Assessing robustness and identifying critical infrastructure in synchromodal transport networks

Kumar Navneet
Preface

It feels immense pleasure in penning my final thoughts on completion of this research. It concludes the journey of my Master of Science in Electrical Engineering, at Technical University of Delft. It has been an enriching experience. This thesis was a continuation of research done by W.J.L. van Dam in the field of Multimodal transport network. I am satisfied with the results I have achieved. My sincere gratitude to my supervisors Zhidong He and Prof. Piet Van Mieghem for their critical inputs and support throughout the research phase. I am grateful to have friends who have constantly supported me. I am thankful to my parents and sister who have had my back throughout everything. Without whom this journey would not have even begun. It’s their faith in me that kept me going. Thank you.

Kumar Navneet
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Abstract

A synchromodal transport network is a relatively new concept of transportation which aims to create more efficient and sustainable transportation plans. It utilizes different modes of transport, i.e., roadways, railways, and waterways synchronously. For synchromodal transportation, these modes do not only depend upon each other for transportation but also cross each other’s links at several points, thus creating interdependence among each other. And the study of several other interdependent networks suggests that failure or disruption of interdependent point can have catastrophic effect on both networks. Thus, identification of critical interdependent infrastructure in a synchromodal transport network is an interesting research area. Further, the identification of critical infrastructure not only help in creating a better mitigation plan but also helps in prioritizing the maintenance/recovery.

First, The Dutch synchromodal transport network is analyzed on the basis of two characteristics 1) Transportation 2) Network topology. Second, in order to identify the criticality of infrastructure, an indicator in term of total system cost for transportation is define which will be known as node criticality. Then two varieties of networks i.e. underlying and overlying network is used to calculate several centrality metrics in order to identify the most influential node in the transport network. Finally, a relationship is established between the node criticality and centrality metric in order to quickly identify the critical infrastructures.

Then, a systematic framework is designed using the concepts from above. After applying the framework on the Dutch synchromodal transport network, several insightful results were observed. First, it is observed that the node criticality exhibits a power-law distribution. Second, a strong correlation between overlying centrality metrics such as weighted degree centrality and eigenvector centrality with node criticality is observed. Lastly, the geographical location top five interdependent pair with respect to node criticality value is found to be in the neighborhood of Rotterdam, which is one of the busiest port of Europe [33]. It would be great if the accuracy of this result can be tested against a real-world example.
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List of Symbols

\[ G(N, L) \] Graph consisting of a set of nodes \( N \) and a set of nodes \( L \)
\[ N \] A set of \( N \) number of nodes
\[ L \] A set of \( L \) number of links
\[ D \] Demand matrix
\[ w_\ell \] weight on a link \( \ell \)
\[ s \] Source of a trip
\[ d \] Destination of a trip
\[ C_G \] Total system cost for a transport network \( G \)
\[ TC_\ell \] Traveling cost on a link \( \ell \)
\[ x_\ell \] Flow on a link \( \ell \)
\[ \alpha \] and \( \beta \) Model parameter for bureau of public roads (BPR) function
\[ p_n \] Probability of choosing a path \( n \)
\[ P \] Path between two nodes
\[ \beta' \] Logit scale parameter
\[ t_{\ell,0} \] Free flow travel time on a link \( \ell \)
\[ d_\ell \] Spatial length of a link \( \ell \)
\[ v_\ell \] Average speed of traveling on a link
\[ \eta_G \] Robustness indicator of a network \( G \)
\[ \omega_i \] Node criticality of node \( i \)
\[ A \] Adjacency matrix
\[ k_i \] Degree of a node \( i \)
\[ DC \] Degree centrality
\[ WDC \] Weighted degree centrality
\[ BC \] Betweenness centrality
\[ BC_{OD} \] Subset betweenness centrality for a set of origin \( O \) and destination \( D \) pairs
\[ EC \] Eigenvector centrality
\[ IC \] Information centrality
\[ CFBC \] Current flow betweenness centrality
\[ LC \] Pseudo inverse laplacian
\[ AoN \] All or nothing assignment
\[ MSP \] Model split assignment
\[ UE \] User equilibrium assignment
\[ SO \] System optimum assignment
Introduction

The transportation network is one of the oldest networks which is driving the socio-economic development of the world. Modern transport networks consist of four different networks i.e. Road, Rail, Water, and Airways. All these networks have different properties, modes of travel, travel time and traveling cost. Each network contributes differently to the transportation of goods and people. Among these networks, the road network has the highest contribution. In 2015, more than 90% people used road network and 71% of goods were transported through road network in EU-28 countries. This over-utilization of road network results in congestion, noise, and high emission. So, it is important to increase the utilization of other networks to reduce the overall emission and congestion from the road network. One way to do this by implementing “Synchromodal Transport Network”.

In this Chapter, the research goal and thesis outline are described. In Section 1.1, motivation and scope of the research are briefly described. In Section 1.2, related research with respect to the synchromodal transport network is described. Afterward, in Section 1.3, research questions are formulated. Finally, the outline of the thesis is presented in Section 1.4

1.1. Motivation and Scope

A synchromodal transport network, as defined by Sarah Pfoser et al., is “evolution of inter- and co-modal transport concepts, where stakeholders of the transport chain actively interact within a cooperative network to flexibly plan transport processes and to be able to switch in real-time between transport modes tailored to available resources. The shipper determines in advance only the basic requirements of the transport such as costs, duration, and sustainability aspects. Thus, transport processes can be optimized and available resources sustainably and fully utilized” [60] [29]. Synchromodal transport network involves a structured, efficient and synchronized combination of two or more modes of transportation [76]. Synchromodal transport not only improves the cost of freight but also decreases the emission from transportation sector [20].

Synchromodal transport network is also a type of interdependent network where two or more transport modes are connected and transfer the traffic between each other via Terminals. The interdependent network is a part of network science which studies the interaction between two or more complex networks. A fundamental property of interdependent networks is that failure of nodes in one network may lead to failure of dependent nodes in other networks [10]. Interdependence is a common feature of many real-world networks [15] such as interdependence between power grid and communication network, transport network and financial network. People rely on each individual network, but natural disasters such as hurricane, power outage, terrorist attack etc have shown that the effect cascaded throughout the interdependent network [90] [71] [58] [32] [10]. One such example is power blackout in Italy. On 28 September 2003, a shutdown of power stations directly led to the failure of nodes in the Internet communication network, which in turn caused further breakdown of power stations that lead to the outage of both electrical and telecommunication network [71]. This motivates us to analyze the vulnerability arises due to interdependence in Synchromodal Transport Network.

The synchromodal transport network has two types of dependency within the network:

- **Interconnection Nodes** - These nodes are responsible for transferring freight from one mode to
another. For example, terminals which connect and transfer goods between roadways, railways, and waterways.

- **Interdependent Nodes** - These nodes independently exist in a single network but any disruption of such node can affect other networks. For example, bridges and tunnels

In this research, we analyze the criticality of interdependent nodes in a synchronodal transport network.

### 1.2. Related research

The Synchronodal freight transportation was introduced and successfully piloted in the Netherlands, 2010 [1]. Since then the researchers are studying its applicability and possibilities of full-scale implementation.

Currently, the main focus of research is towards the implementation and benefits of synchronodal transport network. In the research by Behzad Behdani et al., [4], An overview of synchronodal transport network along with its component is studied. This research also proposed a mathematical model for synchronodal service design. Other researches such as [94], [47], [36] investigated the design and applicability of synchronodal transport network. The case studies done by [94] and [47] show that the synchronomodality improves transport service level, capacity utilization, travel cost and $CO_2$ emissions, etc.

There are very few studies such as [20], [19] that study the robustness of synchronodal transport network and compared it against other transport networks. But to our knowledge, there seems to be no research that studies the effect on robustness due to perturbation of infrastructure on synchronodal transport network, except for Master’s thesis by Ir. W. J. L. van Dam [85]. In this thesis, he studied the robustness of the Dutch freight transport network for synchronodal transport. We are going to extend his work by analyzing the robustness of the Dutch synchronodal transport network against the disruption of interdependent nodes.

### 1.3. Research Questions

The main goal of this research is to:

*Identification of critical infrastructure by accessing the robustness of The Dutch synchronodal transport network*

This goal can be achieved by answering the following questions:-

1. What is the Dutch synchronodal transport network?
2. Is there any method to quantify the criticality of infrastructure in the transport network?
3. If yes, how can these methods be implemented in the Dutch synchronodal transport network to identify the critical infrastructures?

### 1.4. Thesis outline

First, the introduction of Dutch synchronodal transport network is presented in Chapter 2. In this Chapter, different layers of the Dutch synchronodal transport network is briefly explained. The main objective of this Chapter is to identify different characteristics of the modes of the transport network i.e. roadways, railways and waterways and introduce their network topology.

In Chapter 3, different traffic assignment models, such as All-or-Nothing, Model split assignment, User equilibrium, and system optimum, are explained in detail. These are the standard traffic assignment models which are extensively used by transport engineers for analyzing the transport network. These traffic assignment techniques are implemented on the Dutch synchronodal transport network, so the related algorithm for each of these traffic models are described as well.

Chapter 2 and 3 give basic information that is required to answer the research question. Chapter 4 proposes and describes a framework for identifying the critical infrastructure in a transport network. This chapter introduces two important concepts, i.e., node criticality indicator and topological centrality
metrics. Later, node criticality indicator and topological centrality metrics are utilized to identify the critical infrastructure in the Dutch synchromodal transport network.

Chapter 5 will first give an overview of the simulation setup for the robustness assessment of the Dutch synchromodal transport network and later explains the application of the framework in the context of each traffic assignment models, that are, All-or-Nothing, Model split assignment, User equilibrium assignment and system optimum assignment. Chapter 5 also explains the numerical calculation related to node criticality and overlying topological centrality metric for each traffic assignment models. Next, in Chapter 6 all the results of the simulations will be discussed in detail. Finally, in Chapter 7 the conclusion of the research is presented.
2.1. Introduction

Synchromodal transport network is a type of multi-layered transport network, which coherently utilizes two or more modes of the transport network for moving freight from one point to another. These modes of transport network can either be railways, waterways, roadways or airways network. Whereas, Terminals are used for switching between these modes while transporting freight from one point to another.

Synchromodal transport network utilizes road network, rail network and waterways for transportation, and the infrastructures of these networks cross each other multiple times. This makes synchromodal transport network an interdependent network. The interdependent network is a network of networks where one network depends on and supports another network [15]. Interdependent networks have two qualitatively different kind of links: connectivity links and dependency links [10] [15]. The connectivity links are the links which connect the nodes in the same network whereas dependency links are the links which connect the nodes of two different networks. Any disturbance to these links can adversely affect the performance of the network. To understand and measure the effectiveness of the disturbance due to interdependency, it is important to understand the network itself.

Dutch Synchromodal transport network comprises of roadways, railways and waterways network. These networks are already being extensively used by logistic companies for hinterland transportation [85] [94]. Out of these networks, only the road network can independently transport from start to end whereas the other two networks depend on the road network for last-mile transportation. All of these networks can be characterized by different properties, such as capacity, travel time and cost, and there is a lot of dissimilarity between these networks with respect to these characteristics.

In this chapter, the Dutch Synchromodal transport network is described briefly. Dutch Synchromodal transport network comprises of roadways, railways and inland waterways network. These networks are already being used separately for hinterland transportation [85] [94] by logistic companies. So, all of these networks i.e road, rail and water are already well developed with different capabilities and characteristics. In the sections below, each of these networks will be described with respect to their transportation properties and their network topological properties. Especially, in the network topology, only those infrastructure is taken into consideration which is going to be utilized by Synchromodal transport network.

2.2. Origin and Destination pairs

The first step to create any transport network is to identify the traffic generating areas and estimate the amount of traffic generated by these areas, also known as Demand. The Demand estimation is done on the basis of socio-economic characteristics of the targeted area, more on this in chapter 3. After estimating a demand, a transport network can be planned for the entire region.

Netherlands uses NUTS (French: Nomenclature des unités territoriales statistiques) regions forecasting freight demand and route planning [18]. The NUTS standard is developed by the European Union(EU), is a hierarchical system for dividing up the economic territory of the EU for the purpose of regional statistics, socio-economic analysis, and framing regional policies. NUTS divides The territories of Netherlands into 3 levels, where each level is of different size and used for different analytical
purpose [22]. The third level of NUTS, also known as NUTS-3, divides the Netherlands into 40 regions. These regions form the base layer of the synchronomodal Transport Network and demand will be estimated for these regions.

The Netherlands uses BasGoed model to estimating the demand generated by these regions [66] [85]. The BasGoed model produces a $N \times N$ demand matrix $D$, where $N$ is the number of regions. In the demand matrix, row $s$ represents the origin and column $d$ represent the destination of the freight. Whereas the entries of the matrix $D_{sd}$ represents the number of containers, in tonnes/business day (assuming 253 business days per year), between these $s$ and $d$ pairs, also known as OD-pairs [85].

### 2.2.1. Graph model of OD-pair

In the graphical representation of OD-pairs, these 40 regions will be represented by a set of Nodes ($\mathcal{N}_{OD}$) located at the centroid of the geographical region, as shown in Figure 2.1. The nomenclature used for these nodes is taken from NUTS nomenclature. The first two alphabets represent the country and next three numbers are the three classes of NUTS, for example, a centroid with ID NL112- represent the Netherlands and is 1,1 and 2 represent IDs of NUTS-1, NUTS-2, and NUTS-3 respectively.

![Figure 2.1: NUTS-3 regions](image)

The amount of incoming and outgoing containers from each node is shown in Figure 2.2. NL339 has the highest amount of in-coming and out-going containers. This is due to the location of the port of Rotterdam in NL339 and this port is one of the busiest port of EU [33].

As each region is represented by centroid, the cargo must travel to the centroid in order to reach the destination. The OD-pair layer is a disconnected network only having nodes without any links. These nodes will be connected by the links from different transport networks.
2.3. Roadways

A road is a paved way on land which connects two locations and allow transporting of goods or people along it. The road network consists of different elements which facilitates the movement of vehicle, shown in table 2.1.

