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Realizing Steady Supply to a Treatment Plant from Multiple Sources

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Abstract: In sewer systems sewage from different areas is often treated in a shared Waste Water Treatment Plant (WWTP). Currently the flows from different areas are usually determined by needs local to that area. During dry weather this may result in large variations in the flow into the WWTP. There are two reasons why this may be undesirable. Due to design peculiarities of some WWTP's this may disrupt the treatment process and necessitate the use of additional energy and chemicals. In other cases areas are connected to the same pressurized transport pipe line, so energy costs may be higher when multiple stations use the line at the same time. Due to the daily variation in the sewage flow from domestic and light industrial sources, limits on temporary in system storage and due to limitations on the range of discharges the pumps can deliver, minimizing the flow variations can be a complex problem. Under the assumption of a periodic inflow sufficient conditions for the existence of a solution are given. The conditions imply the existence of a repeatable pattern of a length less than a day.

Keywords: Environmental engineering; Waste treatment; Scheduling algorithms.

1. INTRODUCTION

In river deltas, polders and other areas with little natural relief, sewer systems are highly dependent on pumps for the transport of sewage over longer distances. For shorter distances (several city blocks), gravity drives the flow. Transport to a Waste Water Treatment Plant (WWTP) is usually by pressurized pipeline. More details on Dutch sewer systems can be found in NLingenieurs Sewer Systems Workgroup (2009). Often several areas with their own local sewer system at village or city district level share a WWTP. If they also share part of the pipeline to the WWTP then it may save energy if we avoid running the pumps at the same time. If the WWTP is sensitive to flow change then coordinating the running of the pumps will improve the efficiency of the WWTP. General information on the control of sewer systems can be found in Marinaki and Papageorgiou (2005); Ocampo-Martinez (2010); van Nooijen and Kolechkina (2013); García et al. (2015).

2. PRACTICAL PROBLEM STATEMENT

For a group of five large sewer systems that discharge to the same WWTP very large inflow variations under dry weather circumstances were disrupting the biological processes at the WWTP. The responsible organizations decided to investigate the possibility of reducing those variations. Limits on local storage, variation in the inflow into the sewer system over the day and limits on realizable pump flows make the problem non-trivial. This paper not discuss the design of a practical control system for this problem. It will deal only with establishing sufficient conditions for a solution to the coordination problem to exist within the constraints imposed. The importance of this demonstration lies in the fact that, depending on the specific constraints, the problem itself may very well either unsolvable or NP complete.

To show the problem may be NP complete we reduce a version of it to a multiple subset sum problem. Suppose we have m pumps that have a fixed capacity q_i that can be either on or off. Moreover, we have a fixed time step Δt and we can store n time steps worth of (stepwise constant) inflow $q_{\mathrm{in},i}$ in each system. Finally, suppose that for all $i = 1, 2, \ldots, m$ we have

$$n_i = \frac{\sum_{k=1}^{n} q_{\mathrm{in},i}(k)}{q_i} \in \mathbb{N} \tag{1}$$

with $n_i < n$. Now define

$$q_{\mathrm{tgt}} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{k=1}^{n} q_{\mathrm{in},i}(k) \tag{2}$$

To obtain an an outflow that discharges all inflow we need to find $x_{ik} \in \{0, 1\}$, $i = 1, 2, \ldots, m$, $k = 1, 2, \ldots, n$ such that we

$$\text{maximize} \sum_{i=1}^{m} \sum_{k=1}^{n} q_i x_{ij} \tag{3}$$

subject to

$$\sum_{i=1}^{m} q_i x_{ik} \le q_{\mathrm{tgt}},\ k = 1, 2, \ldots, n \tag{4}$$

$$\sum_{k=1}^{n} x_{ik} \le n_i,\ i = 1, 2, \ldots, m \tag{5}$$

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which is the multiple subset sum problem with n_i identical objects of weight q_i, see for example Caprara et al. (2000).

3. ABSTRACT PROBLEM STATEMENT

3.1 General statement

We have m pairs (V_i, Q_i), $i = 1, 2, \ldots, m$ where V_i is a closed, bounded, non-negative interval in $\mathbb{R}$ that represents the lower and upper bounds on the volume of sewage that can be stored in sewer system i and Q_i is a closed, bounded, non-negative interval in $\mathbb{R}$ that represents the lower and upper bounds on the range of flows that the pumping station for sewer system i can generate. Each pumping station will either be off or be generating a flow in Q_i. Each system has an inflow given by a non-negative integrable function $q_{\text{in},i}$. We are looking for a set of non-negative integrable functions q_i, together with a set of starting volumes $v_{0,i}$ such that for all $t \geq 0$

$$q_i(t) \in \{0\} \cup Q_i \tag{6}$$

and

$$v(t) = v_{0,i} + \int_{\tau=0}^{t} q_{\text{in},i}(\tau) - q_i(\tau)\, d\tau \in V_i \tag{7}$$

such that the variation over time of the inflow to the WWTP,

$$q_{\text{wwtp}}(t) = \sum_{i=1}^{m} q_i(t) \tag{8}$$

is minimal.

