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FATIGUE AND FRACTURE OF MARITIME STRUCTURES

by

J.J.W. Nibbering

Lectures prepared for the first WEGEMT in Newcastle upon Tyne,

5 - 21 September 1978.

Sessions Local Strength and Fracture, 12 September.

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Part I : Fatique. Part II : Brittle fracture.

Part III: Fracture mechanics and fracture control.

Summary of lectures on LOCAL STRENGTH ANALYSIS

Local strength in the sense of "fracture-strength" of welded structures is dependent on:

- a. The type of loading.
 (Static strength; dynamic strength: fatigue and impact).
- b. The temperature.(Ductile ↔ brittle behaviour).
- c. The environment.(Corrosion fatigue).
- d. The geometry of the structure.(Stress/strain concentration, state of stress (macro-micro)).
- e. The size, type, number and orientation of defects.
- f. The quality of the material in the weld zones.
- g. The presence of residual stresses.

Indirectly local strength depends on the extent, thoroughness and reliability of non-destructive testing and material control. The link between testing and the foregoing items a to g has to be made by "fracture mechanics" embedded in a sound philosophical base.

- Aspects are: Fail safe ↔ safe life.

 (Crack initiation, propagation, arresting, significance of defects, fitness for purpose).
 - Danger of fatigue for brittle fracture.
 - Fatigue calculations for service loading. (Influence of mean stresses, overloads, combination of high and low frequency stresses, residual stresses, crack closure, cathodic protection).
 - High strength low strength steels.
 - Influence of high-heat input welding on toughness of fine grain steels.
 - Hot-straining embrittlement at intersections of welds.
 - Strength of flame-cut edges.
 - Evaluation of acceptance tests. (Charpy, C.O.D., D.W.T., Robertson, Wells wide plate).
 - Lamellar tearing.

PART I: FATIGUE

1. Introduction

Design in connection to cracking and fracture of structures should:

- a be a <u>fully integrated process</u>. For instance the loading parameters and the permissible values of stress, strain and crack length should not be treated separately as they are interdependent. This will be made more clear in sections 2 and 3.
- b be a process in which every step must be defined in statistical terms (probabilities, confidence limits). This is not purely a consequence of the unavoidable statistical description of sea-induced loads, but also of the - not exactly to define - "capability" of structures and of weaknesses in theories and suppositions.
- c be 100% realistic. This means that it should not deal with models made of homogeneous, isotropic, ideally elastic materials, but with man-made, welded constructions, containing misalignments, defects, residual stresses and locally damaged (embrittled) materials. (Figures 1, 2, 3). In this connection it is emphasized that defects are always present in welded structures and that these defects have to be looked upon as cracks. The consequence is that design for fatigue consists of calculations for crack-growth.
- d include parameters playing a role in <u>destructive</u> and <u>non-destructive</u> testing. The more sophisticated the control of construction methods and materials is, the closer the designer may reduce his margins of safety.
- e consider the whole "environment" as "loading", including corrosive action, low temperatures and eventually possibilities of inspection and reparation.
- f incorporate <u>finite-element calculations</u> combined with <u>fracture-mechanics</u>. For instance, what we like to know is how the stress field at the most critical points depends on the length, depth and orientation of <u>local cracks</u>, and what is the influence of combinations of local axial and bending deformations in triaxial stress conditions.

A practical observation is that increasing the accuracy of the best part of an analysis from for instance 90% to 95% often means at least doubling the relevant effort. When the extra quantity of work involved would have been put into weaker parts of the problem, the overall reliability of the design analysis might have been improved a lot more.

Uneconomical and time-consuming approaches occur everywhere in the design procedure. For instance there exist sophisticated fatigue-calculations of which the reliability is not better than that of very straight-forward simple approaches. The main fault in the sophisticated methods is that parameters which cannot be put in statistical figures are either left out from the calculation or - just the opposite - taken into account in a completely "overdone" way. It will be seen that "crack closure" is one example. Another one is, that all kinds of load aspects are considered to be "random", while they are not or only in a weak sense (fig. 2c). Perhaps the main one is the use of Miner's rule for unsteady loading instead of methods for calculating crack growth.

On the other hand there are a number of fortunate factors, of which advantage can be taken in fatigue calculations, (see fig. 4). Point 4b is illustrated in figures 5, 6 and 7. It is shown that cyclic stress data for structural discontinuities can also be obtained with the aid of strain gauges at critical points. This method has the advantage that the combined effect of the external load components becomes known. In fig. 7 it can be seen that the endurance of relatively complicated structures such as conservative tanker longitudinals correlates rather well with that of simpler models if plotted on the basis of local strains. When it is realised that the stress state in the different types of specimens was very different, the result is

satisfying; it could hardly be improved by applying fracture mechanics, (see section 5). Of course the correlations have been obtained for constant amplitude loading, but it is not unreasonable to expect similarly good results for service-conforming loading. Favourable experiences with strain gauges have also been reported by Gassner and Haibach /23/.

2. State of affairs

2.1. Historical

For many years shipbuilders have shown little interest in fatigue problems. In 1962 Yuille succeeded in bringing the matter to the surface with a pertinent RINA paper /9/. The essence of his reasoning was: 'Do fatigue cracks develop in ships or not? If not, it has no sense to measure all wave-induced stresses. For then only the highest absolute values of stress are of interest in connection with the strength of ships'.

With the aid of fig. 8 Yuille tried to demonstrate that indeed ships are not in danger of fatigue. It does not invalidate the great merits of his paper when it is said that the figure was not absolutely convincing. The load data belonged to the 'OCEAN VULCAN', being a ship of rather conservative design. The fatigue test data were not very accurate. (This was not Yuille's fault. Only very few data on the fatigue behaviour of structures were available). Yuille's argument that in the 'OCEAN VULCAN' fatigue cracks had never developed, was offset by Vedeler's information /10/. Once 129 cracks had been observed in a $4\frac{1}{2}$ year old tanker.

In order to clarify the matter Nibbering /11/ made use of Bennet's data on longitudinal wave bending of two fast dry cargo ships, the CANADA (9085 tons dwt; 19.5 knots) and the MINNESOTA (7260 tons; 19 knots), /13/, /14/. He corrected the frequency distribution of the longitudinal wave bending stresses for:

- a. slamming and whipping, (see fig. 18);
- b. changes in temperature and loading condition;
- c. changes in water pressure on the bottom;
- d. influence of corrosion;
- e. actual peak to peak values, (see fig. 10).

He also converted the cumulative frequency distribution into a line each point of which represented the complete load history of the ships. For these fast ships the correction for slamming proved to be very important, (fig. 11). At that time the first results of axial fatigue experiments with mild steel tanker longitudinals had become available. When they were compared with the deck 'fatigue loading line', (fig. 11), it was evident that fatigue cracks might occur. Yet the outcome was not too pessimistic, for the structures tested incorporated severe discontinuities (fig. 12), and even with these only small cracks could develop.

The type of presentation in fig. 11 becomes far more complicated when it concerns structures such as the hatch corners of modern ships. For these, the vertical longitudinal wave bending is no more the dominant type of loading.

Torsion, horizontal bending and transverse loads have become equally important. When static strength is considered it is logical to look for the most unfavourable combination of the various load components. But for fatigue strength the number of times that various possible combinations occur is important. These frequency distributions are difficult to estimate. The reader is referred to work of Meek and co-workers /2/ and a paper of Alte /22/ for more information.

Wave induced stresses in ships are commonly presented as cumulative frequency distributions (figures 8, 11). This may be the most convenient way for load

experts, but it is certainly not what fatigue experts prefer. Figures 18 and 10 illustrate two relevant aspects. Another, the influence of changes of mean stresses (still water stresses) is equally important, (see sections 3, 4). Apart from this, fatigue specialists are still in difficulties when asked to predict the fatigue behaviour of a ship for which frequency distributions of loads are available.

The method of fig. 11 is primitive. Somewhat better is a rule such as that due to Palmgren-Miner for estimating the fatigue life of a ship with the aid of results of constant load tests. Others rely upon results of programmed load tests, while the more pessimistic require service-conforming loading preferably with prototypes!

2.2. The crack initiation-propagation dilemma

In ships there are hundreds of kilometres of welds and many thousands of weld crossings. Although non-destructive testing is widely applied, weld defects will nevertheless be present in ships. Some will be too small to detect; others escape notice because of their unfavourable position and orientation. Many defects are sharp notches. When locally the cyclic stresses are high enough, fatigue cracks will often start growing after a relatively small number of cycles. In the structural specimens of fig. 7 at 200 N/mm² a crack started after 1600 cycles, reached a length of 4 mm after 8500 cycles and 20 mm after 20,000 cycles. In mechanically notched plates a 1 mm crack developed after 8000 cycles and a 20 mm one after 40,000 cycles. The growth from 1 to 20 mm took 32,000 cycles. The logarithms of 40,000 and 32,000 are practically equal: 4.6 and 4.5. Consequently neglection of the number of cycles necessary for the initiation of the crack hardly influences the position of the Wöhler-curves for 20 mm crack length.

From the foregoing the conclusion may be drawn that for shipbuilding, experimental data about the resistance of structures to crack propagation are of primary interest. The advantage of neglecting the initiation time is that the influence of a number of difficult-to-grasp parameters is excluded (scatter!). Furthermore calculations of crack growth under constant and variable cyclic load conditions can be made with more confidence than the calculation of life time up to crack initiation. For many it will be attractive that such an approach is on the safe side.

It should be emphasized that the approach is only advocated for structural details which are frequently found in ships. For these, expensive and time-consuming measures in order to increase the resistance to crack initiation will generally not be justified. But for structural details which are not numerous such as hatch corners of containerships and bulk carriers, measures to improve the fatigue strength pay greatly.

High quality welding and post-weld treatments may result in considerable benefits. One need not only consider grinding and planing. Undercuts are also highly improved by TIG-welding or by peening, see Hotta et al. /15/; Kanazawa /20/; Gurney /17/ and Reemsnyder /18/. Takahashi /16/ observed an increase in fatigue strength of submerged arc welds from 160 to 320 N/mm² (repeated loading) due to TIG-welding of the undercuts. Harrison et al. /19/ found an increase from 110 to 260 N/mm². Deep grinding proved to be far more expensive than peening and TIG-welding.

Sometimes it is wrongly stated that these improvements have little effect. The argument that for welded redundant structures such as ships 90% of the fatigue life of a structural detail consists of crack propagation is only true in the case of every day workmanship. Figure 11 gives an idea of how much may be gained by machining of butt welds. A most successful approach is to improve the endurance limit so much that cracks simply are not able to initiate; then double amplitudes of stress in the order of magnitude of 200 N/mm² (repeated loading) may be tolerated millions of times instead of

a few 100,000 times. It makes the difference between no cracking during the ship's lifetime or cracking after one year. Ships of the first type (with excellent structural details) should be considered as 'safe-life' structures. For when, despite great care, a crack starts after many years, it will propagate quickly. The cause is that the material in the line of the crack path will have been damaged by the previous large number of load cycles. (See also This contrasts with the case when cracks start early in the lifetime. Then the surrounding material is still relatively sound and the crack propagates slowly. It can be detected in time and repaired; the structure is 'fail-safe'. Apart from this, it is a general rule that the milder the stress concentration in a structure, the smaller the number of cycles for crack propagation will be in relation to total lifetime and vice versa. If the bar discussed previously had a semi-circular notch instead of a saw-cut, a crack would only have started after some 70,000 cycles. The growth from 1 to 20 mm would have taken less than 32,000 cycles, making at the maximum 102,000 cycles in total. Only 33% of this number has been spent for propagation, while for the sharply notched bar it was $32,000 \div 40,000 = 80\%$.

2.3. Load aspects

For stationary states of the sea the quasi-static stresses (peak-trough) conform well to the Rayleigh-distribution. The Rayleigh-parameter E is equal to 8 times the area of the stress spectrum (E = the R.M.S. of the stress-ranges).

Neglecting for a while the vibratory-stresses, we may estimate the cyclic loading of a maritime structure as follows:

a. Define representative sea-states for the route concerned all over the year with the aid of oceanographer's books.

In order to keep things simple these sea-states are sometimes characterized only by the R.M.S.-values of the wave amplitudes or heights. It will be clear that the shape of the wave-spectrum, and particularly the position of the peak relative to the peak(s) of the R.A.O.-spectrum determines to a large extent the resulting stress spectrum. This can be taken into account . by introducing first and higher moments of the spectral curves. But the use of one or two standard shapes and a few different positions of it in horizontal direction may give sufficiently accurate results. It has now sense to differentiate very far. More important is to dispose of reliable figures about the probability of occurrence of the spectra.

b. The multiplication of wave- and R.A.O.-spectra gives stress spectra. Eight times the area of these spectra is equal to the R.M.S.-value of the stress ranges. All R.M.S.-values for the whole life of the structure will have a frequency of occurrence more or less conforming to known statistical distributions (Gauss, Weibull etc.). Then the same is true for the frequency

distribution of the stress ranges themselves. So far things have had nothing to do with fatigue.

The commonly made next step is now to use Miner's rule for calculating the

fatigue-life ($\sum n/N = 1$), (fig. 13).

The first problem then is that the stresses obtained in the foregoing are 'nominal stresses'. These might be used in fatigue-calculations but only when fatigue-curves (Wöhler, S-N) are available for the joints for which we like to know the fatigue-life. If not, 'hot-spot' stresses have to be calculated, or measured at structural models or real structures with strain gauges. Then these stress-values may be used in connection to fatigue-data for butt-welds, fillet-welds etc. to be found in the literature.

One should not have the illusion that the answer obtained has a high accuracy. Sometimes it will be much on the safe side, in others unsafe. The weaknesses are particularly present on the loading side and on the side of the fatiguelife calculation. Yet it is very well possible to improve the calculation process essentially without making it too complicated. The rough rule of Palmgren-Miner can be dismissed and load data and fatigue calculations can

be more logically connected in crack growth calculations starting from N.D.T.-determined defect lengths, (fig. 21). This is not new; many experts all over the world favour that approach. In this method the influence of the sequence of loading can largely be incorporated, but fig. 21a shows a problem. In this connection it should be realized that wave induced stresses are not purely random. This becomes clear when representative wave spectra are studied over the year. Heavy storms occur particularly in autumn and winter and less in summer. Temperature stresses change from day to night and are most severe in spring and summer. Also - looking daily - they depend largely on the position of the sun. In the North Sea storms mostly come from western directions. Contrary to typhoons, they grow gradually in strength and die out similarly. Tide streams are very regular. (fig. 19).

Some of these aspects of loading are very low-frequent and as such determine the level of mean stresses. Now in connection to mean stresses the commonly hold opinion is that they hardly need to enter in fatigue-calculations (fig. 14). The argument is that in welded structures there exist residual stresses of yield point magnitude. Due to that the average level of the stresses is supposed to be above zero. (If so, when Miner's rule is used it would be reasonable to take fatigue data obtained for repeated loading (R = 0)).

This line of thinking is more or less right for hypothetical structures subjected to constant amplitude, constant mean-stress loading. But even then it is conservative. For only as long as cracks are small, their tips will be within the residual stress field. At greater lengths they leave that field and propagate under conditions mainly determined by the external loading. Apart from that, the presence of a crack will cause local relief of the residual stresses.

In marine structures the loading is neither constant amplitude nor constant mean stress. Early in the life of a structure stormy weather may occur during which the sum of the cyclic (quasi-static) stresses, vibratory stress es and mean stress may approach the yield point, leading to yielding at 'hot spots', (fig. 15, 16). This will relieve the residual stresses largely. Moreover when cracks are already present, local yielding at a crack tip creates a zone in which in the unloaded condition compressive stresses are present. On the whole the situation improves drastically. Perhaps most important of all is that in the absence of residual stresses new parts of cracks will be able to close during the compressive part of the loading cycles. What this means for the fatigue-life is illustrated in fig. 17. It shows that after crack formation it is no longer the range of the stresses (double amplitude) which is responsible for crack growth, but the tensile part of the cycle. This is already valid for cracks of 5 mm-in length. It should be realized that extreme compressive loads will only reduce slightly the foregoing favourable influences, just because of the phenomenon of crack

There are other arguments for not neglecting mean stresses, and changes and sequences of these. Figure 18 shows in a simplified form what may happen during 24 hours. Vibratory stresses add to the fatigue-damage in two ways: they increase the <u>number</u> of cyclic stresses, and they enlarge appreciably the <u>range</u> of the quasi-static stresses. In ships the latter is far more important than the former. In offshore structures it may be different.

3. Shortcomings of Miner's rule

In section 2 emphasis has been laid on the non-random character of sea-in-duced loads - particularly for the aspect of sequence of loads - and on the importance of changes of mean stress which may occur.

The present section will show that when using Miner's rule these influences cannot properly be taken into account.

3.1. Shifts of mean stress

Figure 19 gives an idea of progressive simplifications of service loads. The value of each simplification in connection to fatigue-life predictions will be discussed later. Here the lower part of the figure serves as an introduction to figures 20 and 22.

From the viewpoint of Miner's rule figures 20-I and 20-II are identical. They both lead to the same fatigue-damage. But, when crack-growth calculations are carried out, the two 'programmes' lead to entirely different results. This can be easily understood from the relation da/dn = $C.(\Delta K)^m$, (fig. 21). For repeated loading $\Delta K \simeq \sigma \sqrt{\pi}a$ applies (for central through cracks in axially loaded plates). When cracks are absent or very small it also applies to alternating loading, but in case of cracks longer than a few mm's.

 $\Delta K \simeq \frac{\sigma}{2} \sqrt{\pi}a$ should be used as a consequence of crack closure during compression.

When applying first repeated loading (fig. 20-II) and taking m = 4 (for convenience) we get da/dn $\simeq \sigma^4.a^2$. During the following alternating loading, da/dn drops to $(\sigma/2)^4.a^2$. During this stage a will be larger than during the first stage. But as σ is reduced to $\sigma/2$, da/dn is much smaller than during the first phase.

When the experiment starts with alternating loading (fig. 20-II), there is hardly any crack closure effect because initially there is no crack. Thus da/dn $\simeq \sigma^4 a^2$ as for repeated loading.

For one actual case the calculations resulted in: \rightarrow 3 + 15 = 18 mm (fig. 20-II) \rightarrow 4 + 21 = 25 mm (fig. 20-I).

3.2. Sequence of loads

Another case which is not accounted for in calculations with Miner's rule is shown in fig. 22. When A is below the fatigue limit of the structure concerned, it does not give rise to crack extension (A/ πa < K fatigue limit; $a = -\frac{1}{2}$ initial defect length). So, in the situation of fig. 22-L-crack growth cannot only take place when B is working ($\sigma_2/\pi a$ > K fatigue limit).

In fig. 22-II, B causes the same amount of cracking as in fig. 22-I. But after that A may add to the crack extension. This will be so when $A\sqrt{\pi}$ (defect + crack) is greater than K fatigue limit.

4. Influence of yield point

4.1. Constant-amplitude loading

As long ago as 1949 Weck /24/ stated that trying to improve the fatigue strength of welded structures by the use of higher strength steels is fruit-less. Many references can be cited which confirm this statement, but there are also many which are more optimistic.

Figure 23 from /25/ by Munse and La Motte Grover illustrates the situation. For 2×10^6 cycles the upper line is even too optimistic. When the single result at 35 KSI and 110 KSI UTS is neglected, all data fall between 16 and 28 KSI. This is about the scatter width for mild steel. For 10^5 cycles the picture is clearly better.

Gurney /17/ found little or no advantage for 10 higher strength steels with UTS from 430 to 750 N/mm² when used for non-load carrying fillet welds, (pulsating tension).

Fisher et al. /26/ carried out nearly 400 bending tests on welded beams for 3 grades of steel. He concluded that 'steels with yield points between 250 and 700 N/mm² did not exhibit, any significant difference in fatigue strengths'.

What is the reason that an increase in static strength is not accompanied by an increase in cyclic strength? (See figures 24 and 25). At the tip of a

fatigue crack, cyclic elastic + plastic straining occurs. As a first approximation the plastic deformation energy per cycle, being the product of local stress and plastic strain, may be thought to be responsible for the fatigue damage of the material at the crack tip. Comparing steels of different strengths the local stresses will be higher and the local plastic strains smaller, the stronger the steel. The product of both will be more or less the same, independent of yield point (strain energy history per cycle). The capacity of various steels for absorbing cyclic deformation energy is also more or less constant, since a high strength steel has a lower ductility than a low strength steel. As both energy history per cycle and capacity are more or less independent of yield point, the same will apply to the number of cycles to cracking.

In welded structures the residual stresses must also be considered. They will be higher the higher the yield point. This may have an adverse effect, especially in the early stages of crack development. A final aspect is that the welding of higher strength steels requires more care than that of mild steel. In this connection Harrison /19/ may be quoted: 'Small sharp defects at the weld periphery, derived from the welding slag (slag intrusions), are responsible for the low fatigue strength of high-yield steels', (see also /29/).

Figure 26 shows results obtained in Germany. There is a distinct advantage for St. 52 gr. DH36 at high stresses for both P min/P max = 0 and -1. Above some 500,000 cycles St. 42 gr. A behaves better. The German results are for full fracture of the specimens. In Belgium crack growth was recorded which made it possible to construct Wöhler curves for various crack lengths. They have been corrected for the restricted width of the specimens in order to make them directly of use for ships. Both for 1 mm and 20 mm crack length the tendency is similar to that of the German experiments although definitely less pronounced. Taking all the Belgian results together there remains little advantage for St. 52 DH36, (figure 17).

The results of French experiments were even more pessimistic. No difference has been found between St. 42 gr. A and St. 52 gr. DH36. This applies to the results for small and large cracks and to P min/P max = 0 and $-\frac{1}{2}$. Dutch results were better, especially for greater crack lengths. The specimens were much larger and more 'structural' than those used in France, (fig. 27-28).

Therefore it is believed that taking all the results together, a 10 to 15% advantage may be obtained by using Fe 510. But this is only true for constant amplitude - constant mean stress loading.