<table>
<thead>
<tr>
<th>Element</th>
<th>Function</th>
</tr>
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<tbody>
<tr>
<td>Road Section</td>
<td>Facilitate movement of trucks between other elements</td>
</tr>
<tr>
<td>Road intersection</td>
<td>Connect more than two road sections</td>
</tr>
<tr>
<td>Bridge</td>
<td>Facilitate crossing of other infrastructure</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Facilitate crossing of other infrastructure</td>
</tr>
<tr>
<td>Railway crossing</td>
<td>Facilitate crossing of other infrastructure</td>
</tr>
<tr>
<td>(Un)loading Area</td>
<td>Makes (un)loading of trucks possible</td>
</tr>
</tbody>
</table>

Table 2.1: The elements of the road network in the Netherlands [85]

A road section is further sub-divided into three different categories based on their usage and functionality [80]:-

**Flow roads** - It allows a large amount of traffic to travel with high speed over a large distance. These type of road does not have any traffic lights, have physical separation for traffic from the opposite direction, controlled accesses, and graded separated junction. It includes motorways and expressways with a maximum speed of 100-130 km/hr.

**Access roads** – It provides access to the destination. On these roads, fast traffic mixes with general traffic including pedestrian. These roads facilitate ingress and egress from vehicles as well as loading and unloading. The speed of vehicle varies from 30-60 km/hr.

**Distributor roads** – This road types connect Access roads and Flow roads. It has a flow function on the road section and exchange function on the intersections. Speed of vehicle varies between 50-80 km/hr.

The Netherlands has a total length 3,046 km of flow roads, a total length 1,428 km access road
and a total length 126,025 km of other roads [85] [53] [67], shown in Figure 2.3. Out of the three-transportation mode studied in this research road is the most dominant mode for freight transportation. In 2015, 82% of the total domestic freight were transported via road [11]. This is due to the fact that the road network is denser than other network in European union and in most of the cases only network available for last mile traveling [50] [48].

Figure 2.3: Map of Netherlands with Distributor and Flow roads [57]

2.3.1. Network topology of the road network

The road network can be represented by bi-directed or un-directed network $G_{\text{road}}(N_{\text{road}}, L_{\text{road}})$. Where $N_{\text{road}}$ is the set of nodes representing the junctions, the end-points or the crossings points, and connecting different segments of a road network, whereas $L_{\text{road}}$ is a set of links representing the route segments.

The set of links $L_{\text{road}}$ consist the links from different categories of road, that are the Flow road links $L_{FR}$, the Access road links $L_{AR}$ and the Distributor road links $L_{DR}$. The links $L_{FR}$ are always connected via the nodes from the set $N_{\text{road}}$, whereas the links from set $L_{AR}$ and $L_{DR}$ always have one end connected to a node from the set $N_{OD}$ and other end connected a node from the set $N_{\text{road}}$. Thus, for simplicity, the links $L_{AR}$ and $L_{DR}$ are represent by a super-set of links $L_{OD}$, given by equation (2.1):

$$L_{OD} = L_{AR} \cup L_{DR}$$  

(2.1)

The links from the set $L_{OD}$ is further simplified as a single direct link connecting $N_{\text{road}}$ and $N_{OD}$ and spreading in different direction from $N_{OD}$, as shown green dotted lines in Figure 2.4.

Now, a complete transport network comprising of road network can be represented by graph $G_{\text{road}}(N_{\text{road}}, L_{\text{road}})$, where $N_{\text{road}}$ and $L_{\text{road}}$ are given by equations (2.2) and (2.3) respectively.

$$N_{\text{road}} = N_{\text{road}} \cup N_{OD}$$  

(2.2)
2.4. Inland Waterways

Inland waterways (IW) are networks of navigable water bodies, such as rivers and canals. Inland waterways network has different elements facilitating transportation, navigation, and flow control and crossing of different transportation infrastructures. A brief functionality of these elements is shown in Table 2.2.

Water sections are links that facilitate transportation. In Europe these water sections are divided into different classes, each class is divided on the basis of capacity to accommodate vessels and pushed convoys of certain sizes [31]. Out of the classes shown in Figure 2.7a, class III and higher are used for container transportation [85]. So, these are the only classes of interest for this research.

The Netherlands has the densest waterways in Europe [68]. Netherlands has a total of 5046 km of waterways out of which 4800 km of waterways are suitable for freight transport [6]. Inland waterways are economical and environmental-friendly, it becomes a viable alternative to road and rail transport system on European corridor [84]. Although, its utilization for domestic freight transport within The Netherlands is very less. It only contributes to 18% of total domestic freight transport.

Figure 2.4: Graphical representation of a road network

\[ L_{\text{road}} = L_{\text{FR}} \cup L_{\text{OD}} \]

In \( G_{\text{road}}(N_{\text{road}}, L_{\text{road}}) \), there are 1008 numbers of nodes in the set \( N_{\text{road}} \) and 2728 numbers of links in the set \( L_{\text{road}} \). Figure 2.5 shows the degree distribution of \( G_{\text{road}} \), the nodes with very high degree are the nodes from the set \( N_{\text{road}} \). In fact, the highest degree of 42 is for the nodes \( FN_{L}339 \) and \( FN_{L}411 \), and these nodes also have the highest and the second highest total demand (i.e., incoming and outgoing) respectively. Further, Figure 2.6 shows a relationship between the degree and the total demand of nodes in the set \( N_{OD} \), which suggests that the demand from a region is related to the density of links from that region.
Figure 2.5: Degree distribution of Dutch road network

Figure 2.6: The Spearman’s correlation between degree and total Demand of nodes in $\mathcal{N}_{DD}$
<table>
<thead>
<tr>
<th>Element</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Section</td>
<td>Facilitate movement of barges between other elements</td>
</tr>
<tr>
<td>Water Intersection</td>
<td>Connect more than two waterway sections</td>
</tr>
<tr>
<td>Bridge</td>
<td>Facilitate crossing of other infrastructure</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Facilitate crossing of other infrastructure</td>
</tr>
<tr>
<td>Water retaining structures</td>
<td>Regulate the water flow and the water level</td>
</tr>
<tr>
<td>Locks</td>
<td>Facilitate movement of barges between different water levels</td>
</tr>
<tr>
<td>Quay</td>
<td>Makes (un)loading of barges possible</td>
</tr>
</tbody>
</table>

Table 2.2: The elements of the road network in the Netherlands [85]

2.4.1. Network topology of the inland waterways
Inland waterway is represented by a bidirected or undirected network $G_{IWW}(N_{IWW}, L_{IWW})$. Where $L_{IWW}$ is a set of links representing different water sections and $N_{IWW}$ is a set of nodes connecting these links.

Figure 2.7b shows the inland waterways network $G_{IWW}$. It can be observed here that unlike roadways, inland Waterways do not directly connect to the OD-pairs. $IWW$ utilize freight terminals and road network for end-to-end container transportation.

![Map of waterways of The Netherlands](image1)

![Map of inland waterways network used in the research](image2)

Figure 2.7: Inland waterways of The Netherlands

In $G_{IWW}(N_{IWW}, L_{IWW})$, there are 316 numbers of nodes in the set $N_{IWW}$ and 696 numbers of links in the set $L_{IWW}$. Figure 2.8 shows the degree distribution of $IWW$ network combined with freight terminals and the links from set $L_{OD}$. There are 17 regions which remains disconnected even after combining $IWW$ network, terminals and $L_{OD}$. Thus, $IWW$ network cannot form a complete independent transport network.

2.5. Railways
Railways are the means of transport, where vehicles travel on an omnidirectional track. A railways network consists of many elements which help in transportation, management, and control of the in-
Figure 2.8: Degree distribution of Dutch Inland waterways network
2.6. Synchromodal Transport Network

Synchromodal transport network will be formed by combining the above mentioned OD-pair, road network, rail network and water network. After combining, synchromodal transport network not only forms a complete independent transport network but also bring the best characteristics from each layer, such as completeness from the road network, economical and environment-friendliness from the water network and high-speed travel possibility from the rail network.

Apart from these networks, Freight terminals are also added to the synchromodal network. Although, freight terminals are already utilized as an interconnection point for railway and IWW transport network. In synchromodal transport network, these interconnection nodes will enable one the important feature of synchromodality i.e. real-time switching between different transport modes. Further, synchromodality also introduces interdependency between the networks, and the nodes representing these interdependencies are called Interdependent nodes.

### 2.6.1. Interconnection nodes: Freight terminals

In general, a freight terminal is an open system of material flow with two interfaces a) quayside with loading and unloading of a ship and b) land-side where containers are (un)loaded on and off trucks and trains [77]. In a synchromodal transport network, these terminals act as a router which will connect and route containers between two or more modalities. A terminal can be classified into three types: rail terminal, water terminal, and trimodal terminal [85]. A trimodal terminal connects all three modalities whereas a rail/water terminal is connected to the road network only. A spatial overview of available terminals of The Netherlands is given in Figure 2.11. There are 44 terminals shown in Figure 2.11, out of which 14 are trimodal, 2 rail terminal and 28 are water terminals.
2. Dutch Synchromodal Transport Network

(a) A map of railway network as per usage

(b) A map rail network used in this research

Figure 2.9: Railways tracks in Netherlands

Figure 2.10: Degree distribution of Dutch rail network
Network topology of the freight terminals
Freight terminal can be represented a group of sub-graphs $G_{terminal}$, where each $G_{terminal}$ has a node $N_{terminal}$ representing the terminals and has a set of links $L_{terminal}$ connected to $N_{terminal}$ at one end and to a node from other modality at another end. A trimodal node has 3 set of links connected to it whereas a water/rail terminal has only 2 set of links connected.

![Figure 2.11: Map of Netherlands with Terminals](image)

2.6.2. Interdependent Nodes
Synchromodal transport network is a multi-layer network where the infrastructure of one modality crosses other. Whenever there is such a crossing, an extra infrastructure in build to insure the continuous or accident-free traffic flow on each modality. These infrastructures can broadly categories into three types: bridges, tunnels, and railway crossing.

These infrastructures are considered as interdependence point because any disruption of such infrastructure affects two or more modality. For example, if road bridge crossing a waterway is disrupted then traffic on both modalities either needed to stop on both modalities or slowed down on one of them till the bridge is functional again.

Network topology of the interdependent nodes
The interdependent nodes are represented by a set nodes $N_{interdependency}^m$, i.e.

$$N_{interdependency}^m \subset N^m \text{ where } \forall m \in \{road, rail, IWW\}$$ (2.4)

There are 398 interdependent nodes in the dutch synchromodal transport network, as shown in Figure 2.12. These interdependent nodes are created by adding a node in both modalities at the geographical crossing of the links. Thus, these 398 interdependent nodes represent 199 crossing points in the synchromodal transport network.
2.6.3. Network topology of the Synchromodal transport network

Synchromodal transport network is complex graph network represented by $G(N, L)$, where $N$ and $L$ are a super-set of nodes and links given by equation (2.5) and 2.6 respectively.

$$N = N_{\text{road}} \cup N_{\text{rail}} \cup N_{\text{IWW}} \cup N_{\text{terminal}}$$  \hspace{1cm} (2.5)

$$L = L_{\text{road}} \cup L_{\text{rail}} \cup L_{\text{IWW}} \cup L_{\text{terminal}}$$  \hspace{1cm} (2.6)

Table 2.4 represents the count of nodes and links in the Dutch synchromodal transport network. All Count of the links shown in table 2.4 are bi-directional except for the OD-pair links.

<table>
<thead>
<tr>
<th></th>
<th>Rail</th>
<th>IWW</th>
<th>Road</th>
<th>Terminal</th>
<th>OD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>173</td>
<td>316</td>
<td>1008</td>
<td>44</td>
<td>40</td>
<td>1541</td>
</tr>
<tr>
<td>$L$</td>
<td>368</td>
<td>698</td>
<td>2728</td>
<td>202</td>
<td>692</td>
<td>4688</td>
</tr>
</tbody>
</table>

Table 2.4: The count of nodes and links of Dutch synchromodal transport network

Figure 2.13 shows the degree distribution of synchromodal graph network $G$. The distribution is similar to $G_{\text{road}}$ except the count of smaller degree up to 4, have increased. This is because of the railways, and IWW have a maximum degree of 4 while terminals have a maximum degree of 3.
2.7. Summary

In this chapter, the Dutch Synchromodal transport network is explained as a multilayered graph network $\mathcal{G}$ with nodes $\mathcal{N}$ and links $\mathcal{L}$. Each section of the chapter has two-part:

- First part that briefly explains the infrastructure, transportation characteristic related to each layer. Anyone who wants to know more about these infrastructures can refer to chapter 4 and 5 of Mater’s thesis of Ir W. J. L. Van Dam [85]

- Second part explains the graphical overview of the layers, i.e., representation of links and nodes for each layer. The symbols used in this part for the graph network will be used throughout the research

The important concepts discussed in this chapter are the definition of synchromodal transport network, interdependent and interconnection nodes, network topology.
3

Traffic Assignment Models

3.1. Introduction
Traffic assignment is one of the key components of transportation planning and forecasting. In fact, it is the fourth step of travel demand forecasting, sometimes also known as “Four-step model”, following trip generation, trip distribution and mode choice [81]. Traffic assignment models relate travel demand to infrastructure supply, simulating route choice and network conditions, resulting in traffic flows, congestion, travel time and emissions [7]. These models not only help transportation engineers, in performing a cost-benefit analysis of future network but also help them in analyzing the performance of the current network.

Traffic assignment models take input from previous steps of the "Four-step model", so it is important to understand the entire process of the travel demand forecasting. In section 3.2, a brief introduction to the processes involved in travel demand forecasting method is given. Section 3.2 also gives a brief overview of a transportation network design. In the sections from 3.3 to 3.6, different types of traffic assignment models along with their algorithm are described.