3.2 Simplified problem

We assume that the inflows are periodic with period T_{p} and that the solution should result in a constant inflow into the WWTP, from mass conservation it follows that in that case we must have

$$q_{\text{wwtp}} = \frac{1}{T_{\text{p}}} \int_{\tau=0}^{T_{\text{p}}} \sum_{i=1}^{m} q_{\text{in},i}(\tau)\, d\tau \tag{9}$$

4. CONDITIONS FOR EXISTENCE OF A SOLUTION

4.1 Road map

We will start by deriving conditions that are sufficient for a solution to exist when $\inf Q_i = 0$ for all i. Next we derive conditions that show we can keep the separate districts within the allowed volume range in case $\inf Q_i > 0$. We will then show that the simplified problem reduces to a problem of optimal use of a rectangular piece of material to create constrained smaller rectangles. Finally some conditions will be given that guarantee existence of a solution of the simplified problem.

4.2 Basic assumptions

We are considering only dry weather circumstances. During heavy precipitation events other rules apply. The design of sewer systems is almost always such that the installed pumping capacity exceeds the maximum dry weather flow. We will therefore assume that q_{in} is bounded

$$\|q_{\text{in},i}\|_\infty < \infty \tag{10}$$

and that

$$\|q_{\text{in},i}\|_\infty < \sup Q_i \tag{11}$$

Usually, the pumping stations are designed for local operation, the pump starts when a certain water level in the wet well is exceeded and pump stops when the level drops below a second, lower levels. In other words, we may assume that there is sufficient local storage to run the pumps a reasonable time.

4.3 Existence of a solution with zero lower bound on pump capacity

A necessary condition for the existence of a solution is that the equivalent one district case, with volume

$$V_{\text{total}} = \sum_{i=1}^{m} V_i(t) \tag{12}$$

and flow range

$$Q_{\text{total}} = \sum_{i=1}^{m} Q_i(t) \tag{13}$$

should have a solution. Here addition is interval addition. The following lemma provides a condition for the existence of a solution for the one district case that is verifiable by computer.

Lemma 1. Given a pair (V, Q), a starting volume interval V_0 , a bounded periodic inflow q_{in} with period T_{p} and a time step Δt such that $n = T_{\text{p}}/\Delta t$ is a positive integer, if

$$\bar{q} = \frac{1}{T_{\text{p}}} \int_{\tau=0}^{T_{\text{p}}} q_{\text{in}}(\tau)\, d\tau \in Q \tag{14}$$

$$\|q_{\text{in}}\|_\infty < \sup Q \tag{15}$$

and

$$V_0 + \int_{\tau=0}^{k\Delta t} (q_{\text{in}}(\tau) - \bar{q})\, d\tau \subseteq \tag{16}$$
$$[\inf V + \Delta t \sup Q, \sup V - \Delta t \sup Q]$$

for $k = 0, 1, 2, \ldots, n$ then a constant outflow

$$q(t) = \bar{q}_{\text{in}} \tag{17}$$

will keep the stored volume between the bounds specified by V.

Proof.

The condition implies that the volume will be within the bounds $[\inf V + \Delta t \sup Q, \sup V - \Delta t \sup Q]$ at the end of a time step. The boundedness of q_{in} (Equation 15) together with the periodicity of the inflow places the solution in V for all t.

Next we consider multiple districts.

Lemma 2. If we have m districts and there is a constant flow solution for the separate districts then there is a solution such that the sum of the flows is constant and equal to q_{wwtp} as defined in Equation 9.

Proof.

This follows immediately from the definitions.

4.4 Existence of a solution with a non-zero lower bound on pump capacity

If there is a solution for a district with the outflow equal to the mean inflow but the mean inflow is lower than $\inf Q$ then we need to examine whether a solution with non-constant outflow exists for that district. Given the fact that we need to combine this with outer districts to get a sum of outflows that is constant, we need to examine how much room we have to shift the time intervals that the pump for a given district is on. Following the reasoning of Lemma 1, we see that Equation 15 together with

$$V_{0,i} + \int_{\tau=0}^{k\Delta t} \left(q_{\text{in}}\left(\tau\right) - \bar{q}_{\text{in},i}\right) d\tau \subseteq \tag{18}$$
$$\left[\inf V_i + \Delta t\left(\bar{q}_{\text{in},i} + \sup Q_i\right), \sup V_i - \Delta t\left(\bar{q}_{\text{in},i} + \sup Q_i\right)\right]$$

for $k = 0, 1, 2, \ldots, n$ will keep the stored volume between the bounds specified by V_i for any q_i that satisfies

$$\int_{\tau=k\Delta t}^{(k+1)\Delta t} \left(\bar{q}_{\text{in},i} - q_i\right) d\tau = 0 \tag{19}$$