4.2. Influence of mean stresses and random loading

Many experts hold the opinion that for welded structures the range (or double amplitude) of cyclic loading determines the fatigue behaviour; the mean stresses are thought to be of small importance. This is related to the presence of residual stresses of yield point magnitude. The upper peak of the cyclic stress is always close to the yield point provided that the cyclic stresses are rather small. But, as discussed before, in highly stressed structures, local yielding will cause a reduction in the magnitude of the residual stresses and the mean stress will soon become equal to zero at points of stress concentration. It may seem that for these cases mean stresses again are unimportant, but that is not correct. Severe cyclic loading combined with tensile mean stresses may cause more plastic deformation during the tensile than the compressive part of the cycle; little or no crack closure is possible when the stresses are compressive, (fig. 29). From this it may be concluded that:

- a. Mean stresses are more important for crack propagation than for crack initiation.
- b. The adverse effect of mean stresses will not only appear in high stress loading but also in mixed loading (high and low).
- c. Steels with high yield point will suffer less from mean stresses than mild steel.

Unwelded specimens.

In fig. 17 Wöhler's curves for 1 mm crack length show that the results for axial alternating and axial repeated loading are well 'in line' but for 20 mm crack length they lie wide apart. It is evident that from the viewpoint of crack initiation the mean stresses are insignificant. The more the cracks increase in length however, the greater the influence of mean stress on the rate of propagation. At 5 mm crack length, the position of the various curves is between those for 1 and 20 mm. Speaking in terms of cyclic stresses it can be said that $+168/-168 \text{ N/mm}^2$ was equivalent to $195/0 \text{ N/mm}^2$ (N = 10^5 ; St. 52 DH36).

Welded specimens.

All data lead to the conclusion that there is little or no influence of mean stress.

Apparently the welding stresses have great effect, so long as the cracks are within their region.

4.3. Influence of shifts of the mean

Figure 30 gives results for specimens of Fe 410 and Fe 510 (St. 42, St. 52), subjected to (high) repeated and alternating constant loads and to a programme as indicated. All results correspond to a testing-time of 50,000 cycles. They support well the foregoing discussions:

- le. The initiation of cracks is only governed by the double amplitude of stress and not by the stress ratio R (data for 1 mm crack length).
- 2e. The yield point of the steel has little effect on the conventional fatigue strengths (R = 0; R = -1), both for the initiation period as for the propagation stage. But the effect of yield point is large in case of regular shifts of the mean. Miner's prediction is very optimistic for Fe 410 and pessimistic for Fe 510 (see 3e).
- 3e. The influence of shifts of the mean is large and contrary to Miner's hypothesis, (Fe 410: 150 N/mm²; Fe 510: 250 N/mm²; Miner: 200 N/mm²).
- 4e. Sequence effects are important. For, when all groups of 1000 repeated loads would have been brought together into one group of 25,000 cycles followed by a similar one for alternating loading, the differences in fatigue stress for both steels would be less spectacular. Figures 18 and 19 show how mean stresses may shift in ships.

From investigations in Darmstadt, Buxbaum /38/ concluded that changes of mean stress are important when:

- their frequency of occurrence is smaller than 1/20 of that of the main cyclic stresses;
- the amplitude of the mean stresses is greater than the RMS of the amplitudes of the main cyclic stresses.

For those who still like to apply Miner's rule for the whole lifetime of a ship the present author suggests the use, as a simple significant stress, of the sum of the range and mean stress instead of the range only (range = double amplitude). Records of service stresses should be treated correspondingly; in that way cumulative frequency distributions of stresses represent much better the fatigue loading of a ship than is the case if stress ranges alone are used.

5. Crack propagation and fracture mechanics (for arguments see fig. 21b)

The Paris-Erdogan law /31/:
$$\frac{da}{dn} = C(\Delta K)^{m}$$

has been generally accepted as an effective tool for the evaluation of results of fatigue crack propagation experiments and for prediction purposes. With the passage of time four aspects have become clear:

- a. m is not constant;
- b. da/dn K plots consist of three branches instead of one;
- c. m and C are very much influenced by scatter and by inaccuracies of curve fitting;
- d. the type of fatigue testing should be taken into account /32/, (constant load, constant strain or constant net stress tests).

Gurney /34/ discovered some dependency of m on the yield point, but Maddox /35/ could not confirm this.

Crooker and Lange /33/ brought together a lot of experimental results from published literature. For many steels with yield points varying from 350 to 2000 N/mm², most of the m-values were between 2·2 and 4·4, (fig. 31).

The results of Belgian experiments have led to m-values varying from 3 to 6 for repeated axial loading, and from 1 to 3 for repeated bending loading. But there is no need for great disappointment, since fig. 32 shows that all results for repeated loading, (two steels, three plate thicknesses), conform extremely well to a linear relation between log C and m.

This fortunate result was also found in Great Britain /46/ and Japan /47/. It is not surprising that the data for alternating loading in fig. 32 lie apart. Crack closure during compression is mainly responsible for a relatively slower rate of crack propagation.

How well results of experiments at different loads may correspond after careful testing, crack measuring and analysis is illustrated in figures 39 and 40.

The results of structural specimens cannot be adequately analysed by plotting da/dn as function of ΔK . As an alternative, the number of cycles for crack growth from f.i. 5 to 8 mm can be plotted as a function of representative ΔK values. The 5-8 mm crack length has been chosen for various reasons. Such small cracks do not change the overall geometry of the structure. The influence of differences in local geometry, i.e. weld shape, undercuts etc. is suppressed. The same applies to the effect of small incidental variations in the rate of crack propagation.

Figure 33 gives a summary of all formulae tried. For the structural specimens several formulae have been adopted for illustrating the various possible approaches rather than that the author is convinced of their usefulness. The main reason for the fracture mechanics approach is that we urgently need a method for comparing the results of experiments with notched plates on the one hand and structural specimens on the other.

The main difficulty is whether a structural specimen such as in fig. 7 should be conceived as a wide plate containing a substantial notch, or as a small plate (bracket) having a very small initial side notch (the undercut of the weld). In the first case the stress to be used for the stress intensity parameter K is the nominal stress; in the second case it is a local stress. How local is something which has to be concluded from the comparisons in fig. 35. Generally speaking the various curves II seem to conform best. The position of the strain gauges 2 the output of which has been used as stresses in the particular AK formula is such that these have the significance of local 'nominal' stresses. The curves III, for local 'peak' stresses are clearly worse. This is fortunate because it is what would be expected from a fracture mechanics point of view. It is remarkable that curves I, which consider the specimens as a whole, are close to curves II. This could be fortuitous, but it may also be interpreted as an indication that an 'overall' approach is not too sensitive to structural parameters (square notch, triangular bracket, presence of flanges).

A fourth approach in which the triangularity of the stress distribution in the brackets has been taken into account, has also led to satisfactory results. For clarity the curves have not been included in the figures.

Figure 34 gives all the notched plate data. The important, but unwelcome fact

appearing in this figure is that the groups of curves I and III do not coincide. Yet both are for repeated loading and the difference between the type of loading (bending or axial) has been accounted for by using appropriate fracture mechanics formulae. There remains the difference in the stress gradient causing unequal stress/strain histories at the crack tips. That apart, it should be observed that generally in fatigue bending the loading is 'purer' than in axial loading, where some secondary bending is often unavoidable which may be responsible for 5 to 10% higher stresses at the notches.

As a conclusion to this section the reader is asked to recall what was said in section 1. Good correlations were also obtained between the results of all structural specimens by plotting these as a function of local stresses (fig. 7). The position of the curves for the structural specimens in relation to that for the unwelded notched specimens is particularly valuable, because experiments with the latter are relatively inexpensive.

6. Estimation of the fatigue life of ship structures by experiment and/or calculation

6.1. Ships and aircraft

The greatest need for accurate predictions of fatigue life is in aircraft design and construction. This is primarily associated with the fact that weight savings are of great importance, which has led to the use of high strength light alloys. Unfortunately these are very notch sensitive. Crack growth under cyclic loading is rapid and critical crack lengths are in the order of magnitude of only a few centimetres.

Summarising, the following differences exist between ships and aircraft in connection with fatigue:

- le. For ships, critical crack lengths are an order of magnitude greater than for aeroplanes, (see section 2.2).
- 2e. Due to this, and in view of the presence of weld defects in ships, attention should be paid mainly to crack propagation, (see section 2.3).
- 3e. Cracks in ships can easily be discovered before becoming critical which reduces the need for accurate calculations.
- 4e. For aeroplanes reliable predictions with the aid of experiments (for instance flight simulation with prototypes) are required not only from a safety point of view but are also justified economically in view of the large number of aircraft of one type.
- 5e. Fatigue data for welded structures show more scatter than data for riveted structures, (defects, residual stresses, weld deformations, differences in composition and mechanical properties of weld, heat affected zone and parent metal). This means that predictions of crack initiation for highly stressed components such as hatch corners of 'open' ships and details of offshore structures cannot be made very accurately.
- 6e. The loading of ships is more complicated than that of aircraft, (see sections 2.1 and 2.4). This is a handicap both for experiments and for calculations. (The use of strain gauges giving data about real structures under service conditions may be indispensable, see section 2.4).
 - 7e. The environment of ships' structures is very corrosive, (water, air, cargo).

The foregoing points indicate why predictions of fatigue life in shipbuilding cannot be made as accurately as in aircraft building, and also why the need is not so great! This does not mean that the present situation in shipbuilding is satisfactory, as witnessed by the extensive use which is still made of the Palmgren-Miner rule.

6.2. Procedures for predicting the fatigue life of ships

Calculations using Miner's rule may be looked upon as a rough method for the prediction of the fatigue life of ships' structures. It is sometimes thought that a good method is the testing of real structures with service conforming

loading. But as Freudenthal /39/ says 'This is almost as much an over-simplification of the problem as the use of constant amplitude tests at the most damaging level of the spectrum'.

The point is that we must know how severe the loading is in connection with fatigue as compared to other possibilities. This can only be established by applying a great number of the latter in testing. Even then the information thus obtained is only really valuable when the probability of occurrence of each of the service-conforming loads used can be indicated.

Another procedure consists of a systematic analysis of the influence of all possible load parameters on the fatigue life. In the aircraft industry, many people have been working along this line for years. Yet Schijve, in an informative and comprehensive paper on cumulative damage /40/ is not very optimistic, despite the valuable work carried out for instance in Darmstadt by Gassner, Haibach and co-workers /41/.

In the marine field the situation is even worse, as will be clear from what has been said about the differences between aeroplanes and ships.

A third procedure which appears sophisticated and modern is random loading. Apart from other objections it must be emphasized that a ship is not a randomly loaded structure. For instance, summer and winter conditions and ballast and loaded conditions are often well defined. Of course a combination of the deterministic and random parts of the complete load spectrum would be an excellent approach (see fig. 19), but simplifications of that procedure are thought to be justified in many cases.

A logical approach is to see whether it is possible to use the load data more or less in the form they came forward from the before described analysis, viz. as R.M.S.-values of double amplitudes of stresses for short periods (f.i.

6.3. Use of R.M.S.-stress values

12 hours). Indeed, it would be most welcome when the fatigue-damage caused by a short-term packet of varying sea-induced loads would be equivalent to the damage caused by constant amplitude loading with the same number of cycles and a double amplitude equal to the R.M.S. of the ranges (fig. 19). Apart from the obvious advantages of simlicity and time-saving, this approach includes the cycles of small amplitude (below the fatigue-limit) which become effective in connection to crack propagation above certain lengths. On the other hand the few high peaks of the spectrum, of which the influence is rather beneficial than damaging, are excluded. Paris proposed such a procedure already in 1962 /42/. Swanson et al. have found a favourable support from experiments /43/. Others, like Schijve /44/ are not enthusiastic. Figure 36 indicates that some value like 1.2/E might be more logical than \sqrt{E} . This has an enormous effect on the calculated fatigue-life. (It will be seen later that even higher constants are required). When Q is known, calculations of crack <u>lengths</u> with da/dn = $C(\Delta K)^m$ will certainly give more reliable results than Miner's rule. For, sequence effects and changes of mean stresses can now be taken into account. It should be realized that the method may lead to far too optimistic results when data for different weather conditions are mixed. Then the Rayleigh distribution no longer applies. But this is not the worst. As stormy periods are far less frequent than periods of better weather, mixing of the data will lead to the complete elimination of the high stresses occurring during storms. This can best be understood by considering a frequency distribution of stress-amplitudes like the one in fig. 37 from /48/. It may be read as a line which indicates how often specific stresses (ranges) have been exceeded in the period concerned. It may also be used as a histogram. When we look at the interval 103 to 104 cycles, a value of 35 N/mm2 has been exceeded 10^4 times and 50 N/mm^2 10^3 times. Consequently there were $10^4 - 10^3 = 9000$ cycles lying between 35 and 50 N/mm². Roughly said, there were 9000 cycles of on the average 42.5 N/mm². But taking into account that the horizontal scale is logarithmic, there were 9000 cycles of on the average 37.5 N/mm². However, from the viewpoint of fatigue crack propagation (and fatigue damage), the

stress values for the interval 1000 - 2000 cycles (close to 50 N/mm^2) are about three times as effective as the stress values for the interval 9000 - 10,000 cycles (close to 35 N/mm^2). (This follows from da/da = $C.(\sigma/a)^m$. For m = 3 is $(50/35)^3 \simeq 3$).

When a corresponding correction is made, the representative stress value is 40 N/mm^2 instead of 37.5 N/mm^2 . Obviously the error becomes smaller the smaller the intervals of N be.

A possible - and not so bad - way of doing fatigue calculations could be by taking blocks of:

and carrying out crack-growth calculations with these values. Even this simple method will yield more reliable results than can be obtained with Miner's rule. Completely wrong would be an approach in which all data are mixed. The 600,000 cycles of 8 N/mm² and the 90,000 of 26 N/mm² would dominate all the other values, even when the R.M.S. or a higher power for the stress values is taken.

$$600,000 \times 8^{2} = 384 \times 10^{5}$$

$$90,000 \times 26^{2} = 610 \times 10^{5}$$

$$9000 \times 40^{2} = 144 \times 10^{5}$$

$$900 \times 55^{2} = 25 \times 10^{5}$$

$$90 \times 68^{2} = 5 \times 10^{5}$$

$$9 \times 80^{2} = 0,5 \times 10^{5}$$

$$700,000 \qquad 1168 \times 10^{5}$$

R.M.S. =
$$\sqrt{\frac{1168 \times 10^5}{7 \times 10^5}} = 13 \text{ N/mm}^2$$
.

700,000 Cycles of 13 N/mm² will give no crack growth at all, even at the 'hottest spots'. But the high-stress blocks of the first table may certainly give crack extension at serious weld defects in areas of high stress concentrations in corrosive circumstances. Figure 38 shows what is essentially wrong in this method.

It is interesting to compare the 'block' method with an approach in which the R.M.S.-values of the records obtained at sea are used. From fig. 7 in /48/ the following information can be drawn:

∕E N/mm²	frequency of occurrence	corresponding N
21	23	9000
17.5	90	36,000
14 :	200	80,000
10.5	340	135,000
7	430	172,000
3.5	300	120,000

It will be immediately clear that this load-programme consisting of \times E stress values and corresponding numbers of cycles, will not lead to any cracking. It is clearly less severe than the 'block' programme discussed before (see table).

Even when these values would be enlarged in accordance with fig. 36 (factor 1.2),

the result of crack growth calculations would still remain too optimistic. The significant value $\sqrt{2}E$ might be a satisfactory calculation tool. Another possibility worth investigating is a triangular short term distribution as shown in fig. 19. It has the same R.M.S. as the short term Rayleigh distribution and $\sqrt{3}E$ represents the average of the one tenth highest values. But this triangular distribution is more useful for experimental work than for calculations. For the Rayleigh distributions themselves can very well be used for calculating crack growth. The sequence of the individual stress ranges needs not be completely random, but can be defined in a realistic way. One can go very far by introducing corrections for high tensile loads based on calculations of crack opening displacement (C.O.D. = $K^2/E.\sigma_v$). Then the influence of

yield point is taken into account. But without additional experiments the influence of residual (compressive) stresses due to overloading and of strain hardening in the plastic zone near the crack tip is still difficult to quantify especially in case of welding stresses. It is the author's opinion that for practical purposes the adverse influence of the welding stresses may be considered to be compensated for by the local compressive stresses due to high tensile loads and the Elber effect on crack closure /49/, (see section 7). Then only the influence of the high tensile loads on crack closure remains in the calculations. In other words, the influence of large shifts of the mean combined with alternating stormy and calm periods.

7. Crack closure and the Elber effect

In the preceding sections crack closure has already been discussed several times. It has been shown that in alternating loading the crack is closed during the greater part of the compressive load (figures 43, 44). Elber /49/ has discovered that a crack may already close prior to zero load in the tension region. This is explained in fig. 42. Originally it was thought that the shear lips at the plate surface (fig. 45) were mainly responsible for 'premature'closure. Then the phenomenon would only be important for thin plates. But figures 46 and 47 show that in thicker plates (15-30 mm) the phenomenon is equally significant. The 'effective' load is between 70 and 80% of the applied load.

8. Corrosion fatigue

It has already been realized many years ago that the environment has a great influence on the fatigue behaviour of metals.

For maritime structures the seawater and atmosphere may indeed cause substantial reductions in lifetime. Accurate numbers have been lacking for a long time for two reasons:

- a. Ships are always well protected by painting and cathodic protection.

 Due to that there was little need for corrosion fatigue data.
- b. When such data became of interest for offshore structures they could only come forward after several years, because the experiments had to be carried out at cyclic frequencies corresponding to those in practice.

Even nowadays the information does not yet meet out wishes. Most experiments have been carried out in the domain between 10^3 and 10^6 cycles, while lifetimes up to 10^8 are real.

On the other hand it is often said that high stresses have little effect on corrosion fatigue and that the real need in in the very low-stress region. This is only partly true. Figure 18 illustrates the point. When tests would be carried out at frequencies in the order of magnitude of 0.0001 Hz, a great influence of environment on high-stress cycles might become manifest. It would be interesting to compare these results with tests in which alternatively peaks and long rest-periods occur.

Fortunately those data which have already become available are not so dramatic as had been expected by many. The reductions in lifetime due to seawater are

in the order of magnitude of one half. On a logarithmic scale it is very little, which means that it can easily be compensated for by slightly increased scantlings of the structure. Figures 39 and 40 show some results obtained by the author. They are certainly not alarming. But they are for unwelded specimens. When welds are present electrochemical actions become possible which may have a detrimental effect on lifetime. Yet the first results of a large European testing program with welded components are in line with the mentioned 50% reduction in lifetime.

Another disturbing influence may be stress corrosion in case of high still water loads. A lot of additional factors play a role like temperature, fouling, pollution, oxygen and hydrogen content, refreshment of the water by waves and streams, pH number, the alternation of circumstances in the splash zone and last but not least painting and cathodic protection, either passive or active.

It seems that cathodic protection is mainly effective in preventing crack initiation /51/. So it should be installed already during or immediately after construction at sea.

As said before, corrosion-fatigue-testing is time-consuming because the frequency of testing should correspond to reality. Haibach /50/ mentions that testing time may be reduced by a factor 20 at the maximum by omitting the very small stress values.

Another possibility is testing at higher stress levels than the real ones. But this has also its limitations. Above certain stresses the crack tip moves so fast that the corrosive medium has insufficient time to interact. (But see end of this chapter).

There are other methods for reducing testing time. Instead of S-N curves, da/dN- Δ K curves are constructed. What is needed are accurate measurements of crack growth. Then it is possible to precrack a plate at high frequency (say 10 Hz). Next the frequency is lowered to 0.1 Hz or 0.2 Hz and crack growth is observed (C.O.D.-measurements can be of help). After 0.5 mm crack extension the frequency is increased to 10 Hz again for about 2 mm crack growth. Then it is lowered again to 0.1 Hz for another 0.5 mm etc.

When the high-frequency testing is carried out in air, it may even be possible to have a check on crack growth afterwards when studying the crack surface. The combination of all low-frequent data permits the construction of a da/dn- Δ K curve (fig. 41).

For the time being such results must be considered as giving rise to large scatter in case of real structures at sea. This means that risks of corrosion fatigue must be expressed in statistical terms for design purposes. What risks may be taken depends not only on the consequences of failure for people, environment, structures and profits, but to a great extent on amount and reliability of inspections and possibilities of reparation.

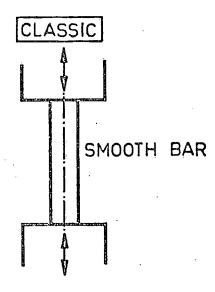
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ONLY AXIAL

(NO BENDING OR TWISTING)

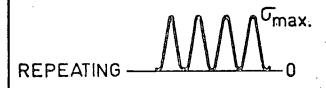
UNIAXIAL

CONSTANT AMPLITUDE

Tmax.

ALTERNATING OR

LOAD



σ_{min.}

CONSTANT FREQUENCY ENVIRONMENT: DRY AIR

PRACTICE

RANDOMLY LOADED, WELDED STRUCTURES

2ª MATERIAL DIFFERENCES.

PLATE, STIFFENER, WELD HEAT AFFECTED ZONE, STRAIN HARDENED AND/OR AGED.

2b. STRUCTURE VERSUS BAR.

COMPLEX AND THREE-DIMENSIONAL DISCONTINUITIES. DEVIATING SCANTLINGS. DEFECTS DUE TO WELDING AND CUTTING. RESIDUAL WELDING STRESSES.

CONSEQUENTLY:

STRESS CONCENTRATIONS.
TRIAXIAL STATES OF STRESS.
COMBINATIONS OF:
AXIAL LOADING, BENDING, SHEAR AND TORSION.

MARGINS:

INTENSITY OF NON DESTRUCTIVE TESTING AND OUALITY-CONTROL. HEAT-TREATMENTS. INSPECTIONS AND REPARATIONS.

2c EXTERNAL LOADS.

COMBINATIONS OF LOAD-TYPES
(BENDING-TORSION etc.) WITH VARYING

PHASE DIFFERENCES.