3.2. Travel demand forecasting
Travel Demand Forecasting is a multi-stage process to predict future transport demand when establishing transport plan within a fixed budget. The forecasting result gives quantitative input to evaluate the supply strategy of transport facilities and land use planning [16]. It gives assistance to transport engineers and policymakers in intelligent decision making for network planning & designing, performance improvement and policymaking for transportation. The type of network design parameter which can be extracted from the forecasting results are the following: -

- The determination of road width
- The calculation of traffic signal timings
- The setting of user charges
- A road closer scheme
- The provision of new transport service
- The construction of new infrastructure etc

Travel demand forecasting method is conventionally known as “four-step process”, shown in figure 3.1. Traditionally, this model considers the steps in sequential order but that does not suggest that the decision made by travelers are in the same sequence rather than simultaneously. Further, this model consists of other hidden steps that are land-use planning, time-of-day modeling, direction factoring, transit person/vehicle trip table generation, etc, which are input to one of these four steps. Out of these hidden steps, it is important to understand: what is land-use planning? as it will give an overview of a transport network and its parameters. In the below subsections, firstly, land-use planning will be explained along with associated transport network parameter such as nodes, links, and weights. Then in the subsequent subsections, steps involved in "Four-step process" will be briefly explained.
3.2.1. Land-use planning

Land-use planning is the process of selecting and segmenting a target area, these areas can be at a national level or regional level. These areas include all the developed land, with under-developed areas which are supposed to be developed by the target year of forecasting. The imaginary line representing the boundaries of the target area is called as "Cordon line". While drawing cordon lines, future growth, political jurisdiction, census area boundaries, and natural boundaries are taken into account. Then the targeted area is divided into zones, known as transportation analysis zones (TAZ). The purpose of such a sub-division is to facilitate the spatial quantification of land use and economic factors which influence travel pattern [56]. An example of such sub-division of The Netherlands, also known as NUTS region, is shown in figure 3.2.

A transport network will be laid on the targeted area, connecting houses, buildings, and offices inside the TAZ and connecting other TAZs as well. A transport network consists of set of Nodes ($\mathcal{N}$) and set of links ($\mathcal{L}$). A link represents a section of a path (roadway, railway, etc) connecting nodes, while nodes are points where two or more link meets. Links can have different characteristics which can be utilized for transport network analysis, these characteristics can be:

- Link length - The spatial distance between two nodes
- Link travel time - Time required to travel between two nodes
- Link travel cost - It is the cost of traveling on the link which may include the price of fuel, vehicle depreciation cost, etc
- Link Capacity - Maximum number vehicle that can travel on the link, simultaneously (Maximum flow)

In the transport network, TAZs are represented by zone centroids, they are connected to nodes by a link referred to as a connector. These centroids represent the center of activity for the zone and are used for loading the trips on the network. Trips are defined as a person or vehicle traveling from one centroid to another with no intermediate stops [81]. These centroids are also known as Origin and destination pairs or OD pairs. Representation of a transport network is shown in figure 3.3

3.2.2. Trip generation analysis

After studying the activity of the zone and its socioeconomic characteristics, the next task is to quantify the number of the trip each zone will produce or attract. The process of quantification is known as a trip generation. There are different characteristics of land-use that influence the trip generation, such as:

- density/intensity of land-use is related to the average number of trips per day, based on experience with the type of land use at hand [2]
3.2. Travel demand forecasting

There are three main techniques that are used for trip generation analysis i.e. a) Cross-classification b) Multiple regression analysis and C) experienced based analysis. Since these techniques are beyond the scope of this research thus will not be discussed.

3.2.3. Trip distribution analysis

After getting the number of trips to and from each traffic analysis zone, the next task is to determine how these trips will be distributed among these zones. This analysis is known as trip distribution analysis. The Logit model [44] and The gravity model [24] are the two popular models used for trip distribution analysis [2]. The output of trip distribution analysis is a trip table, also known as an Origin-Destination demand matrix or an OD matrix.

OD matrix is a $s \times d$ matrix, where row $s$ represent the origin and column $d$ represent the destination, and the values of matrix $(D_{sd})$ represent the number of trips produced by zone $s$ which will travel to zone $d$. The units of the values of the OD matrix can be in per hour, per day or per year scale depending upon the type of travel demand analysis to be done.

3.2.4. Mode choice analysis

Trip distribution analysis gives the number of trips starting and ending between OD pairs. Now, mode choice analysis will determine what type of transportation mode (for example a bus or a car) will be chosen for traveling between these OD pair. Mode choice analysis commonly uses logit model [39] to determines the probability of choosing a type of transportation system. Mode choice models estimate
how many people will use public transit and how many will use private automobiles [2], whereas for freight transportation these models can be used to estimate the choice between different transportation systems such as roadways, railways, waterways, etc. The estimation of choices is influenced by different factors such as travel cost, level of service and travel time, etc.

3.2.5. Traffic assignment analysis

Trip generation gives the number of trips from each zone, trip distribution gives how these trips are distributed among the OD pairs, mode choice tells what will be the preferred mode of travel. Now, the traffic assignment will determine the selection of a route for each trip. The choice of routes is made on the basis of a number of criteria such as travel time, distance of travel, generalized cost of traveling, etc. Out of these criteria travel time is often considered as sole criterion as travel cost and length can be considered as a function of time in most cases [21]. These criterions are often known as travel resistances/impedance, Traffic assignment models choose a route in such a way that either of these resistances is least for the selected route. These resistances are the cost of traveling on a link.

Traffic assignment process reproduces the pattern of vehicular movements which would be observed when the travel demand represented by the OD matrix is assigned on the transportation system. The major aims of traffic assignments procedure are:

- To estimate the volume of traffic on the links of the network
- To obtain the aggregate network measures such as total system travel time, total distance covered by the vehicle, etc
- To estimate travel cost between OD pairs
- To analyze the travel pattern of each OD pair
- To obtain reasonable link flow and to identify heavily congested links

There are different types of traffic assignment models available such as All-or-Nothing (AoN), incremental assignment, capacity restraint assignment, user equilibrium assignment (UE), system optimum assignment (SO), etc. In the sections below, AoN, UE, SO and model split (MSP) traffic assignment is discussed in details, as these assignment techniques are implemented for analyzing the Dutch synchronomodal transport network.

Once all the steps of travel demand forecasting are complete, it will give an overview of the volume of traffic that a transportation system can expect to serve in the future. Transport network planner uses
3.3. All or nothing (AoN) assignment

All or nothing (AoN) assignment is the simplest traffic assignment model. This assignment assumes that there is no congestion in the transport network. All trips between OD pairs consider the same attribute, i.e. travel impedance, for the chosen route. These trips perceive and weigh the route in the same way. This means that travel impedance is fixed for all trips, i.e. every traveler will always choose the same route while traveling between any OD pair.

AoN traffic assignment method assumes the following:

- All trip-makers have prior knowledge of the travel cost on the links and always choose the fastest route
- Cost of traveling on a link is fixed and does not vary with congestion

The traffic assignment process of AoN assignment on a transport network \( G \) with a set of nodes \( \mathcal{N} \) and a set of links \( \mathcal{L} \) is shown in figure 3.4. There are \( s \) numbers of origins and \( d \) numbers of destinations identified by trip generation analysis in the network \( G \). And \( D_{sd} \) is the trip generation rate or demand between each \( s \) and \( d \) OD-pairs.

A total system cost \( C_G^{AoN} \) for AoN traffic assignment can be calculated using equation (3.2).

\[
C_G^{AoN} = \sum_s \sum_d \sum_{\ell} D_{sd} \times C_{sd,\ell} \mathbb{1}(\ell, \mathcal{P}_{sd} \subseteq \mathcal{L}) 
\]

Where \( C_{sd,\ell} \) is cost of traveling on the link \( \ell \).

One can choose either link length, link travel time or link travel cost (definitions in section 3.2.1) as cost of traveling on the link. For AoN assignment, if link travel time selected then it will be free-flow travel time\(^2\) on that link, as links are congestion-free in AoN assignment.

AoN assignment has a limitation that it ignores the congestion by assigning all the traffic on the best route, thus produces irrational flow pattern. It also fails to reflect the availability of other similar routes with same or marginal higher travel cost.

However, AoN assignment is best suited for an uncongested transport network having fewer route choice with high dissimilarity in the cost of traveling. This assignment is also used for identifying the desired path which will be utilized in the absence of congestion, this makes AoN assignment an important component of other traffic assignments models.

---

1 Design parameter can be related to physical property of the links such as surrounding environment and landscape, physical protection, Amenities etc [42]
2 The free flow travel time of a link is calculated as the link length divided by the free flow speed [46]
3.4. Modal split (MSP) assignment

A mode-split assignment is a combination of mode choice analysis and traffic assignment [79]. In this model, first, a set of feasible paths or different modes of traveling (such as train, trucks, ship, etc.) is identified, then the trips are distributed among these paths or modes. The distribution of the trips is based on the associated utility value of each path or mode. Here, utility value can either be the cost of traveling or the level of service (LOS) or the comfort of traveling, which can be selected on the type of analysis. For example, in this research, Cost of traveling is selected as a utility value for traveling on different feasible paths.

The Multinomial logit model [45] is one of the methods that is used for calculating a relative attractiveness or a utility value of a path [5]. This model uses path enumeration where path choice probabilities depends upon path specific cost of travelling [82], i.e:

\[ p_n = \frac{e^{-\beta'C_n}}{\sum_{\forall m \in P} e^{-\beta'C_m}} \]  \hspace{1cm} (3.3)

where \( p_n \) is the probability of choosing a path \( n \), \( C \) is the cost of traveling on a path, \( P \) is choice set of paths and \( \beta' \) logit scale parameter.

For a transport network \( G(N,L) \) with an OD-matrix \( D_{sd} \) where \( s \) is the number of origins and \( d \) is the number of destinations, the MSP assignment can be performed using following steps:

Step 1: For each OD pair \( s \) and \( d \), determine a set of \( k \)-shortest paths \( \mathcal{P}_{sd} = \{\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_k\} \). Where \( \mathcal{P}_k \subseteq L \), having \( \ell \) number of links and the cost of travelling on the link \( \ell \) is given by \( C_\ell \)

Step 2: \( \forall \mathcal{P}_k \in \mathcal{P}_{sd} \), calculate the cost of travelling \( C_{sd,\mathcal{P}_k} \) for the path \( \mathcal{P}_k \), using equation (3.4)

\[ C_{sd,\mathcal{P}_k} = \sum_{\forall \ell \in \mathcal{P}_k} C_\ell \]  \hspace{1cm} (3.4)

Step 3: \( \forall \mathcal{P}_k \in \mathcal{P}_{sd} \), calculate the path choice probability \( p_{\mathcal{P}_k} \) for the path \( \mathcal{P}_k \), using multinomial logit model:

\[ p_{\mathcal{P}_k} = \frac{\exp\left(-\beta'C_{sd,\mathcal{P}_k}\right)}{\sum_{\forall \mathcal{P}_l \in \mathcal{P}_{sd}} \exp\left(-\beta'C_{sd,\mathcal{P}_l}\right)} \]  \hspace{1cm} (3.5)

Step 4: \( \forall \mathcal{P}_k \in \mathcal{P}_{sd} \), find the portion of demand \( D_{sd,\mathcal{P}_k} \) assign to the \( \mathcal{P}_k \), using equation (3.6)

\[ D_{sd,\mathcal{P}_k} = p_{\mathcal{P}_k} \times D_{sd} \]  \hspace{1cm} (3.6)

Step 5: repeat for all OD pairs

After completing MSP assignment, The total system cost \( C_G^{MSP} \) can be calculated using equation (3.7).

\[ C_G^{MSP} = \sum_s \sum_d \sum_{\forall \mathcal{P}_k \in \mathcal{P}_{sd}} D_{sd,\mathcal{P}_k} \times C_{sd,\mathcal{P}_k} \]  \hspace{1cm} (3.7)

Similar to AoN, this assignment also does not consider the congestion in the network but it utilizes the availability of other similar routes with same or marginal higher travel cost. However, mode choice analysis can also be combined with traffic assignment methods such as User equilibrium, which consider the congestion in the network [26].

3.5. User equilibrium (UE) assignment

So far the traffic assignment algorithms consider that a link has unlimited capacity and traffic flow with a free flow travel time, thus ignoring the effect of congestion in the network. Whereas in reality link travel time is a function of flow on the link. As flow increases towards the capacity of a link, the travel time on the link increases. This means traffic condition worsen and congestion starts to develop on the link.

\(^{3}\)LOS is used to analyze highways by categorizing traffic flow and assigning quality levels of traffic based on performance measure like speed, density, delay etc
3.5. User equilibrium (UE) assignment

A relationship between link travel time and flow with respect to congestion can be given by equation (3.8) [51], and this relationship can be seen in the figure 3.5.

\[ t_\ell(x_\ell) = t_{\ell,0} \left[ 1 + \alpha \left( \frac{x_\ell}{c_\ell} \right)^\beta \right] \]  

(3.8)

Where \( t_\ell(x_\ell) \) is the travel time on a link \( \ell \) for flow \( x_\ell \), \( c_\ell \) is capacity of the link and \( t_{\ell,0} \) is the free flow travel time, given by equation (3.9). \( \alpha \) and \( \beta \) are the model parameter, for which the value of \( \alpha = 0.15 \) and \( \beta = 4 \) are generally used [51] [56].

\[ t_{\ell,0} = \frac{d_\ell}{v_\ell} \]  

(3.9)

where \( d_\ell \) is the distance and \( v_\ell \) is average speed of the vehicle on the link \( \ell \) when there is no traffic.

So the minimum cost path computed prior to the trip assignment will not be the best path after the trips are assigned. Further, the choice of a route, in a congested network, depends on the user’s perception of the shortest path. This type of traffic behavior is well modeled by John Wardrop [93], also known as Wardrop equilibrium models [13]. Wardrop equilibrium models predict the steady state traffic flow evolved from successive route adjustment with respect to cost of traveling. These models are frequently used by transportation planner for predicting the real-life network [13].