We see that if the separate districts satisfy Equation 18 then we will have enough freedom to build a repeatable pattern of pumping within a time step Δt. It then becomes a question of filling a rectangle with horizontal side Δt and vertical side

$$\bar{q}_{\text{in,total}} = \frac{1}{T_{\text{p}}} \int_{\tau=0}^{T_{\text{p}}} \sum_{i=1}^{m} q_{\text{in},i}\left(\tau\right) d\tau \tag{20}$$

with rectangles $R_{i,j}$ (of which there are $n_i > 0$ for district i) in such a way that for $i = 1, 2, \ldots, m$

$$\sum_{j=1}^{n_i} R_{i,j} = \bar{q}_{\text{in},i}\Delta t \tag{21}$$

and the height of $R_{i,j}$ lies in Q_i for $j = 1, 2, \ldots, n_i$. There is literature on this type of problem as it occurs in many industries, albeit mostly for fixed size rectangles, see for example Dyckhoff (1990).

If $\bar{q}_{\text{in,total}} \in Q_i$ for all i then it is clear that there is a solution. If this is not the case then we first reduce the problem by creating rectangles for all pumps i that cannot run together with another pump, in other words where

$$\bar{q}_{\text{in,total}} - \inf Q_i < \min_{j \neq i} \inf Q_j \tag{22}$$

We are then left with a subset of pumps that we need to fit into a somewhat narrower rectangle. So, in principle we can consider just the problem where some pumps can run either solo or in combination with another pump and at least one can run only in combination with another pump.

5. EXISTENCE OF A SOLUTION FOR A SPECIFIC CASE

In the case that provided the reason for this study there will be either just one or at most two pumps that cannot provide $\bar{q}_{\text{in,total}}$.

Lemma 3. Suppose the sewer systems satisfy Equation 18. Let $\mathscr{J}$ be the set of all sets of pumps that can produce $\bar{q}_{\text{in,total}}$. If there is a subset J such that each pump occurs in exactly one element of J and there is a flow setting for the pumps $I \in J$ such that

$$\forall i_1, i_2 \in I \Rightarrow \frac{\bar{q}_{\text{in},i_1}}{q_{i_1}} = \frac{\bar{q}_{\text{in},i_2}}{q_{i_2}} = \lambda\left(I\right) \tag{23}$$

then a repeatable pattern exists.

Proof.

Let group I run for time $\lambda\left(I\right)\Delta t$, this discharges $\bar{q}_{\text{in},i}\Delta t$ from sewer system i for each $i \in I$ and $\lambda\left(I\right)\bar{q}_{\text{in,total}}\Delta t$ at flow rate $\bar{q}_{\text{in,total}}$ to the WWTP, each pump occurs exactly once in a group I so the total volume discharged is $\bar{q}_{\text{in,total}}\Delta t$.

The values in Table 1 show that for an average flow of $1840\text{m}^3/\text{h}$ only system 4 cannot run solo. We can run 1,3,5

Table 1. Pump station data

	Mean inflow	Pump capacity	
		lower bound	upper bound
	(m^3/h)	(m^3/h)	(m^3/h)
System 1	881.4	1000	4250
System 2	293.5	1060	2000
System 3	263.8	1600	3280
System 4	124.7	300	1200
System 5	182.6	550	1900

solo and pair systems 2 and 4 as required by Lemma 3. Check on pairing of 2 and 4. We need to solve

$$\frac{300 + 900x}{124.7} = \frac{1060 + 940y}{293.5}$$
$$300 + 900x + 1060 + 940y = 1840$$
$$900x + 940y = 480$$
$$\frac{780}{124.7} - \frac{1060}{293.5} = 940\left(\frac{1}{124.7} + \frac{1}{293.5}\right)y$$
$$y = 0.246$$
$$x = 0.276$$

6. CONCLUSION

The question of the existence of a pumping strategy for a group of pumping stations linked to the same WWTP, that evens out the daily pattern of inflow into the sewer systems, is non-trivial. Under the assumption of a periodic inflow there are computer verifiable sufficient conditions that reduce the question of the existence of a pumping strategy over the whole day to the existence of a repeatable pattern of a length less than a day. For a simple case conditions for the existence of a solution are given.

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