SUPERPOSITION PROBLEM— MM. AM

IN CORROSIVE ENVIRONMENT.
CATHODIC PROTECTION

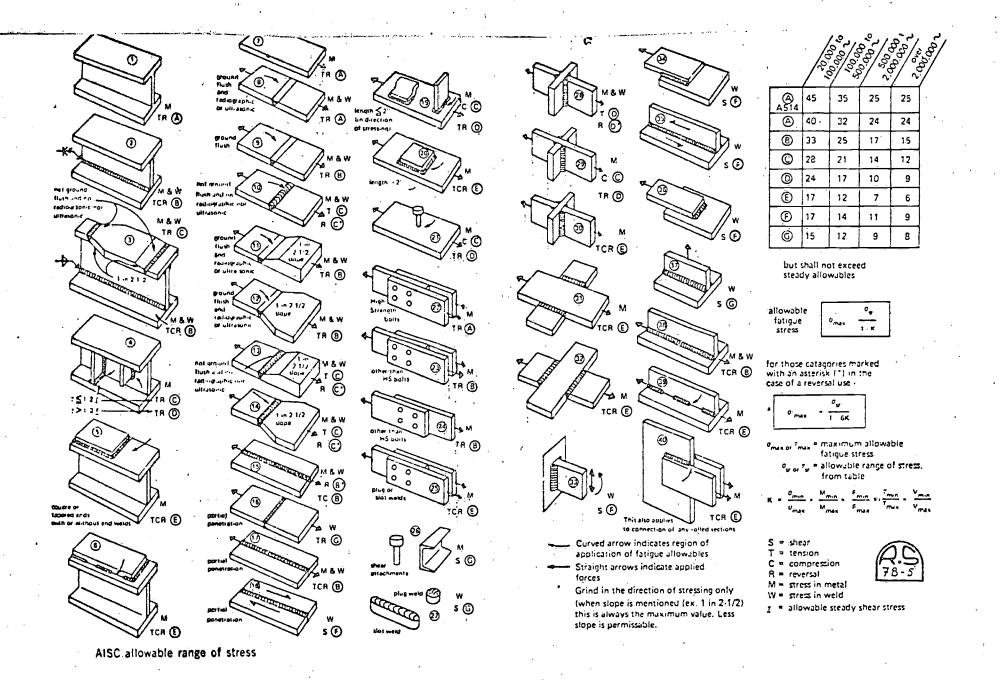


FIG. 3

"FAVORABLE" CIRCUMSTANCES.

- a. FINITE ELEMENTS CALCULATION.
- b. THE GREATEST PRINCIPAL STRESS GOVERNS THE FATIGUE DAMAGE.
- c. SERVICE LOADS CAN BE REPRESENTED BY STATISTICAL PARAMETERS.
- e. CRACKS CLOSE IN COMPRESSION

 TENSILE STRESSES MAINLY
 RESPONSIBLE FOR CRACK GROWTH.
- f. IN MILD STEEL THE CRITICAL CRACK-LENGTH IS LARGE (EXCEPT IN CASE OF BRITTLE FRACTURE).

CRACKS DEVELOP PERPENDICULAR
TO LARGEST MAIN STRESS.
COMPLICATIONS ARE: SECUNDARY
BENDING AND TORSION.

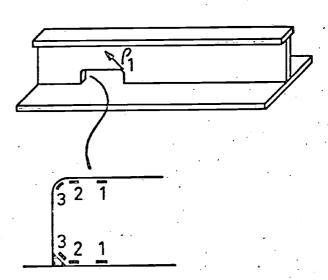
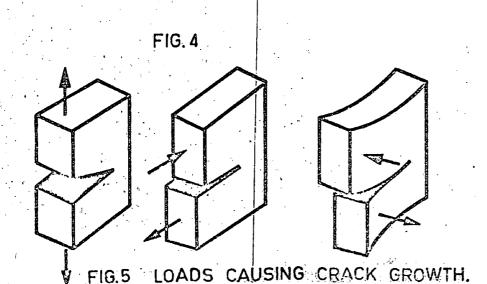


FIG. 6



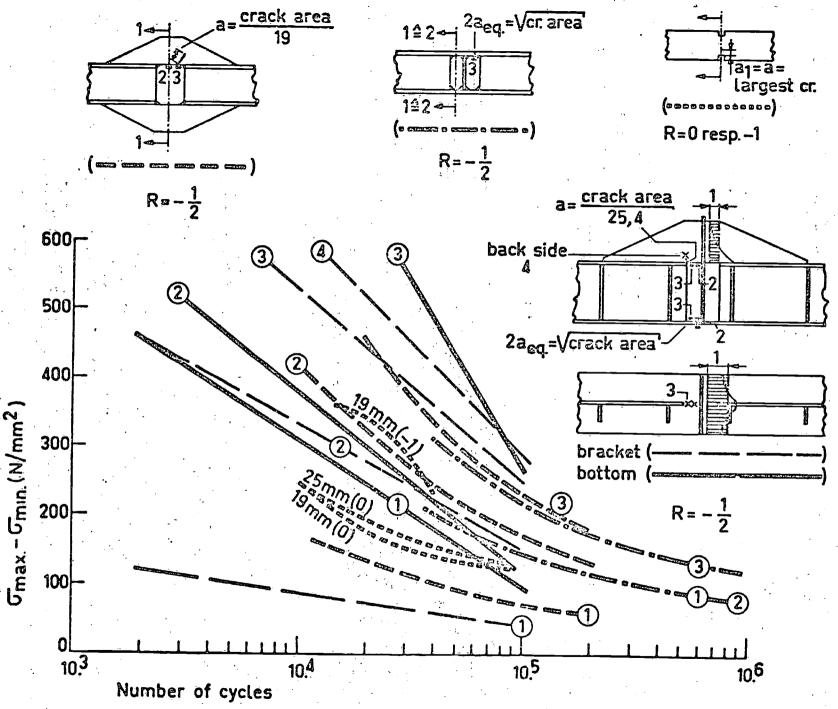
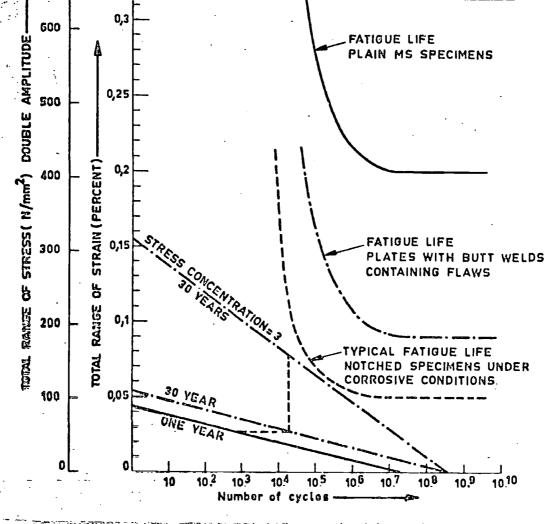
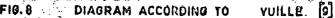


FIG.7 RESULTS FOR 5mm CRACK-LENGTH IN DIFFERENT SPECIMENS OF St.52 (DATA FOR SMALL SPECIMENS CORRECTED FOR FINITE WIDTH).





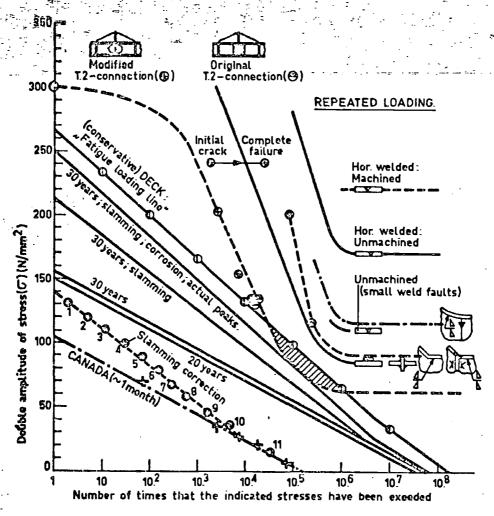


FIG.11 CONFRONTATION OF THE CYCLIC LOADING OF A SHIP WITH FATIGUE-DATA OF WELDED DETAILS [11] (1963).

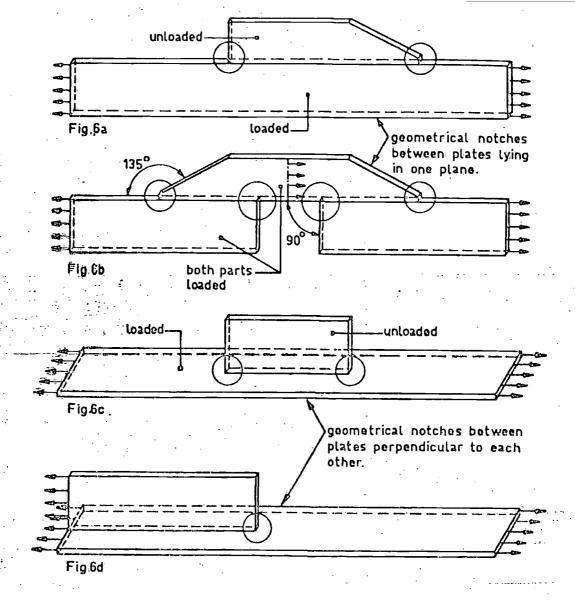


FIG.12 TYPICAL STRUCTURAL DISCONTINUITIES.

$$(1-1-2)+(2-3)+(3-4)$$
 is less serious than $(1-4)$

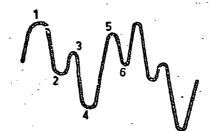


FIG.10 REAL PEAK TO PEAK VALUES.

THE RULE OF PALMGREN-MINER IS:

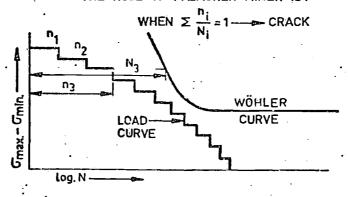


FIG.13 PALMGREN-MINER

Stresses and deformations at discontinuities with stress/strain concentration = 3

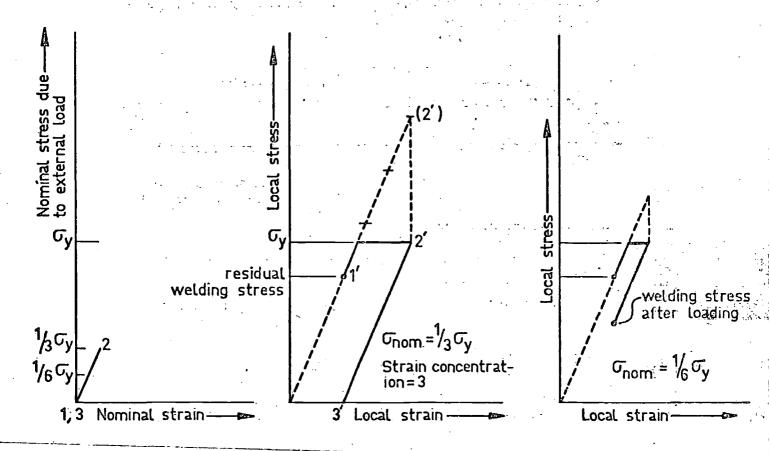
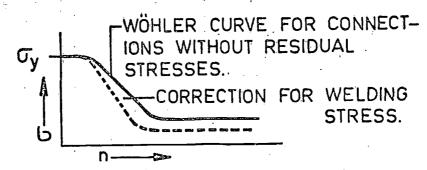


FIG.15 THE DISAPPEARANCE OF WELDING STRESSES BY HIGH LOADS.

INFLUENCE RESIDUAL STRESSES.

a. CONSTANT AMPLITUDE:

LARGE INFLUENCE AT SMALL LOADS SMALL INFLUENCE AT LARGE LOADS



DISADVANTAGE: HIGH STRENGTH STEELS PENALISED.

b. VARYING AMPLITUDE WWW.WM

SEE INFLUENCE PEAK STRESSES

FIG.14

INFLUENCE OF EXTREME LOADS

MM

1. TENSION PEAK

P FICE

- a. STRESS-FREE AT POINTS OF STRESS CONCENTRATION.
- b. HIGH COMPRESSIVE STRESSES IN UNLOADED CONDITION AT CRACK TIPS
- c. BLUNT CRACK.

DISADVANTAGE

ADVANTAGE

a. LESS CRACK CLOSURE.

b. STRAIN HARDENING.

2. COMPRESSION PEAK

CRACK CLOSURE IMPROVED CONSEQUENTLY:

NO HIGH RESIDUAL TENSILE
STRESSES AT NOTCH TIP

FIG. 16



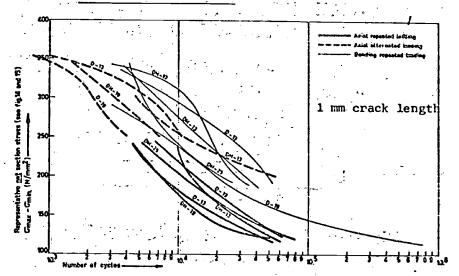


Fig. 17a. No effect of crack closure.

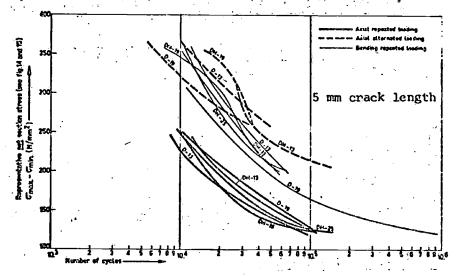


Fig. 17b. Important effect of crack closure.

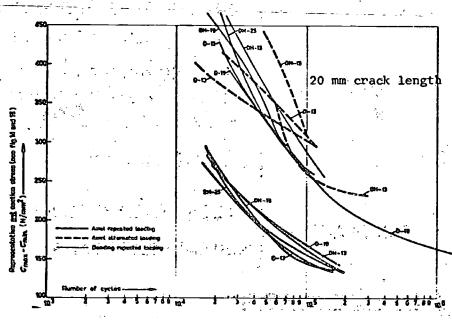


Fig. 17c. Large effect of crack closure.

FIG.17 WÖHLER CURVES FOR VARIOUS CRACK LENGTHS, UNWELDED NOTCHED SPECIMENS.

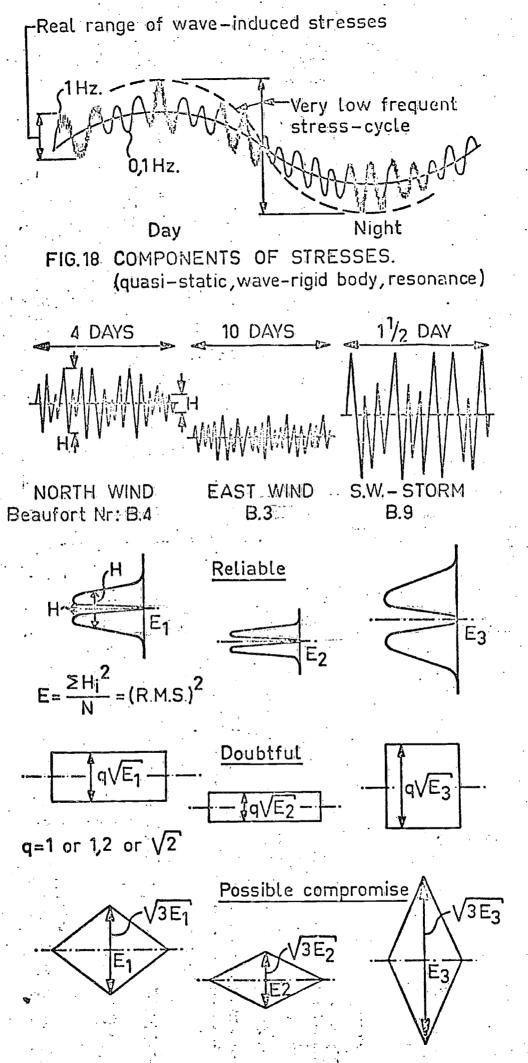


FIG.19 POSSIBLE SIMPLIFICATION OF RECORDS OF WAVE INDUCED STRESSES.

INFLUENCE OF AVERAGE STRESS ON CRACK-CLOSURE

(NA->LITTLE CRACK CLOSURE) (NA->FULL CRACK CLOSURE)

$$K_{avg}A_{II} = C \sqrt{\pi.10} = 5.6C$$
 $K_{avg}A_{II} = \frac{C}{2} \sqrt{\pi(10+16)}' = 4.6C$

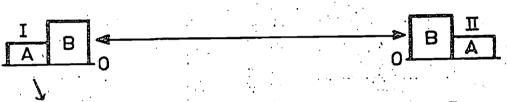
$$K_{\text{avgB}_{\text{I}}} = C \sqrt{\pi (10 + 4 + \frac{21 - 4}{2})} = 8,5 C$$

25 mm

$$\frac{da}{dN} = c(K)^{3,5}$$

MINER'S RULE:

INFLUENCE OF SEQUENCE NEGLECTED



WHEN A IS BELOW THE FATIQUE -LIMIT ONLY B

CAUSES FATIQUE (MINER)

IN CRACK-GROWTH CALCULATIONS A CRACK

CAUSED BY B IN CASE I MAY BE EXTENDED BY A

II:
$$K_A = C_A \sqrt{11'(5+10)} = 7C_A$$

CRACK DUE TO B

FIG. 22

FIG. 20

CRACK EXTENSION PER LOAD CYCLE =
$$\frac{\Delta a}{\Delta n}$$
 ($\Delta n=1$).

$$\frac{da}{dn} = c(\Delta K)^{m} \text{ (PARIS, ERDOGAN)}$$

$$\frac{da}{dn} = c(\Delta K)^{m} \text{ (PARIS, ERDOGAN)}$$

$$\rightarrow$$
 LOG $\frac{da}{dn}$ = LOG c + m LOG ΔK

AFTER A CRACK OF A FEW mm HAS DEVELOPED:

$$\frac{da}{dn} \approx c.(Kmax)^m$$
 (FOR Kmin ≤ 0) $\frac{1}{2}$ 0-LINE

(FOR STRESS-RELIEVED STRUCTURES)

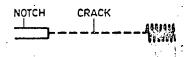


FIG. 21

LONG NOTCH CRACK

THE CRACK THAT DEVELOPED AT THE SHORT NOTCH HAS PROPAGATED IN MATERIAL THAT HAS BEEN FATIGUEDAMAGED MORE THAN THE MATERIAL IN LINE OF THE LONG NOTCH.

THE CRACKGROWTH-METHOD HAS NOT THE INTENTION TO REDUCE THE QUALITY-LEVEL BY ADMITTING DEFECTS, BUT TO RECOGNIZE THAT DEFECTS SMALLER THAN 5 TO 10 mm. WILL OFTEN ESCAPE N.D.T. CONTROL.

```
COMPARE:

5 mm DEFECT + 5 mm CRACK-GROWTH

NITIATION: 200.000 CYCLES

PROPAGATION: 150.000 CYCLES

10 mm DEFECT + 5 mm CRACK-GROWTH

NITIATION: 70.000 CYCLES

PROPAGATION: 80.000 CYCLES

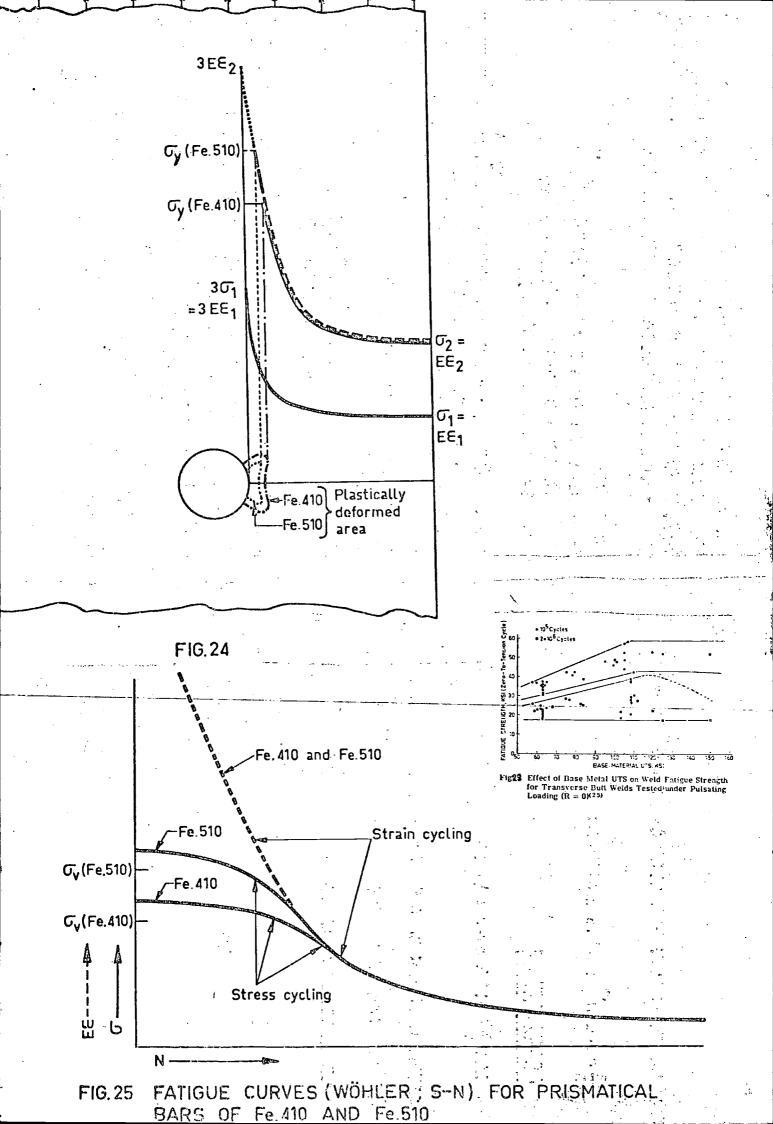
FOR A CRITICAL LENGTH OF 15 mm

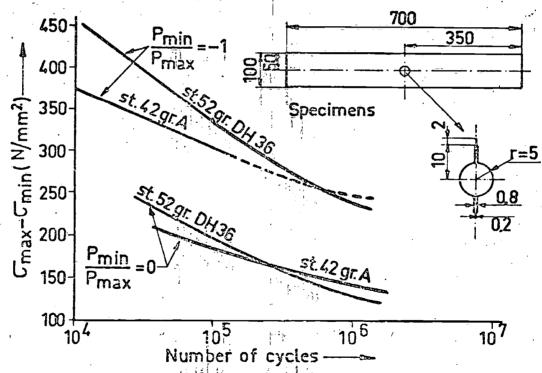
THE NUMBERS ARE:

350.000 + 80.000 = 430.000 RESP. 150.000

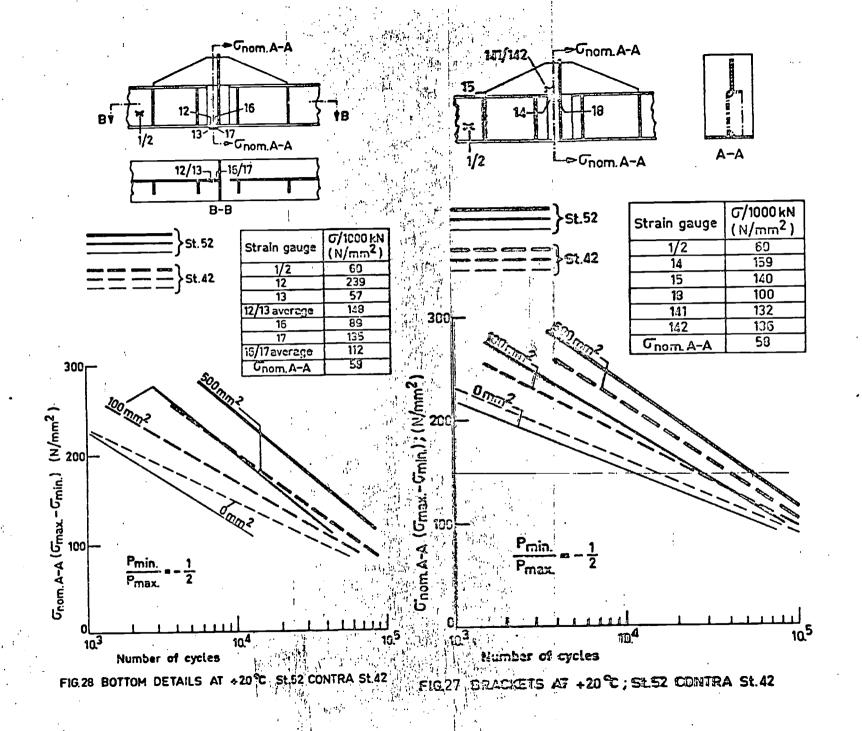
NEARLY A FACTOR 3
```

FIG. 21^b





Number of cycles ————
FIG 26 RESULTS OF GERMAN EXPERIMENTS WITH
CENTRALLY NOTCHED PLATES UNDER
AXIAL LOADING.