User equilibrium traffic assignment is based on Wardrop’s first principle. According to Wardrop’s first principle, drivers in a congested network choose their route selfishly, a behavior that is captured by the Nash equilibrium of the underlying non-cooperative game [14]. If it is assumed that the driver has perfect knowledge of the travel cost on a network and choose the best route according to Wardrop’s first principle, the behavioral assumption will lead to a deterministic user equilibrium [43]. The UE assignment can be stated as a non-linear optimization problem, given by equation (A.1)

Minimize \[ Z(x_\ell) = \sum_\ell \int_0^{x_\ell} t_\ell(x_\ell) dx \]

subject to \[ \sum_k f^{sd}_k = D^{sd} \quad \forall s, d \]

\[ x_\ell = \sum_s \sum_k t^{sd}_l f^{sd}_k \quad \forall l \]

\[ f^{sd}_k \geq 0 \quad \forall s, d, k \]

\[ x_\ell \geq 0 \quad l \in \mathcal{L} \]  

(3.10)
where $k$ is the path, $x_\ell$ is equilibrium flows in link $\ell$, $t_\ell$ is travel time on link $\ell$, $f_k^{sd}$ is flow on path $k$ connecting OD pairs $s,d$, $D_{sd}$ trip rate between $s,d$ and $\delta_k^{sd}$ can be defined by equation (3.11).

$$\delta_k^{sd} = \begin{cases} 1 & \text{if } l \in k \\ 0 & \text{otherwise} \end{cases} \quad (3.11)$$

In UE, when the flow pattern satisfies these constraints no motorist can better off by unilaterally changing routes [43]. At the User equilibrium, all possible routes between all OD-pair have an equal travel time for all the routes with the flow, whereas the routes which do not have any flow have a substantially large travel time. The UE problem is convex because the link travel time functions are monotonically increasing function, and the link travel time a particular link is independent of the flow and other links of the networks [43].

3.5.1. Algorithm for UE
UE assignment is a convex optimization problem due to monotonically increasing link travel time, as discussed in section 3.5. To reach user equilibrium, one can formulate an iterative framework with a step-size parameter $\lambda$ [9], as shown in figure 3.6. On the basis of selection of an optimal $\lambda$ value, There are two algorithms implemented in literature, i.e Frank-Wolfe algorithm [30] [40] [49] and Method of successive average algorithm [72] [37]. In this research, Method of Average(MSA) algorithm is be used, as this algorithm is easy to implement for multiple OD-pair and is considered to be robust than Frank-Wolfe algorithm [88] [64] [59] [41].

MSA algorithm starts the iteration with a higher value of step-size $\lambda$ and gradually decrease the $\lambda$ per iteration. This strategy helps the algorithm to avoid endless oscillations because of the large step size and helps to achieve equilibrium quickly.

for a transport network $G$, the user equilibrium flow $x_\ell^*$ on a link $\ell$, can be calculated using MSA, as shown in Algorithm 1. Consider the transport network $G$ has $N$ number of nodes, $L$ number of links and a trip generation rate of $D_{sd}$ for $s$ and $d$ OD-pair.

A system cost($C_{G}^{UE}$) can be calculated using equation (3.12).

$$C_{G}^{UE} = \sum_{\ell \in L} x_\ell^* \times t_\ell(x_\ell^*) \quad (3.12)$$

Where $t_\ell(x_\ell^*)$ is the travel time on link $\ell$ at equilibrium flow $x_\ell^*$.

Algorithm 1: UE assignment using MSA

**Input:** Transport network $G$($N, L$) and demand matrix $D_{sd}$

**Result:** flows at the equilibrium i.e. $x_\ell^* \forall \ell \in L$

1. Find initial set of path $P_{sd}$ $\forall s - d$ pairs, by applying AoN assignment with link travel time as $t_{\ell,0}$ and $x_\ell = 0 \forall \ell \in L$ ;
   /* a feasible solution */

2. Update $x_\ell \rightarrow x_\ell = D_{sd} \forall \ell \in P_{sd}$ ;

3. while Convergence factor $\geq m$ or iteration $\leq r$ do
   4. Update the link travel time $t_\ell(x_\ell^*)$ using equation (3.8) ;
   5. Find new a set of path $P'_{sd}$ $\forall s - d$ pairs with updated link travel time $t_\ell(x_\ell)$ using AoN;
   6. update the flow to $x_\ell^* \forall \ell \in P'_{sd}$ ;
   7. update $x_\ell \rightarrow x_\ell = x_\ell + (1/r)(x_\ell^* - x_\ell)$;
   8. Calculate convergence factor $m = \frac{(\Sigma x - \Sigma x^*)}{\Sigma x^*}$;
   9. $r \rightarrow r + 1$;

end

3.6. System Optimum (SO) assignment
The system optimum traffic assignment is based on Wardrop’s second principle. According to the second principle, driver co-operate with each other in order to minimize to system travel time. SO can be considered as model in which congestion is minimized when drivers are told which routes to use [3]. This model can help transport engineer and planner to manage the traffic in such a way that an
3.6. System Optimum (SO) assignment

Start with some feasible link flow solution \( x \)

Calculate the link travel times \( t(x) \)

Find the shortest paths between all origins and destinations

Find the all-or-nothing link flows \( x^* \) corresponding to these shortest paths

Choose \( \lambda \in [0, 1] \) and update \( x \leftarrow \lambda x^* + (1 - \lambda)x \)

Check for equilibrium

Yes

End

No

Figure 3.6: Iterative framework for solving complex optimization problem using a step-size parameter \( \lambda \).

optimum social equilibrium can be achieved. This equilibrium can be achieved by solving equation (3.13).

\[
\begin{align*}
\text{Minimize} \quad Z(x) &= \sum_{\ell} x_{\ell} t_{\ell}(x_{\ell}) \\
\text{subject to} \quad &\sum_{k} f_{k}^{sd} = D^{sd} \quad \forall s, d \\
&x_{\ell} = \sum_{s} \sum_{d} \sum_{k} f_{k}^{sd} \delta_{k,l} f_{k}^{sd} \quad \forall \ell \\
&f_{k}^{sd} \geq 0 \quad \forall s, d, k \\
x_{\ell} \geq 0 \quad l \in \mathcal{L}
\end{align*}
\]  

(3.13)

where \( k \) is the path, \( x_{\ell} \) is equilibrium flows in link \( \ell \), \( t_{\ell}(x_{\ell}) \) is travel time on link \( \ell \), \( f_{k}^{sd} \) is flow on path \( k \) connecting OD pairs \( s, d \) and \( D^{sd} \) trip rate between \( s, d \) pair.
3.6.1. Algorithm for SO

To apply system optimum traffic assignment on a network, one can use the same UE assignment algorithm with an updated link travel time \( t'_{\ell}(x_{\ell}) \), given by equation (3.14). The updated link travel time, also known marginal route cost, is used for achieving SO equilibrium [62] [28].

\[
t'_{\ell}(x_{\ell}) = t_{\ell}(x_{\ell}) + \frac{\partial t_{\ell}(x_{\ell})}{\partial x_{\ell}} x_{\ell}
\]  

(3.14)

where \( t_{\ell}(x_{\ell}) \) is given by equation (3.8) and \( x_{\ell} \) is the flow on a link \( \ell \).

For a transport network \( G \), the equilibrium flow \( x_{\ell}^* \) on a link \( \ell \) at SO-equilibrium, can be calculated using Algorithm 2. The transport network \( G \) has \( N \) number of nodes, \( L \) number of links and a trip generation rate of \( D_{sd} \) for \( s \) and \( d \) OD-pairs.

A total system cost for SO traffic assignment is given by equation (3.15).

\[
C_{SO}^G = \sum_{\ell \in L} x_{\ell}^* \times t_{\ell}(x_{\ell}^*)
\]

(3.15)

Where \( t_{\ell}(x_{\ell}^*) \) is the travel time on a link \( \ell \) at equilibrium flow \( x_{\ell}^* \), calculated using Equation 3.8.

**Algorithm 2: SO assignment using MSA**

**Input:** Transport network \( G(N, L) \) and demand matrix \( D_{sxd} \)

**Result:** flows at the equilibrium i.e. \( x_{\ell}^* \ \forall \ell \in L \)

1. Find initial set of path \( P_{sd} \ \forall \ s - d \) pairs, by applying AoN assignment with link travel time as \( t_{\ell,0} \) and \( x_{\ell} = 0 \ \forall \ell \in L \);
   /* a feasible solution */

2. Update \( x_{\ell} \rightarrow x_{\ell} = D_{sd} \ \forall \ell \in P_{sd} \);

3. **while** Convergence factor\( \geq m \) or iteration \( \leq r \) **do**
   4. Update the link travel time \( t'_{\ell}(x_{\ell}) \) using equation (3.14);
   5. Find new a set of path \( P'_{sd} \ \forall \ s - d \) pairs with updated link travel time \( t'_{\ell}(x_{\ell}) \) using AoN;
   6. update the flow to \( x_{\ell}^* \ \forall \ell \in P'_{sd} \);
   7. update \( x_{\ell} \rightarrow x_{\ell} = x_{\ell} + (1/r)(x_{\ell}^* - x_{\ell}) \);
   8. Calculate convergence factor \( m = \frac{\sum x - \sum x^*}{\sum x^*} \);
   9. \( r \rightarrow r + 1 \);

10. **end**
3.7. Summary

This chapter gives a brief introduction of a transportation network and concepts related to its planning and designing. The main focus of this chapter is to introduce the traffic assignment models, which can be used for analyzing the performance, robustness and resilience of the network. All or nothing (AoN), model split (MSP), User equilibrium (UE) and System optimum (SO) are the traffic assignment model discussed in this chapter and these will later utilize to model the traffic flow on the synchronomodal transport network. Other important concepts discussed in this chapter are System cost ($C_G$) of a network $G$, Demand/trip generation rate, travel impedance, Origin destination pairs, travel demand modeling, etc.

An example of how to implement these models of traffic assignment can be found in the Appendix.
Framework for critical infrastructure identification

4.1. Introduction
Transportation system is one of the most critical systems of a country, which faces many threats to its critical infrastructure ranging from structural failure to malicious attacks [83]. Critical infrastructures are the asset, system or part, whose disturbance or destruction would cause a significant impact on the entire system performance [35]. Further, if one can identify critical infrastructures a priori, a better transportation network and transportation plan can be designed.

There are two ways to identify a critical infrastructure: a) By checking the effect on network performance under the removal of the infrastructure and b) Heuristically via the graph topological properties of infrastructure. In this chapter, first, we are going to design a general framework then we are going to apply this framework for different traffic assignment techniques.

4.2. A general framework
Transportation engineer uses system cost to analyze a fully functional transport network. The system cost can be calculated using an OD-matrix under a specific traffic assignment technique, as explained in Chapter 3. Any degradation or disruption of a transport network element will have an impact on the original system cost. On the basis of the severity of the impact on the system cost, the criticality of an element can be quantified. Further, according to network science, robustness of a network is related to its underlying topology and services [86].

In Figure 4.1, a framework is proposed for the identification of critical infrastructure in a transport network. This framework takes a predefined Demand matrix as an input and applies to a transport network $G$, using different traffic assignment models. There can be two types of transport network $G$:

a. A transport network with original parameter

b. A transport network with degraded parameter (see network parameter degradation in section 4.2.1)

First, the framework will calculate the original system cost $C_G$. Then the element of the network is disrupted which will form a perturbed network $G'$ and the new system cost $C_{G'}$ will be calculated. Using $C_G$ and $C_{G'}$, the framework will calculate a robustness indicator ($\eta_G$), as defined in section 4.2.2. The robustness indicator will help in analyzing the network robustness under perturbation and in identifying critical infrastructures.

Further, this framework will establish a relationship between the topological centrality metrics (defined in section 4.3) and the robustness indicator. This relationship will help in identifying a suitable topological centrality which can identify critical infrastructures in the network, quickly [38].
4. Framework for critical infrastructure identification

Figure 4.1: A framework for identifying critical infrastructure in a transport network
4.2.1. Network perturbation

Any challenges or disruption that causes a degradation in the performance of a network is known as network perturbation [86]. In a transport network, these challenges can be categories into three major types:

1. Elemental disruption - This challenge can be complet/partial failure of a network element. For example closer of a bridge or tunnel due to maintenance.

2. Network parameter degradation - In a transport network, every link has an associated property such as cost of traveling, capacity, etc which drives the performance of the network. Any degradation of such parameters can be considered as parameter degradation challenges. For example, closing a lane of multi-lane highway, due to maintenance, will reduce the capacity of the highway and increase the congestion in the network.

3. Combined disruption - This is the combination of elemental disruption and parametric degradation challenge. A real-life example of such disruption could be the failure of a bridge on a visibility impaired highway.

In this research, elemental disruption is considered as a prime network perturbation and the impact on the network performance due to this perturbation will be measured. These elements can be bridges, tunnels, junction points, etc, and represented as nodes in the transport network. The node which has a high impact on the network performance will be identified as critical infrastructure for the network. The elemental disruption is applied for two types of transport network:

a. Elemental disruption on a transport network having original link properties. This will assist in the identification of critical infrastructure under the normal operating condition of any transport network.

b. Elemental disruption on a transport network with degraded link properties. This disruption will focus on the identification of critical infrastructure for a multi-modal transport network, in which the degradation of link properties of single-mode (such as roadways, railways or waterways) will have effect on the entire network. Transport network with degraded link properties describes the real-life scenarios such as failure of a road bridge over a frozen waterway, ongoing maintenance of road network near a railway crossing, etc. Further, the result from this analysis can assist in identifying the significance of a modality for synchronomodal transport network.

4.2.2. Network performance indicator

The total cost of travelling $C_G$ for a transport graph $G(N, L)$ can be used as a performance measure, which is defined as:

$$C_G = \sum_{\ell \in L} x_{\ell} \times C_{\ell}(x_{\ell})$$  \hspace{1cm} (4.1)

where $x_{\ell}$ is the total flow on the link $\ell$ and $C_{\ell}(x_{\ell})$ is the cost of traveling on the link $\ell$ for the total flow $x_{\ell}$.

