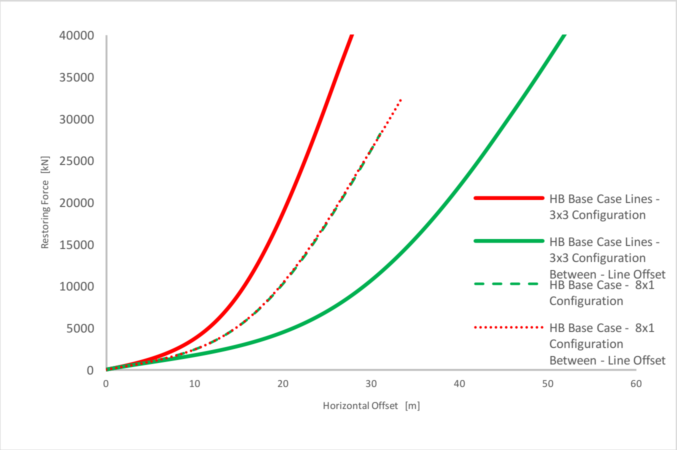


Figure V-4 Restoring force curves for in-line and in between line offset

VI. CONCLUSIONS

For the method reconstruction it can be concluded that the three parts that form the overall method together, perform well and will not be the cause of some deviating results.

To prove the robustness of the method a parameter study is executed. It can be concluded that for most variations good extrapolations can be made. The most remarkable exception is the case when a mooring system has a large stiffness difference for in-line and in-between-line translations.

An alternative approach is studied for the POT (GPD) method. The GEV distribution can provide a good fit for annual load data. To make better use of the provided data it is possible to incorporate time-depend GEV parameters that takes seasonal variations into account.

In the final part of this study a case study is executed to study the applicability of the five dominant load parameters. For a 3X3 mooring configuration it is possible to produce reliable extrapolations for four of the five load parameters with the proposed methodology. These parameters are vertical turret load, horizontal turret load, line tension and turret bending moment. For the fifth parameter (horizontal offset) some adaption of the proposed method can be done. This can be done by including directionality.

The answer to the research question is: It is possible to produce reliable 100-year extrapolations for shallow water by fitting responses from quasi-dynamic simulations of environmental data to a generalized Pareto distribution for the most dominant load parameters,

with or without some adaptions to the methodology depending on the mooring lay-out.

VII. RECOMMENDATION FOR FURTHER WORK

A. *Joint occurrence*

In (Oostra 2015) the following conclusions is stated:

> "Since only the extrapolations based on dataset II and dataset II-refined are within reasonable distance of the result from the conventional method, the conclusion can be made that focusing on the maximum Hs case within a threshold exceeding period is not sufficient."

This statement says that the best performing dataset is the dataset with a result close to the conventional method. It will be valuable to get a good insight in the differences and dependences to the environmental characteristics between the conventional method of creating a 100-year design environment with associated wind, waves and current and the response based approach.

Analysis of dependence (Fontaine, et al. 2013) and joint probabilities can be done in two steps. The first step is to describe the marginal PDF of the main variables. These are wind, wave and current. The empirical distributions can now be fitted by probability distribution models like GPD or GEV.

The second step described in (Fontaine, et al. 2013) is to analyse the dependence between variables, using three methods:

- "Statistical test of significance of correlation: Bravais–Pearson and Spearman.

- Scatter plot between two variables (X1 and X2) and analysis of main conditional statistical parameters (mean and standard deviation) of one variable versus the other.
- Principal Component Analysis (PCA) that allows for the analysis of dependence between more than two variables. " (Fontaine, et al. 2013)

The dependence analysis resulted in following conclusions:

- "Independence of sub-surface currents versus wind and sea states.
- Independence between sea state systems.
- Independence of wind and swell systems.
- Dependence of wind and wind sea (intensity and direction).
- Dependence of Hs and Tp in every sea state system. " (Fontaine, et al. 2013)

Applying this study will result in a better insight in the sensitivity of both methodologies to variations in the environment on the final extrapolations.

B. Climate change

The trend nowadays is to require longer return periods (10,000 years) for the assessment of mooring configurations. When longer return periods are required it will be necessary to study the influence of climate change on extreme return levels.

Global warming will result in increased temperature all over the world. This will result in a sea level rise, and probably in an increase of the frequency and severity of extreme events.

C. Cancel out arbitrary decisions

The reliability of extreme return levels with the peak-over-threshold approach is highly depending on the engineer using it. It would be an improvement to minimize the influence of the user on the decisions made through the process. It is possible to study the possibility to improve or automate the threshold selection procedure and its influence on the final extrapolations.

VIII. BIBLIOGRAPHY

Battjes, J.A. 1979. "Encounter probability of extreme structural response values based on multi-parameter descriptions of the physical environment." *BOSS conference proceedings.*

Beal, Robert C. 1991. *Directional ocean wave spectra.* Baltimore: The Johns Hopkins University Press.

Bowers, J.A., I.D. Morton, and G.I. Mould. 2000. "Directional statistics of the wind and waves." *Elsevier* 13-30.

Bruyn, Nathan De. 2016. "Application of extreme value theory for the assessment of turret moored floating structures." MSc Thesis, Delft University of Technology, Delft.

Coles, Stuart. 2001. *An Introduction to Statistical Modeling of Extreme Values.* London.

Coles, Stuart G., and Jonathan A. Tawn. 1994. "Statistical Methods for Multivariate Extremes: An Application to Structural Design." *Journal of the Royal Statistical Society. Series C (Applied Statistics)* (Wiley for the Royal Statistical Society) 1-48.

Dekking, F.M., C. Kraaikamp, H.P. Lopuhaä, and L.E. Meester. 2005. *A modern introduction to probability and statistics.* Delft: Springer.

DNV.GL. 2015. *Offshore standard position mooring (DNVGL-OS-E301).* Standard, DNV.GL.

Fontaine, E., P. Orsero, A. Ledoux, R. Nerzic, M. Prevosto, and V. Quiniou. 2013. "Reliability analysis and Response Based Design of a moored FPSO in West Africa." *Structural Safety* (Elsevier) 82-96.

Klemens, Ben. 2009. *Modeling with data (tools and techniques for scientific computing).* Princeton: Princeton university press.

Marin. 2008. *Dynfloat Master.* Manual, Marin, Wageningen: Marin.

Méndez, F. J., M. Menéndez, A. Luceno, and I.J. Losada. 2006. "Analyzing Monthly Extreme Sea Levels with a Time-Dependent GEV Model." *Journal of atmospheric and oceanic technology* 24: 894-911.

Menéndez, Melisa, Fernando J. Méndez, Cristina Izaguirre, Alberto Luceño, and Inigo j. Losada. 2009. "The influence of seasonality on estimating return values of significant wave height." *Coastal Engineering* (Elsevier) 56: 211-219.

Oostra, Kay. 2015. *On the use of hindcasts in SPM design.* Delft.

Silva, R.M.C, J.M. Vasconcellos C.E. Parente, B.P. Jacob, and A.C. Fernandes. 2000. "Review of design criteria for deepwater risers and mooring systems in a multidirectional environment." *Proceedings of the tenth international offshore and polar engineering conference.* Seattle: The international offshore and polar engineers. 8-14.

Tromans, Peter S., and Luc Vanderschuren. 1995. "Response Based Design Conditions in the North Sea: Application of a New Method." *OTC 7683.* Houston: Offshore technology conference. 387-398.