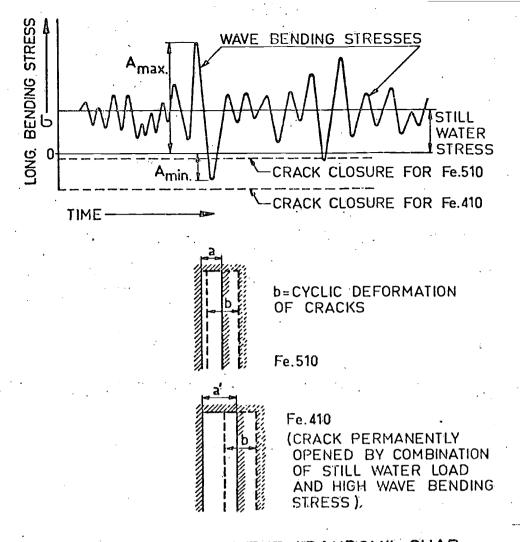


FIG.29 INFLUENCE OF THE "RANDOM"-CHAR-ACTER OF SHIPLOADS ON THE DEFORMATIONS AT CRACKS.

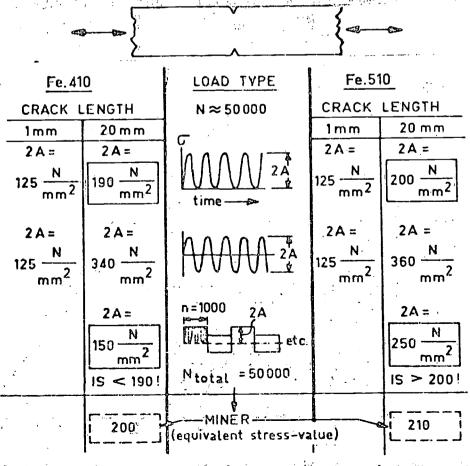


FIG.30 EFFECT OF MEAN STRESSES AND MINER'S RULE.

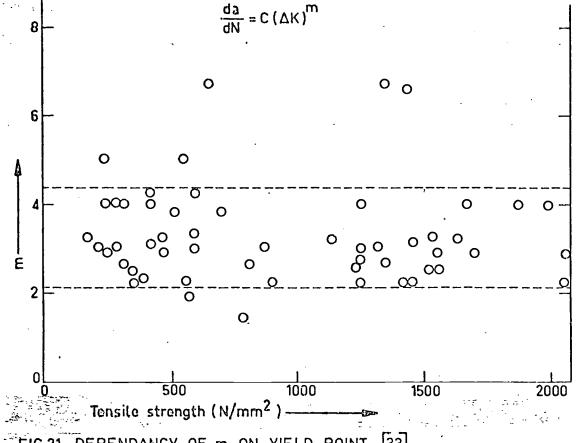
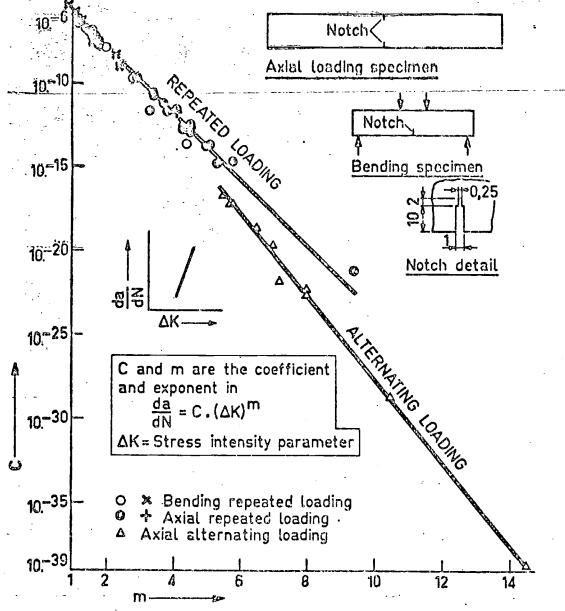


FIG.31 DEPENDANCY OF m ON YIELD POINT [33]



RELATIONS BETWEEN C AND m FOR TWO STEELS AND THREE PLATE-THICKNESSES.

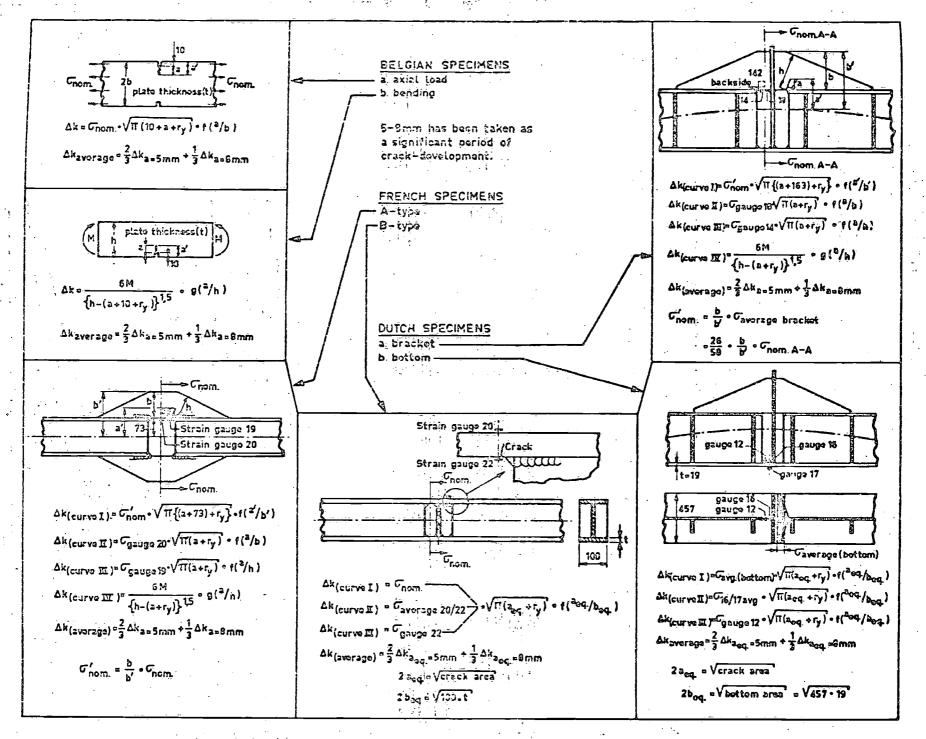


FIG. 33 SUMMARY OF FORMULAE FOR THE DETERMINATION OF ΔK -VALUES.

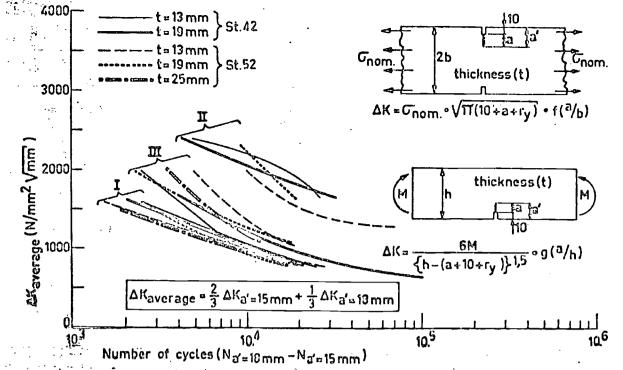


FIG.34 AK-An CURVES FOR AXIAL REPEATED LOADING (I), AXIAL ALTERNATING LOADING (II) AND BENDING REPEATED LOADING (II):

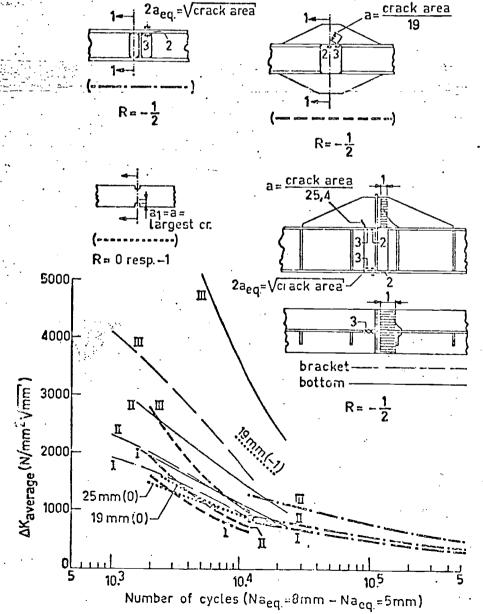
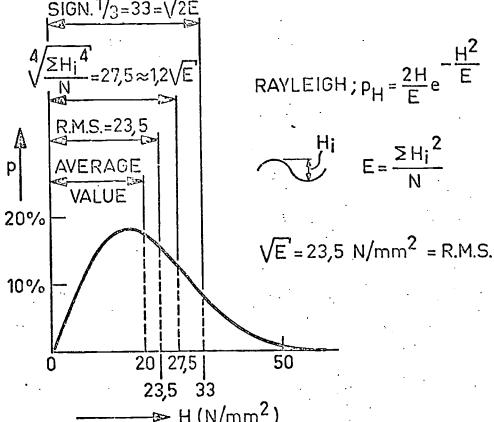


FIG.35 COMPARISON BETWEEN ΔK-ΔN CURVES FOR DIFFERENT TYPES
OF SPECIMENS
ΔK IS THE AVERAGE VALUE DURING CRACK GROWTH FROM 5mm
TO 8mm. FOR σ IS TAKEN RESPECTIVELY THE STRESS AT THE
LOCATIONS 1,2 OR 3



→ H (N/mm²)

THE PARIS FORMULA $\frac{da}{dN} = c(\Delta K)^{m} =$

 $c(\sigma\sqrt{\pi a})^m$ WITH m=3 TO 4 INDICATES THAT A R.M.S. STRESS IS NOT SUITABLE FOR REPRESENTING RANDOM LOADING

(damage is not proportional to stress squared)

FIG.36 RAYLEIGH DISTRIBUTION AND REPRES-

CRACK GROWTH CALCULATION: $\frac{da}{dN} = c(\Delta K)^{rn}$ IDEAL

CALCULATE INFLUENCE OF EACH CYCLE ON CRACK GROWTH.

NEEDED: STANDARD-LOAD HISTORY



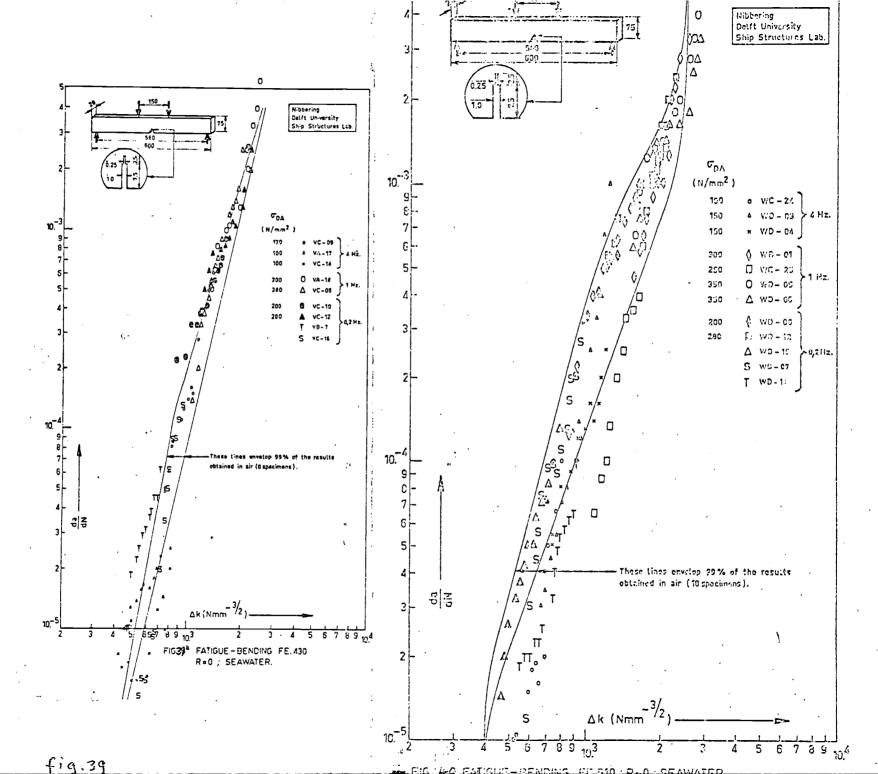
BUILT UP FROM MEASUREMENTS AT SEA, INFORMATION ABOUT THE WEATHER IN VARIOUS SEASONS, LOADED-BALLAST CONDITIONS. SPEEDS(CONTAINERSHIP---TANKER; HARBOUR TIMES).

PROBLEMS:

- 1. HOW REPRESENTATIVE
- 2. LOAD DATA ARE MOSTLY GIVEN IN STATISTICAL TERMS
- 3. MANY UNKNOWNS:

INFLUENCE OF PEAKS, CHANGES OF MEAN ETC. ON PLASTIC ZONES, RESIDUEL STRESSES, CRACKCLOSURE (ELBER-EFFECT ETC.)

FIG. 36^a



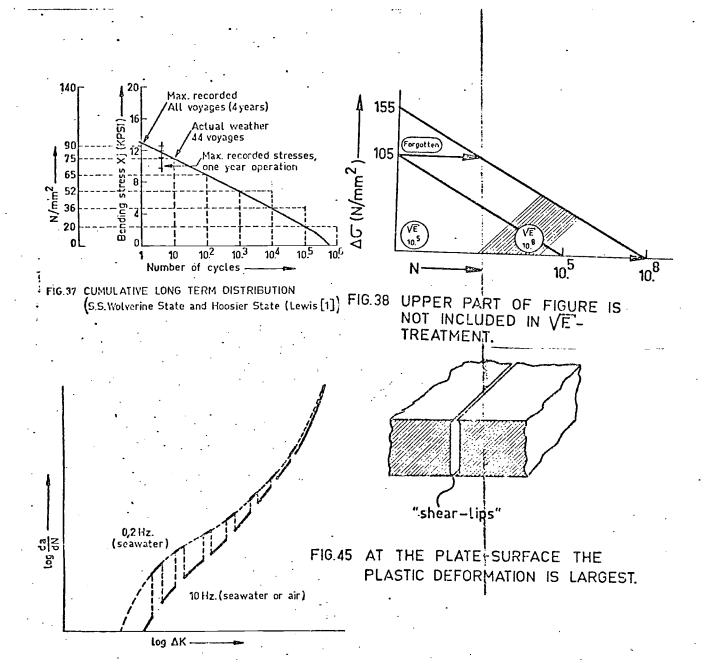
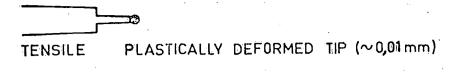
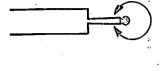


FIG41 Accelerated corrosion fatigue testing.





AFTER UNLOADING THE CRACK
TRIES TO REMAIN OPEN BUT THE
SURROUNDING MATERIAL THAT ONLY
HAS BEEN DEFORMED ELASTICALLY,
DOES NOT ALLOW THIS.
THE CRACK TIP MATERIAL IS
COMPRESSED PARTLY PLASTICALLY,
PARTLY ELASTICALLY.
AFTER THE CRACK HAS PROPAGATED,
THE MATERIAL AT THE CRACKSURFACE REMAINS COMPRESSED
IN THE UNLOADED CONDITION.
THE CRACK CLOSES BEFORE THE
LOAD BECOMES ZERO

SUBSEQUENT PLASTIC ZONES.

FIG.42_CRACK_CLOSURE AND THE ELBER-EFFECT.

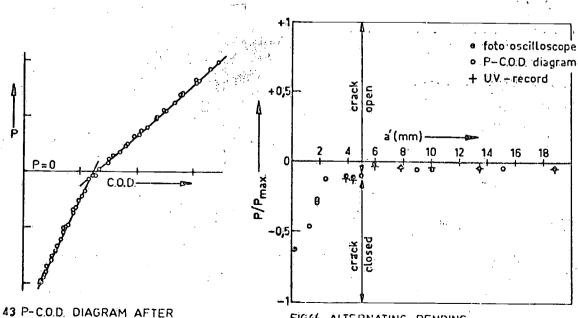
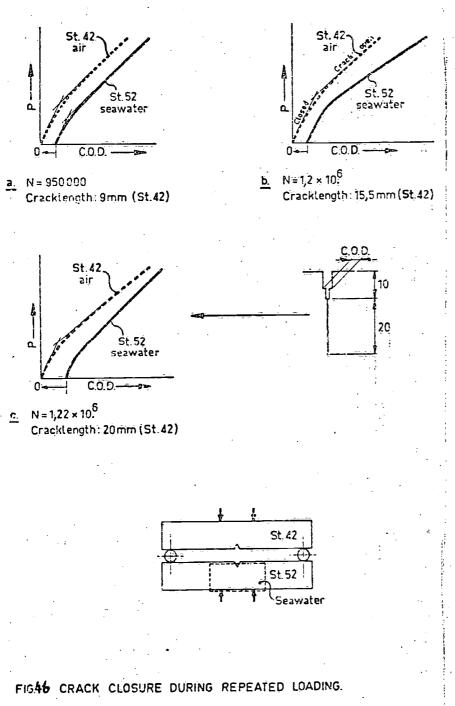


FIG.43 P-CO.D. DIAGRAM AFTER
37100 CYCLES FOR CRACKLENGTH 10 mm

FIGA4 ALTERNATING BENDING PART OF COMPRESSIVE LOAD DURING WHICH CRACK REMAINS OPEN.



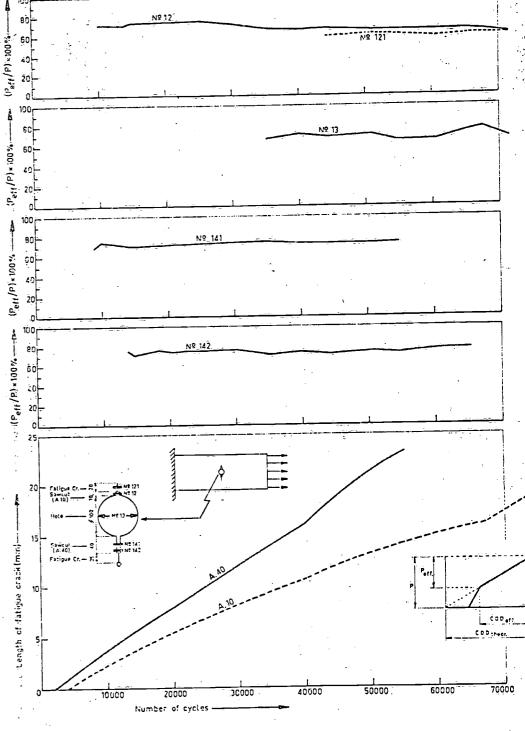


FIG.47 CRACK CLOSURE IN REPEATED AXIAL LOADING.

PART II: BRITTLE FRACTURE

1. The danger of notches

The danger of notches can best be visualized by comparing the behaviour of three strips of plating as in fig. 1.

The strip with the hole has about the same strength, but a smaller overall elongation as compared to the values of the full strip. There are two yield regions, a small one A when the material transverse of the hole yields, and a normal one B for the rest of the strip. The hole has an elastic stress concentration factor 3. It is supposed that in the elasto-plastic and fully plastic range the strain concentration remains equal to 3. When the nominal stress passes $1/3 \, \sigma_{_{\rm V}}$, the material at the edge of the hole comes in the

plastic range. But it does not behave in the same way as the material of the full strip. It will deform practically linear with the nominal stress σ ,

because it is surrounded by purely elastically deforming material. The plastic zone at the hole increases gradually until finally all material transverse of the hole has come into the plastic state. From then on yielding in that part of the plate occurs in the normal way. This means that when the average stress there becomes equal to σ , the deformation suddenly increases from about 0.1 to 1%.

This does not appear in the stress-strain curve because this yielding occurs only over a small length of the whole bar. When the nominal stress is further increased to $\sigma_{_{_{\mbox{V}}}}$, the rest of the strip starts yielding in the normal way.