Similarly, for any perturbed network $G'$, the new total cost of traveling $C_{G'}$ can be calculated using the equation (4.1). Now, the effect in performance under perturbation can be measured by the normalized increment of the total cost of traveling, which is defined as robustness indicator and given as:

$$\eta_G = \frac{C_{G'} - C_G}{C_G}$$  \hspace{1cm} (4.2)

Further, the impact of disruption of any node $i \in N$ on the performance of the network can be calculated by the normalized increment in the total cost of traveling, is defined as node criticality. The node criticality $\omega_i$ for a transport element $i$ is a special case of robustness indicator ($\eta_G$) which gives the effect on the performance of a network under single node disruption, which is given by equation (4.3)

$$\omega_i = \frac{C_{G \setminus i} - C_G}{C_G}$$  \hspace{1cm} (4.3)
4.3. Topological centrality metrics

A network topology specifies how items, called nodes, are interconnected or related to other nodes by links, as defined by P. Van Mieghem et al [86]. While the network centrality quantifies the importance of a node in the network with respect to the surrounding nodes [73] [69]. Centrality metrics are utilized to identify critical infrastructures in a complex network. For example, A. B. M. Nasiruzzaman et al [52] has study different centrality metrics and try to find critical node in a power grid network. Following are the definition of few of the centrality metrics that will be utilized in this research:

- **Degree Centrality (DC)** - Degree centrality measures the centrality of a node with respect to the number of links incident upon the node, also known as the degree of the node. A network $G$ with $N$ nodes and $L$ links can mathematically represent by an Adjacency matrix $A_{N \times N}$ where each element $A_{ij}$ represents the link between nodes $i$ and $j$. If there is link between nodes $i$ and $j$ then $A_{ij} = 1$ otherwise $A_{ij} = 0$. Now the degree $k_i$ of node $i$ can be calculated as:

$$k_i = \sum_{j=1}^{N} A_{ij} \quad (4.4)$$

and the corresponding degree centrality ($DC_i$), can be defined as [27]:

$$DC_i = \frac{k_i}{N - 1} \quad (4.5)$$

The value of $DC_i$ varies between 0 to 1 where 0 represents a disconnected node and 1 represents a node which is connected to all other nodes.

The above-mentioned degree centrality measure is for an unweighted network, which can be extended for a weighted network $G(N,L)$ with a link weight of $w_\ell$ for any link $\ell$. In this case, the value of adjacency matrix $A_{ij} = w_\ell$ for a link $\ell$ and otherwise 0 and the corresponding weighted degree centrality $WDC_i$ is calculated using equations 4.4 and 4.5.

- **Closeness centrality (CC)** - Closeness centrality measures the average distance of a node to all other nodes. For a graph network $G(N,L)$, the distance $d_{ij}$ between a node $i$ and $j$ is defined as the smallest sum of link weight throughout all the possible paths between $i$ to $j$ in a weighted graph, or the minimum number of links traversed in an unweighted graph, and the corresponding Closeness centrality $CC_i$ for a node $i$ is defined as [27]:

$$CC_i = \frac{N - 1}{\sum_{j \in \mathbb{N}, j \neq i} d_{ij}} \quad (4.6)$$

A node with the highest Closeness centrality, is the node that is closest to all other nodes in the network in term of the distance and the distance is the function of link weight in the weighted network.

- **Betweenness centrality (BC)** - For a graph network $G(N,L)$ with link weight $w_\ell$, the betweenness centrality $BC_i$ of a node $i$ is the sum of the fraction of all-pairs shortest paths that pass through node $i$, defined as [27]:

$$BC_i = \sum_{m, n \in \mathbb{N}} \frac{\sigma(m, n \mid i)}{\sigma(m, n)} \quad (4.7)$$

where $\sigma(m, n \mid i)$ represents the number of shortest path between node $m$ and $n$ that passes through node $i$ and $\sigma(m, n)$ represents the total number of shortest path between nodes $m$ and $n$.

The betweenness centrality can also be calculated only for a set of nodes instead of all possible node pair. Then the between centrality of a node $i$ will represent the importance with respect to the set of nodes [25]. For example, in the case of transport network, betweenness centrality $BC_{OD}$ can be calculated only for the Origin and Destination (OD) nodes and can be calculated as

$$BC_{OD} = \sum_{m \in \mathbb{N}, n \in \mathbb{N}} \frac{\sigma(m, n \mid i)}{\sigma(m, n)} \quad (4.8)$$
where $s$ is the set of origin nodes and $d$ is the set of destination nodes.

- **Eigenvector Centrality (EC)** Eigenvector centrality measures the influence of a node in the network [8]. The Eigenvector centrality $EC_i$ of node $i$ is defined by the $i^{th}$ element of vector $x$ defined by the equation:

$$Ax = \lambda x$$

(4.9)

where $A$ is the adjacency matrix of the graph $G$ with eigenvalue $\lambda$. By virtue of the Perron–Frobenius theorem, there is a unique solution $x$, all of whose entries are positive, if $\lambda$ is the largest eigenvalue of the adjacency matrix $A$ [54]. If a node has high Eigenvector centrality, then it is more accessed than the other nodes.

- **Information centrality (IC)** - Information centrality, as defined in [78], is the harmonic average of the information associated with the path from node $i$ to the other nodes and given by:

$$IC_i = \frac{N}{\sum_{j=1}^{n} \frac{1}{m_{ij}}}$$

(4.10)

where $In_{ii} = \infty$ and $In_{ij} = \sum_{k=1}^{P} \frac{1}{d_{ij}(s)}$ here $P$ total number of path between $i$ and $j$ and $d_{ij}(s)$ is the sum of all the link weight in the path $s$.

- **Current flow betweenness centrality (CFBC)** - Current flow betweenness centrality of node $i$ is given by the total sum of electrical current that flows through it, when considering all node pairs as source-sink pairs of a unit current flow [34]. This centrality is also known as random walk centrality and the algorithm to calculate the Current flow betweenness centrality $CFBC_i$ for a node $i$ is given described in [55].

- **pseudo inverse Laplacian (LC)** pseudo inverse of any node $i$ is the $i^{th}$ diagonal element of the pseudo inverse of the Laplacian matrix $Q_{n \times n}$ [87].

All the above-mentioned centrality metrics utilizes the link weight except for degree centrality $DC_i$. While calculating these metrics for a transport network, two types of link weight are selected: 1) cost of traveling $C_\ell$ on a $\ell$ without any flow and 2) The amount of flow $x_\ell$ on a link $\ell$. On the basis of the selection of link weight, there two categories of metrics i.e. underlying metrics and overlying metrics are calculated.

### 4.4. Underlying and Overlying metrics

The underlying metrics are computed on the structural topology of the network i.e. the link weight $w_\ell$ is selected as the cost of traveling $C_\ell$ on the link $\ell$. Thus, these centrality metrics are not affected by the type of traffic assignment.

However, the overlying metrics are computed for the network with link weight $w_\ell$ as:

$$w_\ell = \begin{cases} 
  x_\ell & \text{if there is a flow on the link } \ell \\
  0 & \text{otherwise}
\end{cases}$$

(4.11)

When a transport network is loaded with traffic, the links which lie on the path selected during the traffic assignment will have flow on it while rest of the link will have no flow on it. For calculating the overlay centrality metric the links which have no flow will be assigned with weight $W_\ell = 0$. Figure 4.2 shows the configuration of a link weights before and after AoN traffic assignment on the network. Tables 4.1 and 4.2 shows the values of centrality metrics for underlying and overlying network. One can observe the changes in the values of centrality metrics for overlaying and underlying network.

Let us take an example of weighted degree centrality $WDC$, if we want to identify the critical node from the underlying $WDC$ metric values the node $N4$ and $N2$ will have same criticality while after traffic one can easily identify that node $N4$ is more critical than $N2$. 


Figure 4.2: Example of a network displaying the link weight before and after traffic assignment.

**Fig A: Before Traffic Assignment**

**Fig B: After Traffic Assignment**

<table>
<thead>
<tr>
<th>Node</th>
<th>DC</th>
<th>WDC</th>
<th>CC</th>
<th>BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>0.16</td>
<td>0.16</td>
<td>0.32</td>
<td>0.00</td>
</tr>
<tr>
<td>N1</td>
<td>0.66</td>
<td>1.50</td>
<td>0.43</td>
<td>0.33</td>
</tr>
<tr>
<td>N2</td>
<td>0.50</td>
<td>1.00</td>
<td>0.40</td>
<td>0.00</td>
</tr>
<tr>
<td>N3</td>
<td>0.66</td>
<td>1.66</td>
<td>0.43</td>
<td>0.33</td>
</tr>
<tr>
<td>N4</td>
<td>0.66</td>
<td>1.00</td>
<td>0.55</td>
<td>0.66</td>
</tr>
<tr>
<td>D1</td>
<td>0.16</td>
<td>0.16</td>
<td>0.38</td>
<td>0</td>
</tr>
<tr>
<td>D2</td>
<td>0.16</td>
<td>0.16</td>
<td>0.32</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1: Underlying Centrality metrics for the network shown in Figure 4.2A

<table>
<thead>
<tr>
<th>Node</th>
<th>DC</th>
<th>WDC</th>
<th>CC</th>
<th>BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>0.20</td>
<td>0.16</td>
<td>0.75</td>
<td>0.00</td>
</tr>
<tr>
<td>N1</td>
<td>0.40</td>
<td>0.5</td>
<td>2</td>
<td>0.75</td>
</tr>
<tr>
<td>N2</td>
<td>0.00</td>
<td>0.00</td>
<td>2</td>
<td>1.85</td>
</tr>
<tr>
<td>N3</td>
<td>0.40</td>
<td>0.33</td>
<td>2</td>
<td>0.49</td>
</tr>
<tr>
<td>N4</td>
<td>0.60</td>
<td>0.66</td>
<td>2</td>
<td>0.30</td>
</tr>
<tr>
<td>D1</td>
<td>0.20</td>
<td>0.16</td>
<td>0.75</td>
<td>0</td>
</tr>
<tr>
<td>D2</td>
<td>0.20</td>
<td>0.16</td>
<td>0.75</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.2: Overlying Centrality metrics for the network shown in Figure 4.2B
Robustness assessment of the Dutch synchromodal transport network

5.1. Introduction
The framework is applied to the Dutch synchromodal transport network. The four traffic assignment technique i.e. All-or-Nothing (AoN), Model split assignment (MSP), User-equilibrium assignment (UE) and System-optimum assignment (SO) are used in the framework. The algorithm for these assignment techniques are described in chapter from section 3.3 to 3.6.

In section 5.2, a brief overview of network configurations of synchromodal transport network will be given. Section 5.3 describes network perturbation strategies. Finally, from sections 5.4.1 to 5.4.4 describe the assumptions, simulation and calculation related to each traffic assignment techniques, i.e., All or Nothing (AoN), Modal split (MSP), User equilibrium (UE) and System optimum (SO).

5.2. Dutch synchromodal transport network
The data of the dutch synchromodal transport network (DSTN) is taken from the previous research [85], done by Ir W. J. L. Van Dam. As described in section 2.6, The DSTN $G(N, L)$ is a combination of three modalities i.e roadways ($G_{road}$), railways ($G_{rail}$) and inland waterways ($G_{WW}$) with freight terminal($G_{terminal}$), where the set of nodes $N$ and links $L$ is given as:

$$\begin{align*}
N &= N_{road} \cup N_{rail} \cup N_{WW} \cup N_{terminal} \\
L &= L_{road} \cup L_{rail} \cup L_{WW} \cup L_{terminal}
\end{align*}$$

(5.1)

There are $N = 1541$ number of nodes in the set $N$ including $N_{OD} = 40$ number of origin-destination (OD) nodes and $N_{IDP} = 398$ interdependent nodes. And $L_w = 4688$ number of bi-direction links in the set $L$ where $w$ represents the link weight. The link weights are the attribute of a link which is characterizes the modality.

5.2.1. Link weights

Link Length
For all the modalities, Length $d_\ell$ of a link $\ell$ is the spatial distance of the link between two nodes. In the simulation, $d_\ell$ is always measured in kilometers(km).

Link travel time
In general, link travel time is the time required to travel between two nodes. A link can have two type of travel time:

- Free flow travel time $(t_{\ell,0})$ of a link $\ell$ is defined as $t_{\ell,0} = \frac{d_\ell}{v_\ell}$, where $d_\ell$ is the distance and $v_\ell$ is the average speed of the vehicle on the link when there is no congestion.
Travel time \((t_\ell(x_\ell))\) is the travel time on link \(\ell\) for a flow of \(x_\ell\) and given as:

\[
t_\ell(x_\ell) = t_{\ell,0} \left[ 1 + \alpha \left( \frac{x_\ell}{c_\ell} \right)^\beta \right]
\]

(5.2)

Where \(c_\ell\) is capacity of the link and \(\alpha\) and \(\beta\) are the model parameter.

Table 5.1 presents the configuration of the parameters \(\alpha\), \(\beta\), \(v_\ell\), and \(c_\ell\) for different modalities (or link types) that are used in the simulation. The average speed \(v_\ell\) (in km/hr) for different modalities are mentioned in the previous research [85].

The link capacity is generally defined as the maximum number of vehicles that can travel on the link simultaneously. This definition of capacity is more related to a road network rather than a rail or water network. For example, on rail link there cannot be more than one train traveling at the same time in same direction. Similarly for water network the capacity is define on the basis of carrying-capacity of a ship. So, the road capacity [11] [75], the water capacity [92], the rail capacity [63] are the approximated value of capacities converted in terms of weight. Further, assigning different values of capacity for each modalities is inline with the assumption that in synchromodal transport network link attribute characterizes a modality.

<table>
<thead>
<tr>
<th>Link of Modality</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(v_\ell) (km/hr)</th>
<th>(c_\ell)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow roads</td>
<td>0.15</td>
<td>4</td>
<td>60</td>
<td>2300(vehicle/hr)×2(tons/vehicle)</td>
</tr>
<tr>
<td>OD/terminal roads</td>
<td>0.15</td>
<td>4</td>
<td>30</td>
<td>2300(vehicle/hr)×2(tons/vehicle)</td>
</tr>
<tr>
<td>Railways</td>
<td>0.15</td>
<td>8</td>
<td>90</td>
<td>2200(tons/train)×2(trains/hr)</td>
</tr>
<tr>
<td>Waterways</td>
<td>0.15</td>
<td>4</td>
<td>15</td>
<td>1200(tons/ship)×5(ships/hr)</td>
</tr>
</tbody>
</table>

Table 5.1: The configuration of parameters of link travel time for different modalities

**Link travel cost**

Travel cost on a link is the cost of traveling on the link which may include the price of fuel, vehicle depreciation cost etc. The general cost of traveling on any link \(l\), except for an OD-link \(L_{OD}\) and terminal link \(L_{terminal}\), of a modalities \(m\) is given as [85]:

\[
TC_\ell = \kappa_m \times d_\ell + \tau_m \times t_{\ell,0}, \quad \forall \ell \in \{L \setminus \{L_{OD}, L_{terminal}\}\}
\]

(5.3)

Where \(\kappa_m\) and \(\tau_m\) are the unit costs per unit distance and per unit time respectively. The general travel cost for OD links given as:

\[
TC_\ell = \kappa_{road} \times d_\ell + 1.5 \times \kappa_{road} \times t_{\ell,0}, \quad \forall \ell \in L_{OD}
\]

(5.4)

Whereas the general cost of traveling on a terminal link connected via road \((L_{terminal}^{road})\) is given by equation (5.5) and for the links connected via either rail \((L_{terminal}^{rail})\) or water \((L_{terminal}^{water})\) is given by equation (5.6)[85].

\[
TC_\ell = 1.5 \times \kappa_{road} \times d_\ell + 1.5 \times \kappa_{road} \times t_{\ell,0} + a \times f^{-b}, \quad \forall \ell \in L_{terminal}^{road}
\]

(5.5)

\[
TC_\ell = a \times f^{-b}, \quad \forall \ell \in \{L_{terminal}^{water}, L_{terminal}^{rail}\}
\]

(5.6)

Where \(a\) and \(b\) are the transshipment cost and \(f\) is the annual number of container transported through each terminal.