Fracture of the strip occurs when at the edge of the hole the strain has become equal to some 30 to 50% (depending on the relation between hole diameter and plate thickness). In the material transverse of the hole, the average elongation then is between 30/3 = 10 and 50/3 = 17%. Due to that the net stress in that section is close to the tensile strength of the material. So a reduction in overall strain is the only serious consequence of the presence of the hole. Mostly the elongation is even more than one third of that of the uniform bar despite the fact that the strain concentration is 3. For a plate with a sharp notch the situation is very different. When all material transverse of the notch starts yielding, the notch-tip material has to deform largely due to the high strain concentration. For instance 1% average strain in the notched section (at the end of the yield region) may cause locally-at-the-tip-some-10-to-20%_deformati<u>on. Such a high</u> strain does not cause fracture in the case of a round hole in a thin plate. For in that situation the ductility of the material at the edge of the hole is practically as large as that of a uniform test bar. The reason is that it can contract freely in the thickness direction. Figure 2 shows that this is not possible at a notch tip. There the material is in a triaxial state of stress, particularly when the plastic zone is small as compared to plate thickness (fig. 3, plane strain condition). This state of stress causes an increase of the local yield point and a reduction of the ductility. The thicker the plate, the severer the state of stress is and the smaller the local ductility. When it becomes smaller than the mentioned 10-20%, the plate fails at the moment the net stress becomes σ_{v} , (fig. 1c). The dramatic consequence is a

very low overall elongation to fracture, because the material ahead and abaft of the notched section is still in the elastic condition.

There is a very simple, but seldom or never applied method for restoring the strength and ductility of the notched plate. Figure 4 shows the principle. When in the notched section the total amount of material (plate + doubler) is more than elsewhere, yielding will first take place outside the notched section and only at a higher nominal stress in that section. Then the overall elongation of the plate prior to yielding (and fracture) of the notched section is about 1% instead of the 0.1% of fig. 1c. This method is only effective in uniaxially loaded structures. It is far better than the usual ring-

type doublings, which mostly contain too little material.

The same principle can be applied to hatch corners (fig. 5). Again yielding of the material transverse of the corner should be prevented. At the corner itself no extra material is needed. It would only add to the triaxial state of stress. Figure 6 shows that also at other weak sections in a ship, the local structure can be protected against premature yielding. For that is what should be aimed at at bad structural details.

The adverse effect of a triaxial tensile state of stress can be visualized with the aid of Mohr's circles (in principle!), (fig. 7).

In plane stress the shear stress is equal to

$$\frac{\rho_3 - \rho_1}{2} = \frac{\rho_3 - 0}{2} = \frac{1}{2}\rho_3.$$

In plane strain it is $\frac{\rho_3-\rho_1}{2}$, being much smaller than $\frac{1}{2}\rho_3$. Due to that in

plane strain ρ_3 may sooner pass σ_B than $\tau = \frac{\rho_3 - \rho_1}{2}$ will pass τ_{flow} . Then the material fractures before yielding.

In plane stress $\tau = \frac{\rho_3}{2}$ will always pass first τ_{flow} before ρ_3 can reach σ_B , which means that the material is able to yield. After yielding τ_{flow} rises due to strain hardening. At a certain moment it

becomes greater than $\sigma_{B}/2$. Then plastic deformation is no longer possible and fracture occurs.

2. Experimental research

Brittle fractures have largely occurred during and some time after World War II. They were <u>low</u>-stress fractures, which means that the stress-strain behaviour of the particular structural details was even worse than that in fig. 1c. The nominal fracture stress was lower than yield point, say $\frac{1}{2}\sigma_{\rm v}$ or 2/3 $\sigma_{\rm v}$.

One might call these fractures elastic ones for plastic deformation seemed to be absent. (We nowadays know that locally always a small amount of plastic deformation occurs prior to fracture).

For a long time the causes were not clear. It was obvious that the replacement of riveting by welding was essential, but the insight in the consequences was lacking. There were several possibilities:

- a. The presence of residual welding stress (some ships and structures fractured in unloaded condition!).
- b. Bad structural details (imitations of riveted ones): hatch corners, interconnections of longitudinals.
- c. Bad welding (defects).
- d. Steel not very weldable.
- e. Steel too brittle.

An enormous amount of research has been carried out in the U.S.A. and later in Great Britain, Japan and other countries. But only about after 1957 the picture became clear.

- Ad a. We nowadays know that the welding stresses are only the last straws. When a structure has not any ductility left due to the use of bad material, inferior welding and a bad design, the welding stresses indeed lower the strength. But often they have had a beneficial effect. Many fractures have been arrested when they were forced to leave the bad weld region under the influence of welding stresses, (fig. 13).
- Ad b. The original hatch-corner construction of Liberty-ships was of an extremely inferior design, (fig. 8).

 The most critical point is the deck where three severe discontinuous members coincide. This results in a high stress concentration factor,

very steep gradients and a severe state of triaxial tension. The transverse hatch end beam contributes to this particularly because it opposes any transverse and vertical (lateral) contraction of the deck at the corner. Deck and longitudinal coaming are also mutually restrained.

The remedies for the structural faults of fig. 8 are as follows:

- 1. Reduce the stress concentrations and steepness of stress gradients in the longitudinal members. This can be done by rounding the corners and restoring the continuity (see fig. 9).
- 2. Ensure that the points of highest stresses do not occur where the structure is rigid. The rounded deck corner in fig. 9 is very successful in this respect.
- 3. Avoid concentrations of welding.

The structure in fig. 9 corresponds more or less with the so-called A.B.S. design which particularly has been applied in Victory ships. Practical experience as well as laboratory tests have indicated that this design was very satisfactory if compared with the Liberty-corner given in fig. 8. But a really 'well' designed structure would be one in which special attention is paid to various details such as the shape of rounded edges, welding details, connection between coaming flanges, a separation between coamings and girders etc. The well known flexible 'Kennedy' type of corner is a good example.

- Ad c. Bad welding has largely contributed to the fracture in ships. A brittle fracture always starts at a defect. Mostly of course in defects at places of high stresses and high restraints (triaxiality). But a fracture can only start in a weld, when the weld metal is of inferior quality (f.i. high hydrogen content).
- Ad d. The latter is not always caused by the electrodes, but may be due to the steel to be welded. High carbon amounts were present in the war-time steels. That made that also the Heat-Affected Zones of these steels could be of very low quality.
- Ad e. The steels were not only unsuitable for welding, they were also 'brittle'. But the word brittle is misleading because it is used in connection to various phenomena of fracturing, (surface appearance: cristalline/fibrous, speed of fracturing, deformation prior to fracture, cleavage/shear, stable/unstable, high stress/low stress). Static tests with notched plates of American war-time steel always behaved very satisfactorily up to very low temperatures. But as soon as a fracture started in one or another way at quite normal temperatures, the fractures propagated easily at high speed (up to 2 km/sec.). The cause was that the steels used were very strain-rate sensitive. When a fracture propagated the material ahead of the momentary crack tip deformed very locally and very fast. The high speed of deformation caused an increase of yield point and a corresponding reduction in ductility. Again (as in case of high triaxiality) the metal fractured before it could flow. But in the case of crack propagation, speed and triaxiality work together in raising the yield point.

When the temperature (another factor that influences yield point and ductility) is sufficiently low, the fracture can be largely of the cleavage type, with cristalline appearance. Only at the surface shear lips may form because there the state of stress is biaxial. In ships the fractures have mostly a chevron-type appearance. The chevrons

are - very momentary - stops and starts.

A crack arrests when the shear lips at the surface gradually increase in thickness, and thus an increasing amount of fracture energy is consumed in shearing. This is an order of magnitude greater than the energy needed for cleaving.

About 1957 Mylonas and Drucker /2/ succeeded in provoking low-stress brittle fractures at realistic temperatures in unwelded steel. All previous experiments

with hundreds of specimens up to lengths of 10 m, widths of 3 m and thicknesses of 30 mm, containing mechanically-made sharp notches, had resulted in 'high-stress' brittle fractures, with nominal stresses equal to yield point. All these years the influence of welding had been clearly underestimated! Mylonas's specimens were also unwelded, but his intention was to demonstrate that the main cause of low-stress brittle fractures was 'exhaustion of the ductility' of the steel by forming, welding, ageing etc. and that the residual stresses were of secondary importance. His specimens had deep side notches. They were sidely compressed up to 5% and after that pulled to fracture at low temperature.

The criticism with respect to these experiments is that the damaging treatment was very unrealistic and will certainly have resulted in residual, tensile

stresses at the notches' tips.

About that time Wells /1/ obtained very-low-stress fractures in welded plate specimens. It may sound astonishing but up to that time laboratory experiments with welded plates or structural specimens had also always resulted in rather high-stress fractures (see §3). Nominal stresses were at least 2/3 of yield point, while it was suspected that in ships fractures had occurred at much lower stress levels. After Wells's successful experiments it was discovered that similar ones had already been carried out by Kennedy in 1945. But they had not attracted attention. Wells's experiments were in fact a follow-up of Greene's work /1/, who more or less haphazardly obtained one low-stress fracture among a large number of high-stress ones in welded plates.

Some in Ghent had also shown the way with his tensile tests with strain-aged steels /1/.

Wells's specimen was simply made by hutt-welding two plates, in the edges of which saw-cuts were present (fig. 10). The material at the tips of the notches plastically deformed during welding (several passes) at critical temperatures (100°-400°C). This strain-ageing resulted in a complete exhaustion of ductility. Typical results are shown in fig. 11.

The full lines are those obtained with Robertson crack-arrest tests./3/.
Robertson had discovered that brittle fractures in steel cannot propagate at very low stresses (<> 80 N/mm²) and not above—certain temperatures, called transition temperatures. The latter depended mone or less on the nominal stress. The Robertson test is still important for estimating crack-arrest temperatures. (In a plate under tension a crack is started by impact at a notch where the metal is cooled with liquid nitrogen).

Wells's results fitted beautifully in Robertson's diagram (fig. 11). Region III is that of partial fractures initiated under the influence of a low load and residual stresses.

In Region II the load is so much higher that the cracks could not arrest.

-Kihara and Masubuchi /l/ repeated Wells's experiments and included specimens with notches made <u>after</u> welding. The difference in transition temperature between these and the specimens with notches made <u>before</u> welding was 50 to 100°C. Again it was evident that the <u>damage</u> to the metal inflicted by welding was the dominating influence. The <u>residual</u> stresses were of secondary importance.

It should be realized that Wells's conditions are very well possible in ships. Thousands of weld crossings between plates and stiffeners are present.

3. Fatigue and brittle fracture

In section 2 it has been mentioned that full-scale experiments with structural specimens gave better results than expected from service fractures. Figure 12 illustrates this for bottom longitudinals (dotted line, American war-steel, /6/, Irwin, Campbell).

The transition from 'low'-stress (not so low!) fractures to full-yield fractures occurs at -18° C. This temperature is much lower than that of interest for ships (seawater -5° C).

Nibbering /7/ supposed that this difference in behaviour was due to differen-

ces in loading conditions between the laboratory experiments and practice. The American experiments consisted of one pull to fracture, while ship structures are subjected to cyclic loading. The obvious question now is, whether in practice brittle fractures have initiated from fatigue cracks. This was difficult to answer. The reason was that the brittle part of ships' fractures largely overruled the fatigue part. An investigation by Nibbering /8/ revealed how easy fatigue cracks could be overlooked, but also that the initiation of brittle cracks from small fatigue cracks is possible in ships, (fig. 13). In general small fatigue cracks are more dangerous than large ones; the tip of a small crack is still situated in the - possibly bad - weld zone. Figure 12 shows results from experiments with bottom longitudinals with and without small fatigue cracks (fig. 14).

The dotted curve left refers to specimens made of Al-killed mild steel, fabricated in 1960. It is interesting to see that it is some 20°C 'better' than war-steel (and/or 'better' weldable). The full curve on the right is for specimens containing fatigue cracks. The shift of some 30°C due to fatigue, brings the results even on the right-hand side of the American specimens. When these latter would also have been subjected to cyclic loading prior to the tensile test to fracture, their transition temperature might have been some $-18^{\circ} + 30^{\circ} = +12^{\circ}\text{C}$. This is indeed a normal temperature for ships. The line-dot curve on top of the figure refers to specimens tested after 1970 which were made of Nb-containing fine grain steel (Lloyd's grade DH 36 Nb) of higher strength. The behaviour was excellent.

Apart from the foregoing, fig. 15 shows that all 'low-stress' fractures in fig. 12 are in fact high stress fractures. For the bottomplate starts yielding over the full width at about 2/3 of yield load, (midfig. 15). The interruption of the longitudinals at the bulkhead and the presence of the bracket cause a raise in the neutral axis of the specimens (fig. 16) resulting in bending in a vertical plane. Due to that the bottomplate was subjected to axial stresses and bending stresses.

There is another unpleasant factor that adds to the possibility of crack initiation at fatigue cracks. The high sharpness of those cracks makes the cracktip-metal extremely sensitive to high speed loading, (in case of metals which are highly strain-rate sensitive like ships' steels). The point is illustrated in fig. 18.

From the experiments it could be concluded that strain rates corresponding to about $10,000~\rm Hz$, were required for suppressing any plastic deformation at the crack tip below about $-10^{\circ}\rm C$. Frequencies like that cannot be caused by seainduced loads, not even when the forebottom of a ship is slamming on the seasurface. The only possibility is that accidental cargo-drops or impacts of rolling hatch-covers against supports and similar phenomena occur close to fatigue cracks.

In order to obtain an idea about this, impact-tests with full-scale fatigue-cracked specimens were carried out. The procedure consisted in subjecting the specimens to a small static load at low temperature and submitting them to light impact loads; when the specimens did not fracture the static load was increased. The result of the investigation was that two out of four specimens fractured at a load which was nearly one third of yield load. The other two fractured at about 60% of yield load.

For acceptance testing and application of fracture mechanics, see part III.

References

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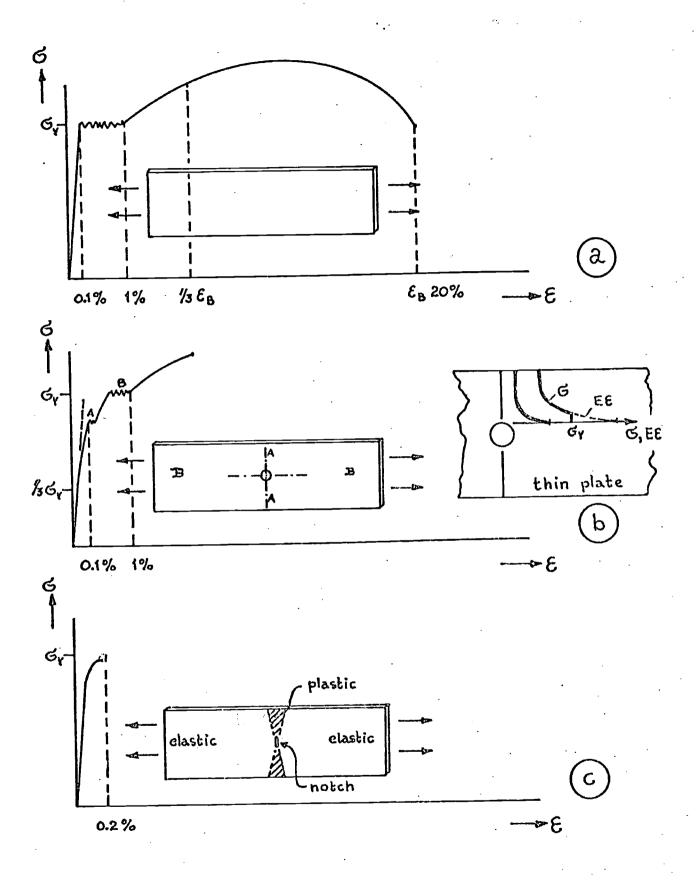
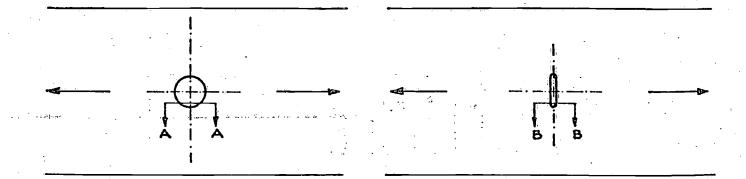
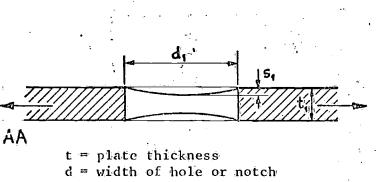


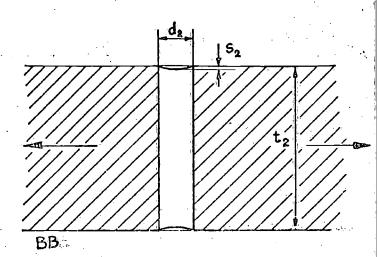
FIG. 1





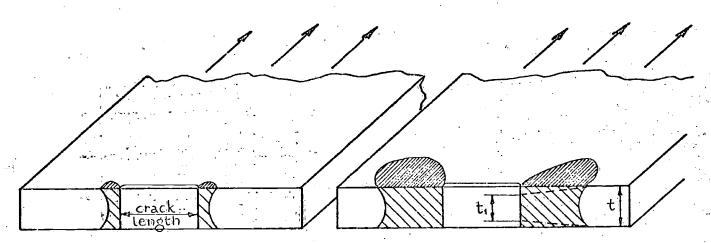
s = local contraction

Hole in thin plate.



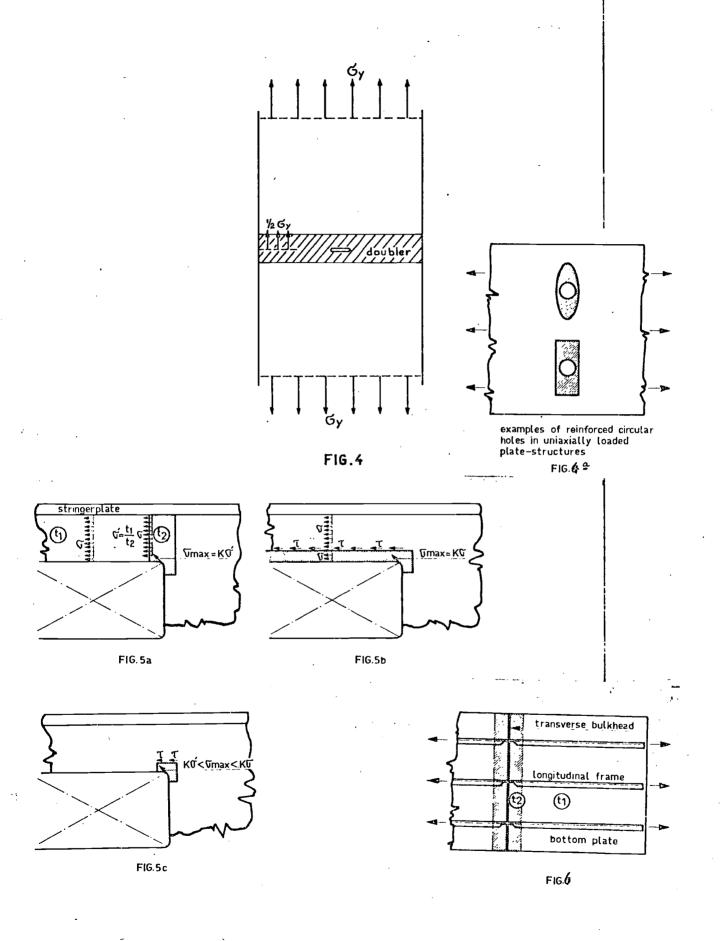
Notch in thick plate.

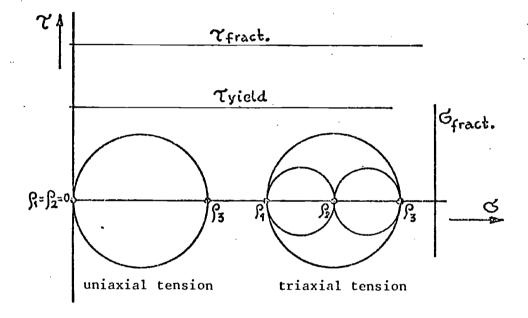
FIG. 2 Difference in contraction at a hole and a notch $(\frac{s_1}{t_1} >> \frac{s_2}{t_2})$.



Small plastic zone (plane strain). No contraction possible in thickness direction.

Large plastic zone (plane stress).
t.l = reduced thickness at notch tip.





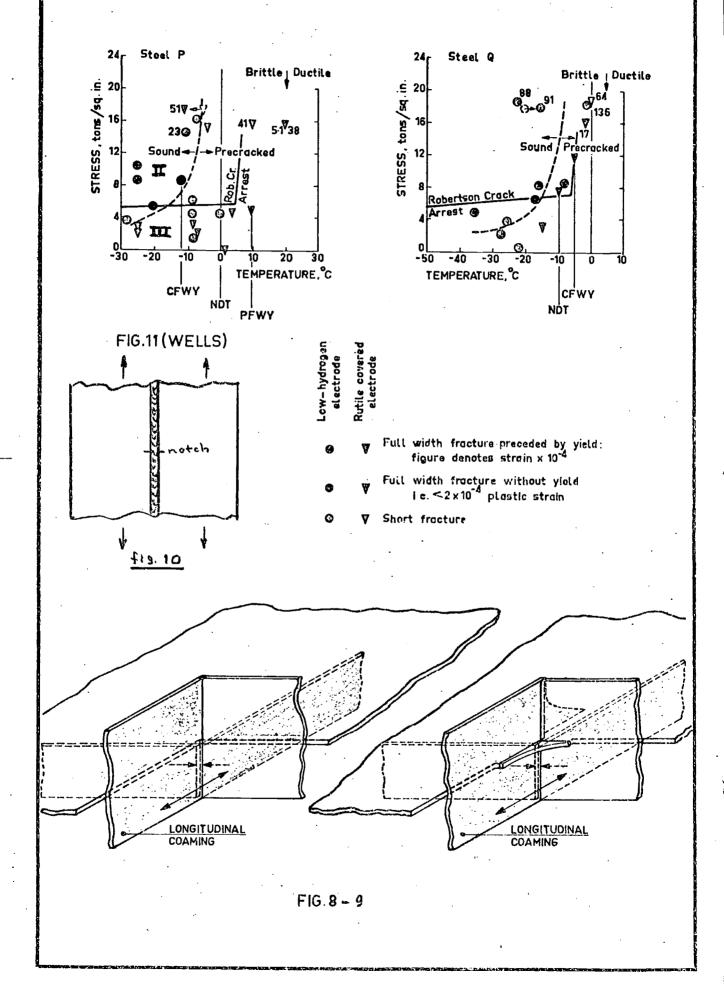
Yielding:
$$\frac{\rho_3 - \rho_1}{2} > \tau_{yield}$$

$$\rho_3 < \sigma_{fract}.$$
Fracture: $\frac{\rho_3 - \rho_1}{2} < \tau_{yield}$

$$\rho_3 = \sigma_{fract}.$$

FIG.7

Simplified explanation of influence of state of stress on yielding and fracture.



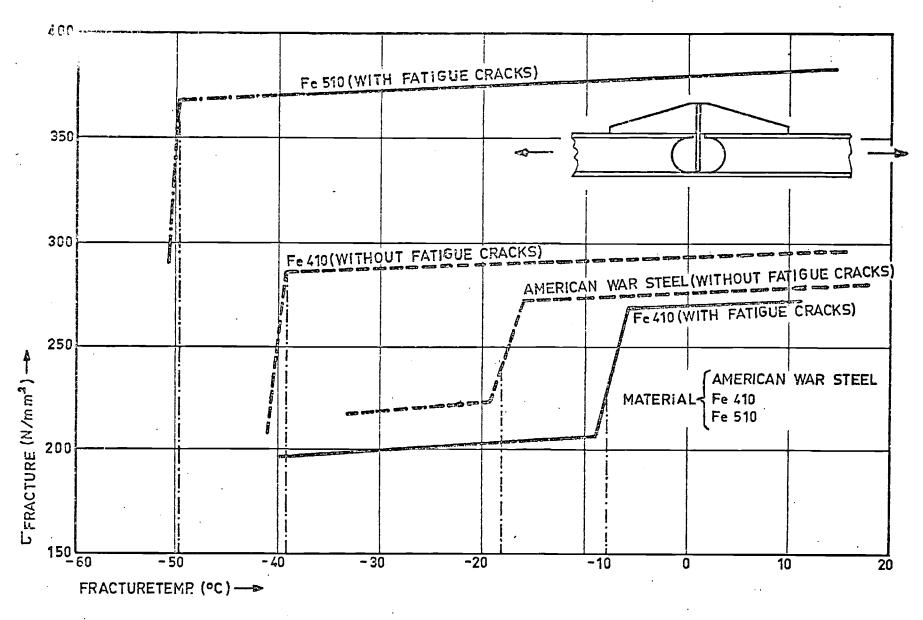


FIG.12 IMPROVEMENT OF SHIP STEEL SINCE 1940 AS REFLECTED BY FULL SCALE TESTS.

Fig.13. Ship fracture started at small fatigue-crack at about +4 °C [47].

 $\gamma_{i}, i\gamma_{i}$

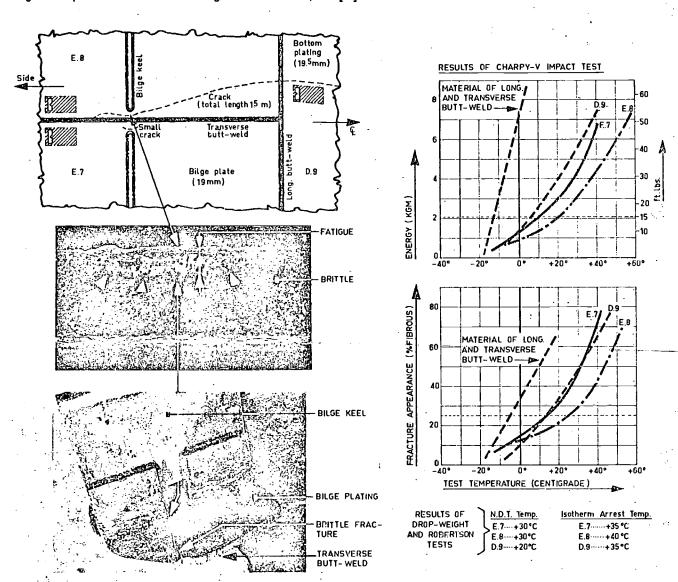
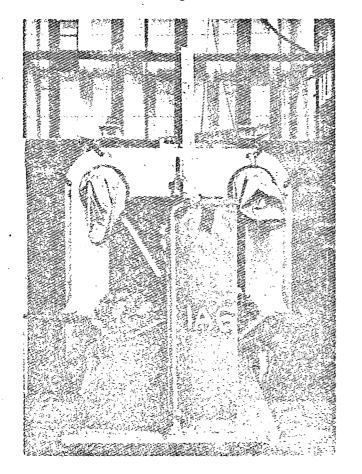
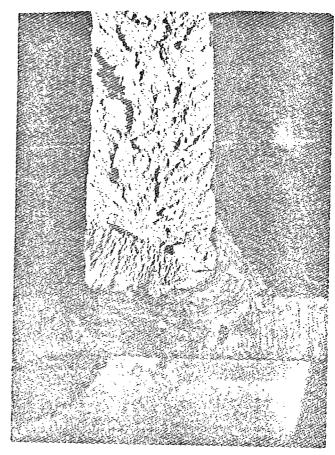
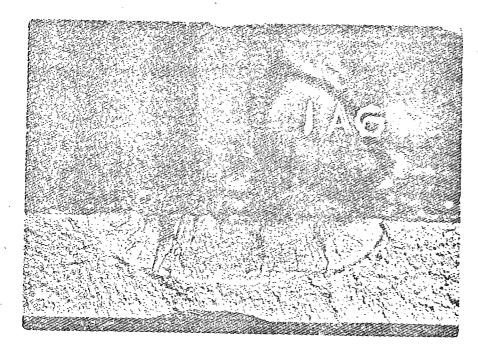
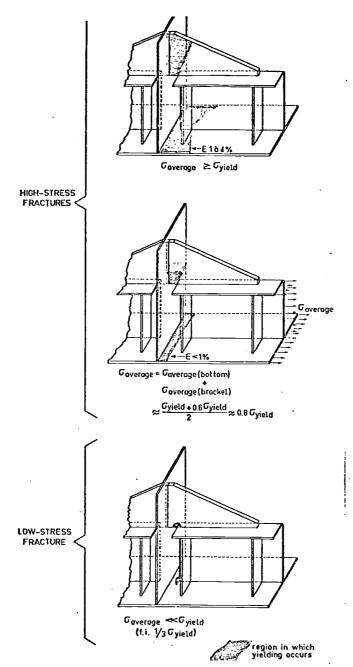


Fig. 14. Brittle fractures started from fatigue-cracks.



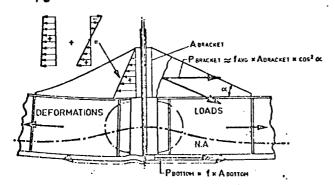






Fig/5. Plastic zone in case of high stress and low stress fractures

Fig (6 Illustration of inefficient use of bracket material.



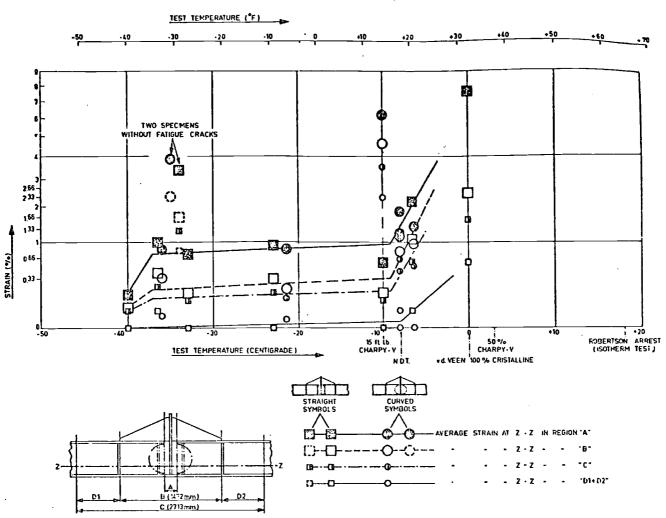


Fig. 1%. Strain at fracture for different gauge-lengths of specimens containing fatigue-cracks [44], [45].

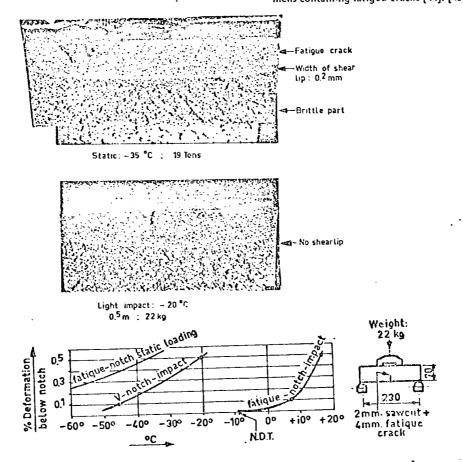
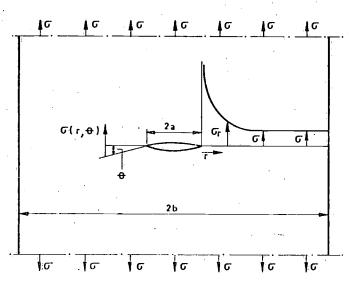


Fig. 8. Difference in cracking behaviour between specimens loaded statically and dynamically. Influence of notch acuity.

Introduction to fracture mechanics

Most introductions on fracture mechanics start with energy-approaches.

The strain energy released at a unit extension of an existing crack (e.g. extended by saw-cutting) is called G. At the moment the released energy becomes larger than the energy the material close to the crack tip can absorb (by deforming) an unstable fracture starts. In other words: the strain energy release rate has become critical (G_c) ; the resistance of the material to crack extension was no longer sufficient. This resistance – often called the fracture toughness – is the same as G_c .



$$\sigma(r,\theta) = \sigma \times f_1(a,b,\text{etc.}) \times f_2(r,\theta)$$

$$\downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow$$
load notch position of geometry
$$\sigma(r,\theta)$$
Stress intensity
$$= f_3(r) \times f_4(\theta) =$$

$$= \frac{1}{\sqrt{2\pi r}} \times f_4(\theta)$$
for $\theta = 0$ is
$$f_4(\theta) = 1 \quad f_2(r,\theta) =$$

$$\sigma_r = \frac{K}{\sqrt{2\pi r}} \leftarrow \frac{1}{\sqrt{2\pi r}}$$

For a through-thickness notch in an infinite wide plate is:

$$K = \sigma \times f_1(a) = \sigma \sqrt{\pi a} \to \sigma_r = \frac{\sigma \sqrt{\pi a}}{\sqrt{2\pi r}} = \sigma \sqrt{\frac{a}{2r}}$$

Fig. 14. The stress intensity parameter K.

It can be shown that

$$G = \frac{\pi a \sigma^2}{F}$$
, (see 2)

in which: $\sigma =$ nominal stress

a =half crack length

E = Young's modulus.

The foregoing has only been mentioned for the sake of completeness. For there is a more simple way to attack fracture problems by only looking to what happens at the tip of a notch or crack.

But we should forget thinking about stress concentrations. They depend too much on the sharpness of the notch and on the amount of plasticity occurring at the notch's tip. It is much simpler to say: what happens exactly at the tip is not feasible, but it is determined by the state of stress and strain in the immediate vicinity of the tip. When we are able to characterize that state we have obtained a very efficient tool for solving crack problems.

The stresses $\sigma_{(r,\theta)}$ near the crack-tip depend on:

- 1. The nominal stresses $(\sigma) \rightarrow \sigma_{r,\theta} = c_1 \cdot \sigma$;
- 2. The geometry and position of the crack.

For a centre-line through crack perpendicular to uniaxial load stress (Fig. 14) crack length 2a is the only geometric parameter that counts.

It can be shown that $\sigma_{r,\theta} = c_2 \sqrt{a}$.

When I and 2 are combined, the stress field around the crack tip is completely defined by $\sigma\sqrt{a}$. Historically $\sigma\sqrt{\pi a}$ has become usual and has been designated by K. (For other cracks, e.g.: part-thickness ones, instead of $\sqrt{\pi a}$ other expressions are valid).

For pure-elastic problems, where linear elastic fracture mechanics apply K suffices and $\sigma_{r,0}$ is not of any interest. But in cases of small scale yielding we need expressions for the size of the plastic zone, and for this we need to know the actual stress distribution. It can be shown that

$$\sigma_{r,\theta} = \frac{K}{\sqrt{2\pi r}} f(\theta)$$

For

$$\theta = 0, f(\theta) = 1 \rightarrow \sigma_r = \frac{K}{\sqrt{2\pi r}}$$

The use of K will be explained a little more. When a tensile test is carried out with a plate as in figure 14, K will rise in proportion to the load $(K = \sigma \sqrt{na})$, as long as no extensive yielding occurs. (This will be discussed further on). When a fracture develops the nominal fracture stress might be called the fracture

strength of the plate. But it is not a measure for the fracture toughness of the plate-material. For with another initial notch, a different fracture strength would have been found. Consequently fracture stress is not a material property as in the case of an unnotched bar. A good material property is K_c , the K-value at the moment of fracturing. For, one will observe from the test results that $\sigma_1 \sqrt{\pi a_1}$ of the first plate is equal to $\sigma_2 \sqrt{\pi a_2}$ of the second one (σ_1 and σ_2 are the nominal stresses at fracture). In other words: fracture develops when K reaches a critical value K_c , which is called the fracture toughness.

Nowadays K and K_c have largely replaced G and G_c mentioned in the beginning of the appendix, the relation between both is $K^2 = E.G.$ (plane stress) or $K^2 = E.G./1-v^2$ (plane strain), see figure 15.04 2

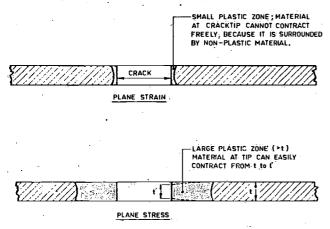


Fig. 15. Plane strain and plane stress.

Instead of K_c , the symbol K_{lc} is often used. The subscript I refers to fractures perpendicular to the notch plane like cleavage fractures in the case of steel. For all materials the fracture toughness is a function of thickness. The larger the thickness the smaller K_c . At a certain thickness a minimum K_c is found which is called K_{lc} : the plane strain fracture toughness. In fact only this value is a real material property. (For lower strength steels only at a particular temperature). Apart from the large capacity for plastic deforming, and the temperature dependancy of mild steel, the strong influence on it of fabrication procedures like cold forming, welding, flame cutting is of utmost significance. It makes that K_R -values for unimpaired parent metal have little practical value (see figure 18).

One might object that they could be useful for estimating the crack-arresting ability of a steel. But then another fact should be realized viz. the large dependency of K_{Ic} on loading speed. For crack-arresting only K_{IDc} -values are of interest, (D = dynamic), but in most cases the so called crack arrest

transition temperature has a greater practical value,

Correction for plasticity

For most industrial materials the initiation of a crack is preceded by more or less plastic deformation at the crack tip.

As long as the plastic zone is small (in relation to plate thickness) the stress state at the crack tip is strongly triaxial (plane strain). Then the average yield point in the plastic zone is about $\sqrt{3} \times \sigma_{\text{yield}}$. The size of the plastic zone can be calculated by realizing that at the edge of that zone (at $r = r_y$ in Fig. 16) the stress is equal to the local yield point. But that is not sufficient, it should also be taken into account that in the elastic (full line) and the elasto-plastic (dotted line) situation the stresses should be equivalent. It can be shown that this results in a size of plastic zone S to be equal to $2r_y$. (See (19.17)

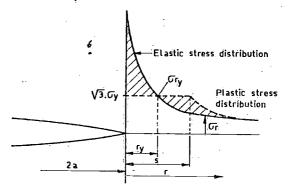


Fig. 16. Plastic zone in plane strain condition $(\sigma y_{local} = \sqrt{3} \cdot \sigma y_{normal}; \sigma y = yield point).$

As said before, r_{ν} can be calculated.

$$\rightarrow \ln$$

$$\sigma_r = \frac{K}{\sqrt{2\pi r}}$$

one should substitute $\sigma_y \sqrt{3}$ for σ_r and r_y for r.

$$\rightarrow r_{y} = \frac{K^{2}}{6\pi\sigma_{y}^{2}} \qquad \text{(plane strain)}.$$

For plane stress

$$\sigma_r = \sigma_y \to r_y = \frac{K^2}{2\pi\sigma_y^2}$$

In the plastic condition the stresses are cut off to the level of the local yield point, but the strains keep obeying more or less the formula:

$$E \cdot \varepsilon_r = \frac{K}{\sqrt{2\pi r}}$$
 ($E \cdot \varepsilon_r$ now replaces σ_r).

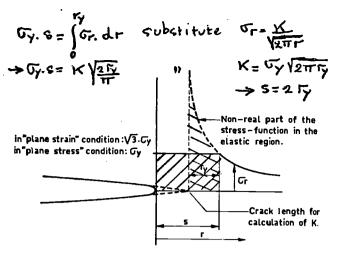


Fig. 17. Virtual crack-length in presence of plastic zone.

Fracture becomes rather a consequence of exhausting the capacity for *deformation*, than of surpassing a certain critical stress-value.

K should be corrected for the presence of a plastic zone. In figure 17 (from [7]) is illustrated that K is larger than in the fully elastic condition, because a notch with a plastic zone has a *virtual* crack length larger than the notch's length $\rightarrow K = \sigma \sqrt{\pi(a+r_y)}$.

The K-concept is of great value for fatigue-problems, (see 1.3), and fracture problems of very high strength materials. With steels of moderate strength fracture practically always occurs at nominal stresses close to or exceeding yield point. It will be clear that then the K-concept has not any significance. It should be replaced by the C.O.D.-concept. C.O.D. = crack opening displacement at the notch's tip; it is discussed in section 2.6.

C.O.D.-measurements

The attractiveness of the C.O.D.-concept is that its physical meaning is quite clear: it is the deformation (extension) of the material at the tip of a notch or crack. Wells [7] introduced this concept.

For lower strength steels of moderate thicknesses it is much more realistic to specify required fracture toughnesses in terms of critical C.O.D.'s than in terms of K_{Ic} or G_{Ic} . These steels owe their structural usefulness mainly to their large plastic deformability, so that a ductility-requirement is a logical consequence.

As discussed before critical C.O.D.'s for these steels should be in the order of magnitude of 0.2 to 0.4 mm.

Even for cracks as large as 100 mm in length, these values can only occur at nominal stresses equal to or larger than yield point.

This can be illustrated as follows (see Fig. 12).

For a crack with a plastic zone $2r_y$, the virtual crack length is about $2(a+r_y)$.

From

$$\delta_x = \frac{4\sigma}{E} \sqrt{a^2 - x^2}$$

the C.O.D. at crack tip can be calculated by sub-

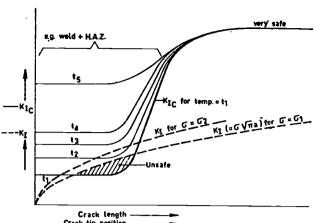


Fig 18. Relative importance of crack-length and crack position in welded region. (See which fig. 19) stituting a+r, for a and a for x

$$\rightarrow \text{C.O.D.} = \frac{4\sigma}{E} \sqrt{r_y^2 + 2a \cdot r_y};$$

when C.O.D. = 0.4 and $\sigma = 30 \text{ kg/mm}^2 \rightarrow r_y = 35 \text{ mm}$. When linear elastic fracture mechanics would be valid for this situation, the nominal stress should be equal to the one calculated from

$$r_y = \frac{a\sigma^2}{2\sigma_y^2}$$
 (see appendix)

For a steel with a yield point of 35 kg/mm² this leads to

$$\sigma = \sqrt{\frac{2r_y \cdot \sigma_y^2}{a}} = \sqrt{\frac{2 \cdot 35 \cdot 35^2}{50}} = 41 \text{ kg/mm}^2$$

This is larger than yield point indeed, and linear elastic fracture mechanics do certainly not apply.

For a C.O.D. of 0.3 mm, $r_y = 22$ mm and $\sigma_{\text{calc}} = 33$ kg/mm². This is also too close to σ_y for valid calculation results.

One disadvantage of C.O.D._c as a fracture criterion is, that it is probably not independent of crack length. For from C.O.D. = $\frac{4\sigma}{E} \sqrt{r_y^2 + 2a \cdot r_y}$

it can be seen that when $\sigma \simeq \sigma_y$, the larger the crack, the larger the C.O.D. at a certain size of plastic zone. Now, as is known [7], the size of the plastic zone (in relation to plate thickness) is a measure for the stress state in that plastic zone, $(r_y \ge t \rightarrow \text{fully plane stress})$ (see Fig. 15)). Consequently the larger a crack the larger the C.O.D. needed for plane stress.

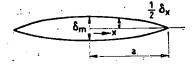
Nevertheless, in view of the many unknown factors involved, and for cracks or defects between 10 and 100 mm length, one single C.O.D.-value will suffice. It has the advantage that for very thick plates, Wells requirement of $r_y \ge t$ does not lead to unrealistically high toughness values. A C.O.D. of say 0.3 mm indeed represents a quite satisfactory deformability. The fact that it corresponds to a situation of plane strain in very thick plates is not objectionable at all.

Summary C.D.D.

For a crack without a plastic zone applies

$$\delta_{x} = \frac{4\sigma}{E} \sqrt{a^2 - x^2}$$





A crack with a plastic zone has a virtual length

$$a' = a + r_y$$

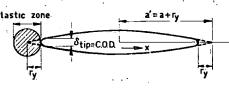


Fig. 12. The C.O.D. concept.

(1) Becomes:

$$\delta_x = \frac{4\sigma}{E} \sqrt{(a+r_y)^2 - x^2} \tag{2}$$

For x = 0 is

$$\delta_{\dot{x}} = \delta_{\dot{m}} \rightarrow \delta_{\dot{m}} = \frac{4\sigma}{F}(a + r_{\dot{y}})$$

This results in:

$$\mathbf{r}_{\mathbf{y}} = \frac{E \cdot \delta_{m}}{4\sigma} - a \tag{3}$$

so the plastic zone can be calculated when δ_m has been measured.