Table 5.2 presents the values of the unit cost per unit of distance \(\kappa_m\) for modalities \(m\), the unit cost per unit of time \(\tau\) for modalities \(m\) and the transshipment cost parameter \(a\) and \(b\) for terminals. And
Parameters | Values
---|---
$\kappa_{\text{road}}$ | 0.038 €/tonne/km
$\kappa_{\text{rail}}$ | 0.004 €/tonne/km
$\kappa_{\text{IW}}$ | 0.004 €/tonne/km
$\tau_{\text{road}}$ | 3.98 €/tonne/hr
$\tau_{\text{rail}}$ | 1.0 €/tonne/hr
$\tau_{\text{IW}}$ | 0.13 €/tonne/hr
$a$ | 26.285
$b$ | 0.146

Table 5.2: The values of the parameters of link travel cost $T C_j$ [85]

5.2.2. OD matrix
The OD-matrix for the simulation is taken from BasGoed [66] [85]. Figure B.10 shows the in-between demand for the 40 regions of the Netherlands. In the simulation, the container movement inside the region is not considered so the diagonal values of the OD-matrix are zero.

Figure 5.1: The amount of container transported domestically in the Netherlands

Further, the traffic assignment models consider that the given demand matrix only exists for the modeling period, say for a peak hour of a day. This type of assumption leads to the steady-state analysis of traffic assignment over the considered period [17].

5.3. Network perturbation strategies
As mentioned in section 4.2.1, there are three types of perturbation which are considered for the robustness assessment of a transport network. The strategies for each type of perturbation is defined below.

Element disruption
As described in Chapter 4, a single element disruption is considered i.e. one element is removed and its effect on the performance is measured. During the simulation, only nodes of the DSTN are
considered for removal. In the DSTN, these nodes can be selected from any of the modalities except for the OD-nodes. The OD nodes are the centroids representing an entire region thus their removal is not considered.

Whenever a node is removed, all the links incidence to the node is removed as well, representing a complete failure of the node. In the DSTN, there are some nodes which affect only one modality after removal whereas some nodes, such as interdependent nodes, can affect more than one modalities. So, the DSTN is subjected to two types of node removal strategies:

• **Node removal among interdependent pair** - As mentioned in section 2.6.2, Interdependent nodes are at the crossing points of the modalities. So, all the nodes representing that crossing point will be removed from the network. For example, a rail bridge crossing a road network is removed then the links of the road network underneath that bridge will be removed as well. And there are 199 such interdependent pair exists in the DSTN.

• **Node removal among all nodes** - In this strategies, a node is removed from the DSTN without considering the interdependence i.e. removal of an infrastructure crossing node affects one modality only. For example, if a road bridge crossing waterway is removed then only the links of the road network are removed. For each traffic assignment, one by one 1501 nodes are removed and corresponding node criticality is calculated.

\[ G(N, L, w) \] is a graph network with \( N \) nodes and \( L \) links and \( G \) is loaded with a link weight \( w \). And the total system cost of traveling on this network is given by \( C_G(\mathcal{A}) \), where \( TA \) represent the traffic assignment technique that is used for the flow assignment. Algorithm 3 explains how the total system cost \( C_G(\mathcal{A})(w) \) will be calculated after removal of each infrastructure \( i \). The list of nodes to be removed is given by \( N_i \). If “Removal of interdependent pair” strategy is used then each element \( i \) of the list \( N_i \) has two interdependent nodes else each element \( i \) will have a single node.

#### Algorithm 3: Total system cost calculation after elemental disruption

**Input:** \( G(N, L, w) \) with demand matrix \( D_{sxd} \) and \( N_i \)

**Result:** \( C_G^TA(\mathcal{A})(w) \) and \( C_G^TA_i(\mathcal{A})(w) \) corresponding to original and perturbed network respectively

1. Using traffic assignment \( TA \) assign flow to the network \( G(N, L, w) \);  
2. Calculate the total cost of traveling \( C_G^TA(\mathcal{A})(w) \);  
3. for \( i \) in \( N_i \) do  
   4. Remove \( i \) from the network \( G(N, L, w) \);  
   5. Create a new network \( G/\mathcal{I} \);  
   6. Using traffic assignment \( TA \) assign the flow to the network \( G/\mathcal{I}(N/\mathcal{I}, L/\mathcal{I}, w) \);  
   7. Calculate the total cost of traveling \( C_G^TA_i(\mathcal{A})(w) \) after removal of \( i \) ;  
4. end

**Network parameter degradation**

For this perturbation strategy, the link travel time will be degraded i.e the travel time on a link will be increased from the original setup as described in Section 5.2.1. The two link travel time \( t_{f,0} \) and travel time \( t_x(x, ) \) can be degraded by decreasing the average speed \( v_x \) and by decreasing the capacity \( c_x \) respectively. For UE and SO assignments, travel time \( t_x(x, ) \) is increased by decreasing the capacity of a link. For AoN and MSP assignments, free-flow travel time \( t_{f,0} \) is increased by decreasing the average velocity on a link.

The network parameter degradation is applied on a single modality or on a combination of two. The average speed \( v_t \) is decreased from 30% to 90% with an interval of 10% whereas the capacity \( c_t \) will be decreased from 20% to 80% with an interval of 20%. The purpose of network parameter degradation is to understand the importance of a modality in synchromodal transport network.

Travel cost \( TC_G \) degradation is applied to the DSTN on for AoN and MSP assignments. Travel cost \( TC_G \) is a function of free flow travel time \( t_{f,0} \), as given by equations (5.3), (5.4) and (5.5), thus by degrading the \( t_{f,0} \), new \( TC_G \) is calculated.

\[ G(N, L, w) \] is a synchromodal transport network with \( N \) nodes and \( L \) links and \( G \) is loaded with a link weight \( w \). And the total system cost of traveling on this network is given by \( C_G^TA(\mathcal{A})(w) \), where \( TA \)
represent the traffic assignment technique that is used for the flow assignment. Algorithm 4 explains the calculation of the total system cost $C_{GA}^T(w_{P}^m)$ after degrading the link weight $w$ of modalities $m$ by $p\%$. The parameter degradation is applied on 6 combinations of modalities i.e. road ($Ro$), rail ($Ra$), water ($Wa$), road-rail ($RoRa$), water-road ($WaRo$) and water-rail ($WaRa$).

Algorithm 4: Total system cost calculation after network parameter degradation

\begin{algorithm}
\begin{algorithmic}[1]
\Require $G(N,L,w)$ with demand matrix $D_{sd}$
\Ensure $C_{GA}^T(w_{P}^m)$ corresponding to each $m$ with degradation of $p\%$ and the original cost $C_{GA}^T(w)$
\State Using traffic assignment $TA$ assign flow to the network $G(N,L,w)$;
\State Calculate the total cost of traveling $C_{GA}^T(w)$;
\For {$m$ in [Ro, Ra, Wa, RoRa, WaRo, WaRa]} do
\State \For {$p=0.2$ to $0.9$ step $0.1$} do
\State Update link weight $w_{P}^m\forall \ell \in L_{m} \rightarrow w_{P}^m = w^m(1-p)$;
\State Using traffic assignment $TA$ assign flow to the network $G(N,L,w_{P}^m)$;
\State Calculate the total cost of traveling $C_{GA}^T(w_{P}^m)$;
\EndFor
\EndFor
First, the network parameter is degraded using Algorithm 4 and then the elemental disruption is applied to the updated network using Algorithm 3. Combined disruption will help in understanding the criticality of an interdependent node when one of the modalities is underperforming. A real-world example of this could be the failure of a railway bridge over a road network, and the road network congested and operating to its maximum capacity.

For any traffic assignment, the network considers a continuous flow of container between modalities i.e. there is no delay at the freight terminals. Further, all traffic assignment technique utilizes Dijkstra shortest path algorithm [12] for finding the shortest path between Origin and destinations.

In the simulation, it is considered that the traffic can flow in both directions on any link of the synchromodal transport network. This simplification of synchromodal transport network is done due to the limitation of available data, as stated in the previous research [85]. Further, in this research, a complete failure of a node is considered i.e. all incidence link will be removed.

In the simulation, after every elemental disruption a new network is created by removing the element from the old network, and re-assignment of traffic is done on the new traffic. But in a real-world scenario, the traveler does not re-plan their journey from the start but readjust from the point of disruption. Since this is a steady-state analysis (described in section 5.2.2 and the strategy of re-assignment can be justified by considering the disruption occurs after the peak-hour and has effect till the next peak hour.

5.4. Performance indicator for different traffic assignments

5.4.1. AoN assignment

AoN traffic assignment is described in section 3.3, it is a congestion-free assignment technique and the total system cost after assignment is given by:

\[ C_{GA}^{AoN} = \sum_s \sum_d \sum_{l} D_{sd} \times C_{sd,l} \quad \forall l \in P_{sd} \text{ and } P_{sd} \subseteq L \]  \hspace{1cm} (5.7)

where $C_{sd,l}$ is the cost of traveling on a link $l$ between OD-pairs $s - d$ and $P_{sd}$ is the shortest path between OD-pair $s - d$.

In the AoN assignment, cost of traveling $C_{l}$ on a link will be equal to the weight $w$ on that link, which can either be link length $d_{l}$, free-flow travel time $t_{f,l}$ or travel cost $TC_{l}$. Node criticality $\omega_{AoN,l}$ can be calculated using Equation (5.7) and the Algorithm 3, and given by:

\[ \omega_{AoN,l}(w) = \frac{C_{GA}^{AoN}(w) - C_{GA}^{AoN}(w)}{C_{GA}^{AoN}(w)} \]  \hspace{1cm} (5.8)

where $C_{GA}^{AoN}(w)$ is the total system cost for original DSTN and $C_{GA}^{AoN}(w)$ is the total system cost of perturbed network due to removal infrastructure $i$. 

As described in section 4.4, the overlaying centrality metric is calculated by finding the overlaying network topology after traffic flow assignment. In AoN assignment, a link will have flow only when it belongs to a shortest path \( P_{sd} \), thus flow on the link will be given as:

\[
x_{\ell} = \begin{cases} 
\sum_s \sum_d D_{sd} & \text{if } \ell \in P_{sd} \\
0 & \text{otherwise}
\end{cases}
\] (5.9)

### 5.4.2. MSP assignment

As described in section 3.4, MSP traffic assignment splits the demand \( D_{sd} \) into \( k \)-shortest path using a Multinomial logit model. Multinomial logit model uses scale parameter \( \beta' \) and cost of traveling on the \( k \)-shortest paths to determine the probability of choice \( p_{p} \) of any path \( p \in k \). Then the demand on path \( p \) is \( D_{sd,p} = p_{p} \times D_{sd} \). And the total system cost for this assignment is given by:

\[
C^{\text{MSP}}_{G} = \sum_s \sum_d \sum_{p \in k} D_{sd,p} \times C_{sd,p}
\] (5.10)

Where \( C_{sd,p} \) is the total cost of travel on path \( P \). Similar to AoN the total cost of traveling is replaced by the sum of weights \( w \) \( \forall \ell \in P \). On the basis of the number of the shortest path selected between OD-pairs, the two variant of MSP assignment are simulated that are:

- Model split assignment with 2 shortest paths (\( MS2P \))
- Model split assignment with 3 shortest paths (\( MS3P \))

The three values of \( \beta' \) i.e., \( \beta' = 0.1, \beta' = 0.5, \beta' = 1.0 \) are assigned to the Multinomial logit model for both \( MS2P \) and \( MS3P \) assignments. Further, for both \( MS2P \) and \( MS3P \) the shortest paths are node-disjoint shortest paths. The intuition behind selecting node-disjoint shortest paths as follows:

1. As per the definition of Synchromodality containers/goods have to switch modes (i.e. rail, water or road) in real-time throughout their journey. So, splitting the demand on the basis of strict mode choice will not fit the purpose
2. It is observed during the simulation that the road network is very dominant. To increase the chance of selection of a node from different modalities this strategy seems to fit the purpose

Now, for MSP assignments there are 3 sets of node criticality for both \( MS2P \) and \( MS3P \) corresponding to the values of \( \beta' \). The node criticality will be calculated by using Equation (5.10) and Algorithm 3 and given as:

\[
\omega^{\text{MSP},i}_{\ell}(w) = \frac{C^{\text{MSP}}_{G \setminus i}(w) - C^{\text{MSP}}_{G}(w)}{C^{\text{MSP}}_{G}(w)}
\] (5.11)

Similar to AoN, the overlying centrality metric will be calculated by assigning the flow \( x_{\ell} \) on all the links of the network, where \( x_{\ell} \) is given as:

\[
x_{\ell} = \begin{cases} 
\sum_s \sum_d D_{sd,P} & \text{if } \ell \in P, \forall P \in k \\
0 & \text{otherwise}
\end{cases}
\] (5.12)

### 5.4.3. UE assignment

The UE assignment will be done using the Algorithm 1, as described in section 3.5.1. The total system cost for this assignment is calculated by:

\[
C^{\text{UE}}_{G} = \sum_{\ell \in G} x_{\ell}^{*} \times t_{\ell}(x_{\ell}^{*})
\] (5.13)

Where \( t_{\ell}(x_{\ell}^{*}) \) is the travel on link \( \ell \) at the equilibrium flow \( x_{\ell}^{*} \). The node criticality for UE assignment is given by:

\[
\omega^{\text{UE},i}_{\ell}(w) = \frac{C^{\text{UE}}_{G \setminus i}(w) - C^{\text{UE}}_{G}(w)}{C^{\text{UE}}_{G}(w)}
\] (5.14)

The output of UE assignment algorithm is the flow on a link. So, for creating overlying network topology for UE assignment, the equilibrium flow \( x_{\ell}^{*} \) will be used.
5.4.4. SO assignment
The SO assignment will be done using the Algorithm 2, as described in section 3.6.1. The total system cost for this assignment is calculated by:

$$C_s^{SO} = \sum_{\ell \in L} x_\ell^* \times t_\ell(x_\ell^*)$$ (5.15)

Where $t_\ell(x_\ell^*)$ is the travel on link $\ell$ at the equilibrium flow $x_\ell^*$. The node criticality for SO assignment is given by:

$$\omega_{SO}(w) = \frac{C_s^{SO}(w) - C_s^{SO}(w)}{C_s^{SO}(w)}$$ (5.16)

The output of SO assignment algorithm is the flow on a link. So, for creating overlying network topology for UE assignment, the equilibrium flow $x_\ell^*$ will be used.
Observations and discussion

In this chapter, the results of the simulation are discussed. Sections 6.1, 6.2, 6.3 and 6.4 explain the results from each traffic assignment i.e. All or Nothing (AoN), Mode split assignment (MSP), User equilibrium (UE) and System optimum (SO) respectively. Section 6.5 presents the identification of critical infrastructures within the Dutch synchromodal transport network.

Further, each of the four sections has three sub-parts. In the first part, node criticality $\omega_i$ and the robustness against elemental disruption is discussed. In the second part, the results with respect to network parameter degradation and combined disruption (discussed in section 8) are explained. In the last part, the underlying and overlying topological metrics relationship with node centrality is discussed.

6.1. All or Nothing

6.1.1. Node criticality in AoN assignment

Figure 6.1 shows the distribution node criticality in Dutch synchromodal transport network under AoN assignment with different link weights. Interestingly, the distribution for all three weight type matches a power-law distribution which suggests that there are a few nodes whose removal significantly increases the total cost $C$ compare to other nodes.

The distribution of node criticality $\omega_i^{AoN}$ is fitted by a power-law probability density function (PDF) $f_{\omega}(W)$ where node criticality $W = \omega$ is a random variable, and the PDF is given by:

$$f_{\omega}(W) \approx x^{-k} \quad (6.1)$$

Where the exponent $k$ manifests the robustness of the network i.e. a larger value of $k$ represents a better robustness.

Figures 6.1a, 6.1b and 6.1c shows that when time is selected as link weight, the network shows higher robustness against node perturbation than distance and travel cost. This suggests that the system’s performance optimizes the total system cost when link weight is time. The travel cost is a function of both time and distance(see section 5.2.1), the PDF exponent $k$ reflects this relationship.

Further, for all three weights, i.e., time, travel cost and distance, the removal among interdependent pair shows higher criticality than removal among all nodes.

6.1.2. Effect of network parameter degradation: AoN

The effect of perturbation of travel time $t_{f,0}$ and travel cost $TC_f$ are considered for the investigation of the Dutchsynchromodal transport network. Figure 6.2 shows the effect perturbation on the total system cost $C^{AoN}$ and the flow $x_i$, under the degradation of free-flow travel time for different modalities. Figure 6.2c shows that fractional degradation of free-flow time on Inland waterways (IW) has no effect on $C^{AoN}$, this is due to the fact that IW has no flow even at the original free-flow travel time. Figure 6.2b shows that initial assignment put some flow on the rail network but soon after the first degradation, all the flow are transferred to the road network and $C^{AoN}$ increases.

In the case of the road network, $C^{AoN}$ increases after each degradation of link travel time. A similar trend of increment of $C^{AoN}$ can be observed when the degradation of travel time is applied on two
6. Observations and discussion

Observations and discussion

Figure 6.1: Distribution of node criticality $\omega_{\text{AoN}}$ for both single node and interdependent pair removal for AoN assignment under different link weights. The PDF is fitted by a function $f_{\omega}(W)$. Both the axis are on a log scale.
6.2. MSP assignment

6.2.1. Node criticality and robustness

Figure 6.6 shows the distribution of node criticality \( \omega^\text{MSP} \) for MSP traffic assignment for different logit scale parameter i.e. \( \beta' = [1.0, 0.5, 0.1] \) and different link weights i.e. Link length \( d_e \), free-flow travel time \( t_{f,e} \) and travel cost \( T_{C,e} \). All the distribution shown in Figure 6.6 resembles a power-law distribution. Similar to AoN, the distributions are fitted with a power-law probability density function \( f_\omega(W) \sim x^{-k} \), where \( k \) manifests the robustness of the network.

The network performance is better when the traffic assignment is done with free-flow travel time, irrespective of \( \beta' \). The performance of the network improves as \( \beta' \) decreases for a particular link weight. This implies that if the traffic is distributed equally among the path irrespective of the cost then the system perform better but in real-world the choices of paths are based on the cost of traveling. So, we choose \( \beta' = 1 \) for further analysis of the MSP assignment.

---

\(^1\)The result analyzed here are only for model split between 2 paths i.e. MS2P. MS3P results are not discussed because most of the OD pairs does not have three disjoint paths between them.
6. Observations and discussion

(a) Perturbation of travel time of the road network

(b) Perturbation of travel time of the rail network

(c) Perturbation of travel time of the water network

(d) Perturbation of travel time of the road-rail network

(e) Perturbation of travel time of the road-water network

(f) Perturbation of travel time of the rail-water network

Figure 6.2: Bar plot shows the flow on modalities versus the fraction of time after perturbation. The cyan line shows the total system cost \( c_G^{\text{ADN}}(t_{(\beta)}) \) after perturbation.
Figure 6.3: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ when network parameter $Travel time$ is perturbed for road network. The naming convention for interdependent pair: mode1_id-mode2_id where RI represent road, WI represent water and SI represent railways.
6. Observations and discussion

Figure 6.4: Bar plot shows the load on modalities versus the fraction of cost after perturbation. The cyan line shows the total system cost $C_{\text{load}}(T_C)$ after perturbation.
6.2. MSP assignment

<table>
<thead>
<tr>
<th>Metric</th>
<th>Distance</th>
<th>Time</th>
<th>Travelcost</th>
</tr>
</thead>
<tbody>
<tr>
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<td>BCoo</td>
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<tr>
<td>IC</td>
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</tr>
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<td>-0.00</td>
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<tr>
<td>LC</td>
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<td>-0.25</td>
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(a) Underlying topological centrality VS node criticality for interdependent pairs removal

<table>
<thead>
<tr>
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</thead>
<tbody>
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<td>-0.20</td>
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</tr>
<tr>
<td>LC</td>
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<td>-0.15</td>
<td>-0.54</td>
</tr>
</tbody>
</table>

(b) Underlying topological centrality VS node criticality for single node removal

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</thead>
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<tr>
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<td>0.21</td>
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<td>0.89</td>
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<tr>
<td>LC</td>
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</table>

(c) Overlying topological centrality VS node criticality for interdependent pairs removal

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<th>Metric</th>
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<th>Time</th>
<th>Travelcost</th>
</tr>
</thead>
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<td>-0.29</td>
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<tr>
<td>LC</td>
<td>0.78</td>
<td>0.39</td>
<td>0.59</td>
</tr>
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</table>

(d) Overlying topological centrality VS node criticality for single node removal

Figure 6.5: The Spearman correlation coefficient between topological centrality metric and node criticality for different link weights under AoN assignment.
The effect of network parameter perturbation

Figure 6.2.2. suggests that the network under MSP assignment are more sensitive towards perturbations. Relatively low value but as the fraction of perturbation increases, node criticality increases as well. This perturbation is applied on the road network. Initially, the node criticality under MSP assignment have and thus demand is divided in such a way that the cost compared to the other disjoint path. And after perturbation the newly selected paths are comparable model assigns the probability of choosing a path, the best path may have got a very high probability compared to the other modalities. This may have happened because the MSP algorithm used in the research, is choosing two disjoint paths for traffic assignment. So, initially when the multinomial logit time increases for these modalities. This may have happened because the MSP algorithm used in the research, is choosing two disjoint paths for traffic assignment. So, initially when the multinomial logit time increases for these modalities.

Figure 6.7 shows the effect of time perturbation on traffic flow for different modalities and the effect on the total system cost $C_{T}^{MSP}$. Like AoN, the time perturbation of the road network has the worst effect on the cost $C_{T}^{MSP}$. Whereas for inland waterways and railways the cost decreases as the free-flow travel time increases for these modalities. This may have happened because the MSP algorithm used in the research, is choosing two disjoint paths for traffic assignment. So, initially when the multinomial logit model assigns the probability of choosing a path, the best path may have got a very high probability compared to the other disjoint path. And after perturbation the newly selected paths are comparable and demand is divided in such a way that the cost $C_{T}^{MSP}$ decreases. The travel cost perturbation shown in Figure 6.9 has a similar effect on the synchronomodal network as the time perturbation.

Figure 6.8 shows a heat map of node criticality for removal among interdependent pairs when time perturbation is applied on the road network. Initially, the node criticality under MSP assignment have relatively low value but as the fraction of perturbation increases, node criticality increases as well. This suggests that the network under MSP assignment are more sensitive towards perturbations.
(a) Perturbation of travel time of the road network

(b) Perturbation of travel time of the rail network

(c) Perturbation of travel time of the water network

(d) Perturbation of travel time of the road-rail network

(e) Perturbation of travel time of the road-water network

(f) Perturbation of travel time of the rail-water network

Figure 6.7: Bar plot shows the flow on modalities versus the fraction of time after perturbation. The cyan line shows the total system cost $C^{\text{MSP}}_G(t_{\text{av}})$ after perturbation.
Figure 6.8: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ when network parameter $\gamma$ is perturbed for the road network. The naming convention for interdependent pair: mode1_id-mode2_id where RI represent road, WI represent water and SI represent railways. MSP assignment for $\gamma = 1.0$. 

Observations and discussion
Figure 6.9: Bar plot shows the load on modalities versus the fraction of cost after perturbation. The cyan line shows the total system cost $C_{MS}^{MP}(TC)$ after perturbation (MSP assignment for $\beta^i = 1.0$).
6.2.3. Topological centrality metric analysis

Figures 6.10 shows the Spearman’s correlation between node criticality and different centrality metrics described in section 4.3, where the columns represent the type of link weight i.e. link length, free-flow travel time and travel cost assigned to the network while calculating the node centrality $\omega_R$.

The results are similar to AoN, i.e., for underlying centrality metric OD subset betweenness centrality ($BC_{OD}$) has a better correlation coefficient. Whereas, for overlying centrality metric weighted degree centrality ($WDC$), Eigenvector centrality ($EC$), closeness centrality ($CC$) and current flow betweenness centrality $CFBC$ has better correlation coefficients.

![Correlation coefficients for different centrality metrics](image)

(a) Underlying topological centrality VS node criticality for interdependent pairs removal

(b) Underlying topological centrality VS node criticality for single node removal

(c) Overlying topological centrality VS node criticality for interdependent pairs removal

(d) Overlying topological centrality VS node criticality for single node removal

Figure 6.10: The Spearman correlation coefficient between topological centrality metric and node criticality for different link weights under MSP assignment with $\beta^T = 1.0$

6.3. UE Assignment

6.3.1. Node criticality and robustness

Figure 6.11 shows the distribution of node centrality $\omega^{UE}_i$ under UE traffic assignment. The distribution resembles a power-law distribution and fitted with a probability density function $f_\omega(W) \sim x^{-k}$, where
$K$ signifies the robustness of the network. The robustness of the network is less when interdependent pairs are removed, as compared to removal among all nodes. Thus, interdependent pairs are more critical compared to single node.

6.3.2. Effect of network parameter perturbation
For UE assignment, the network robustness is investigated under link capacity perturbation. Figure 6.12 shows the effect of capacity perturbation on the traffic flow at UE equilibrium and total system cost when the perturbation is applied to different modalities i.e. road, rail, water. In UE assignment, as capacity reduces the total system cost increases, this relationship between perturbation and system cost remain same for all modalities, unlike AoN or MSP where only the perturbation on road network causes an increment in the total system cost. Although, road network still causes the highest decrements in the system performance for same fraction of capacity perturbation as compared to other modalities. As described in section 3.5, this assignment considers the effect of congestion for traffic assignment thus capacity perturbation reduces the performance for all modalities including the dutch synchromodal transport network.

6.3.3. Topological centrality metrics
Figures 6.14 shows the Spearman's correlation between node criticality and different centrality metrics as described in section 4.3, where the columns represent the type of removal i.e. Interdependent pair and single-node. In the case of UE assignment, the correlation coefficients for different centrality metrics are lower than the AoN and MSP assignments. Although, the centralities which have shown good correlation in AoN or MSP, also show a better correlation in UE assignment.

UE assignment is closer to real-world traffic assignment as compared to AoN and MSP, UE assignment distribute the traffic among all paths having comparable cost while in AoN assignments all the traffic is assigned to a single best route. Thus in case of UE assignment the flow is more distributed among the links, this may have caused the reduction in correlation coefficients.

6.4. SO Assignment
All the results of SO assignment have similar characteristics as UE assignment. SO assignment shows better robustness as compared to UE assignment, for both interdependent pair and single-node. Figure 6.15 shows the distribution of node criticality under SO assignment.

Figure 6.16 shows the effect of the capacity perturbation on different modalities. Compared to
Figure 6.12: Bar plot shows the load on modalities versus the fraction of cost after perturbation. The cyan line shows the total system cost $C_{SE}^E(T_r(x^*_r))$ after perturbation.
6.4. SO Assignment

Figure 6.13: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ when network parameter Capacity is perturbed for road network. The naming convention for interdependent pair: mode1_id-mode2_id where RI represent road, WI represent water and SI represent railways.
6. Observations and discussion

Observations and discussion

Figure 6.14: The Spearman correlation coefficient between topological centrality metric and node criticality for interdependent pairs and single node removal for SO assignment

Figure 6.15: Distribution of node criticality $\omega_i^{SO}$ for both single-node and interdependent pair removal for the UE assignment. The PDF is fitted by a function $f_\omega(W)$. Both the axis is on a log scale.