Substituting x = a in (2) \rightarrow

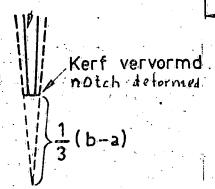
$$\delta_{\text{tip}} = \text{C.O.D.} = \frac{4\sigma}{F} \sqrt{r_y^2 + 2a \cdot r_y}$$
 (4)

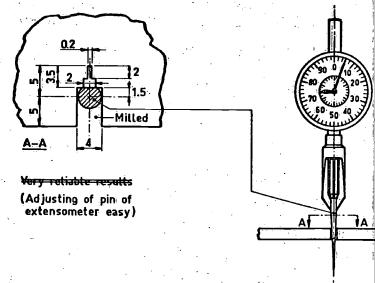
(relation $\delta_{tip} \leftrightarrow r_y$)

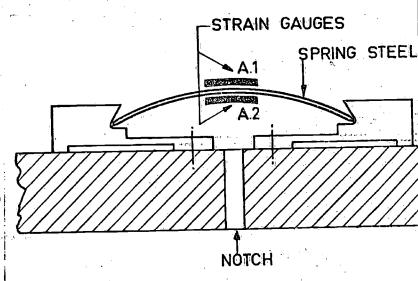
Substitution (3) in (4) \rightarrow

C.O.D. =
$$\sqrt{\delta_m^2 - \left(\frac{4a \cdot \sigma}{E}\right)^2}$$

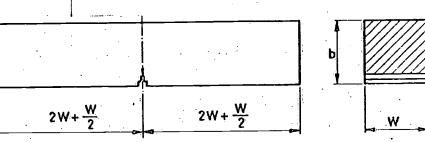
(Relation C.O.D. $\leftrightarrow \delta_m$)

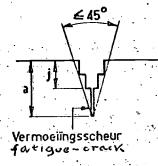






MEASUREMENT OF C.O.D.





Afmetingen proefstuk b = W $a = \frac{W}{3}$ $j = \frac{a}{2}$ Opleglengte is 4W.

2. The strain energy release rate G

Figure 19 is a model of a plate from which the strain energy release rate G can be understood. As mentioned in 1 it is the strain energy released when an existing crack is extended one unit. When the springs are situated at distances of one unit, f.i. 1 cm, the crack extension has the effect that at each crack tip an additional spring tries to open the crack.

The spring shortens, which means that strain energy is released. Other springs pulling at the crack are also reduced in length. The total energy is used for deforming the material at the crack tip. In other words for increasing the C.O.D. and the plastic zone.

It should be realized that it does not make any difference whether the crack extends by fracturing or by saw-cutting. For at the moment fracture occurs, the deformation capacity (ductility) of the material at the crack tip is reduced to practically zero. Only the surface energy has to be overcome, which for steels — is small as compared to the energy previously used for plastically deforming the crack-tip material.

Relation between G and K (fig. 21)

$$G = \frac{1}{\Delta a} \int_{0}^{\Delta a} \frac{1}{2} \sigma_{r} \cdot 2\eta_{r} dr; \qquad \sigma_{r} = \frac{K}{\sqrt{2\pi r}}; \qquad \eta_{r} = \frac{2\sigma}{E} \sqrt{a^{2} - x^{2}}$$
a becomes $a + \Delta a$; in the domain Δa is $x = a + r$

$$+ G = \frac{1}{\Delta a} \int_{0}^{\Delta a} \frac{1}{2} \frac{K}{\sqrt{2\pi r}} \cdot 2 \cdot \frac{2\sigma}{E} \cdot \sqrt{2a(\Delta a - r)}$$
with $\sigma = \frac{K}{\sqrt{\pi a}}$

$$+ G = \frac{2K^{2}}{\pi \cdot \Delta a \cdot E} \int_{0}^{\Delta a} \frac{\sqrt{(\Delta a - r)}}{\sqrt{r}}$$
Substitute $\sqrt{r} = \sqrt{\Delta a} \cdot \sin\phi$

$$+ G = \frac{1}{\Delta a} \frac{K^{2}}{E\pi} \cdot 4 \cdot \int_{0}^{\pi/2} \sqrt{\Delta a(1 - \sin^{2}\phi) \cdot \Delta a} \cdot \cos\phi \cdot d\phi$$

$$= 4 \frac{K^{2}}{E \cdot \pi} \int_{0}^{\pi/2} \cos^{2}\phi \cdot d\phi = \frac{4K^{2}}{E \cdot \pi} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$

$$+ \int_{0}^{\pi/2} \frac{K^{2}}{E \cdot \pi} \cdot \frac{\pi}{2} \cdot \frac{\pi}{2} \cdot \frac{\pi}{2}$$
(For plane strain: $G_{1} = \frac{K_{1}^{2}(1 - \nu^{2})}{E}$).

The energy released during enlarging the crack one cm, increases with crack length.

At a certain moment this energy becomes larger than is necessary for cracking another cm. In other words the strain energy release rate becomes larger than the fracture toughness (in terms of energy). The strain energy release rate G has become $\underline{\text{critical}} \rightarrow G_{c}$ and equal to the fracture toughness.

At the same time the stress intensity factor K has become equal to K the critical K or corresponding fracture toughness.

The determination of K_{c} and G_{c} by experiments often causes disappointments.

Structural steel is so tough that a precracked plate always fractures at nominal stress close to yield point. In the crack section, the material has fully yielded , which means that the plastic zone has become (infinitely) large.

Then the formulae $K = \sigma \sqrt{\pi}a$ and $G = \frac{\pi a \sigma^2}{E}$ are no longer valid. They may only be used for plastic zones smaller than $\frac{1}{2}a$. For at $r = \frac{1}{2}a$, σ_r has become equal to σ . (Follows from $\sigma_r = \frac{K}{\sqrt{2\pi}r}$ and $K = \sigma \sqrt{\pi}a$).

At very low temperatures steel becomes so brittle that valid K_{1} or G_{1} ex-

periments can be carried out. Then it becomes clear that they are real material properties. For it does not make any difference whether the plates have small or large cracks before testing. The product $\sigma_{\rm fract} \times \sqrt{a}$ will always be the same.

Welding may spoil the properties of the material to such an extent (see 3: Practical cases), that the weld zone may fracture brittle at service temperatures. Shock loading can have the same effect. (See chapter Brittle Fracture and Fatigue). It may enlarge the yield point so much that the material cannot deform plastically and fractures in an elastic manner. This is what happens during brittle crack propagation.

Relation between G and C.O.D.

In section 1 it has been found:

$$\delta_{\text{tip}} = \text{C.O.D.} = \frac{4\sigma}{E} \sqrt{r_y^2 + 2a} \cdot r_y$$

$$= \frac{4\sigma}{E} \sqrt{r_y(r_y + 2a)}$$

$$\approx \frac{4\sigma}{E} \sqrt{r_y(2r_y + 2a)} \qquad (r_y \le a)$$
with $K = \sigma \sqrt{\pi(a + r_y)} \rightarrow \text{C.O.D.} = \frac{4K}{E} \sqrt{\frac{2r_y}{\pi}}$
Substitute $r_y = \frac{K^2}{2\pi\sigma_y^2} \rightarrow \text{C.O.D.} = \frac{4K^2}{\pi E.\sigma_y}$ and with $K^2 = E.G$

$$\Rightarrow G = \frac{\pi}{4} \cdot \text{C.O.D.} \sigma_y$$

The factor $\frac{\pi}{4}$ is partly due to the approximation made, that $r_y + 2a \approx 2r_y + 2a$. For greater r_y 's, $\frac{\pi}{4}$ changes into 1.

Burdekin and Stone /9/, /16/ have shown that for the Dugdale-Muskhelishvilli model (fig. 20) the same applies for small ry-values. (In this model the crack is supposed to have a length (a+s) and surrounded by two stress-fields, one internal and one external, consisting of stresses of on the part S of the crack). From these considerations it follows that

$$G = (C.0.D.) \cdot \sigma_y$$

is most likely.

3. Minimum required toughness

Irwin and Wells have stated that a material has sufficient fracture toughness when it can deform so much at crack tips that the plane stress condition is obtained.

In other words: the plastic zone S at the crack tip should become at least two times the plate thickness (see fig. 15) before fracture starts. The point is that when a material has not fractured in the dangerous plane strain condition, it will be able to deform largely in the plane stress condition. Either it fractures before S=2t or it can only fracture at S>>2t. For steel this fact is responsible for the rather abrupt transition from brittle to ductile behaviour. For when it fractures at S<2t at temperature T1, a slight increase in temperature might bring the material at the crack tip in the plane stress condition. Then it can fracture only at a much higher load and corresponding S>>2t.

S = 2t can be translated into terms of C.O.D. for

$$r_y = \frac{K^2}{2\pi\sigma_y^2} = \frac{EG}{2\pi\sigma_y^2} = \frac{E\sigma_y(C.O.D.)}{2\pi\sigma_y^2}$$

$$\Rightarrow \quad \text{C.O.D.}_{\text{crit.}} = \frac{2\pi\sigma_{y}.t}{E}$$

So, C.O.D. crit. is not dependant on crack length and related to plate thickness. The latter is less reasonable than it seems. For thick plates the C.O.D.'s crit.

For 100 mm plate it is about 1 mm. In practice the critical values are in the order of magnitude of 0.2 mm.

Apart from that S = 2t as separation between plane strain and plane stress is too pessimistic, in /5/, /12/ and /13/ it has been found that S = t is sufficient.

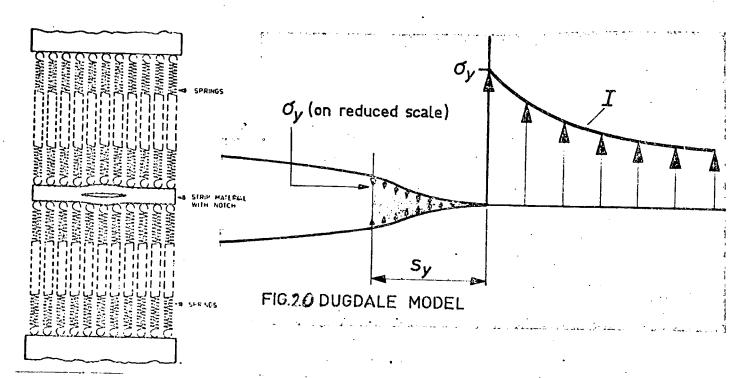


Fig. 19. Model of plate with notch.

Brittle fracture testing.

The most well-known and established test is Charpy-V-notch impact testing. In figure 1 its morits and shortcomings are listed.

The value of the test may be improved by including fracture appearance, notch. root contraction and by instrumentation. But for welded joints with their heterogeneous material structure the test will remain inadequate and its re sults will always be subject to controversial interpretations. Apart from that there is little sense in doing tests of which the shortcomings have to be corrected theroughly when these shortcomings can easily be overcome by

improving the test itself. This does not mean that Charpy-testing should be rejected completely. Work tarried out by members of the I.I.W. Working Group 2912: "Brittle fracture tests for weld metal" /#/ has shown that for those cases where existing Charpy specifications can easily be met, and plate thicknesses are below

35 mm, more cophisticated testing is not necessary. When a value higher th 35 Nm/cm² at the lowest service temperature is obtained at any place of a welded joint the structure will be safe. This may be concluded from figure The 35 Nm/cm² Charpy transition temperature has been plotted against the transition temperature obtained with a dynamically loaded, sharp noteled, welded specimen of full plate thickness satisfying a realistic ductility

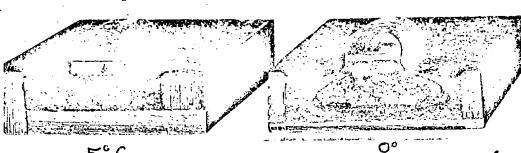
The so-called "Hiblink" test may be looked upon as a procedure in which severe service conditions are simulated. As such it provides a good testcase for judging the reliability of the Charpy-test. It should be observed that the results in the top left part of the figure presumably owe their favourable position to the fact that they have not been taken from the centre of the plate (half plate thickness). Figure 3 shows the effect: only the lowest values, which are for half-plate thickness, correspond to the Miblink

A basically different test is the static C.O.D.-test. There is little correlation between its results and Charpy-date; C.O.D. results are often on the optimistic side. This is quite logical and does not invalidate the test at all for statically (or quasi-statically) loaded structures. (Most lower strength steels are very sensitive to impact loading in presence of sharp notches). The problem for the structural designers is whether for instance wave impact is a quasi-static or a dynamic phenomenon. Measurements on a "siamming" ship have given support to the idea of quasi-static loading 🔊. Another point is that apart from external dynamic loading there is the possibility of internal dynamic loading. In figure 5 some cases are indicated. They all have occurred in practice as well as in laboratories. The solutions are indicated in Figure 6. Taking figures 1, 4, 5 and 6 rogether it appears that a most realistic and practical way of testing is fatigue loading at low temperature. It embraces static, cyclic and dynamic loading, embrittlement, pop-in, hererogeneous character of welded joints and casiness of testing and interpretation (see figure 7).

In a way static C.O.D.-loading also incorporates embrittlement. For, for wolded joints the specimens have to be compressed on the sides in order to climinate the welding stresses and facilitate the development of a small fatigue-crack. The latter itself adds to the strain hardening at the tip of the crack. "evertheless pop-ins are hardly possible. The stipulated minimum C.O.D.-values require such high loads that practically always complete fractures develop.

Another thing is that precompressing is an unrealistic form of appreciable demage of the metal. Figure 8 shows results of tests carried out in the Delft Ship Structures Laboratory. A test program embracing multi-pass S.A.-welding is underway. It can be seen that in the region of interest (C.O.D.'s between 0,2 and 0,3 mm) there may be a shift in permissible lowest service temperature of some 15°C. But the welding engineer is not only penalised by this shift (as compared to heat-treated structures). For, the critical C.O.D. thus found is used in calculations of critical crack length with a stress-value equal to the sum of the load stresses and the <u>ransile</u> welding stresses. In other words in a multi-run weld, the metal at half plate thickness is spoiled by compressive deformation, deprived of its protective compressive residual stresses and calculated with non-existing tensile residual stresses in two respects: critical length and fatigue-growth. Such a procedure leaves little of one argument for C.O.D.-testing: "that a full-thickness weld is required because it conforms best to service conditions".

Finally it should be observed that the utmost safety can only be obtained by using steels which are able to arrest brittle cracks at low service temper aturus. That even then some problems are left is indicated in signe fig. 4

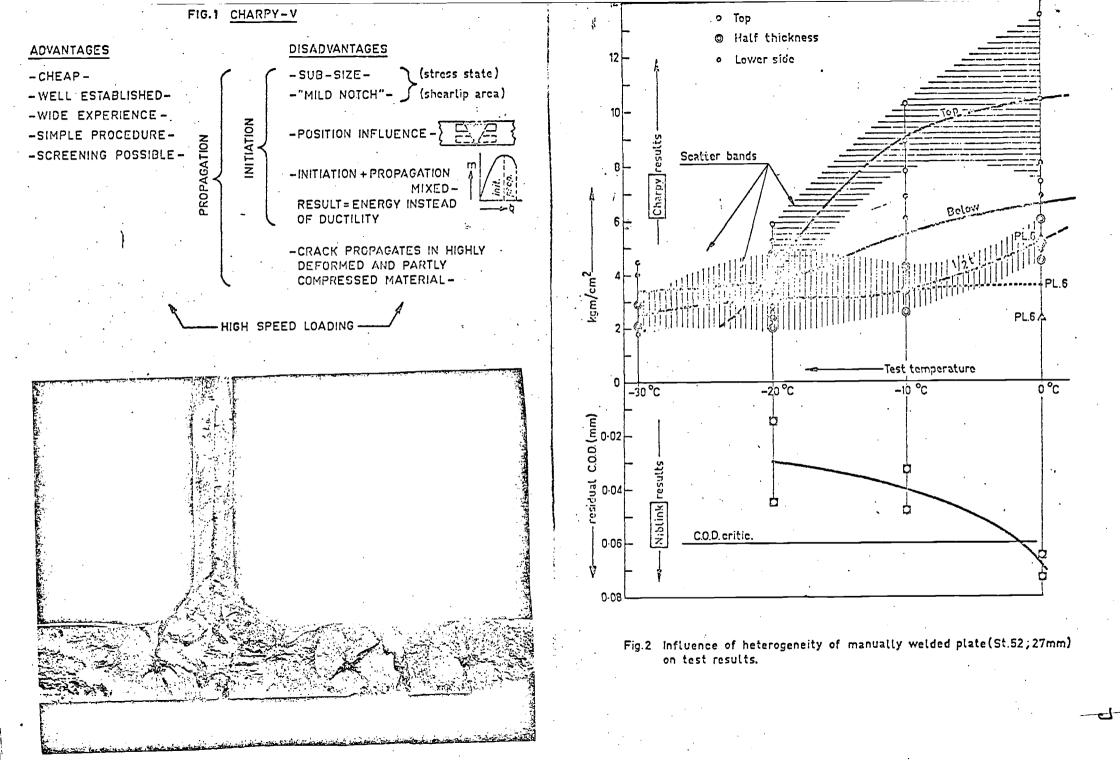




+5°C

-500

Drop Weight test (see fisb)





DISADVANTAGES OF CHARPY ELIMINATED IN :

STATIC C.O.D.-TEST

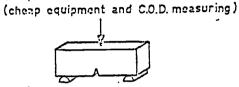
2. DYNAMIC NIBLINK-TEST

OBJECTION: LOW SPEED STRESSING.

External or internal

dynamic phenomena occur in practice.

BUT: WHAT IS THE PROBABILITY?



3. DYNAMIC C.C.D.

(expensive equipment and C.O.D. recording)

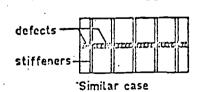
(stepwise increased drop-weight loading)

BASIC PHILOSOPHY: 100 % PREVENTION OF CRACK INITIATION IS POSSIBLE.

FIG. 4

FIG.5 INTERNAL CAUSES OF DYNAMIC LOADING: "POP-IN" AT LOCALLY EMBRITTLED NOTCH-TIPS.

a. HOT-STRAINING EMBRITTLEMENT + HIGH RESIDUAL STRESSES (GREENE-WELLS [4]):





ಾnd weld tip of defect may be plastically deformed by 2nd weld at "ageing" temperatures.

b. FATIGUE-EMBRITTLEMENT [13] (plastic zones)

c. HYDROGEN-EMBRITTLEMENT

d. a.+ b. PROBABLY MOST DANGEROUS [3]; [13]

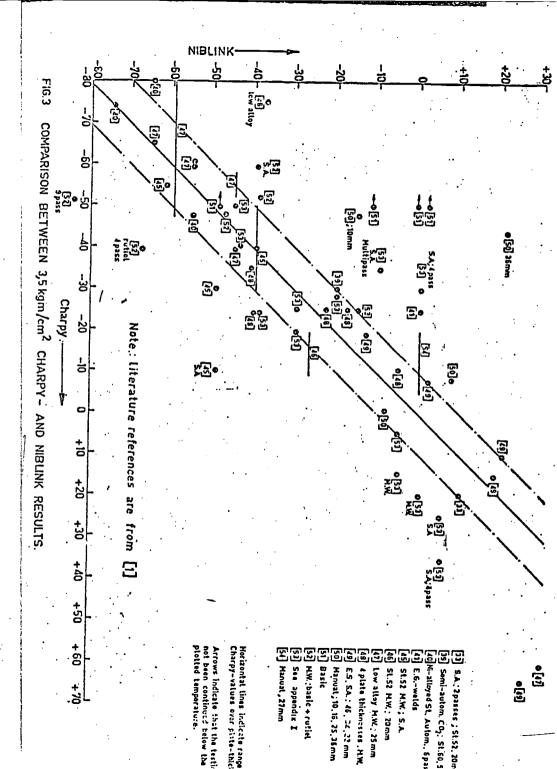


FIG.6 SOLUTIONS:

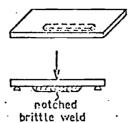
- 100% NON-DESTRUCTIVE TESTING OF WELD CROSSINGS.
- WELD AND BASE METAL MUST BE ABLE TO ARREST SHORT CRACKS.

FUNDAMENTAL TEST:

PELLINI-DROP WEIGHT TEST

HANDICAPS:

-NOT FOR THICK PLATES--COARSE GRAINED ZONES IMPROVE BY BRITTLE WELDING-



3. APPLY: FATIGUE TESTING AT LOW TEMPERATURES. .

FIG.9 BEST PHILOSOPHY: CRACK-ARRESTING STEELS

REQUIRED: LARGE SCALE EXPERIMENTS (Robertson long plate)

GOOD INDICATION: DROP WEIGHT TEAR TESTS.

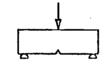


FIG.7 FATIGUE TESTING AT LOW TEMPERATURES.

ADVANTAGES:

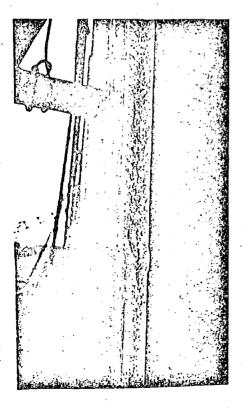
- 1. REALISTIC LOADING, NOTCH AND EMBRITTLED ZOME
- 2. INCLUDES DYNAMIC POP-INS (brittle steps)

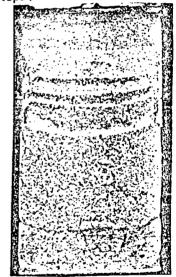
UNRELIABLE : CHARPY-TESTS [3]

PROBLEMS: NOT COMPLETELY SAFE FOR TRANSVERSE WELDS WHEN WELDING STRESSES CANNOT FORCE CRACKS

OUT OF WELDED REGION:

- -IN CASE OF POST WELD TREATMENT-
- -HIGH HEAT INPUT WELDING [3] -
- -NOMINAL STRESSES APPROACHING YIELD POINT-





3. RESULT IS CRITICAL CRACK LENGTH

--- kcr. or C(nom.)cr. or E(hom)cr.