6.5. Identification of critical interdependent pairs

Figure 6.19 shows the top 5 critical interdependent pair with respect to node criticality $\omega_i$. For both UE and SO assignments the top critical interdependent infrastructure lies around in the vicinity of NL339 and we know that this region has the highest total demand (see Figure 2.2). In case of AoN and MSP
6.5. Identification of critical interdependent pairs

Figure 6.16: Bar plot shows the load on modalities versus the fraction of cost after perturbation. The cyan line shows the total system cost $c_\text{S}_G(T_r(x+\epsilon))$ after perturbation.
Figure 6.17: Rows of heat-map represent interdependent pair and shows their respective criticality $\omega_i$ when network parameter capacity is perturbed for road network. The naming convention for interdependent pair: mode1_id-mode2_id where RI represent road, WI represent water and SI represent railways.
assignment, out of top five infrastructures selected on the basis of node criticality, 3 for AoN and 4 for MSP lies in the vicinity of NL339. Logically, this observation is quite valid as NL339 has the highest total demand so any infrastructure near to this region must have higher traffic flow compared to other infrastructure. Thus, the choice of node criticality seems valid.

In addition to that a road bridge over the water network, with ID is 1023, as shown in figure 6.19, is identified as critical infrastructures under all four traffic assignments. Interestingly, this bridge lies between the two regions, those are NL339 and NL411, which has highest total demand.
Figure 6.19: Location of top 5 bridges with highest node centrality under each traffic assignment.
Conclusion

In this thesis, the Dutch synchromodal transport network is used as a case study to formulate a methodology to identify critical infrastructures in synchromodal transport network. Section 7.1 summarizes the case study and the results obtained at the end. Section 7.2 presents the possible future research ideas that came across during this study.

7.1. Conclusion

The research started with the analysis of different components of the Dutch synchromodal transport network. The components which are analyzed in the study are Origin-destination (OD) nodes, road network, rail network, inland waterways, terminals and crossing points such as bridges, tunnels, etc. The purpose of this analysis is to understand the transport network and its topology. The two take-up point from the analysis are: a) Out of the three transport network i.e. roadways, railways and waterways only road network is connected to all OD-nodes, b) The total demand from an OD-node has a relationship with the degree of the node. After understating the network, the next aim of the research is to identify different methods that can quantify the criticality of an infrastructure.

In transportation engineering, travel demand forecasting is used to estimate the expected volume of traffic on a transportation system. The output of travel demand forecasting is total system cost, which is a function of traffic flow on the links and cost of traveling on those links. The cost of traveling is the intrinsic property of a link and can either be link length, travel time on the link or traveling cost on the link. There are several traffic assignment methods which can be used to determine the flow on a link.

In this research, All-or-Nothing (AoN) assignment, Model split (MSP) assignment, User equilibrium (UE) assignment and System optimum (SO) assignment are used to determining the flow on a link. AoN and MSP assignment consider that link has infinite capacity thus ignore the congestion in the network. While UE and SO consider that capacity of a link is limited and as soon as traffic flow on a link reaches to its maximum capacity the travel time will increase exponentially exhibiting congestion. Thus the flow produced by UE and SO are closer to the real-world scenario of a transport network.

The transport network is designed in such a way that the total system cost remains minimum. If there is any change in the transport network i.e. removal/addition of a link or a junction the total system cost will change. In the research, this phenomenon of change of total system cost for any perturbation in the transport network is used to determine the criticality of a node. First, the flow is assigned to the transport network using a traffic assignment model and the total system cost for that flow is calculated. Then a node is removed from the network and once again the flow is assigned using the same traffic assignment model and a new total system cost is calculated. The relative change in the total system cost due to removal of the node determines the criticality of that node and is defined as the node criticality.

This research also utilizes another measure, which comes from network science, to determine the importance of a node in a network and these measures are known as topological centrality metrics. Degree centrality, closeness centrality, betweenness centrality, eigenvalue centrality, etc are a few examples that are used to determine the criticality of node in the transport network. Centrality metrics are calculated on two variant of network. First on the underlying network which calculates the centrality
of a node on the basis of structural topology. Second on the overlying network which calculates the centrality of node on the basis of the flow.

A systematic framework is designed that utilizes both the node centrality and topological centrality metrics to determine the criticality of a node under a given traffic assignment. Now, this framework is applied to the Dutch transport network to access the robustness and to identify critical infrastructures in the network.

The application of the framework produces several insightful results:

- Irrespective of the type traffic assignment or the selection of network parameter, The distribution of node criticality exhibits a power-law behavior. This is an interesting result which suggests that the synchromodal transport network already tends to a robust state against single node failure. We observed that for all the traffic assignments, i.e., AoN, MSP, UE and SO, node criticality for interdependent infrastructure tend to attain a higher value as compared to single-node. This solidifies the importance of considering interdependence for the robustness analysis of the synchromodal transport network. These finding of the node criticality will not only help in creating a better risk mitigation plan but also help in prioritizing the maintenance of transport infrastructures.

- We observe that the link parameter degradation for the road network produces a catastrophic growth in the total system cost, irrespective of the traffic assignment. This suggests that the other two modalities, i.e., inland waterways and railways fail to cater to the excess traffic load due to the unavailability of road network. Thus, we can say that in order to maintain the performance of the synchromodal transport network one must take care of road network.

- The choice of intrinsic link parameters such as average speed, unit cost per unit distance and unit cost per unit time for a modality affects its traffic handling behavior.

- We observed that the overlying centrality metrics are strongly correlated to the node criticality as compared to underlying centrality metric, this implies that the importance of a node is related to the flow passing through that node. We observed that Weighted degree centrality, Eigenvalue centrality, closeness centrality, and current-flow betweenness centrality exhibit higher Spearman’s correlation coefficients as compared to other metrics. Transport engineers can use these centrality matrices to identify critical infrastructure quickly.

The Dutch synchromodal transport network already exhibits robustness against single node failure but its performance decrease with the decrease in the performance of road network. In order to improve the performance of synchromodal transport network we recommend to improve the capabilities of waterways and railways.

7.2. Future research

There are some potential directions of research that are identified during the current work.

7.2.1. Considering delay at terminals

In the current research, a delay at the terminal is not taken into account, i.e., as soon as a container reaches the terminal node it will be transferred to other modality. So this research can be further extended by including the delay at a terminal which will produce more realistic result.

7.2.2. Better traffic assignment

The algorithm used to implement UE and SO traffic assignment produces the steady-state assignment. So it would be great to compare the results from the framework for dynamic traffic assignments. MSP assignment used in this research selects disjoints paths which produce unrealistic results if on a path is very large compared to others. A better way to implement this by selecting K-shortest path between OD-nodes.

7.2.3. Centrality metric

It is observed during this research that overlying centrality produces a better result than underlying centrality at least for the current research set-up. One can extend this concept to the real-world transport network and check if it produces similar results.
In network science, these centrality metrics are sometimes developed for specific network characteristics. For example, current flow betweenness is developed for electrical networks and we have seen that it produces good results in case of transportation networks. One can either further improve an existing centrality metric for transport networks or try to formulate a better one. Researchers suggest looking into percolation centrality for transport networks especially in the case of dynamic traffic assignment. The results from the correlation between node centrality and topological centrality metrics can be used to analyze the cascading failure effect on the synchromodal transport network.

7.2.4. **Network topology of synchromodal transport network**

In the current research, the links of the network do not have any directional restriction whereas in real-world it has a directional restriction. For example, multi-lane highways where one lane is used to travel in one direction. A similar study can be performed on such a network.
A.1. All or Nothing

In All or Nothing, trips between OD pair is assigned to a single path with minimum cost of traveling. Let us consider transport network with two OD node $o$ and $d$ and the corresponding demand between these OD pair is $D_{od} = 5$. There are three possible paths $L_1, L_2, L_3$ between these OD pair, as shown in figure A.1. The cost of traveling on these links are the length of the link, i.e., $L_1 = 2, L_2 = 3, L_3 = 2$. Now, we will assign the traffic using AoN algorithm as mentioned in Section 3.4:

1. Find step is to determine the shortest path on the network. For the given network $L_1$ and $L_2$ are the two shortest path between the OD pairs

2. Assign the demand on the selected path. AoN algorithm can choose either $L_1$ or $L_2$ and then assign the $D_{od}$ to the link. AoN does not give priority to any route it will just select on at random.

3. Finally the cost of traveling will be calculated as $C_{od} = D_{od} \times L_1$, which will be equal 10 in for this network

A.2. Model split assignment algorithm

Figure A.2 represent a transport network with two OD-nodes $o - d$ and the demand between this OD pair is given by $D_{od} = 10$. This network has 5 nodes connected via 7 links. Let us follow the step mentioned in section 3.4 and calculate the cost of traveling on the network

- First we find 2-shortest path on the network. The two path $P_1$ and $P_2$ are the shortest paths between $O = o - d$ pair and are highlighted with green and red color respectively

![Figure A.1: An example of simple transport network to solve AoN assignment](image)
A. Application of traffic assignment algorithm

Now the cost of traveling on each path is calculated by simply adding the link weights of the path and is given as \( C_{P1} = L_2 + L_3 + L_4 = 4 \) and \( C_{P2} = L_6 + L_7 = 5 \).

The probability of choosing each path is calculated using multinomial logit model as given by Equation (3.5) defined in section 3.4. Probability of choosing \( P_1 \) is \( p_1 = 0.73 \) and choosing \( P_2 \) is \( p_2 = 0.27 \). For calculating the probability, the scale parameter \( \beta \) is used as 1.

Calculate the portion of demand which will be assigned to each path by \( D_{od,P1} = D_{od} \times p_1 \) and \( D_{od,P2} = D_{od} \times p_2 \) which is equal to 42.7.

A.3. UE assignment

Figure A.3 shows a transport network with two OD-nodes and two possible paths \( L_1 \) and \( L_2 \) between these two nodes. Each of these paths has weight as link travel time, i.e., \( t_{od} = 2 + x_1 \) and \( t_{od} = 1 + x_2 \) which is a function of the flow \( x_1 \) and \( x_2 \) on the link \( L_1 \) and \( L_2 \) respectively. To get the user equilibrium flow for this network, we will solve the following equations:

Minimize

\[
Z(x_l) = \sum_{\ell} x_{\ell} \int_0^{x_{\ell}} t_{\ell}(x_{\ell}) \, dx
\]

subject to

\[
\sum_k f_{kd}^{k} = D_{od}^{s} \quad \forall s, d
\]

\[
x_l = \sum_{\ell} \sum_{s} g_{lkd}^{\ell} f_{kd}^{k} \quad \forall l
\]

\[
f_{kd}^{k} \geq 0 \quad \forall s, d, k
\]

\[
x_l \geq 0 \quad l \in L
\]

The minimization function \( Z(x_l) \) can be expanded for this case as:

\[
Z(x_l) = \int_0^{x_1} t_1 \, dx + \int_0^{x_2} t_2 \, dx
\]

Now after minimize the equation (A.2) under the constraint \( x_1 + x_2 = D_{od} = 10 \), we obtain equilibrium flow \( x_1^* = 5.5 \) and \( x_2^* = 4.5 \). Notice that at the equilibrium flow the travel time on both link will have...
Figure A.3: An example of transport network used for calculating UE and SO assignment

same value, in this case \( t_1 = t_2 = 6.5 \), which is also the main goal of UE assignment. Now the total cost of traveling for the transport network is

\[
C = x_1 \times t_1 + x_2 \times t_2 = 5.5 \times 6.5 + 4.5 \times 6.5 = 65
\]  
(A.3)

A.4. SO assignment

We will transport network as shown in Figure A.3 and defined in Section A.3. For obtaining system optimum assignment we will solve the following equation:

Minimize \( Z(x_\ell) = \sum x_\ell t_\ell(x_\ell) \)

subject to

\[
\sum_{k} f_{s,d}^{x,k} = D_{s,d} \quad \forall s, d
\]

\[
x_\ell = \sum_{k} \sum_{d} \sum_{k} \delta_{x,k} f_{s,d}^{x,k} \quad \forall \ell
\]

\[
f_{s,d}^{x,k} \geq 0 \quad \forall s, d, k
\]

\[
x_\ell \geq 0 \quad l \in \mathcal{L}
\]  
(A.4)

The minimization function \( Z(x_\ell) \) can be expanded for this case as:

\[
Z(x_\ell) = x_1 \times t_1 + x_2 \times t_2
\]  
(A.5)

Now after minimize the equation (A.5) under the constraint \( x_1 + x_2 = D_{od} = 10 \), we obtain equilibrium flow \( x_1^* = 4.25 \) and \( x_2^* = 5.75 \). Now the total cost of traveling for the transport network is

\[
C = x_1 \times t_1 + x_2 \times t_2 = 4.25 \times 6.25 + 5.75 \times 6.75 = 65.37
\]  
(A.6)
Other results
### Figure B.1: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ for time parameter perturbation of rail

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Fraction of time

$\omega_i$ for time parameter perturbation of rail
Figure B.2: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ for time parameter perturbation of road-rail
Figure B.3: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ for time parameter perturbation of water
Figure B.4: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ for time parameter perturbation of water-rail
Figure B.5: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ for time parameter perturbation of water-road.
Figure B.6: Rows of heat-map represent interdependent pair and show their respective criticality \( \omega_i \) for time parameter perturbation of rail.
Figure B.7: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ for time parameter perturbation of road-rail
Figure B.8: Rows of heat-map represent interdependent pair and shows their respective criticality $\omega_i$ for time parameter perturbation of water.
Table B.9: Interdependent nodes

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Figure B.9: Rows of heat-map represent interdependent pair and show their respective criticality \( \omega_i \) for time parameter perturbation of water-rail.
Figure B.10: Rows of heat-map represent interdependent pair and show their respective criticality $\omega_i$ for time parameter perturbation of water-road.
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