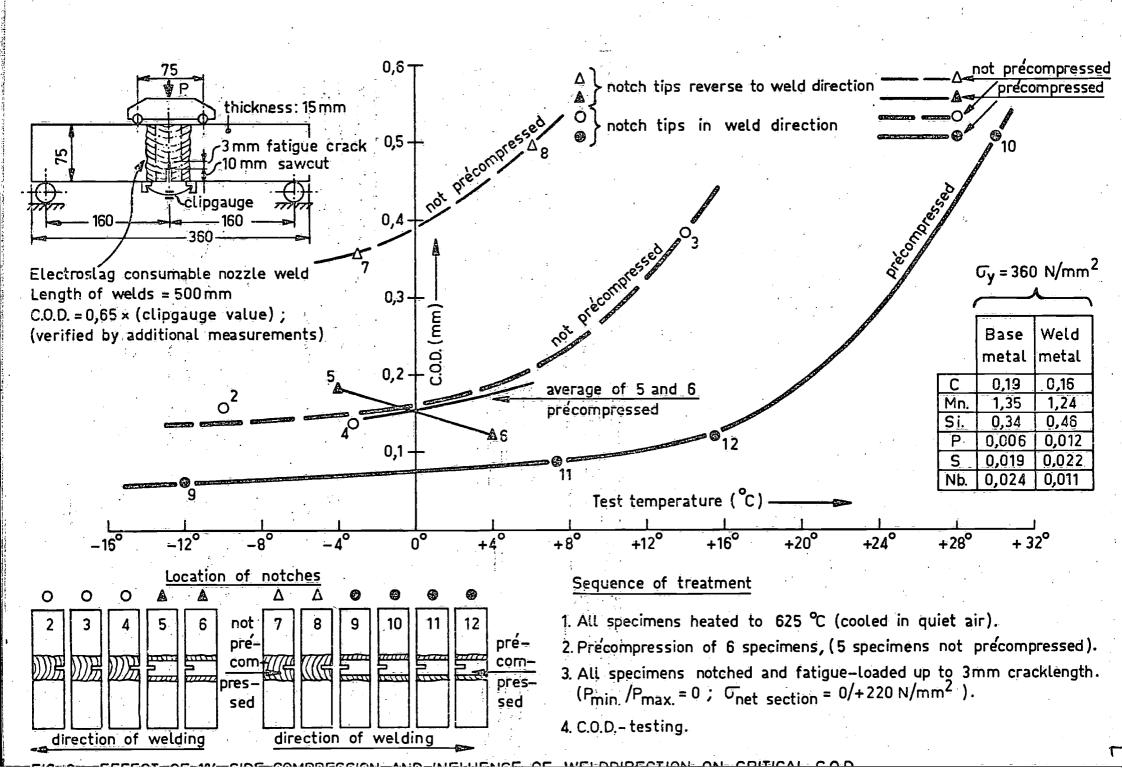
4. LARGE PART OF SPECIMEN IS TESTED.

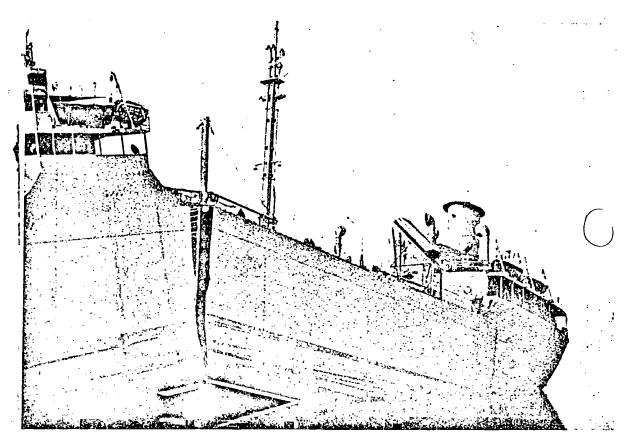


C.O.D. **NIBLINK** one initiation point

(crack "finds" point of lowest material quality)

- 5. BRITTLE STEP FORMING AT SMALL CRACK LENGTH OCCURS AT NIL-DUCTILITY-TEMPERATURE.
- 5. NO C.O.D. MEASUREMENTS NOR PROBLEMS WITH ROTATION POINT.





5. Practical cases.

5.1 General

Cracks in ships are either fatigue-cracks or brittle cracks.

Fatigue-cracks are most common but the danger involved is small.

Brittle cracks are very scarce nowadays, but when they occur, the ship obviously is in danger.

Both types of cracks generally start at defects due to welding or flame cutting especially when these defects are situated at geometrical stress concentrations.

1.1 Defects

In ships with their enormous amounts of welds in between 10 and 1000 km in length, weld defects can never be completely avoided, despite intense nondestructive control.

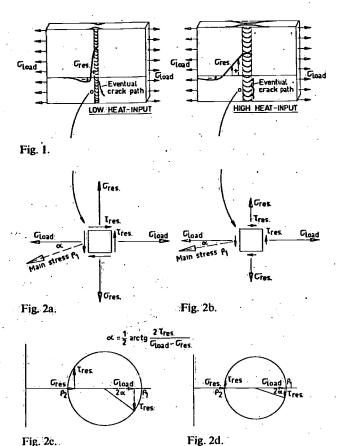
The most important defects are under-cuts, lack of fusion, slag-inclusions, incomplete penetration and cracks in welds and heat-affected zones, (Fig. 5).

In principle one can achieve that not any defect will develop into a large brittle fracture. For this, it is necessary that proper weld and parent materials are

chosen and welding methods are avoided, which excessively destroy the originally sound parent material in a relatively extended zone. Even the mere use of parent material of such high quality that any eventual brittle crack will be arrested, is mostly completely satisfactory. Then the quality of the welds and the H.A. Zones would be of secondary importance from a safety-point of view, and this could lead to substantial reductions in cost of welding, production and quality control. Unfortunately when high heat input-welding methods like one pass submerged-arc, electrogas or electroslagwelding is applied, eventual cracks starting in the welds or heat-affected zones, are not always leaving the welded region under the influence of the residual welding stresses, as is the case with multipass-welding. (See 1.2).

1.2 Residual stresses

The stress field set up with high heat input welding has a much smaller gradient than the one created by low heat input welding, (Fig. 1). Due to that the shear stresses in planes parallel to the weld are also smaller,



and so are the corresponding normal stresses, of which the direction differs 45° from the weld line.

small).

Largest main stress nearly in

line with loading-stresses (α

Largest main stress deviates a

with loading-stresses.

As fracture will always develop in a direction perpendicular to the main tensile stress, it will deviate more from the weld line, the higher the shear stresses be.

In figures 2a and 2b the stress situation for a small element situated in the H.A.Z. is indicated; an estimate of the angle α between the load stresses (nominal stresses) and the main tensile stresses is made in fig. 2c and 2d. It should be reminded that the shear stresses τ_{res} can only exist when the residual tensile stresses (σ_{res}) also vary along the weld line. This is valid for hand-welding with many stops and starts, but far less true for automatic and semi-automatic welding, especially with welds made in one pass.

Residual stresses have often been blamed for causing brittle fracture. This was right for ships constructed during and shortly after World War II, when low-stress brittle fractures were very common and the steel was often not good enough to arrest them. But nowadays residual stresses are rather beneficial. The ships' steel is so much better than formerly, that eventual cracks started in a weld or H.A.Z. are mostly arrested in the parent plate before attaining a critical length. (This is the started with the parent plate before attaining a critical length.)

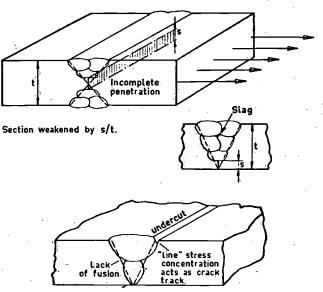


Fig. 5. Main weld defects.

It is unfortunate that in the specifications of steels reliable crack arrest tests are not required. The desired properties are now more or less obtained by requiring certain Charpy-energies at certain temperatures. The correlation between these transition temperatures and the Robertson crack arrest temperature is not only poor, (the latter is on the average some 40°C higher than the 3.5 kgm/cm² Charpy temperature), but the scatter is also large. For individual steels the difference between 3.5 kgm/cm² Charpy and Robertson crack. arrest temperature can be 0°C to 80°C. (See Fig. 7, obtained from [2] by Drs. H. C. van Elst). This does not mean that the Charpy-test has no use anymore in this respect. Verbraak and Van Elst have already long ago propagandized to use it for quality control of steels with specific composition, made according to a fixed procedure. For such a well defined case, the relation between the Robertson crack-arrest temperature and for instance the Charpy-energy at that temperature shows little scatter and can be used for quality

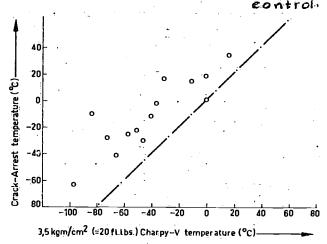


Fig. 7. Relation between 20 ft. lbs. charpy-transition temperature and crack-arrest temperature (Van Elst [2]).

5,2 High heat input welding

There are however some points which need special attention. Firstly again the case of high heat input welds. It is very well possible, and demonstrated in [1], that fatigue-cracks propagate completely in the weld or H.A.Z. (see Fig. 4). In the case considered a

3 mm wide (or narrow!) coarse grained zone existed of very poor notch toughness along an E.G.-weld in a 34 mm plate. Notwithstanding the narrowness of this zone, high stress-low cycle fatigue cracks indeed propagated in that zone over quite a distance (up to 120 mm). The large danger involved can be appreciated when it is known that the fatigue-crack "jumped" forth in a brittle way over a small distance several times (5-10 mm) before it finally developed into a complete brittle fracture which also ran all along the weld line! The test-temperature was -20° C, the nominal fracture stress 24 kg/mm². The steel was extremely good (E-quality), as is obvious from the Charpy V notch—

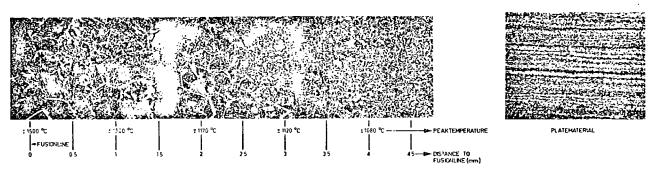


Fig. 3a. Extreme grain coarsening in H.A.Z. of E.G.-welded fine grain steel, (Nb-containing normalised).

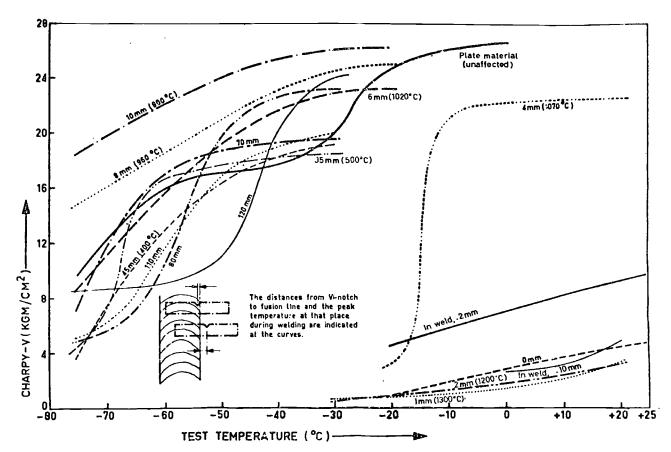


Fig. 3b. Charpy V values of the H.A.Z., St. 52+Nb (thickness: 34 mm) and the E.G.-weld [5].

energy which was 16 kgm/cm² at -60°C! The reduction in quality in the H.A.Z. due to the high heat input of the E.G.-welding is shown in figures 3a and 3b. The crack paths are shown in figure 4.

In terms of "safe" temperature the steel has been spoiled nearly 100°C. Similar dangerous situations can also occur with more normal welding methods – especially when the number of passes is not very high – or when one side butt welding has been applied. Figure 5 shows some examples.

It will be clear, that cracks developing at the points of incomplete penetration will not easily leave that vertical plane, as long as the cross-section remains reduced by that lack of penetration. A similar situation may occur at undercuts and long lacks of fusion, (Fig. 5).

It is evident that transverse butt welds in ship decks, sides and bottom are more critical in this respect than longitudinal butt welds. But generally speaking only really long defects are dangerous. Most of the standards and specifications as to defects are too pessimistic. On the other hand it is well recognized that these specifications have mainly the purpose of guaranteeing a satisfactory level of workmanship, thus having a function in quality control which of course can never be dispensed with.

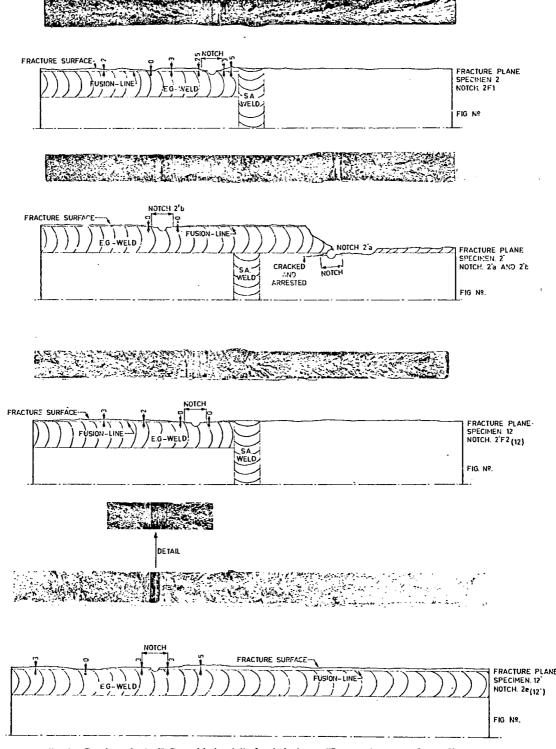
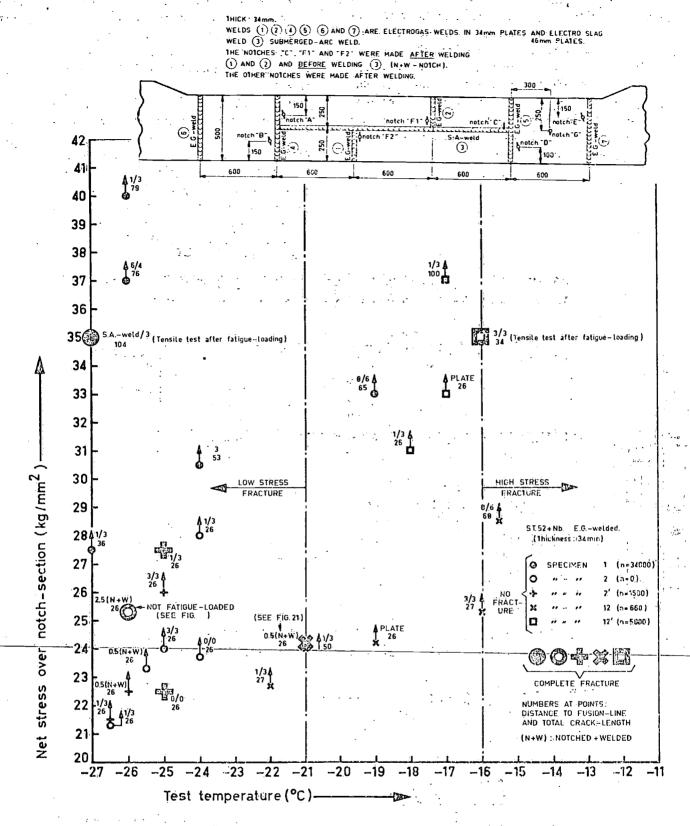


Fig. 4. Crack paths in E.G.-welded axially loaded plates. (For specimen see figure 8).



SE S. Final observations

Fig. 9. Net fracture-stress as function of temperature [5].

Figure 10 is a good illustration of the usefulness of the C.O.D.-concept. There is a clear separation between the "bad" and "good" results, which contrasts sharply with figure 9. The figure however demonstrates also how important welding parameters may be, and that these may completely overshadow the influence of other parameters like crack length. This has already been discussed before as far as the position of the cracks relative to the fusion line concerns. But in this final section the attention is drawn to such an apparent-

ly secondary factor like sequence of welding. The point quite to the right:

$$0 (N+W) 28; n = 10,000$$

represents a partial fracture, which started at a notch, which was made in the H.A.Z. of a transverse E.G. weld *prior* to the subsequent longitudinal submerged

arc welding. The distance between the notch and the S.A.-weld was so small that the material at the notch tip during the S.A.-welding was strained plastically to and fro at a temperature between 300 and 500°C. This caused hot straining embrittlement, which once more reduced the quality of the H.A.Z. at the tip concerned appreciably. (Greene-Wells' embrittlement).

A similar effect may occur at weld defects close to weld-crossing (Fig. 13). The transition temperature of the transverse (first) weld can be increased some 60°C, when the defect is at a critical distance from the (second) longitudinal weld.

Stip (0.01mm)

The unfortunate dilemma for the structural engineer is, that good engineering practice is a welding sequence as indicated in figure 13. For then, there is the smallest chance that weld defects occur. But whenever they occur, the situation may be far more dangerous than when defects are present in a *last* made transverse weld. For cases like the former, two solutions are available:

- 1. Non destructive testing of all weld crossings, (also where stiffeners cross butt-welds).
- 2. Require that the weld metal and H.A.Z. have a

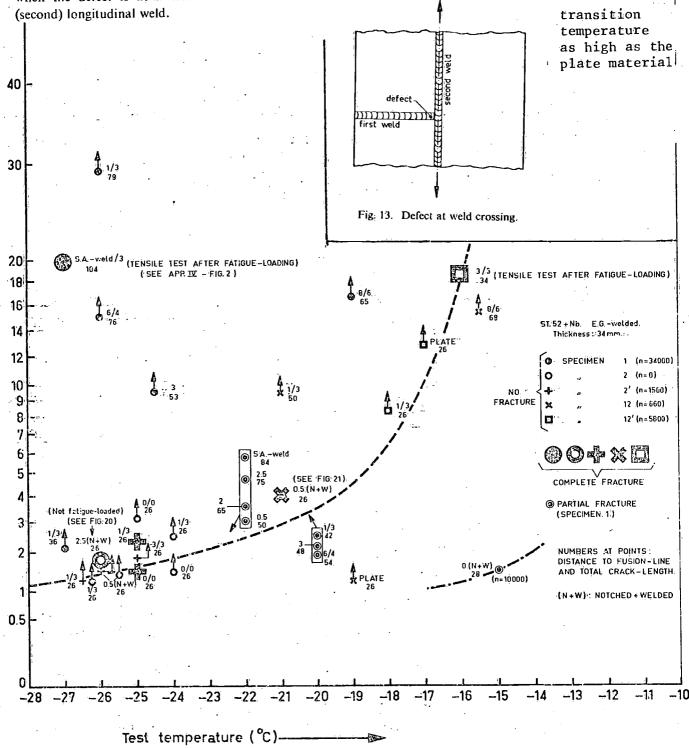


FIG. 19 DEFORMATIONS AT THE NOTCH'S TIPS AT THE MOMENT THE FRACTURE OCCURED GIVEN AS FUNCTION OF TEMPERATURE.

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A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks

J. R. RICE

Assistant Professor of Engineering, Brown University, Providence, R. I:

A line integral is exhibited which has the same value for all paths surrounding the tip of a notch in the two-dimensional strain field of an clastic or deformation-type clastic-plastic material. Appropriate integration path choices serve both to relate the integral to the near tip deformations and, in many cases, to permit its direct evaluation. This averaged measure of the near tip field leads to approximate solutions for several strain-concentration problems. Contained perfectly plastic deformation near a crack tip is analyzed for the plane-strain case with the aid of the slip-line theory. Near tip stresses are shown to be significantly elevated by hydrostatic tension, and a strain singularity results varying inversely with distance from the tip in centered fan regions above and below the tip. Approximate estimates are given for the strain intensity, plastic zone size, and crack tip opening displacement, and the important role of large geometry changes in crack blunting is noted. Another application leads to a general solution for crack tip separations in the Barenblatt-Dugdale crack model. A proof follows on the equivalence of the Griffith energy balance and cohesive force theories of clastic brittle fracture, and hardening behavior is included in a model for plane-stress yielding. A final application leads to approximate estimates of strain concentrations at smooth-ended notch tips in elastic and elastic-plastic materials.

Introduction

the determination of concentrated strain fields near notches and cracks, especially in nonlinear materials. An approximate analysis of a variety of strain-concentration problems is carried out here through a method which bypasses this detailed sofution of boundary-value problems. The appreach is first to identify a line integral which has the same value for all integration paths surrounding a class of notch tips in two-dimensional deformation fields of linear or nonlinear clastic materials. The choice of a near tip path directly relates the integral to the locally concentrated strain field. But alternate choices for the path often permit a direct evaluation of the integral. This knowledge of an averaged value for the locally concentrated strain field is the starting point in the analysis of several notch and crack problems discussed in subsequent sections.

All results are either approximate or exact in limiting cases. The approximations suffer from a lack of means for estimating errors or two-sided bounds, although lower bounds on strain magnitudes may sometimes be established. The primary interest in discussing neulinear materials lies with elastic-plastic behavior in metals, particularly in relation to fracture. This behavior is best modeled through incremental stress-strain relations. But no success has been met in formulating a path integral for incremental plasticity analogous to that presented here for elastic materials. Thus a "deformation" plasticity theory is employed and the phrase "clastic-plastic material" when used here will be understood as denoting a nonlinear elastic material exhibiting a linear Hookean response for stress states within a yield surface and a nonlinear hardening response for those outside.

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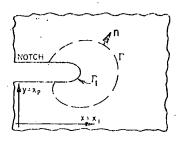


Fig. 1. Flot surfaced notch in two-dimensional deformation field (all stresses depend only on x and y). Γ is any curve surrounding the notch tip, Γ_t denotes the curved notch tip.

Path Independent J Integral. Consider a homogeneous body of linear or nonlinear clastic material free of body forces and subjected to a two-dimensional deformation field (plane strain, generalized plane stress, antiplane strain) so that all stresses σ_{ij} depend only on two Cartesian coordinates $x_1(=x)$ and $x_2(=y)$. Suppose the body contains a notch of the type shown in Fig. 1, having flat surfaces parallel to the x-axis and a rounded tip denoted by the arc Γ_{ij} . A straight crack is a limiting case. Define the strain-energy density W by

$$W = W(x, y) = W(\varepsilon) = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij}, \qquad (1)$$

where $\mathbf{e} = [\epsilon_{ij}]$ is the infinitesimal strain tensor. Now consider the integral J defined by

$$J^{s} = \int_{\Gamma} \left(W dy - \mathbf{T} \cdot \frac{\partial \mathbf{u}}{\partial x} ds \right). \tag{2}$$

Here Γ is a curve surrounding the notch tip, the integral being evaluated in a contractockwise sense starting from the lower flat notch surface and continuing along the path Γ to the upper flat surface. It is the traction vector defined according to the outward normal along Γ , $\Gamma_i = \sigma_{ij}n_{ji}$ u is the displacement vector, and ds is an element of arc length along Γ . To prove path independent, consider any closed curve Γ^* enclosing an area A^* in a